# Series N°3 : Kinematics

## Ex 1 :

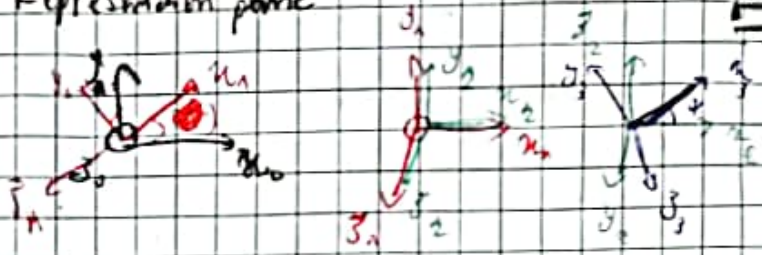
$R_0 \xrightarrow[\text{Rotation } \theta \text{ (selon } z_0)]{} R_1 \xrightarrow[\text{translation } \overrightarrow{O_1O_2}]{} R_2 \xrightarrow[\text{Rotation } \psi \text{ (selon } z)]{} R_3$

+ translation $\overrightarrow{O_0O_1}$

$\vec{\Omega}^0_{1/R_0R_1} = \begin{pmatrix} 0 \\ 0 \\ \dot{\theta} \end{pmatrix}$ $\quad \vec{\Omega}^1_{2/R_1R_2} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ $\quad \vec{\Omega}^2_{3/R_2R_3} = \begin{pmatrix} 0 \\ 0 \\ \dot{\psi} \end{pmatrix}$

Matrice de passe

Representation plane



$$P_{0\to1} = \begin{bmatrix} \cos(x_1x_0) & \cos(x_1y_0) & \cos(x_1z_0) \\ \cos(y_1x_0) & \cos(y_1y_0) & \cos(y_1z_0) \\ \cos(z_1x_0) & \cos(z_1y_0) & \cos(z_1z_0) \end{bmatrix}$$

cos directeur

$\pi/2 - \theta$ $\quad 90$ $\quad \pi/2 + \theta$ $\quad 90$ $\quad 0$ $\quad 90$ $\quad 90$

$$P_{0\to1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R$ selon $y$ $\begin{bmatrix} & 0 & \\ 0 & 1 & 0 \\ & 0 & \end{bmatrix}$

$R$ selon $x$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & & \\ 0 & & \end{bmatrix}$

$R$ selon $z$ $\begin{bmatrix} & & 0 \\ & & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$P_{1\to2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_{2\to3} = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

1) The angular velocity :

a) $\vec{\Omega}^0_{2/R_1}$ (respect to $\to 0$)

expression

$$\vec{\Omega}^{\circ}_{2/R_0} = \vec{\Omega}^{\circ}_{1/R_0} + \vec{\Omega}^{1}_{2/R_1}$$

$$= \begin{Bmatrix} 0 \\ 0 \\ \dot{\theta} \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \dot{\theta} \end{Bmatrix}$$

$$\rightarrow = P_{1\to 2}\, \vec{\Omega}^{2}_{1/R_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \dot{\theta} \end{Bmatrix}$$

b) $\vec{V}^{\circ}(O_1)/R_0 = \dfrac{d^{\circ} O_0 O_1}{dt}/R_0$ # Base Mobile

$$= \frac{d^{\wedge} O_0 O_1}{dt}/R_1 + \vec{\Omega}^{\circ}_{1} \wedge \overrightarrow{O_0 O_1}/R_1$$

$$\overrightarrow{O_0 O_1}/R_1 = \begin{Bmatrix} d \\ 0 \\ 0 \end{Bmatrix} \qquad \vec{\Omega}^{\circ}_{1/R_1} = \begin{pmatrix} 0 \\ 0 \\ \dot{\theta} \end{pmatrix}$$

$$V(O)/R_1 = \frac{d}{dt}\left( \begin{matrix} 1 \\ 0 \\ 0 \end{matrix} + \int \begin{pmatrix} 0 \\ 0 \\ \dot{\theta} \end{pmatrix} \wedge \begin{pmatrix} d \\ 0 \\ 0 \end{pmatrix} \right) = \begin{pmatrix} 0 \\ \dot{\theta} d \\ 0 \end{pmatrix}$$

2) $\vec{V}(O_2)/R_1 = \dfrac{d^{\circ} \overrightarrow{O_1 O_2}}{dt}/R_1$

$$= \frac{d}{dt} \begin{Bmatrix} 0 \\ 0 \\ h(t) \end{Bmatrix} \quad \text{avec } \overrightarrow{O_1 O_2}/R_1 = \begin{Bmatrix} 0 \\ 0 \\ h(t) \end{Bmatrix}$$

1) b) $\vec{V}(M)/R_1 = \dfrac{d^{\circ} \overrightarrow{O_0 M}}{dt}/R_1$

$$= \frac{d^{\wedge} \overrightarrow{O_0 M}}{dt}/R_1 + \vec{\Omega}^{\circ}_{1}/R_1 \wedge \overrightarrow{O_0 M}/R_1$$

$$\overrightarrow{O_0 M}/R_1 = \overrightarrow{O_0 O_1}/R_1 + \overrightarrow{O_1 O_2}/R_1 + \overrightarrow{O_2 M}/R_1$$

$$\begin{Bmatrix} d \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ h(t) \end{Bmatrix} + \begin{Bmatrix} r\cos\psi \\ r\sin\psi \\ 0 \end{Bmatrix}$$

$$\overrightarrow{O_2 M}/R_2 = \begin{Bmatrix} r \\ 0 \\ 0 \end{Bmatrix}/R_2$$

$$O_2 M/R_2 = P_{3\to 2}\, \overrightarrow{O_2 M}/R_3$$

$$\begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} r \\ 0 \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} r\cos\psi \\ r\sin\psi \\ 0 \end{Bmatrix} \rightarrow \overrightarrow{O_2 M}/R_1 = P_{2\to 1}\, \overrightarrow{O_2 M}/R_2$$

