TTK4130 Modeling and Simulation

Assignement 2

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1a) The definitions of SO(3):

$R^T R = R R^T = I$

$R^T = R^{-1}$

$\mathrm{Det}(R) = \pm 1$

SO(3) is Non-Abelian

Seeing as R_1 must span $\mathbb{R}^3$ using three unit vectors of length 1, then the residual unknown parameters must be 0, except for a_{33}, which must be 1. This gives us that R_1 has three unit vectors of exact length 1.

$$\underline{\underline{R_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}}$$

R_2 is a bit trickier, but applying the same principles, we can calculate the first vector $\vec{b_1}$.

$$|\vec{b_1}| = \sqrt{\left(\frac{5}{13}\right)^2 + x^2 + \left(\frac{12}{13}\right)^2} = 1$$

$$1 - \left(\frac{5}{13}\right)^2 - \left(\frac{12}{13}\right)^2 = x^2 = 0$$

$$\vec{b_1} = \begin{bmatrix} \frac{5}{13} & 0 & \frac{12}{13} \end{bmatrix}^T$$

and $\vec{b_2}$ must be

$$\vec{b_2} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$$

$\vec{b_3}$ must be the cross-product of $\vec{b_1}$ and $\vec{b_2}$, or orthogonal to both.

$$\begin{bmatrix} \frac{5}{13} \\ 0 \\ \frac{12}{13} \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$$

$$= \begin{bmatrix} b_{12}b_{23} - b_{13}b_{22} \\ b_{13}b_{21} - b_{11}b_{23} \\ b_{11}b_{22} - b_{12}b_{21} \end{bmatrix} = \begin{bmatrix} 0 - \frac{12}{13} \\ 0 - 0 \\ \frac{5}{13} - 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{12}{13} \\ 0 \\ \frac{5}{13} \end{bmatrix}$$

$$\vec{b_3} = \begin{bmatrix} -\frac{12}{13} \\ 0 \\ \frac{5}{13} \end{bmatrix}$$

$$R_2 = \begin{bmatrix} \frac{5}{13} & 0 & -\frac{12}{13} \\ 0 & 1 & 0 \\ \frac{12}{13} & 0 & \frac{5}{13} \end{bmatrix}$$

b)

$$R_1 R^a_b = R_2$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} = \begin{bmatrix} \frac{5}{13} & 0 & -\frac{12}{13} \\ 0 & 1 & 0 \\ \frac{12}{13} & 0 & \frac{5}{13} \end{bmatrix}$$

$m_{21} = \frac{5}{13}$ $\quad m_{11} = 0$ $\quad m_{31} = \frac{12}{13}$

$m_{22} = 0$ $\quad m_{12} = 1$ $\quad m_{32} = 0$

$m_{23} = -\frac{12}{13}$ $\quad m_{13} = 0$ $\quad m_{33} = \frac{5}{13}$

$$R^a_b = \begin{bmatrix} 0 & 1 & 0 \\ \frac{5}{13} & 0 & -\frac{12}{13} \\ \frac{12}{13} & 0 & \frac{5}{13} \end{bmatrix} \begin{pmatrix} \approx \vec{b}_2^{\,T} \\ \approx \vec{b}_3^{\,T} \\ \approx \vec{b}_1^{\,T} \end{pmatrix}$$

The columns correspond to the vectors $\vec{b_1}, \vec{b_2}$ and $\vec{b_3}$ in R_2

c) $(u^a)^T v^a = (u^b)^T v^b \quad \forall\ u, v \in \mathbb{R}^3$

The resulting ~~values~~ scalars w^a and w^b must be the same number and it relates to the relation between the two vectors regardless of what basis they are in, or the length of the vectors multiplied by the cosine of the angle between them. This does of course not change if one multiplies with a rotational matrix which has been done here.

modsim Ass2 (forts.)

1d)

$$(u^a)^\times \cdot v^a = (R^a_b u^b)^\times \cdot R^a_b v^b$$

$$= R^a_b u^{b\times} \underbrace{R^{aT}_b \cdot R^a_b}_{I} v^b$$

$$= R^a_b u^{b\times} R^a_b v^b$$

$$= R^a_b (u^b \times v^b)$$

$$\square$$

$$= u^a \times v^a$$

2a) See picture of script.
The 3D-simulation is fairly reasonable, it spins in exactly the way one might expect

b) DCM and Euler Angles seem to do the same. Euler angles are a bit easier to work with, and DCM was a bit more "complex" I guess.
otherwise, see pics of script.

3a) $R = R_{K,\theta} = \cos\theta\, I + \sin\theta\, K^{\times} + (1-\cos\theta) K K^{T}$

Show that $K = RK$

We are rotating something around K, which means that the rotation won't change anything in terms of the objects position along the K-axis

If $K = RK$, then:

$i \neq Ri$

$j \neq Rj$

(for non-zero rotational matrix R)

notat:

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \vec{V}^{?}_{\vec{\omega}K} = K \Rightarrow K^T = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$KK^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{K,\theta} = \begin{bmatrix} \cos\theta & 0 & 0 \\ 0 & \cos\theta & 0 \\ 0 & 0 & \cos\theta \end{bmatrix} + \begin{bmatrix} 0 & -\sin\theta & 0 \\ \sin\theta & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1-\cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} (= R_3)(= R_{K,\theta})$$

$$R_{K,\theta} K = K = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \square$$

3b) Made a script, pls have a look.