Fizică Curs 3

Sisteme materiale continue

Fenomene ondulatorii

Ex: - unde electromagnetice (radiații γ, radiații x, UV, Optic, ~~Optic~~, IR

$\xrightarrow{x}$ $\xleftarrow{\gamma}$

unde radio MW, RW) → nu au nevoie de un mediu suport

FM (frequency modulated) — AM (amplitude modulated)

unde $\underbrace{\text{ultra ~~scurte~~ scurte}}_{FM}$, $\underbrace{\text{scurte, medii, lungi}}_{AM}$

VIAVGOR

- unde mecanice (unde sonore, valurile, undele seismice
- unde termice
- unde de materie (ipoteza „de Broglie", fizica cuantică)

} au nevoie de un mediu material pt. a se propaga

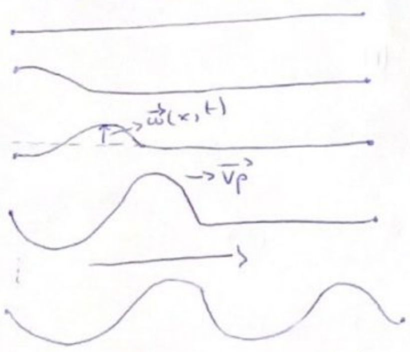
Unda - o perturbație (locală) a mediului care se transmite în timp și spațiu

Clasificarea undelor

După relația între direcția de propagare și cea de oscilație a mărimilor ce caracterizează unda:

- unde longitudinale ($\vec{v}$ propagare ∥ dir. de oscilație
(unde sonore - de presiune -)

- unde transversale (undele ELM) $\vec{u}(x,t) \perp \vec{v_p}$



După mărime (mărimile) ce caract. unda:
- unde scalare
- unde vectoriale (undele electromagnetice $\vec{E}$, $\vec{B}$)
- unde tensoriale S → tensor de rang 0, $\vec{v}(v_x, v_y, v_z)$ → rang 1

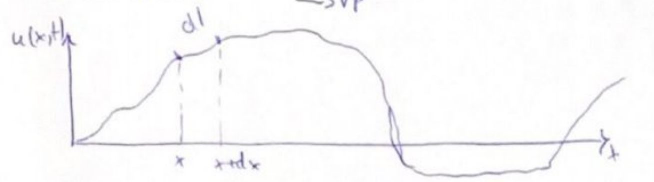
$$t^2 \begin{pmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{pmatrix}$$

După dimensiunea mediului în care se propagă unde:

- unde 1D
- — " — 2D
- — " — 3D

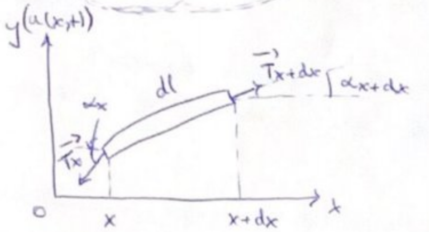
Ec. de propagare a undelor

- transversale



După dir. de propagare

- unde progresive
- — " — regresive
- — " — staționare



α_{x+dx}, α_x - unghiuri mici

$|\vec{T}_{x+dx}| \simeq |\vec{T_x}| \simeq T$

Pe Ox: $dF_x = T\cos\alpha_{x+dx} - T\cos\alpha_x$

$\cos\alpha_{x+dx} \simeq \cos\alpha_x \simeq 1 \qquad dF_x \simeq 0$

Pe Oy: $dF_y = T(\sin\alpha_{x+dx} - \sin\alpha_x)$

$\sin\alpha_{x+dx} \simeq tg\,\alpha_{x+dx}$

$\sin\alpha_x \simeq tg\,\alpha_x$

$dF_y = T(tg\,\alpha_{x+dx} - tg\,\alpha_x)$

$dF_y = T\left(\frac{du}{dx}\Big|_{x+dx} - \frac{du}{dx}\Big|_x\right)$ (1)

Fie S - aria secțiunii firului

ρ - densitatea firului

dm - masa porțiunii dl a firului

$S = \frac{dm}{dV} = \frac{dm}{S \cdot dl}$

$dF \simeq dF_y = dm \cdot \frac{d^2u(x,t)}{dt^2} = \rho S\, dl\, \frac{d^2u(x,t)}{dt^2}$ (2)

(1) + (2) => $T\left(\frac{du}{dx}\Big|_{x+dx} - \frac{du}{dx}\Big|_x\right) = \rho \cdot S \cdot \frac{d^2u(x,t)}{dt^2}\, dx$ (*)

unghiuri mici: $dl \simeq dx$

$f(x) = \frac{du}{dx}\Big|_x$

$f(x+dx) = \frac{du}{dx}\Big|_{x+dx} = \frac{du}{dx}\Big|_x + \frac{1}{1!}\frac{d}{dx}\left(\frac{du}{dx}\right)\Big|_x + \frac{1}{2!}\frac{d}{dx}\left(\frac{du}{dx}\right)\Big|_x (d_x)^2 + \ldots$

$$(*) \Rightarrow \frac{T}{S}\left(\frac{du}{dx}\Big|_x + \frac{d^2u}{dx^2}\Big|_x dx - \frac{du}{dx}\Big|_x \right) = \rho \cdot \frac{d^2u}{dt^2} dx$$

$$\boxed{\overline{V} = \frac{T}{S} \text{ - efortul unitar}}$$

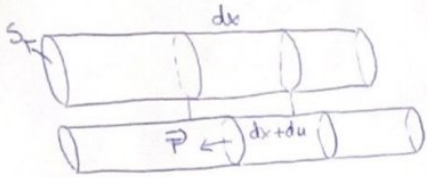
$$\overline{V} \cdot \frac{d^2u}{dx^2} \cancel{dx} = \rho \frac{d^2u}{dt^2} \cancel{dx} \Rightarrow \boxed{\frac{d^2u}{dx^2} - \frac{\rho}{\overline{V}} \cdot \frac{d^2u}{dt^2} = 0}$$ ec. undelor mecanice transversale

$$\left\langle \frac{\rho}{\overline{V}} \right\rangle_{SI} = \frac{kg}{m^3} \cdot \frac{m^2}{N} = \frac{kg}{m} \cdot \frac{s^2}{kg \cdot m} = \frac{s^2}{m^2}$$

$$\boxed{\frac{d^2u}{dx^2} - \frac{1}{v_t^2} \cdot \frac{d^2u}{dt^2} = 0}$$ ec. undelor mecanice transversale

viteza de propagare a undelor mecanice (elastice) transversale este: $v_t = \sqrt{\frac{\sigma}{\rho}}$

* Unde longitudinale



deformare cu du

alungire relativă: $\boxed{\varepsilon = \frac{du}{dx}}$ elongația longitudinală

$$\boxed{dm = \underbrace{dx \cdot S}_{dV} \cdot \rho}$$

(nu se pierde sau se câștigă masă)

$$dF = dm \cdot a = dx \cdot S \cdot \rho \cdot \frac{d^2u}{dt^2}$$

E - modulul lui Young

S - aria secțiunii

$$\boxed{\begin{aligned} \Delta l &= \Delta u \\ l &= \Delta x \end{aligned}}$$

legea lui Hooke: $\frac{1}{E} \cdot \frac{F}{S} = \frac{\Delta u}{\Delta x}$

$$\Rightarrow F = E \cdot S \cdot \frac{du}{dx}$$

$$\Rightarrow dF = E \cdot S \cdot \frac{d}{dx}\left(\frac{du}{dx}\right) dx$$

astfel: $E \cdot S \frac{d}{dx}\left(\frac{du}{dx}\right) dx = dx \cdot S \cdot \rho \cdot \frac{d^2u}{dt^2} \Rightarrow E \frac{d^2u}{dx^2} - \rho \frac{d^2u}{dt^2} = 0 \Rightarrow$

$$\Rightarrow \frac{d^2u(x,t)}{dx^2} - \frac{1}{v_l^2} \cdot \frac{d^2u(x,t)}{dt^2} = 0$$

$\Rightarrow \boxed{v_l = \sqrt{\frac{E}{\rho}}}$ viteza undelor longitudinale

E - modulul lui Young ; ρ - densitatea firului / barei

Δ - laplasian

Ec. undelor mecanice (elastice)

-transversale

$$\frac{d^2u}{dx^2} - \frac{1}{v_t^2} \cdot \frac{d^2u}{dt^2} = 0$$

$u(x,t) \to$ elongația

u - funcție de undă

$$|\vec{T}|_x \simeq |\vec{T}|_{x+dx} \simeq |\vec{T}|$$

Pe ox: $\cos\alpha_x \simeq \cos\alpha_{x+dx} = 1$

oy: $\sin\alpha_x \simeq tg\,\alpha_x = \frac{du}{dx}\Big|_x$

$\sin\alpha_{x+dx} \simeq tg\alpha_{x+dx} = \frac{du}{dx}\Big|_{x+dx}$

$$\frac{d^2u}{dx} - \frac{1}{v_t^2} \cdot \frac{du}{dt^2} = 0 \qquad v_t = \sqrt{\frac{E}{\rho}}$$

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} - \frac{1}{v^2} \cdot \frac{d^2u}{dt^2} = 0$$

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz} - \frac{1}{v^2} \cdot \frac{d^2u}{dt^2} = 0$$

$$\boxed{\Delta u - \frac{1}{v^2} \cdot \frac{d^2u}{dt^2} = 0}$$ ecuația undelor tridimensionale

Δu = laplacianul lui u

$$\Delta = \vec{\nabla}^2 = \left(\frac{d}{dx}\vec{i} + \frac{d}{dy}\vec{j} + \frac{d}{dz}\cdot\vec{k}\right) \cdot \left(\frac{d}{dx}\cdot\vec{i} + \frac{d}{dy}\vec{j} + \frac{d}{dz}\cdot\vec{k}\right) =$$

$$\left(\begin{array}{l} \vec{\nabla}p = \overrightarrow{grad\,p} = \frac{dp}{dx}\vec{i} + \frac{dp}{dy}\vec{j} + \frac{dp}{dz}\vec{k} \\ \vec{\nabla}\cdot\vec{E} = \underbrace{div\vec{E}}_{scalar} = \frac{dE}{dx} + \frac{dE}{dy} + \frac{dE}{dz} \end{array}\right)$$

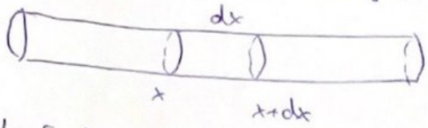
$$= \frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}$$

Fizică curs 4

Energia undelor mecanice

(densitatea volumică de energie, flux de energie, densitatea fluxului de energie, intensitatea undei)

Densitatea volumică de energie



ρ - densitatea barei (bară omogenă - densitate const)

S - suprafața secțiunii transversale

$dV = S \cdot dx$

$dE_c = dm \frac{v_p^2}{2} \Rightarrow dE_c = \overbrace{\rho \cdot S \cdot dx}^{dV} \cdot \frac{v_p^2}{2}$

viteza segmentului infinitezimal de bară de lungime dx este v_p

$v_p = \frac{du}{dt}$

$$\frac{dE}{dV} = \boxed{\rho_{E_c} = \rho \cdot \frac{v_p^2}{2}}$$

Legea lui Hooke:

$\frac{F}{S} = E \cdot \frac{\Delta l}{l_0} \rightarrow \varepsilon$ - alungirea relativă

efortul unitar

$F = E \cdot S \cdot \frac{\Delta l}{l_0}$

$F = -k \cdot \Delta l$

$k = \frac{E \cdot S}{l_0}$

E_p elastică $= -L_{el} = +\frac{k \Delta l}{2}$

E_p elastică $= \frac{E \cdot S}{l_0} \cdot \frac{\Delta l^2}{2} = \frac{E \cdot S \cdot l_0}{2} \cdot \frac{\Delta l^2}{l_0^2}$

dE_p elastică $= \frac{E \cdot \overbrace{S \cdot dx}^{dV}}{2} \cdot \varepsilon^2 \Rightarrow \frac{dE_p}{dV} = \rho_{E_p} = \frac{E \cdot \varepsilon^2}{2}$

Densitatea de energie totală:

$$\boxed{\rho_{E\,totală} = \rho_{E_c} + \rho_{E_p} = 2\rho_{E_c} = 2\rho_{E_p} = \rho \cdot v_p^2 = E \cdot \varepsilon^2}$$

$$\boxed{\rho_{E_c} = \rho_{E_p}}$$

Ec. undelor longitudinale:

$$\frac{d^2u}{dx^2} - \frac{1}{v_l^2} \cdot \frac{d^2u}{dt^2} = 0 \Rightarrow \cancel{\frac{d^2u}{dx^2} = \frac{E}{\rho}\frac{d^2u}{dt^2}} \quad \frac{d^2u}{dx^2} = \frac{\rho}{E}\frac{d^2u}{dt^2}$$

$v_l = \sqrt{\frac{E}{\rho}}$

$$\frac{d}{dx}\left(\frac{du}{dx}\right) = \frac{\rho}{E} \cdot \frac{d}{dt}\left(\frac{du}{dt}\right)^{v_p}$$

temă

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Flux de energie

Energia este transportată de o undă fără a avea asociat și un transport de masă. (spre deosebire de energia transportată de un corp)

Viteza cu care se transmite energia prin intermediul unei unde este chiar viteza de propagare a undei v_u ($\neq v_p$).

