Set N°4

Ex 1: $T = \frac{1}{2} m \dot{x}^2$

case 1:

$U = \frac{1}{2} k \left(x - S_1(t)\right)^2$

$D = \frac{1}{2} \alpha \dot{x}^2$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} + \frac{\partial D}{\partial \dot{x}} = 0$$

$$m\ddot{x} + k\left(x - S_1(t)\right) + \alpha \dot{x} = 0$$

$$m\ddot{x} + \alpha \dot{x} + kx = k S_1(t)$$

$$m\ddot{x} + \alpha \dot{x} + kx = F(t)$$

$$\ddot{x} + \frac{\alpha}{m}\dot{x} + \frac{k}{m} x = \frac{k}{m} S_0 \cos(\Omega t)$$

$$x(t) \simeq x_p(t) = A\cos(\Omega t + \varphi)$$

$$\bar{x}(t) = A e^{j(\Omega t + \varphi)}$$

$$\overline{S_1(t)} = S_0 e^{j\Omega t}$$

$$\bar{\dot{x}}(t) = A j \Omega e^{j(\Omega t + \varphi)}$$

$$\overline{\ddot{x}(t)} = -\Omega^2 A e^{j(\Omega t + \varphi)}$$

$$-\omega^2 A e^{j(\Omega t + \varphi)} + 2\delta A j \Omega e^{j(\Omega t + \varphi)} + \omega_n^2 A e^{j(\Omega t + \varphi)} = \frac{k}{m} S_0 e^{j\Omega t}$$

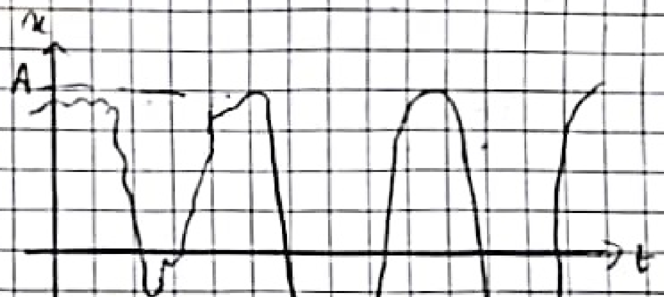
$$A e^{j\varphi}\left(\omega_n^2 - \Omega^2 + 2j\delta\Omega\right) = \frac{k}{m} S_0$$

$$A\left(\omega_n^2 - \Omega^2 + 2j\delta\Omega\right) = \frac{k}{m} S_0 e^{-j\varphi}$$

$$A\sqrt{(\omega_n^2 - \Omega^2)^2 + (2\delta\Omega)^2} = \frac{k}{m} S_0$$

$$A = \frac{k S_0}{m\sqrt{(\omega_n^2 - \Omega^2)^2 + (2\delta\Omega)^2}}$$

$$\varphi = \arctan\left(\frac{-2\delta\Omega}{\omega_n^2 - \Omega^2}\right)$$



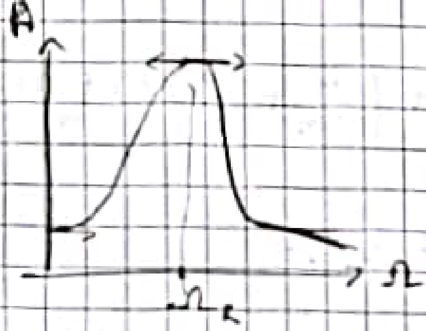
$$\frac{dA}{d\Omega} = \frac{k S_0}{2m}\left[2(-2\omega)(\omega_n^2 - \Omega^2) + 8\delta\Omega\right]$$

$$\propto \frac{1}{\left[(\omega_n^2 - \Omega^2)^2 + (2\delta\Omega)^2\right]^{3/2}}$$

$$= \frac{kS_0 \, 2\Omega \left[(\omega_n^2 - \Omega^2) - 2\delta^2 \right]}{m \left[\qquad \right]^{3/2}}$$

* $\Omega = 0$

* $\omega_n^2 - \Omega^2 - 2\delta^2 = 0 \Rightarrow \Omega_R = \sqrt{\omega_n^2 - 2\delta^2}$



$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} + \frac{\partial D}{\partial \dot{x}} = F_{ex} \frac{\partial x_i}{\partial x}$$

case 2 : course

case 3 :

