ECE 310

Digital Signal Processing

Spring, 2021, ZJUI Campus

Lecture 25

Topics:

- ✓ Introduction to digital filter design

Educational Objectives:

- ✓ Understand FIR filters, IIR filters, their filter structures and characteristics
- ✓ Understand generalized linear phase

Digital Filter Design

$$x_a(t) \longrightarrow \boxed{H_a(\Omega)} \longrightarrow y_a(t)$$

A desired $H_a(\Omega)$

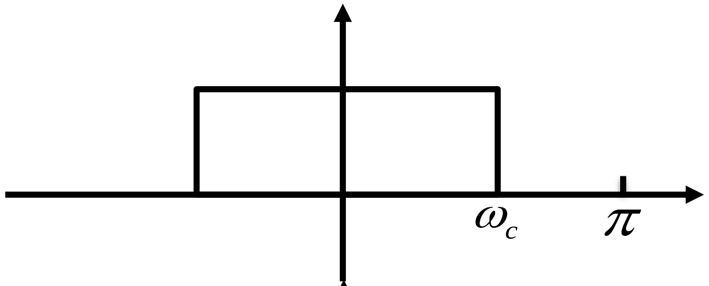
$$\downarrow$$

A desired $H_d(\omega) = H_a(\frac{\omega}{T})$

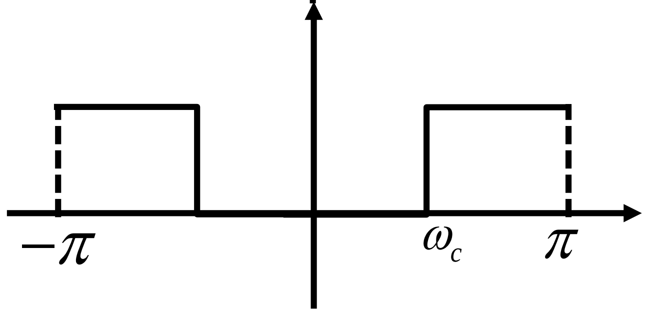
$$x[n] \longrightarrow \boxed{H_d(\omega)} \longrightarrow y[n] \qquad \begin{cases} |H_d(\omega)|: \ \text{magnitude response} \\ \angle H_d(\omega): \ \text{phase response} \end{cases}$$

Classification of Filters

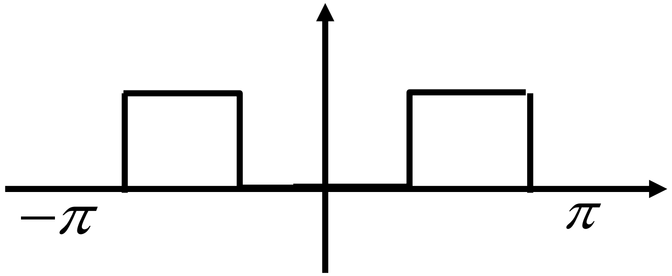
-Low-pass filter (LP)



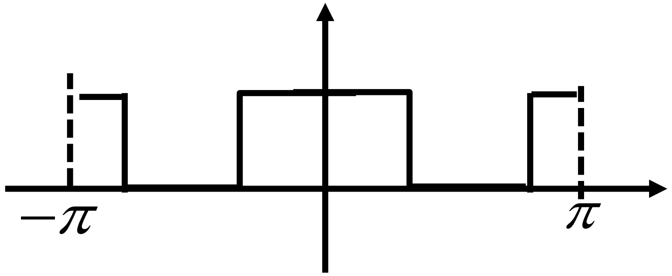
-High-pass filter (HP)



-Band-pass filter (BP)



-Band-stop filter (BS)



Passband, Stop Band and Transition Band

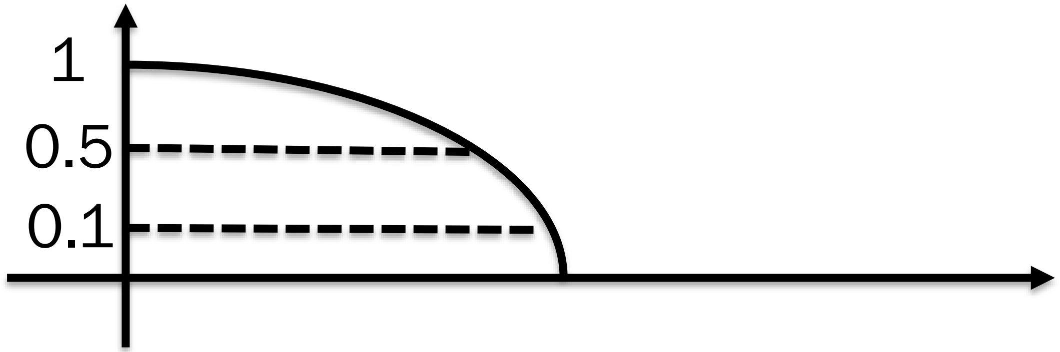
Magnitude: dB (Decibel)

$$20\log_{10}|H_d(\omega)| \quad or \quad 10\log_{10}|H_d(\omega)|^2$$

0db

-6db

-20db



Generalized Linear Phase (GLP)

$$\angle H_d(\omega) = -\omega M$$ linear phase

Generalized LP:

