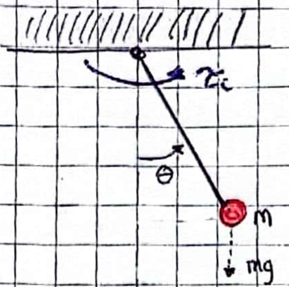


$$\tau_c - mgl\sin\theta = I_o\ddot{\theta} \qquad I_o = ml^2$$

Despejando $\ddot{\theta}$

$$\ddot{\theta} + \frac{mgl\sin\theta}{ml^2} = \frac{\tau_c}{ml^2}$$

$$\ddot{\theta} + \frac{g}{l}\sin\theta = \frac{\tau_c}{ml^2}$$

Expresión en el espacio de estados:

- $q_1 = \theta$
- $q_2 = \dot{q}_1 = \dot{\theta}$
- $\dot{q}_2 = \ddot{q}_1 = \ddot{\theta}$

Tomando el modelo lineal:

$$\ddot{\theta} + \frac{g}{l}\theta = \frac{\tau_c}{ml^2}$$

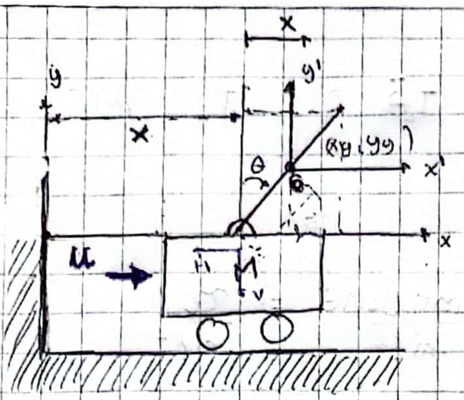
- asumiendo que para pequeñas oscilaciones $\operatorname{sen}\theta \approx \theta$

$$\dot{q}_2 + \frac{g}{l}q_1 = \frac{\tau_c}{ml^2}$$

$$\dot{q}_2 = -\frac{g}{l}q_1 + \frac{\tau_c}{ml^2}$$

$$\begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -g/l & 0 \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1/ml^2 \end{pmatrix}\tau_c$$

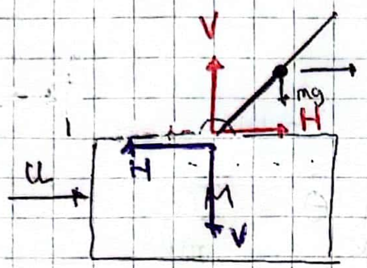
$$\theta = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$$



$$x_g = x + l\sin\theta$$

$$y_g = l\cos\theta$$

Para la varilla:



Movimiento rotacional:

$$I\ddot{\theta} = \text{Vertical}\cdot l\sin\theta + \text{Horizontal}\cdot Hl\cos\theta$$

$$I\ddot{\theta} = Vl\sin\theta + Hl\cos\theta \qquad [3.9]$$

Horizontal Motion:

$$H = \frac{m\,d^2(x + l\sin\theta)}{t^2}$$

$H = m\cdot a \qquad a = \frac{d^2x}{dt}$

$$H = m\ddot{x} + m\frac{d}{dt}\left(l\cos\theta\cdot\dot{\theta}\right)$$

$$H = m\ddot{x} + ml\frac{d}{dt}\left(\cos\theta\cdot\dot{\theta}\right)$$

$$H = m\ddot{x} + ml\left[(-\sin\theta\cdot\dot{\theta})\dot{\theta} + \cos\theta\cdot\ddot{\theta}\right]$$

$$H = m\ddot{x} - ml\sin(\theta)\cdot\dot{\theta}^2 + ml\cos(\theta)\cdot\ddot{\theta} \qquad [3.10]$$

Vertical Motion $\sum F = m.a$

$$m \cdot \frac{d^2}{dt}(l\cos\theta) = V - mg \quad [3.11]$$

Para el carro:

linealización: θ so small

$\sin\theta \cong \theta$
$\cos\theta \cong 1$
$\theta \cdot \dot{\theta}^2 = 0$

$$M\ddot{x} = u - H \quad [3,12]$$

u: Fuerza aplicada

H: la que se opone al movimiento

From: [3.9] $I\ddot{\theta} = Vl\sin\theta - Hl\cos\theta$

$$I\ddot{\theta} = Vl\theta - Hl \quad \text{eq. } [3-13]$$

From [3.10] $H = m\ddot{x} - ml\underbrace{\sin\theta \cdot \dot{\theta}^2}_{0} + ml\cos(\theta)\ddot{\theta}$

$$H = m\ddot{x} + ml\ddot{\theta}$$

$$H = m(\ddot{x} + l\ddot{\theta}) \quad \text{Eq. } [3.14]$$

From [3.11]

$$m\frac{d^2}{dt}(l\cos\theta) = V - mg$$

$$0 = V - mg \quad \text{Eq. } [3,15]$$

From [3,12 and 3,14]

$$M\ddot{x} = u - H \quad \wedge \quad m(\ddot{x} + l\ddot{\theta}) = H$$

remplaza 3.14 en 3.12

$$M\ddot{x} = u - m(\ddot{x} + l\ddot{\theta})$$

$$M\ddot{x} = u - m\ddot{x} - ml\ddot{\theta}$$

$u = (M+m)\ddot{x} + ml\ddot{\theta}$ eq. [3.16]

From eqs: (3.13), (3.14), (3.15)

$J\ddot{\theta} = Vl\theta - Hl$; $H = m(\ddot{x} + l\ddot{\theta})$; $0 = V - mg$

$V = mg$

$J\ddot{\theta} = mgl\theta - m(\ddot{x} + l\ddot{\theta})l$

$J\ddot{\theta} = mgl\theta - ml\ddot{x} - ml^2\ddot{\theta}$

$J\ddot{\theta} = mgl\theta - l(m\ddot{x} + ml\ddot{\theta})$ or $(J + ml^2)\ddot{\theta} + ml\ddot{x} = mgl\theta$

Taking:

$(M+m)\ddot{x} + ml\ddot{\theta} = u$ [1]

$(J + ml)\ddot{\theta} + ml\ddot{x} = mgl\theta$ [2]

Hallar el espacio de estados:

Despejando $\ddot{x}$ de [1]

$$\ddot{x} = \frac{u - ml\ddot{\theta}}{M+m}$$

Despejando $\ddot{\theta}$ de [2]

$$\ddot{\theta} = \frac{mgl\theta}{I+ml} - \frac{ml\ddot{x}}{I+ml}$$

Variables de estado:

$q_1 = x$ $\qquad$ $q_4 = \theta$

$q_2 = \dot{q}_1 = \dot{x}$ $\qquad$ $q_5 = \dot{q}_4 = \dot{\theta}$

$q_3 = \dot{q}_2 = \ddot{q}_1 = \ddot{x}$ $\qquad$ $q_6 = \dot{q}_5 = \ddot{q}_4 = \ddot{\theta}$

remplazando en [1] y [2] →

$$\dot{q}_2 = \frac{-ml}{M+m}q_6 + \frac{u}{M+m} \qquad \dot{q}_5 = \frac{mgl}{I+ml}q_4 - \frac{ml}{I+ml}q_3$$

DD | MM | AA

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-ml}{M+m} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{-ml}{I+ml} & \frac{mgl}{I+ml} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M+m} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot u$$

$$\begin{bmatrix} \theta \\ X \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix}$$