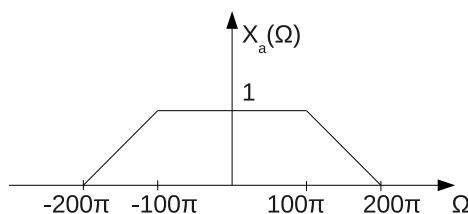


Homework 7

Prof. Zhi-Pei Liang

Due: April 2, 2021

1. The sequence $x[n] = \cos\left(\frac{\pi}{3}n\right)$, $-\infty < n < \infty$ was obtained by sampling the continuous-time signal $x_a(t) = \cos(\Omega_0 t)$, $-\infty < t < \infty$ at a sampling rate of 1000 samples/sec. What are two possible values of Ω_0 that could have resulted in the sequence $x[n]$?
2. The continuous-time signal $x_a(t) = \sin(10\pi t) + \cos(20\pi t)$ is sampled with a sampling period T to obtain the discrete-time signal $x[n] = \sin\left(\frac{\pi}{5}n\right) + \cos\left(\frac{2\pi}{5}n\right)$
 - a) Determine a choice for T consistent with this information.
 - b) Is your choice for T in part (a) unique? If so, explain why. If not, specify another choice of T consistent with the information given.
3. The continuous-time signal $x_a(t) = \cos(400\pi t)$ is sampled with a sampling period T to obtain a discrete-time signal $x[n] = x_a(nT)$
 - a) Compute and sketch the magnitude of the continuous-time Fourier transform of $x_a(t)$ and the discrete-time Fourier Transform of $x[n]$ for $T = 1$ ms.
 - b) Repeat part (a) for $T = 2$ ms.
 - c) What is the maximum sampling period T_{max} such that no aliasing occurs in the sampling process?
4. The continuous-time signal $x_a(t)$ has the continuous-time Fourier transform shown in the figure below. The signal $x_a(t)$ is sampled with sampling interval T to get the discrete-time signal $x[n] = x_a(nT)$. Sketch $X_d(\omega)$ (the DTFT of $x[n]$) for the sampling intervals $T = 1/100, 1/200$ sec.



5. Let $x[n] = x_a(nT)$. Show that the DTFT of $x[n]$ is related to the FT of $x_a(t)$ by

$$X_d(\omega) = \frac{1}{T} \sum_{\ell=-\infty}^{\infty} X\left(\frac{\omega + 2\ell\pi}{T}\right)$$

where $X_d(\omega)$ is the DTFT of $x[n]$ and $X(\Omega)$ the FT of $x_a(t)$.