Series 2: Statics

Ex 1:

$\Sigma F = \Sigma F_x + \Sigma F_y$

a) $\tan\theta = \frac{600}{800} = \frac{3}{4} \Rightarrow \theta = 36{,}86°$

$tg\,\beta = \frac{480}{900} \Rightarrow \beta = 28{,}07°$

$tg\,\gamma = \frac{900}{560} \Rightarrow \gamma = 58{,}1°$

$\Sigma \vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3}$

$$= \begin{cases} F_1 \cos\theta \\ F_1 \sin\theta \end{cases} + \begin{cases} F_2 \sin\beta \\ -F_2 \cos\beta \end{cases} + \begin{cases} -F_3 \cos\gamma \\ -F_3 \sin\gamma \end{cases}$$

b) $\Sigma \vec{F} = \Sigma \vec{F_x} + \Sigma \vec{F_y}$

$$= \begin{cases} F_1 \cos(40) \\ F_1 \sin(40) \end{cases} + \begin{cases} F_2 \cos(70) \\ F_2 \sin(70) \end{cases} + \begin{cases} -F_3 \cos(35) \\ F_3 \sin(35) \end{cases}$$

Ex 2:

$\vec{F}_{net} = \vec{P} + \vec{T_1} + \vec{T_2}$

$$\begin{cases} x: T_1 \sin(15) - T_2 \cos(15) \\ y: -P + T_1 \cos(15) - T_2 \sin(15) \end{cases}$$

$$T_1 = T_2 \frac{\cos(15)}{\sin(15)}$$

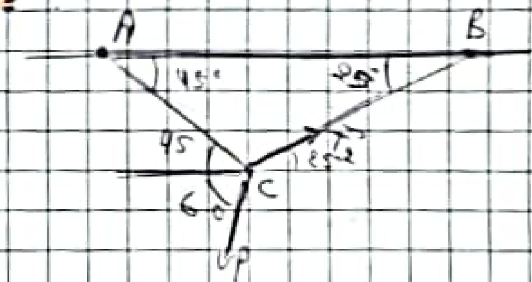
$$-P + T_2 \frac{\cos^2(15)}{\sin(15)} - T_2 \sin(15) = 0$$

$$T_2 \left(\frac{\cos^2(15) - \sin^2(15)}{\sin(15)} \right) = P$$

$$T_2 = \frac{mg \sin(15)}{\cos^2(15) - \sin^2(15)} = 597{,}717\ N$$

$$T_1 = T_2 \frac{\cos(15)}{\sin(15)} = 2230{,}71\ N$$

Ex 1:



$\Sigma \vec{F} = \vec{0}$ $\quad \vec{T_1} + \vec{T_2} + \vec{P} = 0$

$$\begin{cases} -T_1 \cos(45) \\ T_1 \sin(45) \end{cases} + \begin{cases} T_2 \cos(25) \\ T_2 \sin(25) \end{cases} + \begin{cases} -P\cos 60 \\ -P \sin 60 \end{cases}$$

$-\frac{\sqrt{2}}{2} T_1 + T_2 \cos(25) = 250$ — (1)

$\frac{\sqrt{2}}{2} T_1 + T_2 \sin(25) = 500 \sin 60$ (2)

(1) + (2) $\Rightarrow T_2 \cos(25) + T_2 \sin(25) = 500(\sin 60 + \cos 60)$

$$T_2 = \frac{500(\sin 60 + \cos 60)}{\cos(25) + \sin(25)}$$

$T_2 = 513.95\ N$

$T_1 = (-250 + T_2 \cos(25)) \frac{2}{\sqrt{2}}$

$T_1 = 305.18\ N$

Ex 2:

$\Sigma \vec{F} = \vec{0}$

$\vec{T}_{AB} + \vec{T}_{AD} + \vec{T}_{AC} + \vec{P} = \vec{0}$

$A\begin{pmatrix} 0 \\ 480 \\ 0 \end{pmatrix}$ $\quad B\begin{pmatrix} -320 \\ 0 \\ 360 \end{pmatrix}$ $\quad C\begin{pmatrix} 450 \\ 0 \\ 360 \end{pmatrix}$

$D\begin{pmatrix} 250 \\ 0 \\ -360 \end{pmatrix}$

$\vec{AB} = \begin{pmatrix} -320 \\ -480 \\ 360 \end{pmatrix}$ $\quad \vec{AD} = \begin{pmatrix} 250 \\ -480 \\ -360 \end{pmatrix}$

