Chapter 3
The three-phase electric power

Introduction

Three-phase electric power definition

- Includes three single phase,
- Judicious choice of RMS values and phase lags,
- Choix judicieux de la topologie (étoile -triangle)

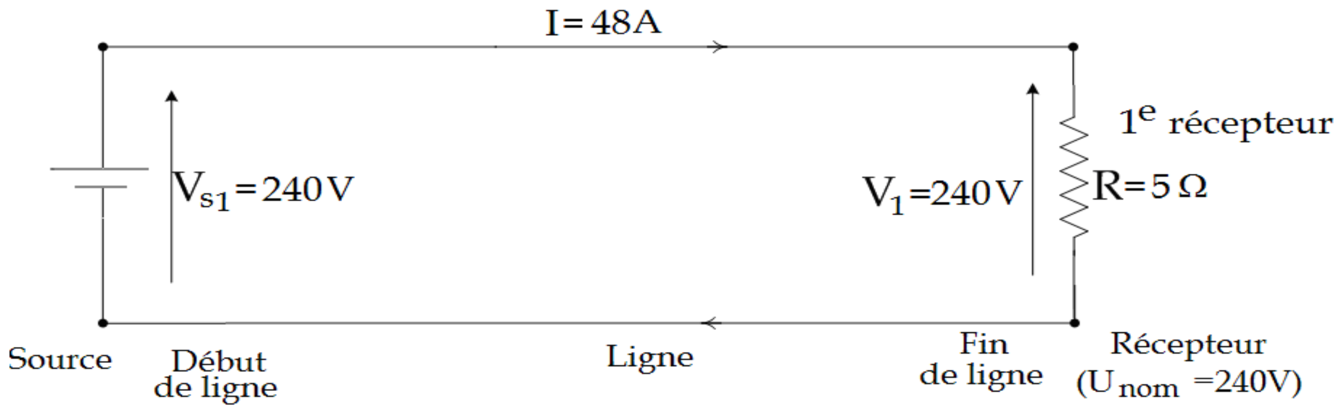
Benefits of the three-phase system

- Transportation is more cost-effective:
  - The yield is better,
  - The cost of wires is lower.
- Three-phase devices (three-phase motors) offer much higher performance than conventional motors

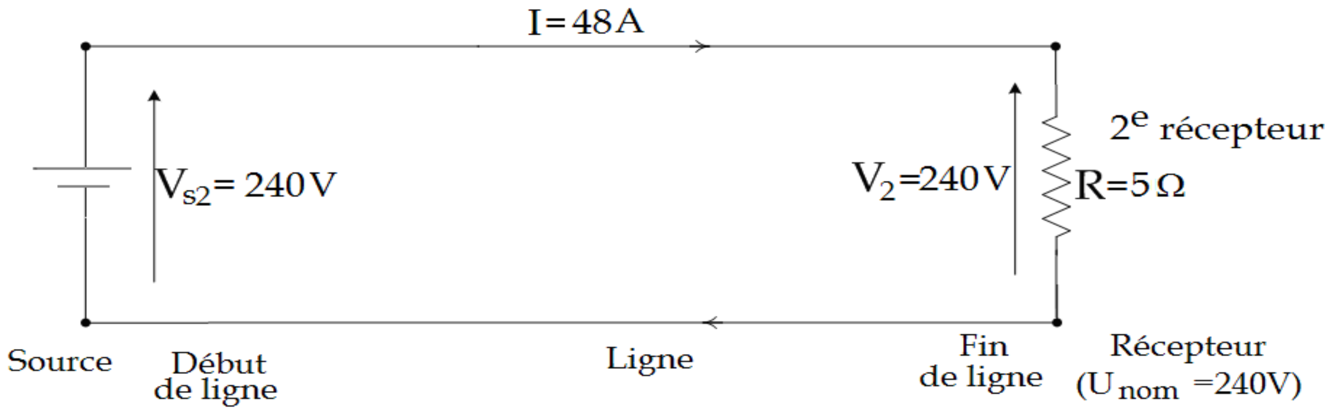
Introduction

Example in DC: Two disjointed networks

Let two DC sources supplying two receivers.



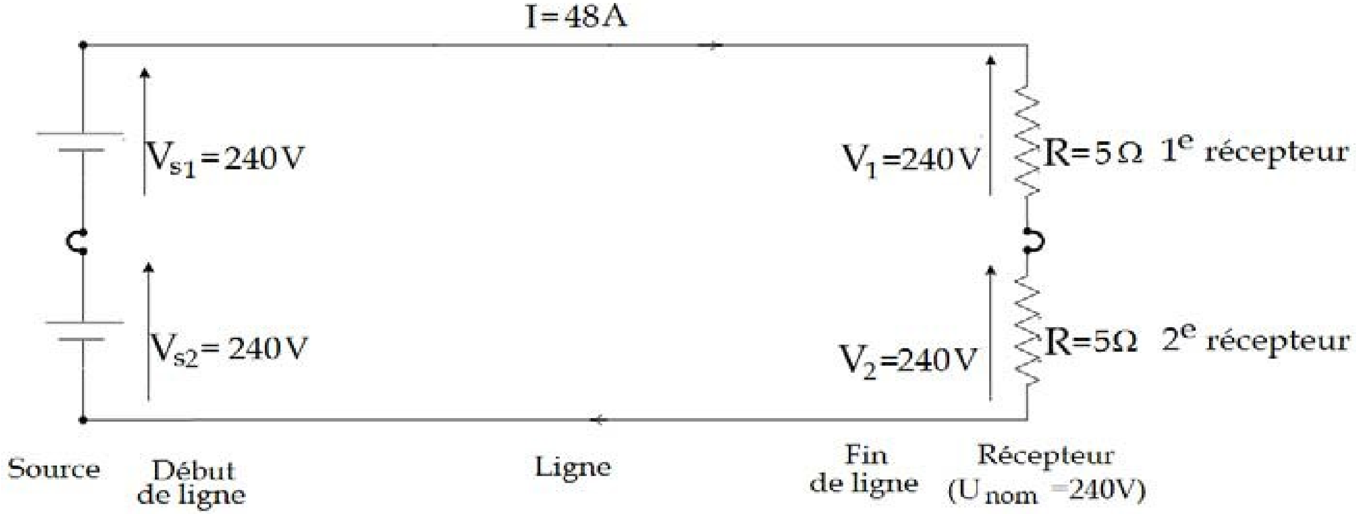
- Voltage at the receiver = 240 V
- Nbre of wires = 4
- Each wire must be capable to transport a current of 48 A.



Introduction

Example in DC: Another topology.

We propose a new topology:

