Imp) # Introduction to Histogram:

Histogram of an image provides a global description of the apparance of an image.

The information obtained from histogram is innermost and hence histogram modeling is important in digital image processing (DIP).

By the defination,

the histogram of an image represents the relative frequency of occurance of various gray level in an image.

Histogram of an image can be plotted as:

(i) The X-axis has the gray level and Y-axis has the number of pixel in each gray level.

(ii) The X-axis represent the gray level and Y-axis represents the probability of occurance of that gray level.

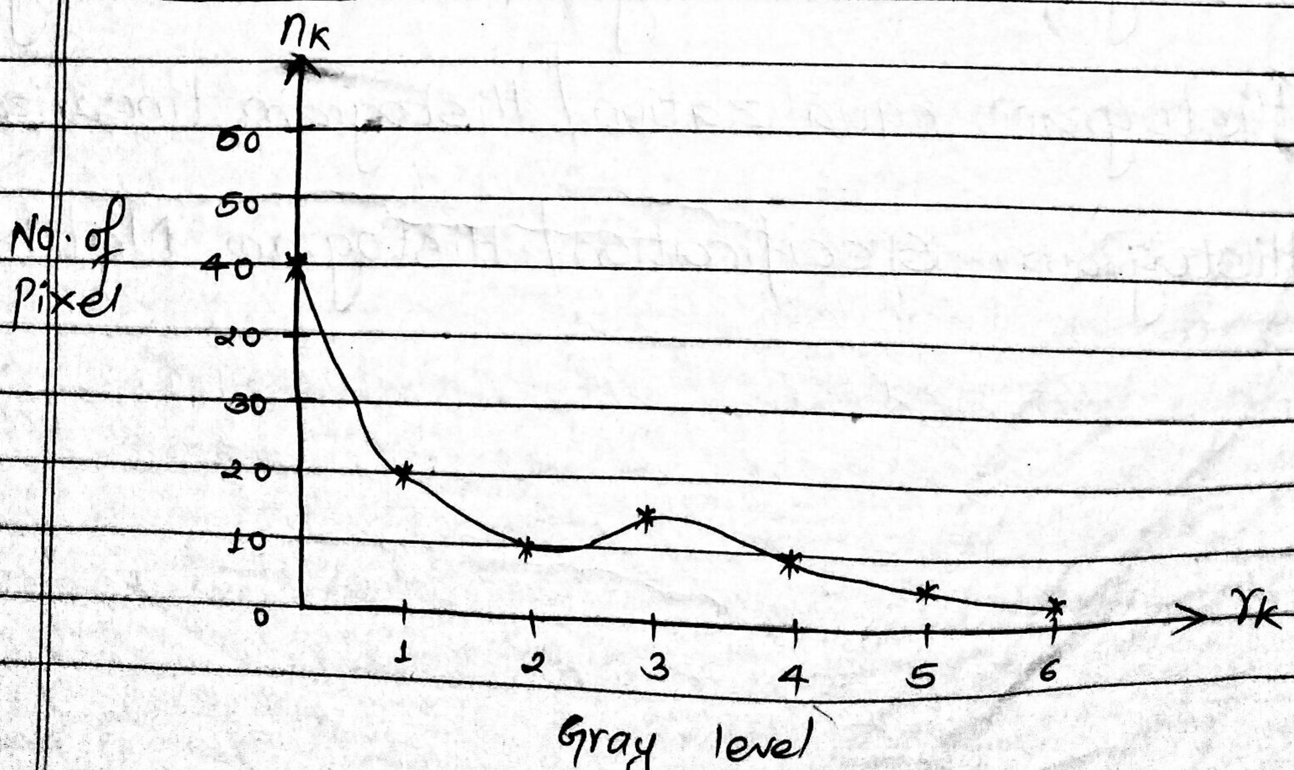
The method of histogram is shown as below:

