

Corrigé Série 1

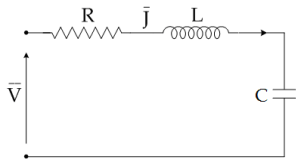
Exercices 5, 6, 7 et 8

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Solution Exercice 5

Soit le circuit RLC série suivant :



$$v(t) = 200\sqrt{2}\sin(100\pi t) \quad (1)$$

Données : $R = 10\Omega$, $L = 0.1H$, $C = 200\mu F$

1) $\omega = 2\pi f = 100\pi \implies f = 50\text{ Hz}$, $V_m = 200\sqrt{2}\text{ V}$, $V = 200\text{ V}$.

2) En représentation complexe :

$$\bar{V} = 200e^{j*0} = 200\text{ Volt.}$$

$$\bar{Z}_t = R + jX, \quad \text{avec } R = 10\Omega \quad \text{et} \quad X = L\omega - \frac{1}{C\omega} = 15.50\Omega$$

D'où

$$\bar{Z}_t = (10.00 + 15.50j)\Omega \quad (2)$$

Comme $X > 0$ donc le circuit est inductif.

3) On a

$$\bar{Z}_t = \frac{\bar{V}}{\bar{J}} \longrightarrow \bar{J} = \frac{\bar{V}}{\bar{Z}_t} = \frac{200}{10.00 + 15.50j} = \frac{200(10.00 - 15.50j)}{100 + (15.50)^2}$$

$$\longrightarrow \bar{J} = 5.88 - 9.11j \quad [A]$$

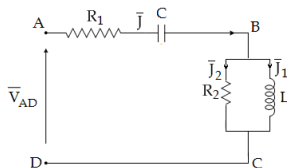
$$\bar{J} = |\bar{J}|e^{j\varphi_J} \rightarrow |\bar{J}| = \sqrt{(5.88)^2 + (9.11)^2} \simeq 10.8A$$

$$\varphi = \arctan\left(-\frac{9.11}{5.88}\right) = -57 \sim -60$$

$$j(t) = 10.8\sqrt{2}\sin(100\pi t - \frac{\pi}{3})$$

solution exercice 6

Soit le circuit suivant :



Données : $V_{AD} = 200 \text{ V}$, $R_1 = 8 \Omega$, $L\omega = \frac{1}{C\omega} = 10 \Omega$, $R_2 = 12 \Omega$.

1)

$$\begin{aligned}\bar{Z}_{BC} &= \frac{\bar{Z}_L \bar{Z}_R}{\bar{Z}_R + \bar{Z}_L} = \frac{jL\omega R_2}{R_2 + jL\omega} \\ &= \frac{120j}{12 + 10j} = \frac{120j(12 - 10j)}{12^2 + 10^2} \\ \bar{Z}_{BC} &= 4.92 + 5.90j[\Omega]\end{aligned}$$

2) On a $\bar{V}_{AD} = V_{AD}$ car cette tension est choisie comme origine des phases.

Solution exercice 6

$$\bar{V}_{AD} = V_{AD} = \bar{V}_{AB} + \bar{V}_{BC} = \left(R_1 + \frac{1}{jC\omega}\right) \bar{J} + \bar{Z}_{BC} \bar{J}$$

D'où

$$\begin{aligned} \bar{J} &= \frac{V_{AD}}{R_1 + \frac{1}{jC\omega} + \bar{Z}_{BC}} = \frac{200}{8 - 10j + 4.92 + 5.90j} = \frac{200}{12.92 - 4.10j} \\ &= 200 \frac{12.92 + 4.10j}{12.92^2 + 4.10^2} = 14.06 + 4.46j \quad [A] \end{aligned}$$

3)

$$\bar{V}_{AB} = \left(R_1 + \frac{1}{jC\omega}\right) \bar{J} = (8 - 10j)(14.06 + 4.46j) = 157.1 - 104.9j$$

$$V_{AB} = \sqrt{157.1^2 + 104.9^2} = 188.9 \text{ V}$$

Solution exercice 6

$$\bar{V}_{BC} = \bar{Z}_{BC} \bar{J} = (4.92 + 5.90j)(14.06 + 4.46j) = 42.9 + 104.9j$$

$$V_{BC} = \sqrt{42.9^2 + 104.9^2} = 113.3 \text{ V}$$

Vérification des égalités : $V_{AD} = \bar{V}_{AB} + \bar{V}_{BC}$ et $\bar{J} = \bar{J}_1 + \bar{J}_2$. On a :

$$\bar{V}_{AB} + \bar{V}_{BC} = 157.1 - 104.9j + 42.9 + 104.9j = 200 \text{ V} = V_{AD}$$

La première égalité est vérifiée. On a :

$$\bar{V}_{BC} = R_2 \bar{J}_2 = jL\omega \bar{J}_1$$

$$\bar{J}_1 = \frac{\bar{V}_{BC}}{jL\omega} = \frac{42.9 + 104.9j}{10j} = 10.49 - 4.29j$$

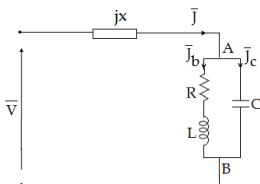
$$\bar{J}_2 = \frac{\bar{V}_{BC}}{R_2} = \frac{42.9 + 104.9j}{12} = 3.57 + 8.74j$$

$$\longrightarrow \bar{J}_1 + \bar{J}_2 = 14.06 + 4.45j = \bar{J}$$

La deuxième égalité est vérifiée.

