502_HW2

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Q1

$$f(\beta_1) = \sum_{i=1}^{n} (Y_i - (\beta_0 + \beta_1 X_i))^2$$

$$\frac{\partial f}{\partial \beta_1}(\hat{\beta}_1) = 0$$

$$-2\sum_{i=1}^{n} X_i Y_i + 2\beta_0 \sum_{i=1}^{n} X_i + 2\hat{\beta}_1 \sum_{i=1}^{n} X_i^2 = 0$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} X_i Y_i - \beta_0 \sum_{i=1}^{n} X_i}{\sum_{i=1}^{n} X_i^2}$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} X_i (Y_i - \beta_0)}{\sum_{i=1}^{n} X_i^2}$$

$$\frac{\partial^2 f}{\partial \beta_1^2}(\hat{\beta}_1) = 2\sum_{i=1}^{n} X_i^2 > 0$$

Thus, the LSE is a least squares minimizer.

Q2

(a)

$$E[\hat{\varepsilon}] = E[Y - \hat{Y}]$$
$$= E[Y - X\hat{\beta}]$$
$$= E[Y - X\beta]$$
$$= E[\varepsilon]$$
$$= 0$$

(b)

$$H^2 = X\cancel{(X^TX)^{-1}X^TX}(X^TX)^{-1}X^T \qquad H^T = X((X^TX)^T)^{-1}X^T$$
$$= X(X^TX)^{-1}X^T = H \qquad\qquad = X(X^TX)^{-1}X^T = H$$

$$\mathrm{Cov}(\hat{\varepsilon}) = \mathrm{Cov}(Y - HY)$$
$$= \mathrm{Cov}((I-H)Y)$$
$$= (I-H)\,\mathrm{Cov}(Y)\,(I-H)^T$$
$$= (I-H)\,\sigma^2 I\,(I-H)$$
$$= (I^2 - 2IH + H^2)\,\sigma^2 I$$
$$= \sigma^2(I-H)$$

(c)

```r
n <- 12
p <- 3
sigma <- 1
X <- matrix(rnorm(n * p), n, p)
X <- cbind(1, X)
H <- X %*% solve(t(X) %*% X) %*% t(X)
Cov <- sigma^2 * (diag(nrow(H)) - H)
D <- diag(diag(Cov)^(-0.5))
(Corr <- round(t(D) %*% Cov %*% D, 2))
```