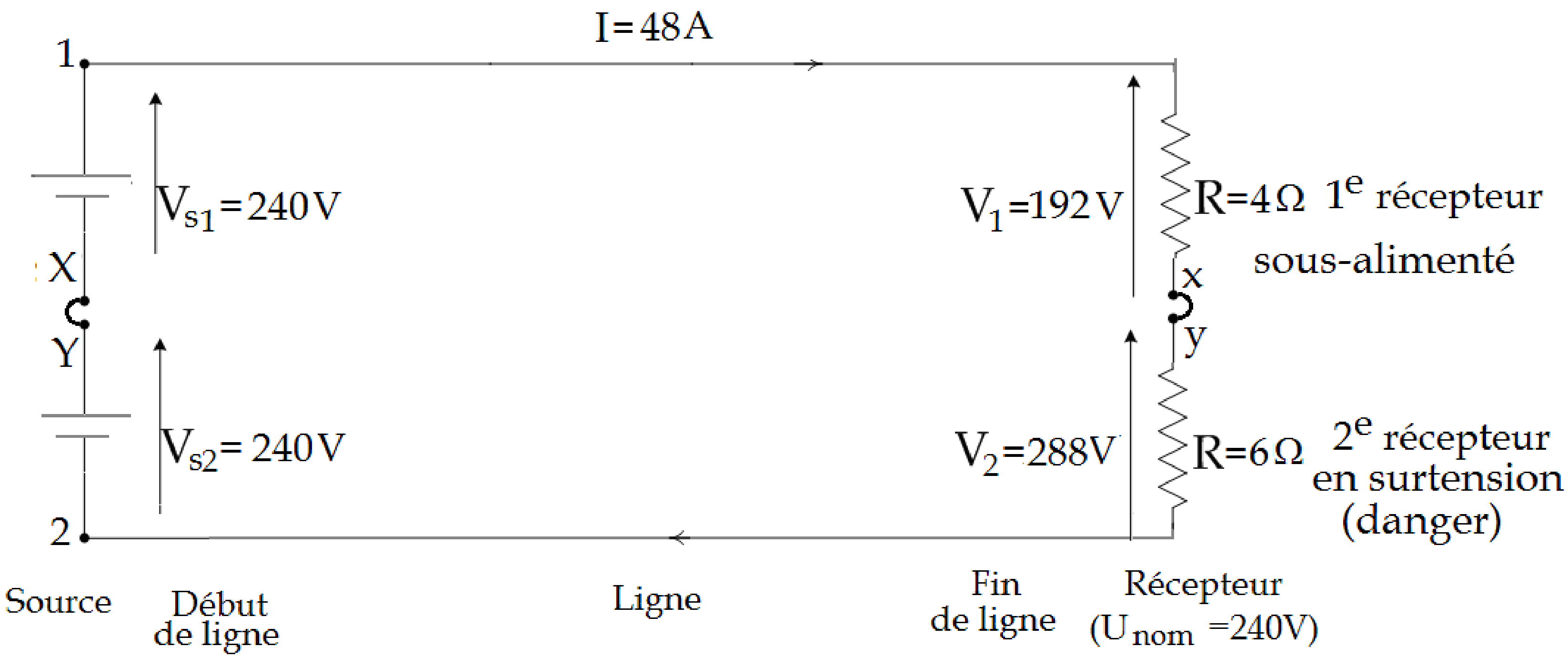


- The voltage at the receiver remains the same,
- We save 02 wires!!!
- Remaining wires transport the same current (48 A)

Is the battle won? NO, because the receivers may be different! What happens in this case?

Introduction

Example in DC: Different Receivers

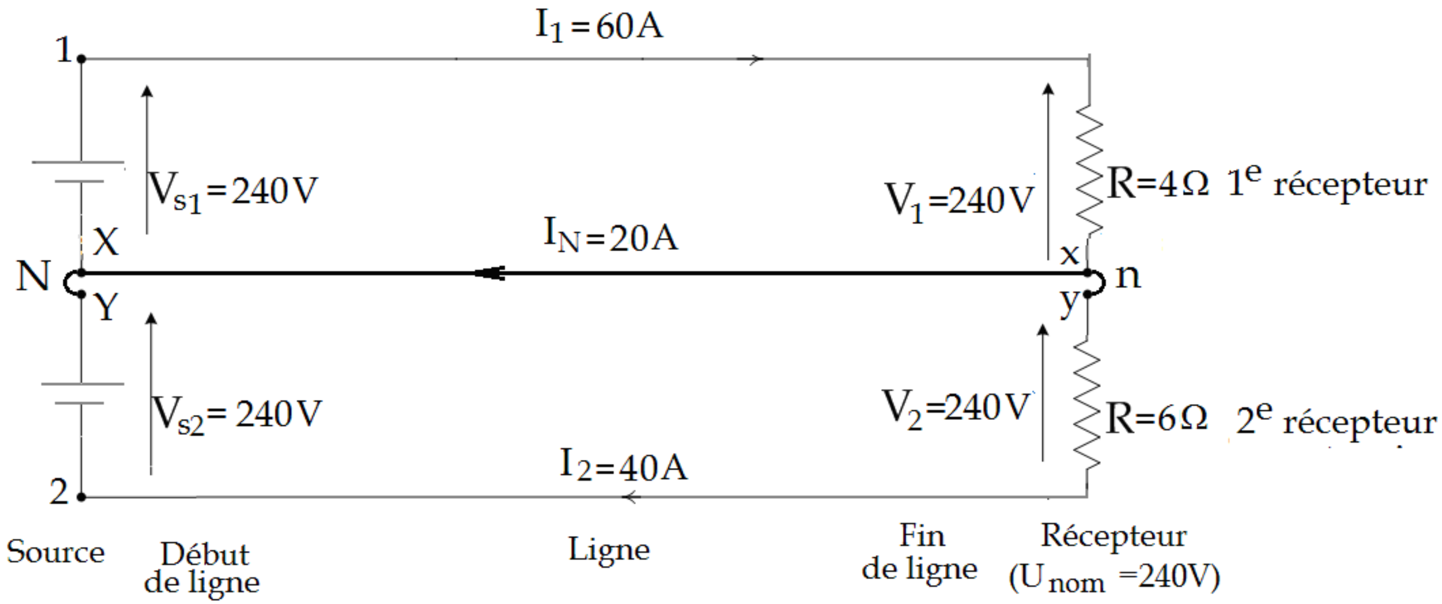


- In this example the current doesn't change (48 A)
- The first receiver (4Ω) is under-powered (192 V),
- The second receiver (6Ω) is over-powered (288 V),
- Solution? Use one (01) supplementary wire !!!

Introduction

Example in DC: Different receivers

- The use of supplementary wire known as ***«Neutral»***



- We connect point ***«N»*** consisting of the connexion ***X-Y*** in the side of sources to the point ***«n»*** in the side of receiver consisting of the connexion ***x-y***.
- => Receivers become again independant,
- => Voltages at the receivers are correct now

Introduction

Example in DC: Different Receivers

- A current is flowing in the Neutral wire $I_N = I_1 - I_2$. Is it a coincidence?
- NO, if one of the two generators was inversed, we would get a sum of currents $I_N = I_1 + I_2 = 100A$,
- Then, the choice for the direction of the power supply is done on purpose,
- In an electric network, we will have necessarily $V_{s1} = V_{s2}$

In conclusion:

$$(V_2 - V_N) = -(V_1 - V_N)$$

Introduction

Example with two sine sources

- In order to minimis the current I_N, at every instant t, voltages must be in opposit phase:

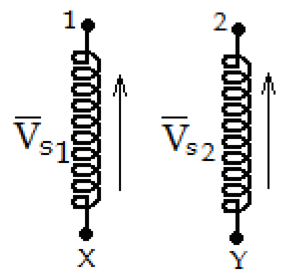
$$v_1(t)-v_X(t)=-\left(v_2(t)-v_Y(t)\right)$$

- and then, by eliminating the term $\sqrt{2}e^{j\omega t}$:

$$\tilde{V}_{s1}=\left(\tilde{V}_1-\tilde{V}_X\right)=\tilde{V}_s e^{j\varphi}$$

$$\tilde{V}_{s2}=\left(\tilde{V}_2-\tilde{V}_Y\right)=-\tilde{V}_s e^{j\varphi}=\tilde{V}_s e^{j(\varphi+\pi)}$$

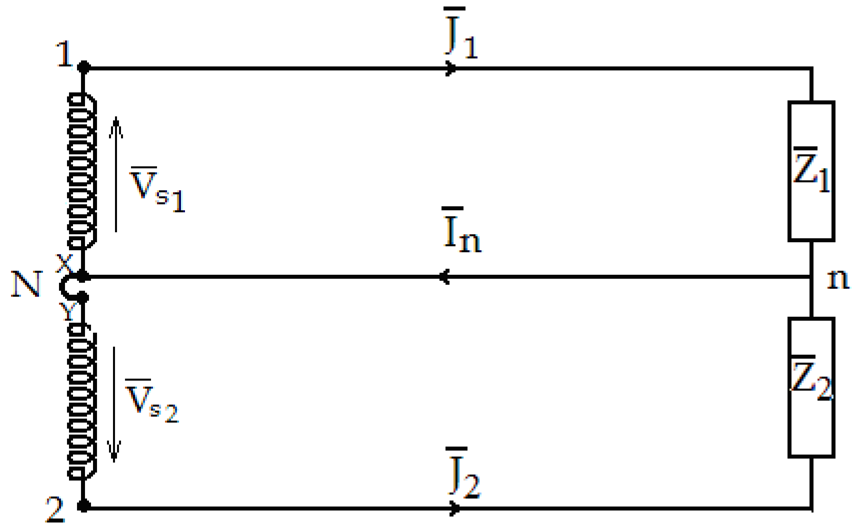
- We use the following symboles to represent the alternators constituing the two sources:



Introduction

Exemple with two sine sources

- We, then, obtain the following network:



- The currents are:

$$\tilde{I}_1 = \frac{\tilde{V}_{s1}}{\tilde{Z}_1} \quad \text{et} \quad \tilde{I}_2 = \frac{\tilde{V}_{s2}}{\tilde{Z}_2}$$

