UNIVERSITÀ DI PISA
DIPARTIMENTO DI INGEGNERIA CIVILE E INDUSTRIALE

8 Gennaio 2025

ESAME DI MECCANICA I - *Corso di Laurea in Ing. Biomedica*

ESAME DI MECCANICA TEORICA ED APPLICATA - *Corso di Laurea in Ing. Robotica e dell'automazione*

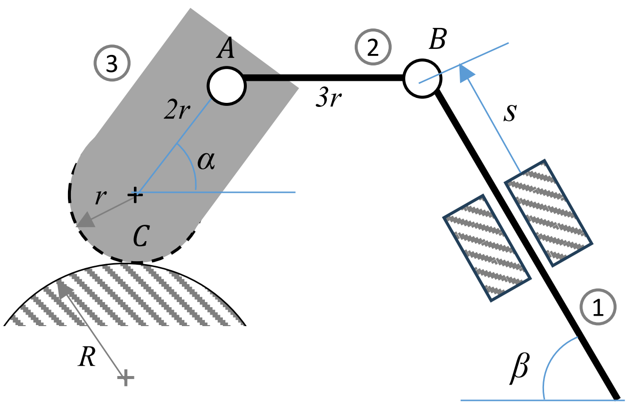
*COGNOME*____________________ *NOME*____________________ *MATRICOLA*_____________ *CDL*____________

Esercizio 1

Si consideri il meccanismo in figura, costituito da 3 corpi. Sia nota la configurazione del meccanismo nell'atto di moto considerato, e la velocità $\dot{s}$ del corpo 1, costante nel tempo:

1) Fare l'analisi geometrica dei vincoli e valutare il tipo di rotolamento in C affinché il sistema abbia 1gdl.
2) Definire il moto assoluto dei corpi 1 e 3 e il moto relativo 1-2 e 2-3.
3) Scrivere l'eq.ne di chiusura delle velocità in forma vettoriale e scalare, in forma parametrica.
4) Risolvere graficamente il problema delle velocità.
5) Risolvere in forma parametrica e numerica il problema delle velocità.
6) Valutare tutti i centri delle velocità, assoluti e relativi.
7) Valutare l'accelerazione assoluta di C.
8) Scrivere l'eq.ne di chiusura delle accelerazioni.

<u>Dati</u>:
$\alpha = 45°$, $\beta = 60°$, $r = 30$ cm,
$R = 110$ cm, $\dot{s} = 6$ cm/s

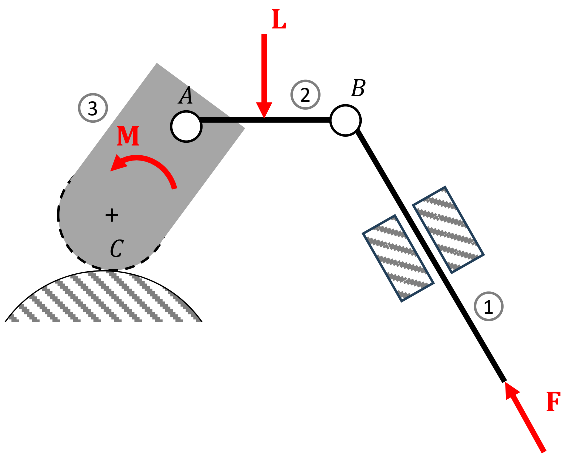


Esercizio 2

Il meccanismo dell'Esercizio 1 è caricato dall'esterno con un momento noto **M** ed una forza nota **L**. Per garantire l'equilibrio statico, si applica una forza **F** di direzione nota ed intensità incognita, mostrata in Figura.

Applicare il PSE, e valutare per ogni caso:

1) Forze reattive e forza **F** in forma parametrica.
2) DCL definitivi in forma numerica.
3) l'asse centrale della coppia prismatica tra il corpo 1 e il telaio.