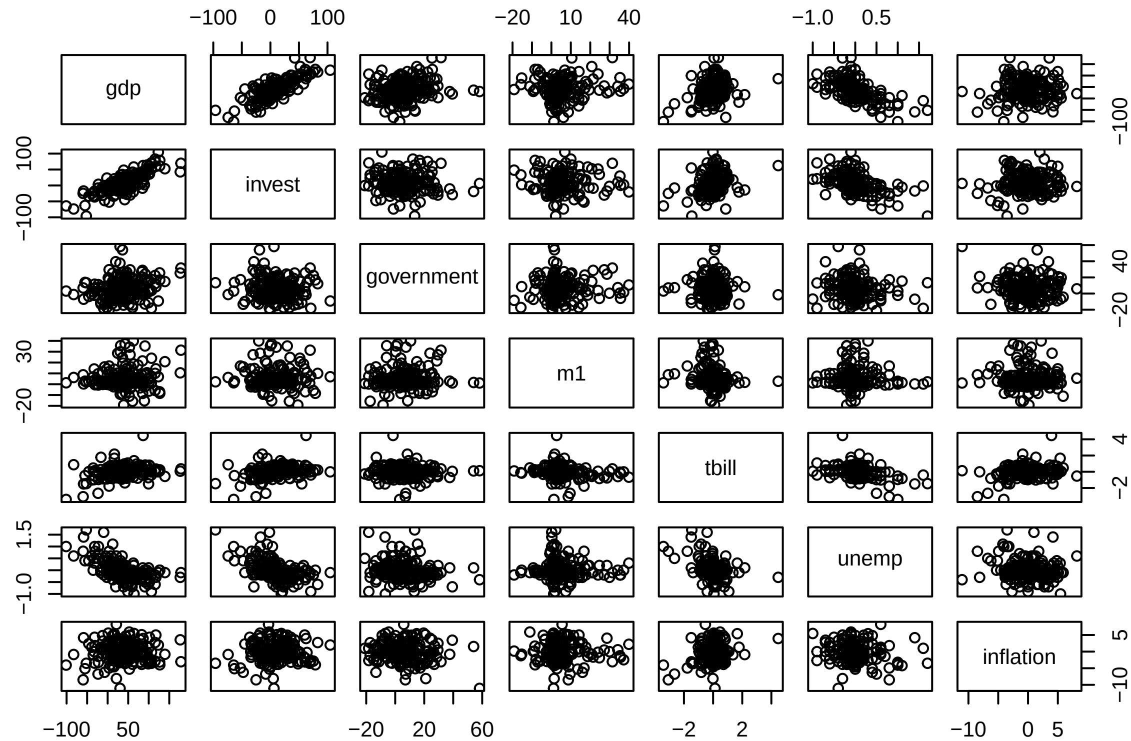
```
##        [,1]  [,2]  [,3]  [,4]  [,5]  [,6]  [,7]  [,8]  [,9] [,10] [,11] [,12]
##  [1,]  1.00 -0.11 -0.10 -0.12 -0.08 -0.17 -0.06 -0.08 -0.08 -0.05 -0.19 -0.17
##  [2,] -0.11  1.00 -0.26 -0.05  0.17  0.16 -0.26  0.09 -0.19 -0.07 -0.11 -0.39
##  [3,] -0.10 -0.26  1.00 -0.06  0.12  0.06 -0.21  0.03 -0.07 -0.23 -0.24 -0.21
##  [4,] -0.12 -0.05 -0.06  1.00 -0.14 -0.32  0.03 -0.11 -0.06  0.06 -0.23 -0.19
##  [5,] -0.08  0.17  0.12 -0.14  1.00 -0.34 -0.06 -0.45 -0.36 -0.18  0.26  0.13
##  [6,] -0.17  0.16  0.06 -0.32 -0.34  1.00  0.27 -0.25  0.07  0.24 -0.47 -0.15
##  [7,] -0.06 -0.26 -0.21  0.03 -0.06  0.27  1.00 -0.07 -0.25 -0.47  0.06 -0.07
##  [8,] -0.08  0.09  0.03 -0.11 -0.45 -0.25 -0.07  1.00 -0.20 -0.25  0.04  0.10
##  [9,] -0.08 -0.19 -0.07 -0.06 -0.36  0.07 -0.25 -0.20  1.00  0.09  0.51 -0.30
## [10,] -0.05 -0.07 -0.23  0.06 -0.18  0.24 -0.47 -0.25  0.09  1.00 -0.43  0.59
## [11,] -0.19 -0.11 -0.24 -0.23  0.26 -0.47  0.06  0.04  0.51 -0.43  1.00 -0.05
## [12,] -0.17 -0.39 -0.21 -0.19  0.13 -0.15 -0.07  0.10 -0.30  0.59 -0.05  1.00
```