- and then:

$$\tilde{I}_n = \tilde{I}_1 + \tilde{I}_2 = \frac{\tilde{V}_{s1}}{\tilde{Z}_1} + \frac{\tilde{V}_{s2}}{\tilde{Z}_2} = \tilde{V}_{s1}\left(\frac{1}{\tilde{Z}_1} - \frac{1}{\tilde{Z}_2}\right)$$

- If $\tilde{Z}_1 = \tilde{Z}_2 = \tilde{Z}$ then $\tilde{I}_n = 0$ => neutral wire becomes useless.
- What if we use three sources instead of two? => three-phase

Definitions and general principles

Three-phase source or Three-phase alternator

- We propose three sources such as:
  - Amplitudes are equal,
  - The phases are equally distributed => $\Delta\varphi=2\pi/3$
- = >We, then, obtain a **balanced** source:

$$\begin{cases} \tilde{V}_{s1}=V_s e^{j\varphi} \\ \tilde{V}_{s2}=V_s e^{j\varphi} e^{-j2\pi/3} \\ \tilde{V}_{s3}=V_s e^{j\varphi} e^{+j2\pi/3} \end{cases}$$

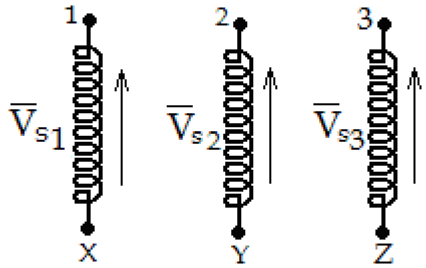
- The source is said ***balanced*** because the amplitudes are equal and the vectors show a **symetric star** in the complex plan.
- In this course, an alternator will be always considered to be **balanced**.
- In addition, we consider $\varphi=0$

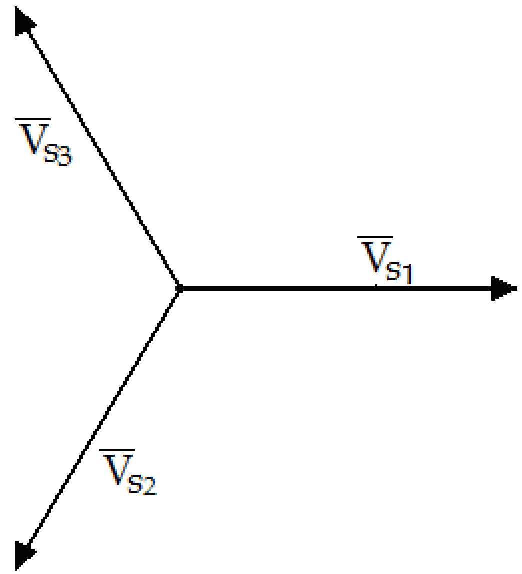
Definitions and general principles

Three-phase source or Three-phase alternator

We, then, obtain:

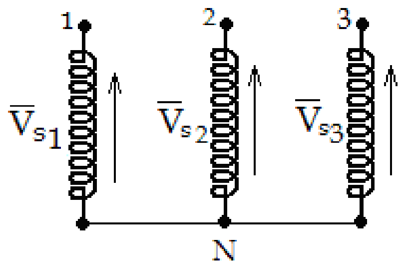
$$\begin{cases} \tilde{V}_{s1} = V_s \\ \tilde{V}_{s2} = V_s e^{-j2\pi/3} \\ \tilde{V}_{s3} = V_s e^{+j2\pi/3} \end{cases}$$



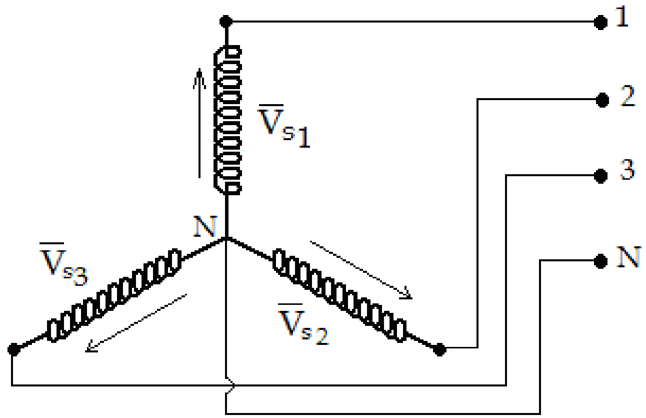


When we link the points ***X***, ***Y*** et ***Z*** in one single point called neutral ***N*** , we obtain ***Star Connection***:

- Receivers are connected to terminals 1,2,3 and N. **How?**



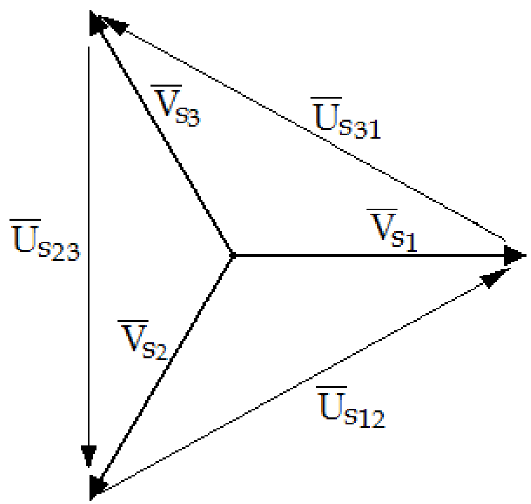
$\Leftrightarrow$



Definitions and general principles

Simple and compound voltages

- **Simple Voltages:**
  - The voltage at alternator terminals.
  - The voltage between a source (a phase) and the neutral.
  - We are talking about $\tilde{V}_{s1}$, $\tilde{V}_{s2}$ and $\tilde{V}_{s3}$
- **Compound Voltages:**
  - The voltage between two sources (two phases):
  - $\tilde{U}_{s12}=\tilde{V}_{s1}-\tilde{V}_{s2}=U_s e^{j\pi/6}$
  - $\tilde{U}_{s23}=\tilde{V}_{s2}-\tilde{V}_{s3}=U_s e^{-j\pi/2}$
  - $\tilde{U}_{s31}=\tilde{V}_{s3}-\tilde{V}_{s1}=U_s e^{j5\pi/6}$
  - We show that $U_s=\sqrt{3}V_s$. For instance $V_s=220V$ => $U_s=380V$
- The compound voltages system is:
  - Balanced (same amplitude)
  - Symetric (same phase lag)

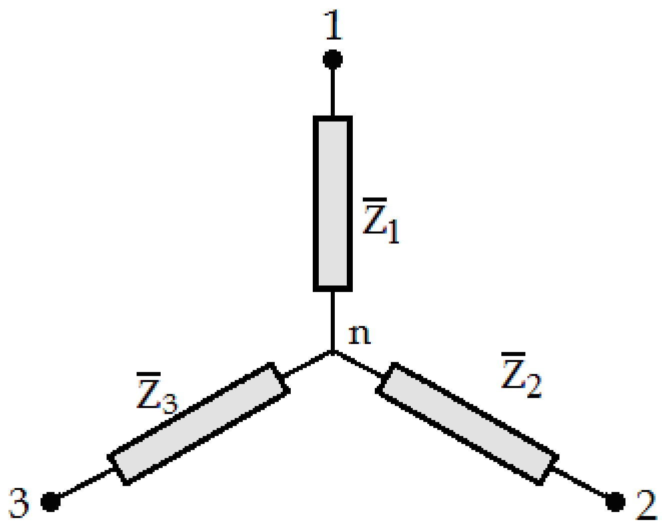


Definitions and general principles

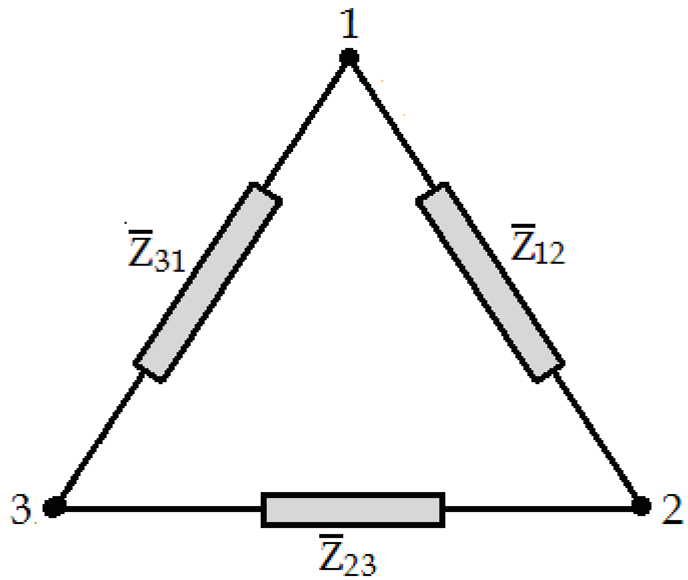
Three-phase Receiver

We have 03 receivers to connect. We denote those receivers as ($\tilde{Z}_1$, $\tilde{Z}_2$ et $\tilde{Z}_3$) or ($\tilde{Z}_{12}$, $\tilde{Z}_{23}$ et $\tilde{Z}_{31}$) We can:

- Either connect between $1-N$, $2-N$ et $3-N$ => ***Star Connection***,
- soit les connecter entre $1-2$, $2-3$ et $3-1$ => ***Delta Connection***.



