**Imploding Detonation Waves in Self-gravitating Ideal Gas**

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| ***Abstract:*** *In the present paper, the propagation of strong spherical imploding detonation waves in a reacting ideal gas under the effect of self-gravitation of the non-homogeneous medium has been investigated, by the help of Chester-Chisnell-Whitham (CCW) theory. It is considered that detonation wave is initially Chapman-Jouguet. Initial taking the density distribution law as* *power decreasing with distance, the analytical expressions for the detonation velocity just behind the front along with other flow variables are derived. Neglecting the effect of overtaking disturbances the variation of non-dimensional detonation velocity, the pressure and density with the propagation distance have been calculated numerically. The effect of change in density parameter at different Alfven Mach number on the convergence of detonation front have been discussed through graphs in details. Finally, it is found that density parameter and Alfven Mach number of gas have a significant role on propagation of strong spherical detonation front in reacting ideal gas with gravitation effect on all the post-flow variables. The software MATLAB have been used for computation of the problem.*  ***Key Word****:**Self-gravitating gas, Strong detonation waves, CCW theory.* |

1. **Introduction**

Study of the propagation of detonation waves in reacting gases have great importance in the research field of astrophysics and shock dynamics. Many scientists have considered the problem of motion of detonation waves in different type of homogeneous and non-homogeneous media. Welsh1, Nigmatulin2 and Tiepel3 have assumed the shock wave by a contracting detonation wave front moving into a uniform combustible ideal gas. In magnetogasdynamics, Vishwakarma4 have extended the problem of Nigmatulin2. Verma and Singh5,6 have investigate the problem of detonation wave propagation of Tiepel3 in non-homogeneous atmosphere. Chester-Chisnell-Witham (CCW) theory7–9 have been applied by Tyl and Wlodarczyk10 for theoretical investigation on the concentric detonation waves in gaseous explosive mixtures. Effect of self-gravitation on the adiabatic propagation of shock waves for weak and strong shock having different type of symmetries in pure and dusty gases have been studied by Gangwar11–13.

In the present paper, the propagation of imploding detonation waves having spherical symmetry have been studied when the ideal gas under the effect of self-gravitation excluding the effect of flow behind the front. The method of CCW is used to investigate the problem. The detonation is a Chapman-Jouguet front i.e. it. travels with sonic speed relative to the burst gas, which determines the law of convergence. The values of the pressure and internal energy in the undisturbed fluid have been neglected in comparison to their values in the disturbed gas i.e. strong detonation wave. The constant amount of heat is produced during the detonation process and by adding this, the basic flow equations only be corrected.

1. **Fundamental equations, boundary conditions and analytical expressions**

The Fundamental equations for one dimensional unsteady, adiabatic flow of an ideal reacting gas can be written as Tyle10,13,14



 



where *u, p, ρ, m, G* and *E* are the particle velocity, the pressure, density, mass of the radius *r*, Gravitational constant and internal energy per unit mass of the ideal gas, respectively and ‘*a*’ is the local speed of sound in ideal gas is given by



where *γ* is the ratio of specific heats of the of the gas. The equation of state for ideal gas is given by



where is the gas constant, and *cp* and c*v* are the specific heats of the gas at constant pressure and volume, respectively. *T* is the temperature of the medium. The internal energy per unit mass of the gas in this case may be given as



The density distribution law of the gas ahead of detonation front is taken to be power varying and given as



where is the constant and the is the ratio of radius ‘’ to the internal radius of the detonation front ‘*R*’ and ‘’a positive integer and is known as density parameter.

Let  and  represent the undisturbed values of flow velocity, density, pressure, and internal energy per unit mass of ideal gas just ahead of the detonation front, and , and  be the modified values of respective quantities at any point across the passage of the detonation front. The jump conditions across the strong detonation in ideal gas detonation front in this case may be written as Whitham9, Tyl and Wlodarczyk10, Vishwakarma and Vishwakarma 4







wheredenote the velocity of detonation front and heat energy released per unit mass, respectively. The indices ‘a’ refers to the sates just ahead the detonation front.

The detonation front is assumed to be in Chapman-Jouguet state. Chapman-Jouguet condition requires that the flow ahead of the shock front will be in sonic state and, in the shock-fixed coordinates, i.e.



where the indices *‘CJ’* denote the Chapman-Jouguet state.

The boundary conditions across the detonation front in this case are











Using the equation sand and - the boundary conditions across the strong detonation front having power decreasing density distribution may be written in the term of velocity of burnt gas











Under the CJ-condition 15,16 the detonation velocity in terms of Alfven ah number *MCJ* is given by



where 

From equation, the mass inside the sphere of radius r is given by



At the equilibrium state, of the gas is assumed to be specified by the condition,  Therefore, from equation, the equilibrium condition prevailing the front of the shock can be written as



On solving the equation and putting the value of m from equation, we have



From the above expression it is observed that for the positive initial pressure the value of density parameter is of the range17. The local speed of sound in the ideal gas having self-gravitation effect with power decreasing density distribution can be given by using the equations and we have:



Using the equations, and , we get



The characteristic form of the fundamental basic equations for converging shock *i. e.* the form in which equation contains derivatives in only one direction in (*r, t*) plane is



Equation divided by  and using the equations after simplifying, we have



where .

Numerical integrating the differential the equation and using boundary conditions- we get the variation of , , and with propagation distance *η*.

