TEXAS A&M UNIVERSITY
Engineering

Thesis Defense:
A Formal Analysis of Bitcoin Cash

Yu Shen

Outline

- Background
- Bitcoin Cash Backbone Protocol
- Comparison with Real World Network

Cryptocurrencies

#	Name	Price	24h	7d	Market Cap	Volume	Circulating Supply	Last 7 Days
1	Bitcoin BTC	$49,795.58	▲ 8.43%	▼ 8.50%	$924,610,894,312	$53,548,614,604 1,079,658 BTC	18,642,200 BTC	
2	Ethereum ETH	$1,581.71	▲ 9.77%	▼ 12.07%	$179,839,542,025	$23,978,480,535 15,316,682 ETH	114,875,712 ETH	
3	Cardano ADA	$1.30	▼ 2.16%	▲ 19.21%	$41,459,440,348	$10,000,495,100 7,706,300,647 ADA	31,948,309,441 ADA	
11	Bitcoin Cash BCH	$506.79	▲ 8.67%	▼ 19.97%	$9,365,354,777	$3,635,754,277 7,247,188 BCH	18,668,063 BCH	

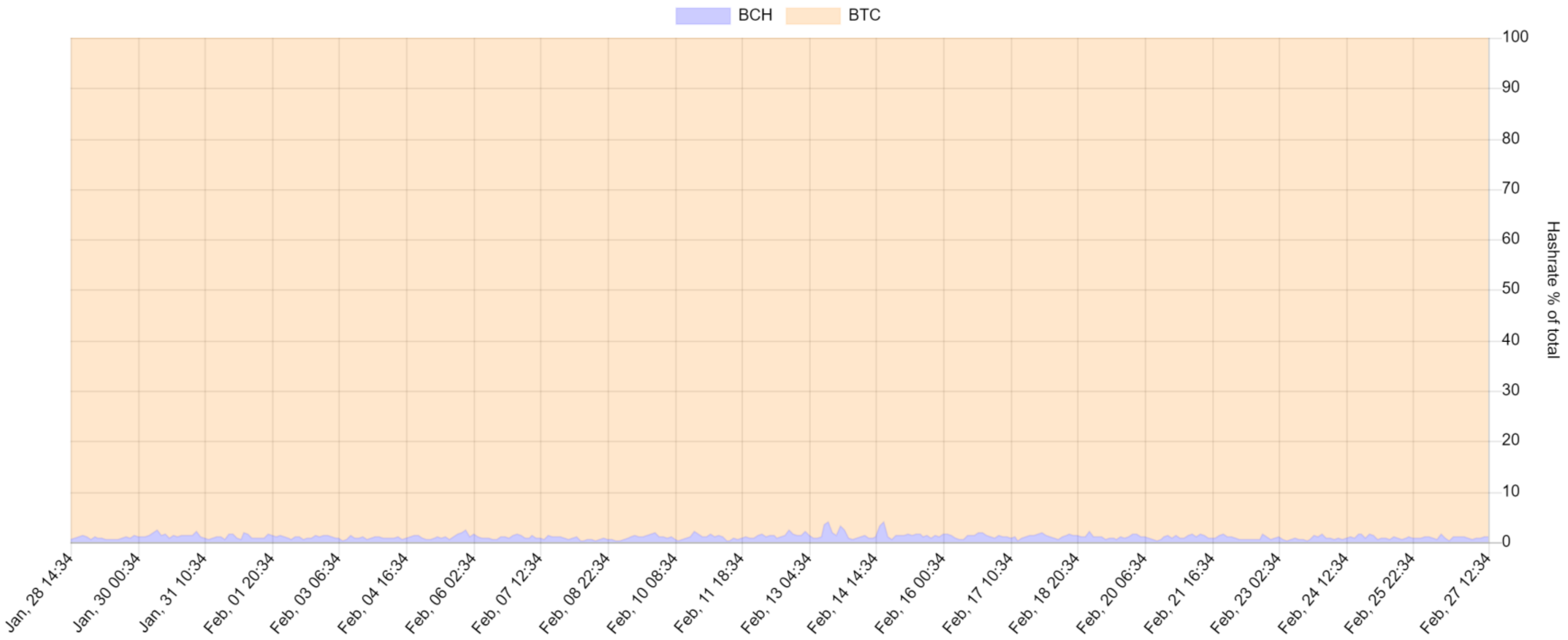
Basic Info about Bitcoin Cash

- A “hard fork” of Bitcoin.
- Was created on Aug. 1 2017.
- Split ratio 1:1.
- Motivation: accommodate an increasing count of transactions.

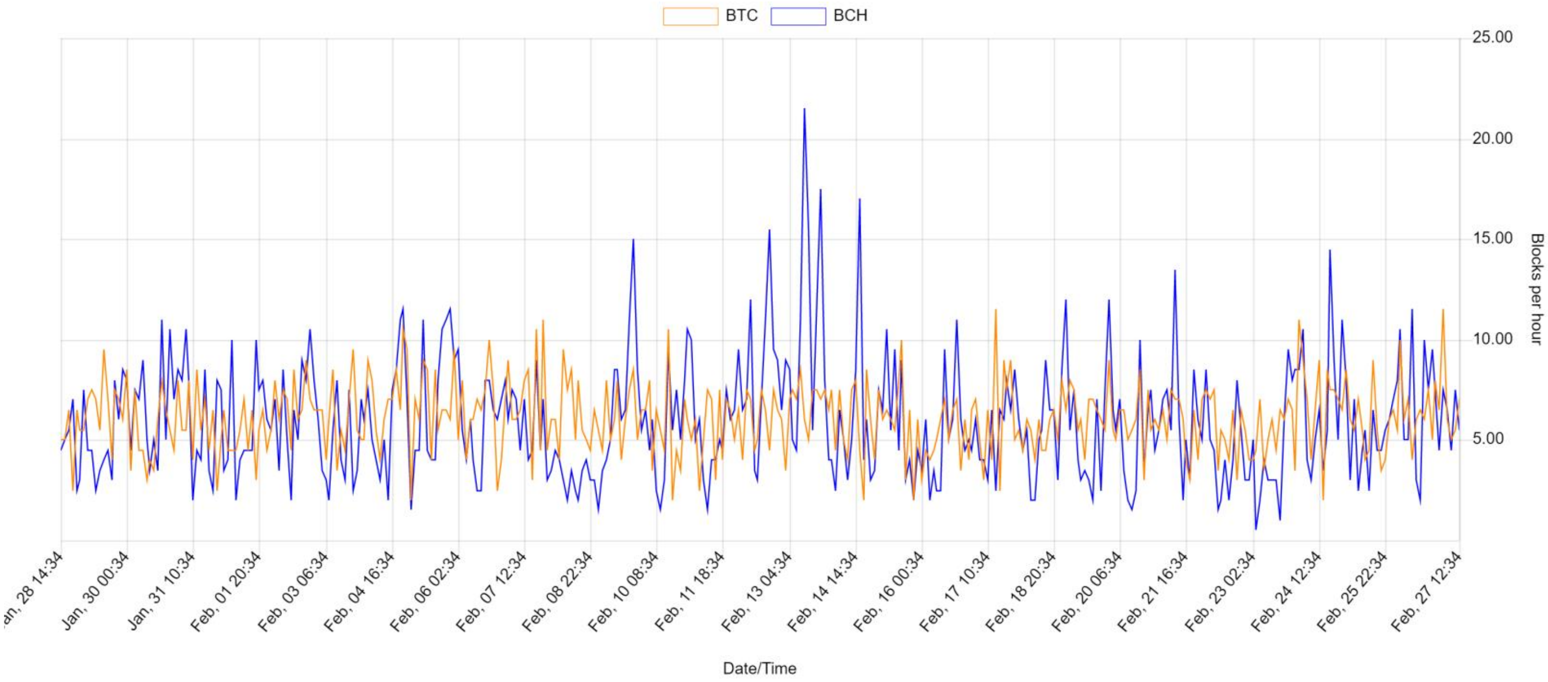
Bitcoin Cash vs. Bitcoin

	Bitcoin	Bitcoin Cash
Ledger Start	Jan 3 2009	Jan 3 2009, split at Aug 1 2017
Mining	Proof-of-Work(SHA-256)	
Block Size Limit	1MB -> 4MB	8MB -> 32MB
Issuance schedule	Initially 50 BTC(BCH) per block, halved every 210,000 blocks	
Block time	10 minutes	
Supply Limit	21,000,000 BTC(BCH)	
Target Recalculation	Every 2 weeks	Every block

