NATIONAL POLYTECHNIC SCHOOL
Department of Integrated Preparatory Classes

Module : Physics 4
2nd year 2024/2025

Set 2
Vibrating string

Exercice1:

1-A string of length $4.35\ m$ and mass $137\ g$ is under a tension of $125\ N$. A standing wave has formed which has seven nodes including the endpoints. What is the frequency of this wave? Which harmonic is it? What is the fundamental frequency?

-Represent the fundamental mode and the first three overtones

2-A string fixed at one end only, is vibrating in its ninth harmonic mode. The speed of a wave on the string is $v = 25.8\ m/s$ and the string has a length of $8.25\ m$. What is the frequency of this wave? What is the wavelength of the wave? What is the fundamental frequency?

-Represent the fundamental mode and the first three overtones

Exercice2:

Two semi-infinite strings positioned along an $x'Ox$ axis are connected at $x = 0$. The string in the region $x < 0$ has a linear mass density μ_1 . The string extending from $0\ to\ +\infty$ has a linear mass density μ_2= $0.25\mu_1$.

An incident wave of amplitude U_0 and angular frequency ω arrives from -∞ and propagates in the direction of increasing x. At $x = 0$, the wave undergoes reflection.

1. Calculate the reflection coefficient at $x = 0$.
2. Show that the resulting wave in the region $x < 0$ varies between two values U_{max} and U_{min} . Determine U_{max} and U_{min} , as well as the positions of the vibration maxima and minima. Calculate the standing wave ratio (SWR).
3. Demonstrate that this system is equivalent to a string terminated at $x = 0$ by a damper, and specify the value of the damping coefficient.
4. In the case where μ_1= μ_2:
   - What happens to the reflection coefficient R (in amplitude) at $x = 0$?
   - Calculate the linear densities of kinetic and potential energy, e_c and e_p , at any point x along the $x'Ox$ axis.
   - Deduce the total energy density and calculate its average value over a wavelength λ.

Exercise3:

A semi-infinite string with a linear mass density μ_1 is stretched along the $x'Ox$ axis between $-\infty$ and $x = 0$. At $x = 0$, it is connected to a second string with a linear mass density μ_2 and length L. At $x = L$, this second string is attached to a rigid wall. The tension in both strings is denoted by T.

An incident wave with angular frequency ω is sent along the string, and at x=0, it undergoes reflection and transmission phenomena.

We write the wave equations in the two regions of space:

$$x < 0 : \qquad u_1(x;t) = A_1 e^{j(wt - k_1 x)} + B_1 e^{j(wt + k_1 x)}$$

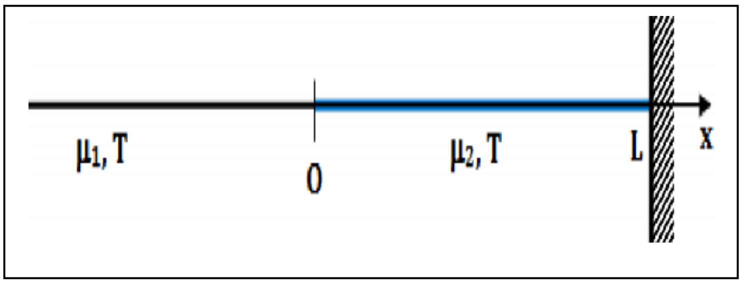
$$x > 0 : \qquad u_2(x;t) = A_2 e^{j(wt - k_2 x)} + B_2 e^{j(wt + k_2 x)}$$

1- Write the continuity equation at $x = 0$ and derive the relationship between A_1, A_2, B_1, and B_2

2-Write the condition at x=L and derive the ratio $\frac{B_2}{A_2}$.

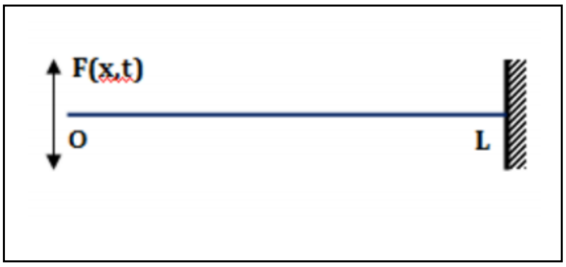
3- Determine the reflection coefficient $R = \frac{B_1}{A_1}$ at $x = 0$.

4- What relationship must exist between L_2 and λ_2 for a node to form at $x = 0$?



Exercise4:

A string of length L and linear mass density μ is stretched horizontally with a tension T. It is fixed at its end at $x = L$ to a rigid support. The end at $x = 0$ is subjected to a sinusoidal force with an amplitude F_0 and angular frequency w . The transverse displacement of a point at position x on the string at time t is denoted as $y(x,t)$.



1-

a. Write the general expression for $y(x,t)$.

b. Provide the boundary conditions at $x = 0$ and $x = L$.

