Chapiter 4
Three-phase Power

The power in three-phase regime

In brief

- The instantaneous power in three phases is equal to the sum of the instantaneous powers on each phase,
- The three-phase active power is equal to the sum of the active power on each phase.
- The three-phase reactive power is equal to the sum of the reactive power on each phase.
- The total apparent power is <u>**different**</u> from the sum of the apparent power on each phase.

Instantaneous Power

Balanced Load

- In the balanced three-phase case, **there is no fluctuating power** as in the single-phase.
- Indeed, as :

$$\begin{cases} v_1(t)=\sqrt{2}\,V\cos(\omega t+\varphi_v) \\ v_2(t)=\sqrt{2}\,V\cos(\omega t+\varphi_v-2\pi/3) \\ v_3(t)=\sqrt{2}\,V\cos(\omega t+\varphi_v+2\pi/3) \end{cases} \text{ et } \begin{cases} j_1(t)=\sqrt{2}\,J\cos(\omega t+\varphi_j) \\ j_2(t)=\sqrt{2}\,J\cos(\omega t+\varphi_j-2\pi/3) \\ j_3(t)=\sqrt{2}\,J\cos(\omega t+\varphi_j+2\pi/3) \end{cases}$$

- then:

$$\begin{aligned} p(t)=p_1(t)+p_2(t)+p_3(t)=2VJ\{ &\cos(\omega t+\varphi_j).\cos(\omega t+\varphi_v) \\ &+\cos(\omega t+\varphi_j-2\pi/3).\cos(\omega t+\varphi_v-2\pi/3) \\ &+\cos(\omega t+\varphi_j+2\pi/3).\cos(\omega t+\varphi_v+2\pi/3)\} \end{aligned}$$

- then:

$$\begin{aligned} p(t)=V\,J\{ &3\cos(\varphi)+\cos(2\omega t+\varphi_i+\varphi_j) \\ &+\cos(2\omega t+\varphi_i+\varphi_j-4\pi/3)+\cos(2\omega t+\varphi_i+\varphi_j+4\pi/3)\}=3\,V\,J\cos(\varphi) \end{aligned}$$

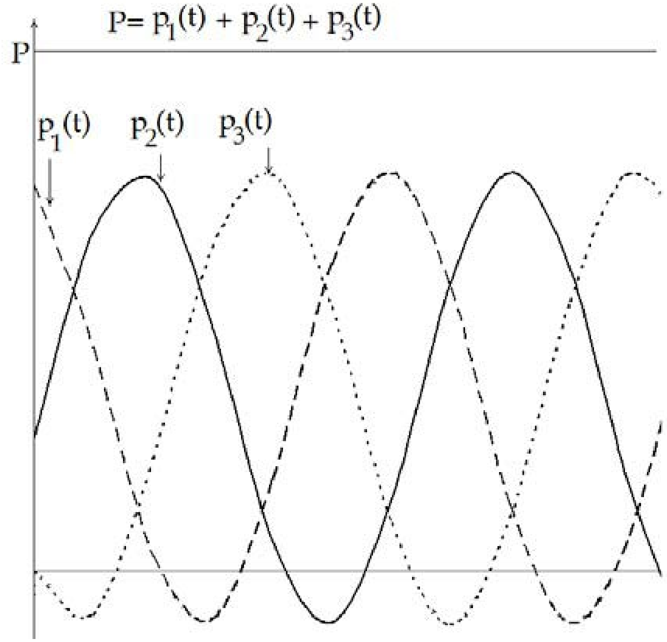
Instantaneous Power

Balanced Load

- Calculations of $p(t)$ give:

$$p(t) = 3\,V\,J\cos(\varphi)$$

- Instantaneous power is constant,
- It equals 3 times the active power in a single phase
- No fluctuating powers whatever the nature of Z.



