University of Illinois
ECE 310 **Profs. Kamalabadi, Katselis, Liang**

Midterm Exam II

7:00-8:30pm, Wednesday, November 6, 2019

Name: ____________________________________

Section: 10:00 AM 12:00 PM 3:00 PM

NetID: _____________

Score: _____________

Problem	Pts.	Score
1	20	
2	6	
3	9	
4	5	
5	8	
6	5	
7	10	
8	10	
9	15	
10	4	
11	8	
Total	100	

Instructions

- You may not use any books, calculators, or notes other than two handwritten two-sided sheets of 8.5" x 11" paper.
- Show all your work to receive full credit for your answers.
- When you are asked to "calculate", "determine", or "find", this means providing closed-form expressions (i.e., without summation or integration signs).
- Neatness counts. If we are unable to read your work, we cannot grade it.
- Turn in your entire booklet once you are finished. No extra booklet or papers will be considered.

(20 Pts.)

1. Mark "True" or "False" for the following statements.

   (a) Zero padding a finite-length signal can change its DTFT. **T/F**

   (b) Suppose that $x[n]$ is bandlimited to $\frac{\pi}{4}$. Then, its DTFT $X_d(\omega) = 0$ for $|\omega| > \frac{\pi}{4}$. **T/F**

   (c) The DTFT of the finite-length signal $x[n] = \cos\left(\frac{\pi}{32}n\right), n = 0, 1, \ldots, 15$ is given by the formula $X_d(\omega) = \pi[\delta(\omega - \frac{\pi}{32}) + \delta(\omega + \frac{\pi}{32})]$. **T/F**

   (d) An LSI system has transfer function $H(z)$. Suppose that the output of the system to the input $x[n] = \cos(\omega_0 n + \phi)$ is $y[n] = |H(e^{\jmath\omega_0})| \cos(\omega_0 n + \phi + \angle H(e^{\jmath\omega_0}))$ for any ω_0, ϕ, where $H(e^{\jmath\omega}) = H(z)|_{z=e^{\jmath\omega}}$. Then, the system $h[n]$ can be either real or complex. **T/F**

   (e) The DTFT $X_d(\omega)$ of a discrete-time signal $x[n]$ is periodic only when $x[n]$ has infinite length. **T/F**

   (f) The 64-point DFT of the finite-length signal $x[n] = \cos(\frac{\pi}{16}n), 0 \le n \le 63$ has only two nonzero elements. **T/F**

   (g) Linear convolution can be computed as a circular convolution via zero-padding. **T/F**

   (h) Circular convolution can be applied to two finite duration sequences with arbitrary lengths. **T/F**

   (i) Let $\{x[n]\}_{n=0}^{5} = \{1, -1, -2, 3, 4, -3\}$. Consider the corresponding 6-point DFT $\{X[k]\}_{k=0}^{5}$. Then, $X[0] = 0$. **T/F**

   (j) FFT is just an efficient algorithm to evaluate DFT. **T/F**

(6 Pts.)

2. A real continuous-time signal $x_c(t)$ is bandlimited to frequencies below 20 kHz, i.e., $X_a(\Omega) = 0$ for $|\Omega| \ge 2\pi(20000)$. Suppose that $x_a(t)$ is sampled with sampling frequency $F_s = 40$ kHz to produce $x[n] = x_a(nT_s)$ and 1000 samples are extracted corresponding to $n = 0, 1, \ldots, 999$. Let $\{X[k]\}_{k=0}^{999}$ be the 1000-point DFT of $\{x[n]\}_{n=0}^{999}$.

   (a) To what continuous-time frequencies do the indices $k = 300$ and $k = 800$ correspond? Provide your answers in Hz.

   (b) What is the spacing between the DFT samples in Hz (continuous-time frequency spacing)?

(9 Pts.)

3. Consider the following signal:

$$x[n] = \delta[n+3] - 3\delta[n+2] + 5\delta[n+1] - 7\delta[n] + 5\delta[n-1] - 3\delta[n-2] + \delta[n-3]$$

Compute the following quantities:

(a) $X_d(0)$

(b) $X_d(\pi)$

(c) $\int_{-\pi}^{\pi} X_d(\omega) d\omega$

(5 Pts.)

4. Let $\{X[k]\}_{k=0}^{7}$ be the 8-point DFT of $x[n] = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Determine the sequence $\{y[n]\}_{n=0}^{7}$ whose DFT is $Y[k] = e^{-j\frac{\pi}{4}k} X[k], \quad k = 0, 1, \ldots, 7.$

(8 Pts.)

5. Give a closed-form expression for $x[n]$, valid for all n, given that its DTFT is $X_d(\omega) = e^{-j\frac{\omega}{3}}$ for $|\omega| \leq \pi$. Your answer should not include complex numbers.

(5 Pts.)

