3170111144 Yuhang Chen

ZHEJIANG UNIVERSITY - UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

ECE 310 Digital Signal Processing

Homework 8

Prof. Zhi-Pei Liang **Due: April 9, 2021**

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1. Assume $x[n]$ is a finite-duration sequence of length 40, and $y[n]$ is obtained by zero-padding $x[n]$ to length 64. That is, $y[n] = x[n]$, for $n = 0, 1, \ldots, 39$, and $y[n] = 0$, $n = 40, 41, \ldots, 63$. Let $\{X[m]\}_{m=0}^{39}$ and $\{Y[m]\}_{m=0}^{63}$ be the DFT of $\{x[n]\}_{n=0}^{39}$ and $\{y[n]\}_{n=0}^{63}$, respectively. Determine all the correct relationships and **justify your answers**.
   (a) $X[0] = Y[0]$
   (b) $X[5] = Y[8]$
   (c) $X[10] = Y[16]$
   (d) $X[12] = Y[18]$
   (e) $X[39] = Y[63]$

2. Given that the DFT of $\{2, 0, 6, 4\}$ is $\{X_0, X_1, X_2, X_3\}$, determine the DFT of $\{2, 1, 0, 3\}$ and express the result in terms of X_0, X_1, X_2, X_3.

   Hint: relate the two sequences using transformations we discussed in class (scaling, time reversal, conjugation, circular shift, ...) and use the corresponding properties of the DFT.

3. a) Find the inverse DFT of the sequence $X[m] = \left\{1, e^{-j\frac{3\pi}{4}}, 0, e^{j\frac{3\pi}{4}}\right\}$ where the first entry of $X[m]$ corresponds to $m = 0$.
   b) Without explicitly computing the inverse DFT sum, find the inverse DFT of the sequence $Y[m] = \left\{1, e^{-j\frac{\pi}{4}}, 0, e^{j\frac{\pi}{4}}\right\}$ where the first entry of $Y[m]$ corresponds to $m = 0$, using your answer to part (a).

4. You are given two sequences $x[n] = [1, 2, 3, 4, 5, 6]$ and $y[n] = [4, 5, 6, 1, 2, 3]$. It is known that $Y[m] = X[m]e^{-j\frac{2\pi}{6}mn_0}$. Find two values of n_0 consistent with this information.

5. Let $X[m]$ be the 8-point DFT of the sequence $x[n] = [1, -1, 2, 3, -3, 0, 0, 0]$. Let $y[n]$ be a finite length sequence whose DFT $Y[m]$ is related to $X[m]$ as $Y[m] = X[m]e^{-j\frac{2\pi}{6}mn_0}$, where $n_0 = 3$. Determine the sequence $y[n]$.

6. Let $X[m]$ be the 6-point DFT of $x[n] = [1, 2, 3, 4, 5, 6]$. Determine the sequence $y[n]$ whose DFT $Y[m] = X[< -m >_6]$.

7. Let $X[m]$ denote the 80-point DFT of $x[n], 0 \leq n \leq 79$. The sequence $y[n]$ is obtained by zero-padding $x[n]$ to length 128. Determine m_0 such that $Y[8] = X[m_0]$.

8. Let $X[m], (0 \le m \le 20)$ and $X_d(\omega)$ respectively be the 21-point DFT and DTFT of a *real-valued* sequence $\{x_n\}_{n=0}^{7}$ that is zero-padded to length 21. Determine all the **correct** relationships and justify your answer.

   (a) $X[19] = X_d(-\frac{4\pi}{21})$.

   (b) $X[2] = X_d^*(-\frac{4\pi}{21})$

   (c) $X[12] = X_d(-\frac{4\pi}{21})$

   (d) $X[4] = X_d^*(-\frac{4\pi}{21})$

P1. $X_d(w) = Y_d(w)$

$X_m = X_d(w)\Big|_{w=\frac{2\pi}{N}m} = X_d\left(\frac{2\pi}{40}m\right)$

$Y_m = Y_d(w)\Big|_{w=\frac{2\pi}{M}m} = X_d\left(\frac{2\pi}{64}m\right)$

Let $X_{m_i} = Y_{m_j}$

$\Rightarrow \frac{2\pi}{40}m_i = \frac{2\pi}{64}m_j$

$\therefore m_i = \frac{40}{64} = \frac{5}{8}m_j$

$\Rightarrow X_{5k} = Y_{8k}$, where $k < 8$

a) $X[0] = Y[0]$: True

b) $X[5] = Y[8]$: True

c) $X[10] = Y[16]$: True

d) $X[12] = Y[18]$: False

e) $X[39] = Y[64]$: False

P2. $\{2,0,6,4\} \xleftrightarrow{DFT} \{X_0, X_1, X_2, X_3\}$

$\quad \| $

$\{x_n\}_{n=0}^{4}$

Let $\{g_n\}_{n=0}^{4} = \{2,1,0,3\}$

$= \left\{\frac{x_{\langle n-1\rangle_4}}{2}\right\}_{n=0}^{4}$

$X_d(w) = \sum_{n=0}^{N-1} x_n e^{-jwn}$

When $x_n' \to \frac{x_n}{2}$

$X_d'(w) = \sum_{n=0}^{N-1} x_n' e^{-jwn}$

$= \frac{1}{2} X_d(w)$

$\therefore \{g_n\}_{n=0}^{4} = \left\{\frac{x_{\langle n-1\rangle_4}}{2}\right\}_{n=0}^{4}$

