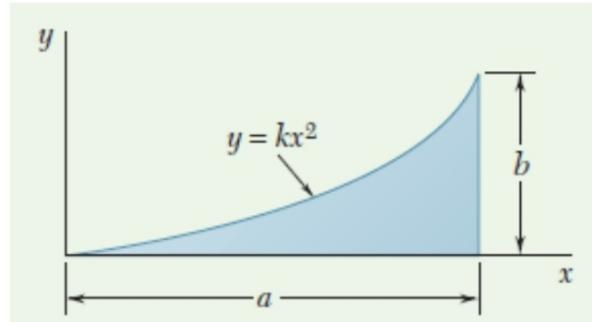


T D 1 :

Exercise 1:

Determine the Area using both horizontal and vertical differential element. ($k = 1$, $a = 1 \text{ m}$, $b = 1 \text{ m}$)

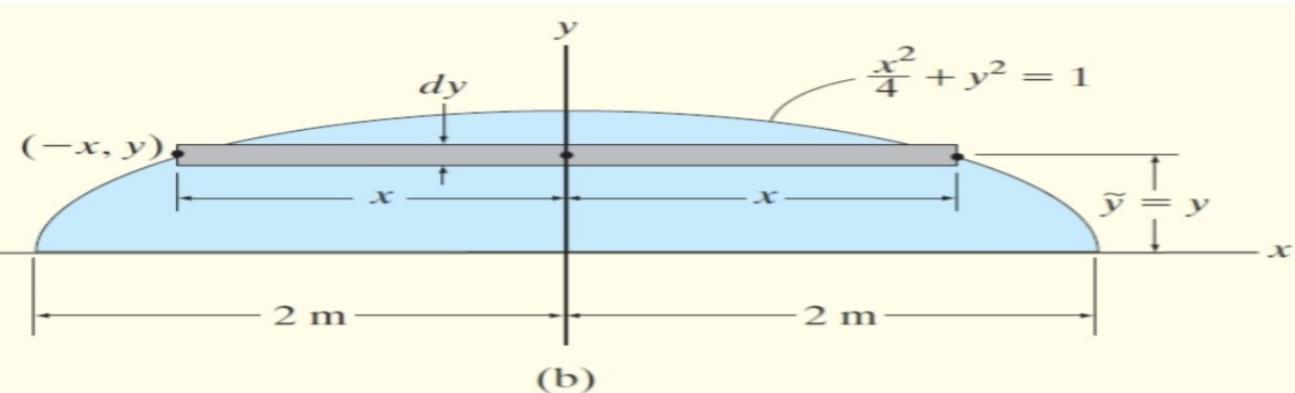
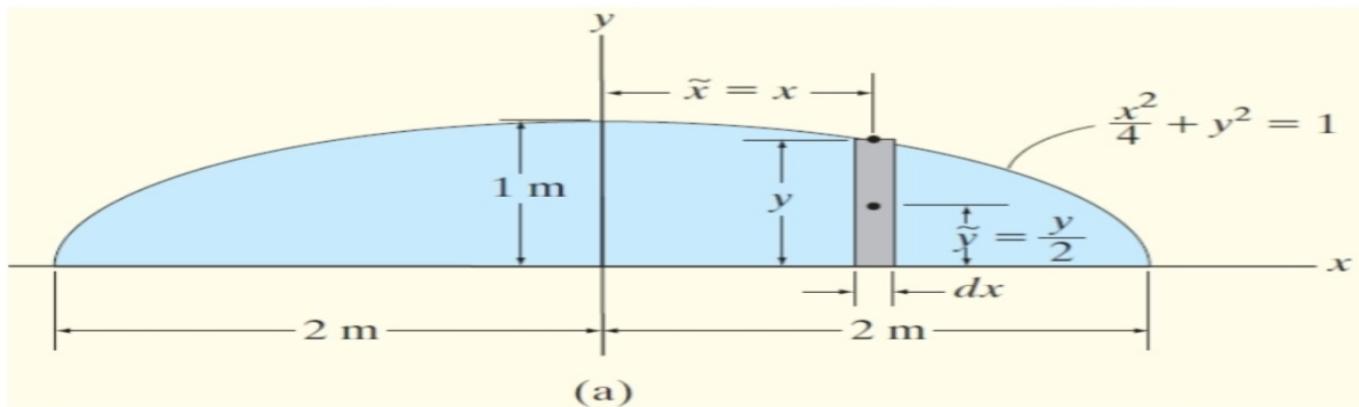


vertical: $A = \int dS = \int y dx = \int_0^1 x^2 dx = \frac{1}{3} [x^3]_0^1$

$$A = \frac{1}{3}$$

Exercise 2 :

Determine the Area using both horizontal and vertical differential element.



$$\text{VII } A = \int dA = \int_{-2}^2 y dx = \int_{-2}^2 \sqrt{1 - \frac{x^2}{4}} dx$$

$$x = 2\cos\theta \Rightarrow dx = -2\sin\theta d\theta$$

$$x = -2 \Rightarrow 2\cos\theta = -2 \Rightarrow \theta = \pi$$

$$x = 2 \Rightarrow \theta = 0$$

$$A = \int_{\pi}^0 \sqrt{1 - \frac{4\cos^2\theta}{4}} (-2\sin\theta) d\theta$$

$$= \int_{\pi}^0 \sqrt{\sin^2\theta} (-2\sin\theta) d\theta$$

$$= \int_{\pi}^0 -2\sin^2\theta d\theta$$

$$\text{we know: } \cos(2\theta) = 1 - 2\sin^2\theta \Leftrightarrow \frac{1 - \cos(2\theta)}{2} = \sin^2\theta$$