$dx = v_u \cdot dt$

În timpul dt prin secțiunea S va trece energia dE: $\left.\begin{array}{l} dE = \rho_E \cdot dV \\ dx = v_u \cdot dt \end{array}\right\} \Rightarrow dE = \rho_E \cdot S \cdot v_u \cdot dt$

$\Rightarrow$ fluxul de energie $\stackrel{def}{=}$ energia transportată de undă în unitatea de timp

$$\phi_E = \frac{dE}{dt} = \rho_E \cdot S \cdot v_u$$

$$\phi_E = E \cdot \varepsilon^2 \cdot S \cdot v_u = \rho \cdot v_p^2 \cdot S \cdot v_u$$

$$\langle \phi_E \rangle_{SI} = 1 \frac{J}{s} = 1\,W$$

Densitatea de flux de energie

$$I = \frac{\phi_E}{S} = \rho \cdot v_p^2 \cdot v_u = E \cdot \varepsilon^2 \cdot v_u \qquad \langle I \rangle_{S.I.} = 1 \frac{J}{s \cdot m^2} = 1 \frac{W}{m^2}$$

Intensitatea undei $\stackrel{def}{=}$ media pe o perioadă a densității de flux de energie.

$$J = \langle I \rangle_T = \left\langle \frac{\phi_E}{S} \right\rangle_T = \langle \rho \cdot v_p^2 \cdot v_u \rangle_T = \rho v_u \langle v_p^2 \rangle_T$$
$(= J)$

$Z_a = \rho \cdot v_u$ - impedanța acustică

$\langle v_p^2 \rangle_T = v_{ef}^2$

$v_{ef} = \sqrt{\langle v_p^2 \rangle_T}$

Unde în fluide - unde de presiune

$$\boxed{\rho_{E_t} = \frac{\Delta p^2}{K_{ad}}}$$

K_{ad} - coeficient de compresibilitate adiabatică

Δp - variația de presiune

Intensitatea undelor în fluide (unde sonore)

$$J = \frac{\langle \Delta p^2 \rangle_T}{\rho \cdot v_u} \qquad J = \frac{p_s^2}{\rho \cdot v_u} \qquad \Rightarrow J = \frac{p_s^2}{Z_a}$$

$p_s = \sqrt{\langle \Delta p^2 \rangle_T}$ - presiune sonoră

~~$Z_a = \frac{1}{\rho v_u}$ impedanța a~~

$Z_a = \rho \cdot v_u$ impedanța acustică

J - intensitatea pt. o undă armonică - rămâne const în timp într-un anumit timp

J - —— " —— amortizată $\rightarrow$ scade în timp

Undă armonică plană progresivă ← unidimensională ~~[illegible]~~

$u(x,t) = A \cdot \cos(\omega t - kx)$ $\qquad \langle k \rangle_{SI} = \frac{rad}{m}$

$(x(t) = A\cos(\omega_0 t + \varphi))$ $\qquad k$ - nr. de undă

$$v_p = \frac{du}{dt} = -\omega A \sin(\omega t - kx)$$

$$v_{ef}^2 = \langle v_p^2 \rangle_T = \langle \omega^2 A^2 \sin^2(\omega t - kx) \rangle_T = \omega^2 A^2 \underbrace{\langle \sin^2(\omega t - kx) \rangle_T}_{\frac{1}{2}}$$

$$\Rightarrow I = \rho \cdot v_w \cdot \frac{\omega^2 A^2}{2} = \frac{1}{2} Z_a \cdot \omega^2 A^2$$

$$I \sim A^2 = const$$

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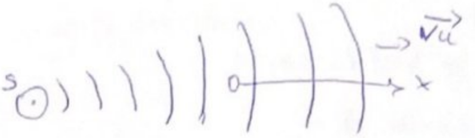
Fizică curs 5

Unde plane



$u(x,t) = A\cos(\omega t - kx)$

$v_p = \dot{u} = -\omega A \sin(\omega t - kx)$

$\rho_{Ec} = \frac{\rho \cdot v_p^2}{2} = \rho \cdot \frac{\omega^2 A^2 \sin^2(\omega t - kx)}{2}$

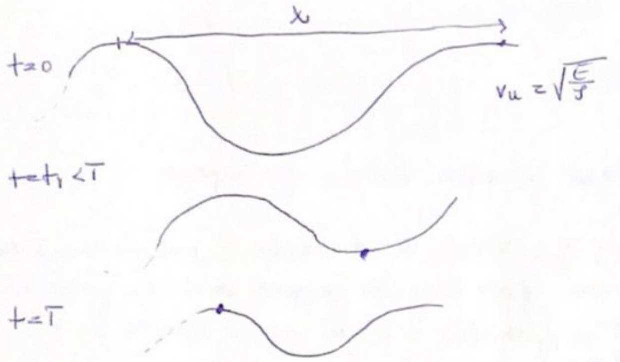
$\varepsilon = \frac{du}{dx} = + Ak\sin(\omega t - kx)$

$\rho_{Ep} = \frac{E \cdot \varepsilon^2}{2} = \frac{E \cdot A^2 \cdot k^2 \sin^2(\omega t - kx)}{2}$

$k = \frac{\omega}{v}$ nr. de undă $\Rightarrow k = \frac{2\pi}{\lambda} - \rho_{Ep} = \frac{E \cdot A^2 \omega^2 \sin^2(\omega t - kx)}{2 v_u^2}$

$\omega = \frac{2\pi}{T} \Rightarrow \frac{\omega}{v_u} = \frac{2\pi}{v_u \cdot T} = \frac{2\pi}{\lambda}$

Pt. o undă distanța pe care o parcurge aceasta într-o perioadă este lungimea de undă $v_u = \frac{\lambda}{T}$



$v_u = \sqrt{\frac{E}{\rho}}$

$\rho_{Ep} = \frac{\cancel{E} A^2 \omega^2 \sin^2(\omega t - kx)}{2 \frac{E}{\rho}} = \frac{\rho A^2 \omega^2 \sin^2(\omega t - kx)}{2}$

$\rho_{E\,totală} = \rho \omega^2 A^2 \sin^2(\omega t - kx) = 2\rho_{Ec} = 2\rho_{Ep}$

Undă sferică (armonică)

$u(r,t) = A(r)\cos(\omega t - kr)$

$A(r) = \frac{A_0}{r}$

$\boxed{u(r,t) = \frac{A_0}{r}\cos(\omega t - kr)}$ Energia totală a undei

Densitatea de En totală a undei scade proporțional cu $\frac{1}{r^2}$

$\rho_{Et} \sim A^2$

$I \sim A^2$

Densit de flux de en

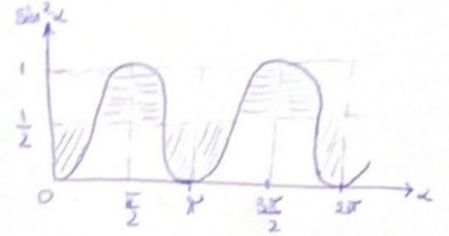
$I = \rho\, v_u \cdot v_p^2 \quad (= E \cdot v_u \cdot \varepsilon^2)$

Intensitatea undei

$$I = \langle \rho v_u v_p^2 \rangle = \rho v_u \langle v_p^2 \rangle$$

Unde armonice plane

$$I = \rho \cdot v_u \langle \omega^2 A^2 \sin^2(\omega t - kx) \rangle_T = \rho v_u \cdot \omega^2 A^2 \langle \sin^2 \underbrace{(\omega t - kx)}_{\text{faza undei}} \rangle$$



$$I = \rho \cdot v_u \cdot \omega^2 A^2$$

$$I = \frac{1}{2} Z_a \omega^2 A^2$$

Pentru undele armonice sferice

$$I = \frac{1}{2} \overset{\rho v_u}{Z_a} \omega^2 A_0^2 \frac{r_0^2}{r^2}$$

Electrostatică și magnetostatică

Electrostatica

- parte a fizicii care studiază sarcinile electrice în repaus

Două sarcini electrice q_1 și q_2 situate la distanța r una de cealaltă vor interacționa prin intermediul unei forțe care este proporțională cu produsul celor 2 sarcini și invers proporțională cu pătratul distanței dintre sarcini (r).

Forța coulombiană de interacțiune între 2 sarcini

$$\boxed{\vec{F}_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}^2} \cdot \frac{\vec{r}_{12}}{r_{12}}}$$

$q_1 \xrightarrow{\vec{r}_{12}} q_2$ $\vec{F}_{12}$

q_1, q_2 pozitive v negative (ambele) (sau) => forța de respingere

q_1, q_2: una pozitivă și cealaltă negativă => forțe de atracție

ε_0 - permitivitatea electrică a vidului (permittivity of free space / vacuum)

$$\varepsilon_0 \approx 8,85 \cdot 10^{-12} \frac{F}{m}$$

$$\frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^9$$

$$\frac{F}{m} = \frac{A^2 s^4}{kg \cdot m^3}$$

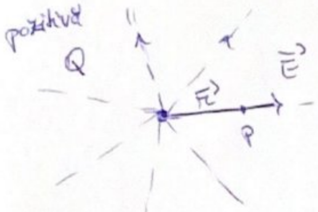
$$[C] = \frac{[Q]}{[U]} = \frac{I^2 \cdot T^2}{I \cdot T [U]} = \frac{I^2 T^2}{[F] \cdot L} = \frac{I^2 \cdot T^2}{M \cdot \frac{L}{T^2}} = \frac{I^2 T^4}{M L^2}$$

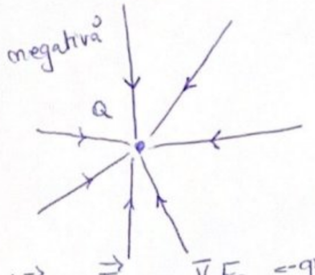
$\langle C \rangle_{SI} = F$ (farad)

Dacă luăm 2 sarcini, una ca o sarcină de probă (q) și o a doua ca o sarcină fixă (Q) atunci observăm că sarcina q simte influența sarcinii fixe la $\forall$ distanță => $\exists$ câmp generat de sarcina Q = câmp electric (vectorial) $\vec{E}$

$\vec{F_e} = q\vec{E}$ $\quad$ $\vec{E}$ - intensitatea câmpului electric

$$\vec{E_Q}(\vec{r}) = \frac{Q}{4\pi\varepsilon_0 r^2}\cdot\frac{\vec{r}}{r}$$





$$E_p(r) = -L_{\vec{F_e}} = -\int \vec{F_e}\cdot d\vec{r} \Rightarrow \vec{F_e} = -\nabla E_p \leftarrow -\text{grad } E_p$$

$$\boxed{E_{p\,\text{electrică}}(r) = \frac{qQ}{4\pi\varepsilon_0 r}}$$

$$\frac{d}{dr}\left(\frac{qQ}{4\pi\varepsilon_0 r}\right) = -\frac{qQ}{4\pi\varepsilon_0}\cdot\frac{1}{r^2}$$

S-a definit potențialul electric V (scalar)

$$\boxed{\vec{E} = -\nabla V} \qquad \boxed{V(r) = \frac{Q}{4\pi\varepsilon_0 r}}$$

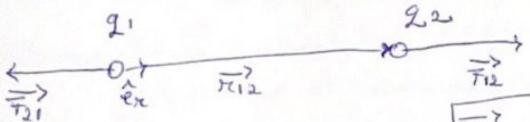
Fizică curs 6

Electrostatică . Magnetostatică

- Legea lui Coulomb -> Forța electrică
- câmpul electric generat de o sarcină
  - potențial electric
  - flux electric
  - Legea lui Gauss pt. câmpul electric
  - Ec. Poisson pt. potențial electric

Magnetostatică

- Legea Biot - Savart
- Câmpul magnetic generat de un conductor liniar
- — " — " — de un conductor circular (spiră) -> Temă
- Legea lui Gauss pt. câmp magnetic
- Legea lui Ampère
- Ec. de continuitate (conservarea sarcinii)



$$\vec{F_{12}} = C\,\frac{q_1 q_2}{r^2}\cdot\frac{\vec{r}}{r} \Rightarrow \boxed{\vec{F_{12}} = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}^2}\cdot\frac{\vec{r_{12}}}{r_{12}}}$$

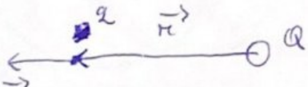
$C = 9\cdot 10^9$ unități SI

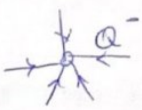
$$C = \frac{1}{4\pi\varepsilon_0}$$

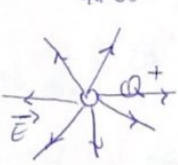
Sarcini de același semn -> forțe de respingere

— " — semne diferite -> forțe de atracție

$$\vec{F_e} = \frac{qQ}{4\pi\varepsilon_0 r^2}\cdot\frac{\vec{r}}{r} = q\cdot\vec{E} \Rightarrow \boxed{\vec{E} = \frac{\vec{F_e}}{q}}$$







$$L = \int_1^2 \vec{F_e}\,d\vec{r} = \cancel{U_1 - U_2}\ E_{p1} - E_{p2}$$

$$dL = \cancel{-dU}\ -dE_p = \vec{F_e}\cdot d\vec{r}$$

$$\boxed{\vec{F_e} = -\nabla E_p} = -\text{grad}\,E_p$$

$$\vec{E} = -\frac{\nabla E_p}{q} = -\nabla\left(\frac{E_p}{q}\right) = -\nabla V$$

$$U = V_2 - V_1$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\cdot\frac{\vec{r}}{r}$$

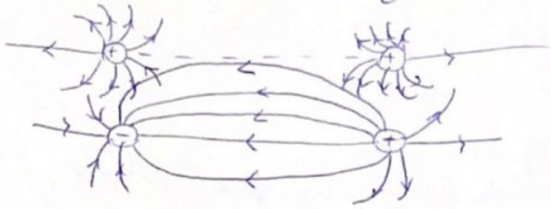
$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$E_p = \frac{qQ}{4\pi\varepsilon_0 r^2}$$

ε_0 - permitivitatea electrică a vidului

$$\varepsilon_0 = 8,854 \cdot 10^{-12} \frac{F}{m}$$

$$\frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^9 \frac{m}{F} = 9 \cdot 10^9 \frac{N \cdot m^2}{C^2} = 9 \cdot 10^9 \, kg \cdot m^3 \cdot s^{-4} A^{-2}$$



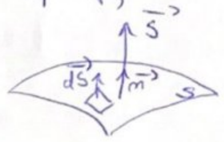
Fluxul electric

Fluxul unei mărim vectoriale $\stackrel{def}{=}$ prod scalar $\overrightarrow{\text{mărime}} \cdot \overrightarrow{\text{supr}}$

prod scalar dintre mărimea vectorială și vectorul suprafață, prin vector suprafață înțelegând un vector care are ca modul mărimea acelei suprafețe, iar direcția și sensul date de de versorul normalei la suprafață

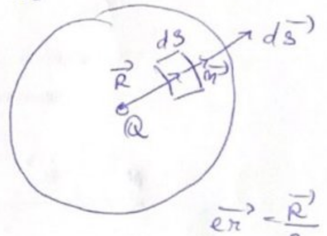
$$d\phi_e = \vec{E} \cdot \vec{dS} = \vec{E} \cdot \vec{n} \cdot dS$$

$$\phi_e = \iint \vec{E} \cdot \vec{dS}$$



Pt. o suprafață închisă (sferă) ce închide sarcina Q

$$\phi_e = \oiint_{sferă} \vec{E} \cdot \vec{dS}$$



$$\phi_e = \oiint_{sferă} \vec{E} \cdot \vec{dS} = \oiint_{sferă} \frac{Q}{4\pi\varepsilon_0 R^2} \vec{e_r} \cdot \vec{e_r} \, dS \Rightarrow$$

$$\vec{e_r} = \frac{\vec{R}}{R}$$

$$\Rightarrow \phi_e = \frac{Q}{4\pi\varepsilon_0 R^2} \cdot \oiint_{sferă} dS = \frac{Q}{4\pi\varepsilon_0 R^2} \cdot 4\pi R^2$$

$$\boxed{\phi_e = \frac{Q}{\varepsilon_0}}$$ Legea lui Gauss pt. câmpul electric (forma integrală)