Star Connection (topology)
Needs 04 wires

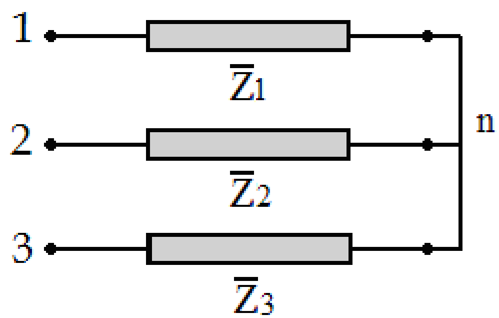


Delta Connection (topology)
Needs 03 wires

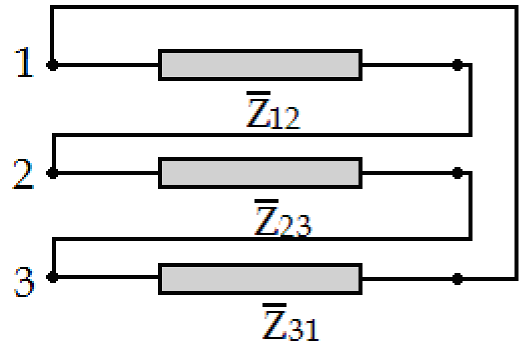
Definitions and general principles

Three-phase Receiver

- Another representation:



Star Connection (topology)



Delta Connection (topology)

- We say that the load is balanced if receivers ($\tilde{Z}_1$, $\tilde{Z}_2$ et $\tilde{Z}_3$) or ($\tilde{Z}_{12}$, $\tilde{Z}_{23}$ et $\tilde{Z}_{31}$) are identical.

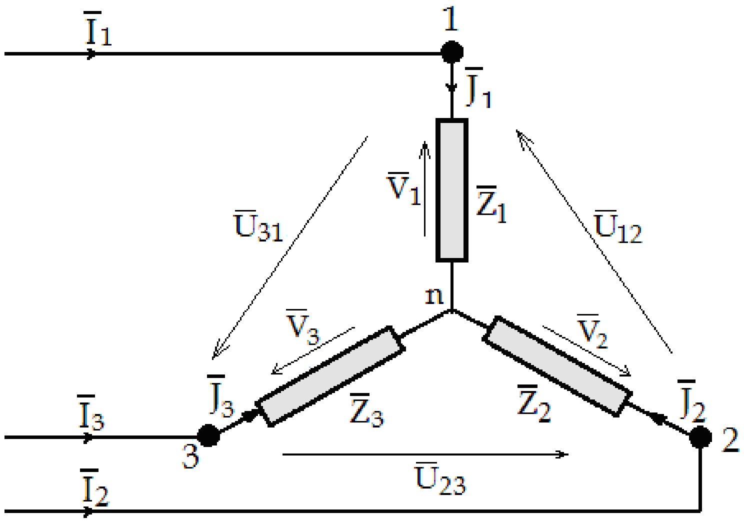
Definitions and general principles

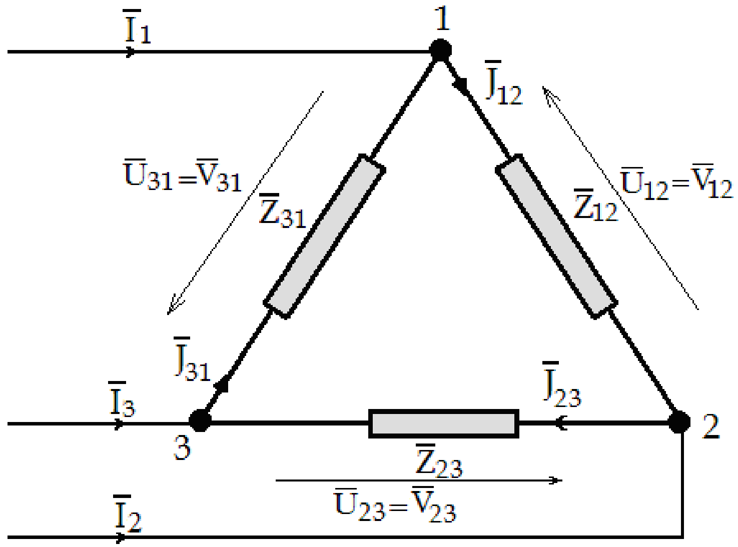
Definitions and notations

- **Simple Currents and voltages:** related to the load
  - **Simple Voltage:** Noted $\widetilde{V}_i$: Voltage across $\widetilde{Z}_i$,
  - **Simple Current:** Noted $\widetilde{I}_{ij}$: Currant flowing through load $\widetilde{Z}_{ij}$.
- **Compound Currents and voltages:** Related to wires (phases)
  - **Compound voltage:** Noted $\widetilde{U}_{ij}$:Voltage between two phases i, j
  - **Compound Current:** Noted I_i: Current flowing in the wire i

