



浙江大学伊利诺伊大学厄巴纳香槟校区联合学院
Zhejiang University-University of Illinois at Urbana Champaign Institute

ECE 448: AI theory and Application

Lecture 12: Stochastic Games, Stochastic Search, and Learned Evaluation Functions

Dr. Hongwei Wang hongweiwang@intl.zju.edu.cn

Prof. Mark Hasegawa-Johnson jhasegaw@illinois.edu

Spring 2023

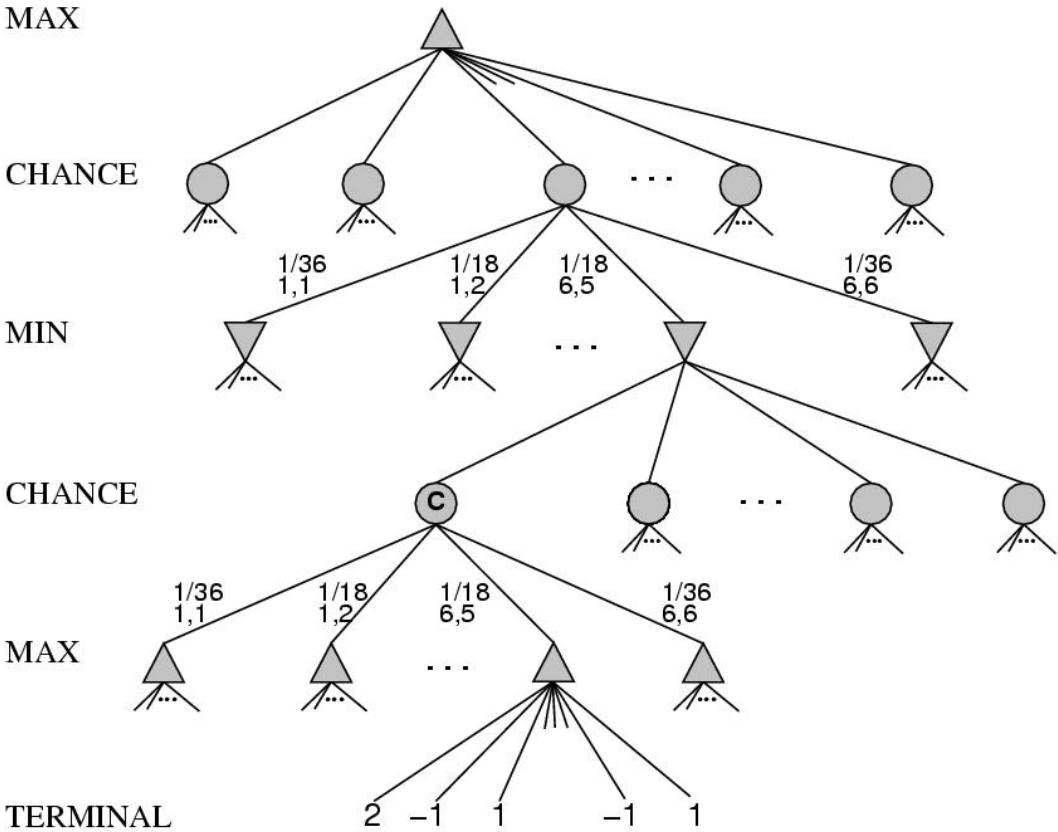
	Deterministic	Stochastic
Perfect information (fully observable)	Chess, checkers, go	Backgammon, monopoly
Imperfect information (partially observable)	Battleship	Scrabble, poker, bridge

1. **Stochastic games: the Expectiminimax algorithm**
2. **Imperfect information**
 - **Minimax formulation**
 - **Expectiminimax formulation**
3. **Stochastic search, even for deterministic games**
4. **Learned evaluation functions**
5. **Case study: Alpha-Go**

1. Stochastic games: the Expectiminimax algorithm
2. Imperfect information
 - Minimax formulation
 - Expectiminimax formulation
3. Stochastic search, even for deterministic games
4. Learned evaluation functions
5. Case study: Alpha-Go

How can we incorporate dice throwing into the game tree?





- **Minimax:**

- **Maximize** (over all possible moves I can make) the
- **Minimum** (over all possible moves Min can make) of the
- Reward

$$Value(node) = \max_{my\ moves} \left(\min_{Min's\ moves} (Reward) \right)$$

- **Expectiminimax:**

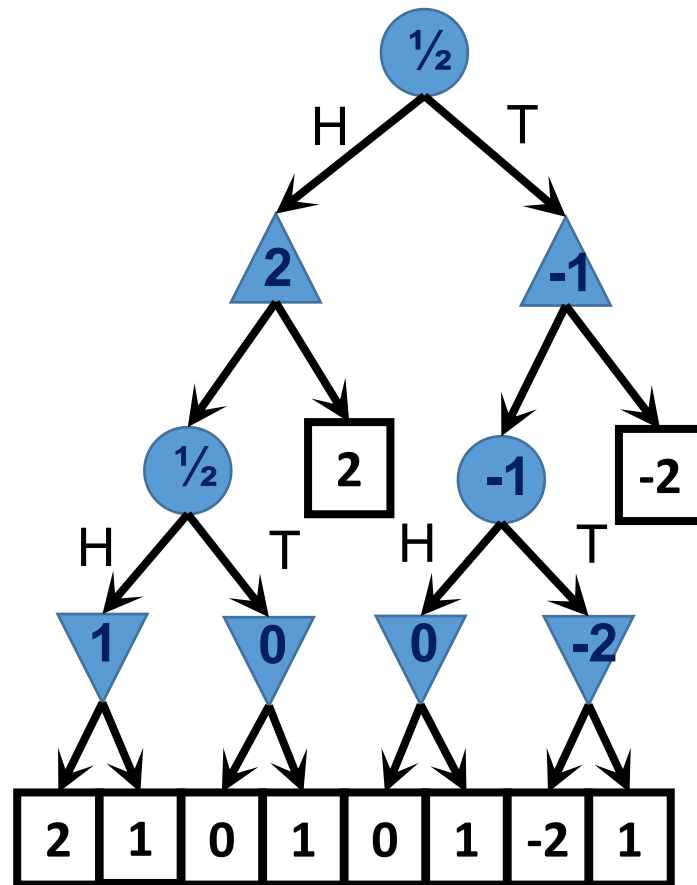
- **Maximize** (over all possible moves I can make) the
- **Minimum** (over all possible moves Min can make) of the
- **Expected** reward

