UNIVERSITÀ DI PISA
DIPARTIMENTO DI INGEGNERIA CIVILE E INDUSTRIALE

24 Gennaio 2025

ESAME DI MECCANICA I - *Corso di Laurea in Ing. Biomedica*

ESAME DI MECCANICA TEORICA ED APPLICATA - *Corso di Laurea in Ing. Robotica e dell'automazione*

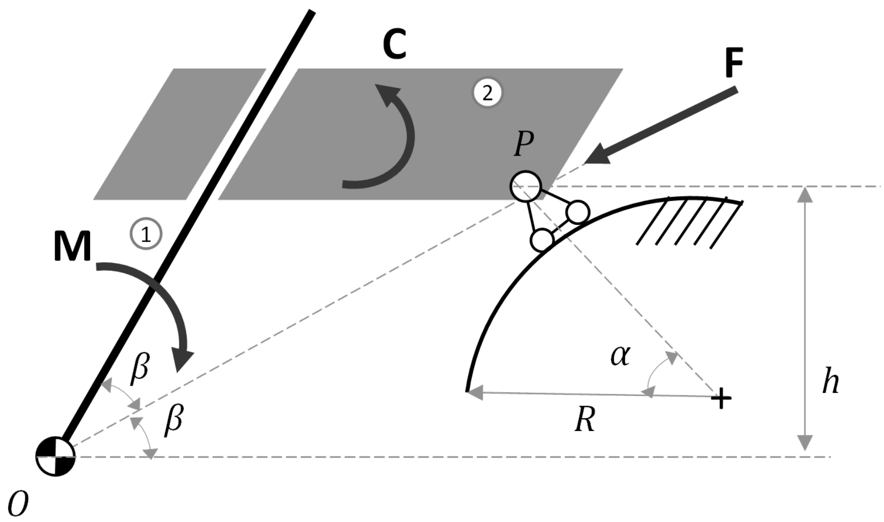
*COGNOME*____________________ *NOME*____________________ *MATRICOLA*______________ *CDL*_____________

Esercizio 1

Si consideri il meccanismo in figura, costituito da 2 corpi. Sia nota la configurazione del meccanismo nell'atto di moto considerato, e la velocità $\dot{\beta}$, oraria e costante, del corpo 1:

1) Fare l'analisi geometrica dei vincoli.
2) Definire il moto assoluto e il moto relativo dei corpi.
3) Scrivere l'eq.ne di chiusura delle velocità in forma vettoriale e scalare, in forma parametrica.
4) Risolvere graficamente il problema delle velocità.
5) Risolvere numericamente il problema delle velocità.
6) Valutare tutti i centri delle velocità, assoluti e relativi.
7) Scrivere l'eq.ne di chiusura delle accelerazioni.
8) Risolvere graficamente il problema delle accelerazioni.

<u>Dati</u>: $\alpha = 45°$, $\beta = 30°$, $R = 100$ cm, $h = 125$ cm, $\dot{\beta} = 5$ rad/s



Esercizio 2

Il meccanismo dell'Esercizio 1 è caricato dall'esterno con un momento noto **C** ed una forza nota **F**, passante per *P*. Per garantire l'equilibrio statico, si applica una coppia incognita **M** al corpo 1.

1) Fare l'analisi fisica dei vincoli
2) valutare se il sistema è complessivamente isostatico ed esternamente isostatico

Applicare il PSE, e valutare per ogni caso:

3) Forze reattive e momento M in forma parametrica
4) DCL definitivi in forma numerica
5) l'asse centrale della coppia prismatica tra il corpo 1 e 2

Dati: F=10 N, C= 15 N/m

Esercizio 3

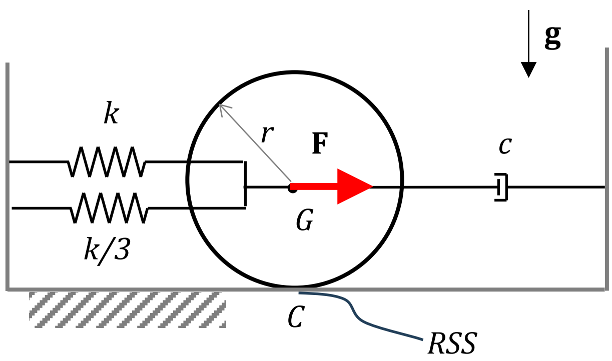
Il disco omogeneo mostrato in figura, collegato al telaio mediante due molle e uno smorzatore, rotola senza strisciare soggetto ad una forza **F** nota armonica.