$\vec{AC} = \begin{pmatrix} 450 \\ -480 \\ 360 \end{pmatrix}$

$\|\vec{AB}\| = 680$ $\quad \|\vec{AC}\| = 750$

$\|\vec{AD}\| = 650$

$$\vec{e}_{AB} = \begin{pmatrix} -8/17 \\ -12/17 \\ 9/17 \end{pmatrix} \Rightarrow \vec{T}_{AB} = T_{AB}\begin{pmatrix} -8/17 \\ -12/17 \\ 9/17 \end{pmatrix}$$

$$\vec{e}_{AC} = \begin{pmatrix} 3/5 \\ -16/25 \\ 12/25 \end{pmatrix} \Rightarrow \vec{T}_{AC} = T_{AC}\begin{pmatrix} 3/5 \\ -16/25 \\ 12/25 \end{pmatrix}$$

$$\vec{e}_{AD} = \begin{pmatrix} 5/13 \\ -48/65 \\ -36/65 \end{pmatrix} \Rightarrow \vec{T}_{AD} = \underset{520}{T_{AD}}\begin{pmatrix} 5/13 \\ -48/65 \\ -36/65 \end{pmatrix} = \begin{pmatrix} 200 \\ -384 \\ -288 \end{pmatrix}$$

$$\begin{cases} -\frac{8}{17}T_{AB} \\ -\frac{12}{17}T_{AB} \\ \frac{9}{17}T_{AB} \end{cases} + \begin{cases} 200 \\ -384 \\ -288 \end{cases} + \begin{cases} \frac{3}{5}T_{AC} \\ -\frac{16}{25}T_{AC} \\ \frac{12}{25}T_{AC} \end{cases} + \begin{cases} 0 \\ P \\ 0 \end{cases} = \begin{cases} 0 \\ 0 \\ 0 \end{cases}$$

$$\begin{cases} -\frac{8}{17}T_{AB} + \frac{3}{5}T_{AC} + 200 = 0 \\ -\frac{12}{17}T_{AB} - \frac{16}{25}T_{AC} - 384 + P = 0 \\ \frac{9}{17}T_{AB} + \frac{12}{25}T_{AC} - 288 = 0 \end{cases}$$

$T_{AB} = 484,54\,N$

$T_{AC} = 54,54\,N$

$P = 768\,N$

Ex 3 :

$A\begin{pmatrix} 0 \\ -1,4 \\ 0 \end{pmatrix}$ $B\begin{pmatrix} -0,78 \\ 0 \\ 0 \end{pmatrix}$ $C\begin{pmatrix} 0 \\ 0 \\ 1,2 \end{pmatrix}$

$D\begin{pmatrix} 1,3 \\ 0 \\ 0,4 \end{pmatrix}$ $E\begin{pmatrix} -0,4 \\ 0 \\ -0,86 \end{pmatrix}$

$\vec{F}_{ext} = \vec{0}$

$\vec{P} + \vec{T}_{AC} + \vec{T}_{AD} + \vec{T}_{AE} + \vec{W} = \vec{0}$

$\vec{AC} = \begin{pmatrix} 0 \\ 1,6 \\ 1,2 \end{pmatrix} \Rightarrow \|\vec{AC}\| = 2$

$\vec{AB} = \begin{pmatrix} -0,78 \\ 1,4 \\ 0 \end{pmatrix} \Rightarrow \|\vec{AB}\| = 1,72$

$\vec{AD} = \begin{pmatrix} 1,3 \\ 1,6 \\ 0,4 \end{pmatrix} \Rightarrow \|\vec{AD}\| = 2,1$

$\vec{AE} = \begin{pmatrix} -0,4 \\ 1,6 \\ -0,86 \end{pmatrix} \Rightarrow \|\vec{AE}\| = 1,86$

$\vec{e}_{AC} = \begin{pmatrix} 0 \\ 0,8 \\ 0,6 \end{pmatrix}$ $\vec{e}_{AD} = \begin{pmatrix} 0,62 \\ 0,76 \\ 0,19 \end{pmatrix}$

$\vec{e}_{AE} = \begin{pmatrix} -0,22 \\ 0,86 \\ -0,46 \end{pmatrix}$ $\vec{e}_{AB} = \begin{pmatrix} -0,44 \\ 0,89 \\ 0 \end{pmatrix}$