în materiale

$$\varepsilon = \varepsilon_r \cdot \varepsilon_0$$

ε_r - permitivitate electrică relativă

$$\langle \varepsilon_r \rangle_{SI} = 1$$

$$\oiint_{S \text{ închisă}} \vec{E} \, d\vec{S} = \frac{Q}{\varepsilon_0}$$

$$Q = \iiint_{V_S} \rho_q \, dV$$

$$\Rightarrow \oint_S \vec{E}\,d\vec{S} = \frac{1}{\varepsilon_0}\iiint_{V_S} \rho_q \, dV \quad (*)$$

$$\oint_S \vec{E}\,d\vec{S} \overset{GO}{=} \iiint_{V_S} \underbrace{\nabla\cdot\vec{E}}_{div\vec{E}}\,dV \quad (**)$$

$$\left.\begin{matrix}(*)\\(**)\end{matrix}\right| \Rightarrow \iiint_V \nabla\cdot\vec{E}\,dV = \frac{1}{\varepsilon_0}\iiint_V \rho_q\cdot dV$$

$$\boxed{\nabla\cdot\vec{E} = \frac{\rho_q}{\varepsilon_0}}$$ forma locală

$$\nabla\vec{D} = \rho_q$$

$$\vec{D} = \varepsilon_0\varepsilon_r\vec{E}$$

$\vec{D}$ - inducția câmpului electric

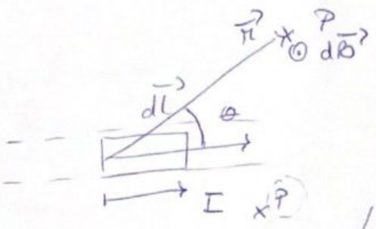
$$\nabla\vec{E} = \frac{\rho_q}{\varepsilon_0} \Rightarrow \nabla(-\nabla V) = \frac{\rho_q}{\varepsilon_0} \Rightarrow -\Delta V = \frac{\rho_q}{\varepsilon_0} \Rightarrow \boxed{\Delta V + \frac{\rho_q}{\varepsilon_0} = 0}$$

Ec. Poisson

Pt. un câmp electrostatic

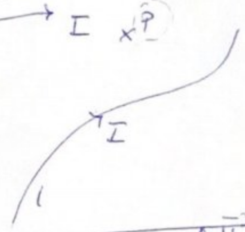
$$\nabla\times\vec{E} = 0 \quad (\nabla\times(-\nabla V) = 0)$$

$(rot\vec{E})$



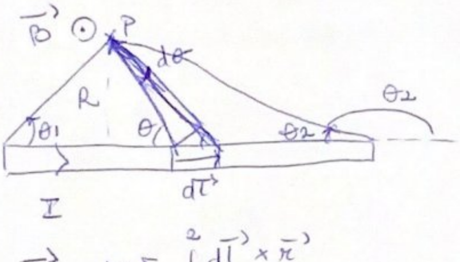
Magnetostatică

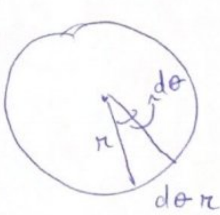
$$\boxed{d\vec{B} = \frac{\mu_0 I}{4\pi}\cdot\frac{d\vec{l}\times\vec{r}}{r^2}}$$ Legea Biot-Savart



$$\boxed{\vec{B} = \int \frac{\mu_0 I}{4\pi}\int\frac{d\vec{l}\times\vec{r}}{r^3}}$$

$$dB = \frac{\mu_0 I}{4\pi r^2}\,dl\sin\theta$$





$d\theta\, r$

$$dl = \frac{r\,d\theta}{\sin\theta}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int_1^2 \frac{d\vec{l}\times\vec{r}}{r^3}$$

$$B = \frac{\mu_0 I}{4\pi}\int_1^2 \frac{dl\sin\theta\cdot r}{r^3}$$

R - cunoscut

$$r = \frac{R}{\sin\theta}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{\theta_1}^{\theta_2}\frac{\cancel{R}\,d\theta}{\sin^{\cancel{2}}\theta}\,\frac{\sin\theta\cdot\cancel{R}}{\cancel{\sin\theta}\,\frac{R^{\cancel{3}}}{\cancel{\sin^3\theta}}} \Rightarrow B = \frac{\mu_0 I}{4\pi R}\int_{\theta_1}^{\theta_2}\sin\theta\,d\theta \Rightarrow B = \frac{\mu_0 I}{4\pi R}(\cos\theta_1 - \cos\theta_2)$$

Cond liniar ∞ $\boxed{B = \frac{\mu_0 I}{2\pi R}}$

Fizică Curs 7

Electrostatica

$$\vec{F_e} = \frac{qQ}{4\pi\varepsilon_0 r^2}\frac{\vec{r}}{r} = q\vec{E}$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\frac{\vec{r}}{r}$$

$$\vec{F_e} = -\nabla E$$

$$E_{pe} = \frac{qQ}{4\pi\varepsilon_0 r}$$

$$\vec{E} = -\nabla V$$

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0 r}}$$

Legea lui Gauss

$$\boxed{\Phi_e = \frac{Q}{\varepsilon_0} \qquad \nabla\cdot\vec{E} = \frac{\rho_q}{\varepsilon_0}} \quad \left(\Phi_e = \oiint_S \vec{E}\,d\vec{S}\right)$$

$$\boxed{\Delta V + \frac{\rho_q}{\varepsilon_0} = 0}$$ Ec. Poisson

Magnetostatica

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\cdot\frac{d\vec{l}\times\vec{r}}{r^3} = \frac{\mu_0 I}{4\pi}\cdot\frac{d\vec{l}\times\vec{e_r}}{r^2}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int_C \frac{d\vec{l}\times\vec{r}}{r^3}$$

Fir conductor liniar ∞ de lung

$$\vec{B} = \frac{\mu_0 I}{2\pi r}$$

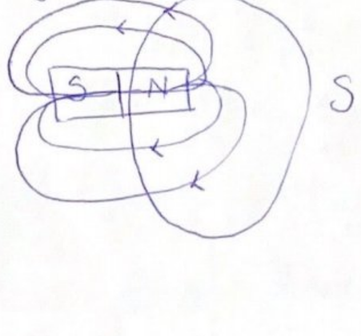
Buclă circulară de rază R

$$\nabla\cdot\vec{B} = \frac{\mu_0 I}{4\pi}\cdot\underbrace{\int_C \frac{\nabla(d\vec{l}\times\vec{r})}{r^3}}_{0}$$

$\nabla\cdot\vec{B} = 0$ nu avem „sarcini" magnetice libere ($\nexists$ monopoli magnetici)

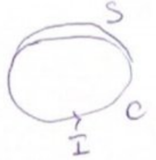
$$\iiint_V \nabla\vec{B}\,dV = 0$$

$$\phi_m = \oiint \vec{B}\cdot d\vec{S} = 0$$



Legea Ampère

$$\oint_C \vec{B}\, d\vec{l} = \mu_0 I$$



$$I = \iint_S \vec{j} \cdot d\vec{S}$$

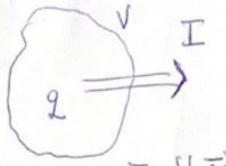
$$\oint_C \vec{B} \cdot d\vec{l} \xrightarrow{\text{Th. Stokes}} \iint_{S_c} \nabla \times \vec{B} \cdot d\vec{S}$$

$$\iint_{S_c} \nabla \times \vec{B} \cdot d\vec{S} = \mu_0 \iint_{S_c} \vec{j} \cdot d\vec{S}$$

$\boxed{\nabla \times \vec{B} = \mu_0 \vec{j}}$ forma locdă lui Ampère

Ec. de continuitate a sarcinii

$I = \frac{dq}{dt}$ prin secțiunea unui circuit



$$I = \iint_S \vec{j}\, d\vec{S}$$

$$q = \iiint_{V_S} \rho_q\, dV$$

$$I = -\frac{dq}{dt} \Rightarrow I + \frac{dq}{dt} = 0$$

$$\oiint_S \vec{j}\, d\vec{S} + \frac{d}{dt} \iiint_{V_S} \rho_q\, dV = 0 \Rightarrow \boxed{\nabla \vec{j} + \frac{d\rho_q}{dt} = 0}$$

$$\iiint_{V_S} \nabla \vec{j}\, dV + \iiint_{V_S} \frac{d\rho_q}{dt}\, dV = 0$$

$$\iiint_{V_S} \left(\nabla \vec{j} + \frac{d\rho_q}{dt}\right) dV = 0$$

$$\iiint_{V_S} \nabla \cdot \vec{A}\, dV = \oiint_S \vec{A}\, d\vec{S} \quad \text{Th. G.O}$$

$$\oint_C \vec{A}\, d\vec{l} = \iint_{S_c} (\nabla \times \vec{A})\, d\vec{S} \quad \text{Th. Stokes}$$

$\nabla \cdot S = \text{grad}\, S = \frac{dS}{dx}\vec{i} + \frac{dS}{dy}\vec{j} + \frac{dS}{dz}\vec{k}$ S-mărime scalară

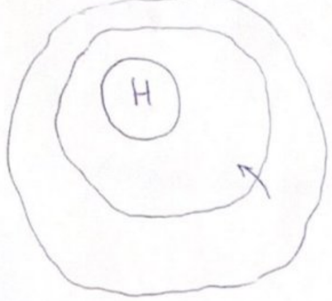
$$\nabla = \frac{d}{dx}\vec{i} + \frac{d}{dy}\vec{j} + \frac{d}{dz}\vec{k}$$

$$\nabla \cdot \vec{V} = \frac{dV_x}{dx} + \frac{dV_y}{dy} + \frac{dV_z}{dz}$$

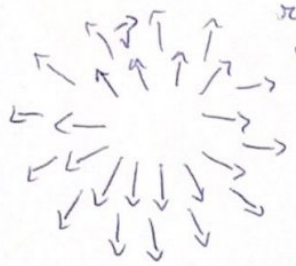
$$\nabla \cdot \vec{V} = \left(\frac{d}{dx}\vec{i} + \frac{d}{dy}\vec{j} + \frac{d}{dz}\vec{k}\right) \cdot \left(V_x \vec{i} + V_y \vec{j} + V_z \vec{k}\right)$$

$$\nabla \times \vec{V} = \text{rot}\, \vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ V_x & V_y & V_z \end{vmatrix} =$$

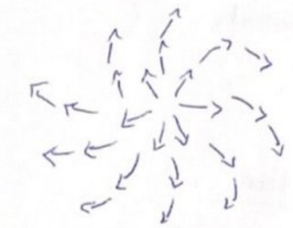
$$= \left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right)\vec{i} + \left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right)\vec{j} + \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\vec{k}$$



gradient de presiune



$rot\,\vec{v} = 0$
$div\,\vec{v} \neq 0$



$rot\,\vec{v} \neq 0$

În regim permanent $\vec{j}$ și ρ_q nu depind de timp.

$$\boxed{\nabla \cdot \vec{j} = 0}$$

Câmpuri electrice și magnetice variabile în timp (electrodinamic)

<u>Legea lui Gauss</u> își păstrează forma

$\nabla \vec{E} = \frac{\rho_q}{\varepsilon_0}$ forma locală $\quad \phi_e = \frac{Q}{\varepsilon_0}$

$\nabla \vec{B} = 0$ forma locală $\quad \phi_m = 0$ $\Big\rangle$ forme integrale

<u>Legea lui Faraday</u> (a inducției electromagnetice)

$$e = -\frac{d\phi_m}{dt}$$

$$\frac{U}{d} = E$$

$$e = \oint_C \vec{E}\,d\vec{l} \;;\quad \phi_m = \iint_{S_C} \vec{B}\,d\vec{S}$$

$$\oint_C \vec{E}\,d\vec{l} = -\frac{d}{dt}\iint_{S_C} \vec{B}\,d\vec{S}$$

$$\iint_{S_C} \nabla \times \vec{E}\,d\vec{S} = \iint_{S_C} \left(-\frac{\partial \vec{B}}{\partial t}\right) \cdot d\vec{S}$$

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$ Forma locală a legii lui Faraday

Legea lui Ampère pt. câmpuri variabile în timp

$\varepsilon_0 \vec{E} \overset{vid}{=} \vec{D}$

$\mu_0 \vec{H} \overset{vid}{=} \vec{B}$

$$\boxed{\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}}$$ Forma locală

$$\oint \vec{B} d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\phi_e}{dt}$$

$$\nabla \times \vec{B} = \mu_0 \left(\underbrace{\vec{j}}_{\text{densitate de curent de conducție}} + \frac{\partial \vec{D}}{\partial t} \right)$$

, $\vec{j}_D = \frac{\partial \vec{D}}{\partial t}$ densitate de curent de deplasare

$$\nabla \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$$

Ec. Maxwell forma locală

$\boxed{\nabla \cdot \vec{E} = \frac{\rho_q}{\varepsilon_0}}$ M I $\nabla \vec{D} = \rho_q$

$\boxed{\nabla \cdot \vec{B} = 0}$ M II $\nabla \cdot \vec{B} = 0$

$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$ M III $\nabla \cdot \vec{D} = -\mu_0 \varepsilon_0 \frac{\partial \vec{H}}{\partial t}$

$\boxed{\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}}$ M IV $\nabla \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$

Ec. Maxwell forma integrală

$$\begin{cases} \phi_e = \frac{Q}{\varepsilon_0} \\ \phi_m = 0 \\ e = -\frac{d\phi_m}{dt} \\ \oint \vec{B} d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\phi_e}{dt} \end{cases}$$

Legi de material

$$\vec{D} = \varepsilon \cdot \vec{E} = \varepsilon_0 \varepsilon_r \vec{E}$$

$\vec{H} = \frac{\vec{B}}{\mu}$ sau $\vec{B} = \mu_0 \mu_r \vec{H}$

Legea lui Ohm

$U = I R$ forma integrală

$$\left.\begin{array}{l} U \sim E \cdot l \\ R = \rho \dfrac{l}{S} \end{array}\right\} \Rightarrow E l = I \cdot \rho \dfrac{l}{S} \Rightarrow E = \rho \cdot j$$

$j = \nabla E$

$\boxed{\vec{j} = \nabla \vec{E}}$ forma locală a legii lui Ohm

μ_0 - permeabilitatea magnetică a vidului

$\mu_0 = 4\pi \cdot 10^{-7} \dfrac{H}{m} \left(\dfrac{T \cdot m}{A} \right)$

$\phi = L I \qquad T \cdot m^2 = H \cdot A$

Fizică curs 8

Ec. Maxwell în vid -> Ec. undelor ~~elementare~~ electromagnetice

forma locală

M#1 $\nabla \cdot \vec{E} = \frac{\rho_q}{\varepsilon_0}$

M#2 $\nabla \cdot \vec{B} = 0$

M#3 $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

M#4 $\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

forma integrală

$\phi_e = \frac{Q}{\varepsilon_0}$

$\phi_m = 0$ (pe o suprafață închisă)

$e = -\frac{d\phi_m}{dt}$

$\oint_C \vec{B}\, d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\phi_e}{dt}$

circulația

În vid, departe de sursele de câmp ($\rho_q = 0$; $\vec{j} = 0$)

$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$

$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \underbrace{(\vec{\nabla} \cdot \vec{\nabla})}_{\Delta} \vec{E}$

$\nabla \times / \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\nabla \times (\nabla \times \vec{E}) = -\nabla \times \frac{\partial \vec{B}}{\partial t} \iff \nabla \cdot \underbrace{(\nabla \cdot \vec{E})}_{M\#1\ 0} - \Delta \vec{E} = -\frac{\partial}{\partial t}(\nabla \times \vec{B})$

$\iff -\Delta \vec{E} = -\frac{\partial}{\partial t}\left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right) \iff \Delta \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$ ec. undelor elementare în vid

$\frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$ ec. undelor elementare în vid

$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ viteza luminii în vid

$\nabla \times / \nabla \times \vec{B} = \mu_0 \underset{0}{\vec{j}} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

$\nabla \times (\nabla \times \vec{B}) = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}(\nabla \times \vec{E}) \iff \nabla \cdot \underbrace{(\nabla \cdot \vec{B})}_{0} - \Delta \vec{B} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \rightarrow \Delta \vec{B} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} = 0$

