

TTK4130 - Modeling and Simulation
Assignment 6
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1a) $\dot{X}_1 = X_1 + X_2 + Z$

$$\dot{X}_2 = Z + u$$

$$0 = \frac{1}{2}(X_1^2 + X_2^2 - 1)$$

We can tell that this isn't an ODE by observing that ~~this is~~ \dot{Z} (time-diff form of Z) isn't given. This is therefore a DAE.

b) We set:

c) $g(X, Z, u) = \frac{1}{2}(X_1^2 + X_2^2 - 1) = 0$

$$\dot{g}(X, Z, u) = \dot{X}_1 X_1 + \dot{X}_2 X_2 = 0$$

$$= X_1(X_1 + X_2 + Z) + X_2(Z + u) = 0$$

This delivers \dot{X} and \dot{Z} , so this ~~is~~ means that the differential index of (1) was 2.

2a)

$$\dot{X} = - \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} X - Z$$

$$\varepsilon \dot{Z} = \frac{1}{10} X - A Z$$

$$A = \begin{bmatrix} x_1^2 & x_2 \\ 0 & x_2^2 \end{bmatrix} + \alpha I$$

We start by rewriting \dot{Z} as

$$\dot{Z} = \frac{1}{\varepsilon} \left(\frac{1}{10} X - \begin{bmatrix} x_1^2 + \alpha & x_2 \\ 0 & x_2^2 + \alpha \end{bmatrix} Z \right)$$

$$\dot{Z} = A^* Z + Bu$$

||

$$\frac{1}{\varepsilon} \begin{bmatrix} -(x_1^2 + \alpha) & -x_2 \\ 0 & -(x_2^2 + \alpha) \end{bmatrix}$$

This have only negative eig-values, so

\dot{Z} is stable $\forall X$

$$\dot{z} = g(x, z)$$

$\frac{\partial g}{\partial z}$ has full rank everywhere

$$F(\dot{x}, x, z, u, t) = \begin{bmatrix} \dot{x} + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x + z \\ \varepsilon \dot{z} - \frac{1}{10} x + A z \end{bmatrix} = 0$$

$$\det\left(\frac{\partial F}{\partial \dot{x} \partial \dot{z}}\right) \neq 0$$

\Rightarrow ODE

b) See the added plots.

We see that decreasing $\varepsilon \rightarrow 0$ makes them more similar.

c) Done

d) This makes sense, since Tikhonov's theorem states that the ODE sol. will be equal to the DAE as $\varepsilon \rightarrow 0$

In c, A becomes singular when $\alpha = 0$, and then the DAE crash. (Around the 1.2-mark)

3a)

$$\dot{X}_1 + u + X_1 + X_2 = 0$$

$$u + X_2 + \dot{X}_2 \dot{X}_1 + \dot{X}_2 u + \dot{X}_2 X_1 + \dot{X}_2 X_2 + u^2 = 0$$

F

$$\frac{\partial F}{\partial \dot{X}} = \begin{bmatrix} 1 & 0 \\ \dot{X}_2 & \dot{X}_1 + X_1 + X_2 + u \end{bmatrix}$$

$$\det\left(\frac{\partial F}{\partial \dot{X}}\right) = \dot{X}_1 + X_1 + X_2 + u = 0 \quad (6a)$$

Jacobian rank-deficient.

\Rightarrow DAE

$$6a) \quad \dot{X}_1 = -X_1 - X_2 - u$$

\downarrow insert into 6b)

$$u + X_2 + \dot{X}_2(-X_1 - X_2 - u) + \dot{X}_2(X_1 + X_2 + u) + u^2 = 0$$

$$X_2 = -u - u^2$$

$$\dot{X}_1 = -X_1 - X_2 - u$$

$$X_2 = -u - u^2$$

X_1 : diff. state.

X_2 : Alg. state

$$b) \quad u + \dot{X}_1 \cancel{X_1} X_1 + \dot{X}_2 X_2 = 0$$

$$u \dot{X}_1 X_1 + \dot{X}_2 u X_2 = 0$$

$$\frac{\partial F}{\partial \dot{x}} = \begin{bmatrix} X_1 & X_2 \\ uX_1 & uX_2 \end{bmatrix}$$

$$\det\left(\frac{\partial F}{\partial \dot{x}}\right) = 0 \Rightarrow \text{DAE}$$

Not possible to determine algebraic or differential states.

$$4a) \quad \dot{x} + u + \tanh(\dot{x}) + xz = 0$$

$$\tanh(2u - z) = 0$$

(=0)

let's say $\dot{x} = v$

$$F(x, v, u, z)$$

$$g(x, v, u, z) = \begin{matrix} v + u + \tanh(uv) + xz = 0 \\ zu - z = 0 \end{matrix}$$

$$b) \begin{bmatrix} \frac{\partial g}{\partial \dot{x}} & \frac{\partial g}{\partial z} \end{bmatrix} = \begin{bmatrix} 1 + \frac{u}{\cosh^2(uv)} & x \\ 0 & -\frac{1}{\cosh^2(2u-z)} \end{bmatrix}$$