Q3

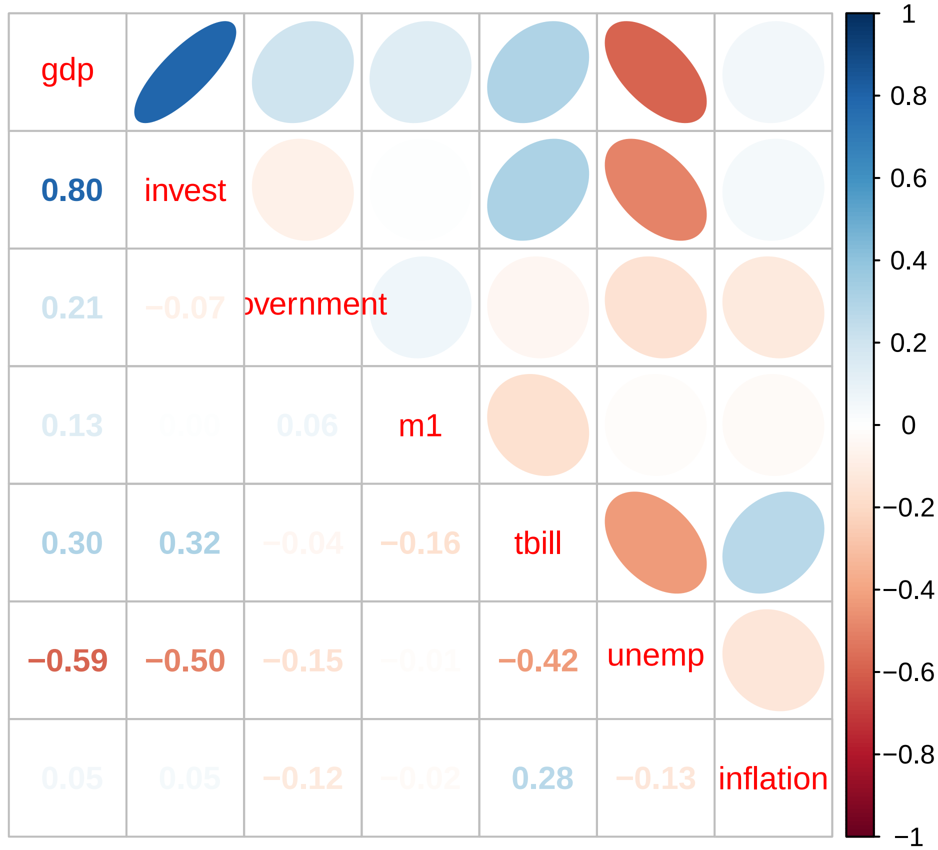
```r
library(AER)
data(USMacroG)
USMacroG.subset <- USMacroG[,c("gdp","invest","government","m1","tbill","unemp",
                               "inflation")]
macro.diff <- na.omit(data.frame(apply(USMacroG.subset, 2, diff)))
attach(macro.diff)
```

(a)

```r
pairs(macro.diff)
```




```
corrplot::corrplot.mixed(cor(macro.diff), upper="ellipse")
```



(b)

```r
fit1 <- lm(tbill ~ gdp)
fit2 <- lm(tbill ~ gdp + inflation)
fit3 <- lm(tbill ~ gdp + unemp)
summary(fit1)$coef[2, 4]
```

```
## [1] 1.084559e-05
```

```r
summary(fit2)$coef[2, 4]
```

```
## [1] 1.362943e-05
```

```r
summary(fit3)$coef[2, 4]
```

```
## [1] 0.2997811
```

For Model 1, the gdp is statistically significant at a 5% level; For Model 2, the gdp is statistically significant at a 5% level in the presence of all the other predictors; For Model 3, the gdp is not statistically significant at a 5% level, but the other predictors are.

For Model 3, the gdp predictor does not add more predictive power to the information already included in the unemp predictor. The gdp predictor is significant in a model that only includes it or with inflation.

(c)

```r
newdata <- data.frame(gdp=60, unemp=0.1)
predict(fit3, newdata=newdata, interval="prediction", level=0.90)
```

```
##           fit       lwr      upr
## 1 -0.02084803 -1.136187 1.094491
```

```r
predict(fit3, newdata=newdata, interval="confidence", level=0.90)
```

```
##           fit        lwr        upr
## 1 -0.02084803 -0.1254977 0.08380161
```

(d)

```r
fullfit <- lm(tbill ~ gdp + invest + government + m1 + unemp + inflation )
summary(fullfit)$coef[6, 1]
```

```
## [1] -0.6122092
```

The change in the Treasury bill rate is predicted to be -0.612 for each unit change in the unemployment rate with no change in other variables.

(e)

```r
anova.fullfit <- anova(fullfit)
(P <- sum(anova.fullfit[1:6, 2]) / sum(anova.fullfit[1:7, 2]))
```

```
## [1] 0.2760125
```

(f)

```r
fit.reduced <- lm(tbill ~ m1 + unemp + inflation)
anova(fit.reduced, fullfit)
```

```
## Analysis of Variance Table
##
## Model 1: tbill ~ m1 + unemp + inflation
## Model 2: tbill ~ gdp + invest + government + m1 + unemp + inflation
##   Res.Df    RSS Df Sum of Sq      F Pr(>F)
## 1    198 81.916
## 2    195 79.797  3    2.1187 1.7258  0.163
```

At a 5% significance level, we fail to reject H0 that beta1=beta2=beta3=0.