$T = \frac{1}{2} m \dot{x}^2$

$U = \frac{1}{2} k x^2$ $\qquad D = \frac{1}{2} \alpha (\dot{x} - \dot{S}_2(t))^2$

$x_1 \qquad x_2$

$(x_1 + x_2)^2$

$(x_1 + x_2)^2$

$(x_1 - x_2)^2$

$(x_1 - x_2)^2$

$(x_1 - S)^2$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} + \frac{\partial D}{\partial \dot{x}} = 0$$

$L = T - U = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} k x^2$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = m\ddot{x} \quad / \quad \frac{\partial L}{\partial x} = -kx$$

$$\frac{\partial D}{\partial \dot{x}} = \alpha (\dot{x} - \dot{S}_2(t))$$

$$m\ddot{x} + kx + \alpha(\dot{x} - \dot{S}_2(t)) = 0$$

$$m\ddot{x} + \alpha\dot{x} + kx = \alpha \dot{S}_2(t)$$

$$\ddot{x} + \underbrace{\left(\frac{\alpha}{m}\right)}_{2\delta}\dot{x} + \underbrace{\left(\frac{k}{m}\right)}_{\omega_n^2} x = \frac{\alpha}{m}\dot{S}_2(t)$$

$\Omega_R = ?$ home work

$x(t) = x_h(t) + x_p(t)$

$x_p(t) \to A\cos(\Omega t + \varphi)$

$\bar{x}(t) = A e^{j(\Omega t + \varphi)}$

$\bar{\dot{x}}(t) = j\Omega A e^{j(\Omega t + \varphi)}$

$\bar{\ddot{x}}(t) = -\Omega^2 A e^{j(\Omega t + \varphi)}$

$$-\Omega^2 A e^{j(\Omega t + \varphi)} + 2\delta j\Omega A e^{j(\Omega t + \varphi)} + \omega_n^2 A e^{j(\Omega t + \varphi)} = 2\delta S_0 \Omega e^{j\Omega t}$$

$$A\left[(\omega_n^2 - \Omega^2) + j2\delta\Omega\right] e^{j\varphi} = 2\delta S_0 \Omega$$

$$A\left[(\omega_n^2 - \Omega^2) + j2\delta\Omega\right] = 2\delta S_0 \Omega e^{-j\varphi}$$

$$A\sqrt{(\omega_n^2 - \Omega^2)^2 + 4\delta^2\Omega^2} = 2\delta S_0 \Omega$$

$$A = \frac{2\delta S_0 \Omega}{\sqrt{(\omega_n^2 - \Omega^2)^2 + 4\delta^2\Omega^2}}$$

$$\varphi = \arctan\left(\frac{-2\delta\Omega}{\omega_n^2 - \Omega^2}\right)$$

$$\frac{dA}{d\Omega} = 2\delta S_0 \frac{\sqrt{\quad} - \frac{2(-2)(\omega_n^2-\Omega^2)+8\delta\Omega}{2\sqrt{\quad}}\cdot\Omega}{\cdots}$$

$$\frac{dA}{d\Omega} = 2\delta S_0 \frac{\frac{(\omega_n^2-\Omega^2)^2+4\delta^2\Omega^2+2\Omega^2(\omega_n^2-\Omega^2)-4\delta^2\Omega^2}{\sqrt{\cdots}}}{\cdots}$$

$$= 2\delta S_0 \frac{(\omega_n^2-\Omega^2)\left[(\omega_n^2-\Omega^2)+2\Omega^2\right]}{\left[(\omega_n^2-\Omega^2)^2+4\delta^2\Omega^2\right]^{3/2}}$$

$$= 2\delta S_0 \frac{(\omega_n^2-\Omega^2)(\omega_n^2+\Omega^2)}{[\cdots]^{3/2}}$$

$$\frac{dA}{d\Omega} = 0 \Rightarrow \Omega_R = \omega_n$$

Ex 2 :

$$T = \frac{1}{2} m \dot{x}^2$$

$$\vec{r}\begin{pmatrix} l\sin\theta \\ l\cos\theta \end{pmatrix} \qquad \vec{dr}\begin{pmatrix} l\cos\theta\, d\theta \\ -l\sin\theta\, d\theta \end{pmatrix}$$

