TTK4130 18.05.2020 10158

1a)

1a) The kinetic energy:

$$T = \frac{1}{2} m \dot{q}^T \dot{q} \qquad q = \begin{bmatrix} q_1 \\ \vdots \\ q_N \end{bmatrix}, \quad q_n = \begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix}$$

Potential energy:

$$V = mg \sum_{k=1}^{N} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} q_k$$

Lagrangian:

$$L = T - V$$

$$L = \frac{1}{2} m \dot{q}^T \dot{q} - mg \sum_{k=1}^{N} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} q_k$$

$$\frac{\partial \mathcal{L}}{\partial \dot{q}} = m\dot{q} \qquad \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial \dot{q}} = m\ddot{q}$$

$$\frac{\partial \mathcal{L}}{\partial q} = -m \begin{bmatrix} g \\ g \\ \vdots \\ g \end{bmatrix} \qquad g = \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} = 0$$

$$\Downarrow$$

$$m\ddot{q} + m\begin{bmatrix} g \\ g \\ \vdots \\ g \end{bmatrix} = 0$$

Constraint: $C(q) = \sum_{k=0}^{N+1} \| q_k - q_{k+1} \|^2 - L^2$

$$= 0$$

We adjust for this:

$$\frac{\partial \mathcal{L}}{\partial q} = -m\begin{bmatrix} g \\ g \\ \vdots \\ g \end{bmatrix} + \lambda \begin{bmatrix} q_0 - q_1 \\ q_1 - q_0 \\ \vdots \\ q_N - q_{N+1} \\ q_{N+1} - q_N \end{bmatrix}$$

$$\Downarrow$$

$$m\ddot{q} + m\begin{bmatrix} g \\ g \\ \vdots \\ g \end{bmatrix} - \lambda \underbrace{\begin{bmatrix} q_0 - q_1 \\ q_1 - q_0 \\ \vdots \\ q_N - q_{N+1} \\ q_{N+1} - q_N \end{bmatrix}}_{q_{diff}} = 0$$

$$m\ddot{q} = -m \begin{bmatrix} g \\ g \\ \vdots \\ g \end{bmatrix} + \lambda \begin{bmatrix} \\ q_{diff} \\ \\ \end{bmatrix}$$

$$C(q) = 0$$

which is an index=3 DAE since $C(q)$ must be differentiated 3 times for it to deliver $\ddot{q}$.

1b) We'll have to perform an index reduction on $C(q)$:

$$\frac{d}{dt}C(q) = \begin{bmatrix} (q_1 - q_2)^T(\dot{q}_1 - \dot{q}_2) \\ \vdots \\ (q_N - q_{N+1})^T(\dot{q}_N - \dot{q}_{N+1}) \end{bmatrix}$$

$$= C(q, \dot{q})$$

Consistency conditions then follow as:

$$C(q) = 0$$

$$C(q, \dot{q}) = 0$$

1c) The force is proportional to λ between the balls, so a ~~[illegible]~~ non-zero λ will correspond to some tension/force between the balls.