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ECE 310 Digital Signal Processing

# Homework 7

**Prof. Zhi-Pei Liang** **Due: April 2, 2021**

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1. The sequence $x[n] = \cos\left(\frac{\pi}{3}n\right), \ -\infty < n < \infty$ was obtained by sampling the continuous-time signal $x_a(t) = \cos(\Omega_0 t), \ -\infty < t < \infty$ at a sampling rate of 1000 samples/sec. What are two possible values of $\Omega_0$ that could have resulted in the sequence $x[n]$?

2. The continuous-time signal $x_a(t) = \sin(10\pi t) + \cos(20\pi t)$ is sampled with a sampling period $T$ to obtain the discrete-time signal $x[n] = \sin\left(\frac{\pi}{5}n\right) + \cos\left(\frac{2\pi}{5}n\right)$

   a) Determine a choice for $T$ consistent with this information.

   b) Is your choice for $T$ in part (a) unique? If so, explain why. If not, specify another choice of $T$ consistent with the information given.

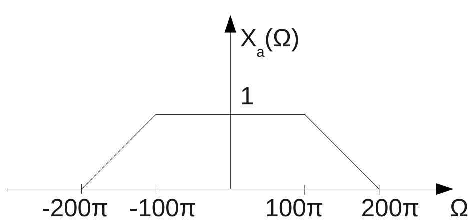
3. The continuous-time signal $x_a(t) = \cos(400\pi t)$ is sampled with a sampling period $T$ to obtain a discrete-time signal $x[n] = x_a(nT)$

   a) Compute and sketch the magnitude of the continuous-time Fourier transform of $x_a(t)$ and the discrete-time Fourier Transform of $x[n]$ for $T = 1$ ms.

   b) Repeat part (a) for $T = 2$ ms.

   c) What is the maximum sampling period $T_{max}$ such that no aliasing occurs in the sampling process?

4. The continuous-time signal $x_a(t)$ has the continuous-time Fourier transform shown in the figure below. The signal $x_a(t)$ is sampled with sampling interval $T$ to get the discrete-time signal $x[n] = x_a(nT)$. Sketch $X_d(\omega)$ (the DTFT of $x[n]$) for the sampling intervals $T = 1/100, 1/200$ sec.




5. Let $x[n] = x_a(nT)$. Show that the DTFT of $x[n]$ is related to the FT of $x_a(t)$ by

$$X_d(\omega) = \frac{1}{T}\sum_{\ell=-\infty}^{\infty} X\left(\frac{\omega + 2\ell\pi}{T}\right)$$

   where $X_d(\omega)$ is the DTFT of $x[n]$ and $X(\Omega)$ the FT of $x_a(t)$.