- Disappearance of vibration phenomena in motors => engine longevity

Active and Reactive power

Any load

- The three-phase active power is equal to the sum of the active powers on each phase.

$$P=\sum_{i=0}^{3} P_i=V_1 J_1 \cos(\varphi_1)+V_2 J_2 \cos(\varphi_2)+V_3 J_3 \cos(\varphi_3)$$

- The three-phase reactive power is equal to the sum of the reactive powers on each phase.

$$Q=\sum_{i=0}^{3} Q_i=V_1 J_1 \sin(\varphi_1)+V_2 J_2 \sin(\varphi_2)+V_3 J_3 \sin(\varphi_3)$$

- where:

$$\varphi_1=\varphi_{v1}-\varphi_{j1},\ \varphi_2=\varphi_{v2}-\varphi_{j2}\ \text{et}\ \varphi_3=\varphi_{v3}-\varphi_{j3}$$

- Total apparent power is different from partial apparent powers: $S\neq\sum_{i=0}^{3} S_i$

Active and Reactive power

Balanced Load

- We get:

$$\begin{cases} V_1 = V_2 = V_3 = V \\ J_1 = J_2 = J_3 = J \\ \varphi_1 = \varphi_2 = \varphi_3 = \varphi \end{cases} \text{ then } \begin{cases} P = P_1 + P_2 + P_3 = 3\, V\, J \cos\varphi \\ Q = Q_1 + Q_2 + Q_3 = 3\, V\, J \sin\varphi \end{cases}$$

- if the load is in a Star fashion

$$\begin{cases} J = I \\ U = \sqrt{3}\, V \end{cases} \text{ then } \begin{cases} P = \sqrt{3}\, U\, I \cos\varphi \\ Q = \sqrt{3}\, U\, I \sin\varphi \end{cases}$$

- if the load is a trdelta fashion

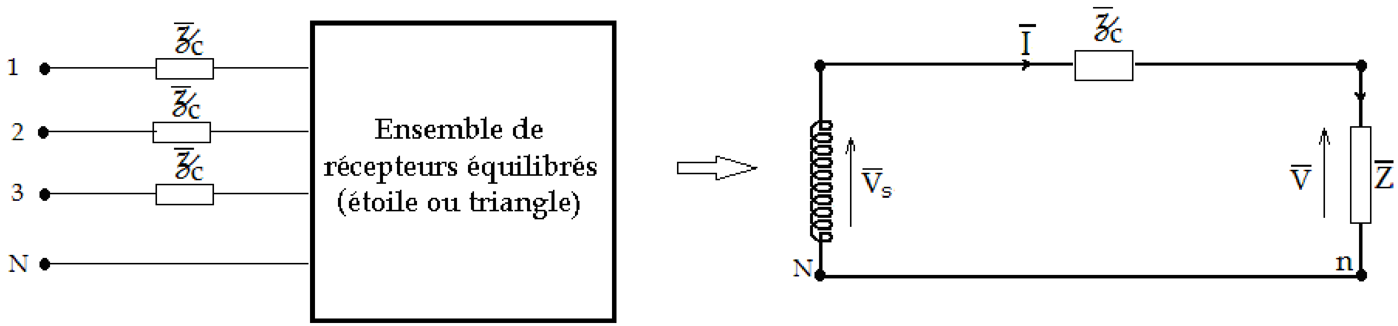
$$\begin{cases} I = \sqrt{3}\, J \\ U = V \end{cases} \text{ then } \begin{cases} P = \sqrt{3}\, U\, I \cos\varphi \\ Q = \sqrt{3}\, U\, I \sin\varphi \end{cases}$$

- We obtain the same expression using the composite values of voltage and current.

Active and Reactive power

Equivalent Star (balanced case)

- As we obtained the same expressions for the power, we will use the notion of equivalent star:
- A balanced triangle installation of current and voltage U corresponds to a star assembly consuming the same energy.
- Consequently, it will be possible to determine the voltage drops as in single-phase.
- We are then interested in a single phase:



Active and Reactive power

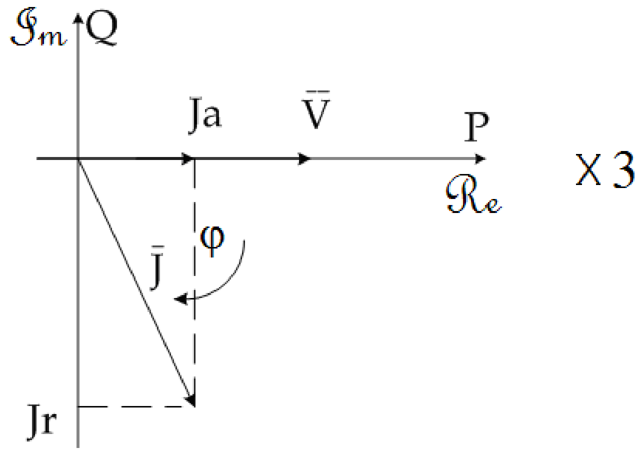
Complex notation (balanced case)

- We return to the definition of apparent power:

$$\widetilde{S} = P + j\,Q = 3\,V\,J\cos\varphi + 3\,j\,V\,J\sin\varphi = 3\,V\,J\,e^{j\varphi} = 3\,\widetilde{V}\,\widetilde{J}^{*}$$

- It is then possible to make a representation in the complex plane using a single phase.

