المدرسة الوطنية العليا لتكنولوجيا الأنظمة المستقلة

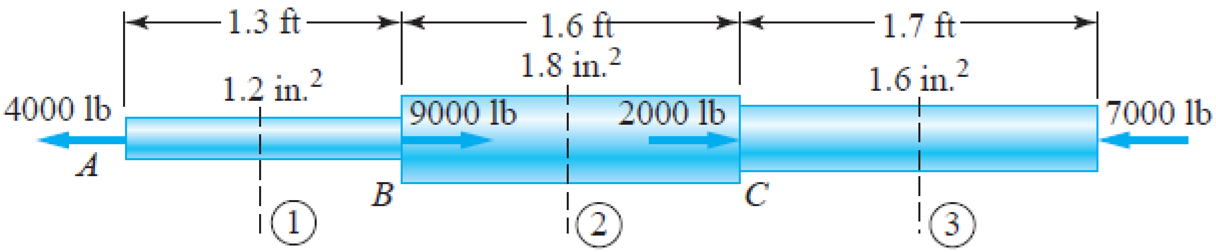
Academic year 2024/2025

2nd Year **Mechanics of Materials (Material Strength)**

T.D N° 7 (Cuts or Method of Sections, Axial Loading)

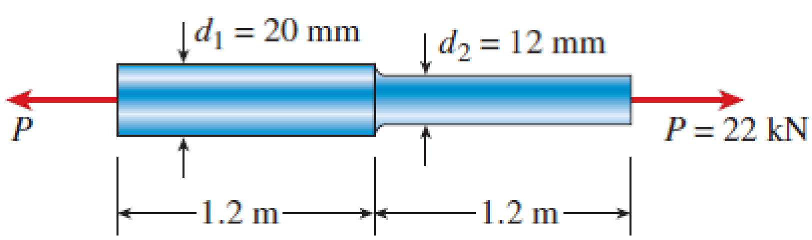
Problem 1 :

The steel bar ABCD is composed of 3 cylindrical segments with different lengths and diameters. Axial loads are applied. Calculate the normal stress in each segment. Draw the normal force N diagram.



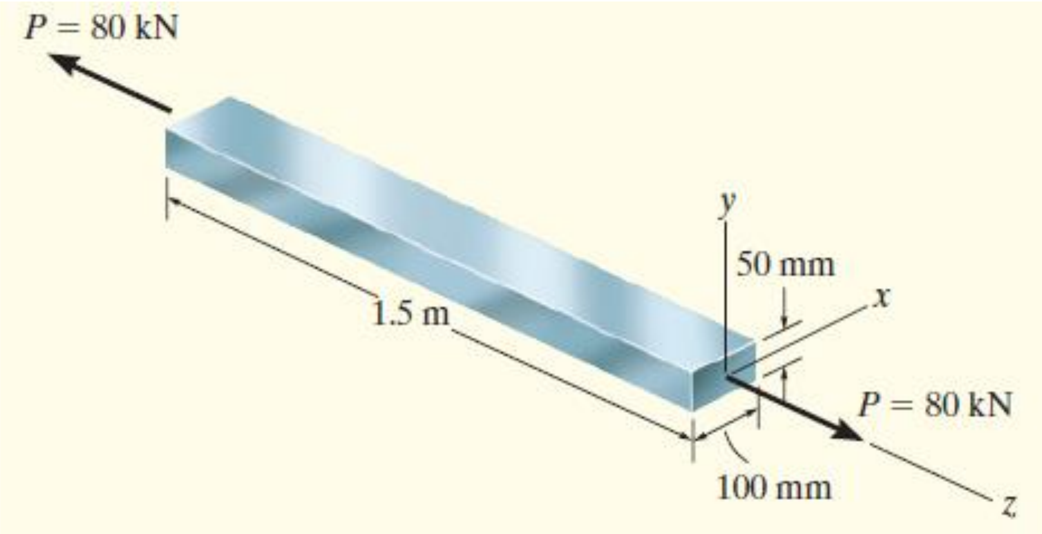
Problem 2 :

A circular steel bar with the modulus of elasticity E = 205 GPa.
(a) What must the elongation be under a tension P = 22 kN?
(b) Now the bar has one diameter and for the same length and volume, what must the elongation be in this case for the same load P?



