浙江大学伊利诺伊大学厄巴纳香槟校区联合学院
Zhejiang University-University of Illinois at Urbana Champaign Institute

# ECE 448: Artificial Intelligence

## Lecture 18: Hidden Markov Models

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- So far, we've mostly dealt with *episodic* environments
  - Exceptions: games with multiple moves, planning
- In particular, the Bayesian networks we've seen so far describe static situations
  - Each random variable gets a single fixed value in a single problem instance
- Now we consider the problem of describing probabilistic environments that evolve over time
  - Examples: robot localization, human activity detection, tracking, speech recognition, machine translation,

# Outline

1. Hidden Markov Models
2. Sum-Product Algorithm for HMMs
3. HMM inference tasks
4. Applications of HMMs

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1. <u>Hidden Markov Models</u>
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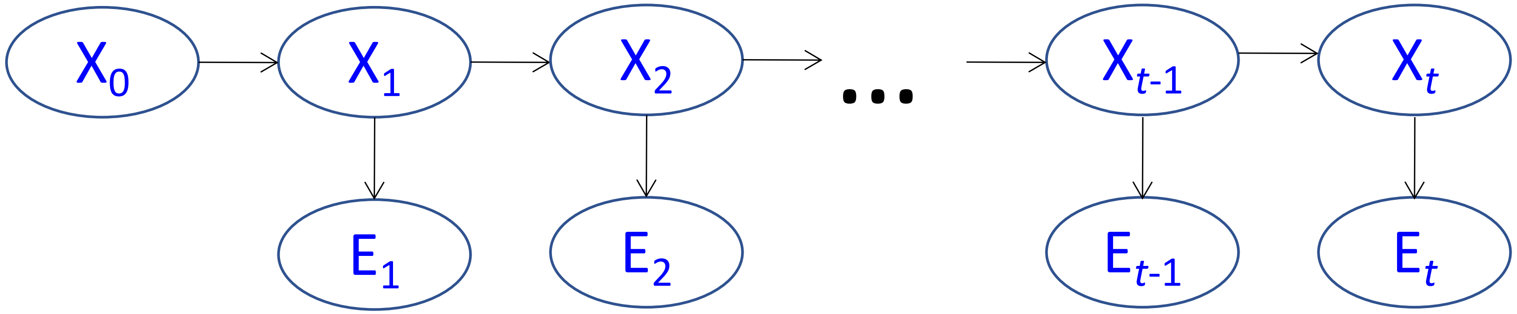
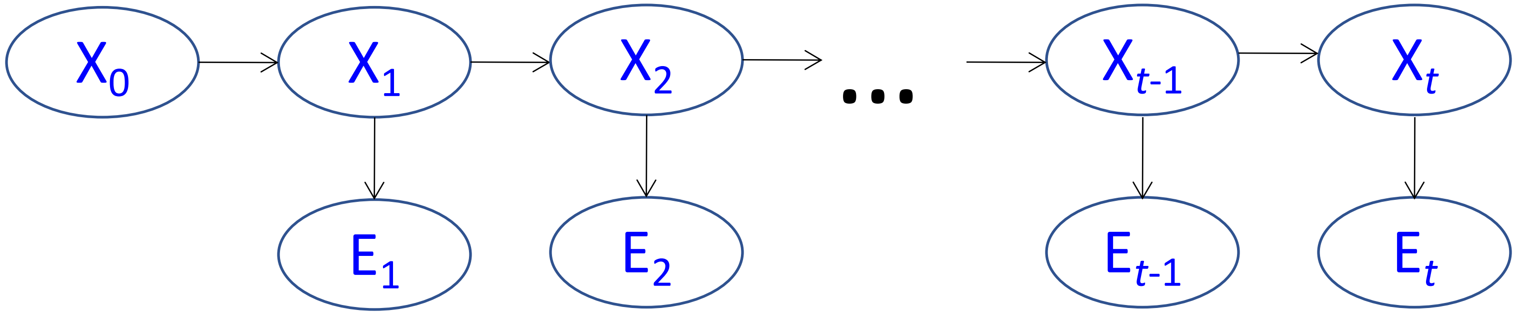
- At each time slice $t$, the state of the world is described by an unobservable variable $X_t$ and an observable *evidence* variable $E_t$
- **Transition model:** distribution over the current state given the whole past history:
  $P(X_t \mid X_0, \ldots, X_{t-1}) = P(X_t \mid \mathbf{X}_{0:t-1})$
- **Observation model:** $P(E_t \mid \mathbf{X}_{0:t}, \mathbf{E}_{1:t-1})$



- **Markov assumption** (first order)
  - The current state is conditionally independent of all the other states given the state in the previous time step
  - What does $P(X_t \mid \mathbf{X}_{0:t-1})$ simplify to?

    $P(X_t \mid \mathbf{X}_{0:t-1}) = P(X_t \mid X_{t-1})$
- Markov assumption for observations
  - The evidence at time *t* depends only on the state at time *t*
  - What does $P(E_t \mid \mathbf{X}_{0:t}, \mathbf{E}_{1:t-1})$ simplify to?

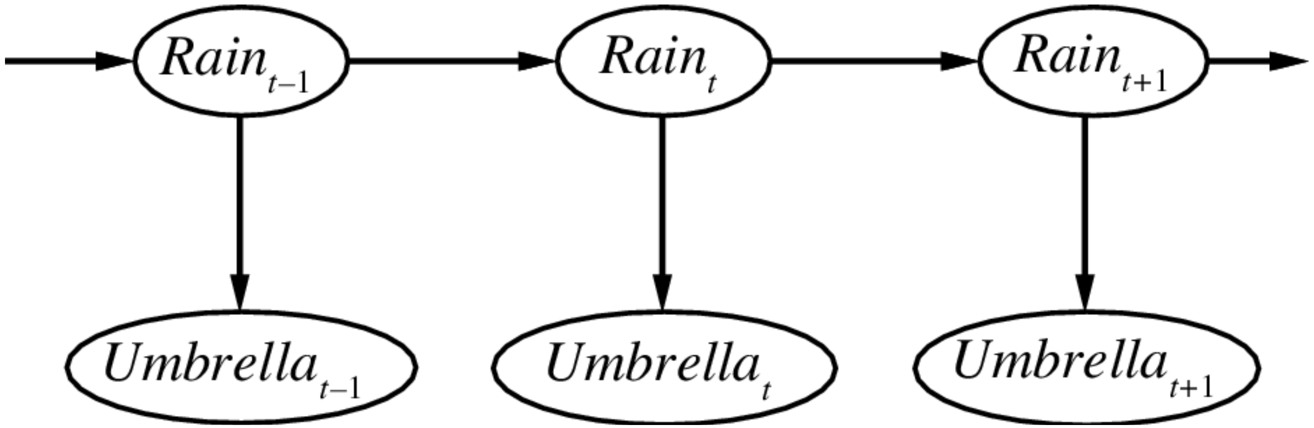
    $P(E_t \mid \mathbf{X}_{0:t}, \mathbf{E}_{1:t-1}) = P(E_t \mid X_t)$