Step(1): Original Image

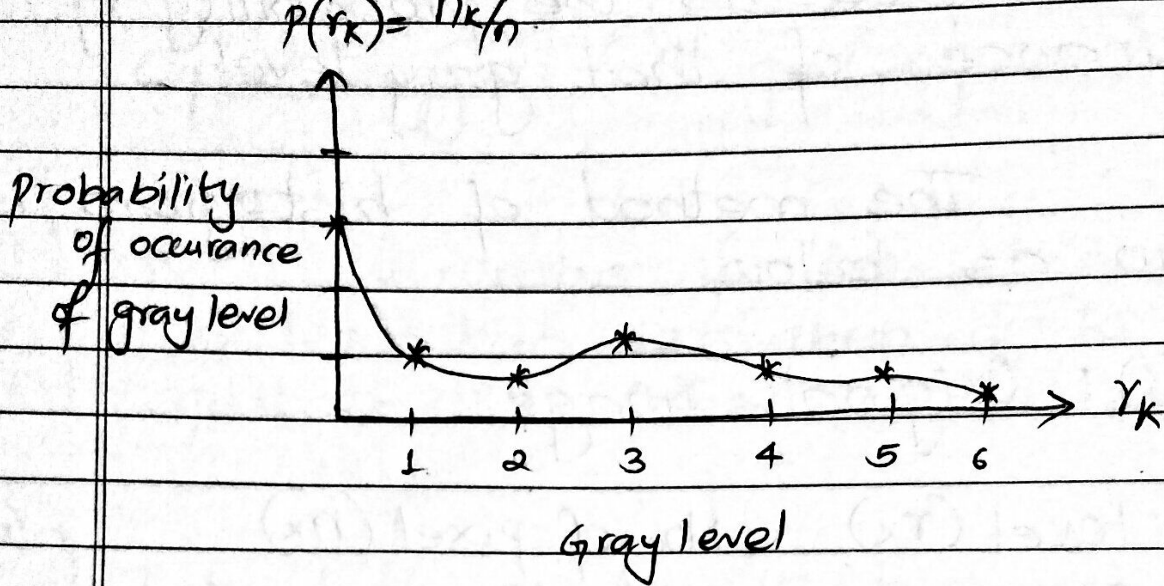
Gray level (r_K)	No. of pixel (n_K)
0	40
1	20
2	10
3	15
4	10
5	3
6	2

Step (2): Histogram Modeling

Method (1):



Method (2):



(fig: Histogram Representation by 2-methods)