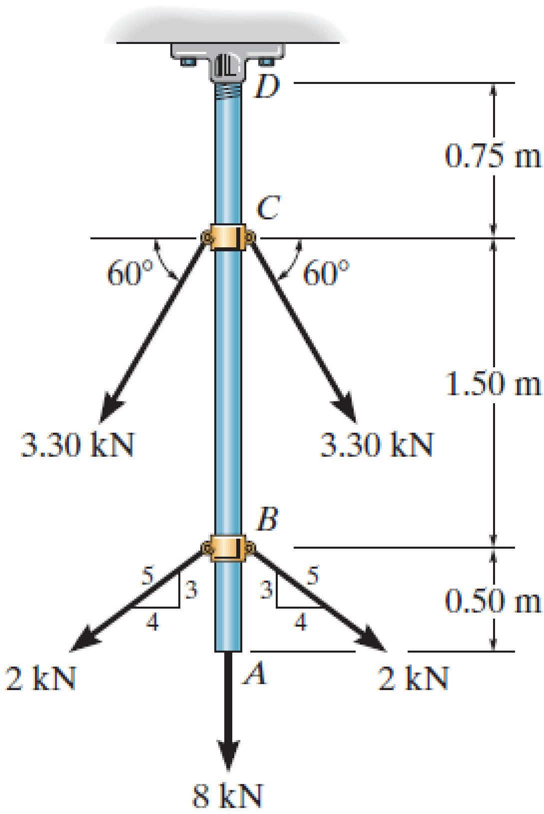
Problem 3 :

A bar made of A-36 steel (E = 200 GPa, ν = 0.32) has the dimensions shown in Fig. If an axial force of P = 80 kN is applied to the bar, determine the change in its length and the change in the dimensions of its cross section. The material behaves elastically.



Problem 4 :

The A992 steel rod (E = 200 GPa) is subjected to the loading shown. If the cross-sectional area of the rod is 60.10^{-6} m^2, determine the displacement of B and A, Neglect the size of the couplings at B, C, and D.

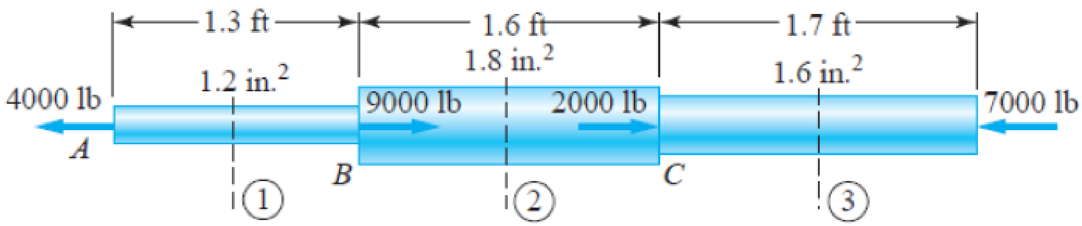


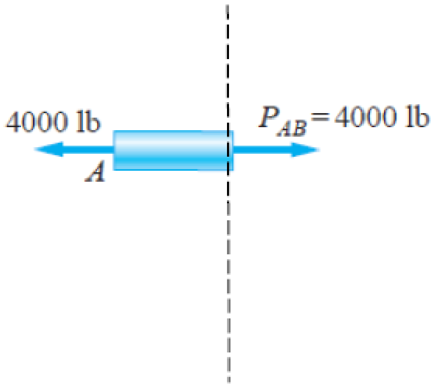
Solution TD 7

Problem 1 :

Pour calculer la contrainte normale dans chaque segment, nous allons appliquer la méthode des coupures.