$$= \begin{Bmatrix} d + r\cos\psi \\ r\sin\psi \\ h(t) \end{Bmatrix}$$

$$V^{\circ}(M)/R_1 = \frac{d}{dt} \begin{Bmatrix} d + r\cos\psi \\ r\sin\psi \\ h(t) \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ \dot{\theta} \end{Bmatrix} \wedge \begin{Bmatrix} r\cos\psi \\ r\sin\psi \\ h(t) \end{Bmatrix}$$

$$= \begin{Bmatrix} -r\dot{\psi}\sin\psi \\ r\dot{\psi}\cos\psi \\ \dot{h}(t) \end{Bmatrix} + \begin{Bmatrix} -\dot{\theta} r\sin\psi \\ \dot{\theta}(d + r\cos\psi) \\ 0 \end{Bmatrix}$$

3) a) $\vec{V}^{\circ}(M)/R_1 = \dfrac{d^{\wedge} \overrightarrow{O_1 M}}{dt}/R_1$ avec $\overrightarrow{O_1 M}/R_1 = \begin{Bmatrix} r\cos\psi \\ r\sin\psi \\ h(t) \end{Bmatrix}$

$$= \frac{d}{dt} \begin{Bmatrix} r\cos\psi \\ r\sin\psi \\ h(t) \end{Bmatrix} = \begin{Bmatrix} -r\dot{\psi}\sin\psi \\ r\dot{\psi}\cos\psi \\ \dot{h}(t) \end{Bmatrix}$$

b) $\vec{V}^{1}(M)/R_2 = P_{1\to 2}\, V^{1}(M)/R_1$

b) $\vec{V}_1(M)/R_0 = P_{1\to 0}\, \vec{V}^{1}(M)/R_1$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} -r\dot{\psi}\sin\psi \\ r\dot{\psi}\cos\psi \\ \dot{h}(t) \end{Bmatrix}$$

4) $\vec{\gamma}^{\circ}(M)_{/R_1} = \dfrac{d^{\circ}\vec{V}^{\circ}(M)_{/R_1}}{dt_{/R_1}}$

$= \dfrac{d^1 V^{\circ}(M)_{/R_1}}{dt_{/R_1}} + \vec{\Omega}^{\circ}_1 \wedge \vec{V}^{\circ}(M)_{/R_1}$

$= \dfrac{d}{dt}\begin{Bmatrix} -(\dot\theta+\dot\psi) r \sin\psi \\ d\dot\theta + (\dot\psi+\dot\theta) r\cos\psi \\ \dot h(t) \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ \dot\theta \end{Bmatrix} \wedge$

$\boxed{\underbrace{(\dot\psi+\dot\theta)}_{f} r \underbrace{\cos\psi}_{g} = f'g + g'f}$

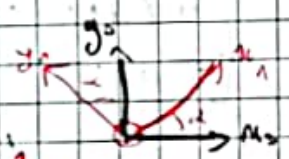
**Ex2 :**

part 1:

$R_1$ is the projection frame → expressed

$R_0 \xrightarrow[\text{Rotation } z\ (\alpha)]{} R_1 \xrightarrow[\text{Rotation } z\ (\beta)]{} R_2$

$\vec{\Omega}^{\circ}_{1/R_0R_1} = \begin{pmatrix} 0 \\ 0 \\ \dot\alpha \end{pmatrix}$ / $\vec{\Omega}^{1}_{2/R_1R_2} = \begin{pmatrix} 0 \\ 0 \\ \dot\beta \end{pmatrix}$



$z_0 = z_1$

$P_{0\to1} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$P_{1\to2} = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

1) a) absolute $\vec{V}^{\circ}(A)_{/R_1} = \dfrac{d^{\circ}\vec{O_0A}}{dt_{/R_1}}$

$= \dfrac{d^1\vec{O_0A}}{dt_{/R_1}} + \vec{\Omega}^{\circ}_{1/R_1} \wedge \vec{O_0A}_{/R_1}$

$\vec{O_0A}_{/R_1} = \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix}$

$\dfrac{d}{dt}\begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dot\alpha \end{pmatrix} \wedge \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \dot\alpha a \\ 0 \end{pmatrix}$

b) $\vec{\gamma}^{\circ}(A)_{/R_1} = \dfrac{d^{\circ}\vec{V}^{\circ}(A)_{/R_1}}{dt_{/R_1}}$

$= \dfrac{d^1 V^{\circ}(A)_{/R_1}}{dt_{/R_1}} + \vec{\Omega}^{\circ}_1 \wedge V^{\circ}(A)_{/R_1}$

$= \dfrac{d}{dt}\begin{pmatrix} 0 \\ \dot\alpha a \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dot\alpha \end{pmatrix} \wedge \begin{pmatrix} 0 \\ \dot\alpha a \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ \ddot\alpha a \\ 0 \end{pmatrix} + \begin{pmatrix} -\dot\alpha^2 a \\ 0 \\ 0 \end{pmatrix}$

$\vec{\gamma}^{\circ}(A)_{/R_1} = \begin{pmatrix} -\dot\alpha^2 a \\ \ddot\alpha a \\ 0 \end{pmatrix}$

2) a) $\vec{V}^{\circ}(B)_{/R_1} = \vec{V}^{\circ}(A)_{/R_1} + \vec{\Omega}^{\circ}_2 \wedge \vec{AB}_{/R_1}$

Repère de l'objet solide

$\vec{\Omega}^{\circ}_{2/R_1} = \vec{\Omega}^{\circ}_{1/R_1} + \vec{\Omega}^{1}_{2/R_1} = \begin{pmatrix} 0 \\ 0 \\ \alpha+\beta \end{pmatrix}$