Soluții ale ec. undelor

Sol. D'Alembert

1D $\frac{\partial^2 \psi}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = 0$

ψ funcția de undă

$\psi = \forall$ componentă a câmpurilor electrice sau magnetice

$u = x - ct$

$w = x + ct$

$\frac{\partial}{\partial w} = \frac{\partial}{\partial x} + \frac{1}{c}\frac{\partial}{\partial t}$

$\frac{\partial}{\partial x} = \frac{1}{2}\left(\frac{\partial}{\partial u} + \frac{\partial}{\partial w}\right)$

$\frac{\partial}{\partial t} = \frac{c}{2}\left(\frac{\partial}{\partial w} - \frac{\partial}{\partial u}\right)$

~~$\frac{\partial}{\partial x} = \frac{\partial}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial}{\partial u}$~~ $\cdot \frac{\partial}{\partial u} = \frac{\partial}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial}{\partial t} \cdot \frac{\partial t}{\partial u}$

$\frac{\partial}{\partial u} = \frac{\partial}{\partial x} \cdot \frac{1}{\frac{\partial u}{\partial x}} + \frac{\partial}{\partial t} \cdot \frac{1}{\frac{\partial u}{\partial t}}$

$\frac{\partial}{\partial u} = \frac{\partial}{\partial x} - \frac{1}{c}\frac{\partial}{\partial t}$

$$\frac{\partial}{\partial u}\left(\frac{\partial \Psi}{\partial w}\right) = \frac{1}{4}\left(\frac{\partial}{\partial x} - \frac{1}{c}\frac{\partial}{\partial t}\right)\left(\frac{\partial \Psi}{\partial x} + \frac{1}{c}\frac{\partial \Psi}{\partial t}\right) = \frac{1}{4}\left(\frac{\partial^2 \Psi}{\partial x^2} + \cancel{\frac{1}{c}\frac{\partial^2 \Psi}{\partial x \partial t}} - \cancel{\frac{1}{c}\frac{\partial^2 \Psi}{\partial t \partial x}} - \frac{1}{c^2}\frac{\partial^2 \Psi}{\partial t^2}\right) =$$

$$\Rightarrow \frac{\partial^2 \Psi}{\partial u \partial w} = 0 \qquad , \Psi = \Psi(u, w)$$

$$\frac{\partial}{\partial u}\left(\frac{\partial \Psi}{\partial w}\right) = 0 \Rightarrow \frac{\partial \Psi}{\partial w} = g'(w) \rightarrow \Psi = g(w) + C'$$ nu depinde de w

$$\frac{\partial}{\partial w}\left(\frac{\partial \Psi}{\partial u}\right) = 0 \Rightarrow \frac{\partial \Psi}{\partial u} = f'(u) \Rightarrow \Psi = f(u) + C'$$ nu depinde de u

$$\Psi = \Psi(u, w) = f(u) + g(w) = f\underbrace{(x - ct)}_{\text{pt unde progresive}} + g\underbrace{(x + ct)}_{\text{pt. unde regresive}}$$

$$\Psi(x, t) = f(x - ct) + g(x + ct)$$

Sol. Fourier

$$\Psi(x, t) = X(x) \cdot T(t)$$

$$\frac{\partial^2 \Psi}{\partial x^2} - \frac{1}{c^2}\frac{\partial \Psi}{\partial t^2} = 0$$

$$T \cdot \frac{\partial^2 X}{\partial x^2} - \frac{1}{c^2} X \frac{\partial^2 T}{\partial t^2} = 0 \Leftrightarrow \frac{1}{X}\frac{\partial^2 X}{\partial x^2} - \frac{1}{Tc^2}\frac{\partial^2 T}{\partial t^2} = 0$$

$$\underbrace{\frac{1}{X} \cdot \frac{\partial^2 X}{\partial x}}_{\text{depinde doar de } x} = \underbrace{\frac{1}{c^2 T}\frac{\partial^2 T}{\partial t^2}}_{\text{depinde doar de } T} = -k^2$$

$$\forall (x, t)$$

$$\frac{\partial^2 X}{\partial x^2} + k^2 X = 0 \qquad \left(\frac{d^2 x}{dt^2} + \omega_0^2 x = 0\right)$$

$$x(t) = A\cos(\omega_0 t + \varphi)$$

$$X(x) = A\cos(kx + \varphi) \quad (*)$$

$$\frac{\partial^2 T}{\partial t^2} + \underbrace{k^2 c^2}_{\omega^2} T = 0$$

$$T(t) = A\cos(\omega t + \varphi) \quad (**)$$

$$\Psi(x, t) = A^2 \cos(kx + \varphi)\cos(\omega t + \varphi)$$

$$\Psi(x, t) = A' \cos(\omega t - kx + \varphi)$$

Sol. UAP

$$\vec{E}(\vec{r}, t) = \vec{E_0} e^{i(\omega t - \vec{k}\vec{r})} + \vec{E_0} e^{i(\omega t + \vec{k}\vec{r})}$$

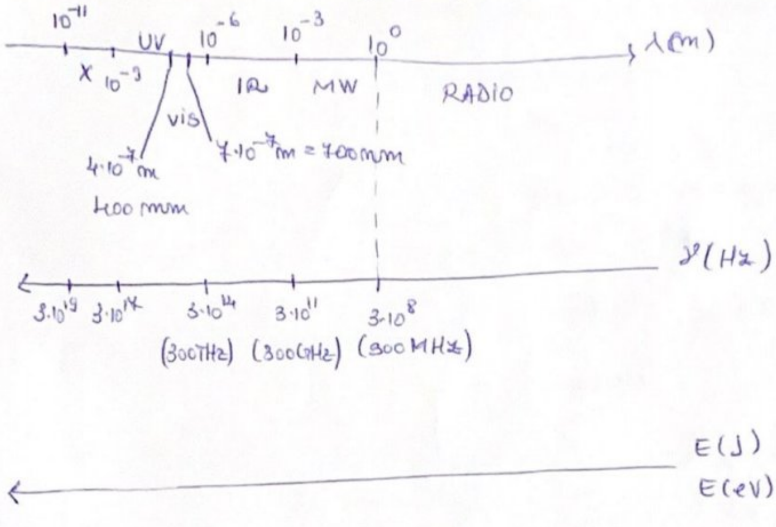
$\omega = 2\pi\nu$ pulsația undei

$k = \frac{\omega}{c}$ nr. de undă

$\vec{k} = k \vec{u}_k$ vector de undă

$\vec{u_k}$ - versorul direcției de propagare a undei

Spectrul undelor electromagnetice



$\nu = \frac{c}{\lambda}$

$E = h\nu$, $h = 6{,}626\cdot10^{-34}$ J·s

Caracteristicile undelor electromagnetice

Transversalitate

1D $\vec{E}(x,t) = \vec{E_0}\, e^{i(\omega t - kx)}$

$\nabla \Leftrightarrow -ik$.

1D $\left(\frac{\partial}{\partial x}\right)$

$\frac{\partial}{\partial t} \Leftrightarrow i\omega$.

3D $\vec{E}(\vec{r},t) = E_0\, e^{i(\omega t - \vec{k}\cdot\vec{r})}$

$\nabla \cdot \Leftrightarrow -i\vec{k}$

(rotor) $\nabla \times \Leftrightarrow -i\vec{k}\times$

$\frac{\partial}{\partial t} \Leftrightarrow i\omega$

$\nabla\cdot\vec{E} = 0 \Rightarrow -i\vec{k}\cdot\vec{E} = 0 \Rightarrow \vec{k} \perp \vec{E}$

$\nabla\cdot\vec{B} = 0 \Rightarrow -i\vec{k}\cdot\vec{B} = 0 \Rightarrow \vec{k} \perp \vec{B}$

$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \Rightarrow -i\vec{k}\times\vec{E} = -i\omega\vec{B}$

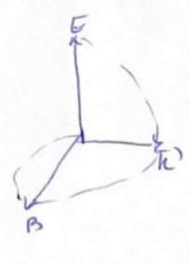
$\vec{k}\times\vec{E} = \omega\vec{B}$ $\quad \vec{k} = k\cdot\vec{u_k}$

$\vec{u_k}\times\vec{E} = \underbrace{\frac{\omega}{k}}_{c}\vec{B} \Rightarrow \vec{B} = \frac{1}{c}\,\vec{u_k}\times\vec{E} \Rightarrow \vec{B}\perp\vec{E}$

Temă

$\vec{u_k}\times\vec{B} = -\frac{1}{c}\vec{E} \Rightarrow \cancel{\vec{E}}\ \vec{E} = -c(\vec{u_k}\times\vec{B})$

$(\vec{k}, \vec{E}, \vec{B})$ - triedru tridreptunghic



Fizică curs 9

$\nabla \vec{E} = \boxed{\operatorname{div} \vec{E} = \frac{\rho_q}{\varepsilon}}$ (mediu dielectric $\varepsilon = \varepsilon_0 \varepsilon_r$) $\vec{D} = \varepsilon \vec{E} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon_0 \vec{E} + \vec{P} =$

$\nabla \cdot \vec{B} = \boxed{\operatorname{div} \vec{B} = 0}$ $= \varepsilon_0 (1 + \chi_e) \vec{E}$

$\nabla \times \vec{E} = \boxed{\operatorname{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t}} \Rightarrow \operatorname{rot} \vec{D} = -\mu \varepsilon \frac{\partial \vec{H}}{\partial t}$

χ_e - susceptivitatea electrică

$\operatorname{rot} \vec{D} = -\frac{1}{c_m^2} \frac{\partial \vec{H}}{\partial t}$

$\vec{P} = \varepsilon_0 \chi_e \vec{E}$

$\vec{B} = \mu \vec{H}$ $\mu = \mu_0 \mu_r$ (permeabilitate magnetică) ? - permitivitate

$\vec{B} = \mu_0 \mu_r \vec{H} = \mu_0 \vec{H} + \vec{M}$

$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$c_m = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}\sqrt{\mu_r \varepsilon_r}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$

$n = \sqrt{\varepsilon_r}$ $\mu_r \simeq 1$ $\varepsilon = \varepsilon_0 \varepsilon_r$ (permitivitatea electrică)

$n = \frac{c}{c_m}$

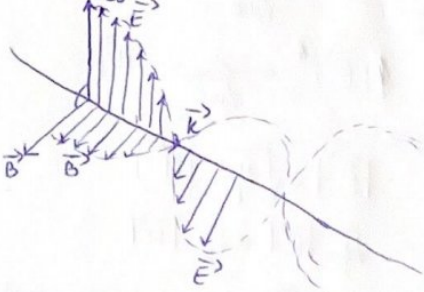
$\nabla \times \vec{B} = \operatorname{rot} \vec{B} = \mu \vec{j} + \mu \varepsilon \frac{\partial \vec{E}}{\partial t}$

$\operatorname{rot} \vec{H} = \vec{j} + \underbrace{\frac{\partial \vec{D}}{\partial t}}_{\vec{j}_D}$

$\vec{j} = \sigma \vec{E}$ legea lui Ohm

$\Delta \vec{E} + \frac{1}{c_m^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$ $a_x = \frac{d^2 x}{dt^2}$

$\Delta \vec{B} + \frac{1}{c_m^2} \frac{\partial^2 \vec{B}}{\partial t^2} = 0$



$$\begin{cases} \vec{E}(\vec{r}, t) = \vec{E}_0 e^{i(\omega t - \vec{k}\vec{r})} \\ \vec{B}(\vec{r}, t) = \vec{B}_0 e^{i(\omega t - \vec{k}\vec{r})} \end{cases}$$

Caracterizarea undelor electromagnetice

$\vec{E} \perp \vec{K}$

Gauss electric

$\nabla \cdot \vec{E} = 0$ (departe de surse)

$\nabla \cdot \longrightarrow -i\vec{K}$

$\Rightarrow -i\vec{K}\vec{E} = 0 \Leftrightarrow \vec{K} \perp \vec{E}$ q.e.d

Gauss magnetic

$\nabla \cdot \vec{B} = 0 \Leftrightarrow -i\vec{K}\vec{B} = 0 \Leftrightarrow \vec{K} \perp \vec{B}$ undele magnetice sunt transversabile

$\nabla \times \longrightarrow -i\vec{k}\times$

$\vec{\nabla}\times\vec{E} = -i\vec{k}\times\vec{E}$

$\frac{\partial \vec{B}}{\partial t} = i\omega\vec{B}$

$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\frac{\partial}{\partial t} \longrightarrow i\omega$

$\Rightarrow -i\vec{k}\times\vec{E} = -i\omega\vec{B}$

$\vec{B} = \frac{\vec{k}\times\vec{E}}{\omega}$

$\vec{B} = \frac{k}{\omega}\,\vec{u_k}\times\vec{E}$

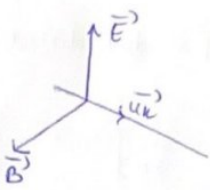
$\boxed{\vec{B} = \frac{1}{c}\vec{u_k}\times\vec{E}}$ $\vec{B}\perp\vec{E}$

în vid

$\vec{k} = k\,\vec{u_k}$

$\frac{k}{\omega} = \frac{1}{c}$

$\boxed{k = \frac{\omega}{c}}$



$\nabla\times\vec{B} = \frac{1}{c_m^2}\frac{\partial \vec{E}}{\partial t}$

$-i\vec{k}\times\vec{B} = \frac{1}{c_m^2}\,i\omega\vec{E}$

$\vec{E} = -c_m^2\,\frac{\vec{k}\times\vec{B}}{\omega}$

$\vec{E} = -c_m^2\,\frac{k}{\omega}\,\vec{u_k}\times\vec{B}$

$\vec{E} = -c_m(\vec{u_k}\times\vec{B})$ $\vec{E}\perp\vec{B}$

$(\vec{E},\vec{B},\vec{k})$ - triedru tridreptunghic

$\vec{E}\times\vec{B} \uparrow\uparrow \vec{k}$

$\vec{k}\times\vec{E} \uparrow\uparrow \vec{B}$

$\vec{k}\times\vec{B} \uparrow\downarrow \vec{E}$

$|\vec{E}| = c_m|\vec{u_k}\times\vec{B}|$

$|\vec{E}| = \frac{1}{\sqrt{\mu\varepsilon}}|\vec{B}|$

$|\vec{E}| = \frac{\mu}{\sqrt{\mu\varepsilon}}|\vec{H}|$

$|\vec{E}| = \sqrt{\frac{\mu}{\varepsilon}}\,|\vec{H}|$

$\sqrt{\varepsilon}\,|\vec{E}| = \sqrt{\mu}\,|\vec{H}|$

$Z = \sqrt{\frac{\mu}{\varepsilon}}$ - impedanța mediului

$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \simeq 376\ \Omega$ impedanța vidului

medii nemagnetice $\mu_r \simeq 1$

$Z = \frac{Z_0}{n}$

<u>Suprafețe echifază, viteza de fază, λ, k</u>

Supr. echifază sunt suprafețele de undă care au aceeași fază.

$\omega t - \vec{k}\cdot\vec{r} = ct$

Supr. echifază cele mai depărtate de sursă formează frontul de undă.