$\vec{T}_{AC} = T_C\begin{pmatrix} 0 \\ 0,8 \\ 0,6 \end{pmatrix}$

$\vec{T}_{AD} = T\begin{pmatrix} 0,62 \\ 0,76 \\ 0,19 \end{pmatrix}$ $T = P$

$\vec{T}_{AE} = T_E\begin{pmatrix} -0,22 \\ 0,86 \\ -0,46 \end{pmatrix}$

$\vec{P} = P\begin{pmatrix} -0,44 \\ 0,89 \\ 0 \end{pmatrix}$

$$\begin{cases} -0,44T \\ 0,89T \\ 0 \end{cases} + \begin{cases} 0 \\ 0,8T_C \\ 0,6T_C \end{cases} + \begin{cases} 0,62T \\ 0,76T \\ 0,19T \end{cases} + \begin{cases} -0,22T_E \\ 0,86T_E \\ -0,46T_E \end{cases} + \begin{cases} 0 \\ -1000 \\ 0 \end{cases} = \begin{cases} 0 \\ 0 \\ 0 \end{cases}$$

$T_E = 325,13\,N$
$T = 377,15\,N$
$T_C = 122,57\,N$

$$\begin{cases} 0,18T - 0,22T_E = 0 \Rightarrow T = 1,16T_E \\ 1,65T + 0,8T_C + 0,86T_E = 1000 \\ 0,19T + 0,6T_C - 0,46T_E = 0 \end{cases}$$

Ex 4 :

$\Sigma\vec{F} = \vec{0} \Rightarrow \vec{R}_A + \vec{R}_B + \vec{R}_C + \vec{F}_1 + \vec{F}_2 = 0$

$tg\,\alpha = \frac{CD}{AD} = \frac{BH}{AH}$ $\alpha = 26,5°$

$BH = \frac{CD}{AD} \cdot AH = \frac{250}{500} \times 150 = 75\,mm$

$tg\,\theta = \frac{HD}{HB} = \frac{350}{75} \Rightarrow \theta = 77,9°$

$$\begin{cases} 0 + R_{Bx} - R_c = 0 \\ R_{Ay} - R_{By} - F_1 - F_2 = 0 \end{cases}$$

$$\begin{cases} R_B \sin\theta - R_c = 0 \quad \Rightarrow R_c = R_B \sin\theta \\ R_A - R_B \cos\theta - F_1 - F_2 = 0 \end{cases}$$

$$\sum \vec{M}_{/A} = 0 \Rightarrow \vec{AA} \wedge \vec{R_A} + \vec{AB} \wedge \vec{R_B} + \vec{AC} \wedge \vec{R_c} + \vec{AF_1} \wedge \vec{F_1} + \vec{AF_2} \wedge \vec{F_2} = 0$$

$$\begin{cases}150 \\ 75\end{cases} \wedge \begin{cases} R_B \sin\theta \\ -R_B \cos\theta \end{cases} + \begin{cases} 500 \\ 250 \end{cases} \wedge \begin{cases} -R_c \\ 0 \end{cases}$$

$$+ \begin{cases} 100 \\ \times \end{cases} \wedge \begin{cases} 0 \\ -F_1 \end{cases} + \begin{cases} 400 \\ \times \end{cases} \wedge \begin{cases} 0 \\ -F_2 \end{cases} = \begin{cases} 0 \\ 0 \end{cases}$$

$$-150 R_B \cos\theta - 75 R_B \sin\theta + 250 R_c - 100 F_1 - 400 F_2 = 0$$

$$R_B(-150\cos\theta - 75\sin\theta) + 250 R_c - 100 F_1 - 400 F_2 = 0$$

$$-104{,}77 R_B + 250 R_c - 100 F_1 - 400 F_2 = 0$$

$$\begin{cases} -104{,}77 R_B + 250 R_c - 200000 = 0 \\ R_B \sin\theta - R_c = 0 \ / \ 0{,}97 R_B = R_c \\ R_A - R_B \cos\theta - 800 = 0 \end{cases}$$

$$\begin{cases} -104{,}77 R_B + 250 R_c = 200000 \\ 0{,}97 R_B = R_c \\ R_A - 0{,}21 R_B = 800 \end{cases}$$