$\vec{AB}_{/R_2} = \begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix}$

$\vec{AB}_{/R_1} = P_{2\to1}\, \vec{AB}_{/R_2}$

$= \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} b\cos\beta \\ b\sin\beta \\ 0 \end{pmatrix}$

$\vec{V}^{\circ}(B)_{/R_1} = \begin{pmatrix} 0 \\ \dot\alpha a \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dot\alpha \end{pmatrix} \wedge \begin{pmatrix} b\cos\beta \\ b\sin\beta \\ 0 \end{pmatrix}$

$$= \begin{pmatrix} -(\dot\alpha+\dot\beta)\, b\sin\beta \\ \dot\alpha a + (\dot\alpha+\dot\beta)\, b\cos\beta \\ 0 \end{pmatrix}$$

$$\rightarrow \vec{V}^{\circ}(B)_{/R_1} = \frac{d\vec{O_0B}}{dt}_{/R_1}$$

$$= \frac{d^{1}\vec{O_0B}}{dt}_{/R_1} + \vec{\Omega}^{\circ}_{1/R_1} \wedge \vec{O_0B}_{/R_1}$$

$$\vec{O_0B} = \vec{O_0A} + \vec{AB}$$

$$\begin{Bmatrix} a \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} b\cos\beta \\ b\sin\beta \\ 0 \end{Bmatrix} = \begin{Bmatrix} a + b\cos\beta \\ b\sin\beta \\ 0 \end{Bmatrix}$$

$$\vec{V}^{\circ}(B)_{/R_1} = \frac{d}{dt}\begin{pmatrix} a+b\cos\beta \\ b\sin\beta \\ 0 \end{pmatrix} + \begin{Bmatrix} 0 \\ 0 \\ \dot\alpha \end{Bmatrix} \wedge \begin{Bmatrix} a+b\cos\beta \\ b\sin\beta \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} -b\dot\beta\sin\beta \\ b\dot\beta\cos\beta \\ 0 \end{Bmatrix} + \begin{Bmatrix} -\dot\alpha b\sin\beta \\ \dot\alpha(a+b\cos\beta) \\ 0 \end{Bmatrix}$$

b) $$\vec{\gamma}^{\circ}(B)_{/R_1} = \vec{\gamma}^{\circ}(A)_{/R_1} + \frac{d\vec{\Omega}^{\circ}_{2}}{dt}_{/R_1} \wedge \vec{AB}_{/R_1} + \vec{\Omega}^{\circ}_{2/R_1} \wedge \left(\vec{\Omega}^{\circ}_{2} \wedge \vec{AB}\right)_{/R_1}$$

$$\vec{\gamma}^{\circ}(A)_{/R_1} = \begin{pmatrix} -\dot a^2 a \\ \ddot\alpha a \\ 0 \end{pmatrix}$$

$$\frac{d^{\circ}\vec{\Omega}_2}{dt/R_1} = \frac{d^{1}\vec{\Omega}_2}{dt/R_1} + \vec{\Omega}^{\circ}_{1} \wedge \vec{\Omega}^{\circ}_{2}$$

$$= \frac{d}{dt}\begin{pmatrix} p \\ 0 \\ \dot\alpha+\dot\beta \end{pmatrix} + \begin{Bmatrix} 0 \\ 0 \\ \dot\alpha \end{Bmatrix} \wedge \begin{Bmatrix} 0 \\ 0 \\ \dot\alpha+\dot\beta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \ddot\alpha+\ddot\beta \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\vec{\Omega}^{\circ}_{2/R_0} \wedge \left(\vec{\Omega}^{\circ}_{2} \wedge \vec{AB}\right)_{/R_1} = \begin{Bmatrix} 0 \\ 0 \\ \dot\alpha+\dot\beta \end{Bmatrix} \wedge \left( \begin{Bmatrix} -(\dot\alpha+\dot\beta)\, b\sin\beta \\ (\dot\alpha+\dot\beta)\, b\cos\beta \\ 0 \end{Bmatrix} \right)$$

$$\begin{Bmatrix} -(\dot\alpha+\dot\beta)^2 b\cos\beta \\ -(\dot\alpha+\dot\beta)^2 b\sin\beta \\ 0 \end{Bmatrix}$$

$$\frac{d^{\circ}\vec{\Omega}_1}{dt/R_1} \wedge \vec{AB}_{/R_1} = \begin{Bmatrix} 0 \\ 0 \\ \ddot\alpha \end{Bmatrix} \wedge \begin{Bmatrix} b\cos\beta \\ b\sin\beta \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} -(\ddot\alpha+\ddot\beta)\, b\sin\beta \\ (\ddot\alpha+\ddot\beta)\, b\cos\beta \\ 0 \end{Bmatrix}$$

$$\vec{\gamma}^{\circ}(B)_{/R_1} = \begin{Bmatrix} -\dot\alpha^2 a - (\ddot\alpha+\ddot\beta) b\sin\beta - (\dot\alpha+\dot\beta)^2 b\cos\beta \\ \ddot\alpha a + (\ddot\alpha+\ddot\beta) b\cos\beta - (\dot\alpha+\dot\beta)^2 b\sin\beta \\ 0 \end{Bmatrix}$$

Part 2:

$$\vec{V}^{\circ}(B)_{/R_0} = P_{1\to 0}\ \vec{V}^{\circ}(B)_{/R_1}$$

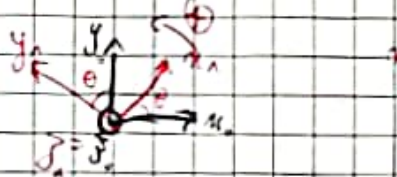
$$= \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} -b(\dot\alpha+\dot\beta)\sin\beta \\ \dot\alpha a + b(\dot\alpha+\dot\beta)\cos\beta \\ 0 \end{Bmatrix}$$

