National Higher School of Autonomous Systems Technology

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Vibrations and waves

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Chapter 3: Acoustic waves in fluids and solids

contents

Chapter 3: Acoustic waves in fluids and solids

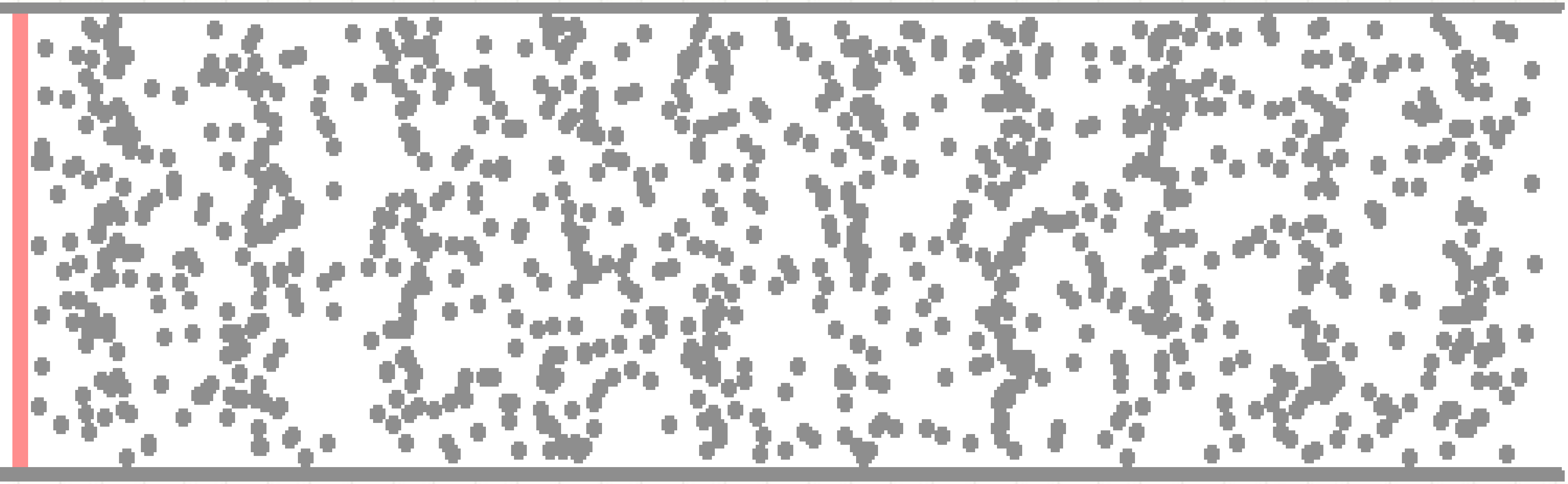
I-Acoustic waves in fluids

1-Introduction

In the previous chapter, we studied the propagation of waves on a vibrating string. Another fundamental example of mechanical waves is represented by acoustic waves, which result from the propagation of small disturbances in a fluid. These acoustic waves can be described either by the displacement of fluid particles or by pressure variations around their average value. The study of this phenomenon also allows us to explore wave propagation in three dimensions, a characteristic absent in the case of waves propagating along a vibrating string.

Chapter 3: Acoustic waves in fluids and solids

1-Introduction



Acoustic waves

Chapter 3: Acoustic waves in fluids and solids

1-Introduction

➢ Sound sources are divided into several categories:

- **Mechanical:** vibrating strings, tuning fork, structural vibrations, impacts, and friction.
- **Aerodynamic:** Human voice, wind, whistling sounds.
- **Electromagnetic:** Speakers, electric arcs.
- **Biological:** Animal sounds…..
- **Natural:** Thunderstorms, earthquakes, ocean waves.

Chapter 3: Acoustic waves in fluids and solids

1-Introduction

- Acoustics is the branch of physics that studies the production, propagation, and perception of sound waves in different media (gases, liquids, solids).
- Sound is a wave produced by a mechanical vibration of the medium, which can be solid, liquid, or gaseous.
- By analogy, sound can be defined as the audible part of the acoustic vibration spectrum, just as light is defined as the visible part of the electromagnetic vibration spectrum.

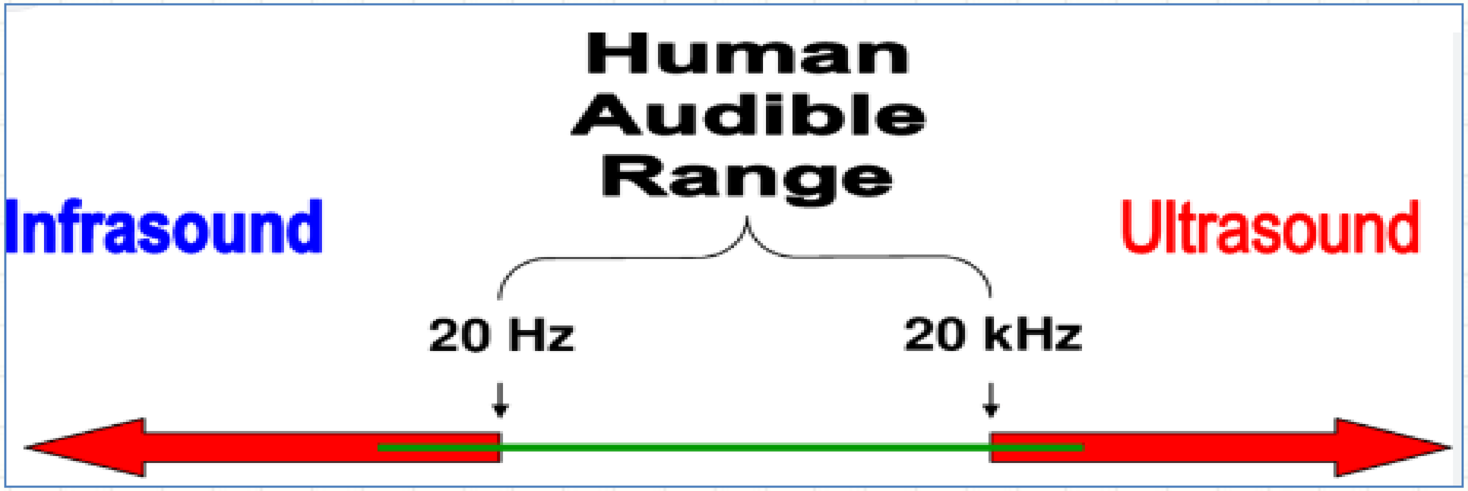
Chapter 3: Acoustic waves in fluids and solids

1-Introduction

- **Infrasound** has a frequency lower than 20 Hz and, due to its low frequency, has the ability to travel long distances through the air, water, or even the ground. It is audible to certain animals, such as elephants, which use it to communicate with each other over distances of up to 10 km.
- **Audible sounds** for the human ear range between 20 Hz and 20 kHz. Low-frequency sounds are called low-pitched sounds, while high-frequency sounds are high-pitched sounds.
- **Ultrasound** has frequencies higher than 20 kHz. It is audible to animals such as bats, cats, and dolphins.

Chapter 3: Acoustic waves in fluids and solids

1-Introduction



Acoustic Frequency Spectrum

Chapter 3: Acoustic waves in fluids and solids

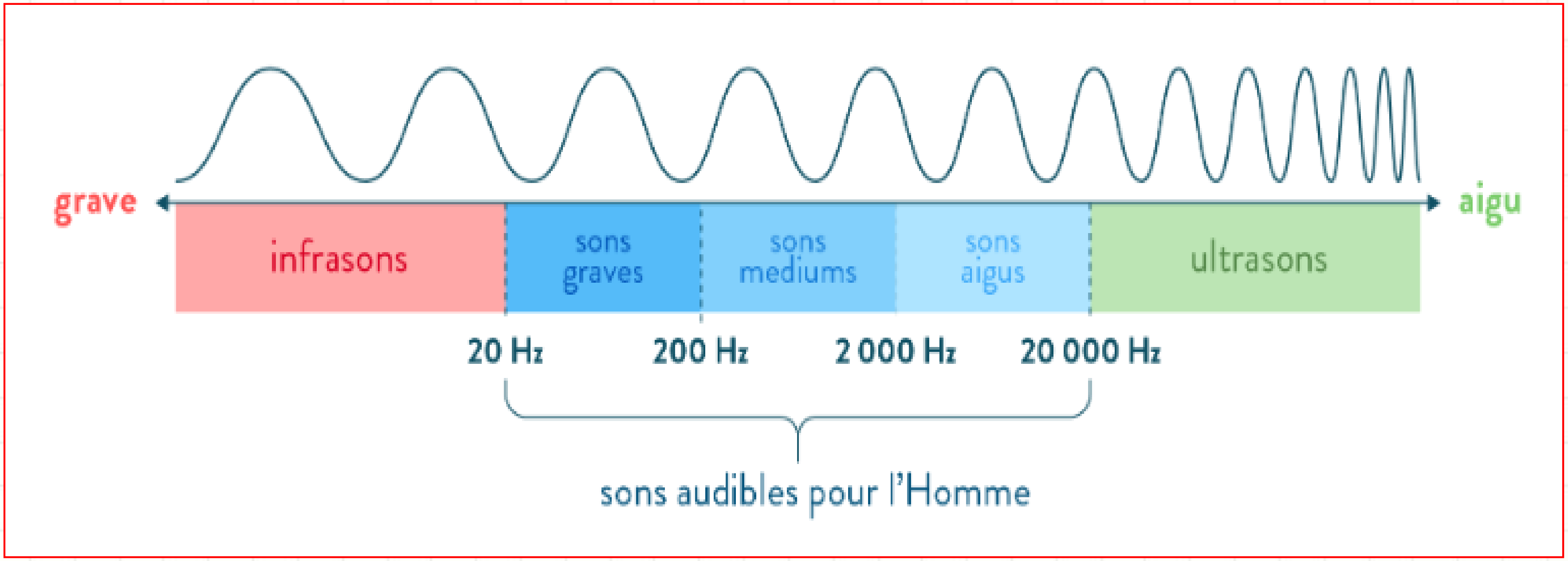
1-Introduction

Audible sound consists of three main frequency bands:

- **Low-pitched sound (son grave)**: A sound with a low frequency (generally below 200 Hz). It is perceived as deep and heavy, like the sound of a drum (tambour) or a rumbling thunder (tonnerre).
- **Midrange sound (Son médium)**: A sound with an intermediate frequency (approximately between 200 Hz and 2000 Hz). It corresponds to the main frequencies of the human voice.
- **High-pitched sound (Son aigu)**: A sound with a high frequency (generally above 2000 Hz). It is perceived as sharp and thin, like a bird's chirp or a whistle (sifflement).