- However, we must not forget that the quantities must be multiplied by 3



Active and Reactive power

Power Factor f_p

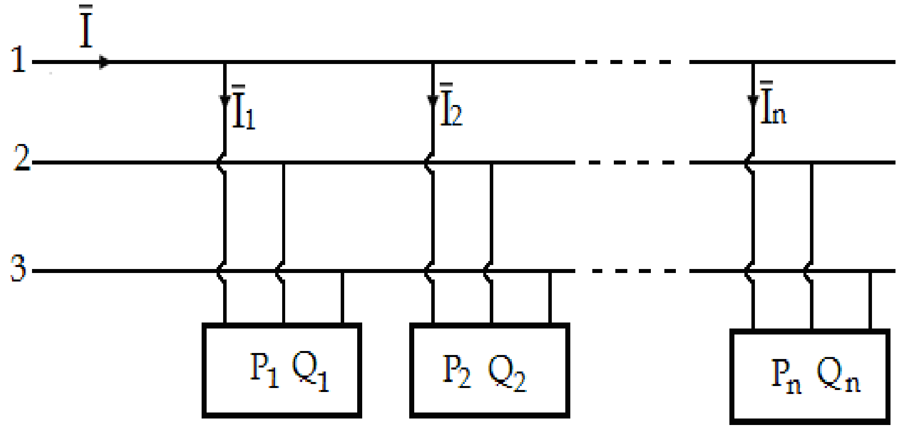
- Nothing changes:

$$f_p = \frac{P}{\sqrt{P^2 + Q^2}}$$

Active and Reactive power

Boucherot's Method

- Let be an installation with several balanced three-phase receivers,
- They can be either triangle or star,
- It is more practical to illustrate the currents on a single phase,



$$P_t=\sum_{i=0}^{n} P_i \text{ et } Q_t=\sum_{i=0}^{n} Q_i \text{ et } S_t=\sqrt{P_t^2+Q_t^2}$$

- Knowing that

$$S_t=3\,VJ=\sqrt{3}\,U\,I\,,$$

- It is possible to determine the current of the line I and the power factor f_p:

$$I=\frac{S_t}{\sqrt{3}\,U} \qquad \text{et} \qquad \cos\varphi_t=\frac{P_t}{S_t}$$

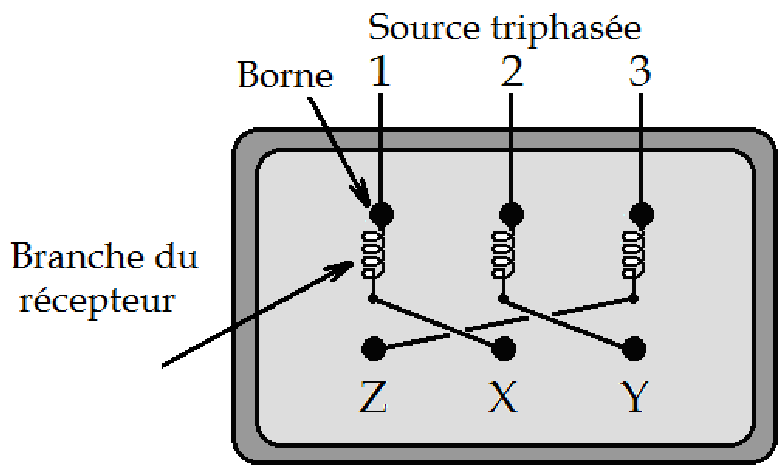
Connection to the network

Three-phase receivers

- The nameplate of certain three-phase equipment bears the indication U_1/U_2 with $U_2=\sqrt{3}U_1$. Example 220 V / 380 V.

- This indicates that the device can be set to operate under these two regimes. How?

