ECE 310

Digital Signal Processing

Spring, 2021, ZJUI Campus

Lecture 5

Topics:

- ✓ Convolution: definition and key properties
- ✓ Calculation of convolution in time-domain

Educational Objectives:

- ✓ Understand why convolution is important for analysis of LSI system
- ✓ Understand the definition and key properties of convolution
- ✓ Know how to evaluate convolution in time domain

Why is Convolution Important for LSI System Analysis?

$$\delta[n] \longrightarrow \boxed{\text{LSI}} \longrightarrow h[\mathrm{n}]$$

Shift-invariance: $\delta[n-k] \longrightarrow h[n-k]$

Linearity: $x_k\delta[n-k] \longrightarrow x_k h[n-k]$

$$\sum_{k=-\infty}^{\infty} x_k\delta[n-k] \longrightarrow \sum_{k=-\infty}^{\infty} x_k h[n-k]$$

$$x[n] \longrightarrow \boxed{y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]}$$

Convolution: Definition

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

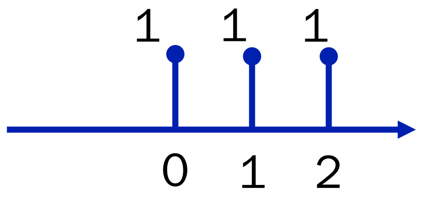
$$y[n] = x[n] * h[n] = h[n] * x[n]$$

Convolution: Evaluation Methods

- Graphical method
- Table method
- Analytical method
- Z-domain method
- FFT method

Graphical Method

Example: $h[n] = \{1, 1, 1\} * \, x[n] = \{1, 2, 3\}$



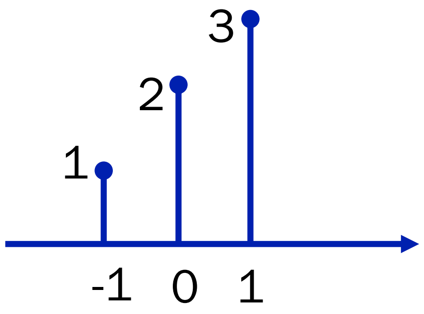


Table Method

	$\boldsymbol{x_{-1}}$	$\boldsymbol{x_0}$	$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	…
$\boldsymbol{h_{-1}}$	$h_{-1}x_{-1}$	$h_{-1}x_0$	$h_{-1}x_1$	$h_{-1}x_2$	
$\boldsymbol{h_0}$	h_0x_{-1}	h_0x_0	h_0x_1	h_0x_2	
$\boldsymbol{h_1}$	h_1x_{-1}	h_1x_0	h_1x_1	h_1x_2	
$\boldsymbol{h_2}$	h_2x_{-1}	h_2x_0	h_2x_1	h_2x_2	
$\boldsymbol{h_3}$	h_3x_{-1}	h_3x_0	h_3x_1	h_3x_2	
…					

Example $\{1,1,1\} * \{1,2,3\}$

	1	1	1
1	1	1	1
2	2	2	2
3	3	3	3

$\{1,3,6,5,3\}$

Analytical Method

Basic formula

$$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a}, \qquad |a| < 1$$

$$\sum_{k=0}^{N} a^k = \frac{1-a^{N+1}}{1-a}$$

* Determining the summation limit!

Analytical method

Example 1: Evaluate $r_1^n u[n] * r_2^n u[n]$

Example 2: Evaluate $x[n] * h[n]$

$$x[n] = \begin{cases} r_1^n, & n \geq 2 \\ r_2^n, & n < 2 \end{cases}$$

$$h[n] = u[n]$$

Properties

- Commutative: $x_1 * x_2 = x_1 * x_2$
- Distributive: $x_1 * (x_2 + x_3) = x_1 * x_2 + x_1 * x_3$
- Associative: $x_1 * (x_2 * x_3) = (x_1 * x_2) * x_3$
- Shifting:

$$\text{Let } x[n] * h[n] = y[n],$$

$$\text{then } x[n-n_0] * h[n] = x[n] * h[n-n_0] = y[n-n_0]$$

$$\begin{cases} \delta[n] * x[n] = x[n] \\ \delta[n-n_0] * x[n] = x[n-n_0] \\ \delta[n]x[n] = x[0]\delta[n] \\ \delta[n-n_0]x[n] = x[n_0]\delta[n-n_0] \end{cases}$$

Properties

Example:

Let $u[n] * 3^n u[n] = \frac{1-3^{n+1}}{1-3} u[n]$,

Find $u[n] * 3^n u[n-2]$

(1) $$\frac{1-3^{n+1}}{1-3} u[n-2]$$

(2) $$\frac{1-3^{n-1}}{1-3} u[n-2]$$

(3) $$9\frac{1-3^{n-1}}{1-3} u[n-2]$$