NATIONAL POLYTECHNIC SCHOOL
Department of Integrated Preparatory Classes

Module : Physics 3
2^{nd} year 2024/2025

Set 4
Forced vibration of damped single degree of freedom systems

Exercise 1:
For the systems shown in *Figure1* and *Figure2*, frames B1 and B2 are rigid and of negligible mass. These frames can either be fixed or subjected to displacements or forces imposed by an external manipulator. For each case, determine the differential equation and the resonance frequency.
For Figure 1:
- **Case 1:** $F(t) = 0, S_2(t) = 0,$ and $S_1(t) = S_0\cos(\Omega t)$
- **Case 2:** $S_1(t) = 0, S_2(t) = 0,$ and $F_1(t) = F_0\cos(\Omega t)$
- **Case 3:** $S_1(t) = 0, F_1(t) = 0,$ and $S_2(t) = S_0\sin(\Omega t)$

For Figure 2:
$S_2(t) = S_0\cos(\Omega t)$
(Hint: It is recommended to perform a change of variable $x - s = y$*)*

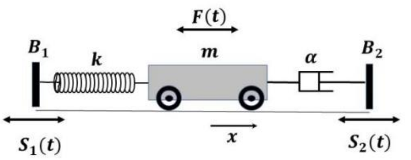


Figure1

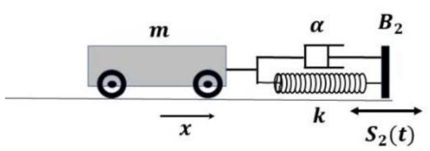
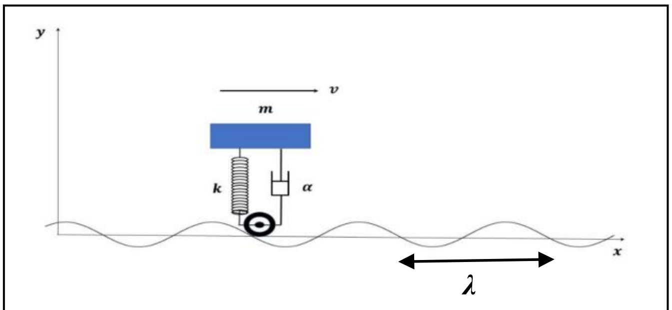


Figure2

Exercise 2:
Figure 4 illustrates a car with a mass m, whose suspension system is represented by a damper with a viscous damping coefficient $\alpha$and a spring with a stiffness constant k. The wheels have negligible masses compared to m. This car moves in the Ox direction along an uneven road, the profile of which is assumed to be sinusoidal $y_1(x) = Y_1 \sin\left(\frac{2\pi x}{\lambda}\right)$ *or* $y_1(t) = Y_1\sin(\Omega t)$,. The car moves along Ox, at a constant speed V , and its vertical position is represented by the variable $y(t)$. It is assumed that there is no lateral movement (along the Oz axis).

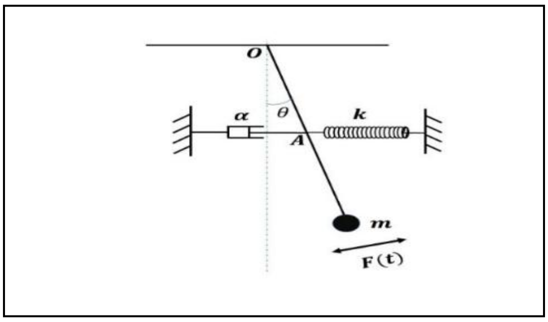
-Establish the differential equation that governs the variations of the y coordinate over time.
- Deduce the amplitude Y of the motion. Only the vertical motion of the car is of interest.



Exercise 3:

In the figure below, a simple pendulum consisting of a point mass m attached to a thin massless rod of length l . At its midpoint $A(OA = l/2)$, it is connected to two rigid walls: one side with a spring of stiffness$K$, and the other with a viscous damper characterized by a damping coefficient α. A sinusoidal force$F(t) = F_0\cos(\Omega t)$ acts on m, with its direction perpendicular to the rod. The equilibrium position is defined by $\theta = 0$.

- Determine the kinetic energy, dissipation energy, and potential energy, and derive the differential equation of motion.
- Calculate the instantaneous power supplied and dissipated, as well as their average values.



Exercise 4:

A cylinder with negligible mass, capable of rotating around its axis at point C, is connected to a fixed vertical support by a spring-damper system, as shown in the figure below. A mass m is welded onto this cylinder at a distance a from point C. At equilibrium, C is at O, the spring is undeformed, and the mass m is positioned vertically. When the cylinder rotates with an angular velocity Ω, the system vibrates horizontally because the center of mass of the system does not coincide with the axis of rotation. The motion of this system is studied as a function of y.

- What is the type of excitation of this system? Determine the equivalent model and specify the expression for the excitation force.
- Find the differential equation of motion in terms of y.
- Determine the expression for the amplitude A of the displacement in steady-state motion.
- To eliminate the cylinder's oscillations regardless of the rotational speed Ω, an additional mass M is added to the cylinder at a distance R from the center C. Determine R as a function of M, m, and a.