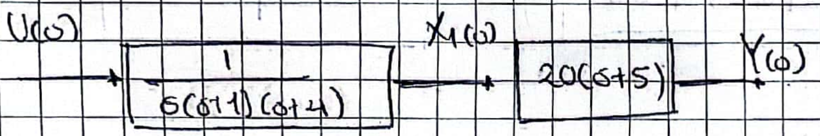
Transcripciones Videos

Ejemplo 12.1 → Norman S. Nise

Control System Engineering 7th Edition

$$G(s) = \frac{20(s+5)}{s(s+1)(s+4)}$$

requerimientos
$$\begin{cases} \%OS = 9.5\% \\ t_s = 0.74\ seg \end{cases}$$



$$\frac{X_1(s)}{U(s)} = \frac{1}{s^3+5s^2+4s}$$

$$X_1(s)\left[s^3+5s^2+4s\right] = U(s)$$

$$\dddot{X}_1 + 5\ddot{X}_1 + 4\dot{X}_1 = u(t)$$

$$\frac{Y(s)}{X_1(s)} = 20(s+5)$$

$$X_1(s)\left[20(s+5)\right] = Y(s)$$

$$20\dot{X}_1 + 100X_1 = y(t)$$

$$X_1 = X_1$$

$$X_2 = \dot{X}_1$$

$$X_3 = \dot{X}_2 = \ddot{X}_1$$

$$\dot{X}_3 = \ddot{X}_2 = \dddot{X}_1$$

$$\dot{X}_3 + 5X_3 + 4X_2 = u(t)$$

$$\dot{X}_3 = -4X_2 - 5X_3 + u(t)$$

$$20X_2 + 100X_1 = y(t)$$

Espacio de estados

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

teniendo en cuenta $\%OS = 9.5\%$:

$$\%OS = e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)} \times 100$$

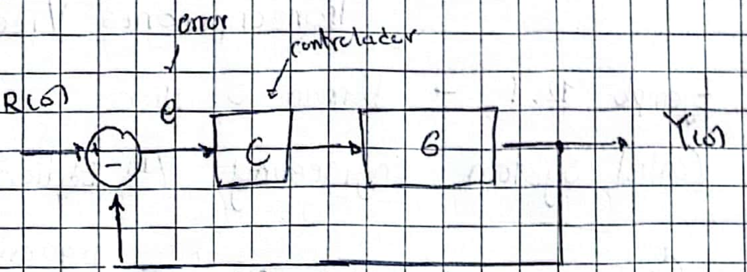
$$0{,}095 = e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)}$$

aplica ln

$$\ln(0.095) = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}}$$

Tipo de sistema

$$U(s) \longrightarrow \boxed{\frac{b_2 s^2 + b_1 s + b_0}{a_3 s^3 + a_2 s^2 + a_1 s + a_0}} \longrightarrow Y(s)$$



Dividiendo

$$U(s) \longrightarrow \boxed{\frac{1}{a_3 s^3 + a_2 s^2 + a_1 s + a_0}} \xrightarrow{X_1(s)} \boxed{\frac{b_2 s^2 + b_1 s + b_0}{1}} \longrightarrow Y(s)$$

(1) (2)

(1)
$$\frac{X_1(s)}{U(s)} = \frac{1}{a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

$$X_1(s)\left(a_3 s^3 + a_2 s^2 + a_1 s + a_0\right) = U(s)$$

$$\downarrow \mathcal{L}^{-1}$$

$$a_3 \dddot{x}_1 + a_2 \ddot{x}_1 + a_1 \dot{x}_1 + a_0 x_1 = u$$

$X_1 = x_1$

$X_2 = \dot{x}_1$

$X_3 = \dot{X}_2 = \ddot{x}_1$

$\dot{X}_3 = \ddot{X}_2 = \dddot{x}_1$

$$a_3 \dot{X}_3 + a_2 X_3 + a_1 X_2 + a_0 X_1 = u$$

$$\dot{X}_3 = -\frac{a_0 X_1}{a_3} - \frac{a_1 X_2}{a_3} - \frac{a_2 X_3}{a_3} + \frac{u}{a_3}$$

Estados espacio:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{a_0}{a_3} & -\frac{a_1}{a_3} & -\frac{a_2}{a_3} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/a_3 \end{bmatrix} u$$

(2)
$$X_1(s) \longrightarrow \boxed{b_2 s^2 + b_1 s + b_0} \longrightarrow Y(s)$$

$$Y(s) = X_1\left(b_2 s^2 + b_1 s + b_0\right) \rightarrow Y(s) = X_1 b_1 s^2 + X_1 b_2 s + X_1 b_0$$

$$\downarrow \mathcal{L}^{-1}$$

$$y(t) = b_2 \overset{X_3}{\cancel{\ddot{x}_1}} + b_1 \overset{X_2}{\cancel{\dot{x}_1}} + b_0 \overset{x_1}{\cancel{x_1}}$$

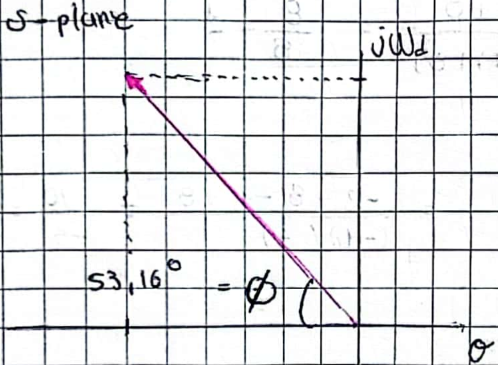
$$y = \begin{bmatrix} b_0 & b_1 & b_2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

$$\left(-2.3539\right)^2 = \left(\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right)^2 \rightarrow 5.5407 = \frac{\xi^2\pi^2}{1-\xi^2}$$

$$\rightarrow 5.5407\left(1-\xi^2\right) = \xi^2\pi^2 \rightarrow 5.5407 - 5.5407\,\xi^2 = \xi^2\pi^2$$

$$5.5407 = \xi^2\left(\pi^2 + 5.5407\right) \rightarrow \xi^2 = \frac{5.5407}{\pi^2 + 5.5407}$$

$$\rightarrow \xi = \sqrt{\frac{5.5407}{\pi^2 + 5.5407}} = 0,5996 \approx 0,6$$



$$s = \sigma + j\omega_d$$

$$s = \xi\omega_n + j\,\omega_n\sqrt{1-\xi^2}$$

teniendo en cuenta

$$t_s = \frac{4}{\sigma} \quad y \quad t_s = 0.74$$

$$\sigma = \frac{4}{t_s} = \frac{4}{0.74}$$

$$\sigma = 5.405 \approx 5.44$$

$$\sigma = \xi\omega_n$$

$$5,44 = 0,6\,\omega_n$$

$$\omega_n = 9,02 \text{ rad/s}$$

$$\xi = \cos\phi$$

$$\phi = \arccos\xi \rightarrow \phi = \arccos(0,6) = 53.16^\circ$$

$$j\omega_d = j\,\omega_n\sqrt{1-\xi^2}$$

$$\omega_d = 9,02\sqrt{1-0,6^2} = 7,22$$

tambien se puede hallar con $\tan\phi = \frac{\omega_d}{\sigma}$ $\quad \omega_d = \tan(53,16^\circ)\cdot 5,44 = 7,22$