

Series 1 =

Ex 3:

1/ Calculer la surface

$$S = \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}$$

$$A(S) = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} dx dy$$

$$\frac{\partial z}{\partial x}(x, y) = 2$$

$$\Rightarrow A(S) = \iint_D \sqrt{1+4} dy$$

$$\frac{\partial z}{\partial y}(x, y) = 3$$

$$= \iint_D \sqrt{14}$$

$$= \sqrt{14} \quad A(D) = \sqrt{14} \cdot 9\pi = 9\sqrt{14}\pi$$

$$2) f(x, y) = 9 - x^2$$

$$D = [0, 2] \times [0, 2]$$

$$A = \int_0^2 \int_0^2 \sqrt{1 + (-2x)^2} dx dy$$

$$A = 2 \int_0^2 \sqrt{1+4x^2} dx$$

$$A = 2 \int_0^{\ln(4+\sqrt{17})} \sqrt{1+5t^2} \cdot e^t dt$$

we put

$$2x = \sinh t = \frac{e^t - e^{-t}}{2}$$

$$A = 2 \int_0^{\ln(4+\sqrt{17})} e^t dt$$

$$dx = e^t dt$$

$$x=0 \rightarrow t=0$$

$$x=2 \rightarrow t = \ln(4+\sqrt{17})$$

$$u = e^t$$

$$u = \frac{1}{4} = 8$$

$$u^2 - 8u - 1 = 0$$

$$(u-4)^2 = 17$$

$$u = 4 \pm \sqrt{17}$$

$$t = \ln(4+\sqrt{17})$$

$$= \int_0^{\ln(4+\sqrt{17})} 2 \left(\frac{e^t + e^{-t}}{2} \right)^2 dt$$

$$= \frac{1}{2} \int_0^{\ln(4+\sqrt{17})} (e^{2t} + e^{-2t} + 2) dt$$

$$= \frac{1}{2} \left[\frac{1}{2} e^{2t} - \frac{1}{2} e^{-2t} + 2t \right]_0^{\ln(4+\sqrt{17})}$$

$$= \frac{1}{2} \left[\frac{(4+\sqrt{17})^2 - (4-\sqrt{17})^2}{2} + 2 \ln(4+\sqrt{17}) \right]$$

$$= \frac{1}{4} \left[(4+\sqrt{17})^2 - (4-\sqrt{17})^2 \right] + \ln(4+\sqrt{17})$$

$$= \frac{1}{4} \times 8 \times 2\sqrt{17} + \ln(4+\sqrt{17})$$

$$= 4\sqrt{17} + \ln(4+\sqrt{17})$$

$$3) A(S) = \iint_D \sqrt{1 + \frac{\cos^2 x}{\sin^4 x}} dx dy$$

$$= \int_0^{\pi/4} \int_0^{\tan x} \left[\frac{1}{\sin x} dy \right] dx$$

$$= \int_0^{\pi/4} \frac{\tan x}{\sin x} dx \Rightarrow \int_0^{\pi/4} \frac{1}{\cos x} dx =$$

$$\int_0^{\pi/4} \frac{\cos u}{\cos^3 u} du = \int_0^{\pi/4} \frac{\cos u}{1 - \sin^2 u} du$$

$$\sin u = t$$

$$dt = \cos u du$$

$$\int_0^{\frac{\sqrt{2}}{2}} \frac{1}{1-t^2} dt$$

$$= \frac{1}{2} \int_0^{\frac{\sqrt{2}}{2}} \left(\frac{1}{1-t} + \frac{1}{1+t} \right) dt$$

$$= \frac{1}{2} \left[\ln \left(\frac{1+t}{1-t} \right) \right]_0^{\frac{\sqrt{2}}{2}}$$

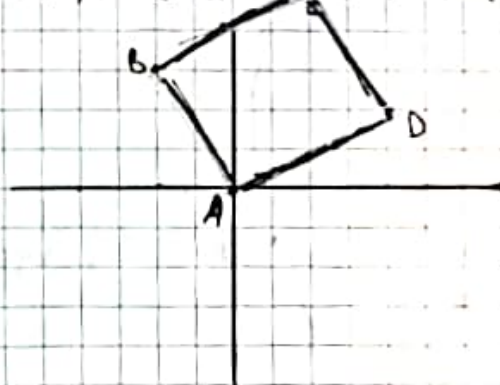
$$= \frac{1}{2} \ln \frac{1+\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}} = \frac{1}{2} \ln \left(\frac{2+\sqrt{2}}{2-\sqrt{2}} \right)$$

$$= \frac{1}{2} \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} = \ln(1+\sqrt{2})$$