- The device contains six terminals allowing the receiver to be configured as a star or triangle.

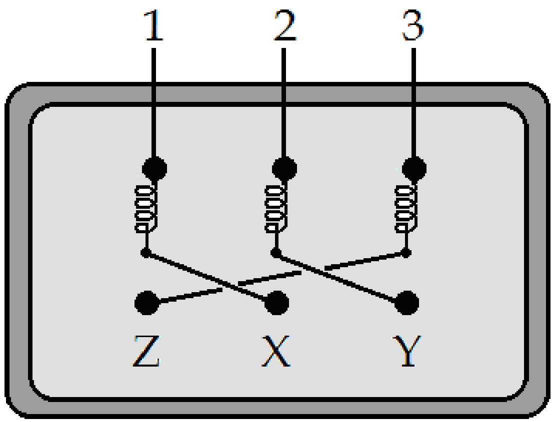


- The receiver branch is designed to work with U_1 (for instance 220 V),

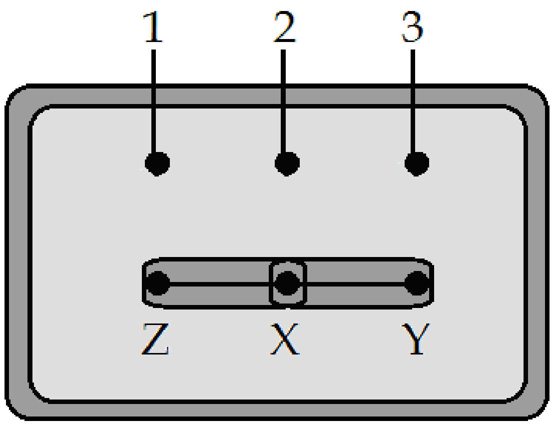
Connection to the network

Three-phase receivers

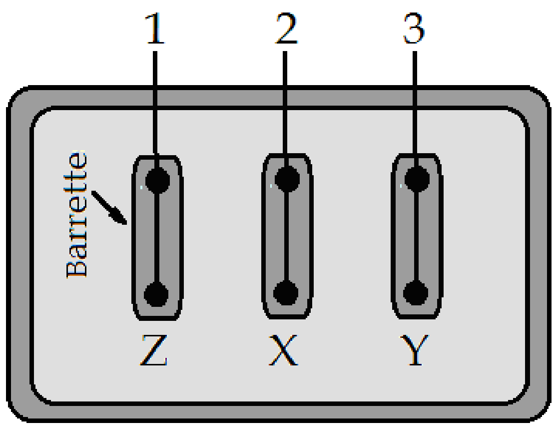
- If the compound voltage of the network $U_{s1}=U_1$ then, the delta topology is the most suitable,
- If $U_{s1}=\sqrt{3}U_1$, then, the star topology is the most suitable,
- We use strips to configure the receiver.