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Esercizio 3

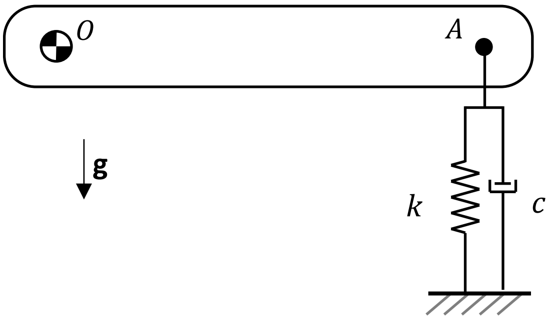
La barra omogenea mostrata in figura è collegata al telaio mediante una cerniera e una molla-smorzatore.

Si vuole studiare la dinamica della barra nell'ipotesi di piccole oscillazioni, in presenza di gravità.

1) Specificare in modo chiaro il sistema di riferimento, la coordinata lagrangiana e le eventuali equazioni di congruenza.
2) Valutare la freccia statica in condizioni di equilibrio statico.
3) Valutare l'equazione del moto, la pulsazione naturale e il fattore di smorzamento.
4) Trovare la legge oraria e rappresentarla graficamente.

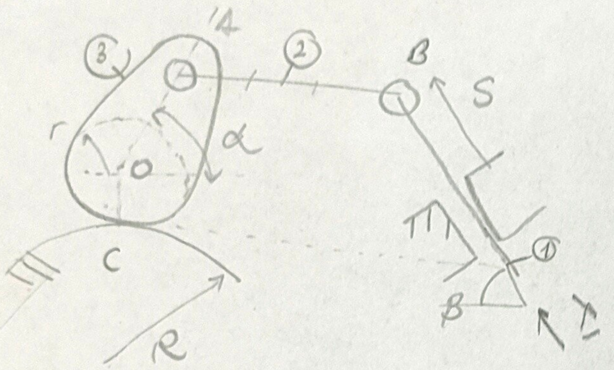
Dati:

m = 2.5 kg, l=0.85 m, k=5 N/m, c = 3 N m/s; posizione iniziale: condizione di equilibrio statico, velocità iniziale di A pari a 0.2 m/s verso il basso.



- CINEMATICA - 8/01/25

$3 \times 3 - \underset{CP}{2} - \underset{CR}{2} \times 2 - \underset{RSS}{2} = 1 \text{ gdl}$



$\alpha = 45$ $\quad \dot{s} > 0$

$\beta = 60$ $\quad \ddot{s} = 0$

$\overline{OA} = a = 2r$

$\overline{AB} = b = 3r$

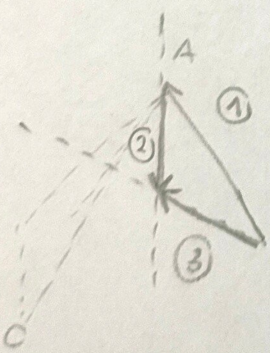
1) $\underline{v}_{A②} = \dot{s}\underline{\lambda} + \dot{\gamma}\underline{k} \wedge \overrightarrow{BA}$

$\underline{v}_{A③} = \underset{\underset{\underline{0}}{\parallel}}{\underline{v}_C} + \dot{\alpha}\underline{k} \wedge \overrightarrow{CA}$

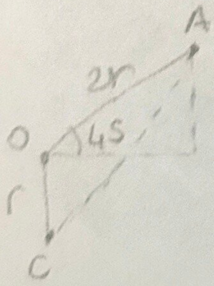
$\boxed{\underset{FFC}{\underline{v}_{A②}} = \underset{FFC}{\underline{v}_{A③}}}$

$\downarrow$

$\| \underset{①}{\dot{s}\underline{\lambda}} + \underset{②}{\dot{\gamma}\underline{k} \wedge \overrightarrow{BA}} = \underset{③}{\dot{\alpha}\underline{k} \wedge \overrightarrow{CA}} \|$



