Chapter 2 (Part 2)

Fluid statics (hydrostatic)

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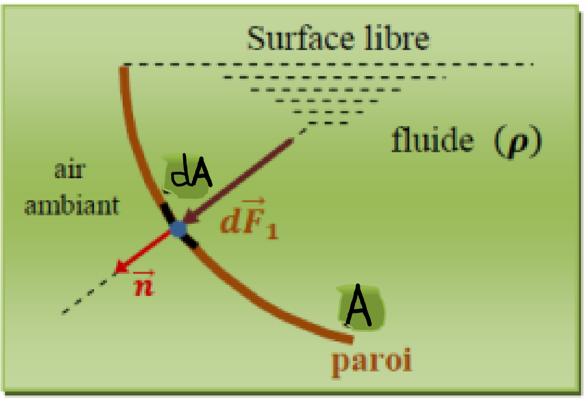
7-Pressure Forces Acting on Submerged Walls

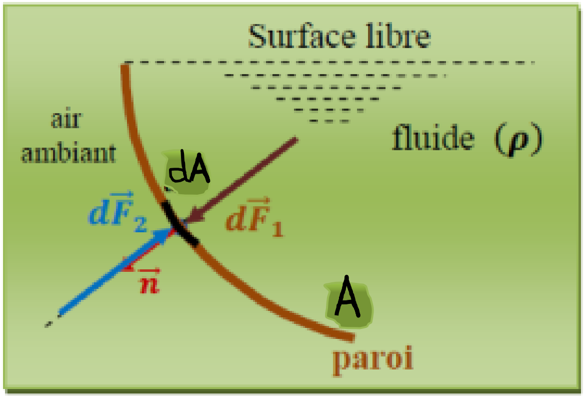
In everyday life, a liquid like water needs to be contained within a container. This container must be able to hold the liquid without cracking or breaking apart. Knowing the pressure forces exerted on the walls of the container is therefore essential for its sizing.



Let M be a point in a fluid at a static pressure P_M and surrounded by an infinitesimal surface area dA with an outward normal n. The fluid exerts an infinitesimal force $\overrightarrow{dF_1}$ on the surface, such that:

$$\overrightarrow{dF_1} = P_M . dA\ \vec{n}$$

Where $\vec{n}$ is the unit vector aligned with the outward normal to surface.



On the atmospheric side, a force $\overrightarrow{dF_2}$ (pressure force that the air in the atmosphere exerts on dA) acts as follows:

$$\overrightarrow{dF_2} = P_{atm} . dA\ (-\vec{n})$$

Thus, it is directly opposed to $\overrightarrow{dF_1}$. The resultant infinitesimal pressure force on the surface dA will be:

$$\overrightarrow{dF} = \overrightarrow{dF_1} + \overrightarrow{dF_2}$$

Let :

$$\overrightarrow{dF} = (P_M - P_{atm}) \cdot dA\ \vec{n}$$

To find the effective (relative) pressure $P_M - P_{atm}$ at point M, we can write the equality of the driving pressures at point M and at a point B located on the free surface of the fluid:

$$P_M + \rho g Z_M = P_{atm} + \rho g Z_B$$

Thus:

$$P_M - P_{atm} = \rho\ g\ (Z_B - Z_M)$$

There is therefore an action on dA as follows:

$$\overrightarrow{dF} = \rho\ g\ (Z_B - Z_M) \cdot dA\ \vec{n}$$

Now, a vectorial integration over the surface A is needed to find the resultant force F. This integration depends on the shape of the surface.

7-1 hydrostatic force on a plane surface

Consider a wall with a surface area A and a center of gravity c, submerged in a liquid and inclined at an angle θ with respect to the horizontal. Let's divide the surface A into sufficiently small elements dA. The pressure force on each element is determined using the following formula:

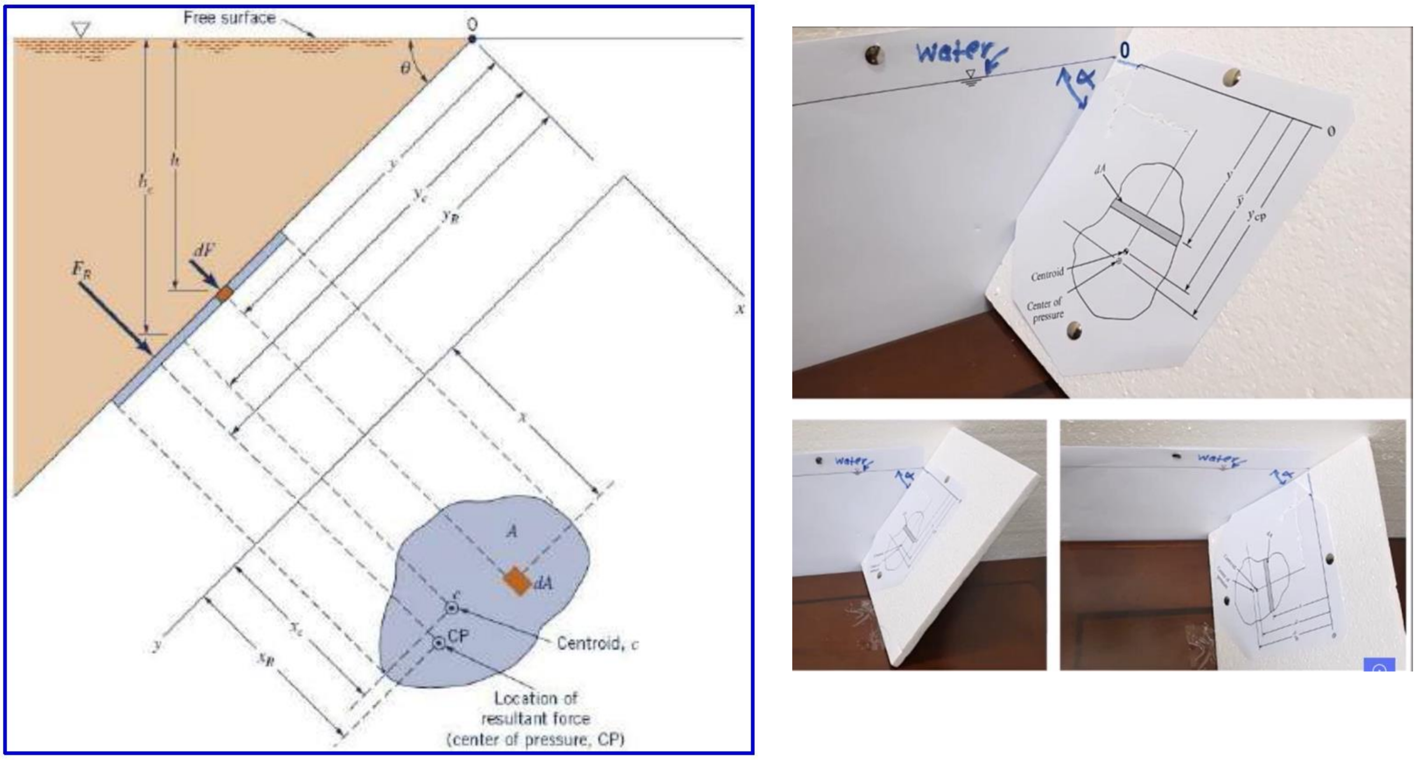


Figure 18. Hydrostatic Pressure Force on an Inclined Plane Surface of Any Shape

$$\mathbf{dF = P.dA = \rho.g.h.dA = \rho.g.y.\sin(\theta).dA}$$

The intensity of the pressure force acting on the surface A is:

$$F_R = \int_A dF = \int_A P.\,dA = \int_A \rho.g.h.dA = \rho.g\int_A y.\sin\theta.\,dA = \rho.g.\sin\theta\int_A y.dA$$

This integral represents the static moment, which is defined as follows:

$$\int_A \mathbf{y}.dA = \mathbf{y_c}.A$$

Where :

$\int_A y \cdot dA = \mathbf{M_s}$ is the static moment of area **A** relative to the horizontal axis O.

Thus, the equation becomes:

$$F_R = \rho.g.sin\theta\, y_c.A \qquad \text{with :} \qquad h_c = y_c\, sin\theta$$

$$F_R = \rho.g.h_c.A$$

Where:

h_c: depth of the center of gravity of the surface

h: depth of the small portion dA relative to a reference axis (often an axis perpendicular to the surface).

y_c: distance between the x-axis and the axis passing through the center of gravity.

y: distance of the small portion dA relative to a reference axis (often an axis perpendicular to the surface).

