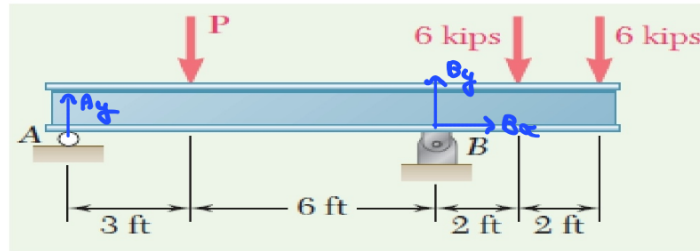


# TD (d)

## Exercise 1:

Three loads are applied to a beam as shown. The beam is supported by a roller at A and by a pin at B. Neglecting the weight of the beam, determine the reactions at A and B when  $P = 15$  kips



$$\sum \vec{F}_{ext} = \vec{0} \Rightarrow \begin{cases} (0x): B_x = 0 \\ (0y): -P - 12 + A_y + B_y = 0 \end{cases}$$

$$\sum \mathcal{M}(\vec{F}) = 0 \Rightarrow -3P + 9B_y - (6 \times 11) - (6 \times 13) = 0$$

$$\Leftrightarrow -3(15) + 9B_y - (6 \times 11) - (6 \times 13) = 0$$

$$B_y = \frac{3 \times 15 + 6 \times 11 + 6 \times 13}{9}$$

$$B_y = 21 \text{ kips}$$

$$-15 - 12 + A_y + B_y = 0$$

$$\Rightarrow A_y = 6 \text{ kips}$$

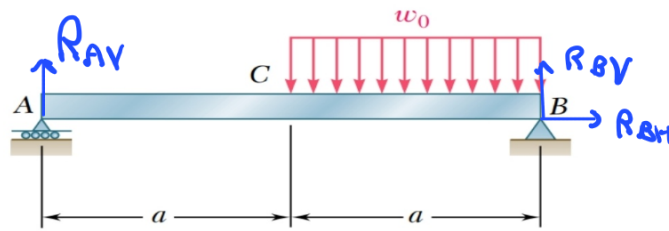
$$A_y = 6 \text{ kips}$$

$$B_y = 21 \text{ kips}$$

$$B_x = 0 \text{ kips}$$

### Exercice 2 :

Déterminer l'expression des réactions  $R_{HB}$  et  $R_{VB}$  des systèmes représentés ci-après.  $q_0$  et  $w_0$  sont les densités de charge (en N/m).



$$\sum \vec{F}_{ext} = 0 \Rightarrow \begin{cases} (0x) : R_{BH} = 0 \\ (0y) : R_{BV} + R_{AV} + a w_0 = 0 \end{cases}$$

$$w_0 = \frac{F}{a} \Rightarrow F = w_0 \cdot a$$

$$\sum M(\vec{F}) = 0 \Rightarrow -w_0 a \left(a + \frac{a}{2}\right) + R_{BV}(2a) = 0$$

