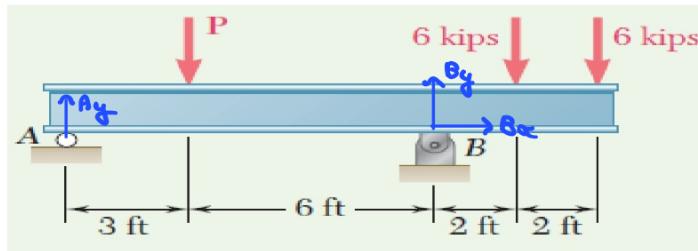


# TD (d)

## Exercise 1:

Three loads are applied to a beam as shown. The beam is supported by a roller at A and by a pin at B. Neglecting the weight of the beam, determine the reactions at A and B when  $P = 15$  kips



$$\sum \vec{F}_{\text{act}} = \vec{0} \Rightarrow \begin{cases} (\text{Ax}): Bx = 0 \\ (\text{Ay}): -P - 12 + Ay + By = 0 \end{cases}$$

$$\sum M(\vec{F}) = 0 \Rightarrow -3P + 9By - (6 \times 11) - (6 \times 13) = 0$$

$$\Leftrightarrow -3(15) + 9By - (6 \times 11) - (6 \times 13) = 0$$

$$By = \frac{3 \times 15 + 6 \times 11 + 6 \times 13}{9}$$

$$By = 21 \text{ kips}$$

$$-15 - 12 + Ay + By = 0$$

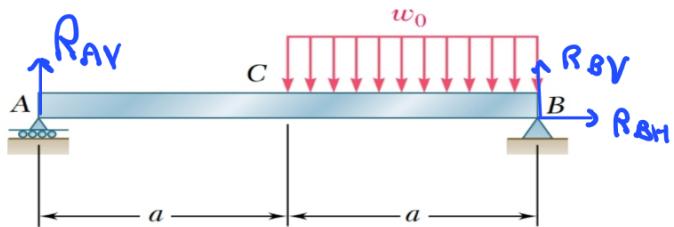
$$\Rightarrow Ay = 6 \text{ kips}$$

$$\left. \begin{array}{l} Ay = 6 \text{ kips} \\ By = 21 \text{ kips} \end{array} \right\}$$

$B_A = 0 \text{ kips}$

### Exercice 2 :

Déterminer l'expression des réactions  $R_{BV}$  et  $R_{AV}$  des systèmes représentés ci-après.  $q_0$  et  $w_0$  sont les densités de charge (en N/m).



$$\sum \vec{F}_{\text{ext}} = 0 \Rightarrow \begin{cases} (\text{Ex}): R_{BV} = 0 \\ (\text{Oy}): R_{BV} + R_{AV} + a w_0 = 0 \end{cases}$$

$$w_0 = \frac{F}{a} \Rightarrow F = w_0 \cdot a$$

$$\sum M(\vec{F}) = 0 \Rightarrow -w_0 a \left( a + \frac{a}{2} \right) + R_{BV}(2a) = 0$$