$\xleftrightarrow{DFT} \left\{\frac{1}{2}X_0, \frac{1}{2}X_1 e^{-j\frac{2\pi}{4}}, \frac{1}{2}X_2 e^{-j\frac{2\pi}{4}\cdot 2}, \frac{1}{2}X_3 e^{-j\frac{2\pi}{4}\cdot 3}\right\}$

P3. $X[m] = \{1, e^{-j\frac{3\pi}{4}}, 0, e^{j\frac{3\pi}{4}}\}$

$\{x[n]\} = \frac{1}{N}\sum_{m=0}^{N-1} X_m e^{j\frac{2\pi}{N}mn}$

$\therefore \{x[0]\} = \frac{1}{4}\sum_{m=0}^{3} 1\, e^{j\frac{2\pi}{4}m0} = \frac{1}{4}$

$\{x[1]\} = \frac{1}{4}\sum_{m=0}^{3} e^{-j\frac{3\pi}{4}} e^{j\frac{2\pi}{4}m\cdot 1}$

$= 0$

$= \frac{1}{4}\left(e^{-j\frac{3\pi}{4}} + e^{-j\frac{\pi}{4}} + e^{j\frac{\pi}{4}} + e^{j\frac{3\pi}{4}}\right)$

$\{x[2]\} = \frac{1}{4}\sum_{m=0}^{3} 0\, e^{j\frac{2\pi}{4}m\cdot 2} = 0$

$\{x[3]\} = \frac{1}{4}\sum_{m=0}^{3} e^{j\frac{3\pi}{4}} e^{j\frac{2\pi}{4}\cdot m\cdot 3}$

$= \frac{1}{4}\left(e^{j\frac{3\pi}{4}} + e^{j\frac{9\pi}{4}} + e^{j\frac{15\pi}{4}} + e^{j\frac{21\pi}{4}}\right)$

$= \frac{1}{4}\left(e^{\frac{3\pi}{4}} + e^{j\frac{\pi}{4}} + e^{j\frac{\pi}{4}} + e^{-j\frac{3\pi}{4}}\right) = 0$

$\therefore$ Inverse DFT:

$\{x[n]\} = \{\frac{1}{4}, 0, 0, 0\}$

$Y[m] = \{1, e^{-j\frac{\pi}{4}}, 0, e^{j\frac{\pi}{4}}\}$

b) $\{Y[n]\}$

$= \{\frac{1}{4}, 0, 0, 0\}$ according to part a.

4. $Y[m] = X[m] e^{-j\frac{2\pi}{6} m n_0}$, $N = 6$

$\updownarrow$ DFT

$\{y_n\}_{n=0}^{5}$

$= \{x_{\langle n+3 \rangle_5}\}_{n=0}^{5}$

$\therefore -n_0 = 3 + 5k$, $k = 0, 1, 2 \cdots N$

i.e. $n_0 = -3$ or $n_0 = 2$

5. $Y[m] = X[m] e^{-j\frac{2\pi}{6} m n_0}$, $n_0 = 3$, $N = 8$

$\updownarrow$ DFT Let $\frac{2\pi}{6} m \cdot 3 = \frac{2\pi}{8} m \cdot k$

$\{y_n\}_{n=0}^{7}$ $\Rightarrow k = 4$

$= \{x_{\langle n-k \rangle_8}\}_{n=0}^{7}$

$= \{x_{\langle n-4 \rangle_8}\}_{n=0}^{7}$

$y_n = [-3, 0, 0, 0, 1, -1, 2, 3]$

6. $Y[m] = X[\langle -m \rangle_6]$, $N = 6$

$= X[\langle -m \rangle_6] e^{j\frac{2\pi}{N} \cdot m \cdot n_0}$, $n_0 = 0$

$= X[\langle -m+0 \rangle_6] e^{0}$

$= X[\langle -m+0 \rangle_6]$

$\updownarrow$ DFT

$\{y_n\}_0^6 = \{x_{\langle -n \rangle_6}\}_{n=0}^{6}$

$\therefore y[n] = [6, 5, 4, 3, 2, 1]$

7. $Y_m = \sum_{n=0}^{127} y_n e^{-j\frac{2\pi}{128} mn}$

$X_m = \sum_{n=0}^{79} x_n e^{-j\frac{2\pi}{80} mn}$

$\therefore$ for $n \in [0, 79]$, $y[n] = x[n]$

for $n \in [80, 127]$, $y[n] = 0$

Let $Y_8 = X_{m_0}$

$\frac{2\pi}{128} \cdot 8 = \frac{2\pi}{80} \cdot m_0$

$\therefore m_0 = 5$

8. $x[n] = [x_0, x_1 \cdots x_7, \overbrace{0, \cdots 0}^{13}]$

$X_m = X_d(w) \big|_{w = \frac{2\pi}{21} m}$

a) $X[19] = X[-2] = X_d(-\frac{4\pi}{21})$, correct

b) real-value, $X[2] = X^*[-2] = X_d^*(-\frac{4\pi}{21})$, correct

c) False, $X[12] = X_d(\frac{24\pi}{21}) \neq X_d(-\frac{4\pi}{21})$

d) False, $X[4] = X_d(\frac{8\pi}{21}) \neq X_d^*(-\frac{4\pi}{21})$