Astrophysics

Conservation Laws

Conservation of linear momentum:
If no external forces,
$$\sum m\vec{v} = \text{constant}$$
- always observed in collisions
- write 1 eqn along each direction or write vector eq

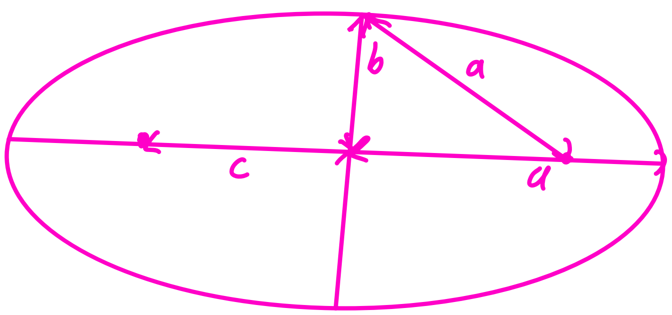
Conservation of angular momentum:
$$\sum \underset{\text{torque}}{\vec{L}} = \sum m(\underset{\text{rotation}}{\vec{r}} \times \vec{v}) = \text{constant}$$
$\vec{r}$: clockwise/anticlockwise
- particle subject to central force only

Conservation of energy:
$$\sum E = \sum KE + GPE = \text{constant}$$
$$KE = \frac{1}{2}kv^2$$
$$GPE = U = -\frac{Gm_1m_2}{r}, \quad G = 6.67408 \times 10^{-11}$$
- GPE always negative, attracting

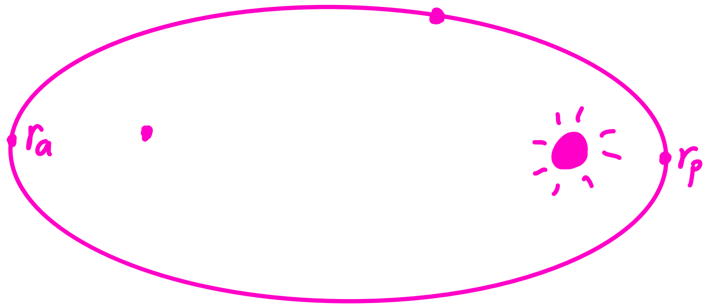
Fundamental Laws

Newton's law of gravity:
2 point masses attract with a force of
$$F = \frac{Gm_1m_2}{r^2}$$
gravity provides centripetal force in circular orbits
$$v = \sqrt{\frac{GM}{r}}$$

Kepler's first law:
The orbit of a planet is an ellipse with the sun at a focus.



$$S_{ellipse} = \pi ab$$
$$e \text{ (eccentricity)} = \sqrt{1 - \frac{b^2}{a^2}}$$
$$c = ea = \sqrt{a^2 - b^2}$$
d



r_p : periapsis, planet closest to sun, planet travel fastest

r_a : apoapsis, planet furthest from sun, planet travel slowest

$$V_p = \sqrt{\frac{GM}{a} \cdot \frac{1+e}{1-e}} = \text{velocity at periapsis}$$

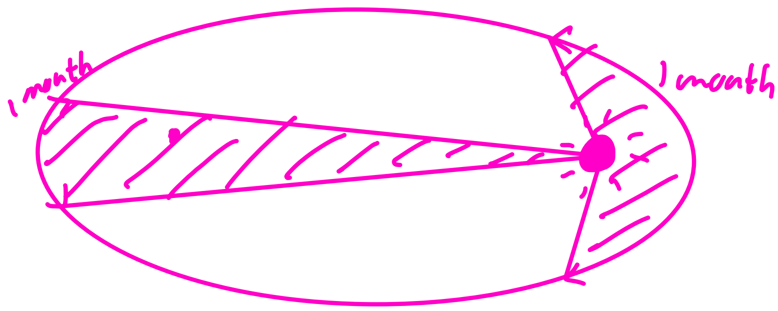
$$V_a = \sqrt{\frac{GM}{a} \cdot \frac{1-e}{1+e}} = \text{velocity at apoapsis}$$

Vis Viva: $KE + GPE = -\frac{GMm}{2a}$ in elliptical orbit

$V = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}$, r = current distance

Kepler's second law:

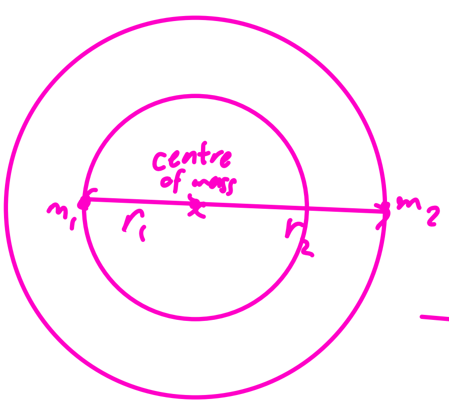
line joining planet and star traces equal area in equal time.



Kepler's third law:

$$\underset{\text{orbit period}}{T}^2 = \frac{4\pi^2}{GM} \underset{\text{greatest radius}}{a}^3$$

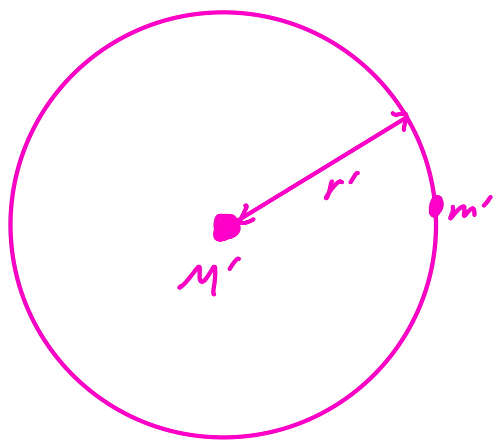
2 body systems



bodys orbit with equal angular velocity, opposite sides of centre of mass

$$\frac{m_1}{m_2} = \frac{r_2}{r_1} = \frac{v_2}{v_1}$$

reduced mass

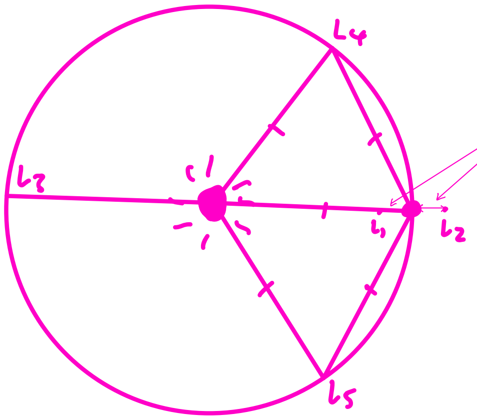


$M' = m_1 + m_2$

$r' = r_1 + r_2$

$m' = \frac{m_1 m_2}{m_1 + m_2}$

Lagrange Points:



$$\sqrt[3]{\frac{mR^3}{3M}}$$

- points of equilibrium
- small mass placed at Lagrange point stays at rest relative to sun and earth

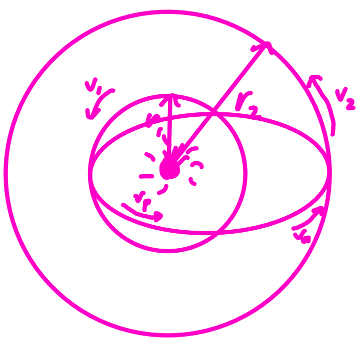
Roche Limit:

- tidal force = gravity
- point of collapse

$$d = \sqrt[3]{\frac{2Mr^3}{m}}$$

Hohmann Transfer Orbit:

- circular ⟶ ellipse ⟶ circular



$$v_1 = \sqrt{\frac{GM}{r_1}}$$

$$v_2 = \sqrt{\frac{GM}{r_2}}$$

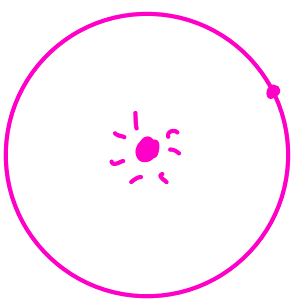
$$v_p = \sqrt{GM\left(\frac{2}{r_1} - \frac{1}{a}\right)}$$

$$v_a = \sqrt{GM\left(\frac{2}{r_2} - \frac{1}{a}\right)}$$

$$a = \frac{r_1 + r_2}{2}$$

Orbit types:

circular:

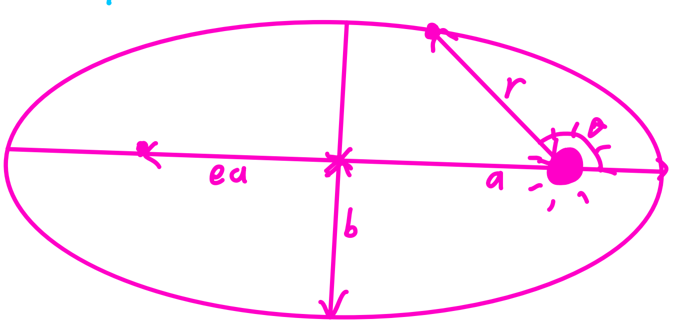


$$e = 0$$

$$E = -\frac{GMm}{2a}$$

$$r = a = b$$

elliptical:

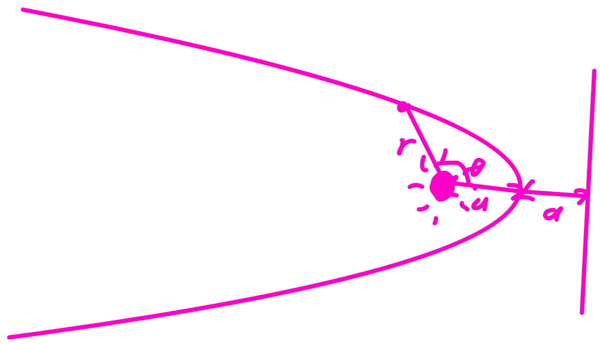


$$0 < e < 1$$

$$E = -\frac{GMm}{2a}$$

$$r = \frac{a(1-e^2)}{1+e\cos\theta}$$

parabolic:



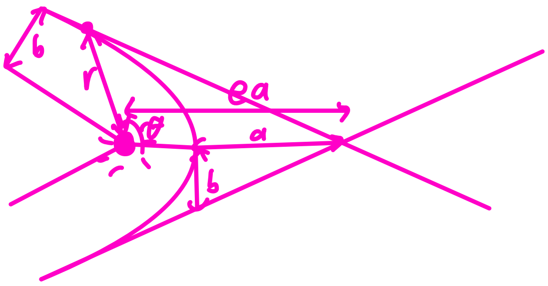
$e = 1$

$E = 0$

$r = \frac{2a}{1+e\cos\theta}$

- escape velocity
- minimum energy for unbounded orbit

hyperbola:

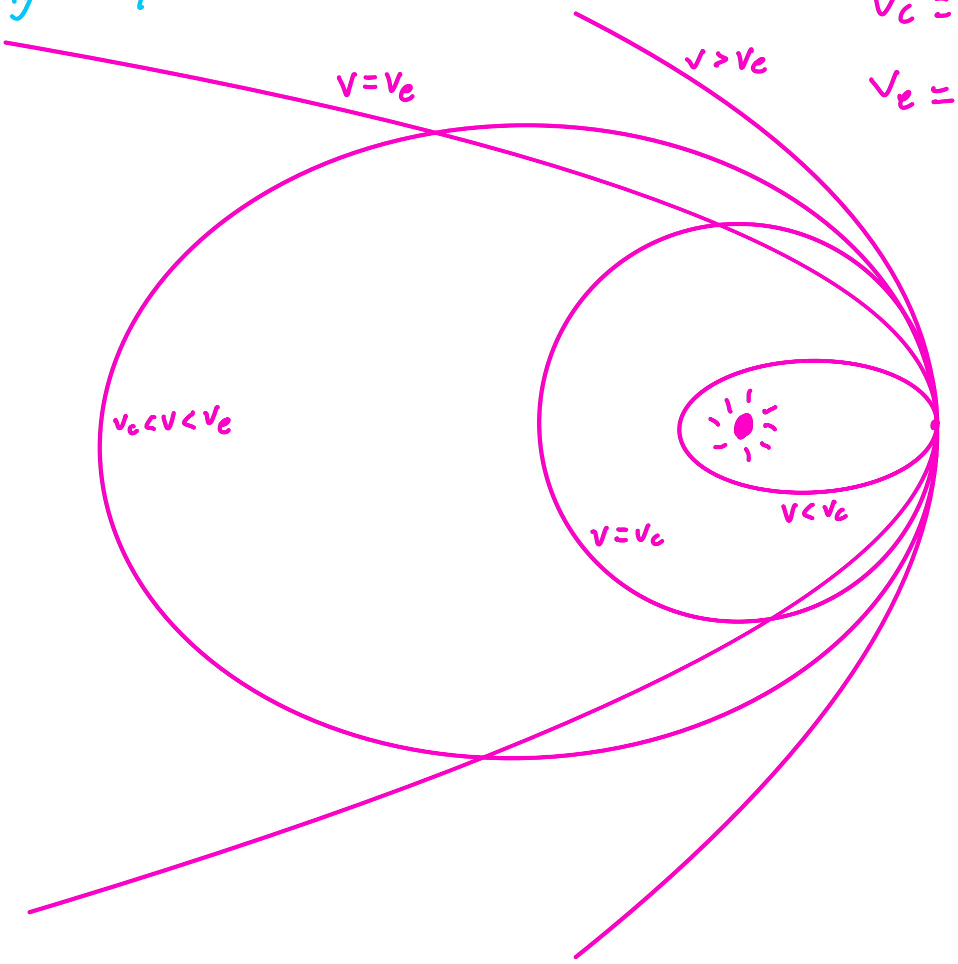


$e > 1$

$E = \frac{GMm}{2a}$

$r = \frac{a(e^2-1)}{1+e\cos\theta}$

Velocity comparison:



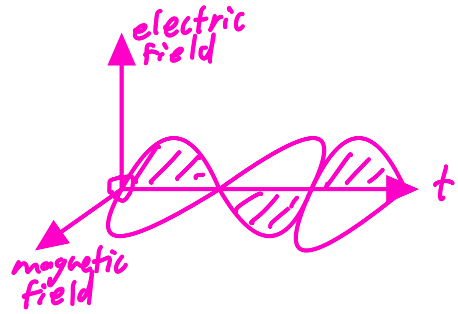
$V_c = \sqrt{\frac{GM}{R}}$

$V_e = \sqrt{\frac{2GM}{R}}$

Light

Definition:

- electromagnetic radiation
- wave-particle duality



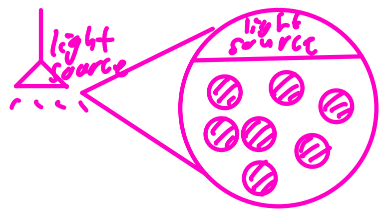
Waves: waves travelling through space at speed c

$c = v\lambda$, λ = wavelength

particles: packets of energy (photons) emitted from objects, quantised

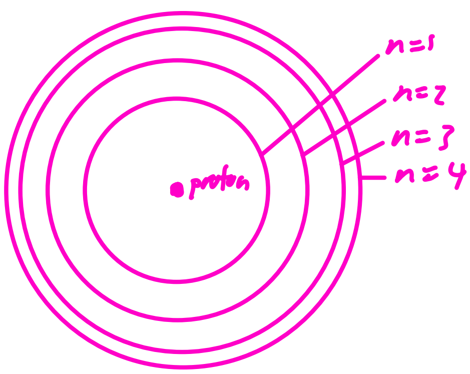
E of 1 photon: $E = hf = \frac{hc}{\lambda}$

$h = 6.626 \times 10^{-34}$ (Planck's constant)

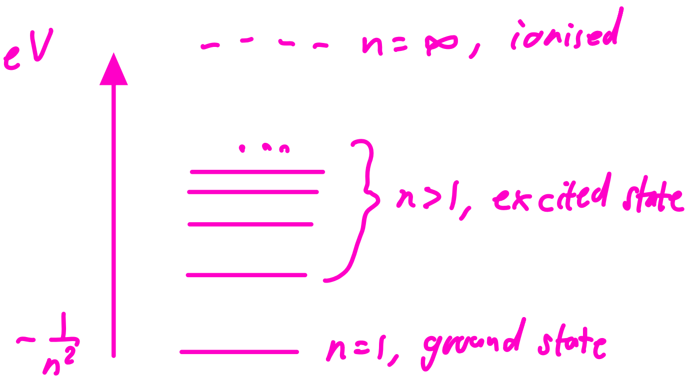


Energy levels:

Hydrogen:



$E_{ionisation} = -13.6\,eV$



Transitions:

Emission:

higher energy level ⟶ lower energy level + photon

- spontaneous: incoherent
- induced: coherent, same direction as received radiation, atom get then release photon as reaction

$$\Delta E = |E_1 - E_2| = hv = \frac{hc}{\lambda}, \quad v = \text{frequency}$$

For hydrogen: $\Delta E = \left|-13.6\,eV\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\right|$

Absorption:

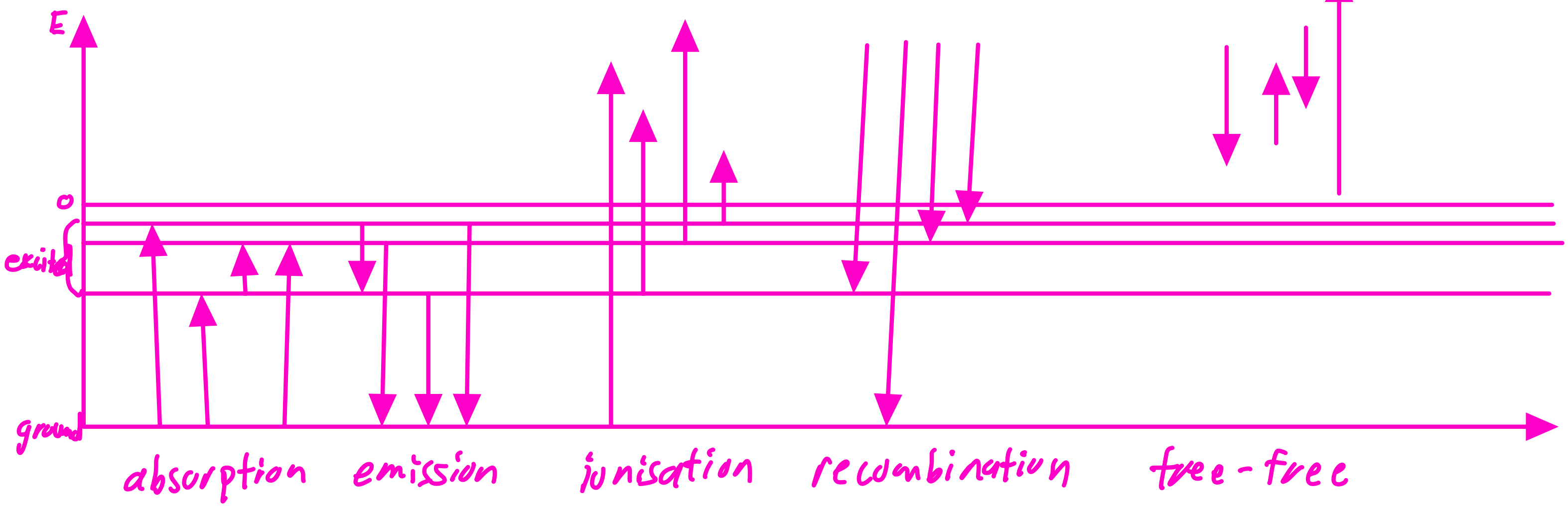
lower energy level + photon ⟶ higher energy level

Ionisation:

- e^- gains enough energy to escape
- extra energy is KE of e^-

Recombination:
- free e^- get trapped in atom

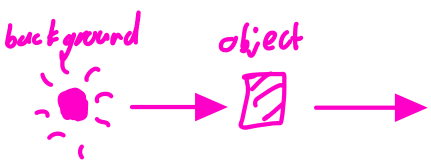
free-free:
- free e^- interact with atom and change energy



Spetrums:

Emission:

- spectrum mainly black
- a few coloured lines
- high energy gas emit photons



- background is continous blackbody radiation
- black lines indicating absorbed light
- object absorb photon

Molecular:
- energy levels only depend on electron orbits in atoms
- in molecules, also rotation and vibration
- vibration transitions in infrared
- rotation transitions in microwave
- narrow bands, many lines

Radiation Laws:

Blackbody:
- object does not reflect or scatter radiation
- entirely absorb and re emit all radiation
- radiation of blackbody depends only on temperature

Planck's Law:

special radiance $= B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \cdot \frac{1}{e^{\frac{h\nu}{kT}}-1} = \frac{2hc^2}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}}-1}$, $k =$ Boltzmann's constant $= 1.380649 \times 10^{-23}$

peak wavelength $= \lambda_{max} = \frac{0.0029}{T}$

when $\lambda \approx \lambda_{max}$, $\frac{hc}{\lambda kT} >> 1$,

$$B_\nu(\nu,t) = \frac{2h\nu^3}{c^2} e^{-\frac{h\nu}{kT}} \approx \frac{2hc^2}{\lambda^5} e^{-\frac{hc}{\lambda kT}}$$

when $\lambda >> \lambda_{max}$, $\frac{hc}{\lambda kT} << 1$,

$$B_\nu(\nu,t) = \frac{2ckT}{\lambda^4} = \frac{2\nu^2 kT}{c^2}$$

Stefan-Boltzmann Law:

radiation $= L = \sigma A T^4$, σ = Stefan-Boltzmann constant $= 5.67 \times 10^{-8}$

$= 4\pi R^2 \sigma T^4$ for star

Photometry:

solid $\angle = \Omega = \frac{\text{area on surface of sphere}}{r^2}$ (steradians)

Power: E released per time

Luminosity: emitted power, given by Stefan-Boltzmann Law (L)

Spectral radiance: emitted power per solid angle, frequency, area, given by Planck Law.

Flux: power through surface

Intensity: flux per area (I)

flux density: intensity per frequency

$$I = \frac{L}{\Omega r^2}$$

$$\text{Flux} = I \underset{\text{area}}{A} \cos\theta$$

Magnitude:

- exponential system
- divide by $\sqrt[5]{100}$ each step
- apparent / absolute magnitude
- more negative $\longrightarrow$ brighter

Apparent magnitude:

- flux received by instrument
- Vega's apparent magnitude set at zero

$$m_1 - m_2 = -\frac{5}{2} \ln\left(\frac{F_1^{\text{flux}_1}}{F_2^{\text{flux}_2}}\right)$$

Absolute magnitude:

- apparent magnitude at fixed 10 parsec away
- actual brightness of star.

$$m - M = 5 \ln\left(\frac{\overset{\text{distance}}{d}}{10}\right)$$

Surface brightness:

- apparent brightness per unit of angular area.

$$SB = \underset{\substack{\text{apparent}\\\text{magnitude}}}{m} + \ln A$$

- all measurements depend on instruments measures which wavelength range

Colour filtering:

Bolometry:

- human eye most sensitive to green light
- visual magnitude m_v
- bolometric magnitude m_{bol}

$m_{bol} = m_v - BC$ (bolometric correction)

$BC = 0$ for F5 class stars

- across all wavelength

UBV:

- measurements using CCD caremas, allowing certain type of light to enter

	bandwidth	average wavelength
U = ultraviolet	66	367
B = blue	94	436
V = visual	88	545
R = red	138	638
I = infrared	149	797

- has average wavelength and bandwidth
- different apparent magnitude in different bands
- usually given V, U-B, B-

$M_U = M_B = M_V$ for A0 type stars

Extinction:

$L = L_0 e^{-\tau}$, τ = optical thickness

Stars

Thermonuclear fusion:

light atom + light atom $\longrightarrow$ heavy atom + E

- energy released from mass lost

$E = mc^2$, $m = m_{reactants} - m_{products}$

- release binding energy of atomic nucleus
- mass of nucleus slightly > mass of p^+ and n
- binding E per nucleon = total bonding E / atomic mas

Proton-Proton chain

Type 1: $^1H + {}^1H \longrightarrow {}^2H + e^+ + V_e$

$^1H + {}^1H + e^- \longrightarrow {}^2H + V_e$

$^2H + {}^1H \longrightarrow {}^3He + y$

$^3He + {}^3He \longrightarrow {}^4He + 2\,{}^1H$

Type 2: $^3He + {}^4He \longrightarrow {}^7Be + y$

$^7Be + e^- \longrightarrow {}^7Li + V_e$

$^7Li + {}^1H \longrightarrow {}^4He + {}^4He$

Type 3: $^7Be + {}^1H \longrightarrow {}^8B + y$

$^8B \longrightarrow {}^8Be + e^+ + V_e$

$^8Be \longrightarrow {}^4He + {}^4He$

mass of star ≤ solar mass

CNO cycles:

$^{12}C + {}^1H \longrightarrow {}^{13}N + y$

$^{13}N \longrightarrow {}^{13}C + e^+ + V_e$

$^{13}C + {}^1H \longrightarrow {}^{14}N + y$

$^{14}N + {}^1H \longrightarrow {}^{15}O + y$

$^{15}O \longrightarrow {}^{15}N + y + V_e$

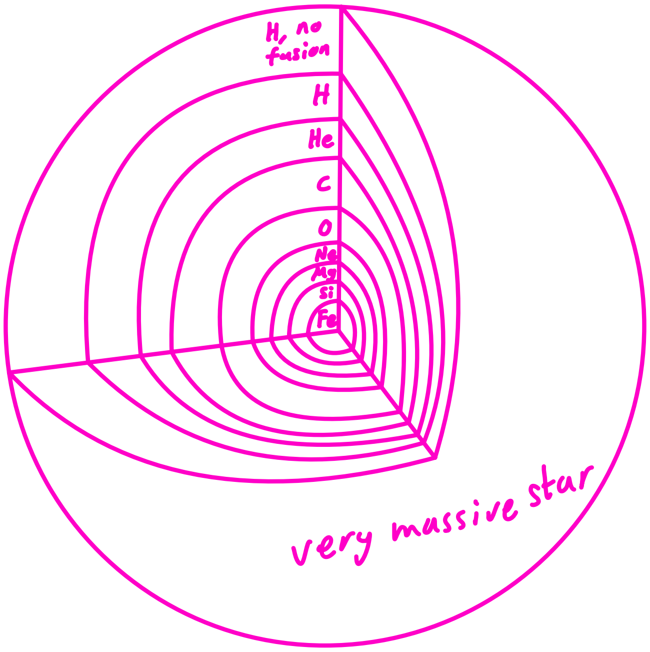
$^{15}N + {}^1H \longrightarrow {}^{12}C + {}^4He$

mass of star > 1.5 solar mass

Triple alpha:

$^4He + {}^4He \longleftrightarrow {}^8Be$

$^8Be + {}^4He \longrightarrow {}^{12}C + y$

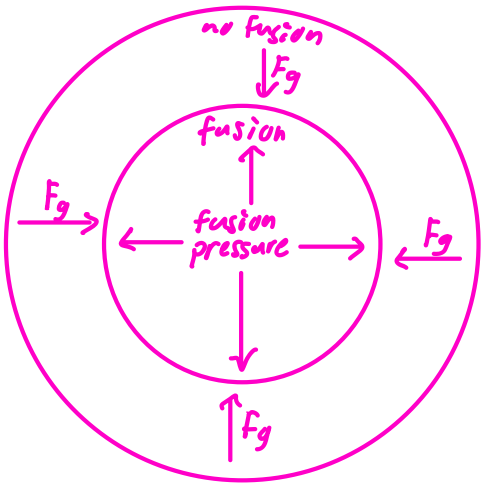


- Fusion stops at iron as further fusion requires, not produces energy
- heavier elements formed by neutron capture, for example in neutron star mergers

Equilibrium:

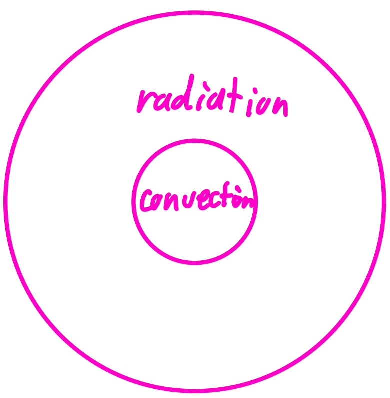
Hydrostatic equilibrium:

- balance between gravity and gas pressure

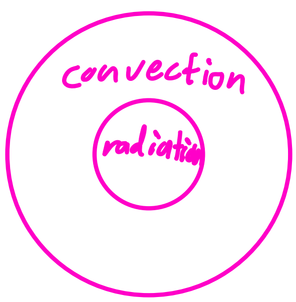


temperature gradient:

- convection: hot gas rises, cools, then fall back again
- radiation: photons travel out and is absorbed



mass > 1.5 solar mass



0.5 solar mass < mass < 1.5 solar mass

mass < 0.5 solar mass

radiation pressure:
- pressure in gas due to momentum of gas particles
- perfect absorber: $P = \frac{I}{c}$
- perfect reflector: $P = \frac{2I}{c}$

state of gas:
- high temperature, near complete ionisation
- obeys ideal gas law

pressure due to gas particles:

$$P = \frac{k\rho T}{\underset{\text{atomic mass}}{\mu}\,\underset{\text{mass of hydrogen}}{m_H}},$$

At higher temperature with radiation pressure,

$$P = \frac{k\rho T}{\mu M_H} + \frac{4\sigma T^4}{3c}$$

At higher pressure $\geq 10^7 kg/m^3$, gas become degenerate,

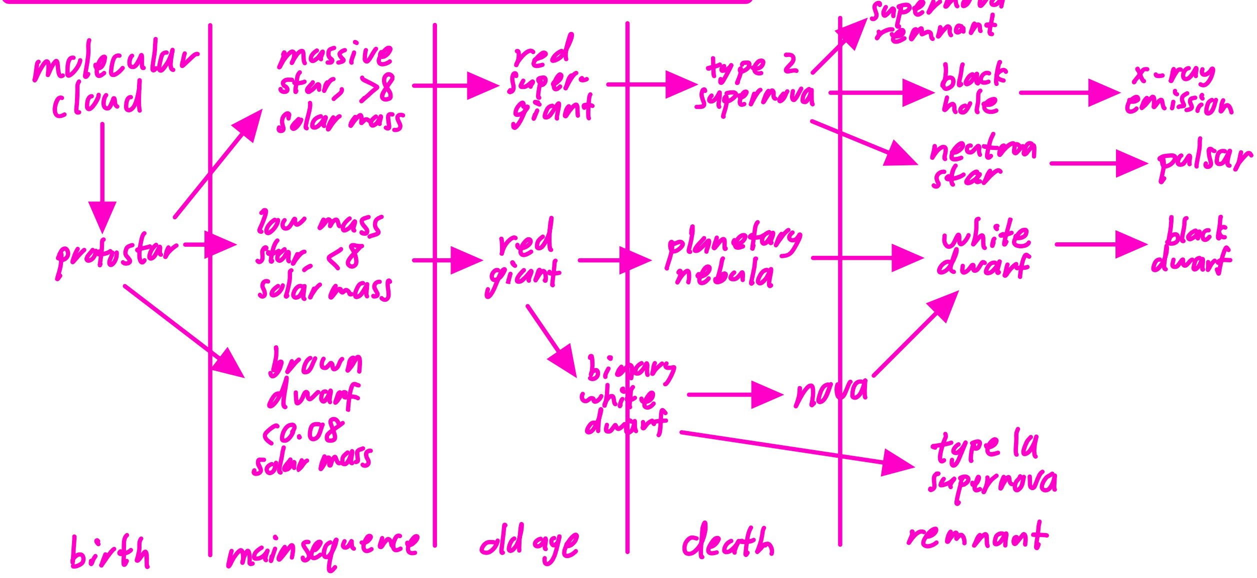
(nonrelativistic) $P = \left(\frac{h^2}{\underset{\text{mass of } e^-}{m_e}}\right)\left(\frac{\rho}{\underset{\text{atomic mass of } e^-}{\mu_e} M_H}\right)^{\frac{5}{3}}$

(relativistic) $P = hc\left(\frac{\rho}{\mu_e M_H}\right)^{\frac{4}{3}}$

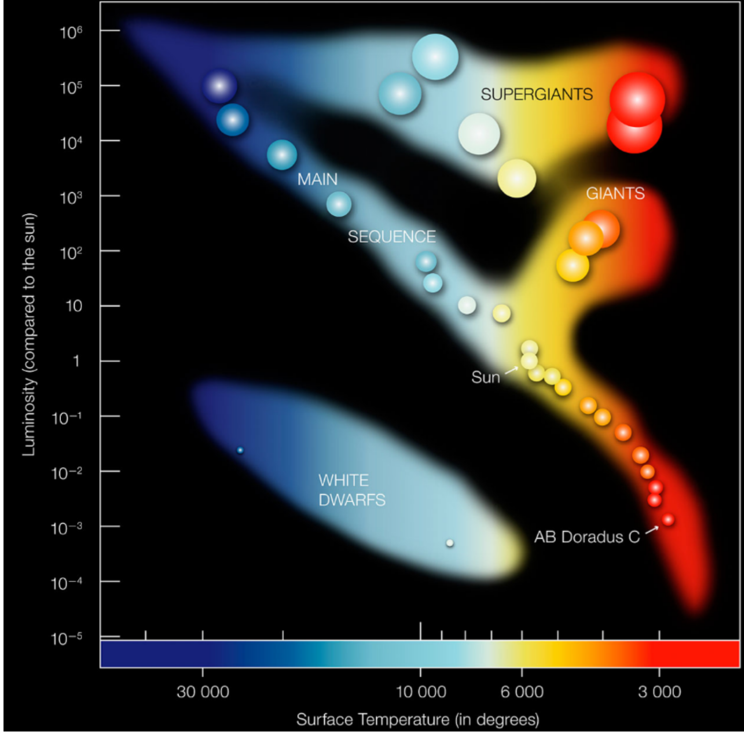
Chandraskhar limit (max mass of white dwarf) = 1.44 solar mass

GPE of star = $-\frac{3GM^2}{5R}$

Stellar evolution:



Hertzsprung-Russell Diagram:
- luminosity (absolute magnitude)
- temperature (spectral class)



Spectral Type	Color	Temperature (K)*	Spectral Features
O		28,000-50,000	Ionized helium,especially heliu
B		10,000-28,000	Helium, some hydrogen
A		7,500-10,000	Strong hydrogen, some ionize metals**
F		6,000-7,500	Hydrogen and ionized metals such as calcium and iron
G		5,000-6,000	Both metals and ionized metal especially ionized calcium
K		3,500-5,000	Metals
M		2,500-3,500	Strong titanium oxide and son calcium

* To convert approximately to Fahrenheit, multiply by 9/5.
** Astronomers regard elements heavier than helium as metals.

