University of Illinois
ECE 310

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Midterm Exam I

7:00-8:30pm, Thursday, October 3, 2019

Name: Solution

Section: 10:00 AM 12:00 PM 3:00 PM

NetID: ____________

Score: ____________

Problem	Pts.	Score
1	20	
2	12	
3	12	
4	5	
5	10	
6	6	
7	15	
8	4	
9	4	
10	12	
Total	100	

Instructions

- You may not use any books, calculators, or notes other than one <u>handwritten</u> two-sided sheet of 8.5" x 11" paper.
- Show all your work to receive full credit for your answers.
- When you are asked to "calculate", "determine", or "find", this means providing closed-form expressions (i.e., without summation or integration signs).
- Neatness counts. If we are unable to read your work, we cannot grade it.
- Turn in your entire booklet once you are finished. No extra booklet or papers will be considered.

(20 Pts.) (+2 pt each)

1. Mark "True" or "False" for the following statements.

   (a) An LSI system specified by the following difference equation: $y[n] - \frac{1}{2}y[n-1] = x[n]$ can be causal or anti-causal. **T**/F

   (b) The input and output relationship of an arbitrary system is completely determined by the system's unit pulse response. T/**F**

   (c) If an LSI system is BIBO unstable, its unit pulse response $h[n]$ must be unbounded. T/**F**

   (d) Let $h[n] = h_1[n] * h_2[n]$ be the unit pulse response (UPR) of two serial subsystems with UPR $h_1[n]$ and $h_2[n]$. If h_1 or h_2 is BIBO *unstable*, h must be BIBO *unstable*. T/**F**

   (e) An LSI system with a finite-length impulse response can be BIBO stable or unstable. T/**F**

   (f) A time-varying system cannot be causal. T/**F**

   (g) Let $\sum_{n=-\infty}^{\infty} x[n]\delta[2^n u[n] - 8] = 4$. Then, $x[3] = 2$. T/**F**

   (h) Suppose that the step response $g[n]$ of an LSI system, i.e., the output of the system to the input $x[n] = u[n]$ has a z-transform with a pole at $z = 1/2$. Then, $H(z)$ has also pole at $z = 1/2$. **T**/F

   (i) If the response $y[n]$ of an LSI system to the input $x[n] = 3^n u[n]$ is unbounded, the system must be BIBO unstable. T/**F**

   (j) The response $y[n]$ of a BIBO *unstable* LSI system to any non-zero input $x[n]$ is always unbounded. T/**F**

(12 Pts.)

2. Determine whether the following systems are linear or nonlinear, shift-invariant or shift-varying, and causal or noncausal.
   Grading: Correct answer = +2 pts.; Incorrect answer = -1 pts. No answer = 0 pts.

   (a) $y[n] = x[|n| + n]$

   provide $x = ax_1 + bx_2$

   $y[n] = x[|n|+n] = ax_1[|n|+n] + bx_2[|n|+n]$

   $\therefore$ Linear

   Let $x[n] = x_1[n-k]$

   $y[n] = x[|n|+n] = x_1[|n|+n-k] \neq y_1[n-k]$ $\Rightarrow$ shift-varying

   Let $n=-1$

   $y[-1] = x[0]$

   depends on future val.s

L / NL	SI / **SV**	C / **NC**

   (b) $y[n] = (0.2)^{|n|}\log[x[n]]$

   $x = ax_1 + bx_2$

   $y[n] = (0.2)^{|n|}\log(x[n])$

   $= (0.2)^{|n|}\log(ax_1[n] + bx_2[n])$

   $\neq ay_1[n] + by_2[n]$

   Let $x[n] = x_1[n-k]$

   $y[n] = (0.2)^n \log(x_1[n-k]) \neq y_1[n-k]$

   $y[n]$ depends on $x[n]$ only

L / **NL**	SI / **SV**	**C** / NC

Q1

(a) Depends on $H(z)$'s ROC.

(b) This system may not be LTI and doesn't have a unit pulse response
e.g. $y[n] = \log(x[n])$

(c) counterexample : $h[n] = u[n]$, bounded but still BIBO unstable

(d) Counter example: HW4 Q4

(e) Since $y[n] = (x * h)[n] = \sum_{i=-\infty}^{\infty} x[n-i]\, h[i]$
If $|x[n]| \leq B \ \forall n$, then $|y[n]| \leq \sum_{i=-\infty}^{\infty} B \cdot |h[i]|$
since $h[n]$ have finite length, then $B \sum_{i=-\infty}^{\infty} |h[i]|$ must be bounded
thus, BIBO stable

(f) Counter example : $y[n] = n\, x[n]$

(g). since $\delta[2^n u[n] - 8]$ is nonzero only at $n = 3$.
we have $\sum_{n=-\infty}^{\infty} x[n] \cdot \delta[2^n u[n] - 8] = x[3] = 4$.