Chapter 3: Acoustic waves in fluids and solids

1-Introduction



Chapter 3: Acoustic waves in fluids and solids

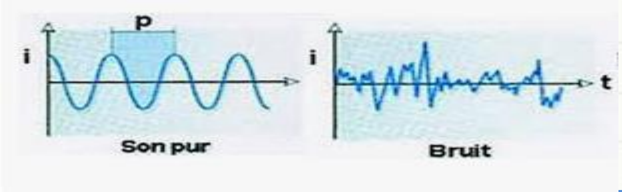
1-Introduction

- Sounds can be classified into several categories based on their structure and frequency composition:

1-**Pure sound** is characterized by a single frequency in the form of a sine wave.

2- **Complex sound** consists of multiple superimposed frequencies. It can be periodic , or aperiodic.

3- **Noise** is a non-periodic sound containing a random mix of frequencies. It can be categorized into different types



Chapter 3: Acoustic waves in fluids and solids

I-Acoustic waves in fluids

2- Wave Equation

To mathematically simplify the study of acoustic waves, we consider the propagation of acoustic waves in a pipe with a cross-section small compared to its length, it is sufficient to take the Ox axis parallel to the axis of the pipe.

Neglecting the influence of viscous forces (fluid friction) and gravity, the only forces acting on an infinitesimal fluid element are pressure forces.

Let us consider a fluid volume element of infinitesimal thickness dx, bounded by two faces of area S.

The sum of the forces applied to the system will then be given by the sum of the pressure forces on the two faces.

- The force exerted by the pressure on a section S of the fluid is given by:

$$F(x,t) = SP(x,t)$$

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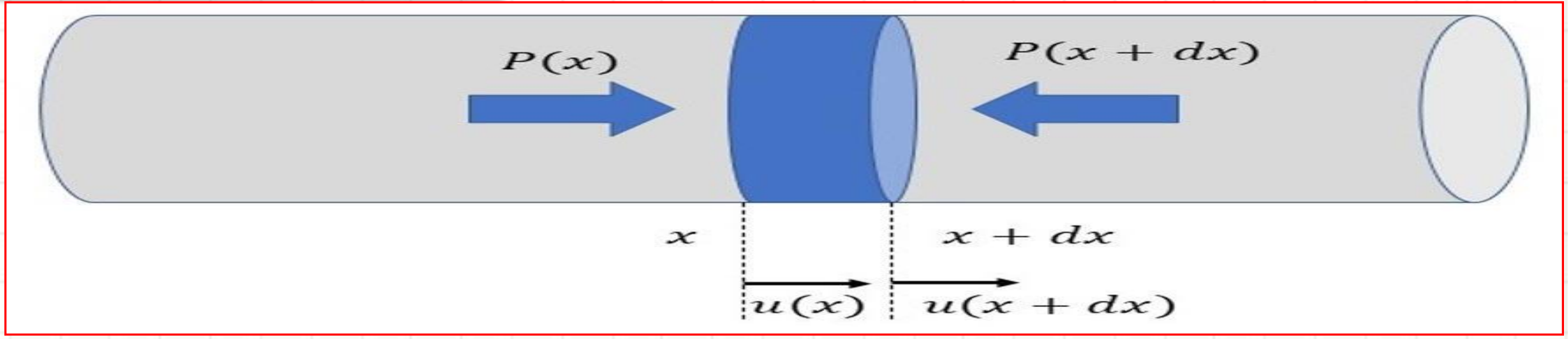
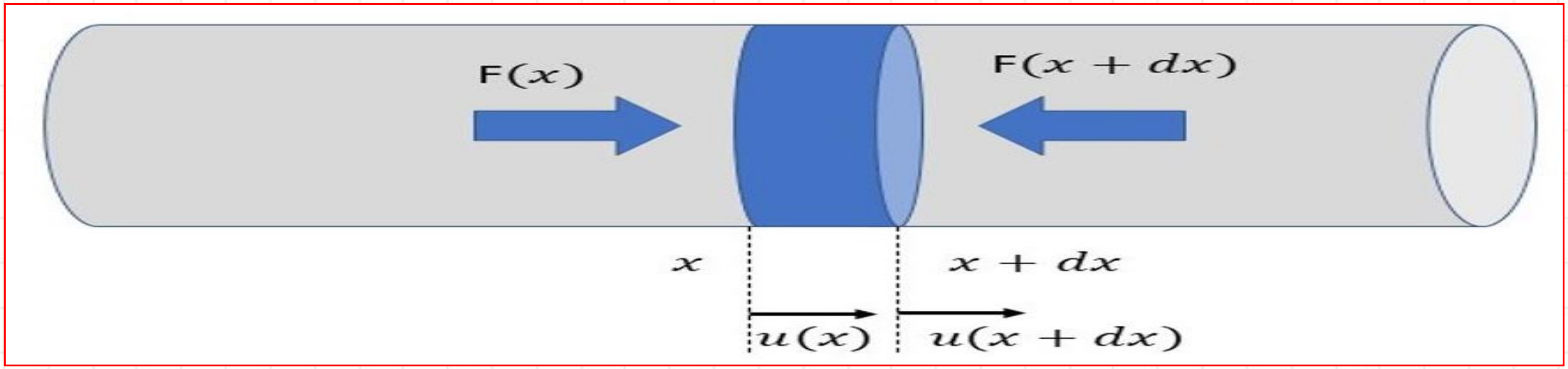
I-Acoustic waves in fluids

2- Wave Equation





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I-Acoustic waves in fluids

2- Wave Equation

Applying Newton's Second Law (Fundamental Principle of Dynamics FPD):

$$\vec{F}(x,t)+\vec{F}(x+dx,t)=\mathrm{dm}\,\vec{a}$$

Projecting this equation along the xxx-axis, we get:

OX: $SP(x,t) - SP(x+dx,t) = dm\frac{d^2u(x,t)}{dt^2}$

$$\Rightarrow S\big(P(x,t) - P(x+dx,t)\big) = dm\frac{d^2u(x,t)}{dt^2}$$

The infinitesimal mass dm can be expressed in terms of the density ρ and the infinitesimal volume $dv = Sdx$:

$$dm = \rho dv \quad \Rightarrow \quad dm = \rho S dx$$

We get: $S\big(p(x,t) - p(x+dx,t)\big) = \rho S dx\frac{d^2u(x,t)}{dt^2}$

Chapter 3: Acoustic waves in fluids and solids

I-Acoustic waves in fluids

2- Wave Equation

We get: $S\big(p(x,t) - p(x+dx,t)\big) = \rho S dx \frac{d^2u(x,t)}{dt^2}$

$$\Rightarrow \quad \frac{\big(p(x,t) - p(x+dx,t)\big)}{dx} = \rho \frac{d^2u(x,t)}{dt^2}$$

$$\frac{\big(p(x+dx,t) - p(x,t)\big)}{dx} = -\rho \frac{d^2u(x,t)}{dt^2} \qquad \Rightarrow \qquad \frac{dp(x,t)}{dx} = -\rho \frac{d^2u(x,t)}{dt^2}$$

Since the fluid is **compressible**, the pressure variation can be linked to the displacement u(x,t) using the compressibility modulus κ or the compressibility coefficient χ:

$$p(x,t) = -\frac{1}{\chi}\frac{du(x,t)}{dx} \quad \text{or} \quad p(x,t) = -\kappa \frac{du(x,t)}{dx} \;, \quad p(x,t) = -\frac{1}{\chi}\frac{\Delta v}{v_0}$$

Where:

χ: Compressibility coefficient

κ: modulus of compressibility

Chapter 3: Acoustic waves in fluids and solids

I-Acoustic waves in fluids

2- Wave Equation

$$\frac{dp(x,t)}{dx} = -\rho\frac{d^2u(x,t)}{dt^2} \quad\Rightarrow\quad -\frac{1}{\chi}\frac{d^2u(x,t)}{dx^2} = -\rho\frac{d^2u(x,t)}{dt^2}$$

We obtain the wave equation:

$$\frac{d^2u(x,t)}{dx^2} - \rho\chi\frac{d^2u(x,t)}{dt^2} = 0 \quad\Rightarrow\quad \frac{d^2u(x,t)}{dx^2} - \frac{1}{v^2}\frac{d^2u(x,t)}{dt^2} = 0$$

Where v is the propagation speed of the acoustic wave.

$$v = \frac{1}{\sqrt{\rho\chi}} \quad \text{or} \quad v = \sqrt{\frac{\kappa}{\rho}}$$

Chapter 3: Acoustic waves in fluids and solids

I-Acoustic waves in fluids

2- Wave Equation

Medium	Speed (m s^{-1})
Gases	
Air (0 °C)	331
Air (20 °C)	343
Helium	965
Hydrogen	1284
Liquids	
Water (0 °C)	1402
Water (20 °C)	1482
Seawater	1522
Solids	
Aluminium	6420
Copper	3560
Steel	5941
Granite	6000
Vulcanised Rubber	54

