TTK4130 - Modeling and Simulatio

Assignement 8

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1a) I have appended all relevant code for this assignement.

b) Semilogarithmic plot of the infinity norm of the residuals:

see: "fig 1: inf norm of residuals"

Comment of results:

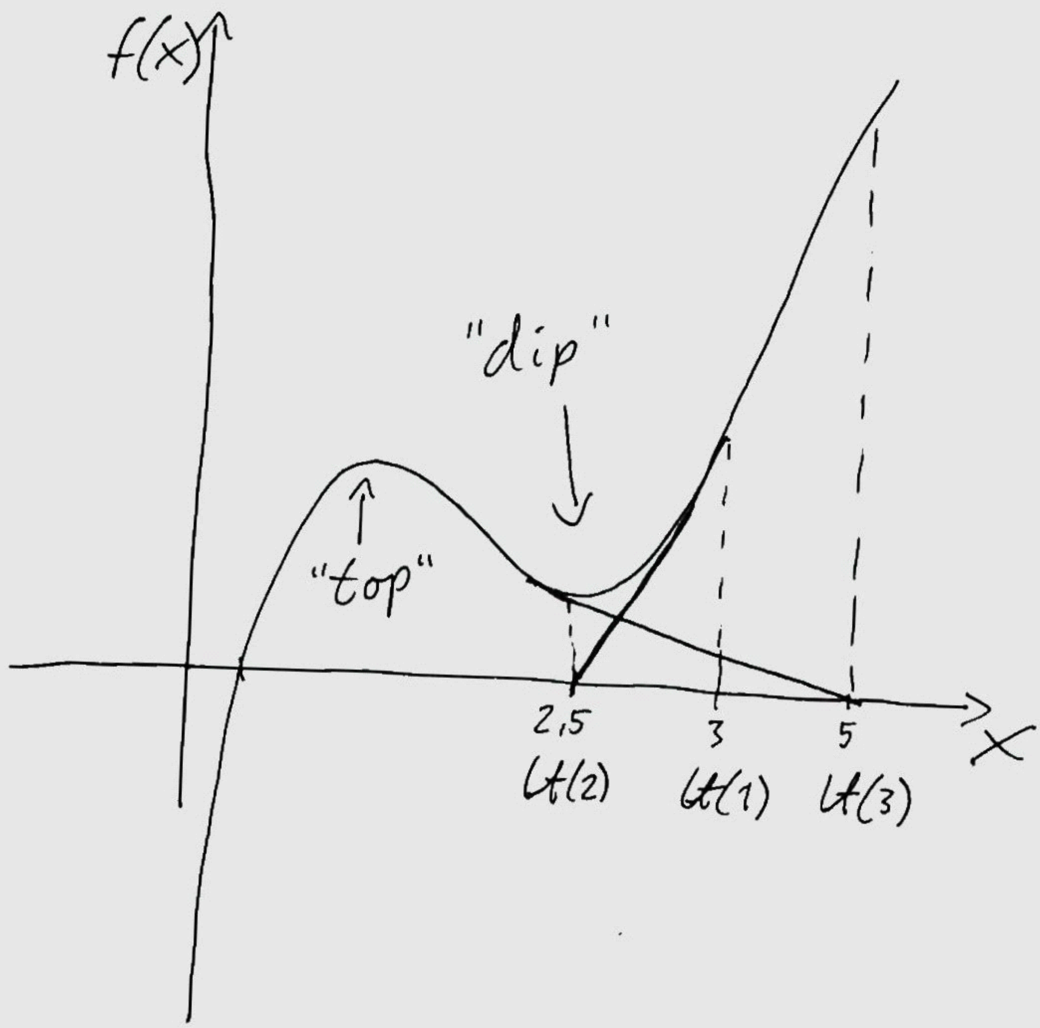
At first, there is a slight increase, however it then seems to decrease quadratically, which I believe fits well with ~~[illegible]~~ the theory

I've added the iteration values, see: "S_b' "

c) plot: "fig 2: Plots of obtained results in 1c)"

$$f(x) = (x-1)(x-2)(x-3)+1$$

This looks something like this.



(Iteration values: "S_c")

One can see that the iteration values "bounce" around the so-called "dip"

The reason for this is that the gradients on both sides of the dip will send the next iteratio[n] to the other side until it manages "to escape the trap" and guess an x-value to the left of the "top" at which point it will converge very quickly.
(This wasn't very scientifically written... sorry...)

If an iteration or initial guess reaches a singular point, the jacobian becomes singular (non-invertible) and Newtons method would fail.
I could add a function that checks if an iteration is near a singular point, and if it is, starts again with a better initial guess.

d) Plot: "fig3: infNorm of ~~#~~ residuals 1d)"
iteration values: "S_d"

The final iteration lands ~~approximately~~ near $\begin{bmatrix} 1 \\ \pi \end{bmatrix}$ which is the closest root according to the hint.
Since our initial guess is so close it converges quite fast.
If we were to guess something else, we would most likely get the same sort of problem we had in 1c, since these functions are nonlinear and contain trigonometric ~~a~~ terms.
The Jacobian at the root is:

$$J_{dr} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

e) (Iteration values: "5_e")

According to Ernesto on the Forumpost "Question 1e Assignement 8", the rate of convergence (r_k) can be found by:

$$r_K = \log(e_{K+1}) - \frac{\log(e_K)}{(\log(e_K) - \log(e_{K-1}))}$$

So let's do that:

Residuals/errors pr. iteration:

k:	value($\cdot 10^3$)	r_K:
1	2.9160	undef
2	0.8640	7,96
3	0.2560	8,89
4	0.0759	6,67
5	0.0225	4,46
6	0.0067	

I'm not really sure how to interpret this, but from the iteration sequence, it seems to converge geometrically or asymtotically

2a) Implicit Euler:

$$X_{K+1} = X_K + \Delta t\, f(X_{K+1}, U_{K+1})$$

$$\underbrace{X_K + \Delta t\, f(X_{K+1}, U_{K+1}) - X_{K+1} = 0}_{\text{Has to be solved for each iteration of IRK}}$$

The code can be found at the end of the document.

b) See plot: "fig 4: Implicit Euler compared to actual solution."