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20192574010

Parcial 2do Corte

Sistemas Dinámicos

1. Para el sistema rotacional de la figura, determine:

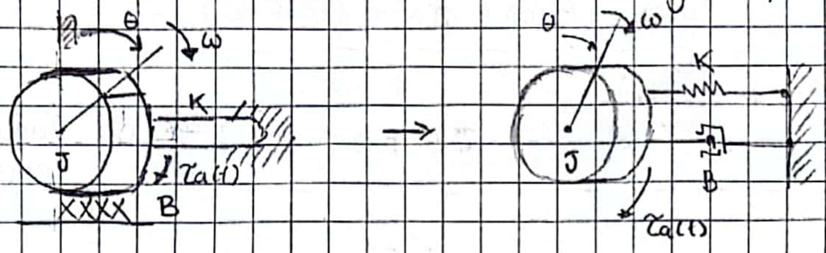
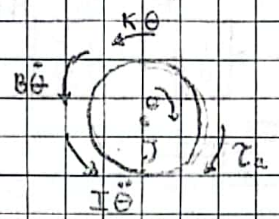


Diagrama de fuerzas:



$$\tau_a = J\ddot{\theta} + B\dot{\theta} + K\theta \quad [1]$$

a.

Representación en el espacio de estados

partiendo de [1]: $\tau_a = J\ddot{\theta} + B\dot{\theta} + K\theta$

$q_1 = \theta$ [2]

$q_2 = \dot{q}_1 = \dot{\theta}$ [3]

$\dot{q}_2 = \ddot{q}_1 = \ddot{\theta}$

$$\tau_a = J \cdot \dot{q}_2 + B \cdot q_2 + K \cdot q_1$$

$$\dot{q}_2 = -\frac{K}{J} q_1 - \frac{B}{J} q_2 + \frac{\tau_a}{J}$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -K/J & -B/J \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/J \end{bmatrix} \tau_a$$

$$\theta = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

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b. Diagrama de bloques

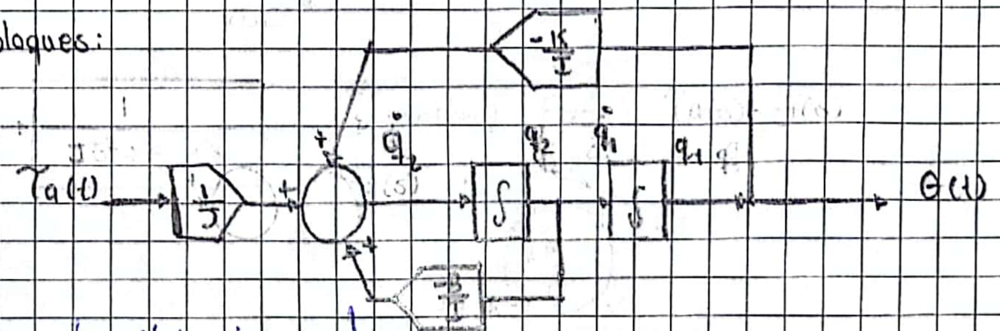
Partiendo de [1] $\tau_a = J\ddot{\theta} + B\dot{\theta} + K\theta$

Aplicando Laplace: $\tau_{a(s)} = J \cdot s^2 \theta(s) + B \cdot s \cdot \theta(s) + K \cdot \theta(s)$

$$\tau_a(s) = \theta(s)\left(Js^2 + Bs + K\right)$$

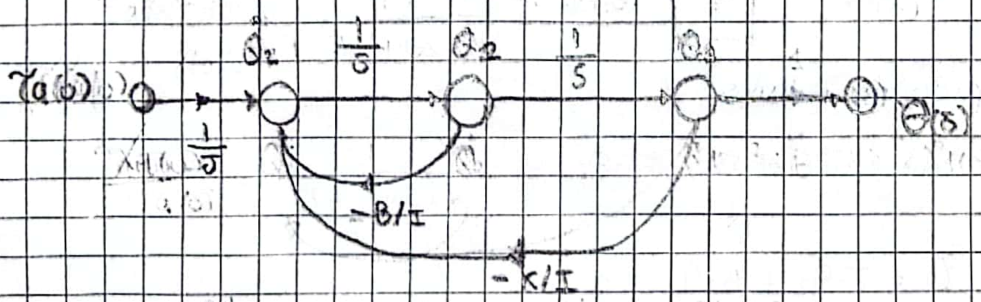
$$G(s) = \frac{\theta(s)}{\tau_a(s)} = \frac{1}{Js^2 + Bs + K}$$ → d. Función de transferencia

