Chapter 01: Functions of Several Variables

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Functions of Several Variables

Let E, F be two sets. The product

$$E \times F = \{(x, y) \mid x \in E \text{ and } y \in F\}.$$

Definition. *A relation R from E to F is a given subset G (or G_R) of $E \times F$.*

We say that y is the ***image*** *of x (x is the* ***pre-image*** *of y) if $(x, y) \in G$.*

Inverse Relation

The inverse relation R^{-1} of R is from F to E defined by

$$yR^{-1}x \Leftrightarrow xRy$$

$$G_{R^{-1}} = s(G_R), \quad \text{where } s = \begin{cases} E \times F \to F \times E, \\ (x, y) \mapsto (y, x) \end{cases}$$

Definition. *A function $f : E \to F$ is a relation from E to F such that each $x \in E$ has at most one image $y \in F$.*

The ***domain*** *of f is*

$$Dom(f) = \{x \in E \mid \textit{the image of } x \textit{ exists}\}.$$

If $x \in Dom(f)$, we denote $f(x)$ its image. When $Dom(f) = E$, f is called a ***map*** *from E to F.*

We define

$$G_F = \{(x, y) \in E \times F \mid x \in Dom(f),\ y = f(x)\}$$

$$\textit{If } F = \mathbb{R}, \quad f : E \to \mathbb{R} \textit{ is a real function}$$

$$\textit{If } F = \mathbb{C}, \quad f : E \to \mathbb{C} \textit{ is a complex function}$$