ZHEJIANG UNIVERSITY - UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

ECE 310 Digital Signal Processing

Homework 9

Prof. Zhi-Pei Liang **Due: April 16, 2021**

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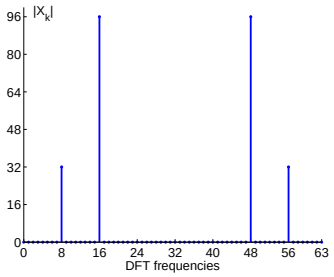
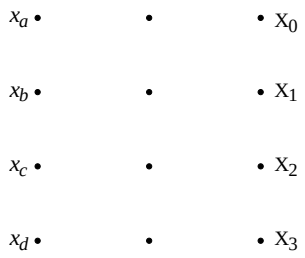
1. A 3.0 sec. segment of $\{x_a(t)\}_{t=0}^{3.0} = \cos(0.2\pi t)$ is sampled at a rate of $1/T = 30$ Hz. The resulting 90 samples are zero padded to 128 and the DFT $\{X[k]\}_{k=0}^{127}$ is computed. Determine k_0 such that $|X[k_0]| \geq |X[k]|$ for $k = 0, 1, \cdots, 63$.

2. Assume that $x_a(t) = \sum_{\ell=1}^{L} A_\ell \cos(\Omega_\ell t)$, where the A_ℓ have positive values. We further assume that $x_a(t)$ is measured at $t = nT$ for $T = 1/8$ second and $n = 0, 1, \ldots, 63$ to obtain $\{x_n\}_{n=0}^{63} = \{x_a(nT)\}_{n=0}^{63}$. The 64-point DFT of $\{x_n\}_{n=0}^{63}$ is represented by $\{X_k\}_{k=0}^{63}$, whose magnitude is shown in the figure below.

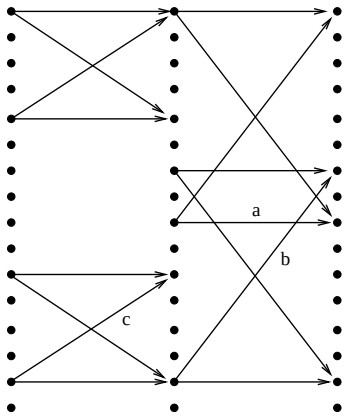


   Determine L, and A_ℓ and Ω_ℓ for $\ell = 1, 2, \ldots, L$.

3. Complete the following signal flow diagram (butterfly structure) of a 4-pt, radix-2, decimation-in-time FFT algorithm. Specify all the connection weights and determine the indexes (a, b, c, and d) of the input signal sequence.



4. The diagram below represents a part of the computation in a 16-point decimation-in-time radix-2 FFT. Indicate the values of the three requested branch weights a, b and c.



5. Determine z_n, the cyclic convolution of x_n and y_n for the following cases:

   (a) $\{x_n\}_{n=0}^{5} = \{1, 2, 3, 4, 5, 6\}$ and $\{y_n\}_{n=0}^{5} = \{1, 0, 0, 1, 0, 0\}$.

   (b) $\{x_n\}_{n=0}^{8} = \{1, 2, 3, 4, 5, 6, 0, 0, 0\}$ and $\{y_n\}_{n=0}^{8} = \{1, 0, 0, 1, 0, 0, 0, 0, 0\}$.

6. The following linear convolution

$$\{x_n\}_{n=0}^{46} * \{h_n\}_{n=0}^{32}$$

   is to be evaluated using the DFT method. Namely,

$$\{x_n\}_{n=0}^{46} * \{h_n\}_{n=0}^{32} = \mathrm{DFT}^{-1}\{\mathrm{DFT}\{x_n\} \cdot \mathrm{DFT}\{h_n\}\}$$

   (a) Determine the minimum number of zeros should be padded to $\{x_n\}$ and $\{h_n\}$, respectively, before the DFTs are applied.

   (b) If the DFTs are to be calculated with a radix-2 FFT algorithm, how many zeros should now be padded to $\{x_n\}$ and $\{h_n\}$, respectively.