National Higher School of Autonomous Systems Technology
Academic year : 2024/2025

Module : Physics 3
2nd year 2024/2025

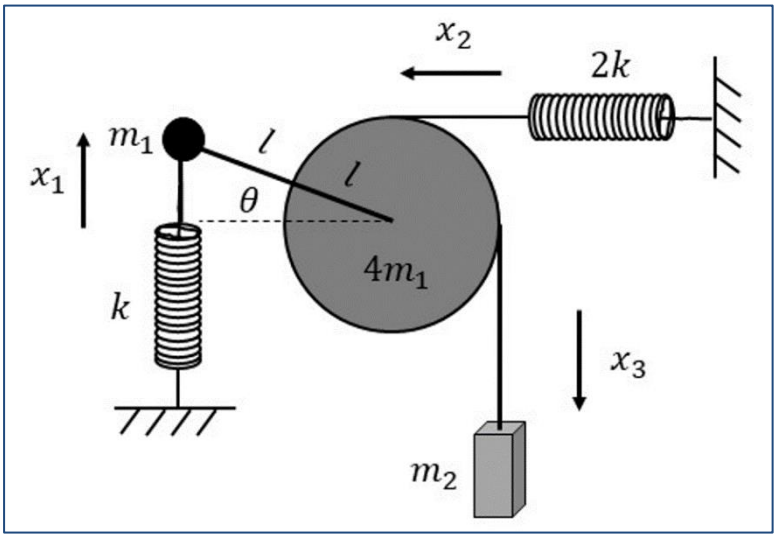
Set N⁰ 2

(Free vibration of undamped single degree of freedom systems)

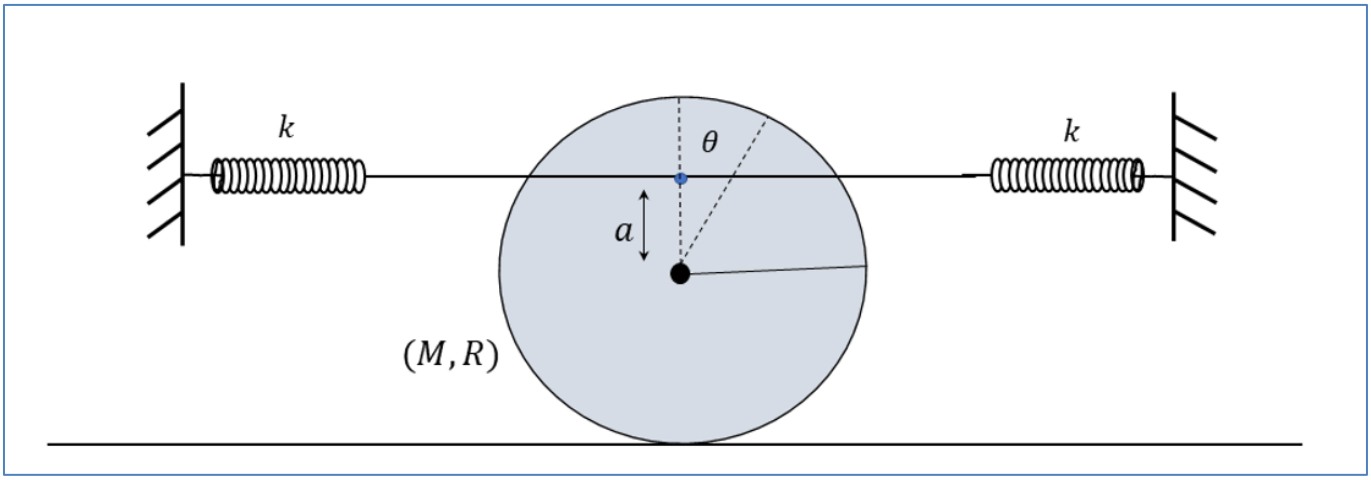
***Exercise* 1:**

In the system shown in the figure below, assume that the pulley can rotate around its center without friction. At equilibrium, the rod is horizontal, and the vertical spring is not deformed. The pulley is displaced from equilibrium by a small angle θ and then released.