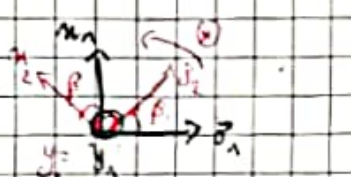
$$\vec{V}^{\circ}(B)_{/R_0} = \begin{Bmatrix} -b(\dot\alpha+\dot\beta)\sin\beta\cos\alpha - \left[\dot\alpha a + b(\dot\alpha+\dot\beta)\cos\beta\right]\sin\alpha \\ -b(\dot\alpha+\dot\beta)\sin\beta\sin\alpha + \left[\dot\alpha a + b(\dot\alpha+\dot\beta)\cos\beta\right]\cos\alpha \\ 0 \end{Bmatrix}$$

**Ex3:**

$R_0 \xrightarrow[\text{rotation } z]{} R_1 \xrightarrow[\text{Rotation } y]{} R_2$

$$\vec{\Omega}_{1/R_0,R_1} = \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \qquad \vec{\Omega}_{2/R_1,R_2} = \begin{pmatrix} 0 \\ \omega_2 \\ 0 \end{pmatrix}$$




$$P_{0\to 1} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad P_{1\to 2} = \begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix}$$

a) $\vec{V}^{\circ}(A)_{/R_0} = \frac{d\vec{O_0A}}{dt/R_0}$ $\quad \vec{O_0A} = \vec{CA}$

$= \frac{d^{\wedge}\vec{CA}}{dt/R_1} + \vec{\Omega}_{1/R_0} \wedge \vec{CA}_{/R_1}$

$\vec{CA} = \vec{CB} + \vec{BA} = \begin{pmatrix}0\\d\\0\end{pmatrix} + \begin{pmatrix}r\cos\beta\\0\\-r\sin\beta\end{pmatrix} = \begin{pmatrix}r\cos(\omega_2 t)\\d\\-r\sin\beta\end{pmatrix}$

$\vec{BA}_{/R_2} = \begin{pmatrix}r\\0\\0\end{pmatrix}$, $\vec{BA}_{/R_1} = \begin{pmatrix}r\cos\beta\\-r\sin\beta\\0\end{pmatrix}$

$\vec{V}^{\circ}(A)_{/R_0} = \frac{d}{dt}\begin{pmatrix}r\cos\omega_2 t\\d\\-r\sin\beta\end{pmatrix} + \begin{pmatrix}0\\0\\\omega_1\end{pmatrix} \wedge \begin{pmatrix}r\cos\omega_2 t\\d\\-r\sin\beta\end{pmatrix}$

$= \begin{pmatrix}-\omega_2 r\sin\omega_2 t\\0\\0\end{pmatrix} + \begin{pmatrix}-\omega_1 d\\\omega_1 r\cos(\omega_2 t)\\0\end{pmatrix}$

$= \begin{pmatrix}-\omega_1 d - \omega_2 r\sin\omega_2 t\\\omega_1 r\cos(\omega_2 t)\\0\end{pmatrix}$

acceleration :

$\vec{\gamma}^{\circ}(A)_{/R_0} = \frac{d^{\circ}\vec{V}(A)_{/R_0}}{dt/R_0}$

$= \frac{d^{\wedge}\vec{V}(A)_{/R_1}}{dt/R_1} + \vec{\Omega}^{\circ}_{1/R_0} \wedge \vec{V}^{\circ}(A)_{/R_1}$

$= \begin{pmatrix}0\\0\\\omega_1\end{pmatrix} \wedge \begin{pmatrix}-\omega_1 d\\\omega_1 r\cos\omega_2 t\\0\end{pmatrix}$

$= \begin{pmatrix}-\omega_1^2 r\cos\omega_2 t\\-\omega_1^2 d\\0\end{pmatrix}$

b) $\vec{V}^{\circ}(B)_{/R_0} = \frac{d^{\circ}\vec{O_0B}}{dt/R_0}$

$= \frac{d^{\circ}\vec{CB}}{dt/R_0} + \vec{\Omega}^{\circ}_{1/R_0,R_1} \wedge \vec{CB}_{/R_1}$

$\vec{CB} = \begin{pmatrix}0\\d\\0\end{pmatrix}$

$\vec{V}^{\circ}(B)_{/R_0} = \frac{d}{dt}\begin{pmatrix}0\\d\\0\end{pmatrix} + \begin{pmatrix}0\\0\\\omega_1\end{pmatrix} \wedge \begin{pmatrix}0\\d\\0\end{pmatrix}$

$= \begin{pmatrix}-\omega_1 d\\0\\0\end{pmatrix} = \begin{pmatrix}-600\\0\\0\end{pmatrix}$

$\vec{\gamma}^{\circ}(B)_{/R_0} = \frac{d^{\circ}\vec{V}(B)_{/R_0}}{dt/R_0}$

$= \frac{d^{\wedge}\vec{V}(B)_{/R_0}}{dt/R_0} + \vec{\Omega}_{1/R_0} \wedge \vec{V}^{\circ}(B)_{/R_0}$

$= \frac{d}{dt}\begin{pmatrix}-\omega_1 d\\0\\0\end{pmatrix} + \begin{pmatrix}0\\0\\\omega_1\end{pmatrix} \wedge \begin{pmatrix}-\omega_1 d\\0\\0\end{pmatrix}$

$\vec{\gamma}^{\circ}(B)_{/R_0} = \begin{pmatrix}0\\\omega_1^2 d\\0\end{pmatrix} = \begin{pmatrix}0\\3000\\0\end{pmatrix}$

c) $\vec{V}^{\circ}(A)_{/R_0} = \vec{V}^{\circ}(B)_{/R_0} + \vec{\Omega}^{\circ}_{2/R_0} \wedge \vec{BA}_{/R_0}$

$\vec{\Omega}^{\circ}_{2} = \vec{\Omega}^{\circ}_{1} + \vec{\Omega}^{1}_{2} = \begin{pmatrix}0\\0\\\omega_1\end{pmatrix} + \begin{pmatrix}\omega_2\\0\\0\end{pmatrix}$

