ECE 310

Digital Signal Processing

Spring, 2021, ZJUI Campus

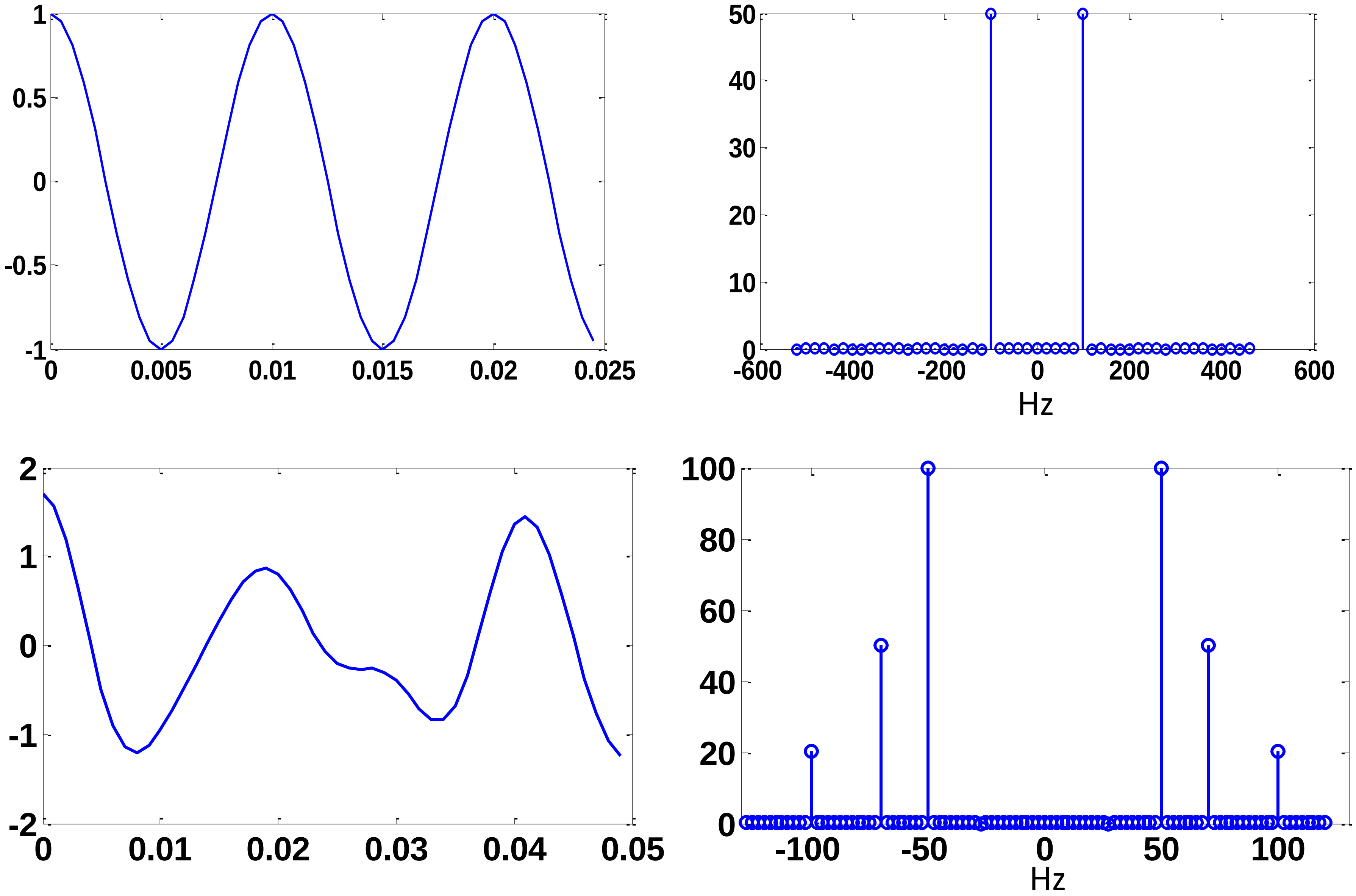
Lecture 22

Topics:

- ✓ Further discussion of DFT Spectral Estimation
- ✓ Fast Fourier Transform (FFT)

Educational Objectives:

- ✓ Understand DFT spectral analysis method
  - Zero-padding effect
  - Windowing effect
  - Resolution limitation
- ✓ Understand the FFT decomposition equation