Well-defined since \uparrow this
 stud has full rank (IFT)

c) We pull a fast $\frac{d}{dt}$ on
 the system from a)

$$\dot{v} + \dot{u} + \frac{\dot{u}v + v\dot{u}}{\cosh^2(uv)} + \dot{x}z + x\dot{z} = 0$$

$$z\dot{u} - \dot{z} = 0$$

$$\dot{V} = - \frac{\dot{u}(1 + 2Xu + \frac{V}{\cosh^2(uv)} + VZ}{1 + \frac{u}{\cosh^2(uv)}} \quad \#$$

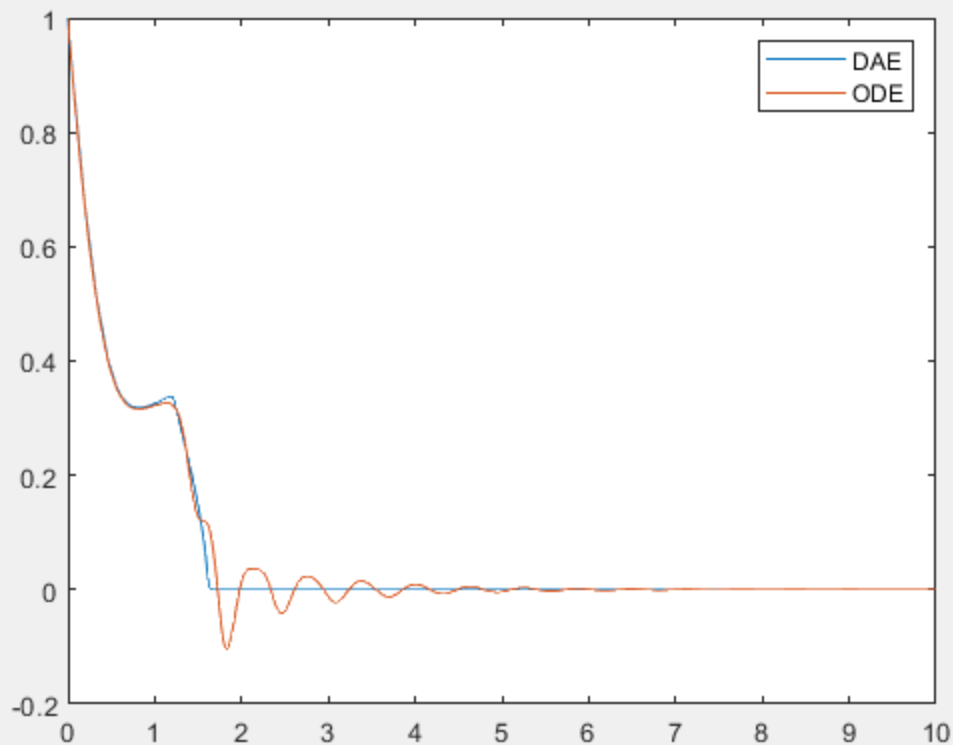
$$Z = Zu$$

$$\dot{X} = V$$

\Rightarrow Ditt index: 2

Figure 1

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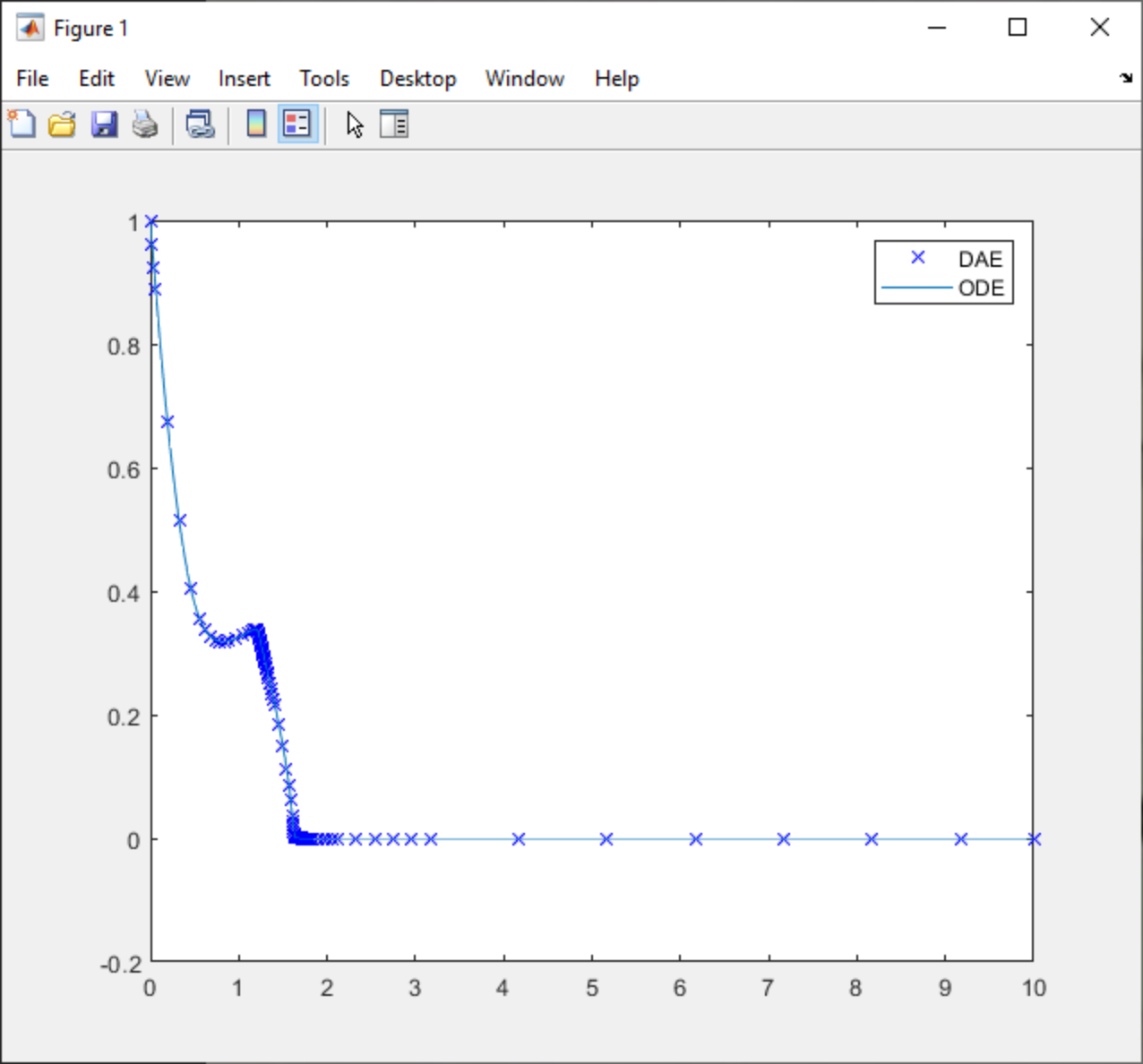




Figure 1



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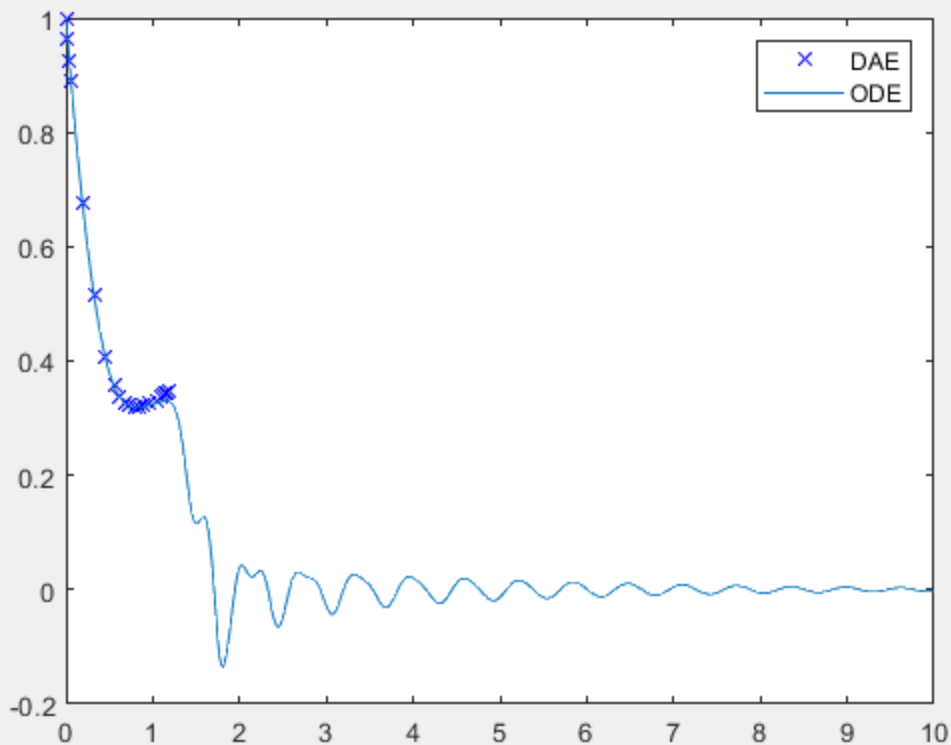


Figure 1

