ZHEJIANG UNIVERSITY - UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

ECE 310 Digital Signal Processing

Homework 4

Prof. Zhi-Pei Liang **Due: March 12, 2021**

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1. Find all the possible ROCs for the following z-transforms and determine the associated inverse z-transform for each case.

   (a) $\dfrac{z^2 - z}{z^2 + 3z + 2}$

   (b) $\dfrac{1}{(1 - \frac{1}{3}z^{-1})(1 - \frac{1}{5}z^{-1})}$

2. Determine whether each of the following transfer functions represents a BIBO stable causal system:

   (a) $H(z) = \frac{z(z-4)}{z^2-5z+6}$

   (b) $H(z) = \frac{z-7}{z^2+1/9}$

   (c) $H(z) = \frac{z+1}{z-1}$

   (d) $H(z) = \frac{z-1}{z^2+j}$

   For each case in which the system is determined to be unstable, find a bounded real-valued input that will produce an unbounded output.

3. The input $x[n] = 2^n\,(u[n] - 3u[n-1])$ to an unknown LSI system produces output $y[n] = (3^n - 2^n)u[n]$. Determine the unit pulse response $h[n]$ assuming the system is causal. Is the system BIBO stable?

4. Two systems with unit-pulse responses

$$h_1[n] = 2u[n] - 2\left(\frac{1}{2}\right)^n u[n], \qquad h_2[n] = \delta[n] - 3\left(\frac{1}{4}\right)^n u[n-1]$$

   are in serial connection.

   (a) For each of the individual systems, as well as for the overall system, determine whether it is BIBO stable.

   (b) Determine the unit pulse response of the overall system.

5. Consider the following difference equation (or LCCDE) system, with zero initial conditions:

$$y[n] = x[n] + 0.5\,x[n-1] - y[n-1] - 0.25\,y[n-2], \quad \text{for } n = 0, 1, 2, \ldots$$

   (a) Find the transfer function and its ROC.

   (b) Find the impulse response of the system.

   (c) Determine the output $y[n]$ given input $x[n] = (-1)^n u[n]$.