Solution exercice 7

Soit le circuit suivant :



$$f = 400\text{Hz}, J_c = 0.1\text{A}, L = 50\text{mH}, R = 25\Omega, C = 0.8\mu\text{F}.$$

1) L'impédance équivalente complexe totale de ce circuit est

$$\bar{Z}_t = jx + \bar{Z}_{AB}$$

$$\frac{1}{\bar{Z}_{AB}} = \frac{1}{\bar{Z}_C} + \frac{1}{\bar{Z}_L}, \quad \text{avec} \quad \bar{Z}_C = 1/jC\omega, \bar{Z}_L = jL\omega$$

$$1/\bar{Z}_{AB} = jC\omega + \frac{1}{R + jL\omega} = \frac{1 - LC\omega^2 + jRC\omega}{R + jL\omega}$$

$$\bar{Z}_{AB} = \frac{R + jL\omega}{1 - LC\omega^2 + jRC\omega} \rightarrow \bar{Z}_t = jx + \frac{R + jL\omega}{1 - LC\omega^2 + jRC\omega}$$

Solution exercice 7

$$\begin{aligned}\bar{Z}_t &= jx + \frac{25 + j * 5 * 10^{-2} * 800\pi}{1 - 5 * 10^{-2} * 8 * 10^{-7} * (800\pi)^2 + j * 25 * 8 * 10^{-7} * 800\pi} \\&= jx + \frac{25 + 125.66j}{0.75 + 0.05j} = jx + \frac{(25 + 125.66j)(0.75 - 0.05j)}{0.75^2 + 0.05^2} \\&= jx + 1.77(25.03 + 92.99j) = 44.30 + j(x + 164.59)\end{aligned}$$

2) Le circuit est résonnant si la partie imaginaire de l'impédance est nulle. Donc si $x = -164.6\Omega$, la réactance est de nature capacitive.

3) $J_c = 0.1A$ et $\bar{J}_c = J_c$ car on choisit \bar{J}_c origine des phases. On a :

$$\begin{aligned}\bar{V}_{AB} &= \bar{Z}_C \bar{J}_c = (R + jL\omega) \bar{J}_b \\ \bar{J}_b &= \frac{1}{jC\omega(R + jL\omega)} \bar{J}_c = \frac{1}{C\omega(-L\omega + jR)} \bar{J}_c \\ &= \frac{-L\omega - jR}{C\omega((L\omega)^2 + (R)^2)} J_c = \frac{-40\pi - 25j}{64\pi * 10^{-5}((40\pi)^2 + (25)^2)} * 0.1\end{aligned}$$

Solution exercice 7

$$\bar{J}_b = -0.381 - 0.076j[A]$$

$$J_b = \sqrt{(0.381)^2 + (0.076)^2} = 0.388A$$

La loi des nœuds donne : $\bar{J} = \bar{J}_b + \bar{J}_c$

$$\rightarrow \bar{J} = -0.381 - 0.076j + 0.1 = -0.281 - 0.076j$$

$$J = \sqrt{(0.281)^2 + (0.076)^2} = 0.291A$$

$$\bar{V} = jx\bar{J} + \bar{V}_{AB} = jx\bar{J} + \frac{\bar{J}_c}{jC\omega}$$

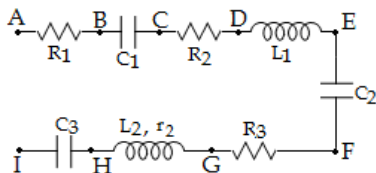
Pour $x = -164.6\Omega$

$$\bar{V} = 164.6j(0.281 + 0.076j) - j \frac{0.1}{0.8 * 10^{-6} * 800\pi} = -12.50 - 3.51j$$

$$V = \sqrt{(12.50)^2 + (3.51)^2} = 12.98V \simeq 13V.$$

Solution exercice 8

Soit le circuit suivant :



Données : $J = 0.1A$, $R_1 = L_1\omega = 60\Omega$, $R_2 = \frac{1}{C_1\omega} = 80\Omega$, $R_3 = \frac{1}{C_2\omega} = 100\Omega$, $r_2 = L_2\omega = 60\Omega$, $\frac{1}{C_3\omega} = 140\Omega$.

