# Chapter 5: General-theorems-of-electric-circuits

## 1. <u>Superposition Theorem:</u>

The Superposition Theorem is a fundamental principle in **linear electrical circuits**. It allows the analysis of circuits with **multiple independent sources** (voltage or current sources) by **considering the effect of each source individually** and then combining the results.

**<u>Example1:</u>** Let be the following circuit

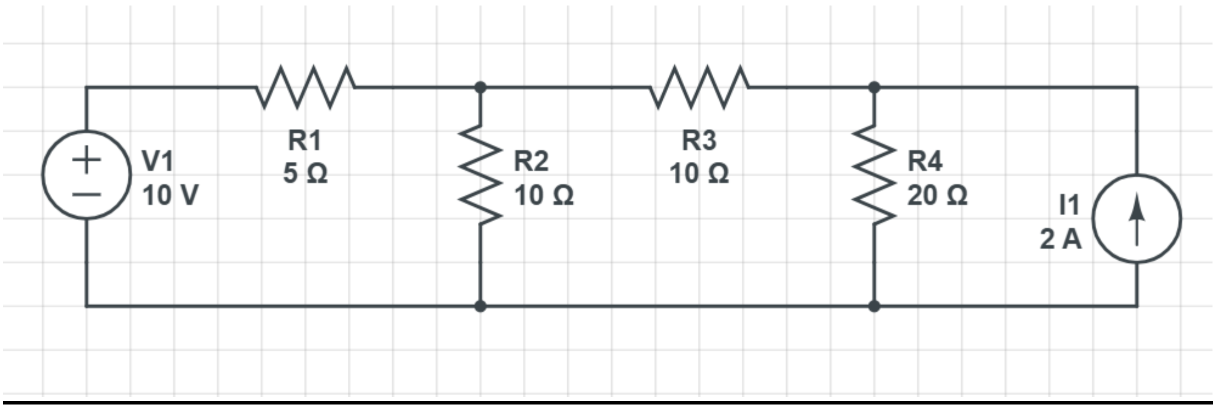


**Figure 1:** Example for superposition theorem

**The steps to follow are:**

1. Put off one source and keep the other on: (let say current source off and replaced by an **open-circuit**):

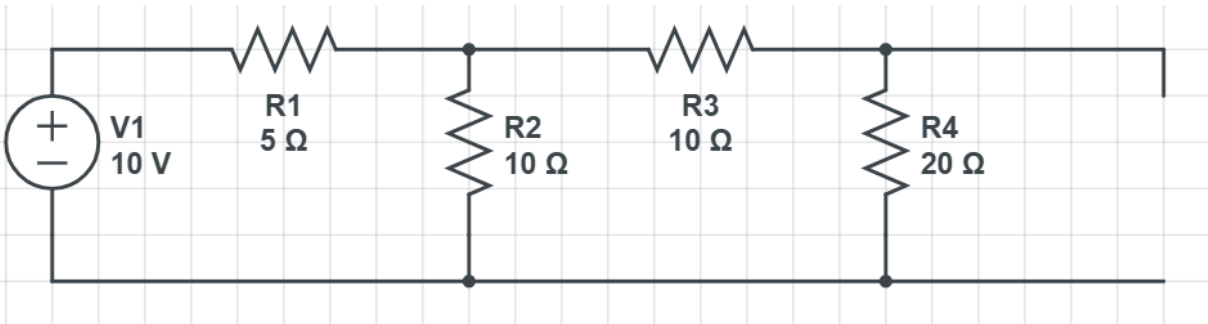


**Figure 2:** Example of Puting off current source

We calculate here currents in each branch of the circuit. R4 is in series with R3 (30 Ohms) and this equivalent resistor is in parallel with R2 (30//10=7.5 Ohms). Finally, we have R1 in series with an equivalent resistor=7.5 Ohms and we obtain 12.5 Ohms. The total current flowing through $R_1$ is $I_1$=10/12.5=0.8 A. Using the loop laws for the first loop (voltage source+$R_1$+$R_2$) we obtain: V1-R1 $I_1$-$V_2$=0. We obtain: $V_2$=6 Volts. We, then, obtain current $I_2$ flowing into $R_2$. $I_2$=6/10=0.6 A. Then, the current flowing in R3 and R4 is 0.8-0.6=0.2 A. Now we obtain all voltages and currents in the circuits caused by the voltage source we go to second step.