Terminal board



« Star » Coupling



« Delta » Coupling

Connection to the network

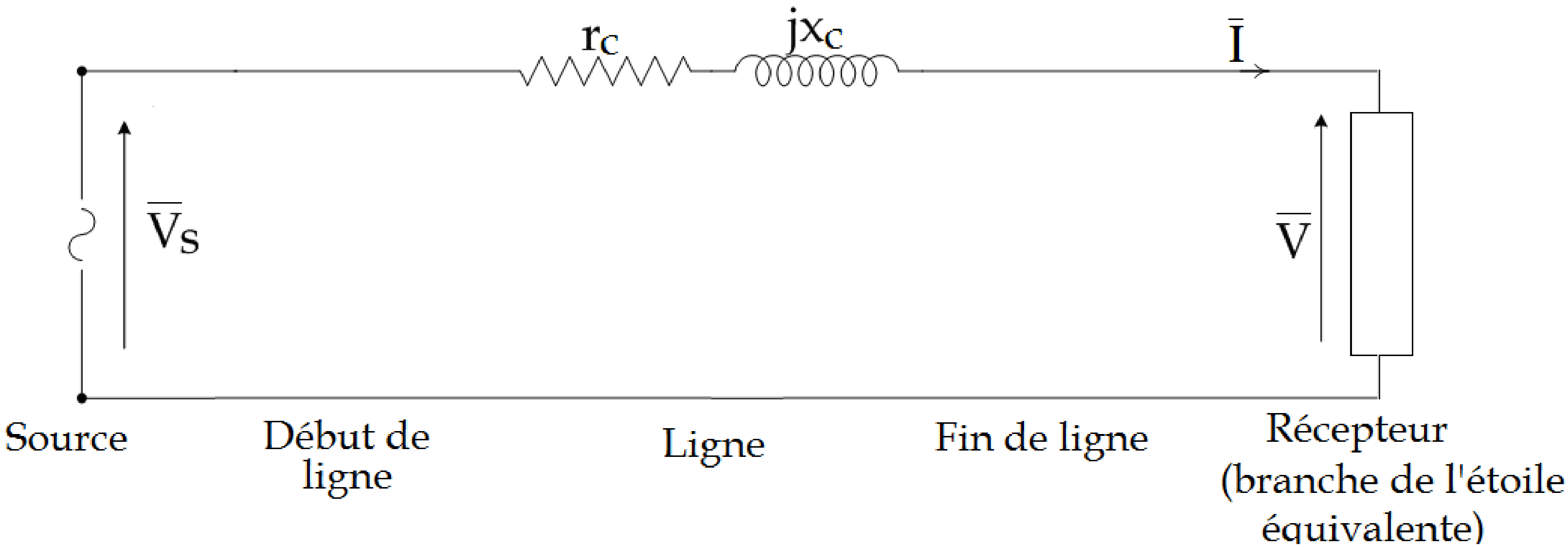
Single phase receivers

- The homes are powered by 220 V / 50 Hz
- Each home is connected to one phase,
- The goal being to balance the three-phase system in order to have a neutral current of small value,
- Semi-industrial installations use compound voltages. They require a three-phase installation.
- It is necessary to respect the indications on the loads.

Voltage Drops

Balanced case

- As the loads are balanced, then we consider only one phase.
- The neutral line carries no current:



Voltage Drop

Balanced case

- We keep the same definition as for the single phase

$$\Delta V = \left|\tilde{V}_s\right| - \left|\tilde{V}\right|$$

- We add a definition for compound voltages:

$$\Delta U = \sqrt{3}\left(\left|V_s\right| - \left|V\right|\right)$$

- We can use the approximate ***Kapp*** formulae to estimate voltage drop:

$$\Delta V = r_c I \cos\varphi + x_c I \sin\varphi$$

Power measurement in 3-phase

Wattmeter Principle

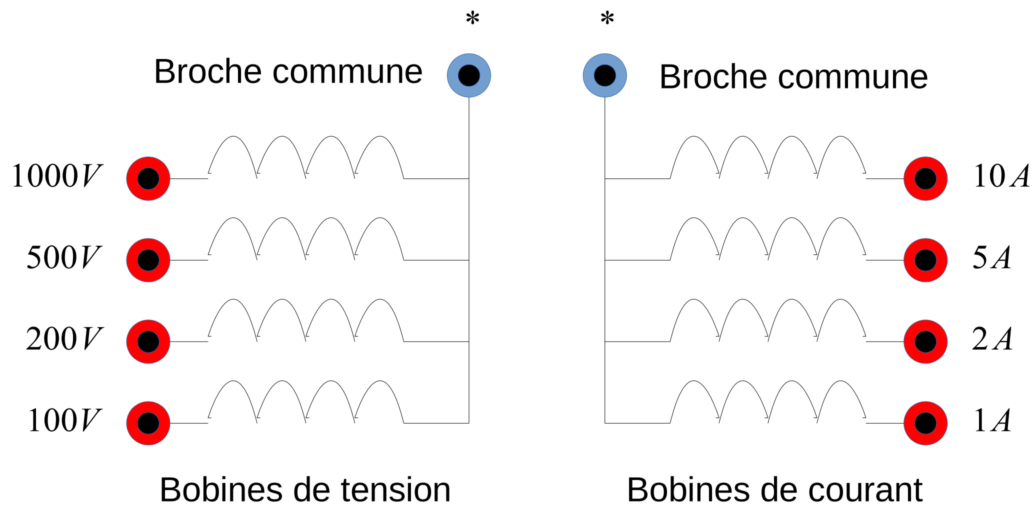
- Appareil qui a une déviation proportionnelle à la puissance active (ie $VJ\cos\varphi$)

