

SERIE N° 2 : FOURIER SERIES

Exercise 1 :

Determine the Fourier series of the 2π -periodic function f defined on $]-\pi, \pi]$ by :

1/ $f : x \mapsto e^x$. 2/ $f : x \mapsto |\sin x|$.

Exercise 2 :

Let $\alpha, \beta, \gamma \in \mathbb{R}$, we denote by f the even and 2π -periodic function defined on $[0, \pi]$ by :

$$f : x \mapsto \alpha x^2 + \beta x + \gamma.$$

1/ Is the function f developable in Fourier series ? In which case, does this series converge uniformly on \mathbb{R} ?

2/ Show that we can determine α, β, γ so that the Fourier series of f reduces to

$$\sum_{n \in \mathbb{N}^*} \frac{\cos nx}{n^2}.$$

3/ Deduct the sum $\sum_{n=1}^{+\infty} \frac{1}{n^2}$.

Exercise 3 :

Let f the 2π -periodic function defined by

$$f : x \mapsto \begin{cases} \cos x, & \text{if } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \\ 0 & \text{if } \frac{\pi}{2} \leq x \leq \frac{3\pi}{2} \end{cases}$$

1/ Draw the graph of the restriction of f to the interval $[-\frac{3\pi}{2}, \frac{3\pi}{2}]$.

2/ Calculate the sum $\sum_{n=0}^{+\infty} \frac{(-1)^n}{4n^2 - 1}$.

Exercise 4 :

Form the Fourier series expansion of the following 2π -periodic functions and deduce for each the indicated sums :

1/ The even function defined on $[0, \pi[$ by $f(x) = 1 - \frac{2x}{\pi}$ and $\sum_{n=0}^{+\infty} \frac{1}{(2n+1)^2}$, $\sum_{n=0}^{+\infty} \frac{1}{(2n+1)^4}$, $\sum_{n=1}^{+\infty} \frac{1}{n^4}$.

2/ The function defined on $[-\pi, \pi]$ by $f(x) = \cosh(\alpha x)$ ($\alpha > 0$) and $\sum_{n=0}^{+\infty} \frac{1}{n^2 + \alpha^2}$, $\sum_{n=0}^{+\infty} \frac{(-1)^n}{n^2 + \alpha^2}$.

Exercise 5 :

Let the function $f : x \mapsto \sum_{n=1}^{+\infty} \frac{\sin nx}{\sqrt{n}}$.

1/ Show that the function f is well defined on \mathbb{R} , 2π -periodic and continuous on $\mathbb{R} - 2\pi\mathbb{Z}$.

2/ Deduce, using Parseval's equality, that $\sum_{n \in \mathbb{N}^*} \frac{\sin nx}{\sqrt{n}}$ is not the Fourier series of a Riemann integrable 2π -periodic function.