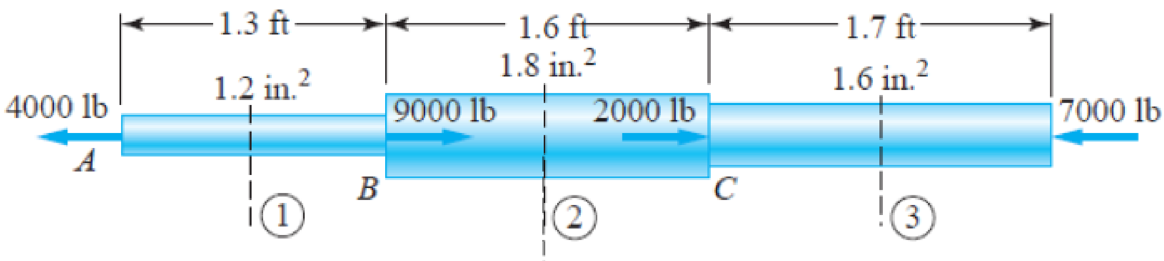
Coupure dans la section (1) :

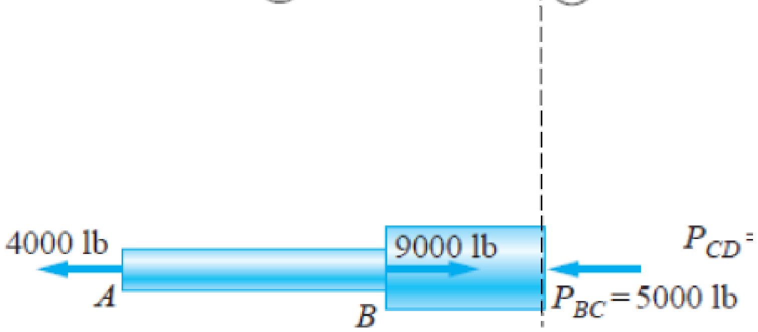




P_{AB} = 4000 lb (Traction)

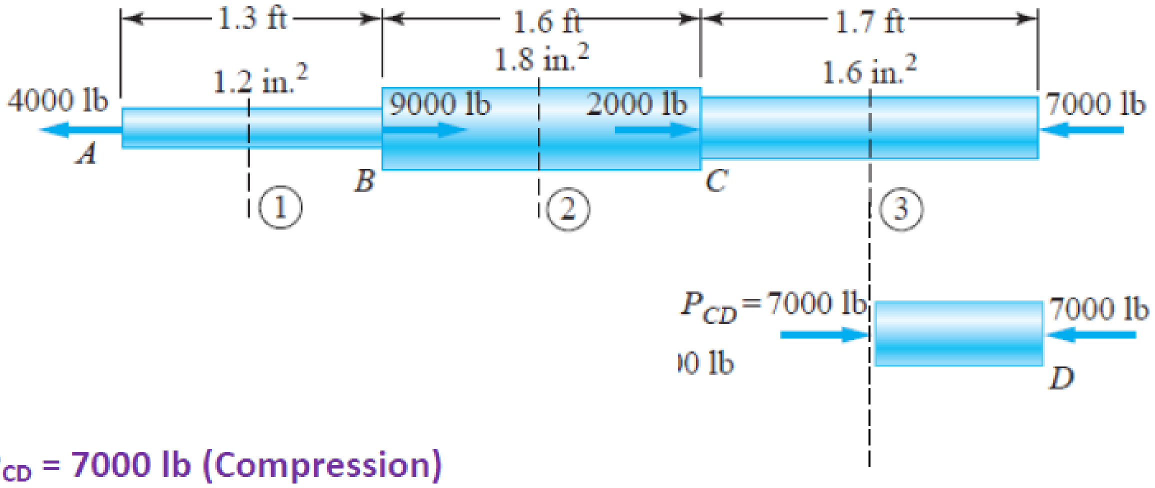
Coupure dans la section (2) :





P_{BC} = 5000 lb (Compression)

Coupure dans la section (3) :



P_{CD} = 7000 lb (Compression)

