ECE 310 Recitation 6 Solution
Thursday Mar 25, 2021

**Concept check**

- √ The Big Picture So Far…
- √ System analysis
  - o Linearity, Shift-Invariance
  - o Causality, BIBO Stability of LSI system
- √ Z-transform: $X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$
  - o ROC and causality
  - o Poles and zeros, 0? ∞?
  - o Inverse z-transform: Partial fraction => ROC based on causality => Table look-up
  - o Important pairs and properties
- √ About δ
  - o $\delta(at) = \frac{1}{|a|}\delta(t)$
  - o $\int_{-\infty}^{\infty} e^{j\omega t} d\omega = 2\pi\delta(t)$
  - o $\sum_{n=-\infty}^{\infty} e^{jn\frac{2\pi}{\tau}t} = \tau \sum_{n=-\infty}^{\infty} \delta(t - n\tau)$
- √ CTFT and DTFT
  - o Important formulas
  - o Important pairs and properties

**Exercise**

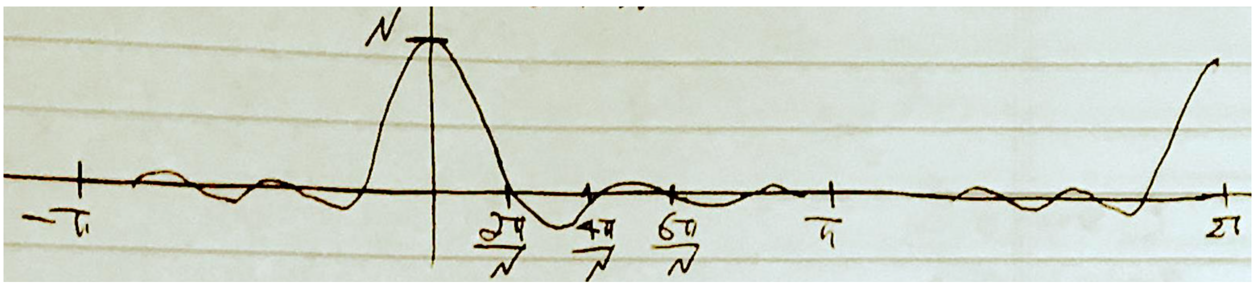
1. (HW2 Q4) Assume that the response of an LTI system to input $x[n] = 3^{-n}u[n]$ is $y[n] = 5^{-n}u[n-1]$. Use the system's properties (linearity and shift invariance) to find $h[n]$, the system's unit pulse response.

   Express the unit pulse $\delta[n]$ with $x[n]$:
   $\delta[n] = 3^{-n}u[n] - 3^{-n}u[n-1] = 3^{-n}u[n] - \frac{1}{3}3^{-n+1}u[n-1] = x[n] - \frac{1}{3}x[n-1]$
   Then utilize the LSI properties of the system:
   $$\begin{aligned} h[n] = \mathcal{H}\{\delta[n]\} &= \mathcal{H}\left\{x[n] - \frac{1}{3}x[n-1]\right\} \\ &= \mathcal{H}\{x[n]\} - \frac{1}{3}\mathcal{H}\{x[n-1]\} \ (linearity) \\ &= y[n] - \frac{1}{3}y[n-1] \ (shift-invariance) \\ &= 5^{-n}u[n-1] - \frac{1}{3}5^{-n+1}u[n-2] \end{aligned}$$

2. (HW5 Q6) $x[n] = (u[n] - u[n-N])/N$, discuss how will the shape of $|X_d(\omega)|$ and $\angle X_d(\omega)$ change as N increases. (Estimate and sketch by hand.)

$$X_d(\omega) = \frac{e^{-j\frac{N-1}{2}\omega}}{N} \cdot \frac{\sin\left(\frac{N}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)},$$

$$|X_d(\omega)| = \left|\frac{\sin\left(\frac{N}{2}\omega\right)}{\mathrm{N}\sin\left(\frac{1}{2}\omega\right)}\right|,\ \angle X_d(\omega) = \begin{cases} -(N-1)\frac{\omega}{2}, & \frac{\sin\left(\frac{N}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)} > 0 \\ -(N-1)\frac{\omega}{2} + \pi, & \frac{\sin\left(\frac{N}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)} < 0 \\ 0, & \frac{\sin\left(\frac{N}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)} = 0 \end{cases}$$




The shape of $\frac{\sin\left(\frac{N}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)}$ is roughly as above. The zero-crossings are $\frac{N}{2}\omega = k\pi, \omega = \frac{2k\pi}{N}$.

So as N increases, there are more zero crossings. The lobes of $|X_d(\omega)|$ become narrower, and the slopes of $\angle X_d(\omega)$ decrease.

3. (fall2019 Q1) T or F

   a. An LSI system specified by the following difference equation: $y[n] - \frac{1}{2}y[n-1] = x[n]$ can be causal or anti-causal.

      True. $y[n] = \frac{1}{2}y[n-1] + x[n]$ is causal while $\frac{1}{2}y[n-1] = y[n] - x[n]$ is anti-causal.

   b. The input and output relationship of an arbitrary system is completely determined by the system's unit pulse response.

      False. The system may not be LTI and doesn't have a unit pulse response, e.g. $y[n] = \log(x[n])$.

4. (fall2019 Q5) calculate the z-transform and corresponding ROC for

$$x[n] = 3^n(u[n-5] - u[n-100])$$

$$X(z) = \sum_{n=5}^{99} (3z)^{-n} = \frac{(3z)^{-5} - (3z)^{-100}}{1 - (3z)^{-1}}, ROC: |z| > 0$$

Note: ROC is $|z|>0$ rather than $|z|>1/3$, because x[n] is a finite impulse.