Tutorial Sheet N°4

Exo1:

1- the representative vector

$(1,1,0,1,-1,0,1,0,0)$

Ex 3:

1- $(e_1, e_{10}, e_3) = \{1,0,0,-1,0,0,0,0,0,-1,0\}$

$(e_4, e_5, e_6, e_8) = \{0,0,0,-1,1,1,0,1,0,0,0\}$

$(e_1, e_{10}, e_{11}, e_2) = \{-1,1,0,0,0,0,0,0,0,0,1,1\}$

2) $c = 11-7+1 = 5$

$(e_1, e_{10}, e_3), (e_5, e_6, e_7)$

$(e_4, e_7, e_8), (e_3, e_{11}, e_2)$

3) cocycle:

$A = \{v_1, v_7\}$ $\Omega^+(A) = (e_1, e_7, e_9, e_{11})$

$\Omega^-(A) = \emptyset$

$B = \{v_1, v_7, v_2\}$

$\Omega^+(B) = (e_3, e_9, e_{11})$

$\Omega^-(B) = (e_4, e_8, e_{10})$

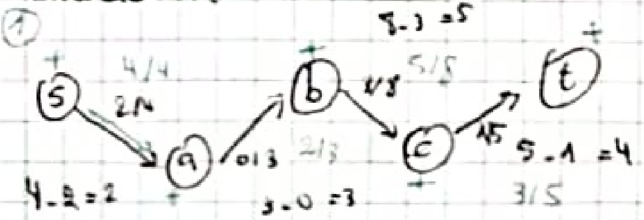
$C = \{v_1, v_2, v_6, v_7\}$

$\Omega^+(C) = \{e_9\}$ $\Omega^-(C) = (e_4, e_8)$

elementary co-cycles $\Omega(A)$, $\Omega(C)$

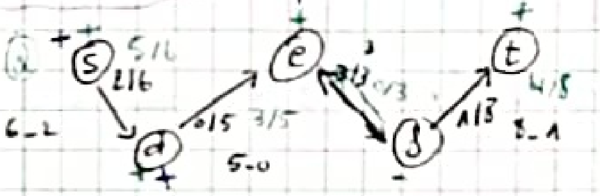
Exercise 4:

(1)



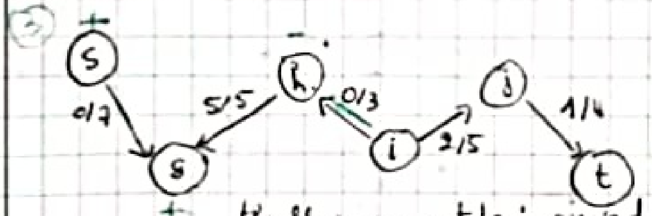
min = 2

(2)

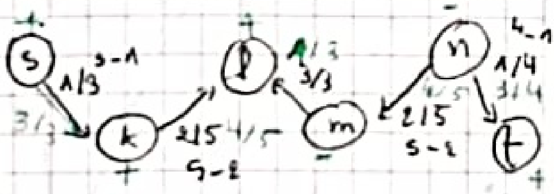


min = 3

(3)

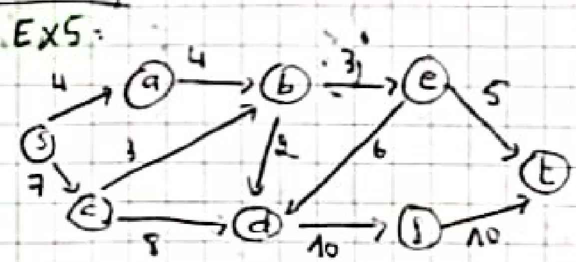


the flow can not be improved

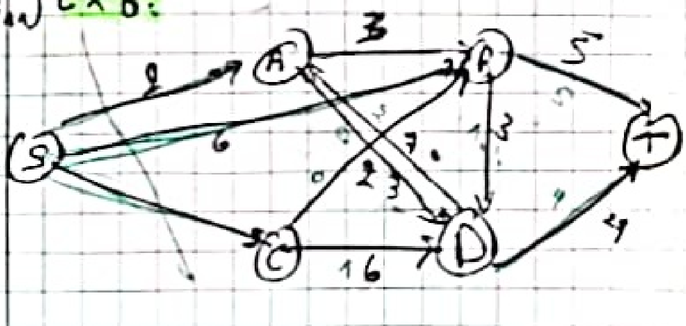


min = 2

Ex5:



Ex6:



S_B_T min=5

S_A_D_T min=2

S_C_D_T min = 1

S_B_D_T min=1

max = 9

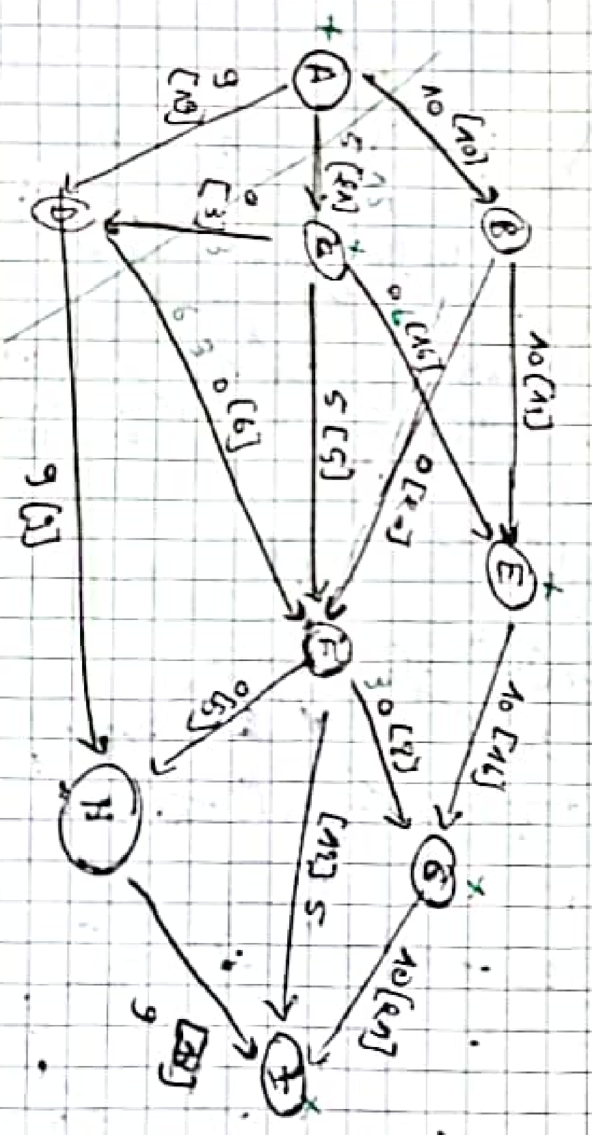
The minimal cut = $\Omega(S)$

$\Omega(S) = \Omega^+(S) \cup \Omega^-(S)$

$= \{+(S,A), +(S,C), +(S,B)\}$

Capacity $(\Omega(A)) = 9$ = maximum flow

Ex 7:



current flow = 24

A_C_E_G_I :

min{21-5, 16-0, 16-10, 21-10}

min{16, 16, 6, 11} = 6

A_C_D_F_G_I :

min{10, 3, 6, 8, 5} = 3

A_D_F_I

min (10, 3, 7) = 3

A_C_E_B_F_H_I :

min(7, 10, 10, 20, 5, 7) = 5

A_D_C_E_B_F_G_I :

min{7, 3, 5, 5, 15, 5, 2} = 2

A_D_C_E_B_F_I

min{5, 1, 3, 3, 13, 4} = 1

A_C_E_B_F_I :

min{2, 2, 2, 2, 3} = 2

flow = 24 + 6 + 3 + 3 + 5 + 2 + 1 + 2 = 46

minimum cut:

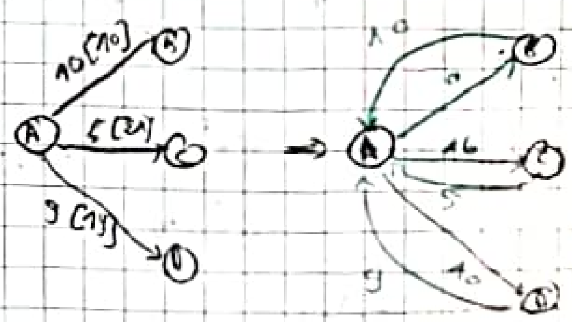
$\Omega(A,D) = \Omega^+ \cup \Omega^-(A,D)$

{AB, AC, DF, DH} ∪ {CD}

its capacity

C = 46

the gap graph

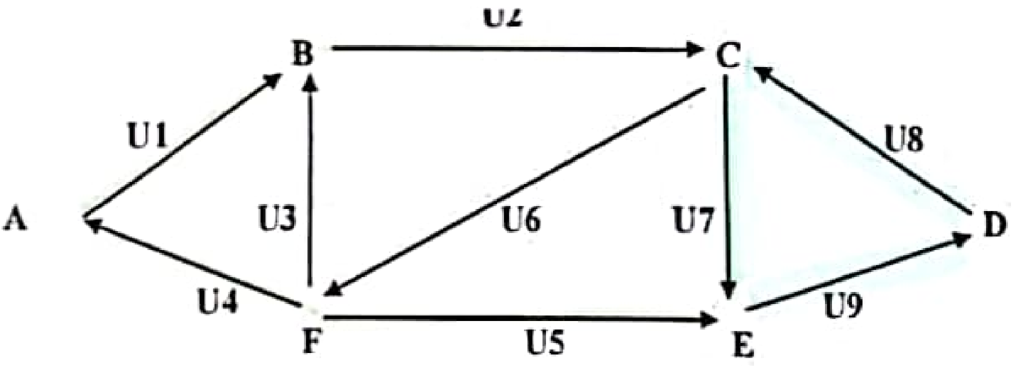


Level: 2nd Year Common Core **Module:** Computer science 3

Academic Year: 2024-2025

Tutorial Sheet N°4 (1/2)

Exercise 1:

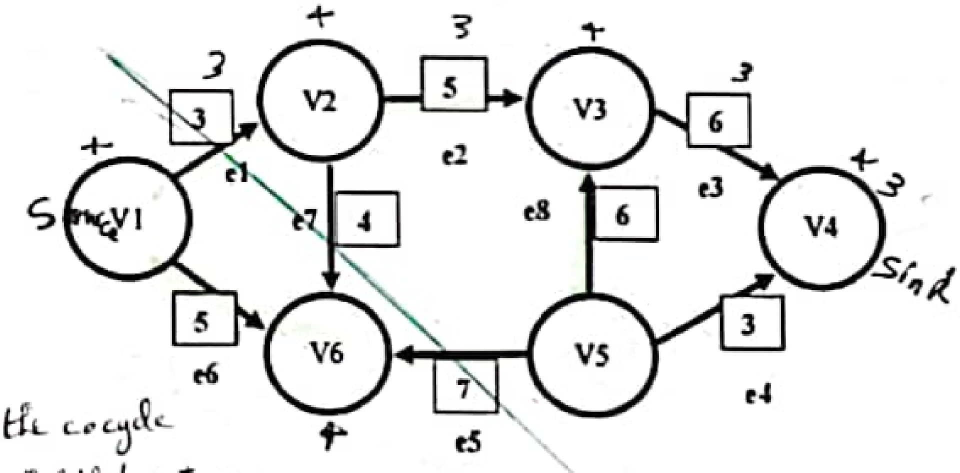


1. What is the representative vector γ of the cycle Γ(U1,U2,U7,U5,U4)?
2. Give the co-cycle Ω(C,D,E). What is its representative vector ω.

