Chapter 4

Dynamics of Real Incompressible Fluids

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1- Introduction

In Chapter 3, to apply Bernoulli's equation, we assumed a perfect fluid. However, the flow of a real fluid is more complex than that of an ideal fluid. This is due to friction forces, caused by the fluid's viscosity, which act between the fluid particles and the walls, as well as between the particles themselves. To solve a real fluid flow problem, a simplified method is used to calculate head losses based on experimental results initiated by the British physicist Osborne Reynolds.

2- Regimes - Reynolds Number

For a long time, hydraulic engineers observed that different flow regimes exist, but Osborne Reynolds studied them experimentally and established the criteria to differentiate them.

2-1 Osborne Reynolds' Experiment

A reservoir supplies water to a horizontal glass pipe equipped with two pressure taps. A valve is used to adjust the flow velocity. A tapered tube with a dye reservoir allows for the visualization of the flow.

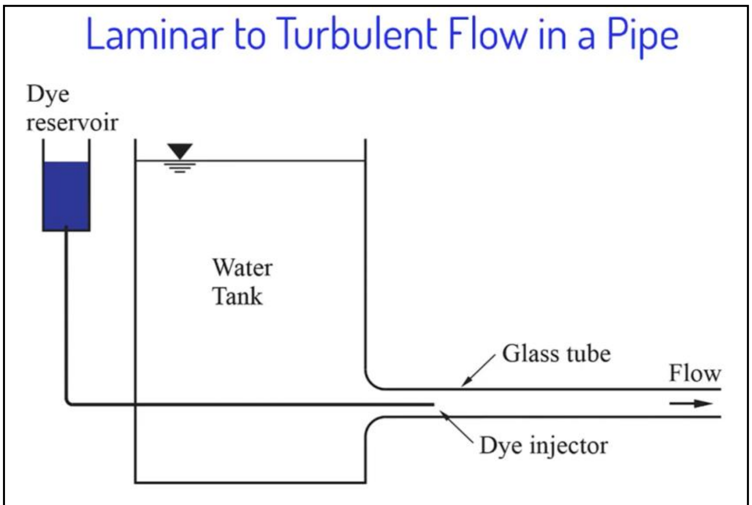


Figure 1: Schematic Diagram of the Reynolds Apparatus.

- At low velocities, the colored stream maintains its individuality all the way to the end. It flows in coaxial cylindrical layers. The pressure drop, Δp, is small. This type of flow is referred to as laminar flow.
- Beyond a certain flow velocity, the colored stream suddenly mixes with the water after traveling a short distance. This marks the transition between laminar and turbulent flow. This is referred to as transitional flow.

- When the velocity is increased further, the colored stream mixes with the flow almost immediately after its introduction. A sudden increase in Δp is observed, and a vortex motion forms in the fluid. This type of flow is referred to as turbulent flow.

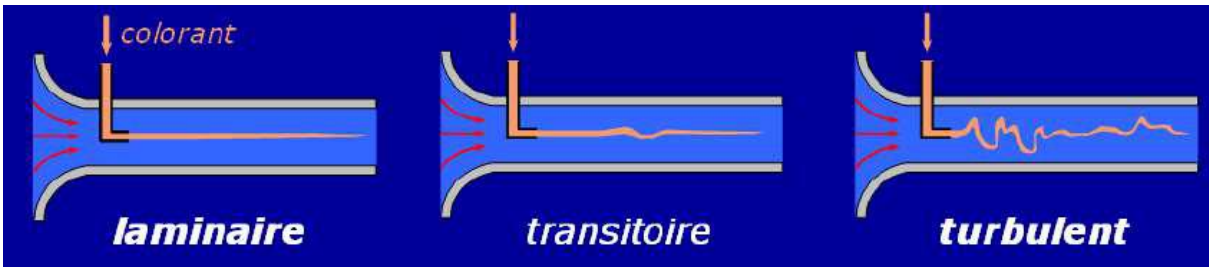


Figure 2 : Flow Regimes

By using various fluids with different viscosities, varying the flow rate, and changing the pipe diameter, Reynolds (1883) demonstrated that the parameter determining whether the flow regime is laminar or turbulent is a dimensionless number known as the Reynolds number, given by the following expression:

$$Re = \frac{\rho.V.D}{\mu} = \frac{V.D}{\nu}$$

D: pipe diameter (in m)

V: average flow velocity (in m/s)

ρ: fluid density (in kg/m³)

μ: dynamic viscosity coefficient (in Pa·s)

ν: kinematic viscosity coefficient (in m²/s)

Empirical Results for Reference:

- If Re < 2000, the flow is laminar.
- If 2000 < Re < 3000, the flow is transitional.
- If 3000 < Re < 100000, the flow is smooth turbulent.
- If Re > 100000, the flow is rough turbulent.

