Epsilite

Double Integrals

Integrate If You Can :)

1. $\iint_D \frac{1}{(x^2+y^2+z^2)^2}\,dxdy,\ D=\{(x,y)\in\mathbb{R}^2:\ y^2<2xz,\ x\geq 0,\ z>0\}.$
2. $\iint_D e^{-(x^2+y^2)+2xy\cos a}\,dxdy,\ D=\{(x,y)\in\mathbb{R}^2:\ 2xy\cos a>(x^2+y^2),\ a>0\}.$
3. $\iint_D \frac{y}{\sqrt{x}}\ln(1-x-y)\,dxdy,\ D=\{(x,y)\in\mathbb{R}^2:\ x>0,\ y>0,\ x+y<1\}.$
4. $\iint_D \frac{(x-y)^a}{((x^2-y^2)^2+1)(x+y)^a}\,dxdy,\ D=\{(x,y)\in\mathbb{R}^2:\ 0<y<x\}.$
5. $\iint_D \frac{(x^2+y^2)}{(y^2-x^2)^{-xy}}\,dxdy,\ D$ bounded with the graphs $y=x,\ xy=a,\ xy=b$, and $y^2-x^2=1$, with $0<a<b$.
6. $\iint_D \sqrt{xy}\,dxdy,\ D=\{(x,y)\in\mathbb{R}^2:\ x^2+y^2\leq xy\}.$
7. $\iint_D e^{\frac{x^3+y^3}{xy}}\,dxdy,\ D=\{(x,y)\in\mathbb{R}^2:\ y^2-2xz\leq 0,\ x^2-2yz\leq 0,\ z>0\}.$
8. $\iint_D (2x^2-y)\,dxdy,\ D=\{(x,y)\in{\mathbb{R}_+}^2\ [1]:\ \frac{x^2}{a^2}+\frac{y^2}{b^2}\leq 1,\ a>0,\ b>0\}.$
9. $\iint_D \frac{y}{x^2+y^2}\,dxdy$ [2], $D=\{(x,y)\in{\mathbb{R}_+}^2:\ x^2+y^2\leq z^2\}.$
10. $\iint_D \frac{dxdy}{\sqrt{(\frac{1}{3}+x^2+y^2)^3}},\ D=\{(x,y)\in\mathbb{R}^2:\ 0<x<\sqrt{2},\ 0<y<\frac{\sqrt{2}}{\sqrt{3}}\}.$
11. $\iint_D xy\,dxdy,\ D=\{(x,y)\in\mathbb{R}^2:\ 4(x-1)^2+9(y-2)^2\leq 36,\ x>y>1\}.$
12. $\iint_D (x+y)^n e^{-(x+y)}\,dxdy,\ D$ is the first positive quarter, with $n\geq -1$.

[1] We define ${\mathbb{R}_+}^2$ to mean $\{(a,b)\in\mathbb{R}^2:\ a\geq 0,\ b\geq 0\}$.
[2] Integrate using 3 distinct methods.

13. $\iint_D \frac{xy}{(1+x^2+y^2)^z}\,dxdy,\ D=\{(x,y)\in\mathbb{R}^2:\ 0<x<1,\ 0<y<1,\ x^2+y^2>1\},\ z\in\{1,2\}.$

14. $\iint_D \frac{\sin(x^2+y^2)}{2+\cos(x^2+y^2)}\,dxdy,\ D=\{(x,y)\in\mathbb{R}^2\ :\ 1<x^2+y^2<4\}.$

15. $\iint_D \frac{y}{x^3}\sin\left(\pi\left(1-\frac{y^2}{x^2}\right)\right)\,dxdy.\ D=\{(x,y)\in\mathbb{R}^2:\ 0<y\sqrt{2}<x,\ 1<x^2-y^2<4\}.$

16. $\iint_D \left(x^2-\frac{y^2}{9}\right)\sin\left(2\arctan\left(\frac{y}{3x}\right)\right)\,dxdy,\ D=\{(x,y)\in\mathbb{R}^2:\ x^2+y^2<1,\ 0<y<x\}.$

17. $\iint_D \cos\left(x^2+y^2-4x+\left(\frac{4}{n}\int_0^n \sin^2 u\ du\right)^2\right)\,dxdy,\ D=\{(x,y)\in\mathbb{R}^2:\ 1<x^2+y^2<4\}.$

18. $\int_0^k\int_0^k \frac{\ln(1+x^2+y^2)}{1+x^2+y^2}\,dxdy.$

19. $\int_0^8\int_{y^{1/3}}^2 \frac{1}{x^4+1}\,dxdy.$

20. $\int_0^1\int_0^1 \frac{x^2-y^2}{(x^2+y^2)^2}\,dxdy.$

21. $\int_0^{\sqrt{2}/2}\int_y^{\sqrt{1-y^2}} \sqrt{1-x^2}\,dxdy.$

22. $\int_0^1\int_0^1 \frac{1}{y\cos x+1}\,dxdy.$