$$A = -2 \int_{\pi}^0 \frac{1}{2} d\theta + 2 \int_0^{\pi} \frac{\cos(2\theta)}{2} d\theta$$

$$= \int_0^{\pi} d\theta + \frac{1}{2} \int_{\pi}^0 2 \cos(2\theta) d\theta$$

$$= [\theta]_0^{\pi} + \frac{1}{2} [\cancel{\sin(2\theta)}]_0^{\pi}$$

$A = \pi$

$$\frac{x^2}{4} + y^2 = 1$$

$$x = 2 \sqrt{1 - y^2}$$

$$\frac{A}{2} = \int_0^1 x \, dy = 2 \int_0^1 \sqrt{1-y^2} \, dy$$

we put $y = \sin \theta \Rightarrow dy = \cos \theta \, d\theta$

$$y=0 \Rightarrow \sin \theta = 0 \Rightarrow \theta = 0$$

$$y=1 \Rightarrow \sin \theta = 1 \Rightarrow \theta = \frac{\pi}{2}$$

$$\frac{A}{2} = 2 \int_0^{\frac{\pi}{2}} \sqrt{1-\sin^2 \theta} \cos \theta \, d\theta$$

$$= 2 \int_0^{\frac{\pi}{2}} \cos^2 \theta \, d\theta = 2 \int_0^{\frac{\pi}{2}} \frac{\cos(2\theta) + 1}{2} \, d\theta$$

$$\frac{A}{2} = \left[\frac{1}{2} \sin 2\theta \right]_0^{\frac{\pi}{2}} + [\theta]_0^{\frac{\pi}{2}} = \frac{\pi}{2}$$

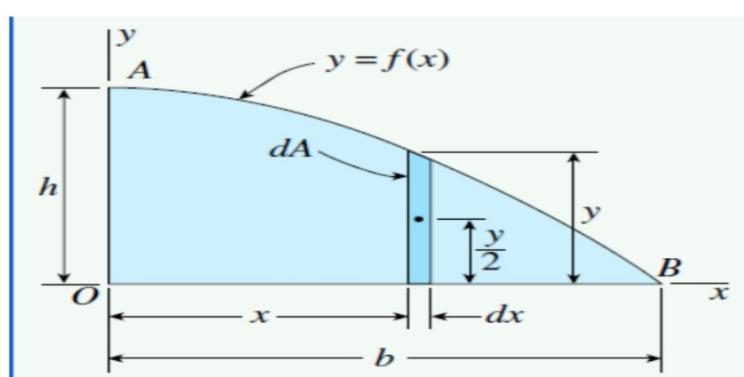
$$A = 2 \times \frac{\pi}{2} \Rightarrow A = \pi$$

Exercice 3 :

A parabolic semi-segment OAB is bounded by the x axis, the y axis, and a parabolic curve having its vertex at A . The equation of the curve is :

$$y = f(x) = h \left(1 - \frac{x^2}{b^2} \right)$$

in which b is the base and h is the height of the semi-segment. Determine the Area using a vertical differential element.



$$dA = y dx$$

$$dA = f(x) dx$$

$$dA = R \left(1 - \frac{x^2}{b^2}\right) dx$$

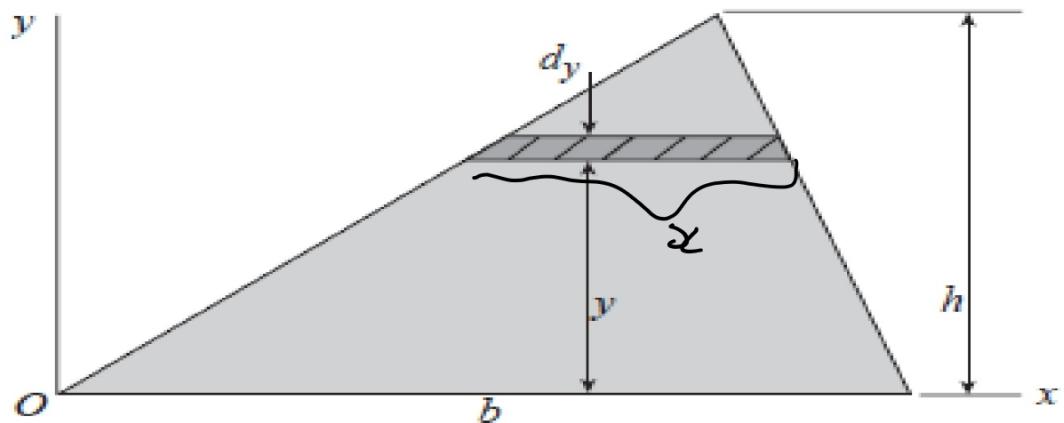
$$A = \int_0^b R \left(1 - \frac{x^2}{b^2}\right) dx$$

$$= Rh - \frac{Rb^3}{3b^2} = Rh - \frac{Rb}{3}$$

$$A = 2 \frac{Rh}{3}$$

Exercise 4 :

Determine the Area of the triangle using a horizontal differential element.



$$dA = x dy$$

d'après Tales:

$$\frac{x}{b} = \frac{h-y}{h}$$

$$x = b \left(1 - \frac{y}{h}\right)$$

$$A = \int_0^R b \left(1 - \frac{y}{h}\right) dy$$
$$= \left[by\right]_0^h - \frac{b}{2h} [y^2]_0^h$$
$$= bh - \frac{bh}{2} = \frac{bh}{2}$$

$$A = \frac{bh}{2}$$