Politecnico di Milano

Financial Engineering

Group 9
Assignement 3

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1 Profit and Loss Impact for an IRS

1.1 P&L Estimation

Bank XX uses a non standard methodology in order to obtain discount factor curve. We have considered market situation on the 15th of Feb 2008 at 10:45 a.m. CET. We supposed that using bank discount factor curve one obtains a 5y swap rate vs 3M equal to 4.06% (versus a mid market rate of 4.042%). We tried to estimate the PL for a trader that pays a 5y vs 3M at 4.045% for 100 Mln. The difference from Mid Rate Fixed leg and the Bank Fixed leg is equal to 1.5 bp.

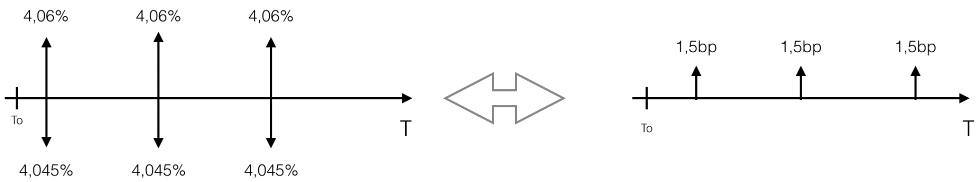

Figure 1: Contract Vs Bank Contract

$$NPV = Notional \cdot 1.5 \sum_{i=1}^{N} \delta(t_{i-1}, t_i) \widetilde{B}(t_0, t_i)$$

Where $\widetilde{B}$ are the discounts calculated by the bank.
The discount curve used by the bank XX is not known so we try to do an approximation of the curve using the bootstrap method. Then we use the discounts that we obtain to calculate the NPV.

$$NPV = Notional \cdot 1.5 \sum_{i=1}^{N} \delta(t_{i-1}, t_i) B(t_0, t_i)$$

Now B are the discounts calculated via bootstrap.
In order to understand if this can be a good approximation we try to create a curve which can be closer to the curve of the bank. Then we compute the error between the two NPV obtained. We observed that the difference between the 5y swap rate of the bank and the mid mkt rate is 1.8bp. So we add this term to all the market rates that we use in the bootstrap and then recompute the discount factor curve using these new rates and finally we recompute the NPV and make the difference with the previous one.

$$NPV = Notional \cdot 1.5 \sum_{i=1}^{N} \delta(t_{i-1}, t_i) B_{shifted}(t_0, t_i)$$

Where $B_{shifted}$ are the discounts calculated shifting the mid market rate.
We observed that the absolute value of the error between the last two NPV is 33.9154€.
Computing the relative error between the error and the second NPV we obtain an error of approximately 5bp which is deemed acceptable.

1.2 Why it is so important to obtain a discount factor curve that reproduces exactly liquid instruments ?

The reason why I want to replicate exactly some pillar instruments is that I do not want to have PnL in my book that is only generated by the model. Continuing on the lines of the above example we can imagine that the bank is forced to liquidate the position, he had a positive mark in his books but if he had to sell it would not realize any gain since he would hypothetically sell at mid (4.042%) or less realizing a loss and not a gain in this situation.

2 Asset Swap Vs Floater

Given the discounting curve vs Euribor 3m on the 15 Feb 2008 at 10:45 C.E.T. and assuming that a 3y bond price for an issuer YY is 98 with an annual coupon equal to 4.6% we have computed the spread of the Asset Swap using the formula:

$$S^{asw} = \frac{C(0) - \overline{C}(0)}{BPV^{flt}}$$

$$C(0) = \bar{c}\sum_{i=1}^{N} \delta(t_{i-1}, t_i)B(t_0, t_i) + B(t_0, t_N)$$

If we are to compare the spread of the ASW, where we obtained 141bp, with the spread of a 3y quarterly floater of YY that trades at par with a spol of 55bps, it seems reasonable to suggest to buy the ASW in order to receive a bigger remuneration.
The NPV immediately after having paid the pull to par of $1-\overline{C}(0)$ turns out to be equal to $1-\overline{C}(0)$, in our case equal to 2%, since all other quantities are unchanged. To estimate the NPV of the swap at other times we can assume that it will be lower than this quantity in the case of rates remaining equal because after t_0^+ we will have less flows. This have to be multiplied also for the probability of default which seems low due to the fact the the bond tared at near par and the spol is relatively low. Everything seems to point out in the direction of buying the ASW but we can ask our self why there is this disparity in the first place. From the table below we can infer this relationship