a) Variante 1 : Tronçon AC :

$$\bar{Z}_{AC} = R_1 + \frac{1}{jC_1\omega} = R_1 - jR_2 = 60 - 80j \quad [\Omega]$$

$$|Z| = \sqrt{60^2 + 80^2} = 100\Omega$$

$$\bar{V}_{AC} = \bar{Z}_{AC}\bar{J}$$

Solution exercice 8

On a

$$\bar{J} = J e^{j \times 23^\circ} = J (\cos 23 + j \sin 23) = 0.092 + j0.039$$

D'où

$$\bar{V}_{AC} = (60 - 80j)(0.092 + j0.039) = 8.64 - 5.02j$$

$$|V|_{AC} = \sqrt{8.64^2 + 5.02^2} = 10 \text{ V}$$

$$\varphi_V = \arctan\left(-\frac{5.02}{8.64}\right) = -30^\circ \rightarrow \varphi = \varphi_J - \varphi_V = 23^\circ + 30^\circ = 53^\circ$$

$$\bar{V}_{AC} = 10 e^{-j30^\circ} \quad (3)$$

$$v(t) = 10\sqrt{2} \sin(100\pi t - 30^\circ) \quad (4)$$

b) Variante 2 : Tronçon BD :

$$\bar{Z}_{BD} = R_2 + \frac{1}{jC_1\omega} = R_2 - jR_2 = 80 - 80j \quad [\Omega]$$

$$|Z| = \sqrt{80^2 + 80^2} = 113.14 \Omega$$

Solution exercice 8

$$\bar{V}_{BD} = \bar{Z}_{BD} \bar{J}$$

On a

$$\bar{J} = J e^{j \times 60^\circ} = J (\cos 60 + j \sin 60) = 0.05 + j0.087$$

D'où

$$\bar{V}_{BD} = 80 (1 - j) (0.05 + j0.087) = 10.93 + 2.96j$$

$$|V|_{BD} = \sqrt{10.93^2 + 2.96^2} = 11.3 V$$

$$\varphi_V = \arctan \left(\frac{2.96}{10.93} \right) = 15^\circ \rightarrow \varphi = \varphi_J - \varphi_V = 60^\circ - 15^\circ = 45^\circ$$

On en déduit

$$\bar{V}_{BD} = 11.3 e^{j15^\circ} \quad (5)$$

$$v(t) = 11.3 \sqrt{2} \sin(100\pi t + 15^\circ) \quad (6)$$

c) Variante 3 : Tronçon CE :

$$\bar{Z}_{CE} = R_2 + jL_1\omega = R_2 + jR_1 = 80 + 60j \quad [\Omega]$$

$$|Z| = \sqrt{80^2 + 60^2} = 100\Omega$$

$$\bar{V}_{CE} = \bar{Z}_{CE}\bar{J}$$

On a

$$\bar{J} = J e^{-j \times 17^\circ} = J (\cos 17 - j \sin 17) = 0.096 - j0.029$$

$$\rightarrow \bar{V}_{CE} = (80 + 60j)(0.096 - j0.029) = 9.42 + 3.44j$$

$$|V|_{CE} = \sqrt{9.42^2 + 3.44^2} = 10V$$

$$\varphi_V = \arctan\left(\frac{3.44}{9.42}\right) = 20^\circ \rightarrow \varphi = \varphi_J - \varphi_V = -17^\circ - 20^\circ = -37^\circ$$

On en déduit

$$\bar{V}_{CE} = 10e^{j20^\circ},$$

$$v(t) = 10\sqrt{2} \sin(100\pi t + 20^\circ)$$

d) Variante 7 : Tronçon GI :

$$\bar{Z}_{GI} = r_2 + jL_2\omega + \frac{1}{jC_3\omega} = r_2 + jr_2 - j\frac{1}{C_3\omega} = 60 - 80j \quad [\Omega]$$

$$|Z| = \sqrt{60^2 + 80^2} = 100\Omega$$

$$\bar{V}_{GI} = \bar{Z}_{GI}\bar{J}$$

On a

$$\bar{J} = J e^{-j \times 17^\circ} = J(\cos 17 - j \sin 17) = 0.096 - j0.029$$

$$\rightarrow \bar{V}_{GI} = (60 - 80j)(0.096 - j0.029) = 3.44 - 9.42j$$

$$|V|_{GI} = \sqrt{3.44^2 + 9.42^2} = 10V$$

$$\varphi_V = \arctan\left(-\frac{9.42}{3.44}\right) = -70^\circ \rightarrow \varphi = \varphi_J - \varphi_V = -17^\circ + 70^\circ = 53^\circ$$

On en déduit

$$\bar{V}_{GI} = 10e^{-j70^\circ} \quad (7)$$

$$v(t) = 10\sqrt{2} \sin(100\pi t - 70^\circ) \quad (8)$$