Formule di Trigonometria

$$\boxed{\sin^2\alpha+\cos^2\alpha=1}\qquad \sin\alpha=\pm\sqrt{1-\cos^2\alpha}\qquad \cos\alpha=\pm\sqrt{1-\sin^2\alpha}$$

$$\cos\alpha=\pm\frac{1}{\sqrt{1+\tan^2\alpha}}\qquad \sin\alpha=\pm\frac{\tan\alpha}{\sqrt{1+\tan^2\alpha}}$$

α	α	sinα	cosα	tanα	cotα
0°	0	0	1	0	∞
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
90°	$\frac{\pi}{2}$	1	0	∞	0

Formule di addizione

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$
$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\sin\beta\cos\alpha$$
$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$
$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

Formule di duplicazione

$$\sin 2\alpha=2\sin\alpha\cos\alpha$$

$$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\ 2\cos^2\alpha-1\\ 1-2\sin^2\alpha\end{cases}$$

$$\tan 2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}$$

Formule parametriche

$$\left.\begin{aligned}\sin\alpha&=\frac{2t}{1+t^2}\\ \cos\alpha&=\frac{1-t^2}{1+t^2}\\ \tan\alpha&=\frac{2t}{1-t^2}\end{aligned}\right\}\quad\left(t=\tan\frac{\alpha}{2}\right)$$

Formule di prostaferesi

$$\sin p+\sin q=2\sin\frac{p+q}{2}\cos\frac{p-q}{2}$$
$$\sin p-\sin q=2\sin\frac{p-q}{2}\cos\frac{p+q}{2}$$
$$\cos p+\cos q=2\cos\frac{p+q}{2}\cos\frac{p-q}{2}$$
$$\cos p-\cos q=-2\sin\frac{p+q}{2}\sin\frac{p-q}{2}$$

Formule di bisezione

$$\sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{2}}$$

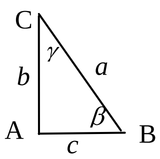
$$\cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2}=\begin{cases}\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}\\ \dfrac{1-\cos\alpha}{\sin\alpha}\\ \dfrac{\sin\alpha}{1+\cos\alpha}\end{cases}$$

Formule di triplicazione

$$\sin 3\alpha=3\sin\alpha-4\sin^3\alpha$$
$$\cos 3\alpha=4\cos^3\alpha-3\cos\alpha$$

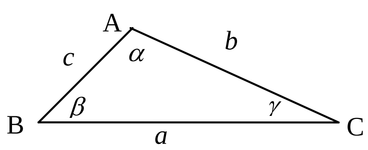
Triangoli rettangoli



$$b=a\sin\beta=a\cos\gamma=c\tan\beta$$
$$a=\frac{b}{\sin\beta}=\frac{b}{\cos\gamma}\ ;\tan\beta=\frac{b}{c}$$

$$c=a\sin\gamma=a\cos\beta=b\tan\gamma$$
$$a=\frac{c}{\sin\gamma}=\frac{c}{\cos\beta}\ ;\tan\gamma=\frac{c}{b}$$

Triangoli qualunque



$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}$$

$$a^2=b^2+c^2-2bc\cos\alpha$$
$$b^2=a^2+c^2-2ac\cos\beta$$
$$c^2=a^2+b^2-2ab\cos\gamma$$

$$Area(\mathrm{ABC})=\frac{1}{2}bc\sin\alpha=\frac{1}{2}ac\sin\beta=\frac{1}{2}ab\sin\gamma$$

Prodotti notevoli e Scomposizioni

prodotti notevoli	
$(a+b)(a-b) = a^2 - b^2$	prodotto di una somma per una differenza
$(a \pm b)^2 = a^2 \pm 2ab + b^2$	quadrato di un binomio
$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$	cubo di un binomio
$(a \pm b)^4 = a^4 \pm 4a^3b + 6a^2b^2 \pm 4ab^3 + b^4$	quarta potenza di un binomio
$(a \pm b)^5 = a^5 \pm 5a^4b + 10a^3b^2 \pm 10a^2b^3 + 5ab^4 \pm b^5$	quinta potenza di un binomio
$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$	quadrato di un trinomio
$(a+b+c)^3 = a^3 + b^3 + c^3 + 3a^2b + 3ab^2 + 3a^2c + 3ac^2 + 3b^2c + 3bc^2 + 6abc$	cubo di un trinomio
$(a+b)(a^2 - ab + b^2) = a^3 + b^3$	particolari prodotti notevoli
$(a-b)(a^2 + ab + b^2) = a^3 - b^3$	

