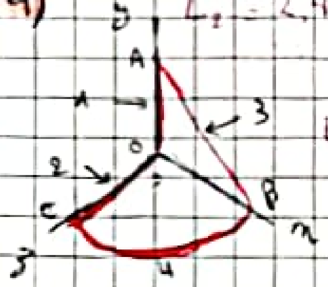
Série 4 : Center of Gravity

Ex 1: $L_1 = 1$ $\quad OG_1 = \begin{pmatrix}0\\1/2\end{pmatrix}$

a) $L_2 = 2{,}4$ $\quad OG_2 = \begin{pmatrix}0\\0\\1{,}2\end{pmatrix}$

$L_3 = 2{,}6$ $\quad OG_3 = \begin{pmatrix}1{,}2\\1{,}2\\0\end{pmatrix}$



$m = \lambda \partial l$ $\quad n = \sigma \partial s$ $\quad x = \rho\, dV$

$$x_G = \frac{\int x\, m}{\int dm} = \frac{\int x\, \lambda \partial l}{\int \lambda \partial l} = \frac{\int x\, \partial l}{\int \partial l}$$

$$= \frac{1}{L} \int x\, \partial l$$

bar OA :

$L = \int \partial l = \partial y \qquad 0 \leq y \leq 1\,m$

$$L_1 = \int_0^1 \partial y = [y]_0^1 = 1m$$

$x_{G_1} = z_{G_1} = 0$ Pas de matière

$$y_{G_1} = \frac{1}{L} \int y\, dl = \frac{1}{L_1} \int y\, dy = 1\left[\frac{y^2}{2}\right]_0^1$$

$$y_{G_1} = \frac{1}{2}$$

bar OC :

$dl = dz$

$$L_2 = \int dl = \int_0^{2,4} dz = [z]_0^{2,4} = 2{,}4$$

$x_{G_2} = y_{G_2} = 0$ Pas de matière

$$z_{G_2} = \frac{1}{L} \int z\, dl = \frac{1}{L_2} \int z\, dz = \frac{1}{2{,}4}\left[\frac{z^2}{2}\right]_0^{2,4}$$

$$= \frac{1}{2{,}4}\left(\frac{2{,}4^2}{2}\right) = \frac{2{,}4}{2} = 1{,}2m$$

Bar AB :

$$l_x = \int dl = \int dx = [x]_0^{2,4} = 2{,}4$$

$$l_y = \int dl = \int dy = [y]_0^1 = 1$$

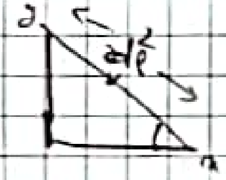
$$x_G = \frac{1}{l_x} \int x\, dl = \frac{1}{l_x} \int x\, dx$$

$$= \frac{1}{l_x}\left[\frac{x^2}{2}\right]_0^{2,4} = \frac{1}{2{,}4}\left(\frac{2{,}4^2}{2}\right) = 1{,}2$$

$$y_G = \frac{1}{l_y} \int y\, dy = \frac{1}{l_y} \int y\, dy = \frac{1}{2}$$

$$l_3 = \sqrt{l_x^2 + l_y^2} = 2{,}6$$

Cas particulier



$$L = \int_0^L dl = L$$

$$x_G = \frac{1}{L} \int x\, dl \qquad dx = dl \cos\theta$$

$$dy = dl \sin\theta$$

$$= \frac{\cos\theta}{L} \int l\, dl \qquad x = l\cos\theta$$

$$= \frac{\cos\theta}{L}\left[\frac{l^2}{2}\right]_0^L = \frac{L\cos\theta}{2} \qquad y = l\sin\theta$$

$$y_G = \frac{1}{L} \int_0^L y\, dl = \frac{1}{2} L \sin\theta$$

Bar CB :

$$dl = r\, d\theta \rightarrow L = \int dl = \int r\, d\theta = r\,[\theta]_0^{\pi/2}$$

$$L_4 = r\frac{\pi}{2} = 1{,}2\pi = 3{,}77$$

$$x_G = \frac{1}{L_4} \int x\, dl = \frac{1}{L_4} \int r^2 \sin\theta\, d\theta \qquad z = r\cos\theta$$

$$x = r\sin\theta$$

$$= \frac{r^2}{L_4} \int \sin\theta\, d\theta = -\frac{r^2}{L_4}[\cos\theta]_0^{\pi/2}$$

$$= \frac{r^2}{1{,}2\pi} = 1{,}52$$

$y_G = 0$ pas de matière

$$z_G = \frac{1}{L_4}\int z\, dl = \frac{1}{L_4}\int r^2 \cos\theta \, d\theta$$

$$= \frac{r^2}{L_4}\left[\sin\theta\right]_0^{\frac{\pi}{2}} = \frac{r^2}{1{,}2\pi} = 1{,}52$$

$$\boxed{X_{G_{sys}} = \frac{\sum X_{G_i} m_i}{\sum m_i}}$$

$$x_{G_{sys}} = \frac{\sum x_{G_i} l_i}{\sum l_i}$$

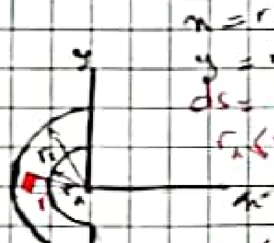
$$\sum l_i = 1 + 2.4 + 2.6 + 3{,}77 = 9{,}77$$

$$x_{G_{sys}} = \frac{0 + 0 + (1{,}2)(2{,}6) + (3{,}77)(1{,}52)}{9{,}77} = 0{,}9$$

$$y_{G_{sys}} = \frac{1(0{,}5) + 0 + (0{,}5)(2{,}6) + 0}{9{,}77} = 0{,}18$$

$$z_{G_{sys}} = \frac{0 + (2{,}4)(1{,}2) + 0 + (3{,}77)(1{,}52)}{9{,}77} = 0{,}88$$

b)