$\vec{\Omega}^{\circ}_{2} = \begin{pmatrix}0\\\omega_2\\\omega_1\end{pmatrix}$

$= \begin{pmatrix}-\omega_1 d\\0\\0\end{pmatrix} + \begin{pmatrix}0\\\omega_2\\\omega_1\end{pmatrix} \wedge \begin{pmatrix}r\cos\omega_2 t\\0\\-r\sin\omega_2 t\end{pmatrix}$

$= \begin{pmatrix}-\omega_1 d - \omega_2 r\sin(\omega_2\theta)\\\omega_1 r\cos(\omega_2 t)\\+r\omega_2\cos(\omega_2 t)\end{pmatrix}$

$\gamma^{\circ}(A) = \frac{d}{dt}V(A) + \Omega^{\circ}_1 V(A)$

$= \frac{d}{dt}\begin{pmatrix}-\omega_2 r\sin(\omega_2 t) - \omega_1 d\\\omega_1 r\cos(\omega_2 t)\\-r\omega_2\cos(\omega_2 t)\end{pmatrix} + \begin{pmatrix}0\\0\\\omega_1\end{pmatrix} \wedge \begin{pmatrix}\cdots\end{pmatrix}$

$= \begin{pmatrix}-\omega_2^2 r\cos\omega_2 t\\-\omega_1\omega_2 r\sin(\omega_2 t)\\r\omega_2^2\sin\omega_2 t\end{pmatrix} + \begin{pmatrix}-\omega_1^2 r\cos(\omega_2\theta\\-\omega_1\omega_2 r\sin(\omega_2 t) - \omega_1^2 d\\0\end{pmatrix}$

$$= \begin{cases} -(\omega_1+\omega_2)^2 r\cos\omega_2 t \\ -2\omega_1\omega_2 r\sin\omega_2 t - \omega_1^2 d \\ r\omega_2^2 \sin\omega_2 t \end{cases}$$

$$\vec{\gamma}^{\circ}(B) = \frac{d^{\circ}}{dt}\begin{Bmatrix} -\omega_1 d \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ \omega_1 \end{Bmatrix} \wedge \begin{Bmatrix} -\omega_1 d \\ 0 \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} 0 \\ -\omega_1^2 d \\ 0 \end{Bmatrix}$$

$$\vec{\gamma}^{\circ}(A)_{/R_0} = \vec{\gamma}^{\circ}(B)_{/R_0} + \frac{d^{\circ}\vec{\Omega}_2}{dt/R_0} \wedge \vec{BA}_{/R_0} + \vec{\Omega}^{\circ}_{2/R_0} \wedge \left(\vec{\Omega}^{\circ}_2 \wedge \vec{BA}\right)_{/R_0}$$

$$= \begin{Bmatrix} -\omega_1^2 d \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ -\omega_1\omega_2 r\sin(\omega_2 t) \\ 0 \end{Bmatrix} + \begin{Bmatrix} -r(\omega_1^2+\omega_2^2)\cos(\omega_2 t) \\ -\omega_1\omega_2 r\sin(\omega_2 t) \\ \omega_2^2 r\sin(\omega_2 t) \end{Bmatrix}$$

$$\frac{d^{\circ}\vec{\Omega}_2}{dt/R_0} = \frac{d^{\wedge}\vec{\Omega}_2}{dt/R_0} + \vec{\Omega}^{\circ}_1 \wedge \vec{\Omega}^{\circ}_2$$

$$= \frac{d}{dt}\begin{Bmatrix} 0 \\ \omega_2 \\ \omega_1 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ \omega_1 \end{Bmatrix} \wedge \begin{Bmatrix} 0 \\ \omega_2 \\ \omega_1 \end{Bmatrix} = \begin{Bmatrix} -\omega_1\omega_2 \\ 0 \\ 0 \end{Bmatrix}$$

$$\frac{d^{\circ}\vec{\Omega}_2}{dt/R_0} \wedge \vec{BA}_{/R_0} = \begin{Bmatrix} -\omega_1\omega_2 \\ 0 \\ 0 \end{Bmatrix} \wedge \begin{Bmatrix} r\cos\cos(\omega_2 t) \\ 0 \\ -r\sin(\omega_2 t) \end{Bmatrix}$$

$$= \begin{Bmatrix} 0 \\ -\omega_1\omega_2 r\sin(\omega_2 t) \\ 0 \end{Bmatrix}$$

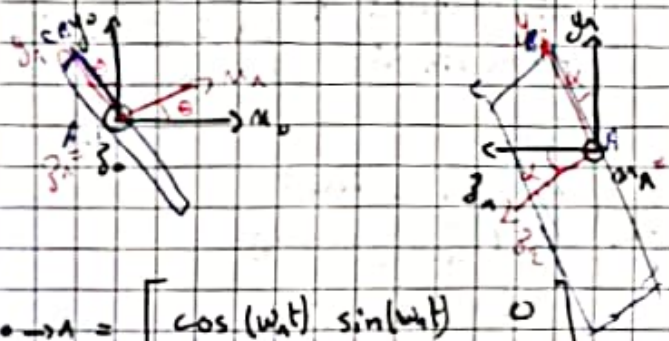
$$\vec{\Omega}^{\circ}_2 \wedge \left(\vec{\Omega}^{\circ}_2 \wedge \vec{BA}\right) = \begin{Bmatrix} 0 \\ \omega_2 \\ \omega_1 \end{Bmatrix} \wedge \begin{Bmatrix} -\omega_2 r\sin(\omega_2 t) \\ \omega_1 r\cos(\omega_2 t) \\ -r\omega_2\cos(\omega_2 t) \end{Bmatrix}$$

$$= \begin{Bmatrix} -r\omega_2^2\cos(\omega_2 t) - r\omega_1\cos(\omega_2 t) \\ -\omega_1\omega_2 r\sin(\omega_2 t) \\ \omega_2^2 r\sin(\omega_2 t) \end{Bmatrix}$$