Ex 4:

$$① f(x, y) = (3x+2y)^2 \sqrt{2y-x}$$

$$A(0,0), B(-2,3), C(2,5), D(4,2)$$



$$A(x_A, y_A), B(x_B, y_B)$$

$$(AB): y = y_B + \left(\frac{y_B - y_A}{x_B - x_A} \right) (x - x_A)$$

$$AD: y = 2 + \frac{1}{2}(x-4) \Leftrightarrow 2y - x = 0$$

$$BC: y = 5 + \frac{1}{2}(x-2) \Leftrightarrow 2y - x = 8$$

$$CD: y = 2 + \frac{-3}{2}(x-4) \Leftrightarrow 2y + 3x = 8$$

$$BA: y = \frac{-3}{2}x \Leftrightarrow 2y + 3x = 0$$

$$u = 2y - x \quad v = 2y + 3x$$

$$g(x) = (3x + 2y)^2 \sqrt{2y - x}$$

$$\text{Jac}(u, v) = \frac{1}{\det(\text{Jac}(x, y))}$$

$$\left| \text{Jac}(x, y) \right| = \begin{vmatrix} 3 & 2 \\ -1 & 2 \end{vmatrix} = 6 - 2 = 4$$

$$\left| \text{Jac}(u, v) \right| = \frac{1}{4}$$

$$V = \frac{1}{4} \int_0^4 \int_0^{4-u} u^2 \sqrt{v} \, du \, dv$$

$$= \frac{1}{4} \int_0^4 u^2 \, du \int_0^{4-u} \sqrt{v} \, dv$$

$$= \frac{1}{4} \left[\frac{u^3}{3} \right]_0^4 \left[\frac{2}{3} v^{3/2} \right]_0^{4-u}$$

$$= \frac{2048}{9} \sqrt{2}$$

$$(2) f(x, y) = \frac{xy}{1+x^2y^2}$$

$$u = xy$$

$$\text{when } u \in [1, 4]$$

$$\text{and } v = x \quad v \in [1, 4]$$

$$\left| \text{Jac} \right| = \begin{vmatrix} y & x \\ 1 & y \end{vmatrix} = y - x$$

$$\left| \text{Jac}(x, y) \right| = \frac{1}{u}$$

$$V = \int_1^4 \int_1^{4/u} \frac{u}{1+u^2} \left| \frac{-1}{v} \right| \, du \, dv$$

$$= \left(\int_1^4 \frac{u}{1+u^2} \, du \right) \left(\int_1^{4/u} \frac{1}{v} \, dv \right)$$

$$= \frac{1}{2} \left[\ln(1+u^2) \right]_1^4 \left[\ln v \right]_1^{4/u}$$

$$= \frac{1}{2} \ln \left(\frac{17}{2} \right) (\ln 4) = \ln(2) \ln \left(\frac{17}{2} \right)$$

Ex 5:

A) finding the Mass

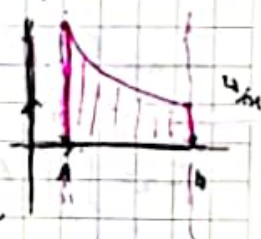
$$M = \iint_D g(x, y) \, dx \, dy \quad D = \begin{cases} 1 < x < 4 \\ 0 < y < 4/x \end{cases}$$

$$M = \int_1^4 \int_0^{4/x} (kx^2) \, dy \, dx$$

$$M = \int_1^4 kx^2 \int_0^{4/x} 1 \, dy \, dx$$

$$M = \int_1^4 kx^2 \left[y \right]_0^{4/x} \, dx$$

$$= \int_1^4 kx^2 \times \frac{4}{x} \, dx = 4k \int_1^4 x \, dx = 30k$$



calculating the centre of mass

$$\bar{x} = \frac{1}{M} \iint_D x f(x, y) \, dy \, dx$$

$$\bar{x} = \frac{1}{30k} \int_1^4 \int_0^{4/x} kx^3 \, dy \, dx$$

$$\bar{x} = \frac{1}{30k} \int_1^4 x^3 \times \frac{4}{x} \, dx$$

$$\bar{y} = \frac{1}{30} \left[\frac{1}{2} n^3 \right]_1^4 = \frac{14}{30} \quad (63)$$

