ECE 310

Digital Signal Processing

Spring, 2021, ZJUI Campus

Lecture 13

Topics:

- ✓ Analysis of LSI systems using z-transform
- ✓ Solution of LCCDE
- ✓ Block diagrams of LCCDE system

Educational Objectives:

- ✓ Know how to find the zero-state response of LCCDE systems
- ✓ Know the structure of H(z) for LCCDE systems
- ✓ Know how to draw block diagram direct form I and direct form II for LCCDE systems

Linear Constant Coefficient Difference Equation

$$y[n]+\sum_{k=1}^{N}a_k y[n-k]=\sum_{k=0}^{N}b_k x[n-k]$$

Shifting property:

$$x[n]\rightarrow X(z)$$

$$x[n-k]\rightarrow z^{-h}X(z)$$

$$y[n]\rightarrow Y(z)$$

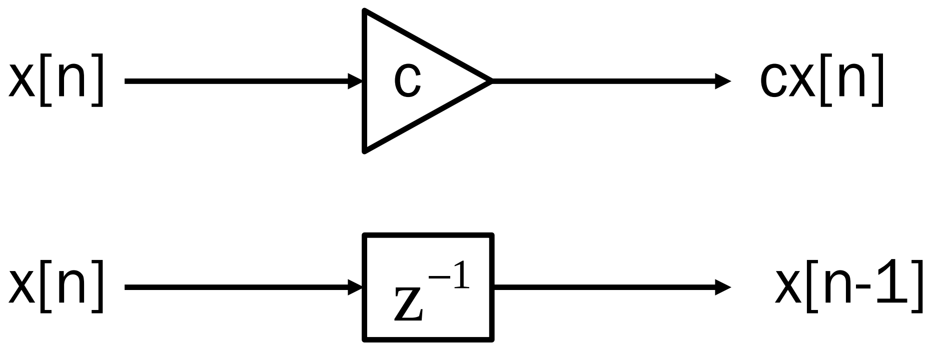
$$y[n-k]\rightarrow z^{-h}Y(z)$$

$$Y(z)+\sum_{k=1}^{N}a_k z^{-k}Y(z)=\sum_{k=0}^{N}b_k z^{-k}X(z)$$

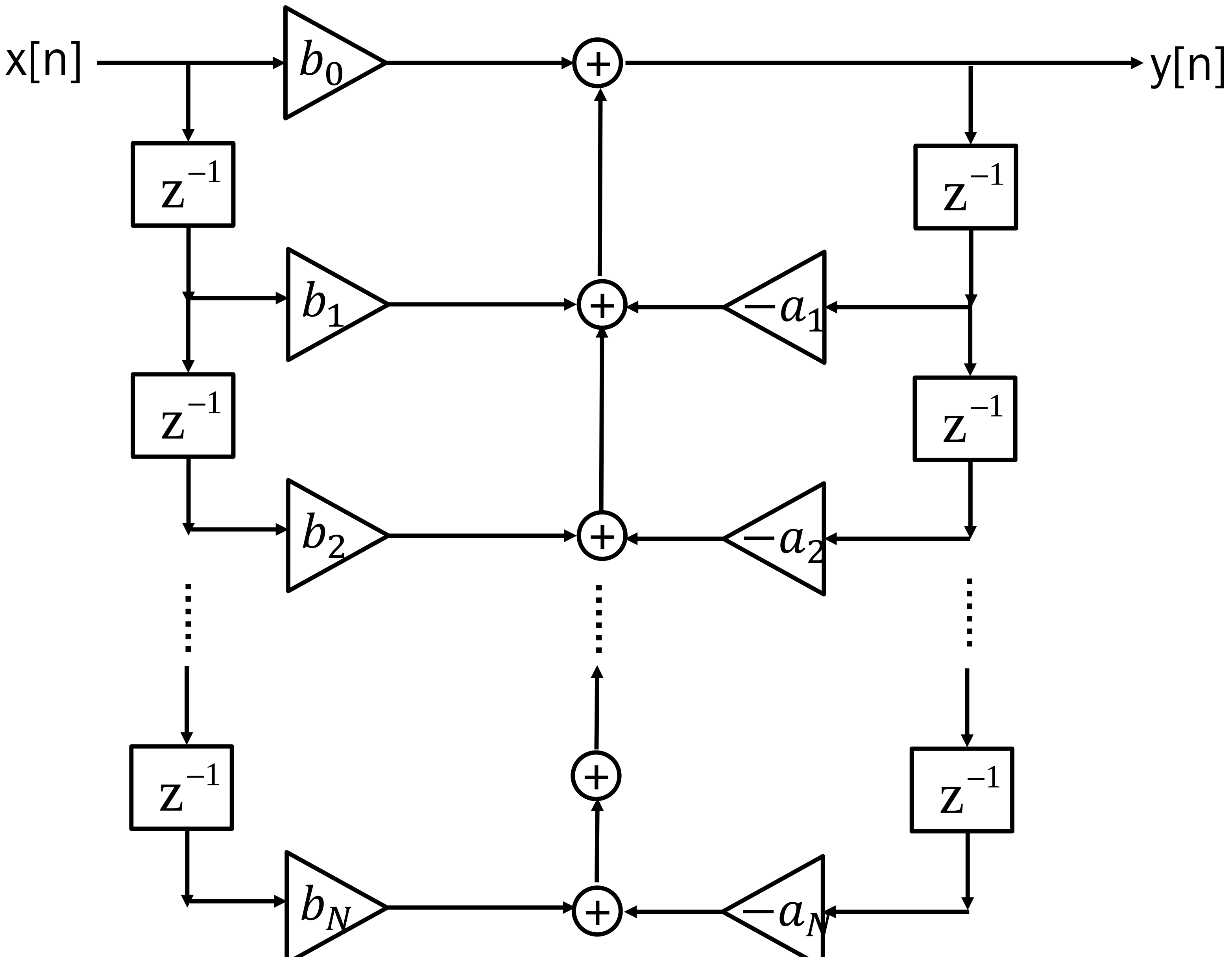
$$Y(z)(1+\sum_{k=1}^{N}a_k z^{-k})=(\sum_{k=0}^{N}b_k z^{-k})X(z)$$

