ECE 310

Digital Signal Processing

Spring, 2021, ZJUI Campus

Lecture 10

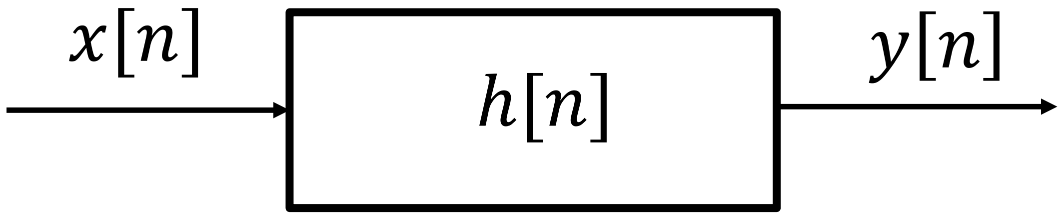
Topics:

- ✓ Analysis of LSI system using z-transform
- ✓ Revisit causality and BIBO stability of LSI systems

Educational Objectives:

- ✓ Understand what is transfer function H(z) and how it is related to unit pulse response h[n]
- ✓ Know how to determine the causality of an LSI system based on H(z) and why
- ✓ Know how to determine the BIBO stability of an LSI system based on H(z)
- ✓ Know how the boundedness of a signal is related to its poles

Transfer of an LSI System



$$y[n] = h[n] * x[n]$$

$$Y(Z) = H(Z)X(Z)$$

$$H(z) = \sum_{n} h[n] z^{-n}$$

Causality of an LSI System

Time-domain Criterion:

$h[n]$ is causal or right-hand sided, i.e., $h[n] = 0, n < 0$

Frequency-domain Criterion:

$ROC_H: \quad |Z| > R$

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

$$= \sum_{k=0}^{\infty} h[k]x[n-k]$$

System's BIBO Stability

A system is BIBO (bounded input, bounded output) stable if

What is bounded signal ?

$$|x[n]| < B$$

Examples:

(a) $\sin(\omega_0 n)$

(b) $e^{-3n}u[n]$

(c) $e^{-3n}u[-n]$

Examples

$$y[n] = 3x[n] + 2x[n-1]$$

$$y[n] = nx[n]$$

BIBO Stability of LSI Systems

- Time-domain criterion:

$$\sum_{n=-\infty}^{+\infty} |h[n]| < \infty$$

- Frequency-domain criterion:

ROC of $H(Z)$ includes the unit circle

BIBO Stability of LSI Systems

Proof of absolute summabilty

$$y[n] = h[n] * x[n]$$

$$= \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

$$|y[n]| = \left| \sum_{k=-\infty}^{\infty} h[k]x[n-k] \right|$$

$$\leq \sum_{k=-\infty}^{\infty} |h[k]x[n-k]|$$

$$= \sum_{k=-\infty}^{\infty} |h[k]| \cdot |x[n-k]|$$

BIBO Stability of LSI Systems

if $|x[n]| < B$

$$y[n] \leq B \sum_{k=-\infty}^{\infty} |h[k]| = \tilde{B} \quad if \sum_{k=-\infty}^{\infty} |h[k]| \text{ is bounded}$$

if $\displaystyle\sum_{k=-\infty}^{\infty} |h[n]|$ is not bounded

$$let\ x[n] = \operatorname{sgn}(h[n_0 - n])\,;\, in\ real\ sequence$$

$$Or\ in\ general\ x[n] = h^*[n_0 - n]/|h[n_0 - n]|$$

BIBO Stability of LSI Systems

$$y[n_0] = \sum_{k=-\infty}^{\infty} h[k]x[n_0 - k] = \sum_{k=-\infty}^{\infty} h[n_0 - k]x[k]$$

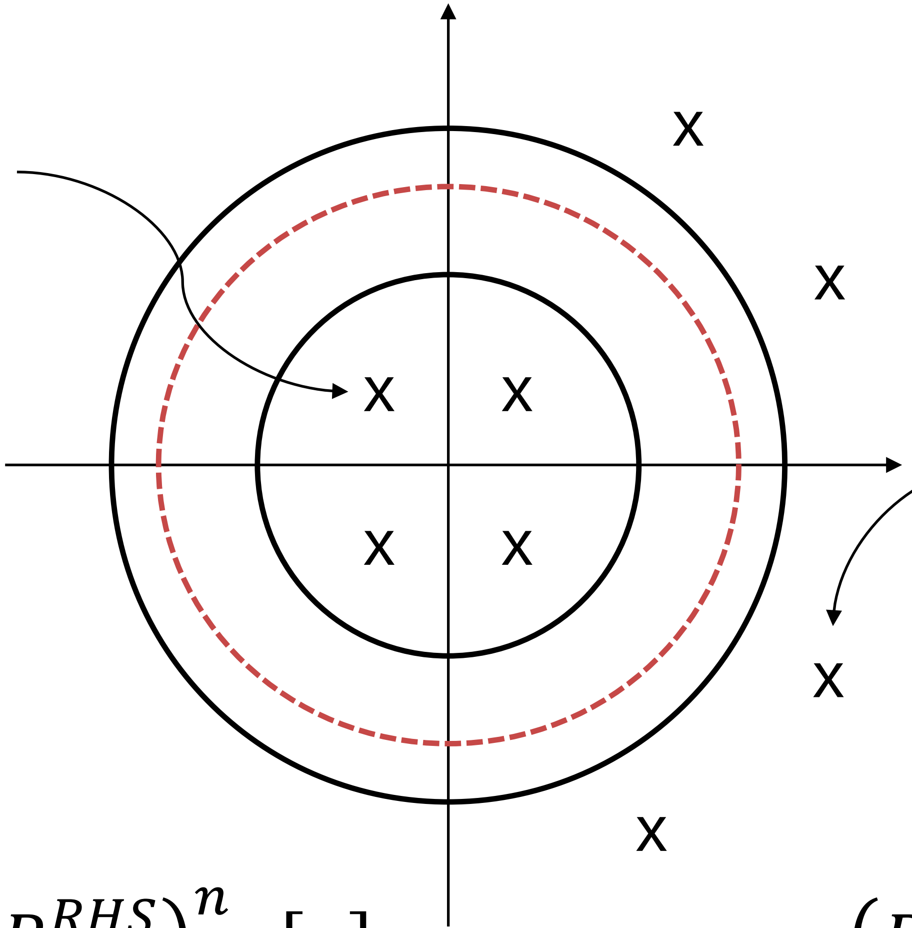
$$= \sum_{k=-\infty}^{\infty} \frac{h^*[n_0 - k]h[n_0 - k]}{|h[n_0 - k]|}$$

$$= \sum_{k=-\infty}^{\infty} |h[n_0 - k]| = \sum_{k=-\infty}^{\infty} |h[k]|$$

ROC of H(z) vs Absolute Summability of *h[n]*

P_i^{RHS}

$|P_i^{RHS}| < 1$



P_i^{LHS}

$|P_i^{LHS}| > 1$

simple poles	$\left(R_i^{RHS}\right)^n u[n]$	$\left(P_i^{LHS}\right)^n u[-n-1]$
multiple poles	$n^l\left(P_i^{RHS}\right)^n u[n]$	$n^l\left(P_i^{LHS}\right)^n u[-n-1]$