$\begin{cases} \dot{\gamma} > 0 \\ \dot{\alpha} > 0 \end{cases}$



$\overrightarrow{CA} = \begin{pmatrix} 2r\cos\alpha \\ 2r\sin\alpha + r \end{pmatrix} = \begin{pmatrix} 2r\sqrt{2}/2 \\ r + 2r\frac{\sqrt{2}}{2} \end{pmatrix} = \begin{pmatrix} \sqrt{2} \\ 1 + \sqrt{2} \end{pmatrix} r$

$\underline{\lambda} = \begin{pmatrix} -\cos\beta \\ \sin\beta \end{pmatrix} = \begin{pmatrix} -1/2 \\ \sqrt{3}/2 \end{pmatrix}$

$$\begin{pmatrix}-1/2\\ \sqrt{3}/2\end{pmatrix}\dot s + \dot\gamma\begin{pmatrix}0\\ -3r\end{pmatrix} = \begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ 0 & 0 & \dot\alpha\\ (\sqrt{2})r & (1+\sqrt{2})r & 0\end{vmatrix}$$

$$= \underline{j}\,\dot\alpha\sqrt{2}r - (1+\sqrt{2})r\dot\alpha\,\underline{i}$$

$$\begin{cases} -1/2\,\dot s = -(1+\sqrt{2})r\dot\alpha\\ \sqrt{3}/2\,\dot s - 3r\dot\gamma = \sqrt{2}r\dot\alpha\end{cases}$$

$$\begin{cases}\dot\alpha = \frac{1}{2}\frac{1}{1+\sqrt{2}}\frac{\dot s}{r} \quad >0\\ \dot\gamma = \left(\frac{\sqrt{3}}{2}\dot s - \sqrt{2}\cancel{r}\frac{1}{2(1+\sqrt{2})\cancel{r}}\dot s\right)\frac{1}{3r}\\ \quad = \left(\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2(1+\sqrt{2})}\right)\frac{1}{3r}\dot s\end{cases}$$

$$\begin{pmatrix}-\cos\beta\\ \sin\beta\end{pmatrix}\dot s + \dot\gamma\begin{pmatrix}0\\ -3r\end{pmatrix} = \begin{pmatrix}-(2\sin\alpha+1)\\ 2\cos\alpha\end{pmatrix}r\dot\alpha$$

$$\begin{cases}\cos\beta\,\dot s = (2\sin\alpha+1)r\dot\alpha\\ \sin\beta\,\dot s - \dot\gamma\,3r = 2r\cos\alpha\,\dot\alpha\end{cases}$$

$$\dot\alpha = \frac{\cos\beta}{2\sin\alpha+1}\frac{\dot s}{r} = \frac{1}{2}\frac{1}{2\frac{\sqrt{2}}{2}+1}\frac{\dot s}{r}$$

$$\dot\gamma = \frac{\sin\beta\,\dot s - 2r\cos\alpha\,\dot\alpha}{3r}$$

$$= \frac{1}{3r}\left(\sin\beta - \frac{2\cancel{r}\cos\alpha}{\cancel{r}}\frac{\cos\beta}{2\sin\alpha+1}\right)\dot s$$

$$= \frac{\dot s}{3r}\left(\sin\beta - \frac{2\cos\beta\cos\alpha}{2\sin\alpha+1}\right)$$

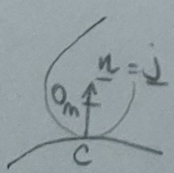
2)

$$\underline{a}_{A②} = \underline{a}_{A③}$$

$$\underline{a}_{A②} = \underset{\ddot s\underline{\lambda}}{\overset{0}{\cancel{\underline{a}_B}}} + \ddot\alpha\,\underline{k}\wedge\overrightarrow{BA} - \dot\alpha^2\overrightarrow{BA}$$

$$\underline{a}_{A③} = \underline{a}_C + \ddot\gamma\,\underline{k}\wedge\overrightarrow{CA} - \dot\gamma^2\overrightarrow{CA}$$

$$\underline{a}_{C③} = \underline{a}_{Cr} = -D\omega_3^2\,\underline{n} = +\frac{Rr}{R+r}\dot\alpha^2\,\underline{j}$$



