ECE 310

Digital Signal Processing

Spring, 2021, ZJUI Campus

Lecture 23

Topics:

- ✓ Further discussion of Fast Fourier Transform (FFT)
- ✓ Fast convolution using FFT

Educational Objectives:

- ✓ Understand the FFT decomposition equation
- ✓ Understand how to construct a butterfly structure for DIT radix-2 FFT
- ✓ Understand the difference between circular convolution and linear convolution

Fast Fourier Transform (FFT)

$$X_m = \sum_{n=0}^{N-1} x_n e^{-j\frac{2\pi}{N}mn} = \sum_{n=0}^{N-1} x_n W_N^{mn}, \qquad m = 0,1 \ldots N-1 \qquad W_N = e^{-j\frac{2\pi}{N}}$$

$$X_m = \sum_{p=0}^{N/2-1} x_{2p} W_{N/2}^{pm} + W_N^m \sum_{p=0}^{N/2-1} x_{2p+1} W_{N/2}^{pm}$$

$$= Y_m + W_N^m Z_m, \qquad m = 0,1 \ldots N-1$$

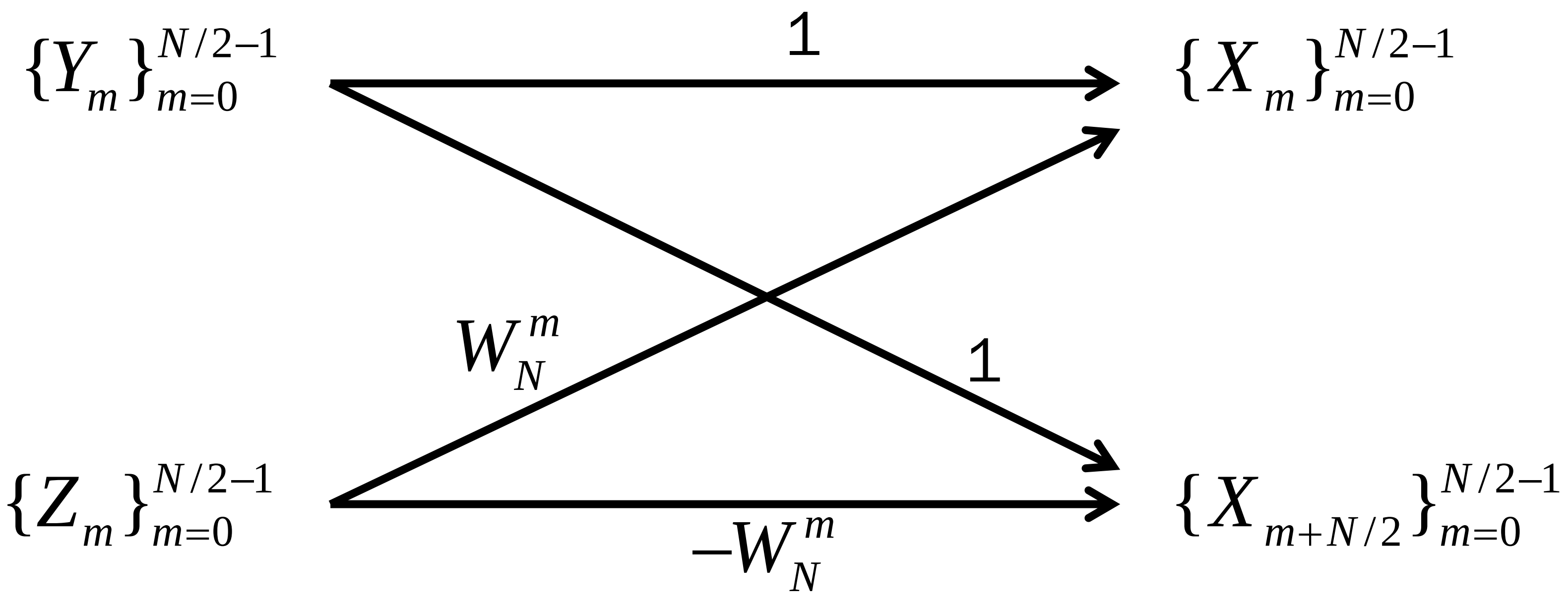
$$\begin{cases} Y_m = \sum_{p=0}^{N/2-1} x_{2p} W_{N/2}^{pm} \\ Z_m = \sum_{p=0}^{N/2-1} x_{2p+1} W_{N/2}^{pm} \end{cases} \qquad m = 0,1 \ldots N/2-1$$