$$\vec{v}\begin{pmatrix} l\cos\dot{\theta} \\ -l\sin\dot{\theta} \end{pmatrix} \qquad \vec{F}\begin{pmatrix} F_0\cos(\omega t)\cos\theta \\ -F_0\cos(\omega t)\sin\theta \end{pmatrix}$$

$$T = \frac{1}{2} m \dot{x}^2$$

$$T = \frac{1}{2} m l^2 \dot{\theta}^2$$

$$U_k = \frac{1}{2}K\left(\frac{l}{2}\theta + \Delta l\right)^2$$

$$U_m = -\int_0^\theta \vec{w}\cdot\vec{dr} = \int_0^\theta mg\, l\sin\theta$$

$$= -mgl\,[\cos\theta]_0^\theta$$

$$= mgl\left[1 - 1 + \frac{\theta^2}{2}\right] = mgl\frac{\theta^2}{2}$$

$$U = \frac{1}{2}K\left(\frac{l}{2}\theta + \cancel{\Delta l}^{\,0}\right)^2 + mgl\frac{\theta^2}{2}$$

$$D = \frac{1}{2}\alpha\left(\frac{l}{2}\dot{\theta}\right)^2 = \frac{1}{8}\alpha l^2\dot{\theta}^2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}}\right) - \frac{\partial L}{\partial\theta} + \frac{\partial D}{\partial\dot{\theta}} = \vec{F}\frac{\partial \vec{r}}{\partial\theta}$$

$$ml^2\ddot{\theta} + \left(K\frac{l^2}{4} + mgl\right)\theta + \frac{1}{4}\alpha l^2\dot{\theta} =$$

$$F_0 l\cos(\omega t)\cos^2\theta + F_0 l\cos(\omega t)\sin^2\theta$$

$$\ddot{\theta} + \boxed{\frac{\alpha}{4m}}^{\,2\delta}\dot{\theta} + \boxed{\left(\frac{k}{4m} + \frac{g}{l}\right)}_{\omega_n^2}\theta = \frac{F_0}{ml}\cos(\omega t)$$

$$\theta(t) = \theta_h(t) + \theta_p(t)$$

$$\Downarrow$$

$$\theta(t) = \theta\cos(\omega t + \varphi)$$

Power Supplied :

$$F = F_0\cos(\Omega t)$$

$$P_s = \vec{F}\cdot\vec{v} \qquad \theta(t) = \theta_0\cos(\Omega t + \varphi)$$

$$v = l\,\dot{\theta}(t)$$

$$v = -\Omega l\theta_0\sin(\Omega t + \varphi)$$

$$P_s = -F_0\Omega l\theta_0\cos(\Omega t)\sin(\Omega t + \varphi)$$

Power dissipated :

$$P_d = \alpha v_A^2 \qquad \alpha\,\vec{v_A}\cdot\vec{v_A}$$

$$= \alpha\left(\frac{l}{2}\dot{\theta}(t)\right)^2$$

$$= \alpha\frac{l^2}{4}\Omega^2\theta_0^2\sin^2(\Omega t + \varphi)$$

$$\langle P_s\rangle = \frac{1}{T}\int_0^T -F_0\Omega l\theta_0\cos(\Omega t)\sin(\Omega t + \varphi)\,dt$$

$$\langle P_s\rangle = -\frac{F_0\Omega l\theta_0}{T}\int_0^T\cos(\Omega t)\sin(\Omega t + \varphi)$$

$$\sin(a) + \sin(b) = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$a + b = 2\Omega t + 2\varphi$$

$$a - b = 2\Omega t \Rightarrow 2a = 4\Omega t + 2\varphi$$

$$\boxed{a = 2\Omega t + \varphi}$$

$$b = \varphi$$

$$\langle P_s\rangle = -\frac{F_0 \Omega l \theta_0}{T}\int_0^T \left[\sin(2\Omega t+\varphi) + \sin\varphi\right] dt$$

$$= \frac{-F_0 \Omega l \theta_0}{2T} T \sin\varphi$$

Sol

$$\theta(t) = \theta_0 \cos(\Omega t+\varphi)$$

$$(\omega_0^2 - \Omega^2) + 2j\delta\Omega = \frac{F_0}{ml\theta_0} e^{j\varphi}$$

$$z = x + jy \quad \begin{cases} \cos\varphi = \frac{x}{\sqrt{x^2+y^2}} \\ \sin\varphi = \frac{y}{\sqrt{x^2+y^2}} \end{cases}$$