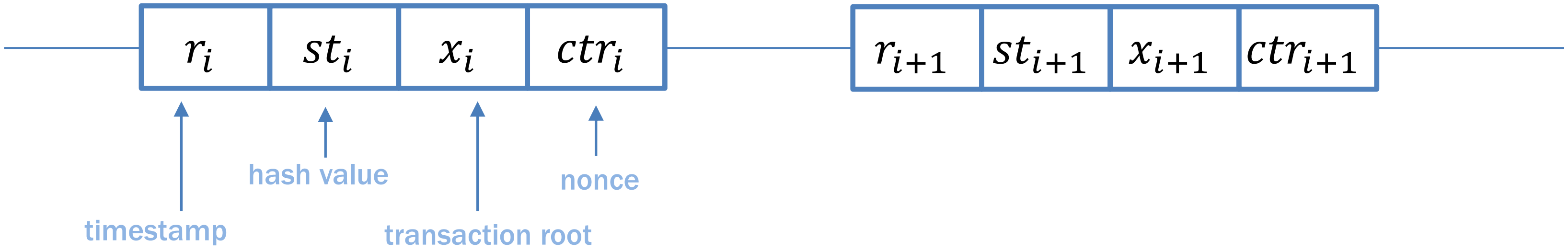
Relative Hashrate in Percentage of Total



Average Number of Blocks per Hour



Blockchain Data Structure



- A block $\langle r, st, x, ctr \rangle$ is valid if it has a small hash value, providing a proof-of-work:

$$H(r, st, x, ctr) < T.$$

- A chain is valid if all its blocks provide a proof-of-work and each block extends the previous one:

For each i, $st_{i+1} = H(r, st, x, ctr)$ and $r_{i+1} > r_i$.

Bitcoin Cash's Target Recalculation Function

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EMA	SMA	ASERT
Aug 1 2017 – Nov 13 2017	Nov 13 2017 – Nov 15 2020	Nov 15 2020 – present

- *Emergency Difficulty Adjustment* (EDA):
  - Bitcoin's DAA + decrease the mining difficulty of Bitcoin Cash by 20%, if the time difference between 6 successive blocks was greater than 12 hours.
- *Simple Moving Average* (SMA):
  - Adjusts the mining difficulty after each block; a moving window of last 144 blocks.
- *Absolutely Scheduled Exponentially Rising Targets* (ASERT)

Bitcoin's Target Recalculation Function

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- The target is recalculated every m blocks.
  - Bitcoin uses $m = 2016$ (approximately two weeks) and calls the period between two recalculation points an *epoch*.
  - If one want to extend the chain of length λm, first determines target T by the last m blocks.
- Informally, if the m blocks were calculated quickly, then increase difficulty (decrease T), otherwise decrease difficulty (increase T).
- Suppose the last m blocks were computed in Δ rounds for target T. If we want to have m blocks in every m/f rounds, set

$$T' = \frac{\Delta}{m/f} \cdot T \ (f = \text{block production rate }).$$

Bitcoin's Target Recalculation Function

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$$T' = \begin{cases} \frac{1}{\tau} \cdot T & if\ \frac{\Delta}{m/f} \cdot T < \frac{1}{\tau} \cdot T \\ \tau \cdot T & if\ \frac{\Delta}{m/f} \cdot T > \tau \cdot T \\ \frac{\Delta}{m/f} \cdot T & otherwise \end{cases}$$

- Bahack's difficulty raising attack:
  - The adversary builds the next epoch all by himself with fake timestamps, resulting in huge difficulty for then next epoch.
  - Works with constant probability.

Bitcoin Cash's Target Recalculation Function

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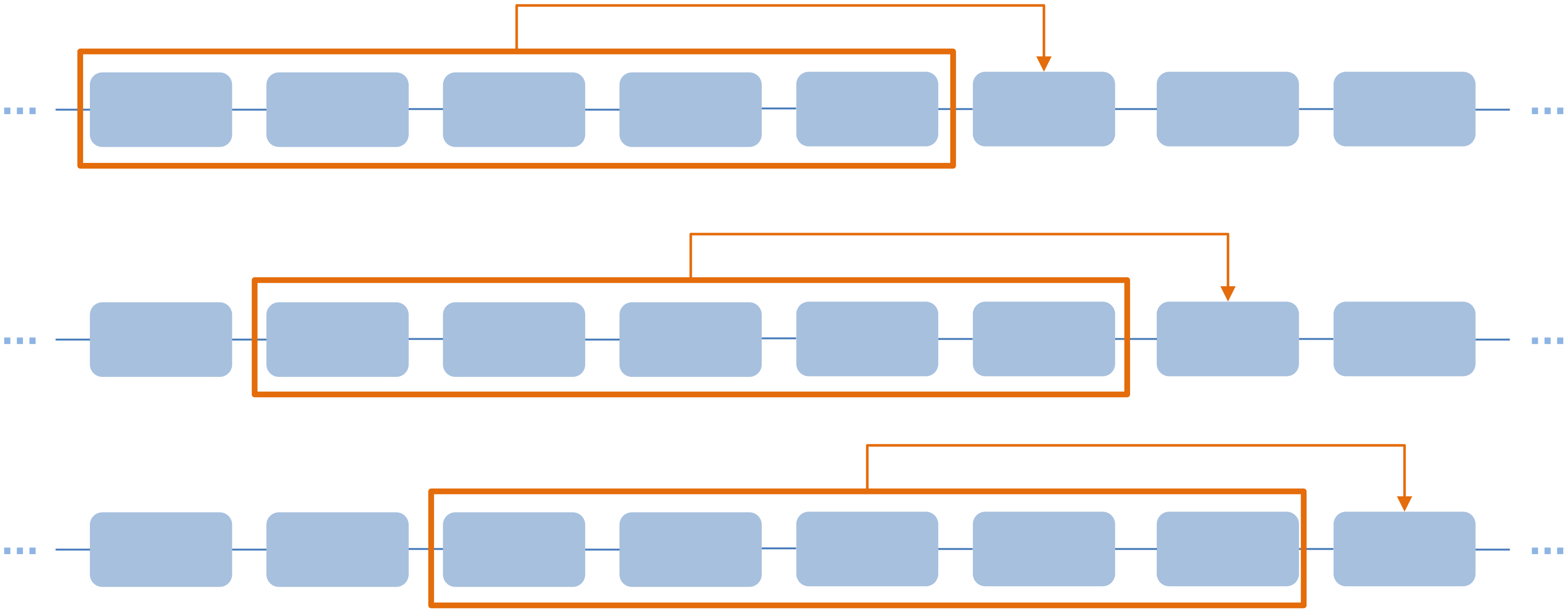
Bitcoin's Target Recalculation Function

$$T' = \begin{cases} \frac{1}{\tau} \cdot T^{avg} & if\ \frac{\Delta}{m/f} \cdot T^{avg} < \frac{1}{\tau} \cdot T^{avg} \\ \tau \cdot T^{avg} & if\ \frac{\Delta}{m/f} \cdot T^{avg} > \tau \cdot T^{avg} \\ \frac{\Delta}{m/f} \cdot T^{avg} & otherwise \end{cases}$$

- *Simple Moving Average* (SMA):
  - Adjusts the mining difficulty after each block
  - A sliding window of last 144 blocks (approximately 1 day).
  - Based on the average target of the 144 blocks.
  - (Epoch-like) m: length of the sliding window.