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0.5
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-1
0
0.005
0.01
0.015
0.02
0.025
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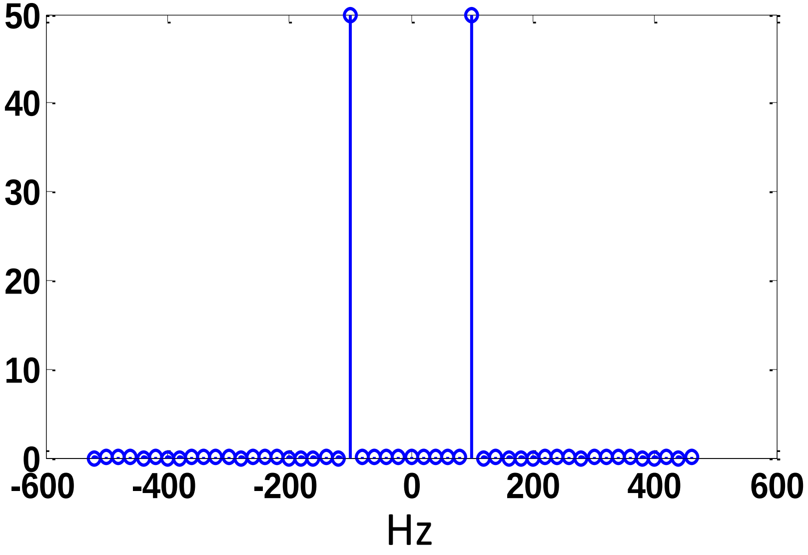
30
20
10
0
-600
-400
-200
0
200
400
600
Hz
2
1
0
-1
-2
0
0.01
0.02
0.03
0.04
0.05
100
80
60
40
20
0
-100
-50
0
50
100
Hz



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10
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-600
-400
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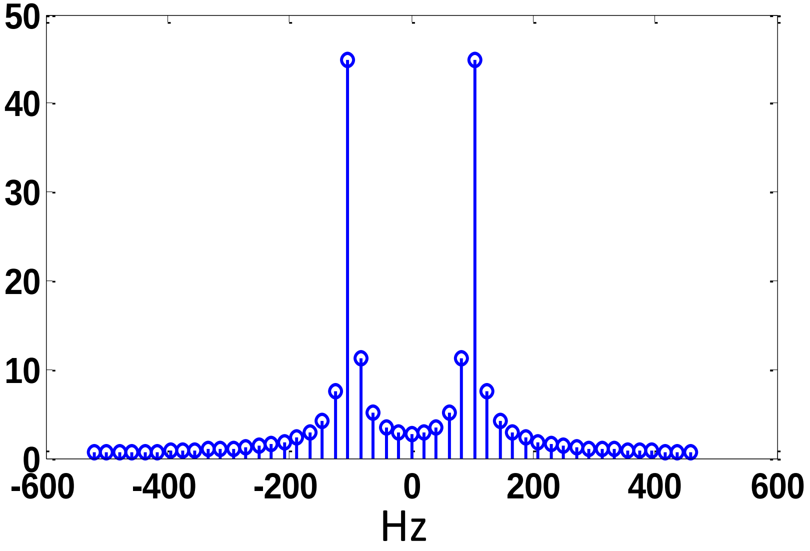

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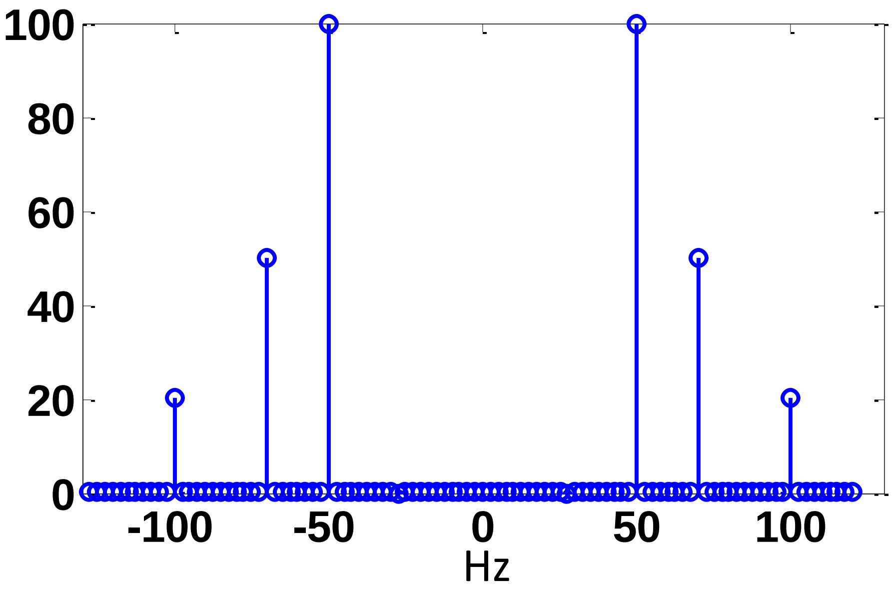

