

7.1D N.22

Ex 1:

$$V_A = 5 \left( 1 - \frac{r^2}{R^2} \right) \text{ m/s}$$

$$V_{\text{max}} \Rightarrow r=0$$

$$V_{\text{max}} = 5 \left( 1 - \frac{0}{R^2} \right) = 5 \text{ m/s}$$

$$V_{\text{avg}} = \frac{V_{\text{max}}}{2} = 2,5 \text{ m/s}$$

$$V_{\text{avg}} = \frac{1}{\text{Surface}} \int_0^R V_A \cdot dS = \frac{1}{2\pi r dr} \int_0^R V_A \cdot dS$$

Ex 2:  $\mu = 8,2 \times 10^{-2} \text{ kg} \cdot \text{s/m}^2$

$$S = Ay^2 + By + C$$

$$V = Ay^2 + By$$

$$V_{\text{max}} = 100 \text{ cm/s}$$

$$100 = A(15)^2 + B(15)$$

$$y = 15 \text{ cm}$$

$$100 = 225A + 15B$$

$$\frac{dV}{dy} = 2Ay + B = 30A + B = 0$$

$$225A + 15B = 100$$

$$30A + B = 0 \Rightarrow B = -30A$$

$$225A + 15(-30A) = 100$$

$$A = -0,44$$

$$30A + B = 0 \Rightarrow B = -30A = -30(-0,44) = 13,2$$

$$B = 13,2$$

$$V = -0,44y^2 + 13,2y$$

$$\frac{dV}{dy} = -0,88y + B$$

$$\bar{E} = \mu \frac{dV}{dy}$$

y (cm)	V (cm/s)	$\frac{1}{dy}$	$\bar{E}$ (Pa)
0	0	13,2	$= 13,2 \mu$ $= 1,09$
5	95,65	8,93	0,75
10	89,3	4,53	0,37
15	100	0	0

Velocity, Viscosity and gradient

Ex 3:

$$H = \frac{4S \cos \theta}{8 \cdot g \cdot d}$$



N.A:

$$H = \frac{4 \cdot 0,515 \cos 130}{13570 \times 9,8 \times 1,5 \times 10^{-3}}$$

$$H = 3,68 \times 10^{-3} \text{ m} = 3,68 \text{ mm}$$

Ex 4:



$$H = \frac{4S \cos \theta}{8 \cdot g \cdot d}$$

$$H = \frac{4 \times 0,074 \cos 0}{1000 \times 9,8 \times 3 \times 10^{-3}} = 0,01 \text{ m} = 10 \text{ mm}$$

Ex 5:

$$1) \frac{dV}{V} = \frac{dP}{\bar{E}}$$

$$dV = \frac{dP}{\bar{E}} \cdot V$$

N.A:  $dV = \frac{20 \times 10^5}{2,25 \times 10^3} \cdot 30 \times 10^{-3}$   
 $dV = 2,6 \times 10^{-5} \text{ m}^3$

2)  $\bar{E} = ?$   $\begin{cases} V=30 \text{ l} \rightarrow 40 \text{ bar} \\ 29,73 \text{ l} \rightarrow 246,8 \text{ bar} \end{cases}$

N.A:  $\frac{dV}{V} = \frac{dP}{\bar{E}} \Rightarrow \bar{E} = \frac{dP}{\frac{dV}{V}}$

N.A:  $\bar{E} = \frac{P_2 - P_1}{\frac{V_2 - V_1}{V_1}} \cdot V_1 = \frac{(246,8 - 40) \times 10^5}{(29,73 - 30) \times 10^{-3}} \cdot 30 \times 10^{-3}$

$$\bar{E} = 2,298 \cdot 10^9 \text{ Pa} = 2,3 \text{ GPa}$$



Ex 10:

$$\beta_c = \frac{1}{V_c} \frac{dV}{dT}$$

$$\beta_c = \frac{1}{9 \times 10^{-3}} \cdot \frac{0,1 \times 10^{-3}}{20 + 273}$$

$$\beta_c = 1,7 \times 10^{-5} \text{ K}^{-1}$$

Ex 7:

$$PV = nRT \Rightarrow PV = \frac{m}{M} RT$$

$$\Rightarrow PV = mRT$$

$$\frac{m}{V} = \rho = \frac{P}{RT} = \frac{10 \times 10^3}{286,7 \times (27 + 273)}$$

$$\rho = 11,68 \text{ kg/m}^3$$

$$\bar{w} = \rho \cdot g = 11,68 \times 9,81 = 114 \text{ N/m}^3$$

$$\bar{v}_s = \frac{1}{\rho} = \frac{1}{11,68} = 0,086 \text{ m}^3/\text{kg}$$

Ex 8:

$$PV = nRT \Rightarrow T = \frac{PV}{nR}$$

$$T = \frac{10^5 \times 40 \times 10^{-3}}{48 \times 10^{-3} \times 286,7} = 280 \text{ K}$$

$$= 17^\circ \text{C}$$

Ex 9:

$$PV = nRT \Rightarrow PV = mRT$$

$$m = \frac{PV}{RT} = \frac{10^5 \times 250}{286,7 \times (25 + 273)}$$

$$m = 299,61 \text{ kg}$$

$$n = \frac{m}{M} = \frac{299,61 \times 10^3}{32}$$

$$n = 9,14 \times 10^3 \text{ mol}$$

Ex 11:

$$\sum \vec{F}_{ext} = \vec{0}$$

$$\vec{F} + \vec{P} = \vec{0}$$

Projection on

$$P \sin \theta - F = 0$$

$$P \sin \theta = \mu A \frac{dV}{dr} = 0$$

$$mg \sin \theta = \mu A \frac{dV}{dr} = 0$$

$$\sin \theta = \frac{\mu A \frac{dV}{dr}}{mg} = 0$$

$$n = 1: \sin \theta = 9,57 \times \frac{\pi (0,243)^2}{4} \cdot \frac{0,3}{0,6 \times 10^{-3}} \times \frac{1}{18 \times 10^3}$$

$$\sin \theta = 0,128 \Rightarrow \theta = 7^\circ$$

Ex 12:

$$F_1 = F_2$$

$$\frac{M_1 A \frac{dV}{d(R-y)}}{d(R-y)} = \frac{M_2 A \frac{dV}{dy}}{dy}$$

$$\frac{M_1}{d(R-y)} = \frac{k M_2}{dy}$$

$$M_1 A \frac{dV}{d(R-y)} = M_2 A \frac{dV}{dy}$$

$$M_2 = k M_1$$

$$\frac{M_1}{d(R-y)} = \frac{k M_1}{dy} \Rightarrow \frac{1}{R-y} = \frac{k}{y}$$

$$\Rightarrow y = kR - ky \Rightarrow y + ky = kR$$

$$y(1+k) = kR \Rightarrow y = \frac{kR}{1+k}$$

$$y = \frac{2}{3} = 0,66 \text{ m}$$