$x = r\cos\theta$
$y = r\sin\theta$
$ds = r\,dr\,d\theta$
$r_1 \leq r \leq r_2$
$\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$

$$S = \iint ds = \iint r\,dr\,d\theta$$

$$= \int r\,dr \int d\theta = \left[\frac{r^2}{2}\right]_{r_1}^{r_2} \left[\theta\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

$$= \frac{r_2^2 - r_1^2}{2}\left(\frac{3\pi}{2} - \frac{\pi}{2}\right) = 14476{,}4 \text{ m}^2$$

$$= \frac{1}{S}\int r^2 dr \int \cos\theta\, d\theta = \frac{1}{S}\left[\frac{r^3}{3}\right]\left[\sin\theta\right]$$

$$= \frac{1}{S}\left(\frac{r_2^3 - r_1^3}{3}\right)(-1-1) = \frac{-903168}{14476{,}4}$$

$$x_G = -62{,}38 \text{ mm}$$

$$y_G = \frac{1}{S}\int y\, ds = \frac{1}{S}\iint r^2 \sin\theta\, dr\, d\theta$$

$$= \frac{1}{S}\int r^2 dr \int \sin\theta\, d\theta = \frac{1}{S}\left[\frac{r^3}{3}\right]\left[-\cos\theta\right] = 0$$

c)

$$V = \iiint r\, d\theta\, dr\, dz$$

$0 \leq \theta \leq 2\pi$
$0 \leq r \leq r(z)$
$0 \leq z \leq h$

$$= \int_0^h \left[\frac{r^2}{2}\right]_0^{r(z)} dz \int d\theta$$

$$= 2\pi \int_0^h \frac{r(z)^2}{2}\, dz$$

$$= \pi \int_0^h \left(50^2 - 2(50)\left(\frac{1}{6}z\right) + \frac{1}{36}z^2\right)$$

$$= \pi\left[50^2 z\right]_0^h - \frac{50}{3}\left[\frac{z^2}{2}\right]_0^h + \frac{1}{36}\left[\frac{z^3}{3}\right]_0^h$$

$$\boxed{V = 383572 \text{ mm}^3}$$

$$\begin{cases} z = 0 \Rightarrow r = r_2 \\ z = h \Rightarrow r = r_1 \end{cases}$$

$$T_\alpha = \frac{r_2 - r_1}{h} = \frac{r(z) - r_1}{h - z}$$

$$r(z) = \frac{r_2 - r_1}{h}(h - z) + r_1$$

$$r(z) = r_2 - r_1 - \frac{(r_2 - r_1)}{h}z + r_1$$

$$r(z) = r_2 + \frac{r_1 - r_2}{h}z$$

$$\boxed{r(z) = 50 - \frac{1}{6}z}$$

$$x_G = \frac{1}{V}\int x\, dV = \frac{1}{V}\int r^2 \cos\theta\, dr\, d\theta\, dz$$

$$x_G = \frac{1}{S}\int x\, dx = \frac{1}{S}\iint r^2 \cos\theta\, dr\, d\theta = \frac{1}{V}\iint r^2\, dr\, dz \int_0^{2\pi} \cos\theta\, d\theta = 0$$

$y_G = \frac{1}{V}\int y\,dV = \frac{1}{V}\int r^2 \sin\theta\, dr\, d\theta\, dz$

$= \frac{1}{V}\iint r^2\, dr\, dz \int \sin\theta\, d\theta$

$= \frac{1}{V}\int_0^h \left[\frac{r^3}{3}\right]_0^{r(z)} \underbrace{\left[-\cos\theta\right]_0^{2\pi}}_{\substack{1-1 \\ 0}} = 0$

$z_G = \frac{1}{V}\int z\, dV$

$= \frac{1}{V}\int z\, r\, dr\, d\theta\, dz$

$= \frac{1}{V}\int \left[\frac{r^2}{2}\right]_0^{r(z)} z\, dz \int d\theta$

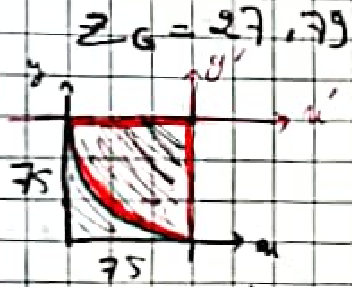
$= \frac{1}{2V}\int \left(50 - \frac{1}{6}z\right)^2 z\, dz\, [\theta]_0^{2\pi}$

$= \frac{1}{2V}\int_0^h 50^2 z - \frac{50}{3}z^2 + \frac{1}{36}z^3\, dz\; 2\pi$

$= \frac{\pi}{V}\left[50^2\left(\frac{z^2}{2}\right) - \frac{50}{3}\left(\frac{z^3}{3}\right) + \frac{1}{36}\left(\frac{z^4}{4}\right)\right]_0^h$

$z_G = 27,79$

d)