Diagrama de bloques:



C. Diagrama de flujo de señal

Teniendo en cuenta las variables de estado establecidas, se tiene:



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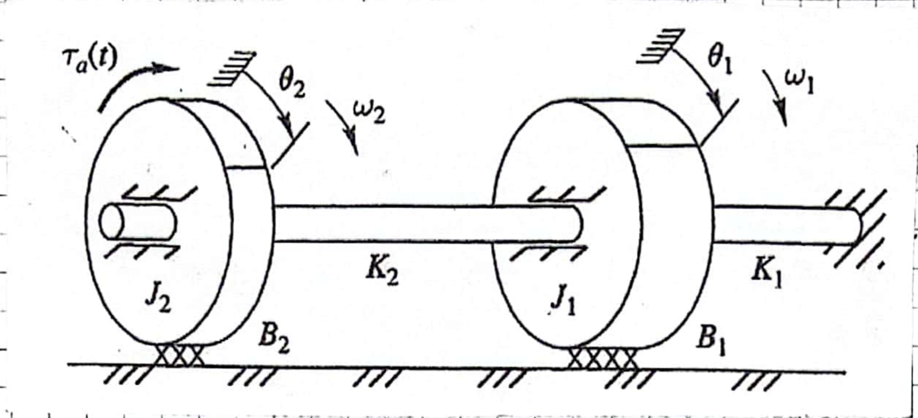
DD	MM	AA

2) Para el sistema rotacional en la figura asuma $\theta_2 > \theta_1$



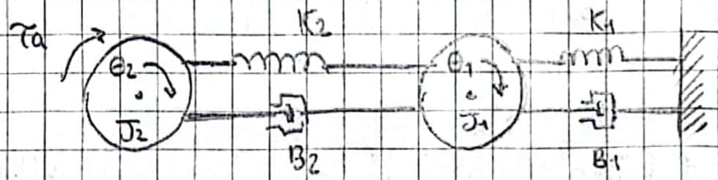
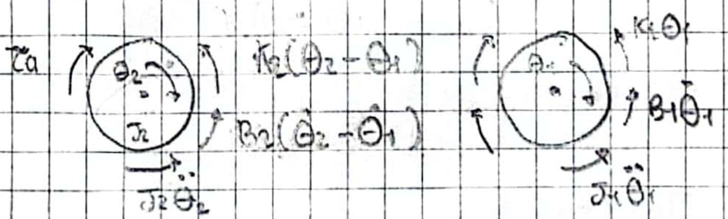


Diagrama de fuerzas:



$$\tau_a = K_2(\theta_2-\theta_1) + B_2(\dot{\theta}_2-\dot{\theta}_1) + J_2\ddot{\theta}_2$$

$$\tau_a = J_2\ddot{\theta}_2 + B_2\dot{\theta}_2 + K_2\theta_2 - B_2\dot{\theta}_1 - K_2\theta_1$$

$$K_2(\theta_2-\theta_1) + B_2(\dot{\theta}_2-\dot{\theta}_1) - K_1\theta_1 - B_1\dot{\theta}_1 - J_1\ddot{\theta}_1 = 0$$

$$-J_1\ddot{\theta}_1 - (B_1+B_2)\dot{\theta}_1 - (K_1+K_2)\theta_1 + B_2\dot{\theta}_2 + K_2\theta_2 = 0$$

