ELECTRICAL FILTERS

INTRODUCTION:

An electrical filter is any **Two-Port** (four-terminal) network capable of selecting the frequencies of a given signal. Such a circuit consists of basic components such as resistors, inductors, and capacitors. The combination of these elements determines the range of frequencies to be selected, meaning that all frequencies outside this range will be significantly attenuated.

The transmission factor **β** is defined as the ratio of the output voltage $\mathbf{V_o}$ to the input voltage $\mathbf{V_i}$:

$$\beta = \frac{v_o}{v_i}$$

The graph representing the variation of **β** as a function of the input signal frequency **f** is shown in **Figure 1** for an ideal filter. The frequencies $\mathbf{f_1}$ and $\mathbf{f_2}$ are called **cutoff frequencies**.

I STUDY OF A REAL FILTER

In practice, it is impossible for a real filter to achieve the ideal response curve shown in **Figure 1**.

Figure 2 represents the response curve of a real filter. It can be observed that a real filter has two main imperfections compared to an ideal filter:

- The transmission factor is **not zero** outside the selected frequency range.
- The transition between the selected frequency range and the other frequencies does **not occur abruptly** but rather gradually.

The **cutoff frequency** is then defined at **-3 dB**. It is the frequency $\mathbf{f_c}$ for which:

β_{max} is the maximum transmission factor.

Calculation of β:

Consider the real filter shown in Figure 3. The input voltage V_i is generated by a sinusoidal generator with an adjustable frequency.

$v_i = V_i \cos w\, t$ et $v_o = V_o \cos(wt + \phi)$

Where ω = 2πf, with f being the generator's frequency.

Applying Kirchhoff's second law, or mesh analysis, we obtain:

$$v_i = Zi_1 + Z'(i_1 + i_2)$$
$$Z'(i_1 + i_2) = -Zi_2 - R_L i_2$$
$$v_o = -R_L i_2$$

From this, we deduce:

$$\beta = \frac{v_o}{v_i} = \frac{Z' R_L}{(Z+Z')R_L + Z(2Z'+Z)}$$

II STUDY OF THE LOW-PASS FILTER

Consider the circuit in **Figure 4a**. Based on the expression of β, we can deduce the equation for the **transmission factor**:

$$\beta = \frac{R_L}{(R_L + 2R) + 2jRC\omega(R_L + R)}$$

The **magnitude** will be: $|\beta| = \frac{R_L}{\sqrt{(R_L+2R)^2+4R^2C^2\omega^2(R_L+R)^2}}$

The **phase shift** will be: $\phi = -arctg(\frac{2RC\omega(R_L+R)}{R_L+2R})$

The **cutoff frequency at -3 dB** is: $f_c = \frac{R_L+2R}{4\pi RC(R_L+R)}$

For a **low-pass filter**, the **transmission factor** is **maximum at low frequencies** and tends to **zero at high frequencies**.

III STUDY OF THE HIGH-PASS FILTER

Consider the circuit in **Figure 4b**. Based on the expression of β, we can deduce the equation for the **transmission factor**:

$$\beta = \frac{-RR_LC^2\omega^2}{2(1 - \frac{R_LRC^2\omega^2}{2} + jC\omega(R_L + R))}$$

The **magnitude** will be: $|\beta| = \frac{RR_LC^2\omega^2}{2\sqrt{(1-\frac{R_LRC^2\omega^2}{2})^2+C^2\omega^2(R_L+R)^2}}$

The **phase shift** will be: $\phi = -arctg(\frac{C\omega(R_L+R)}{1-\frac{R_LRC^2\omega^2}{2}})$

The **cutoff frequency at -3 dB** is: $f_c = \frac{1}{\pi RR_LC}\sqrt{\frac{(R^2+R_L^2+RR_L)+\sqrt{\Delta}}{2}}$

With: $\Delta = (R^2 + R_L^2 + RR_L)^2 + R^2R_L^2$

For a **high-pass filter**, the **transmission factor** is **maximum at high frequencies** and tends to **zero at low frequencies**.

