المدرسة الوطنية العليا لتكنولوجيا الأنظمة المستقلة

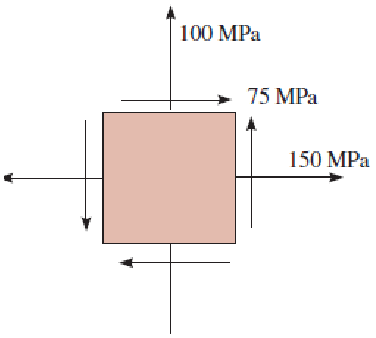
Academic year 2024/2025

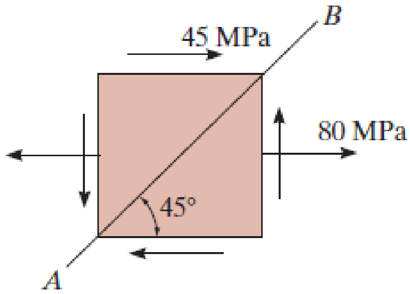
2nd Year **Mechanics of Materials (Material Strength)**

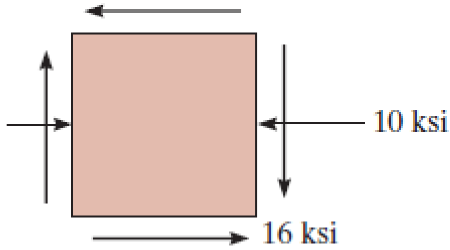
T.D N° 6 (Notion of Stress- Plane Stress Transformation, Principal Stresses and Maximum In-Plane shear Stress)

Problem 1 :

Give the values of the different stresses shown in the following figures

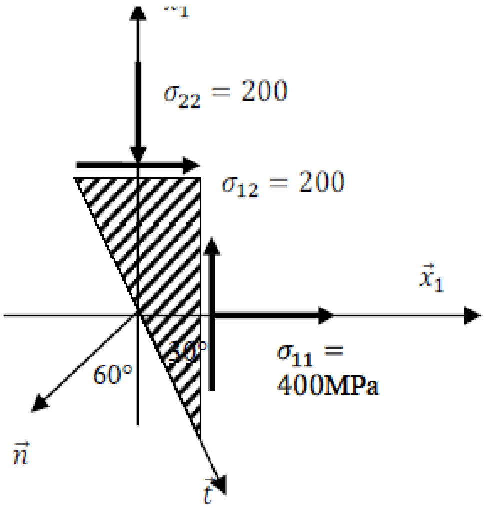






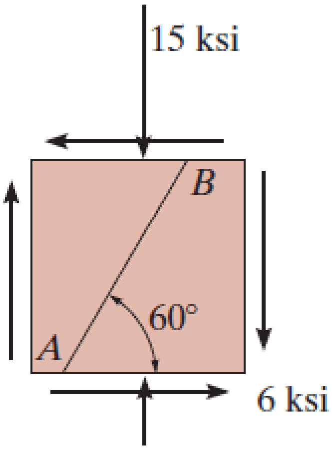
Problem 2 :

Using the direct method (Cauchy's relation), determine the normal and shear stresses on the face inclined with θ = 30°. Draw these stresses on this face.



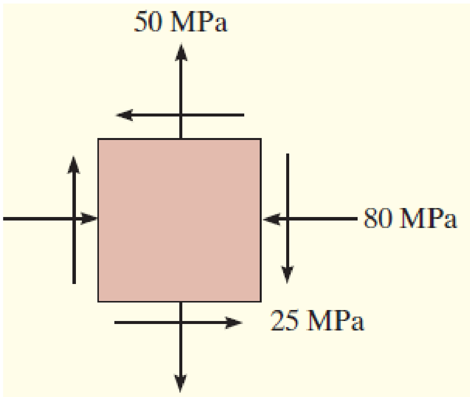
Problem 3:

Determine the normal stress and shear stress acting on the inclined plane *AB*. Solve the problem using the stress transformation equations. Show the results on the sectional element.