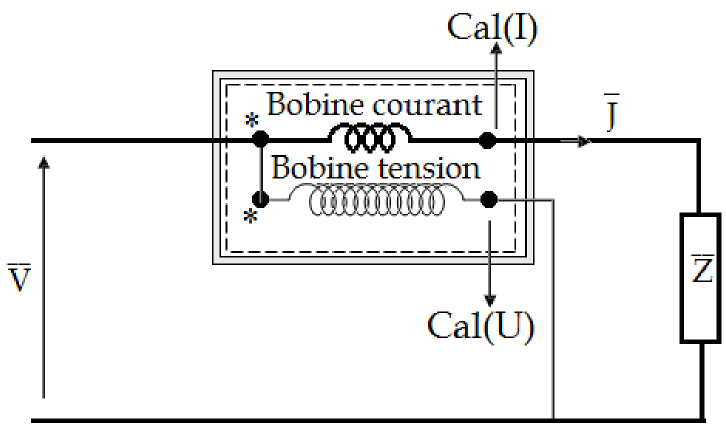
- Il possède donc deux types de circuits (bobines)
  - La bobine courant utilisée comme un ampèremètre (placée en série). Elle possède une faible résistance.
  - Une bobine tension utilisée comme un voltmètre (placée en parallèle). Elle possède une importante résistance interne.

- Chaque bobine correspond à un calibre de tension et de courant.
- Le symbole « * » correspond au point commun à tous les calibres.

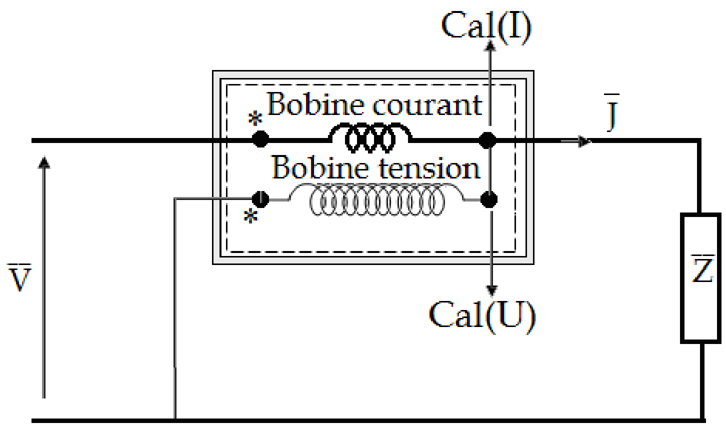
Power measurement in 3-phase

Wattmeter Principle





Upstream circuit



downstream circuit

Power measurement in 3-phase

Single Wattmeter method

- We have in general:

$$P = V_1 J_1 \cos\varphi_1 + V_2 J_2 \cos\varphi_2 + V_3 J_3 \cos\varphi_3 = V\left(J_1 \cos\varphi_1 + J_2 \cos\varphi_2 + J_3 \cos\varphi_3\right)$$

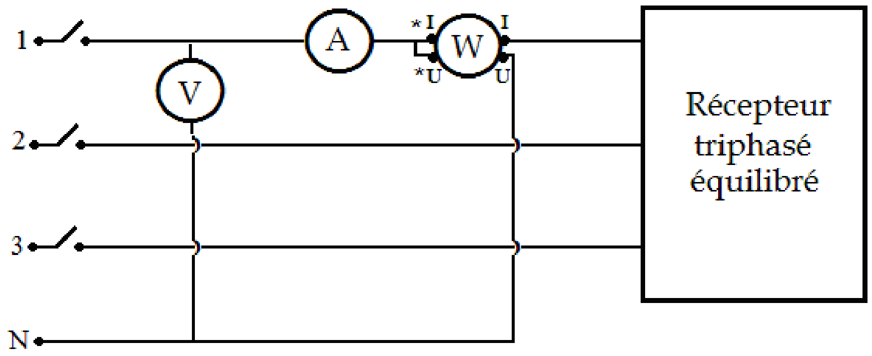
$$Q = V_1 J_1 \sin\varphi_1 + V_2 J_2 \sin\varphi_2 + V_3 J_3 \sin\varphi_3 = V\left(J_1 \sin\varphi_1 + J_2 \sin\varphi_2 + J_3 \sin\varphi_3\right)$$