6. A sequence $x[n]$ with DTFT $X_d(\omega) = e^{-j\frac{\omega^2}{3}}$ is input to an LSI system with unit-pulse response $h[n] = (\frac{1}{3})^n u[n]$. Can the output sequence $y[n]$ be real? Explain.

(10 Pts.)

7. Consider an LSI system characterized by the transfer function:

$$H(z) = 1 + z^{-4}$$

Compute the system response, $y[n]$, to the input

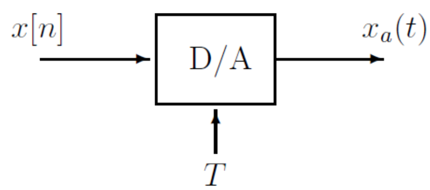
$$x[n] = 3 + 4\cos\left(\frac{\pi}{4}n\right) + e^{j\frac{\pi}{2}n}$$

(10 Pts.)

8. The discrete-time signal $x[n] = \delta[n-3]$ goes through a digital-to-analog (D/A) converter with interpolating interval T. Determine and sketch $x_a(t)$ for the following cases (label your graphs carefully):

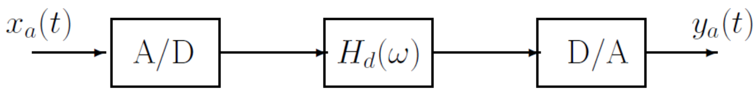
   (a) The D/A is a ZOH

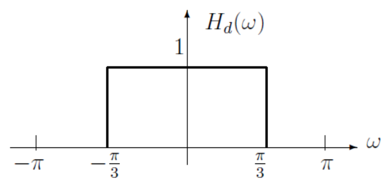
   (b) The D/A is an ideal D/A



(15 Pts.)

9. Consider the processing of a continuous-time signal $x_a(t)$ bandlimited to $\Omega_{\max} = 1000\pi$ rad/sec via the system shown below (where the A/D is an analog-to-digital converter with sampling interval T and the D/A is an ideal digital-to-analog converter with interpolating interval T).





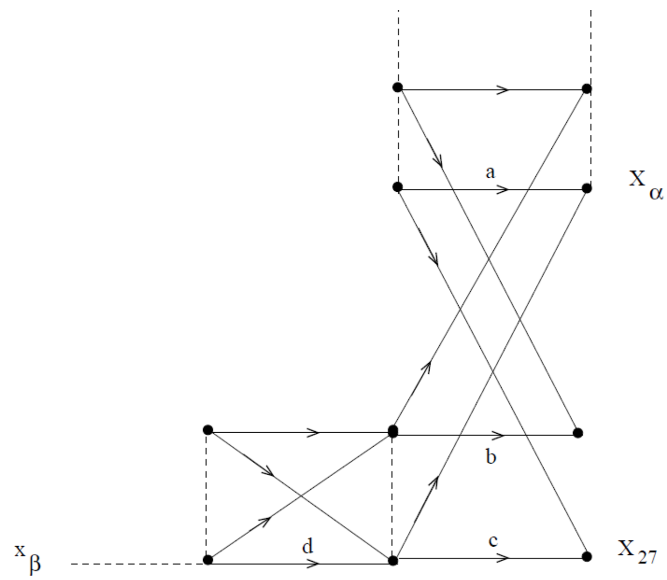
(a) Find the Nyquist interval T_{NYQ} (Nyquist sampling period) associated with the signal $x_a(t)$.

(b) For $T < T_{NYQ}$, is the overall continuous-time system with input $x_a(t)$ and output $y_a(t)$ linear shift-invariant? If so, draw the frequency response $H_a(\Omega)$ of this continuous-time system for $|\Omega| < 1000\pi$ (use T as a parameter in your plot).

(c) Is it possible to find $T > T_{NYQ}$ such that the overall continuous-time system with input $x_a(t)$ and output $y_a(t)$ is linear shift-invariant? If so, what is the largest such value of T?

(4 Pts.)

10. Shown below is part of a radix-2, 32-point DIT FFT. Determine the indices α and β, and the weighting coefficients a, b, c, and d.



(8 Pts.)

11. You are given two finite-duration sequences: $\{x_n\}_{n=0}^{14}$ and $\{h_n\}_{n=0}^{7}$, and you wish to compute their linear convolution using (a) a DFT approach with appropriate zero-padding (namely, $\{x_n\}_{n=0}^{14} * \{h_n\}_{n=0}^{7} = \text{DFT}^{-1}\{\text{DFT}\{x_n\} \cdot \text{DFT}\{h_n\}\}$) and (b) a radix-2, DIT FFT algorithm. What is the minimum number of zeros that must be padded to each of the sequences in order to compute $(x*h)$ using each approach?

DFT Approach: Number of zeros added to $\{x_n\}$ =

DFT Approach: Number of zeros added to $\{h_n\}$ =

FFT Approach: Number of zeros added to $\{x_n\}$ =

FFT Approach: Number of zeros added to $\{h_n\}$ =