ECE 310 Recitation 9
Thursday Apr 15, 2021

Concept check

√ DFT spectral analysis

- Determine the frequency content of a given signal: $x_a(t) = \sum_{i=1}^{M} A_i \cos(\Omega_i t)$, determine $\{\Omega_i, A_i\}_{i=1}^{M}$
- Spectral parameters
  - $X_a(\Omega) \sim X_d(\omega) \sim X_m$
  - Amplitudes: $\frac{A_i}{T}$
  - Frequencies: $m_i \sim \omega_i \sim \Omega_\mathrm{i}$
- Windowing effect:
  - $\hat{x}[n] = x[n]w[n], \widehat{X_d}(\omega) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_d(u) W_d(\omega - u) du$
  - For sinusoidal input $x[n] = A\cos(\Omega_0 nT)$ and length-N rectangular window
  - $\widehat{X_d}(\omega) = e^{-j(\omega-\Omega_0 T)\frac{N-1}{2}} \frac{\frac{A}{2}\sin[(\omega-\Omega_0 T)\frac{N}{2}]}{\sin[(\omega-\Omega_0 T)\frac{1}{2}]} + e^{-j(\omega+\Omega_0 T)\frac{N-1}{2}} \frac{\frac{A}{2}\sin[(\omega+\Omega_0 T)\frac{N}{2}]}{\sin[(\omega+T)\frac{1}{2}]}$
  - Main lobe height: $\frac{AN}{2}$, main lobe width $\frac{4\pi}{N}$, peak location $\omega = \pm\Omega_0 T$

√ FFT

- $\begin{cases} X_m = Y_m + W_N^m Z_m \\ X_{m+N/2} = Y_m - W_N^m Z_m \end{cases}, \begin{cases} Y_m = DFT\{x[2l]\}_{l=0}^{n/2-1}\ (even) \\ Z_m = DFT\{x[2l+1]\}_{l=0}^{n/2-1}\ (odd) \end{cases}, W_N^m = e^{-j\frac{2\pi m}{N}}$
- Butterfly diagram, bit-reverse indexing

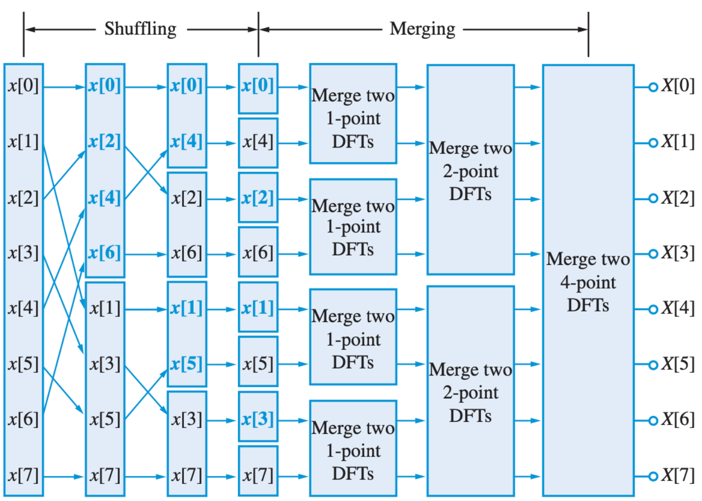

Figure 8.5 The shuffling and merging operations required for recursive computation of the 8-point DFT using the decimation-in-time FFT algorithm.

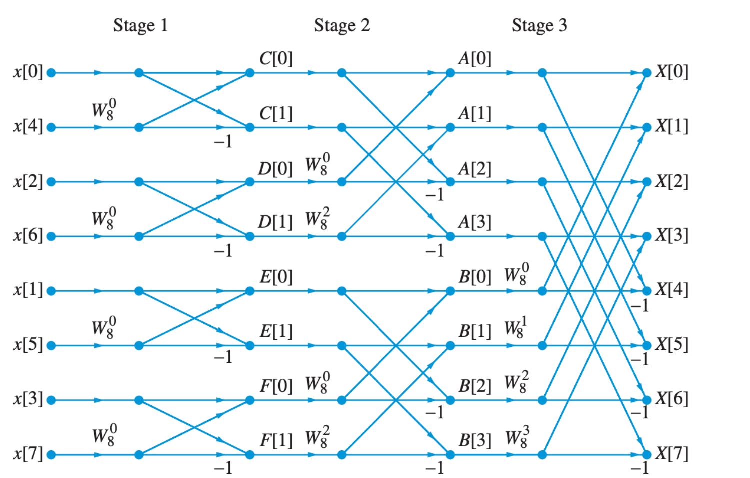

Figure 8.6 Flow graph of 8-point decimation-in-time FFT algorithm using the butterfly computation shown in Figure 8.4. The trivial twiddle factor $W_8^0 = 1$ is shown for the sake of generality.

Exercise

1. A continuous-time signal $x_c(t) = \cos(\frac{\pi}{3}t)$ is sampled at a rate of 30Hz for 12s to produce a discrete-time signal $x[n]$ with length $L = 360$.

   (a) Let $X[k]$ be the length-L DFT of $x[n]$. At what value(s) of k will $X[k]$ have the greatest magnitude?

   (b) Suppose that $x[n]$ is zero-padded to a total length of $L = 512$. At what value(s) of k will $X[k]$ have the greatest magnitude?

   (c) Suppose that $x(t)$ is only sampled for 2s, so the length of $X[k]$ is $L = 60$. At what value(s) of k will $X[k]$ have the greatest magnitude?

   (d) Suppose that the $x[n]$ from (c) is zero-padded to a total length of $L = 64$. At what value(s) of k will $X[k]$ have the greatest magnitude?

2. The diagram below represents a part of the computation in a 16-point decimation-in-time radix-2 FFT. Indicate the values of the three branch weights, d, e, and f.