100
80
60
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20
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-100
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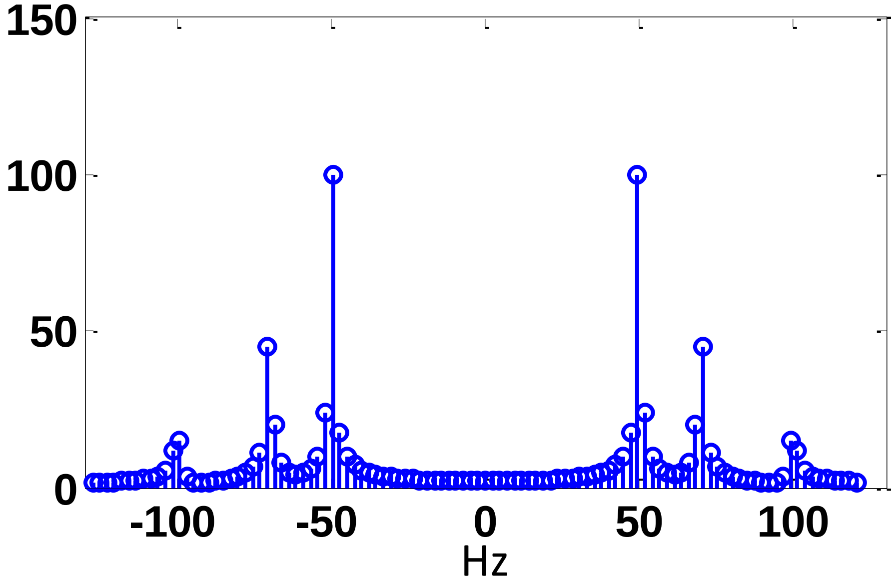
50
100
Hz