Si vuole studiare la dinamica del disco. Siamo in presenza di gravità.

1) Specificare in modo chiaro il sistema di riferimento, la coordinata lagrangiana e le eventuali equazioni di congruenza.
2) Valutare l'equazione del moto in forma canonica
3) Valutare la pulsazione naturale e il fattore di smorzamento in forma parametrica e numerica.
4) Trovare la legge oraria a regime (fare i passaggi) e rappresentarla graficamente.
5) Cosa si intende per condizioni di risonanza? Valutare se il sistema rischia di andare in risonanza, usando/mostrando il diagramma di ampiezza.

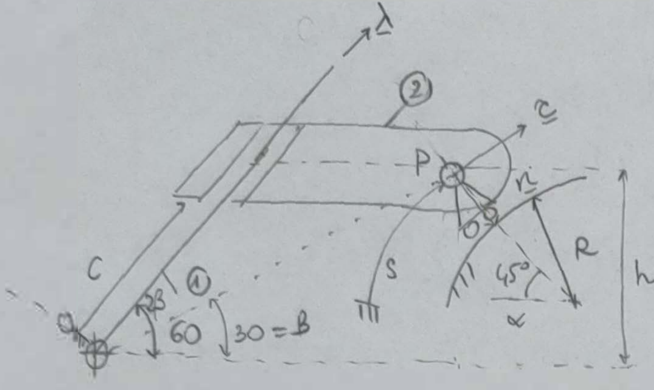
Dati:

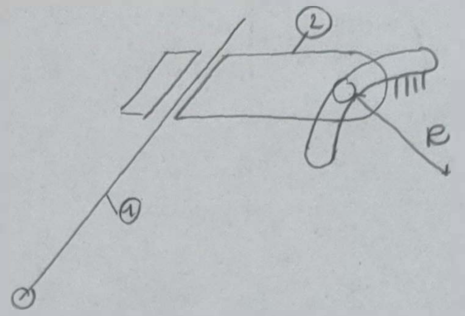
m = 15 kg, r=0.85 cm, k=5 N/m, c = 0.5 N m/s, F=6 cos(0.5 t).



24/01/2025

Esercizio 1 - Cinematica





$$\begin{cases} \dot\theta = -5 \text{ rad/s} \\ r = 0.1 \text{ m} \\ h = 0.125 \text{ m} \end{cases}$$

•) $gdl = 2 \times 3 - 2 - 2 - 1 = 1$ gdl

•) $$\underline{v}_P = \dot s \,\underline{\tau} = \underbrace{\underline{v}_{P②}}_{\Sigma_1} = \underline{v}^{rel} + \underline{v}^{tr} = \dot c \,\underline{\lambda} + \underline{v}_{P①}$$

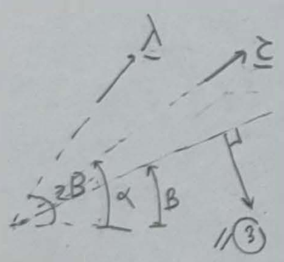
$$\boxed{\underset{①}{\dot s \,\underline{\tau}} = \underset{②}{\dot c \,\underline{\lambda}} + \underset{③}{\underline{\omega} \wedge \overrightarrow{OP}}}$$

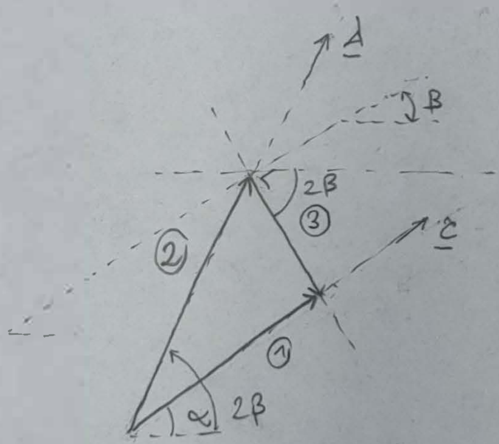
$\underline{\dot s}$ $\underline{\dot c}$?