Definitions and general principles

Definitions and notations



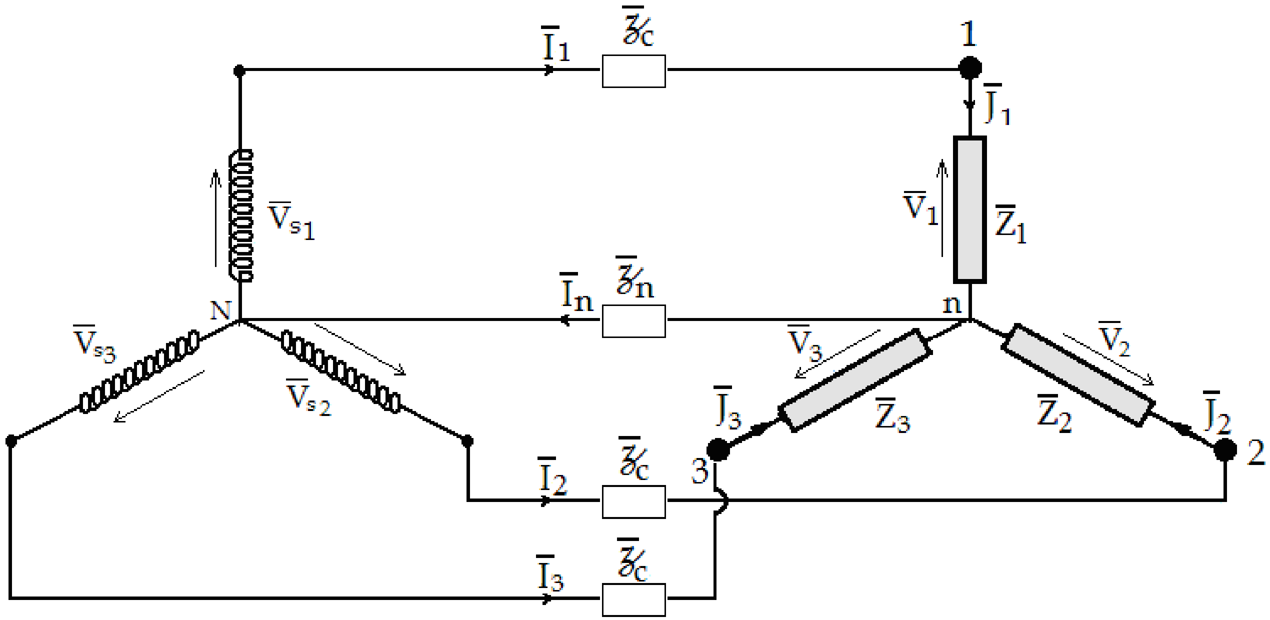


Star connection	Delta connection
$\begin{cases} \tilde{I}_1=\tilde{J}_1 \\ \tilde{I}_2=\tilde{J}_2 \\ \tilde{I}_3=\tilde{J}_3 \end{cases}$ and $\begin{cases} \tilde{U}_{12}=\tilde{V}_1-\tilde{V}_2 \\ \tilde{U}_{23}=\tilde{V}_2-\tilde{V}_3 \\ \tilde{U}_{31}=\tilde{V}_3-\tilde{V}_1 \end{cases}$	$\begin{cases} \tilde{I}_1=\tilde{J}_{12}-\tilde{J}_{31} \\ \tilde{I}_2=\tilde{J}_{23}-\tilde{J}_{12} \\ \tilde{I}_3=\tilde{J}_{31}-\tilde{J}_{23} \end{cases}$ and $\begin{cases} \tilde{U}_{12}=\tilde{V}_{12} \\ \tilde{U}_{23}=\tilde{V}_{23} \\ \tilde{U}_{31}=\tilde{V}_{31} \end{cases}$

Star-Star Connection

General Scheme

- The source and the receiver are both in star topology,
- The **simple** source voltage is directly applied to the load,
- Neutral $«n»$ **may** be linked to $«N»$. it may also not be linked to it.
- Phase wires have an impédance $\tilde{z}_c$ while the neutral has an impédance $\tilde{z}_n$.



Star-Star Connection

Balanced case: Calculations of simple voltages

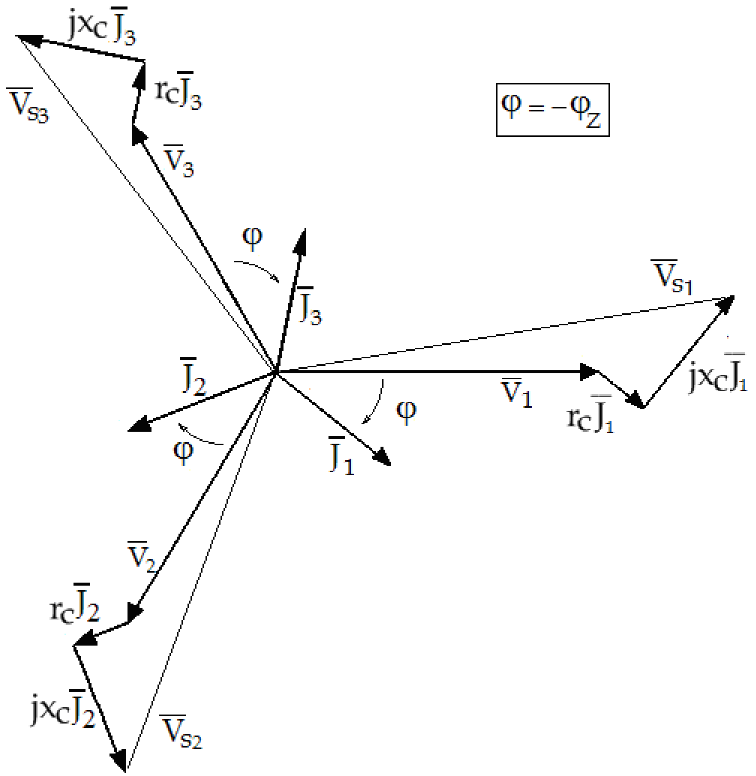
- Let $\tilde{Z}_1=\tilde{Z}_2=\tilde{Z}_3=\tilde{Z}=R+jX$
- Simple voltages across receiver?
  - $$\begin{cases} \tilde{V}_{s1}=\tilde{V}_1+\tilde{z}_c.\tilde{J}_1+\tilde{z}_n\tilde{I}_n=(\tilde{Z}+\tilde{z}_c).\tilde{J}_1+\tilde{z}_n.\tilde{I}_n \\ \tilde{V}_{s2}=\tilde{V}_2+\tilde{z}_c.\tilde{J}_2+\tilde{z}_n\tilde{I}_n=(\tilde{Z}+\tilde{z}_c).\tilde{J}_2+\tilde{z}_n.\tilde{I}_n \\ \tilde{V}_{s3}=\tilde{V}_3+\tilde{z}_c.\tilde{J}_3+\tilde{z}_n\tilde{I}_n=(\tilde{Z}+\tilde{z}_c).\tilde{J}_3+\tilde{z}_n.\tilde{I}_n \end{cases}$$
  - We have:
    - $\tilde{J}_1+\tilde{J}_2+\tilde{J}_3=\tilde{I}_n$
    - $\tilde{V}_{s1}+\tilde{V}_{s2}+\tilde{V}_{s3}=0=(\tilde{Z}+\tilde{z}_c)(\tilde{J}_1+\tilde{J}_2+\tilde{J}_3)+3.\tilde{z}_n.\tilde{I}_n$
  - We deduce that : $(\tilde{Z}+\tilde{z}_c+3\tilde{z}_n).\tilde{I}_n=0$ and then $\tilde{I}_n=0$.
- **In conclusion:** neutral wire is useless in the balanced case!!!

Star-Star Connection

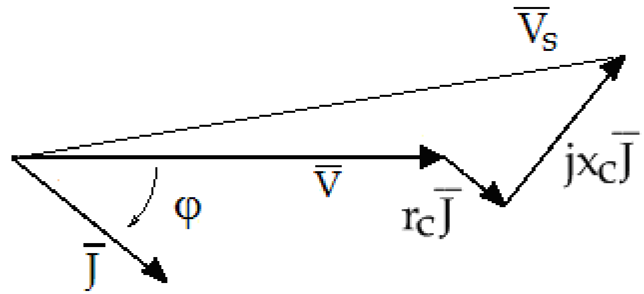
Balanced case: Calculations of simple voltages

As $\widetilde{I}_n = 0$ then:

$$\begin{cases} \widetilde{V}_{s1} = \widetilde{V}_1 + \widetilde{z}_c . \widetilde{J}_1 \\ \widetilde{V}_{s2} = \widetilde{V}_2 + \widetilde{z}_c . \widetilde{J}_2 \\ \widetilde{V}_{s3} = \widetilde{V}_3 + \widetilde{z}_c . \widetilde{J}_3 \end{cases} \Rightarrow \begin{cases} \widetilde{V}_1 = \widetilde{V}_{s1} - \widetilde{z}_c . \widetilde{J}_1 \\ \widetilde{V}_2 = \widetilde{V}_{s2} - \widetilde{z}_c . \widetilde{J}_2 \\ \widetilde{V}_3 = \widetilde{V}_{s3} - \widetilde{z}_c . \widetilde{J}_3 \end{cases}$$



- **Balanced** *Receivers* => Simple voltages ***balanced***.
- => ***symetric network***.
- It's like we have three indépendant networks => It's sufficient to use a single phasor diagram (***Fresnel)***
- In addition, if $z_c \approx 0$ then $\widetilde{V}_{s1} \approx \widetilde{V}_1$, $\widetilde{V}_{s2} \approx \widetilde{V}_2$ and $\widetilde{V}_{s3} \approx \widetilde{V}_3$



Star-Star Connection

Balanced case: *Calculations of compound voltages*

- By using the expressions of simple voltages, we deduce:

$$\begin{cases} \widetilde{U}_{12}=\widetilde{V}_1-\widetilde{V}_2=\widetilde{U}_{s12}-\widetilde{z}_c\left(\widetilde{J}_1-\widetilde{J}_2\right) \\ \widetilde{U}_{23}=\widetilde{V}_2-\widetilde{V}_3=\widetilde{U}_{s23}-\widetilde{z}_c\left(\widetilde{J}_2-\widetilde{J}_3\right) \\ \widetilde{U}_{31}=\widetilde{V}_3-\widetilde{V}_1=\widetilde{U}_{s31}-\widetilde{z}_c\left(\widetilde{J}_3-\widetilde{J}_1\right) \end{cases}$$

