Module : Physics 3

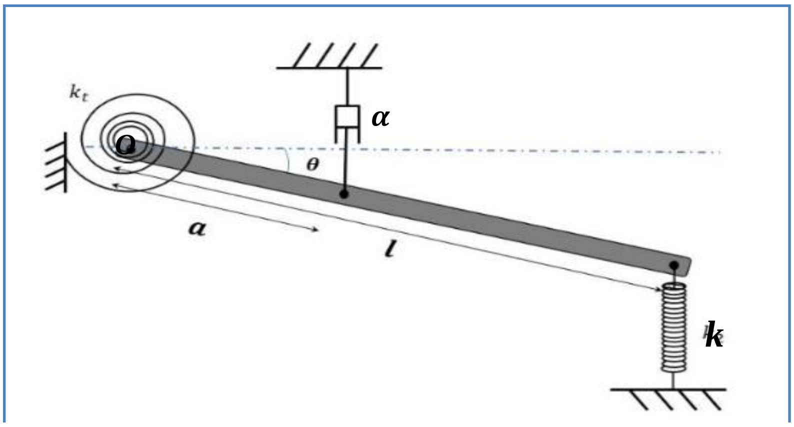
2nd year 2024/2025

Set 3

Damped free vibration of single degree of freedom systems

Exercise 1:

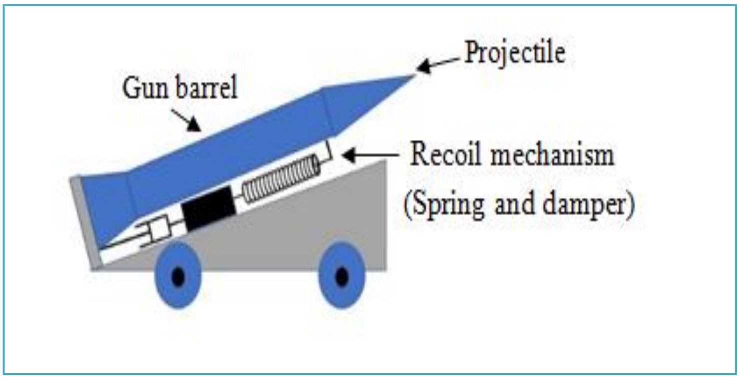
A homogeneous rod of length ***L*** and mass ***M*** can oscillate around a point ***O***, as shown in the figure below. A torsion spring with a constant K_t is placed at point O. At the other end, a linear spring with a constant K is attached, while a damper with a constant α is positioned at a distance **a** from point **O** along the rod. At equilibrium, the rod is horizontal. Determine the natural period of the system.



Exercise 2:

In military cannons, dampers are used to absorb the recoil when it fires. This helps reduce the shock on the cannon, quickly stabilizes the weapon after each shot, and improves the accuracy of subsequent shots. The schematic diagram of cannon is shown in figure below. When the gun is fired, the reaction force pushes the gun barrel in the direction opposite that of the projectile. Since it is desirable to bring the gun barrel to rest in the shortest time without oscillation, the gun barrel and the recoil mechanism have a mass of 500 kg. The maximum recoil distance of the cannon is specified as *0.5 m*. If the initial recoil speed is 10 m/s and the initial position $x(0) = 0$ represents starting position:

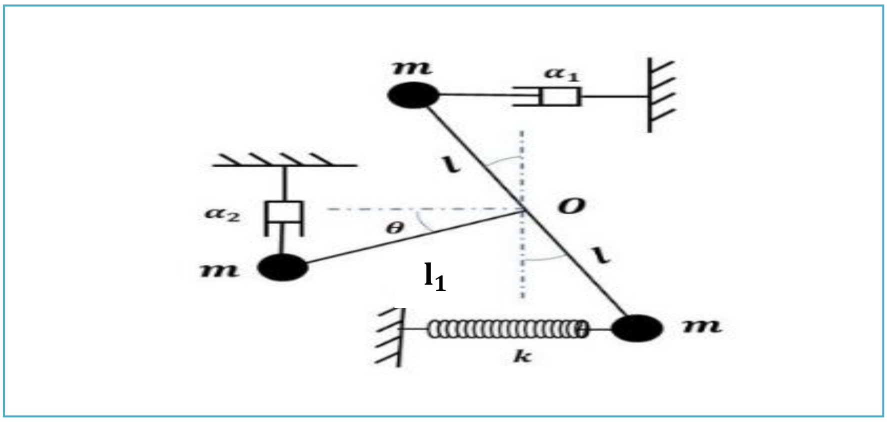
- What is the natural damping regime used in the cannon?
- Calculate the stiffness constant of the rappel mechanism?



Exercise 3:

In the system shown, the two rods with negligible mass are perpendicular. At equilibrium, the rod of length l_1 is horizontal. We release the rod after displacing it from the equilibrium position by a small angle θ.

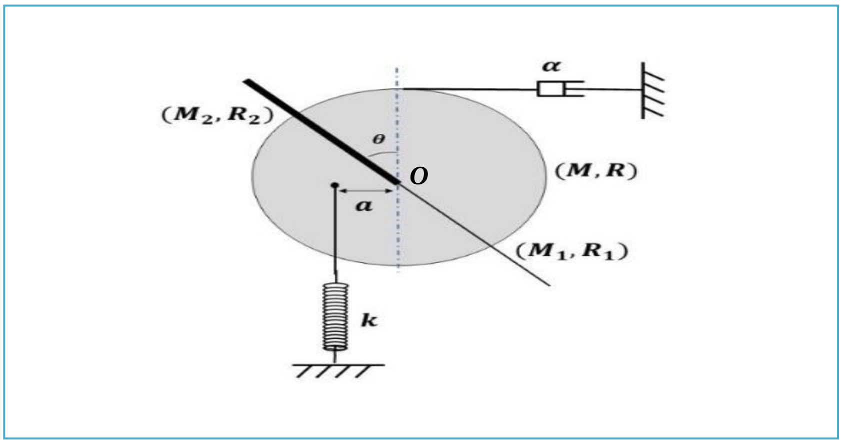
- Calculate the kinetic energy and the potential energy of the system.
- Find the equation of motion using Lagrangian formalism for $l_1 = l$..



Exercise 4;

In the figure below, a homogeneous disk (M, R) can oscillate around point . Two rods with masses M_1 and M_2 and lengths $L_1$and L_2 respectively are welded to the disk along the same diameter. At equilibrium, the rod of length L_1 is vertical and points downward.

- What is the extension of the spring at equilibrium?
- Establish the differential equation of motion for small amplitude oscillations.
- What is the condition for oscillations to occur?



Exercise 5:

In the system shown in the figure below, the two rods of lengths $\boldsymbol{L}$ and $\boldsymbol{3L}$ have negligible mass and can oscillate in the vertical plane as shown in the figure. The two masses located at the top are connected by a negligible mass rod, ensuring that the distance between them remains constant. At equilibrium $\boldsymbol{\theta = \varphi = 0}$.

- Determine the potential energy of the system and the condition for oscillation in the absence of friction.
- Determine the kinetic energy as well as the dissipation function, and deduce the equation of motion for the damped system.
- What are the values of α required to maintain the system in oscillation?
- Calculate the pseudo-period after which the initial amplitude is divided by 4.