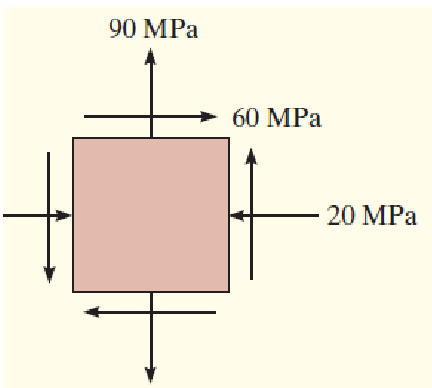
Problem 4:

The state of plane stress at a point is represented on the element shown in Fig.
Determine the state of stress at this point on another element oriented 30° clockwise from the position shown (plane CD). Represent these stresses on the fig.
Calculate the same stresses on the plane BC perpendicular to this plane and compare the results. Represent these stresses on the fig. Are these directions Principal



Problem 5:

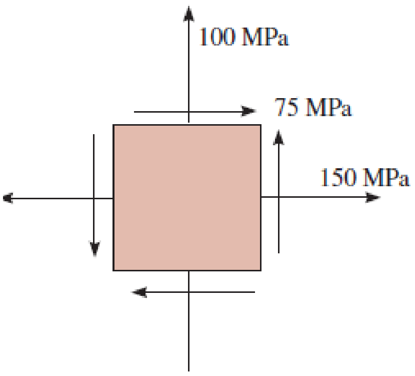
The plane stress state at a point is shown in the following element. Calculate and represent the principal stresses in the figure.
Calculate and represent the maximum shear stress in this case.



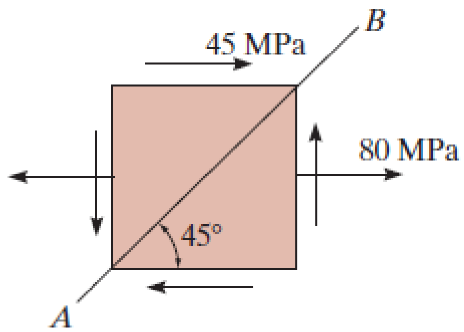
Solution TD 6

La notion de contrainte est basique en RDM. On utilise la relation de cauchy qui est une méthode directe pour avoir le vecteur contrainte sur n'importe quelle face de notre element. L'autre méthode qui est la transformation des équations avec rotation. Les deux méthodes donnent les mêmes résultats ; la méthode de transformation est meilleure car elle donne plus d'informations.

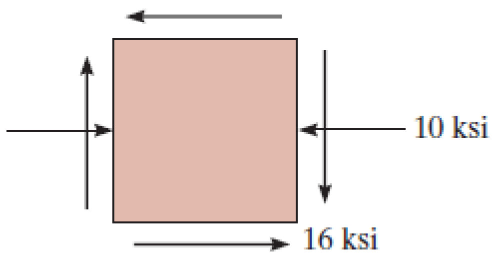
Problem 1



$\sigma_x = 150\text{ MPa} \qquad \sigma_y = 100\text{ MPa} \qquad \tau_{xy} = 75\text{ MPa}$



$\sigma_x = 80\text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = 45\text{ MPa}$



$\sigma_x = -10\text{ ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = -16\text{ ksi}$

Problem 2

STRATEGIE: Nous avons un état plan de contraintes donné. Pour calculer les composantes normale et tangentielle sur une facette donnée, on soit calculer d'abord le vecteur contrainte sur cette méme facette. La relation qui relie un état de contraintes (matrice ou tenseur des contraintes $\bar{\bar{\boldsymbol{\sigma}}}$) et le vecteur contrainte $\vec{\boldsymbol{T}}(\boldsymbol{M},\vec{\boldsymbol{n}})$ sur toute surface de normale $\vec{\boldsymbol{n}}$ est la relation de Cauchy, éq 3.9 du cours :

$$\vec{T}(M,\vec{n}) = \bar{\bar{\sigma}}\vec{n}$$

Une fois, nous avons le vecteur de contrainte, nous pouvons facilement calculer ses composantes en effectuant le produit scalaire du meme vecteur avec les vecteurs unitaires des directions demandées en utilisant les équations comme suit :

