Mécanique rationnelle 1

Mechanics of rigid bodies

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September 2024

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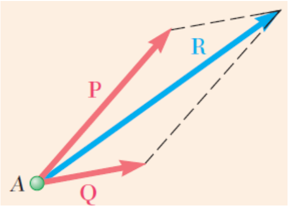
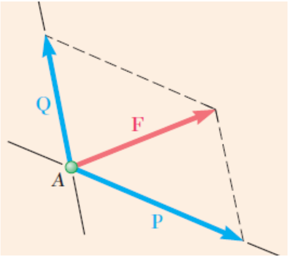
Outline

Resultant and components

Forces are vector quantities characterized by:
a point of application, a magnitude, and a direction.

The resultant (vector addition) can be done according to the parallelogram law.
Any given force acting on a particle can be resolved into two or more components.
The resultant or components can be determined graphically or by trigonometry using the law of cosines and sines.



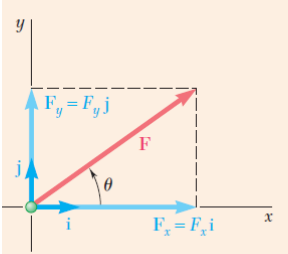


Rectangular Components; Unit Vectors

A force F is resolved into two rectangular components if its components F_x and F_y are perpendicular to each other and are directed along the coordinate axes.

$$F = F_x i + F_y j$$

where:
i and j are the unit vectors along the x and y axes.
F_x and F_y are the scalar components of F (can be positive or negative).



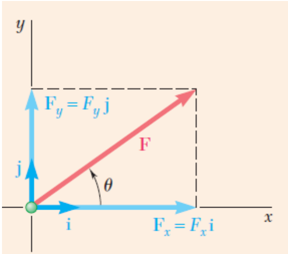
$$F_x = F \cos\theta \;\&\; F_y = F \sin\theta$$

Rectangular Components; Unit Vectors

When the rectangular components F_x and F_y of a force F are given, we can obtain the angle θ defining the direction of the force and the magnitude F of the force from:

$$tan\,\theta = F_y / F_x$$

$$F = \sqrt{F_x^2 + F_y^2}$$



Resultant of Several Coplanar Forces

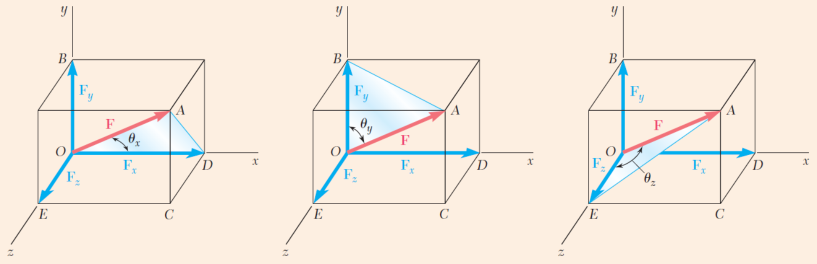
When three or more coplanar forces act on a particle, we can obtain the rectangular components of their resultant R by adding the corresponding components of the given forces algebraically:

$$R_x = \sum F_x \quad R_y = \sum F_y$$

Forces in Space

A force F in three-dimensional space can be resolved into rectangular components F_x, F_y, and F_z. Denoting by θ_x, θ_y, and θ_z, respectively, the angles that F forms with the x, y, and z axes, we have:

$$F_x = \cos\theta_x, F_y = \cos\theta_y, F_z = \cos\theta_z$$



The cosines of θ_x, θ_y, and θ_z are known as the direction cosines of the force F.

Equilibrium of a Particle & Free-Body Diagram

A particle is said to be in equilibrium when the resultant of all the forces acting on it is zero.
The particle remains at rest (if originally at rest) or moves with constant speed in a straight line (if originally in motion).

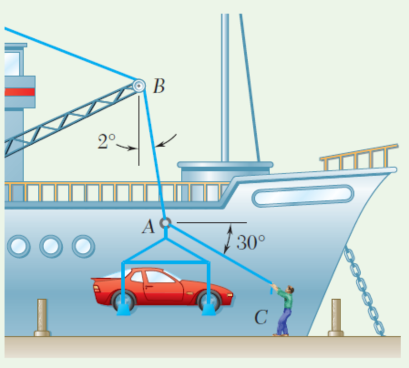
To solve a problem involving a particle in equilibrium, first draw a free-body diagram of the particle showing all of the forces acting on it.

