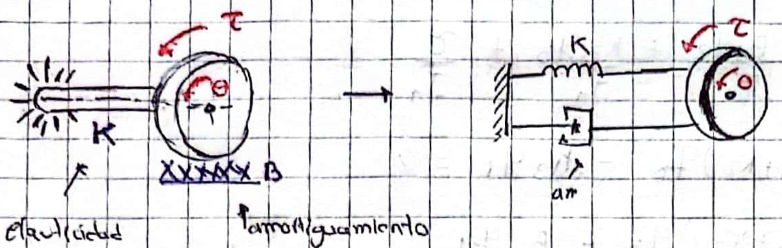
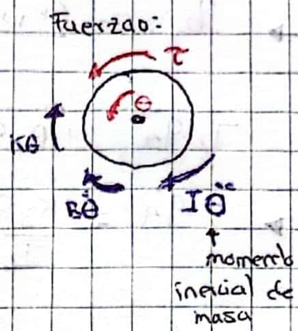
# Sistemas Rotacionales




$I_0 =$ masa inercial

Fuerzas:

$$I\ddot{\theta} + B\dot{\theta} + K\theta = \tau$$

Estado:

$$q_1 = \theta$$

$$q_2 = \dot{q}_1 = \dot{\theta}$$

$$\dot{q}_2 = \ddot{q}_1 = \ddot{\theta}$$

Con $\theta_1 > \theta_2$

Ejercicio:

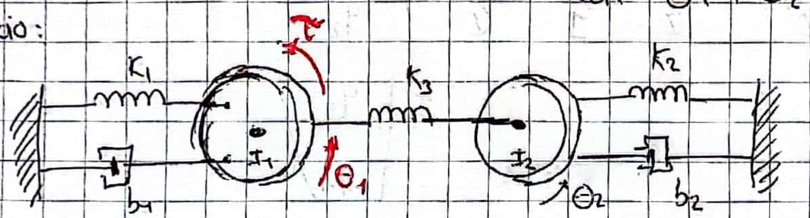
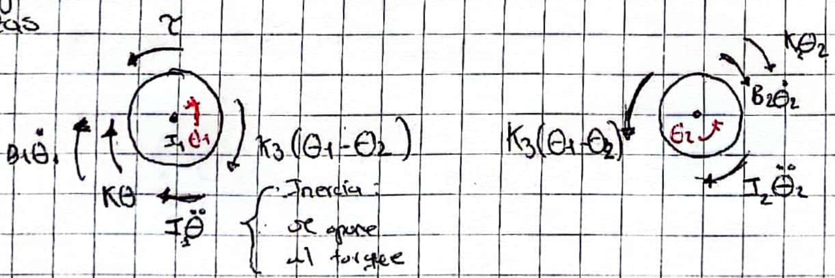



diagrama Fuerzas




$$\tau = I_1\ddot{\theta}_1 + B_1\dot{\theta}_1 + K_1\theta_1 + K_3(\theta_1 - \theta_2) \qquad K_3(\theta_1 - \theta_2) - B_2\dot{\theta}_2 - K\theta_2 - I_2\ddot{\theta}_2 = 0$$

$$[1] \quad \tau = I_1\overset{\dot{q}_2}{\ddot{\theta}_1} + B_1\overset{q_2}{\dot{\theta}_1} + (K_1 + K_3)\overset{q_1}{\theta_1} - K_3\overset{q_3}{\theta_2} \qquad K_3\theta_1 - K_3\theta_2 - B_2\dot{\theta}_2 - K\theta_2 - I_2\ddot{\theta}_2 = 0$$

$$-I_2\ddot{\theta}_2 - B_2\dot{\theta}_2 - (K_3 + K_2)\theta_2 + K_3\theta_1 = 0$$

Variables de estado:

$$q_1 = \theta_1, \quad q_2 = \dot{\theta}_1 = \dot{q}_1 \qquad [2] \quad I_2\overset{\dot{q}_4}{\ddot{\theta}_2} + B_2\overset{q_4}{\dot{\theta}_2} + (K_3 + K_2)\overset{q_3}{\theta_2} - K_3\overset{q_1}{\theta_1} = 0$$

$$\dot{q}_2 = \ddot{\theta}_1, \quad q_3 = \theta_2, \quad q_4 = \dot{\theta}_2 = \dot{q}_3, \quad \dot{q}_4 = \ddot{\theta}_2 = \ddot{q}_3$$

$$\tau = J_1 \ddot{q}_2 + B_1 q_2 + (K_1 + K_3) q_1 - K_3 q_3$$

▶ $$\dot{q}_2 = -\frac{(K_1+K_3)}{J_1} q_1 - \frac{B_1 q_2}{J_1} + \frac{K_3 q_3}{J_1} + \frac{\tau}{J_1}$$