$$Value(node) = \max_{my\ moves} \left(\min_{Min's\ moves} (\mathbb{E}[Reward]) \right)$$

$$\mathbb{E}[Reward] = \sum_{outcomes} Probability(outcome) \times Reward(outcome)$$

- **Expectiminimax:** for chance nodes, sum values of successor states weighted by the probability of each successor

- RANDOM: Max flips a coin. It's heads or tails.
- MAX: Max either stops, or continues.
 - Stop on heads: Game ends, Max wins (value = \$2).
 - Stop on tails: Game ends, Max loses (value = -\$2).
 - Continue: Game continues.
- RANDOM: Min flips a coin.
 - HH: value = \$2
 - TT: value = -\$2
 - HT or TH: value = 0
- MIN: Min decides whether to keep the current outcome (value as above), or pay a penalty (value=\$1).

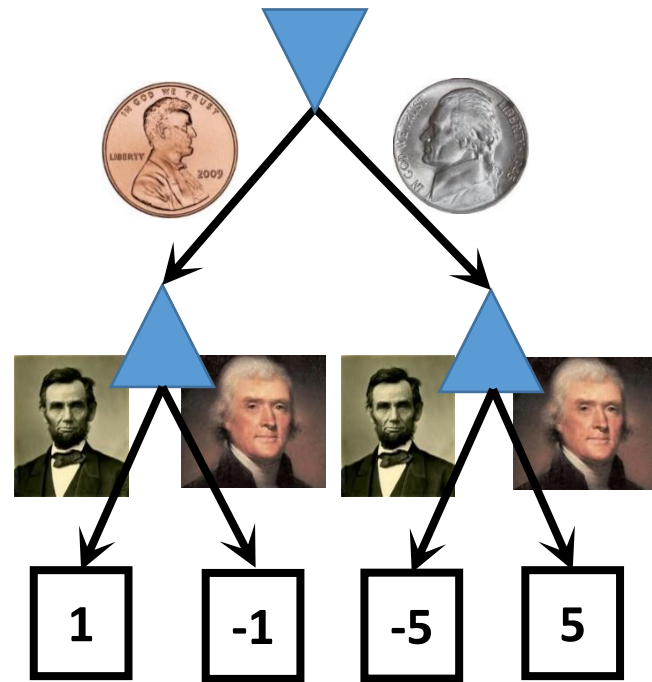


- All of the same methods are useful:
 - Alpha-Beta pruning
 - Evaluation function
 - Quiescence search, Singular move
- Computational complexity is pretty bad
 - Branching factor of the random choice can be high
 - Twice as many “levels” in the tree

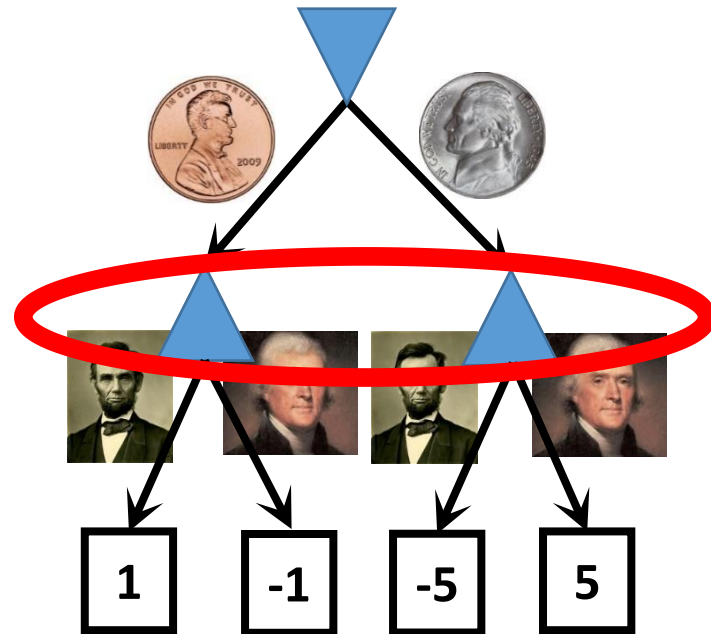
1. Stochastic games: the Expectiminimax algorithm
2. Imperfect information
 - Minimax formulation
 - Expectiminimax formulation
3. Stochastic search, even for deterministic games
4. Learned evaluation functions
5. Case study: Alpha-Go



