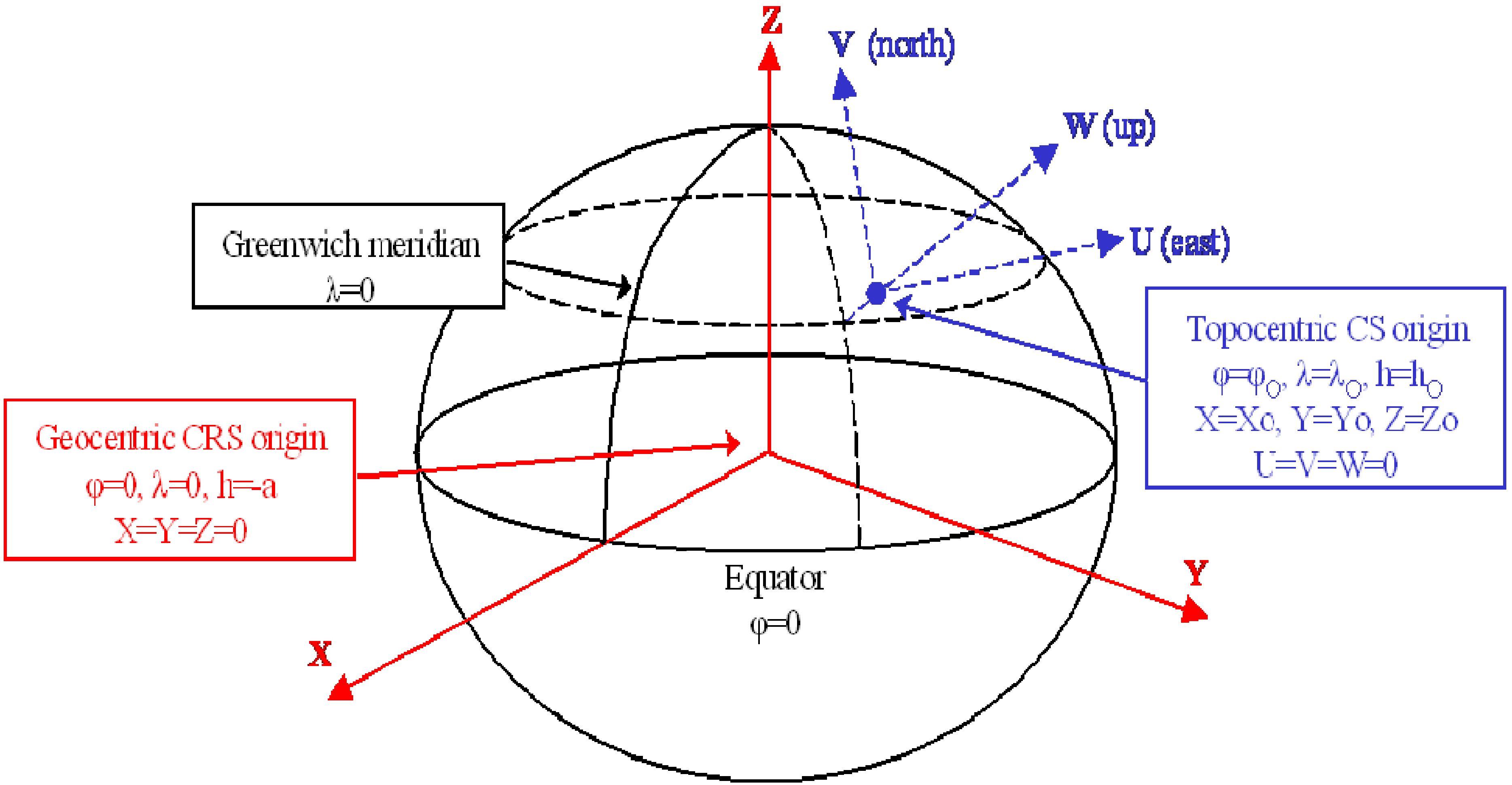
Overview of the Derivation

- Latitude (φ), longitude (λ) and height (L/L/H) are converted to geocentric X, Y, and Z (ECEF)
- An oblique origin for topocentric and orthographic coordinates is chosen at φ_O, λ_O, hgt=0, and the corresponding X_O, Y_O and Z_O are computed
- X/Y/Z are translated and rotated from the geocenter to the oblique origin to create U/V/W (or East/North/Up), also known as topocentric coordinates
- Topocentric U is ellipsoidal orthographic Easting and topocentric V is ellipsoidal orthographic Northing. W is discarded.
- The ellipsoidal orthographic formulas can be simplified with appropriate substitutions as presented herein
- Converting U/V/W to X/Y/Z and then to L/L/H is simply a matter of reversing the computation because all information is retained
- Converting orthographic Easting and Northing to latitude and longitude is more difficult because some information is lost (viz. W)
- Therefore, this presentation takes a numerical, iterative approach to the reverse computation
- Convergence and scale are derived by differentiating the primary equations

EPSG Graphic of X/Y/Z and U/V/W



Ellipsoidal Orthographic

In the scalar equations for U and V in the previous slide substitude X_O, X, Y_O, Y, Z_O and Z for their equivalents from the equations below:

$$X = (\nu + h)\cos\phi\cos\lambda$$

$$Y = (\nu + h)\cos\phi\sin\lambda$$

$$Z = (\nu(1 - e^2) + h)\sin\phi$$

Reduce the result to the simplest form with appropriate sustitutions (including h = 0 so that the plane is tangent to the ellipsoid) and get:

$U = \nu \cos \varphi \sin (\lambda - \lambda_O)$

$V = \nu [\sin \varphi \cos \varphi_O - \cos \varphi \sin \varphi_O \cos (\lambda - \lambda_O)] + e^2 (\nu_O \sin \varphi_O - \nu \sin \varphi) \cos \varphi_O$

See next slide for a full description of the ellipsoidal orthographic forward equations

Orthographic Forward

$E = FE + \nu \cos\varphi \sin(\lambda - \lambda_O)$

$N = FN + \nu [\sin\varphi \cos\varphi_O - \cos\varphi \sin\varphi_O \cos(\lambda - \lambda_O)] + e^2 (\nu_O \sin\varphi_O - \nu \sin\varphi) \cos\varphi_O$

where,

E is Easting, FE is False Easting

N is Northing, FN is False Northing

ν is the prime vertical radius of curvature at latitude φ; $\nu = a /(1 - e^2\sin^2\varphi)^{0.5}$,

ν_O is the prime vertical radius of curvature at φ_O, $\nu_O = a /(1 - e^2\sin^2\varphi_O)^{0.5}$,

e is the eccentricity of the ellipsoid and $e^2 = (a^2 - b^2)/a^2 = 2f - f^2$

a and b are the ellipsoidal semi-major and semi-minor axes,

1/f is the inverse flattening, and

the latitude and longitude of the projection origin are φ_O and λ_O.

The reverse formulas are numerical and iterative.