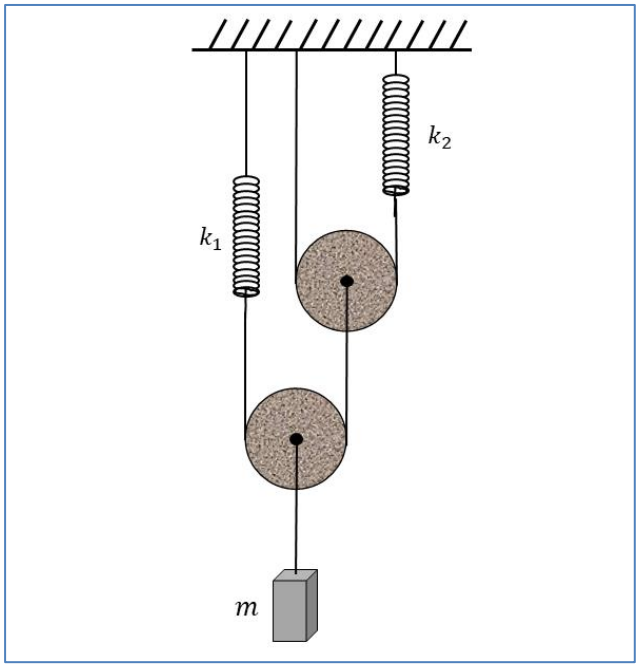
1. Express the potential energy U of the system as a function of θ.
2. Deduce the initial elongation Δl of the spring with stiffness $2k$, and then simplify the expression for U.
3. What value should m_2 have in terms of m_2 so that Δl is zero?
4. Suppose $m_2 = 2m_1$. Find the equation of motion.



Exercise 2: A cylinder of mass M and radius R rolls without slipping and is held by two springs with stiffness constants k_1 and k_2 , as shown in the figure below. Determine the differential equation of motion and deduce the period of the oscillations



Exercise 3:

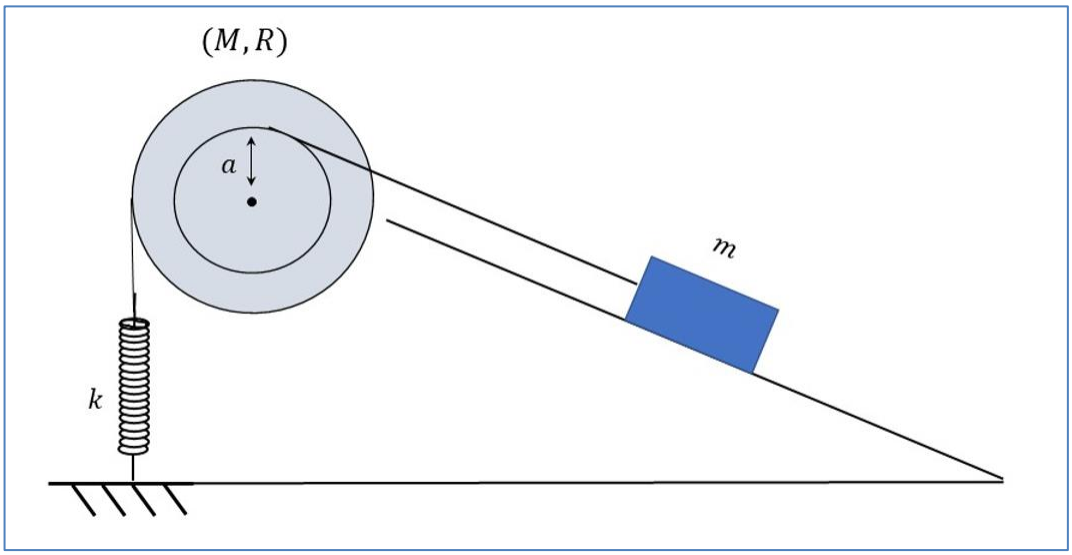


In the mechanical system shown in the figure,
the two pulleys have negligible mass and the same radius ***R***.
- Give the expressions for the kinetic and potential energies of the system as a function of the displacement ***x*** of the mass ***m***.
Then, derive the natural angular frequency of the oscillations.