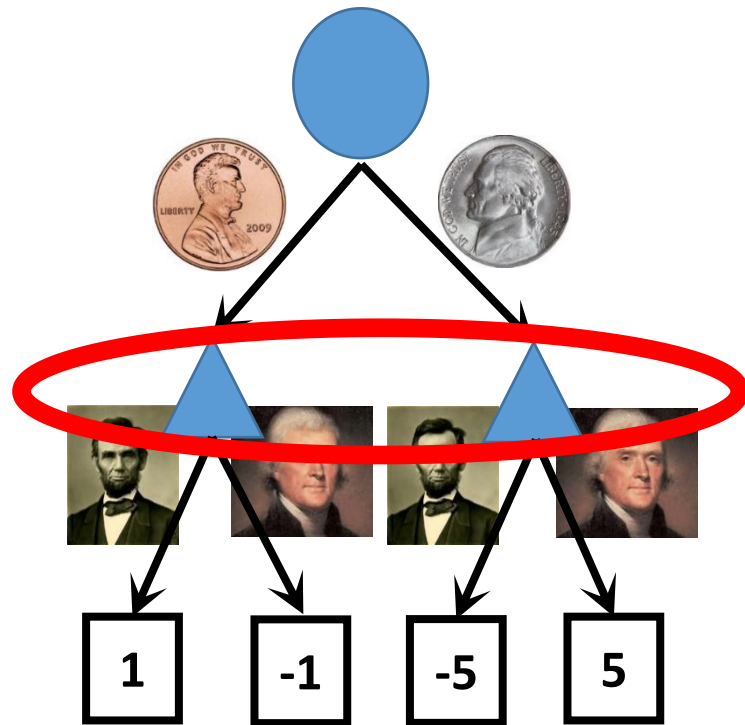
- Min chooses a coin.
- I say the name of a U.S. President.
 - If I guessed right, she gives me the coin.
 - If I guessed wrong, I have to give her a coin to match the one she has.



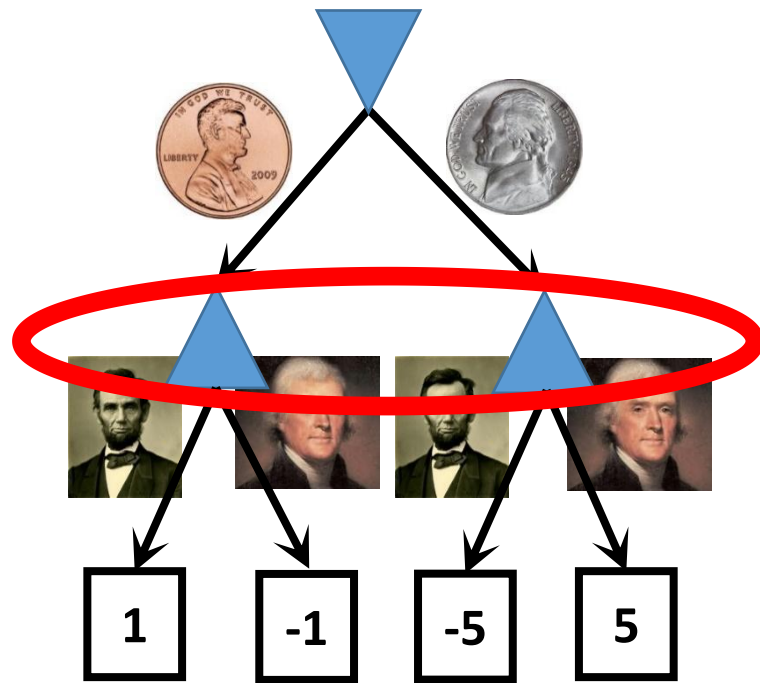
- The problem: I don't know which state I'm in. I only know it's one of these two.



- Expectiminimax: treat the unknown information as random.
- Choose the policy that maximizes my expected reward.
 - “Lincoln”: $\frac{1}{2} \times 1 + \frac{1}{2} \times (-5) = -2$
 - “Jefferson”: $\frac{1}{2} \times (-1) + \frac{1}{2} \times 5 = 2$
- Expectiminimax policy: say “Jefferson”.
- BUT WHAT IF: The chances of “Lincoln” coin and of “Jefferson” coin are not equally likely?



- Suppose Min can choose whichever coin she wants. She knows that I will pick Jefferson – then she will pick the penny!
- Another reasoning: I want to know what is my worst-case outcome (e.g., to decide if I should even play this game...)
- The solution: choose the policy that maximizes my minimum reward.
 - “Lincoln”: minimum reward is -5.
 - “Jefferson”: minimum reward is -1.
- Miniminimax policy: say “Jefferson”.