3- Concept of Head Losses

It can easily be observed that the pressure of a real liquid decreases throughout the length of a pipe in which it flows. This is due to the condition of the pipe's surface and the geometry of the hydraulic circuit.

In other words, a moving real fluid experiences energy losses caused by friction along the pipe walls (linear head losses) or due to obstacles and irregularities in the path (singular head losses).

3-1 Concept of Pipe Roughness

Unlike a smooth surface, a rough surface involves surface irregularities that directly affect friction forces. A rough surface can be considered as composed of a series of elementary protrusions characterized by a height, denoted as k, referred to as "roughness."

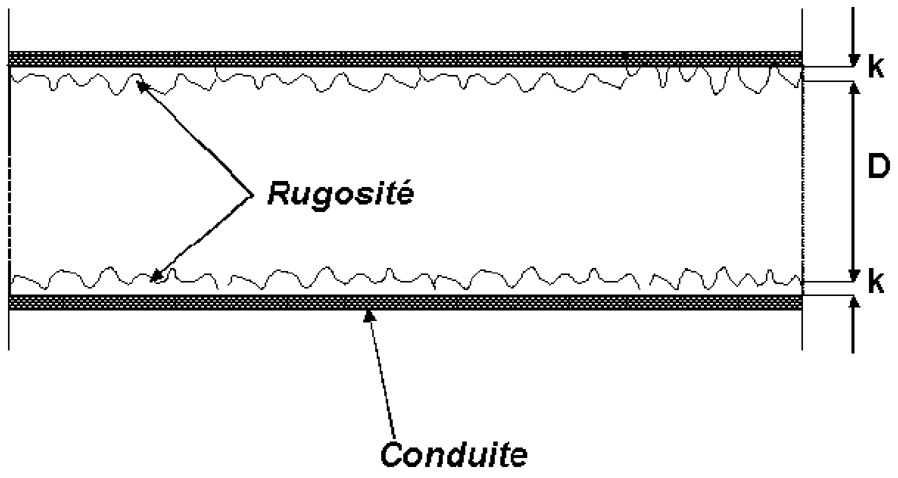


Figure 3 : Pipe Roughness

To compare the roughness relative to the pipe diameter, we introduce the ratio:

$$\varepsilon = \frac{k}{D}$$

3-2 Linear Head Losses

Consider a horizontal cylindrical pipe with a constant diameter D, through which a fluid flows at velocity V. Let us assume that two manometer tubes are placed on the pipe at two locations separated by a distance L, allowing for the measurement of static pressure.

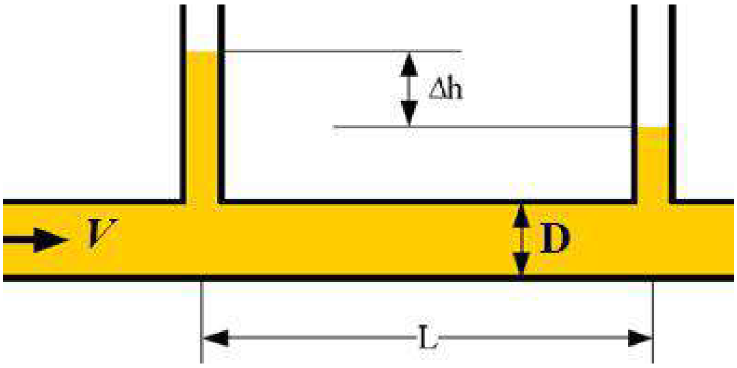


Figure 4 : Pressure Drop Due to Friction

It is observed that the fluid height is greater in the upstream manometer tube compared to the downstream one. The difference between the two levels represents the fluid height corresponding to the head loss Δh.

Linear head losses are proportional to the length L of the pipe, inversely proportional to its diameter D, and proportional to the square of the flow velocity V of the fluid. These head losses are calculated using the Darcy-Weisbach formula:

$$\Delta h_L = \lambda \frac{L.V^2}{2gD}$$ (différence de hauteur)

Ou encore :

$$\Delta p_s = \lambda \frac{\rho.L.V^2}{2D}$$ (différence de pression)

The head loss coefficient λ, or friction factor, is a dimensionless coefficient. The calculation of head losses entirely depends on determining this coefficient λ using empirical formulas that are only applicable under certain conditions.

3-2-1 Head Loss in the Case of Laminar Flow: Re < 2000

In this case, it can be shown that the coefficient λ is solely a function of the Reynolds number Re. The surface condition does not play a role, so λ does not depend on ε (pipe roughness) or the nature of the piping. The coefficient λ is determined by the Poiseuille relation:

$$\lambda = \frac{64}{R_e}$$

We can rewrite **ΔhL** as follows:

$$\Delta h_L = \frac{32.L.V^2}{g.D.R_e} = \frac{32.\mu.L.V}{\rho.g.D^2}$$

It is clear that Δh_L is proportional to the velocity **V**, thus to the flow rate **Q**, and to the kinematic viscosity **ν**.