P1. $T = \frac{1}{1000} < \frac{\pi}{\Omega_{max}}$, $f_s = 1000$

$\therefore \Omega_{max} = 1000\pi$

$\therefore \frac{\pi}{3} < \Omega_0 < 1000\pi$

i.e. $\Omega_0 = \frac{2\pi}{3}, \pi$

P2. a) $T_0 = \frac{\frac{\pi}{5}}{10\pi} = \frac{1}{50}$ s

$x[1] = \sin(\frac{\pi}{5}) + \cos(\frac{2\pi}{5}) = x_a(\frac{1}{50})$

$x[2] = \sin(\frac{2\pi}{5}) + \cos(\frac{4\pi}{5}) = x_a(\frac{2}{50})$

b) No, T can be any value that

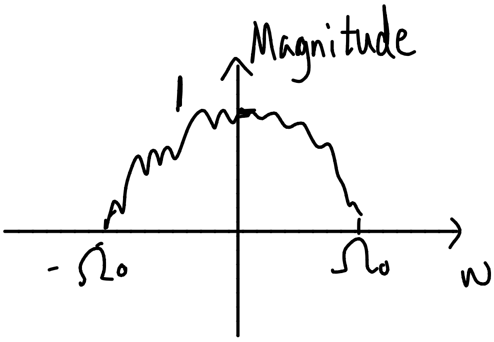
$\forall T < T_0$

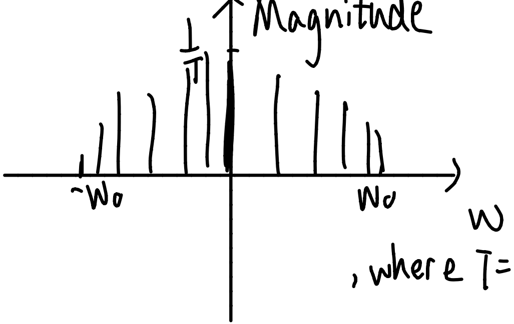
i.e. $T = \frac{1}{100}$ s

By this we get also get

$x[0], x[1], x[2] \ldots$

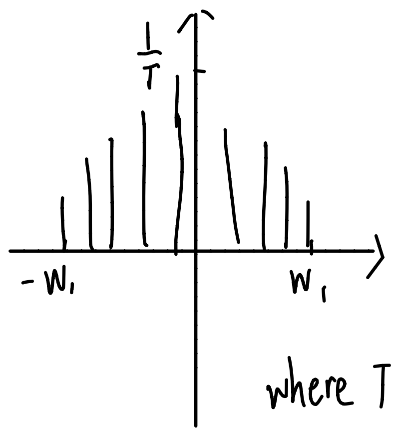
P3. a) $X_d(w) = \frac{1}{T} \sum_{l=-\infty}^{\infty} X_a\left(\frac{400\pi + l\pi}{T}\right)$




, where $T = 1$ms

b)



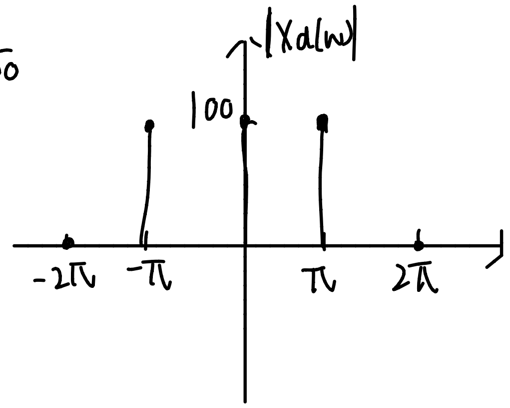

where $T = 2$ms

c) $T < \frac{2\pi}{\Omega_{max}} = \frac{2\pi}{400\pi} = \frac{1}{200}$ s

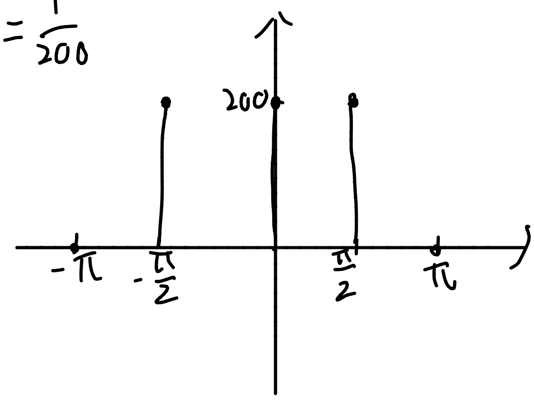
$= 5$ms

P4. $X_d(w) = \frac{1}{T} \sum_{l=-\infty}^{\infty} X_a\left(\frac{w + l\pi}{T}\right)$

$T = \frac{1}{100}$




$T = \frac{1}{200}$




P5. $X_d(w) = \sum_{l=-\infty}^{\infty} x[n] e^{-jwl}$

$= \sum_{l=-\infty}^{\infty} x_a(nT) e^{-jwl}$

$= \sum_{l=-\infty}^{\infty} e^{-jwl} \frac{1}{T} X_a(\frac{w}{T})$

$= \sum_{l=-\infty}^{\infty} \frac{1 - e^{-jwl\cdot\infty}}{1 - e^{-jwl}} \frac{1}{T} X_a\left(\frac{w + 2l\pi}{T}\right) = \frac{1}{T} \sum_{l=-\infty}^{\infty} X_a\left(\frac{w + 2l\pi}{T}\right)$

Hence, proved.