**Ex 4:**

$R_0 \xrightarrow[\text{Rotation } z \to \theta]{} R_1 \xrightarrow[\text{Rotation } x \to \alpha]{} R_2$

$$\vec{\Omega}^{\circ}_{1/R_0} = \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \qquad \vec{\Omega}^{1}_{2/R_1} = \begin{pmatrix} \omega_2 \\ 0 \\ 0 \end{pmatrix}$$



$$P_{0\to 1} = \begin{bmatrix} \cos(\omega_1 t) & \sin(\omega_1 t) & 0 \\ -\sin(\omega_1 t) & \cos(\omega_1 t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_{1\to 2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega_2 t & \sin\omega_2 t \\ 0 & -\sin\omega_2 t & \cos\omega_2 t \end{bmatrix}$$

$O_0B = AB$

$$\vec{V}^{\circ}(B)_{/R_0} = \frac{d^{\circ}\vec{O_0B}}{dt/R_0} = \frac{d^{\circ}\vec{AB}}{dt/R_1}$$

$$= \frac{d^{\wedge}\vec{AB}}{dt/R_1} + \vec{\Omega}^{\wedge}_{0} \wedge \vec{AB}$$

$$\vec{AB} = \begin{Bmatrix} 135\cos(\omega_2 t) \\ 135\sin(\omega_2 t) \end{Bmatrix} \qquad \vec{\Omega}^{\wedge}_{0} = \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix}$$

$$= \frac{d}{dt}\begin{Bmatrix} 0 \\ 135\cos\omega_2 t \\ 135\sin\omega_2 t \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ \omega_1 \end{Bmatrix} \wedge \begin{Bmatrix} 0 \\ 135\cos\omega_2 t \\ 135\sin\omega_2 t \end{Bmatrix}$$

$$= \begin{Bmatrix} 0 \\ -135\omega_2\sin(\omega_2 t) \\ 135\omega_2\cos(\omega_2 t) \end{Bmatrix} + \begin{Bmatrix} -\omega_1 135\cos\omega_2 t \\ 0 \\ 0 \end{Bmatrix}$$

$$\vec{V}^{\circ}(B)_{/R_0} = \begin{Bmatrix} -135\omega_1\cos\omega_2 t \\ -135\omega_2\sin\omega_2 t \\ 135\omega_2\cos\omega_2 t \end{Bmatrix}$$

$$\vec{\gamma}^{\circ}(B)_{/R_1} = \frac{d\vec{V}(B)}{dt}_{/R_1} = \frac{d^{\wedge}\vec{V}(B)}{dt}_{R_1} + \vec{\Omega}^{\wedge}_{2} \wedge \vec{V}(B)$$

$$= \frac{d}{dt}\begin{pmatrix} -135\,\omega_2 \cos\omega_2 t \\ -135\,\omega_2 \sin\omega_2 t \\ 135\,\omega_2 \cos\omega_2 t \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \wedge \begin{pmatrix} -135\,\omega_2 \cos\omega_2 t \\ -135\,\omega_2 \sin\omega_2 t \\ 135\,\omega_2 \cos\omega_2 t \end{pmatrix}$$

$$= \begin{pmatrix} 135\,\omega_1\omega_2 \sin\omega_2 t \\ -135\,\omega_2^2 \cos\omega_2 t \\ -135\,\omega_2^2 \sin\omega_2 t \end{pmatrix} + \begin{pmatrix} 135\,\omega_1\omega_2 \sin\omega_2 t \\ -135\,\omega_2^2 \cos\omega_2 t \\ 0 \end{pmatrix}$$

$$\vec{\gamma}^{\circ}(B)_{R_1} = \begin{pmatrix} 2\times 135\,\omega_1\omega_2 \sin\omega_2 t \\ -135\,(\omega_1^2+\omega_2^2)\cos\omega_2 t \\ -135\,\omega_2^2 \sin\omega_2 t \end{pmatrix}$$

$$\vec{V}^{\circ}(C)_{/R_1} = \vec{V}(B)_{/R_1} + \vec{\Omega}^{\circ}_{2} \wedge \vec{BC}_{/R_1}$$

$$\vec{\Omega}_2 = \vec{\Omega}^{\circ}_{1} + \vec{\Omega}^{\wedge}_{2} = \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix}$$

$$\vec{BC}_{/R_2} = \begin{pmatrix} 0 \\ 0 \\ y_0 \end{pmatrix} \Longrightarrow \vec{BC}_{/R_1} = P_{2\to 1} \cdot \vec{BC}_{/R_2}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega_2 t & -\sin\omega_2 t \\ 0 & \sin(\omega_2 t) & \cos\omega_2 t \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ y_0 \end{pmatrix}$$

$$\vec{BC}_{/R_1} = \begin{pmatrix} 0 \\ -y_0 \sin\omega_2 t \\ y_0 \cos\omega_2 t \end{pmatrix}$$

$$\vec{V}^{\circ}(C)_{/R_1} = \begin{pmatrix} -135\,\omega_1 \cos\omega_2 t \\ -135\,\omega_2 \sin\omega_2 t \\ 135\,\omega_2 \cos\omega_2 t \end{pmatrix} + \begin{pmatrix} \omega_1 \\ 0 \\ \omega_1 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ -y_0 \sin\omega_2 t \\ y_0 \cos\omega_2 t \end{pmatrix}$$

$$\Downarrow$$

$$+ \begin{pmatrix} y_0\omega_1 \sin\omega_2 t \\ -y_0\,\omega_2 \cos\omega_2 t \\ y_0\,\omega_2 \sin\omega_2 t \end{pmatrix}$$

$$\vec{\gamma}^{\circ}(C)_{/R_1} = \vec{\gamma}^{\circ}(B)_{/R_1} + \frac{d\vec{\Omega}^{\circ}_{2}}{dt}_{/R_1} \wedge \vec{BC} + \vec{\Omega}^{\circ}_{2} \wedge \left(\vec{\Omega}^{\circ}_{2} \wedge \vec{BC}\right)$$

$$\frac{d\vec{\Omega}^{\circ}_{2}}{dt}_{/R_1} = \frac{d^{\wedge}\vec{\Omega}^{\circ}_{2}}{dt}_{/R_1} + \vec{\Omega}^{\circ}_{1} \wedge \vec{\Omega}^{\circ}_{2}$$

$$= \frac{d}{dt}\begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \wedge \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix}$$

