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Review of Vedic Mathematics Yavadunam Tavadunikritya Varganicha Yojayet Sutra for Cubing a Number

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Abstract: This paper discloses secrets and logics of the short formula of ancient Vedic Mathematics Yavadunam Tavadunikritya Varganicha Yojayet Sutra in finding the cube of a positive number. Through the formalization process, we have aimed to bridge the gap between ancient mathematical wisdom and modern theoretical foundations, showcasing the timeless utility of Vedic techniques in mathematical calculations. The sutra has been reviewed using the algebraic explanation in terms of decimal number system of the multiplication process by applying Binomial theorem for the positive integral exponent 3. Some more short-cut techniques have been discussed to explain the sutra and it has been found that they are producing the same result. There are shortcomings that have been found in applying the sutra, which has been solved using some other techniques as a special rule. Later the sutra has been discussed for negative integers also with a note. Before coming to the conclusion, we have discussed some limitations of the sutra, although that has been solved in the paper but with a different process. We have discussed the implications of the sutra for modern computational methods and educational practices, suggesting potential areas for further research.

Key Words: Cube of a number, Yavadunam Tavadunikritya Varganicha Yojayet Sutra, Binomial theorem.

1. Introduction

The Vedas are regarded divine in origin. The Vedic mathematics processes are based on both contemporary and ancient mathematical system known as sutras. Each formula describes a mental working concept that may be used to solve a verity of mathematical calculations (Bose, 2021; Ganesh, 2018; Halai, 2018; Kumar, n.d.; Kumar, 2017; Maharaja, 2015). Vedic sutras work successfully for resolving factorization, HCF, LCM, divisions, reciprocals, square and square roots, cubes and cube roots, algebraic equations, multiple simultaneous equations, biquadratic equations, cubic equations, higher degree equations, partial fractions, integrations, differentiation, Pythagoras theorem, Apollonius Theorem, Analytical Conics and so on (Gupta, 2018; Parajuli, 2020; Prasad, n.d.; Selvaraj, et al., 2021)). It consists sixteen sutras and thirteen sub-sutras. It simplifies not only the fundamental arithmetic operations, such as multiplication and division, but also more advanced concepts (Maharaja, 2015; Solanki, 2021). This mathematics technique is rapidly emerging as a tool for various competitive examinations, including CAT, MAT, XAT, SAT, Engineering and banking examinations, where speed and accuracy play a vital role. Vedic mathematics presents several very interesting methods to find the cube of any number in a few seconds (Tekriwal, 2015; Thakur, 2019). Unlike the traditional method, it is easy, interesting, and short (Khatua, 2022). It reduces the time by one-tenth, that's why the Vedic method is far superior to the traditional method of finding cubes.

2. Preliminary Ideas

In the paper we will work under the following basic concepts:

Cube of a Number: When a number is multiplied three times by itself, the new number so obtained is called the cube of that number. For example, $a \text{ x } a \text{ x } a = a^3$ i.e., a^3 is the cube of the number a i.e., the third power of a number is called the cube of that number (James/James, 2001; Shashtri, 2011; Thakur, 2019). Except formal mathematical calculation, there are three methods of Vedic Sutra to find cube of a number: Yavadunam, Anurupyen and Nikhilam sutras as formulae (Shashtri, 2011; Maharaja, 2015; Halai, 2018; Thakur, 2019). In this paper, we will review only Yavadunam sutra, which is stated as follows:

Yavadunam Tavadunikritya Varganicha Yojayet Sutra: The meaning of "Yavadunam Tavadunikritya Varganicha Yojayet Sutra" is "by the deficiency" i.e. it means "whatever the deficiency, lesson by that amount and set up the square of the deficiency" (Shashtri, 2011; Maharaja, 2015; Halai, 2018). This sutra works better when the number to be cubed is near to the base 10^n, for natural numbers n = 1, 2, 3, ... The working rules for this sutra is as follows:

i. Test whether the number is near the base numbers 10, 100, 1000, … or not.
ii. Find the excess or deficit number from the base number.
iii. Then the whole operation will be performed in three parts:
a. In the first part, denoted by LHS, add twice the excess or deficit to the original number.
b. The second part, denoted by middle term, is equal to
= new excess (= number obtained in first part - base) x original excess or deficit.
c. The third part, denoted by RHS, equals 'cube of excess or deficit'.

