

ECE 310 Recitation 8

Concept Check

1. Discrete Fourier Transform (DFT)

$$X_m = \sum_{n=0}^{N-1} x_n e^{-j\frac{2\pi}{N}mn} \leftrightarrow x_n = \frac{1}{N} \sum_{m=0}^{N-1} X_m e^{j\frac{2\pi}{N}mn}, \quad m, n = 0, 1, \dots, N-1$$

2. DFT Properties

- Linearity
- Periodicity
- Conjugate Symmetry
- Time shift
- Duality
- Parseval theorem
- Cyclic Convolution

Exercise

1. **[Fa18 midterm#2]** The length-4 sequences $x_1[n]$ and $x_2[n]$ have DFTs: $X_1[k] = \{1, 2j, -1, 1\}$ and $X_2[k] = \{1, 2, 1, -1\}$, respectively.
 - a) What is the DFT of $x_1[n] + 3x_2[n]$?
 - b) What is the DFT of $e^{j\pi n}x_1[n]$?
 - c) What is $\sum_{n=0}^3 |x_2[n]|^2$?
 - d) What is $x_2[0]$?
 - e) What is $x_1[1]$?
2. Let $X[m]$ be the 10-point DFT of the sequence $x[n] = [1, -1, 2, 3, -3, 4, 0, 0, 0, 0]$. Let $y[n]$ be a finite length sequence whose DFT $Y[m]$ is related to $X[m]$ as $Y[m] = X[m]e^{-j\frac{2\pi}{5}mn_0}$, where $n_0 = 3$. Determine the sequence $y[n]$.
3. Let $X[m]$ denote the 240-point DFT of $x[n]$, $0 \leq n \leq 239$. The sequence $y[n]$ is obtained by zero-padding $x[n]$ to length 256. Determine m_0 such that $Y[32] = X[m_0]$.
4. Prove Cyclic Convolution Property.