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ECE 310 Digital Signal Processing

Homework 9

Prof. Zhi-Pei Liang **Due: April 16, 2021**

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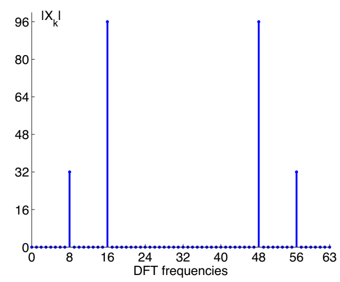
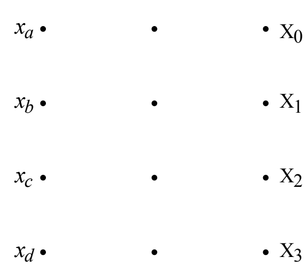
1. A 3.0 sec. segment of $\{x_a(t)\}_{t=0}^{3.0} = \cos(0.2\pi t)$ is sampled at a rate of $1/T = 30$ Hz. The resulting 90 samples are zero padded to 128 and the DFT $\{X[k]\}_{k=0}^{127}$ is computed. Determine k_0 such that $|X[k_0]| \geq |X[k]|$ for $k = 0, 1, \cdots, 63$.

2. Assume that $x_a(t) = \sum_{\ell=1}^{L} A_\ell \cos(\Omega_\ell t)$, where the A_ℓ have positive values. We further assume that $x_a(t)$ is measured at $t = nT$ for $T = 1/8$ second and $n = 0, 1, \ldots, 63$ to obtain $\{x_n\}_{n=0}^{63} = \{x_a(nT)\}_{n=0}^{63}$. The 64-point DFT of $\{x_n\}_{n=0}^{63}$ is represented by $\{X_k\}_{k=0}^{63}$, whose magnitude is shown in the figure below.

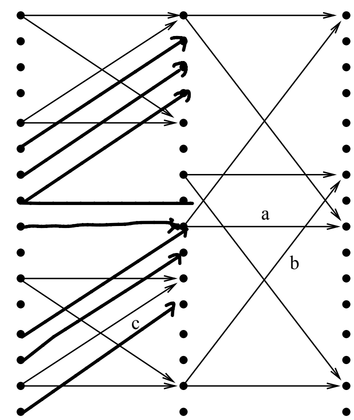


Determine L, and A_ℓ and Ω_ℓ for $\ell = 1, 2, \ldots, L$.

3. Complete the following signal flow diagram (butterfly structure) of a 4-pt, radix-2, decimation-in-time FFT algorithm. Specify all the connection weights and determine the indexes (a, b, c, and d) of the input signal sequence.



4. The diagram below represents a part of the computation in a 16-point decimation-in-time radix-2 FFT. Indicate the values of the three requested branch weights a, b and c.



5. Determine z_n, the cyclic convolution of x_n and y_n for the following cases:

   (a) $\{x_n\}_{n=0}^{5} = \{1, 2, 3, 4, 5, 6\}$ and $\{y_n\}_{n=0}^{5} = \{1, 0, 0, 1, 0, 0\}$.

   (b) $\{x_n\}_{n=0}^{8} = \{1, 2, 3, 4, 5, 6, 0, 0, 0\}$ and $\{y_n\}_{n=0}^{8} = \{1, 0, 0, 1, 0, 0, 0, 0, 0\}$.

6. The following linear convolution

$$\{x_n\}_{n=0}^{46} * \{h_n\}_{n=0}^{32}$$

is to be evaluated using the DFT method. Namely,

$$\{x_n\}_{n=0}^{46} * \{h_n\}_{n=0}^{32} = \mathrm{DFT}^{-1}\{\mathrm{DFT}\{x_n\} \cdot \mathrm{DFT}\{h_n\}\}$$

   (a) Determine the minimum number of zeros should be padded to $\{x_n\}$ and $\{h_n\}$, respectively, before the DFTs are applied.

   (b) If the DFTs are to be calculated with a radix-2 FFT algorithm, how many zeros should now be padded to $\{x_n\}$ and $\{h_n\}$, respectively.

P1. $\{x_n\}_{n=0}^{89} = \{x_a(nT)\} = \cos\frac{0.2}{30}n\pi = \cos\frac{1}{150}n\pi$

$\frac{1}{T} = 30Hz,\ T = \frac{1}{30}s << \frac{2\pi}{2\cdot W} = \frac{2\pi}{2\cdot 0.2s}$

$x_k = \begin{cases} x_n, & \text{when } 0 \le k \le 89 \\ 0, & \text{when } 90 \le k \le 127 \end{cases}$ (zero-p added)

$|X[k_0]| \geq |X[k]|$

$\therefore |X[k_0]|$ is the great magnitude.