We can explain it mathematically also as, if a = original number and d = deviation (excess or deficit from the base), then the whole operation can be summed up as

$$a^3 = (a + 2d) \mid [(a + 2d) - base] \text{ x } d \mid d^3.$$

Example-1: Find the cube of 14.

Solution: Here a = 14 the original Number, which is near to the base 10. Hence, base = 10 and the excess = + 4. Now as per the working rules,

First part or LHS = a + 2d = 14 + 2 × 4 = 22
Second part or Middle term = [(a + 2d) – base] × d = (22 – 10) × 4 = 48
And third part or RHS = $d^3 = 4^3 = 64$

Therefore, combining all the three parts, we get

$14^3 = 22 \mid 48 \mid 64$

Taking one digit from the right most side and adding the remaining to the left, we get

$14^3 = 22 \mid 48 + 6 \mid 4 = 22 \mid 54 \mid 4 = 22 + 5 \mid 4 \mid 4 = 27 \mid 4 \mid 4 = 2744$

which is the required cube of 14.

Example-2: Find the cube of 94.

Solution: Here a = 94, the original number, which is near to the base 100 and deficit = - 6. So, we have

First part = a + 2d = 94 + (2 × – 6) = 94 – 12 = 82
Second part = [(a + 2d) – base] × d = (82 – 100) × –6 = 108
And the third part = $d^3 = -6^3 = -216$

Therefore, combining all the three parts, we get

$94^3 = 82 \mid 108 \mid \mathbf{-216} = 82 \mid 105 + 3 \mid \mathbf{-216} = 82 \mid 105 \mid \mathbf{300 - 216}$

$= 82 \mid 105 \mid 84 = 83 \mid 05 \mid 84 = 830584$

The number of digits in each part taken to write the answer depends on the number of zeros in the base number. In this example, the base is taken as 100, so the number of digits permissible in each part is two. Based on this process, we will review the sutra for more examples and will correlate it with the algebraic representation of the multiplication of numbers.

3. Discussion

Let us discuss and analysis the procedures of the Yavadunam sutra step by step for cubing a number. Since there arise two different case: one for excess and another for deficit. First we analysis the examples and working rules for excess case. In example-1, we have used Yavadunam Sutra and have found the cubes of some numbers. Let us use arithmetical and algebraic methods for the same examples as follows. Using Binomial theorem expansion, we know that

$$14^3 = (14)^3 = (10 + 4)^3 = 10^3 + 3 \text{ x } 10^2 \text{ x } 4 + 3 \text{ x } 10 \text{ x } 4^2 + 4^3$$
$$= 1000 + 1200 + 480 + 64$$

Since the last digit in first three terms is 0 and 4 in the fourth term, so while adding all, the last digit of the answer will be 4, which has only one digit equals to the number of zero in the base 10 near to 14. After writing 4 as the last digit of the answer, we have left the following

$$= 100 + 120 + 48 + 6$$

In which we have to add 6 to its left number i.e.,

$$= 100 + 120 + 54$$

Again the first two terms have zero at the unit place, so adding all we get only 4 at unit's place, which will be placed before 4 in the last answer obtained i.e., 44. Now the left calculation is

$$= 10 + 12 + 5$$

which on adding gives 27. Therefore the cube of the number 14 is 2744. This is the process which has been applied in the working rules of Yavadunam Sutra in three parts.

Similarly we can apply the sutra and verify it for a number having more digits. Let us find the cube of 106, we have

$$106^3 = (100 + 6)^3 = 100^3 + 3 \text{ x } 100^2 \text{ x } 6 + 3 \text{ x } 100 \text{ x } 6^2 + 6^3$$
$$= 10000{,}00 + 1800{,}00 + 108{,}00 + 2{,}16$$

Since the last two digits in first three terms is 00 and 16 in the fourth term, so while adding all, the last two digit of the answer will be 16, which has only two digit equals to the number of zero in the base 100 near to 106. After writing 16 as the last two digit of the answer, we have left the following

$$= 10000 + 1800 + 108 + 2$$
$$= 100{,}00 + 18{,}00 + 1{,}10 \text{ (adding last two terms)}$$

Again the first two terms have zero at the unit and tenth place, so adding all we get only 10 at unit and tenth place, which will be placed before 16 in the last answer obtained i.e., 1016. Now the left calculation is

$$= 100 + 18 + 1$$

which is equal to 119. Therefore the cube of 106 is 1191016.

