Data Challenge 2

Aviation Fuel Mix Optimization

Day 1 | Agenda

Morning (9AM - Noon)

- Challenge 2 Overview + Preview
- Introduction to Linear Programming (1 hour)
  - What is it?
  - Example problem setup
  - Using lpSolve and lpSolveAPI
- Participant Practice Problem (3x 20 minutes)
- Optimizing Fuel Purchasing Introduction + QA (30 minutes)

Afternoon (1PM - 6PM)

- Optimizing Fuel Purchasing Practicum (4 hours)
  - Baseline problem
  - Additional constraints- set1
  - Additional constraints- set2
  - Take Home…
- Discuss plan for next session- 30 mins
- Wrap up - 30 mins

Goals + Objectives

- Formulate a linear programming problem to model a given scenario
- Translate a model to R for optimization
- Create RShiny Interface

Bit of History:

The founders are Leonid Kantorovich, who developed linear programming problems in 1939

George Dantzig, who published the simplex method in 1947. Largely credited for the growth of the discipline

Linear programming arose as a mathematical model developed during World War II to plan expenditures and returns in order to reduce costs to the army and increase losses to the enemy. It was kept secret until 1947.

Linear Programming

Attempts to maximize or minimize an objective function subject to various constraints

To formulate a linear programming problem:

- Define the objective function
- Determine Decision Variables
- Define additional variables (if needed)
- Define the constraint matrix

Maximize Farm Profit: 2 variable to develop intuition

75 acres for two crops: wheat and barley.

Production Cost: $120 per acre for the wheat and $210 per acre for the barley.

Budget: $15000 available for expenses.

Storage Constraint: 4000 bushels.

Avg. Yield: 110 bushels of wheat or 30 bushels of barley (per acre)

Profit: $1.30 and for barley is $2.00 (per bushel)

```
Maximize: (110)(1.30)x + (30)(2.00)y =
143x + 60y [Profit]

subject to:

120x + 210y <= 15000

110x + 30y <= 4000

x + y <= 75

x >= 0 Wheat acreage

 y >= 0 : Barley acreage
```

Maximize Farm Profit: Graphical Intuition

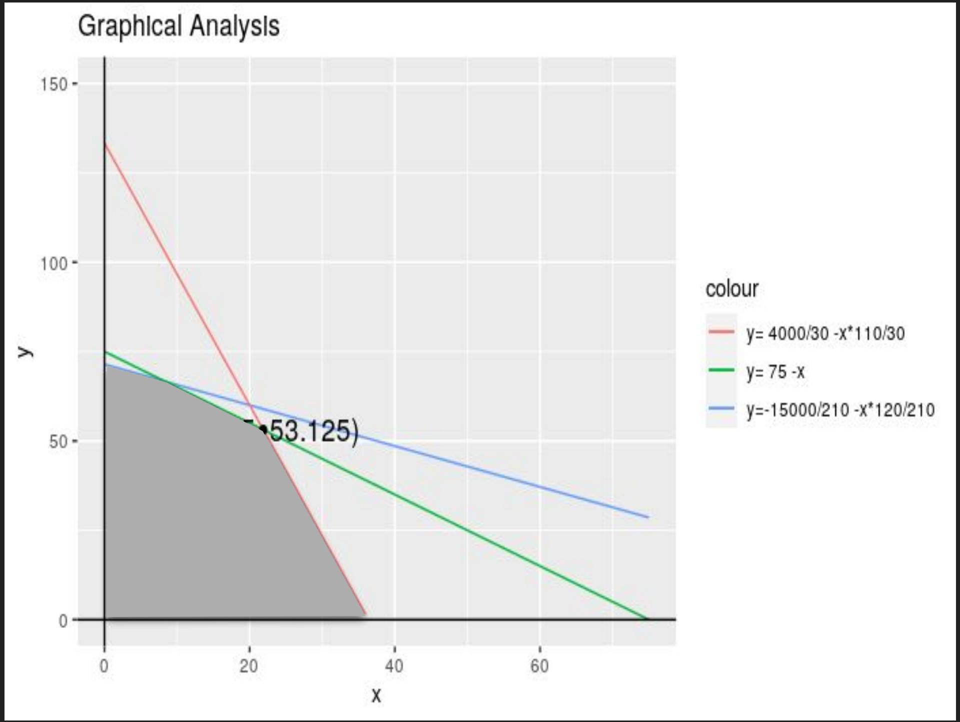
The 3 straight lines depict the straight line representation of the three <= constraints

We are in the right hand upper side of the quadrant such that we only produce +ve bushels of grains!

