ECE 310

Digital Signal Processing

Spring, 2021, ZJUI Campus

Lecture 3

Topics:

- ✓ Discrete-time signals
- ✓ Properties of discrete-time systems

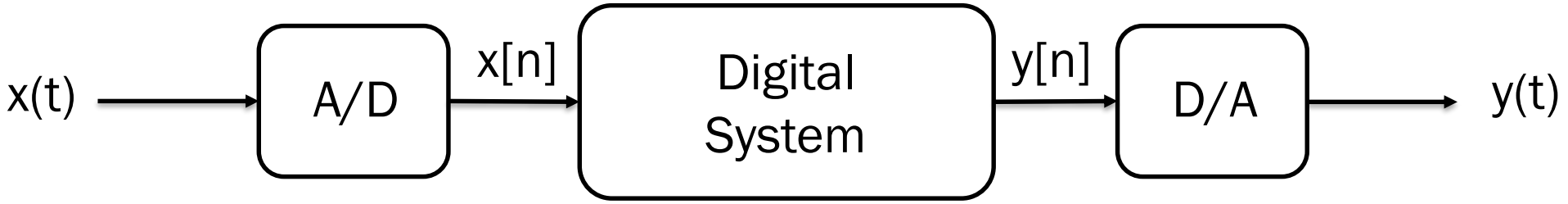
Objectives:

- ✓ Understand linearity and how to determine if a system is linear or not
- ✓ Understand shift-invariance and how to determine if a system is shift-invariant or shift-varying
- ✓ Understand causality and how to determine if a system is causal or not

System Analysis

- **210**

- **310**

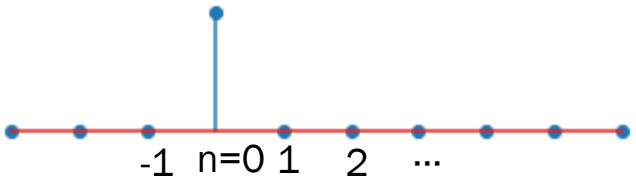


Discrete-Time Signals

Special discrete-time signals:

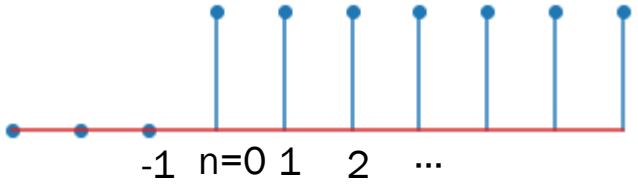
a) Delta Function

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & else \end{cases}$$



b) Step Function

$$\mathrm{u}[n] = \begin{cases} 1, & n \geq 0 \\ 0, & else \end{cases}$$



Representation of Discrete-Time Signals

$$x[n] = \{\ldots, x[-1], x[0], x[1], x[2], \ldots\}$$

$$= \{\ldots x_{-1,}\ x_{0,}\ x_{1,}\ x_{2}\ \ldots\}$$

$$x[n] = \sum_{l=-\infty}^{+\infty} x_l \delta[n-l]$$

System Properties

- **Linearity**
- **Shift-invariance (Time invariance)**
- **Causality**
- **Stability**

