ECE 310

Digital Signal Processing

Spring, 2021, ZJUI Campus

Lecture 26

Topics:

- ✓ Definition of symmetric and anti-symmetric FIR filters
- ✓ Phase behaviors of symmetric and anti-symmetric FIR filters

Educational Objectives:

- ✓ Understand the definition of symmetric FIR filters
- ✓ Understand the definition of anti-symmetric FIR filters
- ✓ Understand the relationship between filter symmetry and GLP

Symmetric FIR Filters

Real: $h_n = h_{N-1-n}, \quad \begin{cases} n = 0,\ldots,\dfrac{N}{2}-1, & \text{N even} \\ n = 0,\ldots,\dfrac{N-1}{2}, & \text{N odd} \end{cases}$

Complex: $h_n = h^*_{N-1-n}, \quad \begin{cases} n = 0,\ldots,\dfrac{N}{2}-1, & \text{N even} \\ n = 0,\ldots,\dfrac{N-1}{2}, & \text{N odd} \end{cases}$

Examples:

a) $h[n] = \{1, 2, 4, 6\}$

b) $h[n] = \{1, 2, 2, 1\}$

c) $h[n] = \{j, -2j, j, 2j, -j\}$

d) $h[n] = \{j, -2j, 1, 2j, -j\}$

Anti-Symmetric FIR Filters

Real: $h_n = -h_{N-1-n},\quad \begin{cases} n = 0,\dots,\dfrac{N}{2}-1, & \text{N even} \\ n = 0,\dots,\dfrac{N-1}{2}, & \text{N odd} \end{cases}$

Complex: $h_n = -h^*_{N-1-n},\quad \begin{cases} n = 0,\dots,\dfrac{N}{2}-1, & \text{N even} \\ n = 0,\dots,\dfrac{N-1}{2}, & \text{N odd} \end{cases}$

Examples:

a) $h[n] = \{1, 2, 4, 6\}$

b) $h[n] = \{1, 2, -2, -1\}$

c) $h[n] = \{j, 2j, 1, 2j, j\}$

d) $h[n] = \{j, 2j, 0, 2j, j\}$

e) $h[n] = \{j, 2j, j, 2j, j\}$

FIR Filter Symmetries vs GLPs

Theorem 1:

Symmetric FIR filter ↔ GLP type 1

$$H_d(\omega) = R(\omega)e^{-j\omega M} \quad , \quad M = \frac{N-1}{2}$$

* For real symmetric FIR filters, $R(\omega)$ is real and even

Theorem 2:

Anti-symmetric FIR filter ↔ GLP type 2

$$H_d(\omega) = R(\omega)e^{j(\frac{\pi}{2}-\omega M)} \quad , \quad M = \frac{N-1}{2}$$