Fast Fourier Transform (FFT)

$$\begin{cases} Y_{m+N/2} = Y_m \\ Z_{m+N/2} = Z_m \end{cases} \qquad m = 0,1 \ldots N/2-1$$

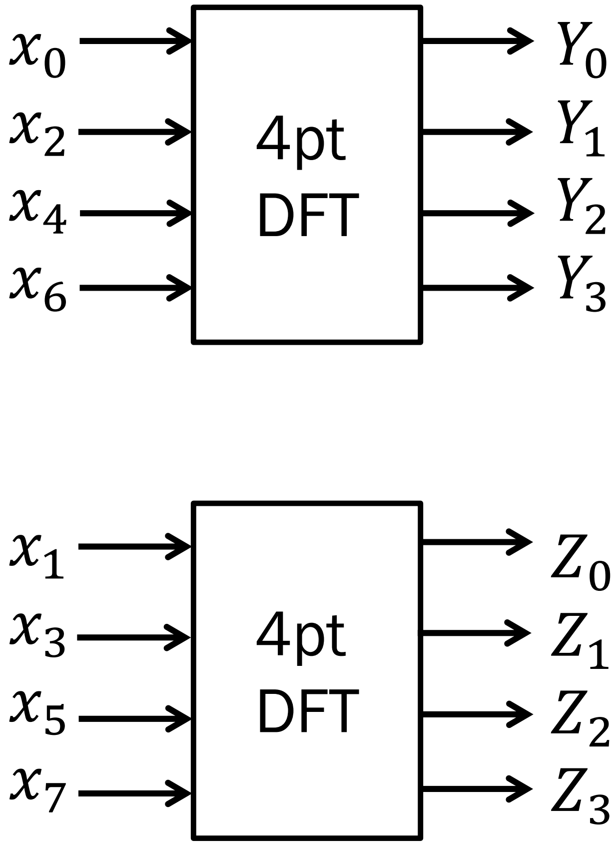
$$\begin{cases} X_m = Y_m + W_N^m Z_m \\ X_{m+N/2} = Y_m - W_N^m Z_m \end{cases} \qquad m = 0,1 \ldots N/2-1$$

