# ECE 310

# Digital Signal Processing

Spring, 2021, ZJUI Campus

# Lecture 14

## Topics:

- ✓ Review of $\delta$-function
- ✓ Review of continuous-time Fourier transform (FT)

## Educational Objectives:

- ✓ Refresh your memory of the definition and properties of $\delta$-function
- ✓ Refresh your memory of FT, key properties and basic transform pairs
- ✓ Get ready for our journey into discrete-time Fourier transform (DTFT) and discrete Fourier transform (DFT)

# $\delta$-Function

$$\int_{-\infty}^{\infty} \varphi(t)\delta(t-t_0)dt = \varphi(t_0)$$

**Key properties:**

- Sampling: $x(t)\delta(t) = x(0)\delta(t)$

  $x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$
- Convolution: $x(t) * \delta(t) = x(t)$

  $x(t) * \delta(t-t_0) = x(t-t_0)$
- Scaling: $\delta(at) = \frac{1}{|a|}\delta(t), \quad a \neq 0$
- Derivative: $\int_{-\infty}^{\infty} \varphi(t)\delta^n(t)dt = (-1)^n \varphi(0)$

# Examples

$$\int_{-\infty}^{\infty} 3\cos(t)\delta(t+3)dt =$$

$$\int_{0}^{\infty} 3\cos(t)\delta(t+3)dt =$$

$$\int_{-\infty}^{\infty} 3\cos(t)\delta(3t-1)dt =$$

# Fourier Transform (FT)

$$x(t) \leftrightarrow X(\Omega),\ \Omega:\ frequency\ rad/\sec \qquad \frac{\Omega}{2\pi} = F\ \text{Hertz}$$

$$X(\Omega) \triangleq \int_{-\infty}^{\infty} x(t)e^{-j\Omega t}dt$$

$$|X(\Omega)|:\ magnitude\ spectrum\ (response)$$

$$\angle X(\Omega):\ phase\ spectrum\ (response)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)e^{+j\Omega t}d\Omega$$

# Key Properties

$$a)\ linearity: ax_1(t)+bx_2(t) \leftrightarrow aX_1(\Omega)+bX_2(\Omega)$$

$$b)\ duality: if\ \ x(t) \leftrightarrow X(\Omega), then\ X(t) \leftrightarrow 2\pi x(-\Omega)$$

$$c)\ sacling: x(at) \leftrightarrow \frac{1}{|a|} X\left(\frac{\Omega}{a}\right)$$

$$d)\ time\ shifting: x(t-t_0) \leftrightarrow X(\Omega)e^{-j\Omega t_0}$$

# Key Properties

e) *frequency shift* : $x(t)e^{j\Omega_0 t} \leftrightarrow X(\Omega - \Omega_0)$

f) *symmetry* : *if* $x(t) \leftrightarrow X(\Omega)$, *then* $x^*(t) \leftrightarrow X^*(-\Omega)$

*special case* : $x^*(t) = x(t) \rightarrow X^*(-\Omega) = X(\Omega)$

$$\mathrm{Re}\{X(\Omega)\} = \mathrm{Re}\{X(-\Omega)\}$$

$$\mathrm{Im}\{X(\Omega)\} = -\mathrm{Im}\{X(-\Omega)\}$$

$$|X(\Omega)| = |X(-\Omega)|$$

$$\angle X(\Omega) = -\angle X(-\Omega)$$

# Key Properties

$g)\ convolution:$

$$x_1(t) * x_2(t) \leftrightarrow X_1(\Omega) X_2(\Omega)$$

$$x_1(t) \cdot x_2(t) \leftrightarrow \frac{1}{2\pi} X_1(\Omega) * X_2(\Omega)$$

# Examples

$$a)\ \delta(t) \leftrightarrow 1$$

$$\int_{-\infty}^{\infty} \delta(t) e^{-j\Omega t} dt = e^{-j\Omega t}\Big|_{t=0} = 1$$

$$b)\ 1 \leftrightarrow 2\pi\delta(\Omega)$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} 2\pi\delta(\Omega) e^{+j\Omega t} d\Omega = e^{+j\Omega t}\Big|_{\Omega=0} = 1$$

$$c)\ e^{j\Omega_0 t} \leftrightarrow 2\pi\delta(\Omega - \Omega_0)$$

# Examples

$$d)\ \cos(\Omega_0 t) \leftrightarrow \pi[\delta(\Omega+\Omega_0)+\delta(\Omega-\Omega_0)]$$

$$\Rightarrow \frac{1}{2}(e^{j\Omega_0 t}+e^{-j\Omega_0 t})$$

$$\Rightarrow \frac{1}{2}(2\pi\delta(\Omega-\Omega_0)+2\pi\delta(\Omega+\Omega_0))$$

