Assignment 4.

1. Case Study: MBS Pricing

On the 15th of February 2008, the reference portfolio of the SPV *Cayman V* has a total notional of € 1 bn: this portfolio can be considered homogeneous. Assume for simplicity that mortgages provide a single payment at the end of the interest period T equal to two years and defaults are independent from interest rates; for the period T the default probability of each mortgage is 6.5%, correlation is 40% and an average recovery of 65% for each mortgage.

a. Value with a Vasicek model the price of a mezzanine Tranche (with subordinations -detachment points- K_d 6% and K_u 9%), considering valid the hypothesis of Large Homogeneous Portfolio. Express the price in percentage units.

b. For the same tranche, estimate the impact of the hypothesis of Large Homogeneous Portfolio for I = 500. Show that the price of the Tranche in relative terms (as a percentage of tranche face value) is well described by the Kullback-Leibler (KL) approximation in the range I = (10, 2 10^4), where the LHP approximation holds. It is required to plot the price varying I (in log scale in the abscissa), with

- the exact solution (up to an I that your computer allows to obtain a price);
- the approximate solution;
- the LHP solution.

c. When pricing the tranche, use a normalization constant more precise than the one arising from the Stirling formula. Do the results differ significantly?

d. [optional] Price the Equity tranche with detachment points K_u 6% varying I in the same range considered at the point b. Is the KL approximation adequate? How can you modify it?

2. Case Study: **Pricing in presence of counterparty risk**

On the 15th of Feb 2008 at 10:45 a.m. CET bank XX buys form ISP a 5y Cliquet option for a 50 MIO € notional. Option payoff yearly payoff (annual bond) at each payment date is

$$\frac{[S(t_i) - S(t_{i-1})]^+}{S(t_{i-1})}$$

with $i = 1, \dots, 5$ .The option is on an equity stock (with no dividends) and constant volatility 20%.
In case of default between $t_{i-1}$and t_i the value that should be considered is in t_{i-1} with payment in t_i. Compute numerically.
What should be the correct price? At what price ISP would try to sell?

[Hint: consider the dynamics of the underlying not of the corresponding forward]

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3. Case Study: Variance-covariance method for VaR & ES in linear portfolio, plausibility check, a simple example of data mining

At the end of the 4th of July 2012 for an equally weighted equity portfolio with: Santander, AXA, Generali, Bayer. Compute daily VaR and ES with a 3y estimation using the dataset provided via Gaussian parametric approach and check VaR order of magnitude with a plausibility check.

Warning:
Pay attention that the trading days of the different stocks are not the same. Add previous day value in case of missing share price.

4. Numerical Exercise
For a Normal distribution and a t-Student with 3 degrees of freedom compute portfolio VaR & ES (with alpha = 99% and 10 business-day time horizon) given a portfolio of 2 stocks with the following data
w = [0.6; 0.4];
volatilities = [15%; 30%];
mu = [3%; 5%];
CorrMatrix = [1, 0.3; 0.3, 1]
with a unitary initial portfolio value. As usual volatilities and mu are p.a. values.

Function signatures
VaR = plausibilityCheck(returns, portfolioWeights, alpha, portfolioValue)
[ES, VaR] = AnalyticNormal(yearlyCovarianceMatrix, yearlyMeanReturns, portfolioWeights, riskMeasureTimeLag, alpha).
[ES, VaR] = AnalyticTstudent(yearlyCovarianceMatrix, yearlyMeanReturns, portfolioWeights, riskMeasureTimeLag, alpha, degreesOfFreedom);

<u>All vectors are column vectors.</u>

*Name of some M*atlab functions:

nchoosek(I,n):	Newton Binomial coefficient
norminv(alpha):	Inverse CDF normal
normpdf(x):	pdf normal
quadgk:	for numerical integration
tinv(alpha,dof):	Inverse cdf t-Student
tpdf(x,dof):	Pdf t-Student
prctile(ptfLosses,100* alpha)	Percentile for a given vector of portfolio Losses

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