$ds = dx\, dy$ $\qquad 0 \leq x \leq 75$

$\qquad 0 \leq y \leq 75$

$x_G = \frac{1}{S}\int x\, ds = \frac{1}{S}\int x\, dx\, dy$

$= \frac{1}{2}(75)$

$y_G = \frac{1}{S}\int y\, ds = \frac{1}{S}\int y\, dx\, dy$

$= \frac{1}{2}(75)$ $\qquad S_1 = 5625$

$G_1 = \begin{pmatrix} 37,5 \\ 37,5 \end{pmatrix}$

$S = \iint r\, dr\, d\theta \qquad \pi \leq \theta \leq \frac{3\pi}{2}$

$0 \leq r \leq 75$

$S_2 = \frac{\pi r^2}{4}$

$x_{G_2} = \frac{1}{S}\int x\, ds = \frac{1}{S}\int r^2\cos\theta\, dr\, d\theta$

$x_{G_2} = \frac{1}{S}\int r^2\, dr \int \cos\theta\, d\theta$

$= \frac{1}{S}\left[\frac{r^3}{3}\right]_0^{75}\left[\sin\theta\right]_\pi^{3\pi/2}$

$G_2 = \begin{pmatrix} -31,83\,m \\ -31,83\,m \\ 0 \end{pmatrix}$

$OG = OO' + O'G = \begin{pmatrix} 75 \\ 75 \\ 0 \end{pmatrix} + \begin{pmatrix} -31,83 \\ -31,83 \\ 0 \end{pmatrix}$

$OG = \begin{pmatrix} 43,17 \\ 43,17 \end{pmatrix}$

$X_G = \frac{\sum m_i x_i}{\sum m_i}$

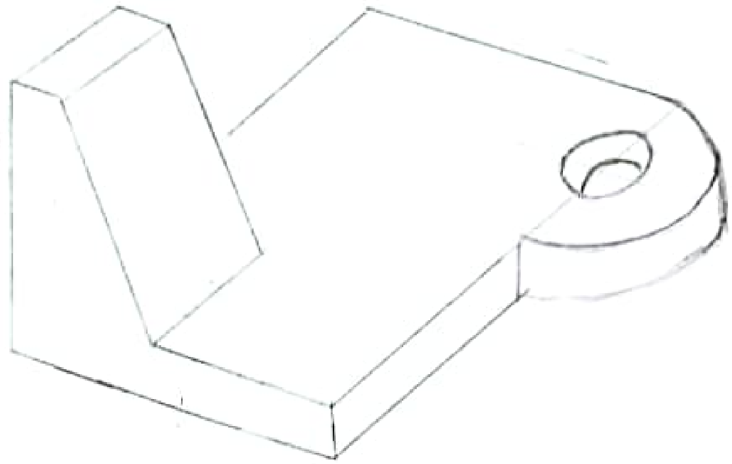
$\sum m_i = \sum S_i = S_1 - S_2 = 1207,14$

$X_G = Y_G = \frac{(5625)(37,5) - (\pi\, 1406,25)(43,17)}{1207,14}$

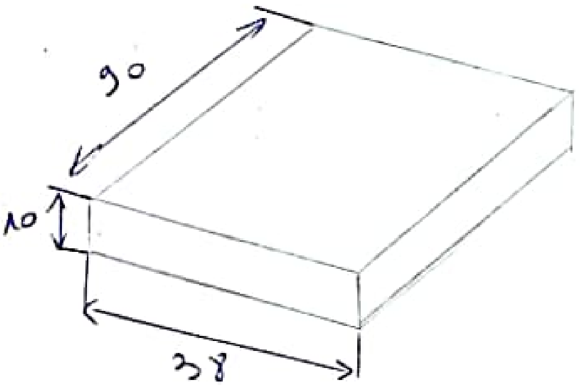
$X_G = Y_G = 16,75\ mm$

$\boxed{X_i = \frac{\iint x_i\, ds}{\iint ds}}$

figure (c)



system (1)



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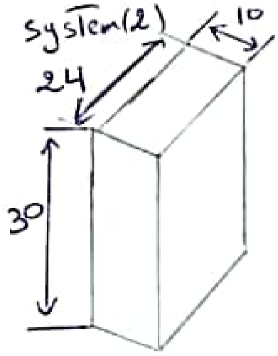
system(2)



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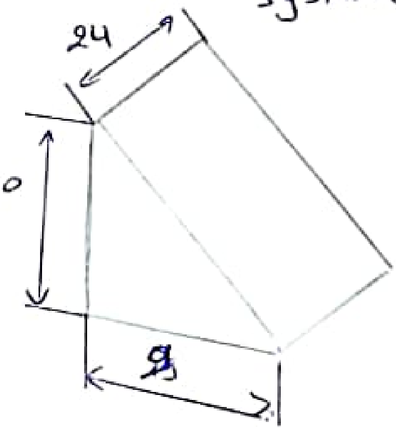
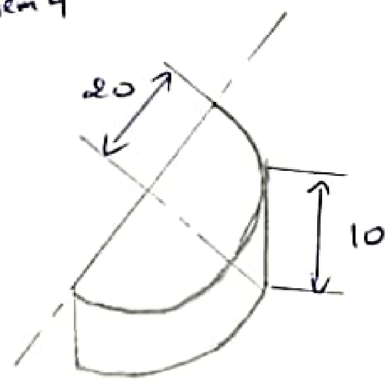
system (3)



