University of Illinois **Fall 2018**
ECE 310 **Profs. Bresler, Katselis, and Shomorony**

Midterm Exam

7:00-8:30PM, Monday, October 15, 2018

Name: ______________________________

UIN: ______________________________

Section: 10:00 AM 3:00 PM 6:00PM Chicago

Score ____________

Problem	Pts.	Score
1	5	
2	5	
3	3	
4	8	
5	6	
6	5	
7	6	
8	5	
9	5	
10	5	
11	5	
12	5	
13	5	
14	3	
15	7	
16	5	
17	5	
18	7	
19	5	
Total		

Please do not turn this page over until told to do so.

You may not use any books, electronic devices, or notes other than **one** handwritten two-sided sheet of 8.5” x 11” paper.

GOOD LUCK!

1. **(5 Pts.)** The output $y[n]$ and input $x[n]$ of a causal system are related by the equation below. Find the values of the impulse response $h[n]$ for the indicated values of n.

$$y[n] = -y[n-6] + 2x[n] + x[n-5]$$

n	0	1	5	6	11	12
$h[n]$						

2. **(5 Pts.)** Let $h[n] = (n-1)^2\,(u[n] - u[n-3])$ for all n. Let $x_1[n] = (-3)^n u[n-1]$. Determine the signal $y[n]$ given by the convolution $y[n] = x_1[n] * h[n]$.

   (a) $y[n] = (-3)^n u[n-1] + (-3)^{n-1} u[n-2] + (-3)^{n-2} u[n-3]$
   (b) $y[n] = (-3)^{n-1} u[n-1] - (-3)^{n-2} u[n-3]$
   (c) $y[n] = (-3)^n u[n] + (-3)^{n-2} u[n-2]$
   (d) $y[n] = (-3)^{n-3} \left((-3)^3 u[n-1] - 3u[n-3] + 4u[n-4]\right)$
   (e) $y[n] = (-3)^n u[n-1] + (-3)^{n-2} u[n-3]$

3. **(3 Pts.)** An LTI system has impulse response $h[n] = \cos(\pi\sqrt{n})u[n]$.
   Is this system BIBO stable? **Yes** **No**

   Why? ____

   ____

4. **(8 Pts.)** For each of the systems with input $x[n]$ and output $y[n]$ in the table, indicate with YES or NO whether the properties indicated apply to the system.

	Linear	Time-Invariant	Causal	Stable
$y[n] = 3y[n-1] + x[n+1]$				
$y[n] = x[2]\cos(x[n])$				
$y[n] = \frac{2}{\lvert n\rvert + 1} y[n-1] + x[n] + \cos(n+3)$				

5. **(6 Pts.)** Given the z-transform pair $x[n] \leftrightarrow X(z) = 1/(1-0.3z^{-1})^3$ with ROC: $|z| > 0.3$, determine the z-transform with ROC of $y[n] = x[n-2] * \left(5^n x[n-1]\right)$

$Y(z)$=

ROC_Y :

6. **(5 Pts.)** The one-sided z-transform of a right-sided sequence $x[n]$ is

$$X(z) = \frac{1}{z^7(z+3)^2}, \qquad |z| > 3$$

Find $x[n]$ for all n.

(a) $n3^{n-9}u[n]$
(b) $(n-8)3^{n-8}u[n-8]$
(c) $(n-8)(-3)^{n-9}u[n-8]$
(d) $(n-6)3^{n-6}u[n-6]$
(e) $(n-6)(-3)^{n-6}u[n-6]$

7. **(6 Pts.)** The one-sided z-transform of a right-sided *real-valued* sequence $x[n]$ is

$$\frac{(1-j)}{1+(1/4+\sqrt{3}/4j)z^{-1}} + \frac{B}{1+(1/4-\sqrt{3}/4j)z^{-1}}, \qquad |z| > 1/2$$

(a) Find B. $B=$

(b) $x[n]$ has the form $x[n] = a\cos(bn+c)d^n u[n]$. Find the unknown constants.

