Single-Phase Power

Definition of Electrical Power

Electrical power is the rate at which electrical energy is transferred or converted by an electrical circuit. It represents the amount of energy per unit time.

- Mathematical expression: **$p(t) = u(t) \times i(t)$**
- Where:
  - $p(t)$ = instantaneous power (watts, W)
  - $u(t)$ = instantaneous voltage (volts, V)
  - $i(t)$ = instantaneous current (amperes, A)
- Unit: 1 Watt = 1 Joule/second = 1 Volt × 1 Ampere
- Energy-Power relationship: $E = \int p(t)\, dt$ (Energy = integral of power over time)

Power Transfer Concept

- Power transfer occurs when electrical energy flows from a source (generator) to a load (receiver).
- Power convention:
  - Generator (source): Supplies/delivers power to the circuit
  - Load (receiver): Absorbs/consumes power from the circuit
- In a complete circuit:
  - $\sum \mathbf{P}_{\mathbf{delivered}} = \sum \mathbf{P}_{\mathbf{absorbed}}$ (Conservation of energy)
- Or equivalently: $\sum \mathbf{P}_{\mathbf{sources}} = \sum \mathbf{P}_{\mathbf{loads}}$

Efficiency

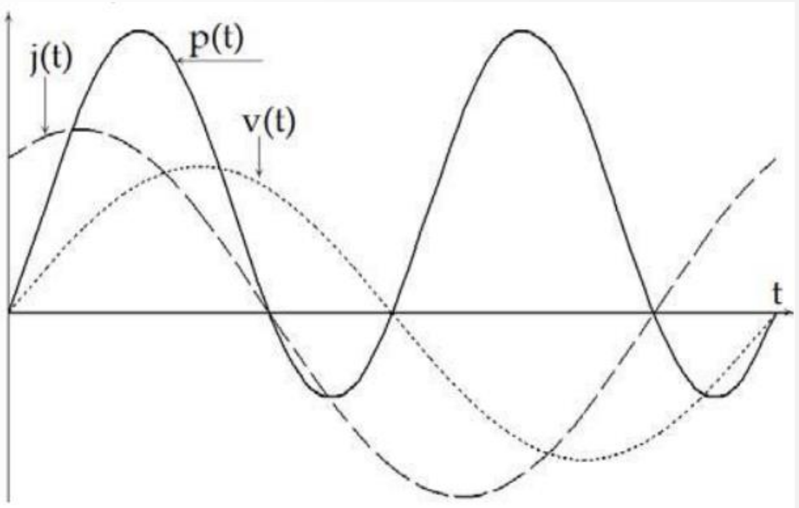
- **Efficiency** is the ratio of useful output power to input power. It measures how effectively a system converts or transfers electrical energy.
- Definition:
  - $\boldsymbol{\eta = P_{out} / P_{in}}$
  - $\boldsymbol{\eta = P_{useful} / P_{supplied}}$
- Expressed as percentage:
  - $\boldsymbol{\eta(\%) = (P_{out} / P_{in}) \times 100\%}$
- Properties:
  - $0 \leq \eta \leq 1$ (or $0\% \leq \eta \leq 100\%$)
  - $\eta = 1$ (100%) → Ideal case (no losses)
  - $\eta < 1$ → Real case (losses present)

Efficiency

- **Applications:**
  - Transformers: η typically 95-99%
  - Electric motors: η typically 80-95%
  - Power supplies: η typically 70-90%

