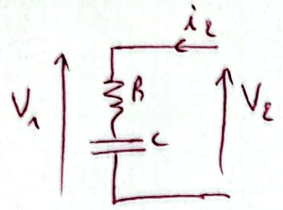
Ex 1:

1) The impedance matrix:

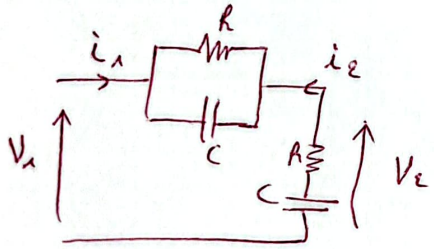
* Suppose that $i_1 = 0$:



$$V_1 = V_2 = \left(R + \frac{1}{jwC}\right) i_2$$

$$z_{12} = \left.\frac{V_1}{i_2}\right|_{i_1=0} \qquad z_{22} = \left.\frac{V_2}{i_2}\right|_{i_1=0}$$

* Suppose that $i_2 = 0$:



$$V_1 = \left[\left(jwC + \frac{1}{R}\right)^{-1} + R + \frac{1}{jwC}\right] i_1$$

$$V_2 = \left(R + \frac{1}{jwC}\right) i_2$$

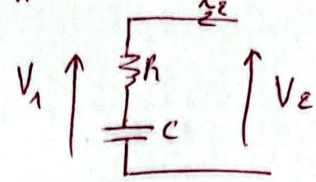
$$z_{11} = \left.\frac{V_1}{i_1}\right|_{i_2=0} \qquad z_{21} = \left.\frac{V_2}{i_1}\right|_{i_2=0}$$

$$\Rightarrow Z = \begin{bmatrix} \frac{R}{1+jwRC} + R + \frac{1}{jwC} & R + \frac{1}{jwC} \\ R + \frac{1}{jwC} & R + \frac{1}{jwC} \end{bmatrix}$$

(0,5) (0,5) (0,5) (0,5)

2) The hybrid matrix:

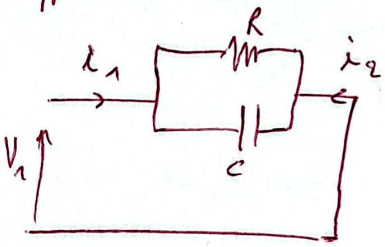
* Suppose that $i_1 = 0$



$$V_1 = V_2 = \left(R + \frac{1}{jwC}\right) i_2$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{i_1=0} \qquad h_{22} = \left.\frac{i_2}{V_2}\right|_{i_1=0}$$

* Suppose that $V_2 = 0$:



$$i_1 = -i_2$$

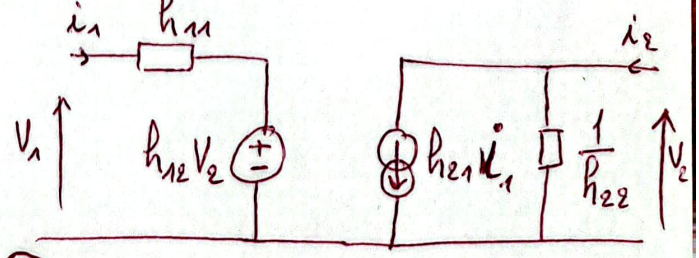
$$V_1 = \left(jwC + \frac{1}{R}\right)^{-1} i_1$$

$$h_{11} = \left.\frac{V_1}{i_1}\right|_{V_2=0} \qquad h_{21} = \left.\frac{i_2}{i_1}\right|_{V_2=0}$$

$$\Rightarrow H = \begin{bmatrix} \frac{R}{1+jwRC} & 1 \\ -1 & \frac{jwC}{1+jwRC} \end{bmatrix}$$

(2)

3)



(1,5)

4) The voltage gain:

We have:

$$V_1 = h_{11} i_1 + h_{12} V_2$$
$$i_2 = h_{21} i_1 + h_{22} V_2$$

$$\Rightarrow i_1 = \frac{1}{h_{21}} i_2 - \frac{h_{22}}{h_{21}} V_2$$

We know that: $V_2 = -R_L i_2 \Rightarrow i_2 = \frac{-V_2}{R_L}$

$$\Rightarrow V_1 = -\frac{h_{11}}{h_{21}} \frac{V_2}{R_L} - \frac{h_{11} h_{22}}{h_{21}} V_2 + h_{12} V_2$$

$$\boxed{\frac{V_2}{V_1} = \frac{h_{21} R_L}{h_{12} h_{21} R_L - h_{11} h_{22} R_L - h_{11}}}$$

(2)

$$\boxed{\frac{V_2}{V_1} = \frac{Z_{21} R_L}{R_L Z_{11} + Z_{11} Z_{22} - Z_{12} Z_{21}}}$$

$$\boxed{\frac{V_2}{V_1} = \frac{Z_B R_L}{Z_A R_L + Z_A Z_B + Z_B R_L}}$$

$$Z_A = \frac{R}{1 + j\omega RC}$$

$$Z_B = R + \frac{1}{j\omega C}$$