Bitcoin Cash's Target Recalculation Function

$$T' = \frac{\Delta}{m/f} \cdot T^{avg}$$



Bitcoin Cash's Target Recalculation Function

- *Absolutely Scheduled Exponentially Rising Targets* (ASERT):
  - Adjusts after each block.
  - Based on the comparison with the calibrated timestamp (the timestamp this block should have if it has the generating rate exactly f).
  - Intrinsically prevents the raising difficulty attack.
  - m: smoothing factor (288 in use, approximately 2 days).

- For v-th block with timestamp r_v, its target is calculated by

$$T' = T_0 \cdot 2^{\left(\frac{r_v - (v-1)/f}{m/f}\right)}$$

Bitcoin Cash's Target Recalculation Function

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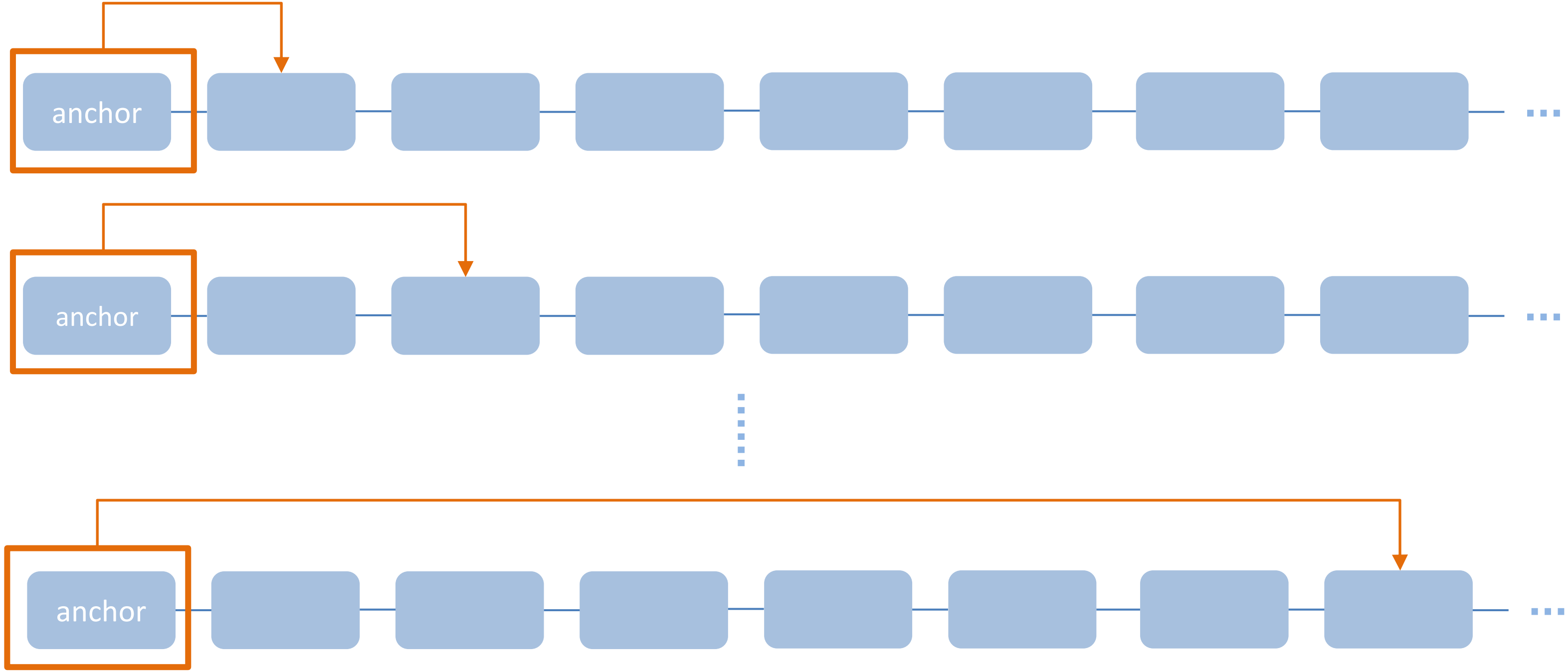

Outline

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- Comparison with Real World Network

This Work

- A follow-up work of the Bitcoin Backbone Protocol ([GKL15, GKL17]).
- First formal analysis of Bitcoin Cash's target recalculation functions.
- New analysis methodology for target recalculation functions in the dynamic setting.

Model

- Time is divided into rounds.
- Bounded Delay Network: Δ round delay.
- A total number of parties n and an adversary that controls t parties
  - Honest parties act independently.
  - Parties controlled by the adversary collaborate.
- Parties communicate by diffusing a message.
  - The adversary can inject messages into a party's incoming message.
  - The adversary can reorder a party's incoming messages.
- Anonymous setting: parties cannot associate a message to a sender.
- Hash function is modeled as a random oracle (RO).

Respecting Environment

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Static | **Permissionless** | **Dynamic**

- It is impossible to achieve desired properties in permissinless setting.
  - If the number of parties increases rapidly, it would generate too many forks (*Consistency* hurts).
  - If the number of parties decreases rapidly, transactions sent to the ledger cannot be confirmed (*Liveness* breaks).
- A dynamic environment: the fluctuation of number of parties is bounded.

Respecting Environment

Static

Permissionless

Dynamic

Definition 1. For $\gamma, \Gamma \in \mathbb{R}^+$, we call a sequence $(n_r)_{r \in \mathbb{N}}$ $(\langle\gamma, \sigma\rangle, \langle\Gamma, \Sigma\rangle)$-*respecting* if it holds that in a sequence of rounds S with $|S| \leq \Sigma$ rounds, $\max_{r \in S} n_r \leq \Gamma \cdot \min_{r \in S} n_r$ and for any consecutive sub-sequence rounds $S' \preccurlyeq S$ with $|S'| \leq \sigma$ rounds, $\max_{r \in S'} n_r \leq \gamma \cdot \min_{r \in S'} n_r$.

- The environment Z can increase or decrease the total number of parties at the beginning of each round, but subject to a constraint.
  - Long term fluctuation: $\forall S, |S| = \Sigma, \max_{r \in S} n_r \leq \Gamma \cdot \min_{r \in S} n_r$.
  - Short term fluctuation: $\forall S', |S'| = \sigma, \max_{r \in S} n_r \leq \gamma \cdot \min_{r \in S} n_r$.
  - Consistent with the recalculation function that adjusts difficulty for each block.

