ECE 310

Digital Signal Processing

Spring, 2021, ZJUI Campus

Lecture 12

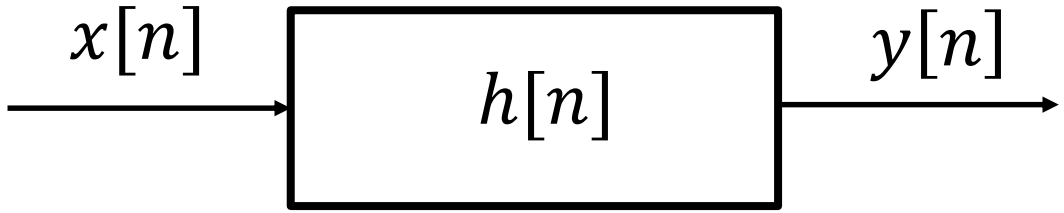
Topics:

- ✓ Analysis of LSI systems using z-transform
- ✓ Determination of unit pulse response using z-transform

Educational Objectives:

- ✓ Know how to determine unit pulse response using z-transform
- ✓ Know how to determine potential choices of ROC for H(z)
- ✓ Get more familiar with inverse z-transform

Determination of Unit Pulse Response



$$H(z) = \frac{Y(z)}{X(z)}$$

$$h[n] = z^{-1}\{H(z)\}$$

Determination of Unit Pulse Response

$$x[n]=(\frac{1}{2})^n u[n],\quad y[n]=(\frac{1}{4})^n u[n]+(\frac{1}{2})^n u[n]$$

$$X(z)=\frac{z}{z-\frac{1}{2}},\qquad |z|>\frac{1}{2}$$

$$Y(z)=\frac{z}{z-\frac{1}{4}}+\frac{z}{z-\frac{1}{2}},\qquad |z|>\frac{1}{2}$$

$$=\frac{z(z-\frac{1}{2})+z(z-\frac{1}{4})}{(z-\frac{1}{4})(z-\frac{1}{2})}=\frac{2z^2-\frac{3}{4}z}{(z-\frac{1}{4})(z-\frac{1}{2})},\qquad |z|>\frac{1}{2}$$

Determination of Unit Pulse Response

$$H(z)=\frac{Y(z)}{X(z)}=\frac{2z^2-\frac{3}{4}z}{(z-\frac{1}{4})(z-\frac{1}{2})}\cdot\frac{z-\frac{1}{2}}{z}=\frac{2z-\frac{3}{4}}{z-\frac{1}{4}}$$

Possible ROCs: $|z|>\frac{1}{4}$ or $|z|<\frac{1}{4}$

$$(|z|>\frac{1}{2})\cap(|z|>\frac{1}{4})=|z|>\frac{1}{2} \qquad (|z|>\frac{1}{2})\cap(|z|<\frac{1}{4})=\text{empty}$$

Valid ROC: $|z|>\frac{1}{4}$,

$$h[n]=2\left(\frac{1}{4}\right)^n u[n]-\frac{3}{4}\left(\frac{1}{4}\right)^{n-1}u[n-1]$$

$$=2\delta[n]-\left(\frac{1}{4}\right)^n u[n]$$

One more example to determine h[n]

$$x[n]=\delta[n-1]-\frac{1}{6}\left(\frac{1}{3}\right)^{n-2}u[n-2],\quad y[n]=\left(\frac{1}{3}\right)^{n-1}u[n-1]$$

$$X(z)=z^{-1}-\frac{1}{6}\frac{1}{z(z-\frac{1}{3})}=\frac{z-\frac{1}{2}}{z(z-\frac{1}{3})},\qquad |z|>\frac{1}{3}$$

$$Y(z)=\frac{1}{z-\frac{1}{3}},\qquad |z|>\frac{1}{3}$$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{1}{z-\frac{1}{3}}\frac{z(z-\frac{1}{3})}{z-\frac{1}{2}}=\frac{z}{z-\frac{1}{2}},$$ possible ROC_H : $|z|>1/2$ or $|z|<1/2$