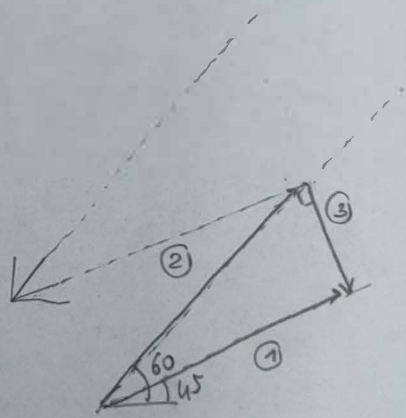
•)

$\dot s > 0$

$\dot c > 0$







$$\underline{\tau} = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} = \begin{pmatrix} \sqrt2/2 \\ \sqrt2/2 \end{pmatrix} \qquad \underline{\lambda} = \begin{pmatrix} \cos(2\beta) \\ \sin(2\beta) \end{pmatrix} = \begin{pmatrix} 1/2 \\ \sqrt3/2 \end{pmatrix}$$

$$\overrightarrow{OP} = \begin{pmatrix} h \,tg(2\beta) \\ h \end{pmatrix} \begin{pmatrix} h/tg\beta \\ h \end{pmatrix} = \begin{pmatrix} \sqrt3 \\ 1 \end{pmatrix} h$$

$$\dot s \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} = \dot c \begin{pmatrix} \cos(2\beta) \\ \sin(2\beta) \end{pmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0 & -\dot\theta \\ h/tg\beta & h & 0 \end{vmatrix}$$

$$\begin{cases} \dot s \cos\alpha = \dot c \cos(2\beta) + \dot\theta h \\ \dot s \sin\alpha = \dot c \sin(2\beta) - \dfrac{h}{tg\beta} \dot\theta \end{cases}$$

$$tg30 = tg\beta = \frac{\sin\beta}{\cos\beta} = \frac{1/2}{\sqrt3/2} = \frac{2}{2\sqrt3} = \frac{\sqrt3}{3}$$

$$\begin{cases} \dot s \dfrac{\sqrt2}{2} = \dot c \dfrac{1}{2} + \dot\theta h \\ \dot s \sqrt2/2 = \dot c \dfrac{\sqrt3}{2} - h \dot\theta \sqrt3 \end{cases}$$

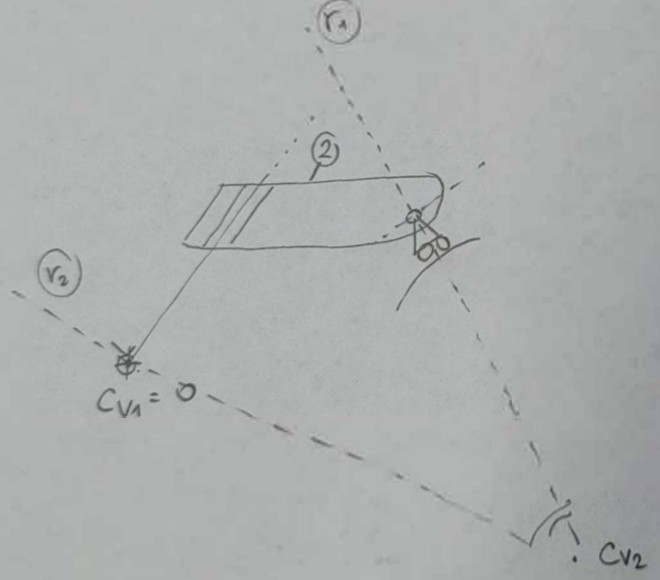
$$0 = \dot c\left(\frac{1}{2} - \frac{\sqrt3}{2}\right) + \dot\theta\, h\,(1+\sqrt3)$$