A: total area of the surface.

dA: a small portion of the surface area of the object.

- ***Position of the Point of Application of the Force (Center of Pressure):***

The force F_R does not act at the center of gravity but at a point called the center of pressure CP. To determine the coordinates of the center of pressure, we consider the moment of the force relative to the x-axis, written as follows:

The moment of the resultant force $\vec{F}$ = $\sum$ of the moments of the elemental forces $\vec{dF}$

This gives:

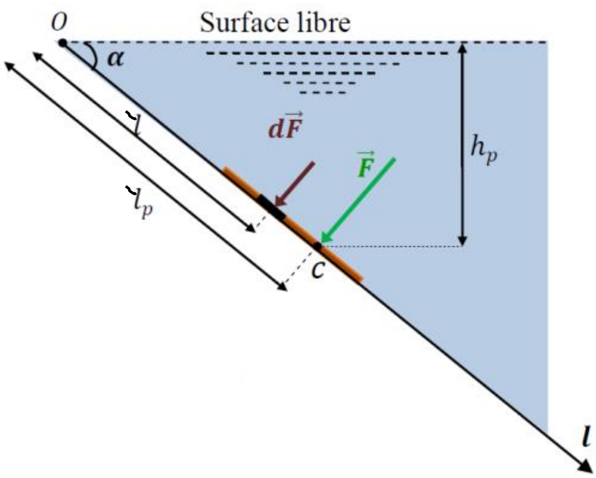
$$F_R \, . \, y_p = \int_A dF.y$$

$$\rho.g.\, y_c.\sin(\theta).A.y_p = \int(\rho.g.\, y \sin(\theta).dA).y$$

$$\rho.g.\, y_c.\sin(\theta).A.y_p = \rho.g.\sin(\theta)\int y^2 dA$$

$$y_c.\, A.y_p = \int y^2 dA$$

$$y_p = \frac{\int y^2 dA}{A.y_c}$$



The integral in the numerator $\int y^2 dA$ is the moment of inertia I_x relative to x:

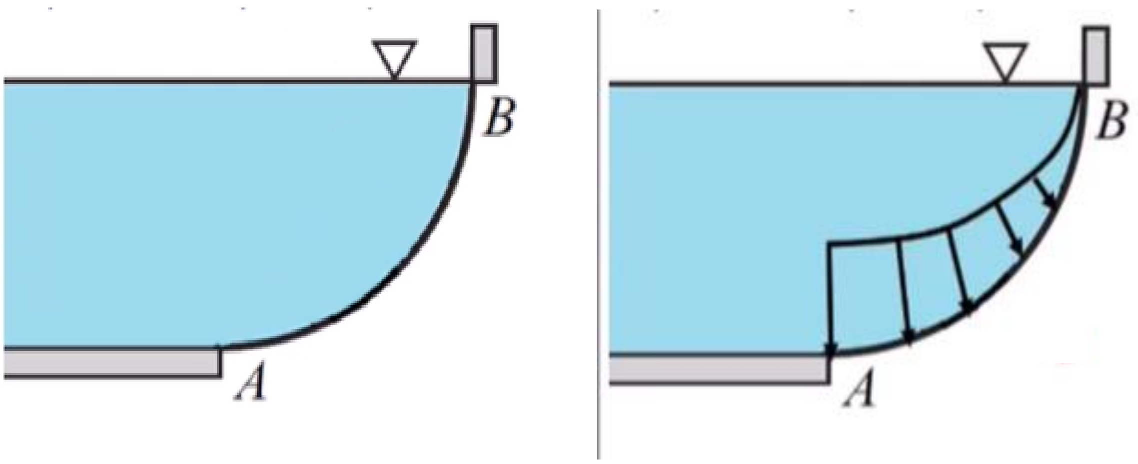
$$y_p = \frac{I_x}{A.y_c}$$

In calculations, it is more convenient to replace the moment of inertia I_x with the moment of inertia I_{xc} relative to the axis parallel to it that passes through the center of gravity of the surface, using the following equation:

$$I_x = I_{xc} + A.\, y_c^2 \qquad \text{and} \qquad y_p = \frac{I_{xc} + A.y_c^2}{A.y_c} = \frac{I_{xc}}{A.y_c} + y_c$$