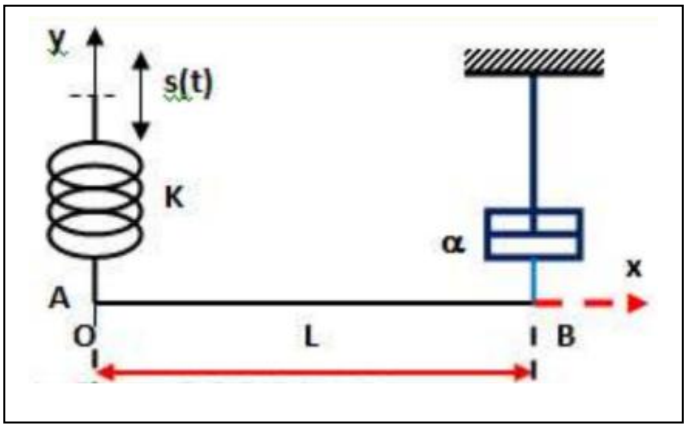
c. Show that the expression for $y(x,t)$. can be written in the form: $y(x,t)$. = f(x).g(t) d- Give the expression for f(x).in terms of F_0, the wave number k, L, and T.

2- Determine the positions of the points of maximum amplitude (antinodes) as a function of the wavelength λ and the length L of the string. What is the distance between two successive nodes?

3-For which angular frequencies is a resonance phenomenon observed?

Exercise5:

A string of length $AB = L = 2m$ and mass $m = 80g$ is subjected to a tension T. Its end A is connected to a spring with stiffness constant k positioned vertically. The other end of the spring is subjected to a vertical sinusoidal displacement with angular frequency $w = 100\pi$ and amplitude s_0 $s(t) = s_0 e^{jwt}$ At $x = L$, the string is connected to a damper with a viscous friction coefficient $\alpha = 0.2N/m$ (see figure).



The tension T of the string is adjusted so that there is no reflection at point B. The string vibrates in the vertical plane along the Oy axis. The displacement of a point at position x on the string is represented by its position $y(x,t)$.

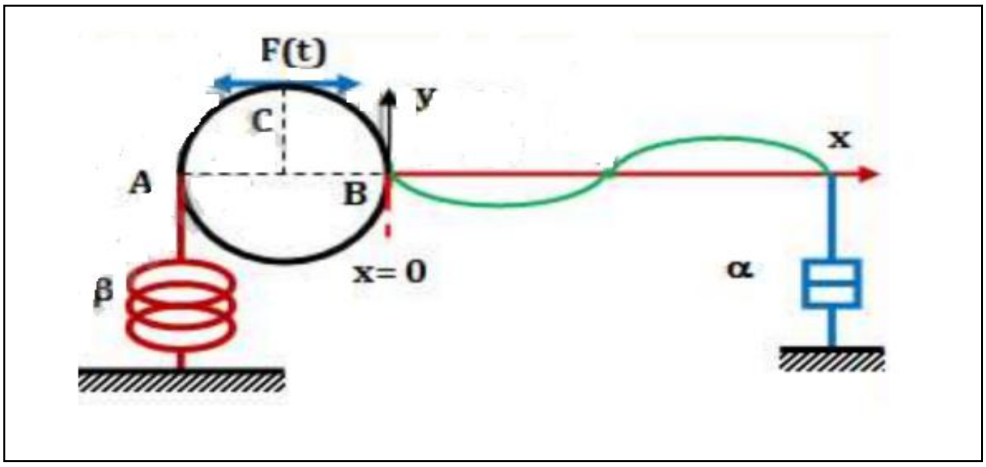
1. Justify without calculation that the displacement is written as: $y(x,t) = y_0 e^{j(w(t-\frac{x}{v})+\varphi)}$
2. Calculate the reflection coefficient at $x = L$. Deduce the values of the string tension T and the phase velocity v.
3. Provide the expression of the displacement $y(x,t)$ for the point at position x on the string. Deduce that the points at $x = 0$ and $x = L$ vibrate in phase.
4. What is the value of the impedance at point A? Determine the amplitude y and the phase φ of the vibration at point A. Deduce the expression of the displacement $y(x,t)$ for a point at position x. In which case is the phase zero? What is the value of y_0 in that case?

Exercise6:

The system shown in the figure below consists of a cylindrical pulley of mass M and radius R. At two diametrically opposite points A and B, located in a horizontal plane at equilibrium, a spring of stiffness β and a string of linear mass density μ are fixed. The string is stretched under a tension T. At point C, located on the surface of the pulley and perpendicular to AB, a tangential force of small amplitude $F(t) = F_0 cos(w\, t)$ is applied. At the other end of the string, a damper with a viscous friction coefficient α is fixed, chosen so that the string supports a progressive sinusoidal wave.

1. Calculate the impedance $Z(x)$ at a point x on the string.
2. What is the input impedance of the string at $x = 0$?
3. Calculate the average power P supplied by the mechanical system to the string.
4. Knowing that at a frequency of oscillation of 10 Hz, $\langle P \rangle$ reaches its maximum value $\langle P \rangle_{max}$, calculate: the stiffness constant β and the linear mass density μ.

Given: $M = 0.5\, kg,$, $T = 10\, N$.



Exercise7:

A string of length L and linear mass density μ is stretched horizontally with a tension T between two rigid walls. The tension of the string is adjusted so that the length of the string equals the wavelength L=λ. A sinusoidal wave of angular frequency ω is created in the string.

1. Show that the displacement at a point x on the string is written as: $y(x,t) = A\, sin(kx).\sin(wt)$
2. Calculate the linear densities of kinetic energy e_c , potential energy e_p , and their time-averaged values . Deduce the average energy density over time .