2. Put off voltage source and keep current source and replace the voltage source by a **short-circuit**):

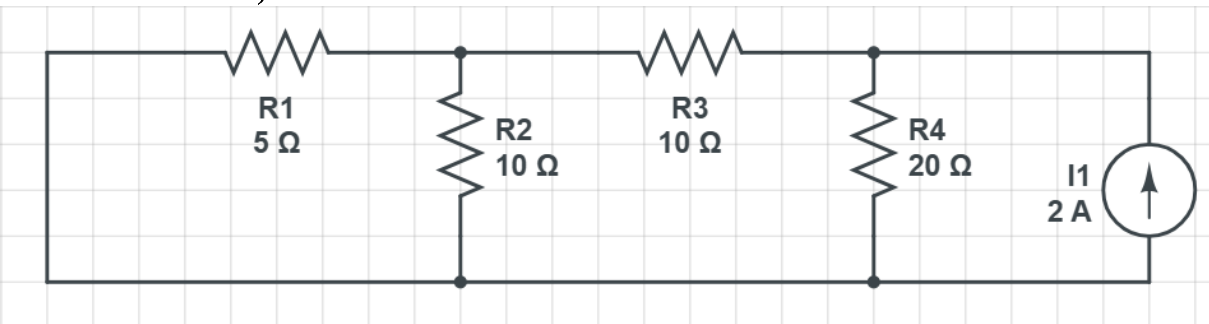


**Figure 3:** Example of Puting off voltage source

Here we get R1 in parallel to R2 (R1//R2=10/3 Ohms). The equivalent resistor is in series with R3 (Req1=10+3.3333=13.33333 Ohms). Finally we have R4//Req1=8 Ohms. The voltage across this equivalent resistor is V4=8*2=16 Volts. We deduce the current flowing into R4=16/20=0.8 A. The current that flows into R3=2-0.8=1.2 A. Using loop's law for R4, R3 and R2 we got V4-V3-V2=0 and thus V2=V4-V3=16-R3*1.2=4 Volts. We deduce the current flowing into R2: $I_{22}$=4/10=0.4 A. Finally, current flowing into R1 equals 1.2-0.4=0.8 A.

3. Last step: make the addition of the currents in each branch. We have to keep the direction of each current.

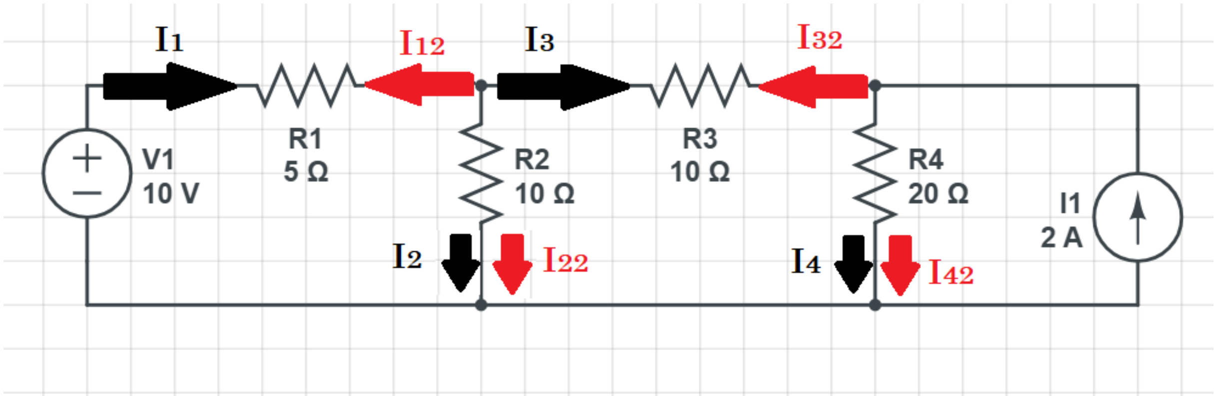

**Figure 4:** Example of currents in all branches (superposition)

We finally get
$J_1=I_1-I_{12}=0.8-0.8=0$ A;
$J_2=I_2+I_{22}=0.6+0.4=1$ A;
$J_3=I_3-I_{32}=0.2-1.2=-1$ A;
$J_4=I_4+I_{42}=0.2+0.8=1$A;

## 2. Millman Theorem :

The Theorem states that in a circuit with multiple voltage sources in parallel, **the voltage at the common node** can be **found by taking a weighted average of the voltage sources**, **weighted by the inverse of their respective resistances**.