$$e)\ \sin(\Omega_0 t) = \frac{1}{2j}(e^{j\Omega_0 t}-e^{-j\Omega_0 t})$$

$$\Rightarrow \pi j(\delta(\Omega+\Omega_0)-\delta(\Omega-\Omega_0))$$

# Examples

$$f)\ x(t) = u(t) - u(t-T)$$

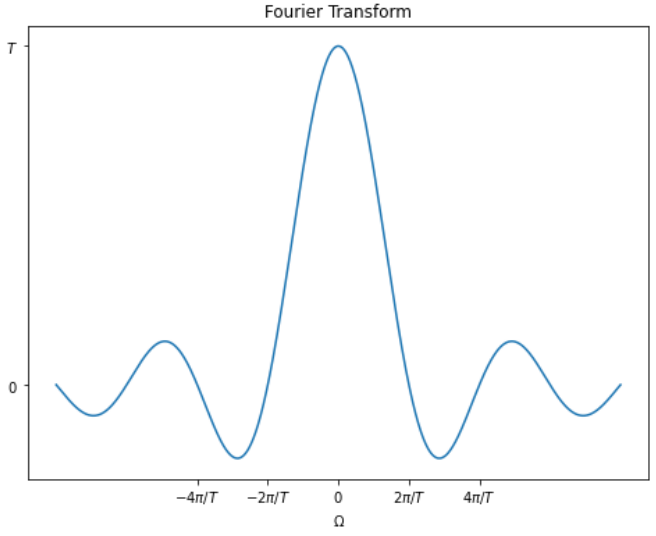
$$\text{consider } x_0(t) = x(t + \frac{T}{2})$$

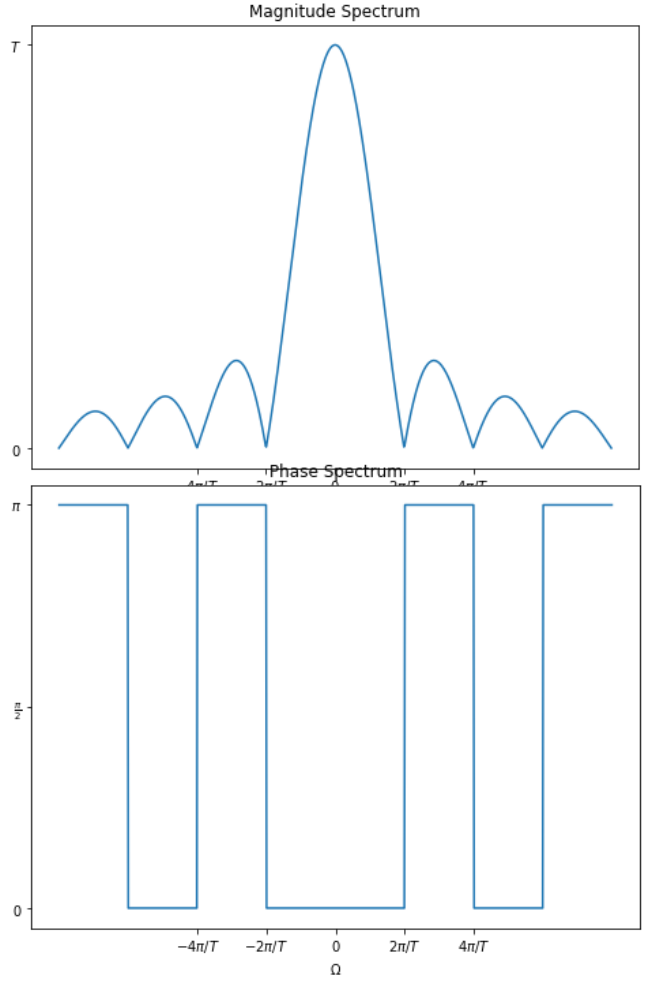
$$\int_{-\infty}^{\infty} x_0(t) e^{-j\Omega t} dt = -\frac{1}{j\Omega} e^{-j\Omega t} \Big|_{-\frac{T}{2}}^{\frac{T}{2}} = -\frac{1}{j\Omega}(e^{-j\Omega\frac{T}{2}} - e^{+j\Omega\frac{T}{2}})$$

$$= -\frac{1}{j\Omega}(-2j\sin(\Omega T/2)) = T\frac{\sin(\Omega T/2)}{\Omega T/2}$$

