N.S.A.S.T **2024/2025**
Second year P.C - Analysis 3

SERIE N° 0 : FUNCTIONS OF SEVERAL VARIABLES

Exercice 1 :
Find the domain and range of the following functions :

$$f_1 : (x, y) \longmapsto \frac{x+y}{xy}, \qquad f_2 : (x, y) \longmapsto \sqrt{4 - x^2 - 4y^2}, \qquad f_3 : (x, y) \longmapsto \arccos(x+y).$$

Exercice 2 :
Find the next limits (if it exists). If the limit does not exist, explain why.

$$\lim_{(x,y)\to(-1,1)} \frac{yx^2}{1+xy^2}, \qquad \lim_{(x,y)\to(2,1)} \frac{x-y-1}{\sqrt{x-y}-1}, \qquad \lim_{(x,y,z)\to(0,0,0)} \frac{xy+yz+zx}{x^2+y^2+z^2}.$$

Exercice 3 :
1/ Use the limit definition of partial derivatives to find $f_x = \frac{\partial f}{\partial x}$ and $f_y = \frac{\partial f}{\partial y}$ of the next functions:

$$f : (x, y) \longmapsto x^2 + y^2 - 2xy, \qquad f : (x, y) \longmapsto \sqrt{x+y}, \qquad f : (x, y) \longmapsto \frac{1}{x+y}$$

2/ In the following, evaluate all the partial derivatives at the given point.

$$\begin{array}{lllll} f & : & (x, y) \longmapsto e^y \sin x, & & (x_0, y_0) = (\pi, 0) \\ f & : & (x, y) \longmapsto \arccos(xy), & & (x_0, y_0) = (1, 0) \\ f & : & (x, y, z) \longmapsto \dfrac{xy}{x+y+z}, & & (x_0, y_0, z_0) = (3, -1, 1). \end{array}$$

3/ Find the four second partial derivatives of the next functions:

$$f : (x, y) \longmapsto e^x \tan y, \qquad f : (x, y) \longmapsto \arctan \frac{y}{x}, \qquad f : (x, y, z) \longmapsto e^{-x} \sin(yz).$$

Exercice 4 :
1/ Find the total differential of the functions

$$f : (x, y) \longmapsto x \cos y - y \cos x, \qquad f : (x, y, z) \longmapsto z^2 + e^y \cos x.$$

2/ In the following, find the function f and use the total differential of f to approximate the quantity.

$$i) \quad (2,01)^2 (9.02) - 2^2.9, \qquad ii) \quad \frac{1-(3,05)^2}{(5,95)^2} + \frac{4}{3}, \qquad iii) \quad \sqrt{(5,05)^2 + (3,1)^2} - \sqrt{34}.$$

Exercice 5 :
1/ In the next, find $\frac{dw}{dt}$ using : **a)** by using the appropriate Chain Rule and **b)** by converting w to a function of t before differentiating.

$$\begin{array}{lllll} i)\ w & = & xy, & x = e^t, & y = e^{-2t} \\ ii)\ w & = & xy \cos z, & x = t, & y = t^2, \quad z = \arccos t \end{array}$$

2/ Using the appropriate chain rule, find $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial s}$ in each case.

$$\begin{array}{rclll} i)\ w & = & x^2+y^2, & x=s+t, & y=s-t \\ ii)\ w & = & x^2-y^2, & x=s\cos t, & y=s\sin t. \end{array}$$

Exercice 6 :

Is the function f of classe $\mathcal{C}^2$ at $(0,0)$, where,

$$f(x,y)=\begin{cases} yx^2\sin\frac{y}{x} & \text{if} \quad x\neq 0 \\ 0 & \text{if} \quad x=0 \end{cases}$$

Exercice 7 :

Compute the Taylor formula with order 4 of the function $f:(x,y)\longmapsto\cos\left(x^2+y^2\right)$ at the point $(0,0)$.

Exercice 8 :

For all functions bellow, identify any extrema of the function by recognizing its given form or its form after completing the square. Verify your results by using the partial derivatives to locate any critical points and test for relative extrema.

$f:(x,y)\longmapsto 5-(x-3)^2-(y+2)^2$, $\quad f:(x,y)\longmapsto\sqrt{x^2+y^2+1}$, $\quad f:(x,y)\longmapsto 10x+12y-x^2-y^2-64.$