第1章 整除
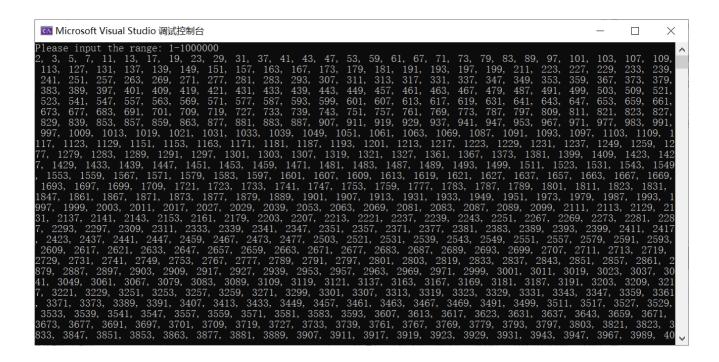
计算证明

1. 计算下面整数对的最大公因子和最小公倍数.

    (1) $(202, 282)$

    (2) $(-666, 1410)$

    (3) $(30, 105, 360)$

    (4) $(8n^2 + 28n + 12, 12n^2 + 30n + 12)$

2. 求下面整数的标准分解式.

    (1) 69

    (2) 200

    (3) 3288

    (4) 22345680

3. 若 $a \in \mathbb{Z}^+ \cup \{0\}$，$a^4 - 3a^2 + 9$ 是质数还是合数?

4. 若 $m - p \mid mn + pq$，求证 $m - p \mid mq + np$.

5. 设 $3 \mid a^2 + b^2$，求证：$3 \mid a$ 且 $3 \mid b$.

6. 设 $a = qn - t$，若 $a \mid pm$，已知 $p - q = t$ 且 $(a, n + 1) = 1$，求证：$a \mid tm$.

7. 求证任意 n 个连续的正整数整数乘积都被 $n!$ 整除. (写出严谨的证明过程)

8. 求证 $12 \mid n^4 + 2n^3 + 11n^2 + 10n,\ n \in \mathbb{Z}$.

9. 证明 n 的标准分解式中次数都是偶数当且仅当 n 是完全平方数.

10. 证明 $\sqrt{5}$ 是无理数. *并将其表示为简单连分数的形式. （*标注表示不作强制要求）

编程练习（基于C/C++）

1. 编写程序使用Eratosthenes筛法打印1 000 000内所有素数及个数，效果如图所示. （*思考：a.对比筛法与普通算法的性能差异；b.递归调用该算法求更大范围素数进行优化；c.求更大的素数（如2^{512}数量级）该方法是否适用？会引入哪些新的问题？）

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Total：

2. 编写程序计算最大公因数和最小公倍数，效果如图所示.

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Microsoft Visual Studio 调试控制台

a=9876
b=6789
gcd(a,b)=3
lcm(a,b)=22349388
```

3. 编写程序实现算术基本定理，效果如下所示.

```
Microsoft Visual Studio 调试控制台

Please input n(n>0): 888
888=2^3*3^1*37^1
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