IV STUDY OF THE BAND-STOP FILTER

Consider the circuit in **Figure 4c**. A **band-stop filter** is obtained by connecting a **low-pass filter** and a **high-pass filter** in parallel. Based on the expression of β, we can deduce the equation for the **transmission factor**:

$$\beta = \frac{1 - R^2C^2\omega^2}{1 - R^2C^2\omega^2 + 2\frac{R}{R_L} + 2jRC\omega(2 + \frac{R}{R_L})}$$

The **magnitude** will be:

$$|\beta| = \frac{|1 - R^2C^2\omega^2|}{\sqrt{(1 - R^2C^2\omega^2 + 2\frac{R}{R_L})^2 + 4R^2C^2\omega^2(2 + \frac{R}{R_L})^2}}$$

The **phase shift** will be:

$$\phi = -arctg\left(\frac{2RC\omega(2+\frac{R}{R_L})}{1-R^2C^2\omega^2+2\frac{R}{R_L}}\right)$$

For **low frequencies**, the **band-stop filter** behaves like a **low-pass filter**, whereas for **high frequencies**, it behaves like a **high-pass filter**. However, by analyzing the expression of the transmission factor, we observe that there exists a specific frequency $\mathbf{f_0}$ at which the transmission factor is **minimal**. This frequency is called the **notch** and is given by:

$$f_0 = \frac{1}{2\pi RC}$$

Additionally, the **band-stop filter** is characterized by a **stop band (rejection bandwidth)**:
$B = f_2 - f_1$

where $\mathbf{f_1}$ and $\mathbf{f_2}$ are the **cutoff frequencies at -3 dB**.

