

Chapter 2
Single-phase electric power

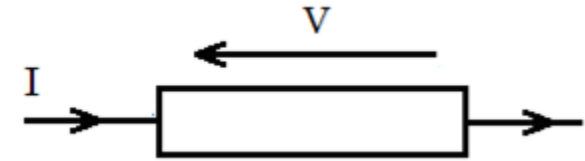
- Cost,
- Environmental aspects,
- Correct use of equipment within the limits of their technical characteristics (rating plate)
- Sizing an installation (cables, transformers, circuit breakers, generators, etc.)

Mechanical and Electrical Efficiency

- For Electric Motor:

- Energy Conversion: electrical \rightarrow mechanical
- Not all electrical power is converted into mechanical power.
- The efficiency of a motor is:

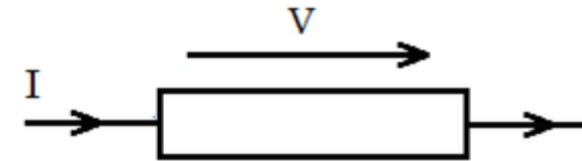
$$\eta_m = \frac{P_{mec}}{P_{elec}} < 1$$



- For a generator:

- Energy Conversion: mechanical \rightarrow electrical
- Not all mechanical power is converted into electrical power.
- The efficiency of a generator is:

$$\eta_g = \frac{P_{elec}}{P_{mec}} < 1$$



Examples

- A continuous motor powered at 200V,
- The plate indicates $P_{mec} = 1,8KW$ and an efficiency of $\eta = 0,8$
- What would be the current absorbed by the load?

$$I = \frac{P_{elec}}{V} = \frac{P_{mec}}{\eta_m \cdot V} = \frac{1800}{0,9 \times 200} = 10A$$

- A genrator with an efficiency of $\eta = 0,8$,
- The nominal electrical power of the generator is 2KW and the corresponding voltage is 200 V
- What is the current generated and what is the mechanical power required to reach the rated speed? (régime nominal)

$$I = \frac{P_{elec}}{V} = \frac{2000}{200} = 10A \quad \text{and} \quad P_{mec} = \frac{P_{elec}}{\eta_g} = \frac{2000}{0,8} = 2500W$$

Instantaneous Power

Consider a dipole traversed by $i(t)$ and maintained under a tension $v(t)$.

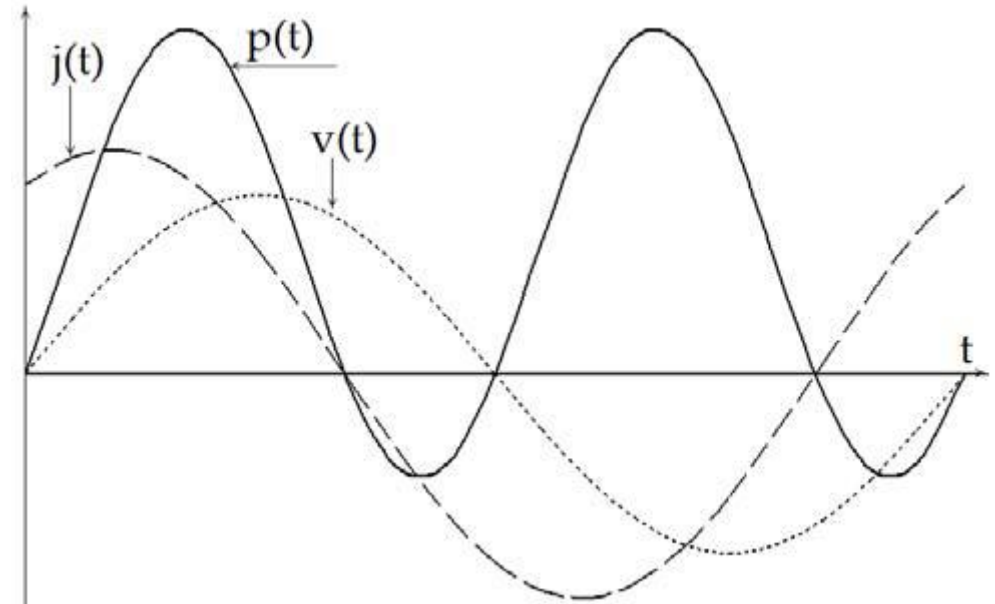
- $i(t) = I\sqrt{2}\sin(\omega t + \varphi_I)$
- $v(t) = V\sqrt{2}\sin(\omega t + \varphi_V)$
- Instantaneous Power $P(t)$ = power in each instant t ,

$$P(t) = v(t) \cdot i(t) = 2 \cdot I \cdot V \sin(\omega t + \varphi_I) \sin(\omega t + \varphi_V)$$

- and thus :

$$P(t) = I \cdot V \cdot \cos(\varphi_V - \varphi_I) + I \cdot V \cdot \cos(2\omega t + \varphi_V + \varphi_I)$$

- $P(t)$ is composed of :
 - a constant power (independent of time) that is positive,
 - a fluctuating power (with 2ω pulsation)