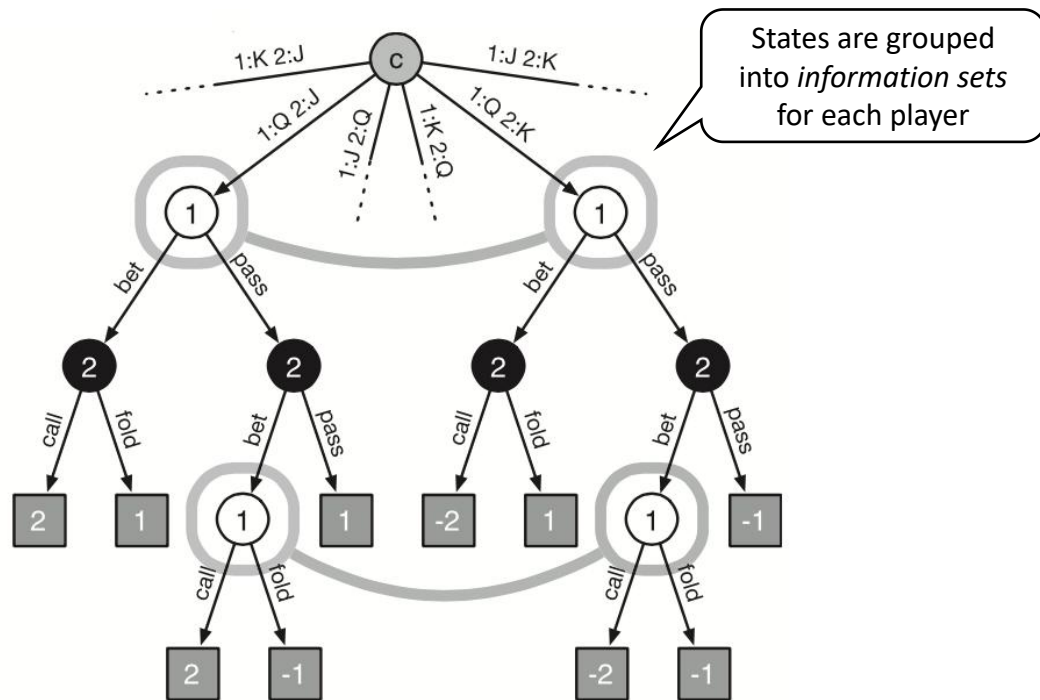
- If you think you know the probabilities of different settings, and if you want to maximize your average winnings (for example, you can afford to play the game many times): **expectiminimax**
- If you have no idea of the probabilities of different settings; or, if you can only afford to play once, and you can't afford to lose: **miniminimax**
- If the unknown information has been selected intentionally by your opponent: use **game theory**

- **Minimax:**
 - **Maximize** (over all possible moves I can make) the
 - **Minimum**
 - (over all possible states of the information I don't know,
 - ... over all possible moves Min can make) the
 - Reward.

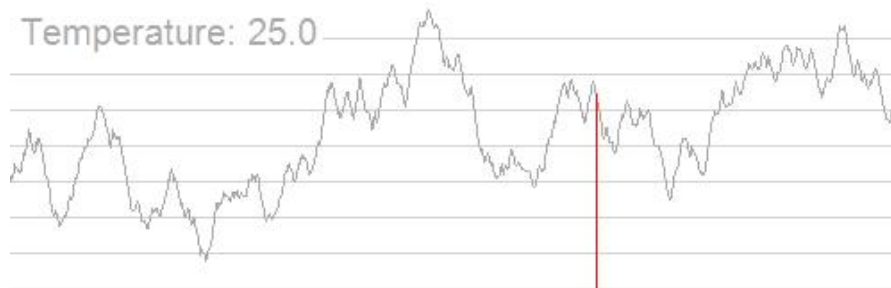
$$Value(node) = \max_{\substack{Max's \\ moves}} \left(\min_{\substack{Min's \\ moves}} \min_{\substack{missing \\ info}} (Reward) \right)$$

Fig. 1. Portion of the extensive-form game representation of three-card Kuhn poker (16).

Player 1 is dealt a queen (Q), and the opponent is given either the jack (J) or king (K). Game states are circles labeled by the player acting at each state ("c" refers to chance, which randomly chooses the initial deal). The arrows show the events the acting player can choose from, labeled with their in-game meaning. The leaves are square vertices labeled with the associated utility for player 1 (player 2's utility is the negation of player 1's). The states connected by thick gray lines are part of the same information set; that is, player 1 cannot distinguish between the states in each pair because they each represent a different unobserved card being dealt to the opponent. Player 2's states are also in information sets, containing other states not pictured in this diagram.



1. Stochastic games: the Expectiminimax algorithm
2. Imperfect information
 - Minimax formulation
 - Expectiminimax formulation
3. Stochastic search, even for deterministic games
4. Learned evaluation functions
5. Case study: Alpha-Go



- The problem with expectiminimax: huge branching factor (many possible outcomes)

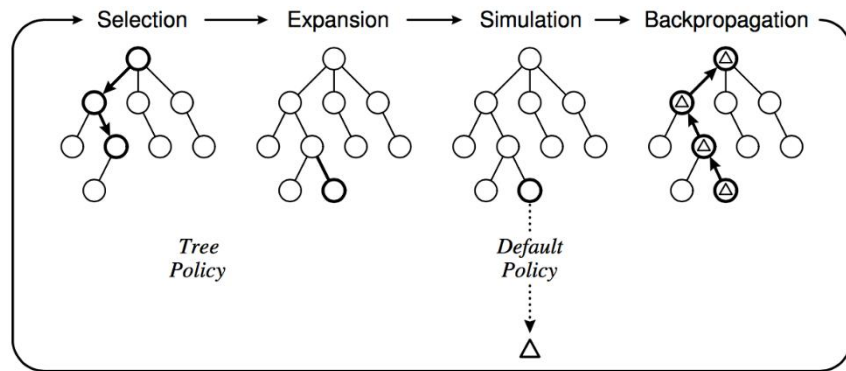
$$\mathbb{E}[\text{Reward}] = \sum_{\text{outcomes}} \text{Probability}(\text{outcome}) \times \text{Reward}(\text{outcome})$$