system 4



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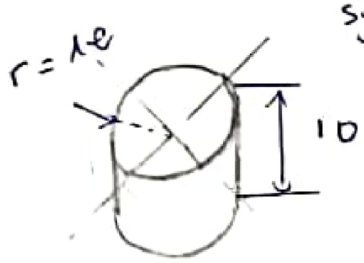
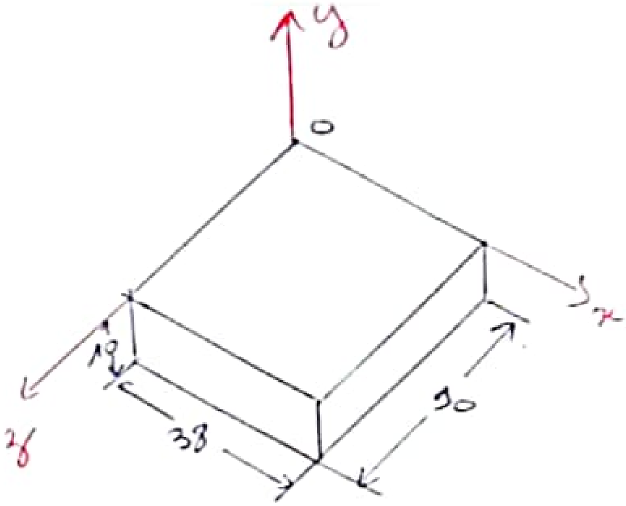
system 5



System (1)



$$V = \iiint dx\, dy\, dz = [x]_0^{38} \cdot [y]_{-10}^{0} \cdot [z]_0^{90}$$

$$= (38)(10)(90) =$$

$$= 34\ 200 \text{ mm}^3$$

$$x_{G_1} = \frac{1}{V}\int x\, dV = \frac{1}{V}\iiint x\, dx\, dy\, dz$$

$0 \leq x \leq 38$

$-10 \leq y \leq 0$

$0 \leq z \leq 90$

$$= \frac{1}{V}\int x\, dx \int dy \int dz$$

$$= \frac{1}{V}\left\{\left[\frac{x^2}{2}\right]_0^{38} [y]_{-10}^{0} \cdot [z]_0^{90}\right\}$$

$$= \frac{1}{\cancel{(38)}\cancel{(10)}\cancel{(90)}}\left\{\frac{(38)^{\cancel{2}}}{2} \cdot \cancel{(10)}\cancel{(90)}\right\} = \frac{38}{2} = 19 \text{ mm}$$

$$y_{G_1} = \frac{1}{V}\int y\, dV = \frac{1}{V}\iiint dx \cdot y\, dy \cdot dz.$$

$$= \frac{1}{V}\left\{[x]_0^{38} \left[\frac{y^2}{2}\right]_{-10}^{0} \cdot [z]_0^{90}\right\}$$

$$= \frac{1}{\cancel{(38)}(10)\cancel{(90)}}\left\{\cancel{(38)}\left\{\frac{0-(-10)^2}{2}\right\} \cdot \cancel{(90)}\right\} = -\frac{10}{2} = -5 \text{ mm}$$

$$z_{G_1} = \frac{1}{V}\int z\, dV = \frac{1}{V}\iiint dx \cdot dy \cdot z\, dz.$$

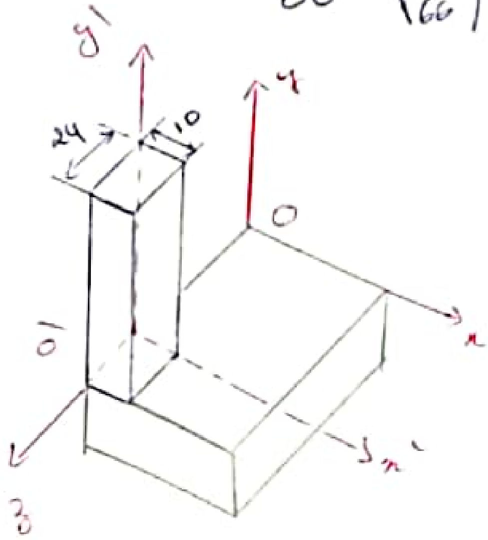
$$= \frac{1}{V}\left\{[x]_0^{38}[y]_{-10}^{0}\left[\frac{z^2}{2}\right]_0^{90}\right\}$$

$$= \frac{1}{\cancel{(38)}\cancel{(10)}(90)}\left\{\cancel{(38)} \cdot \overset{0-(-10)}{\cancel{(10)}}\ \frac{(90)^{\cancel{2}}}{2}\right\} = \frac{90}{2} = 45 \text{ mm}$$

$$OG_1 = \begin{pmatrix} 19 \\ -5 \\ 45 \end{pmatrix} \qquad V_1 = 34200 \text{ mm}^3.$$

_ Système (2)

$$OO' = \begin{pmatrix} 0 \\ 0 \\ 66 \end{pmatrix}$$



$$V_2 = \iiint dx \cdot dy \cdot dz$$
$$= [x]_0^{10} [y]_0^{30} [z]_0^{24}$$
$$= (10)(30)(24) = 7200 \text{ mm}^3$$

$$x'_{G_2} = \frac{1}{V} \int x \, dV = \frac{1}{V} \iiint x \, dx \, dy \cdot dz$$
$$= \frac{1}{V} \left\{ \int x \, dx \int dy \cdot \int dz \right\}$$
$$= \frac{1}{V} \left\{ \left[\frac{x^2}{2}\right]_0^{10} [y]_0^{30} [z]_0^{24} \right\}$$
$$= \frac{1}{(10)(\cancel{30})(\cancel{24})} \left\{ \frac{(10)^2}{2} \cdot (\cancel{30}) \cdot (\cancel{24}) \right\} = \frac{10}{2} = 5 \text{ mm}$$