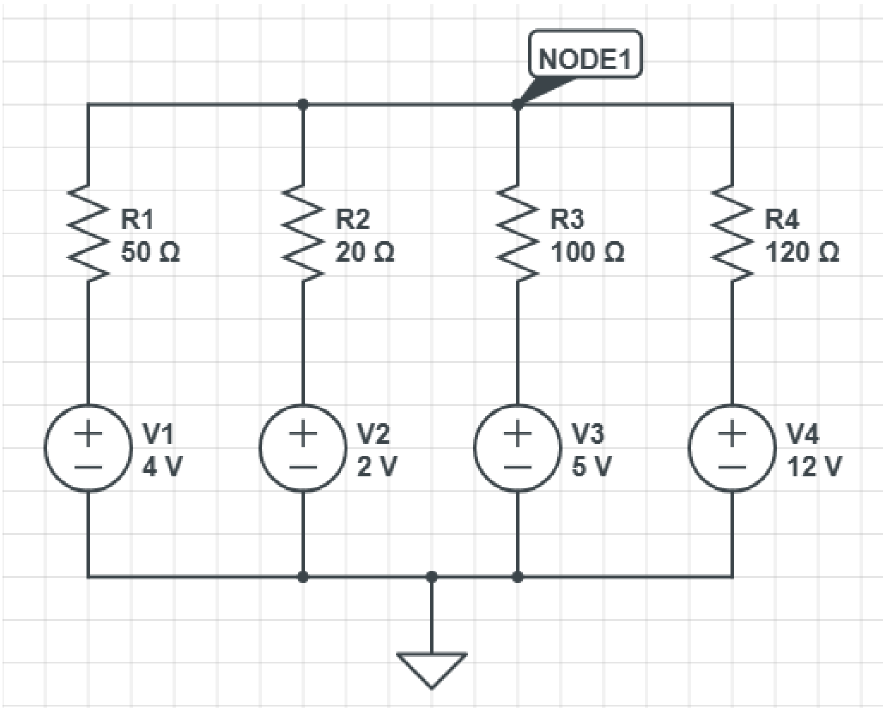


**Figure 5:** Example of Millman theorem

The technique consists of looking through each wire (resistor) the value of voltage it has and then apply the weighted sum of voltages over resistances and divide the whole by the sum of admittances:

$$V = \frac{\sum_{i=1}^{n} \frac{V_i}{R_i}}{\sum_{i=1}^{n} \frac{1}{R_i}}$$

In the example above, it is:

$$V = \frac{\sum_{i=1}^{n} \frac{V_i}{R_i}}{\sum_{i=1}^{n} \frac{1}{R_i}} = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \frac{V_4}{R_4}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} = \frac{\frac{4}{50} + \frac{2}{20} + \frac{5}{100} + \frac{12}{120}}{\frac{1}{50} + \frac{1}{20} + \frac{1}{100} + \frac{1}{120}} = 3.736\, volts$$

# 3. <u>Thevenin Theorem:</u>

**Thevenin's Theorem** is a fundamental principle in electrical circuit analysis that simplifies the analysis of complex linear circuits. It states:

Any linear, two-terminal electrical network consisting of voltage sources, current sources, and resistors can be replaced by an equivalent circuit with a single voltage source (called Thevenin voltage, V_th) in series with a single resistor (called Thevenin resistance, R_th), connected to the load.

### Key Steps to Apply Thevenin's Theorem:

1. **Identify the Portion of the Circuit**:
   - Choose the part of the circuit to simplify, often the section connected to a **specific load.**
2. **Remove the Load**:
   - Temporarily disconnect the load resistor (if any) from the circuit.
3. **Calculate V_th** :
   - Determine the open-circuit voltage across the terminals where the load was connected. This voltage is the Thevenin voltage, V_th .
4. **Calculate R_th**:
   - Turn off all **independent voltage** sources (**replace with short circuits**) and independent current sources (replace with open circuits).
   - Find the equivalent resistance between the two terminals. This resistance is the Thevenin resistance, R_th.