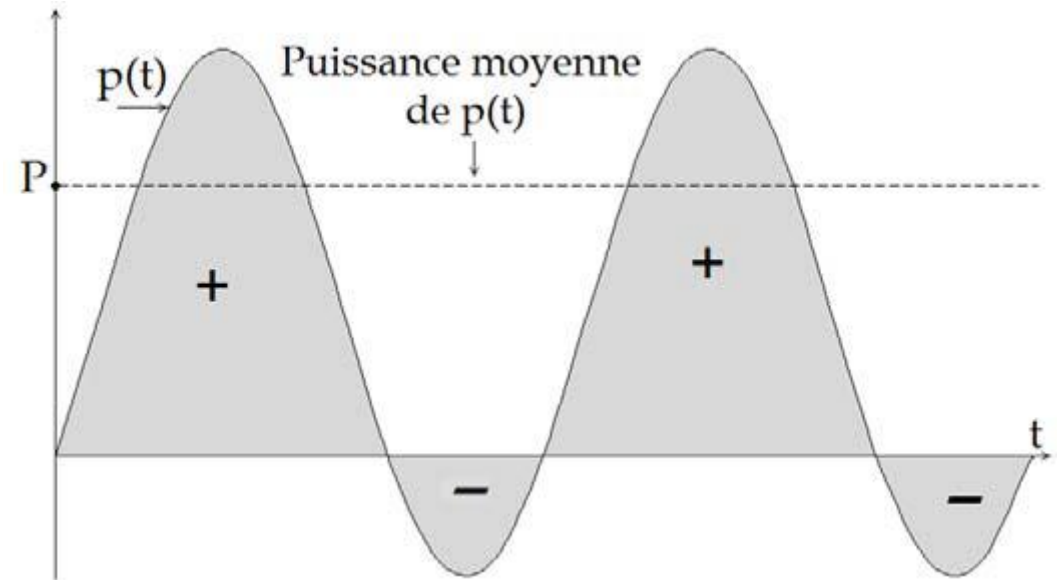


Active power

- Active power =
 - average power over a period,

$$P = \frac{1}{T} \int_0^T P(t) dt$$

- The power absorbed by resistive elements of the circuit.



$$P = \frac{1}{T} \int_0^T P(t) dt = \frac{1}{T} \int_0^T v(t)i(t) dt = V \cdot I \cdot \cos(\varphi_V - \varphi_I) = V \cdot I \cos(\varphi)$$

with $\varphi = \varphi_V - \varphi_I$

- Active power unit: [Watts]

Active power – example

Let a motor powered by sinusoidal voltage of 400V / 50 Hz :

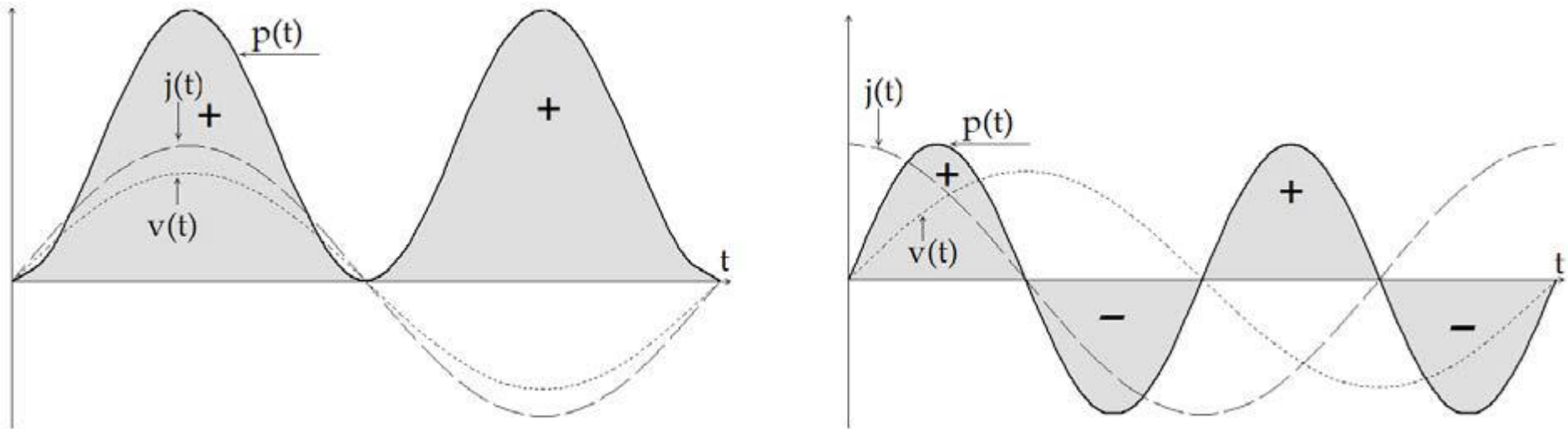
- Useful power of the motor : 7,2 KW,
- Motor efficiency: 0,9 ,
- Power Factor $f_p = \cos\varphi$: 0,8 ,

