502_HW5

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2/16/2022

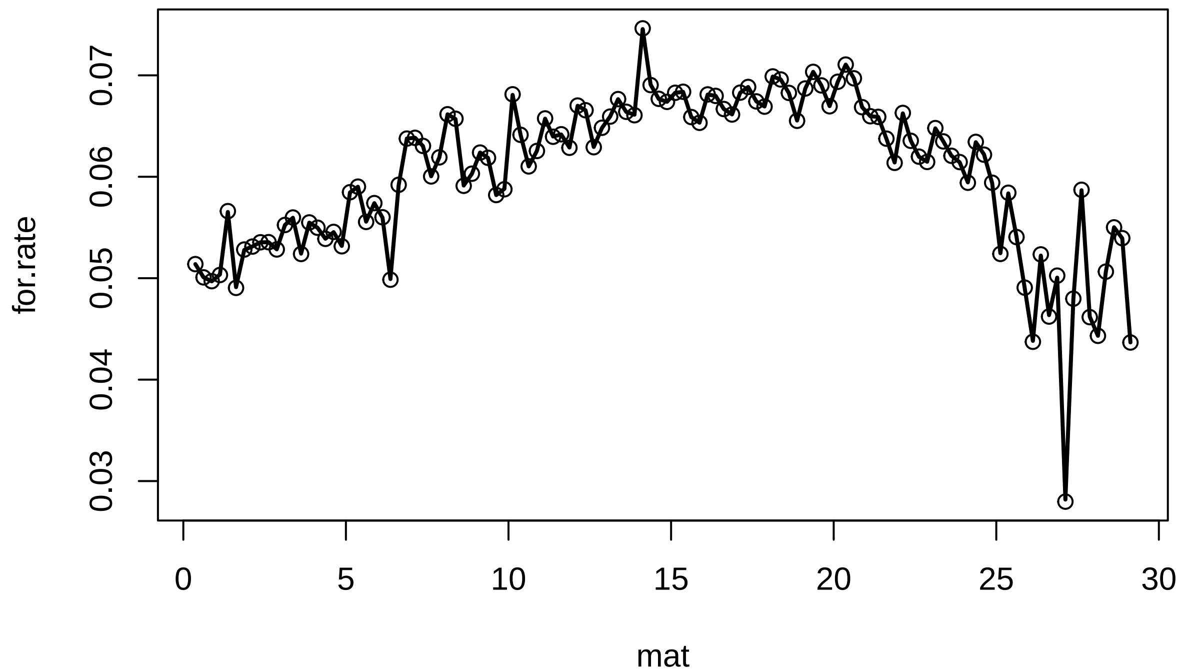
Q1

```
load("Homework 5 Data.Rdata")
strips <- strips[order(strips$T), ]
for.rate <- -diff(log(strips$price)) / diff(strips$T)
mat <- strips$T[-1]
```

(a)

Use all points as knots, and specify spar=0 to interpolate the data.

```
fit.cubic <- smooth.spline(mat, for.rate, all.knots=TRUE, spar=0)
plot(mat, for.rate)
lines(fit.cubic, lwd=2)
```