5. **Reconstruct the Equivalent Circuit**:
   - Replace the original circuit with the Thevenin equivalent circuit: a single voltage source (V_th) in series with a single resistor (R_th).
6. **Reattach the Load**:
   - Connect the load resistor to the Thevenin equivalent circuit and analyze it as needed.

## Advantages:

- It simplifies complex circuits, especially when analyzing the effect of varying load resistances.
- It reduces repetitive calculations in circuits with multiple load changes.

## Example:

For a circuit with a voltage source, resistors, and a load:

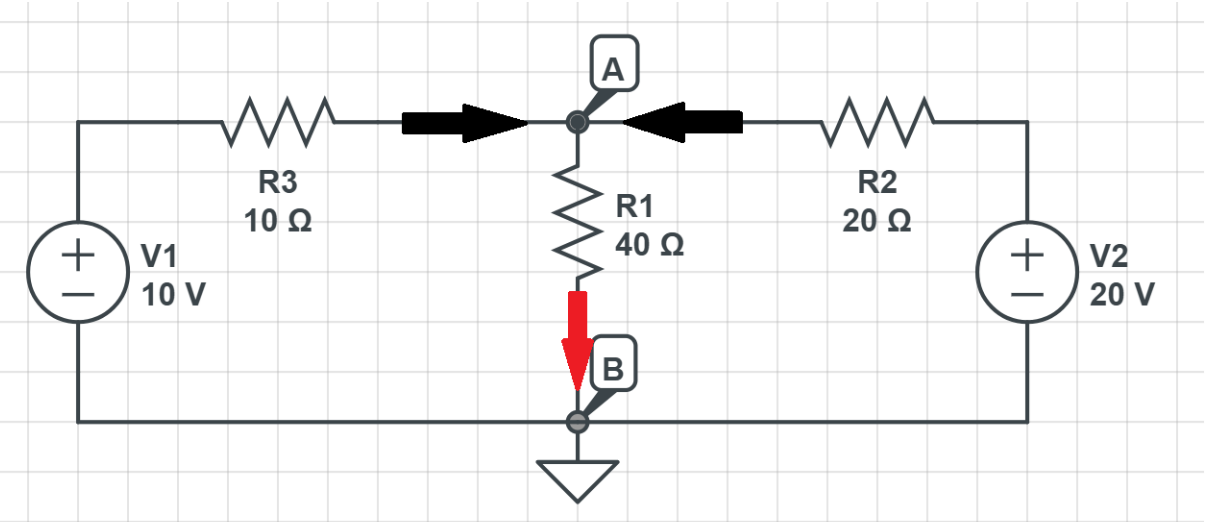


**Figure 6:** Example of Thevenin theorem

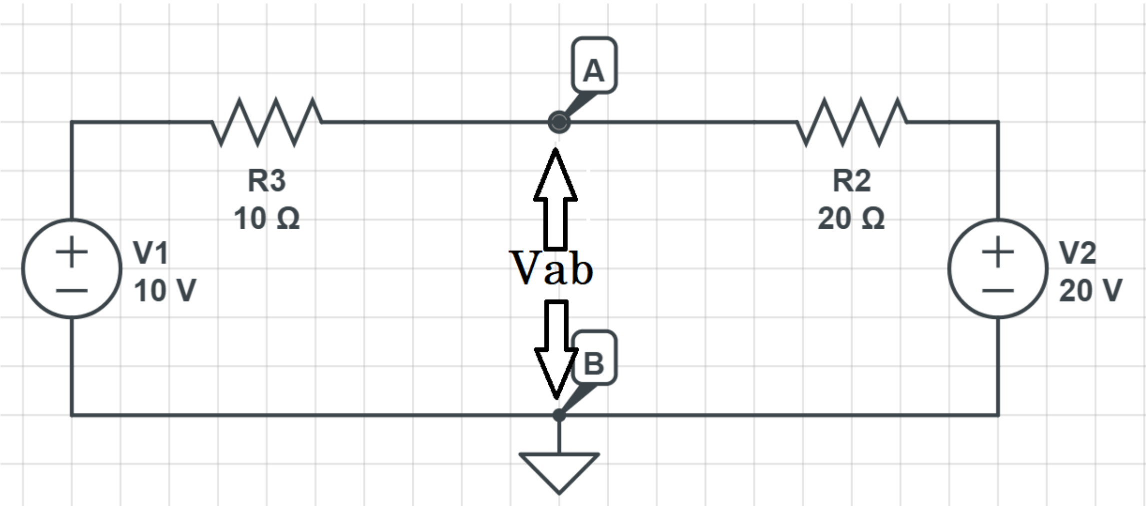


**Figure 7:** Calculating V_th in Thevenin theorem

Using Millman theorem, we get:

$$V_{TH} = V_{ab} = \frac{\frac{V2}{R2} + \frac{V1}{R3}}{\frac{1}{R2} + \frac{1}{R3}} = \frac{40}{3}\ volts$$