What would be the current absorbed by the motor?

$$P_{elec} = P_{mec} / \eta_m = \frac{7200}{0,9} = 8000 \text{ Watts}$$

$$I = \frac{P_{elec}}{V \cos\varphi} = \frac{8000}{0,8 \times 400} = 25 \text{ A}$$

Apparent power and Reactive power



- Let's discuss the cases of a resistance and a capacitor
- In the first case, the power is always **positive**: it flows from ***the source to the load***,
- In the second case, the power is sometimes **positive** and sometimes **negative**: it fluctuates, which means, it oscillates between the source and the generator. In average, the capacitor (same for the inductance) **does not absorb energy!**

Apparent power and Reactive power

Why then be interested in this fluctuating power?

- There really is a flow of power coming and going,
- It is then necessary to provide an additional current in the calculation of the electrical equipment,
- This additional current causes ohmic losses in the conductors => losses at the electricity supplier.

to quantify this fluctuating power, we use the notion of reactive power.

Apparent power and Reactive power

- Consider a circuit consisting of a resistance and a reactance. The Fresnel diagram is given below:
- We have : $\vec{V} = \vec{V}_R + \vec{V}_X$
- Let $\vec{S} = I \cdot \vec{V}$ then $\vec{P} = I \cdot \vec{V}_R$ and finally $\vec{Q} = I \cdot \vec{V}_X$,
- We get the vectorial expression: $\vec{S} = \vec{P} + \vec{Q}$,
- The respective modules are $S = \|\vec{S}\|$, $P = \|\vec{P}\|$ et $Q = \|\vec{Q}\|$
- According to the diagram $S^2 = P^2 + Q^2$

Apparent power and Reactive power

According to the diagram, we have:

- $P = S \cdot \cos\varphi = V \cdot I \cdot \cos\varphi$
 - P represents **active power**
 - The average power absorbed by resistive elements)
- $Q = S \cdot \sin\varphi = V \cdot I \cdot \sin\varphi$
 - Q represents **reactive power**
 - Gives an idea of the power fluctuation between the source and the generator
 - Only exists if there are reactive elements in the circuit which store and release energy.

Apparent power and Reactive power

- $S = V \cdot I$
 - S represents the **apparent power**
 - This is the power that **would appear** to enter the circuit
 - This is the power that a receiver **would consume** if the voltage and current were in phase $\varphi = 0$.
- **Units:**
 - Unit of P : **Watts**
 - Unit of S : **V.A** *Volt-Ampère*
 - Unit of Q : **V.A.R** *Volt-Ampère-Réactif*

Complex Power

Using complex notation, we can obtain the three powers defined previously as follows:

- We define the complex apparent power:

$$\bar{S} = \bar{V} \bar{I}^* \quad \text{with} \quad \bar{S} = P + jQ$$

- The apparent power is obtained as follows :

$$S = |\bar{S}| = V \cdot I$$

- The active power represents the real part of the complex apparent power:

$$P = \Re(\bar{S}) = \Re(\bar{V} \cdot \bar{I}^*) = V \cdot I \cdot \cos(\varphi_V - \varphi_I) = V \cdot I \cdot \cos\varphi$$

- The reactive power represents the imaginary part:

$$Q = \Im(\bar{S}) = \Im(\bar{V} \cdot \bar{I}^*) = V \cdot I \cdot \sin(\varphi_V - \varphi_I) = V \cdot I \cdot \sin\varphi$$

Active and Reactive currents

- Let's start from the basic definition:

$$(\bar{V}\bar{I}^*) = \Re(\bar{V}\bar{I}^*) + j \cdot \Im(\bar{V}\bar{I}^*)$$

- Divide it by \tilde{V} and applying conjugates, we obtain:

$$\bar{I} = \frac{\Re(\bar{V}\bar{I}^*)}{\bar{V}^*} - j \cdot \frac{\Im(\bar{V}\bar{I}^*)}{\bar{V}^*}$$

- Or:

$$\bar{I} = \frac{\Re(\bar{V}\bar{I}^*)}{V^2} \bar{V} - j \cdot \frac{\Im(\bar{V}\bar{I}^*)}{V^2} \bar{V}$$

- We rewrite

$$\bar{I} = \frac{\bar{V}}{V} \left\{ \frac{\Re(\bar{V}\bar{I}^*)}{V} - j \cdot \frac{\Im(\bar{V}\bar{I}^*)}{V} \right\}$$

Active and Reactive currents

- Using the definitions of P and Q , we get:

$$\bar{I} = \frac{\bar{V}}{V} \left(\frac{P}{V} - j \cdot \frac{Q}{V} \right) = I_a + jI_r$$

- with: $I_a = \frac{\bar{V}}{V} \cdot \frac{P}{V}$ and $I_r = \frac{-\bar{V}}{V} \cdot \frac{Q}{V}$