- If only three coplanar forces act on the particle, you can draw a force triangle to express that the particle is in equilibrium. Using graphical methods of trigonometry.
- If more than three coplanar forces are involved, you should use the equations of equilibrium:

$$\sum F_x = 0 \quad \sum F_y = 0$$

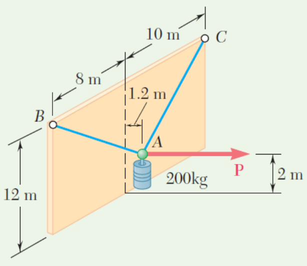
Example 1

In a ship-unloading operation, a $1588kg$ automobile is supported by a cable. A worker ties a rope to the cable at A and pulls on it to centre the automobile over its intended position on the dock. At the moment illustrated, the car is stationary, the angle between the cable and the vertical is 2°, and the angle between the rope and the horizontal is 30°. What are the tensions in the rope and cable?



Equilibrium in Space

A 200-kg cylinder is hung using two cables *AB* and *AC* that are attached to the top of a vertical wall. A horizontal force *P* perpendicular to the wall holds the cylinder in the position shown. Determine the magnitude of *P* and the tension in each cable.

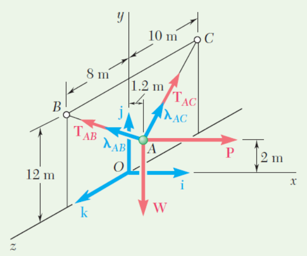


Solution:

$P = 235N$

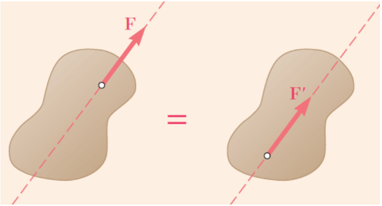
$T_{AB} = 1402N$

$T_{AC} = 1238N$



rigid body equilibrium

- Notion of external and internal forces.
- Principle of transmissibility: the effect of an external force on a rigid body remains unchanged if we move that force along its line of action (Sliding vector).

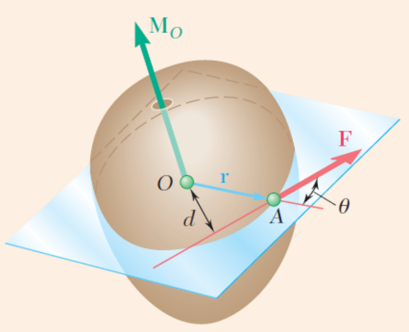


Moment of a Force about a Point

The moment of a force F about a point O:

$$\overrightarrow{M_O} = \overrightarrow{r} \wedge \overrightarrow{F}$$

where $\overrightarrow{r}$ is the position vector drawn from O to the point of application A of the force $\overrightarrow{F}$. Denoting the angle between the lines of action of $\overrightarrow{r}$ and $\overrightarrow{F}$ as θ, we found that the magnitude of the moment of $\overrightarrow{F}$ about O is:



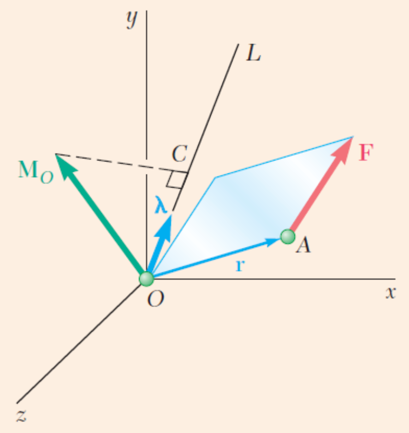
$$M_O = rFsin\theta = Fd$$

where d represents the perpendicular distance from O to the line of action of $\overrightarrow{F}$.

Moment of a Force about an Axis

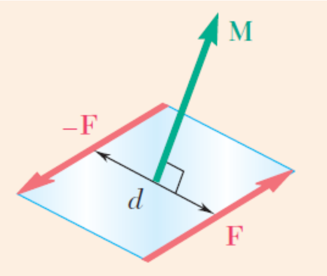
The moment of a force $\overrightarrow{F}$ about an axis OL is the projection $\overrightarrow{OC}$ on OL of the moment $\overrightarrow{M_O}$ of the force $\overrightarrow{F}$, i.e., as the mixed triple product of the unit vector $\overrightarrow{\lambda}$, the position vector $\overrightarrow{r}$, and the force $\overrightarrow{F}$:

$$M_{OL} = \overrightarrow{\lambda}.\overrightarrow{M_O} = \overrightarrow{\lambda}.(\overrightarrow{r} \wedge \overrightarrow{F})$$