Instantaneous Power in Sinusoidal Regime

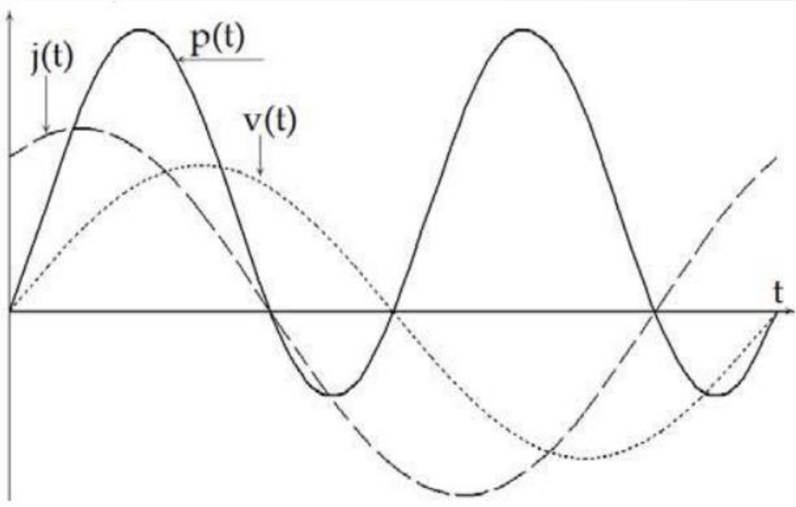
- Consider a dipole traversed by i(t) and maintained under a voltage v(t).
- Given:
  - $i(t) = I\sqrt{2}\sin(\omega t + \varphi\_I)$
  - $v(t) = V\sqrt{2}\sin(\omega t + \varphi\_V)$
- Where I and V are RMS values.
- Instantaneous Power: P(t) is the power at each instant t.
  - **$P(t) = v(t) \times i(t) = 2IV\sin(\omega t + \varphi_I)\sin(\omega t + \varphi_V)$**



Instantaneous Power in Sinusoidal Regime

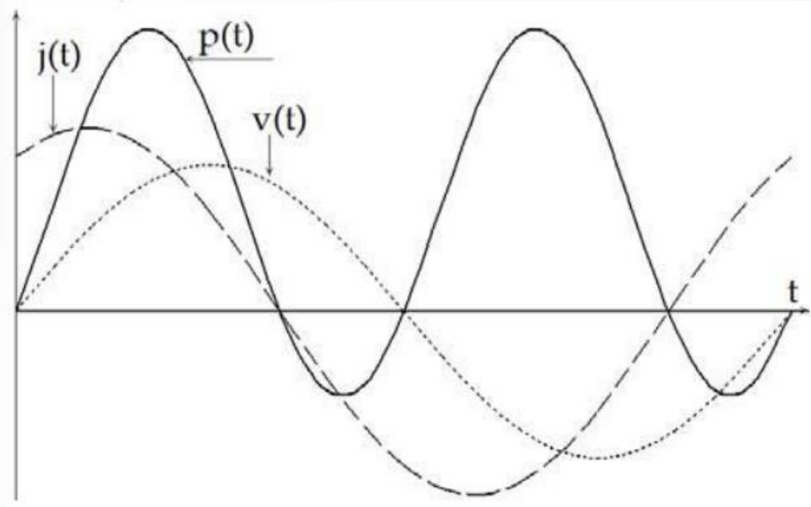
- **$P(t) = v(t) \times i(t) = 2IV \sin(\omega t + \varphi_I) \sin(\omega t + \varphi_V)$**
- Using the trigonometric identity: **$\sin(a) \sin(b) = 1/2[\cos(a-b) - \cos(a+b)]$**
- We obtain:
  - **$P(t) = IV \cos(\varphi_V - \varphi_I) - IV \cos(2\omega t + \varphi_V + \varphi_I)$**
- Or equivalently:
  - **$P(t) = IV \cos(\varphi) - IV \cos(2\omega t + \varphi_V + \varphi_I)$**