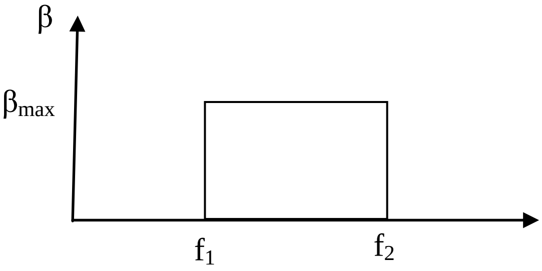

Figure 1

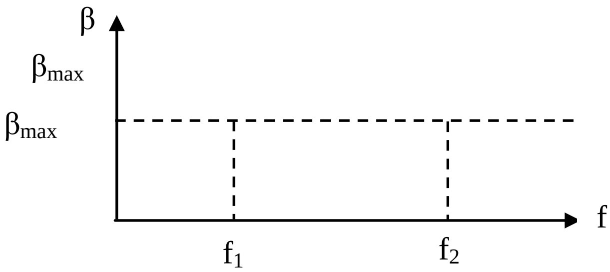

Figure 2

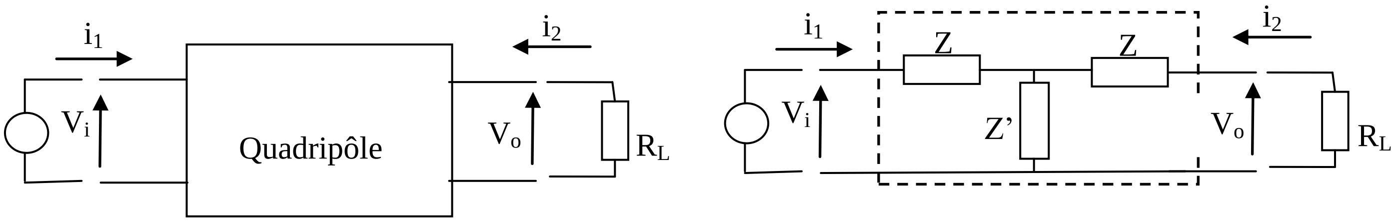

Figure 3

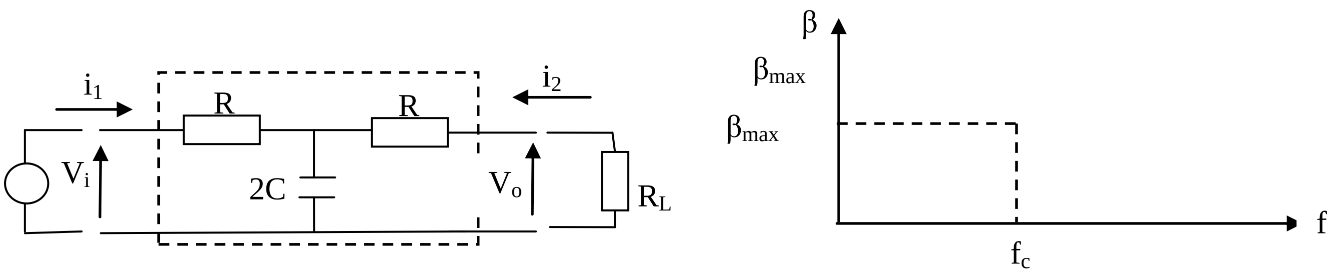

Figure 4a : Low-pass filter

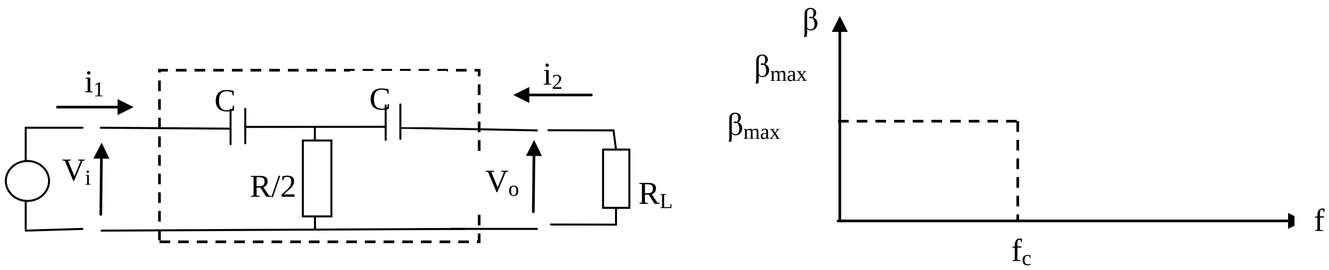

Figure 4b : High-pass filter

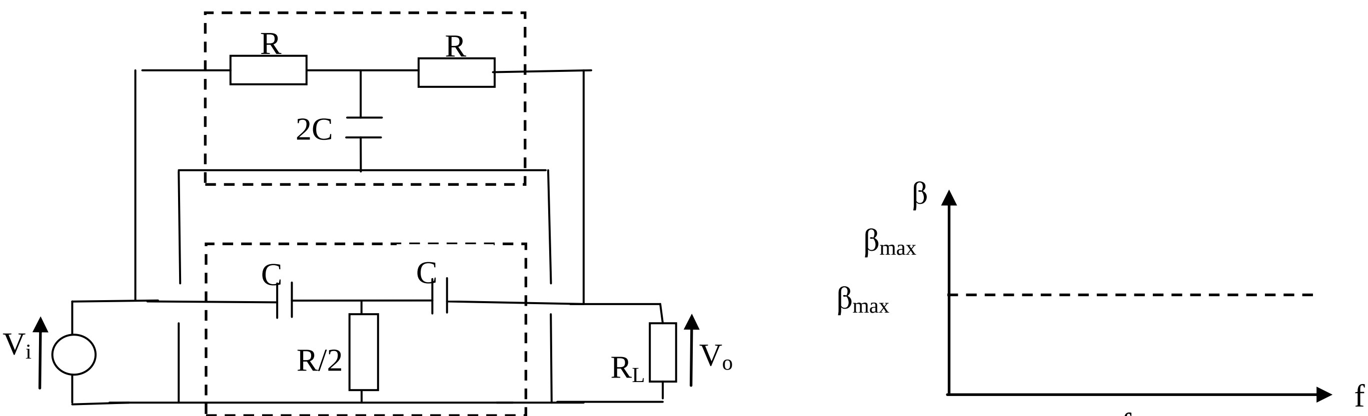

Figure 4c : Band-stop filter