# ECE 310

# Digital Signal Processing

Spring, 2021, ZJUI Campus

# Lecture 6

## Topics:

- ✓ Linear Constant Coefficient Difference Equations (LCCDE)
- ✓ Zero-state response; zero-input response; total response

## Educational Objectives:

- ✓ Understand what is LCCDE: standard delay form and standard advance form
- ✓ Understand what is zero-state response of LCCDE
- ✓ Understand what is zero-input response of LCCDE
- ✓ Understand the iterative method for finding responses of an LCCDE system

# Linear Constant Coefficient Difference Equation (LCCDE)

$$x[n] \longrightarrow \boxed{\text{LSI}} \longrightarrow y[\mathrm{n}]$$

Standard form I (delay form):

$$y[n] + a_1 y[n-1] + \cdots + a_N y[n-N] = b_0 x[n] + b_1 x[n-1] + \cdots + b_N x[n-N]$$

$$y[n] + \sum_{k=1}^{N} a_k y[n-k] = \sum_{k=1}^{N} b_k x[n-k]$$

Standard form II (advance form):

$$y[n] + a_1 y[n+1] + \cdots + a_N y[n+N] = b_0 x[n] + b_1 x[n+1] + \cdots + b_N x[n+N]$$

$$y[n] + \sum_{k=1}^{N} a_k y[n+k] = \sum_{k=1}^{N} b_k x[n+k]$$

# Linear Constant Coefficient Difference Equation (LCCDE)

Example: put an LCCDE in standard form.

$$y[n+1]-\frac{1}{2}y[n-2]=x[n]+\frac{1}{3}x[n-1]$$

# Linear Constant Coefficient Difference Equation (LCCDE)

$$y[n]+\sum_{k=1}^{N}a_k y[n-k]=\sum_{k=1}^{N}b_k x[n-k]$$

Initial conditions: $y[-1],\ y[-2],\ \cdots,\ y[-N]$

**Zero-state response**: Solution to the LCCDE equation with zero initial conditions

**Zero-input response**: Solution to the LCCDE equation with zero input

Warning: Linearity doesn't apply to the total response!

**Trick:** $$y[n] = y_x[n] + y_s[n]$$

Zero-state Zero-input

# Linear Constant Coefficient Difference Equation (LCCDE)

Example: Solving LCCDE by iteration

$$y[n]-\frac{1}{2}y[n-1]=x[n]; \qquad y[-1]=1$$

# Motivation for Z-transform for Analysis of LSI Systems

| ECE 210 | ECE 310 |
|---|---|
| **Linear constant coefficient differential equation** | **Linear constant coefficient difference equation** |
| $\updownarrow$ | $\updownarrow$ |
| **Laplace Transform** | **Z-Transform** |

Example: Determine the unit pulse response of an LSI system that gives

$$y[n] = 3^{-n}u[n] + 5\times 2^{-n}u[n] \text{ for input } x[n] = 2^{-n}u[n]$$