A)

sea $G(s) = \frac{9}{s^2+2s+9}$, aplicando la entrada escalón unitario

hallar su respuesta en el tiempo, usar fracciones parciales

$$\frac{1}{s} \cdot \frac{9}{s^2+2s+9} \rightarrow \frac{1}{s} \cdot \frac{9}{}$$

$$s_1 = -1 + 2\sqrt{2}\,i \qquad s_2 = -1 - 2\sqrt{2}\,i$$

$$(s+1-2\sqrt{2}\,i) \qquad s+1+2\sqrt{2}\,i)$$

$$\frac{9}{s\,(s+1-2\sqrt{2}j)\,(s+1+2\sqrt{2}j)} = \frac{A}{s} + \frac{B}{s+1-2\sqrt{2}j} + \frac{C}{s+1+2\sqrt{2}j}$$

$$9 = A(s+1-2\sqrt{2}j)(s+1+2\sqrt{2}j) + B(s)(s+1+2\sqrt{2}j) + C(s)(s+1-2\sqrt{2}j)$$

$s=0$

$$9 = A(1-2\sqrt{2}j)(1+2\sqrt{2}j)$$

$$9 = A(1-4(2)(-1)) \rightarrow 9 = A(1+8) \qquad A = 1$$

$s = -1+2\sqrt{2}j$

$$9 = B(-1+2\sqrt{2}j)(-1+2\sqrt{2}j+1+2\sqrt{2}j)$$

$$9 = B(-16j - 5.66j)$$

$$B = -0{,}5 + 0{,}18j$$

$s = -1-2\sqrt{2}j$

$$9 = C(-1-2\sqrt{2}j)(-1-2\sqrt{2}j+1-2\sqrt{2}j)$$

$$9 = C(-16 + 5{,}66i)$$

$$C = -0{,}5 - 0{,}18j$$

$$Y(s) = \frac{1}{s} + \frac{-0{,}5+0{,}18j}{s+1-2\sqrt{2}j} + \frac{-0{,}5-0{,}18j}{s+1+2\sqrt{2}j}$$

$$y(t) = 1 + (-0.5 + 0.18j)\,e^{(-1+2\sqrt{2}j)t} + (-0.5 - 0.18j)\,e^{-(1+2\sqrt{2}j)t}$$

$$y(t) = 1 + (-0.5 + 0.18j)(e^{-t} \cdot e^{2\sqrt{2}jt}) + (-0.5 - 0.18j)\,e^{-t} \cdot e^{-2\sqrt{2}jt}$$

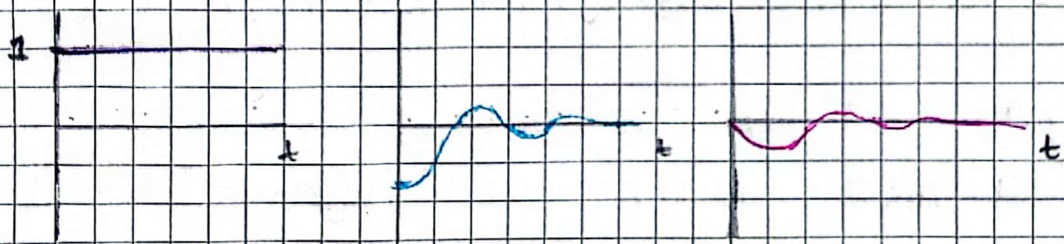
$$y(t) = 1 + -0.5e^{-t} \cdot e^{2\sqrt{2}jt} + 0.18j e^{-t} \cdot e^{2\sqrt{2}jt} - 0.5e^{-t} \cdot e^{-2\sqrt{2}jt} - 0.18j e^{-t} \cdot e^{-2\sqrt{2}jt}$$

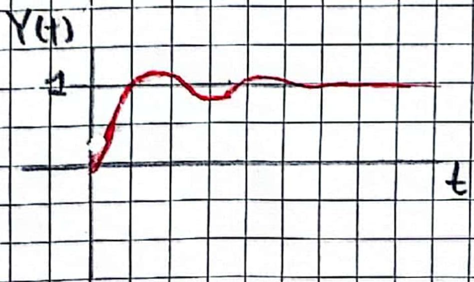
$$y(t) = 1 + -0.5e^{-t} e^{2\sqrt{2}jt} - 0.5e^{-t} \cdot e^{-2\sqrt{2}jt} + 0.18j e^{-t} \cdot e^{2\sqrt{2}jt} - 0.18j e^{-t} \cdot e^{-2\sqrt{2}jt}$$