- An approximate solution: Monte Carlo search

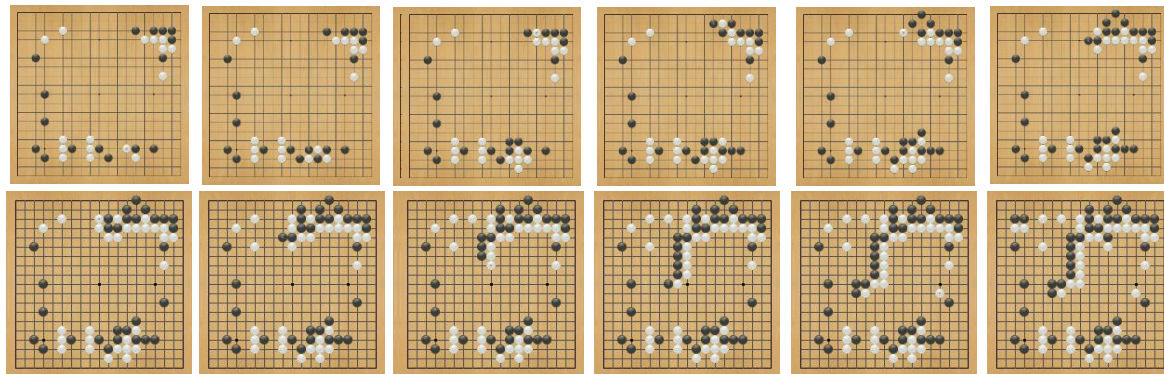
$$\mathbb{E}[\text{Reward}] \approx \frac{1}{n} \sum_{i=1}^n \text{Reward}(i\text{'th random game})$$

- Asymptotically optimal: as $n \rightarrow \infty$, the approximation gets better.
- Controlled computational complexity: choose n to match the amount of computation you can afford.

- What about ***deterministic*** games with deep trees, large branching factor, and no good heuristics – like Go?
- Instead of depth-limited search with an evaluation function, use randomized simulations
- Starting at the current state (root of search tree), iterate:
 - Select a leaf node for expansion using a *tree policy* (trading off *exploration* and *exploitation*)
 - Run a simulation using a *default policy* (e.g., random moves) until a terminal state is reached
 - Back-propagate the outcome to update the value estimates of internal tree nodes



1. Stochastic games: the Expectiminimax algorithm
2. Imperfect information
 - Minimax formulation
 - Expectiminimax formulation
3. Stochastic search, even for deterministic games
4. Learned evaluation functions
5. Case study: Alpha-Go



Training phase:

- Spend a few weeks allowing your computer to play billions of random games from every possible starting state
- Value of the starting state = average value of the ending states achieved during those billion random games



Testing phase:

- During the alpha-beta search, search until you reach a state whose value you have stored in your value lookup table
- Oops.... Why doesn't this work?

Training phase:

- Spend a few weeks allowing your computer to play billions of random games from billions of possible starting states.
- Value of the starting state = average value of the ending states achieved during those billion random games

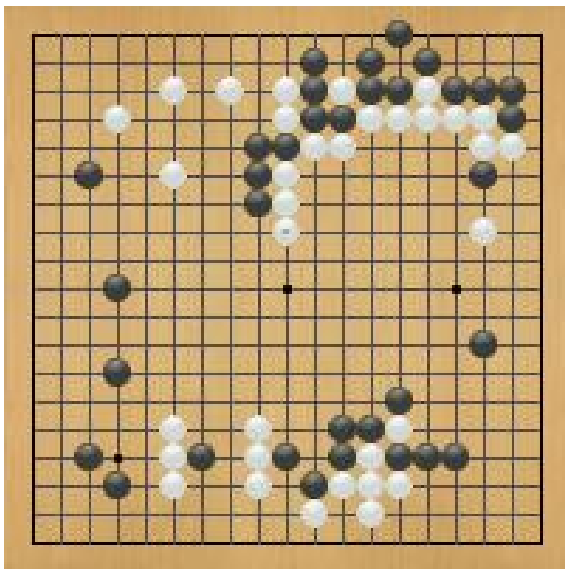
Generalization:

- Featurize (e.g., x_1 =number of  patterns, x_2 = number of  patterns, etc.)
- Linear regression: find a_1 , a_2 , etc. so that $\text{Value}(\text{state}) \approx a_1 * x_1 + a_2 * x_2 + \dots$

Testing phase:

- During the alpha-beta search, search as deep as you can, then estimate the value of each state at your horizon using $\text{Value}(\text{state}) \approx a_1 * x_1 + a_2 * x_2 + \dots$

- Learned evaluation function
 - Pro: off-line search permits lots of compute time, therefore lots of training data
 - Con: there's no way you can evaluate every starting state that might be achieved during actual game play. Some starting states will be missed, so generalized evaluation function is necessary
- On-line stochastic search
 - Con: limited compute time
 - Pro: it's possible to estimate the value of the state you've reached during actual game play