Aplicando Laplace

$$\tau_a = J_2 s^2\theta_2 + B_2 s\theta_2 + K\theta_2 - B_2 s\theta_1 - K_2\theta_1$$

$$\tau_a = \theta_2(J_2 s^2 + B_2 s + K_2) + \theta_1(-B_2 s - K_2) \quad [1]$$

$$-J_1 s^2\theta_1 - (B_1+B_2)s\theta_1 - (K_1+K_2)\theta_1 + B_2 s\theta_2 + K_2\theta_2 = 0$$

$$\theta_1(-J_1 s^2 - (B_1+B_2)s - (K_1+K_2)) + \theta_2(B_2 s + K_2) = 0 \quad [2]$$

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DD	MM	AA

Teniendo la ecuación [1] y [2] se aplica método de cramer

$$\theta_1(-B_2 s - K_2) + \theta_2(J_2 s^2 + B_2 s + K_2) = \tau_a$$

$$\theta_1(-J_1 s^2 - (B_1 + B_2)s - (K_1 + K_2)) + \theta_2(B_2 s + K_2) = 0$$

$$\begin{bmatrix} -B_2 s - K_2 & J_2 s^2 + B_2 s + K_2 \\ -J_1 s^2 - (B_1+B_2)s - (K_1+K_2) & B_2 s + K_2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} T_a \\ 0 \end{bmatrix}$$

$$M = -(B_2 s + K_2)^2 + (J_2 s^2 + B_2 s + K_2)(J_1 s^2 + (B_1 + B_2)s + (K_1 + K_2))$$

$$M_{\theta_1} = T_a \cdot (B_2 s + K_2)$$

$$M_{\theta_2} = T_a \cdot (J_1 s^2 + (B_1 + B_2)s + K_1 + K_2)$$

$$\theta_1 = \frac{M_{\theta_1}}{M} = \frac{T_a \cdot (B_2 s + K_2)}{(J_2 s^2 + B_2 s + K_2)(J_1 s^2 + (B_1+B_2)s + K_1 + K_2) - (B_2 s + K_2)^2}$$

$$[3]\quad \theta_2 = \frac{M_{\theta_2}}{M} = \frac{\tau_a \cdot (J_1 s^2 + (B_1+B_2)s + K_1 + K_2)}{(J_2 s^2 + B_2 s + K_2)(J_1 s^2 + (B_1+B_2)s + K_1 + K_2) - (B_2 s + K_2)^2}$$

de [3] se obtiene θ_2 / τ_a

$$G(s) = \frac{\theta_2(s)}{\tau_a(s)} = \frac{J_1 s^2 + (B_1 + B_2)s + K_1 + K_2}{(J_2 s^2 + B_2 s + K_2)(J_1 s^2 + (B_1+B_2)s + K_1 + K_2) - (B_2 s + K_2)^2}$$

a. Función de transferencia relacionando θ_2 y τ_a

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b. representación en el espacio de estados

$$G(s) = \frac{\Theta_2(s)}{\tau_a(s)} = \frac{J_1 s^2 + (B_1+B_2)s + K_1 + K_2}{(J_2 s^2 + B_2 s + K_2)(J_1 s^2 + (B_1+B_2)s + K_1 + K_2) - (B_2 s + K_2)^2} \cdot \frac{X_1(s)}{X_1(s)}$$

$$\Theta_2(s) = J_1 s^2 X_1 + (B_1+B_2) s X_1 + K_1 X_1 + K_2 X_1 \qquad [4]$$

Primero se debe simplificar:

$$\tau_a(s) = (J_2 s^2 + B_2 s + K_2)(J_1 s^2 + (B_1+B_2)s + K_1 + K_2) - (B_2 s + K_2)^2$$

simplificando por MATLAB

$$\tau_a(s) = J_1 J_2 s^4 + (B_1 J_2 + B_2 J_1 + B_2 J_2) s^3 + (B_1 B_2 + J_1 K_2 + J_2 K_1 + J_2 K_2) s^2 + (B_1 K_2 + B_2 K_1) s + K_1 K_2$$

$$\tau_a(s) = J_1 \cdot J_2 s^4 X_1 + (B_1 J_2 + B_2 J_1 + B_2 J_2) s^3 X_1 + (B_1 B_2 + J_1 K_2 + J_2 K_1 + J_2 K_2) s^2 X_1 + (B_1 K_2 + B_2 K_1) s X_1 + K_1 K_2 X_1 \qquad [5]$$