3-2-2 Head Loss in the Case of Turbulent Flow

Various empirical laws have been proposed based on experimental studies:

- For **3000 < Re < 10⁵**, the flow is considered smooth turbulent, and **λ** depends only on **Re**. The most commonly used law is Blasius' equation:

$\lambda = 0{,}316.(Re)^{-0{,}25}$

- For **Re > 10⁵**, the flow is considered rough turbulent, and **λ** depends only on **ε/d**. The Blench equation can be used:

$$\lambda = 0{,}79\sqrt{\frac{\varepsilon}{d}}$$

3-2-3 Head Loss in the Case of Transitional Flow

For **2000** < **Re** < **3000**, this is the transitional regime between laminar and turbulent flow. **λ** depends on both **ε/d** and **Re**, as shown by Churchill's formula:

$$\frac{1}{\sqrt{\frac{\lambda}{2}}} = 2{,}457.\,Ln\left[\left(\frac{7}{Re}\right)^{0{,}9} + 0{,}27\frac{\varepsilon}{D}\right]$$

3-3 Singular Head Losses

Whenever the flow regime of a fluid is suddenly disturbed—i.e., when the velocity changes rapidly in direction or magnitude—the vortices created lead to additional friction, which adds to the friction caused by viscosity and the pipe walls. These disturbances result in head losses known as singular head losses. The main singular head losses occur at the entrance to the pipe, in areas of contraction or expansion, at bends and branches, and in various components placed along the pipeline (valves, filters, check valves, etc.).

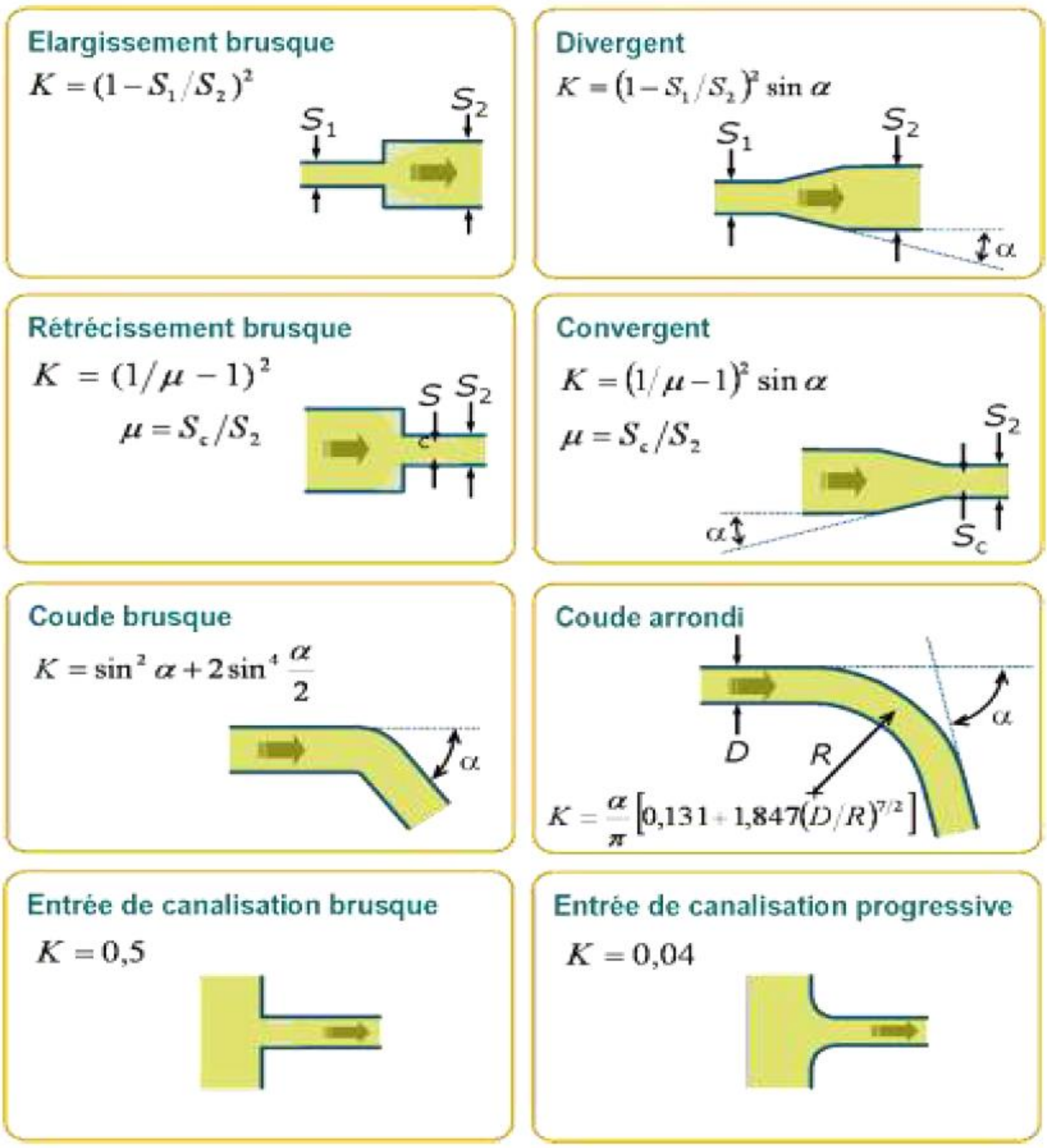
To express these head losses, the following formula is used:

$\Delta h_S = K_s \dfrac{V^2}{2g}$, (height difference) or alternatively

$\Delta p_s = K_s \dfrac{\rho . V^2}{2}$ (pressure difference)

Ks is the singular head loss coefficient, which depends on the geometric singularity and the Reynolds number. The determination of this coefficient is primarily experimental. **Ks** values are provided for various practical configurations in the following table:

Table: Some typical singularities with associated head loss coefficients.



Note: To reduce singular head losses, sharp angles and abrupt changes in section should be avoided. The total head loss between two points in a circuit is:

$$\Delta h = \Delta h_L + \Delta h_S$$

4- Bernoulli's Theorem Applied to a Real Fluid with Work Exchange

It is quite common in hydraulic circuits for a hydromechanical device, placed in a pipe, to enable the transformation of mechanical energy into hydraulic energy (such as a pump) or vice versa (such as a turbine). Thus, the expression for Bernoulli's equation becomes:

$$\frac{v_2^2 - v_1^2}{2} + \frac{P_2 - P_1}{\rho} + g\,(z_2 - z_1) = \Delta h_{12} + \frac{P_{net}}{q_m}$$

The unit of each term in this equation is joules per kilogram (J/kg). This represents Bernoulli's equation applied to a real fluid with work exchange:

$$\Delta h_{12} = \Delta h_L + \Delta h_S$$

Références bibliographiques

1. F.M. White, Fluid Mechanics, Fourth Edition, McGraw-Hill 1998.

2. B. Massey & J. Ward-Smith, Mechanics of Fluids, Eighth edition, Taylor & Francis 2006.

3. Y.A. Çengel & J.M. Cimbala, Fluid Mechanics, Fundamentals and Applications, McGraw-Hill 2006.

4. J.H. Spurk &·N. Aksel, Fluid Mechanics, Second Edition, Springer 2008.

5. F. Durst, Fluid Mechanics, An Intro to the Theory of Fluid Flows, Springer 2008.

6. B.R. Munson, D.F. Young, T.H. Okiishi, W.W. Huebsch, Fundamentals of Fluid Mechanics, Sixth Edition, John Wiley & Sons 2009.

7. P. Chassaing, Mécanique des Fluides, Eléments d’un premier parcours, Troisième Édition CEPAD 2010.

8. S. Amiroudine, J.-L. Battaglia, Mécanique des Fluides, Dunod 2011.

9. J.P. Caltagerone, Physique des Ecoulements Continus, Springer 2013.

10. Dynamique des fluides.Inge L. Ryhming Presse Polytechniques et Universitaires Romandes.

11. Mécanique de fluides – Prépas PC-PSI, Céline Anthoine – Guillaume Levèvre – Samuel Marque- 1999.

12. Mécanique des fluides. Cours, exercices et problèmes corrigés. Classes préparatoires-

Premier cycle universitaire. Christian Grossetête -1999.

13. A.Ameur, "Mécanique des fluides appliquée à l'eau: principe fondamentaux et exercices corrigés", Edition castilla,2009,ISBN:978.2.7135.3026.5.

14. A, Lencastre, "Hydraulique Générale", Edition Eyrolles. 1996 ISBN : 2212018940

15. BEN HAMOUDA. R, "Notions de mécanique des fluides », Centre de Publication Universitaire, Tunisie.2008 ISBN : 978-9973-37-494-3.

16. Bruce R. Munson, Donald. F. Young,Theodore H. Okiishi et Wade W. Huebsch, "Fundamentals of Fluid Mechanics", sixth edition, John Wiley & Sons, Inc. 2009 ISBN 978-0470-26284-9.

17. Carlier M., " Hydraulique Générale et Appliquée", Collection de la Direction des Etudes et Recherches d'EdF, n°14, 1986, Edition Eyrolles.