University of Illinois
ECE 310 **Profs. Kamalabadi, Katselis, Liang**

Midterm Exam I

7:00-8:30pm, Thursday, October 3, 2019

Name: _____

Section: 10:00 AM 12:00 PM 3:00 PM

NetID: _____

Score: _____

Problem	Pts.	Score
1	20	
2	12	
3	12	
4	5	
5	10	
6	6	
7	15	
8	4	
9	4	
10	12	
Total	100	

Instructions

- You may not use any books, calculators, or notes other than one handwritten two-sided sheet of 8.5" x 11" paper.
- Show all your work to receive full credit for your answers.
- When you are asked to "calculate", "determine", or "find", this means providing closed-form expressions (i.e., without summation or integration signs).
- Neatness counts. If we are unable to read your work, we cannot grade it.
- Turn in your entire booklet once you are finished. No extra booklet or papers will be considered.

(20 Pts.)

1. Mark "True" or "False" for the following statements.

   (a) An LSI system specified by the following difference equation: $y[n] - \frac{1}{2}y[n-1] = x[n]$ can be causal or anti-causal. **T/F**

   (b) The input and output relationship of an arbitrary system is completely determined by the system's unit pulse response. **T/F**

   (c) If an LSI system is BIBO unstable, its unit pulse response $h[n]$ must be unbounded. **T/F**

   (d) Let $h[n] = h_1[n] * h_2[n]$ be the unit pulse response (UPR) of two serial subsystems with UPR $h_1[n]$ and $h_2[n]$. If h_1 or h_2 is BIBO *unstable*, h must be BIBO *unstable*. **T/F**

   (e) An LSI system with a finite-length impulse response can be BIBO stable or unstable. **T/F**

   (f) A time-varying system cannot be causal. **T/F**

   (g) Let $\sum_{n=-\infty}^{\infty} x[n]\delta[2^n u[n] - 8] = 4$. Then, $x[3] = 2$. **T/F**

   (h) Suppose that the step response $g[n]$ of an LSI system, i.e., the output of the system to the input $x[n] = u[n]$ has a z-transform with a pole at $z = 1/2$. Then, $H(z)$ has also pole at $z = 1/2$. **T/F**

   (i) If the response $y[n]$ of an LSI system to the input $x[n] = 3^n u[n]$ is unbounded, the system must be BIBO unstable. **T/F**

   (j) The response $y[n]$ of a BIBO *unstable* LSI system to any non-zero input $x[n]$ is always unbounded. **T/F**

(12 Pts.)

2. Determine whether the following systems are linear or nonlinear, shift-invariant or shift-varying, and causal or noncausal.
   Grading: Correct answer = +2 pts.; Incorrect answer = -1 pts. No answer = 0 pts.

   (a) $y[n] = x[|n| + n]$

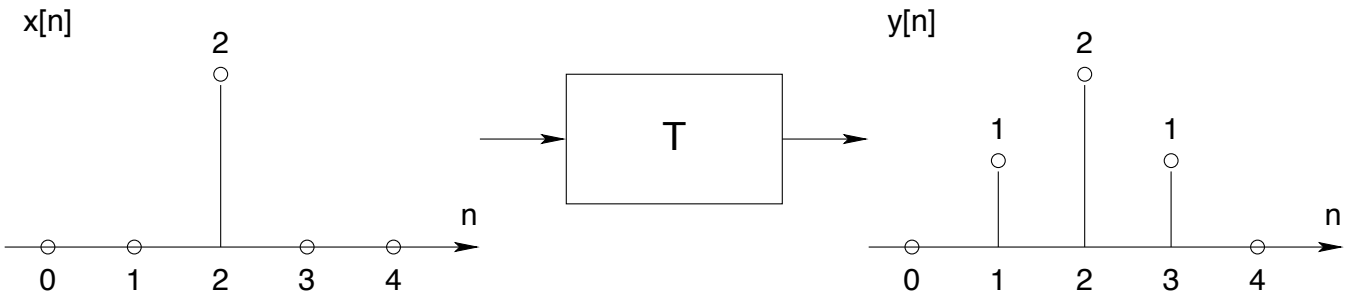
L / NL	SI / SV	C / NC

   (b) $y[n] = (0.2)^{|n|} \log[x[n]]$

L / NL	SI / SV	C / NC

(12 Pts.)

3. Suppose that T is a linear and shift-invariant system. For an input $x[n]$ depicted below we observe $y[n]$ as the output:



(a) (3pts.) Find the unit pulse response of the system T.

(b) (5pts.) Find the output $y[n]$ for $n = -1, 0, 1, 2$ produced by the response of the system to the unit step input $u[n]$.

(c) (4pts.) Is T a causal system? Justify your answer.

(5 Pts.)

4. Calculate the result of the following convolution $\{\underset{\uparrow}{1}, -4, 2, -1, 3, 1\} * \{\underset{\uparrow}{-1}, 1, -1\}$.

(10 Pts.)

5. Calculate the z-transform and corresponding ROC of the following functions:

   (a)

$$x[n] = 3^{-n}\left(u[n-5] - u[n-100]\right)$$

   (b)

$$x[n] = e^{-n^2}u[n-8]u[-n+10]$$

(6 Pts.)

6. Calculate the inverse z-transform for

$$Y(z) = 1 + z^{-100} + \frac{1}{1 - 5z^{-1}}, \qquad \text{ROC: } |z| > 5$$

(15 Pts.)

7. The z-transform of $x[n]$ is given below:

$$X(z) = \frac{z}{z - e^{j\frac{\pi}{3}}} + \frac{z}{z - 0.5}$$

Determine all the valid ROCs of $X(z)$ and for each case, calculate the inverse z-transform.

(4 Pts.)

8. Calculate the DTFT of $x[n] = \{1, \underset{\uparrow}{0}, 0, 2\}$

(4 Pts.)

9. Assume that the z-transform of $x[n]$ is given by

$$X(z) = \frac{z}{z-3}, \qquad |z| > 3.$$

Determine which of the following statements is correct (select one only; -1 pt for incorrect choice):

(a) The DTFT of $x[n]$ is $X(\omega) = \frac{e^{-j\omega}}{e^{j\omega}-3}$

(b) The DTFT of $x[n]$ is $X(\omega) = \frac{e^{-j\omega}}{e^{-j\omega}-3}$

(c) The DTFT of $x[n]$ does not exist

(d) None of the above

(12 Pts.)

10. Consider a *causal* LSI system with transfer function $H(z) = \frac{1-2z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$.

    (a) What is the ROC of $H(z)$?

    (b) Is the system BIBO stable? Justify your answer.

    (c) Find the difference equation (or LCCDE) for this system.

    (d) **True or False**: If the previous system is serially connected to an unstable LSI system with impulse response $\tilde{h}[n] = 2^n u[n]$, then the overall system is BIBO unstable. Justify your answer.