To find R_th we take off voltage source and replace them by short circuits:

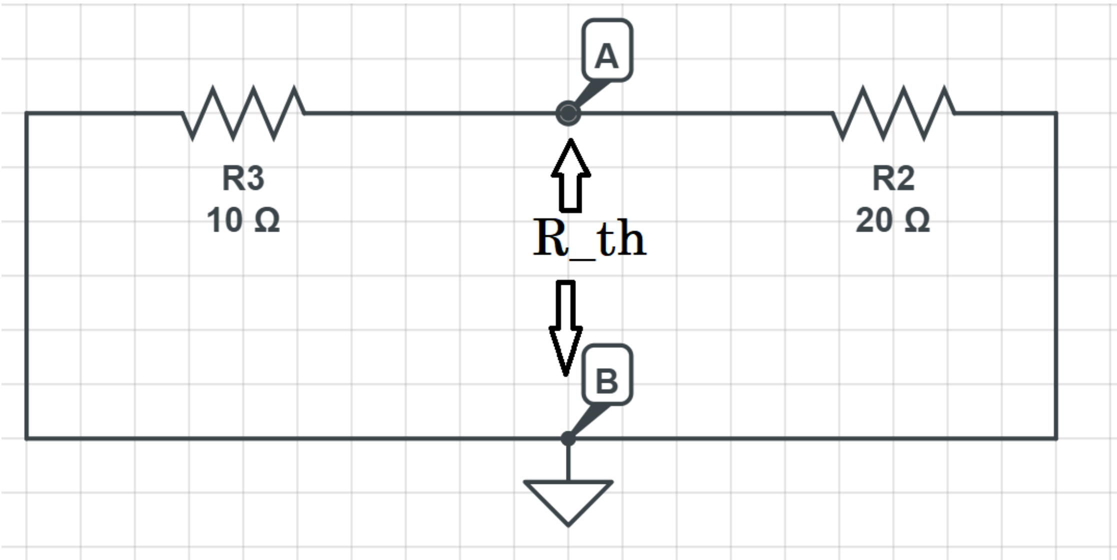


**Figure 8:** Calculating R_th in Thevenin theorem

R_th=R2//R3=$\frac{20}{3}$Ω

Let's replace the circuit by its corresponding Thevenin circuit:

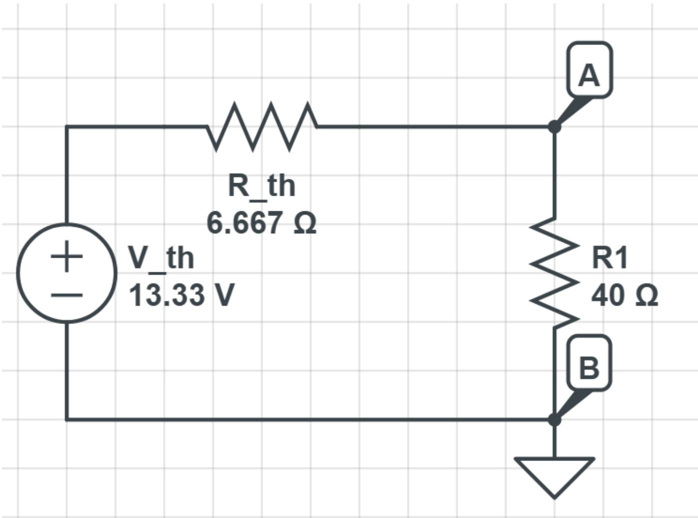


**Figure 9:** equivalent Thevenin theorem circuit

**V_R1=11.43 volts** and **I_R1=0.286 A**

Using classic circuit analyses determine the current and voltage across R1 and check if the correspond to those found with the Thevenin equivalent circuit.