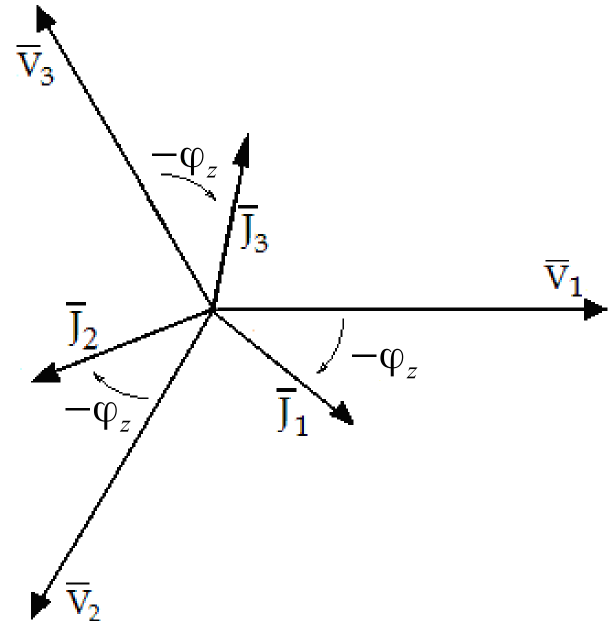
- In addition, if $z_c \approx 0$ then $\widetilde{V}_{s1} \approx \widetilde{V}_1, \widetilde{U}_{s23} \approx \widetilde{U}_{23}$ and $\widetilde{U}_{s31} \approx \widetilde{U}_{31}$

Star-Star Connection

Balanced case: Calculations of simple and compound currents

- Compound currents = Simple currents,

$$\left\{\begin{aligned}
\tilde{I}_1 &= \tilde{J}_1 = \frac{\tilde{V}_1}{\tilde{Z}} = \frac{\tilde{V}_1}{Z} e^{-j\varphi_z} = \frac{V_1}{Z} e^{-j\varphi_z} \\
\tilde{I}_2 &= \tilde{J}_2 = \frac{\tilde{V}_2}{\tilde{Z}} = \frac{\tilde{V}_2}{Z} e^{-j\varphi_z} = \frac{V_1}{Z} e^{-j\varphi_z} e^{-j2\pi/3} \\
\tilde{I}_3 &= \tilde{J}_3 = \frac{\tilde{V}_3}{\tilde{Z}} = \frac{\tilde{V}_3}{Z} e^{-j\varphi_z} = \frac{V_1}{Z} e^{-j\varphi_z} e^{j2\pi/3}
\end{aligned}\right.
\Rightarrow
\left\{\begin{aligned}
& J_1 = J_2 = J_3 = \frac{V}{Z} \\
& \varphi\left(\tilde{J}_1\right) = -\varphi_z \\
& \varphi\left(\tilde{J}_2\right) = -\varphi_z - 2\pi/3 \\
& \varphi\left(\tilde{J}_3\right) = -\varphi_z + 2\pi/3
\end{aligned}\right.$$



- Currents are ***balanced***,
- They are ***symetrical*** *too*.

Star-Star Connection

Unbalnaced case with neutral linked and $\tilde{z}_n \approx 0$

- We suppose $\tilde{z}_c \approx 0$,
- We get

$$\begin{cases} \tilde{V}_{s1} \approx \tilde{V}_1 = \tilde{Z}_1 . \tilde{J}_1 \\ \tilde{V}_{s2} \approx \tilde{V}_2 = \tilde{Z}_2 . \tilde{J}_2 \\ \tilde{V}_{s3} \approx \tilde{V}_3 = \tilde{Z}_3 . \tilde{J}_3 \end{cases} \quad \text{and} \quad \begin{cases} \tilde{U}_{12} = \tilde{V}_1 - \tilde{V}_2 \approx \tilde{U}_{s12} \\ \tilde{U}_{23} = \tilde{V}_2 - \tilde{V}_3 \approx \tilde{U}_{s23} \\ \tilde{U}_{31} = \tilde{V}_3 - \tilde{V}_1 \approx \tilde{U}_{s31} \end{cases}$$

- **Conclusion:** eventhough with unbalanced loads, simple and compound voltages remain balanced and symetrical.

- For currents, we get:

$$\tilde{J}_1 = \frac{\tilde{V}_1}{Z_1} \quad \tilde{J}_2 = \frac{\tilde{V}_2}{Z_2} \quad \tilde{J}_3 = \frac{\tilde{V}_3}{Z_3}$$

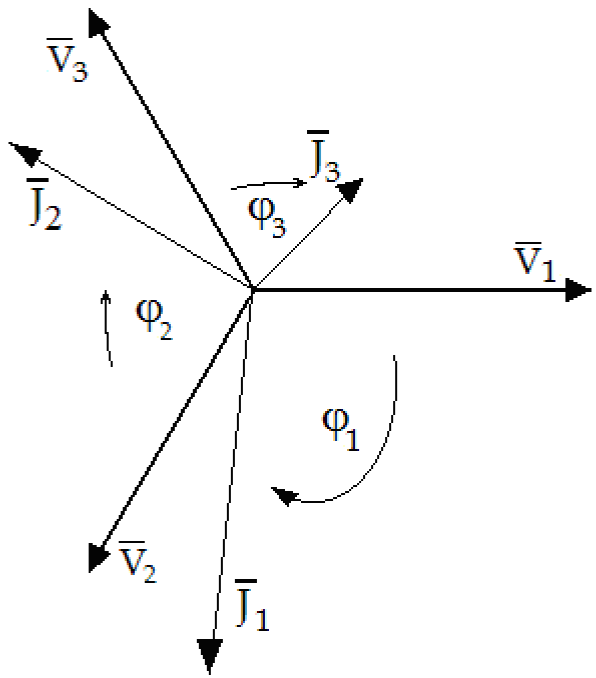
- **Conclusion:** Currents are unbalanced and non symetrical.

Star-Star Connection

Unbalnaced case with neutral linked and $\tilde{z}_n \approx 0$

- The current in the neutral wire not necessiraly null:

$$\tilde{I}_N = \tilde{J}_1 + \tilde{J}_2 + \tilde{J}_3 \neq 0$$



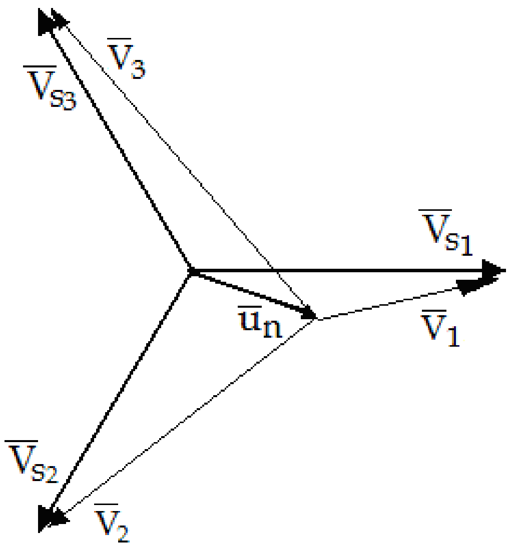
Star-Star Connection

Unbalnaced case with neutral linked and $z_n \neq 0$

- Simple voltages of the source are different than those of the receiver: $\widetilde{V}_{si} \neq \widetilde{V}_i$ because:

$$\begin{cases} \widetilde{V}_{s1} = \widetilde{V}_1 + \widetilde{z}_n \widetilde{I}_n = \widetilde{Z}_1 . \widetilde{J}_1 + \widetilde{z}_n . \widetilde{I}_n \\ \widetilde{V}_{s2} = \widetilde{V}_2 + \widetilde{z}_n \widetilde{I}_n = \widetilde{Z}_2 . \widetilde{J}_2 + \widetilde{z}_n . \widetilde{I}_n \\ \widetilde{V}_{s3} = \widetilde{V}_3 + \widetilde{z}_n \widetilde{I}_n = \widetilde{Z}_3 . \widetilde{J}_3 + \widetilde{z}_n . \widetilde{I}_n \end{cases} \Rightarrow \begin{cases} \widetilde{V}_1 = \widetilde{V}_{s1} - \widetilde{z}_n \widetilde{I}_n \\ \widetilde{V}_2 = \widetilde{V}_{s2} - \widetilde{z}_n \widetilde{I}_n \\ \widetilde{V}_3 = \widetilde{V}_{s3} - \widetilde{z}_n \widetilde{I}_n \end{cases} \Rightarrow \begin{cases} \widetilde{V}_1 = \widetilde{V}_{s1} - \widetilde{u}_n \\ \widetilde{V}_2 = \widetilde{V}_{s2} - \widetilde{u}_n \\ \widetilde{V}_3 = \widetilde{V}_{s3} - \widetilde{u}_n \end{cases}$$

- The voltage at the receiver is moved with $\widetilde{u}_n = \widetilde{z}_n . \widetilde{I}_n$ from that of the source.



