

Assignment 6

Structured Products

1. Case study: Certificate Pricing.

On the 15th of February 2008 at 10:45 C.E.T., the Bank XX issues a certificate whose hedging termsheet is described in the annex.

Determine the upfront X% in the Certificate issue, in a single-curve interest rate modeling setting, knowing that the two counterparties have signed an ISDA with CSA that allows neglecting the counterparty risk. Neglect also the dynamics of interest rates.

Market parameters:

Basket underlyings: ENI $S_0 = 12.3 \text{ Euro}$, AXA $S_0 = 22.1 \text{ Euro}$

Correlation = 0.49%, $\sigma_1 = 20.1\%$, $\sigma_2 = 18.3\%$,

Dividend Yields: $d_E = 3.2\%$, $d_{CS} = 2.9\%$

Discounts: consider values of the 15th of February 2008 at 10:45 C.E.T.

2. Exercise: Pricing Digital option

Verify the difference between the price of a digital option computed according to the Black model and in the case where one takes into account the smile in the curve of the implied volatility.

Dataset:

- Volatility & smile.
- ATM forward
- Notional 10 Mln €
- Digital Payoff 5% Notional
- Expiry: 1y (Act/365)
- Discount curve of the 15th of February 2008 at 10:45 C.E.T.

3. Exercise: Pricing

a. Compute the integral I

$$I = \int_{-\infty}^{\infty} e^{-iz\xi} \frac{1}{\xi^2 + \frac{1}{4}} d\xi$$

for $z = -2.5223\%$, 0 , $+1\%$

- Compute I via FFT. Choose an adequate value for M and, either ξ_1 or dz . Discuss the results for different choices of the FFT parameters.
- Compute I with quadrature.
- Compute I with the residuals technique.

Remark: structure the code with i) a script: runPricingFourier, ii) some functions that compute I (with the two methodologies: FFT & Quadrature) that call iii) an integrand function. It is suggested not to use the parameters in the excel spreadsheet but to deduct them within the code (in order to avoid rounding errors).

Function signatures:

$X = \text{certificatePricing}(\text{underlyings}, \text{certificate}, \text{ratesCurve}, M)$ with

underlyings, struct containing:	S0:	vector of underlyings value at value date
	d:	vector of dividend yields
	sigma:	vector of underlyings volatilities
	corr:	correlation matrix
certificate, struct containing:	setDate:	settlement date (and certificate start date)
	maturity:	Maturity
	alpha:	participation coefficient
	P:	Protection
	spol:	spread over Libor
	flagYearfrac:	flag for yearfrac conv. (floating leg): 2 for Act/360
ratesCurve, struct containing:	Dates	buckets of raw data
	Rates	zero rates at buckets dates
M:		number of simulations

$[\text{priceBlack}, \text{priceSmile}] = \text{priceDigital}(\text{cSelect}, \text{notional}, \text{optionPayoff}, \text{optionStrike}, \text{optionTTM}, \text{discount})$ with

cSelect, struct provided		input dataset
notional:		derivative notional
optionPayoff:		payoff of the digital option (in %)
optionStrike:		strike of the digital option
optionTTM		Time-to-maturity
discount:		discount factor 1y

$I = \text{computeIntegral}(f, \text{moneyness}, \text{modelParams}, \text{numericalParams}, \text{method})$

f		integrand as a function handle, e.g. $f = @(x) 1./(x.^2 + 0.25)$
moneyness:		moneyness of interest as a vector
modelParams		model parameters, e.g. empty
numericalParams:		parameters of the numerical method as a struct, e.g. x1, xN, dx, z1, zN, dz, M for fft x1, xN for quadrature
method:		flag: 0 for quadrature, 1 for fft (default)

Exercise 8.1, Annex:

Indicative Terms and Conditions as of 15th of February 2012

Swap Termsheet

Principal Amount (N):	100 MIO EUR
Party A:	Bank XX
Party B:	I.B.
Trade date:	today
Start Date:	15 Feb 2008
Maturity Date (t):	5 years after the Start Date, subject to the Following Business Day Convention.

Party A pays:	Euribor 3m + 0.30%
Party A payment dates:	Quarterly, subject to Modified Business Convention
Daycount:	Act/360
Party A pays @ Maturity Date:	$(1 - P)$ of the Principal Amount
Protection (P):	95%

Party B pays @ Start Date:	X% of the Principal Amount
Party B pays @ Maturity Date:	Coupon
Participation coefficient (α):	110%
Coupon:	Pays at expiry date the participation coefficient of the performance, if positive, of an equally weighted basket of ENI S.p.A. and AXA S.A.

$$\alpha \cdot (S(t) - P)^+$$

Basket:

$$S(t) = \sum_{n=1}^2 \frac{E_t^n}{E_0^n} \times W_n$$

and:

E_t^n : Value of the n^{th} element of the basket at Maturity Date t ;

E_0^n : Value of the n^{th} element of the basket at Start Date;

W_n : Weight of the n^{th} element of the basket, each one equal to one half..

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