Viteza de fază
(unde progresive)

$\omega dt - \vec{k} d\vec{r} = 0$

1D: $\omega dt - k dx = 0 \Rightarrow v_f = \frac{\omega}{k} = c_m$

pt. unde regresive

$\omega t + \vec{k}\vec{r} = ct$

1D: $\omega t + kx = ct$

$\omega dt + k dx = 0$

$\Rightarrow v_f = -\frac{\omega}{k} = -c_m$

$\omega = 2\pi\nu = \frac{2\pi}{T}$

$k = \frac{\omega}{c_m}$ $\quad \lambda = c_m \cdot T$

$\Rightarrow k = \frac{\omega}{c_m T} \cdot T = \frac{\omega \cdot T}{\lambda} = \frac{2\pi T}{T\lambda} = \frac{2\pi}{\lambda}$

$k = \frac{2\pi}{\lambda} \quad \Rightarrow \lambda = \frac{2\pi}{k}$

$\sqrt{\varepsilon}\,|\vec{E}| = \sqrt{\mu}\,|\vec{H}|$

u = densitatea volumică de energie pt unda electromagnetică

$u = u_{el} + u_{mgn} = \frac{dE}{dV}$

En. undelor electromagnetice. Vector Poynting. Intensit. undelor elm

$u_{el} = \frac{\varepsilon_0 \vec{E}^2}{2} \qquad u_{mgn} = \frac{\mu_0 \vec{H}^2}{2}$

$u_{el} = u_{mgn}$

$u = \varepsilon_0 \vec{E}^2 = \mu_0 \vec{H}^2$

$u = \frac{\varepsilon_0 \vec{E}^2}{2} + \frac{\mu_0 \vec{H}^2}{2}$

Vectorul Poynting $\quad \vec{S} = \vec{E} \times \vec{H} \uparrow\uparrow \vec{k}$

- este un vector care arată direcția de propagare a energiei transportate de unda electromagnetică

Th. Poynting

$$\boxed{-\frac{du}{dt}\int_{V_\Sigma} u\, dV = \oint_\Sigma \vec{S}\, d\vec{\Sigma} + \int_{V_\Sigma} \vec{j}\,\vec{E}\, dV}$$

(Temă)

În absența surselor ($\vec{j} = 0$)

$$-\frac{d}{dt}\int_{V_\Sigma} u\, dV = \oint_\Sigma \vec{S}\, d\Sigma$$

$$-\frac{du}{dt} = \nabla \cdot \vec{S}$$

$$\boxed{\nabla \vec{S} + \frac{du}{dt} = 0}$$

$I \overset{def}{=} \langle |\vec{S}| \rangle = \frac{1}{2Z} \vec{E}^2 \qquad \langle I \rangle_{SI} = \frac{W}{m^2}$

media temporală

Fizică curs 10

Densitatea de energie electromagnetică în vid

$\left(\vec{k} = k \cdot \vec{u_k} \quad , k = \frac{\omega}{c_m} = \frac{n\omega}{c}\right)$

$u = u_{el} + u_{magn} = \frac{\varepsilon \cdot \vec{E}^2}{2} + \mu \frac{\vec{H}^2}{2} = \varepsilon \vec{E}^2 = \mu \vec{H}^2$

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow -i\vec{k} \times \vec{E} = -i\omega \vec{B} \Rightarrow \vec{B} = \frac{k}{\omega} \vec{u_k} \times \vec{E} = \frac{1}{c} \vec{u_k} \times \vec{E}$

$\left(c_m = \frac{1}{\sqrt{\mu \varepsilon}}\right)$

$\left.\begin{array}{l} |\vec{B}| = \frac{1}{c} \cdot \vec{E} \\ \vec{B} = \mu \vec{H} \end{array}\right\} \Rightarrow |\vec{H}| = \frac{1}{c_m \mu} |\vec{E}| \Rightarrow |\vec{H}|^2 = \frac{\varepsilon}{\mu} |\vec{E}|^2$

$\left(\vec{H} = \sqrt{\frac{\varepsilon}{\mu}} \vec{E}\right)$

$\left(Z = \sqrt{\frac{\mu}{\varepsilon}}\right)$

$\vec{S} = \vec{E} \times \vec{H}$ - vectorul Poynting

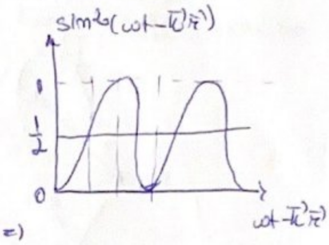
Th. en. electromagnetice

$-\int_{V_\Sigma} \frac{\partial u}{\partial t} dV = \oint_\Sigma \vec{S} d\Sigma + \int_{V_\Sigma} \vec{j} \cdot \vec{E} \, dV \overset{\vec{j}=0}{\Longrightarrow} -\int_V \frac{\partial u}{\partial t} dV = \int_V \nabla \cdot \vec{S} \, dV \Rightarrow$

$\Rightarrow \boxed{\frac{\partial u}{\partial t} + \nabla \cdot \vec{S} = 0}$

$\vec{E} = \vec{E_0} e^{i(\omega t - \vec{k}\vec{r})} =$

$= \vec{E_0} \sin(\omega t - \vec{k}\vec{r})$



$J \overset{def}{=} \langle |\vec{S}| \rangle = \langle |\vec{E} \times \vec{H}| \rangle = \langle \frac{1}{Z} |\vec{E}^2| \rangle$

$J = \frac{1}{2Z} |\vec{E_0}^2| \qquad \langle J \rangle_{SI} = \frac{W}{m^2}$

$\langle u \rangle = \frac{\varepsilon \vec{E_0}^2}{2} = \frac{\sqrt{\mu} \varepsilon}{2\sqrt{\mu}} \vec{E_0}^2 = \sqrt{\mu \varepsilon} \cdot \frac{1}{2Z} \vec{E_0}^2 = \frac{1}{c_m} J \Rightarrow$

$\Rightarrow J = c_m \langle u \rangle$

în vid: $J = c \langle u \rangle$

Impulsul undelor electromagnetice

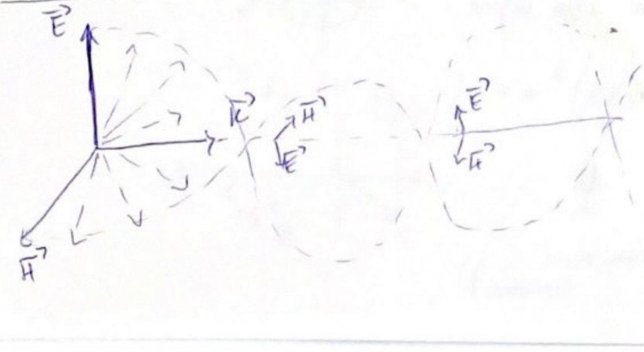
$\vec{g} = \vec{D} \times \vec{B} = \mu \varepsilon \vec{E} \times \vec{H}$

$\langle g \rangle = \frac{\mu \varepsilon}{2Z} \vec{E_0}^2 = \frac{1}{c_m^2} J = \frac{\langle u \rangle}{c_m}$

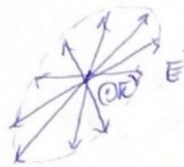
$p_f = \frac{E_f}{c} \qquad E_f = p \cdot c \qquad p = \frac{h}{\lambda}$ p - impulsul fotonului

$E_f = \frac{h}{\lambda} \cdot c = h\nu \Rightarrow \boxed{E_f = h\nu}$ energia fotonului

Polarizarea undelor electromagnetice



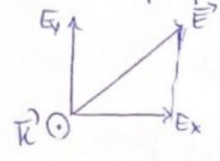
undă nepolarizată



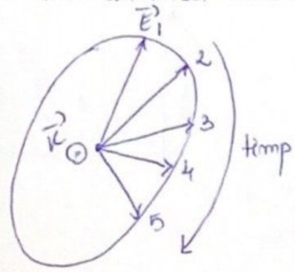
undă parțial polarizată

undă total polarizată (liniar)

Pentru o undă electromag (transversală) dacă se cunoaște dir. de propagare, atunci pentru a det. intensitatea câmpului el. ce ~~...~~ corect. unda, este suficientă cunoașterea a două componente perpendiculare independente.



Dacă în planul transversal direcției de propagare, vectorul intensitate câmp electric oscilează într-un mod anizotrop, spunem că unda este polarizată.



σ^+ (dextrogiră)

σ^- (levogiră)

polarizare eliptică

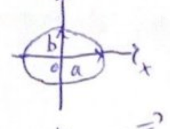
Matematic polarizarea eliptică:

$$\frac{E_x^2}{a^2} + \frac{E_y^2}{b^2} - \frac{2E_xE_y}{ab}\cos\varphi = \sin^2\varphi$$ ecuația unei elipse

φ - defazajul dintre cele 2 unde perpendiculare de aceeași frecvență care se compun pentru a obține unda polarizată eliptic.

$\vec{E} = \vec{E}_x + \vec{E}_y$

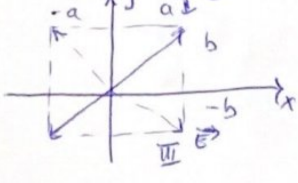
I. $\varphi = \frac{\pi}{2} \Rightarrow \frac{E_x^2}{a^2} + \frac{E_y^2}{b^2} = 1$ ec. unei elipse



II. $\varphi = 0 \Rightarrow \left(\frac{E_x}{a} - \frac{E_y}{b}\right)^2 = 0$

$$\frac{E_x}{a} = \frac{E_y}{b}$$

(polarizare liniară)



$\text{III}.\ \varphi = \pi \quad \frac{E_x}{a} = -\frac{E_y}{b}$

~~Reflexia și refracția undei~~

I. extindere

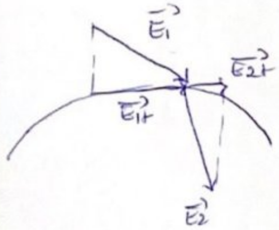
~~$E_x = E_y$~~ $a = b \rightarrow$ polarizare circulară

$$E_x^2 + E_y^2 = R^2$$

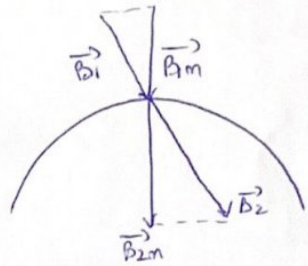
Reflexia și refracția luminii undelor elm

Continuitatea componentelor câmpurilor la suprafața de separare dintre 2 medii transparente.

La trecerea unei unde elm (lumină) printr-o suprafață de discontinuitate care separă cele 2 medii, componentele tangențiale ale intensităților câmpurilor și componentele transversale (normale) ale inducțiilor câmpurilor se conservă.



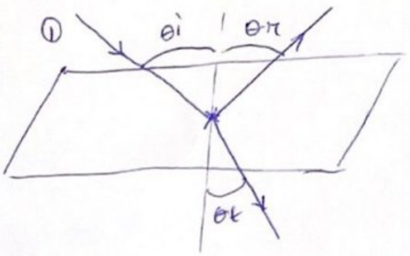
$$E_{1t} = E_{2t}$$
$$H_{1t} = H_{2t}$$



$$B_{1n} = B_{2n}$$
~~$B_1 = B_2$~~
$$D_{1n} = D_{2n}$$

Legile reflexiei și refracției

În general, la întâlnirea unei suprafețe de separare între 2 medii diferite se întâlnesc atât fenomenul de reflexie (întoarcerea undei în mediul din care a provenit), precum și fenomenul de refracție (transmiterea undei în cel de-al doilea mediu-fenomen însoțit de schimbarea direcției de propagare).



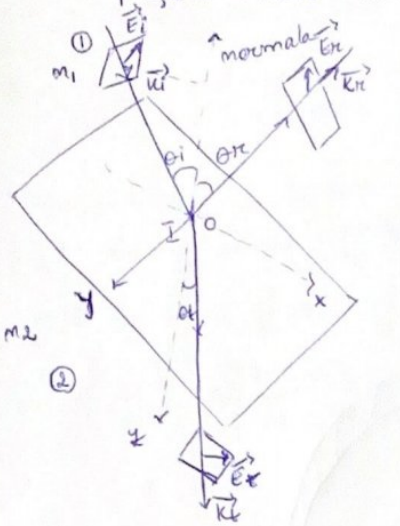
θ_i - incident

θ_r - reflectat

θ_t - transmis

Fizică Curs 11

indice de refracție mai mare - materialul e mai refringent sau mai dens dpdv optic



n_1, n_2 - indicii de refracție

Planul de incidență este planul format de unda incidentă și normala la suprafață.

$\vec{E_i} = \vec{E_{0i}} \cdot e^{i(\omega t - \vec{k_i} \cdot \vec{r})}$

$\vec{E_r} = \vec{E_{0r}} \cdot e^{i(\omega' t - \vec{k_r} \cdot \vec{r})}$

$\vec{E_t} = \vec{E_{0t}} \cdot e^{i(\omega'' t - \vec{k_t} \cdot \vec{r})}$

$k_{ix} = k_i \cos(\frac{\pi}{2} - \theta_i) = k_i \sin\theta_i$

$k_{rx} = k_r \cos(\frac{\pi}{2} - \theta_r) = k_r \sin\theta_r$

$k_{tx} = k_t \cos(\frac{\pi}{2} - \theta_t) = k_t \sin\theta_t$

$E_{0i\,tang}\, e^{i(\omega t - \vec{k_i} \vec{r})} + E_{0r\,tang}\, e^{i(\omega' t - \vec{k_r} \vec{r})} = E_{0t\,tang}\, e^{i(\omega'' t - \vec{k_t} \vec{r})}$

$(\forall)\, t, x, y, z \Rightarrow \boxed{\omega = \omega' = \omega''}$

$k_i = \frac{\omega}{c_{n1}} = \frac{n_1 \omega}{c}$ $\qquad c_{n1} = \frac{c}{n_1}$

$k_r = \frac{n_1 \omega}{c}$

$k_t = \frac{n_2 \omega}{c}$

$k_{ix} \cdot \cancel{x} = k_{rx} \cdot \cancel{x} = k_{tx} \cdot \cancel{x}$

$\cancel{n_1} \sin\theta_i = \cancel{n_1} \sin\theta_r \Rightarrow \boxed{\theta_i = \theta_r}$ Legea reflexiei

$\boxed{n_2 \sin\theta_t = n_1 \sin\theta_i}$ Legea Refracției (Snellius / Snell-Descartes)

Legile reflexiei și refracției

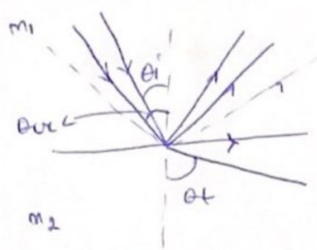
1. Unda incidentă, reflectată și transmisă sunt situate în planul de incidență (același plan).
2. Frecvențele (pulsațiile) undelor incidentă, reflectată și refractată sunt egale.
3. (Legea reflexiei) Unghiul de incidență = unghiul de reflexie. $\theta_i = \theta_r$