$sX_1 = X_2$

$sX_2 = X_3$

$sX_3 = X_4$

reescribiendo [4]

$$\Theta_2(s) = J_1 X_3 + (B_1+B_2) X_2 + (K_1+K_2) X_1 \qquad [6]$$

reescribiendo [5]

$$\tau_a(s) = J_1 \cdot J_2 \cdot s X_4 + (B_1 J_2 + B_2 J_1 + B_2 J_2) X_4 + (B_1 B_2 + J_1 K_2 + J_2 K_1 + J_2 K_2) X_3 + (B_1 K_2 + B_2 K_1) X_2 + K_1 K_2 X_1$$

$$sX_4 = -\frac{(B_1 J_2 + B_2 J_1 + B_2 J_2)}{J_1 J_2} X_4 - \frac{(B_1 B_2 + J_1 K_2 + J_2 K_1 + J_2 K_2)}{J_1 J_2} X_3 - \frac{(B_1 K_2 + B_2 K_1)}{J_1 J_2} X_2 - \frac{K_1 K_2}{J_1 J_2} X_1 + \frac{\tau_a}{J_1 J_2} \qquad [7]$$

Pasando [6] y [7] al dominio del tiempo

$$\Theta_2(t) = J_1 X_3(t) + (B_1+B_2) X_2(t) + (K_1+K_2) X_1(t)$$

$$\frac{dX_4(t)}{dt} = -\frac{(B_1 J_2 + B_2 J_1 + B_2 J_2)}{J_1 J_2} X_4(t) - \frac{(B_1 B_2 + J_1 K_2 + J_2 K_1 + J_2 K_2)}{J_1 J_2} X_3(t) - \frac{(B_1 K_2 + B_2 K_1)}{J_1 J_2} X_2(t) - \frac{K_1 K_2}{J_1 J_2} X_1(t) + \frac{\tau_a}{J_1 J_2}$$

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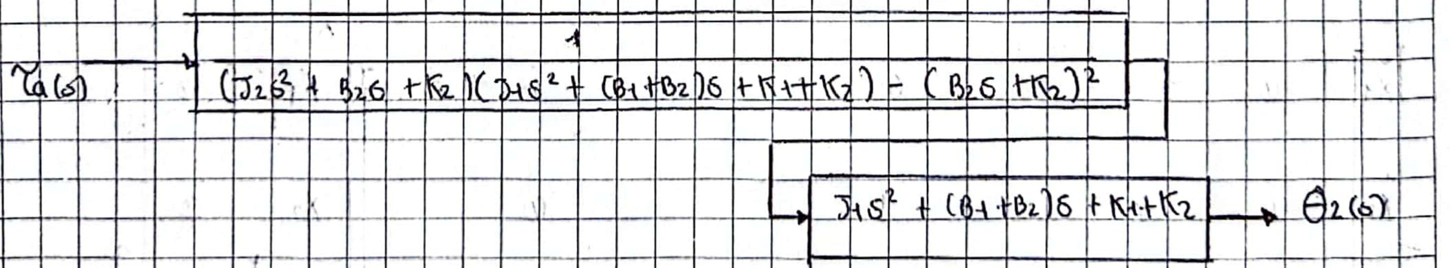
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ \frac{-K_1K_2}{J_1J_2} & \frac{-(B_1K_2+B_2K_1)}{J_1J_2} & \frac{-(B_1B_2+J_1K_2+J_2K_1+J_2K_2)}{J_1J_2} & \frac{-(B_1J_2+B_2J_1+B_2J_2)}{J_1J_2}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 0\\ \frac{1}{J_1J_2}\end{bmatrix}\tau_a$$

