



# ECE 310

# Digital Signal Processing



**Spring, 2021, ZJUI Campus**

# Lecture 17

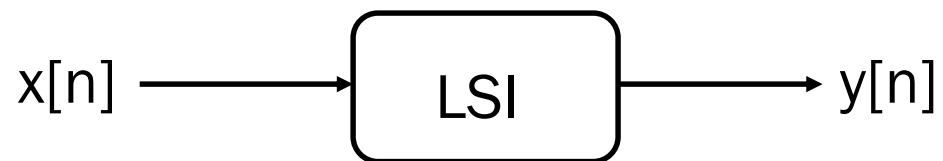
## Topics:

- ✓ Frequency response of stable LSI systems
- ✓ Steady-state analysis of stable LSI systems

## Educational Objectives:

- ✓ Understand what is frequency response
- ✓ Understand how frequency response is related to unit pulse response and transfer function
- ✓ Understand how to calculate system's output for sinusoidal input

# Frequency Response



$$y[n] = x[n] * h[n]$$

$$H_d(\omega) = DTFT\{h[n]\}$$

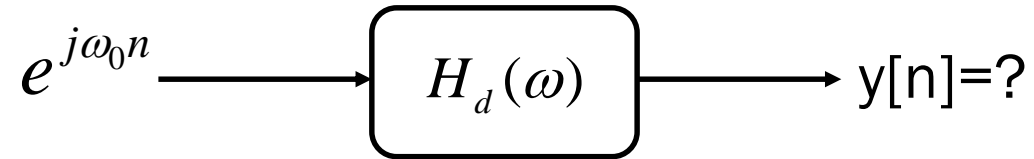
$$Y_d(\omega) = H_d(\omega)X_d(\omega)$$

$|H_d(\omega)|$ : magnitude response

$\angle H_d(\omega)$ : phase response

# Steady-State Analysis of LSI Systems

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X_d(\omega) e^{+j\omega n} d\omega$$



$$y[n] = H_d(\omega_0) e^{j\omega_0 n} = |H_d(\omega_0)| e^{j(\omega_0 n + \angle H_d(\omega_0))}$$

Proof:

# Sinusoidal Function Input

$$\cos(\omega_0 n) \longrightarrow \boxed{H_d(\omega)} \longrightarrow \frac{1}{2} H_d(\omega_0) e^{j\omega_0 n} + \frac{1}{2} H_d(-\omega_0) e^{-j\omega_0 n}$$

$$\sin(\omega_0 n) \longrightarrow \boxed{H_d(\omega)} \longrightarrow \frac{1}{2j} H_d(\omega_0) e^{j\omega_0 n} - \frac{1}{2j} H_d(-\omega_0) e^{-j\omega_0 n}$$

If the system is real-valued,  $h[n]$  is real-valued

$H_d(\omega)$  has Hermitian Symmetry

$$H_d(\omega) = H_d^*(-\omega)$$

$$\text{or } |H_d(\omega)| = |H_d(-\omega)|$$

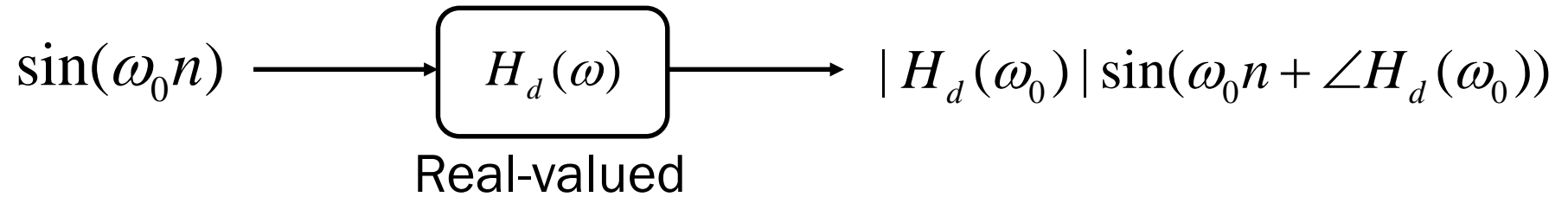
$$\angle H_d(\omega) = -\angle H_d(-\omega)$$