Blockchain Properties

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Common Prefix

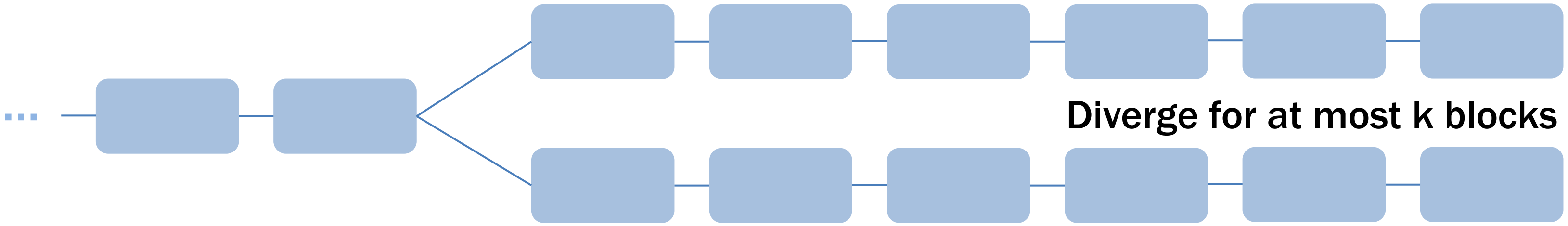
- With parameter $k \in N$, at any round of the execution, if a chain C belongs to an honest party, then for any valid chain C' in the same round such that either diff(C') $>$ diff(C), or diff(C') $=$ diff(C) and head(C')was computed no later than head(C), it holds that $C^{\lceil k} \preccurlyeq C'$ and $C'^{\lceil k} \preccurlyeq C$.

Chain Quality

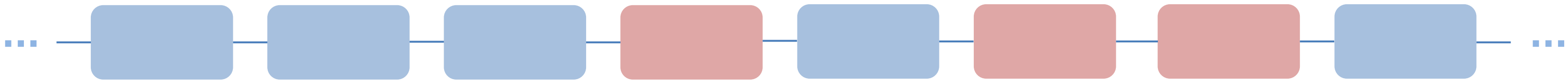
- With parameters $\mu \in R$ and $\ell \in N$, for any party P with chain C in $view_{\Pi,A,Z}$, and any segment of that chain of difficulty d such that the timestamp of the first block of the segment is at least ℓ smaller than the timestamp of the last block, the blocks the adversary has contributed in the segment have a total difficulty that is at most $\mu \cdot d$.

Blockchain Properties

Common Prefix:



Chain Quality:



The percentage of blocks mined by the adversary in the stable blockchain is bounded.

Ledger Properties

A *robust transaction ledger* must satisfy:

Consistency

- For any two honest parties P_1, P_2, reporting $\mathcal{L}_1, \mathcal{L}_2$ at rounds $r_1 \leq r_2$, resp., it holds that the settled part of $\mathcal{L}_1$ is a prefix of $\mathcal{L}_2$.

Liveness

- If a transaction tx is provided to all honest parties for u consecutive rounds, then it holds that for any player P, tx will be in $\mathcal{L}$.

Bitcoin Cash Backbone Protocol

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- In each round r, each party with a chain C_0 performs the following:
  - Receive from the network chains $C_1, C_2, \ldots$
  - Choose the first heaviest chain C among the valid ones in $\{C_0, C_1, C_2, \ldots\}$ (Heaviest means the largest accumulated difficulty).
  - Try to extend the heaviest chain C (Modeled as a Bernoulli trial with a probability of success that depends on the target T).
  - Suppose its last block is the i-th one and equal to (r_i, st_i, x_i, ctr_i) with $st = H(r_i, st_i, x_i, ctr_i)$. Find a ctr such that $H(r, st, x, ctr) < T$. If succeed, let $C \leftarrow C \parallel (r, st, x, ctr)$
  - If $C \neq C_0$(miner extends the chain or switch to another heavier chain), diffuse the new chain C.

Bitcoin Cash Backbone Protocol

Algorithm 4 The Bitcoin Cash backbone protocol in the dynamic setting at round "round" on local state $(st, \mathcal{C})$ parameterized by the *input contribution function* $I(\cdot)$ and the *chain reading function* $R(\cdot)$. The ready flag is **false** if and only if the party was inactive in the previous round.

if ready = **true then**
    $\textsc{Diffuse}(\text{'ready'})$
    $\tilde{\mathcal{C}} \leftarrow \mathsf{maxvalid}(\mathcal{C}$ all chains $\mathcal{C}'$ found in $\textsc{Receive}())$
    $\langle st, x\rangle \leftarrow I(st, \tilde{\mathcal{C}}, \mathtt{round}, \textsc{Input}(), \textsc{Receive}())$
    $\mathcal{C}_{\mathsf{new}} \leftarrow \mathsf{pow}(\mathtt{round}, x, \tilde{\mathcal{C}})$
    if $(\mathcal{C} \neq \mathcal{C}_{\mathsf{new}}) \vee (\text{'}\mathbf{Join}\text{'} \in \textsc{Receive}())$ **then**
        $\mathcal{C} \leftarrow \mathcal{C}_{\mathsf{new}}$
        $\textsc{Diffuse}(\mathcal{C})$ ▷ chain is diffused when it is updated or when someone wants to join.
    end if
    if $\textsc{Input}()$ contains $\textsc{Read}$ **then**
        write $R(\mathbf{x}_{\mathcal{C}})$ to $\textsc{Output}()$
        $\textsc{Diffuse}(\mathsf{RoundComplete})$
    end if
else
    ready $\leftarrow$ **true**
    $\textsc{Diffuse}(\mathsf{Join}, \mathsf{RoundComplete})$
end if

Summary of Parameters

- δ: Advantage of honest parties, $\forall r(t_r/h_r < 1-\delta)$.
- $\gamma, \sigma, \Gamma, \Sigma$: Determine how the number of parties fluctuates across rounds in a period (cf. Definition 1 and Fact 1).
- f: Probability that at least one honest party succeeds generating a PoW in a round assuming h_0 parties and target T_0 (the protocol's initialization parameters).
- m: Smoothing factor (cf. Definition 4).
- τ: Parameter that regulates the target that the adversary could query the PoW with.
- ϵ: Quality of concentration of random variables (cf. Definition 7).
- κ: The length of the hash function output.
- φ: Related to the properties of the protocol.
- L: The total number of rounds in the execution of the protocol.

$$\varphi = \Theta(m) = polylog(\kappa)$$

Proof Roadmap

- Assuming the execution begins with good initial parameters (the initial block production rate is very close to f).
- Consider a sliding window of $\Theta(m)$ rounds.
- If a chain is C is adopted by an honest party, then C satisfies the following with overwhelming probability (in κ):
  - Is never abandoned by honest parties for $\Omega(m/f)$ rounds,
  - Is $O(m/f)$-accurate,
  - Has “good” recalculation points,
  - Has blocks with good targets.

Proof Roadmap

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- Accuracy: no adversarial blocks are present with a timestamp that deviates too much from its real creation time.

Definition 6 (Accuracy). A block created at round u is *accurate* if it has a timestamp v such that $|u - v| \leq \ell + 2\Delta$. A chain is *accurate* if all its blocks are accurate. A chain is *stale*, if for some $u \geq \ell + 2\Delta$, it does not contain an honest block with timestamp $v \geq u - \ell - 2\Delta$.

- Goodness: for a round r, the probability of block generation given current target T_r and number of miners n_r, is very close to the initial block generation rate f.

Definition 5 (Goodness). Round r is *good* if $f/2\gamma(2-\delta)\Gamma^3 \leq ph_r T_r^{\min}$ and $ph_r T_r^{\max} \leq 2\gamma\Gamma^3 f$. A target-recalculation point r is *good* if the target T for the next block satisfies $f/2(2-\delta)\Gamma^3 \leq ph_r T \leq 2\Gamma^3 f$. A chain is *good* if all its target-recalculation points are good.

Random Variables

- D_r: Honest party successfully extends a chain.
  - Sum of the difficulties of all blocks computed by honest parties.
- Y_r: Maximum difficulty among all blocks computed by honest parties.
- Q_r: Isolated successful (consider the Δ round delay).
  - Equal to Y_r when $D_u = 0$ for all $r < u < r + \Delta$ and 0 otherwise.