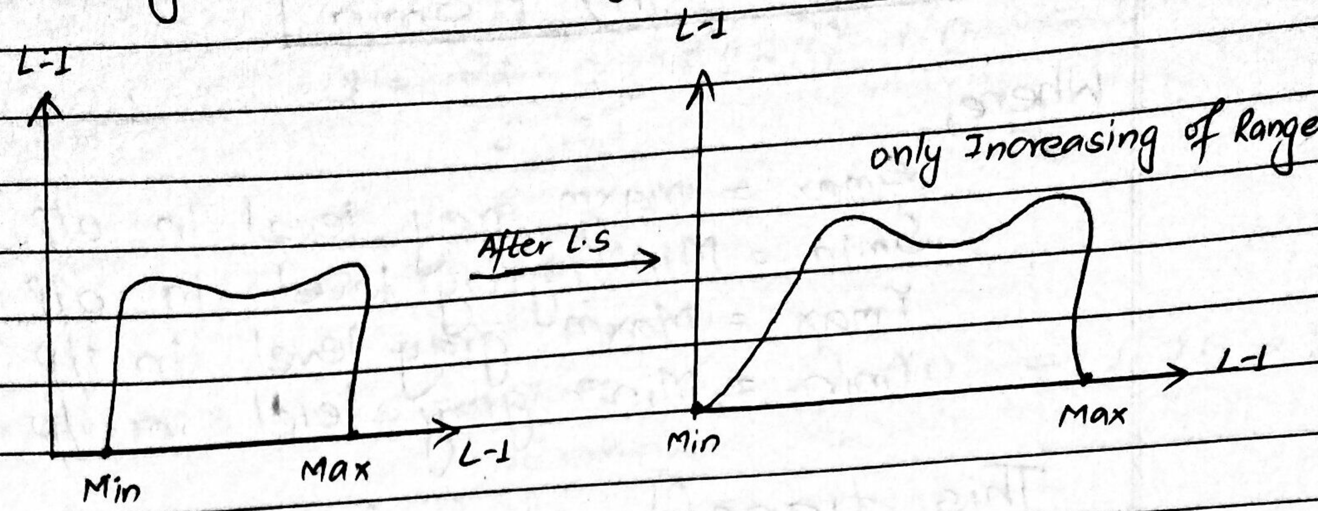
Here,

r_k = k^{th} Gray level
n_k = No. of pixel in k^{th} Gray level
n = Total no. of pixel

Histogram Techniques:

(1) Histogram stretching | Linear stretching
(2) Histogram Equalization | Histogram linearization
(3) Histogram specification | Histogram Matching

1) Histogram stretching:



(fig: Linear stretching)

To increase the dynamic range by using a technique is known as Histogram stretching.

In this method, we donot change basic shape of the histogram but spread it over the entire dynamic range.

For this, we use this technique on the basis of slope. It can be given as:

$$\text{slope}(m) = \frac{S_{max} - S_{min}}{r_{max} - r_{min}} \quad \text{--- (i)}$$

For transformation of histogram stretching then, transformation can be defined as:

$$S = T(r) = \frac{S_{max} - S_{min}}{r_{max} - r_{min}} (r - r_{min}) + S_{min} \quad \text{--- (ii)}$$

$$\boxed{s = m\,(r - r_{min}) + s_{min}}$$

Where,

s_{max} = Max^m gray level in o/p image
s_{min} = Min^m gray level in o/p image
r_{max} = Max^m gray level in i/p image
r_{min} = Min^m gray level in i/p image

This transformation function stretches or increase the dynamic range of given image.

Example: stretch the contrast of histogram over the entire range. [2011 fall]

Gray Level	0	1	2	3	4	5	6	7
No. of pixel	0	0	50	60	50	20	10	0

Now, perform histogram strecthing. Show that the new image has a dynamic range of entire range i.e [0, 7]

Given,

$r_{min} = 2$ $s_{min} = 0$
$r_{max} = 6$ $s_{max} = 7$

Now, We perform transformation function on histogram stretching as:

$$s = T(r) = \frac{s_{max} - s_{min}}{r_{max} - r_{min}}(r - r_{min}) + s_{min}$$