scomposizioni		
$ab + ac = a(b+c)$		raccoglimento totale a fattore comune
$ab + ac + nb + nc = a(b+c) + n(b+c) = (a+n)(b+c)$		raccoglimento parziale a fattore comune
$a^2 - b^2 = (a-b)(a+b)$		differenza di due quadrati
$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$		somma di cubi
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$		differenza di cubi
$a^5 + b^5 = (a+b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$		somma di due potenze di esponente 5
$a^5 - b^5 = (a-b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$		differenza di due potenze di esponente 5
$a^7 + b^7 = (a-b)(a^6 - a^5b + a^4b^2 - a^3b^3 + a^2b^4 - ab^5 + b^6)$		somma di due potenze di esponente 7
$a^7 - b^7 = (a-b)(a^6 + a^5b + a^4b^2 + a^3b^3 + a^2b^4 + ab^5 + b^6)$		differenza di due potenze di esponente 7
$a^2 \pm 2ab + b^2 = (a \pm b)^2$		quadrato di binomio
$a^{2m} \pm 2a^m b^n + b^{2n} = (a^m \pm b^n)^2$		trinomio notevole con esponente pari
$x^2 + sx + p = (x+m)(x+n)$	$m + n = s$ e $m \cdot n = p$	trinomio con somma e prodotto caso $a = 1$
$ax^2 + sx + p$ $ax^2 + mx + nx + p$	• trovare due numeri m ed n tali che: • $m + n = s$ e $m \cdot n = ap$ • si sostituisce $\boldsymbol{sx} \rightarrow \boldsymbol{mx} + \boldsymbol{nx}$ • si effettua un raccoglimento parziale	trinomio con somma e prodotto caso $a \neq 1$
$a^3 \pm 3a^2b + 3ab^2 \pm b^3 = (a \pm b)^3$		cubo di binomio
$a^2 + 2ab + b^2 - c^2 = (a+b)^2 - c^2 = (a+b+c)(a+b-c)$		riduzione a differenza di quadrati
$c^2 - a^2 - 2ab - b^2 = c^2 - (a+b)^2 = (c-a-b)(c+a+b)$		
$a^2 + b^2 + c^2 + 2ab + 2ac + 2bc = (a+b+c)^2$		quadrato di un trinomio
$a^3 + b^3 + c^3 + 3a^2b + 3ab^2 + 3a^2c + 3ac^2 + 3b^2c + 3bc^2 + 6abc = (a+b+c)^3$		cubo di un trinomio

puoi scomporre $(a-b)$ come $(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})$

Tavola dei limiti notevoli

Limiti per $x \to 0$ di $f : \mathbb{R} \to \mathbb{R}$:

$$\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e \tag{1}$$

$$\lim_{x\to 0} \frac{(1+x)^\alpha - 1}{x} = \alpha \quad \alpha \neq 0 \tag{2}$$

$$\lim_{x\to 0^+} x^x = 1 \tag{3}$$

$$\lim_{x\to 0^+} x^{\frac{1}{x}} = 0 \tag{4}$$

$$\lim_{x\to 0} \frac{e^x - 1}{x} = 1 \tag{5}$$

$$\lim_{x\to 0} \frac{a^x - 1}{x} = \log a \tag{6}$$

Funzioni logaritmiche:

$$\lim_{x\to 0^+} x^b \log x = 0 \quad \forall b > 0 \tag{7}$$

$$\lim_{x\to 0} \frac{\log(1+x)}{x} = 1 \tag{8}$$

$$\lim_{x\to 0} \frac{\log_a(1+x)}{x} = \frac{1}{\log a} \tag{9}$$

Funzioni trigonometriche:

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \tag{10}$$

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2} \tag{11}$$

$$\lim_{x\to 0} \frac{1-\cos x}{x} = 0 \tag{12}$$

$$\lim_{x\to 0} \frac{\tan x}{x} = 1 \tag{13}$$

$$\lim_{x\to 0} \frac{\arcsin x}{x} = 1 \tag{14}$$

$$\lim_{x\to 0} \frac{\arctan x}{x} = 1 \tag{15}$$

Limiti per $x \to \infty$ di $f : \mathbb{R} \to \mathbb{R}$:

$$\lim_{x\to \pm\infty} \left(1 + \frac{1}{x}\right)^x = e \tag{16}$$

$$\lim_{x\to +\infty} \frac{a^x}{x^b} = +\infty \quad \forall a > 1, b > 0 \tag{17}$$

$$\lim_{x\to -\infty} a^x |x|^b = 0 \quad \forall a > 1, b > 0 \tag{18}$$

$$\lim_{x\to +\infty} \sqrt[x]{x} = \lim_{x\to +\infty} x^{\frac{1}{x}} = 1 \tag{19}$$

$$\lim_{x\to \infty} \frac{e^x}{x^b} = \infty \tag{20}$$

$$\lim_{x\to +\infty} \frac{\log x}{x^b} = 0 \quad \forall b > 0 \tag{21}$$

$$\lim_{x\to +\infty} \frac{\log x}{e^x} = 0 \tag{22}$$

Forme indeterminate:

$$\frac{0}{0}, \quad \frac{\infty}{\infty}, \quad 0 \cdot \infty, \quad 1^\infty, \quad 0^0, \quad (\pm\infty)^0, \quad +\infty - \infty.$$