7-2 hydrostatic force on a curved surface

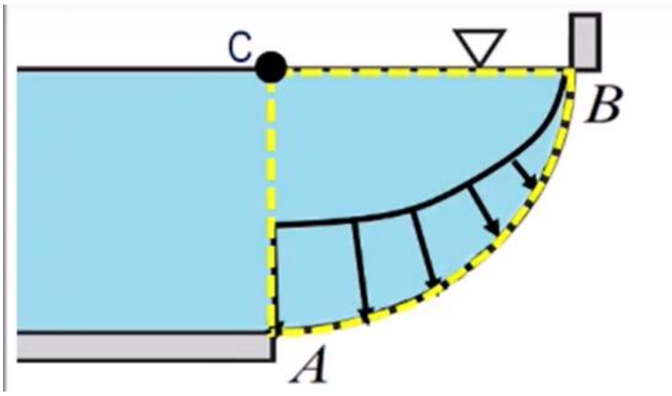
The resultant pressure forces on a curved surface are easier to calculate by breaking down the force into its vertical and horizontal components. Explain via a simple example:

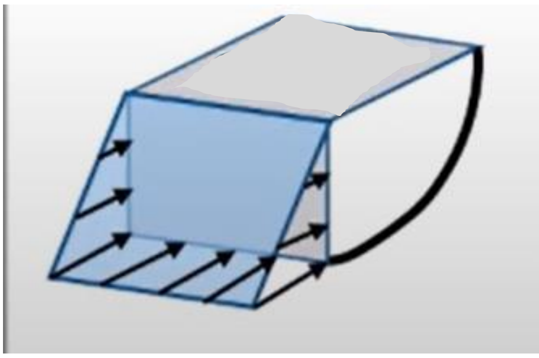


In the case of an immersed curved surface, the main complexity lies in the fact that the pressure forces exerted by the fluid are not uniform or linear along the surface. This means that, unlike a flat surface, a curved surface experiences a distribution of variable forces depending on the depth and shape of the curve.

Indeed, the pressure exerted by the fluid increases with depth, meaning that each point on the curved surface is subjected to a different force based on its position relative to the fluid's free surface. This variation makes it difficult to directly calculate the resultant force, as the forces cannot simply be added together as they would with a flat surface.

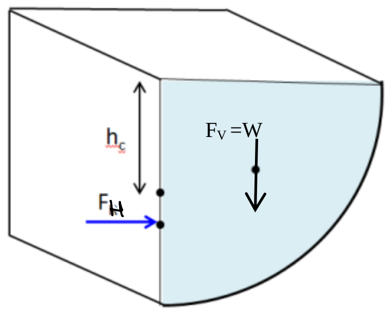
To simplify the calculation of the force exerted by the fluid on a curved surface, we will choose an isolated system (a hypothetical solid) that includes the curved surface but is bounded by the horizontal and vertical projections of this surface. By doing so, we can reduce the problem to simpler forces to analyze





- **Horizontal hydrostatic component :**

The horizontal force exerted on the curved surface corresponds to the pressure force on the vertical projection of the curved surface. This projection corresponds to a submerged flat surface, for which the horizontal force can be calculated by multiplying the average pressure (at the average depth of the projection) by the area of this projection. The horizontal component is the magnitude, and the line of action is found using vertical surface formulas



$$F_H = \rho.g.h_c.A \qquad \text{and} \qquad y_p = \frac{I_{xc}}{A.y_c} + y_c$$

- **Vertical hydrostatic component :**

The vertical component of the hydrostatic force, F_V, is equal to the weight of the liquid located above the curved surface and extending up to the free surface of the fluid (or its extension). This vertical force is therefore determined by the volume of fluid located above the curved surface.

$$F_V = \rho.g.V$$

where V is the volume of the cylinder, calculated as the area of its base multiplied by its depth. The vertical force acts through the centroid of the volume V.

- **Calculation of the Resultant Pressure Force:**

The calculation of the two components, F_H and F_V, allows us to then determine the resultant force F using the following expression:

$$F = \sqrt{F_H^2 + F_V^2}$$

- **Position of the Application Point of the Pressure Force:**

To determine the point of application of the resultant force F on an immersed curved surface, the horizontal component F_H and the vertical component F_V of the force are used. This approach simplifies the calculation by selecting an isolated system that includes the curved surface and its horizontal and vertical projections.