150
100
50
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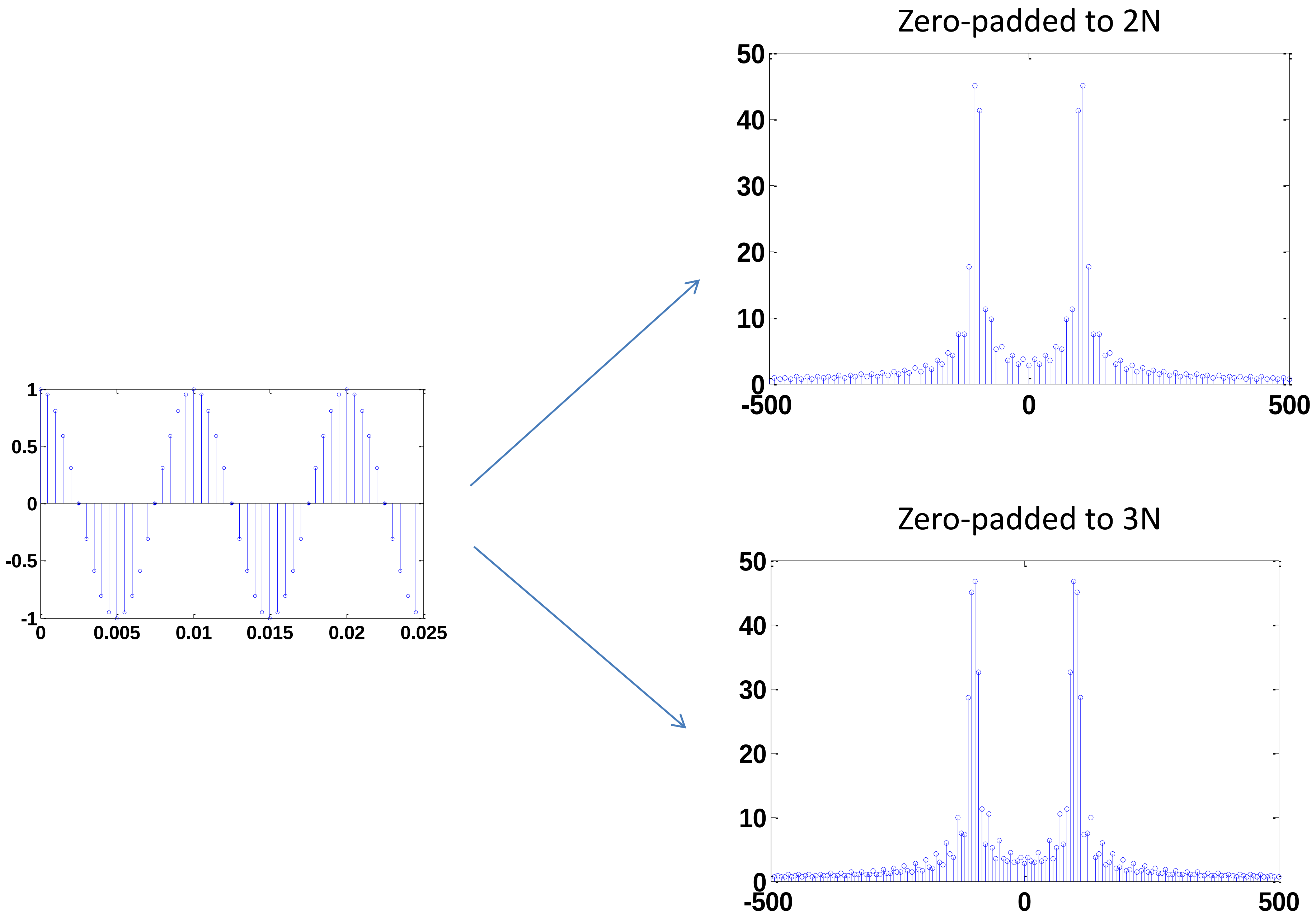