# Example

state

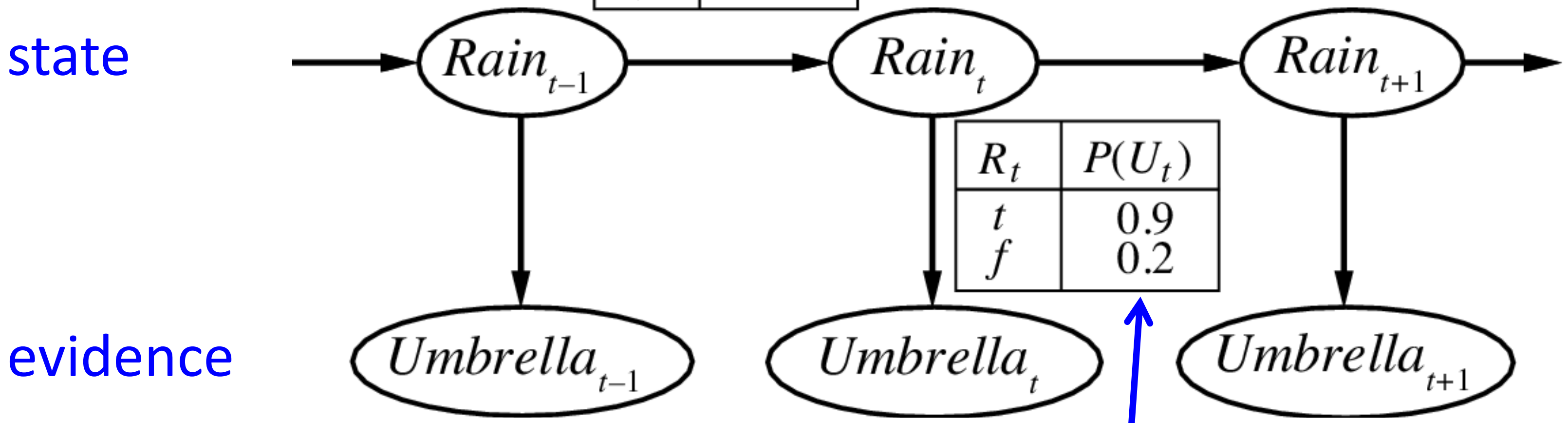
evidence



# Hidden Markov Models

Transition model

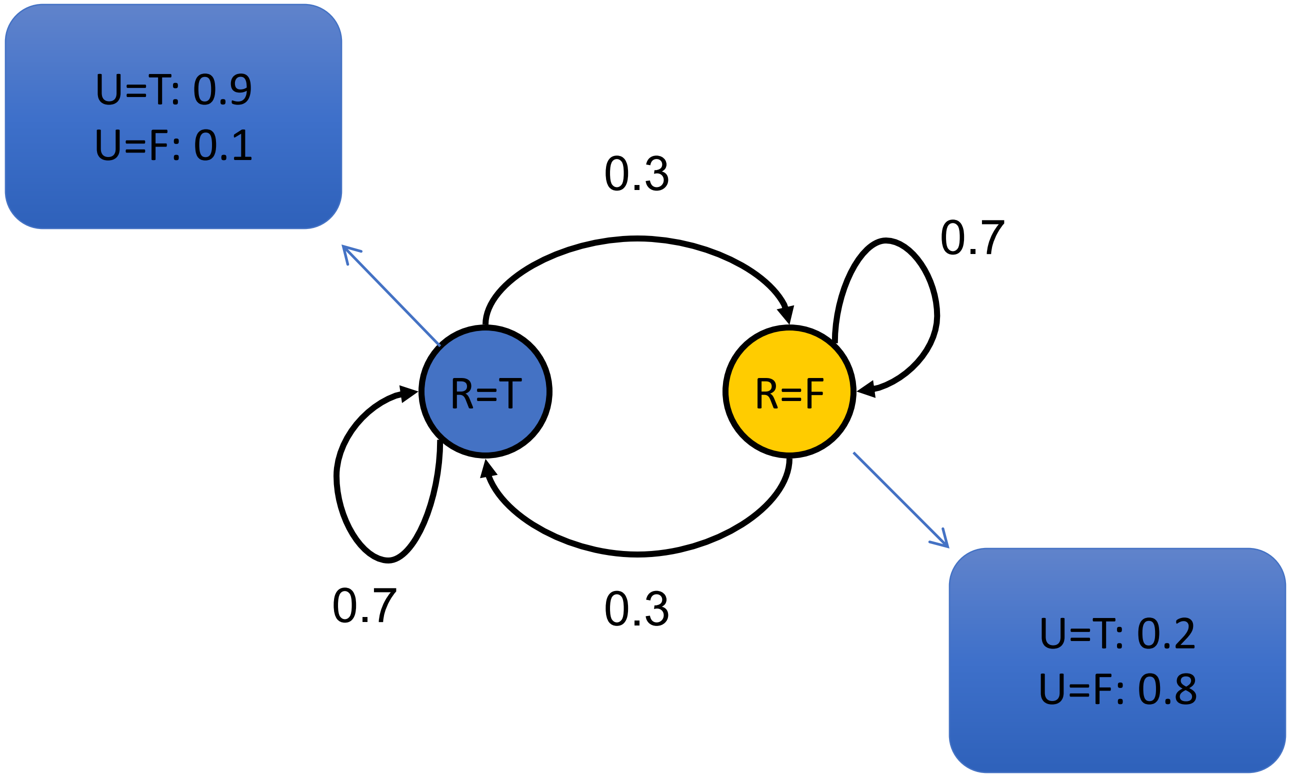
| $R_{t-1}$ | $P(R_t)$ |
|---|---|
| $t$ | 0.7 |
| $f$ | 0.3 |




| $R_t$ | $P(U_t)$ |
|---|---|
| $t$ | 0.9 |
| $f$ | 0.2 |

Observation model

# An alternative visualization




Transition probabilities

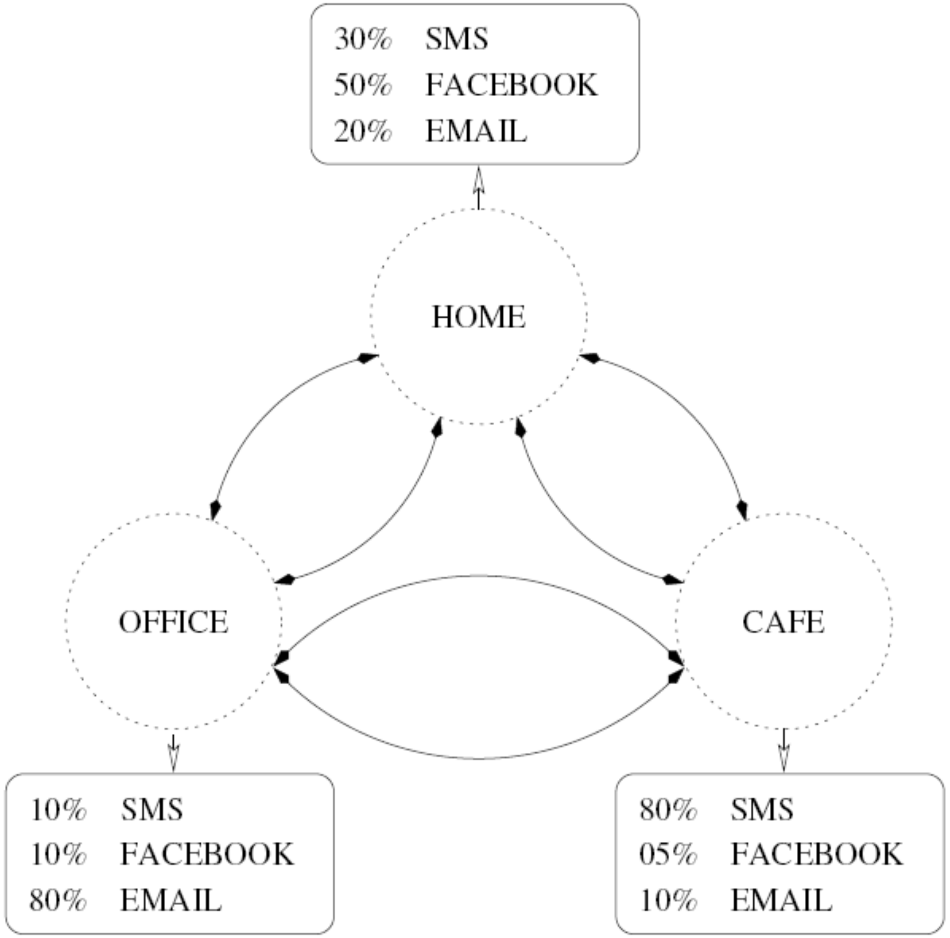
| | $R_t$ = T | $R_t$ = F |
|---|---|---|
| $R_{t-1}$ = T | 0.7 | 0.3 |
| $R_{t-1}$ = F | 0.3 | 0.7 |