$\frac{2\pi k}{128} = \frac{\pi}{150} \Rightarrow k = 1.28 \approx 1$

$2\pi - \frac{2\pi k}{128} = \frac{\pi}{150} \Rightarrow k = 382.72 \approx 383$

$\therefore k_0 = 1$

P2. $x_n = x_a(nT)$

$= \sum_{l=1}^{L} A_l \cos(\Omega_l \frac{n}{8})$

$X_k = \sum_{n=0}^{64-1} x_n e^{-j\frac{2\pi}{N}kn}$

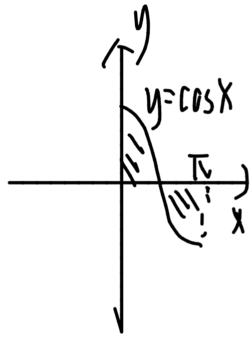
According to the figure,

the DFT spectral is sysmmetrical

$X_0 = X_{63} = 0$

$\therefore \Omega_l \frac{64}{8} = \pi$

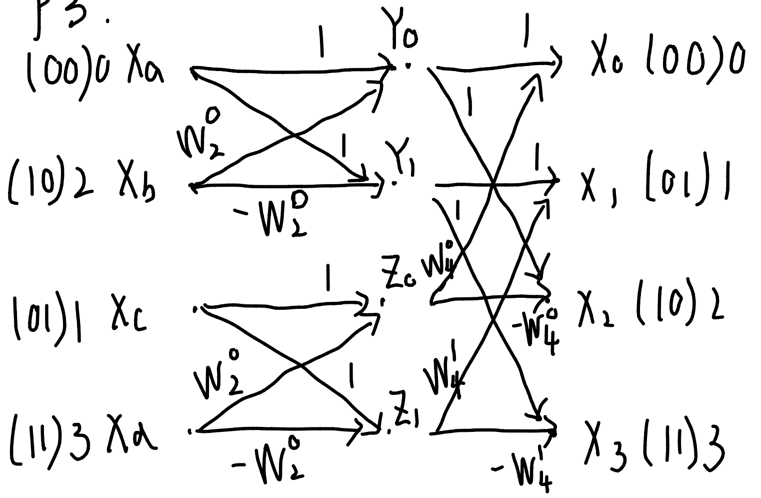
$\Omega_l = \frac{\pi}{8} = 0.125\pi$



$A_l = 32\cdot(2^3 - 1) = 224$

$L = 4.$

P3.



$a = 0,\ b = 2,\ c = 1,\ d = 3$

(Bit-reverse index)

where $W_2^0 = e^{-j\frac{2\pi\cdot 0}{2}} = 1$

$W_4^0 = e^{-j\frac{2\pi\cdot 0}{4}} = 1,\ W_4^1 = e^{-j\frac{2\pi\cdot 1}{4}} = e^{-j\frac{\pi}{2}} = -j$

P4. a. $-W_{16}^0 = -e^{-j\frac{2\pi\cdot 0}{16}} = -1$

b. $W_{16}^6 = e^{-j\frac{2\pi\cdot 6}{16}} = e^{-j\frac{3\pi}{4}} = \frac{\sqrt{2}}{-2} - j\frac{\sqrt{2}}{2}$

c. $W_8^2 = e^{-j\frac{2\pi\cdot 2}{8}} = e^{-j\frac{\pi}{2}} = -j$

P5. $\{x_n\}_{n=0}^{N-1} \circledast \{h_n\}_{n=0}^{N-1} = \{y_n\}_{n=0}^{N-1}$

$= \sum_{k=0}^{N-1} x_k h_{\langle n-k \rangle_N}$

a) $\{x_n\}_{n=0}^{5} \circledast \{y_n\}_{n=0}^{5} = \sum_{k=0}^{5} x_k y_{\langle n-k \rangle_6}$

$n=0, x_0 \circledast y_0 = 1\times1+0+0+4\times1+0+0+6\times1 = 11$

$n=1, x_1 \circledast y_1 = 1\times0+2\times1+3\times0+4\times0+5\times1+6\times0 = 7$

$n=2, x_2 \circledast y_2 = 1\times0+2\times0+3\times1+4\times0+5\times0+6\times1 = 9$

$n=3, x_3 \circledast y_3 = 1\times1+2\times0+3\times2+4\times1+5\times0+6\times0 = 5$

$n=4, x_4 \circledast y_4 = 1\times0+2\times1+3\times0+4\times0+5\times1+6\times0 = 7$

$n=5, x_5 \circledast y_5 = 1\times0+2\times0+3\times1+4\times0+5\times0+6\times1 = 9$

$\therefore \{11, 7, 9, 5, 7, 9\}$

b) $\{x_n\}_{n=0}^{8} \circledast \{y_n\}_{n=0}^{8} = \sum_{k=0}^{5} x_k y_{\langle n-k \rangle_6}$

$x_0 \circledast y_0 = 1\times1+0+0+0+0+0+0+0+0 = 1$

$x_1 \circledast y_1 = 0+2\times1+0+0+0+0+0+0+0 = 2$

$x_2 \circledast y_2 = 0+0+3\times1+0+0+0+0+0+1\times0 = 3$

$x_3 \circledast y_3 = 1\times1+0+0+4\times1+0+0+0+0+0 = 5$

$x_4 \circledast y_4 = 0+2\times1+0+0+5\times1+0+0+0+0 = 7$

$x_5 \circledast y_5 = 3\times1+6\times1 = 9$

$x_6 \circledast y_6 = 0+0+0+4\times1+5\times0+6\times0+0\times1+0 = 4$

$x_7 \circledast y_7 = 0+0+0+0 \quad +5\times1+0+0+0+0 = 5$

$x_8 \circledast y_8 = 0+0+0+0+0+6\times1+0+0+0 = 6$

$\therefore \{1, 2, 3, 5, 7, 9, 4, 5, 6\}$

P6. $L = M+N-1 = 47+33-1 = 79$

a) $\therefore$ minimum number of zeros added to $\{x_n\}$: $79-47=32$

to $\{h_n\}$: $79-33=46$

b) $2^r = 79$

$r = 7$