Confronto di infiniti e infinitesimi:
Se

$$\lim_{n\to\infty} |a_n| = \infty$$

Allora:

$$\log_a n \leq n^b \leq c^n \leq n! \leq n^n \quad \textit{con} \ a, b, c > 1$$

Se

$$\lim_{n\to\infty} |a_n| = 0$$

Allora:

$$\frac{1}{\log_a n} \geq \frac{1}{n^b} \geq \frac{1}{c^n} \geq \frac{1}{n!} \geq \frac{1}{n^n} \quad \textit{con} \ a, b, c > 1$$

INTEGRALI

1) $\int x^p \ dx \ = \frac{x^{p+1}}{p+1} + c \qquad (p \in \mathbb{R}, \quad p \neq -1)$
2) $\int \frac{1}{x} \ dx \ = \ln|x| + c$
3) $\int a^x \ dx \ = \frac{a^x}{\ln a} + c$
4) $\int e^x \ dx \ = e^x + c$
5) $\int \sin x \ dx \ = -\cos x + c$
6) $\int \cos x \ dx \ = \sin x + c$
7) $\int \frac{1}{\cos^2 x} \ dx \ = \tan x + c$
8) $\int \frac{1}{\sin^2 x} \ dx \ = -\cot x + c$
9) $\int \frac{1}{1+x^2} \ dx \ = \arctan x + c$
10) $\int \frac{1}{\sqrt{1-x^2}} \ dx \ = \arcsin x + c$
11) $\int \sinh x \ dx \ = \cosh x + c$
12) $\int \cosh x \ dx \ = \sinh x + c$
13) $\int \frac{1}{\sqrt{1+x^2}} \ dx \ = \mathrm{settsinh} x + c = \ln(x + \sqrt{1+x^2}) + c$
14) $\int \frac{1}{\sqrt{x^2-1}} \ dx \ = \ln|x + \sqrt{x^2-1}| + c$

DERIVATE

1) $D(x^p) = px^{p-1} \quad (p \in \mathbb{R})$
2) $D(a^x) = a^x \ln a$
3) $D(e^x) = e^x$
4) $D(\log_a x) = \frac{1}{x} \log_a e$
5) $D(\ln x) = \frac{1}{x}$
6) $D(\sin x) = \cos x$
7) $D(\cos x) = -\sin x$
8) $D(\tan x) = \frac{1}{\cos^2 x} = 1 + \tan^2 x$
9) $D(\cot x) = -\frac{1}{\sin^2 x} = -1 - \cot^2 x$
10) $D(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$
11) $D(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$
12) $D(\arctan x) = \frac{1}{1+x^2}$
13) $D(\sinh x) = \cosh x$
14) $D(\cosh x) = \sinh x$
15) $D(\mathrm{settsinh} x) = \frac{1}{\sqrt{1+x^2}}$
16) $D(\mathrm{settcosh} x) = \frac{1}{\sqrt{x^2-1}}$

SVILUPPI DI McLAURIN

1) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \frac{x^n}{n!} + o(x^n)$
2) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + ... + (-1)^{n-1}\frac{x^n}{n} + o(x^n)$
3) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + ... + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$
4) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + ... + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$
5) $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + o(x^6)$
6) $\arcsin x = x + \frac{x^3}{6} + \frac{3x^5}{40} + o(x^6)$
7) $\arccos x = \frac{\pi}{2} - \arcsin x$
8) $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} + ... + (-1)^n \frac{x^{2n+1}}{2n+1} + o(x^{2n+2})$
9) $\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + ... + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$
10) $\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + ... + \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$
11) $(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + ... + \binom{\alpha}{n}x^n + o(x^n)$
11a) $\frac{1}{1+x} = 1 - x + x^2 - x^3 + ... + (-1)^n x^n + o(x^n)$
11b) $\frac{1}{\sqrt{1+x}} = 1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \frac{35x^4}{128} + o(x^4)$
11c) $\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5x^4}{128} + o(x^4)$

TRIGONOMETRIA

1) $\sin(p+q) = \sin p \cos q + \cos p \sin q$
2) $\sin(p-q) = \sin p \cos q - \cos p \sin q$
3) $\cos(p+q) = \cos p \cos q - \sin p \sin q$
4) $\cos(p-q) = \cos p \cos q + \sin p \sin q$
5) $\sin(2p) = 2 \sin p \cos p$
6) $\cos(2p) = \cos^2 p - \sin^2 p$
7) $\sin \frac{p}{2} = \pm\sqrt{\frac{1-\cos p}{2}}$
8) $\cos \frac{p}{2} = \pm\sqrt{\frac{1+\cos p}{2}}$
9) $\sin p \cos q = \frac{1}{2}\left[\sin(p+q) + \sin(p-q)\right]$
10) $\sin p \sin q = \frac{1}{2}\left[\cos(p-q) - \cos(p+q)\right]$
11) $\cos p \cos q = \frac{1}{2}\left[\cos(p-q) + \cos(p+q)\right]$
12) $\sin p + \sin q = 2 \sin \frac{p+q}{2} \cos \frac{p-q}{2}$
13) $\sin p - \sin q = 2 \cos \frac{p+q}{2} \sin \frac{p-q}{2}$
14) $\cos p + \cos q = 2 \cos \frac{p+q}{2} \cos \frac{p-q}{2}$
15) $\cos p - \cos q = -2 \sin \frac{p+q}{2} \sin \frac{p-q}{2}$.

