Example: $\int_C \{y^3\,dx + (x^3+3xy^2)\,dy\}$ $\quad \gamma(t) = 3\cos t\,\vec{i} + 3\sin t\,\vec{j}$, $0 \le t \le 2\pi$

$\int_C \ldots = \frac{243\pi}{4}$.

Theorem: Let F be a continuous ~~sed~~ conservative vector field in an open subset D. then

$$\int_C F\cdot d\gamma = \int (\nabla f)\cdot d\gamma = f(\gamma(b)) - f(\gamma(a)),$$

for any ~~of~~ potential function f.

Proof: the proof of this theorem use the chain rule.

$$\int_C F\cdot d\gamma = \int_C \nabla f\cdot d\gamma = \int_a^b (f\circ\gamma)'(t)\,dt$$

Examples: (1) $F:(x,y)\longmapsto 2xy\,\vec{i} + (x^2-y)\,\vec{j}$; $C: (-1,4)\to(1,2)$

$f:(x,y)\longmapsto \left(yx^2 - \frac{y^2}{2}\right)$

(2) $F:(x,y,z)\longmapsto 2xy\,\vec{i} + (x^2+z^2)\,\vec{j} + 2yz\,\vec{k}$, $C: (1,1,0)\to(0,2,3)$

$f:(x,y,z)\longmapsto y(x^2+z^2)$

Remark:

- If D is connected ~~then~~ there is an equivalence in theorem
- If C is closed curve the $\int_C F\cdot d\gamma = 0$

Th (Green's theorem)

Let D be a simply connected subset with a ~~piece~~ piecewise smooth boundary $C=\partial D$ (C lisse par morceaux) C is oriented counterclockwise. If $M,N: D\to\mathbb{R}$ are of class C^1, then

$$\oint_{C=\partial D} M\,dx + N\,dy = \iint_D \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy.$$

Recall that a simple curve is a curve does not cross it ~~self~~ self. $D\subset\mathbb{R}^2$ is simply connected if every simple closed curve in D encloses only points of D.

Example: $\oint_C (y^3\,dx + (x^3+3xy^2)\,dy)$