- If the loads are balanced:

$$P = 3\,V\,J\cos\varphi \qquad \text{and} \qquad Q = 3\,V\,J\sin\varphi$$

- It is then sufficient to consider one phase and then multiply by 3:

$$P = 3 \times lecture$$



- IIt is understood that
  we use the equivalent star diagram when it comes to a triangle topology (i.e. $I = J$)

Power measurement in 3-phase

Single Wattmeter method

- Apparent power measurement:
  - Use a voltmeter and an amperemeter

$$S = 3\,V\,J = 3\,V\,I = \sqrt{3}\,U\,I$$

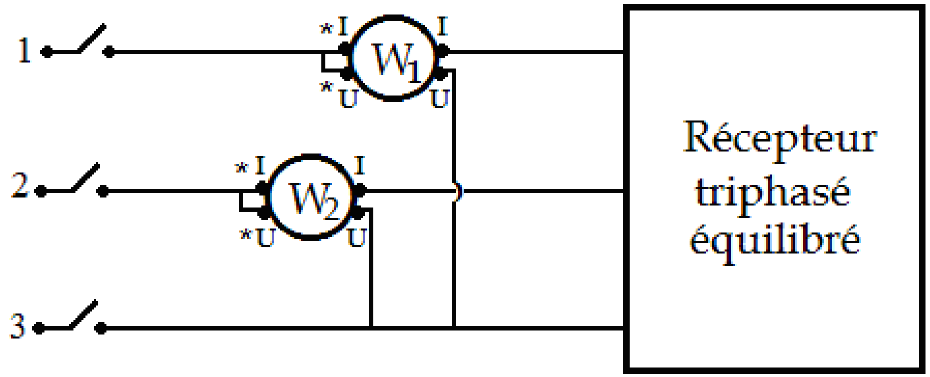
- Ddeduce reactive power:

$$Q = \sqrt{\left(\sqrt{3}\,U\,I\right)^2 - P^2}$$

Power measurement in 3-phase

Two Wattmeters method

- Possibility of measuring active and reactive power,
- Method adapted for a balanced three-phase circuit,
- Take the sign into account because it is algebraic values that are read.



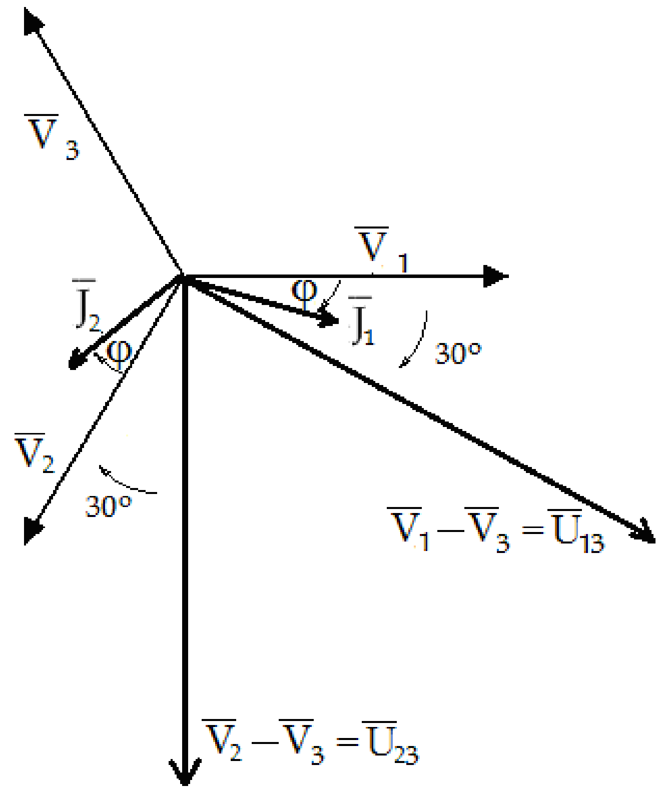
- According to the schematic, we read in the wattmeter and knowing that $I = J$

$$W_1 = U_{13} I_1 \cos\varphi_{u13,\, j1}$$

$$W_2 = U_{23} I_2 \cos\varphi_{u23,\, j2}$$

Power measurement in 3-phase

Two Wattmeters method



- We had:

$$W_1 = U_{13} I_1 \cos\varphi_{u13,\,j1}$$

$$W_2 = U_{23} I_2 \cos\varphi_{u23,\,j2}$$

- But: $\varphi_{u13,\,j1} = \pi/6 - \varphi$ and $\varphi_{u23,\,j2} = \pi/6 + \varphi$
- Then:

$$W_1 = U\,I\cos\left(\frac{\pi}{6} - \varphi\right) \quad \text{and} \quad W_2 = U\,I\cos\left(\frac{\pi}{6} + \varphi\right)$$