Formule parametriche

Posto $\quad t = \tan\frac{x}{2}$:

1) $\sin x = \frac{2t}{1+t^2}$
2) $\cos x = \frac{1-t^2}{1+t^2}$
3) $\tan x = \frac{2t}{1-t^2}$

FUNZIONI IPERBOLICHE

Relazioni fondamentali

1) $\sinh x := \dfrac{e^x - e^{-x}}{2}$
2) $\cosh x := \dfrac{e^x + e^{-x}}{2}$
3) $\tanh x := \dfrac{e^x - e^{-x}}{e^x + e^{-x}} = \dfrac{e^{2x} - 1}{e^{2x} + 1} = \dfrac{1 - e^{-2x}}{1 + e^{-2x}}$
4) $\cosh^2 x - \sinh^2 x = 1$
5) $\cosh 2x = \cosh^2 x + \sinh^2 x$
6) $\sinh 2x = 2 \sinh x \cosh x$

Sviluppi in serie di MacLaurin

Funzione	Sviluppo di McLaurin	Intervallo di convergenza
e^x	$\sum_{n=0}^{+\infty} \frac{x^n}{n!}$	$R = +\infty$
$\frac{1}{1-x}$	$\sum_{n=0}^{+\infty} x^n$	$(-1, 1)$
$\log(1-x)$	$-\sum_{n=1}^{+\infty} \frac{x^n}{n}$	$(-1, 1]$
$\arctan x$	$\sum_{n=0}^{+\infty} \frac{(-1)^n}{2n+1}\ x^{2n+1}$	$[-1, 1]$
$\sin x$	$\sum_{n=0}^{+\infty} \frac{(-1)^n}{(2n+1)!}\ x^{2n+1}$	$R = +\infty$
$\cos x$	$\sum_{n=0}^{+\infty} \frac{(-1)^n}{(2n)!}\ x^{2n}$	$R = +\infty$
$\sinh x$	$\sum_{n=0}^{+\infty} \frac{x^{2n+1}}{(2n+1)!}$	$R = +\infty$
$\cosh x$	$\sum_{n=0}^{+\infty} \frac{x^{2n}}{(2n)!}$	$R = +\infty$
$(1+x)^\alpha$	$\sum_{n=0}^{+\infty} \binom{\alpha}{n} x^n$	$R = 1$

Serie di Fourier

$$f(x) \approx a_0 + \sum_{k=1}^{\infty} a_k \cos\left(k\frac{2\pi}{T}x\right) + b_k \sin\left(k\frac{2\pi}{T}x\right),$$

$$a_0 = \frac{1}{T}\int_0^T f(x)\, dx, \quad a_k = \frac{2}{T}\int_0^T f(x)\cos\left(k\frac{2\pi}{T}x\right)\, dx \quad k \geq 1, \quad b_k = \frac{2}{T}\int_0^T f(x)\sin\left(k\frac{2\pi}{T}x\right)\, dx \quad k \geq 1.$$

$$\int_0^T |f(x)|^2\, dx = Ta_0^2 + \frac{T}{2}\sum_{k=1}^{\infty}(a_k^2 + b_k^2) \quad (\text{ identità di Parseval })$$

Serie Numeriche

❖ Somme parziali

$$\sum_{k=0}^{n} r^k = \frac{1-r^{n+1}}{1-r} \qquad \forall n \in \aleph, \forall r \in \Re, r \neq 1$$

(*progressione geometrica* di ragione *r*)

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \forall n \geq 1$$

(*progressione aritmetica*)

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \qquad \forall n \geq 1$$

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{6}\right)^2 \qquad \forall n \geq 1$$

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = 1 - \frac{1}{n+1} \qquad \forall n \geq 1$$

(*somma telescopica*)

$$\sum_{k=1}^{n} \sin k = \frac{\sin\frac{n}{2} \cdot \sin\frac{n+1}{2}}{\sin\frac{1}{2}}$$

❖ Serie notevoli

$$\sum_{n=1}^{\infty} r^n \quad \begin{cases} \text{diverge a } +\infty & r \geq 1 \\ \text{converge} & -1 < r < 1 \Rightarrow \text{la somma è } \dfrac{1}{1-r} \\ \text{irregolare} & r = -1 \\ \text{diverge} & r < -1 \end{cases}$$

(serie geometrica di ragione *r*)

Se $|r| < 1 \qquad \sum_{n=1}^{\infty} a \cdot r^{n-1} = \frac{a}{1-r} \qquad \sum_{n=k}^{\infty} r^n = \frac{r^k}{1-r}$

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \quad \begin{cases} \text{converge} & p > 1 \\ \text{diverge} & p \leq 1 \end{cases}$$

