**Homework 5**

**Prof. Zhi-Pei Liang**                                                                                    **Due: March 19, 2021**

1. Evaluate the following integrals:

    (a) $\int_{-\infty}^{\infty} (t^2 + 5t - 1)\delta(t)dt =$

    (b) $\int_{1}^{\infty} (t^2 + 5t - 1)\delta(t)dt =$

    (c) $[e^{-t}u(t)] * \delta(5t - 15) =$, where $u(t)$ is a unit step function.

2. Determine the Fourier transform of the following functions:

    (a) $\delta(2t - 3)$
    (b) $\sin(\Omega_0 t + \phi_0)$, where $\Omega_0$ and $\phi_0$ are known real numbers.
    (c) $u(t) - u(t - T)$, where $T$ is a constant.

3. Compute the discrete-time Fourier transform (DTFT) of the following sequence. $x[n] = \alpha^n \sin(\omega_0 n)u[n]$, where $\alpha$ and $\omega_0$ are real constants with $|\alpha| < 1$.
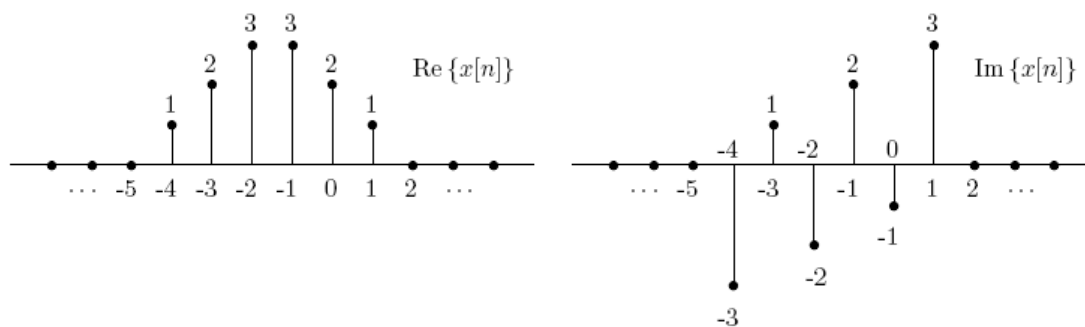
4. Let $X_d(\omega)$ denote the DTFT of the complex valued signal $x[n]$, where the real and imaginary parts of $x[n]$ are given below. Perform the following calculations **without** explicitly evaluating $X_d(\omega)$.

    a) Evaluate $X_d(0)$
    b) Evaluate $X_d(\pi)$
    c) Evaluate $\displaystyle\int_{-\pi}^{\pi} X_d(\omega)d\omega$
    d) Determine and sketch the signal whose DTFT is $X_d^*(-\omega)$

5. Let $x[n]$ be an arbitrary sequence, not necessarily real-valued, with DTFT $X_d(\omega)$. Express the DTFT of the following sequences in terms of $X_d(\omega)$

  a) $x^*[n]$

  b) $x^*[-n]$

6. Consider the complex sequence $x[n] = (u[n] - u[n-N])/N$.

  a) Find closed-form expressions for $|X_d(\omega)|$ and $\angle X_d(\omega)$.

  b) For $N = 5$, plot $|X_d(\omega)|$; How will the shape of $|X_d(\omega)|$ change if $N$ increases.

  c) For $N = 5$, plot $\angle X_d(\omega)$; How will the shape of $\angle X_d(\omega)$ change if $N$ increases.