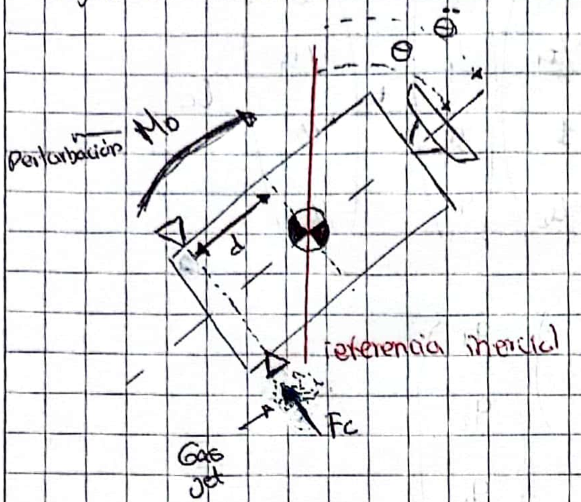
$$J_2 \ddot{q}_4 + B_2 q_4 + (K_3 + K_2) q_3 - K_3 q_1 = 0$$

▶ $$\dot{q}_4 = \frac{K_3 q_1}{J_2} - \frac{(K_3+K_2)}{J_2} q_3 - \frac{B_2 q_4}{J_2}$$

$$\begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{-(K_1+K_3)}{J_1} & -\frac{B_1}{J_1} & \frac{K_3}{J_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K_3}{J_2} & 0 & \frac{-(K_3+K_2)}{J_2} & \frac{-B_2}{J_2} \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 1/J_1 \\ 0 \\ 0 \end{pmatrix} \tau$$

$$\begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix}$$

Ejercicio: Modelo de un satelite




Suma de los momentos Inerciales:

$$\underbrace{F_c \cdot d + M_D}_{u\,:\ \text{entradas al sistema}} = I\ddot{\theta}$$

$$u = I\,\ddot{\theta}$$

$$U(s) = I s^2 \Theta(s)$$

$$\frac{\Theta(s)}{U(s)} = \frac{1}{I s^2}$$

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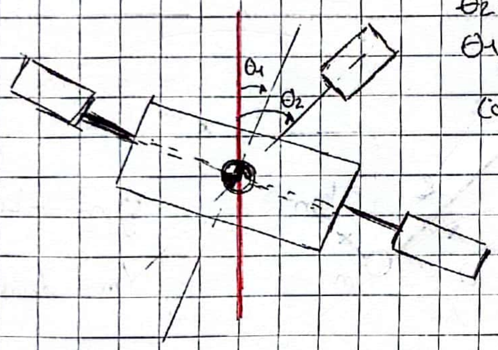
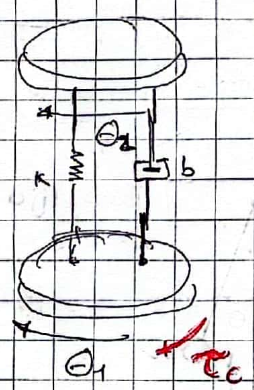
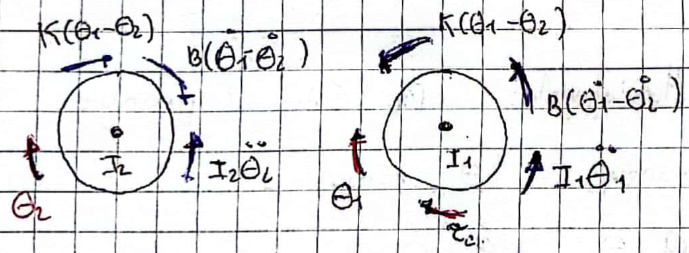
$\theta_2$ = sensor

$\theta_1$ = body

con $\theta_1 > \theta_2$




$$K(\theta_1-\theta_2) + B(\dot{\theta}_1-\dot{\theta}_2) = I_2\ddot{\theta}_2 \quad ; \quad \tau_c - K(\theta_1-\theta_2) - B(\dot{\theta}_1-\dot{\theta}_2) = I_1\ddot{\theta}_1$$

$q_1 = \theta_1$ $\qquad$ $q_3 = \theta_2$

$q_2 = \dot{q}_1 = \dot{\theta}_1$ $\qquad$ $q_4 = \dot{q}_3 = \dot{\theta}_2$

$\dot{q}_2 = \ddot{q}_1 = \ddot{\theta}_1$ $\qquad$ $\dot{q}_4 = \ddot{q}_3 = \ddot{\theta}_2$

$$K(q_1-q_3) + B(q_2-q_4) = I_2\dot{q}_4 \quad ; \quad \tau_c - K(q_1-q_3) - B(q_2-q_4) = I_1\dot{q}_2$$

$$\dot{q}_4 = \frac{K}{I_2}q_1 - \frac{K}{I_2}q_3 + \frac{B}{I_2}q_2 - \frac{B}{I_2}q_4 \quad ; \quad \dot{q}_2 = -\frac{K}{I_1}q_1 + \frac{K}{I_1}q_3 - \frac{B}{I_1}q_2 + \frac{B}{I_1}q_4 + \frac{\tau_c}{I_1}$$

DD MM AA

$$\begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -K/I_1 & -B/I_1 & K/I_1 & B/I_1 \\ 0 & 0 & 0 & 1 \\ K/I_2 & B/I_2 & -K/I_2 & -B/I_2 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 1/I_1 \\ 0 \\ 0 \end{pmatrix} \tau_c$$

$$\begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix}$$