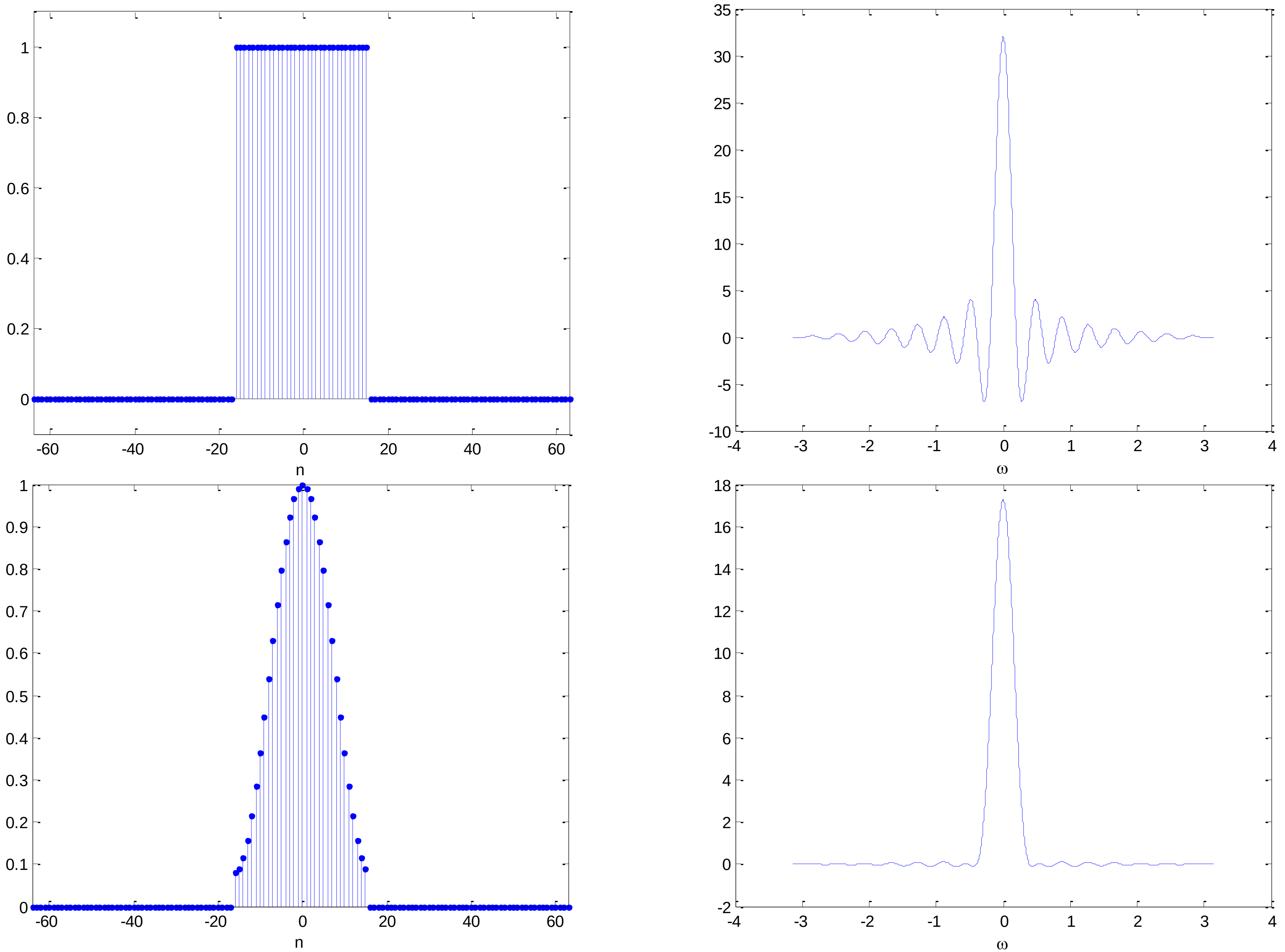
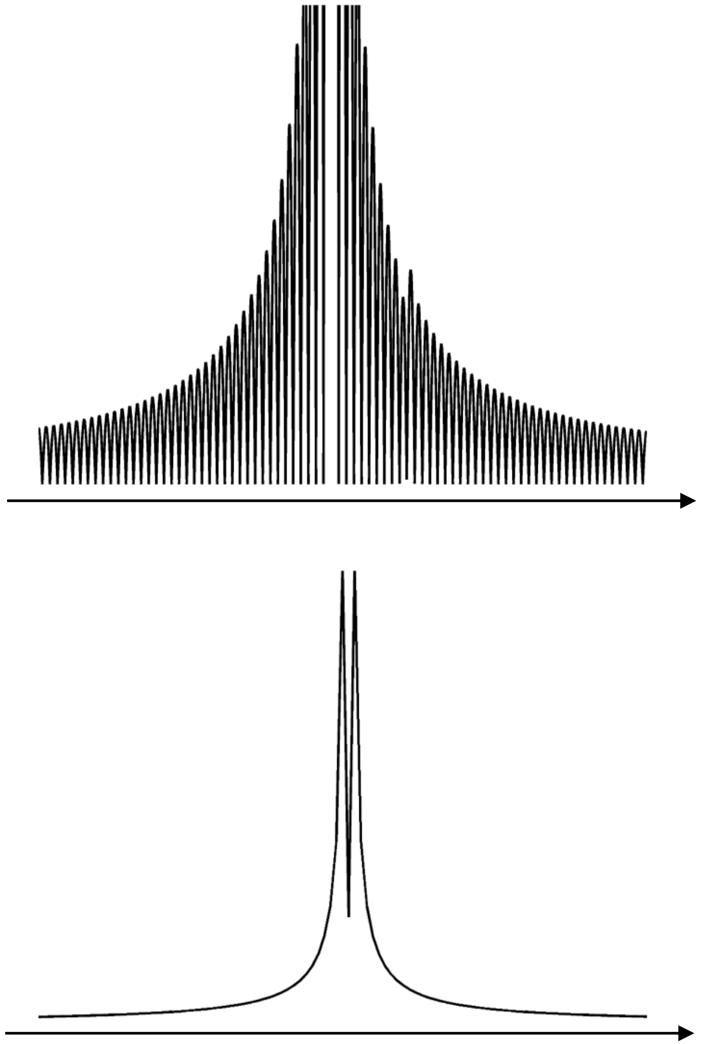
-100
-50
0
50
100
Hz

