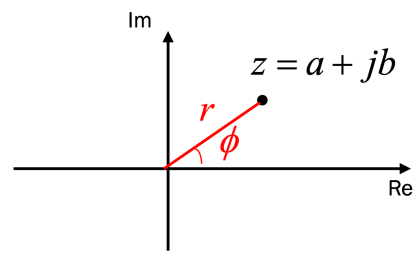
ECE 310 Recitation 1 Solution
Thursday Jan 21, 2021

Concept check



√ Complex number
- Cartesian form and polar form
  - Cartesian form: $z = a + jb, Re(z) = a, Im(z) = b$
  - Exponential/Polar form: $re^{j\phi}, r\angle\phi$
- Magnitude and phase
  - Magnitude: $|z| = r = \sqrt{a^2 + b^2}$
  - Phase: $\arg(z) = \phi = \tan^{-1}\frac{b}{a}$
    - Range $(-\pi, \pi)$
- Important formula
  - Euler formula: $z = re^{j\phi} = r(cos\phi + jsin\phi)$
  - De Moivre's formula: $e^{jn\phi} = (cos\phi + jsin\phi)^n = \cos(n\phi) + j\, sin(n\phi)$
- Operations
  - Conjugation (in Cartesian form and polar form)
  - Addition / subtraction (in Cartesian form)
  - Multiplication / division (in Cartesian form and polar form)

√ Linearity
- $\mathcal{H}\{a_1x_1[n] + a_2x_2[n]\} = a_1\mathcal{H}\{x_1[n]\} + a_2\mathcal{H}\{x_2[n]\}$

√ Time invariance / Shift invariance
- $y[n] = \mathcal{H}\{x[n]\} \Rightarrow y[n - n_0] = \mathcal{H}\{x[n - n_0]\}$

Exercise

1. Simplify the following complex expressions:

   a. j^j

   $= \left(e^{j\frac{\pi}{2}}\right)^j = e^{-\frac{\pi}{2}}$

   b. $\frac{e^{-\frac{j\pi}{6}}}{1-j}$

   $= \frac{e^{-j\pi/6}}{\sqrt{2}e^{-j\pi/4}} = \frac{1}{\sqrt{2}}e^{j\pi/12}$

2. Plot the magnitude and phase of the following functions:

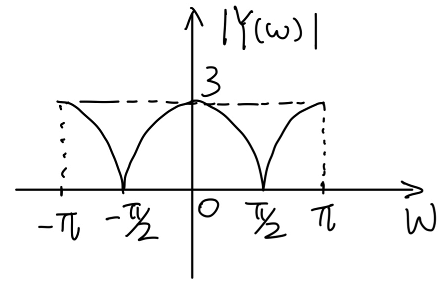
   a. $Y(\omega) = 3jcos(\omega)$

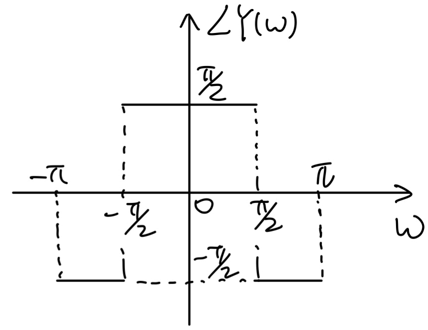
   b. $Y(\omega) = \frac{e^{j\omega/2} - e^{-j3\omega/2}}{2j}$

a. $Y(\omega) = 3j cos(\omega)$

$(j = e^{j\pi/2})$

$$|Y(\omega)| = 3|cos(\omega)|$$
$$\angle Y(\omega) = \begin{cases} \pi/2, & \cos(\omega) \geq 0 \\ -\pi/2, & \cos(\omega) < 0 \end{cases}$$