Observation (emission) probabilities

| | $U_t$ = T | $U_t$ = F |
|---|---|---|
| $R_t$ = T | 0.9 | 0.1 |
| $R_t$ = F | 0.2 | 0.8 |

# Another example

- **States:** X = {home, office, cafe}
- **Observations:** E = {sms, facebook, email}




Transition Probabilities

| | home | office | cafe |
|---|---|---|---|
| home | 0.2 | 0.6 | 0.2 |
| office | 0.5 | 0.2 | 0.3 |
| cafe | 0.2 | 0.8 | 0.0 |

Emission Probabilities

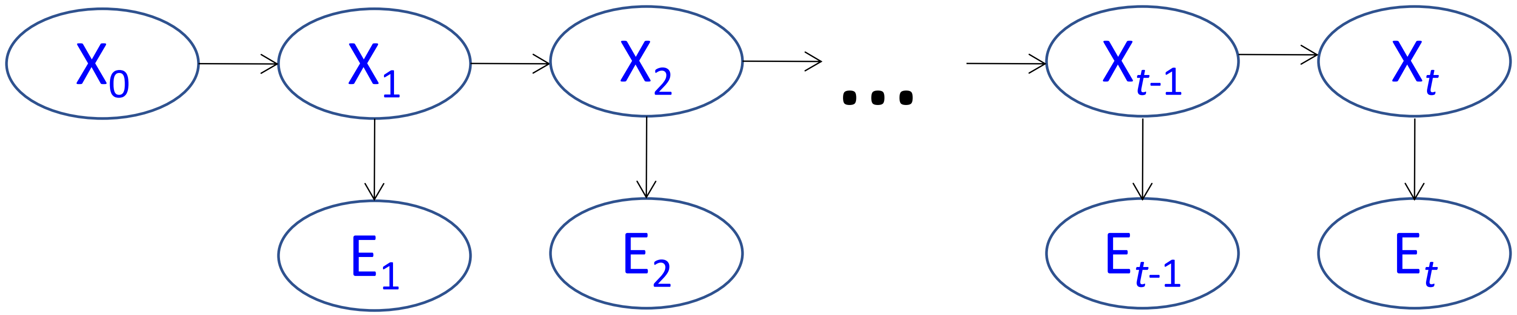
| | sms | facebook | email |
|---|---|---|---|
| home | 0.3 | 0.5 | 0.2 |
| office | 0.1 | 0.1 | 0.8 |
| cafe | 0.8 | 0.1 | 0.1 |

Slide credit: Andy White

# The Joint Distribution

- Transition model: $P(X_t \mid \mathbf{X}_{0:t-1}) = P(X_t \mid X_{t-1})$
- Observation model: $P(E_t \mid \mathbf{X}_{0:t}, \mathbf{E}_{1:t-1}) = P(E_t \mid X_t)$
- How do we compute the full joint $P(\mathbf{X}_{0:t}, \mathbf{E}_{1:t})$?

$$P(\boldsymbol{X}_{0:t}, \boldsymbol{E}_{1:t}) = P(X_0) \prod_{i=1}^{t} P(X_i | X_{i-1}) P(E_i | X_i)$$



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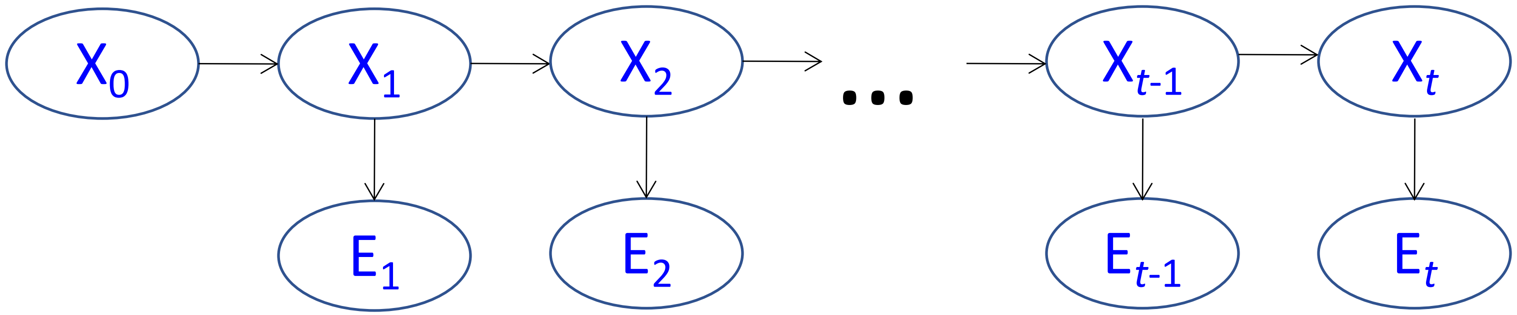
# Review: Bayes net inference

- Inference:
  - Trees: Sum-Product Algorithm (Textbook: “Variable Elimination” Algorithm)
  - Other Nets: Junction Tree Algorithm (Textbook: ”Join Tree” Algorithm)
  - In General: NP-Complete, because clique size = graph size in general
- Parameter learning
  - Fully observed: Count # times each event occurs
  - Partially observed: Expectation-Maximization algorithm
    - Estimate Probability of each event at each time
    - E[# times event occurs] = sum_t(Probability event occurs at time t)

# Sum-Product Algorithm for HMMs

- An HMM is a tree!
- For example, suppose we want to find P(X3|E1,E2,E3)
- Product: P(X0,X1,E1)=P(X0)P(X1|X0)P(E1|X1)
- Sum: P(X1|E1)=P(X1,E1)/P(E1)
- Product: P(X1,X2,E2|E1)=P(X1|E1)P(X2|X1)P(E2|X2)
- Sum: P(X2|E1,E2)=P(X2,E2|E1)/P(E2|E1)
- …