Linearity

- Examples

$(a)\ \ y[n] + \cos[n]\, y[n-1] = x[n]$

$(b)\ \ y[n] = \cos(x[n])$

$(c)\ \ \ y[n] = x(|n|)$

$(d)\ \ y[n] = 5x[n] + 3$

- Definition

$$x_1[n] \rightarrow y_1[n]$$

$$x_2[n] \rightarrow y_2[n]$$

$$ax_1[n] + bx_2[n] \rightarrow ay_1[n] + by_2[n]$$

* a, b constants.
scaling, additive

Linearity: Examples

Example: show system (a) is linear

$$y[n] + \cos(n)\, y[n-1] = x[n]$$

Example: show system (d) is nonlinear

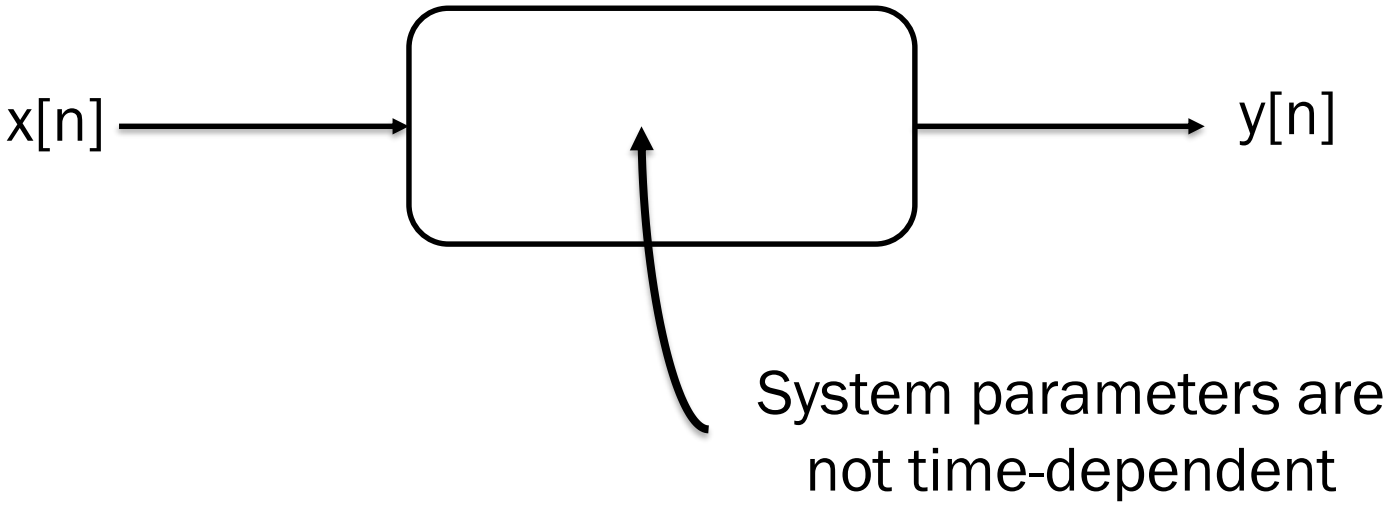
$$y[n] = 5x[n] + 3$$

Shift Invariance

- Definition

If $x[n] \rightarrow y[n]$

then $x[n - n_0] \rightarrow y[n - n_0]$



Shift-Invariance: Examples

Example: show system (a) is time-varying

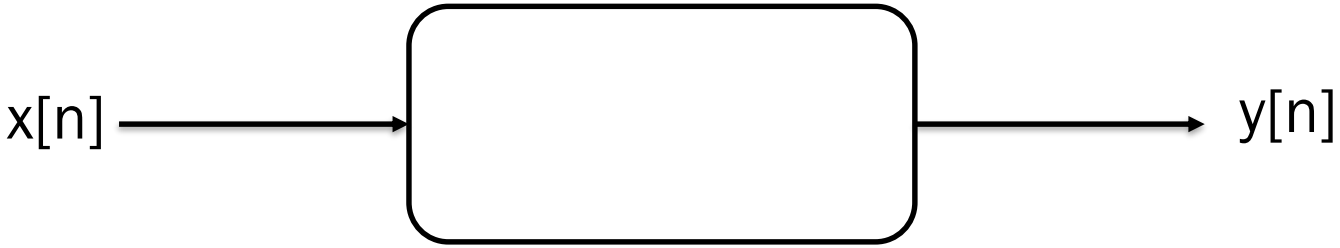
$$y[n] + \cos(n)\, y[n-1] = x[n]$$

Example: show system (b) is time-invariant

$$y[n] = \cos(x[n])$$

Causality

- Definition



- y[n] depends on past and present input x

Causality: Examples

Examples:

$(a)\quad y[n] = \frac{1}{3}(x[n-1] + x[n] + x[n+1])$

$(b)\quad y[n] + 2y[n-1] = x[n]$