Couples

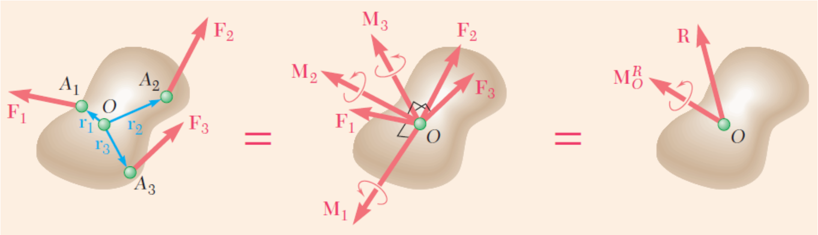
Two forces F and $-F$ having the same magnitude, parallel lines of action, and opposite sense are said to form a couple. The moment of a couple is independent of the point about which it is computed (**free vector**); it is a vector M perpendicular to the plane of the couple and equal in magnitude to the product of the common magnitude F of the forces and the perpendicular distance d between their lines of action.



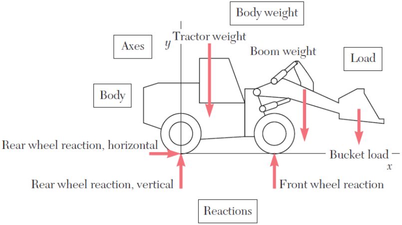
Reduction of a System of Forces to a Force-Couple System

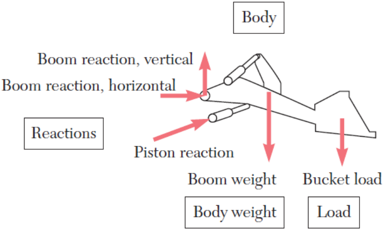
Any system of forces can be reduced to a force-couple system at a given point O by:

- first replacing each of the forces of the system by an equivalent force-couple system at O;
- and then adding all of the forces and all of the couples to obtain a resultant force R and a resultant couple vector M_O^R

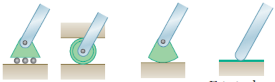
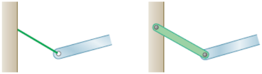
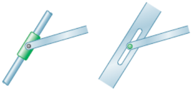
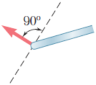


Free-Body Diagrams





EQUILIBRIUM IN TWO DIMENSIONS - Reactions

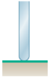
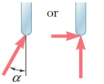
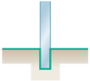
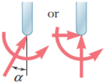
Support or Connection	Reaction	Number of Unknowns
 Rollers, Rocker, Frictionless surface	 Force with known line of action perpendicular to surface	1
 Short cable, Short link	 Force with known line of action along cable or link	1
 Collar on frictionless rod, Frictionless pin in slot	 Force with known line of action perpendicular to rod or slot	1

This rocker bearing supports the weight of a bridge. The convex surface of the rocker allows the bridge to move slightly horizontally.

Links are often used to support suspended spans of highway bridges.

Force applied to the slider exerts a normal force on the rod, causing the window to open.

EQUILIBRIUM IN TWO DIMENSIONS - Reactions

 Frictionless pin or hinge	 Rough surface	 Force of unknown direction	2
 Fixed support		 Force and couple	3

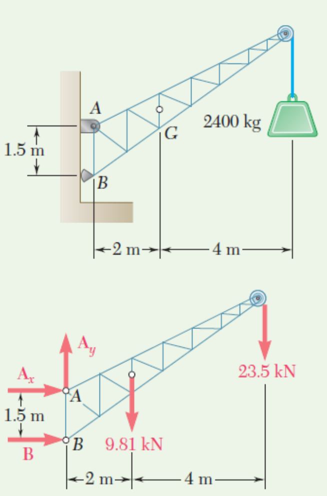
Pin supports are common on bridges and overpasses.

This cantilever support is fixed at one end and extends out into space at the other end.

Example

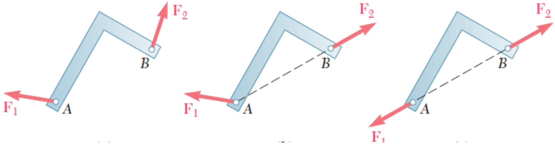
A fixed crane has a mass of $1000kg$ and is used to lift a $2400 - kg$ crate. It is held in place by a pin at A and a rocker at B. The crane's centre of gravity is located at G. Determine the components of the reactions at A and B.