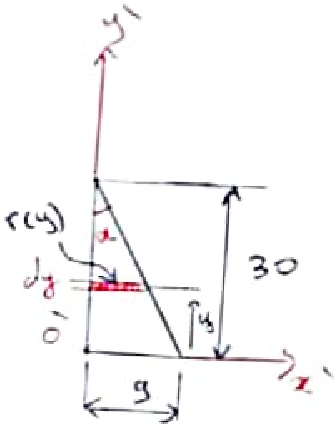
$$0 \leq x \leq 10$$
$$0 \leq y \leq 30$$
$$0 \leq z \leq 24$$

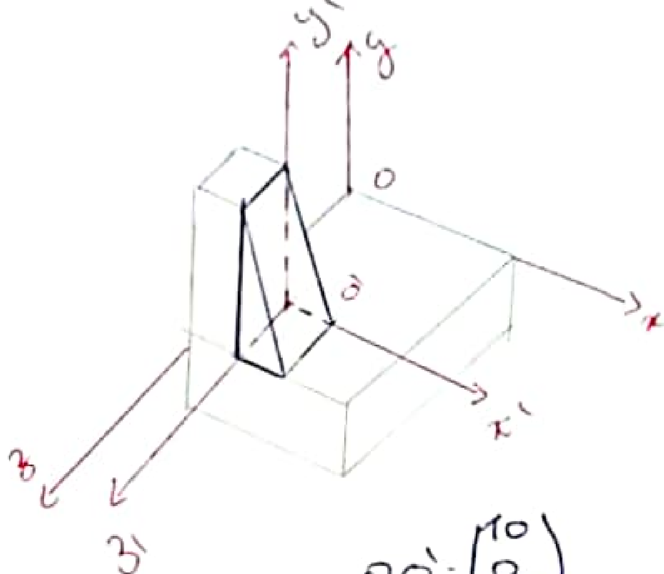
$$y'_{G_2} = \frac{1}{V} \int y \, dV = \frac{1}{V} \iiint y \, dx \cdot dy \cdot dz =$$
$$= \frac{1}{V} \int dx \int y \, dy \int dz = \frac{1}{(10)(30)(24)} \left\{ [x]_0^{10} \cdot \left[\frac{y^2}{2}\right]_0^{30} \cdot [z]_0^{24} \right\}$$
$$= \frac{1}{(\cancel{10}) \cdot (\cancel{30}) \cdot (\cancel{24})} \left\{ (\cancel{10}) \cdot \frac{(30)^2}{2} \cdot (\cancel{24}) \right\} = \frac{30}{2} = 15 \text{ mm}.$$

$$z'_{G_2} = \frac{1}{V} \int z \, dV = \frac{1}{V} \iiint z \, dx \cdot dy \cdot dz = \frac{1}{V} \int dx \int dy \cdot \int z \, dz$$
$$= \frac{1}{(10)(30)(24)} \left\{ [x]_0^{10} [y]_0^{30} \left[\frac{z^2}{2}\right]_0^{24} \right\}$$
$$= \frac{1}{(\cancel{10})(\cancel{30})(\cancel{24})} \left\{ \cancel{10} \cdot \cancel{30} \cdot \frac{(24)^2}{2} \right\} = \frac{24}{2} = 12 \text{ mm}$$

$$O'G_2 = \begin{pmatrix} 5 \\ 15 \\ 12 \end{pmatrix} \; ; \; OG_2 = OO' + O'G_2 = \begin{pmatrix} 0 \\ 0 \\ 66 \end{pmatrix} + \begin{pmatrix} 5 \\ 15 \\ 12 \end{pmatrix} ; \; OG_2 = \begin{pmatrix} 5 \\ 15 \\ 79 \end{pmatrix}$$

System (3)





$$Tg\alpha = \frac{9}{30} = \frac{r(y)}{30-y}$$

$$r(y) = \frac{9}{30}(30-y)$$

$$\boxed{r(y) = 9 - \frac{9}{30}y}$$

$$S_y = r(y)\cdot dy \Rightarrow V = S\cdot dz.$$

$$OO' = \begin{pmatrix}10\\0\\66\end{pmatrix}$$

$0 \le x \le 9$

$0 \le y \le 30$

$0 \le z \le 24$

$$* V_3 = r(y)\cdot dy\cdot dz$$

$$V_3 = \int\left(9 - \frac{9}{30}y\right)dy\cdot\int dz$$

$$= \left(\int 9\,dy - \int\frac{9}{30}y\,dy\right)\int dz = \left([9\times y]_0^{30} - \left[\frac{9}{30}\times\frac{y^2}{2}\right]_0^{30}\right)[z]_0^{24}$$

$$= \left(9\times 30 - \frac{9}{\cancel{30}}\left(\frac{(30)^{\cancel{2}}}{2}\right)\right)(24)$$

$$= \left(9\times 30 - \frac{9\cdot(30)}{2}\right)(24)$$

$$= \frac{9\times 30}{2}\cdot(24) \Rightarrow V_3 = 3240\ mm^3$$

$$* V_3 = \int\overbrace{r(x)\cdot dx}^{S_x}\cdot dz$$

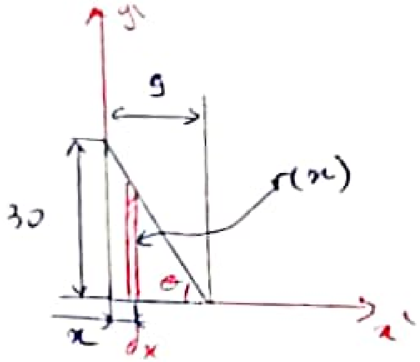
$$= \int\left(30 - \frac{30}{9}x\right)dx\cdot\int dz$$