(0, -1, 0, 0, -1, 1, 0, 0, 0)

Exercise 2:

Let the following graph G be: The capacity of the minimal cut = the maximum flow



The minimal cut = the cocycle associated to the labelled vertices

1. The vector representing the cycle e1 e2 e8 e5 e6 is: none (1,1,0,0,1,-1,0,-1)

(+1 +1 -1 0 -1 +1 0 0) ☐ (+1 +1 0 0 +1 -1 0 0) ☐ (+1 +1 +1 +1 0 0 0 0) ☐

2. The vector representing the cycle e2 e8 e5 e7 is:

(0 +1 0 0 +1 0 -1 -1) ☑ (0 -1 0 0 -1 0 +1 +1) ☐ (0 +1 -1 -1 +1 0 0 0) ☐

The co-cycle associated with the vertex set (V2 V3 V5 V6) is:

=(e1,e6) U (e3,e4) ☒ =(e3, e6) U (e1, e4) ☐ (e2, e7) U (e5, ☐

The representative vector of the co-cycle Ω(V2V3V5V6) is:

(-1, 0 +1 +1 0 -1 0 0) ☒ (+1 0 -1 +1 0 -1 0 0) ☐ (0 -1 0 0 +1 0 -1 ☐

The co-cycle Ω(V2 V3 V5 V6) is elementary:

True ☐ False ☒

The co-cycle Ω(V1 V2 V6) is elementary:

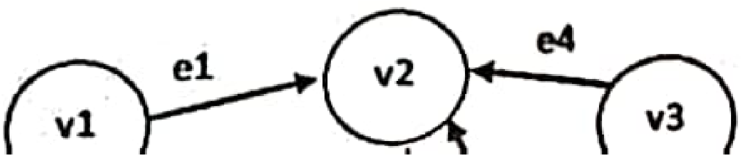
True ☒ False ☐

y applying the FORD-FULKERSON algorithm, the maximum flow in the graph G:

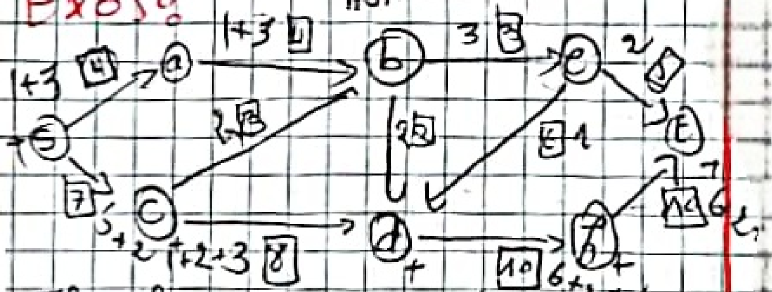
8 ☐ 9 ☐ None of the previous ansv ☒

rcise 3 :

t the following graph G be:



Exo5 :



The chain : { s → c → d → f → t }

{ 7-5 , 8-3 , 10-6 , 12-8 }

{ 2 , 5 , 4 , 4 }

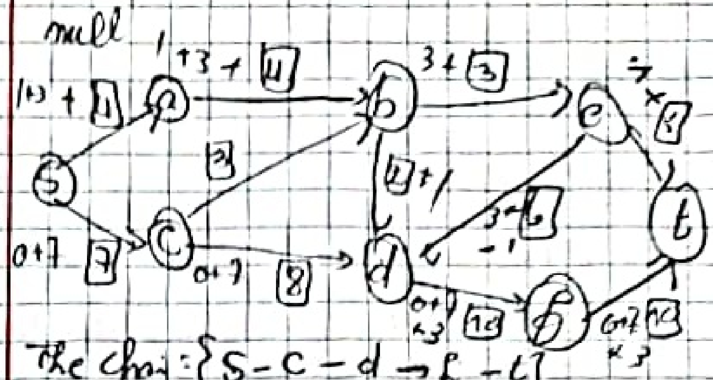
min = 2

The second { s → a → b ← c → d → f → t }

4 - 3   4 - 3   2   8-5   10-8

min = 3

F = 8 + 2 + 1 = 11



The chai : { s - c - d → f - t }

min = 7

The chai : { s - a - b - e - d - f - t }

The min : 3

The chai : { s - a - b - d - e - t }

The min : 1

F = 0+7 + 3 + 1 = 11