

• Class

$$\begin{aligned} \text{Info}(D) &= - \sum_{i=1}^n p_i \log_2(p_i) \\ &= 1(0,5) \\ &= - \left(\frac{1}{14} \log_2 \frac{9}{14} \right) + \left(-\frac{5}{14} \log_2 \frac{5}{14} \right) \\ &= -\frac{9}{14} \log_2 \frac{9}{14} - \frac{5}{14} \log_2 \frac{5}{14} \\ &= -\frac{9}{14} (-0.637) - \frac{5}{14} (-1.485) \\ &= 0.940 \end{aligned}$$

• Feature

$$\begin{aligned} \text{Info}_{\text{age}}(D) &= \sum_{j=1}^V \left| \frac{D_j}{D} \right| \times \text{Info}(D_j) \\ &= \frac{5}{14} I(2,3) + \frac{4}{14} I(4,0) + \frac{5}{14} I(3,2) \\ &= \frac{5}{14} \left[-\frac{2}{5} \log_2 \left(\frac{2}{5} \right) - \frac{3}{5} \log_2 \left(\frac{3}{5} \right) \right] + \frac{4}{14} \left[-\frac{4}{4} \log_2 \left(\frac{4}{4} \right) - \frac{0}{4} \log_2 \left(\frac{0}{4} \right) \right] \\ &= \frac{5}{14} (0.521 + 0.442) + \frac{4}{14} (0) + \frac{5}{14} (0.442 + 0.322) \\ &= \frac{5}{14} (0.963) + \frac{5}{14} (0.764) \\ &= 0.747 + 0.267 = 0.694 \end{aligned}$$

$$\text{Gain}(\text{age}) = \text{Info}(D) - \text{Info}_{\text{age}}(D)$$

$$= 0.940 - 0.694 = 0.246$$

$$\begin{aligned} \text{Info}_{\text{income}}(D) &= \sum_{j=1}^V \left| \frac{D_j}{D} \right| \times \text{Info}(D_j) \\ &= \frac{4}{14} I(2,2) + \frac{6}{14} I(3,1) + \frac{4}{14} I(3,1) \\ &= \frac{4}{14} \left[-\frac{2}{4} \log_2 \left(\frac{2}{4} \right) - \frac{2}{4} \log_2 \left(\frac{2}{4} \right) \right] + \frac{6}{14} \left[-\frac{3}{6} \log_2 \left(\frac{3}{6} \right) - \frac{1}{6} \log_2 \left(\frac{1}{6} \right) \right] + \frac{4}{14} \left[-\frac{3}{4} \log_2 \left(\frac{3}{4} \right) - \frac{1}{4} \log_2 \left(\frac{1}{4} \right) \right] \\ &= \frac{4}{14} + \frac{6}{14} (0.918) + \frac{4}{14} (0.917) \\ &= 0.936 + 0.394 + 0.262 = 0.912 \end{aligned}$$

$$\text{Gain}(\text{income}) = \text{Info}(D) - \text{Info}_{\text{income}}(D)$$

$$= 0.940 - 0.912 = 0.028$$

$$\begin{aligned} \text{Info}_{\text{student}}(D) &= \sum_{j=1}^V \left| \frac{D_j}{D} \right| \times \text{Info}(D_j) \\ &= \frac{7}{14} I(3,4) + \frac{7}{14} I(1,1) \\ &= \frac{7}{14} \left[-\frac{3}{7} \log_2 \left(\frac{3}{7} \right) - \frac{4}{7} \log_2 \left(\frac{4}{7} \right) \right] + \frac{7}{14} \left[-\frac{1}{1} \log_2 \left(\frac{1}{1} \right) - \frac{1}{1} \log_2 \left(\frac{1}{1} \right) \right] \\ &= \frac{7}{14} (0.524 + 0.461) + \frac{7}{14} (0.191 + 0.401) \\ &= \frac{7}{14} (0.985) + \frac{7}{14} (0.592) \\ &= 0.493 + 0.296 = 0.789 \end{aligned}$$

$$\text{Gain}(\text{student}) = \text{Info}(D) - \text{Info}_{\text{student}}(D)$$