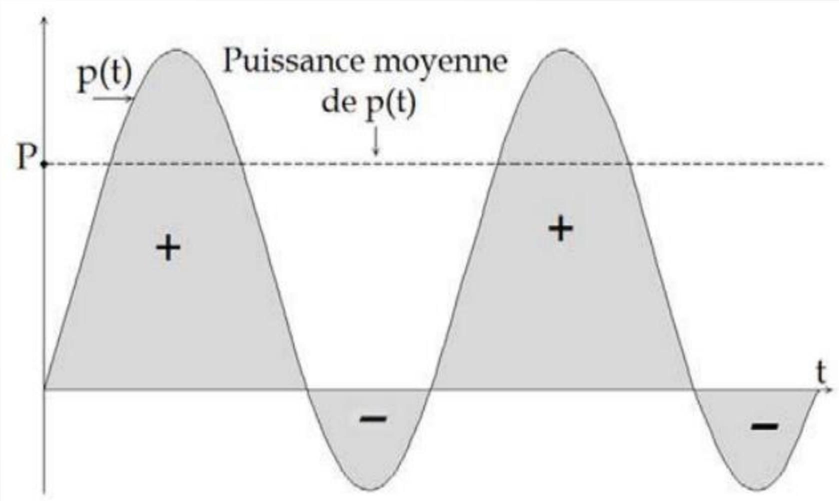
    Where $\varphi = \varphi_V - \varphi_I$ is the phase shift between voltage and current.



Instantaneous Power and active Power

- **$P(t) = IV \cos(\varphi) - IV \cos(2\omega t + \varphi_V + \varphi_I)$**
- **Composition of P(t):** P(t) is composed of two terms:
  - A constant component (DC component):
    - **$P_{avg} = IV \cos(\varphi)$**
    - **P_{avg}** is independent of time, always positive if $-90° < \varphi < 90°$
- A fluctuating component (AC component):
  - (AC component): $IV \cos(2\omega t + \varphi_V + \varphi_I)$
  - (AC component): (oscillates at 2ω pulsation, average value = 0)





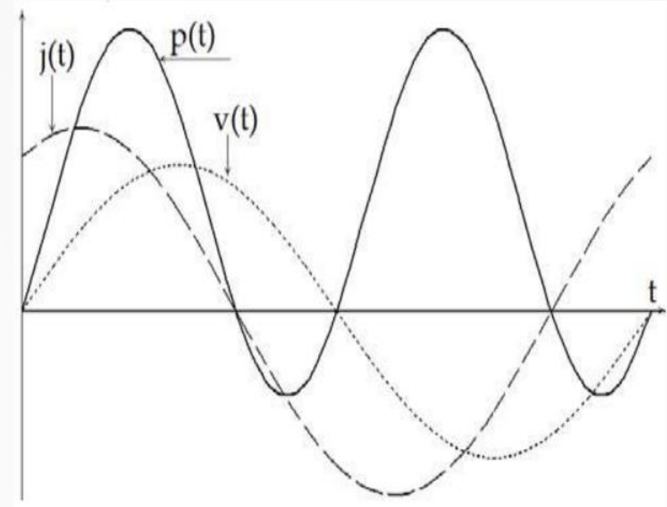
Active power

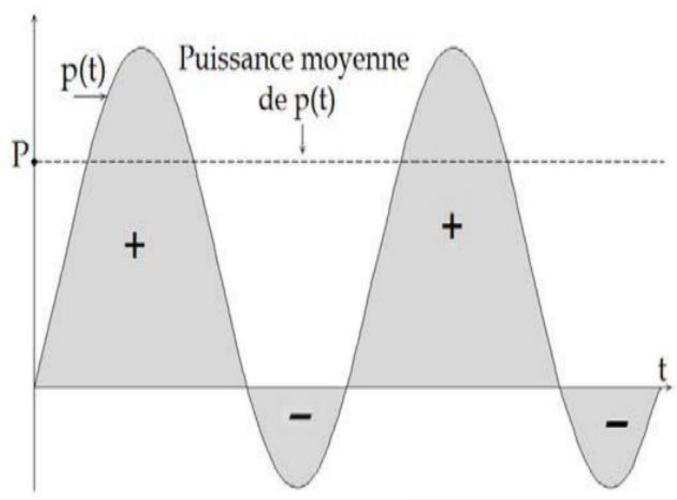
Active power is the average power over one period. It represents the power actually converted into useful work or heat.

