

DT

元

目标

类型含义	类型记号	命题含义	命题记号
单位类型	\top	恒真命题	\top
空类型	\perp	恒假命题	\perp
和类型	$A + B$	析取, 逻辑或	$A \vee B$
积类型	$A \times B$	合取, 逻辑与	$A \wedge B$
函数类型	$A \rightarrow B$	蕴含	$A \Rightarrow B$
	$A \rightarrow \perp$	否定, 逻辑非	$\neg A$

\equiv
↓

判断

Judgemental

$\equiv A$
↓

类型命理

Proposition

Type former

$A + B$

$\frac{A}{A + B} \quad \frac{B}{A + B}$

Deductive Type

消元

$a : A$

False

$\boxed{P(a)} : u$

构造

合成

$a : A : u$

A type Du

$A + B$ type

$\frac{A : u \quad B : u}{A + B : u}$

DT

$$\Gamma, \underline{x:A} + P(x):U$$

1.1

$$f: [a,b] \rightarrow \mathbb{R}$$

$$f_{[a,b]} \text{ 连续}$$

$$\forall x \in [a,b], \forall \varepsilon > 0, \exists \delta > 0. \forall y \in [a,b]$$

term

~~-3/2/3/1/2~~

~~$\delta \varepsilon > 0, \exists \delta > 0. \forall x \in$~~

, $\forall y \in$

C-H 定理

前提

$$|y-x| < \delta \Rightarrow |f(y) - f(x)| < \varepsilon : U$$

$$\Gamma, x: \varepsilon, \delta; y: [a,b] \vdash \square : U$$

$$\Gamma, \varepsilon, \delta, \underline{x}, \underline{y}, + \vdash \square : U$$

DT 1.2
primary (%d, %g)
~~3, 121~~

+em

$\text{pref} = \frac{\text{cstr.-t}}{\Gamma} - \text{if irr} \rightarrow \text{double} \Rightarrow \textcircled{2}$

$\Gamma, (\text{fme-gini cstr}) \vdash \text{pref} : u$

DT

2^{正明}

$x:A \vdash \underline{P(x)} : u$

$p = P(x)$

~~rbe : rbe-t + bmb : u~~

DT 2

~~依'式~~ days function type $(x:A \rightarrow B(x))_h$
~~依'式~~ pair $\underline{\underline{(x:A) \times B(x)}}$

$$A \rightarrow B \sim (x:A) \rightarrow B(x)$$

$$\frac{\Gamma, x:A \vdash B(x) : U}{\Gamma \vdash \boxed{(x:A) \rightarrow B(x)} : U} \dashv_{\text{FISN}}$$

$$\frac{\Gamma, x:A \vdash b : B(x)}{\Gamma \vdash \lambda(x:A). b : \boxed{}} \dashv_{\text{FISN}}$$

$$\frac{\Gamma \vdash b : B \quad \Gamma \vdash a : A}{\Gamma \vdash \boxed{(\lambda(x:A). b) a} \stackrel{\text{def}}{=} b[x \mapsto a] : B} \dashv_{\text{FISN}}$$

$$\frac{\Gamma \vdash f : (x:A) \rightarrow B \quad \Gamma \vdash a : A}{\Gamma \vdash \boxed{f a} : B[x \mapsto a]} \dashv_{\text{FISN}}$$

$$\frac{\Gamma \vdash f : (x:A) \rightarrow B(x) \quad \Gamma \vdash a : A}{\Gamma \vdash \boxed{\forall x. B(x)}} \dashv_{\text{FISN}}$$

DT 例題 28'

$$\frac{(x:A) * B(x)}{(a,b)} \xleftarrow{CH} \frac{\exists x, B(x)}{(a,b)}$$

§2.2

$$\frac{\Gamma, x:A \vdash B(x): u}{\Gamma \vdash (x:A) * B(x) : u^* \text{ PSL}}$$

$$\frac{\Gamma, a:A \vdash b:B}{\Gamma \vdash (a,b):(x:A) * B(x)}$$

构造

$$\frac{\Gamma \vdash p:(x:A) * B(x)}{\Gamma \vdash p.1:A \quad \Gamma \vdash p.2:B[x \mapsto p.1]} \quad \begin{matrix} \text{构造 1} & \text{构造 2} \\ \nearrow & \searrow \end{matrix}$$