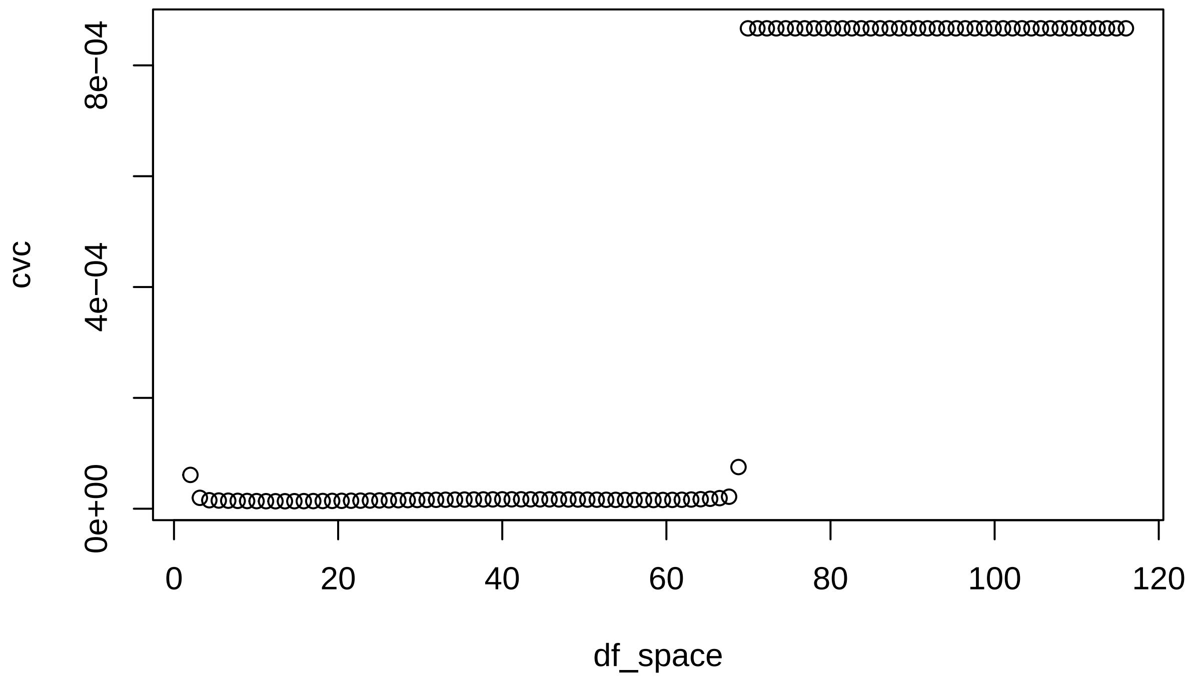


(b)

```
df_space <- seq(from=2, to=116, length.out=100)
cvc <- c()

for (i in df_space) {
   fit.cubic <- smooth.spline(mat, for.rate, df=i, cv=TRUE)
   cvc <- append(cvc, fit.cubic$cv.crit)
}

plot(df_space, cvc)
```



(c)

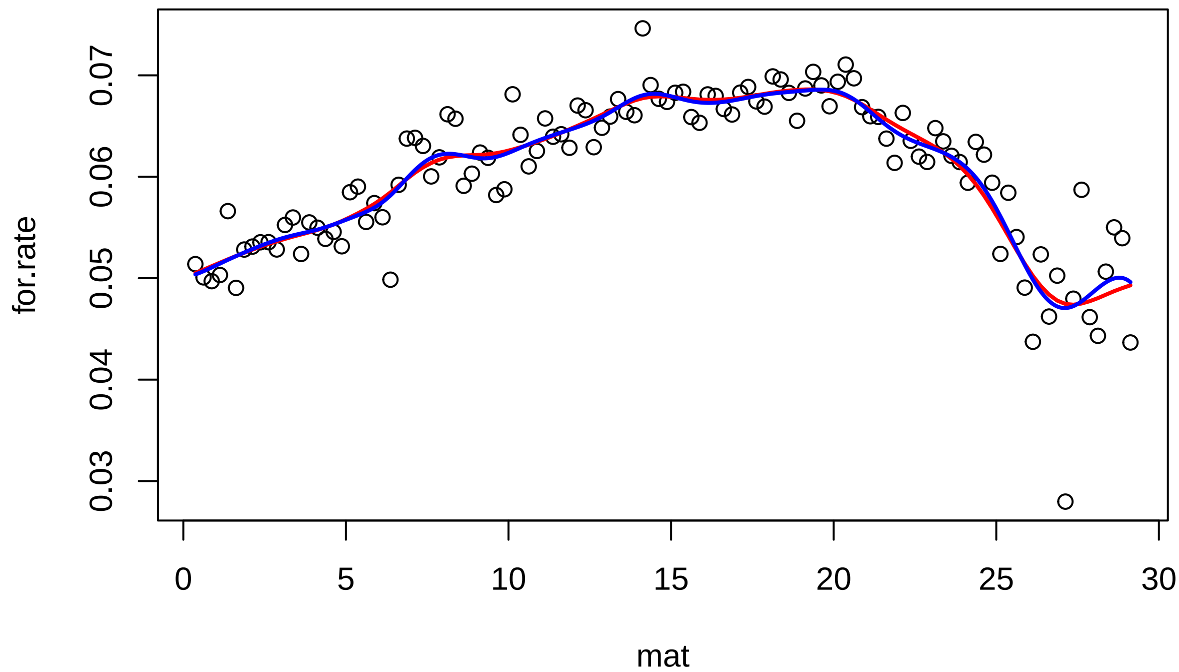
```
fit.cubic <- smooth.spline(mat, for.rate, cv=TRUE)

library(KernSmooth)
```

```
## KernSmooth 2.23 loaded
## Copyright M. P. Wand 1997-2009
```

```
h <- dpill(mat, for.rate)
fit.locpoly <- locpoly(mat, for.rate, bandwidth=h, degree=1)

plot(mat, for.rate)
lines(fit.cubic, lwd=2, col='red')
lines(fit.locpoly, lwd=2, col='blue')
```