- Adversary: consider a set of consecutive adversarial queries J.
  - $A(J)$: sum of the difficulties of all adversarial blocks in J for target at least $T(J)/\tau$
  - $B(J)$: sum of the difficulties of all adversarial blocks in J for target at least $T(J)$

Typical executions

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- For the honest parties:

For any set S of at least ℓ consecutive good rounds,

$$(1-\epsilon)[1-2\gamma\Gamma^3 f]^{\Delta} ph(S) < Q(S) \leq D(S) < (1+\epsilon)ph(S).$$

- For the adversarial parties:

For any set J indexing a set of consecutive adversarial queries and $\alpha(J) = 2(\frac{1}{\epsilon} + \frac{1}{3})\varphi/T(J)$,

$$A(J) < p|J| + \max\{\epsilon p|J|, \tau\alpha(J)\} \text{ and } B(J) < p|J| + \max\{\epsilon p|J|, \alpha(J)\}.$$

- No insertions, copies, predictions.

Typical executions

- Get concentration of the random variables:

Definition 8. [MU05, Chapter 12] A sequence of random variables $X_0, X_1, \ldots$ is a martingale with respect to sequence $Y_0, Y_1, \ldots$, if, for all $n \geq 0$, (1)X_n is a function of $Y_0, \ldots, Y_n$, (2)$\mathbb{E}[|X_n|] < \infty$, and (3) $\mathbb{E}[X_{n+1}|Y_0, \ldots, Y_n] = X_n$.

Theorem 16. [McD98, Theorem 3.15] *Let $X_0, X_1, \ldots$ be a martingale with respect to the sequence $Y_0, Y_1, \ldots$. For $n \geq 0$, let $V = \sum_{i=1}^{n} \mathrm{var}(X_i - X_{i-1}|Y_0, \ldots, Y_{i-1})$ and $b = \max_{1 \leq i \leq n} \sup(X_i - X_{i-1} |Y_0, \ldots, Y_{i-1})$, where $\sup$ is taken over all possible assignments to $Y_0, \ldots, Y_{i-1}$. Then, for any $t, v \geq 0$,*

$$\Pr[(X_n \geq X_0 + t) \wedge (V \leq v)] \leq \exp\left\{ -\frac{t^2}{2v + 2bt/3} \right\}.$$

Theorem 3. *Assuming the Bitcoin Cash backbone protocol runs for L rounds, the event "E is not typical" is bounded by $\mathsf{poly}(L) \cdot e^{-\Omega(\mathsf{polylog}(\kappa))}$.*

Typical executions

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- Accuracy: can be proved by the properties of the typical execution, the honest party would accumulate more difficulties than the adversary party after some rounds.

- Goodness:
  - for SMA, it generally follows the approach in [GKL17], with modifications to overcome the adoption of average targets.
  - However, the previous analysis on goodness is *epoch-based*, which fails in the ASERT function.

“Goodness” in ASERT function

$$T' = T_0 \cdot 2^{\left(\frac{r_v-(v-1)/f}{m/f}\right)}$$

- Observation: the next target in ASERT is w.r.t. timestamp and block height.
- Once we fix a sequence of number of parties:
  - For i-th block with timestamp r, and corresponding number of honest parties h_r, if $r = \frac{i-1}{f} + \frac{m}{f}\log\frac{h_0}{h_r}$ (the *calibrated timestamp*), the i-th block would have block generating rate exactly f.
  - r is a good target recalculation point if

$$\frac{i-1}{f} + \frac{m}{f}\log(2(2-\delta)\Gamma^3 \cdot \frac{h_0}{h_r}) \leq r \leq \frac{i-1}{f} + \frac{m}{f}\log(2\Gamma^3 \cdot \frac{h_0}{h_r})$$

“Goodness” in ASERT function

- A new variable X_i to describe the deviation of *calibrated timestamp*:

$$X_1 = 0 \text{ and } X_{i+1} = X_i + (r_{i+1} - r_i) - \frac{1}{f} - \frac{m}{f}\log(\frac{h_{i+1}}{h_i}) \text{ for } i \geq 0.$$

- Three parts:
  - $(r_{i+1} - r_i)$: the difference of their timestamps;
  - $1/f$: the ideal block interval;
  - (m/f)$\log(h_{i+1}/h_i)$: the influence of the party fluctuation.
- For good target recalculation points, X_i should satisfy

$$-\frac{m}{f}\log 2(2 - \delta)\Gamma^3 \leq X_i \leq \frac{m}{f}\log 2\Gamma^3.$$

“Goodness” in ASERT function

- Problem: we cannot bound the accumulation of the party fluctuation.

Definition 1. For $\gamma, \Gamma \in \mathbb{R}^+$, we call a sequence $(n_r)_{r \in \mathbb{N}}$ $(\langle \gamma, \sigma \rangle, \langle \Gamma, \Sigma \rangle)$-*respecting* if it holds that in a sequence of rounds S with $|S| \leq \Sigma$ rounds, $\max_{r \in S} n_r \leq \Gamma \cdot \min_{r \in S} n_r$ and for any consecutive sub-sequence rounds $S' \preccurlyeq S$ with $|S'| \leq \sigma$ rounds, $\max_{r \in S'} n_r \leq \gamma \cdot \min_{r \in S'} n_r$.

- The sequence can capture exponential growth.
  - The total run time is bounded by a polynomial (in κ), and thus the growth is also polynomially bounded.
- However, this is not enough for term $\frac{m}{f} \log(\frac{h_{i+1}}{h_i})$ in the steps.

"Goodness" in ASERT function

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- A new variable W_i to describe the deviation of a specific *calibrated timestamp* (i.e., *relatively calibrated timestamp*):

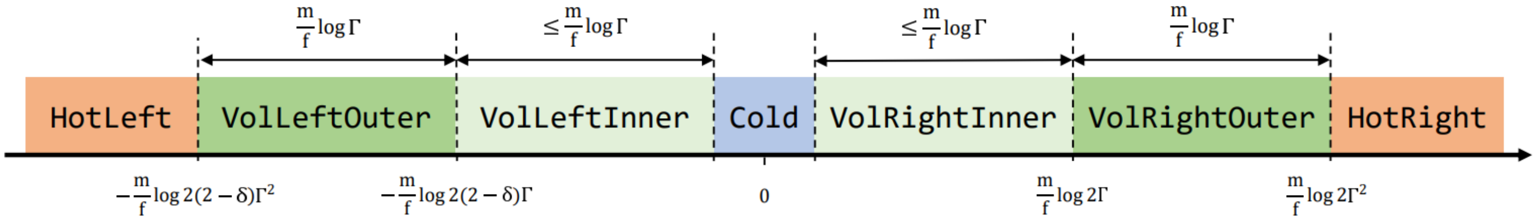
$$W_u = X_u \text{ and } W_{i+1} = W_i + (r_{i+1} - r_i) - \frac{1}{f} \text{ for } i \geq u.$$

- Two parts:
  - $(r_{i+1} - r_i)$: the difference of their timestamps;
  - $1/f$: the ideal block interval.
- For good target recalculation points, W_i should satisfy

$$-\frac{m}{f} \log 2(2 - \delta)\Gamma^2 \leq W_i \leq \frac{m}{f} \log 2\Gamma^2.$$

“Goodness” in ASERT function

- The states based on W_i:



- For good target recalculation points, W_i should satisfy

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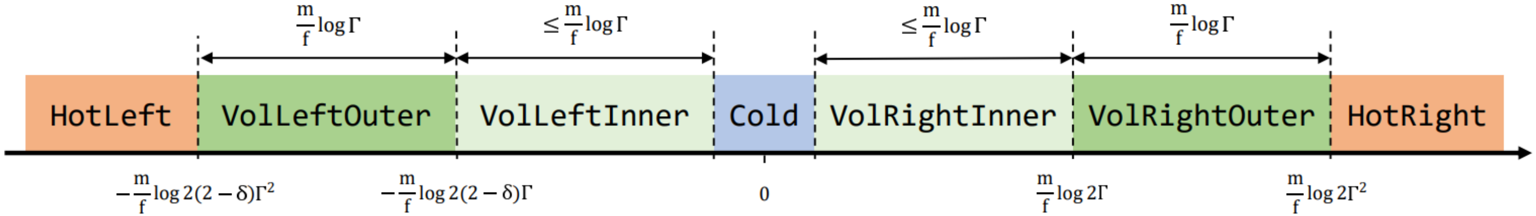
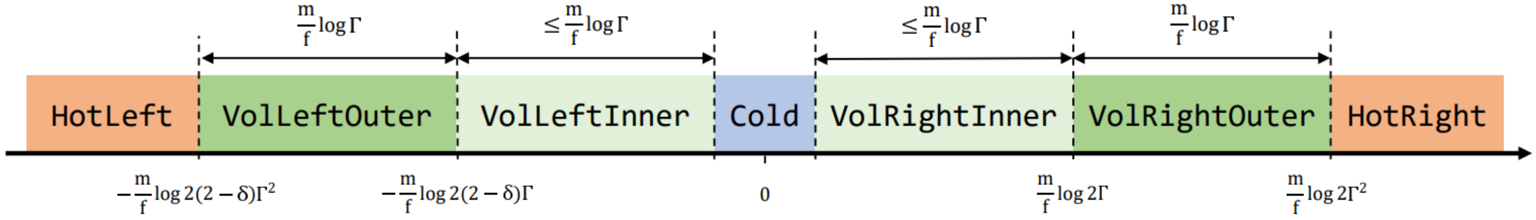
“Goodness” in ASERT function



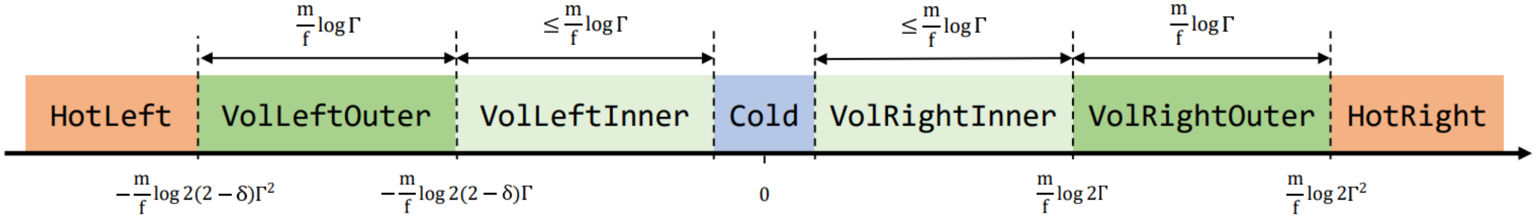
- For blocks $\{B_u, \ldots, B_v\}$ in in a sliding window, it holds that:
  - For a block $B_i, i > u$, with W_i (w.r.t. B_u) in state Cold, we can construct a new sliding window with W_i (w.r.t. B_i) in state VolatileLeftInner, VolatileRightInner or Cold.
  - Extend the analysis of a sliding window from the beginning to the whole execution.

“Goodness” in ASERT function



- For blocks $\{B_u, \ldots, B_v\}$ in in a sliding window, it holds that:
  - If W_u is in state VolatileLeftInner, VolatileRightInner or Cold, the probability of $W_i, i > u$ reaching HotLeft or HotRight is negligible.
  - Never escape to the Hot state (i.e., never break goodness).

“Goodness” in ASERT function



- For blocks $\{B_u, \dots, B_v\}$ in in a sliding window, it holds that:
  - If W_u is in state VolatileLeftInner, VolatileRightInner or Cold, $W_i(i > u)$ will once return to Cold with overwhelming probability.
  - Always feasible to move the sliding window.

Conditions in the analysis

- In order to satisfy the analysis, two conditions on the parameters should be satisfied:
  - We will assume that ℓ is appropriately small compared to the length m of a sliding interval/window:

$$2\ell + 6\Delta \leq \frac{\epsilon m}{2\gamma\Gamma^3 f}.$$

  - The advantage δ of the honest parties over adversarial parties to be large enough to absorb error factors:

$$[1 - 2\gamma\Gamma^3 f]^\Delta \geq 1 - \epsilon \text{ and } \epsilon \leq \delta/8 \leq 1/8.$$

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Real World Network & Parameters

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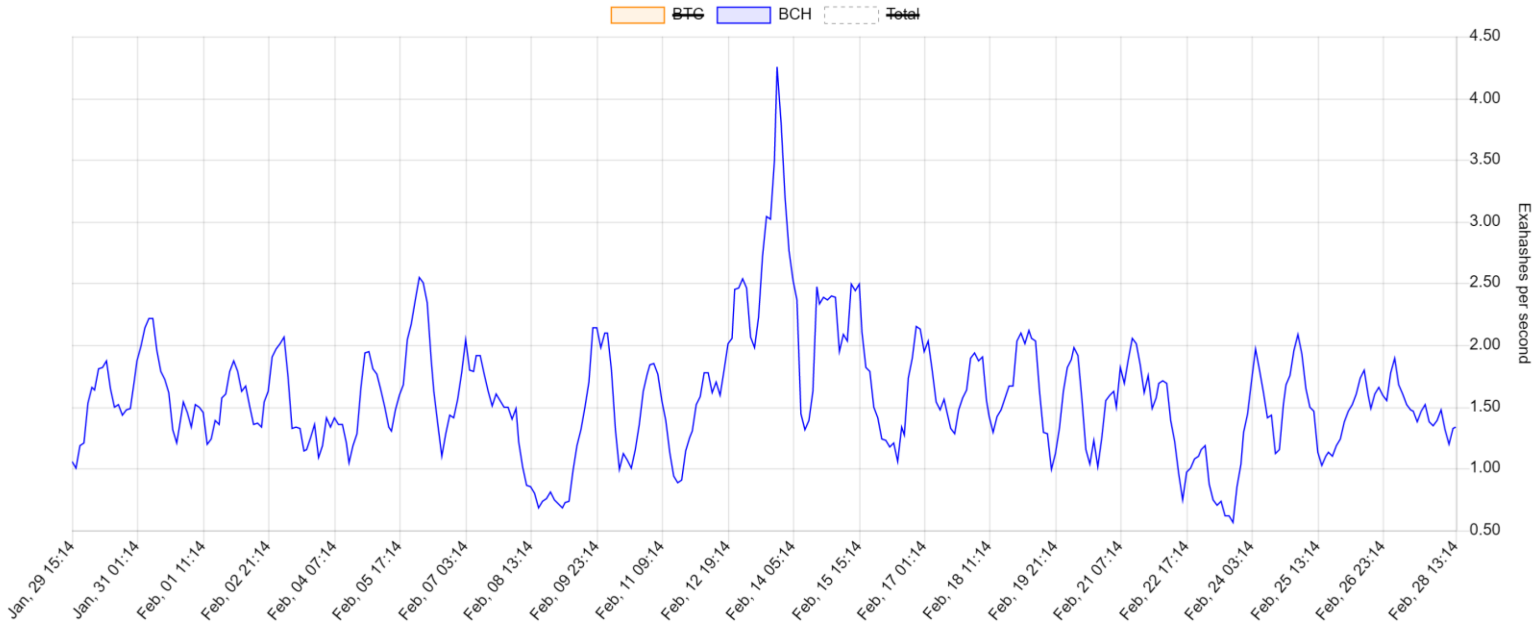
- Party fluctuation (Γ, γ).
  - Extract it from *hashrate.*
- Network delay (Δ).
  - Mainly stems from its multi-hop broadcast and block propagation mechanism.
  - Block propagation time was $> 15s$ in 2014.
- Honest advantage (δ).
- Quality of concentration (ϵ).