$a=$ $b=$ $c=$ $d=$

8. **(5 Pts.)** Find the partial fraction expansion for $X(z) = \dfrac{8z^{-1}}{(1-4z^{-1})(0.5+z^{-1})}$.

$X(z)=$

9. **(5 Pts.)** An LTI system has the impulse response $h[n] = (j)^n u[n-1]$. Determine a difference equation relating the input $x[n]$ and output $y[n]$ of this system.

$y[n]=$

10. **(5 Pts.)** A causal system produces the output $y[n] = \{1, 3, 0, 0, \ldots\}_{n=0}^{\infty}$ when excited by the input signal $x[n] = \{1, 1/2, 0, 0, \ldots\}_{n=0}^{\infty}$. Determine the impulse response of the system.

   (a) $\delta[n] - n(-3)^n u[n]$
   (b) $(-0.5)^n u[n] + \delta[n]$
   (c) $(-0.5)^n u[n] + (-3)^n u[n-1]$
   (d) $5n(-3)^n u[n-1]$
   (e) $(-0.5)^n u[n] + 3(-0.5)^{n-1} u[n-1]$

11. **(5 Pts.)** A system with input $x[n]$ and output $s[n]$ is described by the difference equation

$$s[n] = x[n] + cx[n-1].$$

The output $s[n]$ of the first system is the input to another causal system described by the difference equation:

$$y[n] - 2y[n-1] = s[n].$$

Find the value of c that guarantees that $y[n] = x[n]$.

$c =$

12. **(5 Pts.)** The input and output of a causal system are related by the equation

$$y[n] + 1.5y[n-1] - y[n-2] = x[n]$$

Is this system BIBO stable? **Yes** **No**

Why? ____

____

13. **(5 Pts.)** The causal LTI system with transfer function given below is not BIBO stable. Find a *real valued* bounded input $x[n]$ that produces an unbounded output $y[n]$.

$$H(z) = \frac{1 - 3z^{-3}}{1 - e^{j\pi/3}z^{-1}}$$

(a) $e^{j\pi n/3}u[n] - e^{-j\pi n/3}u[n]$

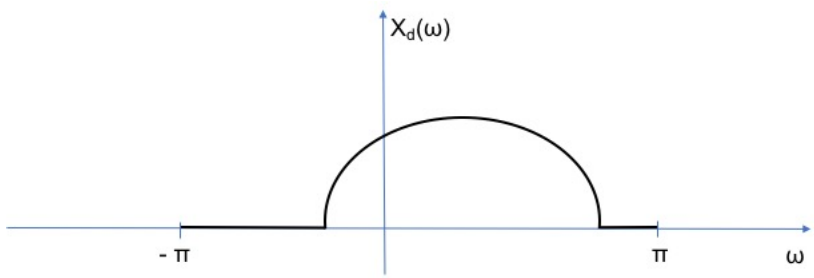
(b) $n\cos(\pi n/3)u[n]$

(c) $e^{j\pi n/3}u[n] + e^{-j\pi n/3}u[n]$

(d) $ne^{j\pi n/3}u[n]$

(e) $\sin(\pi n/6 + \pi/3)$

14. **(3 Pts.)** Let $x[n]$ have the real-valued DTFT given in the plot below. Is $x[n]$ a real-valued sequence?



Yes **No**

Why? ____

____

15. **(7 Pts.)** Let $x[n] = \delta[n] + \delta[n-2] + \delta[n-4]$.

(a) What is the DTFT $X_d(\omega)$ of $x[n]$?

(a) $X_d(\omega) = 1 + e^{-j2w} + e^{j2w}$

(b) $X_d(\omega) = \sin^2(4\omega)$

(c) $X_d(\omega) = e^{-j2w}\,(1 + 2\cos(2\omega))$

(d) $X_d(\omega) = \frac{\sin(3\omega)}{\omega}$

(e) $X_d(\omega) = e^{-j2w}\sin(4\omega)$

(b) **Sketch** the magnitude of $X_d(\omega)$, labeling the axes and “important points” on your sketch.