Which is correct: a) $|z|>1/2$ is a valid ROC

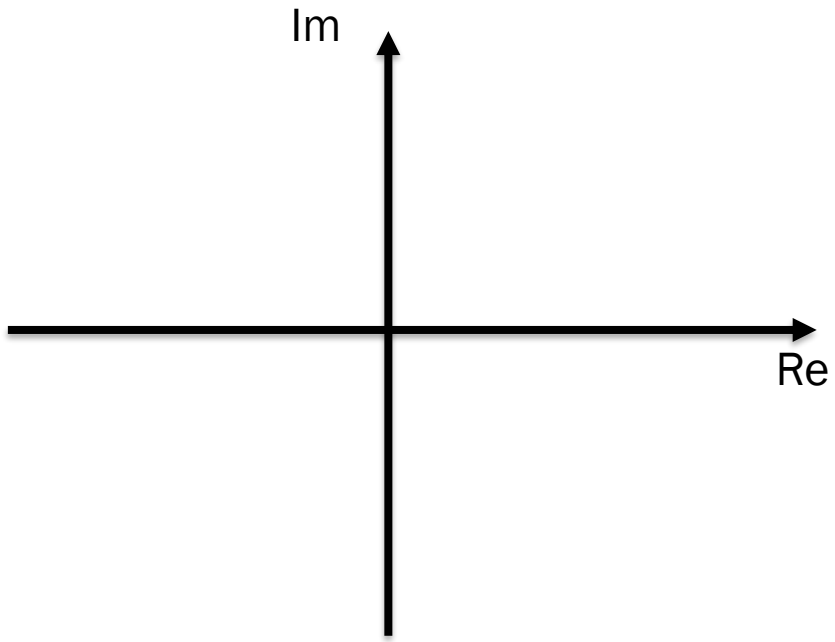
b) $|z|<1/2$ is a valid ROC

c) both $|z|>1/2$ *and* $|z|<1/2$ are valid ROC

More About Possible ROCs

$$X(z) = \frac{z}{(z-2)(z-3)(z-4)}$$

Possible ROCs:



Example

$$\frac{X(z)}{z} = \frac{1}{(z-2)(z-3)(z-4)} = \frac{A}{z-2} + \frac{B}{z-3} + \frac{C}{z-4}$$

$$A = \frac{X(z)}{z}(z-2)\Big|_{z=2} = \frac{1}{-1\times(-2)} = \frac{1}{2}$$

$$B = \frac{X(z)}{z}(z-3)\Big|_{z=3} = \frac{1}{1\times(-1)} = -1$$

$$C = \frac{X(z)}{z}(z-4)\Big|_{z=4} = \frac{1}{2\times 1} = \frac{1}{2}$$

$$X(z) = \frac{\frac{1}{2}z}{z-2} - \frac{z}{z-3} + \frac{\frac{1}{2}z}{z-4}$$

Example

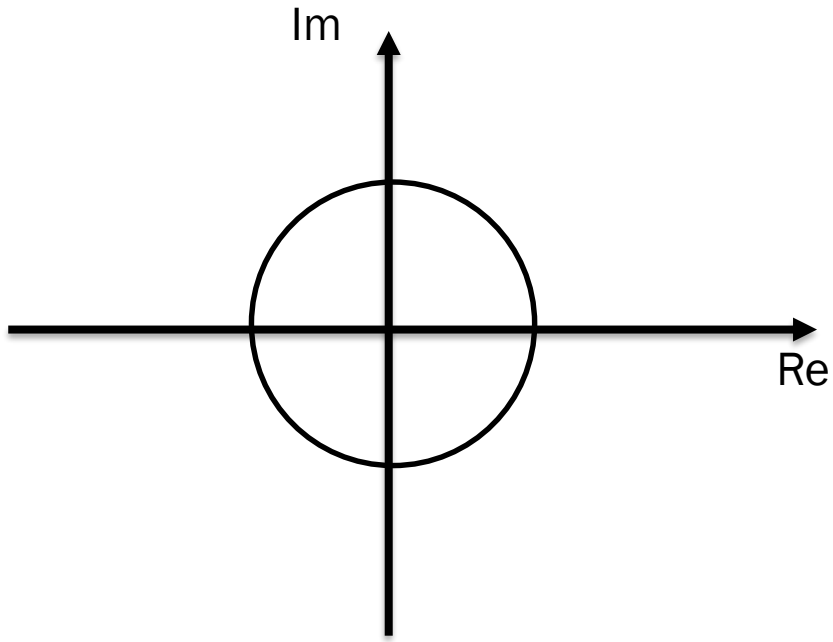
$$X(z)=\frac{\frac{1}{2}z}{z-2}-\frac{z}{z-3}+\frac{\frac{1}{2}z}{z-4}$$