$$\sin\varphi = \frac{-2\delta\Omega m l \theta_0}{F_0}$$

$$\langle P_s\rangle = \frac{\alpha (l\Omega\theta_0)^2}{8} \quad \Big| \; \delta = \frac{1}{8}\frac{\alpha}{m}$$

$$\langle P_d\rangle = \frac{1}{T}\int_0^T \alpha \frac{l^2}{4}\Omega^2\theta_0^2 \sin^2(\Omega t+\varphi)\, dt$$

$$= \frac{\alpha l^2 \Omega^2 \theta_0^2}{4T}\int_0^T \sin^2(\Omega t+\varphi)\, dt$$

$$= C\int_0^T \frac{1-\cos(2(\Omega t+\varphi))}{2} dt$$

$$= \frac{C}{2}\int_0^T 1 - \underbrace{\cos(2\Omega t+2\varphi)}_{0}\, dt$$

$$= \frac{TC}{2} = \frac{\alpha l^2 \Omega^2 \theta_0^2}{8}$$

$$\langle P_s\rangle = \langle P_d\rangle$$

Ex 3:

$$T = \frac{1}{2} m \dot{y}(t)^2$$

$$U = U_k + U_m$$

$$U = \frac{1}{2} k (y(t) + \Delta l - y_A)^2 + mgy$$

$$\left.\frac{dU}{dy}\right|_{\substack{y=0 \\ y_A=0}} = 0$$

$$\frac{dU}{dy} = mg + k(y(t) + \Delta l - y_A)$$

$$mg + k\Delta l = 0 \Rightarrow \Delta l = \frac{-mg}{k}$$

$$D = \frac{1}{2}\alpha(\dot{y}(t) - \dot{y}_A(t))^2 \quad \Big| \; L = T - U$$

$$L = \frac{1}{2} m\dot{y}^2 - \frac{1}{2}k(y+\Delta l - y_A)^2 + mg$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) = m\ddot{y} \quad \Big| \; -\frac{\partial L}{\partial y} = mg + k(y+\Delta l - y_A)$$

$$\frac{\partial D}{\partial \dot{y}} = \alpha(\dot{y} - \dot{y}_A)$$

$$m\ddot{y} + mg + ky + k\underbrace{\Delta l}_{-\frac{mg}{k}} - ky_A + \alpha\dot{y} - \alpha\dot{y}_A = 0$$

$$m\ddot{y} + ky - ky_A + \alpha\dot{y} - \alpha\dot{y}_A = 0$$

$$m\ddot{y} + \alpha\dot{y} + ky = ky_A + \alpha\dot{y}_A$$

$$\ddot{y} + \frac{\alpha}{m}\dot{y} + \frac{k}{m}y = \frac{\alpha}{m}\dot{y}_A + \frac{k}{m}y_A = F$$

$$\boxed{\omega_n = \sqrt{\frac{k}{m}}} \qquad \boxed{\delta = \frac{\alpha}{2m}}$$

$$y_A = y_0 \sin\left(\frac{2\pi x}{\lambda}\right)$$

$$y_A = y_0 \sin(\Omega t)$$

$$\bar{y}_A = y_0 e^{j\Omega t}$$

$$\bar{\dot{y}}_A = y_0 j\Omega e^{j\Omega t}$$

$$y = A\sin(\Omega t+\varphi)$$

$$\bar{y} = Ae^{j(\Omega t+\varphi)}$$

$$-\Omega^2 A e^{j(\Omega t+\varphi)} + 2j\delta\Omega A e^{j(\Omega t+\varphi)} + \omega_n^2 A e^{j(\Omega t+\varphi)} = \frac{\alpha}{m} j\Omega y_0 e^{j\Omega t} + \frac{k}{m} y_0 e^{j\Omega t}$$

$$A\left(\omega_n^2 - \Omega^2 + 2j\delta\Omega\right)e^{j\varphi} = \frac{y_0}{m}\left(K + j\Omega d\right)$$

$$A\sqrt{(\omega_n^2 - \Omega^2)^2 + 4\delta^2\Omega^2} = y_0\sqrt{\omega_n^4 + 4\Omega^2\delta^2}$$

$$A = \frac{\qquad}{\qquad}$$