(serie armonica generalizzata)

$$\sum_{n=2}^{\infty} \frac{1}{n^p \ln^q n} \quad \begin{cases} \text{converge} & p > 1, \forall q \in \Re \\ \text{converge} & p = 1, q > 1 \\ \text{diverge} & p = 1, q \leq 1 \\ \text{diverge} & p < 1, \forall q \in \Re \end{cases}$$

(serie di Abel)

❖ Criterio del confronto asintotico $(a_n \geq 0, b_n \geq 0)$

Se $\lim_{n \to +\infty} \frac{a_n}{b_n} = \lambda \qquad \lambda \in [0, +\infty]$

$0 < \lambda < +\infty \qquad a_n \sim b_n \Rightarrow$ stesso carattere

$\lambda = 0 \qquad$ se b_n converge $\Rightarrow a_n$ converge

$\lambda = +\infty \qquad$ se b_n diverge $\Rightarrow a_n$ diverge

❖ **Criterio del rapporto** $(a_n \geq 0, b_n \geq 0)$

$$\text{Se} \quad \lim_{n\to+\infty} \frac{a_{n+1}}{a_n} = \lambda \quad \lambda \in [0,+\infty] \quad \begin{cases} \lambda > 1 & \text{la serie diverge} \\ \lambda < 1 & \text{la serie converge} \\ \lambda = 1 & \text{nessuna informazione} \end{cases}$$

❖ **Criterio della radice** $(a_n \geq 0, b_n \geq 0)$

$$\text{Se} \quad \lim_{n\to+\infty} \sqrt[n]{a_n} = \lambda \quad \lambda \in [0,+\infty] \quad \begin{cases} \lambda > 1 & \text{la serie diverge} \\ \lambda < 1 & \text{la serie converge} \\ \lambda = 1 & \text{nessuna informazione} \end{cases}$$

❖ **Serie di potenze**

Valide $\forall x \in \Re$,

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \qquad \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \qquad \sinh x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$$

$$\cosh x = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$$

Valide solo per particolari intorni di x

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad \forall x \in (-1;1)$$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n \qquad \forall x \in (-1;1)$$

$$\frac{1}{1-x^2} = \sum_{n=0}^{\infty} x^{2n} \qquad \forall x \in (-1;1)$$

$$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n} \qquad \forall x \in (-1;1)$$

$$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \qquad \forall x \in (-1;1)$$

$$\ln(1-x) = -\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} = -\sum_{m=1}^{\infty} \frac{x^m}{m} \qquad \forall x \in (-1;1)$$

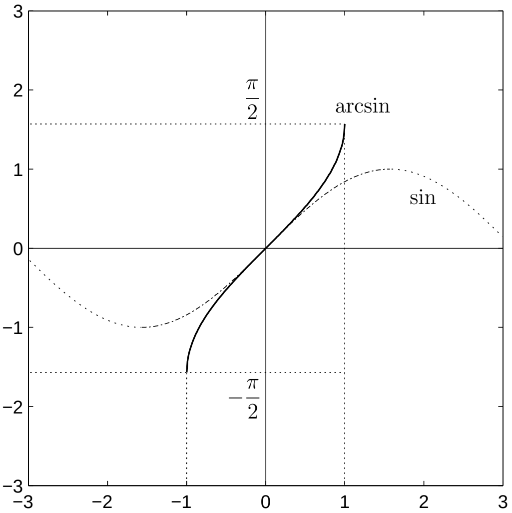
$$\ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} \qquad \forall x \in (-1;1)$$

$$(1+x)^\alpha = \sum_{n=0}^{\infty} \binom{\alpha}{n} x^n \qquad \forall x \in (-1;1)$$

$$\arcsin x = \sum_{n=0}^{\infty} \frac{(2n+1)!!}{(2n+1)(2n)!!} x^{2n+1} \qquad \forall x \in (-1;1)$$