$$= \frac{7}{15}$$

$$\bar{y} = \frac{1}{M} \iint_D y f(m, y) dm dy$$

$$\bar{y} = \frac{1}{30K} \int_1^4 \int_0^{4/n} K n^2 y dy dn$$

$$\bar{y} = \frac{1}{30} \int_1^4 n^2 \left[\frac{1}{2} y^2 \right]_0^{4/n} dn$$

$$\bar{y} = \frac{1}{30} \int_1^4 n^2 \left(\frac{1}{2} \times \frac{16}{n^2} \right) dn$$

$$\bar{y} = \frac{1}{30} \times \frac{16}{2} [3] = \frac{7}{15}$$

the centre of mass $(\bar{x}, \bar{y}) = (14/15, 7/15)$

$$I_x = \iint_D y^2 f(m, y) dm dy$$

$$I_x = \int_1^4 \int_0^{4/n} K n^2 y^2 dy dn$$

$$= \int_1^4 K n^2 \left[\frac{1}{3} y^3 \right]_0^{4/n} dn$$

$$= \int_1^4 4 n^2 \times \frac{1}{3} \frac{64}{n^3} = \frac{256}{3} K \left[\ln n \right]_1^4$$

$$I_x = \frac{256}{3} K \ln 4$$

$$I_y = \iint_D x^2 f(m, y) dm$$

$$I_y = \int_1^4 \int_0^{4/n} K n^2 dy dn$$

$$= \int_1^4 K n^2 [y]_0^{4/n} dn$$

$$= 4K \left[\frac{1}{4} n^4 \right]_1^4 = K [256 - 1]$$

$$= 255K$$

Ex 7:

$m > 0$

$$1) z = 9 - m^2, y = 2 - m^2, y = 0$$

$$z = 0$$

$$\text{Vol}(D) = \iiint_D dm dy dz$$

$$V = \int_0^{\sqrt{2}} \int_0^{2-m^2} \int_0^{9-m^2} dm dy dz$$

$$= \int_0^{\sqrt{2}} \left(\int_0^{2-m^2} (9-m^2) dy \right) dm$$

$$= \int_0^{\sqrt{2}} (9-m^2)(2-m^2) dm$$

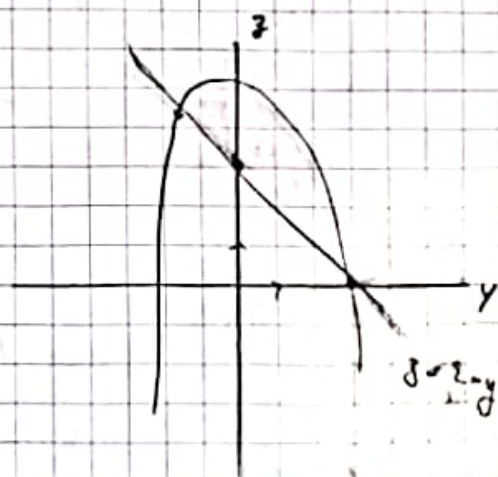
$$= \int_0^{\sqrt{2}} (18 - 2m^3 + m^5 - 9m^2) dm$$

$$= \left[18m - \frac{1}{2} m^4 + \frac{m^6}{6} - 3m^3 \right]_0^{\sqrt{2}}$$

$$= 18\sqrt{2} - 3 \times 2^{3/2} - \frac{1}{2} (4) + \frac{8}{6}$$

$$= 18\sqrt{2} - 6\sqrt{2} - 2 + \frac{4}{3}$$

$$\begin{cases} z = 2 - y \\ z = 4 - y^2 \end{cases} \quad \begin{cases} m=0 \\ m=3 \end{cases} \quad \begin{cases} y=0 \\ y=2 \end{cases}$$