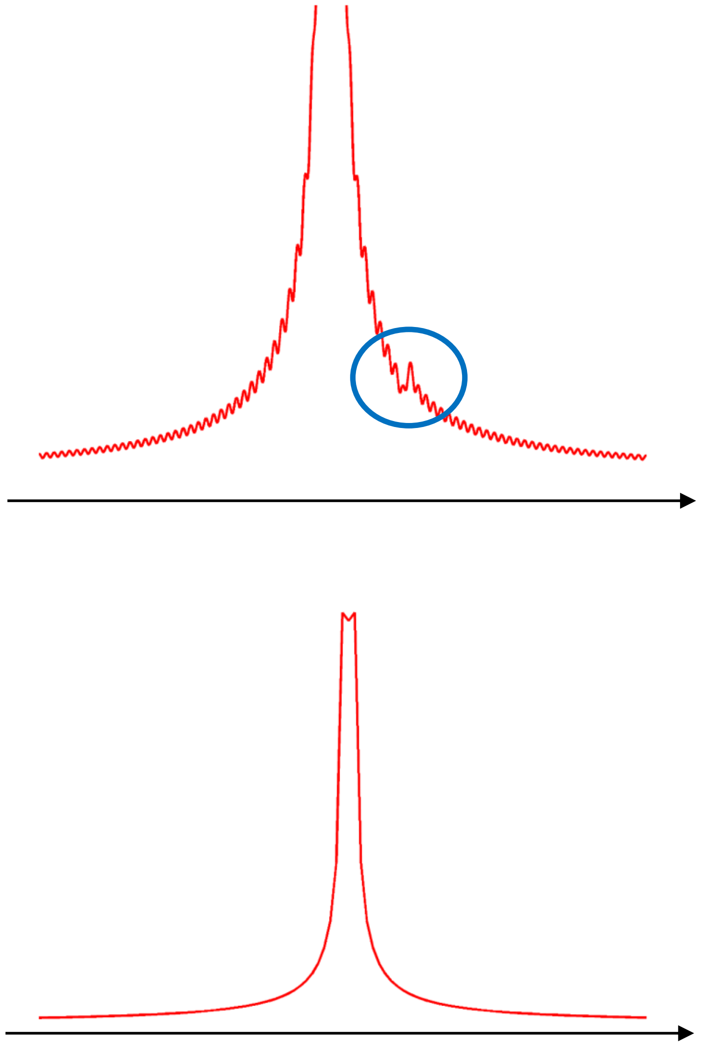


Zero-padded to 2N
Zero-padded to 3N
1
0.5
0
-0.5
-1
0
0.005
0.01
0.015
0.02
0.025
50
40
30
20
10
0
-500
0
500



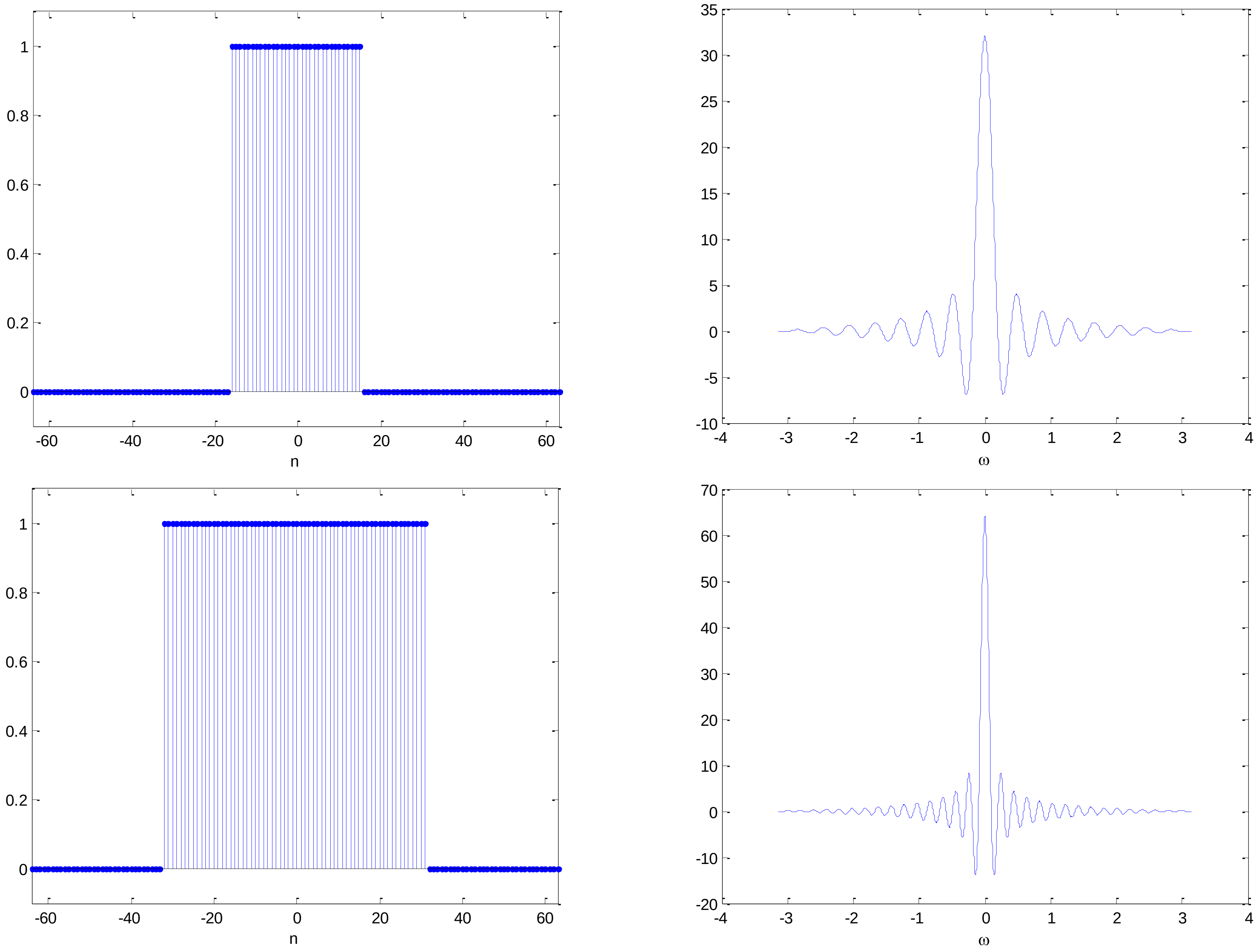
-60 -40 -20 0 20 40 60
n
0 0.2 0.4 0.6 0.8 1
-4 -3 -2 -1 0 1 2 3 4
ω
-10 -5 0 5 10 15 20 25 30 35
-60 -40 -20 0 20 40 60
n
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
-4 -3 -2 -1 0 1 2 3 4
ω
-2 0 2 4 6 8 10 12 14 16 18