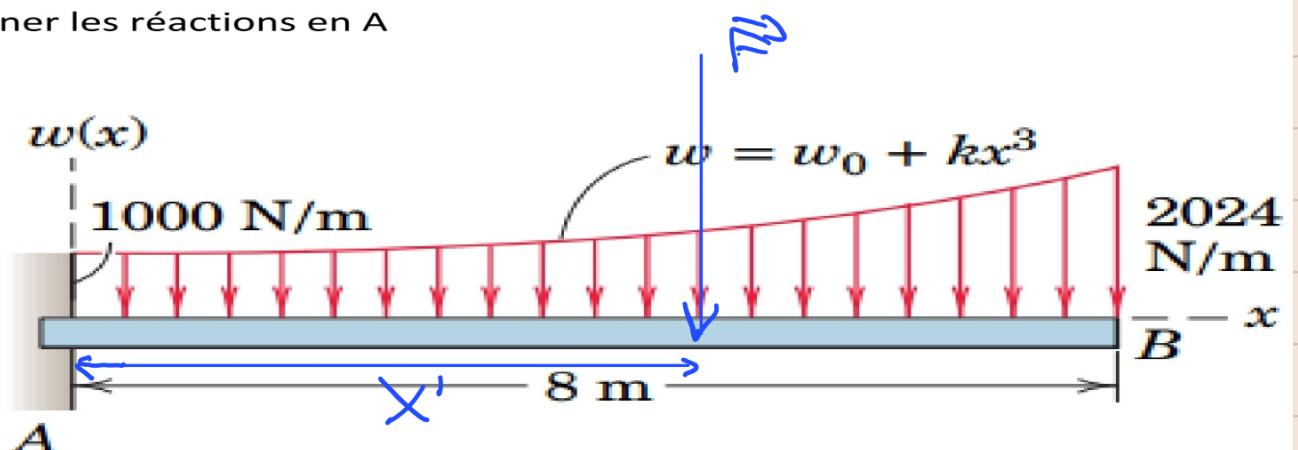
$$R_{BV} = \frac{3a}{4} w_0$$

$$\frac{3a}{4} w_0 + a w_0 + R_{AV} = 0$$

$$R_{AV} = -\frac{7a w_0}{4}$$

### Exercice 3 :

Déterminer les réactions en A



$$w(0) = w_0 = 1000 \text{ N/m}$$

$$w(8) = 1000 + k(8)^3 = 2024 \Rightarrow k = 2$$

$$w(x) = 1000 + 2x^3$$

$$F = \int_0^8 (1000 + 2x^3) dx = \left[ 1000x + \frac{2x^4}{4} \right]_0^8$$

$$F = 10048 \text{ N}$$

$$x' F = \int_0^8 x (1000 + 2x^3) dx$$

$$= \left[ \frac{2}{5} x^5 + 500 x^2 \right]_0^8 = 45107,2 \text{ Nm}$$

$$x' = 4,49 \text{ m}$$

$$\sum \vec{F}_{ext} = \vec{0} \Rightarrow R_A - F = 0$$

$$R_A = F = 10048 \text{ N}$$

$$\sum M_{\vec{P}} = 0 \Rightarrow -F x' + M = 0$$

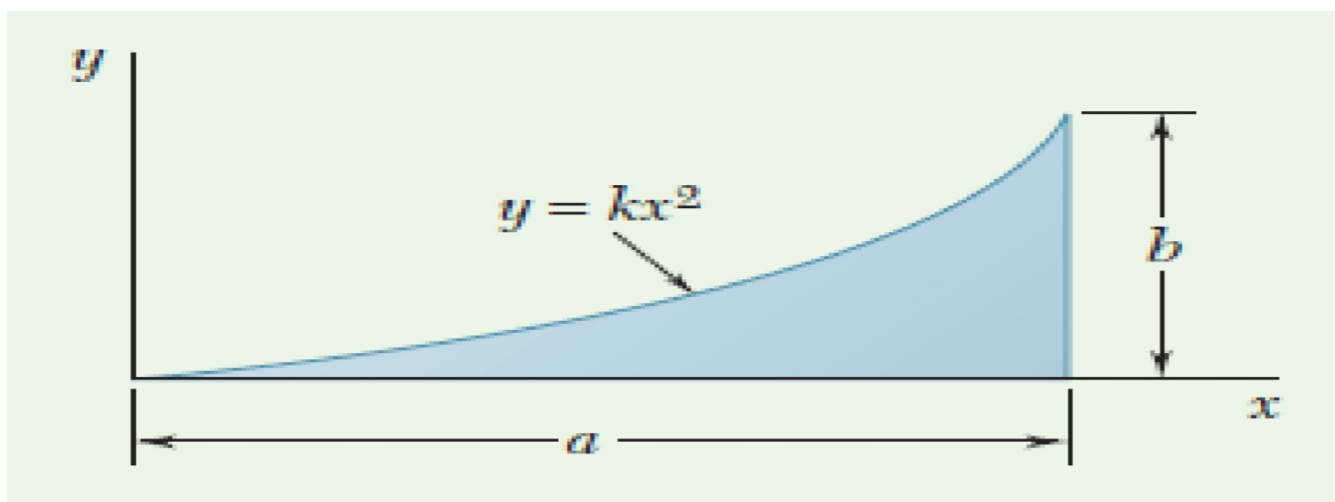
$$M = F x' = 4,49 \times 10048$$

$$M = 45115,52 \text{ Nm}$$

#### Exercise 4 :

Evaluate the dashed surface using a vertical differential element.

Calculer la surface suivante en utilisant un élément différentiel vertical.



$$b = ka^2 \Leftrightarrow \boxed{K = \frac{b}{a^2}} \quad y = \left(\frac{b}{a^2}\right) x^2$$

Ecrire  $x$  en fonction de  $y$ :

$$\boxed{x = \frac{a}{\sqrt{b}} \sqrt{y}}$$

$$\begin{aligned} A &= \int_0^b x \, dy = \frac{a}{\sqrt{b}} \int_0^b (y)^{1/2} \, dy \\ &= \frac{2}{3} \frac{a}{\sqrt{b}} (b)^{3/2} \end{aligned}$$

$$\boxed{A = \frac{2ab}{3}}$$