Recitation 4 Sol

Question 1

a) $Im\{X_d(\omega)\}$

By $x[n]$ is real,

$$X_d(\omega) = X_d^*(-\omega)$$

Since $x[n] = x[-n]$,

$$X_d(\omega) = X_d(-\omega)$$

Therefore,

$$X_d^*(-\omega) = X_d(-\omega)$$

Thus,

$$Im\{X_d(\omega)\} = 0$$

b) $\int_{-\pi}^{\pi} X_d(\omega)d\omega$

By inverse DTFT,

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_d(\omega)e^{j\omega n}d\omega$$

Take $n = 0$ and multiply 2π on both side

$$\int_{-\pi}^{\pi} X_d(\omega)d\omega = 2\pi x[0] = -6\pi$$

c) $X_d(\pi)$

By DTFT,

$$X_d(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

Therefore,

$$X_d(\pi) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\pi n} = \sum_{n=-2}^{2} x[n](-1)^n$$
$$= -1 - 2 - 3 - 2 - 1 = -9$$

d) $\int_{-\pi}^{\pi} |X_d(\omega)|^2 d\omega$

By Parseval's relation, if $x[n]$ has finite energy, the following equation holds.

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X_d(\omega)|^2 d\omega$$

Therefore,

$$\int_{-\pi}^{\pi} |X_d(\omega)|^2 d\omega = 2\pi \sum_{n=-\infty}^{\infty} |x[n]|^2$$
$$= 2\pi(1 + 4 + 9 + 4 + 1) = 38\pi$$

Question 2

$$x[n] = u[n] - u[n-N]$$

When $n = 4$, by DTFT,

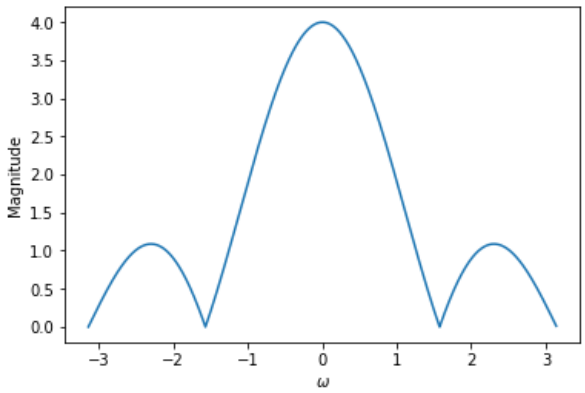
$$X_d(\omega) = \sum_{n=0}^{3} e^{-j\omega n} = \frac{1 - e^{-j\omega \cdot 4}}{1 - e^{-j\omega}} = \frac{e^{-j2\omega}(e^{j2\omega} - e^{-j2\omega})}{e^{-j\frac{\omega}{2}}(e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}})} = e^{-j\frac{3}{2}\omega}\frac{\frac{e^{j2\omega} - e^{-j2\omega}}{2j}}{\frac{e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}}{2j}} = e^{-j\frac{3}{2}\omega}\frac{\sin(2\omega)}{\sin\left(\frac{\omega}{2}\right)}$$

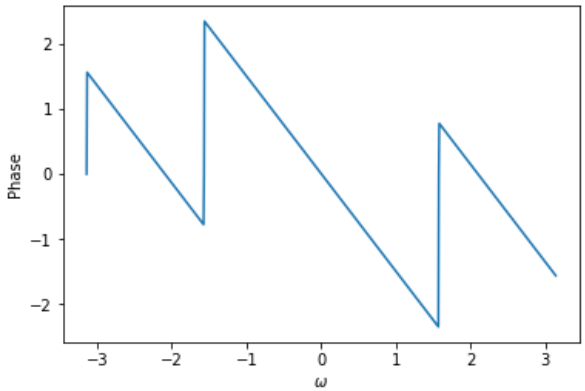
Magnitude:

$$M = \left|\frac{\sin(2\omega)}{\sin\left(\frac{\omega}{2}\right)}\right|$$

Phase:

$$P = -\frac{3}{2}\omega$$





When $n = 5$,

$$X_d(\omega) = \sum_{n=0}^{4} e^{-j\omega n} = \frac{1 - e^{-j\omega \cdot 5}}{1 - e^{-j\omega}} = \frac{e^{-j\frac{5}{2}\omega}(e^{j\frac{5}{2}\omega} - e^{-j\frac{5}{2}\omega})}{e^{-j\frac{\omega}{2}}(e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}})} = e^{-j2\omega}\frac{\frac{e^{j\frac{5}{2}\omega} - e^{-j\frac{5}{2}\omega}}{2j}}{\frac{e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}}{2j}} = e^{-j2\omega}\frac{\sin(\frac{5}{2}\omega)}{\sin(\frac{\omega}{2})}$$

Magnitude:

$$M = |\frac{\sin(\frac{5}{2}\omega)}{\sin(\frac{\omega}{2})}|$$

Phase:

$$P = -2\omega$$