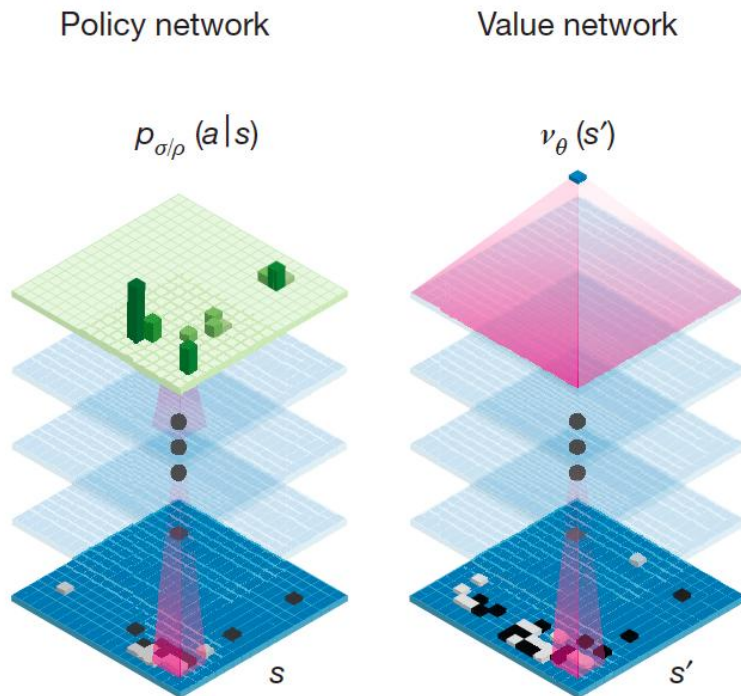


- “Gentlemen should not waste their time on trivial games -- they should play go.”
- -- *Confucius*,
- *The Analects*
- *ca. 500 B. C. E.*

Anton Ninno
antonninno@yahoo.com
roylaird@gmail.com

Roy Laird, Ph.D.

special thanks to Kiseido Publications



- Deep convolutional neural networks

- Treat the Go board as an image
- Powerful function approximation machinery
- Can be trained to predict distribution over possible moves (*policy*) or expected *value* of position

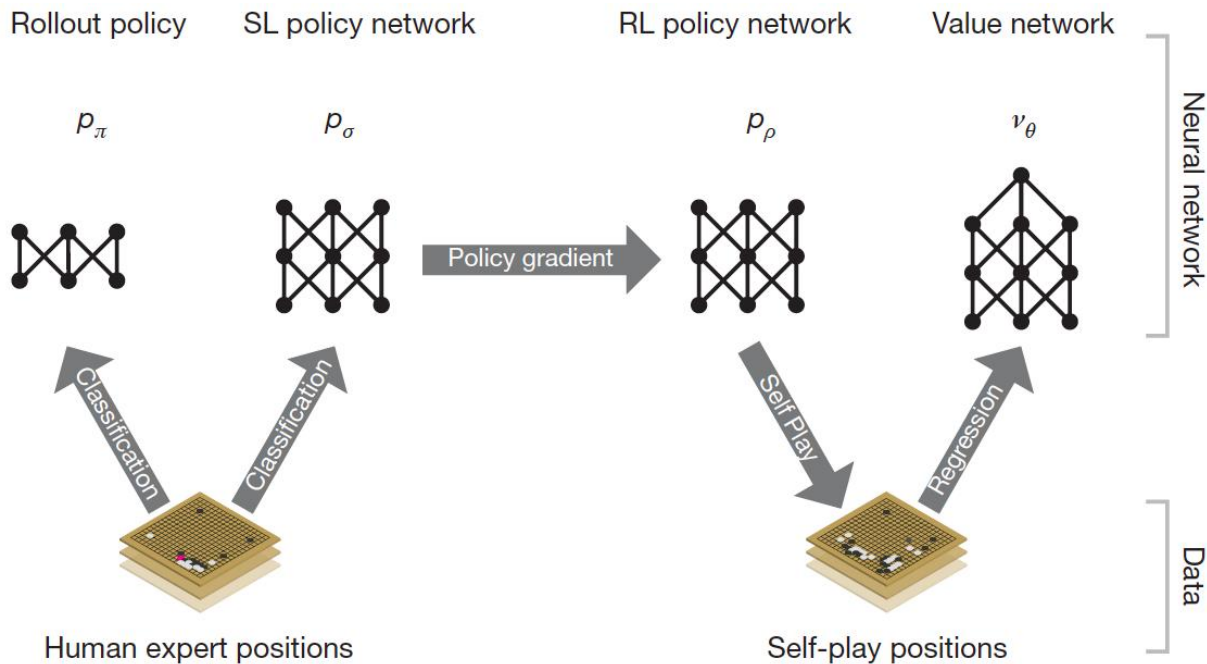
D. Silver et al., [Mastering the Game of Go with Deep Neural Networks and Tree Search](#), Nature 529, January

2016

- SL policy network
 - Idea: perform *supervised learning* (SL) to predict human moves
 - Given state s , predict probability distribution over moves a , $P(a|s)$
 - Trained on 30M positions, 57% accuracy on predicting human moves
 - Also train a smaller, faster *rollout policy* network (24% accurate)
- RL policy network
 - Idea: fine-tune policy network using *reinforcement learning* (RL)
 - Initialize RL network to SL network
 - Play two snapshots of the network against each other, update parameters to maximize expected final outcome
 - RL network wins against SL network 80% of the time, wins against open-source Pachi Go program 85% of the time

- SL policy network
- RL policy network
- Value network
 - Idea: train network for position evaluation
 - Given state s , estimate $v(s)$, expected outcome of play starting with position s and following the learned policy for both players
 - Train network by minimizing mean squared error between actual and predicted outcome
 - Trained on 30M positions sampled from different self-play games

