LSI System.

① $y[n] = \cos(n+10)\, x[n]$

Linearity

$x_1[n] \rightarrow y_1[n] = \cos(n+10)\, x_1[n]$.

$x_2[n] \rightarrow y_2[n] = \cos(n+10)\, x_2[n]$

$a_1 x_1[n] + a_2 x_2[n] \xrightarrow{? \rightarrow \text{Yes!}} a_1 y_1[n] + a_2 y_2[n] = a_1 \cos(n+10)\, x_1[n] + a_2 \cos(n+10)\, x_2[n]$

$\cancel{H\{a_1 x_1[n] + a_2 x_2[n]\}}$

$= \cos(n+10)\,(a_1 x_1[n] + a_2 x_2[n])$

SI:

$x_1[n] \longrightarrow y_1[n] = \cos(n+10)\, x_1[n]$

$x_2[n] = x_1[n-1] \xrightarrow{? \rightarrow \text{No!}} y_1[n-1] = \cos(n-1+10)\, x_1[n-1] = \cos(n+9)\, x_1[n-1]$

$\times$

$y_2[n] = \cos(n+10)\, x_2[n] = \cos(n+10)\, x_1[n-1]$

Causal. Yes.

② $y[n] + \frac{1}{2} y[n-1] = x^2[n]$

Linearity:

$x_1[n] \rightarrow y_1[n] + \frac{1}{2} y_1[n-1] = x_1^2[n]$

$x_2[n] \rightarrow y_2[n] + \frac{1}{2} y_2[n-1] = x_2^2[n]$

$a_1 x_1[n] + a_2 x_2[n] \xrightarrow{? \rightarrow \text{No!}}$

$\cancel{a_1 x_1[n] + a}$

$a_1\left(y_1[n] + \frac{1}{2} y_1[n-1]\right) + a_2\left(y_2[n] + \frac{1}{2} y_2[n-1]\right) = a_1 x_1^2[n] + a_2 x_2^2[n]$

$\neq (a_1 x_1[n] + a_2 x_2[n])^2$

$\cancel{y'[n]}$

SI: $x_1[n] \rightarrow y_1[n] + \frac{1}{2} y_1[n-1] = x_1^2[n]$

$x_2[n] = x_1[n-1] \rightarrow y_1[n-1] + \frac{1}{2} y_1[n-2] = x_1^2[n-1]$

$\longrightarrow y_2[n-1] + \frac{1}{2} y_2[n-1] = x_2^2[n] = x_1^2[n-1]$.

Causality: Yes/ No.

$$y[n] = x^2[n] - \frac{1}{2}\underline{y[n-1]}$$

Past output.
available by Memory

Yes

$\cancel{\frac{1}{2}y[n]}$ $\frac{1}{2}y[n-1] = x^2[n] - y[n]$

$y[n] = 2(x^2[n+1] - y[n+1])$ Not!

③ $y[n] = x[n]\,x[n-1]$

Linearity: $x_1[n] \rightarrow y_1[n] = x_1[n]\,x_1[n-1]$

$x_2[n] \rightarrow y_2[n] = x_2[n]\,x_2[n-1]$

$x'[n] = a_1x_1[n] + a_2x_2[n] \xrightarrow{?} a_1y_1[n] + a_2y_2[n] = a_1x_1[n]\,x_1[n-1] + a_2x_2[n]\,x_2[n-1]$ → No!

$\longrightarrow y'[n] \neq (a_1x_1[n] + a_2x_2[n])(a_1x_1[n-1] + a_2x_2[n-1])$

SI ~~: $x_1[n]$~~

~~Other way~~:

SI: $\cancel{x_1 =}$ $x_1[n] \longrightarrow y_1[n] = x_1[n]\,x_1[n-1]$

$x_2[n] = x_1[n-1] \xrightarrow{?}$ Yes $\cancel{y_2[n] = x_2[n]\,x_2[n-1]}$

$y_2[n] = x_1[n-1]\,x_1[n-2]$

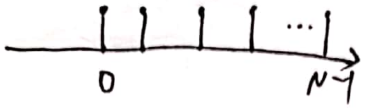
$\parallel$

$\longrightarrow y_2[n] = x_2[n]\,x_2[n-1] = x_1[n-1]\,x_1[n-2]$

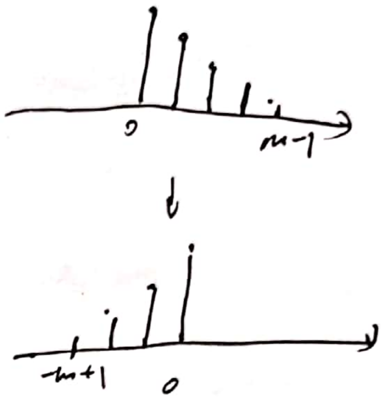
Causality: Yes!

Convolution.

$x[n]$:



$h[n] = a^n$



① No overlap: $n<0$.

$y[n]=0$.

② Partial overlap $1 \le n \le M-2$.

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] = \sum_{k=0}^{n} a^k 1 = \frac{1-a^{n+1}}{1-a}.$$

③ Full overlap: ~~n = M-1~~ $M-1 \le n \le N-1$

$$y[n] = \sum_{k=0}^{M-1} a^k 1 = \frac{1-a^M}{1-a}.$$

④ Partial overlap $N \le n \le M+N-2$.

$$y[n] = \sum_{k=n-N+1}^{M-1} a^k 1 = \underbrace{a^{n-N+1} \frac{1-a^{M+N-n-1}}{1-a}}$$

⑤ $n \ge M+N-1$

$y[n]=0$

$$\Rightarrow \quad y[n] = \begin{cases} 0 & n<0 \\ \dfrac{1-a^{n+1}}{1-a} & 1 \le n \le M-2. \\ \dfrac{1-a^M}{1-a} & M-1 \le n \le N-1 \\ a^{n-N+1}\dfrac{1-a^{M+N-n-1}}{1-a} & N \le n \le M+N-2. \\ 0 & n \ge M+N-1 \end{cases}$$