

Set 5

Free and forced vibration of two degrees of freedom systems

Exercise 1:

In the system shown in Figure 1, the homogeneous rod of mass m and total length $3L$ can oscillate without friction around point O , located at a distance L from one of its ends. The upper end of the rod is connected to a mass via a spring with stiffness k_2 . This mass is also connected to a fixed wall through a spring with stiffness k_1 . The variables x_1 and $x_2 = 2l\theta$ are used to analyze this system.

1. Provide the expressions for the kinetic and potential energies of the system;
2. Derive the differential equations of the system;
3. Calculate the natural angular frequencies by assuming $k_1 = k_2 = k$ and $m_1 = m_2 = m$

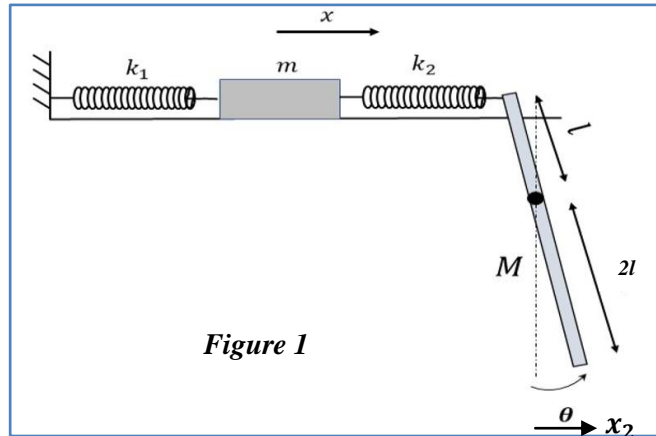


Figure 1

Exercise 2:

In Figure 2, we have outlined a vehicle with its suspension (without dampers). We assume that the springs remain vertical. The mass of the vehicle is m , and its moment of inertia relative to a horizontal axis D passing through the center of gravity G and perpendicular to the plane of the figure is I_0 . The displacement of the center of gravity relative to the equilibrium position is denoted by x (bouncing; pompage). The angle θ (pitch; tangage) made by the chassis with the ground due to rotation around D is assumed to be small. The tilt on the sides (roll) is assumed to be zero.

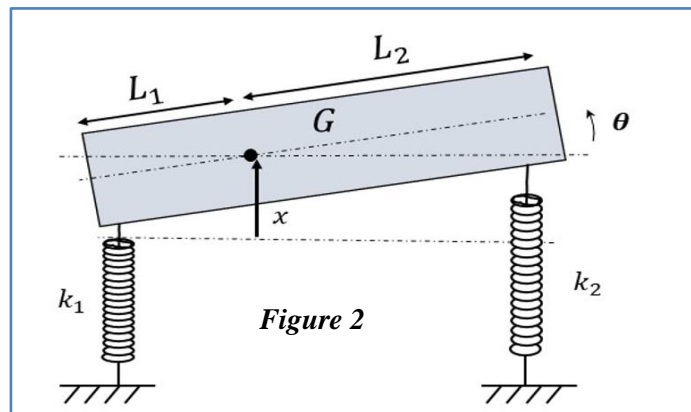
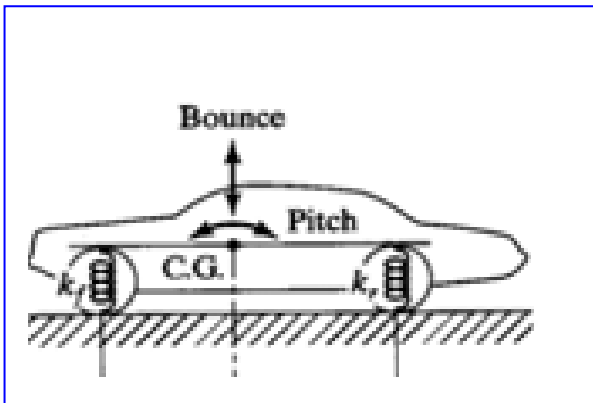


Figure 2

The following values are provided:

- Moment of inertia of the vehicle: $I_0 = mr^2$, $r = 0.9 \text{ m}$, vehicle mass: $m = 1000 \text{ Kg}$, $L_1 = 1 \text{ m}$, $L_2 = 1.5 \text{ m}$
 $k_1 = 18 \text{ KN/m}$ $k_2 = 18 \text{ KN/m}$

Determine the natural frequencies of the system as well as the ratio of amplitudes in each mode.