$$H_d(\omega) = R(\omega)e^{-j\omega M} \qquad \text{(type1)}$$

$$H_d(\omega) = R(\omega)e^{j(\alpha-\omega M)} \qquad \text{(type2)}$$

$R(\omega)$: a real function

α: $\pi/2$

Examples

Determine the phase response of the following filters

$$a)\ \{h_n\}_{n=0}^{2} = \{1, -1, 1\}$$

$$b)\ \{h_n\}_{n=0}^{2} = \left\{-\frac{1}{4},\ 1,\ -\frac{1}{4}\right\}$$

$$c)\ \{h_n\}_{n=1}^{2} = \{1,\ \ -1\}$$

Pole-Zero Filters

$$H(z) = \frac{\sum_{k=0}^{N-1} b_k z^{-k}}{1 + \sum_{k=1}^{N-1} a_k z^{-k}}$$

$$y[n] + a_1 y[n-1] + \ldots + a_{N-1} y[n-N+1] = b_0 x[n] + \ldots + b_{N-1} x[n-N+1]$$

FIR: $H(z) = \sum_{k=0}^{N-1} b_k z^{-k}$

$$h[n] = \{b_0, \ldots, b_{N-1}\}$$

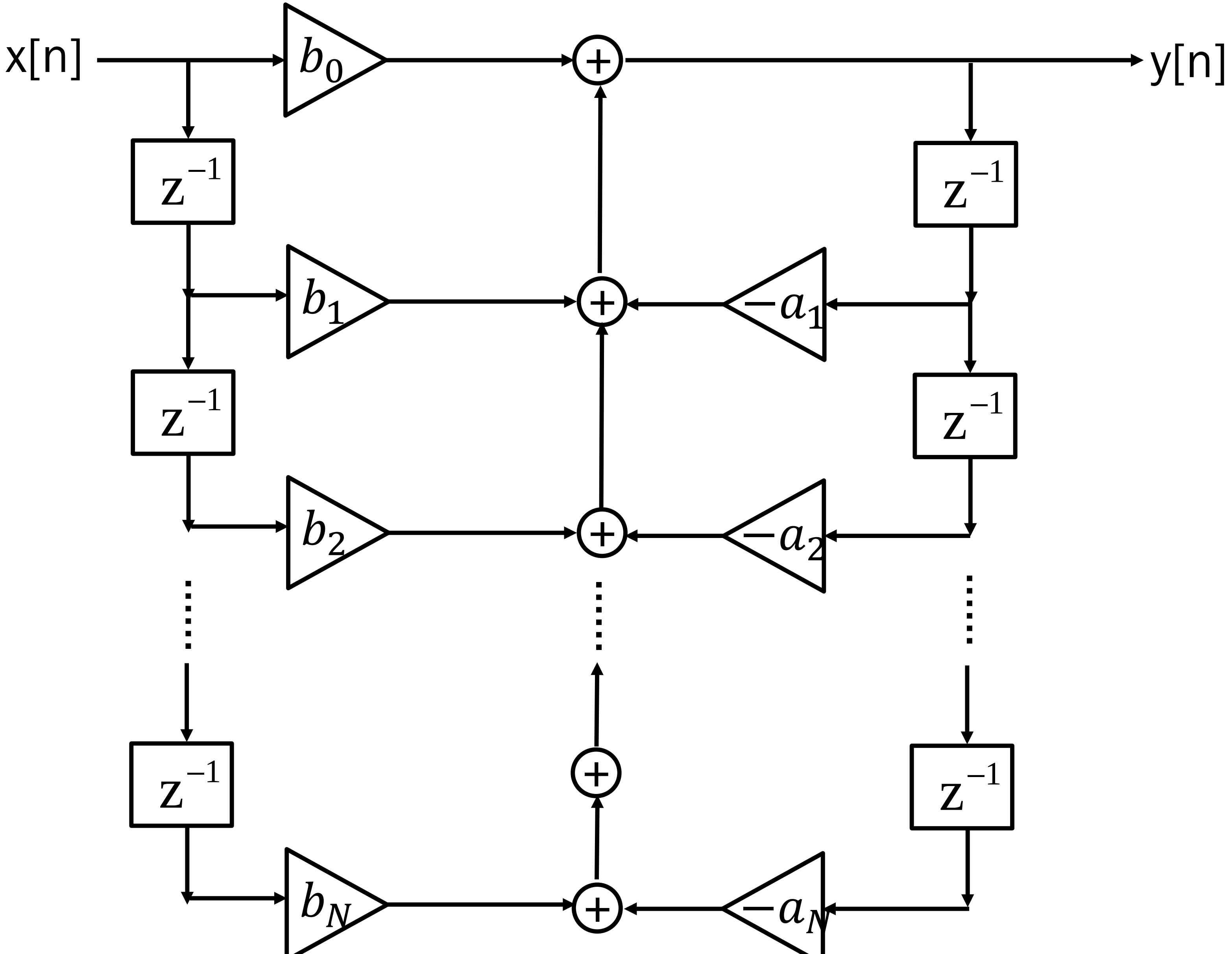
- "good" phase response
- not so "good" magnitude response

IIR: $H(z) = \frac{\sum_{k=0}^{N-1} b_k z^{-k}}{1 + \sum_{k=1}^{N-1} a_k z^{-k}}$

e.g. $h[n] = (\frac{1}{2})^n u[n], H(z) = \frac{z}{z - 1/2}$

- not so "good" phase response
- "good" magnitude response

Direct Form I



Direct Form II