$R_B = 1452{,}11 N$ $\cancel{\theta = 77{,}3}$

$R_c = 1408{,}59 N$

$\cancel{R_A = 1105{,}79}$

$R_A = 1104{,}94 N$

Ex 5 :

$$\sum \vec{F} = 0 \qquad \vec{T_{AD}} + \vec{R_c} + \vec{F} = 0$$

$$\vec{T_{AD}} = \begin{cases} T \sin 30 \\ -T \cos 30 \end{cases} \qquad \vec{F} = \begin{cases} -F \\ 0 \end{cases}$$

$$\vec{R_{cx}} = \begin{cases} R_{cx} \\ R_{cy} \end{cases}$$

$$\begin{cases} \overset{T_x}{T \sin 30} \\ -T \cos 30 \end{cases} + \begin{cases} R_{cx} \\ R_{cy} \end{cases} + \begin{cases} -F \\ 0 \end{cases} = \begin{cases} 0 \\ 0 \end{cases}$$

$$\sum \vec{M}_{/C} = \vec{0} \qquad \vec{CA} \wedge \vec{T} + \cancel{\vec{CC} \wedge \vec{R_c}} + \vec{CB} \wedge \vec{F} = 0$$

(1) $\rightarrow T_y \cdot (250 \cos 30) - T_x (250 \sin 30) - F(200 \sin 30) = 0$

(2) $\rightarrow \begin{cases} -250 \cos 30 \\ 250 \sin 30 \end{cases} \wedge \begin{cases} T_x \\ -T_y \end{cases} + \begin{cases} 200 \cos 30 \\ -200 \sin 30 \end{cases} \wedge \begin{cases} -F \\ 0 \end{cases} = 0$

$$T_y (250 \cos 30) - T_x (250 \sin 30) - F(200 \sin 30) = 0$$

$$T\, 250 \cos^2 30 - T\, 250 \sin^2 30 - 500 \times 200 \sin 30 = 0$$

$$250\, T (\cos^2 30 - \sin^2 30) = 50000$$

$$\boxed{T = 400 N}$$

$R_{cy} = T \cos 30$ $\qquad$ $R_{cx} = F - T \sin 30$

$\boxed{R_{cy} = 346{,}4 N}$ $\qquad$ $\boxed{R_{cx} = 300 N}$

$\boxed{R_c = 458{,}25 N}$

150 / 150 / 400

Ex 6 :

$$R_{Bx} = 0 \Rightarrow R_B = R_{By}$$

$$\sum F_y = R_A + R_B - 300 = 0 \Rightarrow R_A = 300 - R_B$$

$$\sum \vec{M}_{/B} = 0$$

$$\vec{BC} \wedge \vec{F_1} + \vec{BA} \wedge \vec{R_A} + \vec{BD} \wedge \vec{F_D}$$
$$+ \vec{BB} \wedge \vec{R_B} + \vec{BB} \wedge \vec{F_3} = 0$$

$$\begin{Bmatrix} -900 \\ 0 \end{Bmatrix} \wedge \begin{Bmatrix} 0 \\ -50 \end{Bmatrix} + \begin{Bmatrix} d-900 \\ 0 \end{Bmatrix} \wedge \begin{Bmatrix} 0 \\ R_A \end{Bmatrix}$$

$$+ \begin{Bmatrix} -450 \\ 0 \end{Bmatrix} \wedge \begin{Bmatrix} 0 \\ -100 \end{Bmatrix} = 0$$

$$45000 + (d-900) R_A + 45000 = 0$$

$$(d-900) R_A = -90000$$

$$d R_A - 900 R_A = -90000$$

$$d = \frac{900 R_A - 90000}{R_A} \qquad R_A \leq 180N$$

$$d \leq 400$$

$$\sum \vec{M_A} = 0$$

$$R_A = 300 - R_B$$

$$d = \frac{900(300 - R_B) - 90000}{(300 - R_B)}$$

$$R_B \leq 180$$

$$d \geq 150$$

$$\text{so } 150 < d \leq 400$$

Ex 7:

$$\sum \vec{F_{ext}} = \vec{0}$$

$$\vec{F} + \vec{T_D} + \vec{T_E} + \vec{R_A} = 0$$

$$\begin{Bmatrix} 0 \\ -F \\ 0 \end{Bmatrix} + \begin{Bmatrix} -T_D\,0,54 \\ T_D\,0,63 \\ T_D\,0,54 \end{Bmatrix} + \begin{Bmatrix} -0,54 T_E \\ 0,63 T_E \\ -0,54 T_E \end{Bmatrix} + \begin{Bmatrix} R_x \\ R_y \\ R_z \end{Bmatrix} = 0$$