It can be verified using the sutra and its working rules as:

$$106^3 = 106 + (2 \text{ x } 6) \mid 18 \text{ x } 6 \mid 6^3$$
$$= 118 \mid 108 \mid 216 = 118 \mid 108 \mid 2 \text{ (Taking 16 as last two digit for answer)}$$
$$= 118 \mid 108 + 2 \mid \text{ (adding the remaining 2 in its left side)}$$
$$= 118 \mid 110$$
$$= 118 \mid 1 \text{ (taking 10 for the answer to be written in the left side of 16)}$$
$$= 119 \text{ (adding remaining 1 in its left side)}$$

Now writing the three parts of the answer as 1191016 using the working rule of the sutra, we find the cube of 106, which is equal to the above answer.

Let us consider one more example on cubing of the number, we have

$$10036^3 = (10000 + 36)^3 = 10000^3 + 3 \text{ x } 10000^2 \text{ x } 36 + 3 \text{ x } 10000 \text{ x } 36^2 + 36^3$$
$$= 100000000{,}0000 + 1080000{,}0000 + 3888{,}0000 + 4{,}6656$$

Since the last four digits in first three terms is 0000 and 6656 in the fourth term, so while adding all, the last four digit of the answer will be 6656, which has only four digit equals to the number of zero in the base 10000 near to 10036. After writing 6656 as the last four digit of the answer, we have left the following

$$= 100000000 + 1080000 + 3888 + 4$$
$$= 10000{,}0000 + 108{,}0000 + 3892 \text{ (adding last two terms)}$$

Again the first two terms have four zero at the end, so adding all we get only 3892 at the end, which will be placed before 6656 in the last answer obtained i.e., 38926656. Now the left calculation is

$$= 10000 + 108$$

which is equal to 10108. Therefore the cube of 10036 is 1010838926656.

It can also be verified using the procedures of the sutra and its working rules as:

$$10036^3 = 10036 + (2 \text{ x } 36) \mid 108 \text{ x } 36 \mid 36^3, \text{[Since } 10036 + (2 \text{ x } 36) = 10108\text{]}$$
$$= 10108 \mid 3888 \mid 46656$$
$$= 10108 \mid 3888 \mid 4 \text{ (Taking 6656 as last four digit for answer)}$$
$$= 10108 \mid 3888 + 4 \mid \text{ (adding the remaining 4 in its left side)}$$

$= 10108 \mid 3892$
$= 10108 \mid 0$ (taking 3892 for the answer to be written in the left side of 6656)
$= 10108$ (adding remaining 0 in its left side)

Writing the three parts of the answer as 1010838926656 using the working rule of the sutra, we find the cube of 10036, which is equal to the above answer.

From above discussion, we can justify the rules as: since the Binomial expansion of $(x+y)^3$ contains four terms: $x^3, 3x^2y, 3xy^2$ and y^3, where x is the base number of the form 10^n, for natural number n. If x contains 'r' zeros, then the term x^3 will contain at least '3r' zeros, the term $3x^2y$ will contain at least '2r' zeros, the term $3xy^2$ will contain at least 'r' zeros and the last term y^3 will contain no zero, if y has no zero in its last digit. Even if y^3 contains zero in its last digits, it will not affect the working rule.

So obviously the first three terms will contain at least 'r' zeros in their last four places at unit, tenth, hundredth, thousand, … and so on. Therefore the r digit from right side of the fourth term y^3 is written as right hand side of the answer and the remaining digits is added to its left. After removing the r zeros from the first three terms and adding the last two terms, we again find at least r zeros in the first two terms. So the r digits of the third term is written as middle part of the answer to the left of the right hand side number and then r zeros will be removed from the first two terms and the remaining digits in the third term is then added with the first two terms to get the left hand side of the answer. Thus we get the answer of the cube of the given number. These three parts have been used as working rules for Yavadunam sutra for cubing a number in excess case.