$$\longrightarrow \quad H(z)=\frac{Y(z)}{X(z)}=\frac{\sum_{k=0}^{N}b_k z^{-k}}{1+\sum_{k=1}^{N}a_k z^{-k}}$$

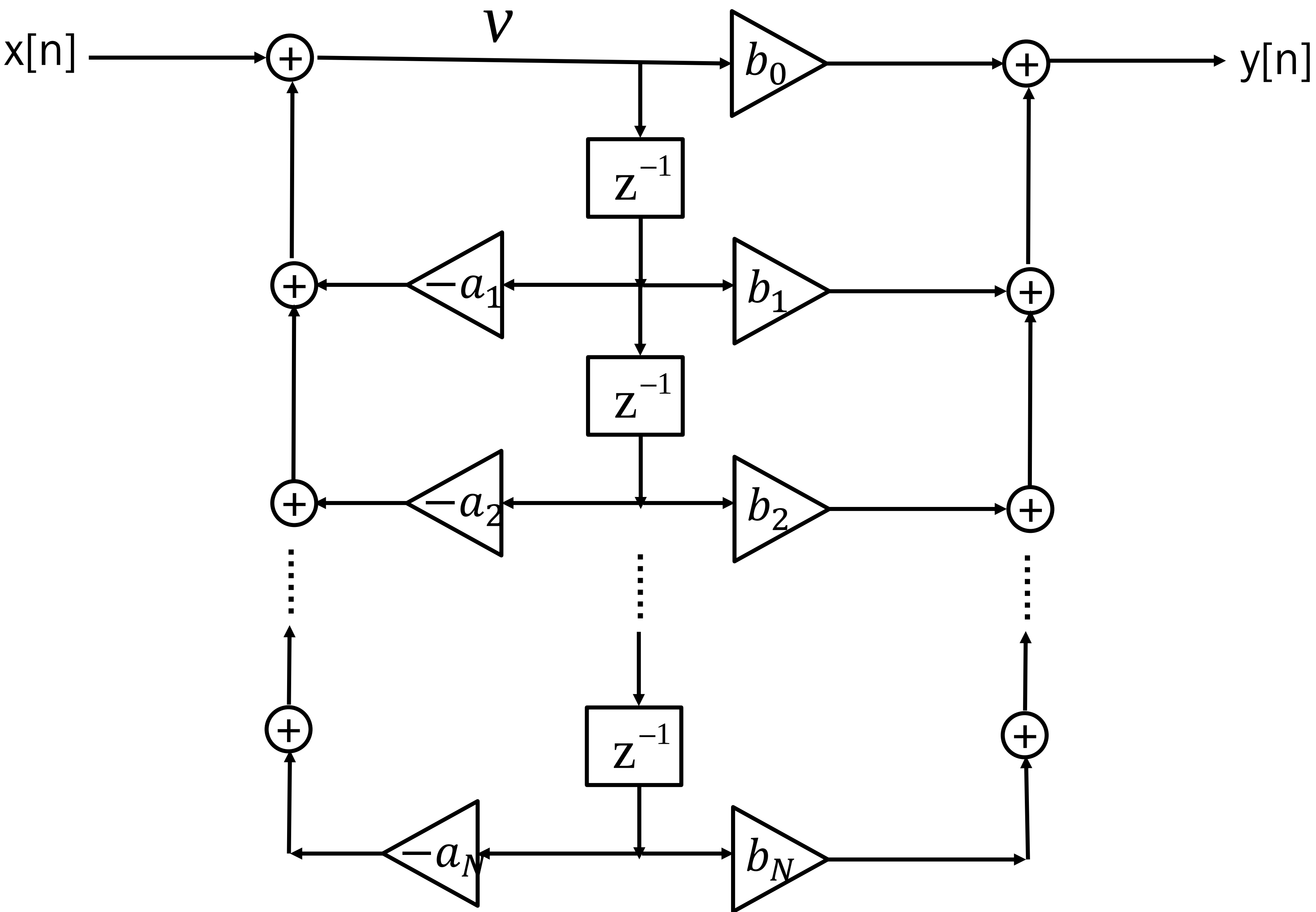
Block Diagram



Direct Form I



Direct Form II



Direct Form II

$$Y(z) = (b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_N z^{-N})V(z)$$

$$= (\sum_{k=0}^{N} b_k z^{-k})V$$

$$V = X(z) - (a_1 z^{-1} + a_2 z^{-2} + \ldots + a_N z^{-N})V$$

$$(1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_N z^{-N})V = X(z)$$

$$V = \frac{X(z)}{1 + \sum_{k=1}^{N} a_k z^{-k}} \qquad Y(z) = \frac{\sum_{k=0}^{N} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} X(z)$$

Example

$$y[n]-5y[n-1]+6y[n-2]=3x[n-1]+5x[n-2], \qquad x[n]=u[n]$$

$$Y(z)-5z^{-1}Y(z)+6z^{-2}Y(z)=3z^{-1}X(z)+5z^{-2}X(z)$$

$$(1-5z^{-1}+6z^{-2})Y(z)=(3z^{-1}+5z^{-2})X(z)$$

$$H(z)=\frac{3z^{-1}+5z^{-2}}{1-5z^{-1}+6z^{-2}}=\frac{3z+5}{z^2-5z+6}, \qquad h[n]=Z^{-1}\{H(z)\}$$

$$X(z)=\frac{z}{z-1}$$

$$Y(z)=\frac{3z+5}{z^2-5z+6}\,\frac{z}{z-1}$$