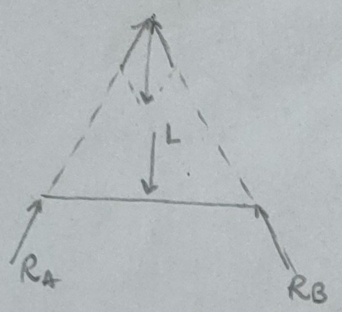
$$\frac{1}{D} = \frac{1}{R_F} - \frac{1}{R_M} = -\frac{1}{R} - \frac{1}{r}$$

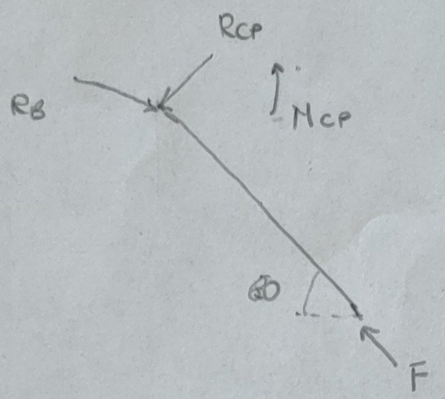
$+O_F$

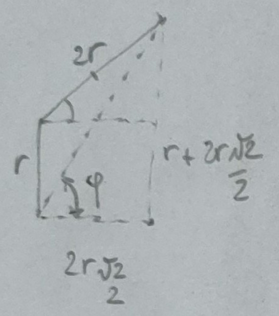
$$D = -\frac{Rr}{R+r}$$

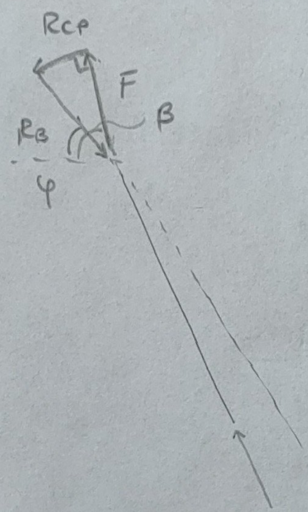
$$\left\| \ddot\alpha\,\underline{k}\wedge\overrightarrow{BA} - \dot\alpha^2\overrightarrow{BA} = \frac{Rr}{R+r}\dot\alpha^2\underline{j} + \ddot\gamma\,\underline{k}\wedge\overrightarrow{CA} - \dot\gamma^2\overrightarrow{CA}\right\|$$

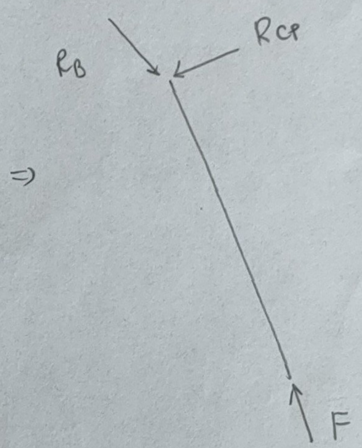
SOL. GRAFICA











$$\operatorname{tg}\varphi = \frac{1+\sqrt{2}}{\sqrt{2}} = 1.707$$

$$\varphi \simeq 59°$$

SOL. ANALITICA

$$\circlearrowleft B \quad -R_C \sin\varphi \, 3\cancel{r} + L \frac{3\cancel{r}}{2} = 0$$

$$R_C = \frac{L}{2\sin\varphi}$$

$$B\circlearrowleft \quad M_{CP} = 0$$

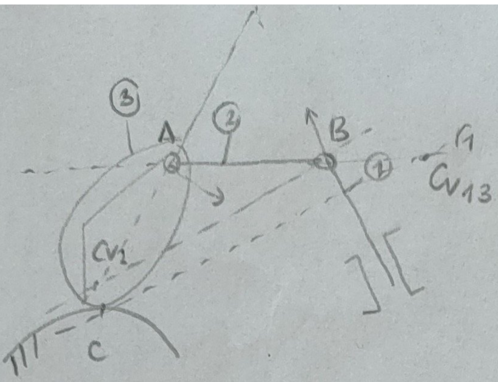
(1) $\underline{F} + \underline{R_{CP}} + R_B = 0$ $\qquad F = R_B \sin(\beta - \varphi)$