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1. Hidden Markov Models
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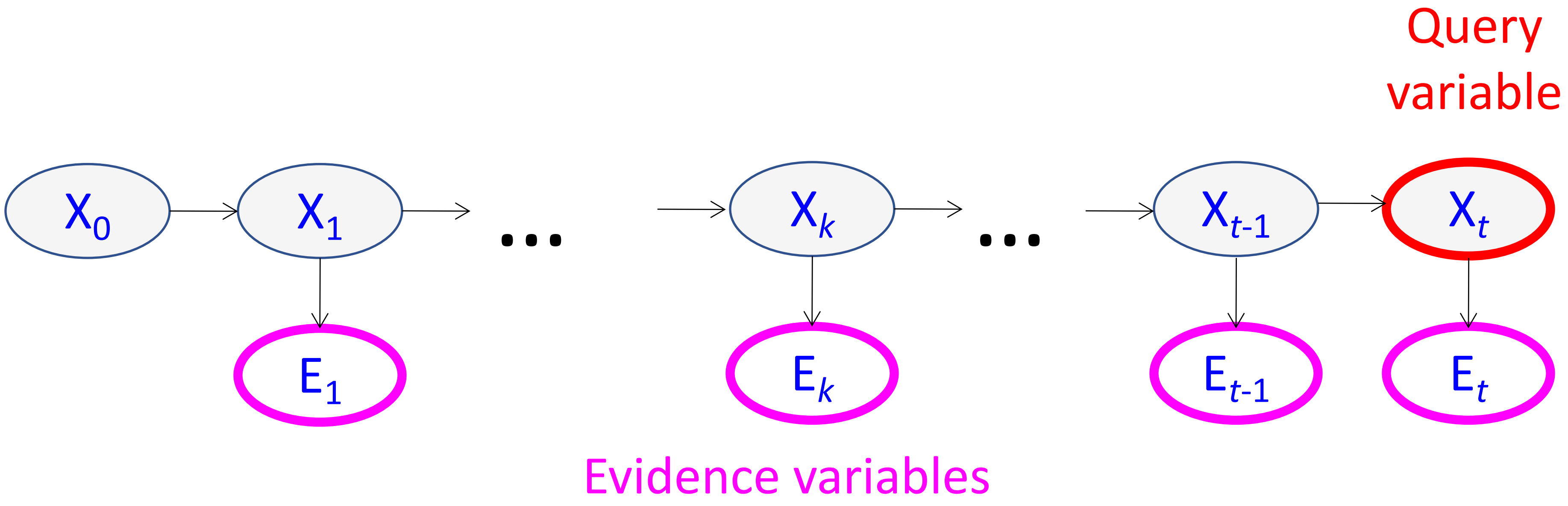
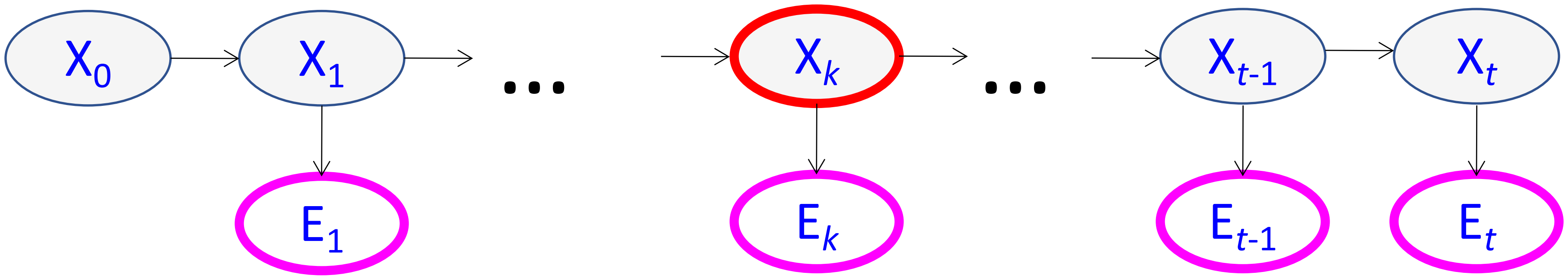
# HMM inference tasks

- **Filtering:** what is the distribution over the current state $X_t$ given all the evidence so far, $\mathbf{e}_{1:t}$ ?
  - The forward algorithm = sum-product algorithm for Xt given e1:t



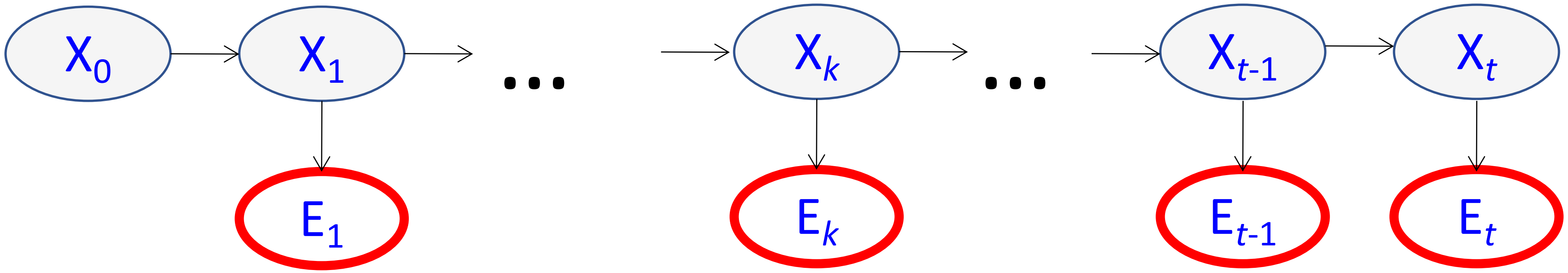
# HMM inference tasks

- **Filtering:** what is the distribution over the current state $X_t$ given all the evidence so far, $\mathbf{e}_{1:t}$ ?
- **Smoothing:** what is the distribution of some state $X_k$ given the entire observation sequence $\mathbf{e}_{1:t}$?
  - The forward-backward algorithm = sum-product algorithm for Xk given e1:t, when 1 < k < t
  - Xk = query variable, unknown, need to consider all its possible values
  - E1:t = evidence variables, known, only need to consider the given values



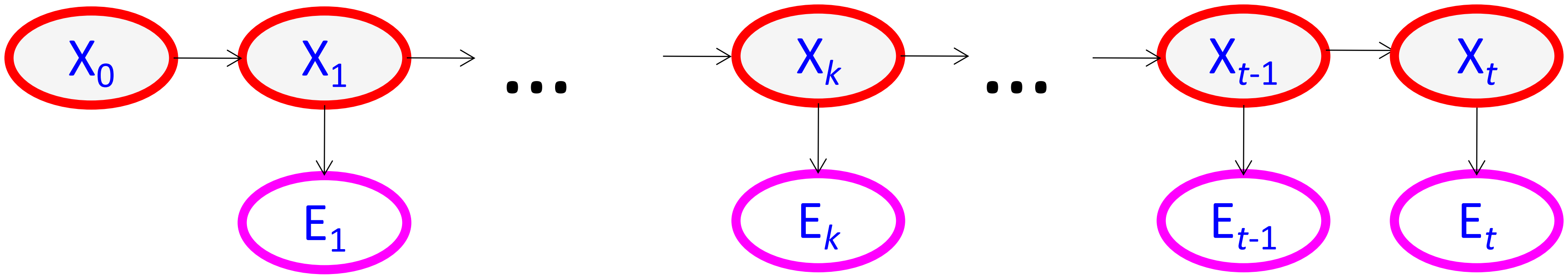
# HMM inference tasks

- **Filtering:** what is the distribution over the current state $X_t$ given all the evidence so far, $\mathbf{e}_{1:t}$ ?
- **Smoothing:** what is the distribution of some state $X_k$ given the entire observation sequence $\mathbf{e}_{1:t}$?
- **Evaluation:** compute the probability of a given observation sequence $\mathbf{e}_{1:t}$



# HMM inference tasks

- **Filtering:** what is the distribution over the current state $X_t$ given all the evidence so far, $\mathbf{e}_{1:t}$
- **Smoothing:** what is the distribution of some state $X_k$ given the entire observation sequence $\mathbf{e}_{1:t}$?
- **Evaluation:** compute the probability of a given observation sequence $\mathbf{e}_{1:t}$
- **Decoding:** what is the most likely state sequence $\mathbf{X}_{0:t}$ given the observation sequence $\mathbf{e}_{1:t}$?
  - The Viterbi algorithm



# HMM Learning and Inference

- Inference tasks
  - **Filtering:** what is the distribution over the current state $X_t$ given all the evidence so far, $\mathbf{e}_{1:t}$
  - **Smoothing:** what is the distribution of some state $X_k$ given the entire observation sequence $\mathbf{e}_{1:t}$?
  - **Evaluation:** compute the probability of a given observation sequence $\mathbf{e}_{1:t}$
  - **Decoding:** what is the most likely state sequence $\mathbf{X}_{0:t}$ given the observation sequence $\mathbf{e}_{1:t}$?
- Learning
  - Given a training sample of sequences, learn the model parameters (transition and emission probabilities)
    - EM algorithm