Example

$$Y(z) = \frac{3z+5}{z^2-5z+6}\frac{z}{z-1}$$

$$\hat{Y}(z) = \frac{Y(z)}{z} = \frac{3z+5}{z^2-5z+6}\frac{1}{z-1} = \frac{3z+5}{(z-2)(z-3)(z-1)}$$

$$= \frac{-11}{z-2} + \frac{7}{z-3} + \frac{4}{z-1}$$

$$Y(z) = \frac{-11z}{z-2} + \frac{7z}{z-3} + \frac{4z}{z-1}$$

How to Handle Zero-Input Response

$$y[n]+\sum_{k=1}^{N} a_k y[n-k]=\sum_{k=0}^{N} b_k x[n-k]$$

$$\text{ICs:}\;\; y[-1],\; y[-2],\; \cdots,\; y[-N]$$

One-sided z-transform: $\quad Y(z)=\sum_{n=0}^{\infty} y[n]z^{-n}$

Shifting property: $\quad$ if $y[n]\rightarrow Y(z)$; then, $y[n-k] \rightarrow z^{-k}Y(z)+\sum_{n=1}^{k} y[-n]z^{n-k}$

$$Y_s(z)=\frac{\sum_{k=1}^{N} a_k \sum_{n=1}^{k} y[-n]z^{n-k}}{1+\sum_{k=1}^{N} a_k z^{-k}}$$

Example

$$y[n]-5y[n-1]+6y[n-2]=3x[n-1]+5x[n-2], \qquad x[n]=u[n]$$

$$y[-1]=-1, y[-2]=2$$

$$Y_s(z)=\frac{-5y[-1]+6y[-1]z^{-1}+6y[-2]}{1-5z^{-1}+65z^{-2}}$$

$$=\frac{5-6z^{-1}+12}{1-5z^{-1}+65z^{-2}}$$

$$=\frac{17-6z^{-1}}{1-5z^{-1}+65z^{-2}}$$