Corrección Parcial 1

1. Diagrama de estados y función de transferencia

$$\dddot{x} + \ddot{x} + 2\dot{x} + x = 2f(t)$$

Diagrama de estados

$$\dddot{x} = -\ddot{x} - 2\dot{x} - x + 2f(t)$$

$q_1 = x$

$q_2 = \dot{q}_1 = \dot{x}$

$q_3 = \dot{q}_2 = \ddot{q}_1 = \ddot{x}$

$\dot{q}_3 = \ddot{q}_2 = \dddot{q}_1 = \dddot{x}$

$$\dot{q}_3 = -q_3 - 2q_2 - q_1 + 2f$$

$$\underbrace{\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}}_{B} \underbrace{f}_{u}$$

$$\underbrace{x}_{y} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}}_{x}$$

Función de transferencia

Aplica Laplace a la ecuación:

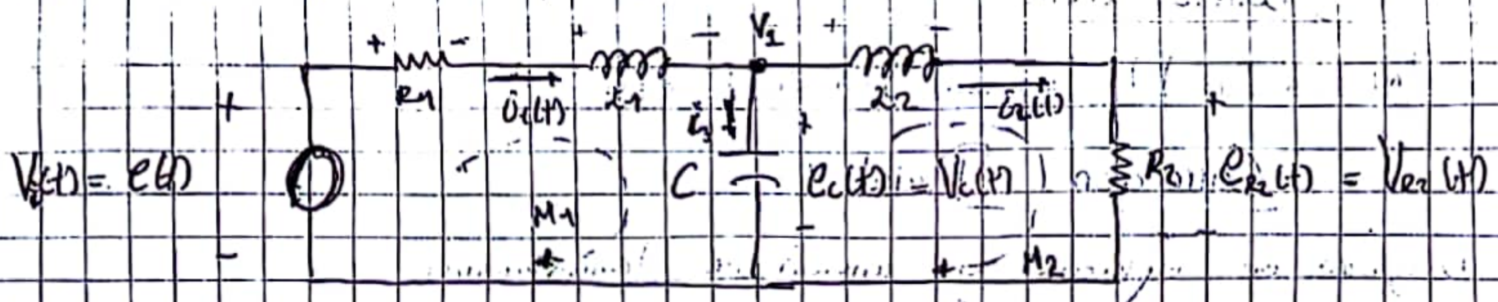
$$\mathcal{L}\{\dddot{x} + \ddot{x} + 2\dot{x} + x\} = \mathcal{L}\{2f(t)\}$$

$$s^3 X(s) + s^2 X(s) + 2s X(s) + X(s) = 2F(s)$$

$$X(s)\left(s^3 + s^2 + 2s + 1\right) = 2F(s)$$

$$\frac{X(s)}{F(s)} = \frac{2}{s^3 + s^2 + 2s + 1}$$

2) Espacio de Estados, salida V_{R2}



Variables de estado:

[1] $V_{L1} = L_1 \dfrac{di_1(t)}{dt}$ [2] $V_{L2} = L_2 \dfrac{d i_2(t)}{dt}$ [3] $i_c = C \dfrac{d V_c(t)}{dt}$

dejar en términos de: i_1, i_2, V_c, V_i

De la Malla 1 M_1:

$$V_i = V_{R1} + V_{L1} + V_c \qquad V_{R2} = i_2 \cdot R_2$$

De la Malla 2 M_2:

$$V_c = V_{L2} + V_{R2}$$

Del nodo V_1: $i_1 = i_3 + i_2$

Para V_{L1} → $V_{L1} = V_i - V_{R1} - V_c$

$$V_{L1} = V_i - R_1 \cdot i_1 - V_c$$ remplazando V_{L1} en [1]

$$\underbrace{\frac{di_1}{dt} = \frac{-R_1}{L_1} i_1 - \frac{1}{L_1} V_c + \frac{V_i}{L_1}}$$