- If we calculate the sum:

$$W_1 + W_2 = 2\,U\,I\cos(\pi/6)\cos\varphi = \sqrt{3}\,U\,I\cos\varphi = P$$

- If we calculate the difference:

$$W_1 - W_2 = 2\,U\,I\sin(\pi/6)\sin\varphi = U\,I\sin\varphi = Q/\sqrt{3}$$

Then: $P = W_1 + W_2$ and $Q = \sqrt{3}(W_1 - W_2)$

Power measurement in 3-phase

Two Wattmeters method

- The method remains valid for measuring active power in the case of unbalance without neutral:
- Indeed as:

$$j_1 + j_2 + j_3 = 0$$

- Then:

$$p(t) = v_1 j_1 + v_2 j_2 + v_3 j_3 = v_1 j_1 + v_2 j_2 + v_3(-j_1 - j_2)$$
$$= j_1(v_1 - v_3) + j_2(v_2 - v_3) = j_1(t) u_{13}(t) - j_2(t) u_{23}(t)$$

- By taken the average value in both sides:

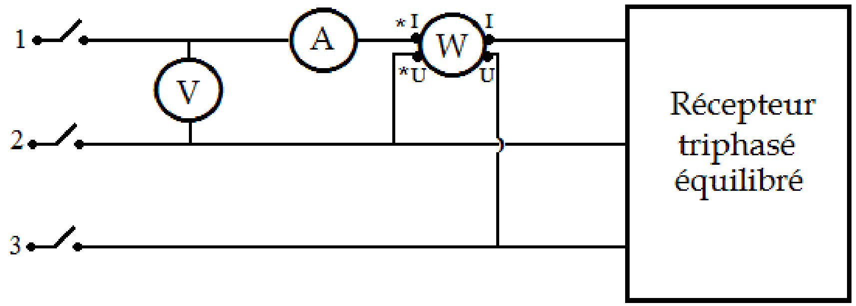
$$P = J_1 U_{13} \cos(\varphi_{u13,\,j1}) + J_2 U_{23} \cos(\varphi_{u23,\,j2}) = W_1 + W_2$$

Power measurement in 3-phase

Boucherot's Method to measure Q

- Allows you to measure the reactive power of a balanced three-phase circuit using a wattmeter.

- Thje following figure illustrates the circuit to use:



- We show that:

$W = U\, I \sin\varphi$ and then $Q = \sqrt{3}\, W$

Power measurement in 3-phase

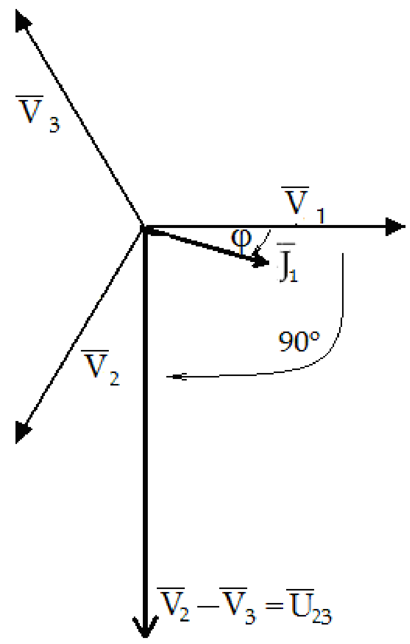
Boucherot's Method to measure Q

- Indeed:

$$W = U_{23} I_1 \cos\varphi_{u23,j1}$$

- But according to Phasor diagram (*Fresnel)*:

$$\varphi_{u23,j1} = \pi/2 - \varphi$$



- and then:

$$W = U I \sin\varphi$$

- and finally:

$$Q = \sqrt{3} W$$

- By using a voltmeter and an amperemeter, we deduce P:

$$P = \sqrt{S^2 - Q^2} \qquad \text{avec} \qquad S = \sqrt{3} U I$$