

Set N2

Ex1:

1) Potential energy

$$U = U_{\text{kin}} + U_{\text{pot}} + U_{\text{kin}} + U_{\text{kin}}$$

$$= \frac{1}{2} k (m_1 + \Delta l)^2 + k (m_2 + \Delta l)^2$$

$$+ 2l m_1 g \sin \theta - m_2 g l$$

$$U_{\text{pot}} = - \int \vec{W}_{m_1} \cdot d\vec{r}_m \quad \vec{W}_{m_1} = \begin{pmatrix} 0 \\ -m_1 g \end{pmatrix}$$

$$\vec{v}_m = \begin{pmatrix} -2l \cos \theta \\ 2l \sin \theta \end{pmatrix} \quad d\vec{r}_m = \begin{pmatrix} 2l \sin \theta \\ 2l \cos \theta \end{pmatrix}$$

$$\int 2m_1 g l \cos \theta d\theta = 2m_1 g l \sin \theta$$

$$\approx 2m_1 g l \theta$$

$$U = \frac{1}{2} k (2l\theta + \Delta l)^2 + k (l\theta + \Delta l)^2$$

$$+ 2l m_1 g \theta - m_2 g l \theta$$

$$U = 2k\theta^2 + k\Delta l^2 + k\Delta l^2 + 2k\theta\Delta l$$

$$+ 2l m_1 g \theta - m_2 g l \theta$$

$$U = 3k\theta^2 + k\Delta l^2 + \theta (2kl\Delta l + 2l m_1 g - m_2 g l)$$

at equilibrium

$$\frac{dU}{d\theta} \Big|_{\theta=0} = 0$$

$$6k\theta^2 + 2k\Delta l^2 + 2l m_1 g - m_2 g l$$

$$\frac{dU}{d\theta} \Big|_{\theta=0} = 0$$

$$2kl\Delta l + 2l m_1 g - m_2 g l = 0$$

$$\Delta l = \frac{m_2 g l - 2l m_1 g}{2k\theta} = \frac{g(m_2 - m_1)}{2k}$$

$$\Rightarrow U = 3k\theta^2 + k \left(\frac{g(m_2 - m_1)}{2k} \right)^2$$

$$U = 3k\theta^2 + \frac{g^2(m_2 - m_1)^2}{4k}$$

$$3) \Delta l = 0 \Leftrightarrow m_2 = 2m_1$$

$$4) m_2 = 2m_1$$

$$U = 3k\theta^2$$

Kinetic energy:

$$T = T_R + T_m + T_n$$

$$= \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{y}^2$$

$$= \frac{1}{4} (4m_1 l^2) \dot{\theta}^2 + \frac{1}{2} m_1 (2l\dot{\theta})^2 + m_1 l^2 \dot{\theta}^2$$

$$= m_1 l^2 \dot{\theta}^2 + 2m_1 l^2 \dot{\theta}^2 + m_1 l^2 \dot{\theta}^2$$

$$T = 4m_1 l^2 \dot{\theta}^2$$

$$L = T - U = 4m_1 l^2 \dot{\theta}^2 - 3k\theta^2$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = 0$$

$$8m_1 l^2 \dot{\theta} + 6k l^2 \theta = 0$$

$$\ddot{\theta} + \frac{3}{4} \frac{k}{m_1} \theta = 0$$

$$\theta(t) = \theta_{\max} \cos(\omega_L t + \phi)$$

Ex 2-

Kinetic energy

$$T = T_r + T_R = \frac{1}{2} M (R\dot{\theta})^2 + \frac{1}{2} \left(\frac{1}{2} MR^2 \right) \dot{\theta}^2$$

$$T = \frac{3}{4} MR^2 \dot{\theta}^2$$

Potential energy

$$U = U_{R_1} + U_{R_2} = \frac{1}{2} [k_1 (\Delta l_1 + R\theta + a\dot{\theta})^2 + k_2 (\Delta l_2 + R\theta + a\dot{\theta})^2]$$

$$\frac{\partial U}{\partial \theta} = \frac{1}{2} \left[k_1 (a+R) (\Delta l_1 + (R+a)\theta) + \right.$$

$$+ 2k_2 (a+R) (\Delta l_2 + (R+a)\theta) \right]$$

$$= (a+R) \left[k_1 (\Delta l_1 + (R+a)\theta) + k_2 (\Delta l_2 + (R+a)\theta) \right]$$

