N.S.A.S.T **2024/2025**
Second year P.C - Analysis 3

SERIE N° 1 : DOUBLE AND TRIPLE INTEGRALS

Exercice 1 :

Evaluate the following integrals :

1/ $I_1 = \iint_D \frac{y}{x^2+y^2}dxdy, \qquad D = \left\{(x,y) \in \mathbb{R}^2 : \quad 1 \leq x \leq 2, \quad x \leq y \leq 2x\right\},$

2/ $I_2 = \iint_D xe^y dxdy, \qquad D = \left\{(x,y) \in \mathbb{R}^2 : \quad x \geq 0, \quad y \geq 0, \quad x+y \leq 4\right\}.$

Exercice 2 :

Find the volume of the solid in the first octant (i.e. $(\mathbb{R}^+)^3$) bounded by the graphs of the equations :

1/ $z = xy, \quad z = 0, \quad y = x, \quad x = 1,$

2/ $1 = x^2 + y^2, \quad x^2 + z^2 = 1,$

3/ $z = x + y, \quad x^2 + y^2 = 4.$

Exercice 3 :

Find the area of the surface given by the equation $z = f(x,y)$ over D, in the following :

1/ $f : (x,y) \longmapsto 12 + 2x - 3y, \qquad D = \left\{(x,y) \in \mathbb{R}^2 : \quad x^2 + y^2 \leq 9\right\}$

2/ $f : (x,y) \longmapsto 9 - x^2, \qquad D = [0,2] \times [0,2]$

3/ $f : (x,y) \longmapsto -\ln|\sin x|, \qquad D = \left\{(x,y) \in \mathbb{R}^2 : \quad 0 \leq x \leq \frac{\pi}{4}, \quad 0 \leq y \leq \tan x\right\}.$

Exercice 4 :

Use a change of variables to find the volume of the solid region lying below the surface $z = f(x,y)$ and above the plane region D.

1/ $f : (x,y) \longmapsto (3x+2y)^2\sqrt{2y-x}$, D is the parallelogram with vertices $(0,0), (-2,3), (2,5), (4,2)$

2/ $f : (x,y) \longmapsto (x+y)^2 \sin^2(x-y)$, D is the square with vertices $(\pi,0), \left(\frac{3\pi}{2}, \frac{\pi}{2}\right), (\pi,\pi), \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

3/ $f : (x,y) \longmapsto \frac{xy}{1+x^2y^2}$, D is bounded by the graphs of $xy = 1, \quad xy = 4, \quad x = 1, \quad x = 4.$

Exercice 5 :

Find the mass, the center of mass and the moments of inertia I_x, I_y of the lamina bounded by the graphs of the equations with given density ρ :

$$\begin{array}{llll}
\mathbf{1/} & y = \sqrt{x}, \quad y=0, \quad x=1, & & \rho(x,y)=ky, \\
\mathbf{2/} & y = \dfrac{4}{x}, \quad y=0, \quad x=1, \quad x=4, & & \rho(x,y)=kx^2, \\
\mathbf{3/} & y = 4-x^2, \quad y=0, & & \rho(x,y)=ky, \\
\mathbf{4/} & y = \sqrt{a^2-x^2}, \quad 0\le y\le x, & & \rho(x,y)=k.
\end{array}$$

Exercice 6 :

Integrate the given function f over the indicate region D.

1/ $f(x,y,z) = 2x-y+z,$ D is bounded by $z=x^2+y^2, \quad z=0, \quad x=0, \quad x=1, \quad y=-2, \quad y=2.$

2/ $f(x,y,z) = \dfrac{z}{\sqrt{x^2+y^2}}, \quad D=\left\{(x,y,z)\in\mathbb{R}^3 : \quad x^2+y^2+z^2\le 1,\ y^2-2xz\le 0,\ 4z^2\le x^2+y^2,\ z\ge 0\right\}$

Exercice 7 :

Use a triple integral to find the volume of the solid Ω bounded by the graphs of the equations :

$$\begin{array}{ll}
\mathbf{1/} & z = 9-x^3, \quad y=2-x^2, \quad y=0, \quad z=0, \quad x\ge 0, \\
\mathbf{2/} & z = 2-y, \quad z=4-y^2, \quad x=0, \quad x=3, \quad y=0.
\end{array}$$

Exercice 8 :

Find the center of mass $(\overline{x},\overline{y},\overline{z})$, the moments of inertia about the y- and the z-axes, of Ω with the density ρ, where

$$\Omega=\left\{(x,y,z)\in\mathbb{R}^3 : \quad x^2+y^2\le z\le\sqrt{4-x^2-y^2}\right\}, \qquad \rho(x,y,z)=kz.$$