Funzioni goniometriche inverse

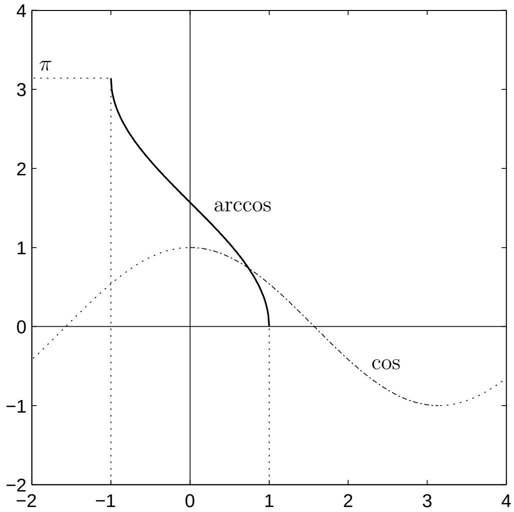
Arcoseno



$y = \arcsin x$

dom $= [-1, 1]$
cod $= [-\pi/2, \pi/2]$

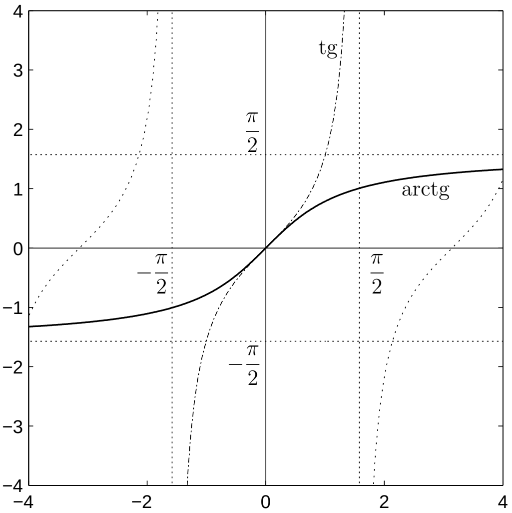
Arcocoseno



$y = \arccos x$

dom $= [-1, 1]$
cod $= [0, \pi]$

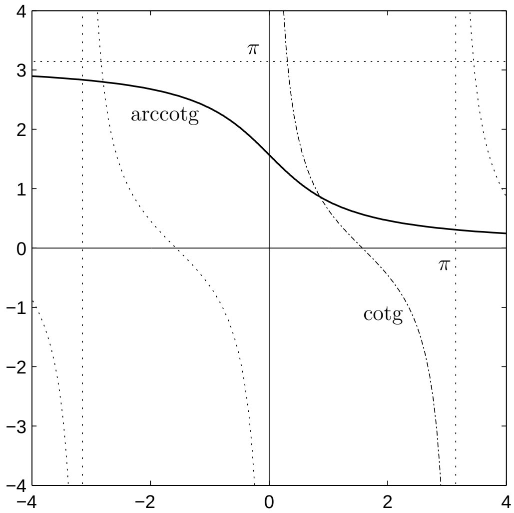
Arcotangente



$y = \operatorname{arctg} x$

dom $= \mathbb{R}$
cod $= \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$

