AB Calculus Integrals as Net Change Review Session Problems

1. For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

(a) Show that the number of mosquitoes is increasing at time $t = 6$.

(b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

(c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

(d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.

Solutions

(a) Since $R(6) = 4.438 > 0$, the number of mosquitoes is increasing at $t = 6$.	1 : shows that $R(6) > 0$
(b) $R'(6) = -1.913$ Since $R'(6) < 0$, the number of mosquitoes is increasing at a decreasing rate at $t = 6$.	$2 : \begin{cases} 1 : \text{considers } R'(6) \\ 1 : \text{answer with reason} \end{cases}$
(c) $1000 + \int_0^{31} R(t)\,dt = 964.335$ To the nearest whole number, there are 964 mosquitoes.	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(d) $R(t) = 0$ when $t = 0$, $t = 2.5\pi$, or $t = 7.5\pi$ $R(t) > 0$ on $0 < t < 2.5\pi$ $R(t) < 0$ on $2.5\pi < t < 7.5\pi$ $R(t) > 0$ on $7.5\pi < t < 31$ The absolute maximum number of mosquitoes occurs at $t = 2.5\pi$ or at $t = 31$. $1000 + \int_0^{2.5\pi} R(t)\,dt = 1039.357$, There are 964 mosquitoes at $t = 31$, so the maximum number of mosquitoes is 1039, to the nearest whole number.	$4 : \begin{cases} 2 : \text{absolute maximum value} \\ \quad 1 : \text{integral} \\ \quad 1 : \text{answer} \\ 2 : \text{analysis} \\ \quad 1 : \text{computes interior critical points} \\ \quad 1 : \text{completes analysis} \end{cases}$

2. For $0 \le t \le 6,$ a particle is moving along the x-axis. The particle's position, $x(t),$ is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1.$ The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2.$

(a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

(b) Find the average velocity of the particle for the time period $0 \le t \le 6.$

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6.$

(d) For $0 \le t \le 6,$ the particle changes direction exactly once. Find the position of the particle at that time.

Solutions

(a) $v(5.5) = -0.45337,\ a(5.5) = -1.35851$ The speed is increasing at time $t = 5.5,$ because velocity and acceleration have the same sign.	$2:$ conclusion with reason
(b) Average velocity $= \frac{1}{6}\int_0^6 v(t)\,dt = 1.949$	$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$
(c) Distance $= \int_0^6 \lvert v(t)\rvert\,dt = 12.573$	$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$
(d) $v(t) = 0$ when $t = 5.19552.$ Let $b = 5.19552.$ $v(t)$ changes sign from positive to negative at time $t = b.$ $x(b) = 2 + \int_0^b v(t)\,dt = 14.134$ or 14.135	$3: \begin{cases} 1: \text{considers } v(t) = 0 \\ 1: \text{integral} \\ 1: \text{answer} \end{cases}$

3. Let R be the region enclosed by the graph of $y = \sqrt{x-1}$, the vertical line $x = 10$, and the x-axis.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 3$.

(c) Find the volume of the solid generated when R is revolved about the vertical line $x = 10$.

Solutions

(a) Area $= \int_1^{10} \sqrt{x-1}\, dx = 18$

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi \int_1^{10} \left(9 - \left(3 - \sqrt{x-1}\right)^2\right) dx$

$= 212.057$ or 212.058

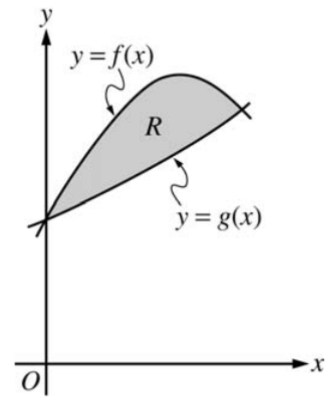
$3 : \begin{cases} 1 : \text{limits and constant} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) Volume $= \pi \int_0^3 \left(10 - \left(y^2 + 1\right)\right)^2 dy$

$= 407.150$

$3 : \begin{cases} 1 : \text{limits and constant} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

4. Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.



(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.

Solutions

The graphs of f and g intersect in the first quadrant at $(S, T) = (1.13569, 1.76446)$.

1 : correct limits in an integral in (a), (b), or (c)

(a) $$\text{Area} = \int_0^S (f(x) - g(x))\,dx = \int_0^S \left(1 + \sin(2x) - e^{x/2}\right) dx = 0.429$$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $$\text{Volume} = \pi \int_0^S \left((f(x))^2 - (g(x))^2\right) dx = \pi \int_0^S \left((1 + \sin(2x))^2 - \left(e^{x/2}\right)^2\right) dx = 4.266 \text{ or } 4.267$$

$3 : \begin{cases} 2 : \text{integrand} \\ \quad \langle -1 \rangle \text{ each error} \\ \quad \text{Note: } 0/2 \text{ if integral not of form} \\ \qquad c\int_a^b \left(R^2(x) - r^2(x)\right) dx \\ 1 : \text{answer} \end{cases}$

(c) $$\text{Volume} = \int_0^S \frac{\pi}{2}\left(\frac{f(x) - g(x)}{2}\right)^2 dx = \int_0^S \frac{\pi}{2}\left(\frac{1 + \sin(2x) - e^{x/2}}{2}\right)^2 dx = 0.077 \text{ or } 0.078$$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$