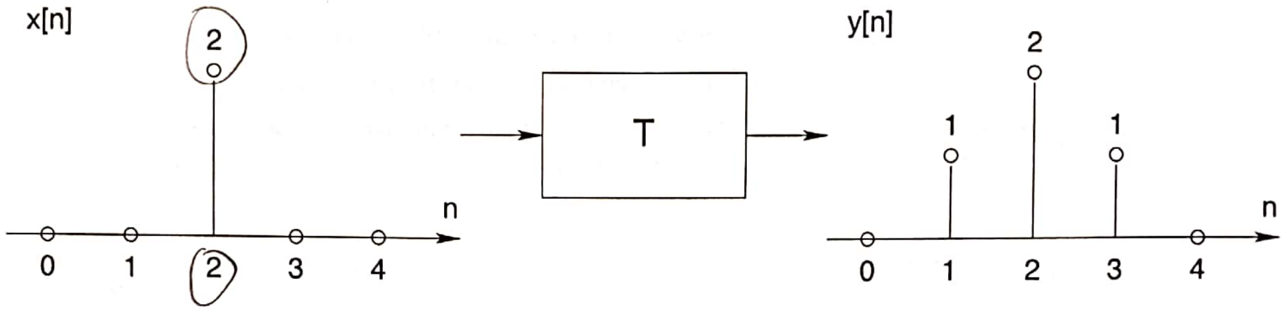
(h). HW2 Q3 : we know $h = g - g_{-1}$ So $H(z) = G(z) - z^{-1} G(z)$
the pole at $z = ½$ does not cancel

(i) BIBO unstable definition : $\exists\, x[n]$ s.t. $\forall n$, $|x[n]| \leq B < \infty$
But $y[n] = (h * x)[n]$ is not bounded

(j) same as (i)

(12 Pts.)

3. Suppose that T is a linear and shift-invariant system. For an input $x[n]$ depicted below we observe $y[n]$ as the output:



(a) (3pts.) Find the unit pulse response of the system T.

$x[n] = 2\,\delta[n-2]$

$y[n] = \delta[n-1] + 2\,\delta[n-2] + \delta[n-3]$

Since LSI, let $\tilde{x}[n] = \frac{1}{2}\,x[n+2] = \delta[n]$

$\tilde{y}[n] = h[n] = \frac{1}{2}\,\delta[n+1] + \delta[n] + \frac{1}{2}\,\delta[n-1]$

+3 for correct answer

−1 for minor mistakes

(b) (5pts.) Find the output $y[n]$ for $n = -1, 0, 1, 2$ produced by the response of the system to the unit step input $u[n]$.

$y[n] = u[n] * h[n] = u[n] * \left(\frac{1}{2}\,\delta[n+1] + \delta[n] + \frac{1}{2}\,\delta[n-1]\right)$

$= \frac{1}{2}\,u[n+1] + u[n] + \frac{1}{2}\,u[n-1]$

$y: \{\cdots 0, \frac{1}{2}, \frac{3}{2}, 2, 2, 2 \cdots\}$

+5 for correct answer

+1 for $u[n] * h[n]$

−1 for minor mistakes

(c) (4pts.) Is T a causal system? Justify your answer.

No, as $\exists\, n = -1 < 0$ s.t. $h[-1] \neq 0$.

+4 for correct answer

+2 if stating what causal system means. (i.e. $h[n] = 0\ \forall n < 0$, or doesn't depend on future val.s.)

(5 Pts.)

4. Calculate the result of the following convolution $\underbrace{\{\underset{\uparrow}{1}, -4, 2, -1, 3, 1\}}_{x_1[n]} * \underbrace{\{\underset{\uparrow}{-1}, 1, -1\}}_{x_2[n]}$.

method 1: $x_2[n] = -\delta[n] + \delta[n-1] - \delta[n-2]$

$x_1[n] * x_2[n] = -x_1[n] + x[n-1] - x[n-2]$

$= \{\underset{\uparrow}{-1}, 5, -7, 7, -6, 3, -2, -1\}$

method 2:

	1	−4	2	−1	3	1
−1	−1	4	−2	1	−3	−1
1	1	−4	2	−1	3	1
−1	−1	4	−2	1	−3	−1

adding the diagonal elements gives

$\{\underset{\uparrow}{-1}, 5, -7, 7, -6, 3, -2, -1\}$

(10 Pts.)