- Write the solutions of $x(t)$ and $\theta(t)$
- What condition must be satisfied to achieve decoupling between x and θ ?
- What are the natural frequencies of bouncing f_b and pitching f_p under this condition?

Exercise 3:

We consider a mechanical system with two degrees of freedom, consisting of two masses m_1 and m_2 , identified by their respective displacements $x_1(t)$ and $x_2(t)$ relative to their equilibrium positions. The mass m_1 is connected to a fixed frame through a spring with stiffness k_1 and a damper with a damping coefficient α . It is subjected to a vertical sinusoidal force with amplitude F_0 and angular frequency Ω . The mass m_2 is connected to two springs: One, with stiffness k_0 , has one end welded to the middle of a massless rod of negligible length

2L. The other, with stiffness k_2 , is attached to a fixed frame. The rod can rotate frictionlessly around one of its extremities attached to the frame.

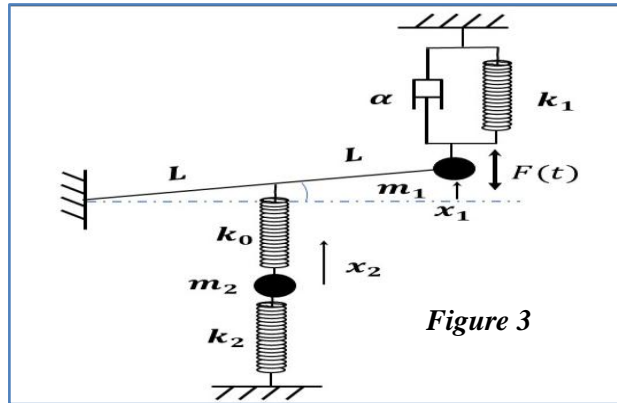


Figure 3

We consider small-amplitude motions and assume: $m_1 = \frac{m_2}{2} = m$ and $k_1 = \frac{k_2}{2} = \frac{k_0}{4} = k$

1. Show that the equations of motion can be expressed as follows:

$$\begin{cases} \frac{M}{2} \ddot{x}_1 + \alpha \dot{x}_1 + K(x_1 - x_2) = F(t) \\ M \ddot{x}_2 + 2Kx_2 + K(x_2 - x_1) = 0 \end{cases}$$

Determine M and K in terms of m and k .

3. a) Write the integro-differential equations in terms of the velocities \dot{x}_1 and \dot{x}_2 .
b) Derive the corresponding electrical equations using the Force-Voltage analogy. Specify the correspondence between the mechanical and electrical elements.
c) Provide the equivalent electrical circuit for the studied mechanical system.
4. a) Calculate the mechanical input impedance $Z_i = \frac{F}{\dot{x}}$
b) Determine the angular frequency for which the mass m_1 remains stationary.

Exercise 4:

In the mechanical system shown in Figure 4, the pulley with mass M and radius R can oscillate frictionlessly around its axis passing through point O .

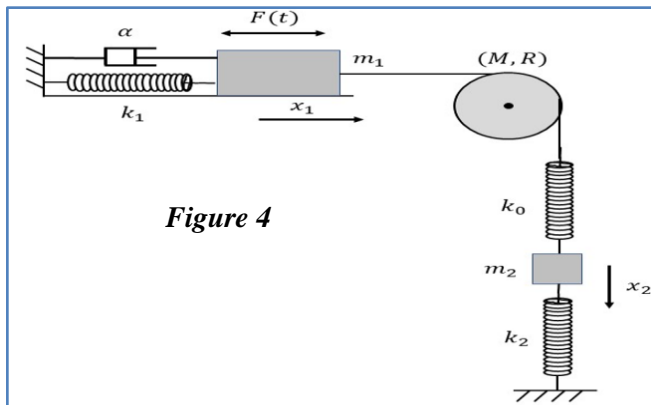


Figure 4

Part 1: Undamped free oscillations ($\alpha = 0$ and $F(t) = 0$).

1. Derive the differential equations of motion.
2. Deduce the equation for natural angular frequencies.

Part 2: Forced Oscillations ($F(t) = F_0 \cos(\Omega t)$ and $\alpha \neq 0$)

Assume the following: $M = 2(m_2 - m_1)$, $m_2 = m$, $k_0 = k_1 = k_2 = k$.

- Derive the new differential equations of motion.
- Calculate the input impedance.
- Determine the velocities $\dot{x}_1(t)$ and $\dot{x}_2(t)$.
- At what angular frequency does mass m_1 remain stationary? What is the amplitude of the oscillations of mass m_2 in this case?