Speed of Sound in some Media

Chapter 3: Acoustic waves in fluids and solids

I-Acoustic waves in fluids

2- Wave Equation

- With some mathematical operations, we can show that the acoustic pressure also satisfies a wave equation

We have : $\frac{dp(x,t)}{dx} = -\rho \frac{d^2u(x,t)}{dt^2} \qquad \Rightarrow \qquad \frac{dp(x,t)}{dx} = -\rho \frac{d\dot{u}(x,t)}{dt}$

We differentiate with respect to x:

$$\frac{d^2p(x,t)}{dx^2} = -\rho \frac{d^2\dot{u}(x,t)}{dxdt} \ldots\ldots\ldots(1)$$

$we\ have \quad p(x,t) = -\frac{1}{\chi}\frac{du(x,t)}{dx}$

We differentiate with respect to t:

$$\frac{dp(x,t)}{dt} = -\frac{1}{\chi}\frac{d\dot{u}(x,t)}{dx}$$

Let's differentiate again with respect to t:

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I-Acoustic waves in fluids

2- Wave Equation

Let's differentiate again with respect to t:

$$\frac{d^2p(x,t)}{dt^2} = -\frac{1}{\chi}\frac{d^2\dot{u}(x,t)}{dxdt} \quad \dots \dots (2)$$

We obtain the wave equation of acoustic pressure p(x,t):

$$(1) and\ (2) \Rightarrow \quad \frac{d^2p(x,t)}{dx^2} - \rho\chi\frac{d^2p(x,t)}{dt^2} = 0 \quad \Rightarrow \quad \frac{d^2u(x,t)}{dx^2} - \frac{1}{v^2}\frac{d^2u(x,t)}{dt^2} = 0$$

Where v is the propagation speed of the acoustic wave.

$$v = \frac{1}{\sqrt{\rho\chi}} \quad \text{or} \quad v = \sqrt{\frac{\kappa}{\rho}}$$

Chapter 3: Acoustic waves in fluids and solids

I-Acoustic waves in fluids

2- Wave Equation

Note:

- In acoustic waves, the pressure wave and the particle displacement wave propagate at the same speed

$$\begin{cases} \dfrac{d^2p(x,t)}{dx^2} - \rho\chi\dfrac{d^2p(x,t)}{dt^2} = 0 \\ \dfrac{d^2u(x,t)}{dx^2} - \rho\chi\dfrac{d^2u(x,t)}{dt^2} = 0 \end{cases} \Rightarrow \quad \boldsymbol{v} = \frac{1}{\sqrt{\rho\chi}} \quad \text{or} \quad \boldsymbol{v} = \sqrt{\frac{\kappa}{\rho}}$$

- The particle displacement in acoustic waves is very small, at most a few millimeters
- In a sound wave in the air is very small. The particle displacement is generally around a micrometer (μm)

Chapter 3: Acoustic waves in fluids and solids

I-Acoustic waves in fluids

3- Acoustic Impedance

- Acoustic impedance is a physical quantity that characterizes the relationship between acoustic pressure and the particle velocity of a sound wave at a given point in a medium. It is defined as:

$$Z(x) = \frac{p(x,t)}{\dot{u}(x,t)}$$

- The unit of acoustic impedance is the Rayl (Ra) or Pa.s/m. This unit is named after John William Rayleigh (1842–1919), a British physicist who made significant contributions to the study of wave phenomena, including acoustics and fluid dynamics.
- Acoustic impedance is essential in the study of the propagation of acoustic waves and their interaction with the media they pass through.

Chapter 3: Acoustic waves in fluids and solids

I-Acoustic waves in fluids

3- Acoustic Impedance

In the case of a sinusoidal progressive plane wave, the acoustic pressure as a function of space and time is expressed as:

$$p(x;t) = p_0\, e^{j(wt-kx)}$$

Where: p_0 is the amplitude of the acoustic pressure wave

The particle velocity is obtained from the pressure using the relation:

$$p(x,t) = -\frac{1}{\chi}\frac{du(x,t)}{dx} \quad \Rightarrow \quad u(x,t) = -\chi \int p(x,t)dx$$

$$\Rightarrow u(x,t) = \frac{\chi p_0}{\mathrm{jk}}\, e^{j(wt-kx)} \quad \Rightarrow \quad u(x,t) = -j\,\frac{\chi p_0}{\mathrm{k}}\, e^{j(wt-kx)}$$

$$\dot{u}(x,t) = \frac{du(x,t)}{dt} \quad \Rightarrow \quad \dot{u}(x,t) = jw\,\left(-j\,\frac{\chi p_0}{\mathrm{k}}\right) e^{j(wt-kx)}$$

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I-Acoustic waves in fluids

3- Acoustic Impedance

$$\dot{u}(x,t) = \frac{\chi w p_0}{k}\; e^{j(wt-kx)} \quad \Rightarrow \quad \dot{u}(x,t) = \frac{p_0}{\rho v}\; e^{j(wt-kx)}$$

Acoustic impedance is given by:

$$Z(x) = \frac{p(x,t)}{\dot{u}(x,t)} \quad \Rightarrow \quad Z(x) = \frac{p_0 e^{j(wt-kx)}}{\frac{p_0}{\rho v}\, e^{j(wt-kx)}} \quad \Rightarrow \quad Z(x) = \rho v$$

This constant value is called characteristic impedance $(Z_c = \rho v)$.

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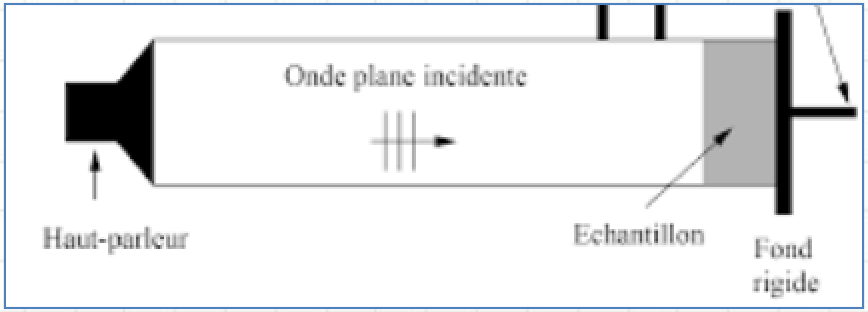
I-Acoustic waves in fluids

3- Acoustic Impedance

- The impedance at a point x in the pipe (tube) with a section area S is defined by:

$$Z(x) = \frac{p(x,t)}{S\,\dot{u}(x,t)}$$



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I-Acoustic waves in fluids

4- Sound intensity

1-Acoustic power (w)

Acoustic power represents the total energy transmitted by a sound source per unit time. It is expressed in **watts (W).**

For a progressive plane wave, the acoustic power at a position x and time t is defined as: $w(x,t) = \vec{F}(x,t).\vec{v}(x.t)$

where:

- $\vec{F}(x,t)$ force applied by the wave (in N)
- $\vec{v}(x.t)$ particle velocity caused by the wave (in m/s)

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I-Acoustic waves in fluids

4- Sound intensity

- In the case of a plane wave, $\vec{F}(x,t)$ and $\vec{v}(x.t)$ are collinear.

$$p(x,t) = p_0 \cos\,(wt - kx$$

$$\dot{\boldsymbol{u}}(\boldsymbol{x},\boldsymbol{t}) = \frac{p_0}{\rho v}\ \cos\,(wt - kx)$$

$$w(x,t) = \vec{F}(x,t).\vec{v}(x.t) \quad \Rightarrow \quad w(x,t) = F(x,t).\dot{\boldsymbol{u}}(\boldsymbol{x},\boldsymbol{t})$$

$$w(x,t) = p(x,t)\,S\,.\frac{p_0}{\rho v}\ \cos\,(wt - kx)$$

$$w(x,t) = \frac{s{p_0}^2}{\rho v}\ \cos^2(wt - kx) \quad \Rightarrow \quad \langle w(x,t)\rangle = w = \frac{s{p_0}^2}{2Z_C}$$

4- Sound intensity

1. Sound Intensity

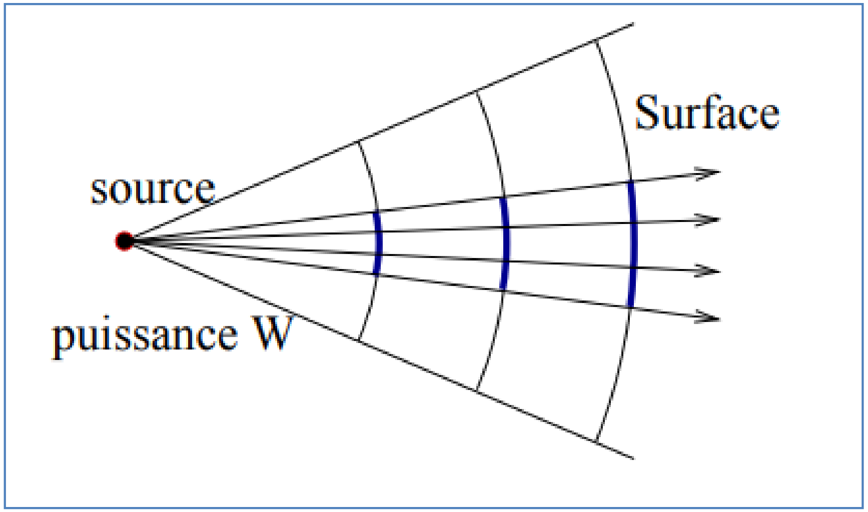
Sound intensity is a physical quantity representing the acoustic power transmitted per unit area. It is expressed in watts per square meter **(W/m²).**

- When a sound source emits an acoustic power , this energy propagates in a three-dimensional space. If the wave propagates uniformly in an isotropic medium (like air), the energy is evenly distributed over a sphere of radius around the source.
- Sound intensity is then defined as: $I = \frac{W}{S}$
- In the case of a plane wave: $I = \frac{W}{S} \quad \Rightarrow \quad I = \frac{{p_0}^2}{2Z_C}$

4- Sound intensity

In the case of spherical propagation, the surface corresponds to a sphere of radius : $S = 4\pi r^2$

Thus: $I = \frac{W}{4\pi r^2}$



- The greater the distance, the lower the sound intensity, since the energy is spread over a larger surface.