Arcocotangente

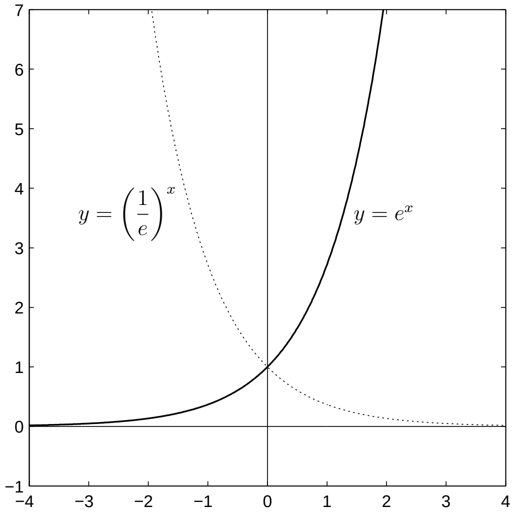


$y = \operatorname{arccotg} x$

dom $= \mathbb{R}$
cod $=]0, \pi[$

Esponenziali e logaritmi

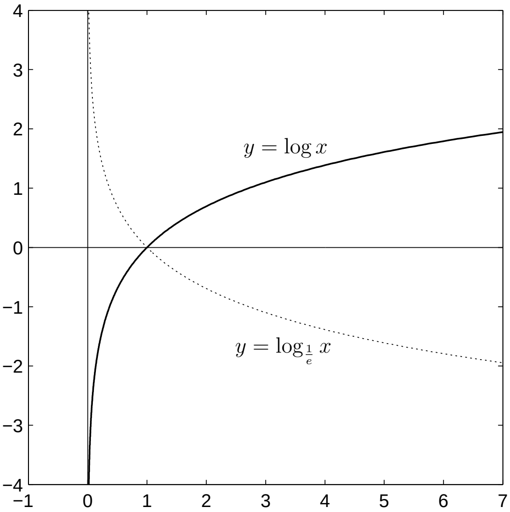
La funzione esponenziale



$y = e^x$; $y = (1/e)^x$

dom $= \mathbb{R}$
cod $= \mathbb{R}^+$

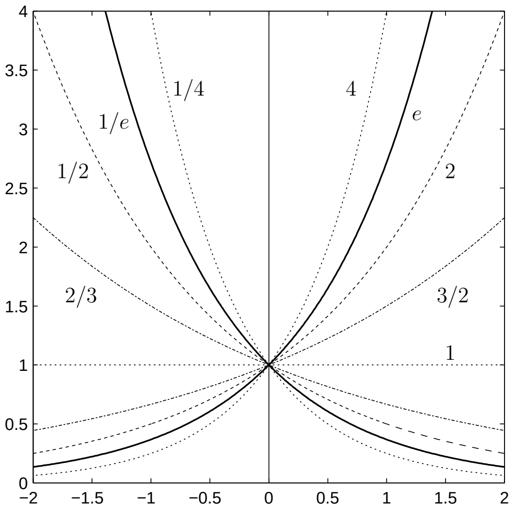
La funzione logaritmo



$y = \log x$; $y = \log_{\frac{1}{e}} x$

dom $= \mathbb{R}^+$
cod $= \mathbb{R}$

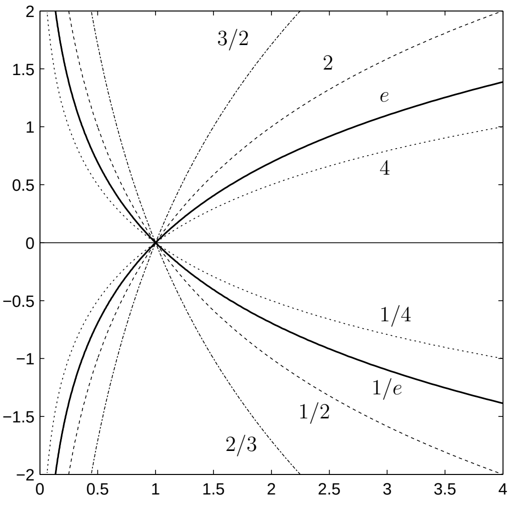
Esponenziali in varie basi



$y = a^x$

$0 < a < 1 \Rightarrow$ funz. decrescente
$a > 1 \Rightarrow$ funz. crescente

Logaritmi in varie basi

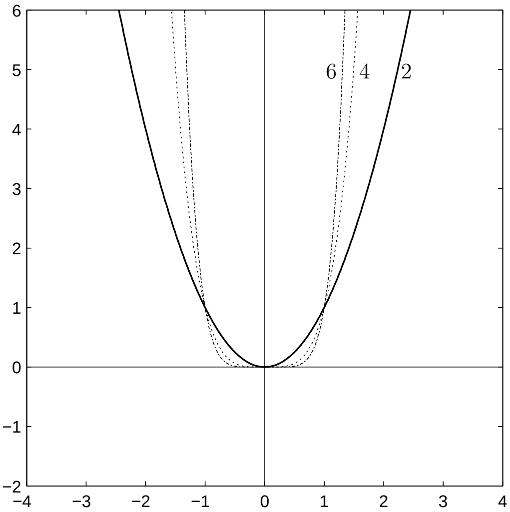


$y = \log_a x$

$0 < a < 1 \Rightarrow$ funz. decrescente
$a > 1 \Rightarrow$ funz. crescente

Potenze, reciproci e radici

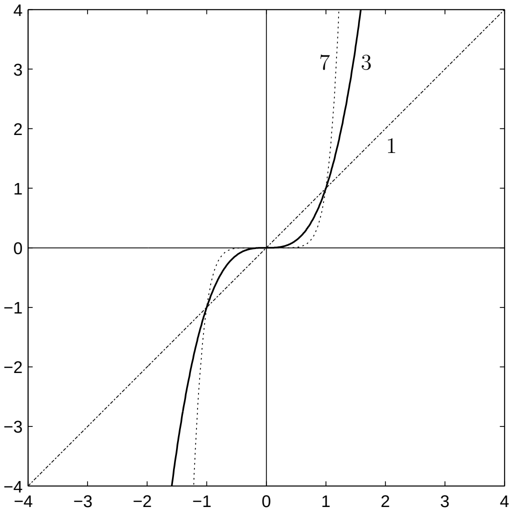
Potenze a esponente pari



$y = x^n$, $n > 0$ pari

dom $= \mathbb{R}$
cod $= \mathbb{R}_0^+$

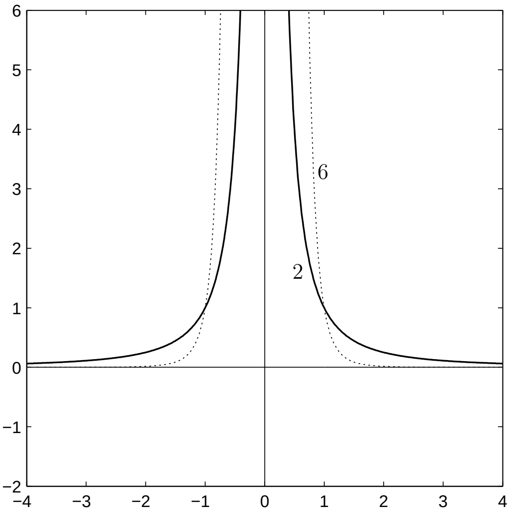
Potenze a esponente dispari



$y = x^n$, $n > 0$ dispari

dom $= \mathbb{R}$