$$= \left[30\cdot x - \frac{30}{9}\cdot\frac{x^2}{2}\right]_0^9\cdot[z]_0^{24}$$

$$= \left[30\times 9 - \frac{30}{\cancel{9}}\left(\frac{9^{\cancel{2}}}{2}\right)\right](24)$$

$$= \frac{30\times 9}{2}\cdot 24 \Rightarrow$$

$$V_3 = 3240\ mm^3$$



$$Tg\theta = \frac{30}{9} = \frac{r(x)}{9-x}$$

$$r(x) = \frac{30}{9}(9-x)$$

$$\boxed{r(x) = 30 - \frac{30}{9}x}$$

$$x'_{G_3} = \frac{1}{V_3}\int x\, dV = \frac{1}{V_3}\int x\, S_x \cdot dz \quad ; \text{ avec } V_3 = \frac{9 \times 30 \times 24}{2}$$

$$= \frac{1}{V_3}\int r(x) \cdot x \cdot dz = \frac{1}{V_3}\int \left(30 - \frac{30}{9}x\right) x \cdot \int dz$$

$$= \frac{2}{9 \times 30 \times 24}\left[30 \cdot \frac{x^2}{2} - \frac{30}{9} \cdot \frac{x^3}{3}\right]_0^9 \left[z\right]_0^{24}$$

$$= \frac{2}{\cancel{9} \times \cancel{30} \times \cancel{24}}\left(\cancel{30} \cdot \frac{9^2}{2} - \frac{\cancel{30}}{\cancel{9}} \cdot \frac{9^3}{3}\right)(\cancel{24})$$

$$= 2\left(\frac{9}{2} - \frac{9}{3}\right) = 2 \cdot \frac{9}{6} = \frac{1}{3}(9) = 3 \text{ mm}$$

$$y'_{G_3} = \frac{1}{V_3}\int y\, dV = \frac{1}{V_3}\int y \cdot S_y\, dz$$

$$= \frac{1}{V_3}\int y\; r(y) \cdot dz = \frac{1}{V_3}\int \left(9 - \frac{9}{30}y\right) y \cdot \int dz$$

$$= \frac{2}{9 \times 30 \times 24}\left[9 \cdot \frac{y^2}{2} - \frac{9}{30} \cdot \frac{y^3}{3}\right]_0^{30} \left[z\right]_0^{24}$$

$$= \frac{2}{\cancel{9} \times \cancel{30} \times \cancel{24}}\left(\cancel{9} \cdot \frac{30^2}{2} - \frac{\cancel{9}}{\cancel{30}}\; \frac{30^3}{3}\right)(\cancel{24}) = 2\left(\frac{30}{2} - \frac{30}{3}\right)$$

$$y_{G_3} = 2 \cdot \frac{30}{6} = \frac{1}{3}(30) = 10 \text{ mm}$$

$$z'_{G_3} = \frac{1}{V_3}\int z\, dV = \frac{1}{V_3}\int z\; S_x\, dx\; dz = \frac{1}{V_3}\int \left(30 - \frac{30}{9}x\right) dx \cdot \int z\, dz$$

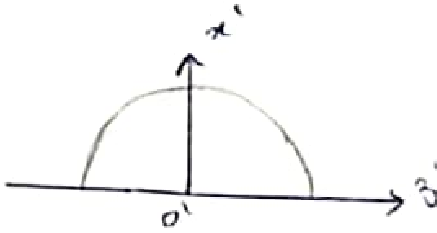
$$= \frac{2}{9 \times 30 \times 24}\left[30x - \frac{30}{9} \cdot \frac{x^2}{2}\right]_0^9 \cdot \left[\frac{z^2}{2}\right]_0^{24} = \frac{2}{9 \times \cancel{30} \times 24}\left[\cancel{30} \cdot (9) - \frac{\cancel{30}}{9}\,\frac{(9)^2}{2}\right]\left[\frac{(24)^2}{2}\right]$$

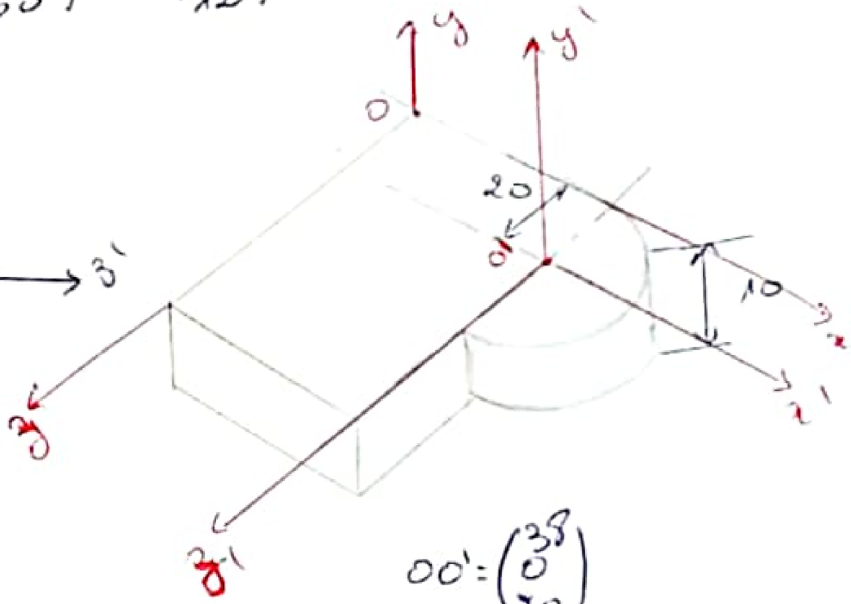
$$z_{G_3} = \cancel{2}\left(1 - \frac{1}{2}\right)\left(\frac{24}{\cancel{2}}\right) = \frac{1}{2}(24) \qquad z_{G_3} = 12 \text{ mm}.$$

