**National High School of Autonomous Systems Technologies**

**Second Year Engineering** | **Subject:Introduction to Autonomous Systems 1** | **Academic year: 2025-2026**

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## Td 2: Boolean Algebra and Simplification of Logic Functions

### Exercise 1

Establish the truth tables for the following functions, then write them in both canonical forms:

1. $F_1 = XY + YZ + XZ$
2. $F_2 = X + YZ + Y\overline{\overline{Z}}\,T$
3. $F_3 = (X + Y) + (\overline{X} + Y + Z)$
4. $F_4 = (\overline{X} + \overline{Z})(X + \overline{T} + Z)Y\overline{Z}$
5. $F_5 = (\overline{X}Y + X\overline{Y})\overline{Z} + (\overline{X}\,\overline{Y} + XY)Z$
6. $F_6 = \overline{X} + YZ$

### Exercise 2

Simplify the following Boolean functions algebraically with a minimal number of operators:

1. $F(A,B,C,D) = A\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,B\,\overline{C}\,D + A\,\overline{B}\,C\,D + A\,B\,\overline{C}\,D$
2. $G(A,B,C,D) = A\,\overline{B}\,\overline{C}\,\overline{D} + A\,B\,\overline{C}\,D + \overline{A}\,B\,C\,D + \overline{A}\,B\,\overline{C}\,\overline{D}$
3. $H(A,B,C,D) = F(A,B,C,D) + G(A,B,C,D)$
4. $K(A,B,C,D) = F(A,B,C,D) \cdot G(A,B,C,D)$

### Exercise 3

Simplify the following functions algebraically.

1. $F_1 = (X + Y)(\overline{X} + Y)$
2. $F_2 = \overline{XY} + XY + \overline{X}Y$
3. $F_3 = XY + \overline{Z} + Z(\overline{X} + \overline{Y})$
4. $F_4 = X(\overline{Y}\,\overline{Z} + YZ) + \overline{X}\,Y\overline{Z} + \overline{X}\,\overline{Y\,Z}$
5. $F_5 = (X + \overline{Y}) + (X\overline{Y} + Z)Z$
6. $F_6 = X\overline{Y} + Z\overline{T} + \overline{X}\,\overline{Y} + \overline{Z}\,\overline{T}$

### Exercise 4

Simplify the following functions algebraically.

1. $F_1 = (X + Y)(\overline{X} + Y)$
2. $F_2 = \overline{XY} + XY + \overline{X}Y$
3. $F_3 = XY + \overline{Z} + Z(\overline{X} + \overline{Y})$
4. $F_4 = X(\overline{Y}\,\overline{Z} + YZ) + \overline{X}\,Y\overline{Z} + \overline{X}\,\overline{Y\,Z}$
5. $F_5 = (X + \overline{Y}) + (X\overline{Y} + Z)Z$
6. $F_6 = X\overline{Y} + Z\overline{T} + \overline{X}\,\overline{Y} + \overline{Z}\,\overline{T}$

**Exercise 5**

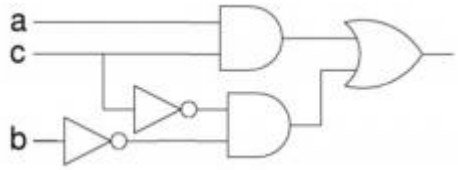
Consider the function F defined by the truth table below:

| A | B | C | F |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

1. Provide the first canonical form of F, then draw the corresponding logic diagram.
2. Simplify the first canonical form of F using the algebraic method. Provide the corresponding logic diagram.
3. Provide the second canonical form of F.

## Exercise 6

Given the following logic circuit:




1. Provide the output logic function of this circuit.
2. Simplify this function using Boolean algebra theorems.
3. Provide the logic circuit of the simplified function.

**Exercise 7**

Simplify the following Boolean functions using the Karnaugh map method.

1. $F(A,B,C) = \overline{A}\,\overline{B}\,C + \overline{A}B\overline{C} + AB\overline{C}$
2. $F(A,B,C) = \overline{A}B\overline{C} + \overline{A}\,BC + AB\overline{C}$
3. $F(A,B,C) = \overline{A}B\overline{C} + \overline{A}BC + \overline{A}BC + A\overline{B}\,\overline{C} + A\overline{B}C$
4. $F(A,B,C) = \overline{ABC} + \overline{A}\,\overline{B}\,C + \overline{A}B\overline{C} + AB\overline{C} + A\overline{B}\,\overline{C} + A\overline{B}C$

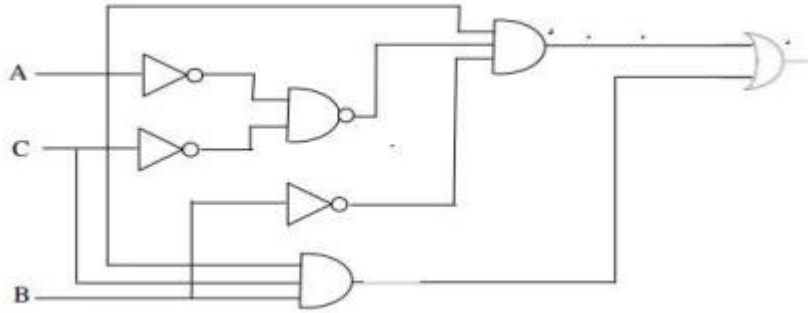
## Exercise 8

Provide the logic diagrams of the following functions, using:

1. $F_1 = (A + \overline{B})CD$
2. $F_2 = A(B + \overline{C}) + \overline{B}C$
3. $F_3 = A\overline{D} + BC$
4. $F_4 = (B + \overline{C})(A + BD)$

## Exercise 9

Simplify the following logic circuit using the algebraic method.



**Exercise 10**

A hot drinks dispenser allows the distribution of coffee or tea, with or without milk, or milk alone. Three buttons control the dispenser: "coffee," "tea," and "milk." To get one of these drinks alone, press the corresponding button. To get a drink with milk, press both the button for the chosen drink and the "milk" button simultaneously. Additionally, the dispenser operates only if a token has been inserted into the device slot. A mistaken operation after inserting the token (e.g., pressing "coffee" and "tea" simultaneously) triggers the token's return. Since milk is free, the token is also returned if only milk is chosen. Calculate and simplify the functions for token return J, coffee distribution C, tea T, and milk L. Note that the token return function may either be active or inactive when no token is inserted into the device.

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