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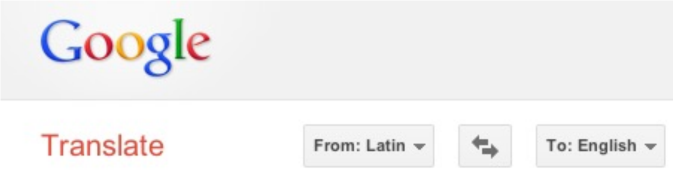
# Applications of HMMs

- Speech recognition HMMs:
  - Observations are acoustic signals (continuous valued)
  - States are specific positions in specific words (so, tens of thousands)

- Machine translation HMMs:
  - Observations are words (tens of thousands)
  - States are translation options

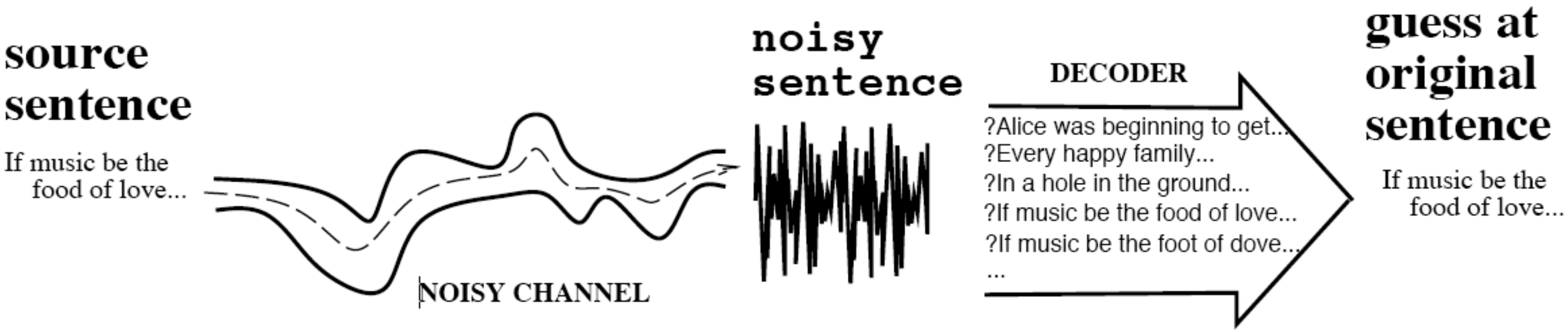



- Robot tracking:
  - Observations are range readings (continuous)
  - States are positions on a map (continuous)