Ex 1

$$\vec{V}^{\circ}(M)_{/R_j} = \vec{V}^{\circ}(M)_{/R_i} + \vec{V}^{\circ}(O_i)_{/R_j} + \vec{\Omega}^{\circ}_{i/R_j} \wedge \vec{O_iM}$$

$$\vec{V}^{\circ}(M)_{/R_1} = \vec{V}^{\wedge}(M)_{/R_1} + \vec{V}^{\circ}(O_1)_{/R_1} + \vec{\Omega}^{\circ}_{1} \wedge \vec{O_1M}_{/R_1}$$

$$\vec{V}^{\wedge}(M)_{/R_1} = \frac{d^{\wedge}\vec{O_1M}}{dt}_{/R_1} = \frac{d}{dt}\begin{pmatrix} r\cos\psi \\ r\sin\psi \\ h(t) \end{pmatrix}$$

$$= \begin{pmatrix} -r\dot{\psi}\sin\psi \\ \dot{r}\cos\psi \\ \dot{h}(t) \end{pmatrix}$$

$$\vec{V}^{\circ}(O_1)_{/R_1} = \frac{d^{\circ}\vec{O_0O_1}}{dt}_{/R_1} = \frac{d^{\wedge}\vec{O_0O_1}}{dt}_{/R_1} + \vec{\Omega}^{\circ}_{1} \wedge \vec{O_0O_1}$$

$$= \frac{d}{dt}\begin{pmatrix} d \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \dot{\theta} \\ 0 \end{pmatrix} \wedge \begin{pmatrix} d \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \dot{\theta}\,d \\ 0 \end{pmatrix}$$

$$\vec{\Omega}^{\circ}_{1} \wedge \vec{O_1M}_{/R_1} = \begin{pmatrix} 0 \\ 0 \\ \dot{\theta} \end{pmatrix} \wedge \begin{pmatrix} r\cos\psi \\ r\sin\psi \\ h(t) \end{pmatrix} = \begin{pmatrix} -r\dot{\theta}\sin\psi \\ r\dot{\theta}\cos\psi \\ 0 \end{pmatrix}$$

$$\vec{V}^{\circ}(M)_{/R_1} = \begin{pmatrix} -r(\dot{\theta}+\dot{\psi})\sin\psi \\ r(\dot{\theta}+\dot{\psi})\cos\psi + \dot{\theta}d \\ \dot{h}(t) \end{pmatrix}$$

$$\vec{\gamma}^{\circ}(M)_{/R_j} = \vec{\gamma}^{\circ}(M)_{/R_i} + \left[\vec{\gamma}^{\circ}(O_i) + \frac{d^{\wedge}\vec{\Omega}^{\circ}_{i}}{dt} \wedge \vec{O_iM} + \vec{\Omega}^{\circ}_{i} \wedge \left(\vec{\Omega}^{\circ}_{i} \wedge \vec{O_iM}\right)\right] + 2\,\vec{\Omega}^{\circ}_{i} \wedge \vec{V_i(M)}$$

$$\vec{\gamma}(M)_{R_0} = \vec{\gamma}^{r}(M)_{R_r} + \left[\vec{\gamma}(O_r)_{R_0} + \frac{d^{0}\vec{\Omega}^{0}_{r}}{dt}\wedge \vec{O_rM} + \vec{\Omega}^{0}_{r}\wedge(\vec{\Omega}^{0}_{r}\wedge\vec{O_rM})\right] + \left[2\vec{\Omega}^{0}_{r}\wedge\vec{V}^{r}(M)_{/R_r}\right]$$

$$\Rightarrow \vec{\gamma}^{r}(M)_{/R_r} = \frac{d^{r}\vec{V}^{r}(M)_{/R_r}}{dt} = \frac{d}{dt}\begin{pmatrix} -r\dot{\psi}\sin\psi \\ r\dot{\psi}\cos\psi \\ \ddot{h}(t)\end{pmatrix}$$

$$= \begin{pmatrix} -r\dot{\psi}^2\cos\psi \\ -r\dot{\psi}^2\sin\psi \\ \ddot{h}(t)\end{pmatrix}$$

$$\Rightarrow \vec{\gamma}^{0}(O_r)_{/R_0} = \frac{d^{0}\vec{V}(O_r)_{/R_0}}{dt} = \frac{d^{r}\vec{V}(O_r)_{/R_0}}{dt}$$

$$+\vec{\Omega}^{0}_{r}\wedge\vec{V}^{0}(O_r)_{/R_0} = \frac{d}{dt}\begin{pmatrix}0\\0\\\dot{\theta}d\end{pmatrix} + \begin{pmatrix}0\\0\\\dot{\theta}\end{pmatrix}\wedge\begin{pmatrix}0\\0\\\theta d\end{pmatrix}$$

$$= \begin{pmatrix}-\dot{\theta}^2 d\\0\\0\end{pmatrix}$$

$$\frac{d^{0}\vec{\Omega}^{0}_{r}}{dt} = \frac{d^{r}\vec{\Omega}^{0}_{r}}{dt_{/R_r}} + \vec{\Omega}^{0}_{r}\wedge\vec{\Omega}^{0}_{r} = \begin{pmatrix}0\\0\\\ddot{\theta}\end{pmatrix}$$