$$P = (1/T) \int_0^T P(t)\,dt = VI\cos(\varphi_{I/V})$$

- $\varphi_{I/V} = \varphi_I - \varphi_V$ = phase shift between voltage and current

- **Physical interpretation:**
- Active power represents the power absorbed by resistive elements of the circuit. It is the only component that performs actual work or produces heat.
- Unit: Watt [W]
- **Properties:**
- $P \geq 0$ when φ is between -90° and +90° (power absorbed)
- $P = VI$ when $\varphi = 0°$ (purely resistive circuit, maximum power transfer)
- $P = 0$ when $\varphi = \pm 90°$ (purely reactive circuit, no energy dissipation)





Example: Motor Power Calculation

Given:

- A motor is powered by a sinusoidal voltage of 400V / 50 Hz with the following characteristics:
  - Useful mechanical power of the motor: $P_{mec} = 7.2$ kW
  - Motor efficiency: $\eta = 0.9$
  - Power factor: $\cos(\varphi) = 0.8$
- Question: What is the current absorbed by the motor?

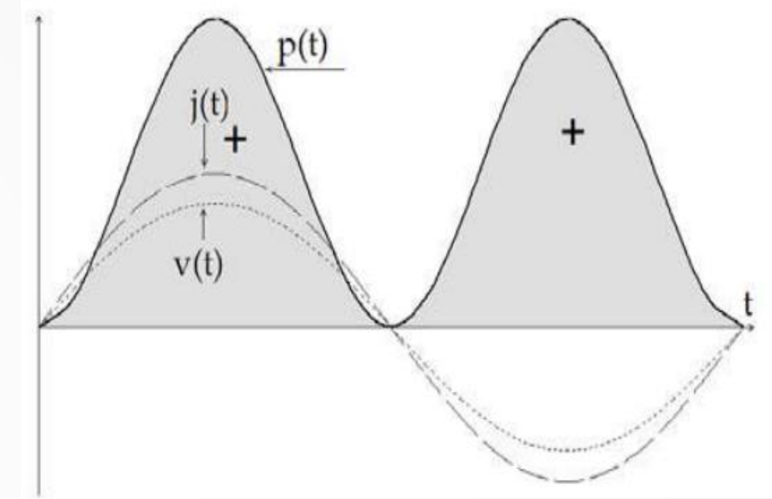
Example: Motor Power Calculation

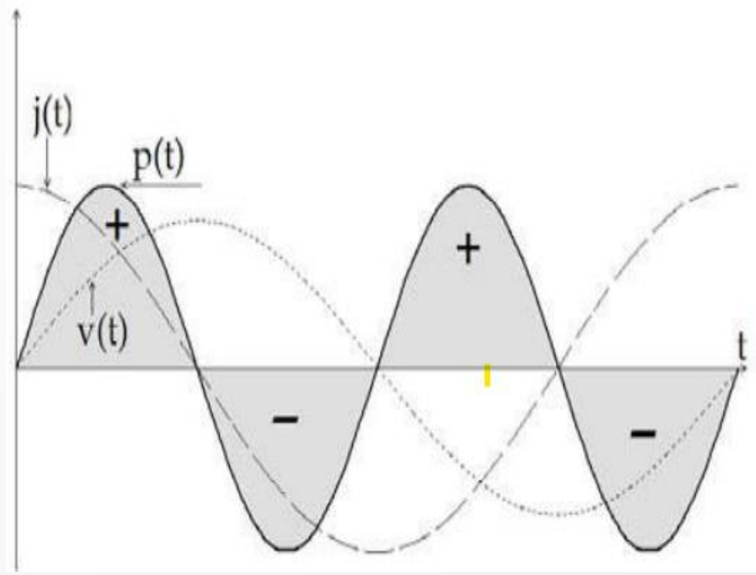
Solution:

- Step 1: Calculate the electrical power absorbed The electrical power absorbed by the motor is:
  - $P_{elec} = P_{mec} / \eta$
  - $P_{elec} = 7200 / 0.9 = 8000$ Watts
- Step 2: Calculate the current Using the active power formula: $P = VI \cos(\varphi)$ Therefore:
  - $I = P_{elec} / (V \cos(\varphi_{I/V}))$
  - $I = 8000 / (400 \times 0.8) = 8000 / 320 = 25$ A

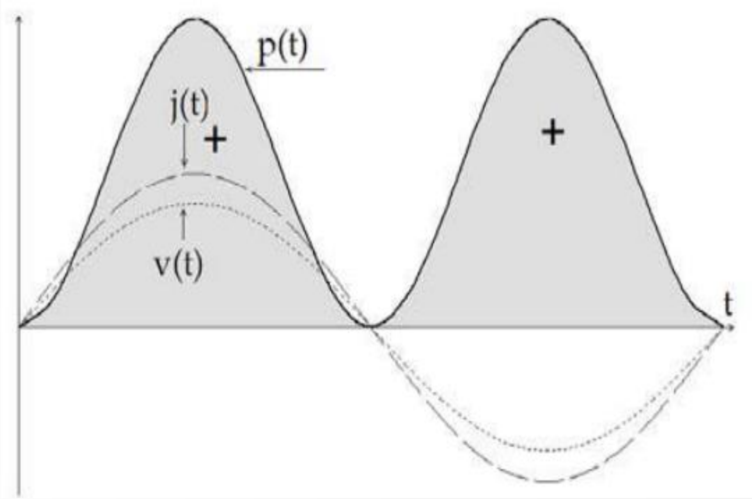
Apparent Power and Reactive Power - Introduction

- Let's discuss the cases of a resistor and a capacitor:
- In the first case (resistor):
  - The power is always positive
  - Power flows continuously from the source to the load
  - Energy is dissipated as heat
- In the second case (capacitor or inductor):
  - The power is sometimes positive and sometimes negative
  - Power fluctuates, oscillating between the source and the load
  - On average, the capacitor (same for the inductance) does not absorb energy





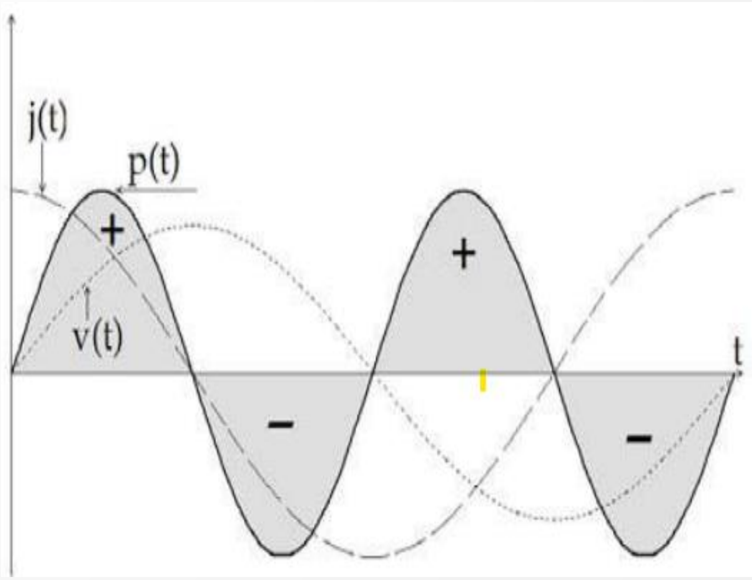
Apparent Power and Reactive Power - Introduction