# 4. Norton Theorem:

**Norton's Theorem** is another fundamental principle in electrical circuit analysis. It is closely related to Thevenin's Theorem but uses a different equivalent circuit representation. It states:

*Any linear, two-terminal electrical network consisting of voltage sources, current sources, and resistors can be replaced by an equivalent circuit with a single current source (called Norton current, I_N) in parallel with a single resistor (called Norton resistance, R_N), connected to the load.*

### Key Steps to Apply Norton's Theorem:

1. **Identify the Portion of the Circuit**:
   - Select the part of the circuit you want to simplify, typically the section connected to a specific load.
2. **Remove the Load**:
   - Temporarily disconnect the load resistor (if any) from the circuit.
3. **Calculate I_N**:
   - Determine the short-circuit current across the terminals where the load was connected. This current is the Norton current, I_N.
4. **Calculate R_N**:
   - Turn off all **independent voltage sources** (replace them with short circuits) and **independent current sources** (replace them with open circuits).
   - Find the equivalent resistance between the two terminals. This resistance is the Norton resistance, R_N.
5. **Reconstruct the Equivalent Circuit**:
   - The original circuit should be replaced with the Norton equivalent: a single current source (I_N) parallel with a single resistor (R_N).
6. **Reattach the Load**:
   - Connect the load resistor to the Norton equivalent circuit and analyze it as needed.

### Relationship Between Thevenin and Norton:

- Thevenin and Norton are duals of each other, and you can easily convert between the two:

### Advantages:

- It simplifies circuit analysis, especially for networks with varying loads.
- It is helpful in analyzing circuits where current sources are more natural or prevalent.

**Example:**

We take the same example used for Thevenin theorem.