$$\dot c\left(\frac{-1+\sqrt3}{2}\right) = \dot\theta\, h\,(1+\sqrt3)$$

$$\Rightarrow \quad \dot c = \dot\theta\, h\left(1+\sqrt3 \quad \frac{2}{\sqrt3-1}\right) = \dot\theta h\, \frac{1+3+2\sqrt3}{\cancel{3-1}}\,\cancel{2} = 2\,(2+\sqrt3)\,h\,\dot\theta$$

$$\dot s = \frac{\dot c\,\cos(2\beta) + \dot\theta h}{\cos\alpha}$$

$$\Rightarrow \quad \dot s = \frac{\dot\theta h\, 2}{\sqrt2}\left(2\,(2+\sqrt3)+1\right) = \dot\theta h\,\sqrt2\,(5+2\sqrt3)$$



C_{V2} : $\perp v_D$ (r1)

$\underline{v}_{O(2)} = \underline{v}^{rel} + \underline{v}^{tr} = \dot s\,\underline{\lambda}$ (r2)

•) $\underline{\omega}_2 = \underline{\omega}^{rel} + \underline{\omega}^{tr} = \underline{\omega}^{tr} = \underline{\omega}_1 = \dot\theta\,\underline{k}$

•) $\underline{a}_{P\,TRAM} = \underline{a}_{P_2\,TR}$

$$\underline{a}_P = \ddot s\,\underline{t} + \frac{\dot s^2}{r}\,\underline{n}$$

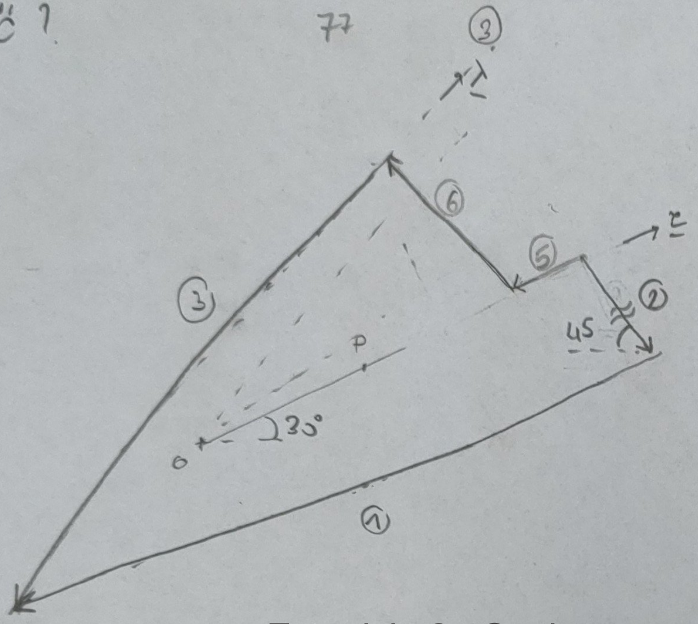
$$\Sigma_{(1)} \quad \underline{a}_{P_2} = \underline{a}^{rel} + \underline{a}^{tr} + \underline{a}^{cor} = \ddot c\,\underline{\lambda} + \ddot\theta\,\underline{k} \wedge \overrightarrow{OP} - \dot\theta^2\,\overrightarrow{OP} + 2\,\overset{\dot\theta\underline{k}}{\underline{\omega}^{tr}} \wedge \overset{\dot s\underline{\lambda}}{\underline{v}^{rel}}$$

① ② ③ ④ ⑤ ⑥

$$\ddot{s}\,\underline{t} + \underbrace{\frac{\dot{s}^2}{r}\,\underline{n}}_{=\,77} = \ddot{c}\,\underline{\lambda} + \underbrace{\ddot{\theta}\,\underline{k}\wedge\vec{OP}}_{\underline{0}} - \underbrace{\dot{\theta}^2\,\vec{OP}}_{=\,6.25} - 2\,\underbrace{\overset{<0}{\dot{\theta}}\,\underline{k}\wedge\overset{>0}{\dot{s}}\,\underline{\lambda}}_{=\,-27.9}$$

$\ddot{s}$?