Now we analysis the examples and working rules for deficit case. In example-2, we have found the cube of 94. Let us explain it algebraically as

$$94^3 = (100-6)^3 = 100^3 - 3 \text{ x } 100^2 \text{ x } 6 + 3 \text{ x } 100 \text{ x } 6^2 - 6^3$$
$$= 1000000 - 180000 + 10800 - 216$$

The right hand side digits or number of the answer must be positive. So we take a number near to 216 to make it positive. Generally we take a base or a sub-base number near to and greater than 216. So we write it as

$$= 1000000 - 180000 + 10500 + (300 - 216)$$
$$= 1000000 - 180000 + 10500 + 84$$

Since in base number there are two zero, so we take two digits from right side to write the answer. Also in first three terms, the last two digits are zero, so after adding we will get 84 in the last two digits of the result i.e., we have after removing the last two digits from each terms

$$= 10000 - 1800 + 105, (\text{RHS} = 84)$$
$$= 8200 + 105, (\text{RHS} = 84)$$

Clearly now the last two digits of the result will be 05 as the first term has two zeros in the unit and tenth places. Thus taking 05 next to 84 in its left side, we get

$$= 82 + 1, (\text{Middle and RHS} = 0584)$$
$$= 83, (\text{Middle and RHS} = 0584, \text{and LHS} = 83)$$

Therefore the final result is 830584 i.e. the cube of 94 is 830584. The same procedures have been followed in the working rules of Yavadunam sutra in finding the cube.

Let us consider one more example on it having more digits. Let us find the cube of the number 9984. We have deficit 9984 -10000 = -16. Thus

LHS Part = 9984 + 2 x (-16) = 9952
Middle Part = -48 x -16 = 768, here new deficit is 9952 - 10000 = -48
RHS Part = $(-16)^3$ = -4096

Thus the cube of the number 9984 is given by the sutra as

$$(9984)^3 = 9952 \mid 768 \mid -4096 = 9952 \mid 767 \mid 10000 - 4096 = 9952 \mid 767 \mid 5904$$

Since there are four zeros in the base number, we take four digits from right side, we get

$= 9952 \mid 767 \mid 0,$ (right hand side four digits 5904)
$= 9952 \mid 0767,$ (right hand side four digits 5904)

Here we have taken 0767 in place of 767, as we have to take four digits. So we have now

$= 9952 \mid 0$

(middle and right hand side each with four digits 07675904, and left hand side is 9952)

Thus the final result is 995207675904, which is the required cube of the number using the sutra. Now let us find the cube of this number algebraically as

$$(9984)^3 = (10000-16)^3 = 10000^3 - 3 \text{ x } 10000^2 \text{ x } 16 + 3 \text{ x } 10000^1 \text{ x } 16^2 - 16^3$$
$$= 10000^3 - 48 \text{ x } 10000^2 + 768 \text{ x } 10000 - 4096$$

$= 10000^3 - 48 \text{ x } 10000^2 + 767 \text{ x } 10000 + 10000 - 4096$
$= 10000^3 - 48 \text{ x } 10000^2 + 767 \text{ x } 10000 + 5904$
$= 10000^2 - 48 \text{ x } 10000^1 + 0767$, (taking last four digits of the cube 5904)
$= 10000^1 - 48$, (taking last eight digits of the cube 0767,5904)
$= 9952$, (taking last eight digits of the cube 0767,5904)

Finally the cube of the number 9984 is 995207675904, which is same as above found using sutra. The procedure followed in above two algebraic methods has been followed in the sutra of Yavadunam sutra in compact form.

We can justify the sutra for this deficit case also in the same manner as has been justified for the excess case. The only difference is that in this case the fourth term is a negative number, which must be made positive by borrowing the near base or sub-base number from its left side term i.e. from the third term and adjust the numbers respectively. It must also kept in mind that the number of zeros in the base or sub-base numbers taken as borrow must have the same number of zeros as the original base number has, otherwise it may produce a wrong result. For example, we can reconsider the example-2 as follows: In it we have, a = 94, which is near to the base 100 and deficit = - 6. So, we have

94^3 = 82 | 108 | – 216 = 82 | 107 + 1 | – 216 = 82 | 107 | 1000– 216
= 82 | 107 | 784 = 82 | 107 + 7| 84 = 82 | 114| 84 = 82 + 1| 14| 84 = 831484

which is wrong. Hence the number of zeros in the borrowing number must be chosen carefully.

In above discussion, we have considered the numbers near to the base only. So we should discuss the examples beyond it to justify the sutra and its working rules for all types of numbers. For it, let us consider some more examples as:

Example-3: Find the cube of 44.