$$\frac{\partial U}{\partial \theta} \Big|_{\theta=0} = 0$$

$$= (R+a) (k_1 \Delta l_1 + k_2 \Delta l_2) = 0$$

$$k_1 \Delta l_1 + k_2 \Delta l_2 = 0$$

$$U = \frac{1}{2} \left[k_1 (\Delta l_1^2 + (a+R)^2 \theta^2 + 2\Delta l_1 (a+R)\theta) \right]$$

$$+ k_2 (\Delta l_2^2 + (a+R)^2 \theta^2 + 2\Delta l_2 (a+R)\theta)$$

$$U = \frac{1}{2} k_1 \Delta l_1^2 + \frac{1}{2} k_1 (a+R)^2 \theta^2$$

$$+ \frac{1}{2} k_2 \Delta l_2^2 + \frac{1}{2} k_2 (a+R)^2 \theta^2$$

$$U = \frac{1}{2} (a+R)^2 (k_1 + k_2) \theta^2 + C$$

$$L = T - U$$

$$L = \frac{3}{4} MR^2 \ddot{\theta}^2 - \frac{1}{2} (a+R)^2 (k_1 + k_2) \theta^2$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = 0$$

$$0 = \frac{3}{2} MR^2 \ddot{\theta} + (a+R)^2 (k_1 + k_2) \theta$$

$$\ddot{\theta} + \frac{2}{3} \frac{(a+R)^2 (k_1 + k_2) \theta}{MR^2}$$

Ex 3:

$$(1) \quad x = \frac{x_1}{2} + \frac{x_2}{4}$$

$$2T_1 = w$$

$$2k_1 m_1 = w$$

$$2T_2 = \frac{w}{2}$$

$$T_2 = \frac{w}{4}$$

$$k_2 m_2 = \frac{w}{4}$$

$$2k_1 m_1 = w$$

$$4k_2 m_2 = w \Rightarrow 2k_1 m_1 = 2k_2 m_2$$

$$U_{k_1} = \frac{1}{2} k_1 (m_1 + \Delta l_1)^2$$

$$U_{k_2} = \frac{1}{2} k_2 (m_2 + \Delta l_2)^2$$

$$T = \frac{1}{2} m \dot{x}^2$$

$$U_T = U_{k_1} + U_{k_2} + U_m$$

$$\text{from (1)} \quad x_2 = \frac{k_2 m_1}{2k_1},$$

$$(2) \rightarrow x = \frac{m_1}{2} + \frac{k_2 m_1}{8k_1}$$

$$m = m_1 \left(\frac{1}{2} + \frac{k_2}{8k_1} \right)$$

$$m = m_1 \left(\frac{4k_2 + k_1}{8k_1} \right) \Rightarrow \bar{m}_1 = \left(\frac{8k_1}{4k_2 + k_1} \right) m$$

$$U_{k_1} = \frac{1}{2} k_1 \left(\frac{8k_2}{4k_2 + k_1} m + \Delta l_1 \right)^2$$

$$\text{from (1)} \quad m_1 = \frac{2k_2 m_2}{k_1}$$

$$(3) \rightarrow m = \frac{k_2 m_2}{k_1} + \frac{m_2}{4}$$

$$m = m_2 \left(\frac{k_2}{k_1} + \frac{1}{4} \right) = m_2 \left(\frac{4k_2 + k_1}{4k_1} \right)$$

$$m_2 = m \left(\frac{4k_1}{4k_2 + k_1} \right)$$

$$\text{put } k_1 = 9k_2$$

$$U_{k_1} = \frac{1}{2} k_1 \left(m \left(\frac{4k_1}{8k_2 + k_1} \right) + \Delta l_1 \right)^2$$

$$U_{K_1} = \frac{1}{2} K_1 \left(\frac{4}{3} n + \Delta l_1 \right)^2$$

$$U_{K_2} = \frac{1}{2} K_2 \left(\frac{4}{3} n + \Delta l_2 \right)^2$$

$$U_m = -mg n$$

$$U_T = K_2 \left(\frac{4}{3} n + \Delta l_1 \right)^2 + \frac{1}{2} K_2 \left(\frac{4}{3} n + \Delta l_2 \right)^2 - mg n$$

$$\frac{dU_T}{dn} = 2K_2 \left(\frac{4}{3} \right) \left(\frac{4}{3} n + \Delta l_1 \right) + K_2 \left(\frac{4}{3} \right) \left(\frac{4}{3} n + \Delta l_2 \right) - mg$$

$$\frac{dU_T}{dn} \Big|_{n=0} = \frac{8}{3} K_2 \Delta l_1 + \frac{4}{3} K_2 \Delta l_2 - mg = 0$$