Now,

When $r=2$: $s = \frac{7-0}{6-2}(2-2) + 0 = 0$

$\therefore \boxed{r=2, \ s=0}$

When $r=3$: $s = \frac{7-0}{6-2}(3-2) + 0 = 1.75 \simeq 2$

$\therefore \boxed{r=3, \ s=2}$

When $r=4$: $s = \frac{7-0}{6-2}(4-2) + 0 = 3.5 \simeq 4$

$\therefore \boxed{r=4, \ s=4}$

When $r=5$: $s = \frac{7-0}{6-2}(5-2) + 0 = 5.2 \simeq 5$

$\therefore \boxed{r=5, \ s=5}$

When $r=6$: $s = \frac{7-0}{6-2}(6-2) + 0 = 7$

$\therefore \boxed{r=6, \ s=7}$

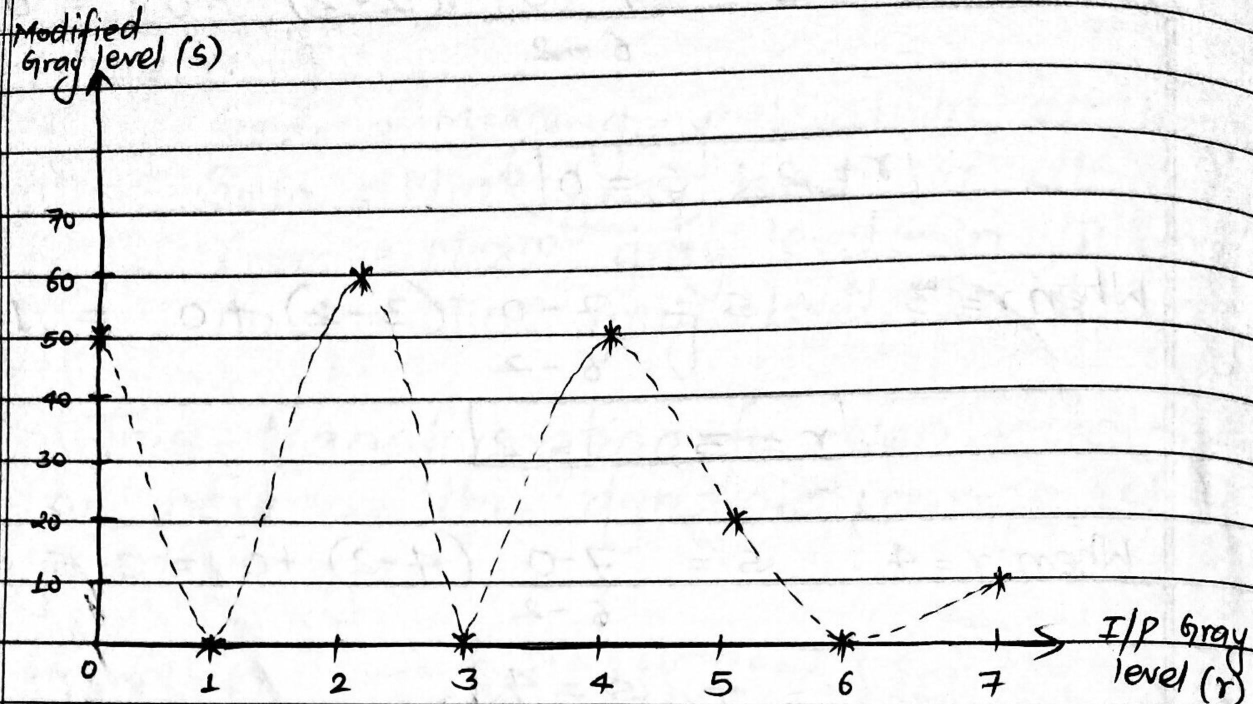
Hence,
Modified histogram or stretched range can be illustrated as:

(S) →

Gray Level	0	1	2	3	4	5	6	7
No. of pixel	50	0	60	0	50	20	0	10

e: $\boxed{s=0}$ ⇒ old Table मा $\boxed{r=2}$ को value हेर्ने त्यो value New Table $s=0$ मा राख्ने and so on

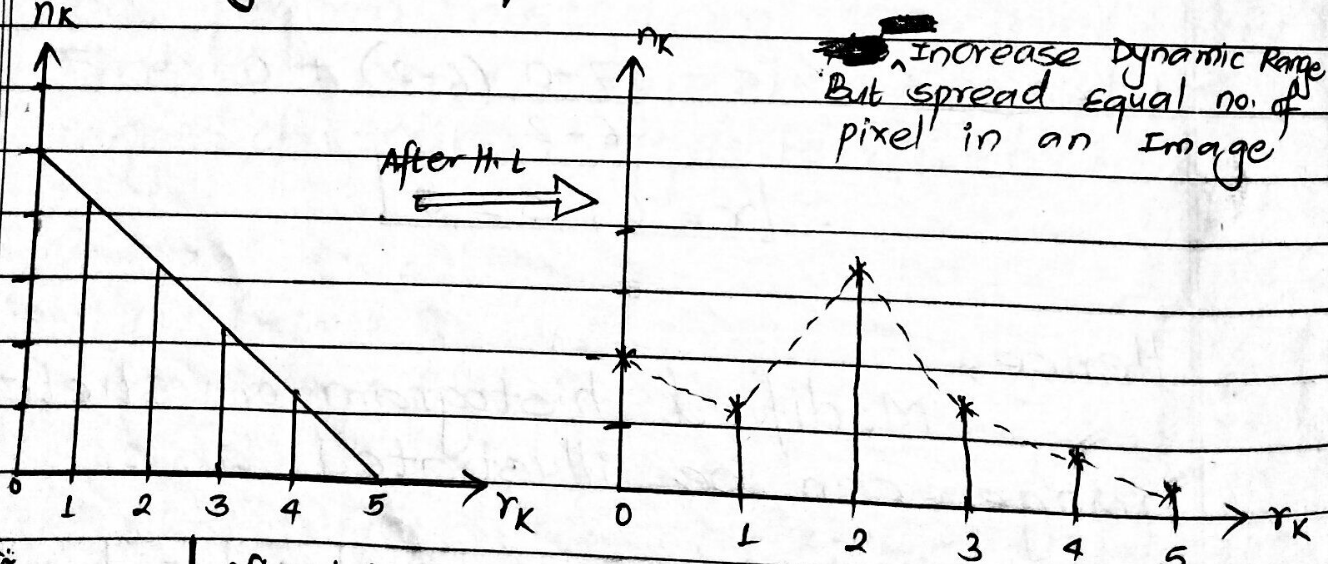
Now, The output histogram image is shown as below:

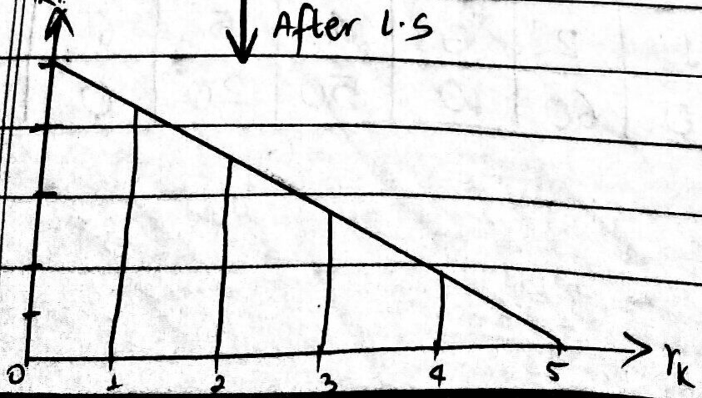


(fig: stretched / Modified / o/p Histogram Image)

(Imp) 2012 fall

② Histogram Equalization:





(fig: Histogram Equalization)

The gray level for continous variable can be characterized by proability density function (Pdf) i.e $P_r(r)$ and $P_s(s)$.

We know that, we need to find the transformation which could be the flat histogram.

Linear stretching is a good technique but shape remain same. There are many applications where we need a flat histogram so ~~that~~ this cannot be achieved by histogram stretching.

Hence, a technique that can be used to obtain a uniform histogram is known as Histogram Equalization or Histogram Linearization.