$(p.1, p.2)$

2.3

$$\begin{array}{c}
 \text{DT} \\
 \frac{(x:A) \rightarrow \sum_{i \in I} B_{ix}}{\prod_{i \in I} (x:A) \times B_{ix}} \\
 \text{the } \underline{f_i} \rightarrow \cancel{\text{the } f_i} \quad \cancel{\text{the } f_i} \\
 \underline{(x:A) \times B_{ix}}
 \end{array}$$

$$\begin{array}{c}
 P \\
 \prod_{i \in I} (x:A) \times B_{ix} \\
 \sum_{i \in I} (x:A) \times B_{ix} \\
 \text{Signature 1}
 \end{array}$$

$$\begin{array}{c}
 \prod_{i \in I} (i:I) X_{ii} \\
 \sum_{i \in I} (i:I) X_{ii} \\
 \text{index}
 \end{array}$$

\rightarrow ~~agr~~ $t : \prod_{i \in I} X_{ii} \equiv \underline{(i:I) \rightarrow X_{ii}}$

$$\begin{array}{c}
 \underline{t : i : X_{ii}} \\
 t_i \quad t_i
 \end{array}$$

$$t : (i:I) \rightarrow X_{ii}$$

$$(x_i)_{i \in I} \leftrightarrow (i:I) \rightarrow x_i$$

C++ oper \overline{i}

$$\begin{array}{c}
 I \\
 (-, -, -, -)
 \end{array}$$

DT
 $(x:A) \rightarrow \{s^{2,3}\}$
 $\underline{\underline{h_1, h_2}}$
 \rightarrow
 ~~x_1, x_2, x_3~~
 \rightarrow
 $\{h_1, h_2\}$
 \rightarrow
 $(x:A) \times B_{1x}$
 \rightarrow
 P_j
 \rightarrow
 $(x:A) \times B_{1x}$
 \rightarrow
 $\sum (x:A) \times B_{1x}$
 $\sum (i:I) X_{(i)}$
 $\pi_{(i:I)} X_{(i)}$
 $\sum (i:I) X_{(i)}$
 index