$O'G_3 = \begin{pmatrix}3\\10\\12\end{pmatrix}$. $\quad OG_3 = OO' + O'G_3$

$$= \begin{pmatrix}10\\0\\60\end{pmatrix} + \begin{pmatrix}3\\10\\12\end{pmatrix} = OG_3 = \begin{pmatrix}13\\10\\72\end{pmatrix}$$

<u>System 4</u> :





$OO' = \begin{pmatrix}38\\0\\20\end{pmatrix}$

$S = r\,dr\,d\theta$

$V_4 = \int S.\,dy$

$$\begin{cases} 0 \leq r \leq 20 \\ 0 \leq \theta \leq \pi \\ -10 \leq y \leq 0 \end{cases}$$

$$\begin{cases} z = r\cos\theta \\ x = r\sin\theta \\ y = y \end{cases}$$

$$V_4 = \iiint r.dr.d\theta\,dy = \int_0^{20} r\,dr \int_0^{10} dy \,.\int_0^{\pi} d\theta$$

$$= \left[\frac{r^2}{2}\right]_0^{20} \cdot \left[y\right]_{-10}^{0} \cdot \left[\theta\right]_0^{\pi} = \frac{(20)^2}{2}.(10).\pi = 2000\pi \,.$$

$$X'_G = \frac{1}{V_4}\iiint x\,dV = \frac{1}{V}\iiint (r\sin\theta)(r\,dr\,d\theta\,dy)$$

$$= \frac{1}{V}\int_0^{20} r^2\,dr \int_0^{\pi} \sin\theta\,d\theta \int_0^{10} dy$$

$$= \frac{1}{V}\left\{\left[\frac{r^3}{3}\right]_0^{20} \left[-\cos\theta\right]_0^{\pi} \left[y\right]_{-10}^{0}\right\}$$

$$= \frac{1}{V}\left\{\left[\frac{(20)^3}{3}\right].\underbrace{\left(-(-1-1)\right)}_{2}(10)\right\}$$

$$= \frac{1}{2000\pi}\;\frac{(20)^3}{3}.20 = \frac{160\,\cancel{000}}{6\,\cancel{000}.\pi} = 8{,}48\ mm$$

$$y'_a = \frac{1}{V_4}\int y\, dV$$

$$= \frac{1}{V_4}\iiint y\; r\,dr\,d\theta\,dy = \frac{1}{V}\left\{\int_{-10}^{0} y\,dy \int_0^{20} r\,dr \int_0^{\pi} d\theta\right\}$$

$$= \frac{1}{V}\left\{\left[\frac{y^2}{2}\right]_{-10}^{0} \cdot \left[\frac{r^2}{2}\right]_0^{20} \cdot \left[\theta\right]_0^{\pi}\right\}$$

$$= \frac{1}{V}\left\{\frac{-(10)^2}{2} \cdot \frac{(20)^2}{2} \cdot \pi\right\}$$

$$= \frac{1}{2000\cancel{\pi}} \times \frac{-(100)}{2} \cdot \frac{400}{2} \cdot \cancel{\pi}$$

$$= \frac{1}{2000} \cdot (-50) \cdot (200) = -5 \text{ mm}.$$

$$z'_a = \frac{1}{V_4}\int z\, dV$$

$$= \frac{1}{V}\iiint (r\cos\theta)(r\,dr\,d\theta)\,dy$$

$$= \frac{1}{V}\int r^2\,dr \int \cos\theta\,d\theta \cdot \int dy$$

$$= \frac{1}{V}\left[\frac{r^3}{3}\right]_0^{20} \underbrace{\left[\sin\theta\right]_0^{\pi}}_{\sin\pi - \sin 0} \left[y\right]_{-10}^{0} = 0$$

$$OG'_4 = \begin{pmatrix} 8{,}48 \\ -5 \\ 0 \end{pmatrix}$$

$$OG_4 = OO' + OG_4$$

$$= \begin{pmatrix} 38 \\ 0 \\ 20 \end{pmatrix} + \begin{pmatrix} 8{,}48 \\ -5 \\ 0 \end{pmatrix} \Rightarrow OG_4 = \begin{pmatrix} 46{,}48 \\ -5 \\ 20 \end{pmatrix}$$