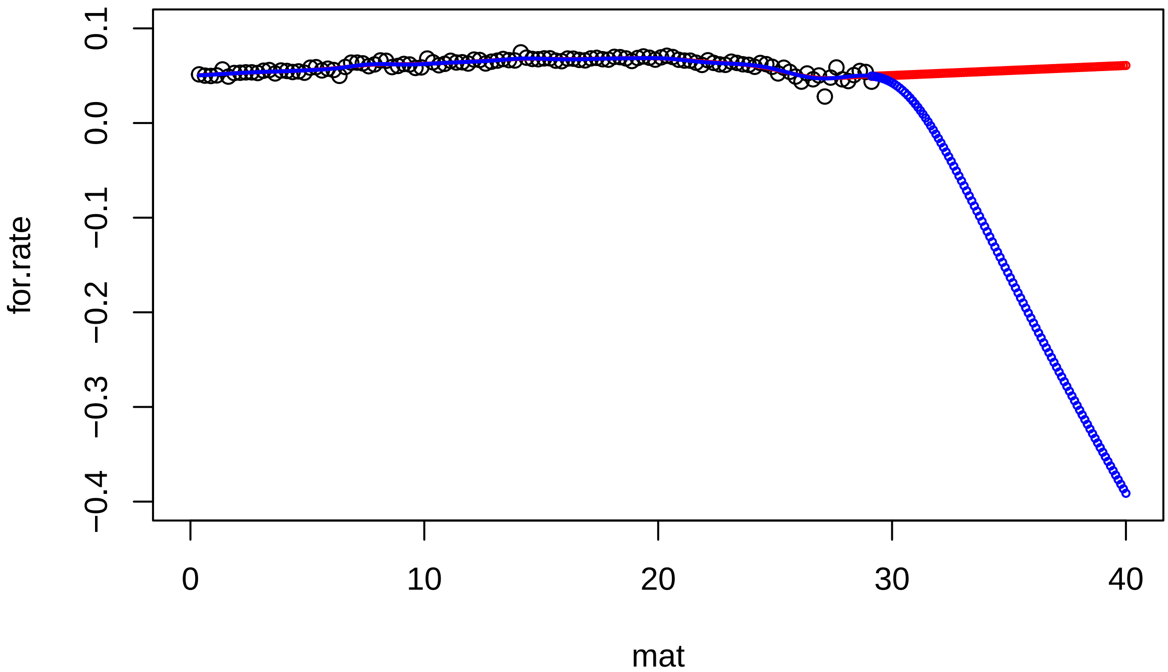
(d)

```
x.grid <- seq(max(mat), 40, len=100)
predict.cubic <- predict(fit.cubic, x.grid)

plot(mat, for.rate, xlim=c(0,40), ylim=c(-0.4,0.1))
lines(fit.cubic, lwd=2, col='red')
lines(fit.locpoly, lwd=2, col='blue')

points(predict.cubic$x, predict.cubic$y, col='red', cex=0.5)

for (x in x.grid) {
   wn <- dnorm(mat, mean=x, sd=h)
   fit <- lm(for.rate~I(mat-x), weights=wn)
   points(x, coef(fit)[1], col='blue', cex=0.5)
}
```



Cubic smoothing spline is more reasonable than local polynomial regression. Forward rates cannot be negative in reality.

Q2

Q2.