# Sinusoidal Function Input

$$\begin{aligned}y[n] &= \frac{1}{2} H_d(\omega_0) e^{j\omega_0 n} + \frac{1}{2} H_d^*(\omega_0) e^{-j\omega_0 n} \\&= \frac{1}{2} [H_d(\omega_0) e^{j\omega_0 n} + (H_d(\omega_0) e^{j\omega_0 n})^*] \\&= \frac{1}{2} [|H_d(\omega_0)| e^{j(\omega_0 n + \angle H_d(\omega_0))} + (|H_d(\omega_0)| e^{j(\omega_0 n + \angle H_d(\omega_0))})^*] \\&= \frac{1}{2} |H_d(\omega_0)| [e^{j(\omega_0 n + \angle H_d(\omega_0))} + e^{-j(\omega_0 n + \angle H_d(\omega_0))}] \\&= |H_d(\omega_0)| \cos(\omega_0 n + \angle H_d(\omega_0))\end{aligned}$$

# Sinusoidal Function Input

By the same analysis



Or

$$\sin(\omega_0 n + \phi_0) \qquad |H_d(\omega_0)| \sin(\omega_0 n + \phi_0 + \angle H_d(\omega_0))$$

# Examples

$$y[n] = x[n] + 2x[n-1]$$

Find  $y[n]$  due to  $x[n] = \cos(\frac{\pi}{2}(n-1)) + 1 + j^n$

a) Check “real-valued”? YES!

b) Check frequency components

$$\omega_1 = \frac{\pi}{2}, \omega_2 = 0$$

$$H_d(\frac{\pi}{2}), H_d(0)$$



# Example

$$H(z) = 1 + 2z^{-1}; ROC_d : |z| > 0$$

$$H_d(\omega) = 1 + 2e^{-j\omega}$$

$$H_d\left(\frac{\pi}{2}\right) = 1 + 2e^{-j\frac{\pi}{2}} = 1 - j2 = \sqrt{5}e^{-j63.45^\circ}$$

$$H_d(0) = 1 + 2 = 3e^{j0}$$

$$y[n] = \sqrt{5} \cos\left(\frac{\pi}{2}(n-1) - 63.45^\circ\right) + 3 + \sqrt{5}e^{j\left(\frac{\pi}{2}n - 63.45^\circ\right)}$$

# Example

$$H_d(\omega) = \cos \omega e^{j\pi \cos \omega}$$

$$x[n] = \cos\left(\frac{\pi}{4}n + 5^\circ\right) + e^{j\frac{\pi}{2}n}$$

Not a real-valued system,  $\angle H_d(\omega)$  does not have odd symmetry

$$x[n] = \frac{1}{2} \left( e^{j(\frac{\pi}{4}n + 5^\circ)} + e^{-j(\frac{\pi}{4}n + 5^\circ)} \right) + e^{j\frac{\pi}{2}n}$$

$$H_d\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} e^{j\pi\frac{\sqrt{2}}{2}}$$

$$H_d\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} e^{j\pi\frac{\sqrt{2}}{2}}$$

$$H_d\left(\frac{\pi}{2}\right) = 0$$

$$y[n] = \frac{1}{2} \frac{\sqrt{2}}{2} e^{j\pi\frac{\sqrt{2}}{2}} e^{j(\frac{\pi}{4}n + 5^\circ)} + \frac{1}{2} \frac{\sqrt{2}}{2} e^{j\pi\frac{\sqrt{2}}{2}} e^{-j(\frac{\pi}{4}n + 5^\circ)}$$

$$= \frac{\sqrt{2}}{2} e^{j\pi\frac{\sqrt{2}}{2}} \cos\left(\frac{\pi}{4}n + 5^\circ\right)$$

$$\neq \frac{\sqrt{2}}{2} \cos\left(\frac{\pi}{4}n + 5^\circ + \pi\frac{\sqrt{2}}{2}\right)$$