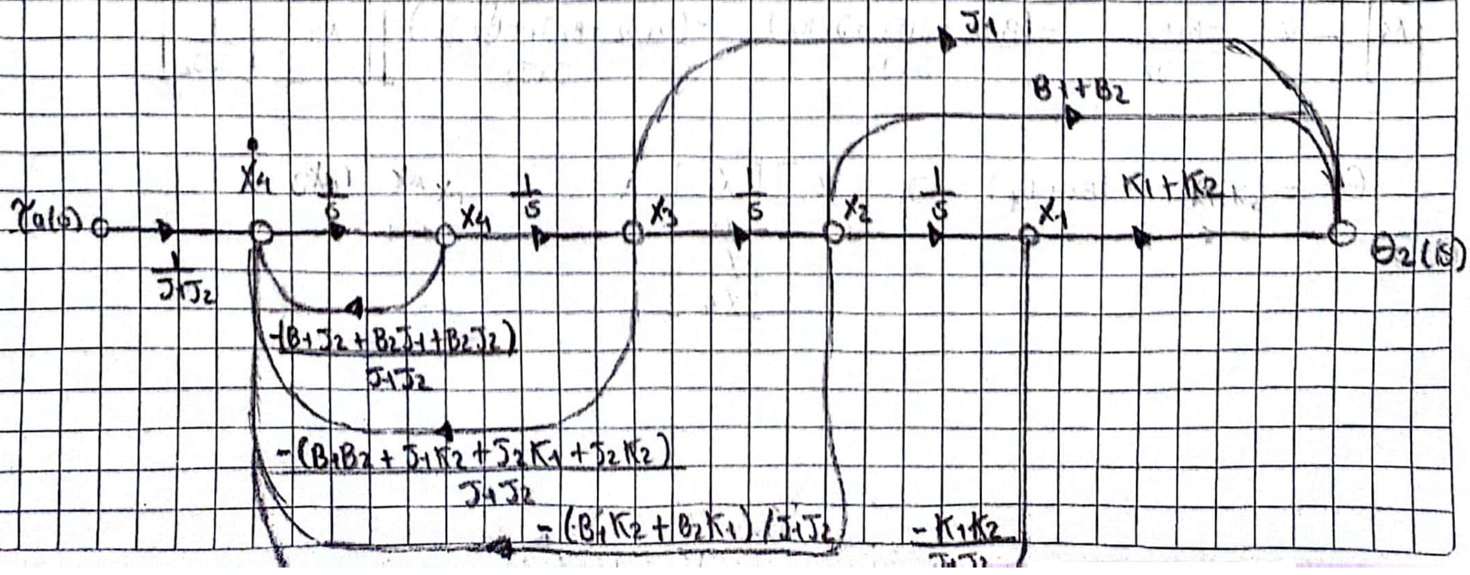
$$\theta_2=\begin{bmatrix}K_1+K_2 & B_1+B_2 & J_1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}$$

c. Diagrama de bloques:

Representando la versión simplificada



d) Diagrama de flujo



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3. Para el sistema del item anterior, mismo requerimiento consideramos $K_1 = 0$

Partiendo de la función de transferencia anterior:

$$G(s) = \frac{J_1 s^2 + (B_1 + B_2)s + K_1 + K_2}{(J_2 s^2 + B_2 s + K_2)(J_1 s^2 + (B_1 + B_2)s + K_1 + K_2) - (B_2 s + K_2)^2}$$

tomando $K_1 = 0$

a. Funcion de transferencia

$$G(s) = \frac{J_1 s^2 + (B_1 + B_2)s + K_2}{(J_2 s^2 + B_2 s + K_2)(J_1 s^2 + (B_1 + B_2)s + K_2) - (B_2 s + K_2)^2}$$

b. Representación en el espacio de estados

tomando las ecuaciones anteriormente desarrolladas, remplazando $K_1 = 0$ en [6] y [7]

$$\Theta_2(s) = J_1 X_3 + (B_1 + B_2) X_2 + K_2 X_1$$

$$s X_4 = -\frac{(B_1 J_2 + B_2 J_1 + B_2 J_2)}{J_1 J_2} X_4 - \frac{(B_1 B_2 + J_1 K_2 + J_2 K_2)}{J_1 J_2} X_3 - \frac{(B_1 K_2)}{J_1 J_2} X_2 + \frac{\tau_a}{J_1 J_2}$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-B_1 K_2}{J_1 J_2} & \frac{-(B_1 B_2 + J_1 K_2 + J_2 K_2)}{J_1 J_2} & \frac{-(B_1 J_2 + B_2 J_1 + B_2 J_2)}{J_1 J_2} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_1 J_2} \end{bmatrix} \tau_a$$

$$\Theta_2 = \begin{bmatrix} K_2 & (B_1 + B_2) & J_1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

