Epsilite

Vectorial Analysis

Exercise 1:

Let $\overrightarrow{F}$ be a vector field defined by $\overrightarrow{F}(x,\ y,\ z) = (2xy,\ x^2 + 3yz^2,\ 2y^3z - 4z^3)$.

1) Determine the scalar potential f of $\overrightarrow{F}$ such that $f(1,\ 1,\ 1) = 0$, using two methods.

Let the field $\overrightarrow{V}$ and the surface of the closed cylinder S defined as

$$\overrightarrow{V}(x,\ y,\ z) = xz\,\overrightarrow{i} + yz\,\overrightarrow{j} + z^2\,\overrightarrow{k}.$$
$$S = \{(x,\ y,\ z) \in \mathbb{R}^3\ /\ x^2 + y^2 = 1,\ 0 \leq z \leq 1\}.$$

2) Calculate the flux of $\overrightarrow{V}$ coming out of S, and find the same result using Ostrogradsky Formula.

Exercise 2:

Let $\overrightarrow{f}(x,\ y,\ z) = (y^2 + z^2,\ -xy,\ -xz)$ be a vector field defined on $D = \{(x,\ y,\ z) \in \mathbb{R}^3\ /\ z > 0\}$.

1) Determine a function $\varphi(z)$ such that $\varphi(1) = 1$ and $\overrightarrow{g}(x,\ y,\ z) = \varphi(z)\overrightarrow{f}(x,\ y,\ z)$ be a rotational field over D.

2) Find a vectorial potential for $\overrightarrow{g}$ such that

$$\overrightarrow{G}(x,\ y,\ z) = (P(x,\ y,\ z),\ Q(x,\ y,\ z),\ 0).$$

Exercise 3:

Let $\overrightarrow{W}$ be a vectorial field defined by

$$\overrightarrow{W}(x,\ y,\ z) = (xy,\ yz,\ -z^2).$$

1) Compute the divergence of $\overrightarrow{W}$.
2) Using Stokes Theorem, calculate the flux φ of $\overrightarrow{W}$ coming out the paraboloid Γ assosiated with $z = x^2 + y^2$ and $0 \leq z \leq 1$.
3) Verify Stokes Theorem for the fallowing case:

$$\overrightarrow{F}(x,\ y,\ z) = (z,\ y,\ x), \text{ and } S = \{(x,\ y,\ z) \in \mathbb{R}^3\ /\ z = x^2 + y^2,\ 0 < z < 1\}.$$

4) Compute, using the Ostrogradsky formula, the flux of the scalar field $\overrightarrow{F}(x,\ y,\ z) = (x^2, y^2, z^2)$ through the closed exterior surface of the unity sphere.

Exercise 4:

Given a vectorial field $\overrightarrow{V}(x,\ y,\ z) = (xg(y,\ z),\ y^2,\ z^2)$ with g being a function of class C^∞ over $\mathbb{R}^2$.

1) Determine g such that a function U exists and satisfies $\overrightarrow{V}(x,\ y,\ z) = \overrightarrow{Curl}\ U(x,\ y,\ z)$.
2) Find the function U such that $\overrightarrow{V}(x,\ y,\ z) = \overrightarrow{Curl}\ U(x,\ y,\ z)$ and $U(x,\ y,\ z) = (0,\ P(x,\ y,\ z),\ Q(x,\ y,\ z))$.
3) Using Stokes Theorem, calculate the flux of $\overrightarrow{V}$ passing through the demi-sphere associated with the equations $z \geq 1$ and $x^2 + y^2 + (z-1)^2 = 1$.

Exercise 5:

Let Γ be a parametrized curve given by

$$\begin{cases} x(t) &= \dfrac{t^2-3}{2} \\ y(t) &= \dfrac{4}{3}t\sqrt{t} \\ z(t) &= 2t+1 \end{cases} \qquad (t \in [0,\ 1])$$

1) Compute the length of Γ.
2) Compute $\displaystyle\int_\Gamma ds$ where $f(x,\ y,\ z) = x + yz - 2$.
3) Compute, in the direct sense, $\displaystyle\int_\Gamma \vec{f}(x,\ y,\ z)\ ds$ in these cases:

1. $\overrightarrow{f}(x,\ y,\ z) = (4xy,\ 3y^2,\ 5z)$ and Γ is the arc linking the points $(0,\ 1,\ 1)$ and $(1,\ 1.5,\ \log 2+1)$ of the trajectory of the movement which the vector speed is $\overrightarrow{V}(t) = (e^t,\ e^{-t},\ 1)$ with $t \in [0,\ b]$ with $b \in \mathbb{R}$ to be determine.

2. $\overrightarrow{f}(x,\ y,\ z) = (0,\ 0,\ y)$ and $\Gamma = \Sigma_1 \cap \Sigma_2$ where Σ_1 is the cylinder of the axis 0_z of radius $R > 0$ and Σ_2 is the plan of the equation $3x - y + z + 1 = 0$.

3. $\overrightarrow{f}(x,\ y,\ z) = (2xy - x^2,\ x^2 + y^2)$ and Γ is the closed curve constituted by the 2 arcs of the parabolas $y = x^2$ and $x = y^2$ in the direct sense using 2 methods.

4. $\overrightarrow{f}(x,\ y,\ z) = (-e^y,\ -ye^x,\ -x^2y)$ and Γ is the part of the paraboloid of the equation $z^2 = x^2 + y^2$ located above the square $[0,\ 1]^2$

5. Compute $\displaystyle\iint_D (2x-4)\ dxdy$ where $D = D_1 \setminus D_2$. We give:
   D_1 is the domain limited by the eclipse of the center $(0,\ 0)$ and of radius $a = 6$ and $b = 4$.
   D_2 is the union of the 2 closed discs of the center $(2,\ 2)$ and $(2,\ -2)$ and radius 1.