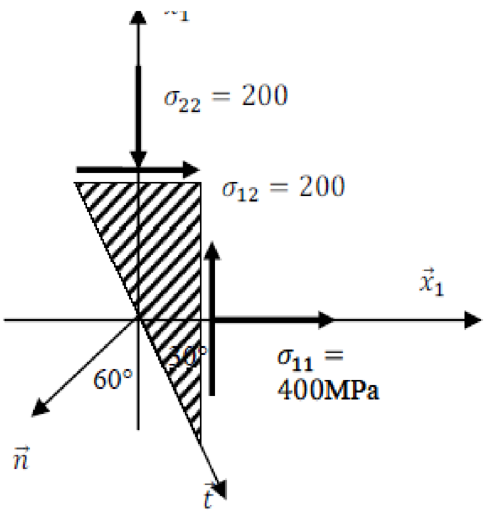
Composante normale : $\boldsymbol{\sigma_n} = \vec{T}(M,\vec{n}).\vec{\boldsymbol{n}}$ (équation 3.3)

Composante tangentielle : $\boldsymbol{\tau} = \vec{T}(M,\vec{n}).\vec{\boldsymbol{t}}$ (équation 3.4)

MODELISATION et ANALYSE:

Pour le tenseur des contraintes dans le plan donné. On a :

$$\bar{\bar{\sigma}} = \begin{pmatrix} \sigma_{11} & \tau_{21} \\ \tau_{12} & \sigma_{22} \end{pmatrix} = \begin{pmatrix} +400 & +200 \\ +200 & -200 \end{pmatrix}$$



Pour le vecteur contrainte sur la facette demandée. On applique la relation de Cauchy :

$$\vec{T}(M,\vec{n}) = \bar{\bar{\sigma}}\vec{n}$$

Le vecteur unitaire normal de cette facette est :

$$\vec{n} = \begin{pmatrix} -\sin\theta \\ -\cos\theta \end{pmatrix} = \begin{pmatrix} -\sin 60° \\ -\cos 60° \end{pmatrix} = \begin{pmatrix} -\dfrac{\sqrt{3}}{2} \\ -\dfrac{1}{2} \end{pmatrix}$$

$$\vec{T}(M,\vec{n}) = \bar{\bar{\sigma}}\vec{n} = \begin{pmatrix} +400 & +200 \\ +200 & -200 \end{pmatrix} \begin{pmatrix} -\dfrac{\sqrt{3}}{2} \\ -\dfrac{1}{2} \end{pmatrix} = \begin{pmatrix} -200\sqrt{3} - 100 \\ -100\sqrt{3} + 100 \end{pmatrix}$$

Pour les composantes : On projette :

Sachant que :

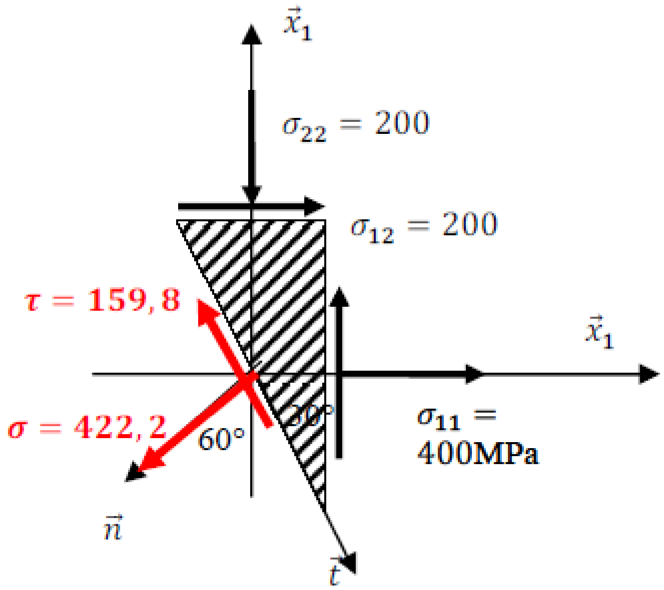
$$\vec{n} = \begin{pmatrix} -\sin 60° \\ -\cos 60° \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} \end{pmatrix} \qquad \vec{t} = \begin{pmatrix} +\sin 30° \\ -\cos 30° \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{pmatrix}$$