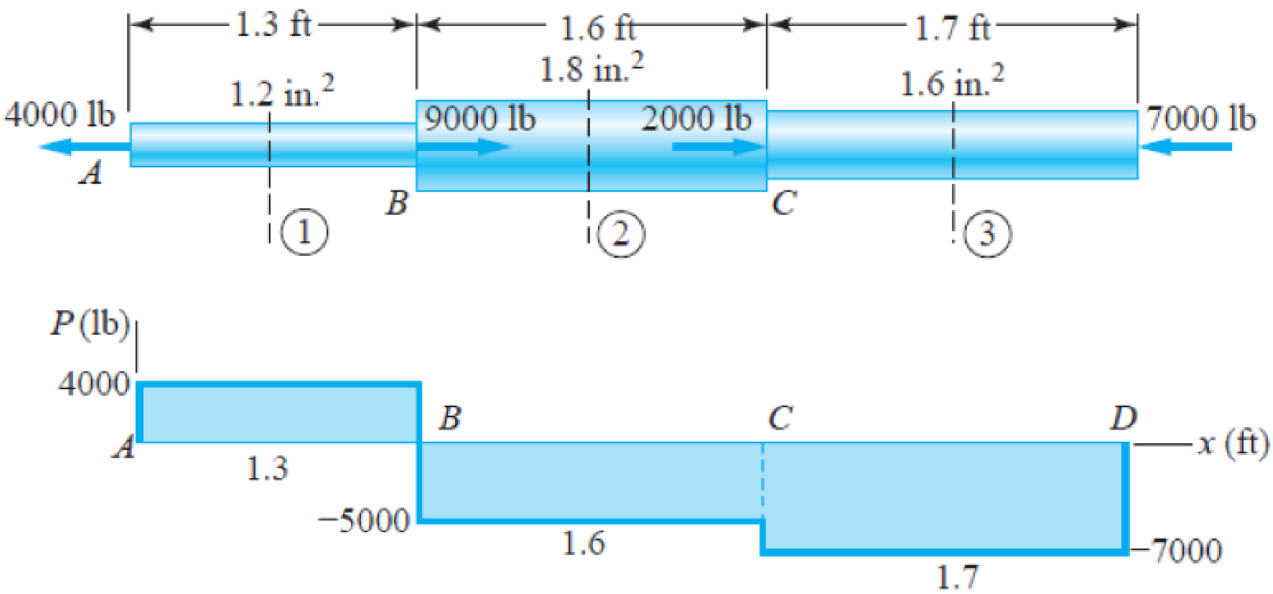
Noter que les forces internes varient d'un segment à un autre, mais la force dans chaque segment est constante.

Les contraintes normales dans les trois segments sont :

$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{4000 \text{ lb}}{1.2 \text{ in.}^2} = 3330 \text{ psi (T)} \qquad \sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{5000 \text{ lb}}{1.8 \text{ in.}^2} = 2780 \text{ psi (C)}$$

$$\sigma_{CD} = \frac{P_{CD}}{A_{CD}} = \frac{7000 \text{ lb}}{1.6 \text{ in.}^2} = 4380 \text{ psi (C)}$$

Le diagramme est le suivant (sachant qu'une traction est positive et que la force normale est constante dans chaque segment)



Problem 2 :

L'allongement total est donné par l'équation 4.22 du cours :

$$\delta_{tot} = \sum_{i=1}^{n} \frac{N_i L_i}{E_i A_i} = \frac{PL}{E}\left(\frac{1}{A_1} + \frac{1}{A_2}\right) = 1.549\ mm$$

Dans le deuxième cas, on a une seule barre, l'allongement est donné par l'équation 4.20 du cours :

$$\delta = \frac{N.L}{E.S}$$

On doit calculer la nouvelle section :

Le volume de la première barre est : $A_1L_1 + A_2L_2 = 0.0005124\ m^3$

La section de la nouvelle barre est :

$(A_1L_1 + A_2L_2)/L_{totale} = (0.0005124)/2.4 = 0.0002135\ m^2$.