$$V = \int_0^3 \int_{-y}^y \int_{-y}^y dz dy dx$$

$$= \int_0^3 \int_0^{2y} (4 - y^2 - 2xy) dy$$

$$= \int_0^3 \int_0^{2y} (-y^2 + y + 2) dy$$

$$= \int_0^3 \left[-\frac{y^3}{3} + \frac{y^2}{2} + 2y \right]_0^{2y} dx$$

$$= \int_0^3 \left(-\frac{8}{3} + 2 + 4 \right) dx = \int_0^3 \frac{10}{3} dx$$

$$V = 10$$

Ex 8:

$$z = \sqrt{x^2 + y^2}, \sqrt{4 - x^2 - y^2}$$

$$dx dy dz \quad x = r \cos \theta$$

$$= r dr d\theta dz \quad y = r \sin \theta$$

$$z = z$$

$$0 \leq r \leq \frac{\sqrt{1+\sqrt{11}}}{2}$$

$$0 \leq \theta \leq 2\pi$$

$$r^2 \leq z \leq \sqrt{4-r^2}$$

$$M = k \iiint z dz d\theta dr$$

$$= \frac{k}{2} \int_0^{2\pi} \int_0^{\sqrt{1+\sqrt{11}}/2} z^2 \Big|_r^{\sqrt{4-r^2}} d\theta dr$$

$$= \frac{k}{2} \int_0^{2\pi} \int_0^{\sqrt{1+\sqrt{11}}/2} (4r - r^3 - r^5) dr d\theta = \frac{k}{2}$$

$$= \frac{k}{2} \int_0^{2\pi} \left(4r - r^3 - r^5 \right) dr d\theta$$

$$= \pi k \left[2r^2 - \frac{1}{4}r^4 - \frac{1}{6}r^6 \right]_0^{\sqrt{1+\sqrt{11}}/2}$$

$$m = \left(2\alpha^2 - \frac{1}{4}\alpha^4 - \frac{1}{6}\alpha^6 \right) \pi k$$

center of mass $(\bar{x}, \bar{y}, \bar{z})$

$\bar{x} = \bar{y} = 0$ because

$$\int_0^{2\pi} \cos \theta d\theta = \int_0^{2\pi} \sin \theta d\theta = 0$$

$$\bar{z} = \frac{k}{m} \iiint z^2 r dz dr d\theta$$

$$= \frac{2k\pi}{3M} \int_0^{\sqrt{1+\sqrt{11}}/2} r [z^3]_r^{\sqrt{4-r^2}} dr$$

$$= \frac{2k\pi}{3M} \int_0^{\sqrt{1+\sqrt{11}}/2} (r(4-r^2)^{3/2} - r^4) dr$$

$$= \frac{2k\pi}{3M} \left[-\frac{1}{2} \times \frac{2}{5} (4-r^2)^{5/2} - \frac{1}{5} r^5 \right]_0^{\sqrt{1+\sqrt{11}}/2}$$

$$= \frac{2k\pi}{3M} \left[-\frac{1}{5} (4-\alpha^2)^{5/2} - \frac{1}{5} \alpha^5 + \frac{32}{5} \right]$$

$$= \frac{2k\pi}{3M} \left(-\frac{1}{5} \alpha^{10} - \frac{1}{5} \alpha^5 + \frac{32}{5} \right)$$

$$\bar{z} = \frac{2k\pi}{k\pi} \times \frac{256 - 8\alpha^{10} - 5\alpha^5}{24\alpha^2 - 3\alpha^4 - 2\alpha^2}$$

$$= 2 \left(\frac{256 - 8\alpha^{10} - 5\alpha^5}{24\alpha^2 - 3\alpha^4 - 2\alpha^2} \right) \quad \alpha = \sqrt{\frac{1+\sqrt{11}}{2}}$$

$$I_{xz} = k \iiint y^2 z dx dy dz$$

$$I_{yz} = k \iiint x^2 z dx dy dz$$

$$I_{xy} = k \iiint z^3 dx dy dz$$

$$I_x = I_{xy} + I_{xz}$$

$$I_y = I_{xy} + I_{yz}$$

$$I_z = I_{xz} + I_{yz}$$