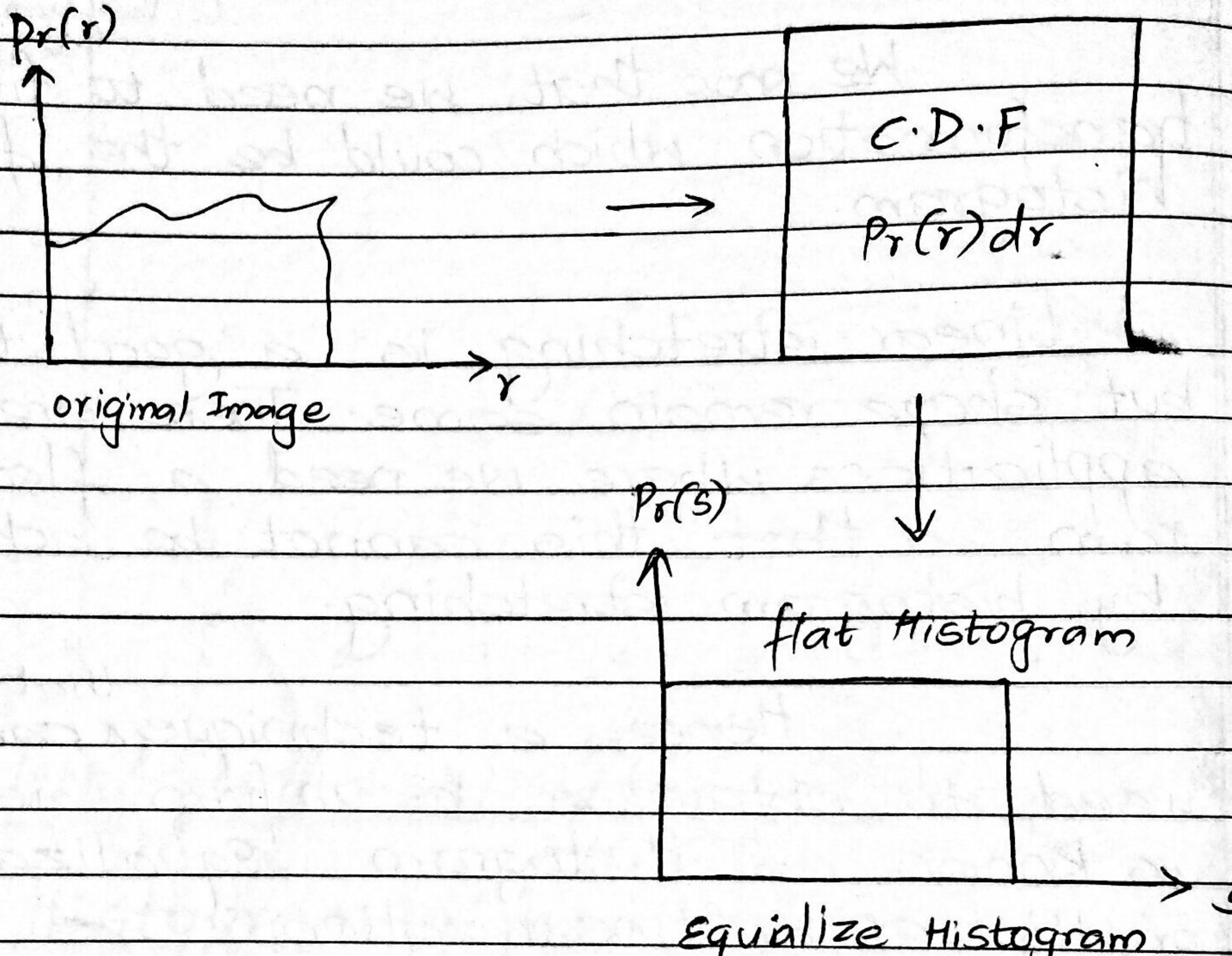
A perfect image is one which has equal number of pixels in all gray level. Therefore, In this technique,

→ image is not only spread over the dynamic range but also to have equal number of pixel in all gray level.

We know that,

$$s = T(r) = \int_0^r P_r(r)\, dr, \quad 0 \leq r \leq 1$$

Above equation shows the transformation function of cumulative pdf (c.d.f) and it can be shown as:



Imp **Example:**

Equalize the given histogram:

Gray level	0	1	2	3	4	5	6	7
No. of pixel	790	1023	850	656	329	245	122	81

Soln: Here,

Step(1):

Gray level	n_K	P.D.F $P_r(r_K) = n_K/n$	C.D.F $S_K = \sum P_r(r_K)$	$7 * S_K$	Rounding off
0	790	0.19	0.19	1.33	1
1	1023	0.25	0.44	3.08	3
2	850	0.21	0.65	4.55	5
3	656	0.16	0.81	5.67	6
4	329	0.08	0.89	6.23	6
5	245	0.06	0.95	6.65	7
6	122	0.03	0.98	6.86	7
7	81	0.02	1	7	7
	n = 4096				