Solution: Here, a = 44 (original number) is near to the base 100. Hence, base = 100 and deficit = 44 – 100 = -56.

LHS Part = a + 2d = 44 + (2 × – 56) = 44 – 112 = – 68
Middle Part = [(a + 2d) – base] × d = (– 68 – 100) × –56 = 9408
RHS Part = d^3 = – 56^3 = – 175616

Combining all the three parts and using the working rule of the sutra, we get

44^3 = – 68 | 9408 | – 175616 (i)
= – 68 | 9407 | 1000000 – 175616 = – 68 | 9407 | 824384
= – 68 | 9407 | 8243 (Taking 84 as the last two digit of the required cube)
= – 68 | 17650 (adding the remaining number in its left)
= – 68 + 176 (Taking 50 before the last two digits 84 of the required cube i.e. 5084)
= 1085084

which is a wrong result as the cube of the number 44 is 85184. It means that the working rule of the sutra fails for the number, which is not near the base.

But after some modification in the process, we can find its exact value i.e. its cube. From (i), we have

= – 68 | 9408– 1756 | – 16 (since we have to take only two digits from right side, we take -16)
= – 68 |7652| – 16 (the remaining number has been added to its left with the negative sign)
= – 68 | 7651 + 1| – 16 (to make the last two digits positive, we borrow 100 from left of -16)
= – 68 | 7651 | 100 – 16
= – 68 + 76 | 51 | 84 (51 and 84 have been taken from middle and right side for the answer)
= – 68 + 76 | 51 | 84 (the remaining number of the middle part has been added to its left)
= 8 | 51 | 84 = 85184

This can be calculated in different way as follows: we have from (i)

44^3 = – 68 |9408| – 175616
= – 68 | 7651 + 1757 | – 175616 = – 68 | 7651 | 175700– 175616
= – 68 | 7651| 175700– 175616 = – 68 | 7651 | 84 = – 68 + 76 | 51 | 84
= – 68+76 | 51 | 84 = 8 | 51 | 84 = 85184

which is also true value of the cube of the number 44.

We can also find the right result of the cube using a **special new rule** as follows: since from (i), we have

44^3 = – 68 |9408| – 175616 = – 68 | 0 + 9408 | – 175616 = – 68 | 0 | 940800 – 175616

(since the base is 100, we multiply the middle term by it and take it in the right side)

= – 68 | 0 | 765184 = – 68 | 7651 (last two digit 84 is taken for the answer)
= – 68 + 76 (last two digit 51 is take in the left of 84 for the answer as 5184)

= 8 | 51 | 84 = 85184

which is the cube of 44. Thus after rearranging the terms, we can find the cube using this sutra.
Let us try the above two processes for another numbers also.

Example: 4. Find the cube of 57.

Solution: Here a = 57 (original number) is nearer to the base 100. Hence base = 100 and the deficit is = − 43.

LHS part = a + 2d = 57 + 2 × − 43 = 57 − 86 = − 29
Middle part = [(a + 2d) − base] × d = (− 29 − 100) × −43 = 5547
RHS part = $d^3 = -43^3 = -79507$

Combining all the three parts and using the sutra working rule, we get

$57^3 = -29\ |5547|\ -79507$ (ii)

Applying the traditional procedures of the sutra

= − 29 | 5546 +1 | − 79507 = − 29 | 5546 |100000 − 79507 = − 29 | 5546 | 20493
= − 29 | 5546+204 | 93 = − 29 |5750| 93 = − 29+57| 50 | 93 = 28| 50 | 93 = 285093

which is a wrong result of the cube of 57. In fact the cube of 57 is 185193.

Following the alternative second method as has been discussed in the previous example-3, we get from (ii)

= − 29 | 5547− 795 | − 07 = − 29 | 4752 | − 07 = − 29 | 4751+1|− 07
= − 29 | 4751 | 100− 07 = − 29 | 4751 | 93 = − 29+47 | 51 | 93 = 18 | 51 | 93 = 185193

which is the right answer of the cube of 57.

We can again apply the procedures that has been applied in previous example as

57^3 = − 29 |5547| − 79507 = − 29 | 5547 | − 79507 = − 29 | 4751+796 | − 79507
= − 29 | 4751| 79600− 79507 = − 29 | 4751 | 93 = − 29 | 4751 | 93 = − 29+47 | 51 | 93
= 18 | 51 | 93 = 185193

which is true value of the cube of 57.