4. Unda incidentă și unda transmisă respectă Legea refracției.

$n_1 \sin\theta_i = n_2 \sin\theta_t$

$n_2 < n_1$

$\theta_t > \theta_i$

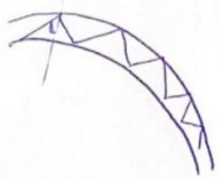


$$\theta_{cr}: \; n_1 \sin\theta_{cr} = n_2 \sin\frac{\pi}{2}$$

$$\sin\theta_{cr} = \frac{n_2}{n_1}$$

$$\theta_{cr} = \arcsin\left(\frac{n_2}{n_1}\right)$$

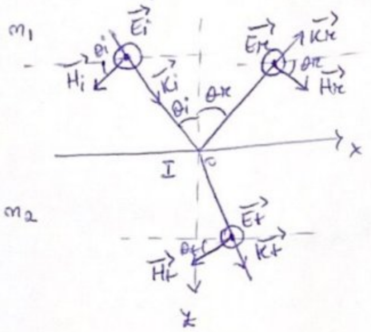
$\theta_i > \theta_{cr} \Rightarrow$ reflexie totală



Relațiile lui Fresnel

Coeficienți Fresnel

Cazul I (polarizarea $\perp$ pe planul de incidență)



$\theta_i = \theta_r$

$$\begin{cases} E_{0i} + E_{0r} = E_{0t} \\ -H_{0i}\cos\theta_i + H_{0r}\cos\theta_r = -H_{0t}\cos\theta_t \end{cases}$$

$$\sqrt{\varepsilon}\, E = \sqrt{\mu}\, H$$

$$H = \frac{1}{Z} E$$

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\begin{cases} E_{0i} + E_{0r} = E_{0t} \quad \Big| \cdot -\frac{1}{Z_2}\cos\theta_t \\ \frac{1}{Z_1} E_{0i}\cos\theta_i - \frac{1}{Z_1} E_{0r}\cos\theta_i = \frac{1}{Z_2} E_{0t}\cos\theta_t \end{cases}$$

$$\left(\frac{1}{Z_1}\cos\theta_i - \frac{1}{Z_2}\cos\theta_t\right)E_{0i} - \left(\frac{1}{Z_1}\cos\theta_i + \frac{1}{Z_2}\cos\theta_t\right)E_{0r} = 0$$

$$\boxed{\left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{Z_2\cos\theta_i - Z_1\cos\theta_t}{Z_2\cos\theta_i + Z_1\cos\theta_t}}$$ coeficientul Fresnel de reflexie în polarizare perpendiculară

medii oarecare

$$\left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{Z_2\cos\theta_i - Z_1\cos\theta_t}{Z_2\cos\theta_i + Z_1\cos\theta_t} \underset{\substack{\text{nemag} \\ \mu_r \cong 1}}{\overset{\text{medii}}{=}} \frac{\frac{1}{n_2}\cos\theta_i - \frac{1}{n_1}\cos\theta_t}{\frac{1}{n_2}\cos\theta_i + \frac{1}{n_1}\cos\theta_t} = \frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t} =$$

$$\left(Z = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_r}{\varepsilon_r}} \cdot \underbrace{\sqrt{\frac{\mu_0}{\varepsilon_0}}}_{Z_0} \overset{\mu_r \cong 1}{=} \frac{1}{n} Z_0\right)$$

$$\frac{\overset{n_1 \sin\theta_i = n_2 \sin\theta_t}{}\ \cancel{n_2}\left(\frac{\sin\theta_t}{\sin\theta_i}\cos\theta_i - \cos\theta_t\right)}{\cancel{n_2}\left(\frac{\sin\theta_t}{\sin\theta_i}\cos\theta_i + \cos\theta_t\right)}$$

$$r_\perp = \frac{\sin\theta_t\cos\theta_i - \cos\theta_t\sin\theta_i}{\sin\theta_t\cos\theta_i + \cos\theta_t\sin\theta_i} = \boxed{\frac{-\sin(\theta_i-\theta_t)}{\sin(\theta_i+\theta_t)}}$$ coef Fresnel pt. medii magnetice

$$\begin{cases} E_{0i} + E_{0r} = E_{0t} & \Big| \cdot \frac{1}{Z_1}\cos\theta_i \\ \frac{1}{Z_1}E_{0i}\cos\theta_i - \frac{1}{Z_1}E_{0r}\cos\theta_i = \frac{1}{Z_2}E_{0t}\cos\theta_t \end{cases}$$

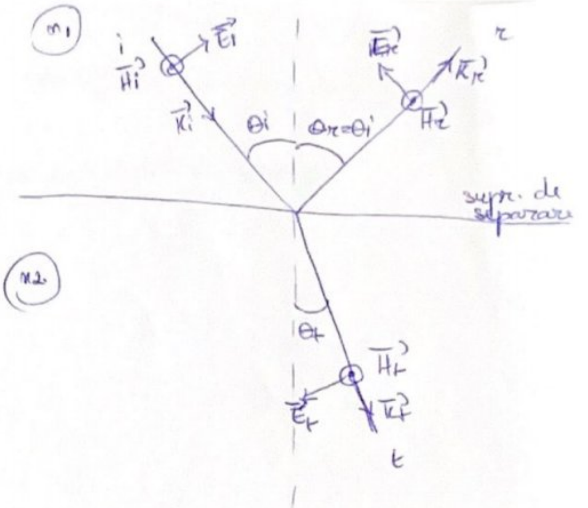
$$\frac{2}{Z_1}E_{0i}\cos\theta_i = E_{0t}\left(\frac{1}{Z_1}\cos\theta_i + \frac{1}{Z_2}\cos\theta_t\right)$$

$$t_\perp = \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2Z_2\cos\theta_i}{Z_2\cos\theta_i + Z_1\cos\theta_t} \overset{\text{medii}}{\underset{\text{nemag}}{=}} \frac{2n_1\cos\theta_i}{n_1\cos\theta_i + n_2\cos\theta_t} = \frac{2\cos\theta_i\sin\theta_t}{\sin(\theta_i+\theta_t)}$$

Fizică curs 12

p (polarized)

Polarizare paralelă



$$\begin{cases} H_{0i} + H_{0r} = H_{0t} \quad \Big| \cdot Z_2 \cos\theta_t \\ E_{0i}\cos\theta_i - E_{0r}\cos\theta_i = -E_{0t}\cos\theta_t \end{cases}$$

$$H_{0i} = \frac{E_{0i}}{Z_1}$$

$$H_{0r} = \frac{E_{0r}}{Z_1}$$

$$H_{0t} = \frac{E_{0t}}{Z_2}$$

$$r_{\parallel} = \left(\frac{E_{0r}}{E_{0i}}\right)_{\parallel} = \frac{Z_1\cos\theta_i - Z_2\cos\theta_t}{Z_1\cos\theta_i + Z_2\cos\theta_t} \xrightarrow[\mu_r \approx 1]{\text{medii nemag}}$$

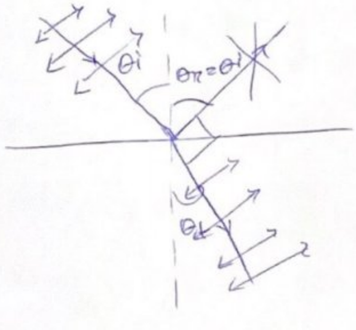
se conservă } comp. tang. ale intensităților câmpurilor
comp. normale ale inducțiilor –"–

$$= \frac{n_2\cos\theta_i - n_1\cos\theta_t}{n_2\cos\theta_i + n_1\cos\theta_t}$$

coef. Fresnel de reflexie în polarizarea paralelă

$$\xrightarrow{n_1\sin\theta_i = n_2\sin\theta_t} \frac{tg(\theta_i - \theta_t)}{tg(\theta_i + \theta_t)} \qquad ?\ (\theta_i + \theta_t) = \frac{\pi}{2}$$

$$t_{\parallel} = \left(\frac{E_{0t}}{E_{0i}}\right)_{\parallel} = \frac{2 Z_2\cos\theta_i}{Z_1\cos\theta_i + Z_2\cos\theta_t} \xrightarrow{\text{medii nemag}} \frac{2 n_1\cos\theta_i}{n_2\cos\theta_i + n_1\cos\theta_t} = \frac{2\sin\theta_t\cos\theta_i}{\cos(\theta_i - \theta_t)\sin(\theta_i + \theta_t)}$$

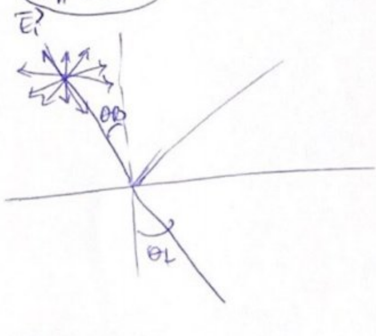


$$\theta_i + \theta_t = \frac{\pi}{2} \qquad \theta_i = \theta_B$$

$$tg(\theta_i + \theta_t) = \infty$$

B - Brewster

$$r_{\parallel} = 0$$

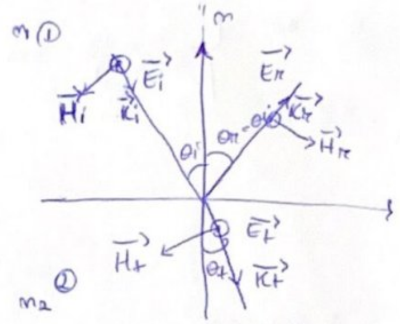


În absența absorbției

R+T = 1 (atunci când avem și absorbție R+T+A = 1)

Fizică Curs 13

$$r_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{Z_2\cos\theta_i - Z_1\cos\theta_t}{Z_2\cos\theta_i + Z_1\cos\theta_t}$$



$$\begin{cases} E_{0i} + E_{0r} = E_{0t} \\ -H_{0i}\cos\theta_i + H_{0r}\cos\theta_i = H_{0t}\cos\theta_t \end{cases}$$

$$\left.\begin{aligned} B &= \mu H = \frac{1}{c_n}\cdot E \\ c_n &= \frac{1}{\sqrt{\mu\varepsilon}} \end{aligned}\right\} \Rightarrow H = \frac{1}{Z}E$$

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

$$t_\perp = \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2Z_2\cos\theta_i}{Z_2\cos\theta_i + Z_1\cos\theta_t} = \frac{2n_1\cos\theta_i}{n_1\cos\theta_i + n_2\cos\theta_t} = \frac{2\cos\theta_i\sin\theta_t}{\sin(\theta_i+\theta_t)}$$

$$r_\perp = \frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t} = \frac{-\sin(\theta_i-\theta_t)}{\sin(\theta_i+\theta_t)}$$

$$r_\parallel = \frac{Z_1\cos\theta_i - Z_2\cos\theta_t}{Z_1\cos\theta_i + Z_2\cos\theta_t} = \frac{n_2\cos\theta_i - n_1\cos\theta_t}{n_2\cos\theta_i + n_1\cos\theta_t} = \frac{tg(\theta_i-\theta_t)}{tg(\theta_i+\theta_t)}$$

$$t_\parallel = \frac{2Z_2\cos\theta_i}{Z_1\cos\theta_i + Z_2\cos\theta_t} = \frac{2n_1\cos\theta_i}{n_2\cos\theta_i + n_1\cos\theta_t} = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i+\theta_t)\cos(\theta_i-\theta_t)}$$

$$I = \langle|\vec{S}|\rangle = \frac{1}{2Z}\left|\vec{E_0}\right|^2$$

$$\vec{E} = \vec{E_0}\, e^{i(\omega t - \vec{k}\vec{r})} = \vec{E_0}\cos(\omega t - \vec{k}\vec{r})$$

__Factor de reflexie și factor de transmisie__

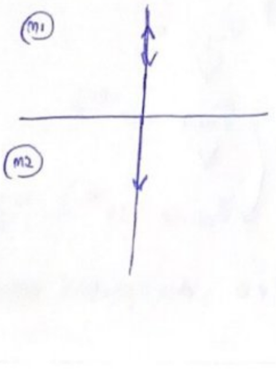
$$R = \frac{\text{fluxul de energie reflectată}}{\text{fluxul de energie incidentă}} = \frac{n_1\cos\theta_i\, E_{0r}^2}{n_1\cos\theta_i\, E_{0i}^2} = r^2$$

$$T = \frac{\text{—"— transmisă}}{\text{—"— incidentă}} = \frac{n_2\cos\theta_t}{n_1\cos\theta_i}\, t^2$$

$R + T = 1$ în absența aborbției

incidență normală aer–sticlă ($n_{aer}^{n_1} \cong 1$, $n_{sticlă}^{n_2} = 1{,}5$)

$$\left(n = \frac{c}{c_n}\right)$$



$$r_\perp = \frac{n_1 - n_2}{n_1 + n_2} = \frac{-0{,}5}{2{,}5} = -0{,}2$$

$R = 0{,}04$

$$r_\parallel = \frac{n_2 - n_1}{n_2 + n_1} = +0{,}2 \Rightarrow R = 0{,}04$$

$R = 0{,}04$ (4% din en. undei incidente se reflectă)

$t_\perp = \frac{2n_1}{n_1+n_2} = \frac{2}{2,5} = 0,8$

$t_\parallel = \frac{2n_1}{n_2+n_1} = 0,8$

$\Big\} \Rightarrow T = \frac{1,5}{1} \cdot 0,64 = 0,96$ (96 % din en. undei incidente se transmite

$R + T = 1$

<u>sticlă - aer</u>

$r_\perp = 0,2$

$r_\parallel = -0,2$

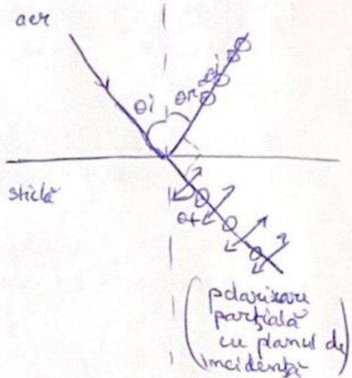
$\Big\} \Rightarrow R = 0,04$

$t_\perp = \frac{3}{2,5} = 1,2$

$t_\parallel = 1,2$

$\Big\} \Rightarrow T = \frac{1}{1,5} \cdot 1,44 = 0,96$

Incidență Brewster aer - sticlă ($n_{aer} \simeq 1$, $n_{sticlă} = 1,5$) (n_1, n_2)



$\theta_i = \theta_B$ (unghi Brewster) $\Downarrow$

reflexie polarizată $\perp$

$r_\parallel = \frac{tg(\theta_i - \theta_t)^{\neq 0}}{tg(\theta_i + \theta_t)} = 0 \Rightarrow tg(\theta_i + \theta_t) = \infty$, $\theta_i + \theta_t = \frac{\pi}{2}$

$R_\perp = r_\perp^2 = \left(\frac{n_1\cos\theta_B - n_2\cos\theta_t}{n_1\cos\theta_B + n_2\cos\theta_t}\right)^2 = \left(\frac{\cos 56° - 1,5\sin 56°}{\cos(56°) + 1,5\sin(56°)}\right)^2$

$n_1 \sin\theta_B = n_2 \sin\theta_t$

$n_1 \sin\theta_B = n_2 \cos\theta_B$

$tg\,\theta_B = \frac{n_2}{n_1}$

$\theta_B = arctg\left(\frac{n_2}{n_1}\right) \simeq 56°$

$R_\perp = \left(\frac{-0,62}{1,8}\right)^2 = (-0,34)^2 = 0,13$

$R_\parallel = r_\parallel^2 = 0$

Componenta $\perp$ se reflectă în procent de 13 % / 0 % $\Big| \Rightarrow$

$\Rightarrow$ reflexie polarizată $\perp$

$\boxed{T_\parallel = 1}$ $\Rightarrow$ Temă

$\boxed{T_\perp \simeq 0,87}$

<u>Unde electromagnetice în medii conductoare</u>

$\sigma_{Ag} = 6,3 \cdot 10^7 \frac{S}{m}$

$\sigma_{Cu} \simeq 5,7 \cdot 10^7 \frac{S}{m}$

$\sigma_{Au} = 4,5 \cdot 10^7 \frac{S}{m}$

$\sigma_{Al} \simeq 3,5 \cdot 10^7 \frac{S}{m}$

$\sigma_{Fe} \simeq 1 \cdot 10^7 \frac{S}{m}$

$\Bigg\}$ ~~conductivitate~~ conductoare

$\sigma_{apă} =$

$\vee$

$\sigma_{aer} = 10^{-15} \frac{S}{m}$

$\vee$

$\sigma_{sticlă} = 10^{-11} - 10^{-15} \frac{S}{m}$

$\Bigg\}$ izolatori

În conductori nu avem sarcini libere $\rho \neq 0$, dar putem avea curenți

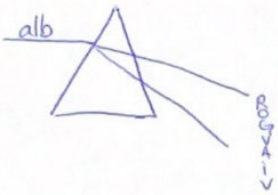
Fizică Curs 15

Teoria clasică a dispersiei

Se numește dispersie variația cu frecvența (lungimea de undă) a parametrilor de material.