$\ddot{c}$?

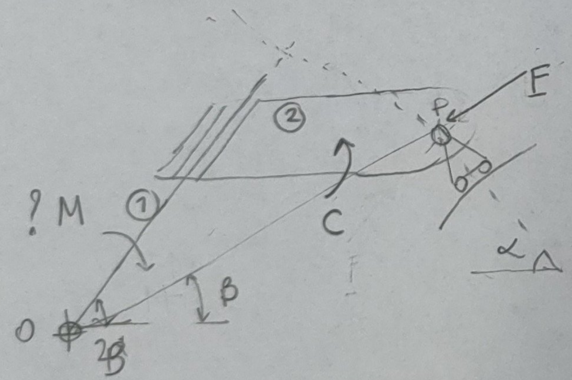


$\ddot{s} < 0$

$\ddot{c} < 0$

Esercizio 2 - Statica

STATICA



PSE

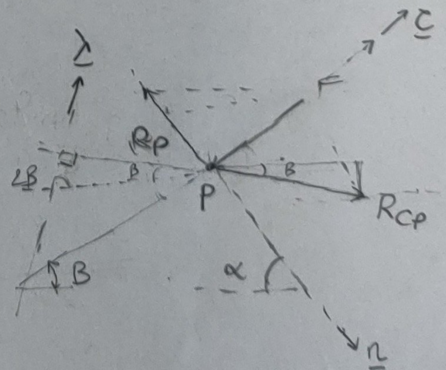
•) Applico $\begin{cases} \underline{C} \text{ noto} \\ \underline{F} \text{ noto} \end{cases}$ ricavo $\underline{M}$

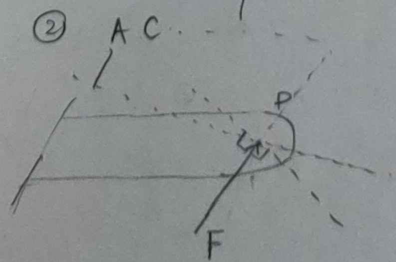
•) SCI sì!

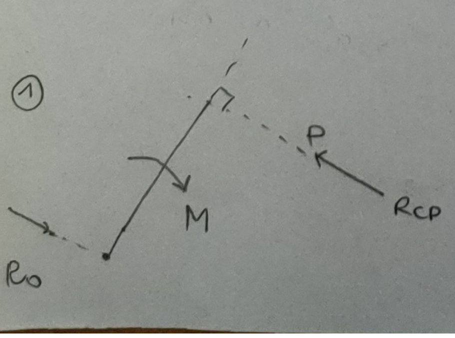
•) SEI $\Rightarrow 3\times 1 - 2 - 1 - 1 = 0$ no!

$\{R_{0x}, R_{0y}, R_P\} + \{R_{CP}, M_{CP}\} + M$

CASO I ($\underline{F}$) → ∄ corpi scarichi







$$\begin{cases} x: & -R_P\cos\alpha - F\cos\beta + R_{CP}\cos\beta = 0 \\ y: & R_P\sin\alpha - F\sin\beta - R_{CP}\sin\beta = 0 \end{cases}$$

$$R_P = \frac{-F\cos\beta + R_{CP}\cos\beta}{\cos\alpha} = \frac{(R_{CP}-F)c\beta}{c\alpha}$$

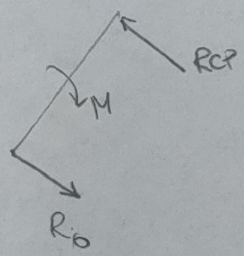
$$(R_{CP}-F)\,c\beta\,tg\alpha - F\sin\beta - R_{CP}\sin\beta = 0$$