Composante normale : $\boldsymbol{\sigma_n} = \vec{T}(M,\vec{n}).\boldsymbol{\vec{n}}$

$$\boldsymbol{\sigma_n} = \begin{pmatrix} -200\sqrt{3} - 100 \\ -100\sqrt{3} + 100 \end{pmatrix} . \begin{pmatrix} -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} \end{pmatrix} = +423\ MPa \quad \text{(dans le sens de } \vec{n}\text{)}$$

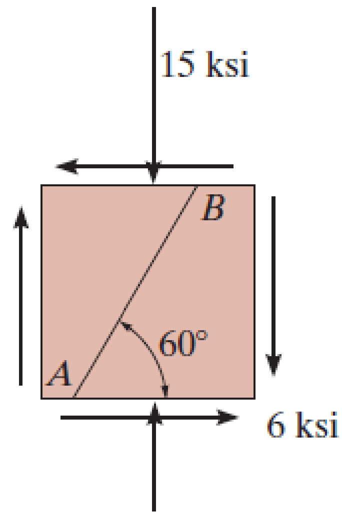
Composante tangentielle : $\boldsymbol{\tau} = \vec{T}(M,\vec{n}).\boldsymbol{\vec{t}}$

$$\boldsymbol{\tau} = \begin{pmatrix} -200\sqrt{3} - 100 \\ -100\sqrt{3} + 100 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{pmatrix} = -159.8\, MPa \quad \text{(sens opposé à } \vec{t}\text{)}$$



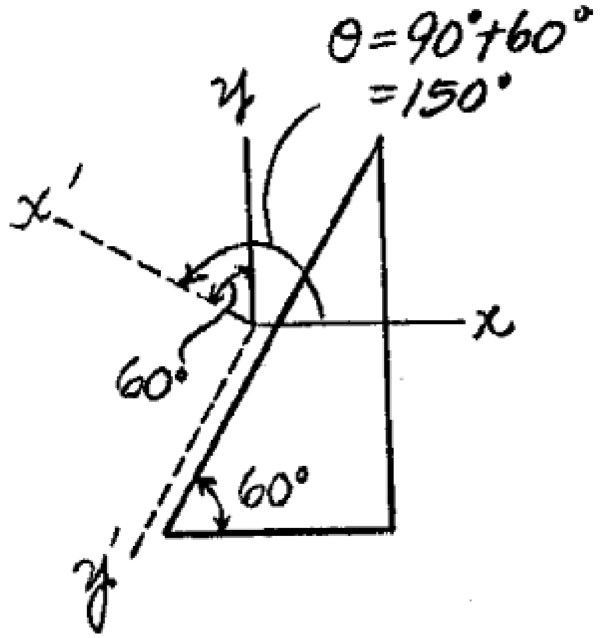
Problem 3 :

Première étape dans ce type de problème c'est d'écrire les différentes contraintes qu'on a en respectant la convention de signe :



$$\sigma_x = 0 \qquad \sigma_y = -15 \text{ ksi} \qquad \tau_{xy} = -6 \text{ ksi}$$

Pour une orientation donnée du plan d'étude, en plus de ces trois contraintes dans le plan initial, on aura besoin de l'angle de ce nouveau plan : (un petit croquis vous permet de le faire : il est clair que l'angle est antihoraire donc il positif et est égal à +150°.



