National Higher School of
Autonomous Systems Technology

epsilite
DEPARTMENT

Nexus Innovators

Probability

1 Exercise

The distribution function of a Weibull distribution is:

$$F(x) = 1 - e^{-\beta x^{\alpha}} \ ; \ x \geq 0$$

Where $\beta \geq 0$ is a certain constant; α a positive integer. Find:

- Its density function f.
- Its mathematical expectation $\mathrm{E}[\mathrm{X}] = \int_0^{\infty} x f(x) dx \ ; \ x \geq 0$.

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(1)

2 Exercise

Let X and Y be two random variables of joint probability density:

$$f(x,y) = \begin{cases} \frac{6}{7}(x^2 + \frac{xy}{2})^2 & : \text{if } 0 < x < 1 \text{ and } 0 < y < 2. \\ 0 & : \text{otherwise.} \end{cases}$$

1. Verify that f is indeed a joint probability density.
2. Determine the marginal probability density of the distribution of the random variable X and that of Y.
3. Calculate $\mathbb{P}(\mathrm{X} \geq \mathrm{Y})$.
4. Calculate the covariance of the couple (X, Y).

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(2)

3 Exercise

The joint density of X and Y is given by:

$$f(x,y) = \begin{cases} 2xy + \frac{3}{2}y^2 & : \text{if } 0 < x < 1,\ 0 < y < 1. \\ 0 & : \text{otherwise.} \end{cases}$$

1. Check that $f(\mathrm{X}, \mathrm{Y})$ is a density function.
2. Find the marginal densities $f_X(x)$ and $f_Y(y)$.
3. Find the conditional densities $f_{X/Y} = y(x),\ f_{Y/X} = x(y)$.
4. Calculate $\mathbb{P}((\mathrm{X}, \mathrm{Y}) \in [0, \frac{1}{2}]^2)$.
5. Find $\mathbb{P}(\mathrm{X} < \mathrm{Y})$.

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(3)

4 Exercise

An archer shoots at n targets. With each shot, he has the probability p of hitting the target and the shots are assumed to be independent. He shoots each target for the first time and we note X the number of targets hit during this first throw. The archer then fires a second time at the remaining targets and Y is the number of targets hit during this attempt. Determine the distribution of the variable $\mathrm{Z} = \mathrm{X} + \mathrm{Y}$.

(4)

5 Exercise

You and an opponent play a video game: in each round, you have a p_1 probability of winning, and your opponent a p_2 probability (independent of you). The first to win a round wins the bet.

- How likely are you to win?
- How likely are you to tie?
- What happens when both players are equally good at the game ($p_1 = p_2$)?

(5)

6 Exercise

- Let n be a natural number ≥ 2, and let X, Y be independent random variables according to the binomial distribution $\mathfrak{B}(n, \frac{1}{2})$. Calculate $\mathbb{P}(\mathrm{X} = \mathrm{Y})$.
- Let $p \in [0, 1]$. Assume that X follows the binomial distribution $\mathfrak{B}(n, p)$, and that Y follows the uniform distribution on $[[0, n]]$.

1. Calculate E(Y) and porve that $\mathrm{Var}(\mathrm{Y}) = \dfrac{n(n+1)}{12}$.
2. We consider the matrix $\mathrm{C} = \begin{bmatrix} \mathrm{X} & \mathrm{X} \\ \mathrm{Y} & \mathrm{Y} \end{bmatrix}$.

   Calculate the probability p_1 that the matrix C is antisymmetric, and the probability p_2 that the matrix C is symmetric.

(6)

7 Exercise

It is assumed that there is a probability equal to p of being checked when taking the train. Mr.A makes n trips per year on this line.

A) We take $p = 0.1$ and $n = 700$.

1. What is the probability that Mr.A will be checked between 60 and 80 times in the year?
2. Mr.A actually always travels without a ticket. In order to take into account the possibility of making several passages with the same ticket, it is assumed that the price of a ticket is 1.12 euros. What minimum fine should the company set so that the fraudster has, over a period of one year, a probability of more than 0.75 of being a loser?

B) We assume that $p = 0.5,\ n = 300$.

- Mr.A still travels without a ticket. Knowing that the price of a ticket is 1.12 euros, what minimum fine should the company set so that the fraudster has, over a period of one year, a probability of more than 0.75 of losing?

(7)