Bitcoin Cash’s Hashrate per Second



Party fluctuation ratio > 8.

Bitcoin Cash’s Daily Average Hashrate

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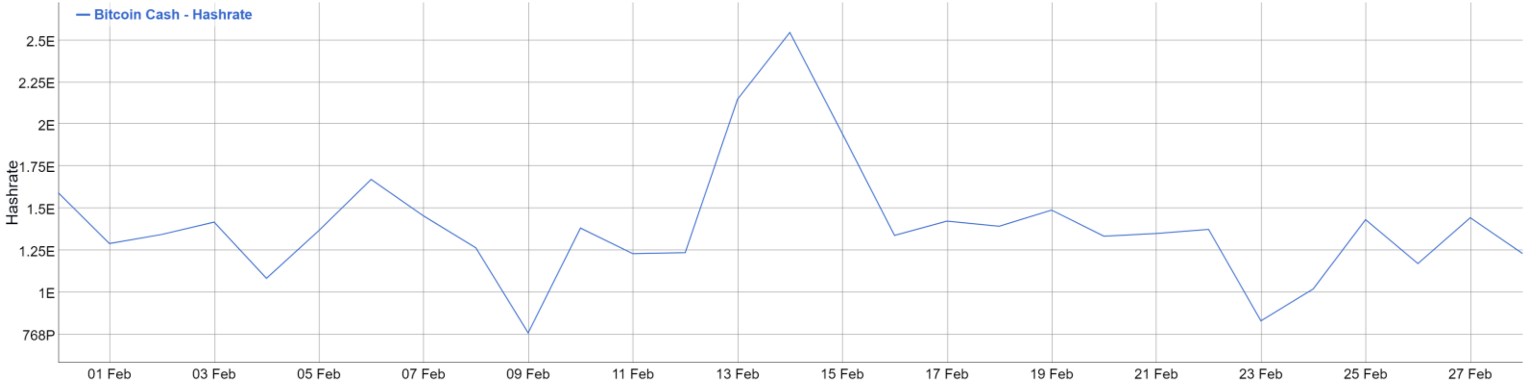
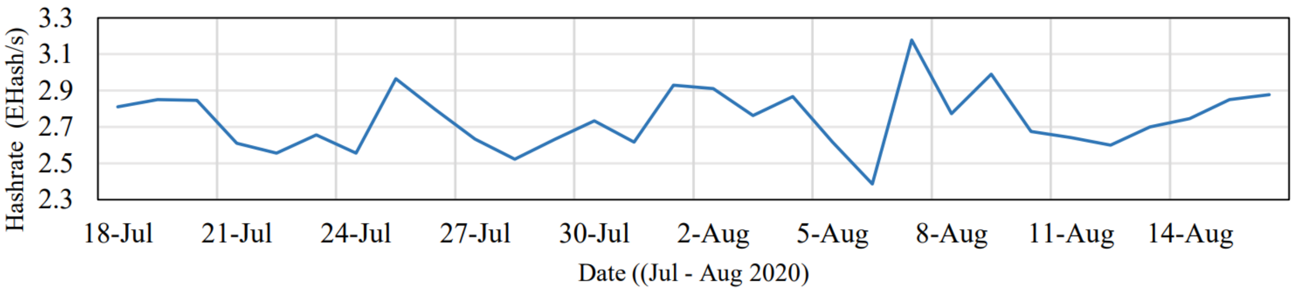


Bitcoin Security under Temporary Dishonest Majority

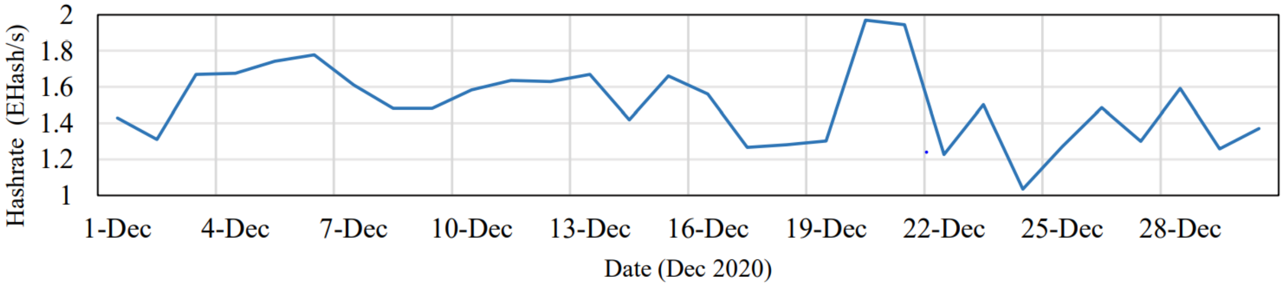
Georgia Avarikioti, Lukas Kaeppeli, Yuyi Wang, Roger Wattenhofer

We prove Bitcoin is secure under temporary dishonest majority. We assume the adversary can corrupt a specific fraction of parties and also introduce crash failures, i.e., some honest participants are offline during the execution of the protocol. We demand a majority of honest online participants on expectation. We explore three different models and present the requirements for proving Bitcoin's security in all of them: we first examine a synchronous model, then extend to a bounded delay model and last we consider a synchronous model that allows message losses.

Bitcoin Cash's Daily Average Hashrate



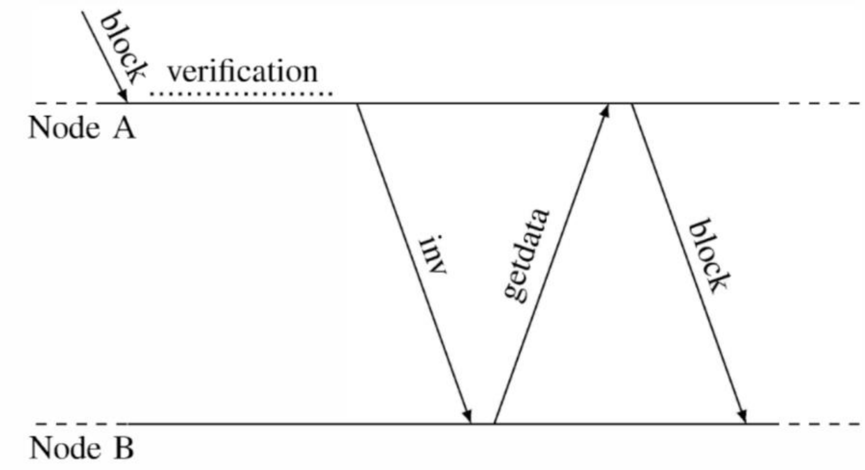
A quiet environment with $\Gamma = 1.398$ **and** $\gamma = 1.057$**.**



An environment of wild fluctuation with $\Gamma = 1.88$ **and** $\gamma = 1.099$**.**

Bitcoin Cash's Block Propagation Time

- Network delay (Δ).
  - Mainly stems from its multi-hop broadcast and block propagation mechanism.