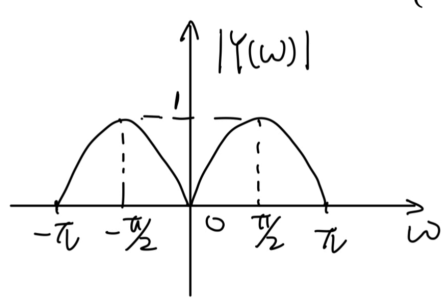


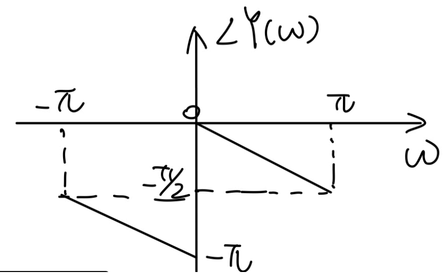


b. $Y(\omega) = \frac{e^{j\omega/2} - e^{-j3\omega/2}}{2j}$

$$Y(\omega) = e^{-j\omega/2}\frac{e^{j\omega} - e^{-j\omega}}{2j} = \sin(\omega)e^{-j\omega/2}$$

$$|Y(\omega)| = |sin(\omega)|$$
$$\angle Y(\omega) = \begin{cases} -\omega/2, & \sin(\omega) \geq 0 \\ -\omega/2 - \pi, & \sin(\omega) < 0 \end{cases}$$





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3. Determine if the following systems are: 1) linear, 2) time-invariant. Justify your statements.
   a. $y[n] = \max(0, x[n])$
   b. $y[n] = x[|n| - n]$
   c. $y[n] = nx[n]$

a. $y[n] = \max(0, x[n])$

1) **Non-linear**: superposition does not hold.

$\mathcal{H}\{a_1x_1[n] + a_2x_2[n]\} = \max(0, a_1x_1[n] + a_2x_2[n])$
$a_1\mathcal{H}\{x_1[n]\} + a_2\mathcal{H}\{x_2[n]\} = a_1 \max(0, x_1[n]) + a_2 \max(0, x_2[n])$
$\mathcal{H}\{a_1x_1[n] + a_2x_2[n]\} \neq a_1\mathcal{H}\{x_1[n]\} + a_2\mathcal{H}\{x_2[n]\}$

One counter example can be: $x_1[n] = -1, x_2[n] = 1$
Then $\mathcal{H}\{x_1[n] + x_2[n]\} = 0$; while $\mathcal{H}\{x_1[n]\} + \mathcal{H}\{x_2[n]\} = 0 = 1$

2) **Time invariant**:
Suppose $y[n] = \mathcal{H}\{x[n]\} = \max(0, x[n])$,
then $y[n - n_0] = \max(0, x[n - n_0])$.
Suppose $x'[n] = x[n - n_0]$,
then $\mathcal{H}\{x[n - n_0]\} = \mathcal{H}\{x'[n]\} = \max(0, x'[n]) = \max(0, x[n - n_0])$
$\mathcal{H}\{x[n - n_0]\} = y[n - n_0]$

b. $y[n] = x[|n| - n]$

1) **Linear**:
$\mathcal{H}\{a_1x_1[n] + a_2x_2[n]\} = a_1x_1[|n| - n] + a_2x_2[|n| - n]$
$= a_1\mathcal{H}\{x_1[n]\} + a_2\mathcal{H}\{x_2[n]\}$

2) **Time-varying**:
Suppose $y[n] = \mathcal{H}\{x[n]\} = x[|n| - n]$,
then $y[n - n_0] = x[|n - n_0| - (n - n_0)]$.
Suppose $x'[n] = x[n - n_0]$,
then $\mathcal{H}\{x[n - n_0]\} = \mathcal{H}\{x'[n]\} = x'[|n| - n] = x[(|n| - n) - n_0]$.
$\mathcal{H}\{x[n - n_0]\} \neq y[n - n_0]$

c. $y[n] = nx[n]$

1) **Linear**:
$\mathcal{H}\{a_1x_1[n] + a_2x_2[n]\} = n(a_1x_1[n] + a_2x_2[n])$
$= a_1nx_1[n] + a_2nx_2[n] = a_1\mathcal{H}\{x_1[n]\} + a_2\mathcal{H}\{x_2[n]\}$

2) **Time-varying**:
Suppose $y[n] = \mathcal{H}\{x[n]\} = nx[n]$,
then $y[n - n_0] = (n - n_0)x[n - n_0]$.
Suppose $x'[n] = x[n - n_0]$,
then $\mathcal{H}\{x[n - n_0]\} = \mathcal{H}\{x'[n]\} = nx'[n] = nx[n - n_0]$.
$\mathcal{H}\{x[n - n_0]\} \neq y[n - n_0]$