$$U_T = K_2 \left[\frac{16}{9} n^2 + 4\Delta l_1^2 + \frac{8}{3} n \Delta l_1 \right] + \frac{1}{2} K_2 \left[\frac{16}{9} n^2 + \Delta l_2^2 + \frac{9}{3} n \Delta l_2 \right] - mg n$$

$$= K_2 \frac{16}{9} n^2 + K_2 \Delta l_1^2 + \frac{16}{18} n^2 K_2 + \Delta l_2^2$$

$$U_T = \frac{8}{3} K_2 n^2 + C$$

$$L = T - U = \frac{1}{2} m \ddot{n}^2 - \frac{8}{3} K_2 n^2$$

$$\frac{d}{dt} \left(\frac{dL}{dn} \right) = m \ddot{n}$$

$$-\frac{\partial L}{\partial n} = \frac{16}{3} K_2 n$$

$$m \ddot{n} + \frac{16}{3} K_2 n = 0 \quad \omega_n = \sqrt{\frac{16}{3m} K_2}$$

$$\ddot{n} + \frac{16}{3m} K_2 n = 0$$

$$EX5 = \textcircled{1}$$

$$\frac{1}{2} K_1$$

$$\frac{1}{2} K_2$$

$$\frac{1}{2} K_3$$

$$1/m_1$$

$$1/m_2$$

$$1/m_3$$

$$1/m_4$$

$$1/m_5$$

$$1/m_6$$

$$1/m_7$$

$$1/m_8$$

$$1/m_9$$

$$1/m_{10}$$

$$1/m_{11}$$

$$1/m_{12}$$

$$1/m_{13}$$

$$1/m_{14}$$

$$1/m_{15}$$

$$1/m_{16}$$

$$1/m_{17}$$

$$1/m_{18}$$

$$1/m_{19}$$

$$1/m_{20}$$

$$1/m_{21}$$

$$1/m_{22}$$

$$1/m_{23}$$

$$1/m_{24}$$

$$1/m_{25}$$

$$1/m_{26}$$

$$1/m_{27}$$

$$1/m_{28}$$

$$1/m_{29}$$

$$1/m_{30}$$

$$1/m_{31}$$

$$1/m_{32}$$

$$1/m_{33}$$

$$1/m_{34}$$

$$1/m_{35}$$

$$1/m_{36}$$

$$1/m_{37}$$

$$1/m_{38}$$

$$1/m_{39}$$

$$1/m_{40}$$

$$1/m_{41}$$

$$1/m_{42}$$

$$1/m_{43}$$

$$1/m_{44}$$

$$1/m_{45}$$

$$1/m_{46}$$

$$1/m_{47}$$

$$1/m_{48}$$

$$1/m_{49}$$

$$1/m_{50}$$

$$1/m_{51}$$

$$1/m_{52}$$

$$1/m_{53}$$

$$1/m_{54}$$

$$1/m_{55}$$

$$1/m_{56}$$

$$1/m_{57}$$

$$1/m_{58}$$

$$1/m_{59}$$

$$1/m_{60}$$

$$1/m_{61}$$

$$1/m_{62}$$

$$1/m_{63}$$

$$1/m_{64}$$

$$1/m_{65}$$

$$1/m_{66}$$

$$1/m_{67}$$

$$1/m_{68}$$

$$1/m_{69}$$

$$1/m_{70}$$

$$1/m_{71}$$

$$1/m_{72}$$

$$1/m_{73}$$

$$1/m_{74}$$

$$1/m_{75}$$

$$1/m_{76}$$

$$1/m_{77}$$

$$1/m_{78}$$

$$1/m_{79}$$

$$1/m_{80}$$

$$1/m_{81}$$

$$1/m_{82}$$

$$1/m_{83}$$

$$1/m_{84}$$

$$1/m_{85}$$

$$1/m_{86}$$

$$1/m_{87}$$

$$1/m_{88}$$

$$1/m_{89}$$

$$1/m_{90}$$

$$1/m_{91}$$

$$1/m_{92}$$

$$1/m_{93}$$

$$1/m_{94}$$

$$1/m_{95}$$

$$1/m_{96}$$

$$1/m_{97}$$

$$1/m_{98}$$

$$1/m_{99}$$

$$1/m_{100}$$

$$1/m_{101}$$

$$1/m_{102}$$

$$1/m_{103}$$

$$1/m_{104}$$

$$1/m_{105}$$

$$1/m_{106}$$

$$1/m_{107}$$

$$1/m_{108}$$

$$1/m_{109}$$

$$1/m_{110}$$

$$1/m_{111}$$

$$1/m_{112}$$

$$1/m_{113}$$

$$1/m_{114}$$

$$1/m_{115}$$

$$1/m_{116}$$

$$1/m_{117}$$

$$1/m_{118}$$