$$\frac{d^{0}\vec{\Omega}^{0}_{r}}{dt}\wedge\vec{O_rM} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\vec{\Omega}^{0}_{r}\wedge(\vec{\Omega}^{0}_{r}\wedge\vec{O_rM}) = \begin{pmatrix}0\\0\\\dot{\theta}\end{pmatrix}\wedge\begin{pmatrix}-r\dot{\theta}\sin\psi\\r\dot{\theta}\cos\psi\\0\end{pmatrix}$$

$$= \begin{pmatrix}-r\dot{\theta}^2\cos\psi\\-r\dot{\theta}^2\sin\psi\\0\end{pmatrix}$$

$$\Rightarrow 2\vec{\Omega}^{0}_{r}\wedge\vec{V}^{r}(M)_{/R_r} = \begin{pmatrix}0\\0\\2\dot{\theta}\end{pmatrix}\wedge\begin{pmatrix}-r\dot{\psi}\sin\psi\\r\dot{\psi}\cos\psi\\\dot{h}(t)\end{pmatrix}$$

$$= \begin{pmatrix}-2r\dot{\theta}\dot{\psi}\cos\psi\\-2r\dot{\theta}\dot{\psi}\sin\psi\\0\end{pmatrix}$$

$$\vec{\gamma}^{0}(M)_{/R_0} = \begin{pmatrix}-r(\dot{\psi}^2+\dot{\theta}^2)\cos\psi - 2r\dot{\theta}\dot{\psi}\cos\psi - \dot{\theta}^2 d\\-r(\dot{\psi}^2+\dot{\theta}^2)\sin\psi - 2r\dot{\theta}\dot{\psi}\sin\psi\\\ddot{h}(t)\end{pmatrix}$$

$O_r = A$

Ex 2 :

$$\vec{V}^{0}(B)_{/R_0} = \vec{V}^{r}(B)_{/R_r} + \vec{V}^{0}(A)_{/R_0} + \vec{\Omega}^{0}_{r}\wedge_{R_0}\vec{AB}$$

$$\vec{V}^{r}(B)_{/R_r} = \frac{d^{r}\vec{AB}}{dt_{/R_r}} = \frac{d}{dt}\begin{pmatrix}b\cos\beta\\b\sin\beta\\0\end{pmatrix} = \begin{pmatrix}-b\dot{\beta}\sin\\b\dot{\beta}\cos\\0\end{pmatrix}$$

$$\vec{V}^{0}(A)_{/R_0} = \frac{d^{0}\vec{OA}}{dt_{/R_0}} + \vec{\Omega}^{0}_{r}\wedge\vec{OA} = \frac{d}{dt}\begin{pmatrix}a\\0\\0\end{pmatrix} + \begin{pmatrix}0\\0\\\dot{\alpha}\end{pmatrix}\wedge\begin{pmatrix}a\\0\\0\end{pmatrix}$$

$$V^{0}(A)_{/R_0} = \begin{pmatrix}0\\\dot{\alpha}a\\0\end{pmatrix}$$

$$\vec{\Omega}^{0}_{r}\wedge\vec{AB} = \begin{pmatrix}0\\0\\\dot{\alpha}\end{pmatrix}\wedge\begin{pmatrix}b\cos\beta\\b\sin\beta\\0\end{pmatrix} = \begin{pmatrix}-\dot{\alpha}b\sin\beta\\\dot{\alpha}b\cos\beta\\0\end{pmatrix}$$

$$\vec{V}^{0}(B)_{/R_0} = \begin{pmatrix}b(-\dot{\alpha}\sin\beta - \dot{\beta}\sin\beta)\\b(\dot{\alpha}\cos\beta + \dot{\beta}\cos\beta) + \dot{\alpha}a\\0\end{pmatrix}$$

$$\vec{\gamma}^{0}(B)_{/R_0} = \vec{\gamma}^{r}(B)_{/R_r} + \left[\vec{\gamma}^{0}(A)_{/R_0} + \frac{d^{0}\vec{\Omega}^{0}_{r}}{dt_{/R_r}}\wedge\vec{AB}\right.$$

$$\left. + \vec{\Omega}^{0}_{r}\wedge(\vec{\Omega}^{0}_{r}\wedge\vec{AB}_{/R_r})\right] + \left(2\vec{\Omega}^{0}_{r}\wedge\vec{V}^{r}(B)_{/R_r}\right.$$

$$\vec{\gamma}^{r}(B)_{/R_r} = \frac{d^{r}(V(B)_{/R_r})}{dt} = \frac{d}{dt}\begin{pmatrix}-b\dot{\beta}\sin\beta - \\ b\dot{\beta}\cos\beta \\ 0\end{pmatrix}$$

$$= \begin{pmatrix}b\ddot{\beta}^2\cos\beta - b\ddot{\beta}\sin\beta\\b\ddot{\beta}^2\sin\beta + b\ddot{\beta}\cos\beta\\0\end{pmatrix}$$

$$\vec{\gamma}^{0}(A)_{/R_0} = \frac{d^{0}(\vec{V}^{0}(A))}{dt}_{/R_r} = \frac{d^{r}\vec{V}^{0}(A)}{dt} + \vec{\Omega}^{0}_{r}\wedge\vec{V}^{0}(A)_{/R_r}$$

$$= \frac{d}{dt}\begin{pmatrix}0\\\dot{\alpha}a\\0\end{pmatrix} + \begin{pmatrix}0\\0\\\dot{\alpha}\end{pmatrix}\wedge\begin{pmatrix}0\\\dot{\alpha}a\\0\end{pmatrix} = \begin{pmatrix}-\dot{\alpha}^2 a\\\ddot{\alpha}a\\0\end{pmatrix}$$