- Then, the current is decomposed to two parts:
 - A part in phase with the tension: **Active Current**,
 - A part in quadrature with the tension: **Reactive Current**.
- In the case $\varphi_V = 0$, which means the tension is taken as phase reference, we, then, get simple expressions:

$$I_a = I \cdot \cos\varphi_I = \Re(\bar{I}) \quad \text{et} \quad I_r = I \cdot \sin\varphi_I = \Im(\bar{I})$$

Power in sinusoidal regime

Active and Reactive currents

General Case	$\varphi_V = 0$ Case
$I_a = \frac{\bar{V}}{V} \cdot \frac{P}{V}$ and $I_a \neq \Re(\bar{I})$ and $P = I_a \cdot \bar{V}^*$	$P = VI_a$ and $I_a = I \cdot \cos\varphi_I = \Re(\bar{I})$
$I_r = \frac{-\bar{V}}{V} \cdot \frac{Q}{V}$, $I_r \neq \Im(\bar{I})$, $Q = -I_r \cdot \bar{V}^*$	$Q = -V \cdot I_r$ et $I_r = I \cdot \sin\varphi_I = \Im(\bar{I})$
$I^2 = I_a^2 + I_r^2$ with I : Apparent Current	

Definition of f_p

- Very significant factor,
- Ratio of active power to apparent power:

$$f_p = \frac{P}{S} = \cos\varphi$$

- It depends on the nature of the dipole attached to the electrical network,
- Ideally $I = I_a$ then $\varphi \approx 0$ and $f_p \approx 1$,
- Slf f_p is bad, the apparent current used is higher for nothing.

Example

Consider an electrical device consuming 1000 W at 200 V. Let us determine the apparent current in the case where the power factor is 0.9 then 0.5:

P = 1000 W	P = 1000 W
V = 200 V	V = 200 V
$f_p = \cos(\varphi) = 0,9$	$f_p = \cos(\varphi) = 0,5$
$I = \frac{P}{V \cdot \cos\varphi} = \frac{1000}{200 \times 0,9} = 5,5A$	$I = \frac{P}{V \cdot \cos\varphi} = \frac{1000}{200 \times 0,5} = 10A$

Consequences of a bad f_p

Higher currents \Rightarrow Inconveniences at the energy supplier:

- Larger alternators and transformers,
- Higher voltage at the start of the line,
- Larger section lines,
- Greater Joule Effect losses,
- Larger control, protection and switching devices.

Actions taken by the power supply company

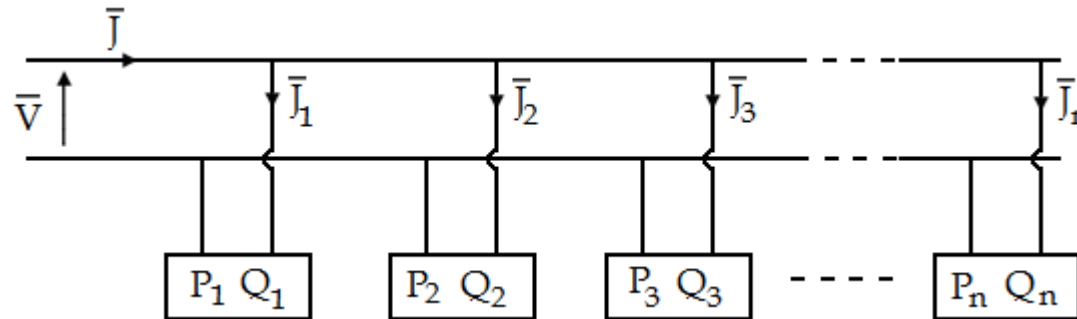
- f_p threshold to be respected,
- The company installs reactive energy meters
- The customer must then pay a tax proportional to their consumption, in addition to the price of the active energy consumed.
- The customer needs to improve his power factor. How ?

How to improve it?

- Avoid running the motors at idle and do not use oversized motors.
 - Motors at idle: $\cos\varphi = 0,1 \rightarrow 0,3$
 - Motor at rated load: $\cos\varphi = 0,8 \rightarrow 0,85$.
- Install compensation systems:
 - Capacitor banks in parallel with the motors,
 - Use of synchronous motors which exhibit capacitive effects under certain conditions.

Problem

- How to carry out the energy assessment of any electrical installation comprising n devices?



- Boucherot's Theorem** states :

« **Whatever the grouping, the total active power involved is equal to the arithmetic sum of the partial active powers, and the total reactive power is the algebraic sum of the partial reactive powers.** »

$$P_t = \sum_{i=1}^n P_i \quad \text{et} \quad Q_t = \sum_{i=1}^n Q_i$$

Boucherot Method

Consequences

- We can calculate the overall power factor:

$$S_t = \sqrt{P_t^2 + Q_t^2} \quad f_p = \frac{P_t}{S_t}$$

- **Be careful:**

$$S_t \neq \sum_{i=1}^n S_i$$

- If the receivers are all in parallel, we can deduce the current absorbed by the installation :
 - Apparent current: $I = \frac{S_t}{V}$
 - Active current: $I_a = \frac{P_t}{V}$
 - Hence the possibility of dimensioning the section of the cables based on the apparent current I .