4- Sound intensity

2. Sound Intensity Level

The human ear does not perceive variations in intensity linearly but logarithmically. That is why the sound intensity level, expressed in decibels (dB), is used:

$$L_I = 10log\frac{I}{I_0}$$

Where:

$I_0 = 10^{-12}$ W/m2 : is the reference intensity corresponding to the threshold of human hearing. I_0 is the lowest or threshold intensity of sound a person with normal hearing can perceive at a frequency of 1000 Hz.

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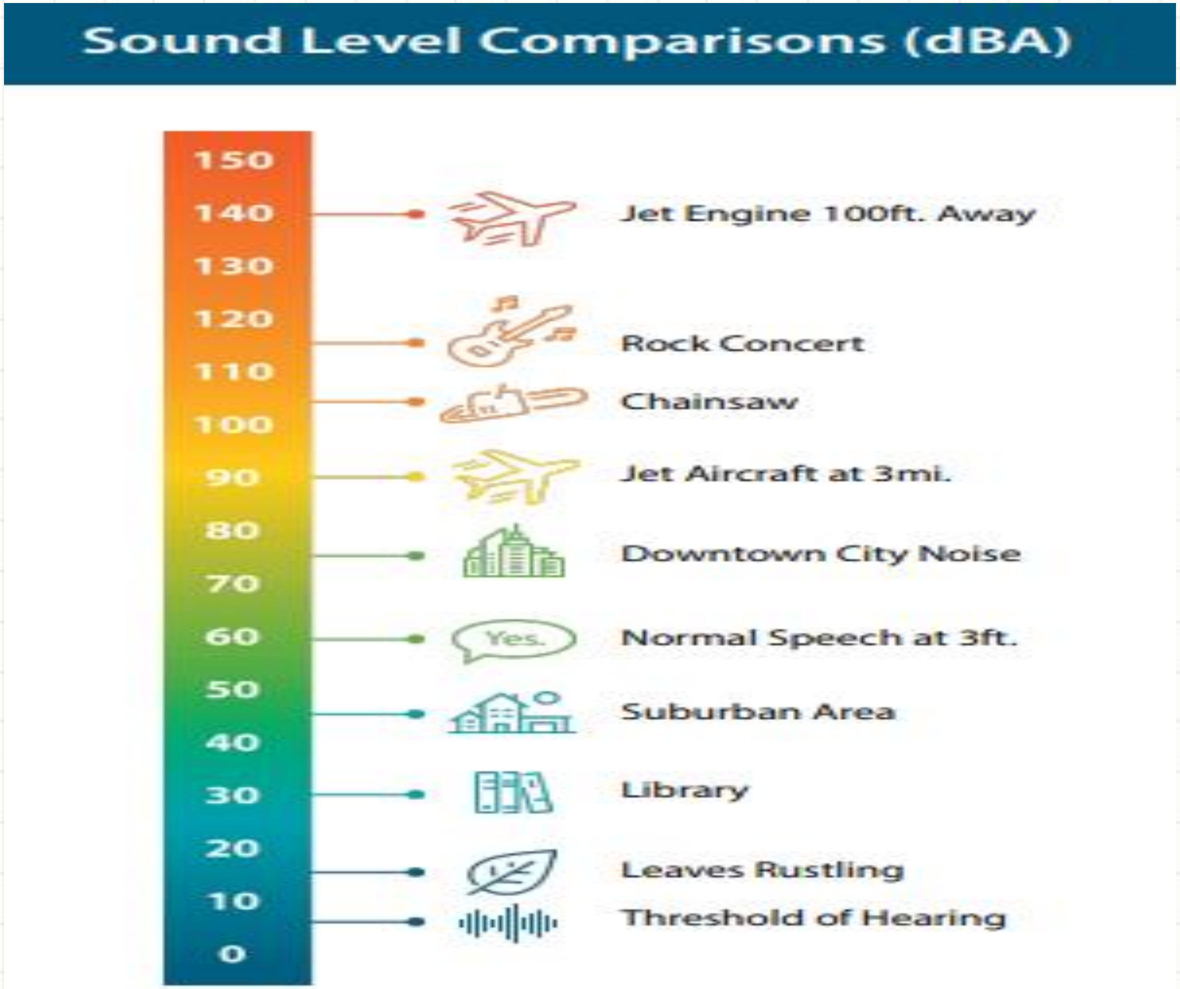
4- Sound intensity



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I-Acoustic waves in fluids

4- Sound intensity

- An increase of 10 dB corresponds to an intensity multiplied by 10.
- The decibel (dB) comes from the bel, named after Alexander Graham Bell, the inventor of the telephone
- The human ear perceives sound logarithmically, so the decibel is a more suitable unit than the watt per square meter.
- You cannot directly sum levels.
- For incoherent sounds, you can sum the intensities.
- To find the total level, you must first sum the intensities of the different sounds, then calculate the corresponding level.

I-Acoustic waves in fluids

4- Sound intensity

Summation of Levels:

We have : $b = log a \quad \Rightarrow \quad a = 10^{b}$

$$L_I = 10\,log\frac{I}{10^{-12}} \quad \Rightarrow \quad I = 10^{-12} \times 10^{\frac{L_I}{10}}$$

Case of 2 levels: - Source 1: Level L_1, intensity I_1

- Source 2: Level L_2, intensity I_2

Using the definition:

$$I_1 = 10^{-12} \times 10^{\frac{L_1}{10}} \quad \text{and} \quad I_2 = 10^{-12} \times 10^{\frac{L_2}{10}}$$

$$I_{tot} = I_1 + I_2 \quad \Rightarrow \quad I_{tot} = 10^{-12}(10^{\frac{L_1}{10}} + 10^{\frac{L_2}{10}})$$

$$L_{tot} = 10\,log\frac{I_{tot}}{10^{-12}} \quad \Rightarrow \quad L_{tot} = 10\log(10^{\frac{L_1}{10}} + 10^{\frac{L_2}{10}})$$

- The total level for N sources is given by:

$$L_{tot} = 10\log(10^{\frac{L_1}{10}} + 10^{\frac{L_2}{10}} + \cdots + 10^{\frac{L_N}{10}})$$

Chapter 3: Acoustic waves in fluids and solids

I-Acoustic waves in fluids

4- Sound intensity

Note:

- Another way to characterize sound is through the sound pressure level (in dB), which is defined by:

$$L_p = 20 log \frac{p}{p_0}$$

Where : $p_0 = 20 \times 10^{-6}$ is the reference pressure corresponding to the threshold of human hearing.

- Or in terms of power W

$$L_w = 10 log \frac{w}{w_0}$$

Where: $w_0 = 10^{-12}$ W is the reference power

Chapter 3: Acoustic waves in fluids and solids

I-Acoustic waves in fluids

5- Reflection and Transmission

Consider two semi-infinite fluid media separated by a flat surface. Let us choose an orthonormal reference frame such that the yOz plane coincides with the separation surface. When an acoustic wave originating from $-\infty$, propagating in the first medium along the x-axis, reaches the separation surface, it gives rise to two waves:

- A reflected wave propagating in the first medium in the negative x-direction.
- A transmitted wave propagating in the second medium in the positive x-direction.

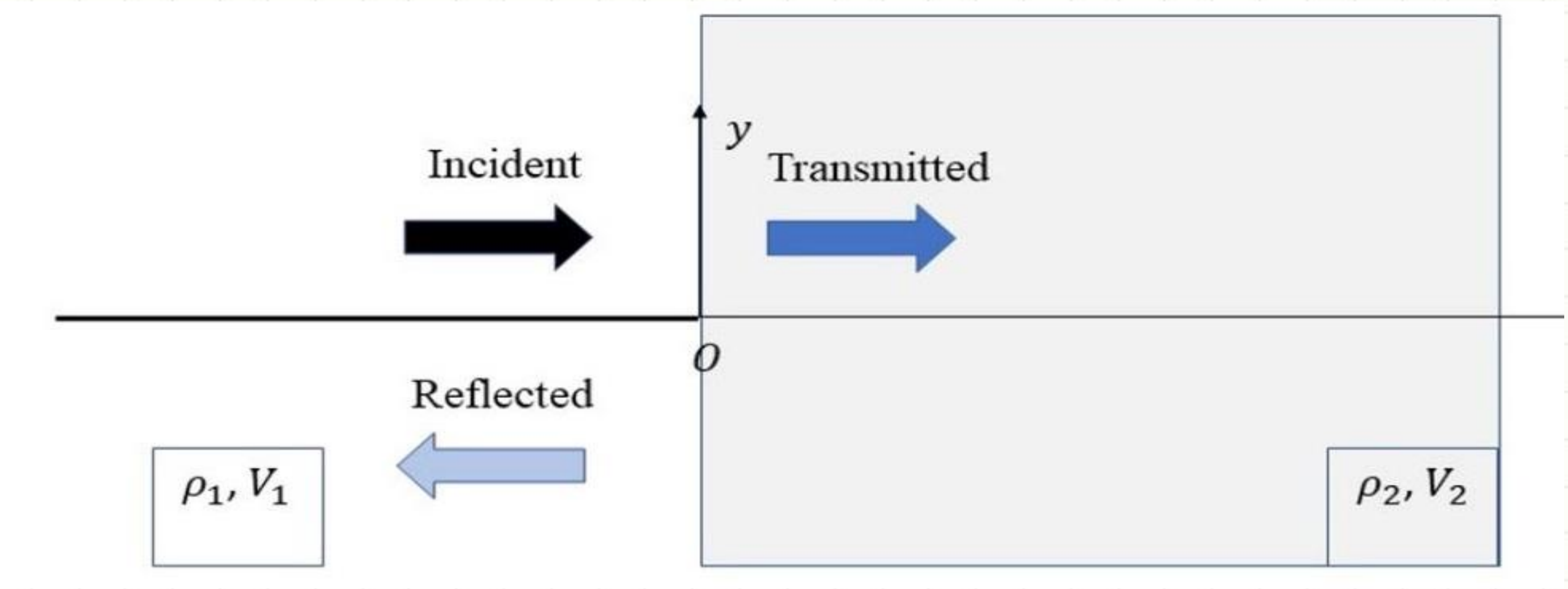
The resulting wave in the first medium ($x \leq 0$ and $x \geq 0$) is described by:

$$x \leq 0 \qquad p_1(x,t) = p_i(x,t) + p_r(x,t)$$

$$= p_i e^{j(wt-k_1x)} + p_r e^{j(wt+k_1x)}$$

$$x \geq 0 \qquad p_2(x,t) = p_t e^{j(wt-k_2x)}$$

5- Reflection and Transmission



Réflexion à une interface fluide-fluide

Chapter 3: Acoustic waves in fluids and solids

I-Acoustic waves in fluids

5- Reflection and Transmission

- The reflection and transmission coefficients in terms of pressure are defined by:

$$R = \frac{p_r}{p_i} \quad \text{and} \quad T = \frac{p_t}{p_i}$$

- The particle velocity :

$$p(x,t) = -\frac{1}{\chi}\frac{du(x,t)}{dx} \quad \Rightarrow \quad u(x,t) = -\chi\int p(x,t)dx$$

$$\dot{u}(x,t) = \frac{du(x,t)}{dt} \quad \Rightarrow \quad \dot{u}(x,t) = \frac{p_0}{\rho v}\, e^{j(wt-kx)}$$

$$\dot{u}(x,t) = \frac{p_0}{Z_C}\, e^{j(wt-kx)}$$

Chapter 3: Acoustic waves in fluids and solids

I-Acoustic waves in fluids

5- Reflection and Transmission

$$\begin{cases} p_1(x,t) = p_i e^{j(wt-k_1x)} + p_r e^{j(wt+k_1x)} \\ \dot{u}_1(x,t) = \dfrac{1}{Z_1}\left(p_i e^{j(wt-k_1x)} - p_r e^{j(wt+k_1x)}\right) \end{cases} \quad and \quad \begin{cases} p_2(x,t) = p_t e^{j(wt-k_2x)} \\ \dot{u}_2(x,t) = \dfrac{1}{Z_2}\left(p_t e^{j(wt-k_2x)}\right) \end{cases}$$

- The continuity conditions for pressure and displacement at x=0 are:

$$\begin{cases} p_1(0,t) = p_2(0,t) \\ u_1(0,t) = u_1(0,t) \end{cases}$$

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I-Acoustic waves in fluids

5- Reflection and Transmission

$$\begin{cases} p_i + p_r = p_t \\ \frac{1}{Z_1}(p_i - p_r) = \frac{1}{Z_2} p_t \end{cases} \Rightarrow \begin{cases} p_i + p_r = p_t \\ p_i - p_r = \frac{Z_1}{Z_2} p_t \end{cases}$$

$$\Rightarrow \begin{cases} 1 + R_p = T_p \\ R_p = \dfrac{Z_2 - Z_1}{Z_2 + Z_1} \\ T_p = \dfrac{2Z_2}{Z_2 + Z_1} \end{cases}$$

5- Reflection and Transmission

- Taking into account the relations $p(x,t)$ and $u(x,t)$ the reflection and transmission coefficients for displacement and velocity of particles are:

$$R_u = \frac{Z_1 - Z_2}{Z_2 + Z_1} \qquad T_u = \frac{2Z_1}{Z_2 + Z_1}$$

$$R_{\dot{u}} = \frac{Z_1 - Z_2}{Z_2 + Z_1} \qquad T_{\dot{u}} = \frac{2Z_1}{Z_2 + Z_1}$$

- $R_{\dot{u}} = R_u$ and $T_{\dot{u}} = T_u$
- Using the relation $I = \frac{{p_0}^2}{2Z_C}$ we obtain the reflection and transmission coefficients for acoustic Intensity:

$$R_I = \left(\frac{Z_1 - Z_2}{Z_2 + Z_1}\right)^2 \quad and \quad T_I = \frac{4Z_1Z_2}{(Z_2+Z_1)^2}$$

See exercise 4 set2

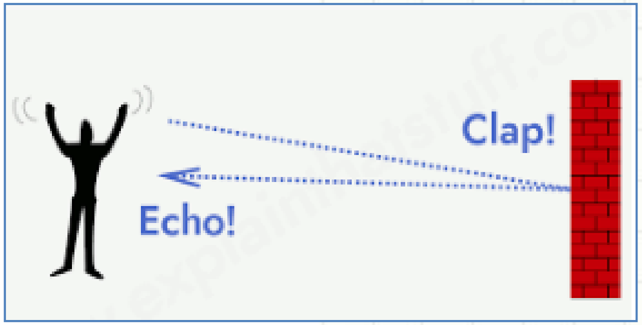
5- Reflection and Transmission

Example 1: Sound in Air Hitting a Solid Wall

Air has a low acoustic impedance ($Z_{air} = 400\ kg \cdot m^{-2} \cdot s^{-1}$).

A concrete wall (béton) has a very high acoustic impedance

($Z_{concrete} \approx 8 \times 10^6\ kg \cdot m^{-2} \cdot s^{-1}$).



- With such a large impedance difference, almost all the sound wave is **reflected**, which explains why we hear an **echo** near a concrete (béton) wall

I-Acoustic waves in fluids

5- Reflection and Transmission

Reflection and transmission in tubes

When a plane acoustic wave propagates in tube of cross-section S , the acoustic flow rate (débit) is defined by:

$d = S\dot{u}$ Where $\dot{u}$: is the particle velocity

- The impedance at a point x of the tube is defined by:

$$Z(x) = \frac{p(x,t)}{S\ \dot{u}(x,t)}$$

For a tube of section S traveled by an incident sinusoidal pressure wave of amplitude P_0 following increasing x , we can show that:

$$Z(x) = \frac{\rho v}{S} = Z_c$$

Where Z_c : is the characteristic impedance of the tube

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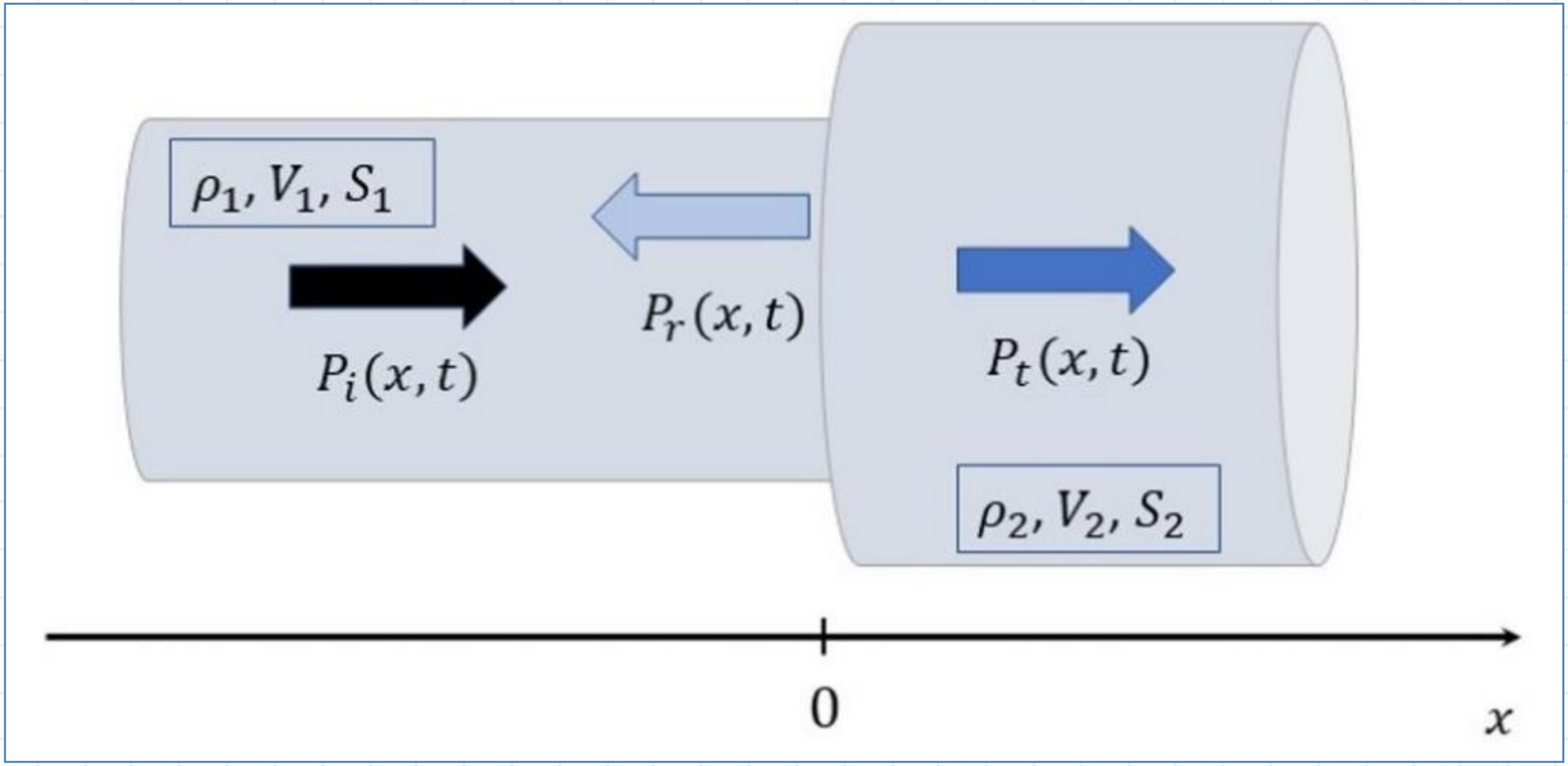
5- Reflection and Transmission