Timeline:

Dynamical timescale:
- fast
- only gravity, no pressure, collapse
- formation of protostar

$$t = \sqrt{\frac{R^3}{GM}}$$

Thermal timescale:
- medium
- radiate away all thermal energy
- contraction of protostar towards main sequence

$$t = \sqrt{\frac{GM^2}{RL}}$$

Nuclear timescale:
- slow
- radiate away all fusion energy
- no more hydrogen
- main sequence

$$t = \frac{0.0007Mc^2}{L}$$

$$L \propto M^{-3.5}$$

Variable stars:

Eclipsing:
- binary systems
- 1 star pass in front of the other

Pulsating:

- expansion/contraction of outer layers

Erupting:

- faint stars ejecting mass

Rotating:

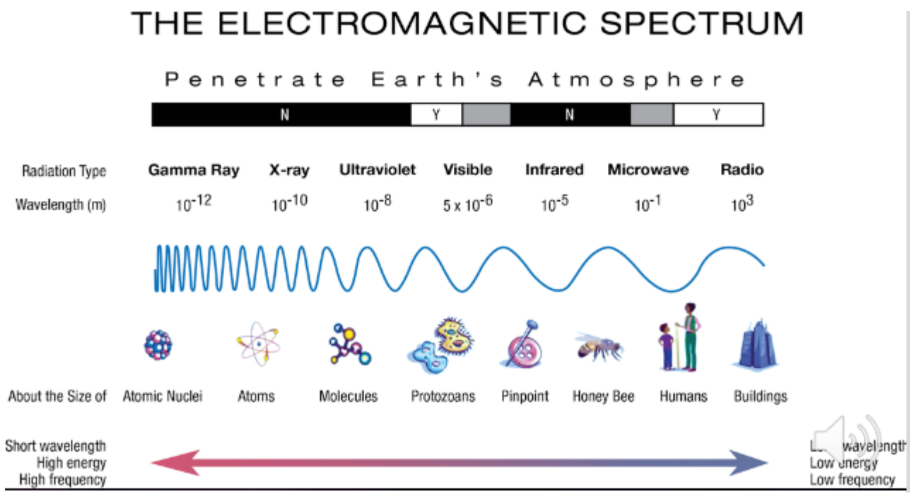
- rotation
- uneven temperature distribution
- uneven brightness

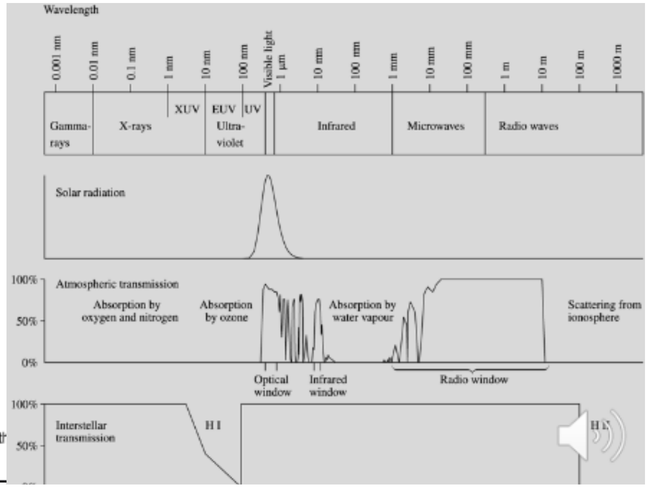
Cepheid:

- compressed by gravity
- becomes opaque, trapping radiation
- gas heats up and expands, becomes transparent
- photons released, gas cools back down, pressure drops
- gas compressed by gravity, cycle repeats

Optics

multi-wavelength astronomy:





Extinction of light:

absorption:
- radiation energy change to heat
- re radiated at infrared

scattering:
- changed direction of light propagation
- reduced intensity along original direction

Rayleigh scattering:
- particle size << wavelength
- oscillating electric field of light wave act on charges in particles
- charges move at same frequency
- particle become small radiating dipole
- scattered light radiated
- shorter wavelength scatter more

Mie scattering:
- particle size ≈ wavelength
- scattering of electromagnetic plane wave by homogeneous sphere
- does not depend on wavelength

21cm line:
- spin-flip e^- transition in hydrogen
- fixed 21cm wavelength
- uniform hydrogen distribution throughout galaxy
- wavelength difference by Doppler effect (redshift)
- gives rotation curve of galaxy
- map motion and presence of hydrogen

Common observations:

γ rays:
- highly energetic events
- supernovae
- gamma ray bursts
- black hole accretion
- black hole collisions
- solar flares

x rays:
- black holes at galactical centre
- supernova remnants
- white dwarf
- hot stars

UV rays:
- interstellar medium
- hot young stars
- white dwarf

Visible light:
- moons
- meteors
- planets
- stars

infrared rays:
- interstellar medium
- comets
- asteroids

microwaves, radio waves:
- cosmic microwave background radiation
- radio galaxies
- quasars
- pulsars
- stars
- galaxys
- 21cm hydrogen line
- can see through interstellar medium

Stellar Systems

Star Systems:

Visual:
- can see both stars actually moving around each other

Astrometric:
- can see 1 brighter star moving around centre of mass

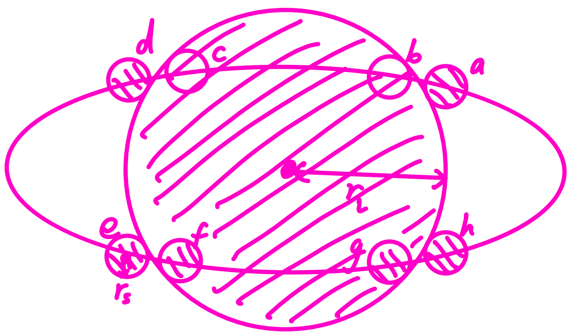
Spectroscopic:
- appear as 1 star even in powerful telescopes
- 2 spectral lines
- Doppler shift (redshift / blueshift)

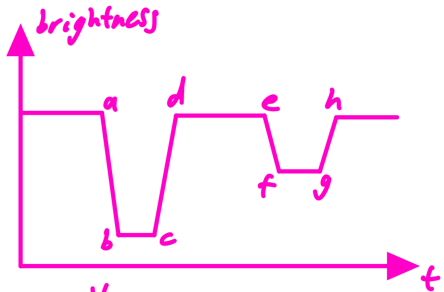
Photometric:
- periodic oscillation in total brightness
- eclipsing each other

~~**Optical double:**~~
- looks like binary star system
- actually single star

Eclipsing light patterns:

Complete eclipse:

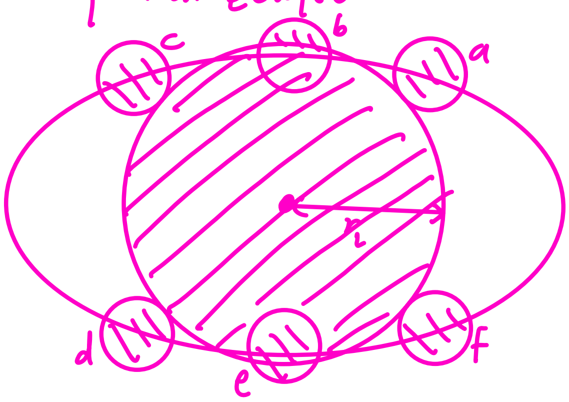


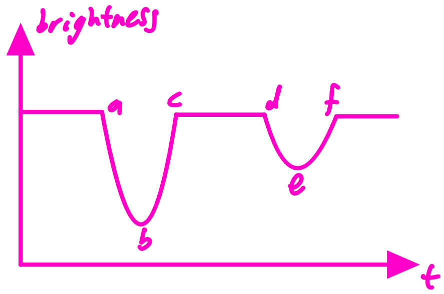


$r_s = \frac{v}{2}(t_b - t_a)$

$r_c = \frac{v}{2}(t_c - t_a)$

partial eclipse:





primary minimum: hotter star behind

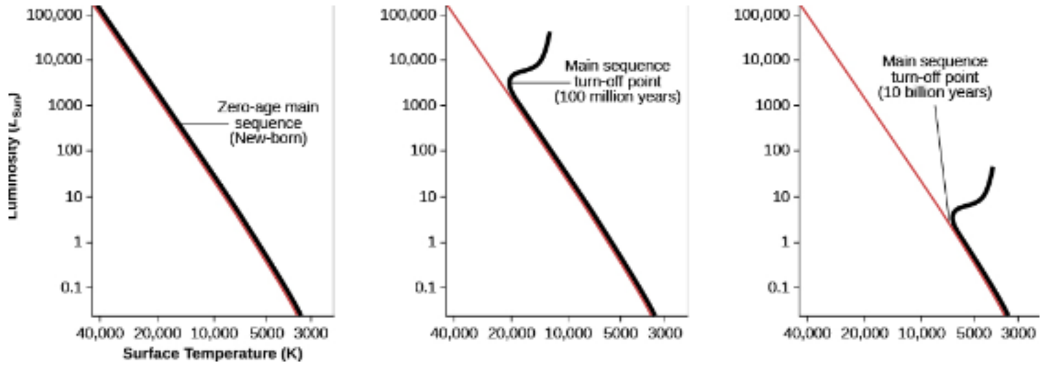
secondary minimum: cooler star behind

open clusters:
- $10 \leq$ number of stars ≤ 1000
- irregular

- gradually dispersed
- generally younger (< 10^9 yrs)

globular cluster:
- 10^5 stars
- globular, spherical symmetry
- generally older

main sequence turn off point:
- each point on main sequence has main sequence lifetime
- age of cluster = main sequence lifetime of turn off point