$$\sum_{i \in I} (i; l) \times_{l; l} \hookrightarrow (i; l) \times X_{l; l} : \mathcal{U}$$

```

class StackManager {
public:
    enum Tag { t1, t2, t3 };
    void* body();
    void* tag();
    void* data();
};

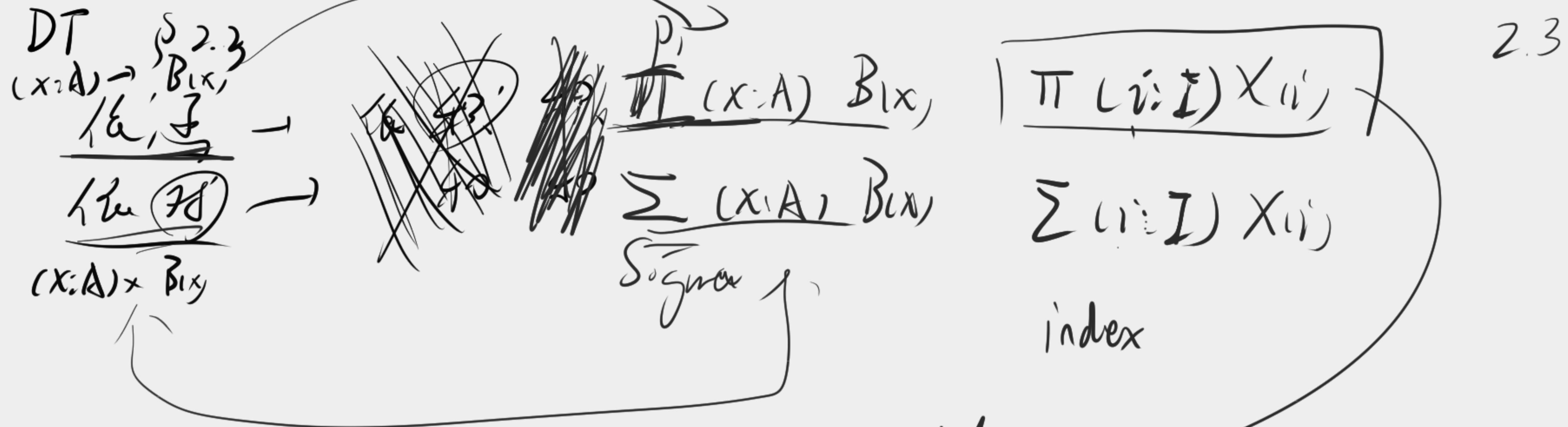
class StackManager {
public:
    enum Tag { t1, t2, t3 };
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    void* tag();
    void* data();
};

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public:
    enum Tag { t1, t2, t3 };
    void* body();
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    void* data();
};

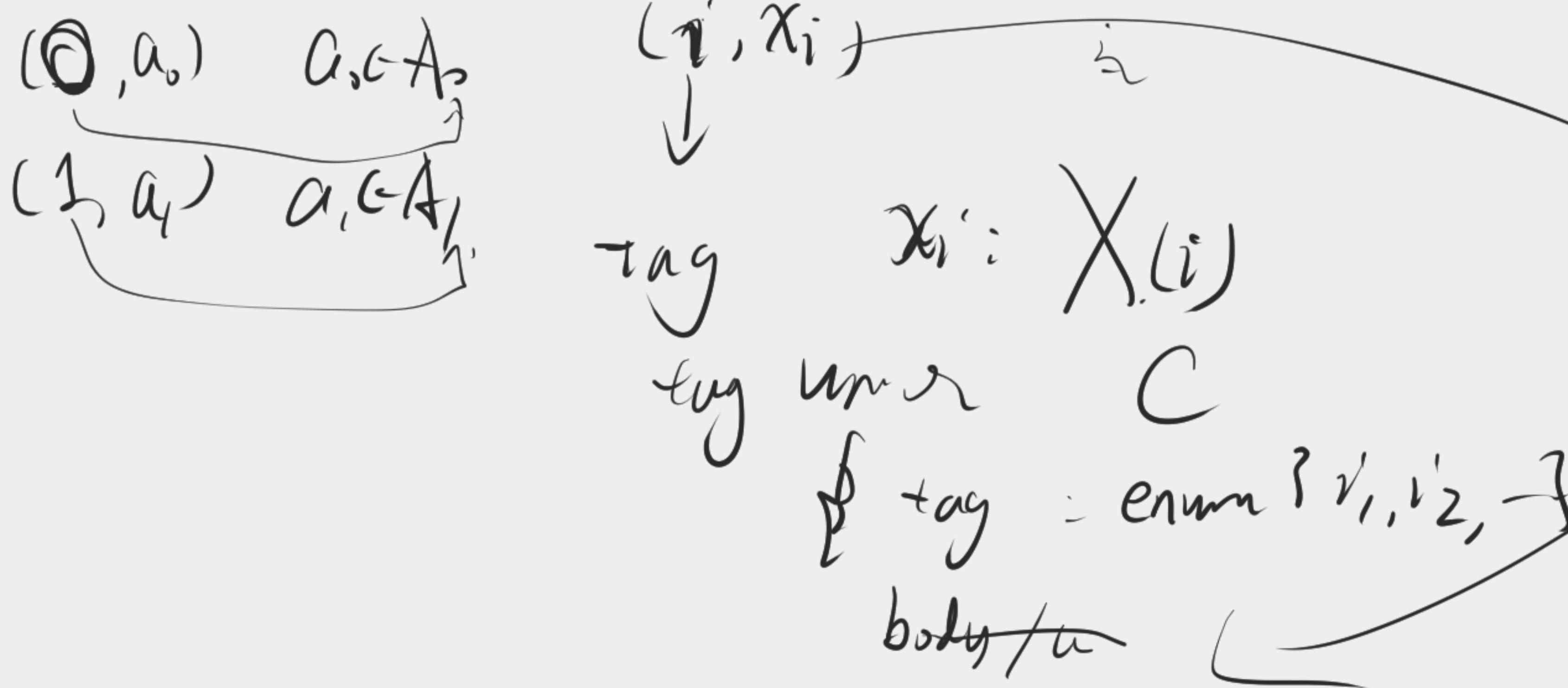
```

The diagram illustrates a class hierarchy for a stack-based memory manager. It shows three levels of inheritance:

- Level 1:** A base class `StackManager` with three protected members: `body()`, `tag()`, and `data()`. This level is represented by a large oval.
- Level 2:** Three derived classes inherit from the base class. Each derived class has its own implementation of the three methods. These are represented by smaller ovals.
- Level 3:** The derived classes contain additional members: `enum Tag { t1, t2, t3 }`. These are shown as labels next to the derived classes.



$$\sum_{i:I} (i:I) X(i) \longleftrightarrow (i:I) \times X(i) : U$$



DT 2.4

从直觉型
到直觉数论
从直觉子类型

$$\frac{\Gamma, x:A \vdash Bx : u}{(x:A) \rightarrow B(u)}$$

$$(x:A) \times B(x)$$

$$\frac{B(x)}{B: ((x:A) \rightarrow u)}$$

$\boxed{\Gamma, x:A \vdash B(x) : u}$

$\Gamma \vdash B : A \rightarrow u$

直觉类型层次

Girard paradox

$\boxed{A \rightarrow u} : u$

$\boxed{A \rightarrow u} : u$

$A : \boxed{u_0 | u_1 | u_2} : u_3$

$A : u_0 \quad u_0 : u_1 \quad A : u_0 = u_1$ hierarchy

$u_0 : u_1 \rightarrow u_0 : u_2, u_0 : u_3$

层叠

$\forall x:A, P(x)$

$\frac{}{\exists x : A, P(x)}$

直觉

公理

$\neg P$

$P, \boxed{A \rightarrow u} : u_1$

u = u

with直觉 A

u_0 in predicate

DT

15:00 纸张

array <T, n = int)