$$BD = \begin{pmatrix} -1,8 \\ 2,1 \\ 1,8 \end{pmatrix} \rightarrow \|\vec{BD}\| = 3,3$$

$$BE = \begin{pmatrix} -1,8 \\ 2,1 \\ -1,8 \end{pmatrix} \rightarrow \|\vec{BE}\| = 2,1$$

$$e_{BD} = \begin{pmatrix} -0,54 \\ 0,63 \\ 0,54 \end{pmatrix} \qquad e_{BE} = \begin{pmatrix} -0,54 \\ 0,63 \\ -0,54 \end{pmatrix}$$

$$\sum \vec{M}_{/A} = \vec{AB} \wedge \vec{T_D} + \vec{AB} \wedge \vec{T_E} + \vec{AC} \wedge \vec{F}$$

$$\begin{Bmatrix} 1,8 \\ 0 \\ 0 \end{Bmatrix} \wedge \begin{Bmatrix} -0,54 T_D \\ 0,63 T_D \\ 0,54 T_D \end{Bmatrix} + \begin{Bmatrix} 1,8 \\ 0 \\ 0 \end{Bmatrix} \wedge \begin{Bmatrix} -0,54 T_E \\ 0,63 T_E \\ -0,54 T_E \end{Bmatrix}$$

$$+ \begin{Bmatrix} 3 \\ 0 \\ 0 \end{Bmatrix} \wedge \begin{Bmatrix} 0 \\ -F \\ 0 \end{Bmatrix}$$

$$\begin{cases} 0 + 0 + 0 \\ -1,8 \times 0,54 T_D + 1,8 \times 0,54 T_E + 0 = 0 \\ 1,8 \times 0,63 T_D + 1,8 \times 0,63 T_D + 3F = 0 \end{cases} \qquad T_E = T_D$$

$$2 \times 1,8 \times 0,63 T_D = 3F$$

$$\boxed{T_D = T_E = 5291\ N}$$

$$-1,08 T_D + R_x = 0$$

$$R_x = 1,08 T_D$$

$$\boxed{R_x = 5714,28N}$$

$$1,26 T_D + R_y - F = 0$$

$$R_y = F - 1,26 T_D =$$

$$\boxed{R_y = -2666,66N}$$

$$\boxed{R_z = 0} \qquad \boxed{R_A = 6305,87N}$$

Ex 8 :

$$\sum \vec{F} = \vec{0}$$

$$\vec{T_1} + \vec{T_2} + \vec{T_3} + \vec{T_4} + \vec{R_A} + \vec{R_D} = 0$$

$$\begin{cases} -T_1 \cos 30 \\ T_1 \sin 30 \end{cases} + \begin{cases} -T_2 \\ 0 \end{cases} + \begin{cases} -T_3 \sin 10 \\ -T_3 \cos 10 \end{cases}$$

$$+ \begin{cases} -T_4 \sin 10 \\ -T_4 \cos 10 \end{cases} + \begin{cases} R_{Ay} \\ R_{Az} \end{cases} + \begin{cases} R_{Dy} \\ R_{Dz} \end{cases} = \begin{cases} 0 \\ 0 \end{cases}$$

$$\sum \vec{M}_{/A} = 0$$

$$\vec{AB_1} \wedge T_1 + \vec{AB_2} \wedge \vec{T_2} + \vec{AC_1} \wedge \vec{T_3} + AC_2 \wedge \vec{T_4}$$
$$+ \vec{AD} \wedge \vec{R_D} = 0$$

$$\vec{AB_1} = \vec{AB} + \vec{BB_1} = \begin{pmatrix} 225 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ r\sin 20 \\ r\cos 20 \end{pmatrix}$$

$$\vec{AB_2} = \vec{AB} + \vec{BB_2} = \begin{pmatrix} 225 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -r \end{pmatrix}$$

$$\vec{AC_1} = \vec{AC} + \vec{CC_1} = \begin{pmatrix} 450 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ r\cos 10 \\ r\sin 10 \end{pmatrix}$$

$$\vec{AC_2} = \vec{AC} + \vec{CC_2} = \begin{pmatrix} 450 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -r\cos 10 \\ -r\sin 10 \end{pmatrix}$$

$$\vec{AD} = \begin{pmatrix} 680 \\ 0 \\ 0 \end{pmatrix}$$