Solution: $B = 107.1kN, A_x = -107.1kN, A_y = 33.3kN$



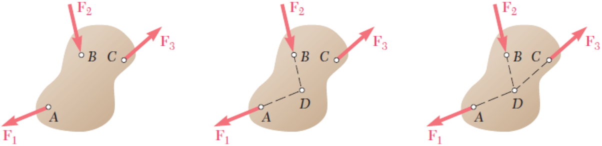
Equilibrium of a Two-Force Body

if a two-force body is in equilibrium, the two forces must have the same magnitude, the same line of action, and the opposite sense.

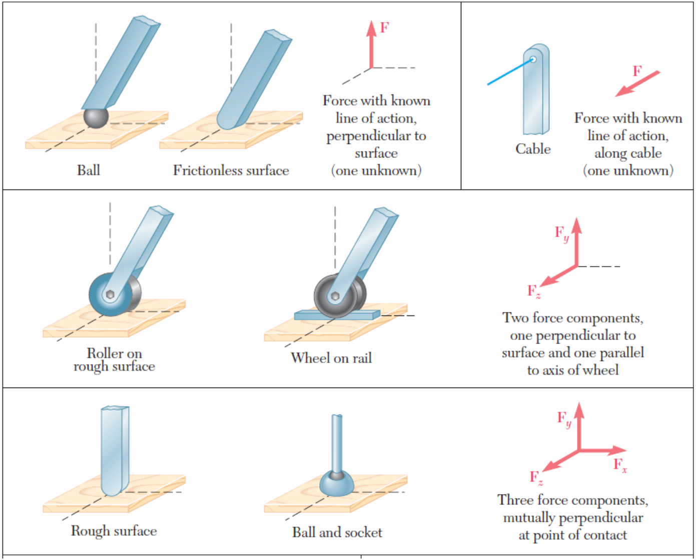


Equilibrium of a Three-Force Body

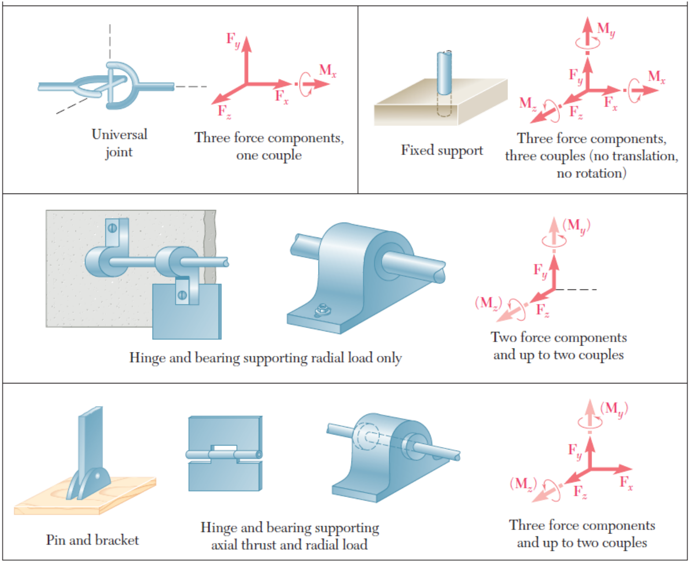
The lines of action of the three forces must be either concurrent or parallel.



EQUILIBRIUM IN THREE DIMENSIONS - Reactions

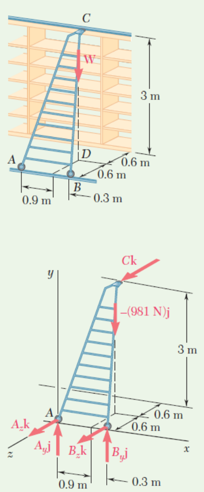


EQUILIBRIUM IN THREE DIMENSIONS - Reactions



Example

A $20-kg$ ladder used to reach high shelves in a storeroom is supported by two flanged wheels A and B mounted on a rail and by a flangeless wheel C resting against a rail fixed to the wall. An $80-kg$ man stands on the ladder and leans to the right. The line of action of the combined weight W of the man and ladder intersects the floor at point D. Determine the reactions at A, B, and C.



Solution:

$$B=(736N)j+(98.1N)k,\ C=(196.2N)k,\ A=(245N)j-(98.1N)k$$