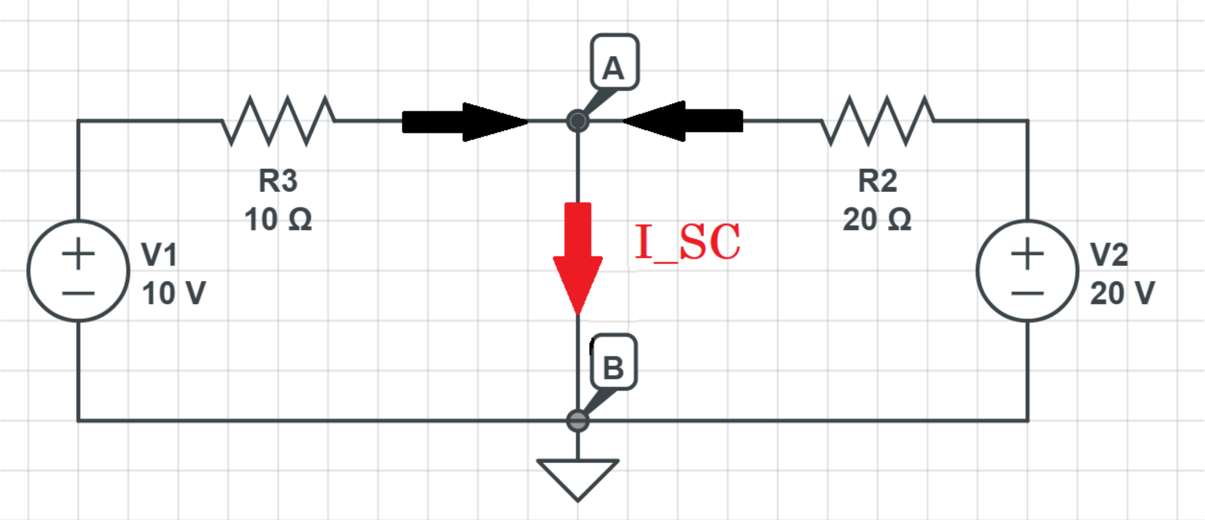


**Figure 10:** Calculating I_SC (short circuit current) in in Norton theorem

From the figure 10, we get that I_N=I1+I2=1+1=2 A

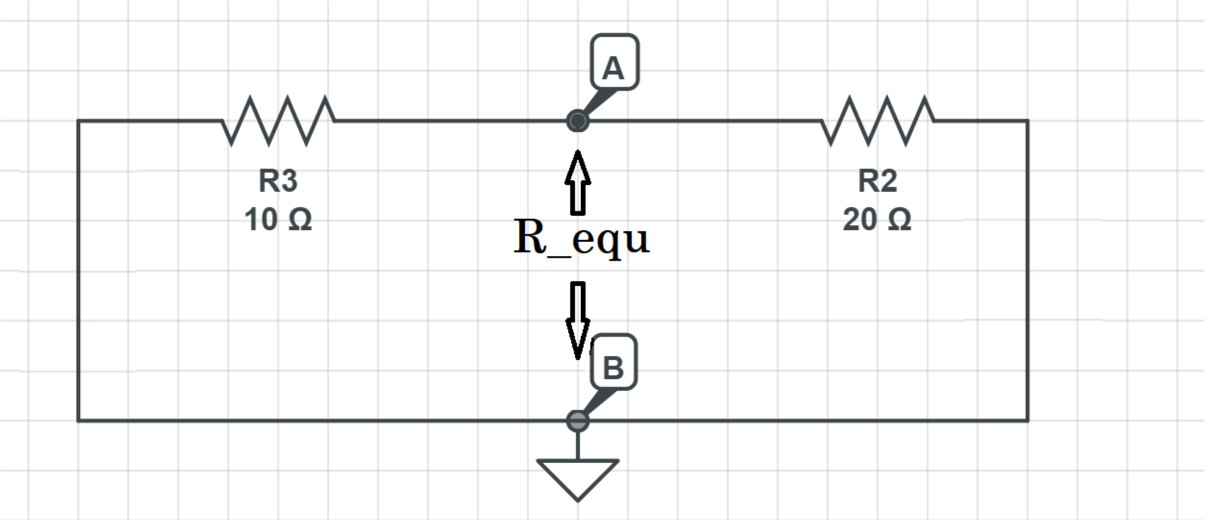


**Figure 11:** Calculating R_equ (short circuit all voltage sources) in in Norton theorem

From figure 11, we get that R_N=R_equ=R2//R3=$\frac{20}{3}\Omega$ (you notice that this is exactly the same resistance found for Thevenin).

Finally, we obtain our equivalent circuit (Figure 12):

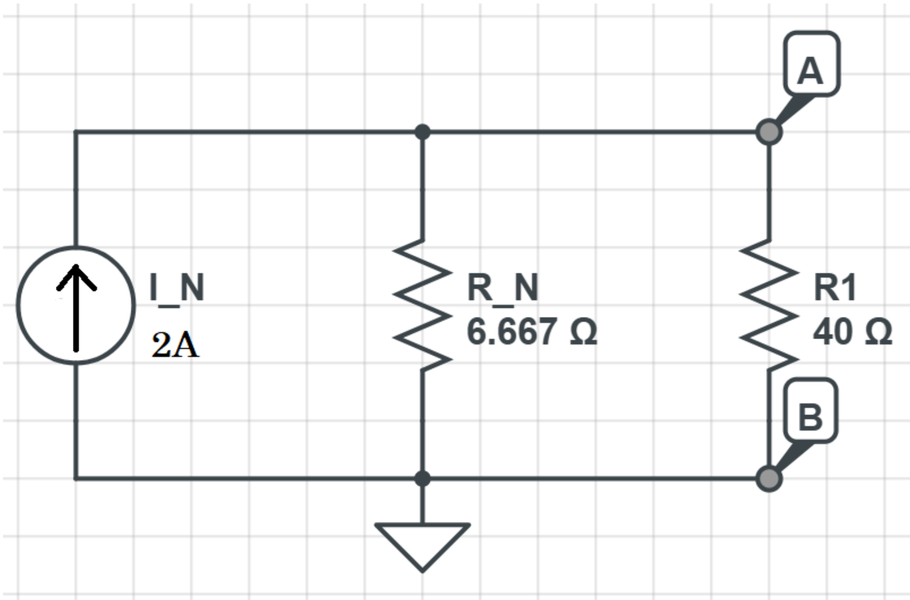


**Figure 12:** Norton equivalent circuit

Using current divider we find the current I1 flowing into the load:

$I_1 = \frac{R\_N}{R_1+R\_N} I\_N$=**0.286 A=**I_R1 (the current found using Thevenin)

V1=I1*R1= **11.43 volts** =V_R1 (voltage calculated using Thevenin circuit).

**<u>Norton-Thevenin equivalence:</u>**

R_N=R_th

I_N=$\frac{V\_th}{R\_th}$