$$ROC: \quad |z|<2$$

$$x[n]=-\frac{1}{2}2^n u[-n-1]$$

$$+3^n u[-n-1]$$

$$-\frac{1}{2}4^n u[-n-1]$$



Example

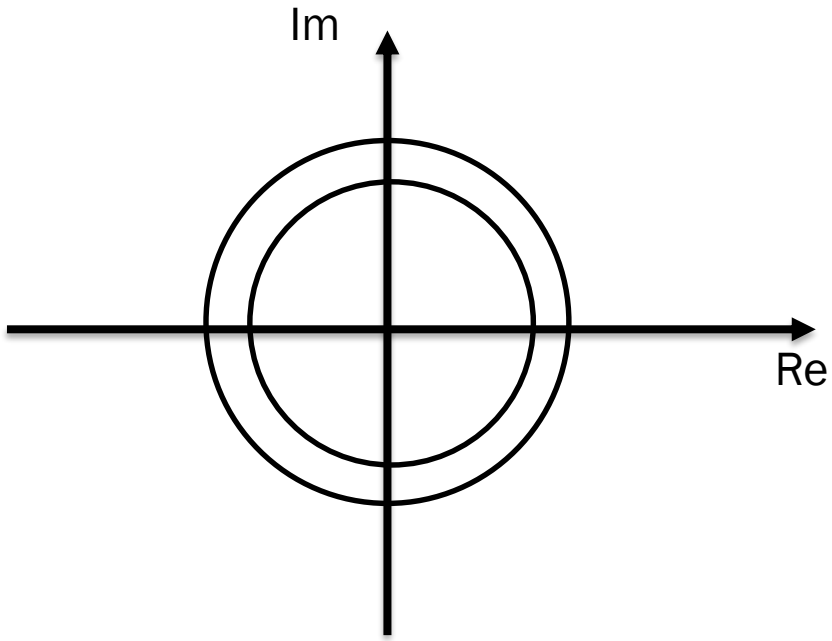
$$X(z)=\frac{\frac{1}{2}z}{z-2}-\frac{z}{z-3}+\frac{\frac{1}{2}z}{z-4}$$

$$ROC:\quad 2<|z|<3$$

$$x[n]=\frac{1}{2}2^n u[n]$$

$$+3^n u[-n-1]$$

$$-\frac{1}{2}4^n u[-n-1]$$



Example

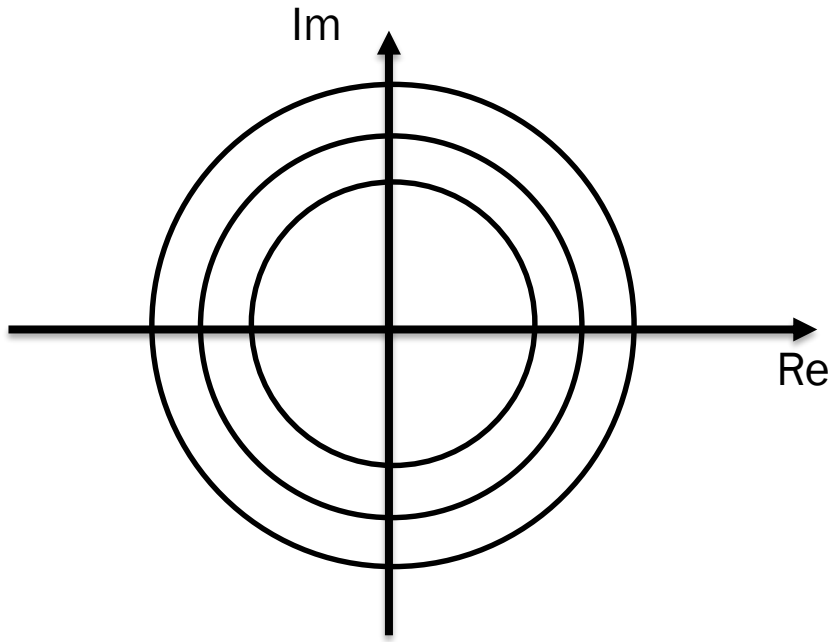
$$X(z)=\frac{\frac{1}{2}z}{z-2}-\frac{z}{z-3}+\frac{\frac{1}{2}z}{z-4}$$

$$ROC:\quad 3<|z|<4$$

$$x[n]=\frac{1}{2}2^n u[n]$$

$$-3^n u[n]$$

$$-\frac{1}{2}4^n u[-n-1]$$



Example

$$X(z) = \frac{\frac{1}{2}z}{z-2} - \frac{z}{z-3} + \frac{\frac{1}{2}z}{z-4}$$

$$ROC: \quad |z| > 4$$

$$x[n] = \frac{1}{2}2^n u[n] - 3^n u[n] + \frac{1}{2}4^n u[n]$$