- We still have: $\begin{cases} \widetilde{U}_{12} = \widetilde{V}_1 - \widetilde{V}_2 \approx \widetilde{U}_{s12} \\ \widetilde{U}_{23} = \widetilde{V}_2 - \widetilde{V}_3 \approx \widetilde{U}_{s23} \\ \widetilde{U}_{31} = \widetilde{V}_3 - \widetilde{V}_1 \approx \widetilde{U}_{s31} \end{cases}$

- **Conclusions:**
  - Simple voltages are unbalanced and non symetrical
  - Compound voltages remain symetrical and balanced

Star-Star Connection

Unbalnaced case with neutral linked and $z_n \neq 0$

- Let's determine displacement voltage $\tilde{u}_n$:
- We prefer to use admittances $\tilde{Y} = 1/\tilde{Z}$:

$$\begin{cases} \tilde{V}_{s1} = \tilde{V}_1 + \tilde{z}_n \tilde{I}_n = \tilde{Z}_1 . \tilde{J}_1 + \tilde{u}_n \\ \tilde{V}_{s2} = \tilde{V}_2 + \tilde{z}_n \tilde{I}_n = \tilde{Z}_2 . \tilde{J}_2 + \tilde{u}_n \\ \tilde{V}_{s3} = \tilde{V}_3 + \tilde{z}_n \tilde{I}_n = \tilde{Z}_3 . \tilde{J}_3 + \tilde{u}_n \end{cases} \Rightarrow \begin{cases} \tilde{Z}_1 . \tilde{J}_1 = \tilde{V}_{s1} - \tilde{u}_n \\ \tilde{Z}_2 . \tilde{J}_2 = \tilde{V}_{s2} - \tilde{u}_n \\ \tilde{Z}_3 . \tilde{J}_3 = \tilde{V}_{s3} - \tilde{u}_n \end{cases} \Rightarrow \begin{cases} \tilde{J}_1 = (\tilde{V}_{s1} - \tilde{u}_n) \tilde{Y}_1 \\ \tilde{J}_2 = (\tilde{V}_{s2} - \tilde{u}_n) \tilde{Y}_2 \\ \tilde{J}_3 = (\tilde{V}_{s3} - \tilde{u}_n) \tilde{Y}_3 \end{cases}$$

- By using junction's law: $\begin{cases} \tilde{J}_1 = (\tilde{V}_{s1} - \tilde{u}_n) \tilde{Y}_1 \\ \tilde{J}_2 = (\tilde{V}_{s2} - \tilde{u}_n) \tilde{Y}_2 \\ \tilde{J}_3 = (\tilde{V}_{s3} - \tilde{u}_n) \tilde{Y}_3 \\ \tilde{I}_N = \tilde{J}_1 + \tilde{J}_2 + \tilde{J}_3 = \tilde{y}_n \tilde{u}_n \end{cases}$

- We obtain:

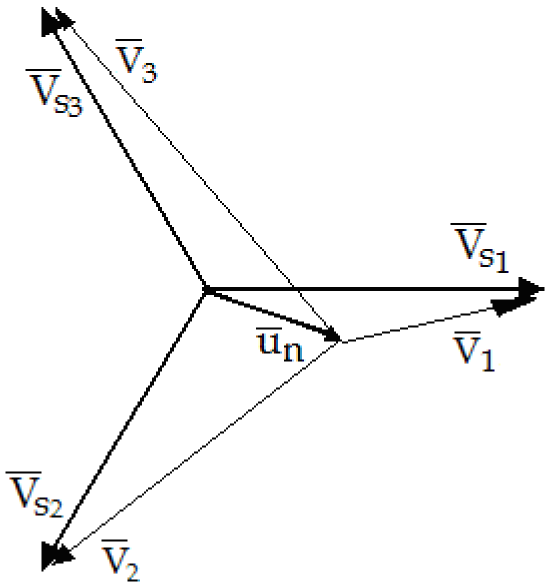
$$\tilde{u}_n = \frac{\tilde{V}_{s1} \tilde{Y}_1 + \tilde{V}_{s2} \tilde{Y}_2 + \tilde{V}_{s3} \tilde{Y}_3}{\tilde{Y}_1 + \tilde{Y}_2 + \tilde{Y}_3 + \tilde{y}_n}$$

Star-Star Connection

Unbalnaced case with neutral linked and $z_n \neq 0$

Comments:

- Voltages at the receiver are no longer balanced,
- Some receivers will be under-powered
- Other receivers will be over-powered!!! There is a serious risk of dammaging them
- **Solution:**
  - We must use a neutral wire of weak impedance.
  - This wire must under no circumstances be broken!!!
  - Indeed, in case this wire is broken, the unbalnace will be accentuated:
  - $\tilde{z}_n \rightarrow \infty \ \Rightarrow \ \tilde{y}_n \rightarrow 0$ Displacement voltage will be bigger :

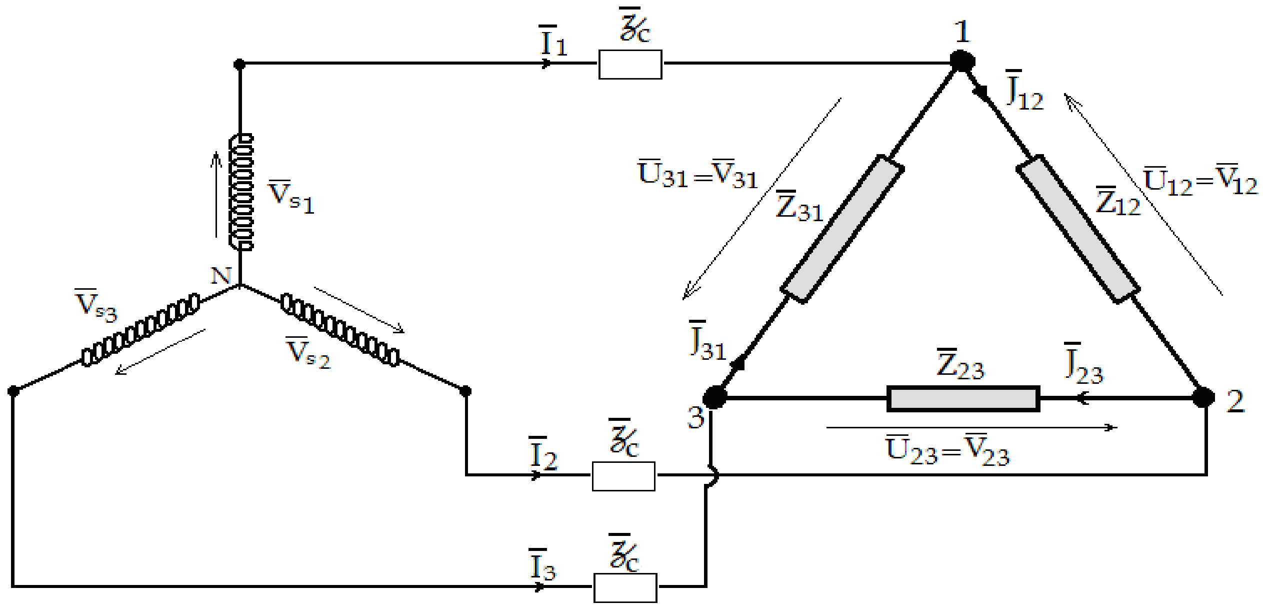


$$\tilde{u}_n = \frac{\tilde{V}_{s1}\tilde{Y}_1 + \tilde{V}_{s2}\tilde{Y}_2 + \tilde{V}_{s3}\tilde{Y}_3}{\tilde{Y}_1 + \tilde{Y}_2 + \tilde{Y}_3}$$