DT §3.1 \equiv_A

同一类型
Identity
相等性
Equality

三元
T = A
Eqs

internalize 内化

28/3
PA

①.

1 : (a:A) → a =_A a

$\Gamma \vdash a:A \quad \Gamma \vdash a':A$

$\frac{}{\Gamma \vdash [a =_A a'] : U}$ $=_A$ 定义

$a = a'$ See

$a = a'$

$\forall x \in a, \neg x \in a'$
 $\forall x \in a', x \in a$

E

$a = a' \quad p: a = b$

$\frac{}{\Gamma, x:A, y:A, p: x =_A y \vdash C(x, y, p) : U}$

$\Gamma \vdash \text{ind}_=(x, y, p, c) : C(x, y, p)$

$\frac{}{z + 3 = 4, z + 3 = 4}$

$\frac{a = b}{z = z}$

$C(x, y, p) : U$

$\frac{\Gamma \vdash c: (z:A) \rightarrow C(x-z, y-z, p) \rightarrow I_z}{J \neq (2a)}$

$c: (z:A) \rightarrow$

$C(x-z, y-z, p) \rightarrow I_z$

$$\frac{DT\ 3.1 \frac{\Gamma, x:A, y:A, p:x \infty y + \underline{C(x,y,p)} : u \quad \Gamma \vdash c:(z:A) \rightarrow C(x,z,l_z)}{\Gamma, x:A, y:A, p:x \infty y + \text{ind}_{=}^{(x,y,p,c)} : \underline{C(x,y,p)}}}{J}$$

3.1.1 3) $\forall a \in A : f(a) = f(a')$

$$\mathcal{V}(C^{(a,a',p)}) \equiv \frac{\|fa - f a'\|}{\|f\|} \cdot u$$

$$C((\mathbb{Z} \cdot A) \rightarrow (\mathbb{C}^{\times})_{\mathbb{Z}}|_{\mathbb{Z}})$$

$$c: (\exists A) \rightarrow \boxed{f z} =_{\beta} \boxed{f z} \quad \text{ind}_{=} (a, a', p, \lambda z. 1_{f z}) : fa =_{\beta} a'$$

indumenta

四月内记

自支

$$z = \bar{z}$$

ℓ_1, ℓ_2, ℓ_3

$$a = a' \rightarrow a' = a$$

$\text{DT } \Gamma, n:N \vdash P(n):U$

①

②

$$\int \begin{array}{c} 1. P_0 : P(0) \quad \Gamma, n:N \vdash P(n):U \quad \Gamma \vdash P_0 : P(0) \\ 2. \frac{\forall n. \quad P(n) \supseteq P(n+1)}{\forall n. \quad P(n)} \end{array} \quad \begin{array}{c} \Gamma, m:N \vdash \text{ind}_N(m, P_0, P_S) : P(m) \\ \Gamma \vdash \odot : (m:N) \rightarrow P(m) \end{array}$$

注記

1) 的法

归纳定理

$\text{add} : I_N \rightarrow I_N \rightarrow N$

$P_S \equiv \lambda_n. \quad n \cdot S(n)$

江川

$I_N : U$ 定義

$0 : I_N$ 指定 0

$\vdash n : N$

$\vdash S(n) : N$ 指定 S

succ

S_m

$$\left\{ \begin{array}{c} \frac{\frac{\frac{\Gamma, n:N \vdash P(n):U}{P(n) \equiv N} \quad \Gamma, n:N \vdash n:N \quad \Gamma \vdash P_S : \frac{I_N \rightarrow I_N \rightarrow N}{N}}{\Gamma, m:N, n:N \vdash \text{ind}_N(m, n, P_S) : N} \quad \text{①}}{\text{②}} \\ \text{②} \quad \text{定義} \end{array} \right.$$