$$R_{CP} = R_B \cos(\beta - \varphi)$$

se $\varphi \simeq 60$ $\quad R_C = \frac{L}{\sqrt{3}} \quad \begin{cases} R_{CY} = \frac{L}{\sqrt{3}}\frac{\sqrt{3}}{2} = \frac{L}{2} \\ R_{CX} = \frac{L}{\sqrt{3}}\frac{1}{2} = \frac{L}{2\sqrt{3}} \end{cases}$

$$R_{CP} = M_{CP} = 0$$

CV :



$Cv_1 \nexists$ $\quad$ $Cv_{12} \equiv B$

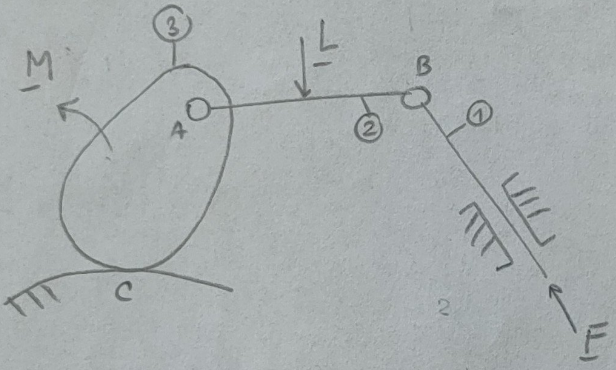
Cv_2 fig $\quad$ $Cv_{23} = A$

Cv_3 C $\quad$ Cv_{13}

$Cv_{13} \Rightarrow \quad Cv_1 \quad Cv_3 \quad Cv_{13} \quad r_1$

$\quad Cv_{23} \quad Cv_{21} \quad Cv_{13} \quad \overrightarrow{AB}$

STATICA



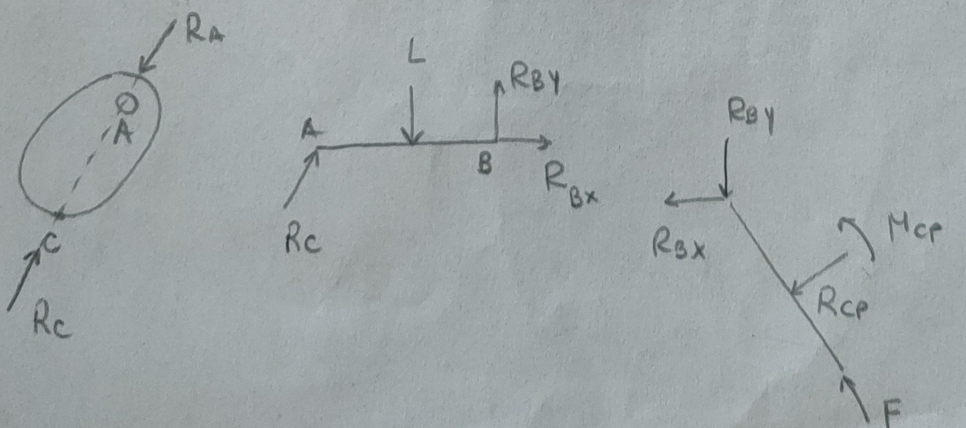
Trovare F con PSE

•) SCI ⇒ 8 reaz. vinc. + 1 F attiva = 9 incognite vs 3×3 Eq^{ni}

•) SNEI

•) corpi scarichi ⇒ se applichiamo il PSE

CASO 1: L ⇒ corpo scarico 3



$R_A = R_C$

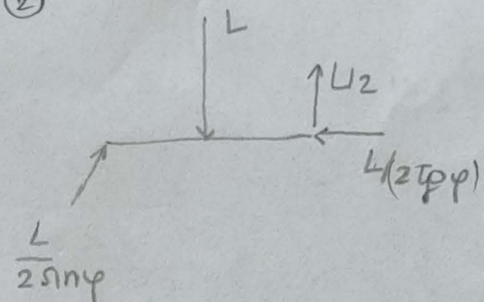
(2)