The horizontal component F_H is calculated using the following formula:

$$\mathbf{F_H = \rho \cdot g \cdot hc \cdot A}$$

where h_c is the depth at the center of pressure of the projected surface, and A is the area of this surface. The line of action of F_H is located at the depth y_p given by:

$$y_p = \frac{I_{xc}}{A.y_c} + y_c$$

The vertical component F_V is equal to the weight of the liquid located directly above the curved surface, up to the free surface. It is calculated as follows:

$$\mathbf{F_V = \rho \cdot g \cdot V}$$

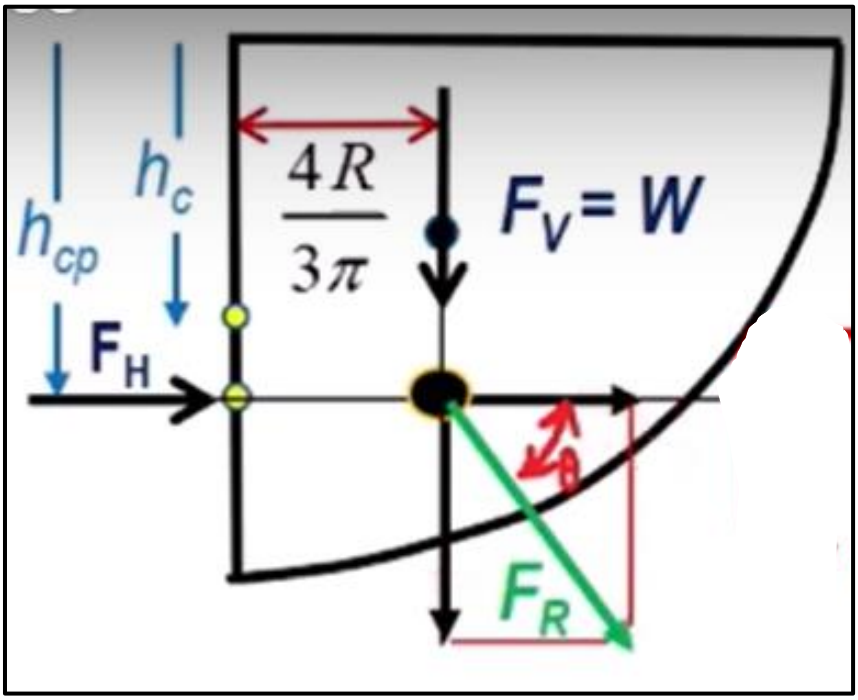
Where V represents the volume of the fluid above the curved surface. The vertical force acts at the center of gravity of this volume. Once F_H and F_V are obtained, the resultant force F can be calculated by:

$$\boldsymbol{F = \sqrt{F_H^2 + F_V^2}}$$

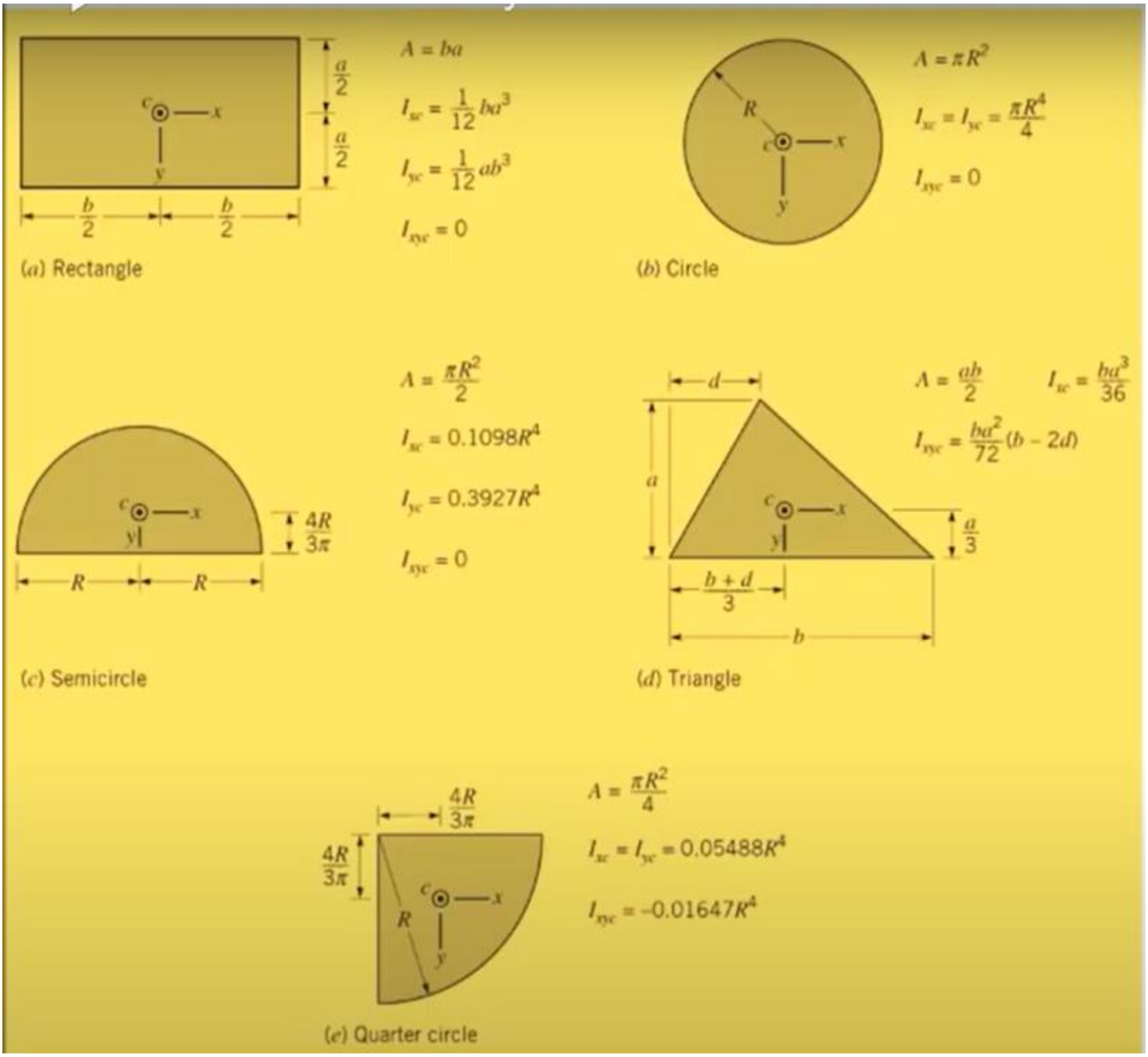
The point of application of F is determined based on the lines of action of F_H and F_V, without the need to establish equations for the curved surface or segment, as the components directly simplify the localization of the point of application. Taking into account that the angle of inclination of the resultant force F with respect to the horizontal is given by the following formula:

$$\theta = arctg\frac{F_V}{F_H}$$

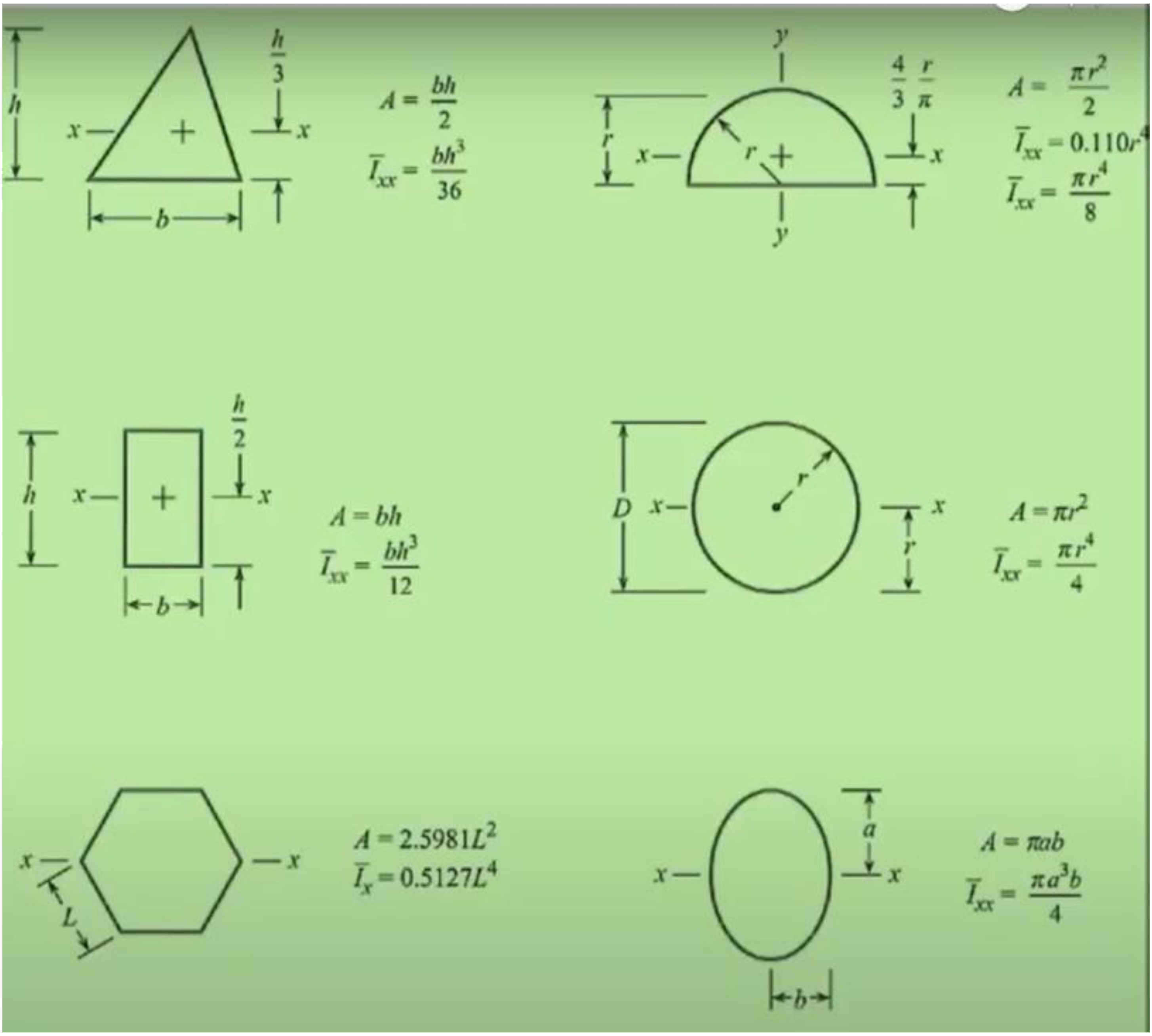
Taking into account all the data and information gathered, the line of action of the force can be represented as follows:



For known geometric shapes, the following figures provide the coordinates of the centroids (centers of gravity), areas, and corresponding moments of inertia I. These data allow for precise determination of the point of application of the force for each shape, thereby simplifying force distribution calculations and facilitating their use in practical engineering and physics applications.



$A = ba$
$I_{xc} = \frac{1}{12} ba^3$
$I_{yc} = \frac{1}{12} ab^3$
$I_{xyc} = 0$
(a) Rectangle
$A = \pi R^2$
$I_{xc} = I_{yc} = \frac{\pi R^4}{4}$
$I_{xyc} = 0$
(b) Circle
$A = \frac{\pi R^2}{2}$
$I_{xc} = 0.1098R^4$
$I_{yc} = 0.3927R^4$
$I_{xyc} = 0$
(c) Semicircle
$A = \frac{ab}{2}$
$I_{xc} = \frac{ba^3}{36}$
$I_{xyc} = \frac{ba^2}{72}(b - 2d)$
(d) Triangle
$A = \frac{\pi R^2}{4}$
$I_{xc} = I_{yc} = 0.05488R^4$
$I_{xyc} = -0.01647R^4$
(e) Quarter circle



A = bh/2
Ī_xx = bh³/36
A = πr²/2
Ī_xx = 0.110r⁴
Ī_yy = πr⁴/8
4r/3π
A = bh
Ī_xx = bh³/12
A = πr²
Ī_xx = πr⁴/4
A = 2.5981L²
Ī_x = 0.5127L⁴
A = πab
Ī_xx = πa³b/4