$E_f = h\nu$ - energia unui foton

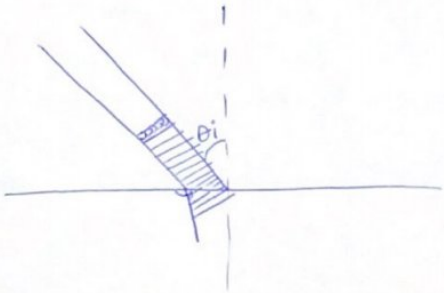
$$E_f = h\nu \stackrel{vid}{=} \frac{hc}{\lambda_0} \stackrel{mediu}{=} \frac{hcn}{\lambda_{mediu}} = \frac{hc}{\underbrace{n\lambda_{mediu}}_{\lambda_0}}$$



$\lambda_{vis} \in (400, 750)$ nm
(vid) VIAVGOR

$\nu_{vis} = \frac{c}{\lambda} \in (400, 750)$ THz
ROGVAIV

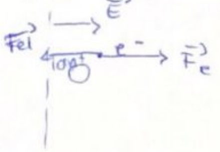
$$\frac{3\cdot 10^8 \text{ m/s}}{4\cdot 10^{-7} \text{ m}} = 0{,}75\cdot 10^{15} \text{ Hz} = 7{,}5\cdot 10^{14} \text{ Hz} = 7500\cdot 10^{11} \text{ Hz}$$



Modelul Lorentz pentru fenomenul de dispersie

Electronii formează un fluid continuu.

Acest model este bun în explicarea fenomenelor de la nivel optic, pt. că într-un volum mediu de λ^3 avem aprox. 10^8 atomi.



Forța Lorentz

Parte electrică

$\vec{F_e} = q\cdot\vec{E}$

Parte magnetică

$\vec{F}_{Lm} = q\vec{v} \times \vec{B}$

General

$$\vec{F_L} = q(\vec{E} + \vec{v}\times\vec{B})$$

$$\vec{E}(\vec{r},\vec{t}) = \vec{E_0}\, e^{i(\omega t - \vec{k}\vec{r})} \simeq \vec{E_0}\, e^{i\omega t}$$

$$\vec{F_e} = e\vec{E_0}\, e^{i\omega t}$$

$$m\ddot{x} + \Gamma\dot{x} + Kx = F_0 e^{i\omega t} \quad , F_0 = eE_0 \qquad \gamma = \frac{\Gamma}{m} \quad \left(\text{La osc. } \gamma = \frac{\Gamma}{2m}\right)$$

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = \frac{eE_0 e^{i\omega t}}{m} \qquad (e = 1{,}6\cdot 10^{-19}\, C)$$

$$x(t) = x_0 e^{i\omega t}$$

$$-x_0\omega^2 e^{i\omega t} + 2i\gamma\omega x_0 e^{i\omega t} + \omega_0^2 x_0 e^{i\omega t} = \frac{eE_0}{m} e^{i\omega t}$$

(+ −)

⇓

Polarizare

$$P = \varepsilon_0(\varepsilon_r - 1)E$$

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$$

$$\vec{D} = \varepsilon\vec{E} + \varepsilon_0\varepsilon_r\vec{E}$$

$$P = D - \varepsilon_0 E = \varepsilon_0(\varepsilon_r - 1)E$$

La nivelul microscopic polarizarea electrică = densitatea de dipol electric (moment dipolar)

$$p = ex \qquad P = N\cdot ex$$

$$\varepsilon_0(\tilde{\varepsilon}_r - 1)\cancel{E_0 e^{i\omega t}} = -Ne\left(-\frac{e}{m}\frac{\cancel{E_0 e^{i\omega t}}}{\omega_0^2 - \omega^2 + i\gamma\omega}\right)$$

$$\tilde{\varepsilon}_r = 1 + \underbrace{\frac{Ne^2}{m(\omega_0^2 - \omega^2 + i\gamma\omega)}}_{\substack{\text{polarizabilitate} \\ (\alpha)}}$$

$$\tilde{\varepsilon}_r = 1 - \chi_e \qquad \tilde{n} = \sqrt{\tilde{\varepsilon}_r} = \sqrt{1 - \tilde{\chi}_e} = 1 - \frac{\tilde{\chi}_e}{2} = n_r + i\underbrace{K}_{\text{absorbție}}$$

$$= 1 + N\alpha$$

$$\tilde{n} = n_r + iK$$

$$\tilde{\chi}_e = \frac{Ne^2}{m\varepsilon_0(\omega_0^2 - \omega^2 + i\gamma\omega)}$$

$$\tilde{n} = 1 + \frac{\tilde{\chi}_e}{2}$$

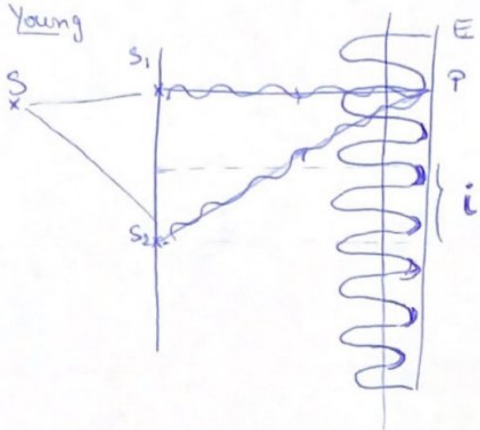
$$n_r = 1 + \frac{Ne^2(\omega_0^2 - \omega^2)}{2m\varepsilon_0[(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2]} = n_r(\omega)$$

$$K = \frac{Ne^2}{2m\varepsilon_0}\frac{\gamma\omega}{[(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2]}$$

Fizică curs 16

Interferența = compunerea a 2 sau mai multe unde

Young



$\vec{E} = \vec{E_1} + \vec{E_2}$

$\vec{E} = \vec{E_0}\, e^{i(\omega t - \vec{k}\vec{r})}$

$$I_{tot} = \frac{1}{2Z}|\vec{E}|^2 = \frac{1}{2Z}\vec{E}\cdot\vec{E}^* = \frac{1}{2Z}(\vec{E_1}+\vec{E_2})(\vec{E_1}^*+\vec{E_2}^*) = \underbrace{\frac{1}{2Z}\vec{E_1}^2}_{I_1} + \underbrace{\frac{1}{2Z}\vec{E_2}^2}_{I_2} + \underbrace{\frac{1}{2Z}(\vec{E_1}\cdot\vec{E_2}^* + \vec{E_2}\cdot\vec{E_1}^*)}_{I_{12}}$$

I_{12} - termenul de interferență

$I_{12} \neq 0 \Rightarrow$ unde coerente

$I_{tot} = I_1 + I_2 + I_{12}$

Condiții de coerență

- aceeași frecvență $\omega_1 = \omega_2 = \omega$
- defazaj constant în timp
- ~~polarizarea undelor să nu fie~~
- cele 2 unde care interferă nu trebuie să fie polarizate perpendicular

$\vec{E_1} = \vec{E}_{01}\, e^{i(\omega t - \vec{k_1}\vec{r_1} + \phi_2)}$

$\vec{E_2} = \vec{E}_{02}\, e^{i(\omega t - \vec{k_2}\vec{r_2} + \phi_1)}$

$$\langle I_{tot} \rangle = \langle I_1 \rangle + \langle I_2 \rangle + \frac{1}{2Z} E_{01} E_{02}\left[e^{i(-\vec{k_1}\vec{r_1} + \phi_1 + \vec{k_2}\vec{r_2} - \phi_2)} + e^{-i(-\vec{k_1}\vec{r_1} + \phi_1 + \vec{k_2}\vec{r_2} - \phi_2)}\right]$$

$$I_{12} = \frac{1}{2Z}\, 2\sqrt{|E_{01}|^2 |E_{02}|^2}\, \cos\left[\underbrace{|\vec{k_2}\vec{r_2} - \vec{k_1}\vec{r_1}|}_{n\Delta r} + \phi_1 - \phi_2\right]$$

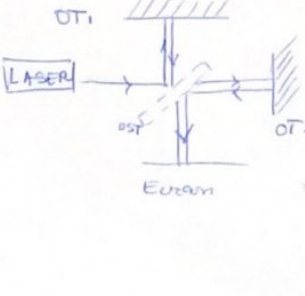
$n\Delta r$ drum optic

$$\cos\alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2}$$

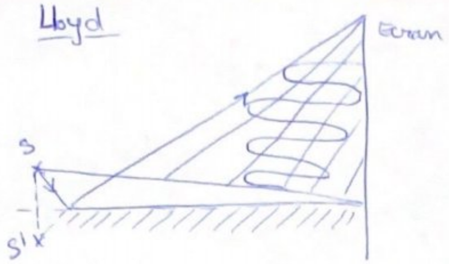
$m \in N$

$k\Delta r = \frac{2\pi}{\lambda_0} n \Delta r = 2m\pi$ pt. maxime

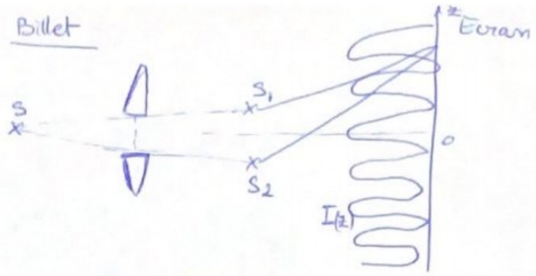
Michelson



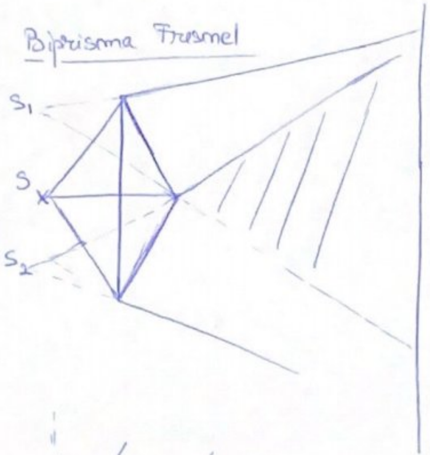
Lloyd



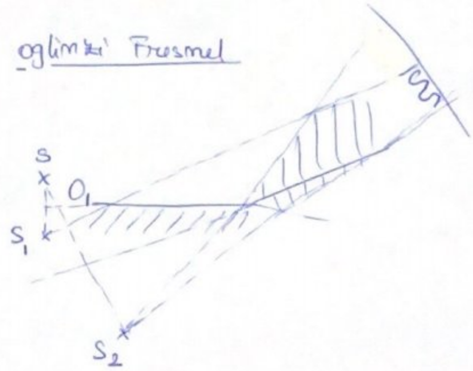
Billet

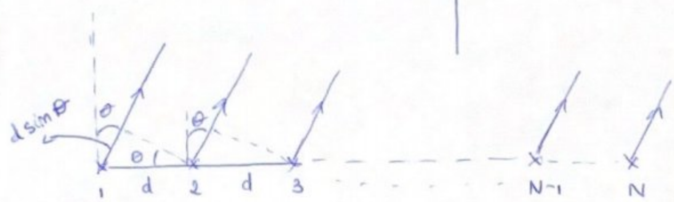


Biprisma Fresnel



oglinzi Fresnel





$$E_j = E_{0j}\, e^{i(\omega t - k r_j)} \quad , r_2 = r_{(1)} - d\sin\theta$$

$$E_{tot}(\theta) = \sum_{j=1}^{m} E_j = E_{01}\, e^{i(\omega t - kr)} \left[1 + e^{ikd\sin\theta} + e^{2ikd\sin\theta} + \dots + e^{i(N-1)d\sin\theta} \right] \Rightarrow$$

$$\Rightarrow E_{tot}(\theta) = E_0 e^{i(\omega t - kr)} \frac{1 - e^{iNkd\sin\theta}}{1 - e^{idk\sin\theta}} = E_0 e^{i(\omega t - kr)} \frac{e^{ikNd\sin\theta} - 1}{e^{ikd\sin\theta} - 1} =$$

$$= E_0 e^{i(\omega t - kr)} \frac{e^{\frac{ikNd\sin\theta}{2}}}{e^{\frac{ikd\sin\theta}{2}}} \frac{\sin\frac{kNd\sin\theta}{2}}{\sin\frac{kd\sin\theta}{2}}$$

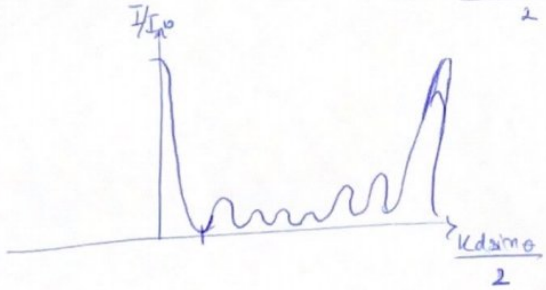
$$I_{tot} = \frac{1}{2Z} E_{tot} \cdot E_{tot}^*$$

$$I_{tot}(\theta) = I_0(0) \frac{\sin^2\frac{Nkd\sin\theta}{2}}{\sin^2\frac{kd\sin\theta}{2}}$$

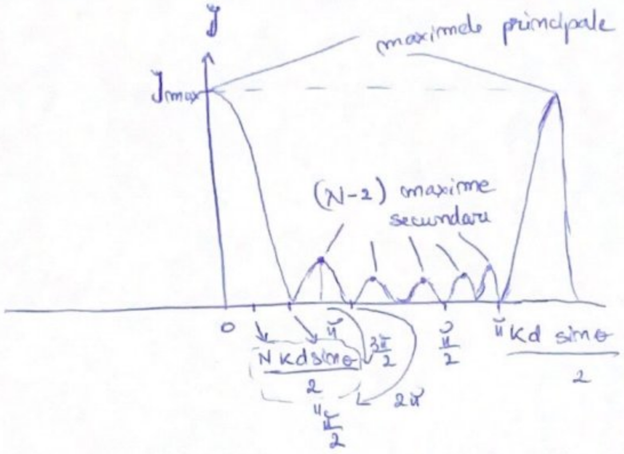
la $\theta \to 0$ $\quad I_{tot}(0) = I_0 \cdot N^2 \frac{\sin^2\frac{Nkd\sin\theta}{2}}{\sin^2\frac{kd\sin\theta}{2}}$