$$= 0.940 - 0.789 = 0.151$$

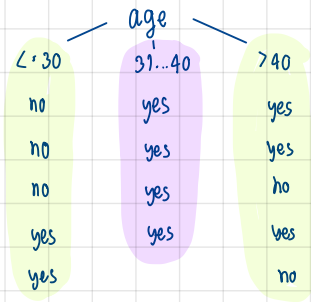
$$\begin{aligned} \text{Info}_{\text{credit}}(D) &= \sum_{j=1}^V \left| \frac{D_j}{D} \right| \times \text{Info}(D_j) \\ &= \frac{8}{14} I(6,2) + \frac{6}{14} I(3,2) \\ &= \frac{8}{14} \left[-\frac{6}{8} \log_2 \left(\frac{6}{8} \right) - \frac{2}{8} \log_2 \left(\frac{2}{8} \right) \right] + \frac{6}{14} \left[-\frac{3}{6} \log_2 \left(\frac{3}{6} \right) - \frac{2}{6} \log_2 \left(\frac{2}{6} \right) \right] \\ &= \frac{8}{14} (0.311 + 0.3) + \frac{6}{14} (0.5 + 0.5) \\ &= \frac{8}{14} (0.611) + \frac{6}{14} (1) \\ &= 0.344 + 0.429 = 0.773 \end{aligned}$$

$$\text{Gain}(\text{credit}) = \text{Info}(D) - \text{Info}_{\text{credit}}(D)$$

$$= 0.940 - 0.773 = 0.167$$

∴ เลือก Gain(age) เป็นตัวเลือก

age	income	student	credit	rating	buys	computer
<=30	high	no	fair		no	
<=30	high	no	excellent		no	
31...40	high	no	fair		yes	
>40	medium	no	fair		yes	
>40	low	yes	fair		yes	
>40	low	yes	excellent		no	
31...40	low	yes	excellent		yes	
<=30	medium	no	fair		no	
<=30	low	yes	fair		yes	
>40	medium	yes	fair		yes	
<=30	medium	yes	excellent		yes	
31...40	medium	no	excellent		yes	
31...40	high	yes	fair		yes	
>40	medium	no	excellent		no	



Recursive Greedy Recursive

F₁ age <= 30

age	income	student	credit	buys
<=30	high	no	fair	no
<=30	high	no	excellent	no
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
<=30	medium	yes	excellent	yes

$$\begin{aligned}
 \text{Info}(D) &= \sum_{i=1}^n p_i \log_2(p_i) \\
 &= I(2,3) \\
 &= -\frac{2}{5} \log_2\left(\frac{2}{5}\right) - \frac{3}{5} \log_2\left(\frac{3}{5}\right) = 0.971
 \end{aligned}$$

$$\begin{aligned}
 \text{Info}_{\text{income}}(D) &= \frac{2}{5} I(0,2) + \frac{1}{5} I(1,1) + \frac{1}{5} I(1,0) \\
 &= \frac{2}{5} \left[-\frac{2}{2} \log_2\left(\frac{2}{2}\right) - \frac{1}{2} \log_2\left(\frac{1}{2}\right) \right] + \frac{2}{5} \left[-\frac{1}{2} \log_2\left(\frac{1}{2}\right) - \frac{1}{2} \log_2\left(\frac{1}{2}\right) \right] + \frac{1}{5} \left[-1 \log_2\left(\frac{1}{5}\right) - 2 \log_2\left(\frac{0}{5}\right) \right] \\
 &= 0.4
 \end{aligned}$$

$$\begin{aligned}
 \text{Gain}_{\text{income}} &= \text{Info}(D) - \text{Info}_{\text{income}}(D) \\
 &= 0.971 - 0.4 = 0.571
 \end{aligned}$$

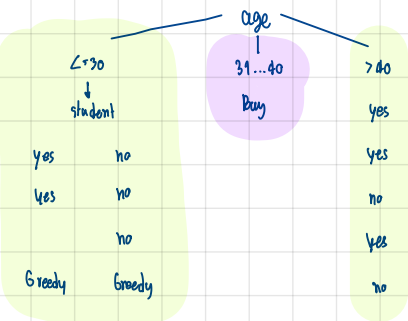
$$\begin{aligned}
 \text{Info}_{\text{student}}(D) &= \frac{3}{5} I(0,2) + \frac{2}{5} I(1,1) \\
 &= \frac{3}{5} \left[-\frac{2}{3} \log_2\left(\frac{2}{3}\right) - \frac{1}{3} \log_2\left(\frac{1}{3}\right) \right] + \frac{2}{5} \left[-\frac{1}{2} \log_2\left(\frac{1}{2}\right) - \frac{1}{2} \log_2\left(\frac{1}{2}\right) \right] \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \text{Gain}_{\text{student}} &= \text{Info}(D) - \text{Info}_{\text{student}}(D) \\
 &= 0.971 - 0 = 0.971
 \end{aligned}$$