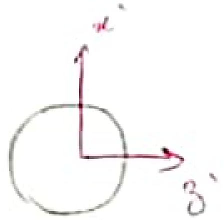
Système 5

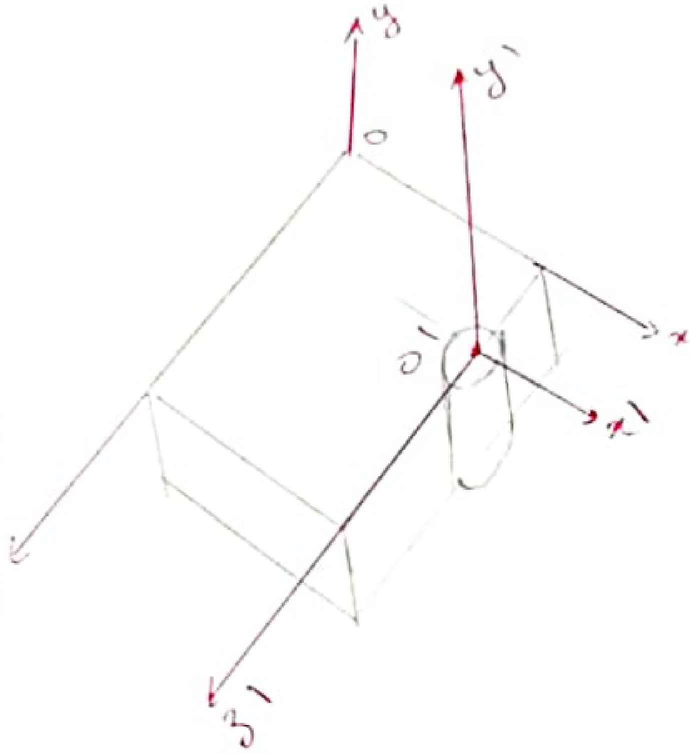
$S=\iint r\,dr\,d\theta$

$0\leq\theta\leq 2\pi$

$0\leq r\leq 12$

$-10\leq y\leq 0$





$V=\int S\cdot dy$

$=\int r\,dr\int d\theta\int dy$

$=\left[\frac{r^2}{2}\right]_0^{12}\left[\theta\right]_0^{2\pi}\left[y\right]_{-10}^{0}$ $\begin{cases} z = r\cos\theta \\ x = r\sin\theta \\ y = y \end{cases}$

$=\frac{(12)^2}{\cancel{2}}\cdot(\cancel{2}\pi)(10)$

$OO'=\begin{pmatrix}38\\0\\20\end{pmatrix}$

$V_5=1440\pi$

$x'_{G_5}=\frac{1}{V_5}\int x\,dV=\frac{1}{V_5}\iiint r\sin\theta\cdot r\,dr\,d\theta\cdot dy$

$=\frac{1}{V_5}\left\{\int r^2\,dr\int\sin\theta\,d\theta\int dy\right\}$

$=\frac{1}{V_5}\left\{\left[\frac{r^3}{3}\right]_0^{12}\underbrace{\left[-\cos\theta\right]_0^{2\pi}}_{0}\left[y\right]_{-10}^{0}\right\}=0$

$y'_{G_5}=\frac{1}{V_5}\int y\,dV=\frac{1}{V_5}\iiint y\,r\,dr\,d\theta\cdot dy$

$=\frac{1}{V_5}\left\{\int r\,dr\int y\,dy\int d\theta\right\}=\frac{1}{V_5}\left\{\left[\frac{r^2}{2}\right]_0^{12}\left[\frac{y^2}{2}\right]_{-10}^{0}\left[\theta\right]_0^{2\pi}\right\}$

$=\frac{1}{V_5}\left\{\frac{(12)^2}{2}\cdot\left(-\frac{(-10)^2}{2}\right)2\pi\right\}$

$=\frac{1}{(12)^2(\cancel{\pi})(10)}\left(\frac{(12)^2}{2}\right)\cdot\left(-\frac{(10)^2}{2}\right)\cdot(2\pi)=\frac{-10}{2}=-5\text{ mm}.$

$$z'_G = \frac{1}{V_5}\int z\,dV = \frac{1}{V_5}\int r\cos\theta \; r\,dr\,d\theta\,dy.$$

$$= \frac{1}{V_5}\int r^2 dr \int \cos\theta\,d\theta \int dy$$

$$= \frac{1}{V_5}\left\{\left[\frac{r^3}{3}\right]_0^{12}\left[\sin\theta\right]_0^{2\pi}\left[y\right]_{-10}^{0}\right\} = 0$$

$$O'G_5 = \begin{pmatrix} 0 \\ -5 \\ 0 \end{pmatrix} \qquad OG_5 = OO' + O'G_5 = \begin{pmatrix} 38 \\ 0 \\ 20 \end{pmatrix} + \begin{pmatrix} 0 \\ -5 \\ 0 \end{pmatrix} = \begin{pmatrix} 38 \\ -5 \\ 20 \end{pmatrix} = OG_5$$

$$V_1 = 34\,200 \quad OG_1 = \begin{pmatrix} 19 \\ -5 \\ 45 \end{pmatrix} \Big/ \quad V_2 = 7200 \quad OG_2 = \begin{pmatrix} 5 \\ 15 \\ 79 \end{pmatrix}.$$

$$V_3 = 3240 \quad OG_3 = \begin{pmatrix} 13 \\ 10 \\ 72 \end{pmatrix} \Big/ \quad V_4 = 6\,283,18 \quad OG_4 = \begin{pmatrix} 46,48 \\ -5 \\ 20 \end{pmatrix}$$

$$V_5 = 4\,523,89\,; \; OG_5 = \begin{pmatrix} 38 \\ -5 \\ 20 \end{pmatrix}.$$

$$\Sigma m_i = V_1 + V_2 + V_3 + V_5 - V_5 = 46\;399,29 \text{ mm}^3.$$

$$X_{G_{syst}} = \frac{(34200)(19) + 7200(5) + 3240(13) + 6283,18\,(46,48) - 4523,89\,(38)}{46\;399,29}$$

$$\boxed{X_{G_{syst}} = 18,27 \text{ mm}}$$

$$y_{G_{sys}} = \frac{34200\,(-5) + 7200\,(15) + 3240\,(10) + 6283,18\,(-5) - (4523,89)(-5)}{46\;399,29}$$

$$\boxed{Y_{G_{syst}} = -0,84 \text{ mm}}$$

$$z_{G_{syst}} = \frac{34200(45) + 7200(79) + 3240(72) + 6283,18(20) - (4523,19)(20)}{46\;399,28}$$

$$\boxed{z_{G_{syst}} = 52,21 \text{ mm}}$$