$$A(\omega_n^2-\Omega^2+2j\delta\Omega)e^{j\varphi}=\frac{y_0}{m}(K+j\ell\Omega)$$

$$A\sqrt{(\omega_n^2-\Omega^2)^2+4\delta^2\Omega^2}=y_0\sqrt{\omega_n^2+4\Omega^2\delta^2}$$

$$A=\frac{\quad}{\quad}$$

Sat N°5

Ex1.

1. $T=\frac{1}{2}m\dot{x}_1^2+\frac{1}{2}I_{/O'}\dot{\theta}^2$

$$\left(I_{\Delta}+M\left(\frac{\ell}{2}\right)^2\right)$$
$$\left[\frac{1}{12}M(2\ell)^2+M\frac{\ell^2}{4}\right]$$

$$T=\frac{1}{2}m\dot{x}_1^2+\frac{1}{2}M\ell^2\dot{\theta}^2$$

$x_2=2\ell\theta$

$\dot{x}_2=2\ell\dot{\theta}$

$$T=\frac{1}{2}m\dot{x}_1^2+\frac{1}{8}M\dot{x}_2^2$$

$$U=\frac{1}{2}Kx_1^2+\frac{1}{2}K_2(x_1+\ell\theta)^2+$$

$$\vec{r}_m=\begin{pmatrix}\frac{\ell}{2}\sin\theta\\ -\frac{\ell}{2}\cos\theta\end{pmatrix}\qquad \vec{dr}_m\begin{pmatrix}\frac{\ell}{2}\cos\theta\, d\theta\\ \frac{1}{2}\sin\theta\, d\theta\end{pmatrix}$$

$$\vec{W}\begin{pmatrix}0\\ -Mg\end{pmatrix}$$

$$U_M=-\int\vec{W}\cdot\vec{dr}_m=\int Mg\frac{\ell}{2}\sin\theta$$

$$=-Mg\frac{\ell}{2}\left[\cos\theta\right]_0^\theta=-Mg\frac{\ell}{2}(\cos\theta-1)$$
$$\left(1-\frac{\theta^2}{2}-1\right)$$

$$=Mg\frac{\ell}{2}\cdot\frac{\theta^2}{2}$$

$$U=\frac{1}{2}K_1x_1^2+\frac{1}{2}K_2\left(x_1+\frac{x_2}{2}\right)^2+\frac{Mg}{16\ell}x_2^2$$

2.

$$\begin{cases}m\ddot{x}_1+K_1x_1+K_2\left(x_1+\frac{x_2}{2}\right)=0\\ \frac{M}{4}\ddot{x}_2+\frac{1}{2}K_2\left(x_1+\frac{x_2}{2}\right)+\frac{Mg}{8\ell}x_2=0\end{cases}$$

$$K_1=K_2=K=\frac{Mg}{2\ell}\qquad M=m$$

$$\begin{cases}m\ddot{x}_1+2Kx_1+\frac{K}{2}x_2=0\\ \frac{m}{4}\ddot{x}_2+\frac{1}{4}Kx_2+\frac{1}{4}Kx_2+\frac{1}{2}Kx_1=0\end{cases}$$

$$\begin{cases}\ddot{x}_1+\frac{2K}{m}x_1+\frac{K}{2m}x_2=0\\ \ddot{x}_2+\frac{2K}{m}x_2+\frac{2K}{m}x_1=0\end{cases}$$

3. $x_1(t)=A_1\cos(\omega t+\varphi_1)\Rightarrow\bar{A}_1e^{j\omega t}$

$x_2(t)=A_2\cos(\omega t+\varphi_2)\Rightarrow\bar{A}_2e^{j\omega t}$

$$\begin{cases}-\omega^2\bar{A}_1+\frac{2K}{m}\bar{A}_1+\frac{K}{2m}\bar{A}_2=0\\ -\omega^2\bar{A}_2+\frac{2K}{m}\bar{A}_2+\frac{2K}{m}\bar{A}_1=0\end{cases}$$

$$\begin{cases}\left(-\omega^2+\frac{2K}{m}\right)\bar{A}_1+\frac{K}{2m}\bar{A}_2=0\\ \frac{2K}{m}\bar{A}_1+\left(-\omega^2+\frac{2K}{m}\right)\bar{A}_2=0\end{cases}$$

$\det=0$

$$\left(-\omega^2+\frac{2K}{m}\right)^2-\left(\frac{K}{m}\right)^2=0$$

$$\left(-\omega^2+\frac{2K}{m}+\frac{K}{m}\right)\left(-\omega^2+\frac{2K}{m}-\frac{K}{m}\right)=0$$

$$\left(-\omega^2+\frac{3K}{m}\right)\left(-\omega^2+\frac{K}{m}\right)=0$$

$$\omega_1=\sqrt{\frac{K}{m}}\qquad \omega_2=\sqrt{\frac{3K}{m}}$$

$$x_1(t) = a_{11}\cos(\omega_1 t + \varphi_1) + a_{12}\cos(\omega_2 t + \varphi_2)$$

$$x_2(t) = a_{21}\cos(\omega_1 t + \varphi_1) + a_{22}\cos(\omega_2 t + \varphi_2)$$

Ex 2 :

$$T = \frac{1}{2} M \dot{x}_1^2 + \frac{1}{2}\left(\frac{1}{2} M R^2\right)\dot{\theta}^2 + \frac{1}{2} m (\underbrace{l\dot{\varphi}}_{x_2} + \dot{x}_1)^2$$

$$T = \frac{3}{4} M \dot{x}_1^2 + \frac{1}{2} m (\dot{x}_1 + \dot{x}_2)^2$$

$$U = \frac{1}{2} k (x_1 + R\theta - S(t))^2 + \frac{1}{2} k (x_1 + x_2)^2 + mgl\frac{\varphi^2}{2}$$

$$U = \frac{1}{2} k (2x_1 - S(t))^2 + \frac{1}{2} k (x_1 + x_2)^2 + \frac{mg}{2l} x_2^2$$

$$T = \frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} m (\dot{x}_1 + \dot{x}_2)^2$$

$$T = m\dot{x}_1^2 + m\dot{x}_1\dot{x}_2 + \frac{1}{2} m\dot{x}_2^2$$

$$U = \frac{1}{8} k (4x_1^2 + S^2 - 4x_1 S) + \frac{1}{2} k (x_1^2 + x_2^2 + 2x_1 x_2) + \frac{1}{2} k x_1^2$$

$$\mathcal{L} = m\dot{x}_1^2 + m\dot{x}_1\dot{x}_2 + \frac{1}{2} m\dot{x}_2^2 - k x_1^2 - k x_1 x_2 - k x_2^2 - \frac{1}{8} k S^2 + \frac{1}{2} k x_1 S$$