Step (2): Representing New Gray Level

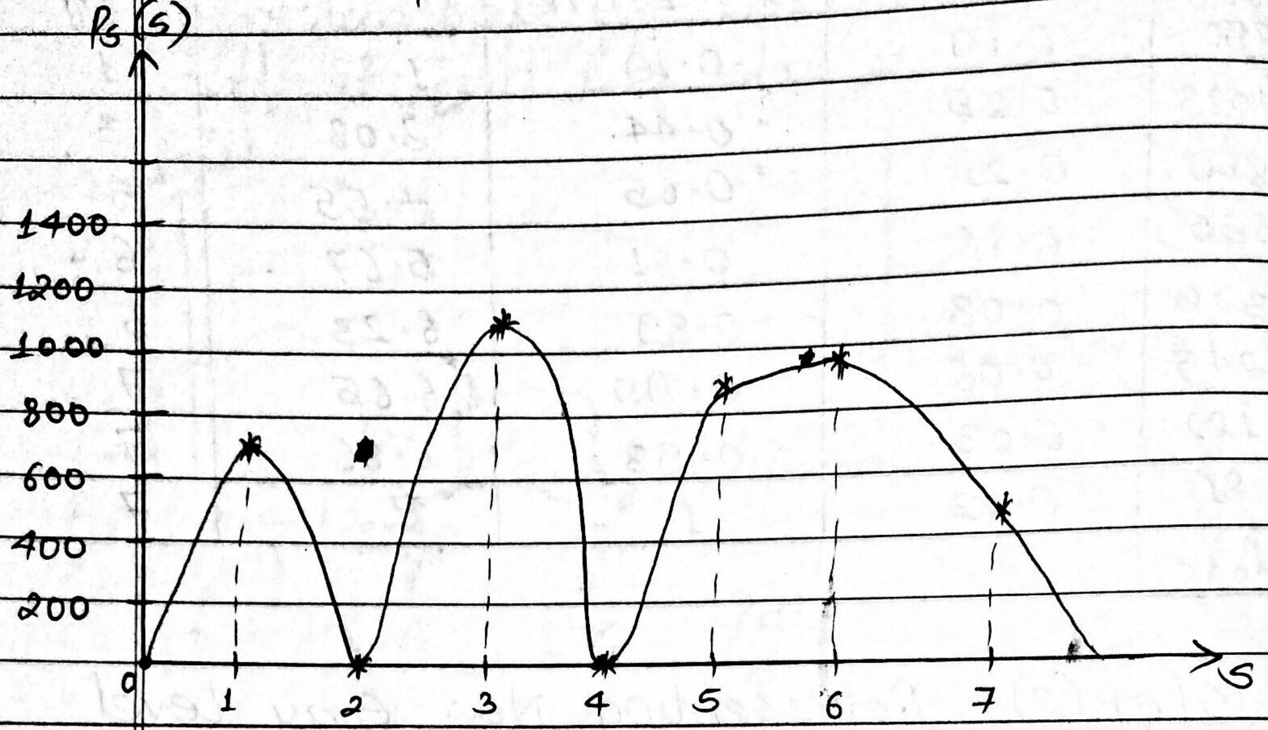
old Gray Level	No. of pixel	New Gray level
0	790	1
1	1023	3
2	850	5
3	656	6 } — 6
4	329	6
5	245	7 } — 7
6	122	7
7	81	7

Step (3): Modified Histogram

Gray level	0	1	2	3	4	5	6	7
No. of pixel	0	790	0	1023	0	850	985	448

(656 + 329)

Hence, Equalized Histogram:



(fig: Equalized Histogram)

③ Histogram specification:

(Th + Nm)

Histogram specification is not interactive because it always gives one result.

i.e an approximation to an uniform histogram.

It is at time desirable to have an interactive methods in which certain gray levels are highlighted.

let, us suppose that $P_r(r)$ is the original pdf (Probability density function) and $P_z(z)$ is the ~~original~~ desired pdf.

