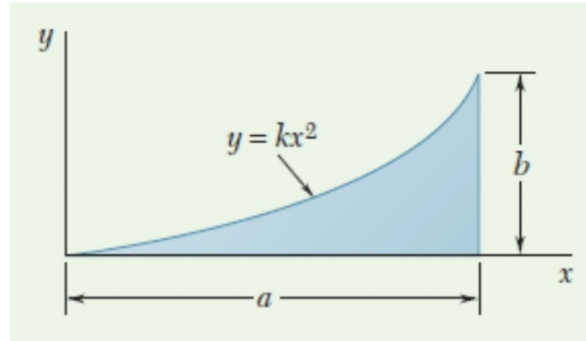


# TD 1 :

## Exercise 1:

Determine the Area using both horizontal and vertical differential element. ( $k = 1$ ,  $a = 1$  m,  $b = 1$  m)

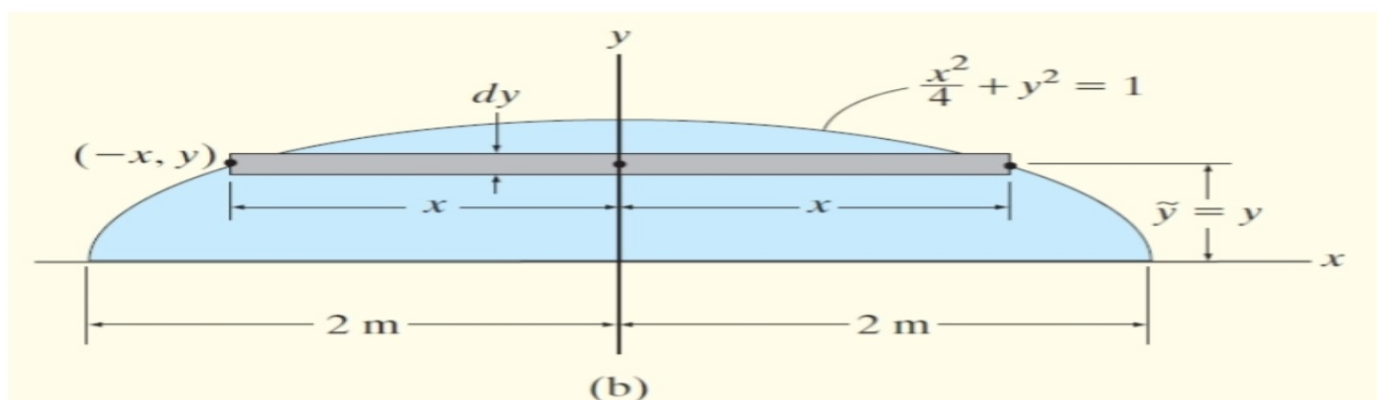
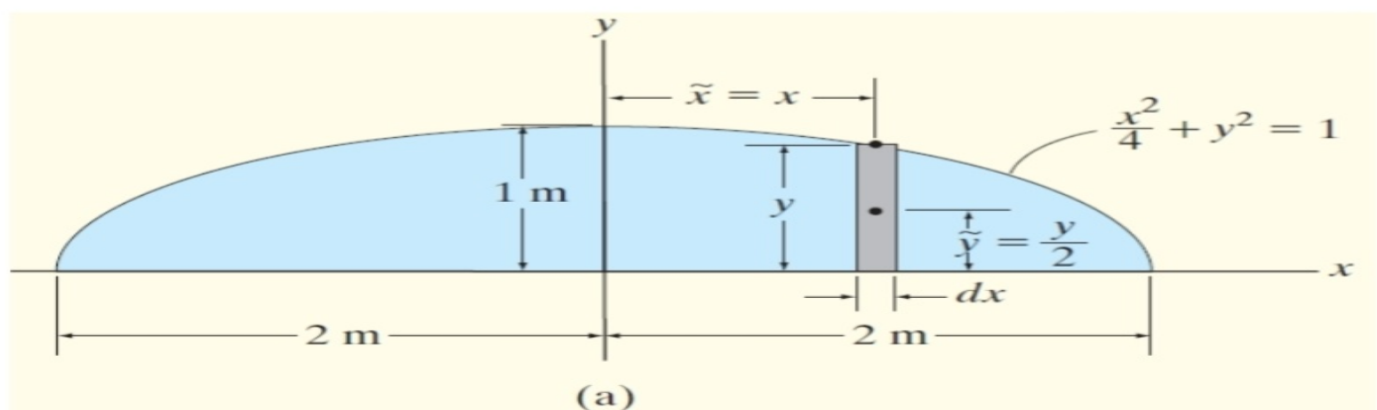