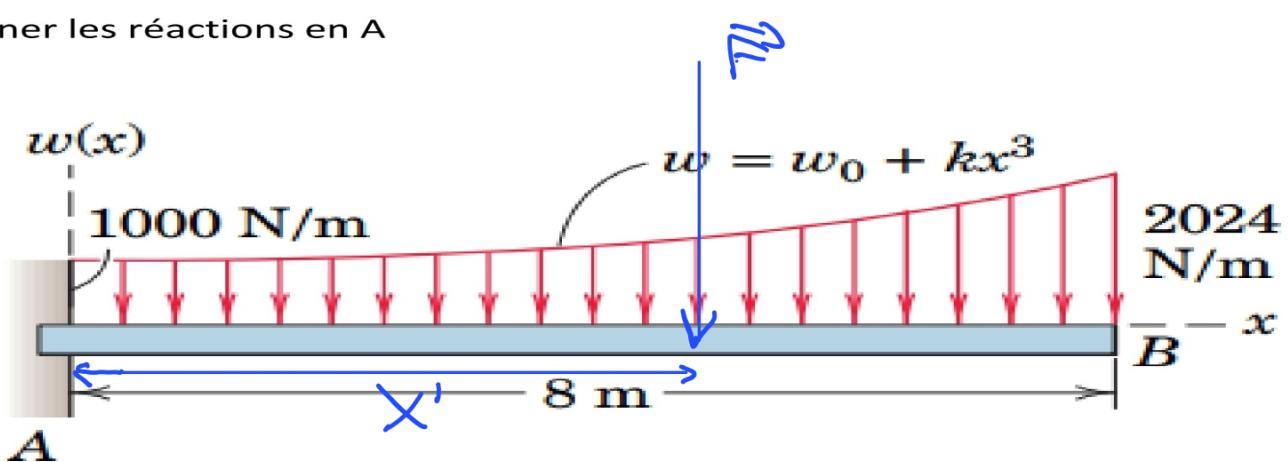
$$R_{BV} = \frac{3a}{4} w_0$$

$$\frac{3a}{4} w_0 + a w_0 + R_{AV} = 0$$

$$R_{AV} = -\frac{7a w_0}{4}$$

### Exercice 3 :

Déterminer les réactions en A



$$\omega(0) = \omega_0 = 1000 \text{ rad/s}$$

$$\omega(8) = 1000 + k(8)^3 = 2024 \Rightarrow k = 2$$

$$\omega(x) = 1000 + 2x^3$$

$$F = \int_0^8 (1000 + 2x^3) dx = \left[ 1000x + \frac{x^4}{2} \right]_0^8$$

$$F = 10048 \text{ N}$$

$$X' F = \int_0^8 x (1000 + 2x^3) dx$$

$$= \left[ \frac{2}{5} x^5 + 500x^2 \right]_0^8 = 45107,2 \text{ Nm}$$

$$X' = 4,49 \text{ m}$$

$$\sum \vec{F}_{ext} = \vec{0} \Rightarrow R_A - F = 0$$

$$R_A = F = 10048 \text{ N}$$

$$\sum M_{ext} = 0 \Rightarrow -Fx' + M = 0$$

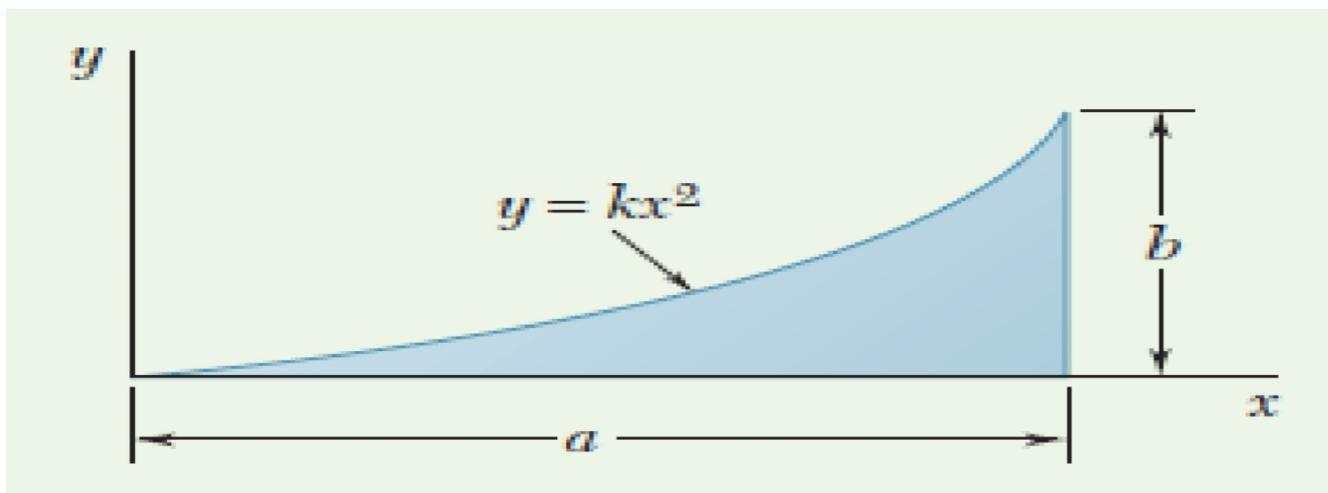
$$M = Fx' = 4,49 \times 10048$$

$$M = 45107,2 \text{ Nm}$$

#### Exercise 4 :

Evaluate the dashed surface using a vertical differential element.

Calculer la surface suivante en utilisant un élément différentiel vertical.



$$b = k\alpha^2 \Leftrightarrow K = \frac{b}{\alpha^2} \quad y = \left(\frac{b}{\alpha^2}\right) \alpha^2$$

Ecrire  $x$  en fonction de  $y$ :

$$x = \frac{\alpha}{\sqrt{b}} \sqrt{y}$$

$$\begin{aligned} A &= \int_0^b x \, dy = \frac{\alpha}{\sqrt{b}} \int_0^b (y)^{1/2} \, dy \\ &= \frac{2}{3} \frac{\alpha}{\sqrt{b}} (b)^{3/2} \end{aligned}$$

$$A = \frac{2 \alpha b}{3}$$