$$\begin{cases} R_C \cos\varphi + R_{Bx} = 0 \\ R_C \sin\varphi - L + R_{By} = 0 \end{cases}$$

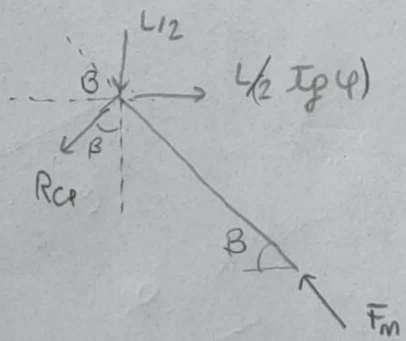
$$-R_C \sin\varphi \; 3r + L \frac{3}{2} r = 0$$

$$\begin{cases} R_{Bx} = -R_C \cos\varphi = -\dfrac{L}{2\,tg\varphi} \\ R_C = \dfrac{L}{2\sin\varphi} \\ R_{By} = L - R_C \sin\varphi \\ \quad = L - \dfrac{L}{2}\dfrac{\sin\varphi}{\sin\varphi} = \dfrac{L}{2} \end{cases}$$

(2)



(1)



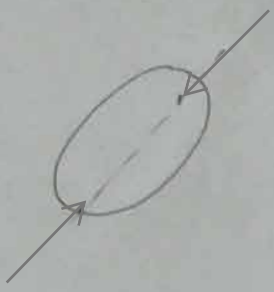
(1) $B \circlearrowleft \quad M_{CP} = 0$

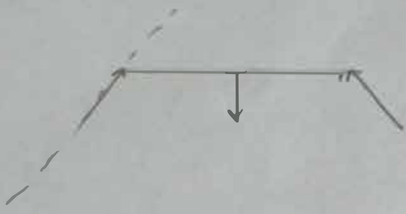
$$\begin{cases} \dfrac{L}{2\,tg\varphi} - R_{CP} \sin\beta - F_m \cos\beta = 0 \\ -L/2 - R_{CP} \cos\beta + F_m \sin\beta = 0 \end{cases}$$

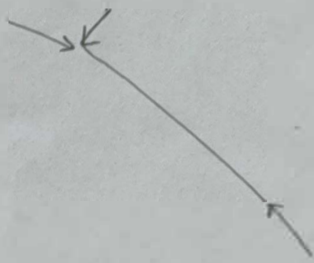
$$\begin{cases} R_{CP} = \dfrac{L/(2\,tg\varphi) - F_m \cos\beta}{\sin\beta} \\ -\dfrac{L}{2} - \left[\dfrac{L}{2\,tg\varphi\; tg\beta} - F_m \dfrac{c^2\beta}{s\beta}\right] + F_m \sin\beta = 0 \end{cases}$$

$$\begin{cases} -\dfrac{L}{2} - \dfrac{L}{2\,tg\beta\, tg\varphi} + \dfrac{F_m \cos\beta \cos\beta + F_m \sin^2\beta}{\sin\beta} = 0 \\ -\dfrac{L}{2} - \dfrac{L}{2\,tg\beta\, tg\varphi} + \dfrac{F_m}{\sin\beta} = 0 \end{cases}$$

$$\begin{cases} F_m = \dfrac{L}{2}\left(1 + \dfrac{1}{2\,tg\beta\, tg\varphi}\right)\sin\beta \\ R_{CP} = \dfrac{L}{2}\dfrac{1}{tg\varphi \sin\beta} - \dfrac{1}{tg\beta}\,\dfrac{L}{2}\left(1 + \dfrac{1}{2\,tg\beta\, tg\varphi}\right)\sin\beta \\ \quad = \dfrac{L}{2}\left[\dfrac{1}{tg\varphi \sin\beta} - \dfrac{\sin\beta}{tg\beta} - \dfrac{\cos\beta}{2\,tg\beta\, tg\varphi}\right] \end{cases}$$