vertical:  $A = \int ds = \int y dx = \int_0^1 x^2 dx = \frac{1}{3} (x^3)_0^1$

$$A = \frac{1}{3}$$

## Exercise 2 :

Determine the Area using both horizontal and vertical differential element.



$$V \setminus \mathcal{A} = \int dA = \int_{-2}^2 y \, dx = \int_{-2}^2 \sqrt{1 - \frac{x^2}{4}} \, dx$$

$$x = 2 \cos \theta \Rightarrow dx = -2 \sin \theta \, d\theta$$

$$x = -2 \Rightarrow 2 \cos \theta = -2 \Rightarrow \theta = \pi$$

$$x = 2 \Rightarrow \theta = 0$$

$$\mathcal{A} = \int_{\pi}^0 \sqrt{1 - \frac{4 \cos^2 \theta}{4}} (2 \sin \theta) \, d\theta$$

$$= \int_{\pi}^0 \sqrt{\sin^2 \theta} (-2 \sin \theta) \, d\theta$$

$$= \int_{\pi}^0 -2 \sin^2 \theta \, d\theta$$

we know:  $\cos(2\theta) = 1 - 2\sin^2 \theta \Leftrightarrow \frac{1 - \cos(2\theta)}{2} = \sin^2 \theta$

$$\mathcal{A} = -2 \int_{\pi}^0 \frac{1}{2} \, d\theta + 2 \int_0^{\pi} \frac{\cos(2\theta)}{2} \, d\theta$$

$$= \int_0^{\pi} d\theta + \frac{1}{2} \int_{\pi}^0 2 \cos(2\theta) \, d\theta$$

$$= [\theta]_0^{\pi} + \frac{1}{2} [\sin(2\theta)]_{\pi}^0$$

$$\boxed{\mathcal{A} = \pi}$$

$$\frac{x^2}{4} + y^2 = 1$$

$$x = 2\sqrt{1-y^2}$$

$$\frac{A}{2} = \int_0^1 x \, dy = 2 \int_0^1 \sqrt{1-y^2} \, dy$$

We put  $y = \sin \theta \Rightarrow dy = \cos \theta \, d\theta$

$$y=0 \Rightarrow \sin \theta = 0 \Rightarrow \theta = 0$$

$$y=1 \Rightarrow \sin \theta = 1 \Rightarrow \theta = \frac{\pi}{2}$$

$$\frac{A}{2} = 2 \int_0^{\frac{\pi}{2}} \sqrt{1-\sin^2 \theta} \cos \theta \, d\theta$$

$$= 2 \int_0^{\frac{\pi}{2}} \cos^2 \theta \, d\theta = 2 \int_0^{\frac{\pi}{2}} \frac{\cos(2\theta) + 1}{2} \, d\theta$$

$$\frac{A}{2} = \left[ \frac{1}{2} \sin 2\theta \right]_0^{\frac{\pi}{2}} + [\theta]_0^{\frac{\pi}{2}} = \frac{\pi}{2}$$

