

ECE 310 Recitation 5

Concept Check

1. $\delta(t)$ function

a) $\int_{-\infty}^{\infty} \delta(t) dt = 1$

b) Key properties:

$$\int_{-\infty}^{\infty} \delta(t - t_0) f(t) dt = f(t_0)$$
$$\int_{-\infty}^{\infty} \delta(\alpha t) f(t) dt = \frac{f(0)}{|\alpha|} \quad (\alpha \neq 0)$$

2. Fourier Transform (Continuous Time)

$$x(t) = \int_{-\infty}^{\infty} \frac{X(\Omega)}{2\pi} e^{j\Omega t} d\Omega \leftrightarrow X(\Omega) = \int_{-\infty}^{\infty} x(t) e^{-j\Omega t} dt$$

3. DTFT

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X_d(\omega) e^{j\omega n} d\omega \leftrightarrow X_d(\omega) = \sum_{-\infty}^{\infty} x[n] e^{-j\omega n}$$

Exercise

1. [Fa18 midterm#1] Consider the sequence

$$\{x[n]\} = \{-1, 2, -3, 2, -1\}$$

Compute the following quantities. (Hint: *You should never have to explicitly compute the DTFT $X_d(\omega)$*)

a) $\text{Im}\{X_d(-\omega)\}$ (Hint: $x[n] = x[-n]$)

b) $\int_{-\pi}^{\pi} X_d(\omega) d\omega$

c) $X_d(\pi)$

d) $\int_{-\pi}^{\pi} |X_d(\omega)|^2 d\omega$

2. Calculate the DTFT of $x[n] = u[n] - u[n - N]$ for $N=4$ and 5. Plot the magnitude and phase spectra.