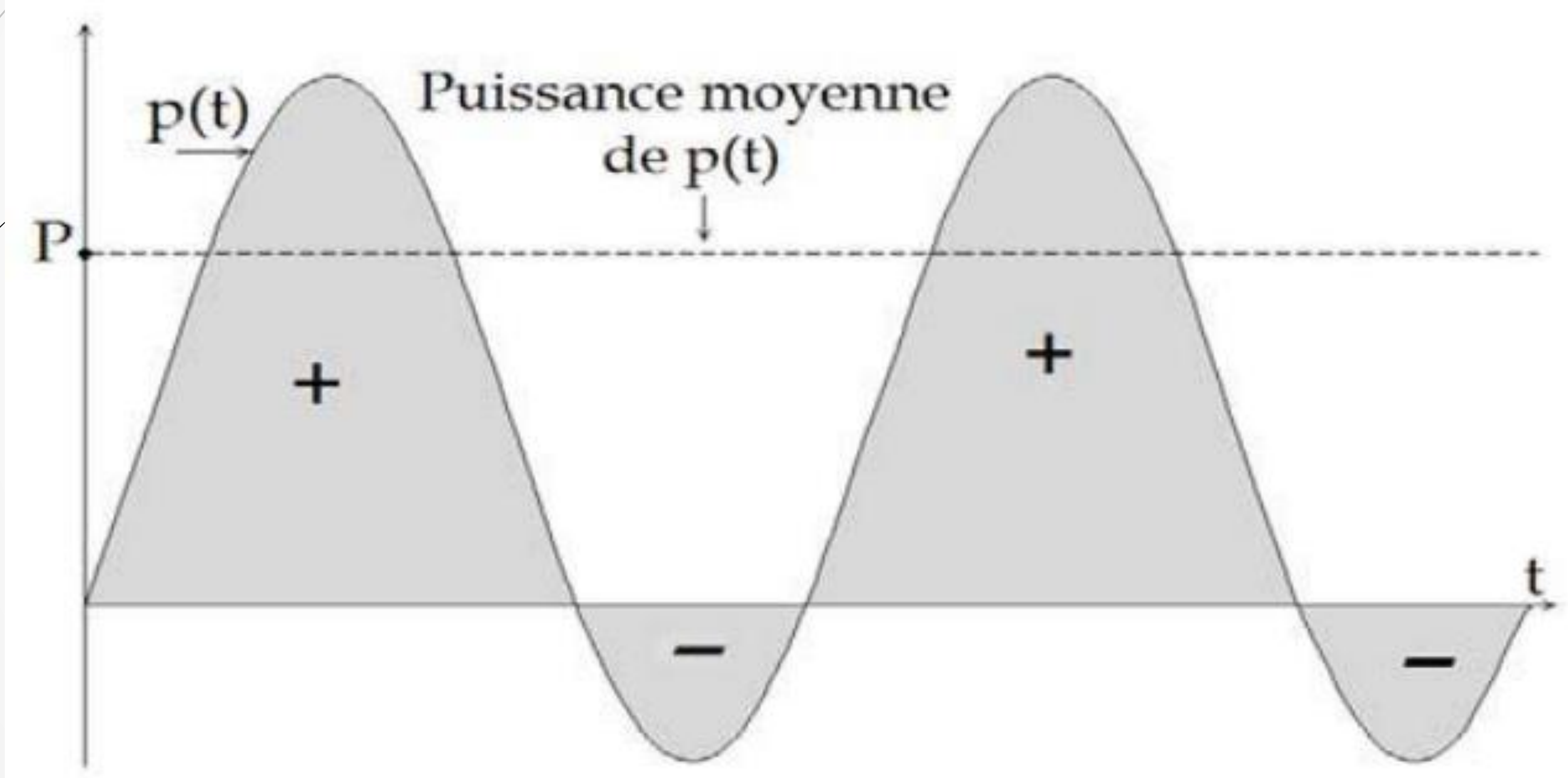
Why be interested in this fluctuating power?