* For real anti-symmetric FIR filters, $R(\omega)$ is real and odd

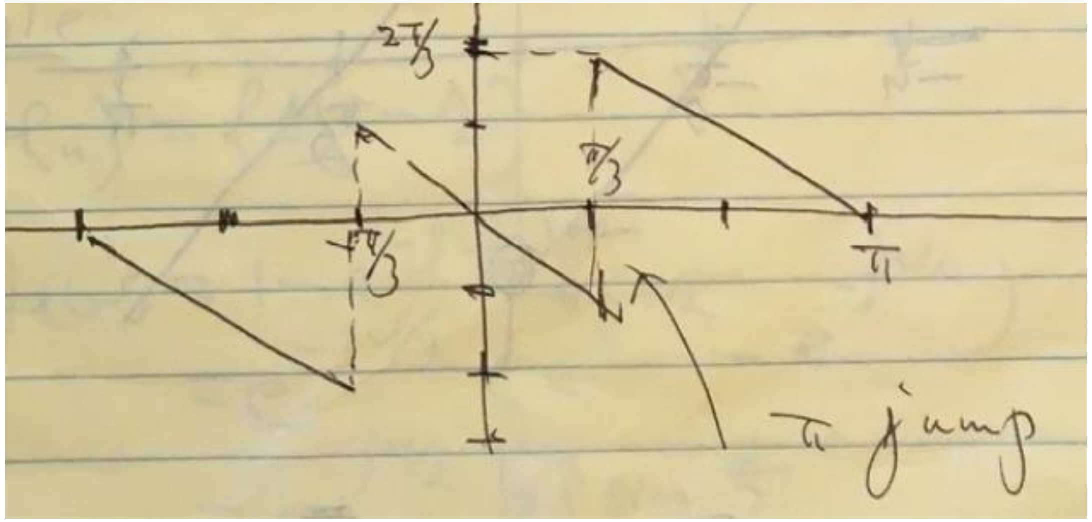
Example

$$\{h_n\} = \{1, -1, 1\}$$

$$H_d(\omega) = 1 - e^{-j\omega} + e^{-j2\omega}$$
$$= e^{-j\omega}(e^{j\omega} - 1 + e^{-j\omega})$$
$$= e^{-j\omega}\underbrace{(2\cos(\omega) - 1)}_{R(\omega)}$$

$$M = \frac{3-1}{2} = 1$$

$$\angle H_d(\omega) = \begin{cases} -\omega, & 2\cos\omega - 1 > 0 \;\Rightarrow\; |\omega| < \dfrac{\pi}{3} \\ -\omega \pm \pi, & 2\cos\omega - 1 < 0 \;\Rightarrow\; \dfrac{\pi}{3} < |\omega| < \pi \end{cases}$$



Type-1 GLP, not LP!

Example

$$\{h_n\} = \{1, \; -1\}$$

$$H_d(\omega) = 1 - e^{-j\omega}$$

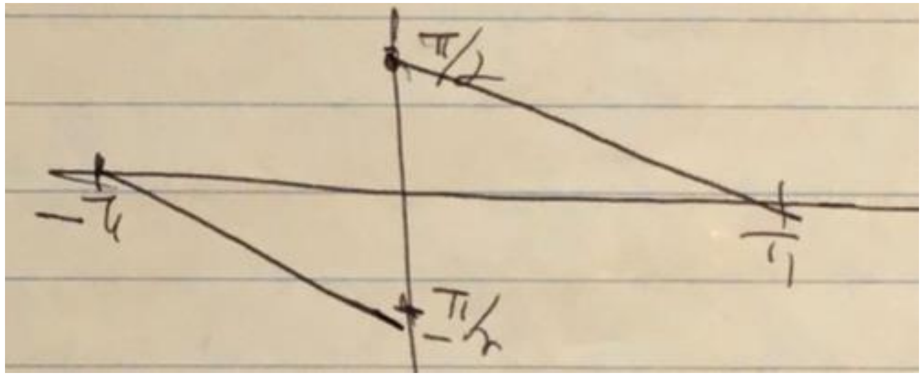
$$= e^{-j\omega/2}\left(e^{j\omega/2} - e^{-j\omega/2}\right)$$

$$= e^{-j\omega/2} 2j \sin\frac{\omega}{2}$$

$$= e^{j\left(\frac{\pi}{2}-\frac{\omega}{2}\right)} \underbrace{2\sin\frac{\omega}{2}}_{R(\omega)}$$

$R(\omega)$ Real, odd!

$$\angle H_d(\omega) = \begin{cases} \dfrac{\pi}{2} - \dfrac{\omega}{2}, & \sin\dfrac{\omega}{2} > 0 \;\Rightarrow\; 0 < \omega < \pi \\ -\dfrac{\pi}{2} - \dfrac{\omega}{2}, & \sin\dfrac{\omega}{2} < 0 \;\Rightarrow\; -\pi < \omega < 0 \end{cases}$$



Type-2 filters can never have LP!

Example

$$\{h_n\} = \{1,\ 2,\ \ -1\}$$

$$\begin{aligned} H_d(\omega) &= 1 + 2e^{-j\omega} - e^{-j2\omega} \\ &= e^{-j\omega}(e^{j\omega} + 2 - e^{-j\omega}) \\ &= e^{-j\omega}(2 + j2\sin\omega) \end{aligned}$$

Not GLP type-2