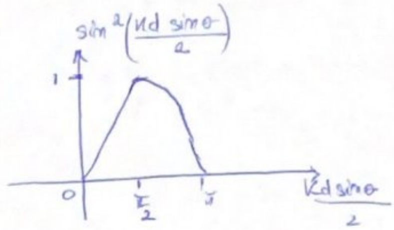
$$\sin^2\frac{Nkd\sin\theta}{2} = \sin\pi = 0$$

$$\sin\theta = \frac{2\pi}{Nkd}$$



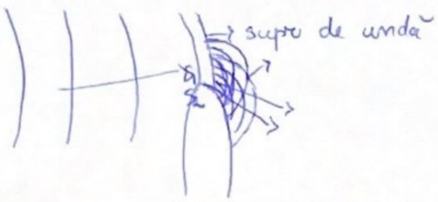
Fizică Curs 17





Difracția

Devierea unei unde de la propagarea rectilinie



Pt. unde armonice sferice

$$E(r,t) = \frac{E_0}{r} e^{i(\omega t - kr)} \qquad \left(J = \frac{1}{2Z} |\vec{E}^2| \right)$$

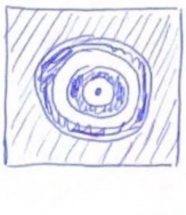
$$J \sim E^2 \qquad J = \frac{J_0}{r^2}$$

$$- r_0) \quad r)$$

Difracție:

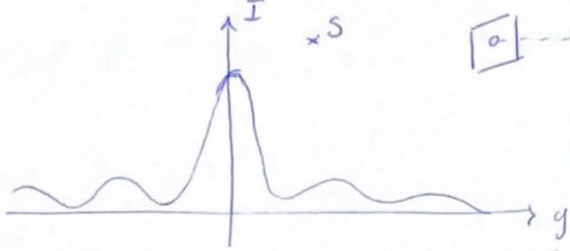
1) în cazul în care sursa și obs. sunt situate aproape de obstacol (near field diffraction) → difracție Fresnel

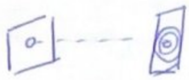
observatorul va vedea în zona obstacolului fâșii circulare luminoase și întunecate.

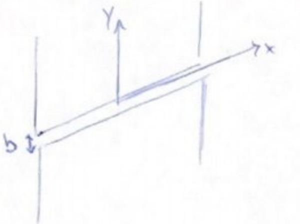


2. Sursa și obs. sunt departe de obstacol (fantă, obiect)
(far field diffraction) -> difracție Fraunhofer

Pe ecran va apărea o figură staționară cu un maxim principal și (situate simetric) maxime și minime secundare.





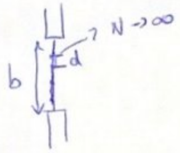


$$E(\theta) = E(0)\,\frac{\sin\frac{kb\sin\theta}{2}}{\frac{kb\sin\theta}{2}}$$

$$I(\theta) = I_0\,\frac{\sin^2\frac{kb\sin\theta}{2}}{\left(\frac{kb\sin\theta}{2}\right)^2}$$

Interf. N-surse

$$I = I_0\,\frac{\sin^2\left(\frac{Nkd\sin\theta}{2}\right)}{\sin^2\left(\frac{kd\sin\theta}{2}\right)}$$



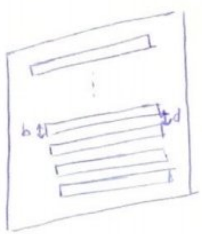
$Nd = b$

$$I = I_0\,\frac{\sin^2\left(\frac{kb\sin\theta}{2}\right)}{\sin^2\left(\frac{kb\sin\theta}{2N}\right)}$$

$$\sin\underbrace{\frac{kb\sin\theta}{2N}}_{f.\ mică} \simeq \frac{kb\sin\theta}{2N}$$

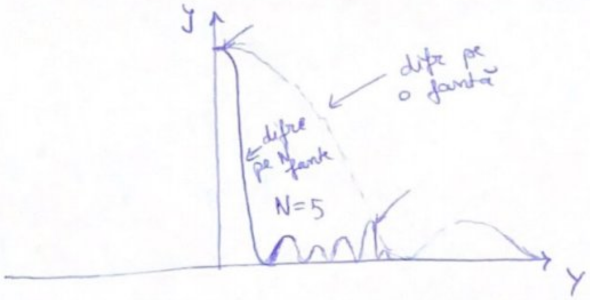
$$I = I_0\,\frac{\sin^2\left(\frac{kb\sin\theta}{2}\right)}{\sin^2\left(\frac{kb\sin\theta}{2N}\right)} \underset{N\to\infty}{\simeq} \underbrace{I_0 N^2}_{I_0}\,\frac{\sin^2\left(\frac{kb\sin\theta}{2}\right)}{\left(\frac{kb\sin\theta}{2}\right)^2}$$

$$\boxed{I \simeq I_0\,\frac{\sin^2\left(\frac{kb\sin\theta}{2}\right)}{\left(\frac{kb\sin\theta}{2}\right)^2}}$$

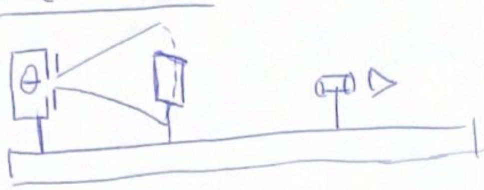


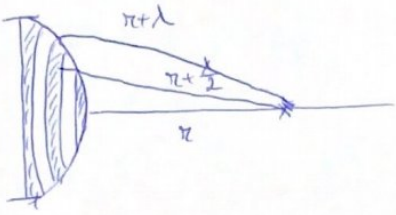
Difracția pe o rețea de fante (b - lărgimea unei fante, d - constanta rețelei)

$$I = I_0 \cdot \frac{\sin^2\left(\frac{kb\sin\theta}{2}\right)}{\left(\frac{kb\sin\theta}{2}\right)^2}\;\frac{\sin^2\left(\frac{Nkd\sin\theta}{2}\right)}{\sin^2\left(\frac{kd\sin\theta}{2}\right)}$$

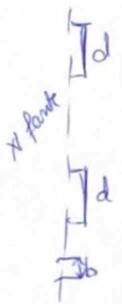

I
difr pe o fantă
difr pe N fante
N=5
y

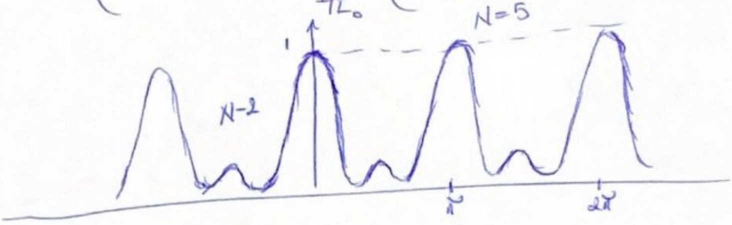
Fizică curs 18





$$I(\theta)=I_0\,\frac{\sin^2\frac{kb\sin\theta}{2}}{\left(\frac{kb\sin\theta}{2}\right)^2}=I_0\,\frac{\sin^2\left(\frac{\pi b\sin\theta}{\lambda}\right)}{\left(\frac{\pi b\sin\theta}{\lambda}\right)^2}$$

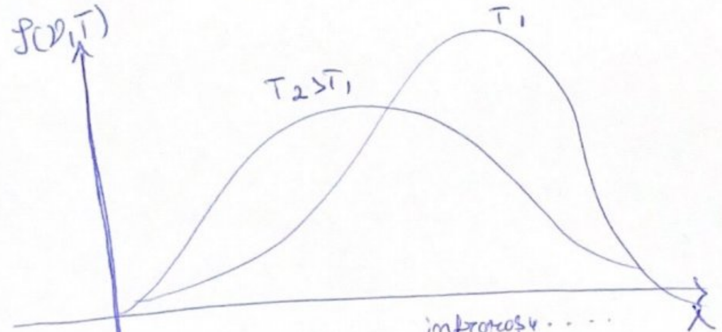




$$I=N^2 I_0;\quad \frac{\sin^2\left(\frac{Nkd\sin\theta}{2}\right)}{\sin^2\left(\frac{kd\sin\theta}{2}\right)}$$

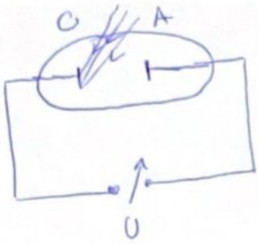
Rețea de difr

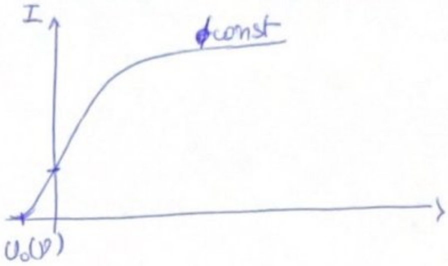
$$I=I_0\,\frac{\sin^2\frac{kb\sin\theta}{2}}{\left(\frac{kb\sin\theta}{2}\right)^2}\cdot\frac{\sin^2\left(\frac{Nkd\sin\theta}{2}\right)}{\sin^2\left(\frac{kd\sin\theta}{2}\right)}$$



$\rho^e(\nu,T)$ = densitate spectrală de energie ⟺

⟺ energie pe unit. de volum pe unit. de frecv.





$\vec{p} = \hbar \vec{k}$

h = const lui Planck

$\hbar$ = — h ———— redusă $= \frac{h}{2\pi}$

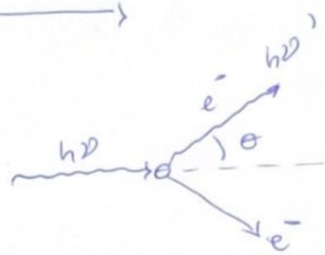
$p = \hbar k = \frac{h}{2\pi} k = h \cdot \frac{1}{\lambda}$

$\lambda = \frac{c}{\nu}$

$\langle u \rangle = c \langle g \rangle$

$\langle g \rangle = \frac{\langle u \rangle}{c}$

pt. foton $p = \frac{E}{c}$

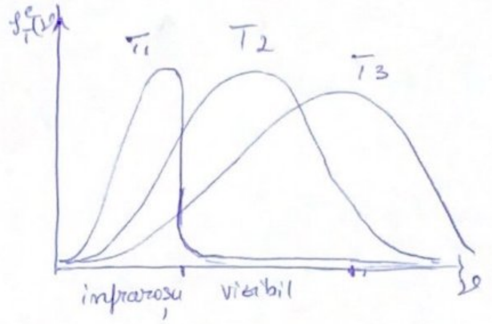


$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

Fizică Curs 19

$\rho_T^e(\nu) \stackrel{def}{=} \dfrac{\text{en. emisă}}{(\text{unit de volum})(\text{unit. de frecv})}$

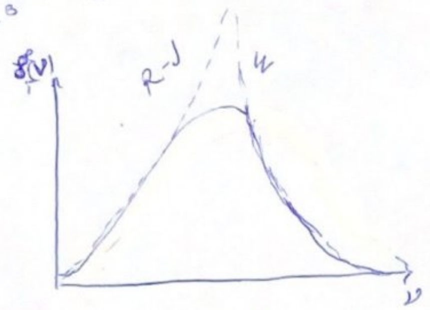
densit. spectrală de energie



$T_1 < T_2 < T_3$

$\rho^{R-J} = \dfrac{8\pi\nu^2}{c^3} k_B T$

Rayleigh-Jeans



$\rho_T^W(\nu) \approx \dfrac{8\pi\nu^2}{c^3} \cdot e^{-\frac{h\nu}{k_B T}}$

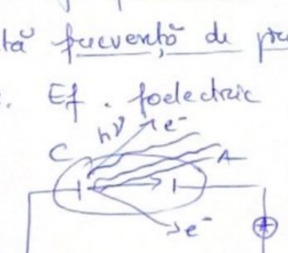
Wien

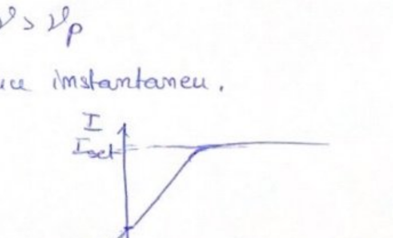
Plank: $\rho_T^{eP}(\nu) = \dfrac{8\pi\nu^2}{c^3} \dfrac{h\nu}{e^{\frac{h\nu}{k_B T}} - 1}$

(1905) Einstein

Legile efectului fotoelectric

1. Curentul fotoelectric de saturație este prop. cu fluxul radiației (luminoase) electromagnetice la o anumită frecvență.
2. En. cinetică maximă a electronilor emiși este prop. cu frecvența (ν) radiației incidente.
3. Ef. fotoelectric nu se produce decât dacă frecv. depășește o anumită frecv. numită frecvență de prag. $\nu > \nu_p$
4. Ef. fotoelectric se produce instantaneu.





$h\nu = L_{extracție} + E_{c\,max}$

ciocnire foton - e^-

$E_{tot\ in} = E_{tot\ finală} \Rightarrow \begin{cases} h\nu + m_0 c^2 = h\nu' + mc^2 \\ \frac{h\nu}{c}\vec{u_k} = \frac{h\nu'}{c}\vec{u_k'} + m\vec{v} \end{cases} \Rightarrow \Delta\lambda = \underbrace{\left(\frac{h}{m_0 c}\right)}_{\Lambda_{compton}}(1-\cos\theta)$

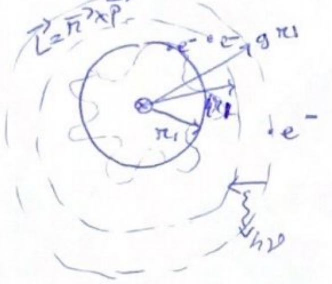
$h = 6,626 \cdot 10^{-34}\ J\cdot s$

$p_{foton} = \frac{h}{\lambda} = \frac{h\nu}{c}$

$E_f = p \cdot c$

$\nu = \frac{c}{\lambda}$

$\vec{L} = \vec{r} \times \vec{p}$



Modelul Bohr (H)

1. e^- se mişcă pe orbite circulare în jurul nucleului.
2. $L = n\hbar$ $\quad L = mrv$

   $\hbar = \frac{h}{2\pi}$
3. $h\nu = E_n - E_m$ $\quad e^-$ ~~nu poate~~ se află în atom în stări staționare cu en. discrete

$mv = p$

$p \cdot r = n\frac{h}{2\pi}$

$\left.\begin{array}{l} p = n\frac{h}{2\pi r} \\ p = \frac{h}{\lambda} \end{array}\right\} \Rightarrow 2\pi r = n\lambda$

Davisson - Germer

Relațiile de Broglie

$\exists$ o undă asociată fiecărei microparticule

$E = h\nu = \hbar\omega$

$p = \frac{h}{\lambda} = \hbar k$

$\vec{p} = \hbar\vec{k}$ → vect. de undă

$E = \sqrt{p^2c^2 + m_0^2c^4}$

$E = pc$ (foton)