- There really is a flow of power coming and going
- It is necessary to provide an additional current in the calculation of electrical equipment
- This additional current causes ohmic losses in the conductors
- These losses affect the electricity supplier

To quantify this fluctuating power, we use the notion of **reactive power.**



Apparent Power, Active power and Reactive Power



Apparent Power

- **Definition:** Apparent power **S** represents the total power that must be supplied by the source to deliver both active and reactive power to the load.
- **The complex representation:**
  - $\underline{S} = \underline{V}\,\underline{I} \Longrightarrow \underline{S} = Ve^{j(\omega t)}Ie^{j(\omega t+\varphi)}$
  - $\Longrightarrow \underline{S} = VIe^{j(2\omega t+\varphi)}$
  - $\Longrightarrow \underline{S} = e^{j(2\omega t)}\left[VIe^{j(\varphi)}\right]$
  - $\Longrightarrow \underline{S} = e^{j(2\omega t)}[VI\cos(\varphi) + jVI\sin(\varphi)]$
- We define the **Apparent power S** is the product of RMS voltage and RMS current.
  - $S = \left|\underline{S}\right| = \left|VIe^{j(2\omega t+\varphi)}\right| = VI$
- **The Fresnel representation of Apparent power $\vec{S}$**:
  - $\vec{S} = |S| \angle\varphi = S\angle\varphi$
- **Unit:** Volt. Ampere [VA]

Active Power

- **Definition:** Active power is the average power consumed by a circuit over one complete period. It represents the real power that is converted into useful work or heat.
- **Mathematical expressions:**
  - $P = (1/T) \int_0^T p(t)\, dt = VI \cos(\varphi)$
  - $P = Re(\underline{S})$
  - $P = S \cos(\varphi)$
  - $P = Re(\underline{Z}) I^2$
  - $P = \frac{V^2}{Re(\underline{Z})}$
- **Unit:** Watt [W]
- **Power Facture: $\cos(\varphi) = \frac{P}{S}$**

Reactive Power

- Definition: Reactive power **Q** represents the amplitude of the power that oscillates between the source and reactive elements (inductors and capacitors). It does not perform any useful work but is necessary for the operation of reactive components.
- Mathematical expression:
  - $\boldsymbol{Q = Im(\underline{S})}$
  - $\boldsymbol{Q = VI\sin(\varphi)}$
  - $\boldsymbol{|Q| = \left|Im(\underline{Z})I^2\right|}$
  - $\boldsymbol{|Q| = \left|\frac{V^2}{Im(\underline{Z})}\right|}$
- **Unit:** Volt-Ampere Reactive [VAR]
- **Reactive factor**: $\boldsymbol{\sin(\varphi) = \frac{Q}{S}}$

Apparent Power, Active power and Reactive Power

$$S^2 = P^2 + Q^2$$

Case 1 - Current as Phase Reference

- When taking current as phase reference:
  - $\varphi = \varphi_V$
- Sign convention for φ:
  - φ > 0: Voltage leads current → Inductive circuit (L dominant)
  - φ < 0: Voltage lags current → Capacitive circuit (C dominant)
  - φ = 0: Voltage and current in phase → Purely resistive circuit
- Power expressions:
  - Active power: $P = VI\cos(\varphi)$
  - Reactive power: $Q = VI\sin(\varphi)$
    - $Q > 0$ when $\varphi > 0$ (inductive)
    - $Q < 0$ when $\varphi < 0$ (capacitive)
  - Apparent power: $S = VI$

Case 1 - Current as Phase Reference

- Advantages of this reference:
  - Natural choice for series circuits (same current everywhere)
  - Direct visualization of impedance angle
  - φ directly represents the impedance phase angle

Case 2 - Voltage as Phase Reference

- When taking voltage as phase reference:
  - $\varphi = \varphi_I$
- Sign convention for φ:
  - $\varphi < 0$: Voltage leads current → Inductive circuit (L dominant)
  - $\varphi > 0$: Voltage lags current → Capacitive circuit (C dominant)
  - $\varphi = 0$: Voltage and current in phase → Purely resistive circuit
- Power expressions:
  - Active power: $P = VI \cos(\varphi)$
  - Reactive power: $Q = VI \sin(\varphi)$
    - $Q < 0$ when $\varphi > 0$ (inductive)
    - $Q > 0$ when $\varphi < 0$ (capacitive)
  - Apparent power: $S = VI$