Exercise4:

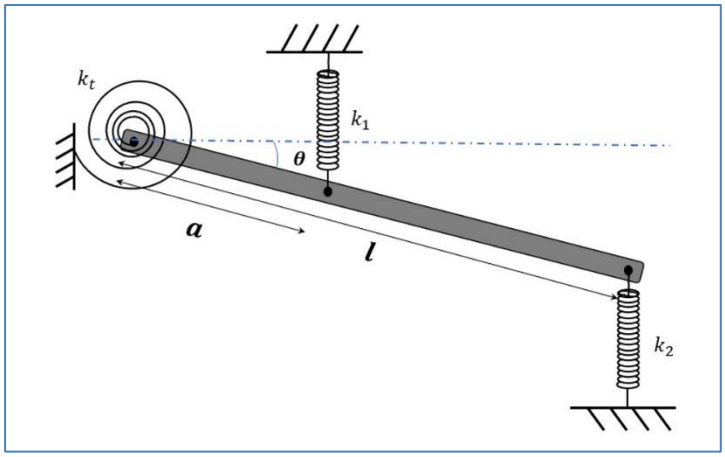
In the system shown in the figure below, the point mass ***m*** can slide on a pulley inclined at an angle ***α*** with respect to the horizontal. It is attached to the groove of a pulley ***(M,R)*** at a distance ***a*** from its center.

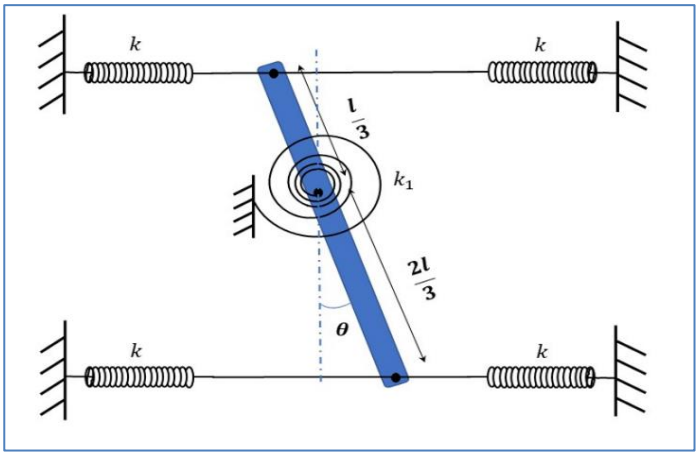
- Determine the potential energy of the system and the extension of the spring at equilibrium, as well as the condition for oscillations if there are any.
- Determine the differential equation of motion.

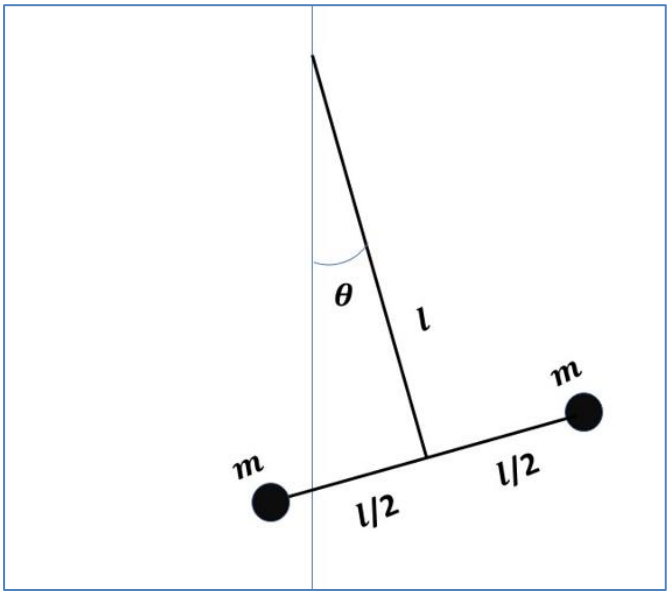


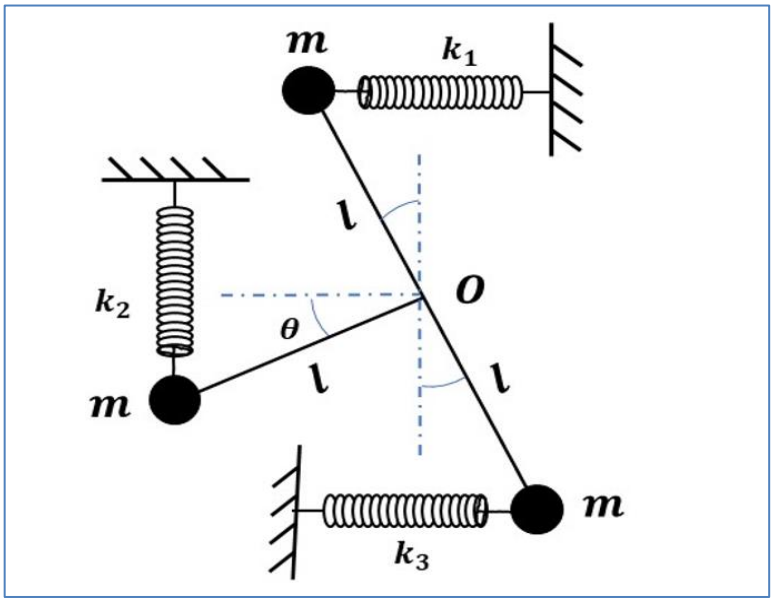
Exercise 5:

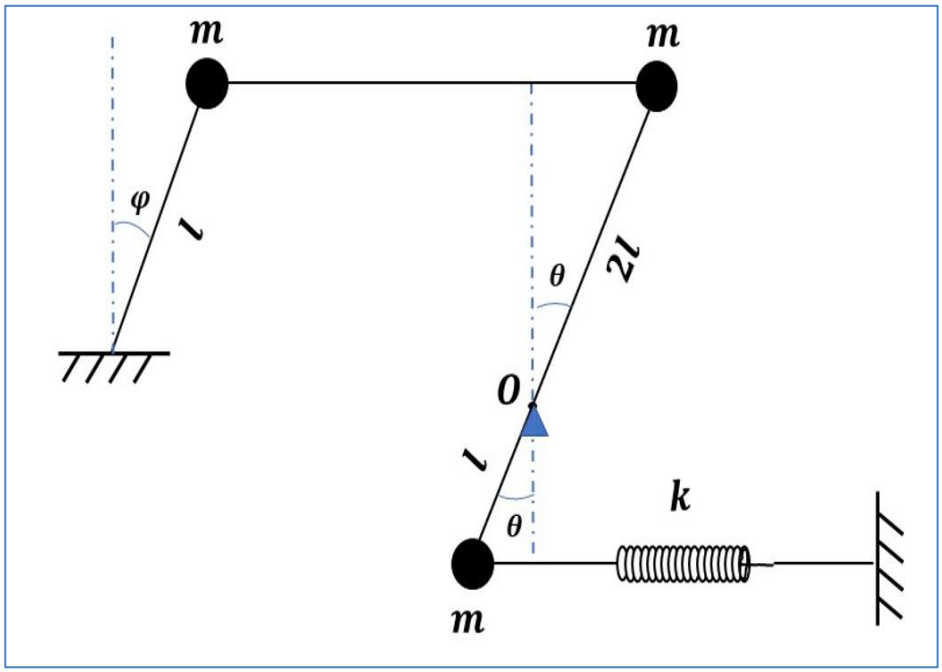
Find the equation of motion for each of the following cases

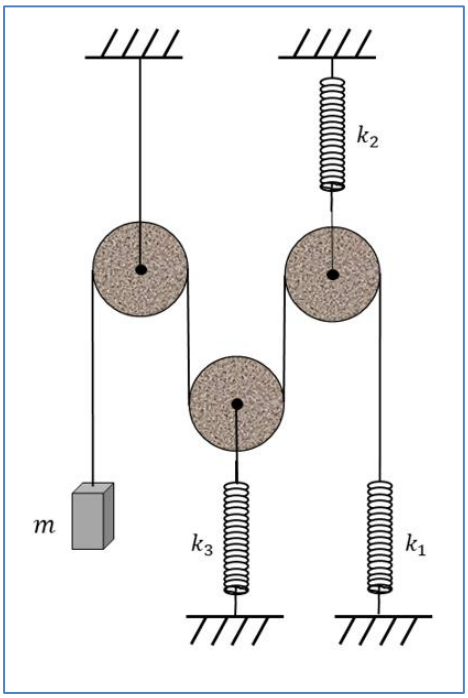

k_t
k_1
θ
a
l
k_2


k
k
$\frac{l}{3}$
k_1
$\frac{2l}{3}$
θ
k
k


θ
l
m
m
$l/2$
$l/2$


m
k_1
l
k_2
O
θ
m
l
l
k_3
m


m
m
φ
l
θ
$2l$
O
l
k
θ
m


k_2
m
k_3
k_1