D. Silver et al., [Mastering the Game of Go with Deep Neural Networks and Tree Search](#), Nature 529, January 2016



D. Silver et al., [Mastering the Game of Go with Deep Neural Networks and Tree Search](#),
Nature 529, January 2016

- Monte Carlo Tree Search
 - Each edge in the search tree maintains *prior probabilities* $P(s,a)$, *counts* $N(s,a)$, *action values* $Q(s,a)$
 - $P(s,a)$ comes from SL policy network
 - Tree traversal policy selects actions that maximize Q value plus exploration bonus (proportional to P but inversely proportional to N)
 - An expanded leaf node gets a value estimate that is a combination of value network estimate and outcome of simulated game using rollout network
 - At the end of each simulation, Q values are updated to the average of values of all simulations passing through that edge

- Monte Carlo Tree Search

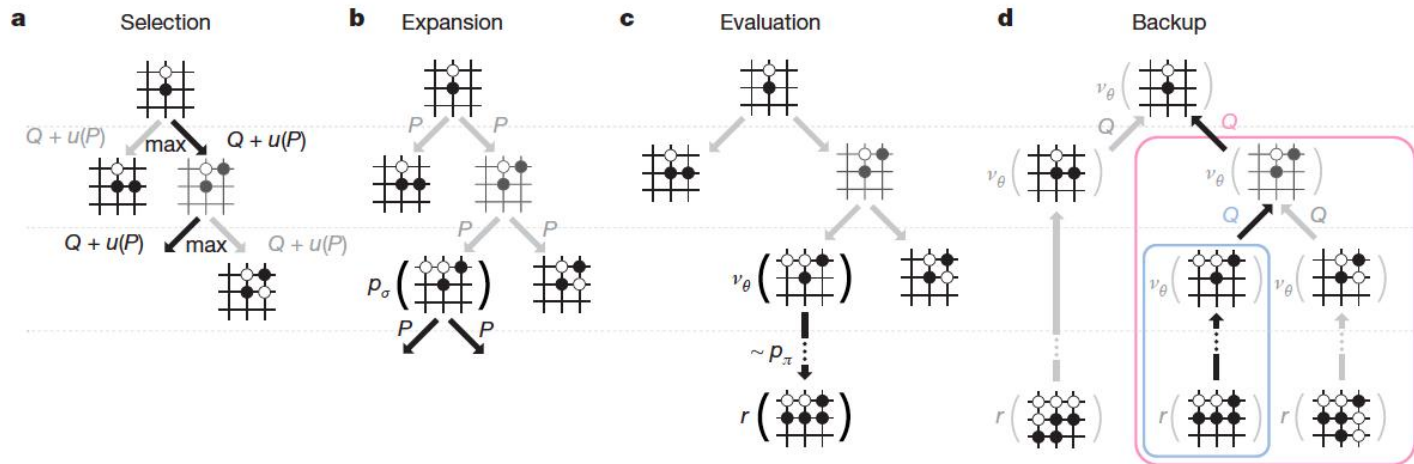


Figure 3 | Monte Carlo tree search in AlphaGo. **a**, Each simulation traverses the tree by selecting the edge with maximum action value Q , plus a bonus $u(P)$ that depends on a stored prior probability P for that edge. **b**, The leaf node may be expanded; the new node is processed once by the policy network p_σ and the output probabilities are stored as prior probabilities P for each action. **c**, At the end of a simulation, the leaf node

is evaluated in two ways: using the value network v_θ ; and by running a rollout to the end of the game with the fast rollout policy p_π , then computing the winner with function r . **d**, Action values Q are updated to track the mean value of all evaluations $r(\cdot)$ and $v_\theta(\cdot)$ in the subtree below that action.

