

**Homework 3****Prof. Zhi-Pei Liang****Due: March 5, 2021**

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1. Show that an LSI system with unit pulse response  $h[n]$  is causal if and only if  $h[n] = 0$  for  $n < 0$ .
2. Show that an LSI system with unit pulse response  $h[n]$  is BIBO-stable if and only if  $\sum_{n=-\infty}^{\infty} |h[n]|$  is bounded (i.e.,  $h[n]$  is absolutely summable).
3. Determine whether each of the following systems that map input signal  $\{x[n]\}$  to output signal  $\{y[n]\}$  is BIBO stable.
  - (a)  $y[n] = x^5[n] + 3$
  - (b)  $y[n] = x[n] * u[n]$
  - (c)  $y[n] = nx[n]$
  - (d)  $y[n] = \frac{x[n]}{x[1]}$
  - (e)  $y[n] = x[n] * h[n]$ , where  $h[n] = \begin{cases} 0 & \text{for } n < 0 \\ 2^{(n+1)^2} & \text{for } 0 \leq n < 100 \\ 0.5^n & \text{for } n \geq 100 \end{cases}$
4. Determine the  $z$ -transform and sketch the ROC for each of the following sequences:
  - (a)  $x[n] = \delta[n+1] - 2\delta[n-2]$
  - (b)  $\{x[n]\} = \{-1, 0, 1, 2, 3\}$   
 $\quad \quad \quad \uparrow$
  - (c)  $x[n] = \left(\frac{1}{2}\right)^{n-1} u[n-2]$
  - (d)  $x[n] = 2\left(\frac{1}{2}\right)^n u[n-2] + 3\left(\frac{1}{3}\right)^{n-3} u[n+3]$
5. Given the  $z$ -transform pair

$$x[n] \longleftrightarrow X(z) = \frac{1}{1 - (1/3)z^{-1}}, \quad \text{with ROC: } |z| > 1/3,$$

use the  $z$ -transform properties to determine the  $z$ -transform and ROC of the following sequences

- (a)  $y[n] = x[n-1]$
- (b)  $y[n] = n^2 x[n]$
- (c)  $y[n] = 2^n x[n]$
- (d)  $y[n] = \cos(\pi n/4) x[n]$
- (e)  $y[n] = (x * u)[n]$
- (f)  $y[n] = (x * h)[n]$  where  $h[n] = x[n-2]$