Similarly following the special new rule as discussed in the previous example-3, we get from (ii) that

57^3 = − 29 |5547| − 79507 = − 29 | 5547 | − 79507 = − 29 | 0+5547 | − 79507
= − 29 | 0 | 554700− 79507 = − 29 | 0 | 475193 = − 29 | 4751 | 93
= − 29+47 | 51 | 93 = 18 | 51 | 93 = 185193

which is the right value of the cube of 57.

Example-5: Find the cube of 453.

Solution: Here a = 453 (original number) is nearer to the base 1000. Hence base = 1000 and the deficit = −547.
LHS part = a + 2d = 453 + 2 × − 547 = 453 − 1094 = − 641
Middle part = [(a + 2d) − base] × d = (− 641 − 1000) × −547 = 897627
RHS part = $d^3 = -547^3 = -163667323$

Combining all the three parts, we get

453^3 = − 641 |897627| − 163667323 (iii)
Using the traditional rule of the sutra, we get
453^3 = − 641 |897627| − 163667323 = − 641|897626+1| − 163667323
= − 641|897626|1000000000 − 163667323 = − 641| 897626|836332677
= − 641| 897626+836332 | 677 = − 641| 1733958 | 677 = − 641+1733|958| 677
= 1092 |958| 677 = 1092958677

which is wrong as the right value of the cube of 453 is 92959677.

Following the same second procedure as has been discussed in example-3, we get

$= -641 \mid 733959 - 163667 \mid -323 = -641 \mid 570292 \mid -323 = -641 \mid 570291+1 \mid -323$
$= -641 \mid 733959 \mid 1000 - 323 = -641 + 733 \mid 959 \mid 677 = -641+733 \mid 959 \mid 677$
$= 92 \mid 959 \mid 677 = 92959677$

which is the required cube of 453.

Applying the same another procedure as has been applied earlier, we get

$453^3 = -641 \mid 897627 \mid -163667323 = -641 \mid 733959+163668 \mid -163667323$
$= -641 \mid 733959 \mid 163668000 - 163667323 = -641 \mid 733959 \mid 677 = -641 \mid 733959 \mid 677$
$= -641 + 733 \mid 959 \mid 677 = -641+733 \mid 959 \mid 677 = 92 \mid 959 \mid 677 = 92959677$

which is true value of the cube of the number 453.

Following the same special new rule as has been discussed above, we get from (iii)

$453^3 = -641 \mid 897627 \mid -163667323 = -641 \mid 0+897627 \mid -163667323$
$= -641 \mid 0 \mid 897627000 - 163667323 = -641 \mid 0 \mid 733959677$
$= -641 \mid 0 + 733959 \mid 677 = -641 + 733 \mid 959 \mid 677 = -641+ 733 \mid 959 \mid 677$
$= 92 \mid 959 \mid 677 = 92959677$

which is the true value of the cube of 453.

Example-6: Find the cube of 564.

Solution: Here a = 564 (original number) is nearer to the base 1000. Hence base = 1000 and the deficit = –436.

First part = a + 2d = $564 + 2 \times -436 = 564 - 872 = -308$
Second part = $[(a + 2d) - \text{base}] \times d = (-308 - 1000) \times -436 = 570288$
Third part = $d^3 = -436^3 = -82881856$

Combining all the three parts we get,

$$564^3 = -308 \mid 570288 \mid -82881856 \quad \text{(iv)}$$

Using traditional process of the sutra, we get

$564^3 = -308 \mid 570288 \mid -82881856 = -308 \mid 570287 + 1 \mid -82881856$
$= -308 \mid 570287 \mid 100000000 - 82881856 = -308 \mid 570287 \mid 17118144$
$= -308 \mid 570287 + 171181 \mid 144 = -308 \mid 741468 \mid 144 = -308 + 741 \mid 468 \mid 144$
$= 433 \mid 468 \mid 144 = 433 \mid 468 \mid 144 = 433468144$

which is wrong result of the cube of 564, whose correct value is 179406144.

Using the second procedure as above, we get from (iv) that

$= -308 \mid 570288 - 82881 \mid -856 = -308 \mid 487407 \mid -856 = -308 \mid 487406 + 1 \mid -856$
$= -308 + 487 \mid 406 \mid 1000 - 856 = -308 + 487 \mid 406 \mid 144 = 179 \mid 406 \mid 144$
$= 197 \mid 406 \mid 144 = 197406144$

which is correct value of the cube of 564.