**III. Results and Discussion**

Under the effect of the self-gravitation the expressions for non-dimensional detonation velocity the pressure behind the detonation wave, and the density across the detonation front  in terms of the nondimensional flow velocity just behind the strong spherical imploding detonation wave may be obtained by using the equation,, and, respectively. The equation has been solved numerically with the help of software MATLEB and calculate the values of all other post front values of parameters. It is observed that velocity of detonation wave , the non-dimensional pressure behind the detonation front, and the density across the spherical detonation wave all flow variables are depend upon the propagation distance , adiabatic index of the gas , density parameter(*w*), and Alfven Mach number . It is clear from the expressions all the flow parameters are not directly depends upon Gravitational constant G as in simple case of implosion spherical shock wave17 Initially taking the flow velocity. The variation of detonation velocity  with the propagation distance , the pressure across the detonation front, and the density across the front  for  have been computed and displayed through Fig. (1)-(6).

It is important to mention here from the equation and for finite positive equilibrium pressure, the constant *w* should obey the inequality 1<*w*<317 It is observed from Fig.(1),(3) and (5) that strong detonation front velocity, pressure across the detonation front, and the density across the front  is decreases with shock converges in the medium having self-gravitational effect with power decreasing initial density distribution. All parameters are depending upon the value of Alfven Mach number  and also decrease with increase in  from 2.2,2.22 and 2.25. The variation of all flow parameter with propagation distance at different density parameters is shown in Fig.(2),(4) and Fig.(6), it is noticed that the value of density parameter  increases from 2.5 to 2.55 the slope of the graph in increases at fix value . It is also observed that strength of detonation wave is also increase with density parameter (*ω*). The results obtained here are compared with the results for the problem of propagation of strong spherical shock waves under the effect of gravitation with variable density excluding effect of overtaking disturbances and the similar nature of change in perturbation of flow parameters are observed.17

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| Fig. 1. Variation of detonation front velocity  with the propagation distance for *MCJ* = 2.2 and 2.22 at *w*=2.5 and *γ*=1.2. | Fig. 2. Variation of detonation front velocity  with the propagation distance for *w=*2.5 and 2.55 at *MCJ* = 2.2 |
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| Fig. 3. Variation of pressurewith the propagation distance  for *MCJ* = 2.2 and 2.22 at *w*=2.5 and *γ*=1.2. | Fig. 4. Variation of pressure with the propagation distance  for *w=*2.5 and 2.55 at *MCJ* = 2.2 |
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| Fig. 5: Variation of density  with the propagation distance  for *MCJ* = 2.2 and 2.22 at *w*=2.5 and *γ*=1.2. | Fig. 5: Variation of density  with the propagation distance  for *w=*2.5 and 2.55 at *MCJ* = 2.2 |

**III. Conclusion**

The well-known theory (CCW) of shock dynamics has been applied for the problem of motion of strong spherical detonation wave front under the effect of self-gravitation of the gas with power varying density distribution. We have neglected the effect of the flow behind the detonation front and maintain the CJ state in the study. Perturbation in all the flow variables like detonation velocity pressure and density across the front have been calculated and explained through graphs.

**References**

1. Welsh, R. L. Imploding shocks and detonations. *J Fluid Mech* 61 (1967).

2. Nigmatulin, R. I. Converging cylindrical and spherical detonation waves. *Journal of Applied Mathematics and Mechanics* **31**, 171–177 (1967).

3. Teipel, I. Detonation waves in pipes with variable cross-section. *Acta Mech* **47**, 185–191 (1983).

4. Vishwakarma, J. P., Nath, G. & Srivastava, R. K. Self-similar solution for cylindrical shock waves in a weakly conducting dusty gas. *Ain Shams Engineering Journal* vol. 9, 1717–1730 (2018).

5. Verma, B. G. & Singh, J. B. Imploding detonation waves. *Def Sci J* **31**, 1–6 (1981).

6. Verma, B. G. & Singh, J. B. Magnetogasdynamic converging spherical detonation waves. *Astrophys Space Sci* **72**, 133–142 (1980).

7. Chester, W. The diffraction and reflection of shock waves. *Quarterly Journal of Mechanics and Applied Mathematics* **7**, 57–82 (1954).

8. Chisnell, R. F. The normal motion of a shock wave through a non-uniform one-dimensional medium. *Proc R Soc Lond A Math Phys Sci* **232**, 350–370 (1955).

9. Whitham, G. B. On the propagation of shock waves through regions of non-uniform area or flow. *J Fluid Mech* **4**, 337–360 (1958).

10. Tyl, J. & Wlodarczyk, E. Thermodynamic characteristic of the detonation products of octogen. *J. Tech. Phys* **24**, 139–146 (1983).

11. Gangwar, P. K. Behavior of dusty real gas on adiabatic propagation of cylindrical imploding strong shock waves. *AIP Conf Proc* **1953**, 130024 (2018).

12. Gangwar, P. K. Entropy production on dusty shock propagation in presence of overtaking disturbances. *AIP Conf Proc* **2220**, 120008 (2020).

13. Gangwar, P. K. Effect of solid dust particles on the motion of cylindrical strong imploding shock wave in self-gravitating real gas. **2451**, 020077 (2022).

14. Yadav, R. P. Effect of overtaking disturbances on the propagation of strong cylindrical shock in a rotating gas. *Modelling, Measurement and Control B* **46**, 1–11 (1992).

15. Li, H., Ben-Dor, G. & Grönig, H. Analytical study of the oblique reflection of detonation waves. *AIAA Journal* **35**, 1712–1720 (1997).

16. Li, H. & Ben-Dor, G. A Modified CCW Theory for Detonation Waves. *Combust Flame* **113**, 1–12 (1998).

17. Yadav, R. P. & Gangwar, P. K. Theoretical study of propagation of spherical converging shock waves in self- gravitating gas. *Modelling, Measurement and Control B* **72**, 39–54 (2003).