# ECE 310

# Digital Signal Processing

Spring, 2021, ZJUI Campus

# Lecture 4

## Topics:

- ✓ Shift-invariance of discrete-time systems
- ✓ BIBO stability
- ✓ Convolution relationship of LSI systems

## Educational Objectives:

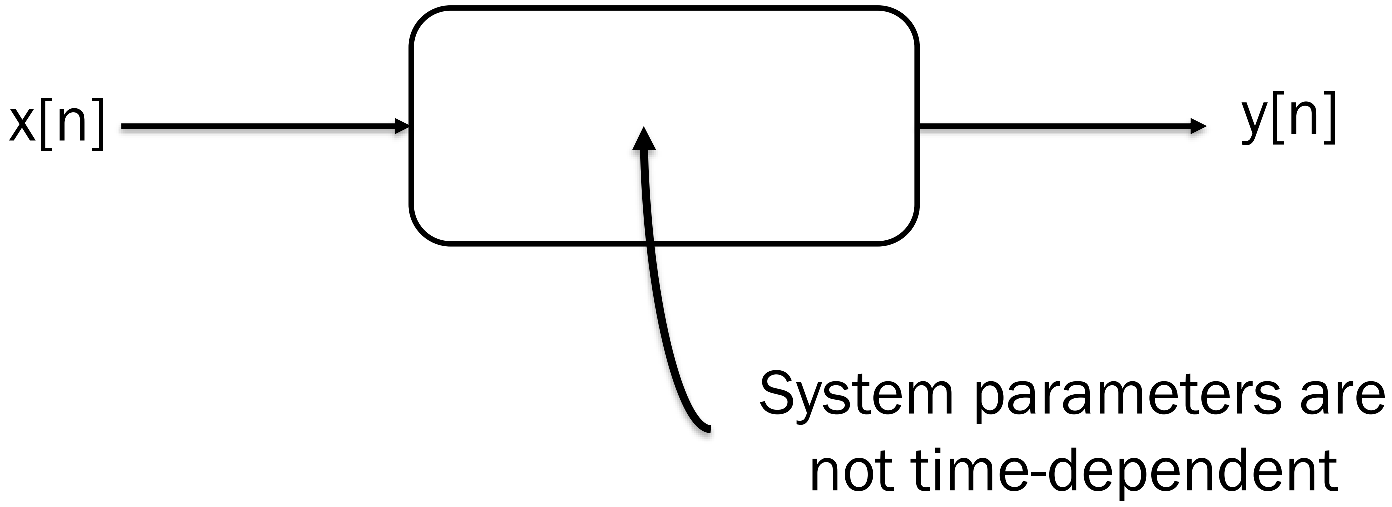
- ✓ Understand shift-invariance and how to determine if a system is shift-invariant or shift-varying
- ✓ Understand BIBO stability and how to determine if a system is BIBO stable or not
- ✓ Understand the convolution relationship of LSI systems
- ✓ Understand how to determine causality and BIBO stability of LSI systems

# Shift Invariance

- Definition

If $x[n] \rightarrow y[n]$

then $x[n - n_0] \rightarrow y[n - n_0]$



# Shift-Invariance: Examples

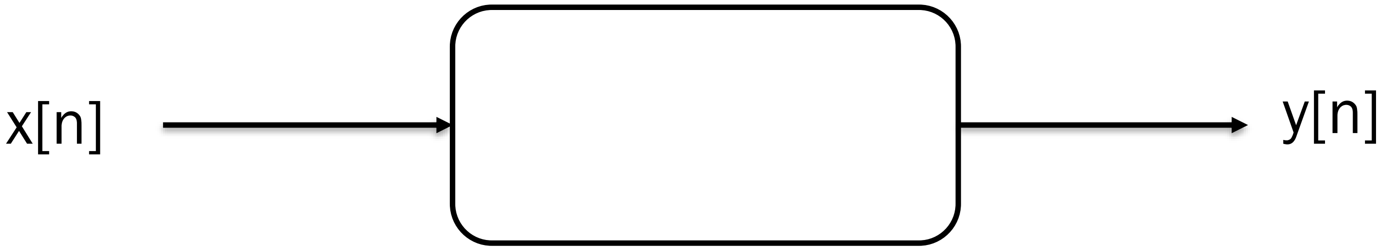
**Example: show system (a) is time-varying**

$$y[n] + \cos(n)\, y[n-1] = x[n]$$

**Example: show system (b) is time-invariant**

$$y[n] = \cos(x[n])$$

# Bounded-Input Bounded Out (BIBO) Stability




**Definition:**

A system is BIBO (Bounded Input, Bounded Output) stable if
**every bounded input** $\longrightarrow$ **bounded output**

- Bounded signal: $|x[n]| < B \Longrightarrow$ finite

# Examples

Example: Determine if the following functions are bounded:

1) $\delta[n]$
2) $e^{-3n}\,u[n]$
3) $e^{3n}u[n]$

Example: Determine if the following systems are BIBO stable:

1) $y[n] = 3x[n] + 2x[n-1]$
2) $y[n] = nx[n]$
3) $y[n] = \frac{1}{x[n]}$

# Convolution Relationship of LSI Systems

## Input-output relationship of LSI system

$$x[n] \longrightarrow \boxed{\text{LSI}} \longrightarrow y[n]$$

Let $x[n] = \delta[n]$, unit pulse; $y[n] = h[n]$, unit pulse response

# Convolution Relationship of LSI Systems

## Convolution Sum!

$$x[n] * h[n] \triangleq \sum_{l=-\infty}^{+\infty} x[l] \cdot h[n-l] = \sum_{l=-\infty}^{+\infty} x[n-l] \cdot h[l]$$

# Causality and BIBO Stability of LSI Systems

- Causality $\Longleftrightarrow$ $h[n] = 0, n < 0$ right-sided signal

$$y[n] = \sum_{l=0}^{+\infty} x[n-l] \cdot h[l]$$

current and past values!

- Stability:

$$\sum_{n=-\infty}^{+\infty} |h[n]| < B$$

absolutely summable!

# More Example on Linearity

Example: $y[n] = \frac{x[n]}{x[1]}$

Let $x_1[n] \rightarrow y_1[n]$: $\qquad y_1[n] = \frac{x_1[n]}{x_1[1]}$ ... (1)

$x_2[n] \rightarrow y_2[n]$: $\qquad y_2[n] = \frac{x_2[n]}{x_2[1]}$ ... (2)

Further assume $x_3[n] = ax_1[n] + bx_2[n] \rightarrow y_3[n]$

we have $y_3[n] = \frac{x_3[n]}{x_3[1]} = \frac{ax_1[n]+bx_2[n]}{ax_1[1]+bx_2[1]}$ ... (3)

Superposition rule requires

$$y_3[n] = ay_1[n] + by_2[n] = a\frac{x_1[n]}{x_1[1]} + b\frac{x_2[n]}{x_2[1]}$$

which is **not** consistent with the result in (3)