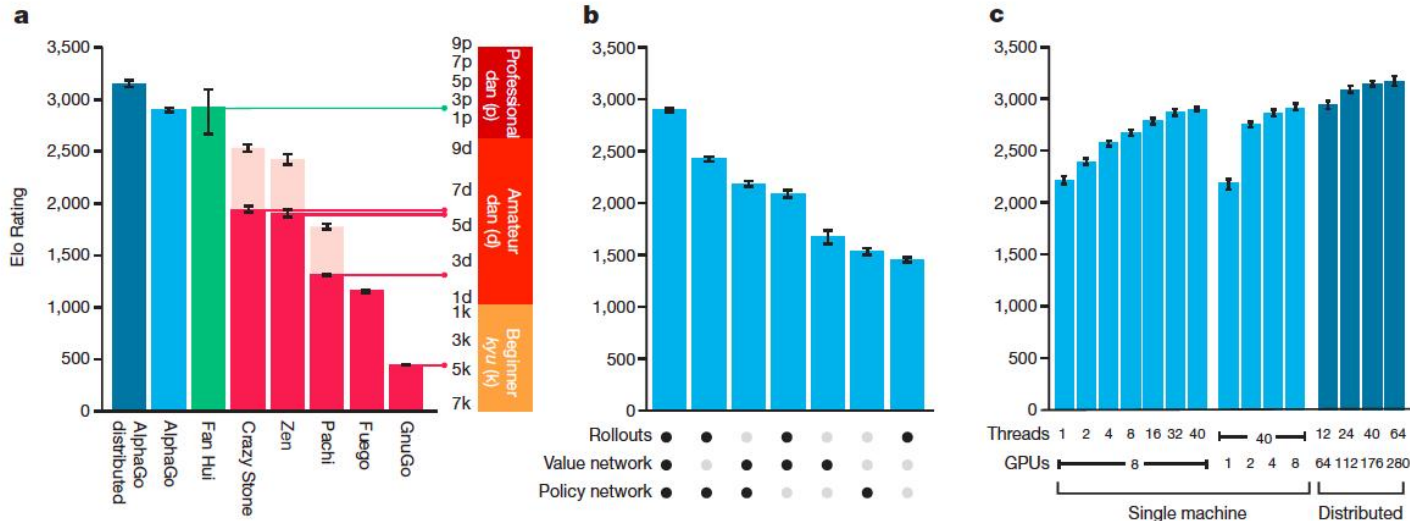


Figure 4 | Tournament evaluation of AlphaGo. **a**, Results of a tournament between different Go programs (see Extended Data Tables 6–11). Each program used approximately 5 s computation time per move. To provide a greater challenge to AlphaGo, some programs (pale upper bars) were given four handicap stones (that is, free moves at the start of every game) against all opponents. Programs were evaluated on an Elo scale³⁷: a 230 point gap corresponds to a 79% probability of winning, which roughly corresponds to one amateur *dan* rank advantage on KGS³⁸; an approximate correspondence to human ranks is also shown,


horizontal lines show KGS ranks achieved online by that program. Games against the human European champion Fan Hui were also included; these games used longer time controls. 95% confidence intervals are shown. **b**, Performance of AlphaGo, on a single machine, for different combinations of components. The version solely using the policy network does not perform any search. **c**, Scalability study of MCTS in AlphaGo with search threads and GPUs, using asynchronous search (light blue) or distributed search (dark blue), for 2 s per move.

WiFi | Panera Bread The computer that

bing.com/videos/search?q=alpha-go&view=detail&mid=768A0740F2736EEB02C9768A0740F2736EEB02C9&FORM=VIRE

tools.beckman.illin Skyward Summary of Lorelei JASA_editor LoReHLT Evaluations Informatics PhD | UNCorpus Kaldi: Kaldi kyoudaimae University of Illinois:

alpha-go



Related searches

- Alphago Transparent Ba...
- Alphago Master
- Lee Sedol Alphago
- Alphago Lose
- Alphago Terminator
- Alphago Live

The computer that mastered Go

YouTube · 805,000+ views · 1/25/2016 · HD · by [nature video](#)

Save View page

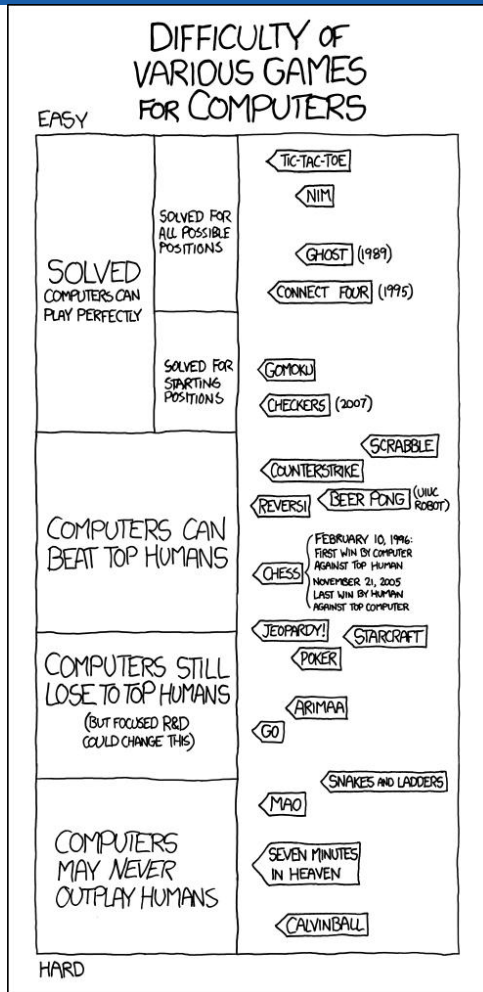
Related videos

Feedback

- Minimax algorithm: Ernst Zermelo, 1912
- Chess playing with evaluation function, quiescence search, selective search:
Claude Shannon, 1949 ([paper](#))
- Alpha-beta search: John McCarthy, 1956
- Checkers program that learns its own evaluation function by playing against itself: Arthur Samuel, 1956 ([Rodney Brooks blog post](#))

- Computers are better than humans:
 - **Checkers:** [solved in 2007](#)
 - **Chess:**
 - State-of-the-art search-based systems now better than humans
 - [Deep learning machine teaches itself chess in 72 hours, plays at International Master Level](#) (arXiv, September 2015)
- Computers are competitive with top human players:
 - **Backgammon:** [TD-Gammon system](#) (1992) used *reinforcement learning* to learn a good evaluation function
 - **Bridge:** top systems use Monte Carlo simulation and alpha-beta search
 - **Go:** computers were not considered competitive until AlphaGo in 2016

- Computers are ~~not~~ competitive with top human players:
 - **Poker**
 - [Heads-up limit hold'em poker is solved](#) (2015)
 - Simplest variant played competitively by humans
 - Smaller number of states than checkers, but partial observability makes it difficult
 - *Essentially weakly solved* = cannot be beaten with statistical significance in a lifetime of playing
 - [CMU's Libratus system beats four of the best human players at no-limit Texas Hold'em poker](#) (2017)



<http://xkcd.com/1002/>

See also: <http://xkcd.com/1263/>



calvinball



Calvinball:

- [Play it online](#)
- [Watch an instructional video](#)