16. **(5 Pts.)** Determine the signal $x[n]$ whose DTFT is $X_d(\omega) = -1 + 3e^{-j(\omega+\pi/2)} + j\sin(4\omega)$. **Note**: The arrow indicates $n = 0$.

(a) $x[n] = \{0.5\pi, 0, 0, 0, \underset{\uparrow}{-1}, 3, 0, 0, -0.5\pi\}$

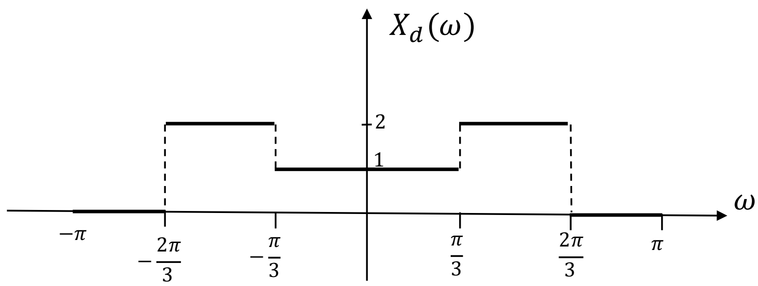
(b) $x[n] = \{0.5, 0, 0, 0, \underset{\uparrow}{-1}, -3j, 0, 0, -0.5\}$

(c) $x[n] = \{0.5j, 0, 0, 0, 1, \underset{\uparrow}{-3j}, 0, 0, 0.5j\}$

(d) $x[n] = \{-2\pi, 0, 0, 0, \underset{\uparrow}{-1}, 3e^{-j\pi/2}, 0, 0, 2\pi\}$

(e) $x[n] = \{0.5\pi j, 0, 0, 0, \underset{\uparrow}{-1}, 3j, 0, 0, 0.5\pi j\}$

17. **(5 Pts.)** The DTFT of $x[n]$ is as shown below. Determine $x[n]$.



(a) $x[n] = \text{sinc}(\pi n)\cos(\pi n/2) + \text{sinc}(2\pi n/3)$

(b) $x[n] = \frac{1}{3}\text{sinc}(\pi n/3) + \frac{1}{3}\text{sinc}(\pi n/6)\cos(\pi n/2)$

(c) $x[n] = 2\text{sinc}(\pi n/3) + \text{sinc}(2\pi n/3)$

(d) $x[n] = \frac{1}{3}\text{sinc}(\pi n/3)\cos(\pi n/3) + 3\text{sinc}(2\pi n/3)$

(e) $x[n] = \frac{1}{3}\text{sinc}(\pi n/3) + \frac{1}{3}\text{sinc}(\pi n/6)\cos(\pi n/5)$

18. **(7 Pts.)** Consider the connection of LTI systems given by Fig. 1, where the impulse response of the first system is $h_1[n] = 2\delta[n] - \delta[n-2]$, and the frequency response of the second system for $|\omega| \leq \pi$ is given by

$$H_2(e^{j\omega}) = \begin{cases} 1 & \text{for } |\omega| \leq \frac{3\pi}{4} \\ 0 & \text{for } \frac{3\pi}{4} < |\omega| \leq \pi \end{cases}$$

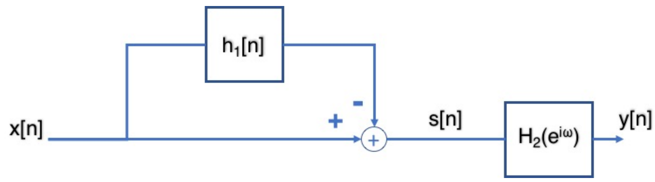


Figure 1: Interconnection of LTI Systems

(a) Find the impulse response of the interconnected system with input $x[n]$ and output $y[n]$

$h[n] =$

(b) Find the frequency response of the interconnected system

$H(e^{j\omega}) =$

19. **(5 Pts.)** A causal system is described by the difference equation

$$y[n] + \frac{\sqrt{3}}{3} y[n-1] = x[n].$$

Determine the response of the system to the input $x[n] = 2\cos(\frac{\pi}{2}n)$, $-\infty < n < \infty$.

$y[n] =$