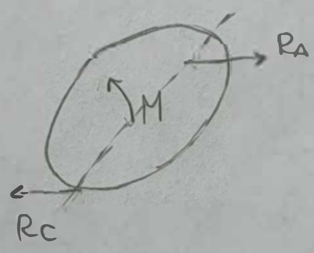






CASO 2 : ↺ $\underline{M}$ ⇒ CORPO SCARICO 2

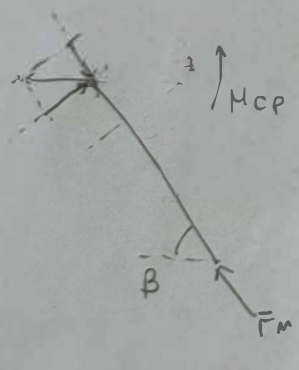
$R_A = R_B$



$R_C = R_A$

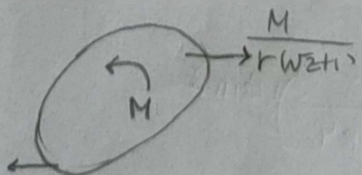
$$C \circlearrowleft \quad -M + R_A \left(2r \frac{\sqrt{2}}{2} + r\right) = 0$$

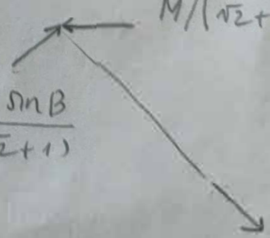
$$R_A = \frac{M}{r(\sqrt{2}+1)}$$



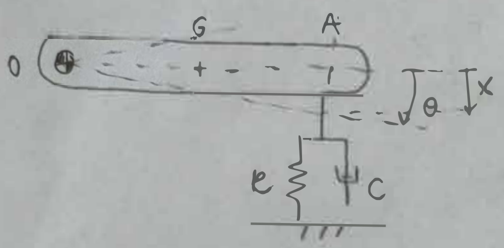
$B \circlearrowleft \quad M_{CP} = 0$

$$\begin{cases} R_A \cos\beta = F_M \\ R_A \sin\beta = R_{CP} \end{cases} \qquad \begin{cases} F_M = \dfrac{M \cos\beta}{r(\sqrt{2}+1)} \\ R_{CP} = \dfrac{M \sin\beta}{r(\sqrt{2}+1)} \end{cases}$$

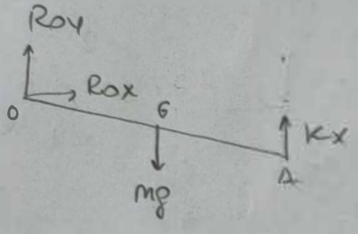




Ep^{ni} CONGRUENZA



CONDIZIONI DI RIPOSO $\Rightarrow$ $\begin{cases} x = l\theta \\ \dot{x} = l\dot{\theta} \\ \ddot{x} = l\ddot{\theta} \end{cases}$



CONDIZIONI STATICHE

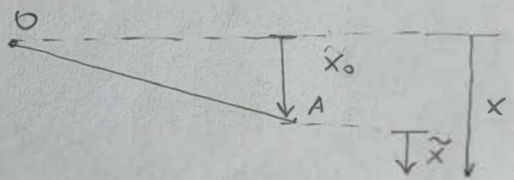
$$m = 2{,}5\ kg$$
$$l = 0.85\ m$$
$$J_G = \frac{ml^2}{12}$$
x_0 = condiz. di ep statico