We can again apply the another previous procedure to get the cube of the number 564 as

$564^3 = -308 \mid 570288 \mid -82881856$
$= -308 \mid 487406+82882 \mid -82881856 = -308 \mid 487406 \mid 82882000 - 82881856$
$= -308 \mid 487406 \mid 144 = -308 + 487 \mid 406 \mid 144 = -308 + 487 \mid 406 \mid 144$
$= 179 \mid 406 \mid 144 = 197 \mid 406 \mid 144 = 197406144$

which is the required value of the cube of the number 564.

Whereas using the special new rule, we get from (iv)

$564^3 = -308 \mid 570288 \mid -82881856 = -308 \mid 0 + 570288 \mid -82881856$
$= -308 \mid 0 \mid 570288000 - 82881856 = -308 \mid 0 \mid 487406144$

= – 308 + 487 | 406 | 144 = – 308 + 487 | 406 | 144 = 179 | 406 | 144
= 197 | 406 | 144 =197406144

which is correct value of the cube of 564.

Example-7: Find the cube of 1552.

Solution: Here a = 1552 (original number) is nearer to the base 1000. Hence base = 1000 and the excess = 552.

First part = a + 2d = 1552 + 2 × 552= 1552 + 1104 = 2656
Second part = [(a + 2d) – base] × d = (2656–1000) × 552 = 914112
Third part = $d^3 = 552^3$ = 168196608

Combining all the three parts and using the rule of sutra, we get

1552^3 = 2656 | 914112 | 168196608 = 2656 |914112+*168196*| 608
= 2656 |1082308| 608 = 2656+1082 |308| 608 = 3738|308| 608 = 3738308608

Whereas in case of excess we don't need the second and the special new rule as have been discussed above. Let us take one more example of excess type as:

Example-8: Find the cube of 159.

Solution: Here a = 159 (original number) is nearer to the base 1000. Hence the base = 100 and the excess = 59.

First part = a + 2d = 159 + 2 × 59 = 159 + 118 = 277
Second part = [(a + 2d) – base] × d = (277–100) × 59 = 10443
Third part = $d^3 = 59^3$ = 205379

Combining all the three parts and using the sutra, we get

159^3 = 277 |10443| 205379 = 277 | 10443 + 2053 | 79 = 277 | 12496 | 79
= 277 + 124 | 96 | 79 = 401 | 96 | 79 = 4019679

which is the required cube of 159.

Example-9: Find the cube of 1465.

Solution: Here a = 1465 which is near to the base 1000. Hence the base = 1000 and the excess = 465.

LHS part = a + 2d = 1465 + 2 × 465 = 1465 + 930 = 2395
Middle part = [(a + 2d) – base] × d = (2395–1000) × 465 = 648675
RHS part = $d^3 = 465^3$ = 100544625

Combining all the three parts as per the sutra, we get

1465^3 = 2395 | 648675 | 100544625 = 2395 | 648675 + 100544 | 625
= 2395 | 749219 | 625 = 2395 + 749 | 219 | 625 = 3144 | 219 | 625
= 3144219625

which is the required cube of 1465.

From above examples and their analysis, it can be stated that the sutra fails for the numbers, when the number is not near about the base and it is a case of deficit one. Whereas the sutra is applicable for all such numbers if the case is of excess type.

Cube of Negative Integers: We can apply the working rules for negative integers also using the formula $(-a)^3 = -(a)^3$. So no special attention and example is required for such cases.

4. Limitations

The sutra works better when the number to be cubed is near to the base numbers 10, 100, 1000, 10000, … etc. The working rule of the sutra fails for the number, which is not near the base, especially in deficit case.

5. Conclusion

From above examples and their analysis we find that the sutra fails for the numbers, when the number is not near about the base especially in case of deficit one. Whereas the sutra is applicable for all such numbers in the case of excess one. Therefore in case of deficit one, we can apply the other three procedures discussed in the examples. We can also use the procedures of sutra for negative numbers after multiplying the cube of its positive number by -1.

6. Future Scope of Research

The special new rule applied as the third method is a hit and trial method. No derivation exists for the process applied in this case. So a scope is left for its algebraic and theoretical proof.

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