Reflection and transmission in tubes



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I-Acoustic waves in fluids

5- Reflection and Transmission

Reflection and transmission in tubes

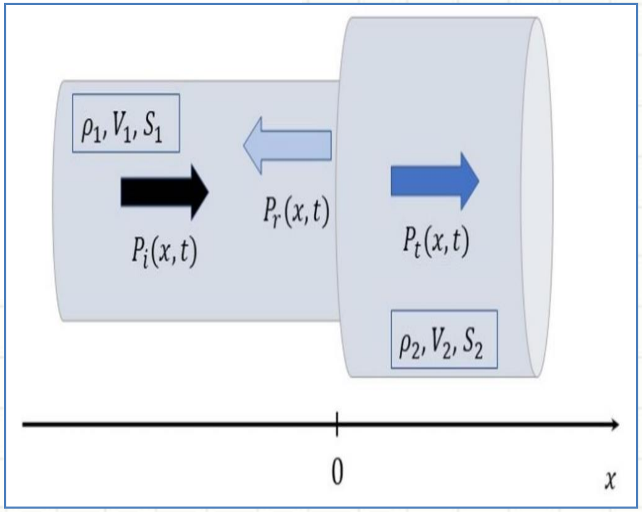
- An incident acoustic pressure wave propagates in a tube of cross-section S_1 filled with a fluid of density ρ_1, connected to a tube (S_2, ρ_2). R_P and T_P at the interface $x = 0$.

$$x \leq 0 \qquad p_1(x,t) = p_i(x,t) + p_r(x,t)$$

$$= p_i e^{j(wt-k_1x)} + p_r e^{j(wt+k_1x)}$$

$$x \geq 0 \qquad p_2(x,t) = p_t e^{j(wt-k_2x)}$$

- The continuity conditions for pressure and flow rate at x=0 are:



I-Acoustic waves in fluids

5- Reflection and Transmission

Reflection and transmission in tubes

➢ The continuity conditions for pressure and flow rate at x=0 are:

$$\begin{cases} p_1(0,t) = p_2(0,t) \\ d_1(0,t) = d_1(0,t) \end{cases} \Rightarrow \begin{cases} p_1(0,t) = p_2(0,t) \\ S_1 u_1(0,t) = S_2 u_2(0,t) \end{cases}$$

➢ Using similar calculations to the previous section, we show that:

$$\begin{cases} \boldsymbol{R_p} = \dfrac{Z_2 - Z_1}{Z_2 + Z_1} \\ \boldsymbol{T_p} = \dfrac{2Z_2}{Z_2 + Z_1} \end{cases}$$

Where $Z_1 = \frac{\rho_1 v_1}{S_1}$: is the characteristic impedance of the tube 1

$Z_1 = \frac{\rho_2 v_2}{S_2}$: is the characteristic impedance of the tube 2

Chapter 3: Acoustic waves in fluids and solids

I-Acoustic waves in fluids

5- Reflection and Transmission

Standing waves in tube

In a tube waves are created at one end of the tube by something that vibrates and travel to the end of the tube and reflects

- A standing wave is created by the waves traveling in each direction.
- As was the case for strings, the length of a tube determines the frequency of a standing wave in the tube.
- There are several complications, however, depending on if one or both ends are closed or open.

$$p(x,t) = p_i(x,t) + p_r(x,t)$$
$$= p_i e^{j(wt-kx)} + p_r e^{j(wt+kx)}$$

Chapter 3: Acoustic waves in fluids and solids

I-Acoustic waves in fluids

5- Reflection and Transmission

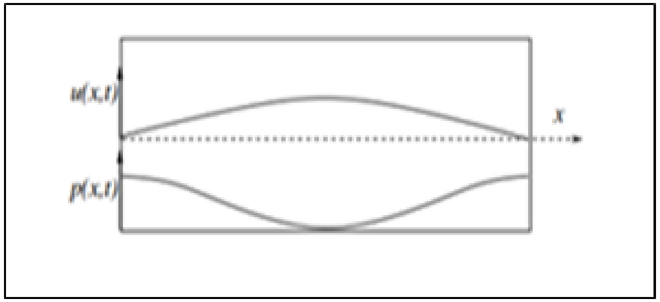
Standing waves in tube

1-Closed tube

we consider a pipe whose two ends, at $x = 0$ and $x = L$, are closed by a rigid wall.

Since the wall cannot be set into motion, the displacement field is naturally zero:

- boundary conditions:



-Displacement

$u(0,t) = 0$ and $u(L,t) = 0$

-Pressure

$\left.\frac{\partial p(x,t)}{\partial x}\right|_{x=0} = 0$ and $\left.\frac{\partial p(x,t)}{\partial x}\right|_{x=L} = 0$

✓ *A displacement node corresponds to pressure antinode, and vice versa.*

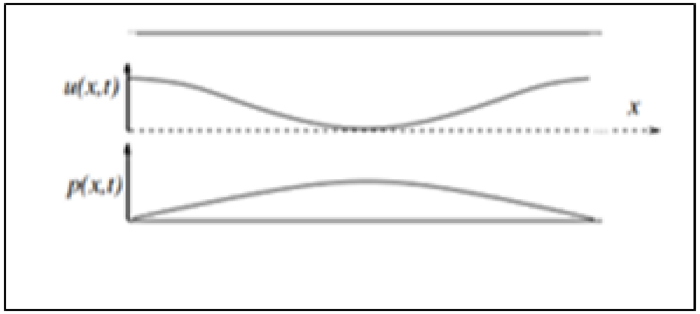
5- Reflection and Transmission

Standing waves in tube

2-Open tube

We now consider a tube open at both ends. The wave is then in contact with a large pressure reservoir at atmospheric pressure. It is therefore natural to impose that the excess pressure is zero:

- boundary conditions:



-Pressure

$p(0,t) = 0$ and $p(L,t) = 0$

Displacement

$$\left.\frac{\partial u(x,t)}{\partial x}\right|_{x=0} = 0 \quad \text{and} \quad \left.\frac{\partial u(x,t)}{\partial x}\right|_{x=L} = 0$$

✓ *A displacement node corresponds to pressure antinode, and vice versa.*

I-Acoustic waves in fluids

5- Reflection and Transmission

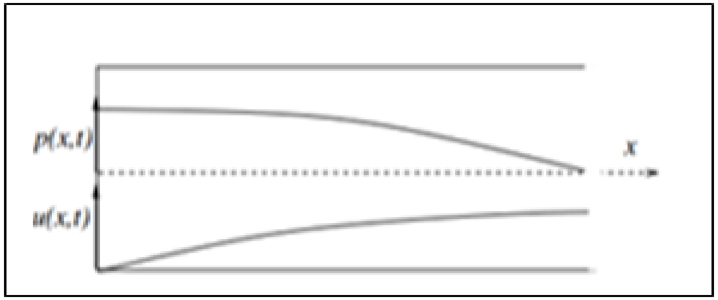
Standing waves in tube

3- Half-open tube

- Let us now consider the last case, the most interesting one. We choose a sound tube where one end, at $x = 0$, is closed, and the other, at $x = L$, is open. boundary conditions:

-Pressure

$$\left.\frac{\partial p(x,t)}{\partial x}\right|_{x=0} = 0 \quad \text{and} \quad p(L,t) = 0$$



Displacement

$$u(0,t) = 0 \qquad \text{and} \qquad \left.\frac{\partial u(x,t)}{\partial x}\right|_{x=L} = 0$$