(a)

$$\bar{Y}_n \sim N(\mu, \frac{V}{n})$$

$$V = \sum_{h=-\infty}^{\infty} r(h)$$

$$= 2\sum_{h=0}^{\infty} r(h) - r(0)$$

$$= 2\sum_{h=0}^{\infty} \frac{6^2}{1-\phi^2}\phi^h - r(0)$$

$$= \frac{26^2}{1-\phi^2}\frac{1}{1-\phi} - \frac{6^2}{1-\phi^2}$$

$$= \frac{4}{0.04} \times \frac{1}{0.4} - \frac{2}{0.64} = 12.5 \quad \frac{V}{n} = \frac{12.5}{100} = 0.125$$

$$\bar{Y} \sim N(\mu, 0.125)$$

$$\frac{\bar{Y}-\mu}{\sqrt{0.125}} \sim N(0,1)$$

$$P(-1.96 < \frac{\bar{Y}-\mu}{\sqrt{0.125}} < 1.96) = 0.05$$

$$P(\bar{Y} - 1.96\sqrt{0.125} < \mu < \bar{Y} + 1.96\sqrt{0.125}) = 0.05$$

$$(-0.393, 0.993)$$

(b)

0 is in $(-0.393, 0.993)$

We fail to reject the null, the data is compatible with the hypothesis that $\mu=0$.

Q3

Q3

(a)

$\phi(z) = 1+0.5z$

(b)

$\phi(z) = 1-5z+8z^2-2z^5$

(a)

```r
Mod(polyroot(c(1,0.5)))
```

```
## [1] 2
```

autoregressive polynomial!=0 for all complex numbers |z|<=1, so there exists a unique causal stationary solution.

(b)

```r
Mod(polyroot(c(1,-5,8,0,0,-2)))
```

```
## [1] 0.3575712 0.3575712 1.7073595 1.3415130 1.7073595
```

There are two roots make autoregressive polynomial=0 with |z|<=1, so there does not exists a causal stationary solution.

Q4

Q4.

(a) For $h \geq 3$

$Cov(Y_{t-h}, e_t + e_{t-1} - e_{t-2}) = 0$ ①

$Cov(Y_{t-h}, Y_t + 0.5Y_{t-1}) = \gamma(h) + 0.5\gamma(h-1)$ ②

① = ②:

$\gamma(h) + 0.5\gamma(h-1) = 0$

(b) let $h = 1$

$Cov(Y_{t-1}, e_t + e_{t-1} - e_{t-2})$

$= Cov(Y_{t-1}, e_{t-1}) - Cov(Y_{t-1}, e_{t-2})$

$= Cov(-0.5Y_{t-2} + e_{t-1} + e_{t-2} - e_{t-3}, e_{t-1}) - Cov(Y_{t-1}, e_{t-2})$

$= 1 - Cov(-0.5(-0.5Y_{t-3} + e_{t-2} + e_{t-3} - e_{t-4}) + e_{t-1} + e_{t-2} - e_{t-3}, e_{t-2})$

$= 1 - 0.5 = 0.5$ ①

$Cov(Y_{t-1}, Y_t + 0.5Y_{t-1}) = \gamma(1) + 0.5\gamma(0)$ ②

① = ②

$\gamma(1) + 0.5\gamma(0) = 0.5$

(c)

$$\begin{cases} \gamma(0)+0.5\gamma(1)=2.75 \\ \gamma(1)+0.5\gamma(0)=0.5 \\ \gamma(2)+0.5\gamma(1)=-1 \end{cases}$$

$$\underset{A}{\begin{pmatrix} 1 & 0.5 & 0 \\ 0.5 & 1 & 0 \\ 0 & 0.5 & 1 \end{pmatrix}} \underset{\gamma}{\begin{pmatrix} \gamma(0) \\ \gamma(1) \\ \gamma(2) \end{pmatrix}} = \underset{c}{\begin{pmatrix} 2.75 \\ 0.5 \\ -1 \end{pmatrix}}$$

$$\gamma = A^{-1}c$$

(d)

$$\gamma(3)+0.5\gamma(2)=0$$

$$\gamma(3)=\frac{5}{24}$$

$$\gamma(4)+0.5\gamma(3)=0$$

$$\gamma(4)=-\frac{5}{48}$$

$$\gamma(5)=\frac{5}{96}$$

```
A <- matrix(c(1,0.5,0,0.5,1,0.5,0,0,1), nrow=3)
c <- matrix(c(2.75,0.5,-1), nrow=3)
solve(A) %*% c
```

```
##            [,1]
## [1,]  3.3333333
## [2,] -1.1666667
## [3,] -0.4166667
```