$$1/m_{119}$$

$$1/m_{120}$$

$$1/m_{121}$$

$$1/m_{122}$$

$$1/m_{123}$$

$$1/m_{124}$$

$$1/m_{125}$$

$$1/m_{126}$$

$$1/m_{127}$$

$$1/m_{128}$$

$$1/m_{129}$$

$$1/m_{130}$$

$$1/m_{131}$$

$$1/m_{132}$$

$$1/m_{133}$$

$$1/m_{134}$$

$$1/m_{135}$$

$$1/m_{136}$$

$$1/m_{137}$$

$$1/m_{138}$$

$$1/m_{139}$$

$$1/m_{140}$$

$$1/m_{141}$$

$$1/m_{142}$$

$$1/m_{143}$$

$$1/m_{144}$$

$$1/m_{145}$$

$$1/m_{146}$$

$$1/m_{147}$$

$$1/m_{148}$$

$$1/m_{149}$$

$$1/m_{150}$$

$$1/m_{151}$$

$$1/m_{152}$$

$$1/m_{153}$$

$$1/m_{154}$$

$$1/m_{155}$$

$$1/m_{156}$$

$$1/m_{157}$$

$$1/m_{158}$$

$$1/m_{159}$$

$$1/m_{160}$$

$$1/m_{161}$$

$$1/m_{162}$$

$$1/m_{163}$$

$$1/m_{164}$$

$$1/m_{165}$$

$$1/m_{166}$$

$$1/m_{167}$$

$$1/m_{168}$$

$$1/m_{169}$$

$$1/m_{170}$$

$$1/m_{171}$$

$$1/m_{172}$$

$$1/m_{173}$$

$$1/m_{174}$$

$$1/m_{175}$$

$$1/m_{176}$$

$$1/m_{177}$$

$$1/m_{178}$$

$$1/m_{179}$$

$$1/m_{180}$$

$$1/m_{181}$$

$$1/m_{182}$$

$$1/m_{183}$$

$$1/m_{184}$$

$$1/m_{185}$$

$$1/m_{186}$$

$$1/m_{187}$$

$$1/m_{188}$$

$$1/m_{189}$$

$$1/m_{190}$$

$$1/m_{191}$$

$$1/m_{192}$$

$$1/m_{193}$$

$$1/m_{194}$$

$$1/m_{195}$$

$$1/m_{196}$$

$$1/m_{197}$$

$$1/m_{198}$$

$$1/m_{199}$$

$$1/m_{200}$$

$$1/m_{201}$$

$$1/m_{202}$$

$$1/m_{203}$$

$$1/m_{204}$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} + \frac{\partial D}{\partial \dot{\theta}} = 0$$

$$U_{m_1} = \frac{3}{2} m_1 g R \dot{\theta}^2$$

$$U_{m_2} = -m_2 g R \dot{\theta}$$

$$U_{m_3} = \frac{1}{2} m_3 g R \dot{\theta} = m_3 g R \ddot{\theta}$$

$$U = \frac{1}{2} m_1 g R \dot{\theta}^2 + m_2 g R \dot{\theta} + \frac{1}{2} k_1 (-R\dot{\theta} + \Delta l_1)^2$$

$$+ \frac{1}{2} k_2 (R\dot{\theta} + \Delta l_2)^2$$

$$\frac{dU}{d\theta} \Big|_{\theta=0} = 0$$

$$\frac{dU}{d\theta} = m_2 g R \dot{\theta} + m_2 g R - k_1 R (\dot{\theta} - \Delta l_1) \\ + k_2 R (\dot{\theta} + \Delta l_2)$$

$$\frac{dU}{d\theta} \Big|_{\theta=0} = m_2 g R - k_1 R \Delta l_1 + k_2 R \Delta l_2 = 0$$

$$U = \frac{1}{2} m_1 g R \dot{\theta}^2 + k_1 (\dot{\theta} - \Delta l_1)^2 + \frac{1}{2} k_2 (\dot{\theta} + \Delta l_2)^2$$

$$T = T_{m_1} + T_{m_2} + T_{m_3} + T_{m_4}$$

$$T_{m_1} = \frac{1}{2} I_{I_B} \dot{\theta}^2 = \frac{1}{2} m_1 R^2 \dot{\theta}^2$$

$$T_{m_2} = \frac{1}{2} I_{I_B} \dot{\theta}^2 = \frac{1}{4} M R^2 \dot{\theta}^2$$

