2a) One can observe that the matrix delivering the differential states

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

is rank deficient and $X_1$ ~~$x_3$~~ is therefore an algebraic state.

$$F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \dot{X} - X + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$

$$\frac{\partial F}{\partial \dot{x}} = \left.\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\right\} \text{Rank deficient}$$

We can write the function as:

$$\dot{x}_2 = x_1 + u \quad (1)$$
$$\dot{x}_3 = x_2 \quad (2)$$
$$x_3 = 0 \quad (3)$$

First we differentiate (3)

$$\dot{x}_2 = x_1 + u \qquad \dot{x}_2 = x_1 + u \quad (1)$$
$$\dot{x}_3 = x_2 \qquad \Rightarrow \quad x_2 = 0 \quad (4)$$
$$\dot{x}_3 = 0$$

Then (4)

$$\dot{x}_2 = x_1 + u$$
$$\dot{x}_2 = 0 \qquad \Rightarrow \quad x_1 = -u \quad (5)$$

Then (5)

$$\dot{x}_1 = -\dot{u}$$

It is therefore of <u>index = 3</u>

2b) $\dot{x} = Ax + bz \quad (1)$

$0 = \frac{1}{2}(x^T x - L^2) \quad (2) \quad L = L(t)$

1. We apply $\frac{d}{dt}$ on (2)

$\dot{x} = Ax + bz \quad (1)$

$0 = \dot{x}^T x - \cancel{2} L(t) \cdot \dot{L}(t) \quad (3)$

2. The consistancy conditions are given by (2) and (3)

$C(x,t) = \frac{1}{2}(x^T x - L^2) = 0$

$C(\dot{x}, x, t) = \dot{x}^T x - \cancel{2} L(t) \dot{L}(t) = 0$

3. $\frac{\partial L}{\partial t}$ must be full rank

By (1) and (3):

$$0 = (Ax + bz)^T x - L(t)\dot{L}(t)$$

b must then be zero

2c) $\det\left(\frac{\partial f}{\partial \dot{x}}\right) = 0$

$$\det\left(\frac{\partial \dot{g}(x)}{\partial t}\right) = \det\left(\frac{\partial^2 g(x)}{\partial t^2}\right) \neq 0$$