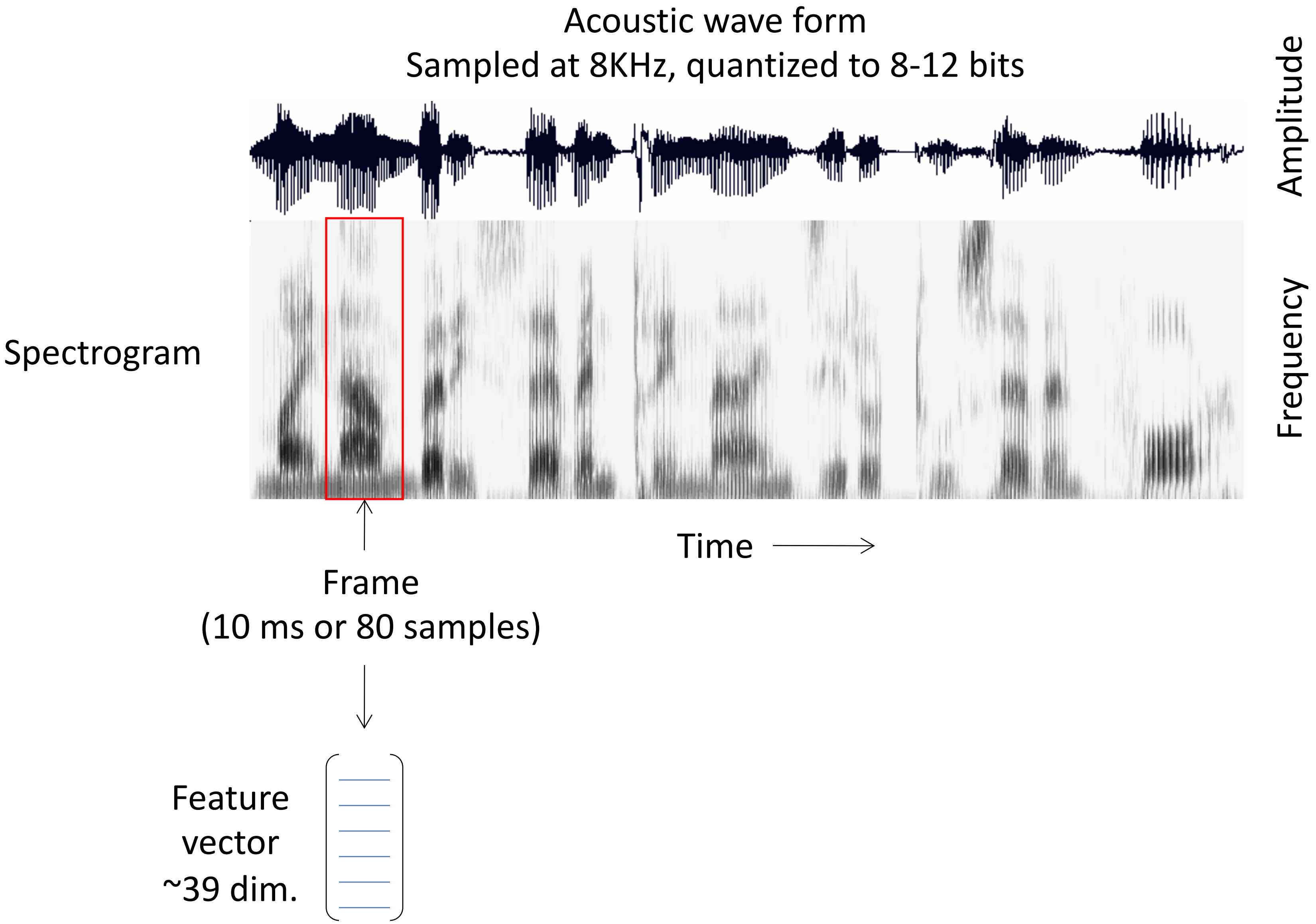
Source: Tamara Berg

# Applications of HMMs: Speech recognition

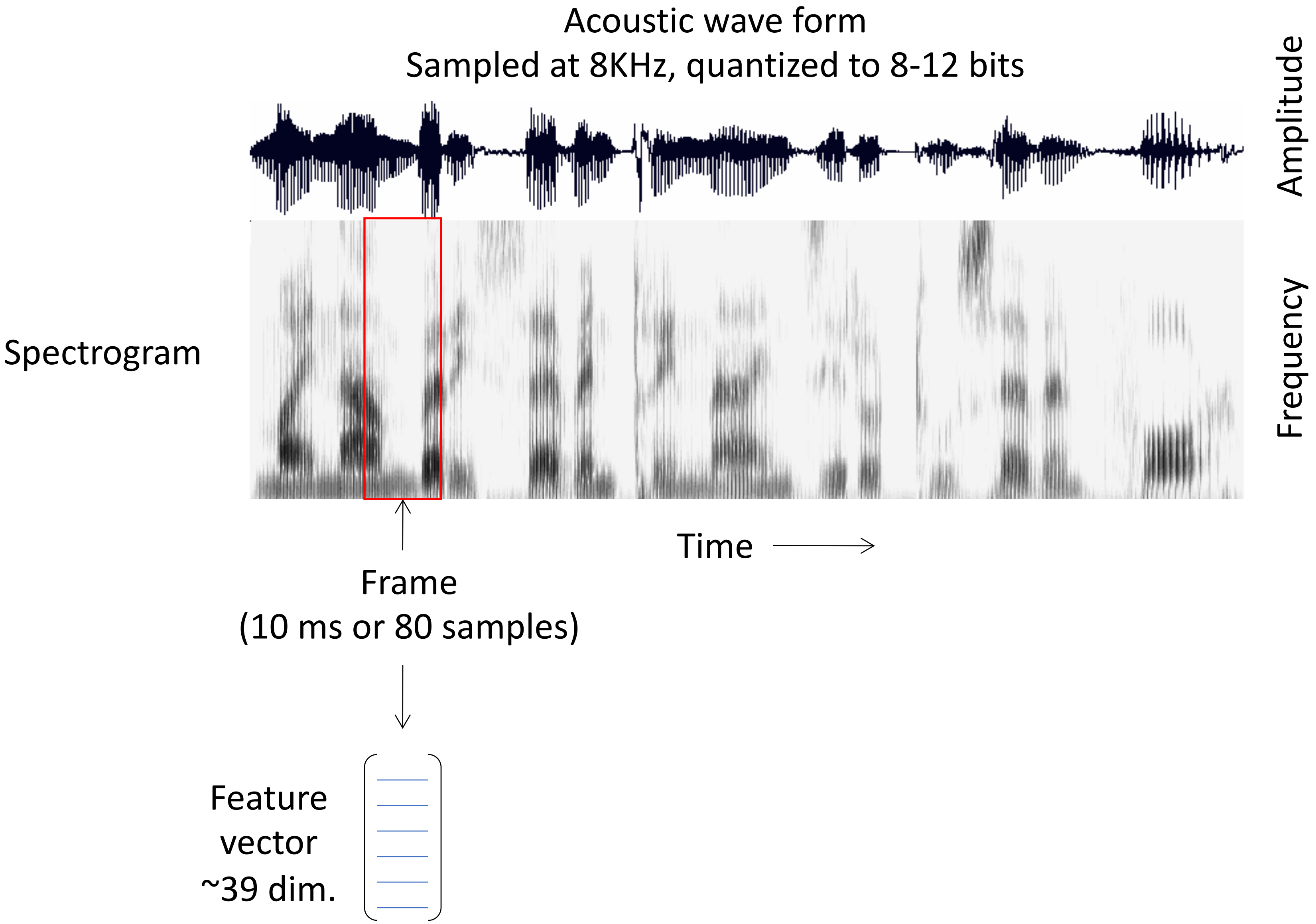
- "Noisy channel" model of speech



# Speech feature extraction



# Speech feature extraction



# Phonetic model

- **Phones:** speech sounds
- **Phonemes:** groups of speech sounds that have a unique meaning/function in a language (e.g., there are several different ways to pronounce “t”)

# Phonetic model

ZJUI

| IPA Symbol | ARPAbet Symbol | Word | IPA Transcription | ARPAbet Transcription |
|---|---|---|---|---|
| [p] | [p] | parsley | [ˈparsli] | [p aa r s l iy] |
| [t] | [t] | tarragon | [ˈtærəgɑn] | [t ae r ax g aa n] |
| [k] | [k] | catnip | [ˈkætnɨp] | [k ae t n ix p] |
| [b] | [b] | bay | [beɪ] | [b ey] |
| [d] | [d] | dill | [dɪl] | [d ih l] |
| [g] | [g] | garlic | [ˈgɑrlɨk] | [g aa r l ix k] |
| [m] | [m] | mint | [mɪnt] | [m ih n t] |
| [n] | [n] | nutmeg | [ˈnʌtmɛg] | [n ah t m eh g] |
| [ŋ] | [ng] | ginseng | [ˈdʒɪnsɨŋ] | [jh ih n s ix ng] |
| [f] | [f] | fennel | [ˈfɛnl̩] | [f eh n el] |
| [v] | [v] | clove | [kloʊv] | [k l ow v] |
| [θ] | [th] | thistle | [ˈθɪsl̩] | [th ih s el] |
| [ð] | [dh] | heather | [ˈhɛðɚ] | [h eh dh axr] |
| [s] | [s] | sage | [seɪdʒ] | [s ey jh] |
| [z] | [z] | hazelnut | [ˈheɪzl̩nʌt] | [h ey z el n ah t] |
| [ʃ] | [sh] | squash | [skwɑʃ] | [s k w a sh] |
| [ʒ] | [zh] | ambrosia | [æmˈbroʊʒə] | [ae m b r ow zh ax] |
| [tʃ] | [ch] | chicory | [ˈtʃɪkɚi] | [ch ih k axr iy ] |
| [dʒ] | [jh] | sage | [seɪdʒ] | [s ey jh] |
| [l] | [l] | licorice | [ˈlɪkɚɨʃ] | [l ih k axr ix sh] |
| [w] | [w] | kiwi | [ˈkiwi] | [k iy w iy] |
| [r] | [r] | parsley | [ˈpɑrsli] | [p aa r s l iy] |
| [j] | [y] | yew | [yu] | [y uw] |
| [h] | [h] | horseradish | [ˈhɔrsrædɪʃ] | [h ao r s r ae d ih sh] |
| [ʔ] | [q] | uh-oh | [ʔʌʔoʊ] | [q ah q ow] |
| [ɾ] | [dx] | butter | [ˈbʌɾɚ] | [b ah dx axr ] |
| [ɾ̃] | [nx] | wintergreen | [wɪɾ̃ɚgrin] | [w ih nx axr g r i n ] |
| [l̩] | [el] | thistle | [ˈθɪsl̩] | [th ih s el] |

**Figure 4.1** IPA and ARPAbet symbols for transcription of English consonants.

| IPA Symbol | ARPAbet Symbol | Word | IPA Transcription | ARPAbet Transcription |
|---|---|---|---|---|
| [i] | [iy] | lily | [ˈlɪli] | [l ih l iy] |
| [ɪ] | [ih] | lily | [ˈlɪli] | [l ih l iy] |
| [eɪ] | [ey] | daisy | [ˈdeɪzi] | [d ey z i] |
| [ɛ] | [eh] | poinsettia | [pɔɪnˈsɛriə] | [p oy n s eh dx iy ax] |
| [æ] | [ae] | aster | [ˈæstɚ] | [ae s t axr] |
| [ɑ] | [aa] | poppy | [ˈpɑpi] | [p aa p i] |
| [ɔ] | [ao] | orchid | [ˈɔrkɨd] | [ao r k ix d] |
| [ʊ] | [uh] | woodruff | [ˈwʊdrʌf] | [w uh d r ah f] |
| [oʊ] | [ow] | lotus | [ˈloʊɾəs] | [l ow dx ax s] |
| [u] | [uw] | tulip | [ˈtulɨp] | [t uw l ix p] |
| [ʌ] | [uh] | buttercup | [ˈbʌɾɚˌkʌp] | [b uh dx axr k uh p] |
| [ɝ] | [er] | bird | [ˈbɝd] | [b er d] |
| [aɪ] | [ay] | iris | [ˈaɪrɨs] | [ay r ix s] |
| [aʊ] | [aw] | sunflower | [ˈsʌnflaʊɚ] | [s ah n f l aw axr] |
| [ɔɪ] | [oy] | poinsettia | [pɔɪnˈsɛriə] | [p oy n s eh dx iy ax] |
| [ju] | [y uw] | feverfew | [fivɚfju] | [f iy v axr f y u] |
| [ə] | [ax] | woodruff | [ˈwʊdrəf] | [w uh d r ax f] |
| [ɚ] | [axr] | heather | [ˈhɛðɚ] | [h eh dh axr] |
| [ɨ] | [ix] | tulip | [ˈtulɨp] | [t uw l ix p] |
| [ʉ] | [ux] | | [] | [] |

**Figure 4.2** IPA and ARPAbet symbols for transcription of English vowels

HMM states in most speech recognition systems correspond to ***subsegments*** *of* ***triphones***

- ***Triphone***: the /b/ in “about” (ax-b+aw) sounds different from the /b/ in “Abdul” (ae-b+d). There are around 60 phones and as many as $60^3$ context-dependent *triphones.*
- ***Subsegments***: /b/ has three subsegments: the closure, the silence, and the release. There are $3\times 60^3$ subsegments of triphones.