# Examples

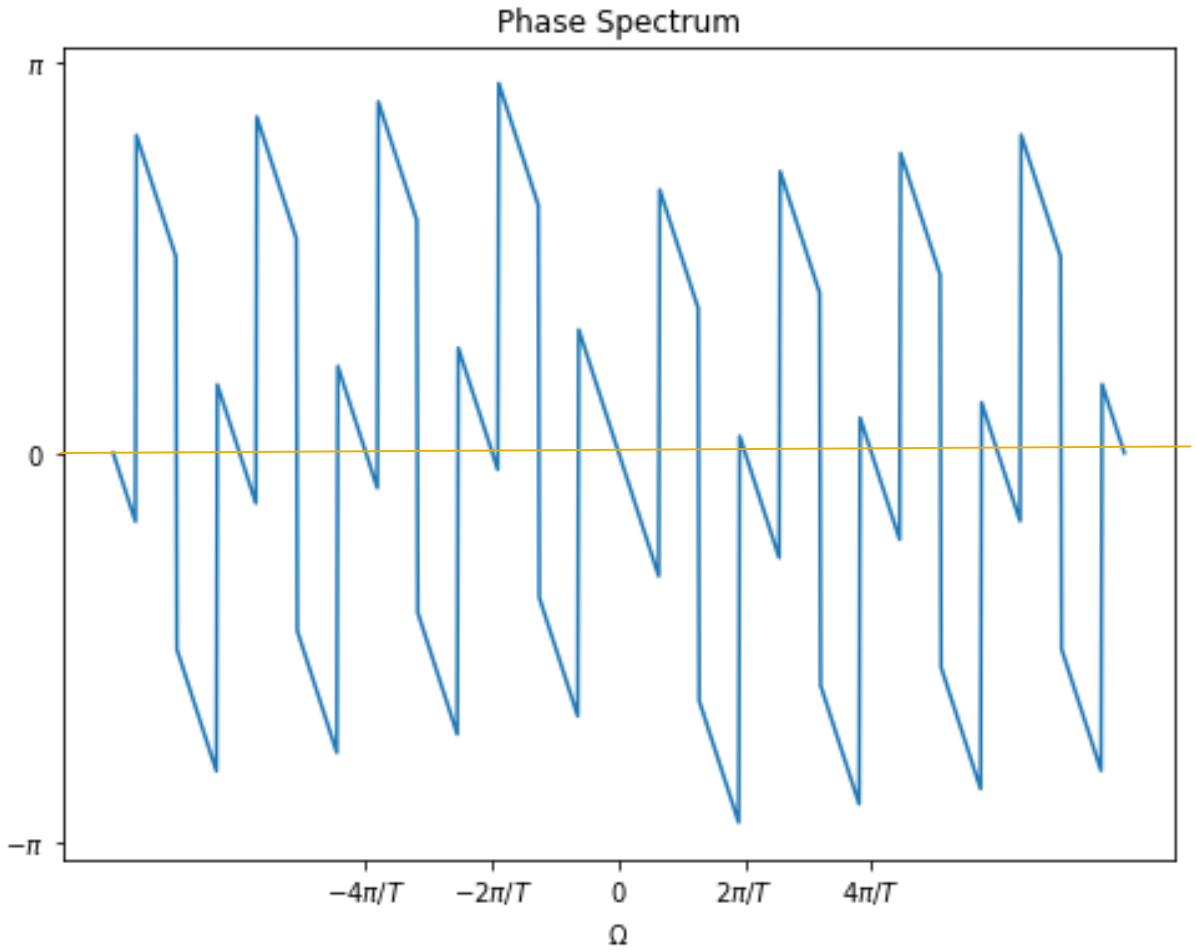
$$X(\Omega) = T \frac{\sin(\Omega T / 2)}{\Omega T / 2}$$



# Examples

$$X(\Omega) = Tsinc(\frac{\Omega T}{2})e^{-\frac{j\Omega T}{2}}$$



# Key Transform Pairs

$$e^{-at}u(t) \leftrightarrow \frac{1}{a + j\Omega}, a > 0$$

$$u(t) \leftrightarrow \frac{1}{j\Omega} + \pi\delta(\Omega)$$

$$\delta(t) \leftrightarrow 1$$

$$1 \leftrightarrow 2\pi\delta(\Omega)$$

$$e^{j\Omega_0 t} \leftrightarrow 2\pi\delta(\Omega - \Omega_0)$$

# Key Transform Pairs

$$\cos(\Omega_0 t) \leftrightarrow \pi[\delta(\Omega - \Omega_0) + \delta(\Omega + \Omega_0)]$$

$$\sin(\Omega_0 t) \leftrightarrow j\pi[\delta(\Omega + \Omega_0) - \delta(\Omega - \Omega_0)]$$

$$rect(\frac{t}{T}) \leftrightarrow T\, sinc(\frac{\Omega T}{2})$$

$$\frac{W}{\pi} sinc(Wt) \leftrightarrow rect(\frac{\Omega}{2W})$$

$$\sum_{n=-\infty}^{\infty} \delta(t - nT) \leftrightarrow \Omega_0 \sum_{n=-\infty}^{\infty} \delta(\Omega - n\Omega_0), \Omega_0 = \frac{2\pi}{T}$$