$$\begin{aligned}
 \text{Info}_{\text{credit}}(D) &= \frac{3}{5} I(1,2) + \frac{2}{5} I(1,1) \\
 &= \frac{3}{5} \left[-\frac{1}{3} \log_2\left(\frac{1}{3}\right) - \frac{2}{3} \log_2\left(\frac{2}{3}\right) \right] + \frac{2}{5} \left[-\frac{1}{2} \log_2\left(\frac{1}{2}\right) - \frac{1}{2} \log_2\left(\frac{1}{2}\right) \right] \\
 &= 0.551 + 0.4 = 0.951
 \end{aligned}$$

$$\begin{aligned}
 \text{Gain}_{\text{credit}} &= \text{Info}(D) - \text{Info}_{\text{credit}}(D) \\
 &= 0.971 - 0.951 = 0.020
 \end{aligned}$$

∴ Gain _{income} > Gain _{student}



Recursive

F₂ age >40

age	income	student	credit	buys
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
>40	medium	yes	fair	yes
>40	medium	no	excellent	no

$$\begin{aligned}
 \text{Info}(D) &= I(3,2) \\
 &= -\frac{3}{5} \log_2 \left(\frac{3}{5} \right) - \frac{2}{5} \log_2 \left(\frac{2}{5} \right) \\
 &= 0.971
 \end{aligned}$$

$$\begin{aligned}
 \text{Info}_{\text{income}}(D) &= \frac{3}{5} I(3,1) + \frac{2}{5} I(1,1) \\
 &= \frac{3}{5} \left[-\frac{3}{5} \log_2 \left(\frac{3}{5} \right) - \frac{1}{5} \log_2 \left(\frac{1}{5} \right) \right] + \frac{2}{5} \left[-\frac{1}{2} \log_2 \left(\frac{1}{2} \right) - \frac{1}{2} \log_2 \left(\frac{1}{2} \right) \right] \\
 &= 0.551 + 0.4 = 0.951
 \end{aligned}$$

$$\begin{aligned}
 \text{Gain}_{\text{income}} &= \text{Info}(D) - \text{Info}_{\text{income}}(D) \\
 &= 0.971 - 0.951 = 0.02
 \end{aligned}$$

$$\begin{aligned}
 \text{Info}_{\text{student}}(D) &= \frac{2}{5} I(1,1) + \frac{3}{5} I(2,1) \\
 &= \frac{2}{5} \left[-\frac{1}{2} \log_2 \left(\frac{1}{2} \right) - \frac{1}{2} \log_2 \left(\frac{1}{2} \right) \right] + \frac{3}{5} \left[-\frac{2}{3} \log_2 \left(\frac{2}{3} \right) - \frac{1}{3} \log_2 \left(\frac{1}{3} \right) \right] \\
 &= 0.4 + 0.551 = 0.951
 \end{aligned}$$

$$\begin{aligned}
 \text{Gain}_{\text{student}} &= \text{Info}(D) - \text{Info}_{\text{student}}(D) \\
 &= 0.971 - 0.951 = 0.02
 \end{aligned}$$

$$\begin{aligned}
 \text{Info}_{\text{credit}}(D) &= \frac{3}{5} I(3,0) + \frac{2}{5} I(1,1) \\
 &= \frac{3}{5} \left[-\frac{3}{5} \log_2 \left(\frac{3}{5} \right) - \frac{0}{5} \log_2 \left(\frac{0}{5} \right) \right] + \frac{2}{5} \left[-\frac{1}{2} \log_2 \left(\frac{1}{2} \right) - \frac{1}{2} \log_2 \left(\frac{1}{2} \right) \right] \\
 &= 0.4
 \end{aligned}$$

$$\begin{aligned}
 \text{Gain}_{\text{credit}} &= \text{Info}(D) - \text{Info}_{\text{credit}}(D) \\
 &= 0.971 - 0.4 = 0.571
 \end{aligned}$$