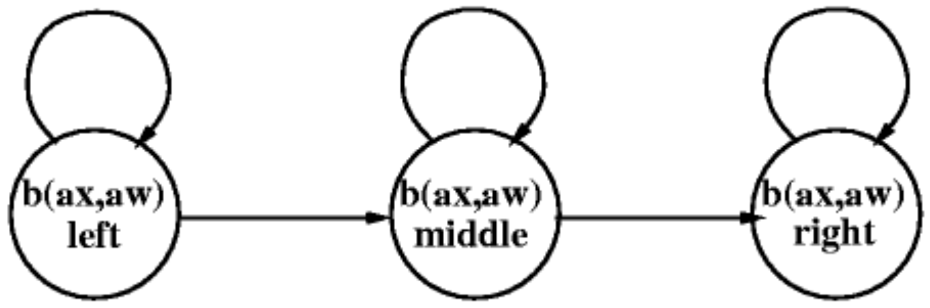



**Figure 7.11** An example of the context-dependent triphone b(ax,aw) (the phone [b] preceded by a [ax] and followed by a [aw], as in the beginning of *about*, showing its left, middle, and right subphones.

# HMM models for words

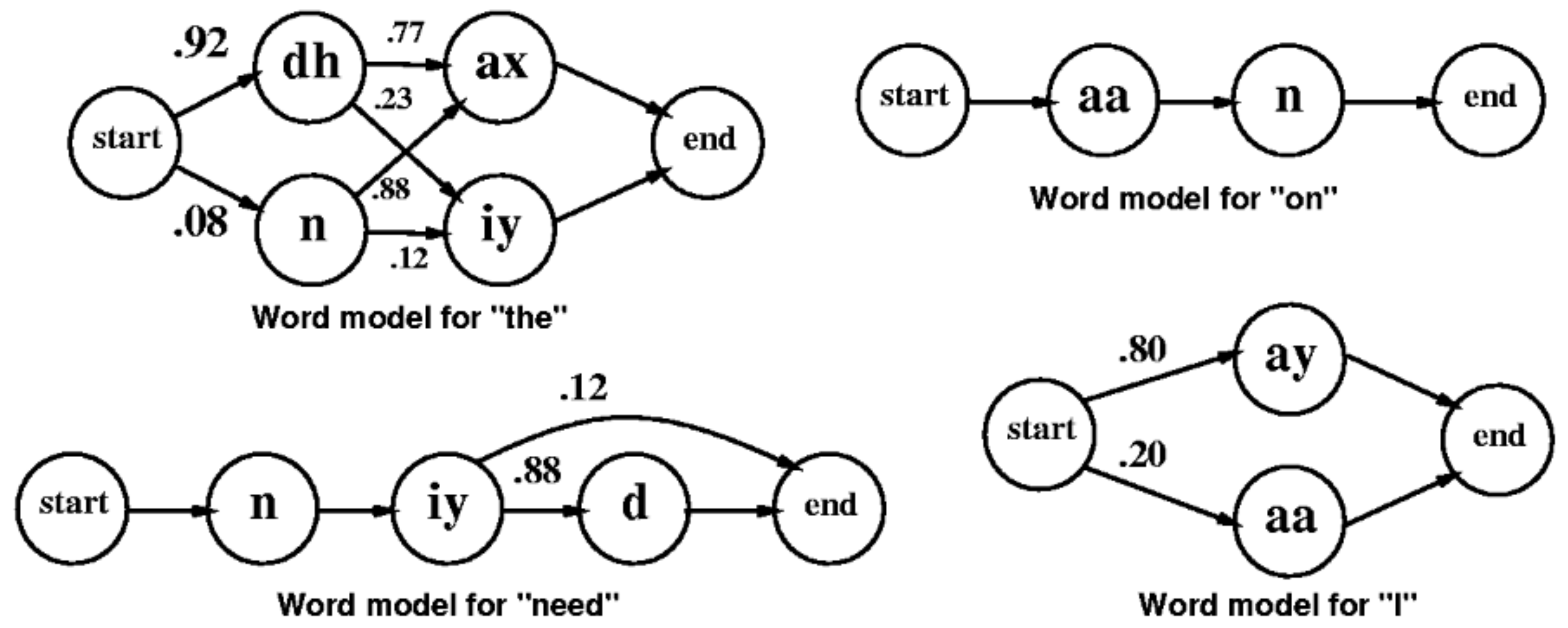
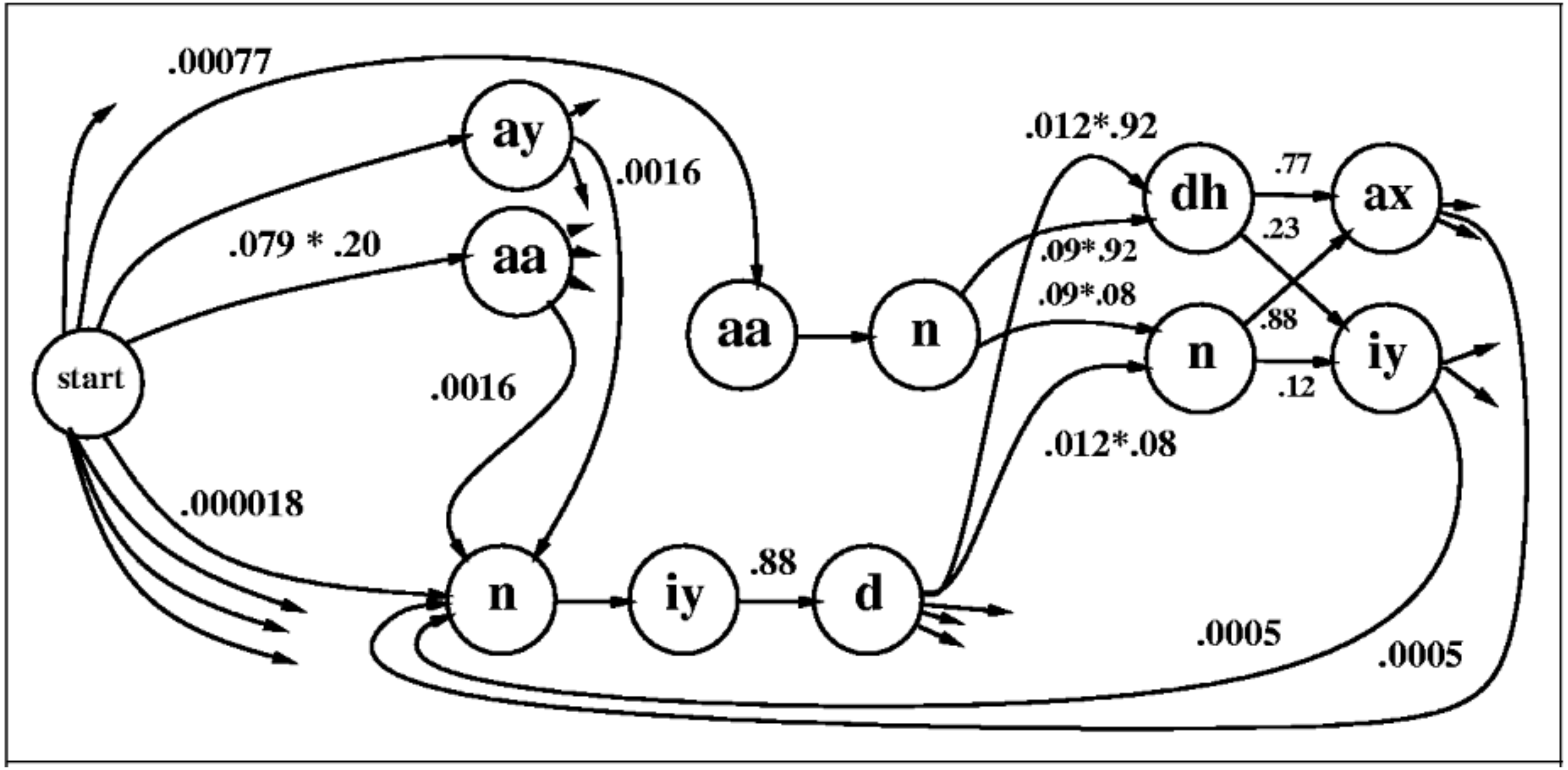



**Figure 7.5** Pronunciation networks for the words *I*, *on*, *need*, and *the*. All networks (especially *the*) are significantly simplified.

