

Homework 6

Prof. Zhi-Pei Liang

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1. A causal LSI system is described by the difference equation: $y[n] - y[n-1] = x[n]$.
 - (a) Determine the system's transfer function $H(z)$
 - (b) Determine the system's unit pulse response $h[n]$
 - (c) Determine the system's frequency response $H_d(\omega)$; is $H_d(\omega) = H(z)|_{z=e^{j\omega}}$? if not, explain why.
2. An LSI system is described by the difference equation

$$y[n] = x[n] + x[n-10]$$

- (a) Compute and sketch its magnitude and phase response
- (b) Determine its output to inputs
 - i. $x[n] = \cos \frac{\pi}{10}n + 3 \sin \left(\frac{\pi}{3}n + \frac{\pi}{10} \right)$
 - ii. $x[n] = 10 + 5 \cos \left(\frac{2\pi}{5}n + \frac{\pi}{2} \right)$

3. The frequency response of an LSI system is

$$H_d(\omega) = \omega e^{j \sin \omega}, \quad |\omega| \leq \pi.$$

Determine the system output $y[n]$ for the following inputs:

- (a) $x[n] = 5 + 10e^{j(\frac{\pi}{4}n + 45^\circ)} + j^n$
 - (b) $x[n] = 5 + 10 \cos(\frac{\pi}{4}n + 45^\circ) + j^n$.
4. The difference equation of a causal LSI system is given by

$$y[n] - \frac{1}{\sqrt{4}}y[n-1] = x[n], \quad -\infty < n < \infty.$$

Determine $y[n]$ for input $x[n] = 10 + \cos(\frac{\pi}{4}) \sin(\frac{\pi}{2}n) + 2(-1)^n$, $-\infty < n < \infty$.

5. The response of a real LSI system for input

$$x[n] = 3 + \cos \left(\frac{\pi}{4}n + 10^\circ \right) + \sin \left(\frac{\pi}{3}n + 25^\circ \right)$$

is

$$y[n] = 9 + 2 \sin \left(\frac{\pi}{4}n + 10^\circ \right).$$

Determine the system response $\tilde{y}[n]$ for input

$$\tilde{x}[n] = 5 + 2 \sin \left(\frac{\pi}{4}n + 15^\circ \right) + 10 \cos \left(-\frac{\pi}{3}n + 25^\circ \right).$$