TTK4130 Modeling and Simulation

Assignement 1

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1a) 1. and 3. are already State space.

2. $\ddot{x} + c\dot{x} + g\left(1 - \left(\frac{x_d}{x}\right)^K\right) = 0$

$x_1 = x$

$x_2 = \dot{x} = \dot{x}_1$

$\ddot{x} = \dot{x}_2$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \left(-\dot{x}_2 - g\left(1 - \left(\frac{x_d}{x_1}\right)^K\right)\right)\frac{1}{c} \\ -cx_2 - g\left(1 - \left(\frac{x_d}{x_1}\right)^K\right) \end{bmatrix}$$

$$= \begin{bmatrix} x_2 \\ -cx_2 - g\left(1 - \left(\frac{x_d}{x_1}\right)^K\right) \end{bmatrix}$$

b) 1. $\dot{x}_1 = 0$ & $\dot{x}_2 = 0$:

$a u_1 = b\sqrt{x_1} \Rightarrow \underline{\underline{x_{10} = \frac{a^2 u_1^2}{b^2}}} > 0$

$u_1(u_2 - x_2) = -c(u_3 - x_2)$

$-x_2 u_1 + u_1 u_2 = -c u_3 + c x_2$

$c u_3 + u_1 u_2 = (c + u_1) x_2 \Rightarrow \underline{\underline{x_{20} = \frac{c u_3 + u_1 u_2}{c + u_1}}}$

2. $\ddot{x} = \dot{x} = 0$

$$\Rightarrow \quad g = g\left(\frac{x_d}{x}\right)^K$$

$$\left(\frac{x_d}{x}\right)^K = 1 \qquad \underline{\underline{x_0 = x_d}} \quad \begin{pmatrix} x_{10} = x_d \\ x_{20} = 0 \end{pmatrix}$$

3. $\dot{x} = 0$ & $\dot{y} = 0$

$$\Rightarrow \quad y = \frac{x}{\ln\sqrt{x^2+y^2}}, \quad x = -\frac{y}{\ln\sqrt{x^2+y^2}}$$

$$y^2 - \frac{yx}{\ln\sqrt{x^2+y^2}} + x^2 + \frac{yx}{\ln\sqrt{x^2+y^2}} = 0$$

$$x^2 + y^2 = 0 \quad \Rightarrow \quad \begin{matrix} x_0 = 0 \\ \underline{\underline{y_0 = 0}} \end{matrix}$$

c) 1. $$\dot{x}_1 \approx \left.\frac{\partial f}{\partial x_1}\right|_{x_{10}, x_{20}} (x_1 - x_{10})$$

$$= -\frac{b}{2}\frac{1}{\sqrt{x_{10}}}(x_1 - x_{10})$$

$$= -\frac{b}{2}\frac{b}{a u_1}(x_1 - x_{10}) = -\frac{b^2}{2 a u_1}\left(x_1 - \frac{a^2 u_1^2}{b^2}\right)$$

$$\dot{x}_2 \approx \frac{\partial f}{\partial x_1}\bigg|_{x_{10},x_{20}} (x_1 - x_{10})$$

$$+ \frac{\partial f}{\partial x_2}\bigg|_{x_{10},x_{20}} (x_2 - x_{20})$$

$$= -\frac{a}{x_{10}^2}\Big(u_1(u_2 - x_{20}) + c(u_3 - x_{20})\Big)(x_1 - x_{10})$$

$$+ \frac{-au_1}{x_{10}} - \frac{ac}{x_{10}}$$

2. $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -cx_2 - g\left(1 - \left(\frac{x_d}{x_1}\right)^k\right) \end{bmatrix}$$

$$\dot{x}_1 \approx \frac{\partial f}{\partial x_2}\bigg|_{x_{10},x_{20}} (x_2 - x_{20})$$

$$= x_2$$

$$\cancel{c}\left(= \frac{-k\left(\frac{x_d}{x_1}\right)^{k-1}}{x_d}\right)$$

$$\dot{x}_2 \approx \frac{\partial f}{\partial x_1}\bigg|_{x_{10},x_{20}} (x_1 - x_{10})$$

$$+ \frac{\partial f}{\partial x_2}\bigg|_{x_{10},x_{20}} (x_2 - x_{20})$$

$$= \cancel{gkx_1 - gkx_d - cx_2}$$

$$\cancel{= gkx_d - gkx_d - cx_2}$$

$$= -\frac{gK}{X_d} X_1 - gK - cX_2$$

3. $\cancel{X_{10} = X_{20}} = 0 = \dot{x} = \dot{y}$

$$\frac{d\dot{x}}{dy} = 1 + \frac{4xy}{(x^2+y^2)\log^2(x^2+y^2)}$$

$$\frac{d\dot{x}}{dx} = \frac{1}{\log(\sqrt{x^2+y^2})} - \frac{x^2}{(x^2+y^2)\log^2(\sqrt{x^2+y^2})}$$

$$\dot{x} \approx y$$

$$\frac{d\dot{y}}{dx} = -1$$

$$\dot{y} \approx -x$$

$$\frac{d\dot{y}}{dy} = 0$$

d) 3.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\det(A) = 1$$

$$\det(A - \lambda I) = \begin{vmatrix} -\lambda & 1 \\ -1 & -\lambda \end{vmatrix}$$

$$= \lambda^2 + 1 = 0$$

$$\Rightarrow \lambda = \pm i$$

<u>Marginally stable</u>

1. $\dot{x}_2 = -\varphi x_1 + \theta$

$\dot{x}_1 = -\psi x_2 + Z$

φ og ψ positiv

$\Rightarrow$ <u>Stable</u>

2.

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 \\ \frac{gK}{x_d} & -C \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -gK \end{bmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} -\lambda & 1 \\ \frac{gK}{x_d} & -\lambda - C \end{vmatrix}$$

$$= \lambda^2 + \lambda C - \frac{gK}{x_d}$$

$$\lambda = \frac{-C \pm \sqrt{C^2 - 4 \cdot 1 \cdot \left(-\frac{gK}{x_d}\right)}}{2}$$

Always a root in r.h.p

$\Rightarrow$ Unstable

(ModSim A1 - forts.)

2a) I observe that $x' = x^2$ curves further and further upwards as time goes and that $x' = \sqrt{|x|}$ flattens out.

b) Mathematically, of course, this doesn't make much sense, so there's probably something wrong with how I use ode45.

$x' = x^2$ has the general solution

$x = \frac{-1}{t + C}$ and the special sol.

$x = -\frac{1}{t-1}$ given $x(0) = 1$

which should just shoot up to infinity and beyond as $t \to 1$, making a really ugly graph.

I got a flat graph after some fiddling with $x' = \sqrt{|x|}$ using ODE45. Not sure why it looks like that, but I'm assuming it's to teach us that "computers have their limits"...

3a) We get something like this:

$$\dot{H} = (b-d)H - bdH^2 - iHZ$$
$$\dot{I} = -(a+d)I + iHZ$$
$$\dot{Z} = aI + rD - nHZ$$
$$\dot{D} = d(H+I) - rD + nHZ$$

and given:

$$\dot{H} + \dot{I} + \dot{Z} + \dot{D} = bH - bdH^2$$

b) See graphs

c) I think in the model with the quarantine, those that die within, respawns as Zombies outside the quarantine. This could probably be fixed adding another parameter:

D_Q = dead in quarantine