$\Gamma, m:N, n:N \vdash \text{ind}_N(m, n, P_S) : N$

$\text{add} \equiv \lambda m n. \quad \text{ind}_N(m, n, P_S)$

$$DT \quad \text{予測} \rightarrow \text{出力} \cdot \text{評価} \quad \xrightarrow{\text{ind}_{IN}(m, p_0, p_0) = p(m)} \\ \underline{P_{inj}: m \quad p_0 \quad p_s}$$

$$\underline{\text{ind}_{IN}(0, p_0, p_0) = p_0 = p(0)} \quad \text{Vergleich}$$

$$p_S: \{n \in IN\} \xrightarrow{\text{P}(n) \rightarrow P(S(n))}$$

$$\text{ind}_{IN}(S(n), p_0, p_S) = p_S \cap \underline{\text{ind}_{IN}(n, p_0, p_S) = p(S_n)}$$

rec $f(m) = m \cdot m$

$$\text{ind}_{IN}(0, p_0, p_S) = p_0$$

$m = 0 \Rightarrow p_0$

$$\text{ind}_{IN}(S(n), p_0, p_S) = p_S \cap \text{ind}_{IN}(n, p_0, p_S)$$

$m = S(n) \Rightarrow p_S \neq f(n)$

$m = 0 \Rightarrow p_0 = n$

$m = S(n) \Rightarrow p_S = \lambda \cdot n \cdot S(n)$

$\overline{SSS(\cancel{0+1})} = SSSS_0 = 4$

DT

$$D : IN \Rightarrow P_0$$

$$S(n) : IN \Rightarrow P_S \text{ n } \text{hd}_1(n, \rightarrow)$$

$$\frac{11260}{W \geq 2} IN$$

$$4 S \xrightarrow{\text{AST}} \text{add } 4 \\ \underline{\lambda - m . Sm} \quad n$$

解释



$$DT \ 3/2.2.1 \quad \forall n. \quad \boxed{n+0 =_N n} \equiv P(n)$$

$$\Gamma, n : \mathbb{N} \vdash P(n) : \mathcal{U} \quad \Gamma + 1, : \frac{0+0 =_N 0}{\equiv 0 =_N 0}$$

$$\Gamma \vdash P_s : (n : \mathbb{N}) \rightarrow P(n) \rightarrow P(S(n))$$

Ind_N

$$\hookrightarrow P_s : (n : \mathbb{N}) \rightarrow n+0 =_N n \rightarrow \frac{S(n) \wedge 0 =_N S(n)}{\equiv S(n+0) =_N S(n)}$$

$\uparrow \quad \uparrow$

$$f \equiv \lambda x. S(x)$$

$$\rightarrow f^{(n+0)} =_N f(n)$$

3/22 3.2.2 ap

$$a =_A a'$$

$\downarrow +$

ap

$$fa =_B fa'$$

Ceq

induction on n

$$\textcircled{1} \quad P_0 : P(0)$$

$$\textcircled{2} \quad P_s : \underline{\hspace{10cm}}$$

32.2

$$m+n =_N n+m$$

DT §3.3 固約 - 消去規則



$\text{ind}_{\frac{\Gamma, x, y : A, P \vdash F}{C(x, y, P) : u}}$

DT

DT

$\text{ind}_{IN} \frac{\Gamma, n : IN \vdash P(n) : u}{}$

$\text{non-DT} \quad \text{rec}_{IN} \frac{\Gamma + P : u}{\Gamma \text{ elim } \in N}$

add

$P \in IN$
recursion

DT 53)

$\times \text{def}$ $\text{Ind}_{(x:A) \times B, xy} \quad \text{Rw' 28'}$

P.2
P.2

$\Gamma, z : (x:A) \times B + C_2, : u \quad \Gamma + C : (a:A) \rightarrow (b:B) \rightarrow C(z \mapsto (a,b)) \quad \Gamma \vdash p : x : A \times B$

$\Gamma \vdash \text{Ind}_x(C, p) : C(z \mapsto p)$

ip2 17/18

$\Gamma + a : A, \Gamma, a : A \vdash b : B$

$\Gamma \vdash \text{Ind}_x(C, (a,b)) \equiv c_a b : C(z \mapsto (a,b))$ ✓

$C_1 = \lambda a. b. a$

$\text{Ind}_x(C, p) \equiv p. 1$

Ind_x

$C_2 = \lambda a b. b$

$\text{Ind}_x(C, p) \equiv p. 2$

52.3 1B

$B \times C$

$\text{Ind}_x(C,) \quad \text{Rw' Ind}_x$

Ind_B

DT § 4 → HoTT

M⁻T^T .

Y^Y X^X

DT

(x:A) ⊢ P(x): u

ind_{II}(λ ,
S_n) →

induction
higher type
w type

ind

rec DT

exist.
elim { free
ind }

○ mu b^b