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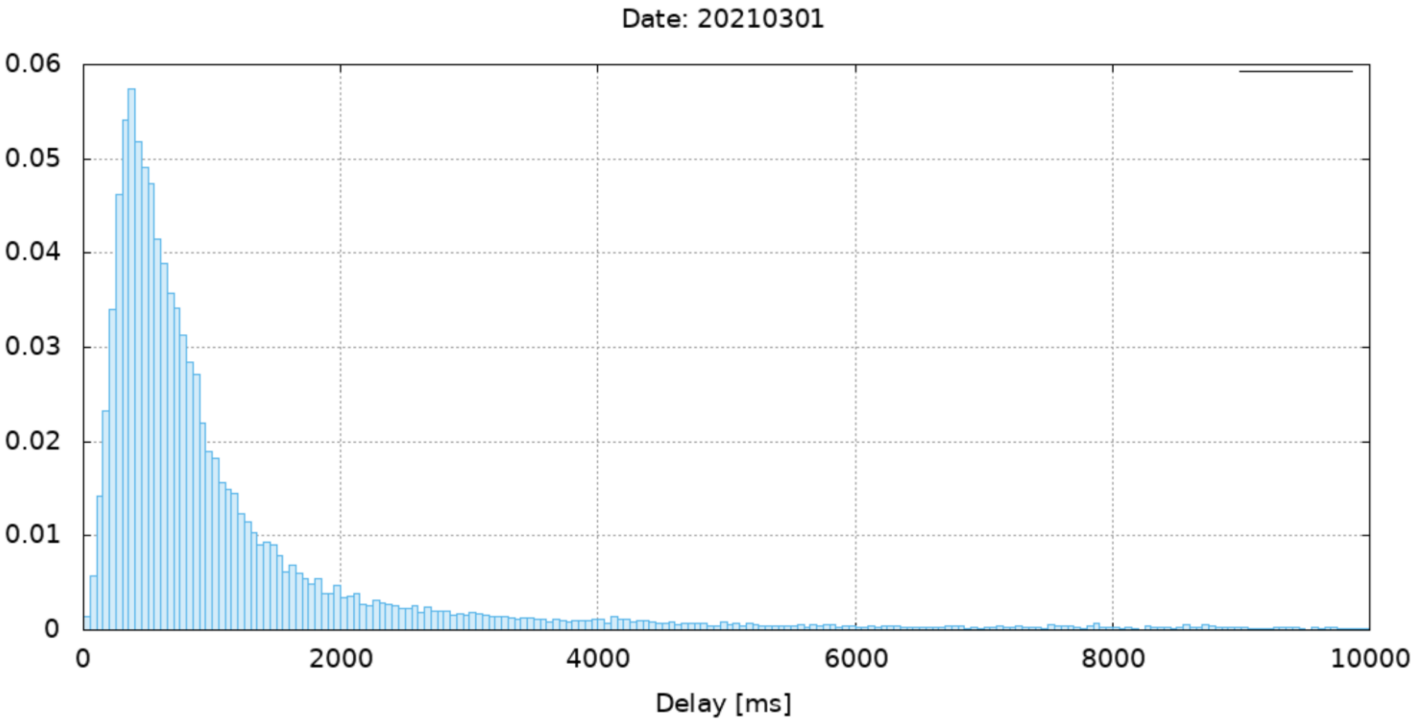

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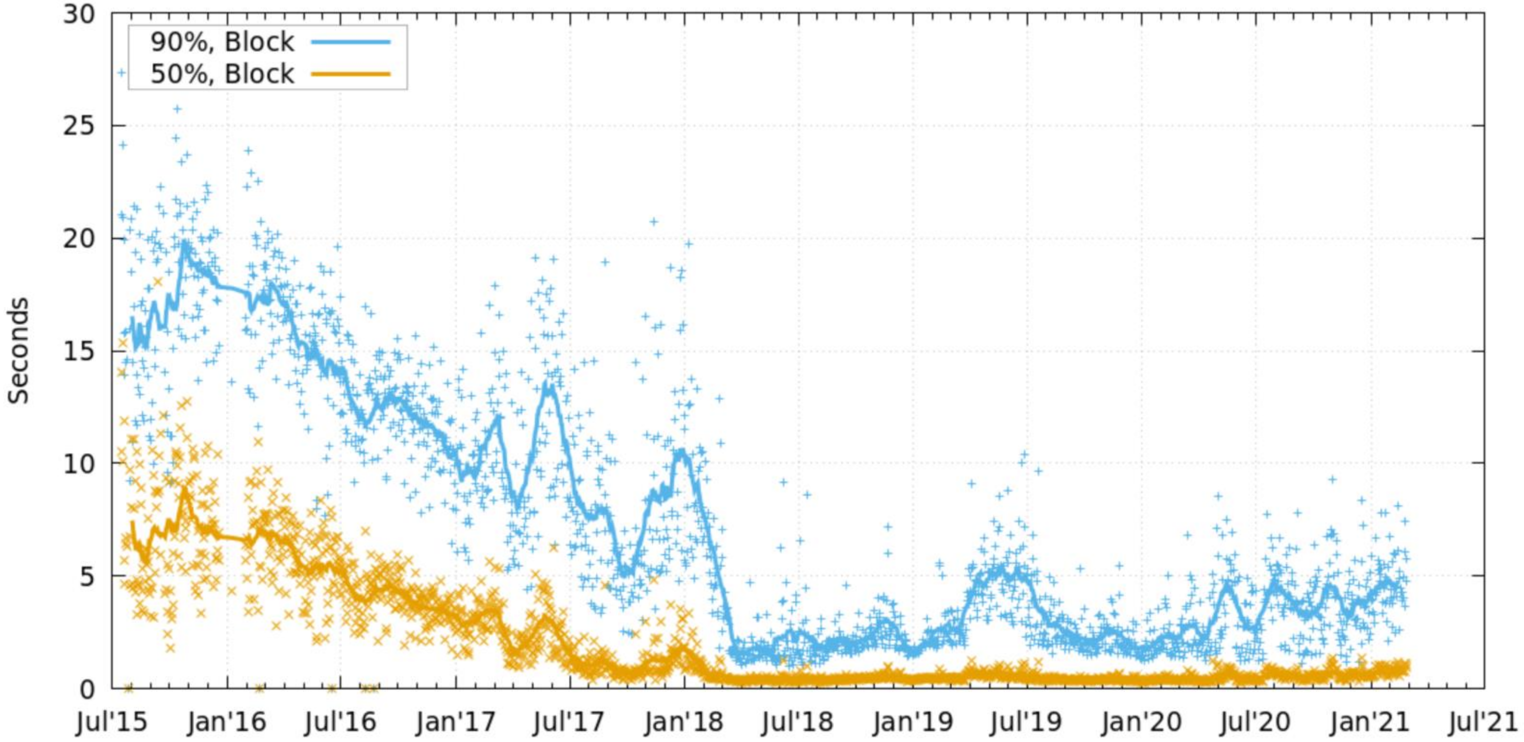

Real World Specs

Parameter	Value
Block generating rate f	0.01 (1 round = 6 seconds)
Network delay Δ	1 (=1 round=6 seconds)
Party fluctuation ratio Γ, γ	1.88, 1.099
Honest advantage δ	0.99
Quality of concentration ϵ	0.123

$$2\ell + 6\Delta \leq \frac{\epsilon m}{2\gamma\Gamma^3 f}.$$

$$[1 - 2\gamma\Gamma^3 f]^{\Delta} \geq 1 - \epsilon \text{ and } \epsilon \leq \delta/8 \leq 1/8.$$

Conclusions

- Under current parameters, the probability to escape to Hot state (break the goodness) is tiny ($< 10^{-9}$).
- Under current parameters, the probability of not returning to Cold state is also tiny ($< 10^{-12}$).
- ASERT is better than SMA, because wilder fluctuation can be inserted into ASERT function.
  - SMA fails when we plugin $\Gamma = 1.88$.
- In order to achieve desired ledger properties, the smoothing factor m should be much larger (approximately several years) to get the ideal ledger properties.

Future/Ongoing Work

- Non-monotonically increasing timestamps
  - In Bitcoin/Bitcoin Cash, the timestamp of a block should be larger than the medium of the last 11 blocks.
  - This work assumes monotonically increasing timestamps.
- Adaptive adversaries

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