5. Calculate the z-transform and corresponding ROC of the following functions:

(a)

$$x[n] = 3^{-n}\left(u[n-5] - u[n-100]\right)$$

method 1:

$$X(z) = \sum_{n=5}^{99} (3z)^{-n}$$

3pt $\rightarrow$ $$= \frac{(3z)^{-5} - (3z)^{-100}}{1 - (3z)^{-1}}$$

2pt $\rightarrow$ ROC: $|z| > 0$

method 2

$$x[n] = 3^{-5}\, 3^{-(n-5)}\, u[n-5] - 3^{-100}\, 3^{-(n-100)}\, u[n-100]$$

$$X(z) = 3^{-5} z^{-5} \frac{z}{z - 1/3} - 3^{-100} z^{-100} \frac{z}{z - 1/3}$$

$$= \frac{(3z)^{-5} - (3z)^{-100}}{1 - (3z)^{-1}}$$

(b)

$$x[n] = e^{-n^2} u[n-8]u[-n+10]$$

$$x[n] = e^{-n^2}\left(\delta[n-8] + \delta[n-9] + \delta[n-10]\right)$$

$$= e^{-8^2}\delta[n-8] + e^{-9^2}\delta[n-9] + e^{-10^2}\delta[n-10]$$

3pt $\rightarrow$ $$X(z) = e^{-8^2} z^{-8} + e^{-9^2} z^{-9} + e^{-10^2} z^{-10}$$

ROC: $|z| > 0$

2pt

(6 Pts.)

6. Calculate the inverse z-transform for

$$Y(z) = 1 + z^{-100} + \frac{1}{1-5z^{-1}}, \qquad \text{ROC: } |z| > 5$$

$$= 1 + z^{-100} + \frac{z}{z-5}$$

$$y[n] = \delta[n] + \delta[n-100] + 5^n u[n]$$

2pt 2pt 2pt

(15 Pts.)

7. The z-transform of $x[n]$ is given below:

$$X(z) = \frac{z}{z - e^{j\frac{\pi}{3}}} + \frac{z}{z - 0.5}$$

Determine all the valid ROCs of $X(z)$ and for each case, calculate the inverse z-transform.

a) ROC: $|z| > 1$: $x[n] = e^{j\frac{\pi}{3}n} u[n] + 0.5^n u[n]$

1pt 2pt 2pt

b) ROC: $1 > |z| > 0.5$, $x[n] = -e^{j\frac{\pi}{3}n} u[-n-1] + 0.5^n u[n]$

1pt 2pt 2pt

c) ROC: $|z| < 0.5$, $x[n] = -e^{j\frac{\pi}{3}n} u[-n-1] - 0.5^n u[-n-1]$

1pt 2pt 2pt

(4 Pts.)

8. Calculate the DTFT of $x[n] = \{1, \underset{\uparrow}{0}, 0, 2\}$

$$X_d(w) = e^{+jw} + 2e^{-j2w}$$

2pt

2pt

(4 Pts.)

9. Assume that the z-transform of $x[n]$ is given by

$$X(z) = \frac{z}{z-3}, \qquad |z| > 3.$$

Determine which of the following statements is correct (select one only; -1 pt for incorrect choice):

(a) The DTFT of $x[n]$ is $X(\omega) = \frac{e^{-j\omega}}{e^{j\omega}-3}$

(b) The DTFT of $x[n]$ is $X(\omega) = \frac{e^{-j\omega}}{e^{-j\omega}-3}$

(c) The DTFT of $x[n]$ does not exist

(d) None of the above

(12 Pts.)

10. Consider a *causal* LSI system with transfer function $H(z) = \frac{1-2z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$.

(a) What is the ROC of $H(z)$?

(b) Is the system BIBO stable? Justify your answer.

(c) Find the difference equation (or LCCDE) for this system.

(d) **True or False:** If the previous system is serially connected to an unstable LSI system with impulse response $\tilde{h}[n] = 2^n u[n]$, then the overall system is BIBO unstable. Justify your answer.

(a) ROC: $|z| > \frac{1}{2}$ - (two poles : $z = \frac{1}{4}$ and $z = \frac{1}{2}$)

+2 for correct answer

(b) Yes, as ROC contains unit circle

+2 for correct answer

+1 if stating the BIBO stable $\Leftrightarrow$ includeds unit circle

(c) $H(z) = \frac{Y(z)}{X(z)} = \frac{1-2z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$

$\Rightarrow (1-\frac{3}{4}z^{-1}+\frac{1}{8}\cdot z^{-2})\,Y(z) = (1-2z^{-1})\,X(z)$

$\Rightarrow \quad y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = x[n] - 2x[n-1]$

+5 for correct answer

−1 for minor mistakes

(d) False. $\tilde{H}(z) = \frac{1}{1-2z^{-1}}$

$H_{overall}(z) = \tilde{H}(z)\cdot H(z) = \frac{1}{1-2z^{-1}} \cdot \frac{1-2z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$

$= \frac{1}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$

ROC doesn't change - So still stable.

+3 for correct answer.

+0 if justification is wrong