Par la suite on applique les équations de transformation pour les contraintes

(remarque : les équations de transformation sont longues, les élèves doivent les ramener pendant les TD)

Stress Transformation Equations:

$\theta = +150°$ (Fig. *a*) $\qquad \sigma_x = 0 \qquad \sigma_y = -15 \text{ ksi} \qquad \tau_{xy} = -6 \text{ ksi}$

We obtain,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

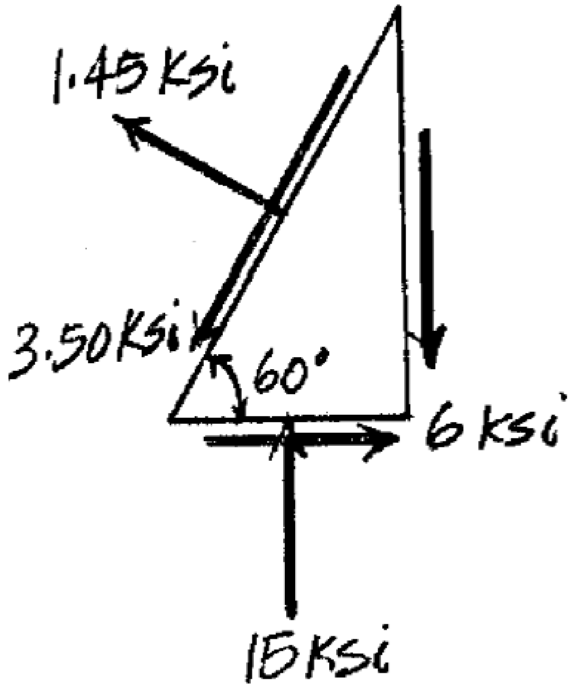
$$= \frac{0 + (-15)}{2} + \frac{0 - (-15)}{2}\cos 300° + (-6)\sin 300°$$

$$= 1.45 \text{ ksi}$$ **Ans.**

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$= -\frac{0 - (-15)}{2} \sin 300° + (-6) \cos 300°$$

$$= 3.50 \text{ ksi}$$ **Ans.**



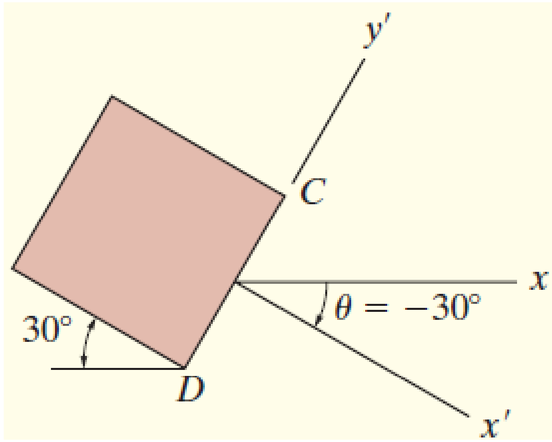
Problem 4:

First of all, we write the different given stresses using the sign convention:

$$\sigma_x = -80 \text{ MPa} \qquad \sigma_y = 50 \text{ MPa} \qquad \tau_{xy} = -25 \text{ MPa}$$

The orientation 30° is clockwise, so it is negative (plane CD) :

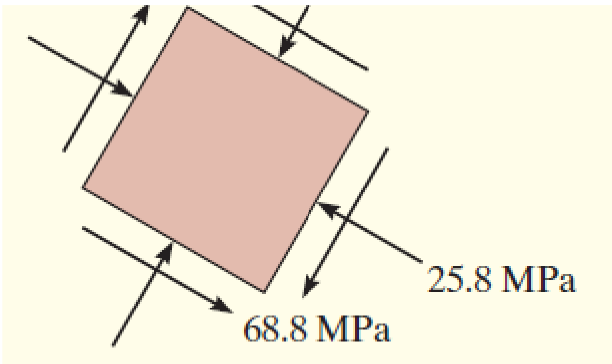
To obtain the stress components on plane C, the positive x' axis must be directed outward, perpendicular to CD, and the associated y' axis is directed along CD. The angle measured from the x to the x' axis is θ = -30° (clockwise). Applying Transformation Eqs. yields:



$$\begin{aligned}
\sigma_{x'} &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \\
&= \frac{-80 + 50}{2} + \frac{-80 - 50}{2}\cos 2(-30°) + (-25)\sin 2(-30°) \\
&= -25.8 \text{ MPa} \qquad \textit{Ans.}
\end{aligned}$$

$$\begin{aligned}
\tau_{x'y'} &= -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \\
&= -\frac{-80 - 50}{2}\sin 2(-30°) + (-25)\cos 2(-30°) \\
&= -68.8 \text{ MPa} \qquad \textit{Ans.}
\end{aligned}$$

On peut représenter ces contraintes sur le plan CD de la figure:



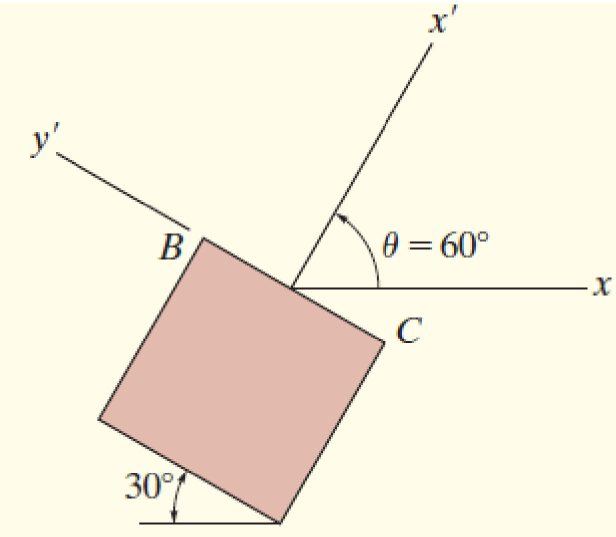
Plane BC. Establishing the x' axis outward from plane BC, Fig. 9–7c, then between the x and x' axes, $\theta = 60°$ (counterclockwise). Applying Eqs. 9–1 and 9–2,* we get

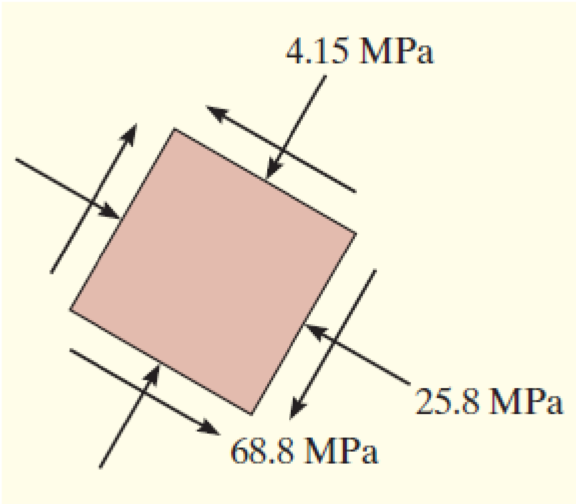
$$\sigma_{x'} = \frac{-80 + 50}{2} + \frac{-80 - 50}{2}\cos 2(60°) + (-25)\sin 2(60°)$$

