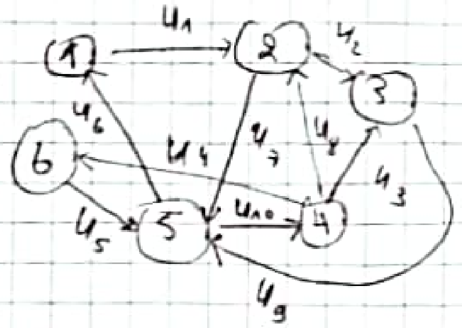
Tutorial Sheet
N° 2

Ex1:

1) a)



b)

v	d
1	2
2	4
3	3
4	4
5	5
6	2

c) a circuit : $3 \to 5 \to 4 \to 3$

$1 \to 2 \to 3 \to 5 \to 1$

2) a) no because we have passed through the vertex "c" twice

b) no because the edge AC appears twice

Ex 2:

1) G1: Symmetric connected eulerian, Hamiltonian

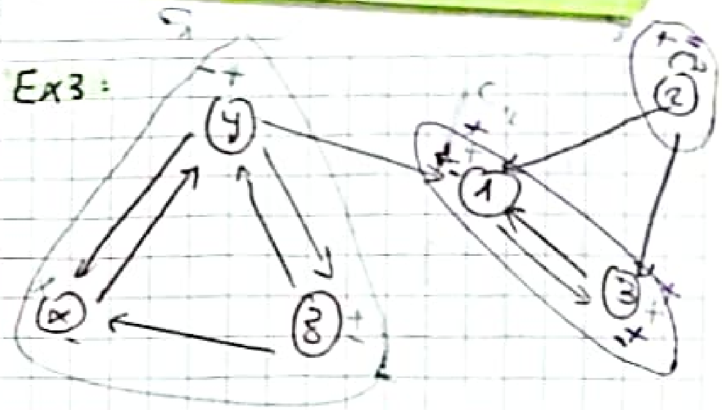
G2: Rooted tree, Bipartite

2) G1 : spanning subgraph (partial) not connected

G2 : induced subgraph

G3 : Spanning, induced subgraph

Ex3:



1- the graph is not strongly connected

2- $C_1 \longrightarrow C_2 \longleftarrow C_3$

Ex4:

$\sum d(x) = 2 * |E| = 2 \times 22 = 44$

we have 5V of degree 3 $(5\times3=15)$

$44 - 15 = 29$

$4 \to x$

$7 \to y$

since we have 10 vertices

$x+y=5$ and $4x+7y=29$

$x = 5-y$

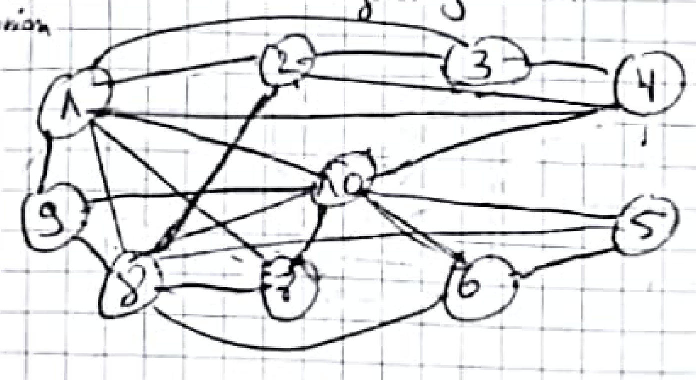
$4(5-y)+7y=29$

$20-4y+7y=29$

$3y=9 \Rightarrow \boxed{y=3 \to 7}$

$\boxed{x=2 \to 4}$

2 vertices of degree 4



Ex5 : number of edges $m = n(n-1)/2 \to$ complete graph

1) $d(x) = n-1$

$\Sigma d(x) = n(n-1) = 2.m$

so $m = \dfrac{n(n-1)}{2}$

2) we suppose that we have a disconnected graph and undirected (2 connected components)

the 1st one is complete and it contains $(n-1)$ vertices

the 2nd one contains one vertex (isolated)

the number of edges is :

$(n-1)(n-2)/2$

So if we add one edge between the 2 componenents, the graph will become connected

Ex 6 :

1) We can represent a graph by :

① An adjacency Matrix

② An incidence matrix

③ Adjacency linked lists

2)

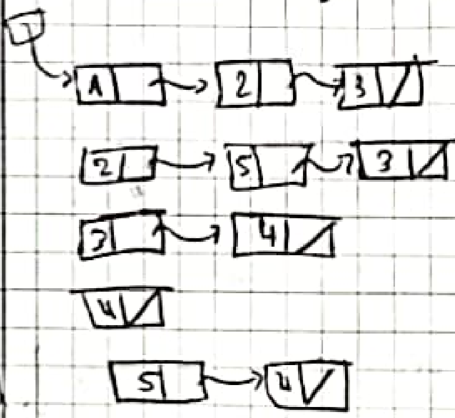
	1	2	3	4	5
1	0	1	1	0	0
2	1	0	1	0	1
3	1	1	0	1	0
4	0	0	1	0	1
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Deff $A_{ij} = \begin{cases} 1 & \text{if there's an edge between } i \text{ and } j \\ 0 & \text{else} \end{cases}$

Incidence matrix :

	e_1	e_2	e_3	e_4	e_5	e_6
1	1	1	0	0	0	0
2	-1	-1	1	1	0	0
3	0	0	-1	0	0	1
4	0	0	0	0	-1	-1
5	0	0	0	-1	1	0

deff $A_{ij} = \begin{cases} 1 & \text{if there's an edge from } i \text{ to } j \\ 0 & \text{if there's no connection between } i \text{ } j \\ -1 & \text{if there's edge from } j \text{ to } i \end{cases}$



Ex 7 :

1) 0 or 2 odd vertices $\Rightarrow$ Eulerian graph

eulerian Cycle $\Rightarrow$ all the vertices have even degree

eulerian path $\Rightarrow$ only 2 vertices " odd degree

*) yes because all the vertices have even degrees

finding one :

{10, 15, 14, 13, 11, 12, 9, 6, 7, 3, 1, 2, 8, 4, 5}

2) Hamiltonian path :

to say that the graph is Hamiltonian, it has to contain at least 2 vertices with Zero plus-degree, so we can't extract Hamiltonian path from it

3) Hamiltonian circuit

(1, 2, 5, 4, 3, 6, 1)

Hamiltonian path:

(4, 3, 6, 5, 1, 2)

the graph is hamiltonian because it admits a Hamitonian circuit