Power calculation: example

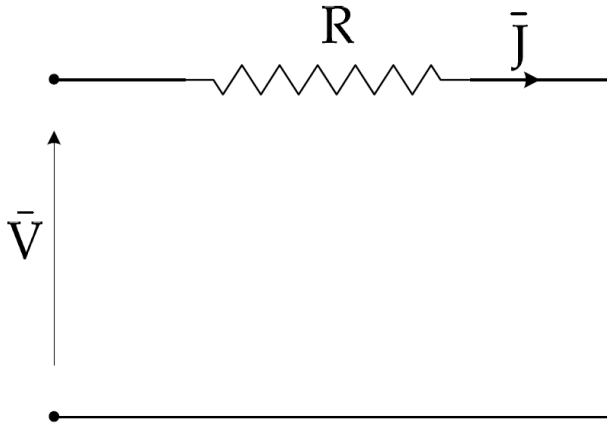
Resistive circuit:

We have:

$$\tilde{Z} = R$$

Active power: $P = \Re(\bar{V}\bar{I}^*) = \Re(\tilde{Z}I^2) = I^2\Re(\tilde{Z}) = R \cdot I^2 = \frac{V^2}{R}$

Reactive power: $Q = \Im(\bar{V}\bar{I}^*) = \Im(\tilde{Z}I^2) = I^2\Im(\tilde{Z}) = 0$

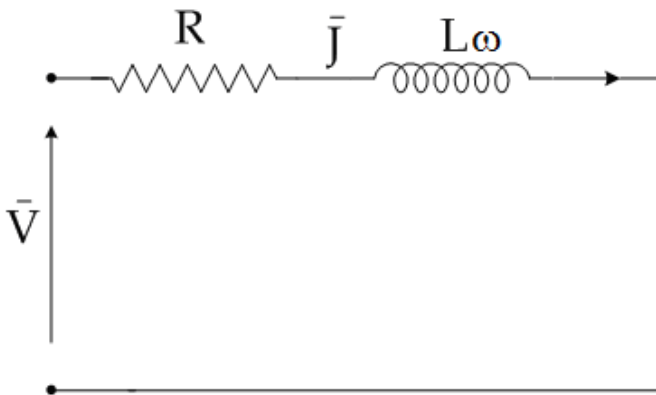


R-L circuit:

We have: $\tilde{Z} = R + jL\omega$

Active power: $P = \Re(\bar{V}\tilde{I}^*) = \Re(\tilde{Z}I^2) = I^2\Re(\tilde{Z}) = R \cdot I^2 = R \frac{V^2}{Z^2}$

Reactive power: $Q = \Im(\bar{V}\tilde{I}^*) = \Im(\tilde{Z}I^2) = I^2\Im(\tilde{Z}) = (L\omega)I^2 = (L\omega) \frac{V^2}{Z^2}$



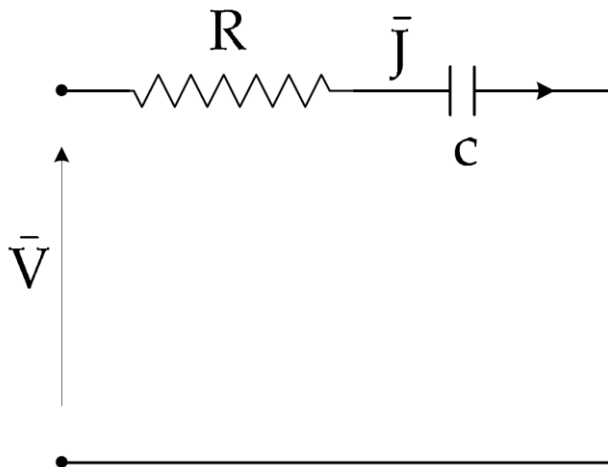
Power calculation: example

R-C circuit

We have: $\tilde{Z} = R - j \frac{1}{C\omega}$

Active power: $P = \Re(\bar{V}\bar{I}^*) = \Re(\tilde{Z}I^2) = I^2\Re(\tilde{Z}) = R \cdot I^2 = R \frac{V^2}{Z^2}$

Reactive power: $Q = \Im(\bar{V}\bar{I}^*) = \Im(\tilde{Z}I^2) = I^2\Im(\tilde{Z}) = \frac{-1}{(C\omega)} I^2 = \frac{-1}{(C\omega)} \frac{V^2}{Z^2}$



R-L-C circuit

- We have:

$$\tilde{Z} = R + j(L\omega - \frac{1}{C\omega})$$

- Active power:

$$P = \Re(\bar{V}\tilde{I}^*) = \Re(\tilde{Z}I^2) = I^2\Re(\tilde{Z}) = R \cdot I^2 = R \frac{V^2}{Z^2}$$

- Reactive power:

$$Q = \Im(\bar{V}\tilde{I}^*) = \Im(\tilde{Z}I^2) = I^2\Im(\tilde{Z}) = (L\omega - \frac{1}{C\omega})I^2 = (L\omega - \frac{1}{C\omega}) \frac{V^2}{Z^2}$$

Power in non-sinusoidal regime

Need for the study of the non-sinusoidal regime

- Impossibility of producing and transporting a purely sinusoidal voltage (alternators, transformers)
- The receiver can also deteriorate the voltage (electronic power converters such as choppers)
- How can these aspects be taken into account in practice?