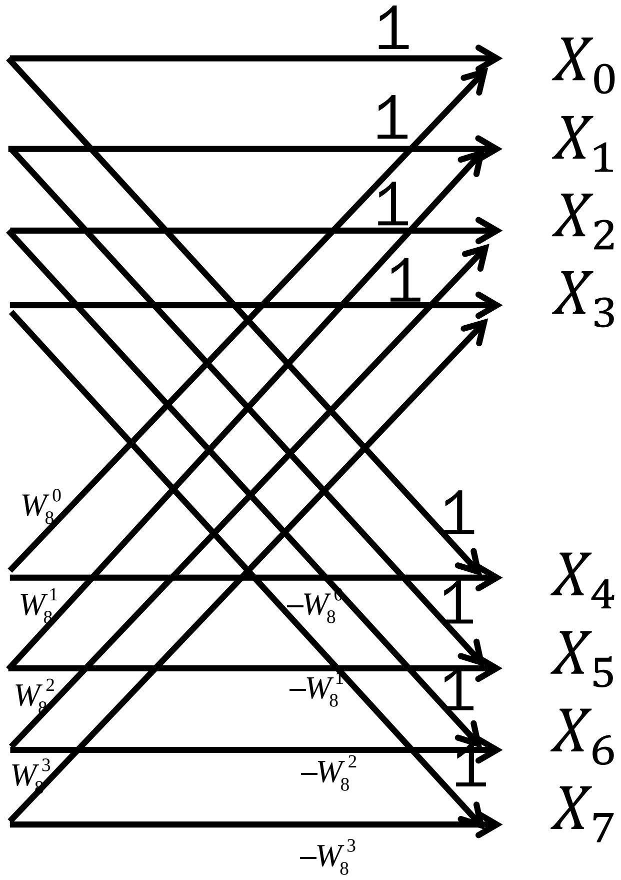
Butterfly diagram



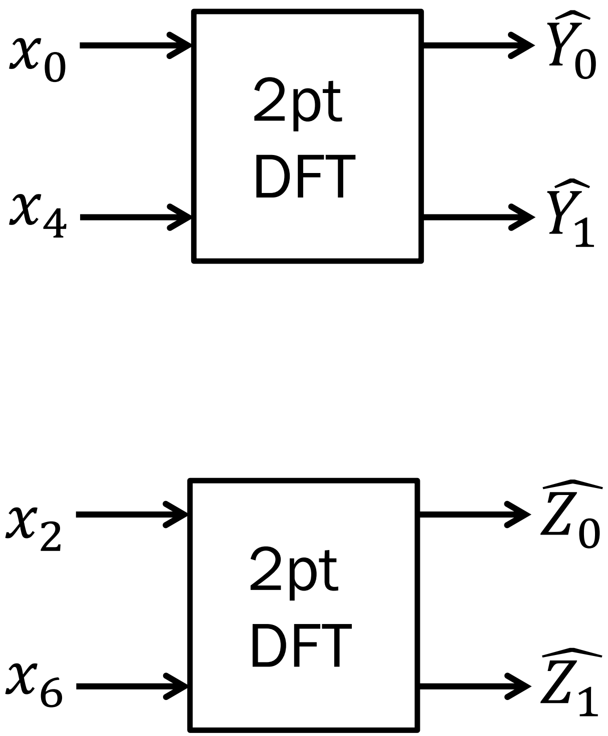
Butterfly diagram

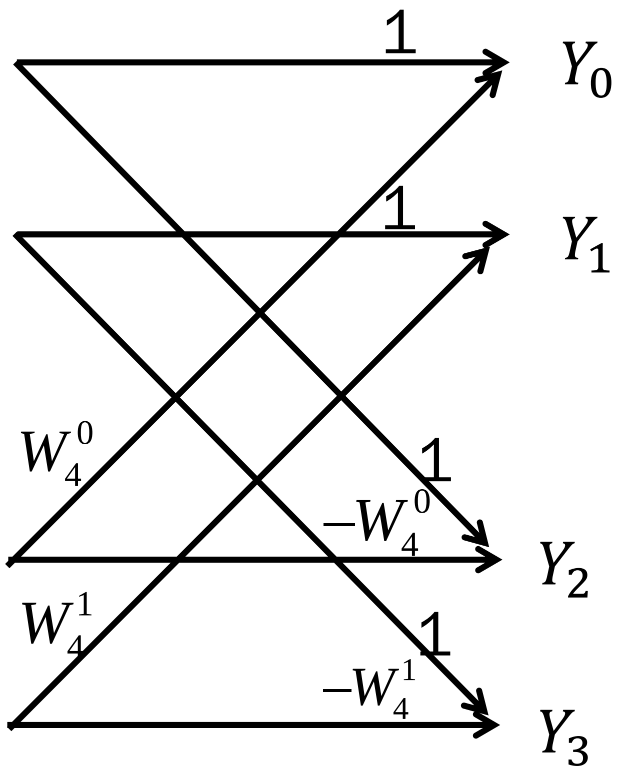
8pt FFT

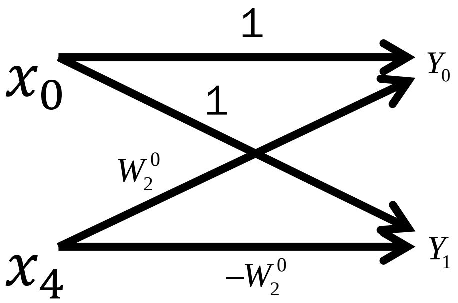




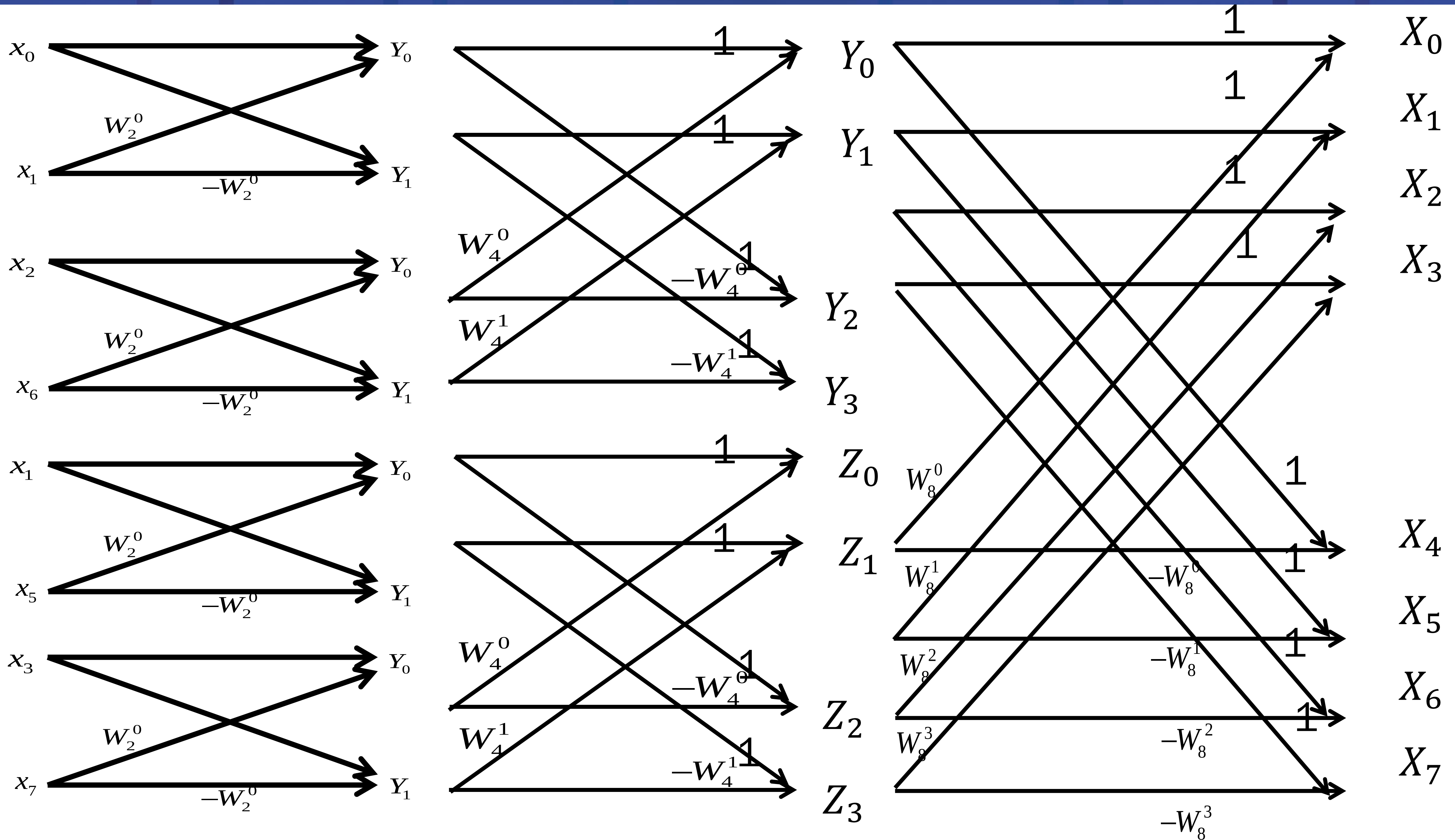
Butterfly diagram







Butterfly diagram

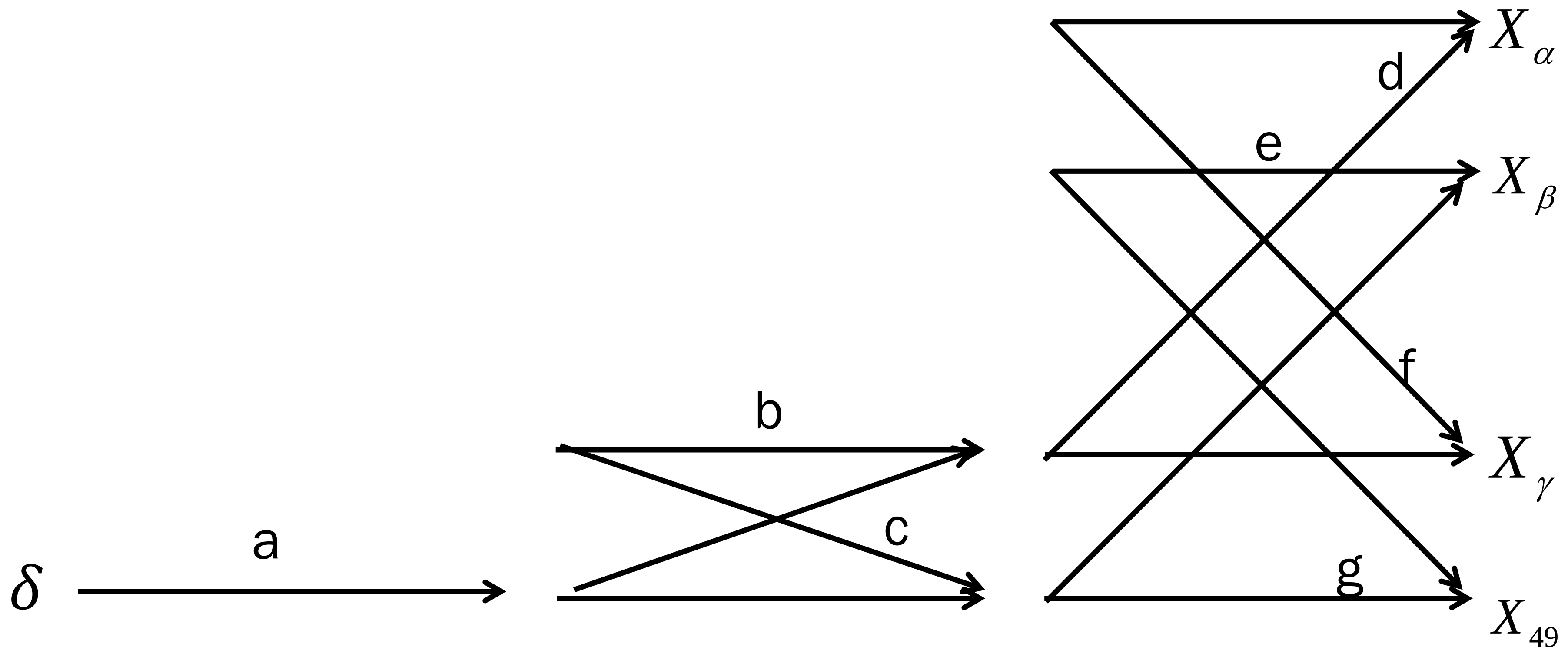


Bit-reverse index

(0) 000	←	000 (0)
(4) 100	←	001 (1)
(2) 010	←	010 (2)
(6) 110	←	011 (3)
(1) 001	←	100 (4)
(5) 101	←	101 (5)
(3) 011	←	110 (6)
(7) 111	←	111 (7)

Example: 64-pt-FFT, DIT, Radix-2

Determine all the connection weights and signal indexes.



Fast Linear Convolution Using FFT

- Linear Convolution

$$\{x_n\}_{n=0}^{N-1} * \{h_n\}_{n=0}^{M-1} = \{y_n\}_{n=0}^{L-1}, \ L = N + M - 1$$

- Circular Convolution

$$\{x_n\}_{n=0}^{N-1} \circledast \{h_n\}_{n=0}^{N-1} = \{y_n\}_{n=0}^{N-1} = \sum_{k=0}^{W-1} x_k h_{<n-k>_N}$$

Fast Linear Convolution Using FFT

$$a)\ \{x_n\} \overset{FFT}{\longleftrightarrow} \{X_m\}$$

$$b)\ \{h_n\} \overset{FFT}{\longleftrightarrow} \{H_m\}$$

$$c)\ \{X_m H_m\}$$

$$d)\ FFT^{-1}\{X_m H_m\}$$