Para V_{L2} → $V_{L2} = V_c - V_{R2}$

$$V_{L2} = V_c - R_2 \cdot i_2$$ remplaza en V_{L2} en [2]

$$\underbrace{\frac{di_2}{dt} = \frac{-R_2}{L_2} i_2 + \frac{1}{L_2} V_c}$$

$$\underbrace{\frac{di_2}{dt} = -\frac{R_2}{L_2} i_2 + \frac{1}{L_2} V_c}$$

Para $i_c = i_3 \rightarrow$ $i_1 = i_3 + i_2$

$i_3 = i_1 - i_2$ remplaza en $i_3 = i_c$ en [3]

$$\underbrace{\frac{dV_c}{dt} = \frac{1}{C} i_1 - \frac{1}{C} i_2}$$

Espacio de estado

$$\begin{bmatrix} \dot{i}_1 \\ \dot{i}_2 \\ \dot{V}_c \end{bmatrix} = \begin{bmatrix} -R_1/L_1 & 0 & -1/L_1 \\ 0 & -R_2/L_2 & 1/L_2 \\ 1/C & -1/C & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ V_c \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} V_i$$

$$V_{R2} = \begin{bmatrix} 0 & R_2 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ V_c \end{bmatrix}$$

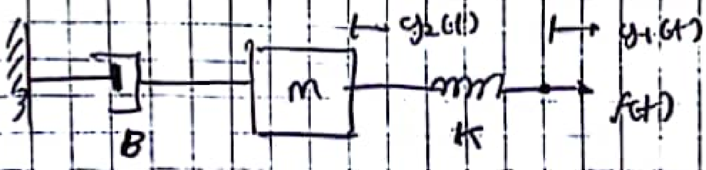
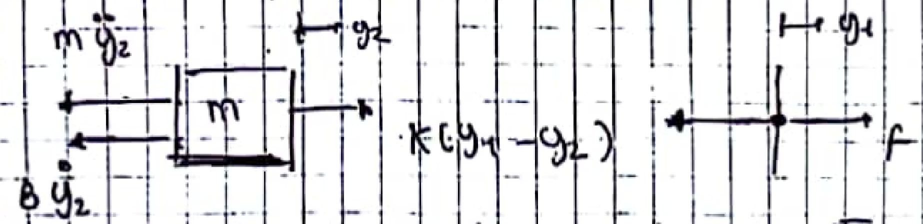


Diagrama de cuerpo libre



Por la segunda Ley de Newton $\quad + \rightarrow y \cdot \sum F = ma$

en y_2: $\quad -B\dot{y}_2 + K(y_1-y_2) = m\ddot{y}_2$ $\qquad$ en y_1: $\quad -K(y_1-y_2)+f=0$

$$\ddot{y}_2 = \frac{-B}{m}\dot{y}_2 + \frac{K}{m}(y_1-y_2) \qquad -Ky_1 + Ky_2 + f = 0$$

Variables de estado

$q_1 = y_1$

$q_2 = y_2$

$q_3 = \dot{q}_2 = \dot{y}_2$

$\dot{q}_3 = \ddot{q}_2 = \ddot{y}_2$

$$\dot{q}_3 = -\frac{B}{m}q_3 + \frac{K}{m}q_1 - \frac{K}{m}q_2 \quad [1]$$

$$-Kq_1 + Kq_2 + f = 0$$

$\rightarrow$ Despejo q_2

$$q_2 = \frac{K q_1}{K} - \frac{f}{K} \qquad \underbrace{q_2 = q_1 - \frac{1}{K}f}_{\text{remplazo en [1]}}$$

$$\dot{q}_3 = -\frac{B}{m}q_3 + \frac{K}{m}q_1 - \frac{K}{m}\left[q_1 - \frac{1}{K}f\right]$$

$$\dot{q}_3 = \frac{-B}{m}q_3 + \frac{K}{m}q_1 - \frac{K}{m}q_1 + \frac{K}{mK}f$$

$$\dot{q}_3 = \frac{-B}{m}q_3 + \frac{1}{m}f$$

Espacio de estados:

$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ 0 & 0 & -B/m\end{bmatrix}\begin{bmatrix}q_1\\ q_2\\ q_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1/m\end{bmatrix}f$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}q_1\\ q_2\\ q_3\end{bmatrix}$$