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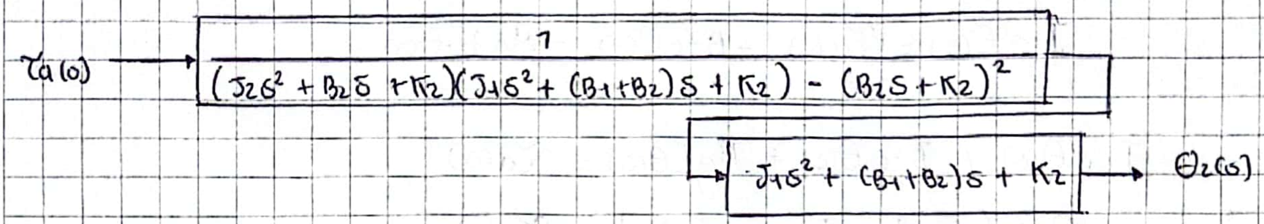
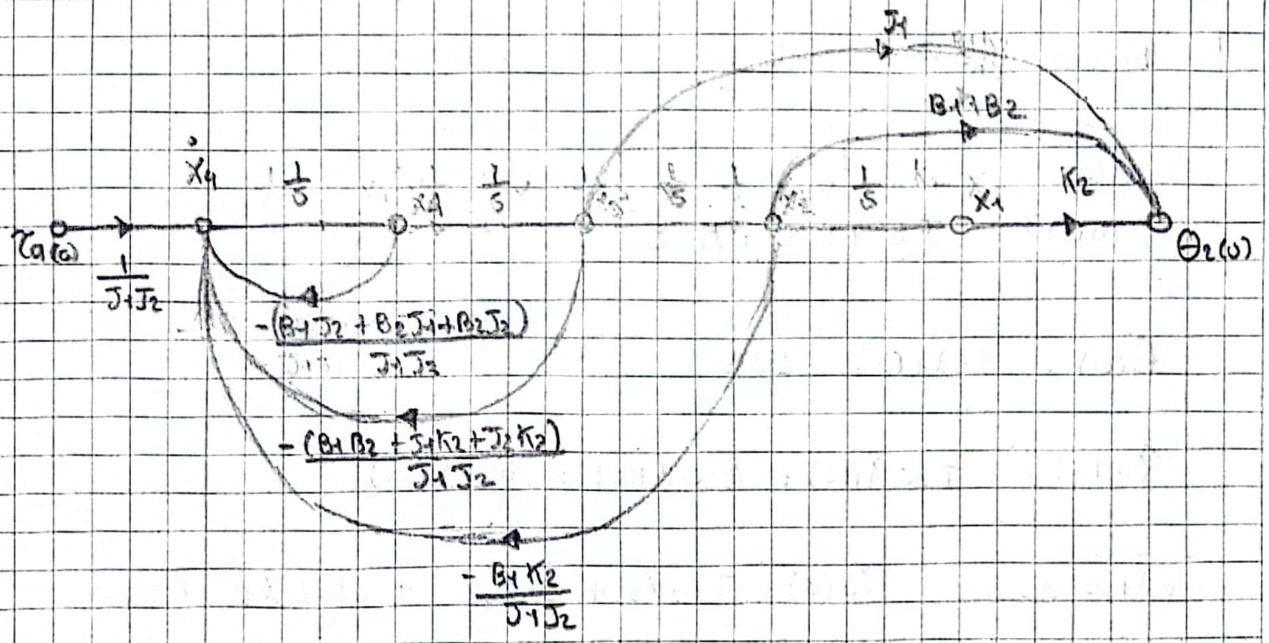
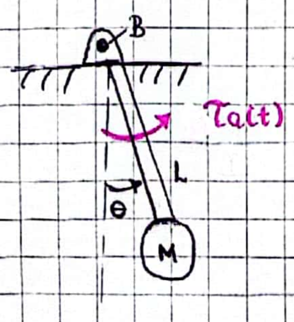
c. Diagrama de bloques