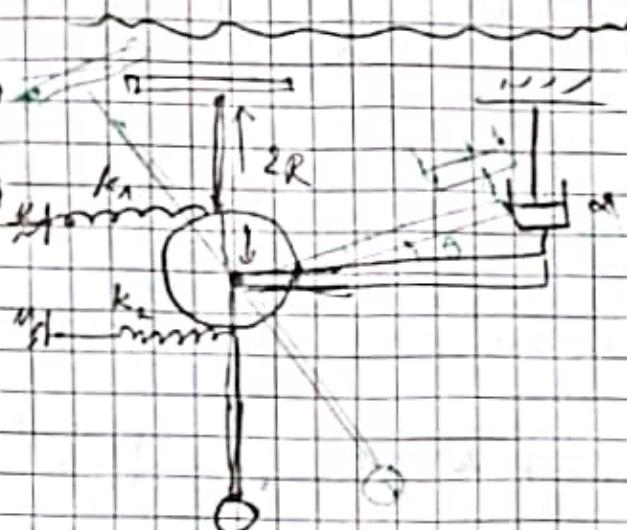
$$T_{m_3} = T_{Rm_3} + T_{Tm_3} = \frac{1}{2} \left(\frac{1}{k_1} m R^2 \right) \dot{\theta}^2 + \frac{1}{2} m (2R)^2$$

$$T_{m_4} = \frac{49}{24} m R^2 \dot{\theta}^2$$

$$T_{m_5} = \frac{1}{2} \left(\frac{1}{3} m_2 R^2 \right) \dot{\theta}^2 = \frac{1}{3} m R^2 \dot{\theta}^2$$

$$\ddot{\theta} + \frac{\alpha}{3m} \dot{\theta} + \frac{2K}{3m} \theta = 0$$

$\frac{2\alpha}{3m}$ w_n^2



$$U = U_{m_1} + U_{m_2} + U_{m_3} + U_{m_4} + U_{R_2}$$

$$U_{m_1} = \frac{1}{2} k_1 (R\dot{\theta} + \Delta l_1)^2$$

$$U_{m_2} = \frac{1}{2} k_2 (R\dot{\theta} + \Delta l_2)^2$$

$$\vec{r}_{m_1} = \begin{pmatrix} 3R \sin \theta \\ -3R \cos \theta \end{pmatrix} \quad \vec{dr}_{m_1} = \begin{pmatrix} 3R \cos \theta d\theta \\ 3R \sin \theta d\theta \end{pmatrix}$$

$$\vec{W}_{m_1} = \begin{pmatrix} 0 \\ -mg \end{pmatrix}$$

$$\vec{W}_{m_1} \cdot \vec{dr}_{m_1} = -mg 3R \sin \theta d\theta$$

$$U_{m_1} = \int \vec{W}_{m_1} \cdot \vec{dr}_{m_1} = mg 3R \int_{0}^{\theta} \sin \theta d\theta \\ = -mg 3R [\cos \theta]_0^\theta \\ = -mg 3R [\cos \theta - 1]$$

$$= \frac{(\cos \theta - 1)}{(1 - \dot{\theta}^2 - 1)}$$

$$T = \frac{57}{8} m R^2 \dot{\theta}^2$$

$$U = \frac{1}{2} \omega (R\dot{\theta})^2$$

$$\frac{d}{dt} \left(\frac{\partial U}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} + \frac{\partial D}{\partial \dot{\theta}} = 0$$

$$L = \frac{57}{8} m R^2 \dot{\theta}^2 - \left(\frac{1}{2} mg R \dot{\theta}^2 + k R^2 \dot{\theta}^2 \right)$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) = \frac{57}{4} m R^2 \ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = -(mgR + 2kR^2)\dot{\theta}$$

$$\frac{57}{4} m R^2 \ddot{\theta} + (mgR + 2kR^2)\dot{\theta} + kR^2 \dot{\theta} = 0$$

$$\ddot{\theta} + \underbrace{\frac{4}{57} \frac{2}{m} \dot{\theta}}_{2\zeta} + \underbrace{\frac{4}{57} \left(\frac{g}{R} + \frac{2k}{m} \right) \dot{\theta}}_{\omega_n^2} = 0$$