✓ *A displacement node corresponds to pressure antinode, and vice versa.*

Chapter 3: Acoustic waves in fluids and solids

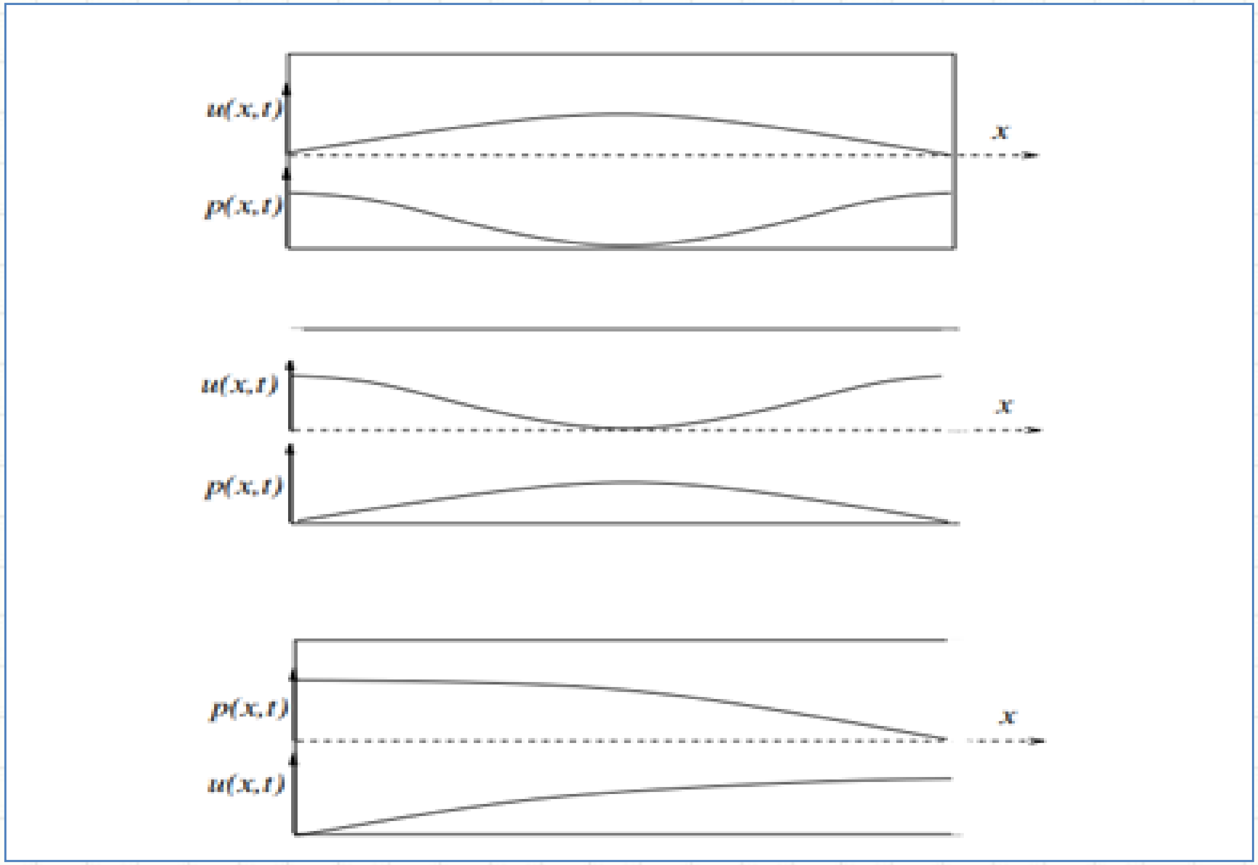
I-Acoustic waves in fluids

5- Reflection and Transmission

1- Closed tube

2- Open tube

3- Half-open tube



Chapter 3: Acoustic waves in fluids and solids

I-Acoustic waves in fluids

5- Reflection and Transmission

Standing waves in tube

Rubens tube

Chapter 3: Acoustic waves in fluids and solids

I-Acoustic waves in fluids

6- Doppler Effect

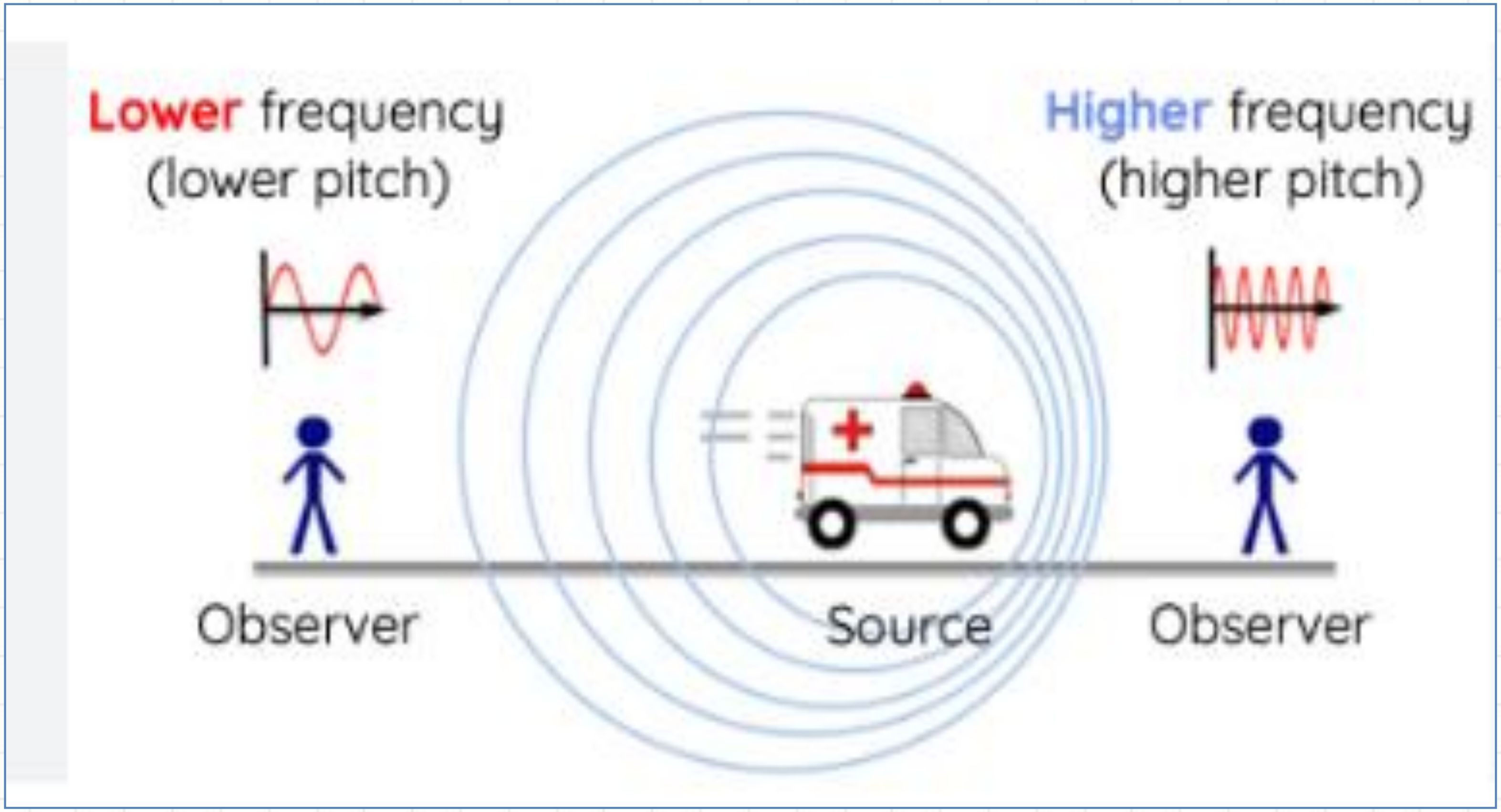
Doppler effect:

- Is the change in frequency of a sound wave (source) perceived by an receiver when the source and the receiver are moving relative to each other. A common example is the sound of an ambulance, which becomes higher-pitched as it approaches and lower-pitched as it moves away. This phenomenon is due to the change in the frequency of the sound signal. The Doppler effect is named after the Austrian physicist Christian Andreas Doppler (1803-1853).
- The Doppler effect also applies to other types of waves, notably electromagnetic waves.

Chapter 3: Acoustic waves in fluids and solids

I-Acoustic waves in fluids

6- Doppler Effect



6- Doppler Effect

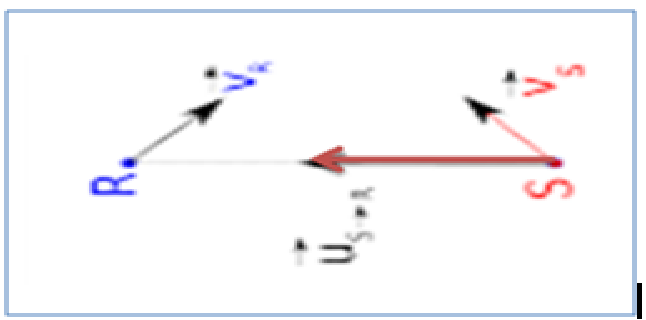
Doppler Effect

Chapter 3: Acoustic waves in fluids and solids

I-Acoustic waves in fluids

6- Doppler Effect

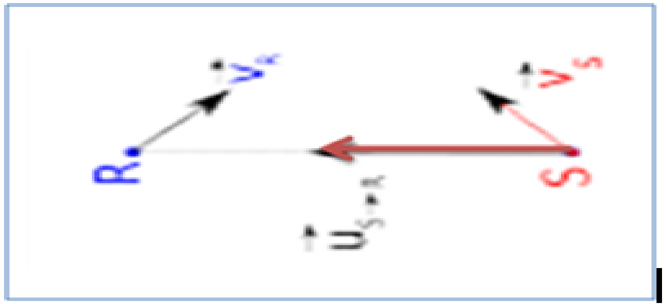
- The frequency emitted by the source (fs) is constant; only the frequency detected by the receiver changes, provided there is a relative motion between the source of sound waves and the receiver.
- Doppler Effect is relative motion between source and observer
- Using a relativistic demonstration, we derive the expression for the Doppler effect.



Chapter 3: Acoustic waves in fluids and solids

I-Acoustic waves in fluids

6- Doppler Effect



$$f_R = f_s \frac{v - \vec{v}_R.\vec{u}_{S-R}}{v - \vec{v}_S.\vec{u}_{S-R}} \quad \Rightarrow \quad f_R = f_s \frac{v \; - \; v_R. \; \cos(\vec{v}_R, \vec{u}_{S-R})}{v \; - \; v_S. \; \cos(\vec{v}_R, \; \vec{u}_{S-R})}$$

where:

- f_R frequency heard by the receiver (Hz)
- f_s frequency of the source (Hz)
- v speed of sound in the medium (m/s)
- v_R speed of the receiver (m/s)
- v_s speed of the source (m/s)
- $\vec{u}_{S-R}$ is the unit vector directed from the source to the receiver.