Power in non-sinusoidal regime

Fourier series

- Each periodic function of frequency f_0 may be decomposed into an infinite series of sinusoidal signals of frequencies multiple of f_0 :

$$v(t) = \frac{V_0}{2} + \sum_{n=1}^{\infty} V_n \sqrt{2} \sin(n\omega t + \varphi_{vn})$$

$$i(t) = \frac{I_0}{2} + \sum_{n=1}^{\infty} I_n \sqrt{2} \sin(n\omega t + \varphi_{in})$$

- In general, in electricity, the mean values V_0 and I_0 are null.
- The first component ($n = 1$) is said «fundamental». The other ones are called «harmonics».
- The effective values are then expressed as follows:

$$V = \sqrt{V_1^2 + V_2^2 + V_3^2 + \cdots + V_n^2} \quad \text{and} \quad I = \sqrt{I_1^2 + I_2^2 + I_3^2 + \cdots + I_n^2}$$

Power in non-sinusoidal regime

Definitions of powers (1/2)

- **Instantaneous power:** Power at each instant t :

$$p(t) = v(t) \cdot i(t)$$

- **Active power:** The definition remains:

$$P = \frac{1}{T} \int_0^T p(t) dt$$

- and then $P = \sum_0^\infty V_n I_n \cos(\varphi_n)$ with $\varphi_n = \varphi_{vn} - \varphi_{in}$
- It is the sum of the active powers transported by each of the sinusoidal components.
- **Reactive power:** It is the sum of the reactive powers carried by each of the sinusoidal components.

$$Q = \sum_0^\infty V_n I_n \sin(\varphi_n) \quad \text{with} \quad \varphi_n = \varphi_{vn} - \varphi_{in}$$

Power in non-sinusoidal regime

Definitions of powers (2/2)

- **Apparent power:** Defined from the effective values of the voltage and current as given previously:

$$S = V \cdot I$$

- **Distorting power D:** Unfortunately we no longer have triangular equality: $S^2 \neq P^2 + Q^2$
- A measure of the importance of the power carried by the harmonics is the distorting power D defined as follows:

$$D = \sqrt{S^2 - P^2 - Q^2}$$

- In the sinusoidal case we would have $D = 0$.

- **Power Factor:** ratio of active power to apparent power: $f_p = \frac{P}{S} =$

$$\frac{P}{\sqrt{P^2 + Q^2 + D^2}}$$

Voltage drops in power cables

Problem

- The arrival voltage is not that delivered by the source.
- The reason: cables resistances:

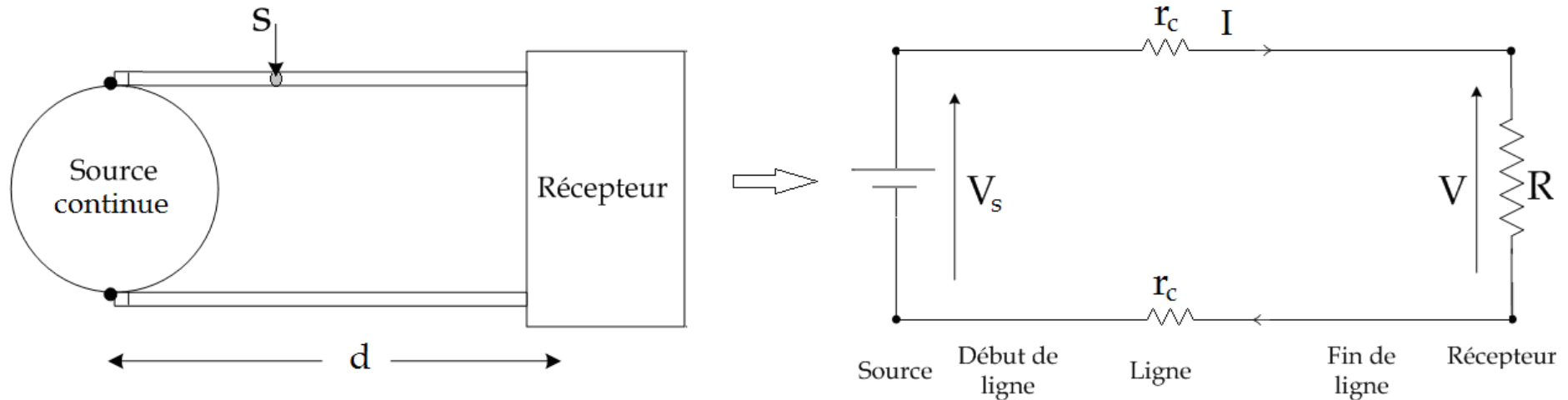
$$R = \rho \frac{l}{S}$$

- where l is the length, S the section ρ the resistivity.
- We define voltage drop:
Voltage drop = source voltage – delivered voltage
- The choice of cable section must be made according to a tolerable voltage drop.