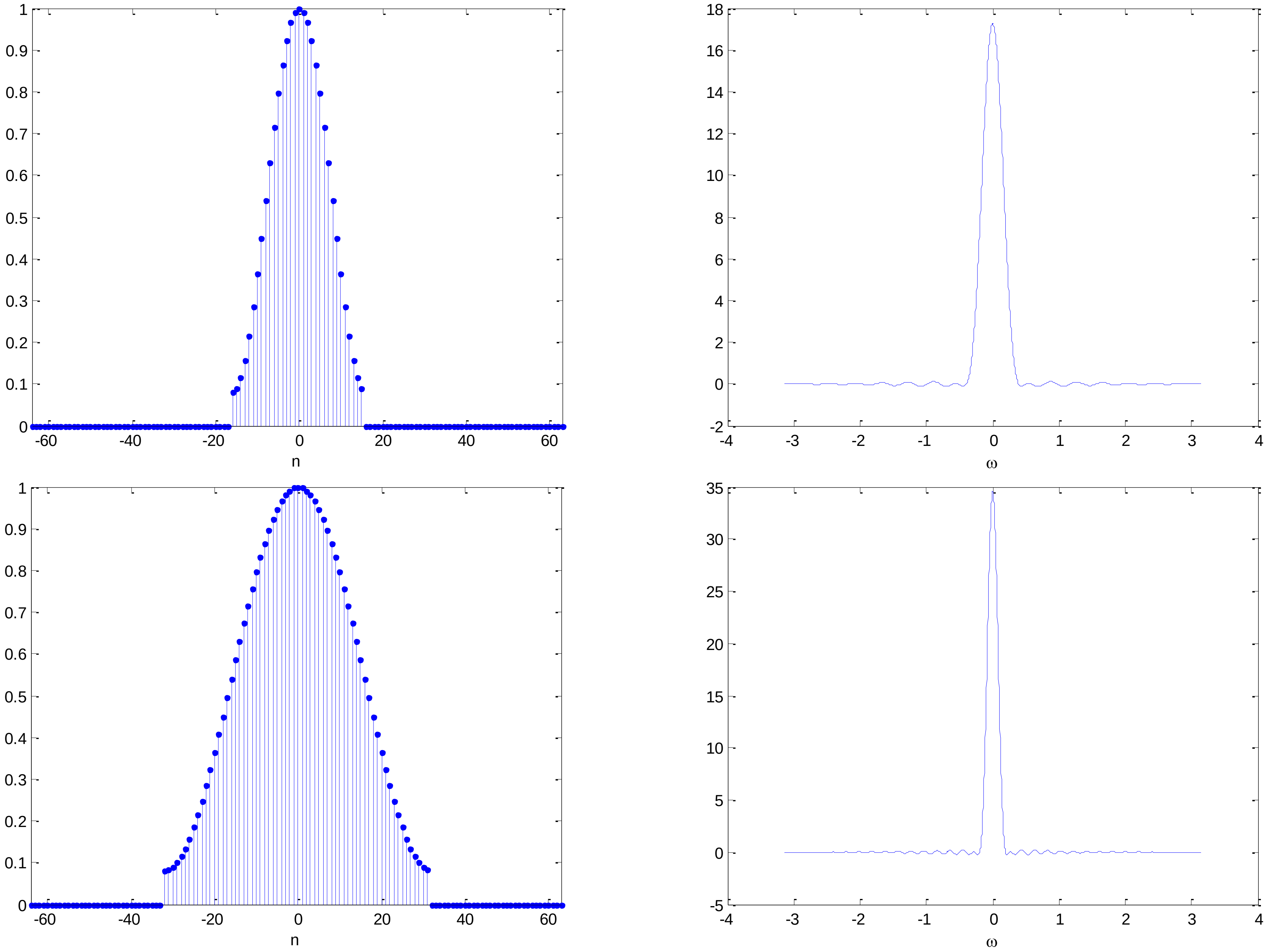


Rectangular window

Hamming window



1
0.8
0.6
0.4
0.2
0
-60 -40 -20 0 20 40 60
n
35
30
25
20
15
10
5
0
-5
-10
-4 -3 -2 -1 0 1 2 3 4
ω
1
0.8
0.6
0.4
0.2
0
-60 -40 -20 0 20 40 60
n
70
60
50
40
30
20
10
0
-10
-20
-4 -3 -2 -1 0 1 2 3 4
ω



n
ω
n
ω

Fast Fourier Transform (FFT)

$$X_m = \sum_{n=0}^{N-1} x_n e^{-j\frac{2\pi}{N}mn}, \qquad m = 0,1\ldots N-1$$

$$x_n = \frac{1}{N}\sum_{m=0}^{N-1} X_m e^{j\frac{2\pi}{N}mn}, \qquad n = 0,1\ldots N-1$$

FFT (Decimation-in-Time, DIT, Radix-2) algorithm:

- ✓ **Requirement:** $N = 2^r, \quad r \text{ integer}$
- ✓ **Computational complexity:** $N\log_2(N)$

Fast Fourier Transform (FFT)

$$N = 2^{14} = 16{,}384$$

$$N^2 = 268{,}435{,}456$$

$$N \log_2 N = 229{,}376$$

$$\text{saving factor} = \frac{268{,}435{,}456}{229{,}376} = 1170$$

Fast Fourier Transform (FFT)

$$X_m = \sum_{n=0}^{N-1} x_n e^{-j\frac{2\pi}{N}mn} = \sum_{n=0}^{N-1} x_n W_N^{mn}, \qquad m = 0,1 \ldots N-1 \qquad W_N = e^{-j\frac{2\pi}{N}}$$

$$X_m = \sum_{p=0}^{N/2-1} x_{2p} W_{N/2}^{pm} + W_N^m \sum_{p=0}^{N/2-1} x_{2p+1} W_{N/2}^{pm}$$

$$= Y_m + W_N^m Z_m, \qquad m = 0,1 \ldots N-1$$

$$\begin{cases} Y_m = \sum_{p=0}^{N/2-1} x_{2p} W_{N/2}^{pm} \\ Z_m = \sum_{p=0}^{N/2-1} x_{2p+1} W_{N/2}^{pm} \end{cases} \qquad m = 0,1 \ldots N/2-1$$

Fast Fourier Transform (FFT)

$$\begin{cases} Y_{m+N/2} = Y_m \\ Z_{m+N/2} = Z_m \end{cases} \qquad m = 0,1 \ldots N/2-1$$

$$\begin{cases} X_m = Y_m + W_N^m Z_m \\ X_{m+N/2} = Y_m - W_N^m Z_m \end{cases} \qquad m = 0,1 \ldots N/2-1$$