$$\dot{x}_0 = 0.5\ m/s$$

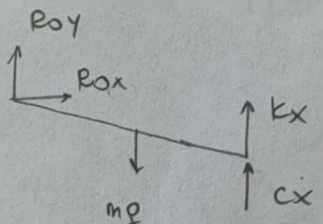
$\Rightarrow$ O $\circlearrowleft$ $mg\frac{l}{2} - kxl = 0$

$$x = x_0 + \tilde{x}$$

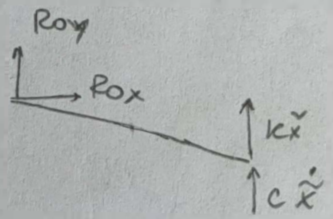
$x_0 = x = \frac{mg}{2k}$ $\Rightarrow$ FRECCIA STATICA



$\Rightarrow$ CONDIZIONI DINAMICHE $\Rightarrow$



e



con $\tilde{x} = 0$ DALLA CONFIG DI EQ STATICO

O $\circlearrowleft$ $lk\tilde{x} + lc\dot{\tilde{x}} = -J_O\ddot{\theta}$

$$\underline{M}_O^{(ma)} = -J_O\ddot{\theta}\,\underline{k} + m\,\cancel{\vec{OG} \wedge \underline{a}_O}^{\,0}$$

$$J_O\ddot{\theta} + lc\dot{\tilde{x}} + lk\tilde{x} = 0$$

$$\frac{J_O}{l}\ddot{\tilde{x}} + lc\dot{\tilde{x}} + lk\tilde{x} = 0$$

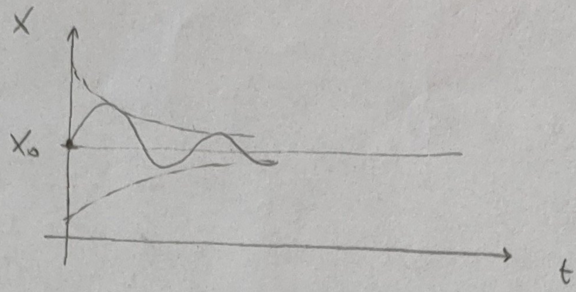
$$J_O\ddot{\tilde{x}} + l^2c\dot{\tilde{x}} + l^2k\tilde{x} = 0 \longleftrightarrow \frac{J_O}{l^2}\ddot{\tilde{x}} + c\dot{\tilde{x}} + k\tilde{x} = 0$$

$$\omega_n = \sqrt{\frac{k}{J_O}l^2} = 2.45\ rad/s$$

$$\xi = \frac{c}{2\sqrt{km}} = \frac{c}{2\sqrt{k\frac{J_O}{l^2}}} = \frac{cl}{2\sqrt{kJ_O}} = 0.34 \qquad \frac{cl^2}{2\sqrt{kl^2 \cdot J_O}}$$

$$\tilde{x} = A\, e^{-\xi \omega_n t} \sin(\omega_s t + \varphi)$$

$$x = x_0 + \tilde{x}$$



$$\begin{cases} x(0) = x_0 \\ \dot{x}(0) = v_0 > 0 \end{cases}$$

$\downarrow$

$$\begin{cases} \tilde{x} = 0 \\ \dot{\tilde{x}} = v_0 \end{cases} \rightarrow \begin{cases} 0 = A \sin\varphi \Rightarrow \varphi = 0 \\ A(-\xi\omega_n) \cancel{\sin(\varphi)} + A \cos(\omega_s \cancel{t} + \varphi)\, \omega_s = v_0 \end{cases} \Rightarrow A\,\omega_s = v_0$$

$$A\,\omega_s = v_0$$

$$A = \frac{v_0}{\omega_s}$$

$\downarrow$

$$\theta_{max} = \frac{A}{l}$$

$$J_0 = J_G + m\,\overline{OG}^2$$

$$\overset{!}{=} \frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2 = ml^2\left(\frac{1}{12} + \frac{1}{4}\right) = \frac{ml^2}{3}$$

$$\omega_s = \omega_n \sqrt{1 - \xi^2}$$