Voltage drops in power cables

Problem

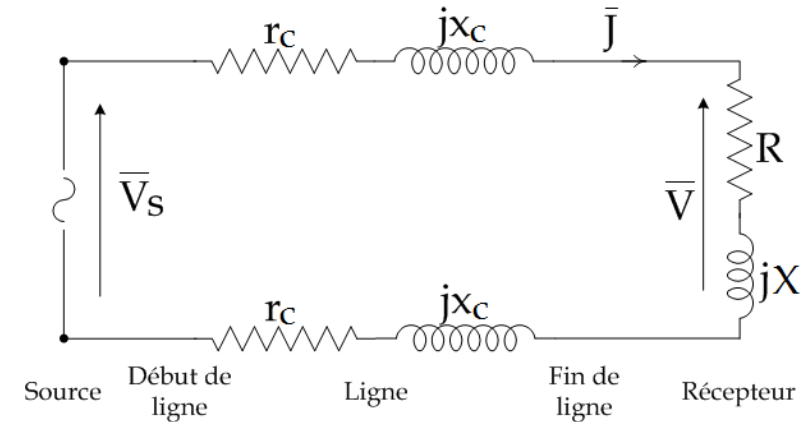
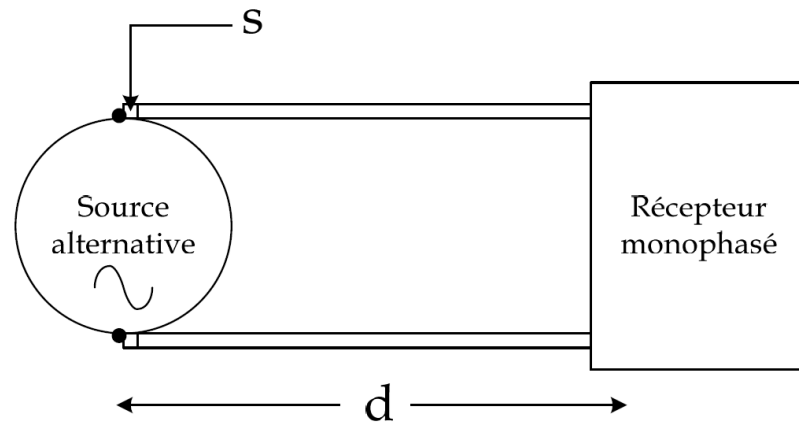
- Example en DC: $V = V_s - 2 \cdot r_c \cdot I$



- In AC, other aspects must be taken into account:
 - Magnetic effects between neighboring cables => mutual induction.
 - Capacitive effects between conductors (cables – earth – pylons).

Voltage drops in power cables

Modeling



- **Source:** $\tilde{V}_S = \tilde{E}_S - \tilde{Z}_S \cdot \tilde{I} \approx \tilde{E}_S$ because $\tilde{Z}_S \approx 0$
- **Cables:** Two cables of length d and section S
 - cables resistance $r_c = \rho \frac{d}{S}$
 - cables reactance $x_c = l_c \omega - \frac{1}{C_c \omega} \approx l_c \omega$
 - l_c is given by the manufacturer and depends on the distance between the cables.

Voltage drops in power cables

Modeling

- **Receiver:**

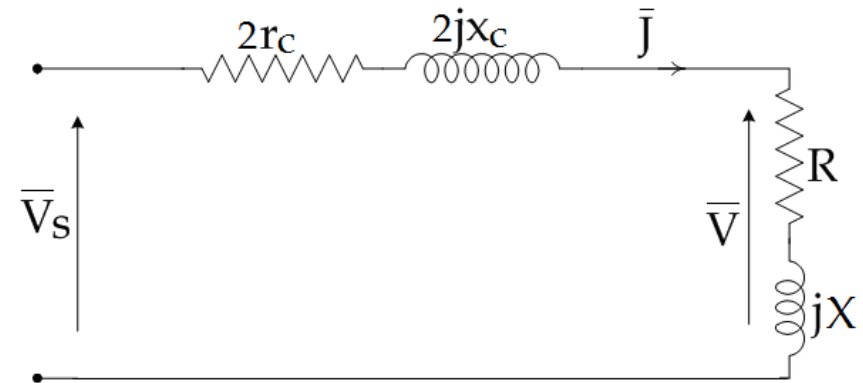
- Consumes power P under the voltage V with a power factor $\cos\varphi$.
- Presents an impedance $\tilde{Z} = R + jX$
- The two representations are equivalent

- **Balance sheet:**

$$\bar{V} = \bar{V}_S - 2(r_C + jx_C) \cdot \bar{I} = \bar{V}_S - 2\tilde{z}_C \bar{I}$$

