

Concept check

✓ Complex number

○ Cartesian form and polar form

- Cartesian form: $z = a + jb, \text{Re}(z) = a, \text{Im}(z) = b$
- Exponential/Polar form: $re^{j\phi}, r\angle\phi$

○ Magnitude and phase

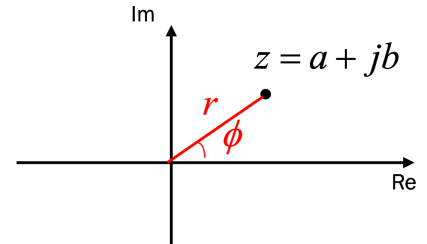
- Magnitude: $|z| = r = \sqrt{a^2 + b^2}$
- Phase: $\arg(z) = \phi = \tan^{-1} \frac{b}{a}$
 - Range $(-\pi, \pi)$

○ Important formula

- Euler formula: $z = re^{j\phi} = r(\cos\phi + j\sin\phi)$
- De Moivre's formula: $e^{jn\phi} = (\cos\phi + j\sin\phi)^n = \cos(n\phi) + j\sin(n\phi)$

○ Operations

- Conjugation (in Cartesian form and polar form)
- Addition / subtraction (in Cartesian form)
- Multiplication / division (in Cartesian form and polar form)



✓ Linearity

- $\mathcal{H}\{a_1x_1[n] + a_2x_2[n]\} = a_1\mathcal{H}\{x_1[n]\} + a_2\mathcal{H}\{x_2[n]\}$

✓ Time invariance / Shift invariance

- $y[n] = \mathcal{H}\{x[n]\} \Rightarrow y[n - n_0] = \mathcal{H}\{x[n - n_0]\}$

Exercise

1. Simplify the following complex expressions:

- a. j^j
- b. $\frac{e^{-j\pi/6}}{1-j}$

2. Plot the magnitude and phase of the following functions:

- a. $Y(\omega) = 3j\cos(\omega)$
- b. $Y(\omega) = \frac{e^{j\omega/2} - e^{-j3\omega/2}}{2j}$

3. Determine if the following systems are: 1) linear, 2) time-invariant. Justify your statements.

- a. $y[n] = \max(0, x[n])$
- b. $y[n] = x[|n| - n]$
- c. $y[n] = nx[n]$