$$y(t) = 1 - 0.5e^{-t}\left[e^{2\sqrt{2}jt} + e^{-2\sqrt{2}jt}\right] + 0.18j e^{-t}\left[e^{2\sqrt{2}jt} - e^{-2\sqrt{2}jt}\right]$$

$$y(t) = 1 - 0.5e^{-t} \cdot 2[\cos(2\sqrt{2}t)] + 0.18j e^{-t} \cdot 2j[\sin(2\sqrt{2}t)]$$

$$y(t) = 1 - e^{-t}[\cos(2\sqrt{2}t)] - 0.36e^{-t}[\sin(2\sqrt{2}t)]$$





B) $G(s) = \frac{9}{s^2+9}$, aplicando de entrada escalon unitario

$$Y(s) = \frac{1}{s} \cdot \frac{9}{s^2+9} = \frac{9}{s(s+3i)(s-3i)} = \frac{A}{s} + \frac{B}{s+3i} + \frac{C}{s-3i}$$

$$9 = A(s+3i)(s-3i) + B(s)(s-3i) + C(s)(s+3i)$$

con $s = 0$ $\quad 9 = A(3i)(-3i) \quad 9 = A(9) \quad A = 1$

con $s = 3i$ $\quad 9 = C(3i)(3i+3i) \quad 9 = C(3i)(6i) \quad 9 = (-18)C$

$C = \frac{9}{-18}$ $\quad C = -\frac{1}{2}$

con $s = -3i$

$9 = B(-3i)(-3i-3i)$ $\quad 9 = B(-3i)(-6i)$ $\quad 9 = B(-18)$

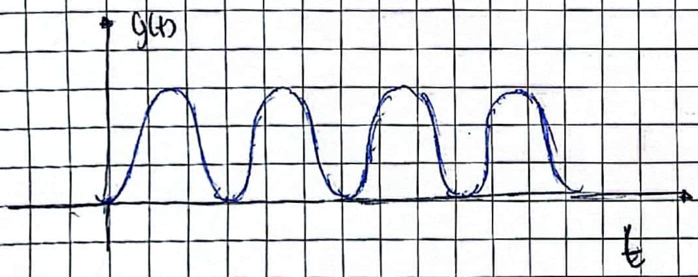
$B = \frac{9}{18} = \frac{1}{2}$

$$Y(s) = \frac{1}{s} + \frac{1}{2(s+3i)} - \frac{1}{2(s-3i)}$$

$$y(t) = 1 + \frac{1}{2}e^{-3it} - \frac{1}{2}e^{3it}$$

$$y(t) = 1 - \frac{1}{2}\left[e^{3it} + e^{-3it}\right]$$

$$y(t) = 1 - \cos(3t)$$



Ejercicio: Hallar la representación en el espacio de estados

$$G(s) = \frac{9}{s^2 + 9s + 9}$$

Tomando $G(s) = \frac{Y(s)}{X(s)}$ y teniendo en cuenta que el orden del sistema es de grado 2

$$\vec{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$$

$$\frac{Y(s)}{X(s)} = \frac{9 \cdot X_1(s)}{(s^2+9s+9)X_1(s)} \rightarrow \quad Y(s) = 9X_1(s) \quad [1]$$

$$X(s) = s^2 X_1(s) + 9s \cdot X_1(s) + 9X_1(s)$$

asumiendo

$$[2] \quad s \cdot X_1(s) = X_2(s) \quad \Rightarrow \quad X(s) = s \cdot s X_1(s) + 9s \cdot X_1(s) + 9X_1(s)$$

$$[3] \quad X(s) = s X_2(s) + 9X_2(s) + 9X_1(s)$$

pasando las ecuaciones [1], [2] y [3] al dominio del tiempo:

$$[1,1] \quad y(t) = 9x_1(t) \qquad [2,1] \quad \frac{dX_1(t)}{dt} = x_2(t)$$

$$[3,1] \quad X(t) = \frac{dX_2(t)}{dt} + 9X_2(t) + 9X_1(t) \quad , \text{ despeja } X_2'(t)$$

$$X_2'(t) = -9X_1(t) - 9X_2(t) - X(t)$$

DD	MM	AA

Representación en el espacio de estados:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -9 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} x(t)$$

entrada

$$y = \begin{bmatrix} 9 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$