- **Voltage drop:**

$$\Delta V = V_S - V = |\tilde{V}_S| - |\tilde{V}| \neq 2|\tilde{z}_C \tilde{I}|$$



Determination of voltage drop

Exact expression

- Voltage divider:

$$\tilde{V} = \frac{\tilde{Z}}{\tilde{Z} + \tilde{Z}_c} \cdot \tilde{V}_S = \frac{R + jX}{R + 2r_c + j(X + 2x_c)} \cdot \tilde{V}_S$$

- and then

$$V = \frac{\sqrt{R^2 + X^2}}{\sqrt{(R + 2r_c)^2 + (X + 2x_c)^2}} V_S$$

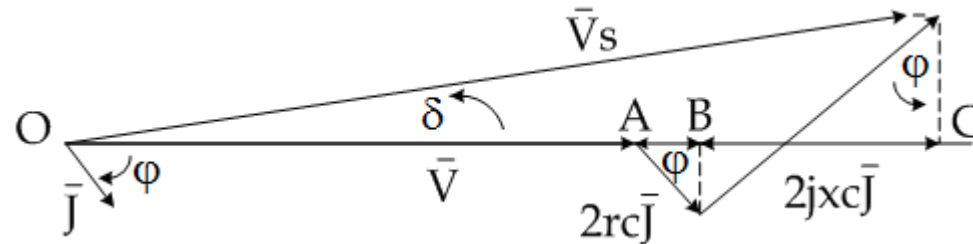
- finally

$$\Delta V = V_S - V = V_S - \frac{\sqrt{R^2 + X^2}}{\sqrt{(R + 2r_c)^2 + (X + 2x_c)^2}} V_S$$

Determination of voltage drop

Approximated expression

- For an inductive load : $\tilde{V}_s = \tilde{V} + 2r_c\tilde{I} + 2x_c\tilde{I}$

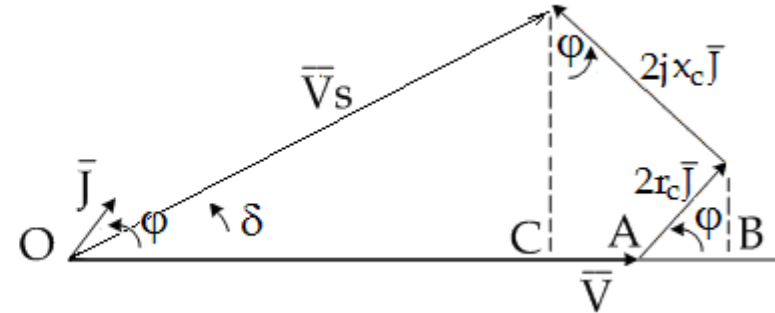
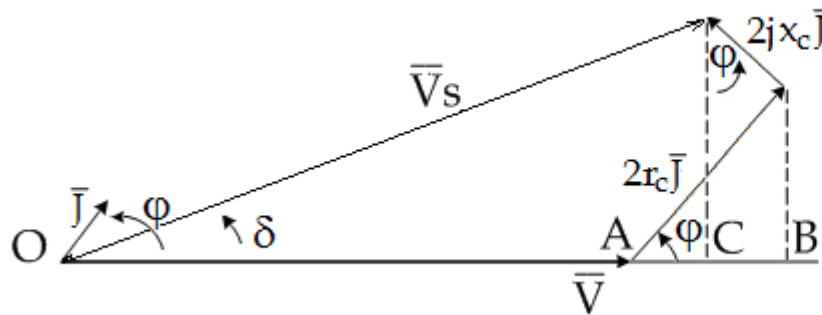


- We suppose ΔV is small comparing to V_s and V
- We, then, can write $V_s \approx V_s \cos \delta$
- Then, $\Delta V \approx AC = AB + BC = 2r_c I \cos \phi + 2x_c I |\sin \phi|$
- $\Delta V \approx 2r_c I \cos \phi + 2x_c I |\sin \phi|$

Determination of voltage drop

Approximated expression

- For a capacitive load: $\tilde{V}_S = \tilde{V} + 2r_c\tilde{I} + 2x_c\tilde{I}$
 - We suppose ΔV is small comparing to V_S and V



- We, then, can write $V_S \approx V_S \cos \delta$
- $\Delta V = V_S - V \approx OC - OA = AC = AB - CB = 2r_c I \cos \phi - 2x_c I |\sin \phi|$
- $\Delta V \approx 2r_c I \cos \phi - 2x_c I |\sin \phi|$

Determination of voltage drop

Expression in percentage

- In general, voltage drop is expressed in percentage,
- Voltage drop (%) = $\frac{\Delta V}{V}$ where V is the voltage at the receiver,
- Practical limits:
 - -2,5% to 5% for lighting circuits,
 - 10% for power circuits.

Voltage drop

Rendement d'une ligne (Yield of a line)

Ratio of the active power of the load compared to the active power of the entire circuit:

$$\eta = \frac{V \cdot I \cos \varphi}{V \cdot I \cos \varphi + 2r_c I^2}$$