



# ECE 310

# Digital Signal Processing



**Spring, 2021, ZJUI Campus**

# Lecture 6

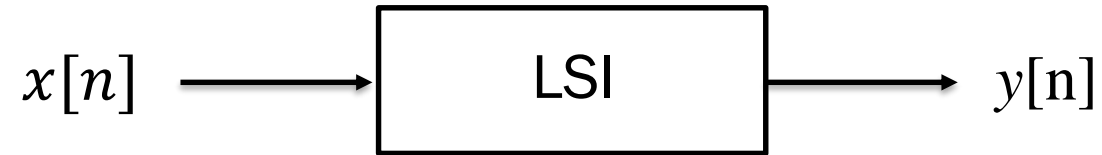
## Topics:

- ✓ Linear Constant Coefficient Difference Equations (LCCDE)
- ✓ Zero-state response; zero-input response; total response

## Educational Objectives:

- ✓ Understand what is LCCDE: standard delay form and standard advance form
- ✓ Understand what is zero-state response of LCCDE
- ✓ Understand what is zero-input response of LCCDE
- ✓ Understand the iterative method for finding responses of an LCCDE system

# Linear Constant Coefficient Difference Equation (LCCDE)



Standard form I (delay form):

$$y[n] + a_1 y[n-1] + \cdots + a_N y[n-N] = b_0 x[n] + b_1 x[n-1] + \cdots + b_N x[n-N]$$

$$y[n] + \sum_{k=1}^N a_k y[n-k] = \sum_{k=1}^N b_k x[n-k]$$

Standard form II (advance form):

$$y[n] + a_1 y[n+1] + \cdots + a_N y[n+N] = b_0 x[n] + b_1 x[n+1] + \cdots + b_N x[n+N]$$

$$y[n] + \sum_{k=1}^N a_k y[n+k] = \sum_{k=1}^N b_k x[n+k]$$

# Linear Constant Coefficient Difference Equation (LCCDE)

Example: put an LCCDE in standard form.

$$y[n+1] - \frac{1}{2} y[n-2] = x[n] + \frac{1}{3} x[n-1]$$

# Linear Constant Coefficient Difference Equation (LCCDE)

$$y[n] + \sum_{k=1}^N a_k y[n-k] = \sum_{k=1}^N b_k x[n-k]$$

Initial conditions:  $y[-1], y[-2], \dots, y[-N]$


**Zero-state response:** Solution to the LCCDE equation with zero initial conditions

**Zero-input response:** Solution to the LCCDE equation with zero input

**Warning:** Linearity doesn't apply to the total response!

**Trick:**

$$y[n] = y_x[n] + y_s[n]$$



Zero-state      Zero-input

# Linear Constant Coefficient Difference Equation (LCCDE)

Example: Solving LCCDE by iteration

$$y[n] - \frac{1}{2} y[n-1] = x[n]; \quad y[-1] = 1$$

# Motivation for Z-transform for Analysis of LSI Systems

## ECE 210

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Linear constant coefficient **differential** equation



Laplace Transform

## ECE 310

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Linear constant coefficient **difference** equation



Z-Transform

Example: Determine the unit pulse response of an LSI system that gives

$$y[n] = 3^{-n}u[n] + 5 \times 2^{-n}u[n] \text{ for input } x[n] = 2^{-n}u[n]$$