ZHEJIANG UNIVERSITY - UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

ECE 310 Digital Signal Processing

Homework 10

Prof. Zhi-Pei Liang **Due: April 23, 2021**

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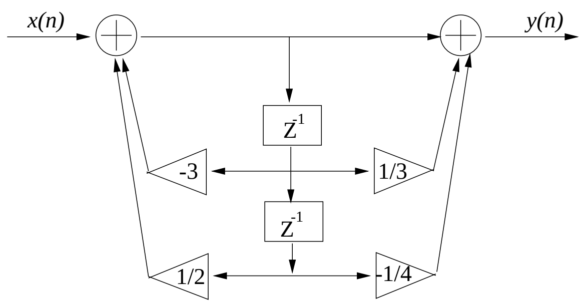
1. The transfer functions of three LSI systems are given below. For each system, determine if it is an FIR or an IIR filter (justify your answer).

   (a) $H(z) = 1 + z^{-1} + 7z^{-6}$

   (b) $H(z) = \dfrac{z^2 + 3z + 2}{z + 1}$

   (c) $H(z) = \dfrac{z + 1}{z^2 + 3z + 2}$

2. Derive the transfer function and the corresponding difference equation for the following block diagram



3. Draw a Direct Form I block diagram for the system in Problem 2.

4. Draw a block diagram implementation (in direct form I and II, respectively) of the system described by

$$y[n+1] - 2y[n] - 5y[n-1] = x[n] + 8x[n+1] - 2x[n-1]$$

5. The frequency response of a GLP filter can be expressed as $H_d(\omega) = R(\omega)e^{j(\alpha - M\omega)}$ where $R(\omega)$ is a real function. For each of the following filters, determine whether it is a GLP filter. If it is, find $R(\omega)$, M, and α, and indicate whether it is also a linear phase filter.

   (a) $\{h_n\}_{n=0}^{2} = \{2, 1, 2\}$

   (b) $\{h_n\}_{n=0}^{2} = \{1, 2, 3\}$

   (c) $\{h_n\}_{n=0}^{2} = \{-1, 3, 1\}$

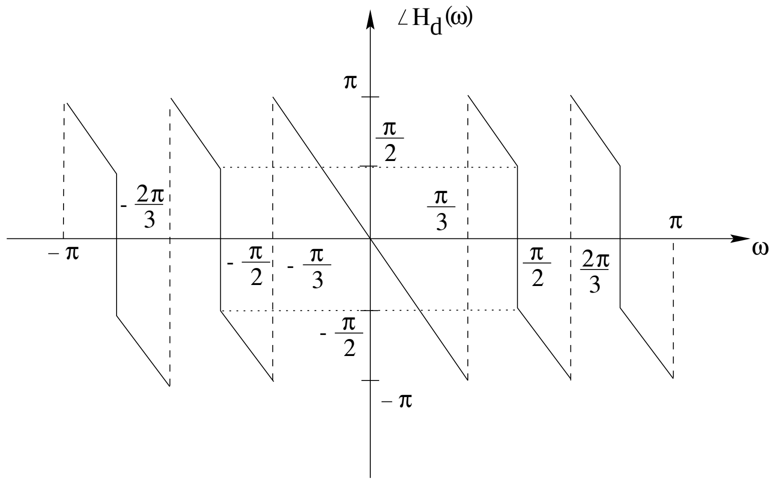
   (d) $\{h_n\}_{n=0}^{4} = \{1, 1, 1, -1, -1\}$

   (e) $\{h_n\}_{n=0}^{2} = \{1, 0, -1\}$

   (f) $\{h_n\}_{n=0}^{3} = \{2, 1, 1, 2\}$

   In each case, the remaining terms of the unit pulse response of the filter are zero.

6. Given the following phase response $\angle H_d(\omega)$ of a generalized linear-phase FIR filter, answer the following questions. Explain you answers.



(a) Is the filter (i) type-1 GLP, (ii) type-2 GLP, or (iii) neither type-1 GLP nor type-2 GLP?

(b) Determine the filter length from the phase plot.

(c) Can you characterize the filter as (i) possibly low-pass, (ii) possibly high-pass, (iii) neither high-pass nor low-pass, or (iv) the given information is insufficient to make any of the preceding statements? (Specify **all** correct answers).

(d) Determine $H_d(\frac{\pi}{2})$.

7. The frequency response of a length-N symmetric or antisymmetric FIR filter with unit pulse response $h[n]$ can be expressed as

$$H_d(\omega) = R(\omega)e^{j\left(\alpha-\left(\frac{N-1}{2}\right)\omega\right)}.$$

For **ONE** of the following, show that

(a) for symmetric $h[n]$ with N even,

$$R(\omega) = 2\sum_{n=0}^{\frac{N}{2}-1} h[n]\cos\left(\omega\left(\frac{N-1}{2}-n\right)\right)$$

(b) for symmetric $h[n]$ with N odd,

$$R(\omega) = h\left[\frac{N-1}{2}\right] + 2\sum_{n=0}^{\frac{N-3}{2}} h[n]\cos\left(\omega\left(\frac{N-1}{2}-n\right)\right)$$

(c) for antisymmetric $h[n]$ with N even,

$$R(\omega) = 2\sum_{n=0}^{\frac{N}{2}-1} h[n]\sin\left(\omega\left(\frac{N-1}{2}-n\right)\right)$$

(d) for antisymmetric $h[n]$ with N odd,

$$R(\omega) = 2\sum_{n=0}^{\frac{N-3}{2}} h[n]\sin\left(\omega\left(\frac{N-1}{2}-n\right)\right)$$