Any point in the shaded area could be a possible solution. Why?

The lpsolve routine chooses 21.875, 53.125

To Do: Use any point in the gray area and outside to see if you can beat the algorithm!!



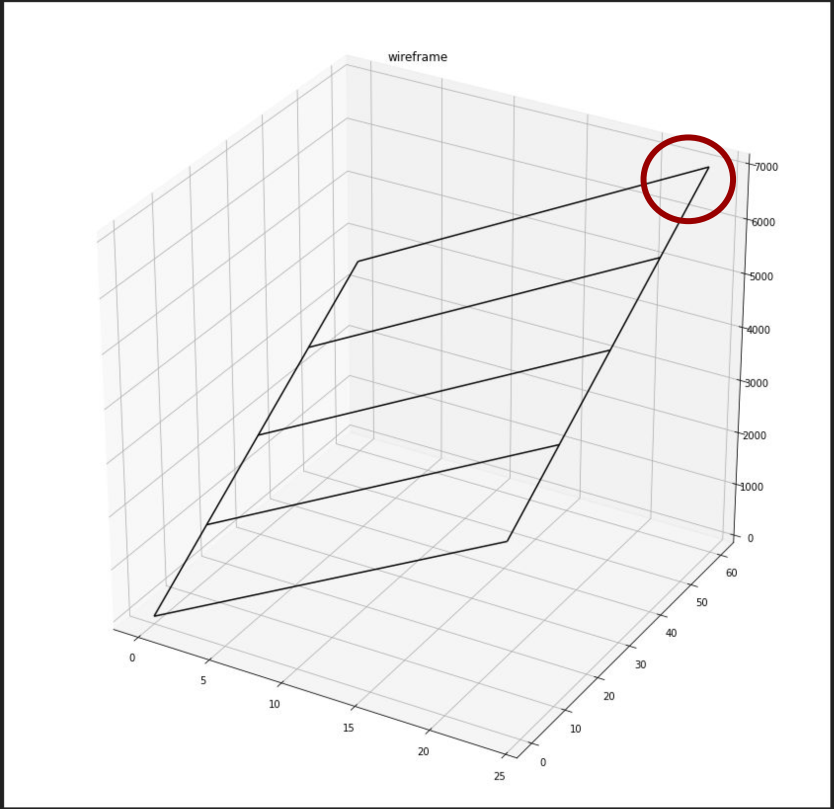
The Optimization Surface

The wireframe is the solution surface within the constraint area

As you see the solution surface is a smooth linear plane

The plot is within the valid constraint region beyond which the optimization problem is no longer viable

The top pointy edge is the highest value achieved via optimization



Linear Programming in R: lpSolve

- Define objective function
- Define the constraints:
  - Define constraint matrix
  - Define constraints’ sign direction
  - Define right hand side of constraints
- Optimize the problem
- Retrieve Solutions

```
obj.fun <- c(1,2,3)

constr <- matrix(c(1,1,1), ncol=3, byrow=TRUE)
constr.dir <- c(">=")
rhs <- c(5)

prod.sol <- lp("min", obj.fun , constr , constr.dir,
rhs,compute.sens=TRUE)

prod.sol #Prints the output of objective function

prod.sol$solution #Retrieves optimal variable values
```

Linear Programming in R: lpSolve

Maximize: (110)(1.30)x + (30)(2.00)y = 143x + 60y [Profit]

subject to:

120x + 210y <= 15000

110x + 30y <= 4000

x + y <= 75

x >= 0 Wheat acreage

y >= 0 : Barley acreage

```
obj.fun <- c(143,60)

constr <- matrix(c(120,210,
                   110, 30,
                     1,1,
                     1,0,
                     0,1), ncol=2, byrow=TRUE)
constr.dir <- c("<=", “<=”,”<=”,”>=”,”>=”)
rhs <- c(15000,4000,75,0,0)

prod.sol <- lp("max", obj.fun , constr , constr.dir,
rhs,compute.sens=TRUE)

prod.sol #Prints the output of objective function

prod.sol$solution #Retrieves optimal variable values
```

Linear Programming in R: lpSolveAPI

The lpSolveAPI package is an R interface to 'lp_solve',a Mixed Integer Linear Programming (MILP) solver with support for purelinear, (mixed) integer/binary, semi-continuous and special ordered sets(SOS) models