# Putting words together



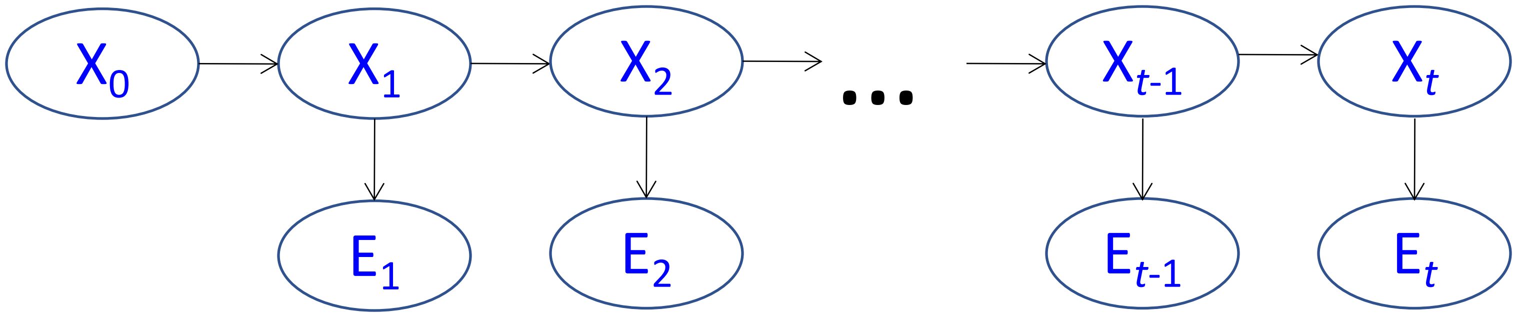

- Given a sequence of acoustic features, how do we find the corresponding word sequence?

# The Viterbi Algorithm



$$\max_{\boldsymbol{X}_{0:t}} P(\boldsymbol{X}_{0:t}, \boldsymbol{E}_{0:t})$$
$$= \max_{X_t} P(E_t|X_t) \max_{X_{t-1}} P(X_t|X_{t-1}) P(E_{t-1}|X_{t-1}) \max_{X_{t-2}} \ldots$$

Complexity changes from O{N^T} to O{TN^2}



# Decoding with the Viterbi algorithm

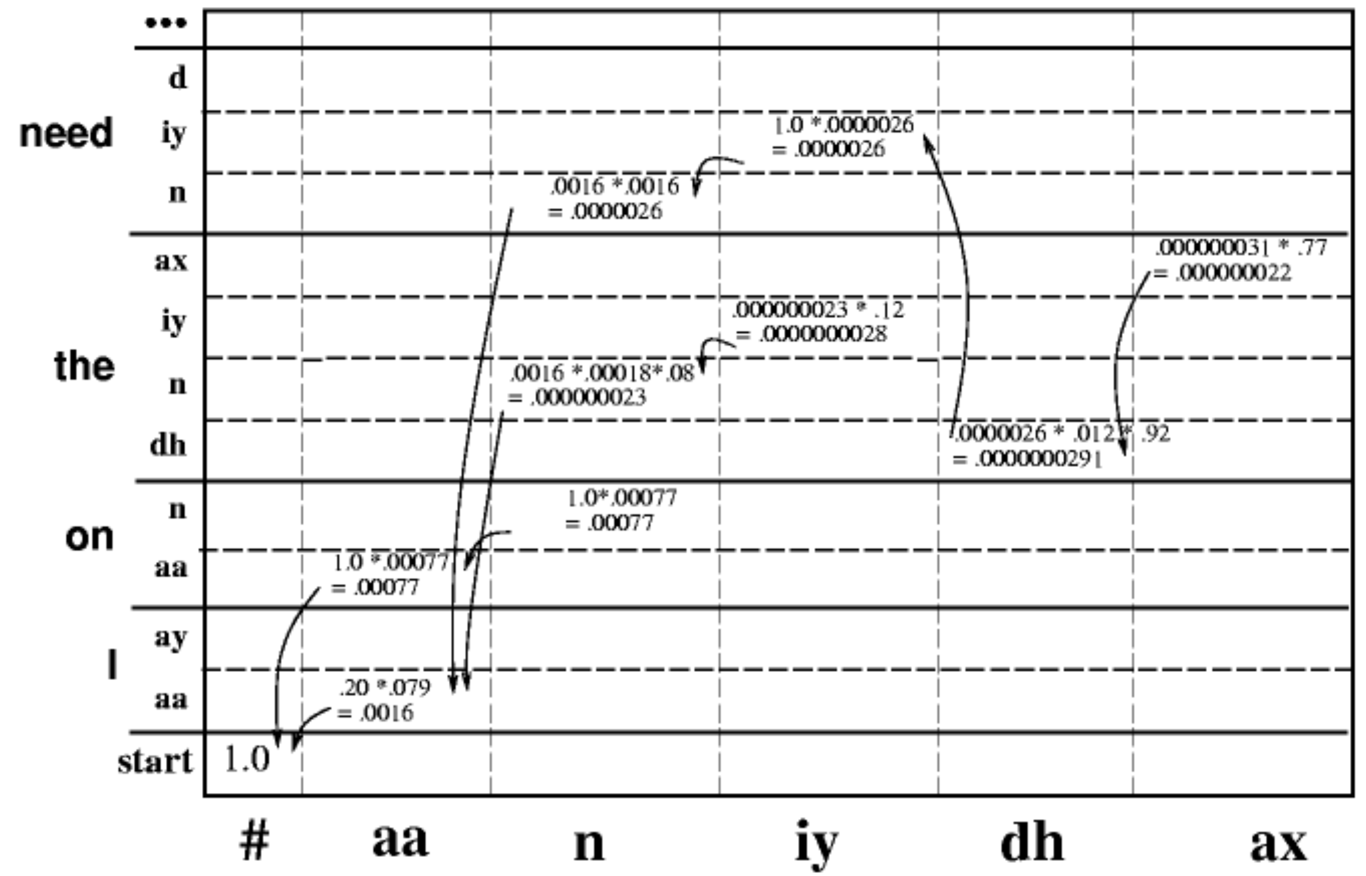



**Figure 7.10** The entries in the individual state columns for the Viterbi algorithm. Each cell keeps the probability of the best path so far and a pointer to the previous cell along that path. Backtracing from the successful last word (*the*), we can reconstruct the word sequence *I need the*.

# For more information

- CS 447: Natural Language Processing
- ECE 417: Multimedia Signal Processing
- ECE 594: Mathematical Models of Language
- Linguistics 506: Computational Linguistics
- D. Jurafsky and J. Martin, “Speech and Language Processing,” 2nd ed., Prentice Hall, 2008