$$= -4.15 \text{ MPa} \qquad \textit{Ans.}$$

$$\tau_{x'y'} = -\frac{-80 - 50}{2}\sin 2(60°) + (-25)\cos 2(60°)$$

$$= 68.8 \text{ MPa} \qquad \textit{Ans.}$$





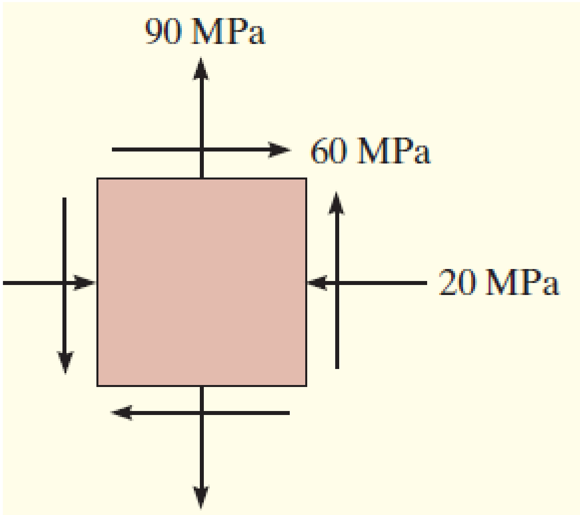
Non ces directions ne sont pas principales car le $\tau_{x'y'}$ n'est pas nul.

Problem 5:

SOLUTION

From the established sign convention,

$$\sigma_x = -20 \text{ MPa} \qquad \sigma_y = 90 \text{ MPa} \qquad \tau_{xy} = 60 \text{ MPa}$$



Orientation of Element. Applying Eq. 9–4,

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{60}{(-20 - 90)/2}$$

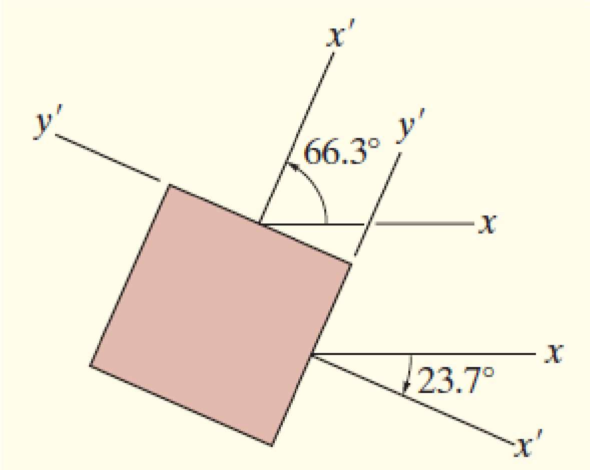
Solving, and referring to this first angle as θ_{p_2}, we have

$$2\theta_{p_2} = -47.49^\circ \qquad \theta_{p_2} = -23.7^\circ$$

Since the difference between $2\theta_{p_1}$ and $2\theta_{p_2}$ is 180°, the second angle is

$$2\theta_{p_1} = 180^\circ + 2\theta_{p_2} = 132.51^\circ \qquad \theta_{p_1} = 66.3^\circ$$

In both cases, θ must be measured positive *counterclockwise* from the x axis to the outward normal (x' axis) on the face of the element, and so the element showing the principal stresses will be oriented as shown in Fig. 9–11*b*.



(b)

The principal plane on which each normal stress acts can be determined by applying Eq. 9–1 with, say, $\theta = \theta_{p_2} = -23.7°$. We have

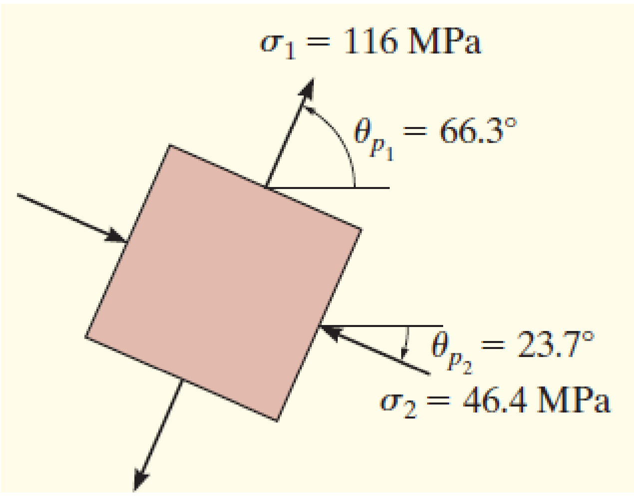
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{-20 + 90}{2} + \frac{-20 - 90}{2}\cos 2(-23.7°) + 60\sin 2(-23.7°)$$

$$= -46.4 \text{ MPa}$$

L'application pour $\sigma_{y'}$ donne : 116 MPa

On trace les contraintes principales sur le schéma sachant que la contrainte maximale σ_1 est suivant y'.