More details [here]…

LpSolve API supports in building the Linear Programming problem iteratively with additional constraints and new decision variables

```
# Solution with lpSolveAPI -------------------
# Let's try to solve the problem again using lpSolveAPI
# Use lpSolveAPI
library(lpSolveAPI)
# Set 4 constraints and 3 decision variables
lprec <- make.lp(nrow = 4, ncol = 3)
# Set the type of problem we are trying to solve
lp.control(lprec, sense="min")
# Set type of decision variables
set.type(lprec, 1:3, type=c("integer"))

# Set objective function coefficients vector C
set.objfn(lprec, C)

# Add constraints
add.constraint(lprec, A[1, ], "<=", B[1])
add.constraint(lprec, A[2, ], "<=", B[2])
add.constraint(lprec, A[3, ], "<=", B[3])
add.constraint(lprec, A[4, ], ">=", B[4])

# Display the LPsolve matrix
Lprec
solve(lprec)
get.objective(lprec)
```

Transportation Problem- Lp Transport

A set of customers with a specified demand must be satisfied by another set of supplier with certain capacities (“supply”)

Transport Cost

Minimize Cost:

	C1	C2	C3	C4	**Supply**
S1	*1*	*2*	*3*	*4*	**100**
S2	*4*	*3*	*2*	*1*	**300**
S3	*1*	*5*	*3*	*2*	**400**
Demand	**100**	**100**	**200**	**400**	

```
library(lpSolve)
# specifying cost matrix
cost.mat <- matrix(nrow=3,ncol=4)
cost.mat[1,] <- 1:4
cost.mat[2,] <- 4:1
cost.mat[3,] <- c(1,5,3,2)
# this is a minimization problem
direction = "min"

# capacity may not be exceeded
row.signs <- rep("<=",3)
row.rhs <- c(100,300,400)

# demand must be satisfied
col.signs <- rep(">=",4)
col.rhs <- c(100,100,200,400)

# solve and assign lp object
solution <- lp.transport(cost.mat = cost.mat,
                         direction = direction,
                         row.signs = row.signs,
                         row.rhs = row.rhs,
                         col.signs = col.signs,
                         col.rhs = col.rhs)
solution$solution
```

Example Problem 3

A company has to decide its production levels for the next 2 quarters. The anticipated widget demand for those months are 1000, 1400, 1700, 1500, 1800, and 1200 units respectively. The maximum production per month is 1400 units. Widgets produced one month can be delivered either that same month or stored in inventory and delivered at some other month. It costs the company $3 to produce a widget in standard production and $2 to carry one unit in inventory from one month to the next. Through additional man-hours, up to 500 additional units can be produced per month but, in this case, the company incurs an additional cost of $7/unit. Formulate as a linear program the problem of determining the production levels so as to minimize the total costs.

Optimizing Fuel Purchasing Practicum

A company that produces aircraft biofuel is planning a new product called FC (Fuel-Corn). The table below shows the total quarterly demand in tonnes (t) for the coming years as communicated by their customers.

FC demand (T)

Q1	Q2	Q3	Q4
1,200	1,100	1,300	1,000

Fuel corn is made of a combination of Fuel and Corn. The supply schedule dictates that we purchase these raw materials on a bimonthly schedule. The costs per tonne of Fuel and Corn for every two month period in the coming year is listed below.

	B1	B2	B3	B4	B5	B6
Fuel ($/t)	2	2.5	2	1	1.5	3
Corn ($/t)	1.5	1	2	1	2	2.5

FC composition is obtained by mixing 35% of Fuel and 65% of Corn. The life of Fuel is of four consecutive months and the life of Corn, six (i.e., if we buy Fuel in early January, we cannot use it in early May). We just buy Fuel and Corn at the beginning of each two-month period and make the deliveries of FC at the beginning of each quarter. For simplicity, we assume that one can buy, mix and sell the same day. In addition, the plant manager has told us that in any two-month period, we cannot buy more Fuel than triple of Corn.

Develop a model to minimize the cost of production.

Optimizing Fuel Purchasing Practicum

Questions / Prompts:

- What are the **decision variables** in the problem?
- What are the problem constraints?
- Do we need any slack variables?
- How can we formulate the Q1 constraint? Future constraints?
- How can we formulate no more than triple fuel than corn?
- What about the degradation of fuel?
- Is this constraint sufficient?
- Is this constraint too restrictive?
- We could purchase additional corn in March to not only cover our Q2 needs, but also stockpile for our anticipated Q3 delivery. How can we ensure that the extra fuel from March's order is not double allocated?
    - We can envision March + May + July > Q3 delivery, but what if some of March was already used for the Q2 delivery? How do we ensure that it is not over allocated?

Homework / Assignment Ask

- Reformulate the model so that a 25% discount is applied to any purchases over 1000 tons