I-Acoustic waves in fluids

6- Doppler Effect

- in one dimension :

$$f_R = f_s \frac{v \mp v_R}{v \mp v_s}$$

1- When the source is moving towards a stationary receiver: $f_R = f_s \frac{v}{v - v_s}$

2- When the source is moving away from a stationary receiver: $f_R = f_s \frac{v}{v + v_s}$

3- When the receiver is moving towards a stationary source: $f_R = f_s \frac{v + v_R}{v}$

4- When the receiver is moving away from a stationary source: $f_R = f_s \frac{v - v_R}{v}$

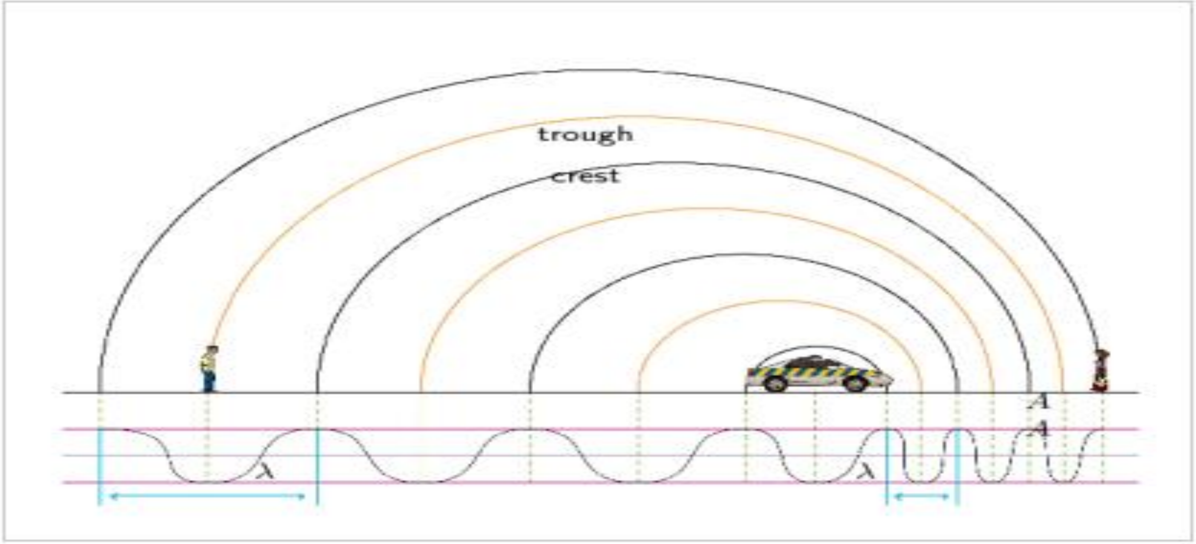
Chapter 3: Acoustic waves in fluids and solids

I-Acoustic waves in fluids

6- Doppler Effect

1- When the source is moving towards a stationary receiver: $f_R = f_s \frac{v}{v - v_s}$

- The frequency detected by the receiver is higher than the frequency of the source (wavelength becomes shorter) because more wave-fronts are detected per second. ($f_R > f_s$).
- Frequency is inversely proportional to wavelength

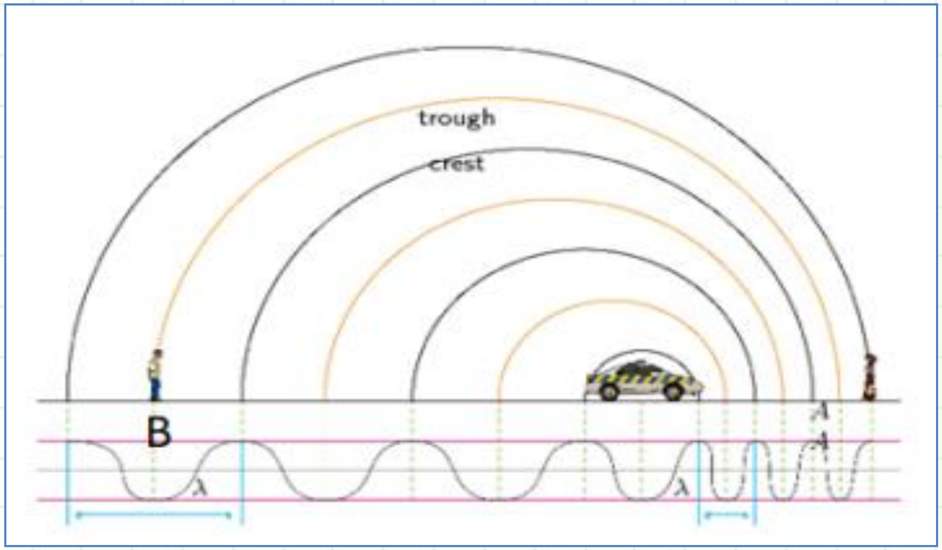


I-Acoustic waves in fluids

6- Doppler Effect

2- When the source is moving away from a stationary receiver: $f_R = f_s \frac{v}{v+v_s}$

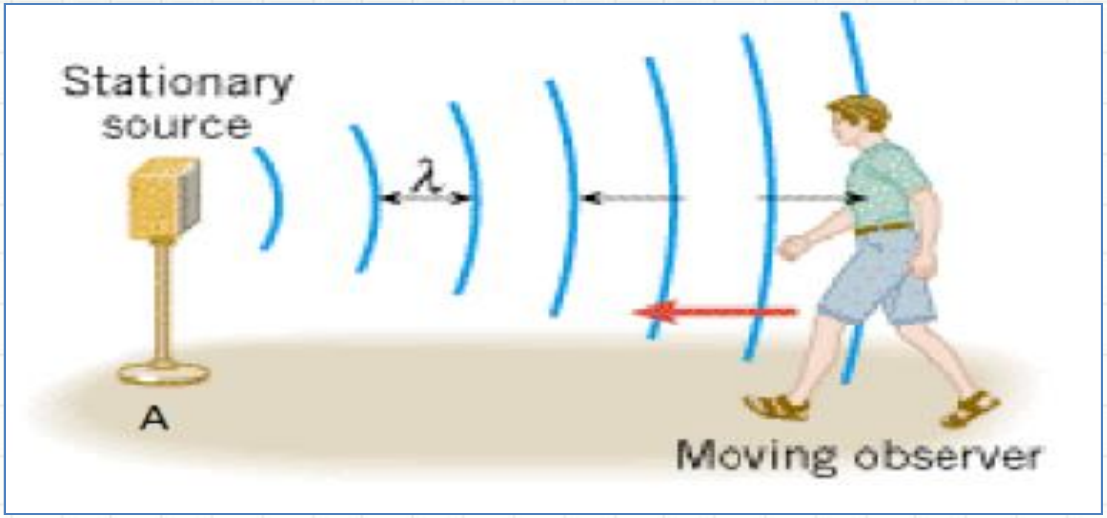
➢ The frequency detected by the listener is lower than the frequency of the source (wavelength becomes longer)



6- Doppler Effect

3- When the receiver is moving towards a stationary source: $f_R = f_s \frac{v+v_R}{v}$

- the frequency detected by the receiver is higher than the frequency of the source

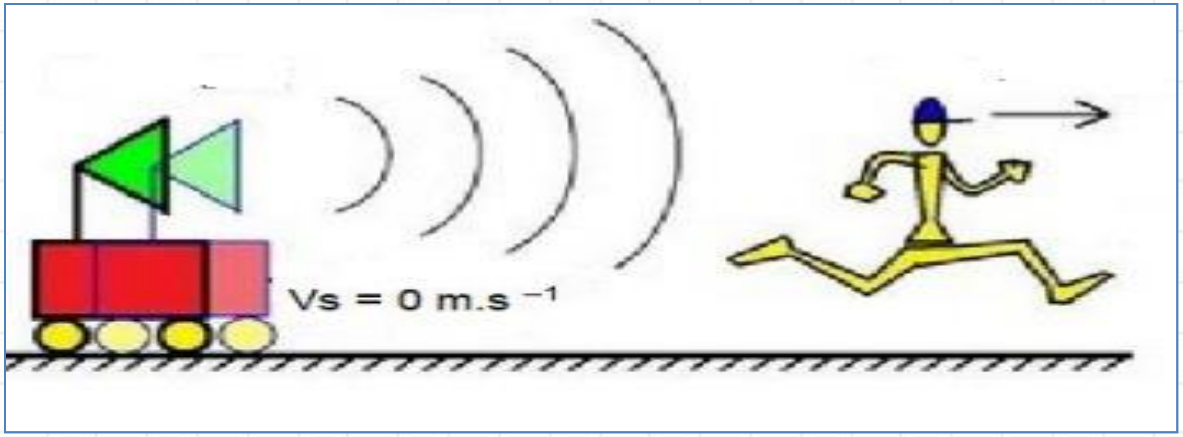


I-Acoustic waves in fluids

6- Doppler Effect

4- When the receiver is moving away from a stationary source: $f_R = f_s \frac{v - v_R}{v}$

- the frequency detected by the receiver is lower than the frequency of the source



6- Doppler Effect

Example

An ambulance is moving away from a stationary listener with a velocity of 25 m/s. The frequency emitted by the ambulance is 450 Hz. Calculate the frequency that the listener will detect, if the speed of sound in air is 340 m/s.

Chapter 3: Acoustic waves in fluids and solids

I-Acoustic waves in fluids

7-Acoustic Energy

Acoustic energy density

As with vibrating strings, instead of the system's dynamics, we can consider the mechanical energy contained in the sound wave. Assuming the system is conservative, this energy consists of a kinetic part and a potential part.

1- Kinetic energy

$$dE_k = \frac{1}{2} dm V^2 \qquad \Rightarrow \qquad dE_k = \frac{1}{2}\rho dv\left(\frac{\partial U}{\partial t}\right)^2$$

- Thus the kinetic energy per unit volume (density of kinetic energy) is:

$$e_k = \frac{dE_k}{dv} \qquad \Rightarrow \qquad e_k = \frac{1}{2}\rho\left(\frac{\partial u}{\partial t}\right)^2$$

7-Acoustic Energy

2- Potential energy

The volumetric density of potential energy of the fluid is given by:

$$E_p = \int -p dv$$

We have $dp = -\frac{1}{\chi}\frac{\Delta v}{v_0} \Rightarrow dv = -\chi v_0\, dp$

$$\Rightarrow \quad E_p = \int -p(-\chi v_0)\, dp$$

$E_p = \chi v_0 \int p dp \Rightarrow e_p = \frac{E_p}{v_0} \Rightarrow e_p = \frac{1}{2}\chi p^2$

- The total acoustic energy density is:

$$e_T = e_p + e_k$$