On trouve :

$$\delta = \frac{N.L}{E.S} = \frac{22.10^3 X2.4}{205.10^9 X0.0002135} = 1.2\ mm$$

Problem 3 :

The normal stress in the bar is

$$\sigma_z = \frac{N}{A} = \frac{80(10^3)\ \text{N}}{(0.1\ \text{m})(0.05\ \text{m})} = 16.0(10^6)\ \text{Pa}$$

From the table in the back of the book for A-36 steel $E_{st} = 200$ GPa, and so the strain in the z direction is

$$\epsilon_z = \frac{\sigma_z}{E_{st}} = \frac{16.0(10^6)\ \text{Pa}}{200(10^9)\ \text{Pa}} = 80(10^{-6})\ \text{mm/mm}$$

The axial elongation of the bar is therefore

$$\delta_z = \epsilon_z L_z = [80(10^{-6})](1.5\ \text{m}) = 120\ \mu\text{m} \qquad \textit{Ans.}$$

Using Eq. 8–9, where $\nu_{st} = 0.32$ as found in the back of the book, the lateral contraction strains in *both* the x and y directions are

$$\epsilon_x = \epsilon_y = -\nu_{st}\,\epsilon_z = -0.32[80(10^{-6})] = -25.6\ \mu\text{m/m}$$

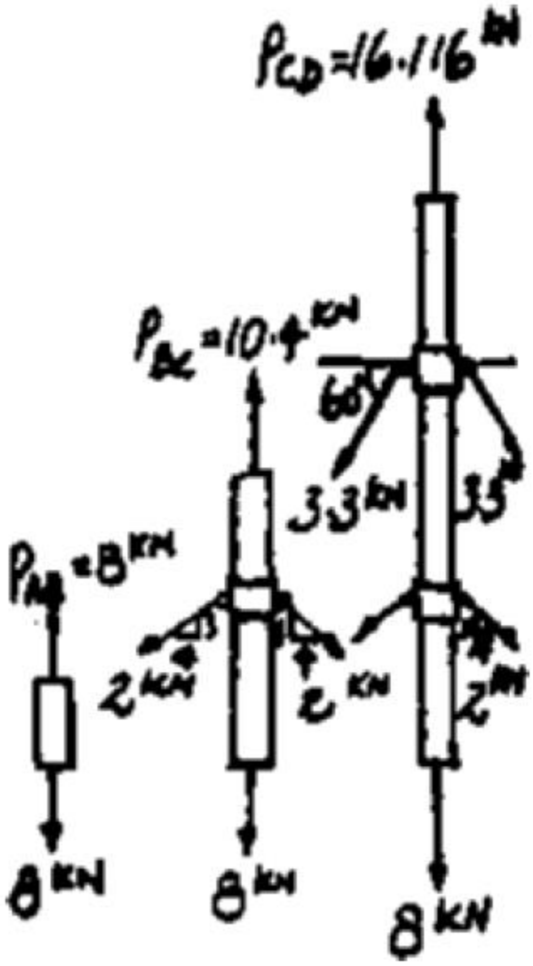
Thus the changes in the dimensions of the cross section are

$$\delta_x = \epsilon_x L_x = -[25.6(10^{-6})](0.1\ \text{m}) = -2.56\ \mu\text{m} \qquad \textit{Ans.}$$

$$\delta_y = \epsilon_y L_y = -[25.6(10^{-6})](0.05\ \text{m}) = -1.28\ \mu\text{m} \qquad \textit{Ans.}$$

Problem 4 :

La méthode des coupes permet d'avoir les forces normales dans les trois coupes



Le déplacement du B c'est la somme de l'allongement des deux barres BC et CD

$$\delta_B = \Sigma\frac{PL}{AE} = \frac{16.116(10^3)(0.75)}{60(10^{-6})(200)(10^9)} + \frac{10.4(10^3)(1.50)}{60(10^{-6})(200)(10^9)}$$

$$= 0.00231 \text{ m} = 2.31 \text{ mm}$$

Le déplacement de A est aussi :

$$\delta_A = \delta_B + \frac{8(10^3)(0.5)}{60(10^{-6})(200)(10^9)} = 0.00264 \text{ m} = 2.64 \text{ mm}$$