$$\sum_{i=1}^{N} \mathbb{E}\left[D_{0,i}NPV(\tau)(\mathbb{I}_{t_i<\tau<t_{i+1}})\right] = \sum_{i=1}^{N} \delta_{i-1}\mathbb{E}\left[D_{0,i}(s^{spol} - s^{asw})\mathbb{I}_{t_i<\tau}\right]$$

If we think the market has not misplaced the ASW spread and the above relationship does in fact hold then we will better off with just the floater and not the synthetic one, since it's readily available on the market.
We found a very similar discussion in *Understanding Credit Derivatives and Related Instruments* Ch. 5.

FLOATER VS ASSET SWAP PACKAGE

		Floater	**Asset Swap Package**
no default	$\mathbf{t}_0$	**-1**	$-\overline{C}(0) - (1 - \overline{C}(0))$
no default	$\mathbf{t}_i$	$\delta(L(t_{i-1}, t_i) + S^{spol})$	$\delta(L(t_{i-1}, t_i) + S^{asw})$
no default	$\mathbf{t}_N$	$\mathbf{1}+\delta(L(t_{N-1}, t_N) + S^{spol})$	$\mathbf{1}+\delta(L(t_{N-1}, t_N) + S^{asw})$
default	$\mathbf{t}_i$	π	$\pi + NPV(\tau)$

3 CDS Bootstrap

In order to bootstrap the credit curve for a single name product we should invert the following relationship between the fee leg and contingent leg of a CDS. This is the formula that also consider the accrual term.

$$(1-\pi)\sum_{i=1}^{N} e(t_0, t_{i-1}, t_i) = \overline{s}\sum_{i=1}^{N} \delta(t_{i-1}, t_i)\overline{B}(t_o, t_i) + \overline{s}\sum_{i=1}^{N} \frac{\delta(t_{i-1}, t_i)}{2} e(t_0, t_{i-1}, t_i)$$

After some passages, we can get an iterative formula to compute the probability to survive until t_N with the information of the credit curve up to t_{N-1}

$$P(t_0, t_N) = \frac{\sum_{i=1}^{N-1} \Pi_i e(t_0, t_{i-1}, t_i) + \Pi_N B(t_0, t_N) P(t_0, t_{N-1}) - \overline{s}\sum_{i=1}^{N-1} \delta_{t_{i-1}} B(t_0, t_i) P(t_0, t_i)}{\left(\Pi_N + \overline{s}\delta_{t_{N-1}}\right) B(t_0, t_N)}$$

where $\Pi_i = 1 - \pi - \overline{s}\frac{\delta_{t_{i-1}}}{2}$ if we consider accruals or $\Pi_i = 1 - \pi$ if we neglect them.
Once we have the probability to survive for every t_i until T (maturity) we can easily compute the intensities λ, also known as Hazard rates, using the following formula:

$$P(t, T) = e^{-\int_t^T \lambda(s) ds}$$

To see the effect of considering or not accruals in the bootstrap we repriced the CDS rates in both cases.
Clearly we can again consider the formula to recomputed CDS spreads with or without accruals, if we use the survival probabilities that we got without accruals and reprice the spreads with the formula that considers it then we should obtain again exactly the same spreads. In both cases the error in terms of prices we get is well below the 1bp threshold.
It the following picture we can show the errors done in neglecting the accruals:

λ_{approx}	λ_{exact}
0.003992	0.004000
0.004863	0.004875
0.005750	0.005766
0.006645	0.006665
0.006859	0.006882

At the end we use the Jarrow & Turnbull (1995) model and we obtain :