Star – Delta Connection

Introduction

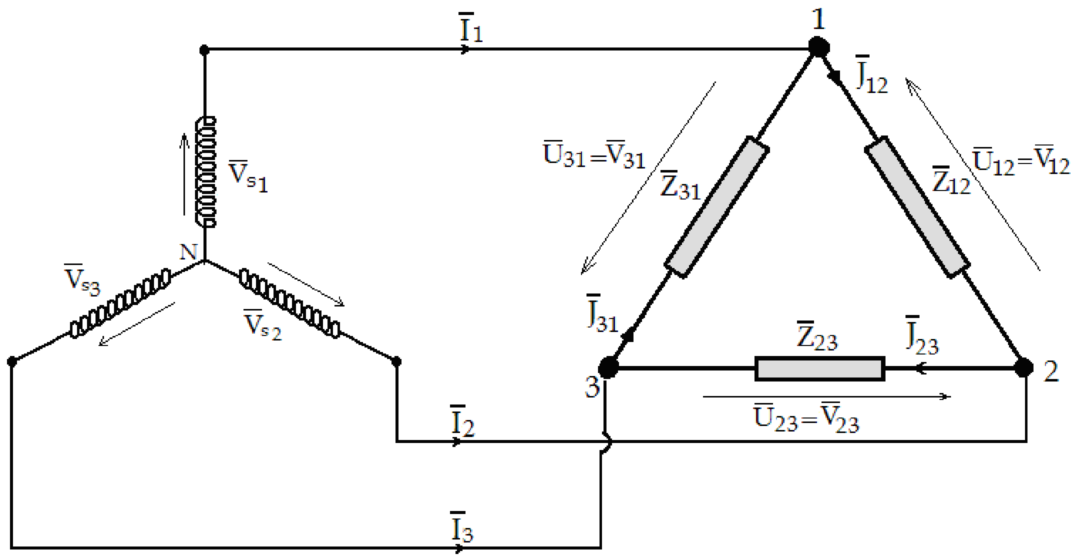
- Loads are under compound voltages pf the source,
- Neutral is not used,
- Loads may be balanced or not.



Star – Delta Connection

Compound / simple voltages at the receiver

- We suppose in the following that $\tilde{z}_c = 0$. We, then, obtain:



$$\tilde{V}_{12} = \tilde{U}_{12} = \tilde{U}_{s12}, \qquad \tilde{V}_{23} = \tilde{U}_{23} = \tilde{U}_{s23}, \qquad \tilde{V}_{31} = \tilde{U}_{31} = \tilde{U}_{s31}$$

- Voltages are symetrical and balanced

Star – Delta Connection

Compound / simple currents at the receiver

- Ohm's law across each load is expressed:

$$\begin{cases} \tilde{V}_{12} = \tilde{Z}_{12}\tilde{J}_{12} \\ \tilde{V}_{23} = \tilde{Z}_{23}\tilde{J}_{23} \\ \tilde{V}_{31} = \tilde{Z}_{31}\tilde{J}_{31} \end{cases} \Rightarrow \begin{cases} \tilde{J}_{12} = \dfrac{\tilde{V}_{12}}{\tilde{Z}_{12}} \\ \tilde{J}_{23} = \dfrac{\tilde{V}_{23}}{\tilde{Z}_{23}} \\ \tilde{J}_{31} = \dfrac{\tilde{V}_{31}}{\tilde{Z}_{31}} \end{cases} \quad \text{and by definition :} \quad \begin{cases} \tilde{I}_1 = \tilde{J}_{12} - \tilde{J}_{31} \\ \tilde{I}_2 = \tilde{J}_{23} - \tilde{J}_{12} \\ \tilde{I}_3 = \tilde{J}_{31} - \tilde{J}_{23} \end{cases}$$

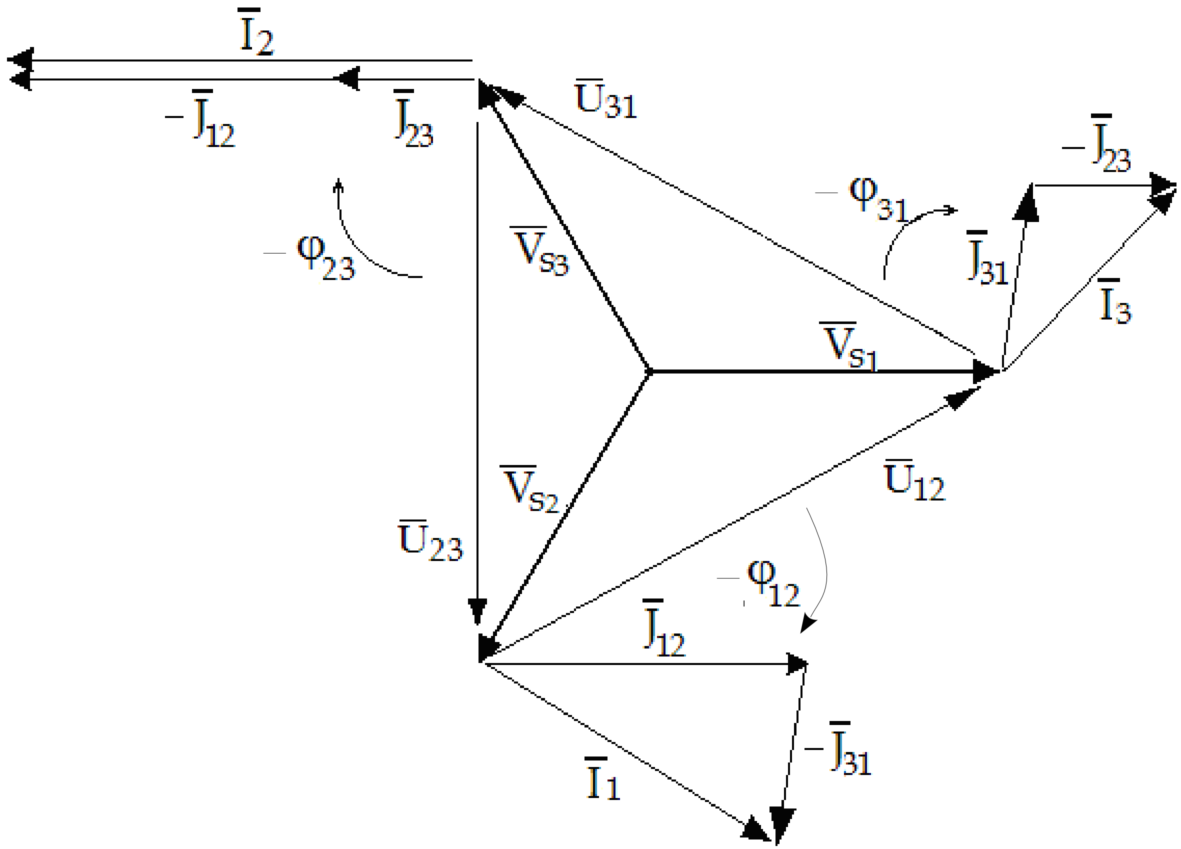
- We notice that we still have:

$$\tilde{I}_1 + \tilde{I}_2 + \tilde{I}_3 = 0$$

- It doesn't mean that the current are balanced.

Star – Delta Connection

Vectors Representation – unbalanced case



Star – Delta Connection

Unbalanced case

- We get in this case the simple currents balanced

$$\begin{cases} \tilde{V}_{12}=\tilde{Z}_{12}\tilde{J}_{12} \\ \tilde{V}_{23}=\tilde{Z}_{23}\tilde{J}_{23} \\ \tilde{V}_{31}=\tilde{Z}_{31}\tilde{J}_{31} \end{cases} \Rightarrow \begin{cases} \tilde{J}_{12}=\dfrac{\tilde{V}_{12}}{\tilde{Z}_{12}}=\dfrac{\tilde{U}_{12}}{\tilde{Z}}=\dfrac{\tilde{U}_{12}}{Z}e^{-j\varphi_Z} \\ \tilde{J}_{23}=\dfrac{\tilde{V}_{23}}{\tilde{Z}_{23}}=\dfrac{\tilde{U}_{23}}{\tilde{Z}}=\dfrac{\tilde{U}_{23}}{Z}e^{-j\varphi_Z} \\ \tilde{J}_{31}=\dfrac{\tilde{V}_{31}}{\tilde{Z}_{31}}=\dfrac{\tilde{U}_{31}}{\tilde{Z}}=\dfrac{\tilde{U}_{31}}{Z}e^{-j\varphi_Z} \end{cases}$$

- As simple currents are balanced, so it is the case for compound ones
- We show that:

$$I=\sqrt{3}J$$

Star – Delta Connection

Vectors Representation – unbalanced case