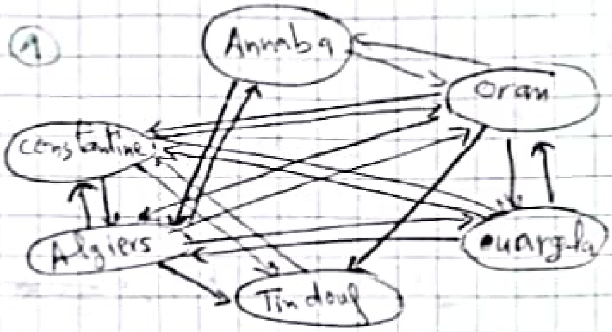
Tutorial Sheet N°1

Ex 1:

(1)



(2) constantine - Annaba - Algiers - Tindouf - Ouargla

(3)

	Prede	succe	neighbors	degree
constantine	oran - Tindouf Algiers - ouargla	oran, Algiers, Tindouf, ouargla	oran, Alg Tindouf ouargla	8
Annaba	oram Algiers	oran, Algiers	oran Alg	4
oran	const, Annaba Alg. ouargla	const, Annaba Alg, ouargla Tindouf	const, Annaba Alg, ouargla Tindouf	9
Algiers	const, Annaba oran, ouargla	const, Annaba oran, Tindouf ouargla	const, Annaba oran, ouargla Tindouf	9
Tindouf	Annaba, Alg oran	const	Annaba Alg, oran const	4
ouargla	const, oran algiers	const. oran, Algiers	const. oran, Algiers	6

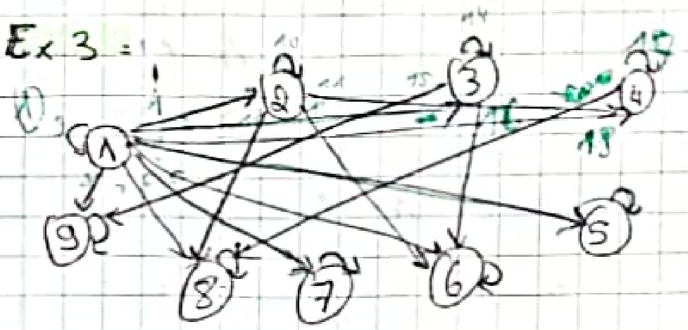
(4) this graph is antisymetric because there is an edge from Algiers to tindouf and there's no edge from tindouf to Algiers.

(5) to make a graph antisymetric we need to delete one edge

Ex 2:

Ex 3:

(1)



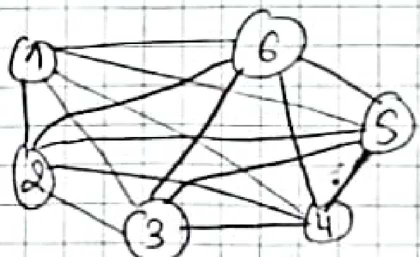
(2) set of prime numbers is the set of vertices with only 2 predecessors (in degree = 2)

$\{2, 3, 5, 7\}$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	-1	-1	-1	-1	-1	-1	-1	-1	-1 1	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	-1 1	-1	-1	-1	0	0
3	0	1	0	0	0	0	0	0	0	0	0	0	0	-1 1	-1
4	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0

Ex 4:

1)



$n(n-1)/2$

$6 \times 5/2 = 15$

15 edges, every day we have 3 sub-groups so 5 days

2) d1 (1, 2, 3), (4, 5, 6) one day

Ex5:

2) completed graph K6

Ex6:

1) Adjacency matrix:

$$A=\begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 3 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 4 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 5 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 6 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 7 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \end{array}$$

Incidence matrix at the edges:

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}
1	-1	-1	-1	0	0	0	0	0	0	0	0
2	1	0	0	1	0	0	0	1	0	1	0
3	0	0	0	-1	-1	0	1	0	1	0	0
4	0	0	0	0	1	-1	0	0	0	0	0
5	0	0	0	0	0	1	-1	-1	0	0	0
6	0	0	1	0	0	0	0	0	0	-1	1
7	0	1	0	0	0	0	0	0	-1	0	-1

2) Longest simple path:

$\underbrace{1-7}_{e_2}\underbrace{-3}_{e_9}\underbrace{-4}_{e_5}\underbrace{-5}_{e_6}\underbrace{-3}_{e_7}\underbrace{-2}_{e_4}$

4) one circuit: $3 \xrightarrow{e_5} 4 \xrightarrow{e_6} 5 \xrightarrow{e_7} 3$

* a lot of cycles

ex: 2 – 1 – 7 – 6 – 2

5) non elementary cycles:

1 – 7 – 3 – 4 – 5 – 3 – 2 – 1

Ex7:

$G_1 - G_2 - \ldots - G_{13} \to$ simple and connected

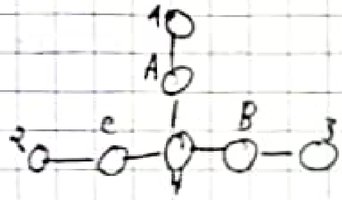
$G_1 - G_2 \to$ complete K4

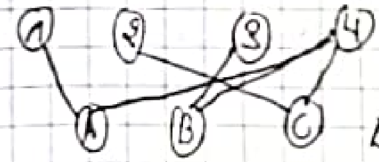
$G_1 \ldots G_{10} \to$ regular

$G_{11} - G_{12} - G_{13} \to$ trees

$G_6 \to$ Bi-partite

G_{11}:



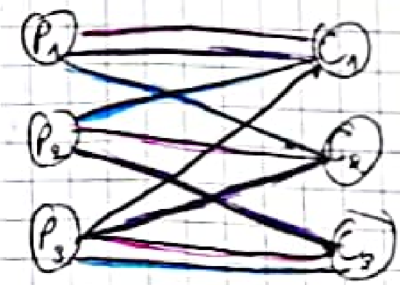


so it's Bipartite

$G_5, G_6, G_8, G_9, G_{10}$ non-linear

Ex8:

1- representing the graph



– each edge represents 1 hour

2- Bi-partite graph

Ex 9 :

to know if it's possible we need to check if these graph are Eulerian or not. if yes then it's possible if no then it's not.

G1 ⇒ no (all the vertices have odd degrees)

G2 ⇒ yes (we have just two vertices with odd degrees)

G3 ⇒ no (more than two vertices have odd degrees)

G4 ⇒ yes (just two vertices with odd degrees)

G5 ⇒ no (more than two vertices with odd degrees)

Ex10 :

G1 : Eulerian and Hamiltonian

Eulerian because all vertices have an even degree and the graph is connected

Hamiltonian because it passes once and only once through all the vertices

G2 : not Eulerian because more than one vertex ~~has~~ odd degree

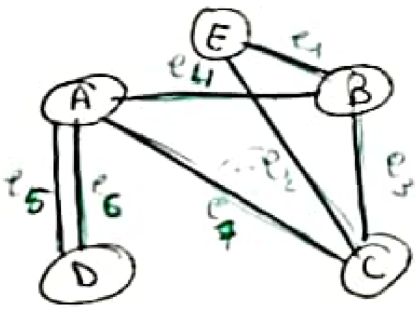
Hamiltonian because it admits a Hamiltonian cycle

G3 : not Eulerian because it's not connected

not Hamiltonian because we can't go through all the vertices

G4 : not Eulerian because it's not connected

not Hamiltonian because we can't go through all the vertices



since all the vertices have even degrees except one vertex and the graph is connected

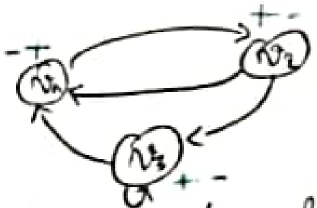
and it has no Eulerian cycles since not all vertices has even degrees

. an Example of an Eulerian path

$(E) \xrightarrow{e_1} (E) \xrightarrow{e_2} (C) \xrightarrow{e_3} (B) \xrightarrow{e_4} (A) \xrightarrow{e_5} (D) \xrightarrow{e_6} (A) \xrightarrow{e_7} (C)$

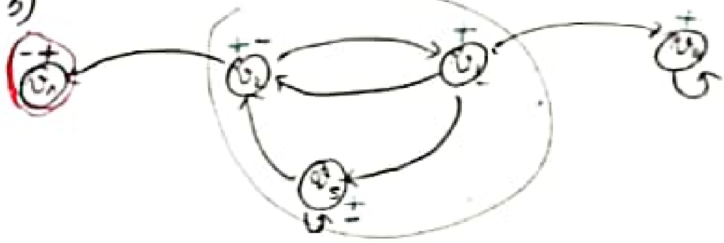
Ex 12 :

a)



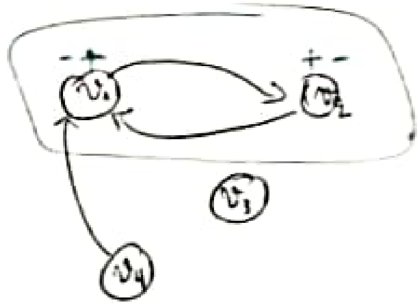
v_1, v_2, v_3 are strongly connected components

b)



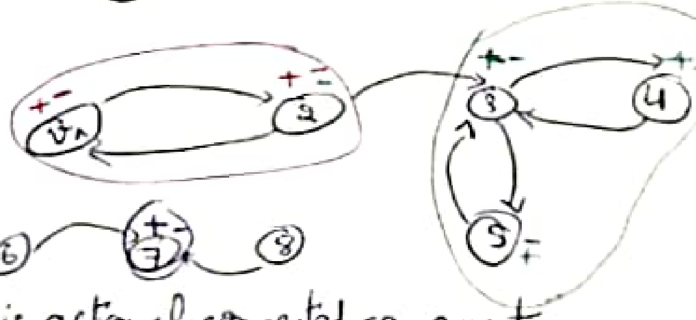
v_2, v_3, v_5 are strongly connected components

c)



v_1 and v_2 are strongly connected components

d)



v_7 is a strongly connected component

v_3, v_4 and v_5 are strongly connected components

v_1 and v_2 are strongly connetted components