λ_{JT}
0.004000
0.004428
0.004857
0.005285
0.005571

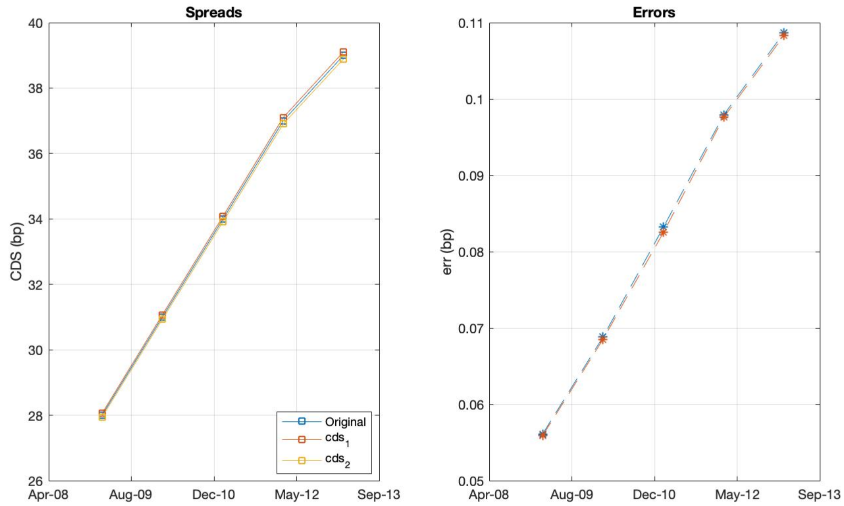

Figure 2: Error Accrual

This approximate result come from the fact that if we assume the CDS paying constant continuous protection until default we have that in a time $(t, t+dt)$ the protection paid is $\overline{s}dt$

$$FeeLeg = \overline{s} \cdot \mathbb{E}\left[\int_0^T D(t_0,t)1_{\tau>t}dt\right] = \overline{s}\int_0^T P(t_0,t)B(t_0,t)1_{\tau>t}dt$$

while for the contingent leg we have

$$ContingLeg = \mathbb{E}\left[(1-\pi)D(t_0,\tau)1_{\tau<t}\right] = (1-\pi)\int_0^T \mathbb{E}\left[D(t_0,t)1_{\tau\in dt}\right] = (1-\pi)\int_0^T \frac{dP(t_0,t)}{dt}B(t_0,t)1_{\tau>t}dt$$

If we assume $P(t_0,t)$ comes from an exponential model with constant intensity we have $P(t_0,t) = \exp(\lambda(t-t_0))$ and hence the contingente eg became:

$$ContingLeg = (1-\pi)\lambda\int_0^T P(t_0,t)dtB(t_0,t)1_{\tau>t}dt$$

which is the JT approximation. This model is accurate for short maturities and it's convinient as a rule of thumb since it does not require

equaling the contingent and the FeeLeg we get that

$$\lambda = \frac{\overline{s}}{1-\pi}$$

4 Equivalence of ASW and CDS spreads

We can understand that $NPV(t)$ is independent of the default of the underlying bond of the ASW, being $NPV(t)$ the npv of the ASW at time t. This is because at t_0^+ on the contract is written to