Suppose that, histogram equalization is first applied on the original image.

i.e

$$S = T(r) = \int_0^r P_r(r)\, dr$$

Histogram equalization of desired image 'z' is that:

$$V = G(z) = \int_0^z P_z(r)\, dz$$

Now,

The inverse process $z = G^{-1}(s)$ which have the desired pdf that provide histogram specification.

i.e $z = G^{-1}(s) = G^{-1}(T(r)) = G^{-1}\int_0^r P_r(r)\, dr$

Example:

Matching the given histogram:

Histogram for image (a):

Gray Level	0	1	2	3	4	5	6	7
No. of pixel	790	1023	850	656	329	245	122	81

Histogram for image (b):

Gray level	0	1	2	3	4	5	6	7
No. of pixel	0	0	0	614	819	1230	819	614

Soln: Here,

Step(1):

Equalized Histogram of Image (a):

Gray level	0	1	2	3	4	5	6	7
No. of pixel	0	790	0	1023	0	850	985	448

Now,

Equalized Histogram for Image (b) is:

Gray level	n_K	P.D.F $P_r(r_K) = n_K/n$	C.D.F $S_K = \sum P_r(r_K)$	$S_K * 7$	Rounding off
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	614	0.149	0.149	1.05	1 Image(a) 790
4	819	0.20	0.35	2.45	2
5	1230	0.30	0.65	4.55	5
6	819	0.2	0.85	5.77	6
7	614	0.15	1	7	7
	$n = 4096$				

Equalized Histogram of Image (b) is:

Gray level	0	1	2	3	4	5	6	7
No. of pixel	0	614	819	0	0	1230	819	614

Step(2):

To obtain histogram specification, we apply inverse transformation comparing both equalized histogram.

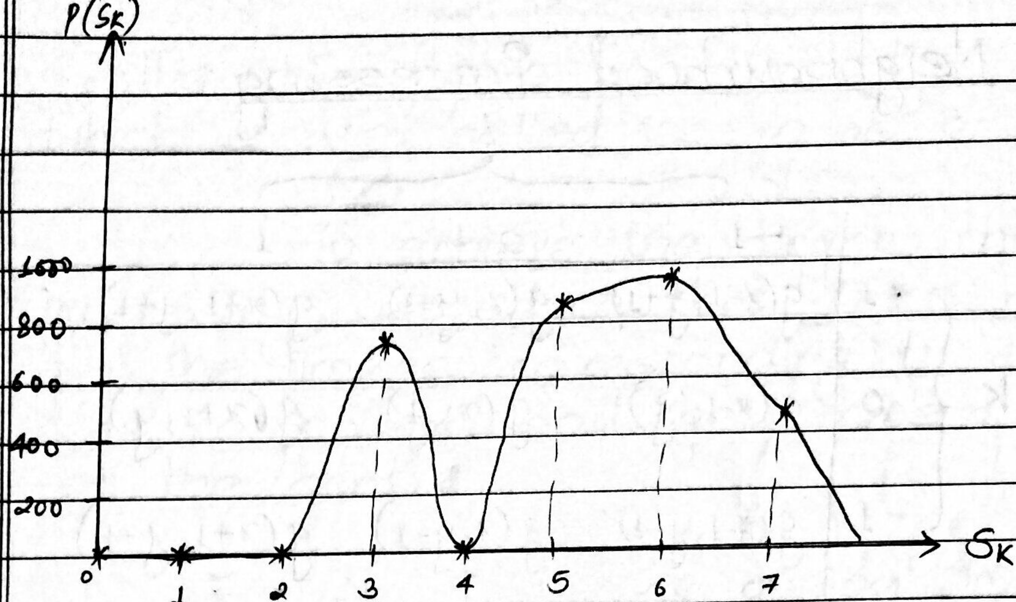
Image (b) → Gray level → Rounding off no.

Then, Image (a) → Same Gray level

Then, No. of pixel of Image (a)

Gray level	0	1	2	3	4	5	6	7
No. of pixel	0	0	0	790	0	850	985	448

Hence,
The matched or resultant histogram is shown as below:



(fig: Specified Image)

Spatial Operations:

Many image enhancement techniques are based in spatial operations. Most of the spatial operations are performed on the basis of local neighbourhood of the input pixel.

Some of the example of spatial operation for image enhancement are as below:


Esc
Tab
CapsLk
Shift
Fn
Alt
Ctrl
Enter
Num Lock
Home
End
PgUp
PgDn
Ins
Del
intel
CORE i5