Star populations:

Population I:
- young stars
- contain a lot of metals from remains of older stars

Population II:
- old stars
- mostly H and He, little metals

Population III:
- hypothetical
- first stars, made of H and He only
- died quickly, ejecting their components

Cosmology

Big Bang Theory:

Hubble's Law:

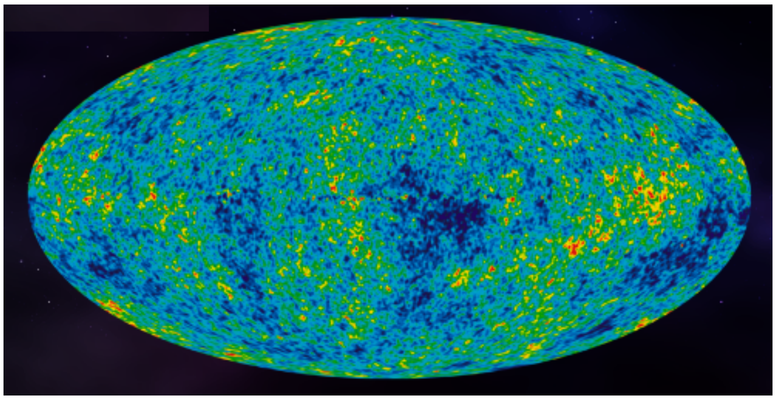
- objects recede with velocity proportional to distance

  $V = Hd$, H = Hubble constant = 70

  Hubble time (age of universe) = 1.44×10^{10} yrs

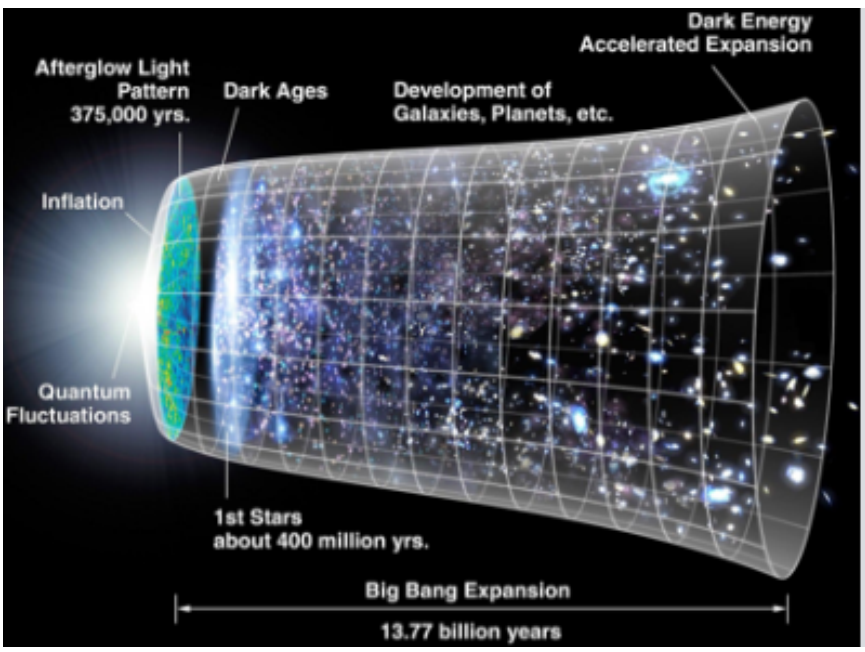
Cosmic Microwave Background Radiation:

- constant microwave background of 2.73K



Big Bang Theory:

- universe started from singularity
- rapid, then slow, then accelerate expansion
- space itself expanding
- finite observable universe
- light from far objects redshifted
- microwave background from leftover radiation



Expansion of universe:

proper distance:

- distance between 2 objects at certain time
- changes as universe expands

comoving distance:

- fixed distance unit
- defined to be current proper distance

$$d = a\underbrace{d_0}_{\text{comoving distance}},\quad a = \text{expansion factor}$$

redshift:

observed wavelength > source wavelength due to expansion of space

Generally:

$$z + 1 = \frac{1}{a} = \frac{\lambda_{received}}{\lambda_{source}}$$

relativistic ($v \approx c$):

$$v = \frac{(1+z)^2 - 1}{(1+z)^2 + 1} \cdot c$$

non-relativistic ($v \ll c$):

$$z = \frac{v}{c},\quad d = \frac{cz}{H}$$

Luminosity distance:

$$I = \frac{L}{4\pi d_0^2 (1+z)^2}$$

Friedmann Model:

Geometry of universe:

$k = 0$:

- flat, ordinary Euclidean space
- no edge, no centre, infinite volume

$k = 1$:

- closed
- finite volume, but no edge, no centre

$k = -1$:

- open
- no edge, no centre, infinite volume

Dark Energy:

- unknown
- causes expansion of universe
- energy in vacuum
- theoretical

Universe components:

Effects on expansion constant, a:

Matter:

$$a = \left(\frac{3}{2}H\right)^{\frac{2}{3}} t^{\frac{2}{3}}$$

Radiation:

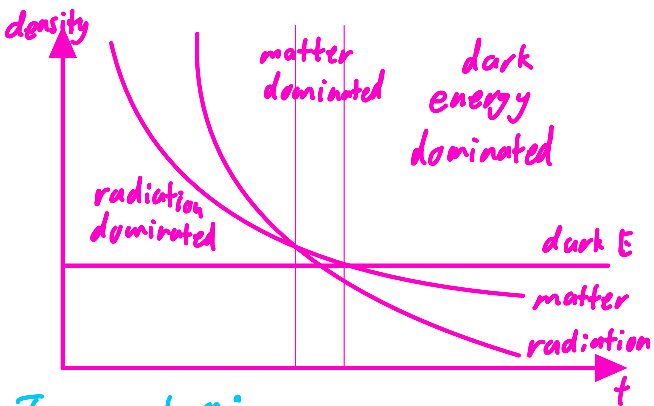
$$a = \sqrt{2H_0 t}$$

Dark E:

$$a = a e^{Ht}$$

Proportionality table:

	matter	radiation	dark E
$\rho \longleftrightarrow a$ relation	$\rho \propto a^{-3}$	$\rho \propto a^{-4}$	$\rho \propto$ constant
$a \longleftrightarrow t$ relation	$a \propto t^{\frac{2}{3}}$	$a \propto t^{\frac{1}{2}}$	$a \propto e^{t}$



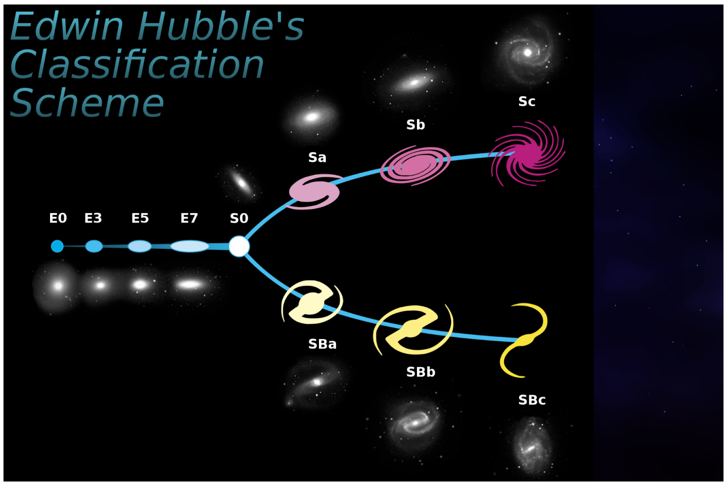
Temperature:

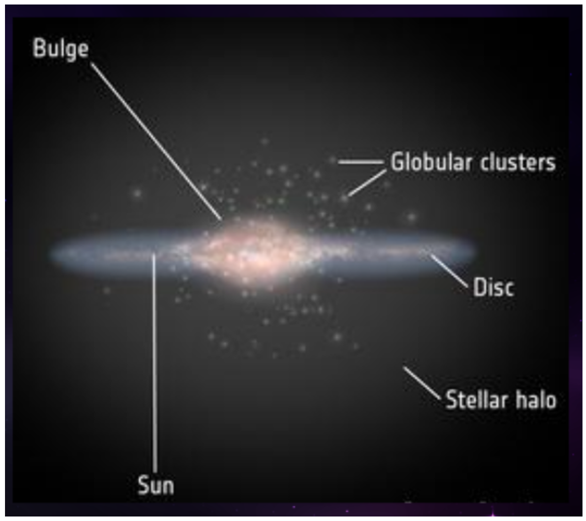
E density $= u = \frac{4\sigma}{c} T^4$

$aT =$ constant $= T_0 =$ initial temperature

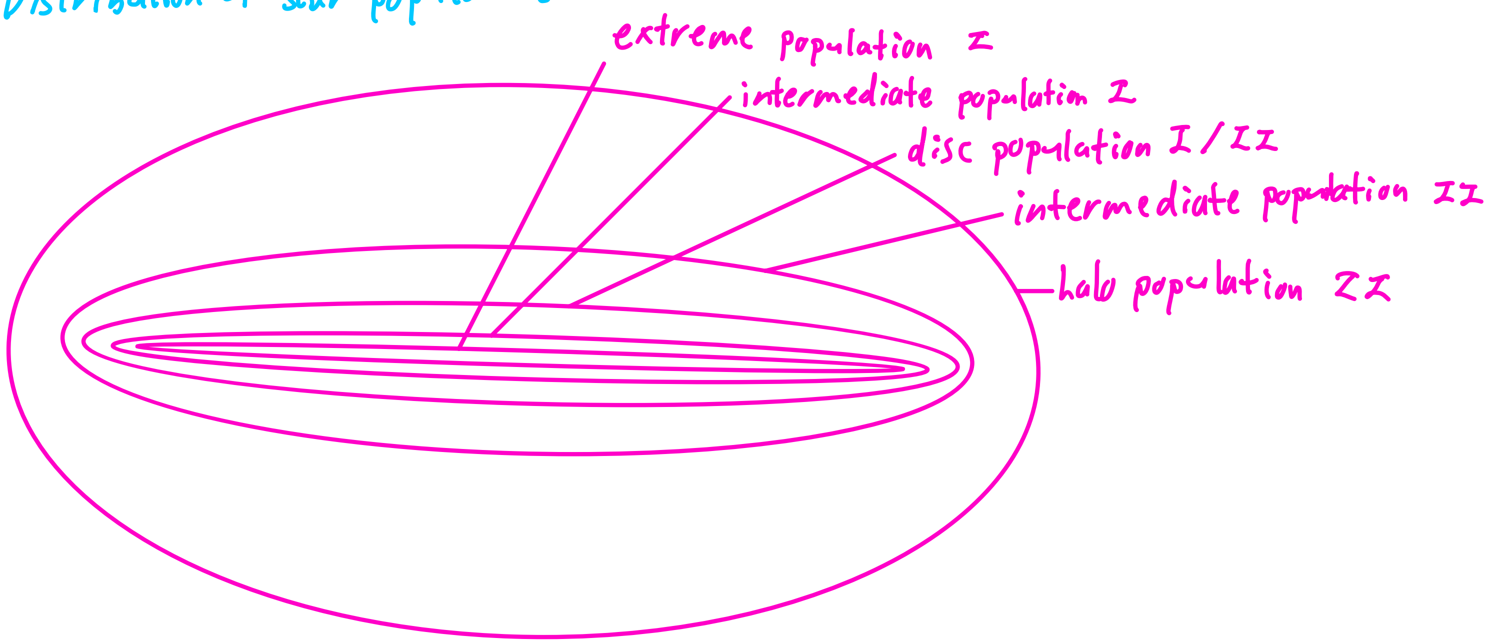
Galactic Astrophysics

Spiral Galaxys:





Distribution of star populations:



bulge, halo: older stars

spiral arms: younger stars

density wave theory:

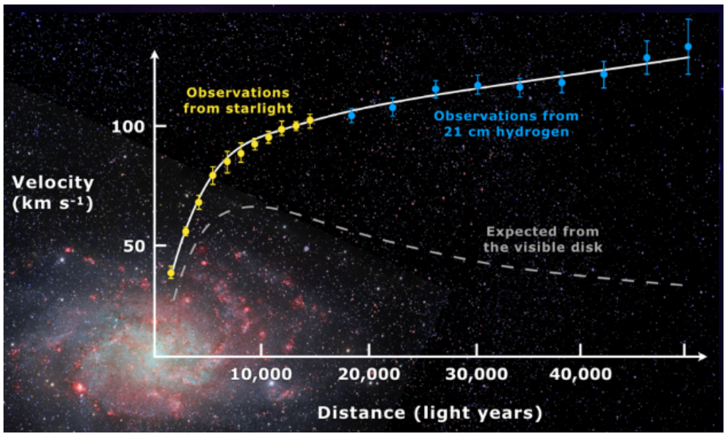
if spiral arms are single objects:

- near centre, higher angular velocity
- winds faster
- spiral arms disappear???

Spiral arms are waves

- stars rotate around centre
- density of stars uneven, form waves (spiral arms)
- objects move through density waves and are compressed

galaxy rotation curve:



inner region
- like rigid body
- angular velocity independent of distance

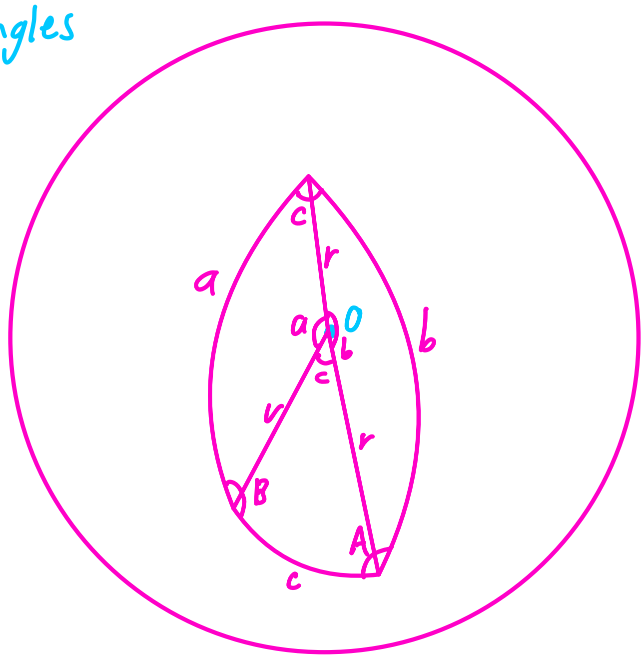
outer region
- assume galaxy has mass concentrated at centre
- according to visual observations
- velocity decrease with distance
- actual velocity ~ constant
- extra mass in outer region (dark matter)

Coordinates and time

Spherical trig:

great circles
- plane of circle pass through centre of sphere
- shortest distance between 2 points

angles



distance $= r\theta$

$\angle A + \angle B + \angle C > 180$

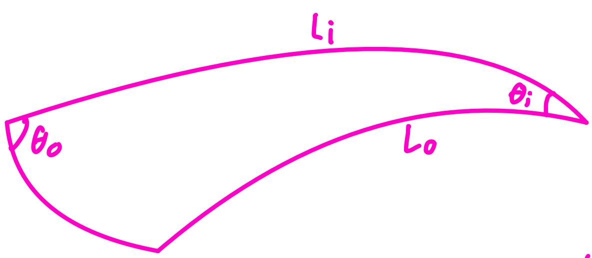
$S_{\triangle ABC} = r^2(\angle A + \angle B + \angle C - 180)$

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

$\cos c = \cos a \cos b + \sin a \sin b \cos C$

$\cos C = -\cos A \cos B + \sin A \sin B \cos c$

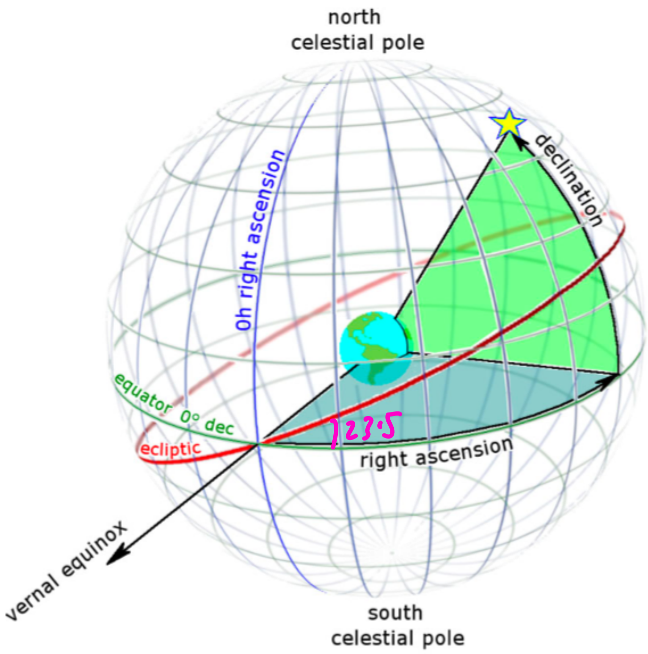
cotangent 4 part rule



$\cos L_i \cos \theta_i = \cot L_o \sin L_i - \cot \theta_o \sin \theta_i$

Spherical coordinates:

Celestial sphere



- imaginary sphere centred on Earth
- contain projection of stars

celestial equator: projection of Earth's equator

ecliptic: projection of Sun's orbit

∠ between celestial equator and ecliptic = Earth's axis tilt = 23.5

vernal equinox: intersection of celestial equatior and ecliptic

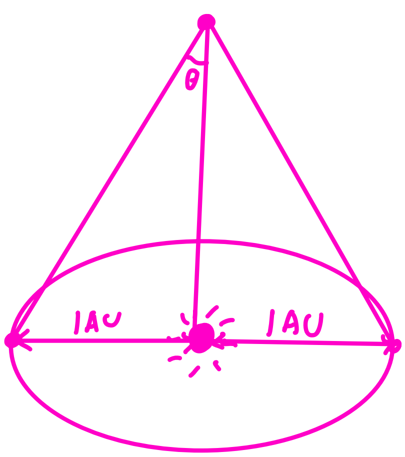
- where Sun rises
- First Point of Aries
- in Pisces

- stars are points on celestial sphere

declination: angle above celestial equation

right ascension: angle from Vernal Equinox, in direction of Sun moving

parallax

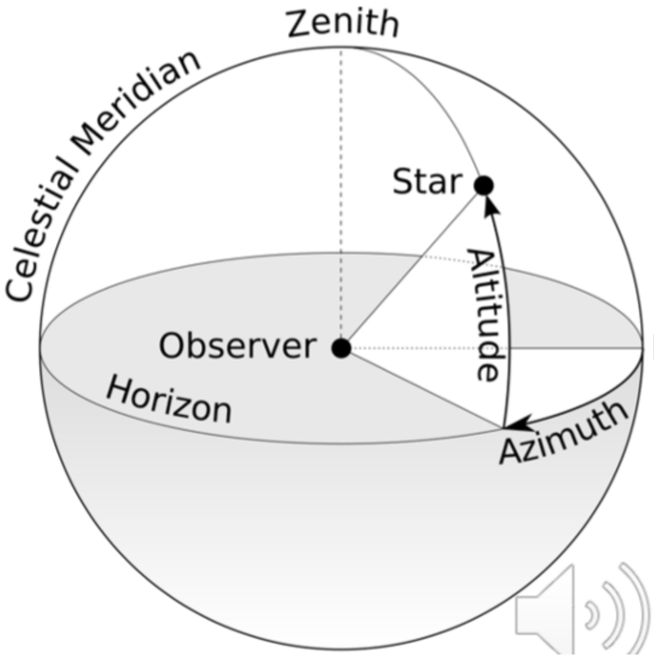


parallax = θ

distance (in parsec)

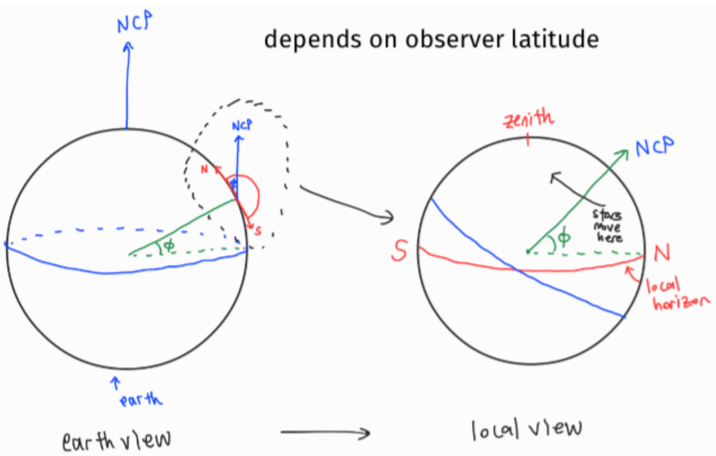
$= \frac{1}{\text{parallax}}$ (in arc seconds)

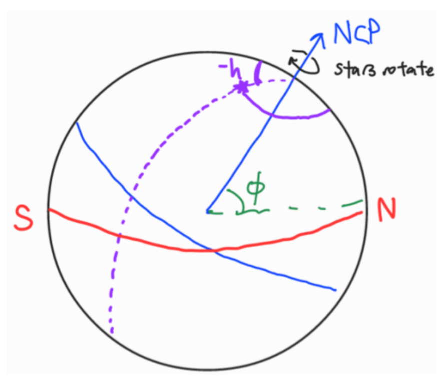
Alt-Azimuth Coordinates



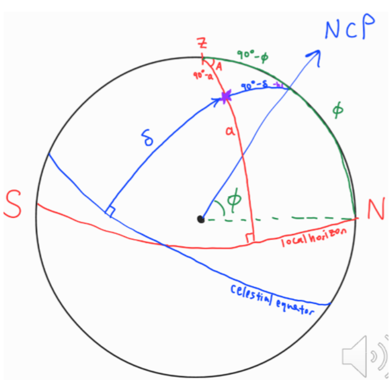
Azimuth: angle clockwise from North
Altitude: angle upwards from local horizon
zenith: directly above observer
meridian: line through zenith, North
- object culminates (crosses meridian) twice a day
- marks highest and lowest altitude

hour angle: time since object crossed meridian above horizon





Coordinate conversion



draw master diagram, then solve using spherical trig

Time:

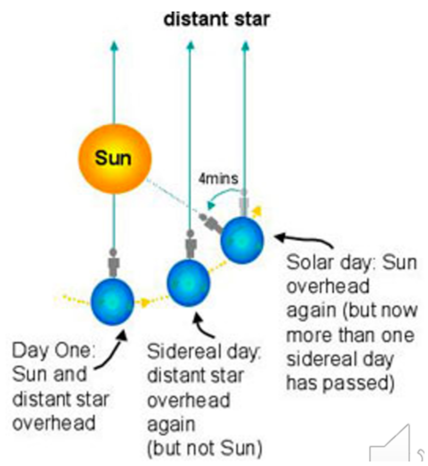
Days

- Earth's rotation around axis and around Sun are in same direction

Solar day: time between sun overhead (24h)

Sidereal day: time between distant star overhead (23h 56min 4s)

- for every round around the Sun there is 1 more sidereal day than solar day



time zones

civil time
- Universal Coordinated Time
- Greenwich Mean Time
- solar time at 0 longtitude (Greenwich, England)

local solar time
- noon = sun cross meridian above horizon
- depend on longtitude

local sidereal time
- defined based on distant celestial object
- hour angle of vernal equinox
- sun move slower than distant stars

Data Analysis

Calculation:

variance

population variance $= \sigma^2$

$$\sigma^2 = \frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}$$

N = population size
μ = population mean

= mean of square − square of mean

$$= \frac{1}{N}\sum_{i=1}^{N} x_i^2 - \left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)^2$$

sample variance $= s^2$

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

$\bar{x}$ = sample mean
n = sample size

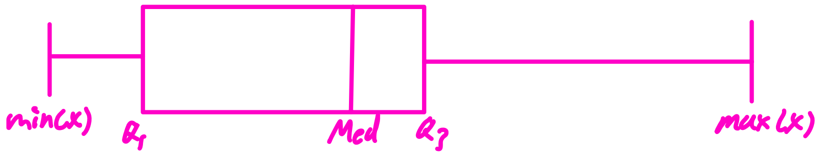
$$= \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x\right)^2\right]$$

1D data

MENU >> 3.stats >> 1:1-VARIABLE >> (enter values) >> OPTN >> 3:1-VARIABLE CALC

$\bar{x}$ = mean
Σx = sum
Σx^2 = sum of square
$\sigma^2 x$ = variance
$\sigma x = \sqrt{\text{variance}}$
$s^2 x$ = sample variance
$s x = \sqrt{\text{sample variance}}$
n = number of data
min(x) = minimum
Q_1 = 1st quartile
Med = median
Q_3 = 3rd quartile
max(x) = maximum

box plot:



2D data

MENU >> 3:stats >> (choose regression equation) >> (enter data) >> OPTN >> 3: 2-VARIABLE CALC
-similar to 1D data

Regression

MENU >> 3:stats >> (choose regression equation) >> (enter data) >> OPTN >> 4:REGRESSION CALC

-find coefficients in regression equation

$$r = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2 \cdot \sum_{i=1}^{n}(y_i-\bar{y})^2}} = \frac{\sum_{i=1}^{n} x_i y_i - \frac{1}{n}\sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} y_i}{\sqrt{\left(\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2\right)\left(\sum_{i=1}^{n} y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2\right)}}$$

accurate correlation if $r \approx 1, -1$

Spreadsheet

MENU >> 4:SPREADSHEET

Performing operation on data in column A, results go to B:

1. enter data in column A
2. go to cell B1
3. click OPTN
4. click 1: FILL FORMULA
5. type in formula in terms of A1, for example (A1 × 5)^2
6. type in range, for example B1:B10
7. press = twice

Graph plotting:

Linearisation:

convert given or found equation into the form $y = mx + c$,

y in terms of dependent variable

x in terms of independent variable

m, c in terms of possibly unknown constants

find unknown constants from slope and intercept of plotted graph

general graph plotting:

- correct points
- good scale
- large enough graph
- good line / curve of best fit
  - ↳ must pass through centroid