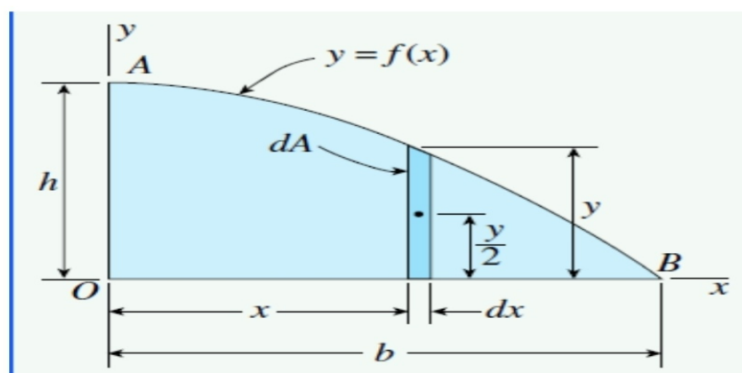
$$A = 2 \times \frac{\pi}{2} \Rightarrow \boxed{A = \pi}$$

### Exercise 3 :

A parabolic semi-segment  $OAB$  is bounded by the  $x$  axis, the  $y$  axis, and a parabolic curve having its vertex at  $A$ . The equation of the curve is :

$$y = f(x) = h \left( 1 - \frac{x^2}{b^2} \right)$$

in which  $b$  is the base and  $h$  is the height of the semi-segment. Determine the Area using a vertical differential element.



$$dA = y dx$$

$$dA = f(x) dx$$

$$dA = R \left( 1 - \frac{x^2}{b^2} \right) dx$$

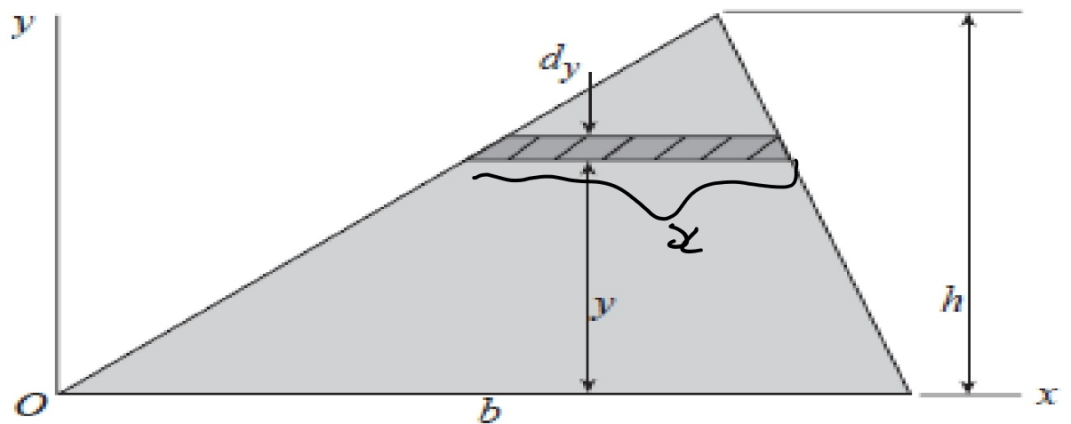
$$A = \int_0^b R \left( 1 - \frac{x^2}{b^2} \right) dx$$

$$= Rb - \frac{Rb^3}{3b^2} = Rb - \frac{Rb}{3}$$

$$A = 2 \frac{Rb}{3}$$

#### Exercise 4 :

Determine the Area of the triangle using a horizontal differential element.



$$dA = x dy$$

d'après Tales:

$$\frac{x}{b} = \frac{h-y}{h}$$

$$x = b \left( 1 - \frac{y}{h} \right)$$

$$A = \int_0^R b \left( 1 - \frac{y}{R} \right) dy$$

$$= \left[ by \right]_0^R - \frac{b}{2R} \left[ y^2 \right]_0^R$$

$$= bR - \frac{bR}{2} = \frac{bR}{2}$$

$$A = \frac{bR}{2}$$