d. Diagrama de flujo de señal



4. Para el sistema rotacional en la Figura, determine:



a. La función de transferencia relacionando θ y τ_a

- Tomando como referencia el modelamiento dado por la ecuación de Euler – Lagrange y usando el modelo lineal para pequeñas oscilaciones:

$I = ML^2$

$$I\ddot{\theta} = \tau_a - B\dot{\theta} - MgL\theta \qquad [1]$$

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aplicando Laplace:

$$I \cdot s^2 \Theta(s) = \tau a(s) - B \cdot s\Theta(s) - MgL \cdot \Theta(s)$$

$$I \cdot s^2 \Theta(s) + B \cdot s\Theta(s) + MgL\Theta(s) = \tau a(s)$$

$$\Theta(s) \left[I \cdot s^2 + B \cdot s + MgL \right] = \tau a(s)$$

$$\frac{\Theta(s)}{\tau a(s)} = \frac{1}{I \cdot s^2 + Bs + MgL}$$ Función de transferencia

b. Representación en el espacio de estados

$$\frac{\Theta(s)}{\tau a(s)} = \frac{1}{I \cdot s^2 + B \cdot s + MgL} \cdot \frac{X_1(s)}{X_1(s)}$$

$$\Theta(s) = 1 \cdot X_1(s) \quad [2]$$

$$\tau a(s) = I \cdot s^2 X_1(s) + B \cdot s X_1(s) + MgL X_1(s)$$

$sX_1 = X_2$ $\qquad \tau a(s) = Is \cdot X_2 + B \cdot X_2 + MgL \cdot X_1 \quad [3]$

$$sX_2 = -\frac{B}{I} X_2 - \frac{MgL}{I} X_1 + \frac{\tau a}{I} \quad [4]$$

pasando al dominio del tiempo [4] y [2]

$$\frac{dX_2(t)}{dt} = -\frac{B}{I} X_2(t) - \frac{MgL}{I} X_1(t) + \frac{\tau a(t)}{I}$$

$$\Theta(t) = X_1(t)$$

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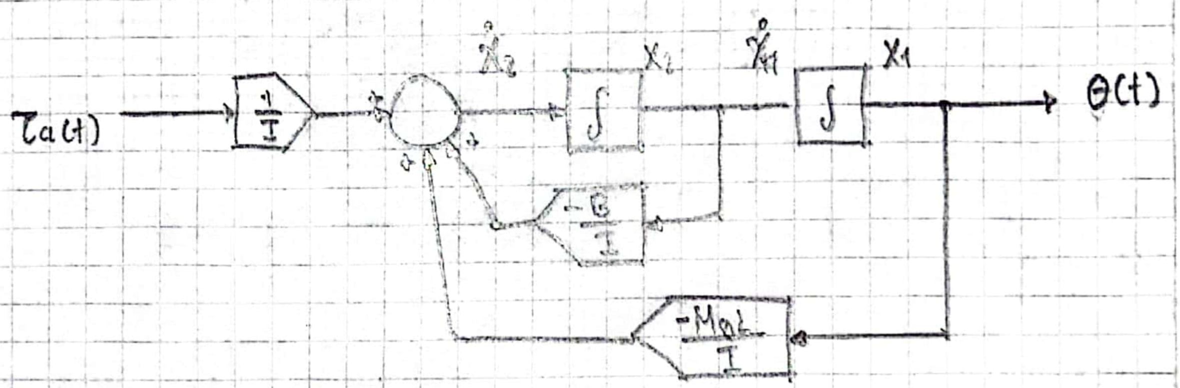
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{Mgl}{I} & -\frac{B}{I} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{I} \end{bmatrix} \tau_a$$

$$\theta(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

c. Diagrama de bloques



d. Diagrama de flujo de señal