cod $= \mathbb{R}$

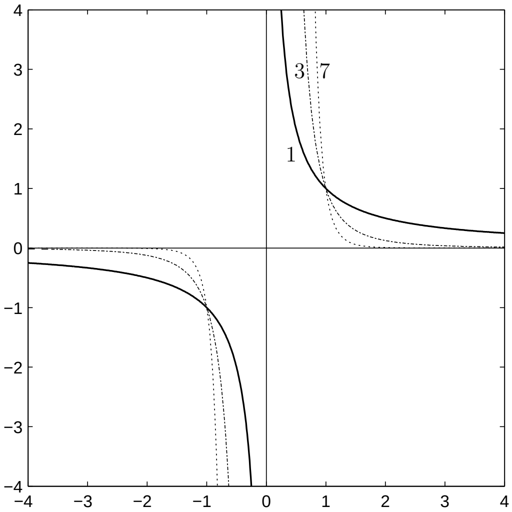
Reciproci di potenze pari



$y = \dfrac{1}{x^n}$, $n > 0$ pari

dom $= \mathbb{R} - \{0\}$
cod $= \mathbb{R}^+$

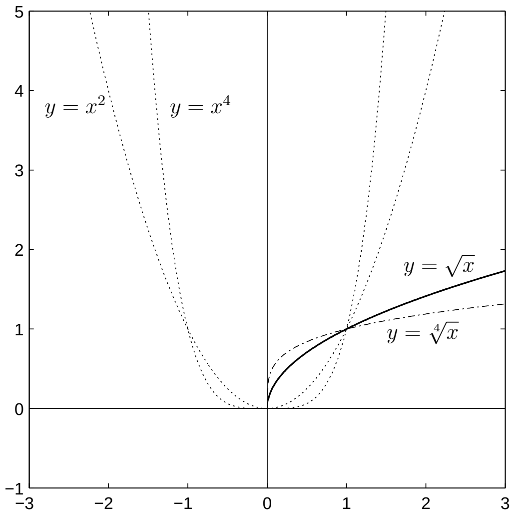
Reciproci di potenze dispari



$y = \dfrac{1}{x^n}$, $n > 0$ dispari

dom $= \mathbb{R} - \{0\}$
cod $= \mathbb{R} - \{0\}$

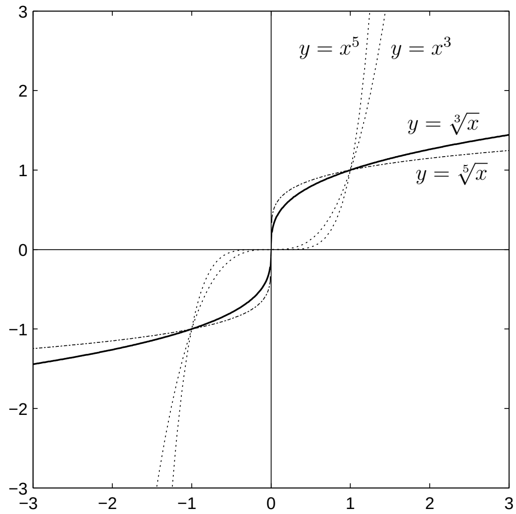
Radici di indice pari



$y = \sqrt[n]{x}$, $n > 0$ pari

dom $= \mathbb{R}_0^+$
cod $= \mathbb{R}_0^+$

Radici di indice dispari

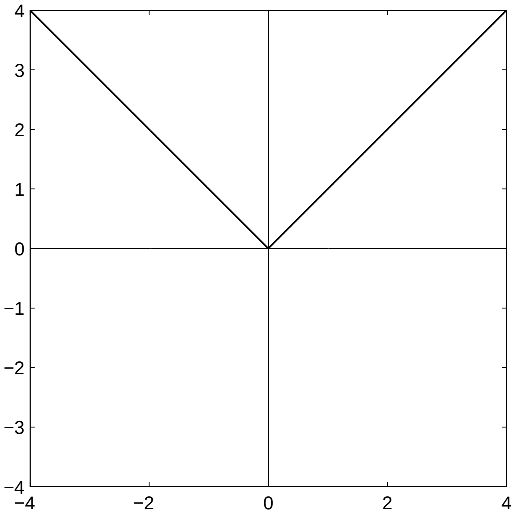


$y = \sqrt[n]{x}$, $n > 0$ dispari

dom $= \mathbb{R}$
cod $= \mathbb{R}$

Valore assoluto e segno

La funzione valore assoluto

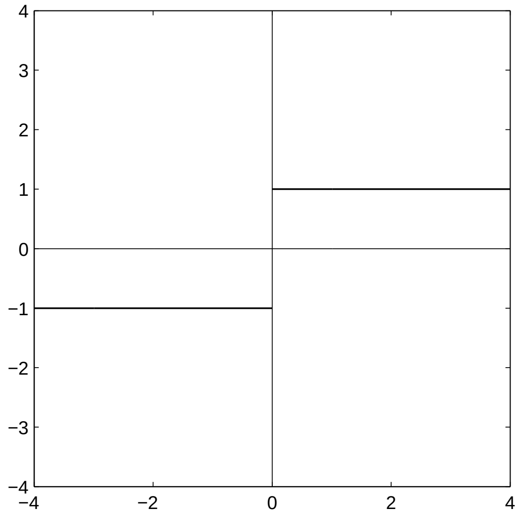


$y = |x|$

dom $= \mathbb{R}$
cod $= \mathbb{R}_0^+$

$$|x| = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$$

La funzione segno



$y = \operatorname{sign} x$

dom $= \mathbb{R} - \{0\}$
cod $= \{-1, 1\}$

$$\operatorname{sign} x = \begin{cases} 1 & \text{se } x > 0 \\ -1 & \text{se } x < 0 \end{cases}$$