N.H.S.A.S.T **2025/2026**
Second year P.C - Analysis 3

SERIE N° 0 : FUNCTIONS OF SEVERAL VARIABLES

Exercice 1 :

1/ In the following relations, determine whether z is a function of x and y :

$$zx^2 + 3y^2 - xy = 0, \qquad xz^2 + 2xy - y^2 = 0.$$

2/ In the following, determine the values of the function f at given points :

i) $f : (x,y) \longmapsto \ln|x+y|,\qquad (0,-1),(e,3e),$

ii) $f : (x,y) \longmapsto \displaystyle\int_x^y (2t-3)\,dt,\qquad (4,-1),(2,1),$

iii) $f : (x,y,z) \longmapsto \dfrac{xy}{z},\qquad (2,3,9),(1,0,1),(-2,3,4).$

Exercice 2 :

1/ Describe the domain and range of the following functions :

$$f_1 : (x,y) \longmapsto x\sqrt{y}, \qquad f_2 : (x,y) \longmapsto e^{xy}, \qquad f_3 : (x,y) \longmapsto \arcsin\left(\frac{y}{x}\right),$$

$$f_4 : (x,y) \longmapsto \frac{y}{\sqrt{x}}, \qquad f_5 : (x,y) \longmapsto \frac{xy}{x-y}, \qquad f_6 : (x,y) \longmapsto \sqrt{4-x^2-4y^2}.$$

2/ Describe the level curves of the following functions, then sketch the level curves for the given c-values :

i) $z = x+y,\qquad c=-1,0,2,$ *ii)* $z = x^2+4y^2,\qquad c=-2,0,4,$

iii) $z = e^{2xy},\qquad c=1,2,4,$ *iv)* $z = \dfrac{x}{x^2+y^2},\qquad c=-2,2,\dfrac{1}{2},1.$

Exercice 3 :

1/ Find the next limits (if it exists). If the limit does not exist, explain why.

$$\lim_{(x,y)\to(0,0)} \frac{x+y}{y+x^2}, \qquad \lim_{(x,y)\to(1,1)} \frac{2x-y-1}{\sqrt{2x-y}-1}, \qquad \lim_{(x,y,z)\to(0,0,0)} \frac{xy+yz^2+xz^2}{x^2+y^2+z^2}.$$

2/ Discuss the continuity of the functions :

$$f_1(x,y) = \begin{cases} \dfrac{x^2+2xy^2+y^2}{x^2+y^2} & \text{if} \quad (x,y)\neq(0,0) \\ 0 & \text{if} \quad (x,y)=(0,0) \end{cases} \qquad f_2(x,y) = \begin{cases} \dfrac{1-\cos(xy)}{x^2+y^2} & \text{if} \quad (x,y)\neq(0,0) \\ 0 & \text{if} \quad (x,y)=(0,0) \end{cases}$$

Exercice 4 :

1/ Use the limit definition of partial derivatives to find $f_x = \frac{\partial f}{\partial x}$ and $f_y = \frac{\partial f}{\partial y}$ of the next functions :

$$f : (x,y) \longmapsto x^2 e^{2y}, \qquad f : (x,y) \longmapsto 2y^2\sqrt{x}, \qquad f : (x,y) \longmapsto \ln\left(x^2+y^2\right).$$

2/ In the following, evaluate the first partial derivatives at the given point.

$$f \ : \ (x,y) \longmapsto e^y \cos x, \qquad (x_0,y_0) = (0,0)\,,$$
$$f \ : \ (x,y,z) \longmapsto \sqrt{3x^2+y^2-2z^2}, \qquad (x_0,y_0,z_0) = (1,-2,1)\,.$$

3/ Find the four second partial derivatives of the next functions:

$$f:(x,y) \longmapsto \ln(x-y)\,, \qquad f:(x,y) \longmapsto \sqrt{2x^2+y^2}, \qquad f:(x,y,z) \longmapsto 2xe^y - 3ye^{-x}.$$

Exercice 5 :

In the following, find the function f and use the total differential of f to approximate the quantity.

$$i) \quad (2,01)^2\,(9.02) - 2^2.9, \qquad ii) \quad \frac{1-(3,05)^2}{(5,95)^2} + \frac{4}{3}, \qquad iii) \quad \sqrt{(5,05)^2 + (3,1)^2} - \sqrt{34}.$$

Exercice 6 :

1/ In the next, find $\frac{dw}{dt}$ using : **a)** by using the appropriate Chain Rule and **b)** by converting w to a function of t before differentiating.

$$i)\ w = xy, \qquad x = e^t, \qquad y = e^{-2t}$$
$$ii)\ w = xy\cos z, \qquad x = t, \qquad y = t^2, \qquad z = \arccos t$$

2/ Using the appropriate chain rule, find $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial s}$ in each case.

$$i)\ w = x^2+y^2, \qquad x = s+t, \qquad y = s-t$$
$$ii)\ w = x^2-y^2, \qquad x = s\cos t, \qquad y = s\sin t.$$

Exercice 7 :

Is the function f of classe $\mathcal{C}^2$ at $(0,0)$, where,

$$f(x,y) = \begin{cases} yx^2 \sin \frac{y}{x} & \text{if} \quad x \neq 0 \\ 0 & \text{if} \quad x = 0 \end{cases}$$

Exercice 8 :

Compute the Taylor formula with order 4 of the function $f\,:\,(x,y) \longmapsto \cos(x^2+y^2)$ at the point $(0,0)$.

Exercice 9 :

For all functions bellow, identify any extrema of the function by recognizing its given form or its form after completing the square. Verify your results by using the partial derivatives to locate any critical points and test for relative extrema.

$$f:(x,y) \longmapsto 8-(x-1)^2-(y+1)^2\,, \quad f:(x,y) \longmapsto \sqrt{4x^2+y^2+4}, \quad f:(x,y) \longmapsto 10x+12y-x^2-y^2-64.$$