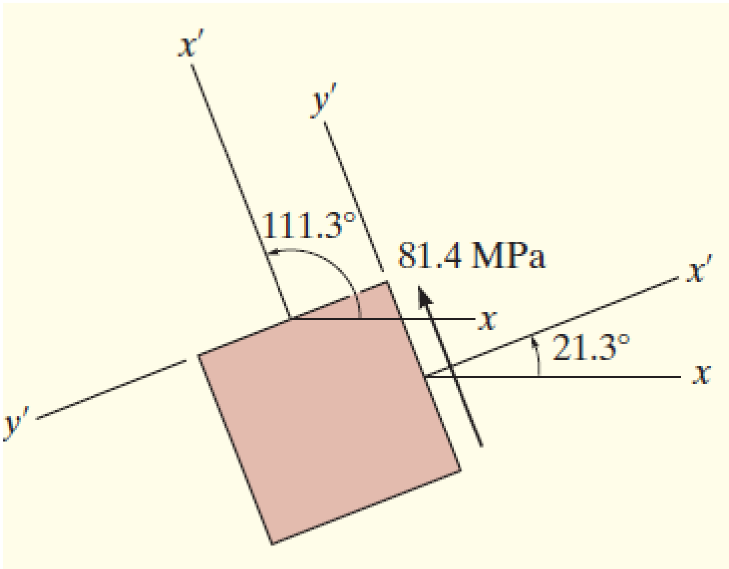
Orientation of Element. Since $\sigma_x = -20$ MPa, $\sigma_y = 90$ MPa, and $\tau_{xy} = 60$ MPa, applying Eq. 9–6, the two angles are

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(-20 - 90)/2}{60}$$

$$2\theta_{s_2} = 42.5^\circ \qquad \theta_{s_2} = 21.3^\circ$$

$$2\theta_{s_1} = 180^\circ + 2\theta_{s_2} \qquad \theta_{s_1} = 111.3^\circ$$

Remarquer que la contrainte de cisaillement se trouve à +- 45° des contraintes principales.



Pour calculer la valeur max du cisaillement, on applique l'équation :

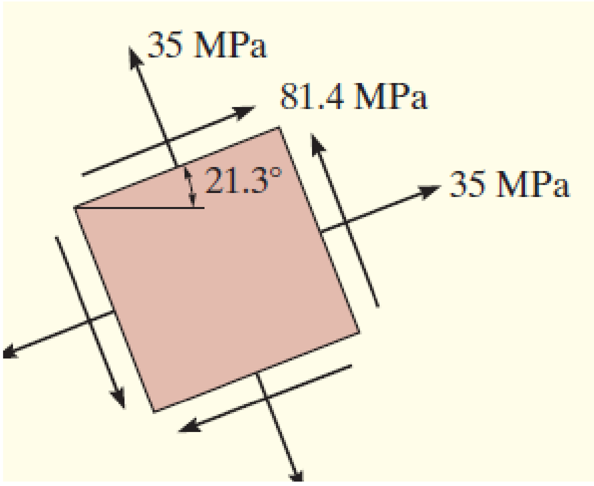
substituting $\theta = \theta_{s_2} = 21.3^\circ$ into Eq. 9–2. We have

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{-20 - 90}{2}\right)\sin 2(21.3^\circ) + 60\cos 2(21.3^\circ)$$

$$= 81.4 \text{ MPa}$$

La représentation de cette valeur maximale est la suivante :


35 MPa
81.4 MPa
21.3°
35 MPa