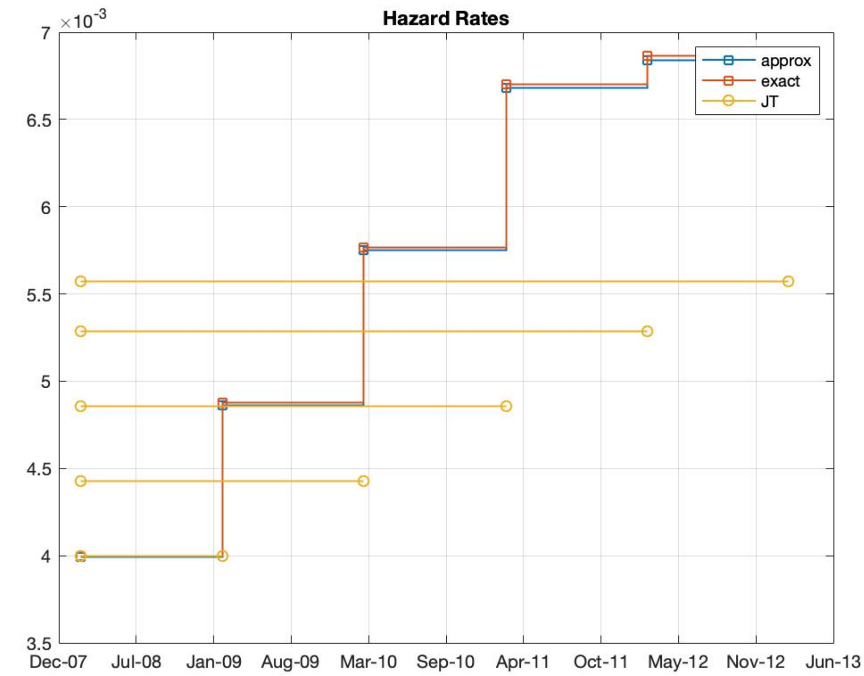

Figure 3: λ_{approx} vs λ_{exact} vs λ_{JT}

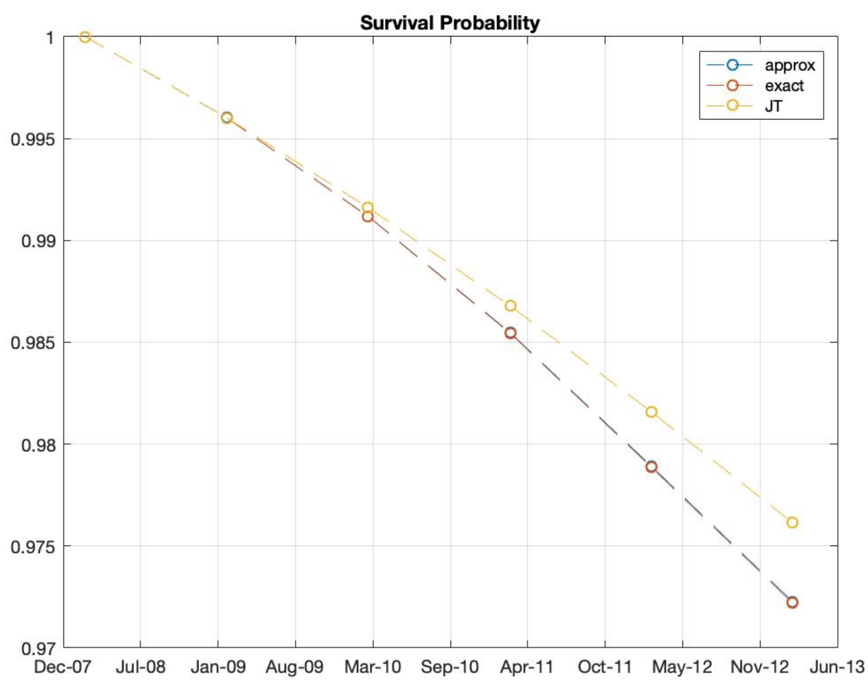

Figure 4: Survaival Probabilities

pay quantities that only depends on the i.r. considered.
However the question is about the equivalence of the ASW spread and the CDS spread. The main point is that a ASW is written to convert a fixed coupon into a floating coupon bond, given that we write the paying ASW and buy fixed coupon bond (ASW package). Lets imagine that if the company defaults we are forced to sell the ASW, clearly since we understood that ASW are mainly written to convert fixed into floaters, in case of default there will be little or no request of ASW referencing a defaulted company, and hence we would expect little demand of the remaining ASW that would drive down the price $NPV(t)$.
In order to answer we need to provide the following statement:
At valuation time t we can find the following equivalence between the cash flows of the of ASW package and the CDS

$$\mathbb{E}_t\left[\Pi^{ASWpk}(t)\right] = \mathbb{E}_t\left[\Pi(t)^{CDS}\right] + \mathbb{E}_t\left[\mathbb{I}_{\tau\leq T} D(t,\tau) NPV(\tau)\right]$$

Assuming independence between the default probabilities and the NPV we can find a bound for the difference since $\mathbb{E}[\mathbb{I}_{\tau\leq T}] < 2\%$ and hence

$$|\mathbb{E}[\Pi^{ASWpk}(t) - \Pi^{CDS}(t)]| < 8bps$$

From this framework follows the equivalence between ASW and CDS spread.