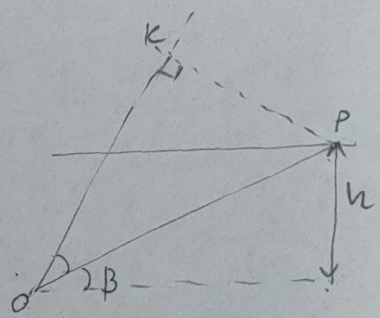
$$R_{cp}(c\beta\, tg\alpha - s\beta) - F\, c\beta\, tg\alpha - F\sin\beta = 0$$

$$R_{cp} = \frac{c\beta\, tg\alpha + s\beta}{c\beta\, tg\alpha - s\beta} F = \frac{tg\alpha + tg\beta}{tg\alpha - tg\beta} F$$

$$R_p = (R_{cp} - F)\frac{c\beta}{c\alpha} = \left(\frac{tg\alpha + tg\beta}{tg\alpha - tg\beta} - 1\right) F \frac{c\beta}{c\alpha}$$

$$= \frac{tg\alpha + tg\beta - tg\alpha + tg\beta}{tg\alpha - tg\beta} F \frac{c\beta}{c\alpha} = \frac{2\sin\beta}{\cos\alpha}\frac{F}{tg\alpha - tg\beta}$$





$$o\circlearrowleft) = -M + R_{cp}\, b = 0$$

$$M = R_{cp}\,\overline{ok}$$

$$\overline{OP} = \frac{h}{\sin\beta}$$

$$\overline{ok} = \overline{OP}\cos\beta = \frac{h}{\sin\beta}\cos\beta = \frac{h}{tg\beta}$$

$$M = \frac{tg\alpha + tg\beta}{tg\alpha - tg\beta}\,\frac{h}{tg\beta}\, F$$

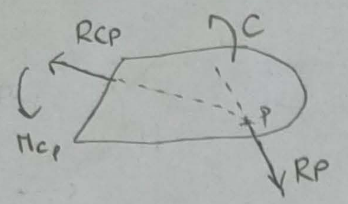
$F = 10$ N $\Rightarrow$ R_p = ... (N)

Da risolvere anche numericamente

R_{cp} =.... (N)

M = ... (N m)

CASO 2 ($\underline{C}$) → ∄ CORPI SCARICHI

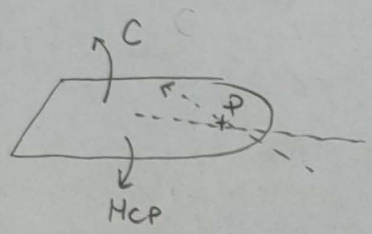


⇓

$\underline{R}_{CP} + \underline{R}_P = \underline{0}$

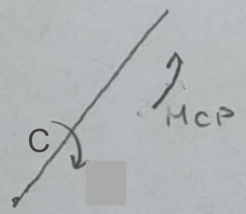
⇓

$\underline{R}_{CP} = -\underline{R}_P = \underline{0}$ poiché aventi direzioni ≠



$R_{CP} = R_P = 0$

$C = M_{CP}$



$R_O = 0$

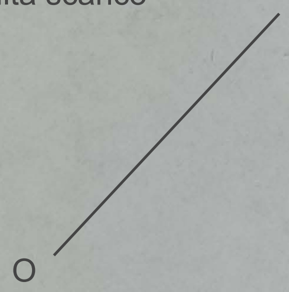
⇓

poiché $\underline{R}_{CO} = 0$

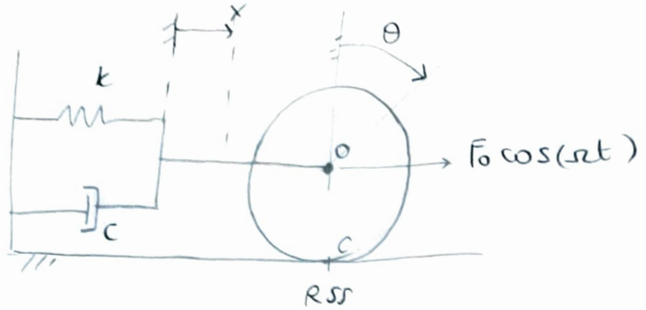
∄ A.C.

$C = 15$ N/m

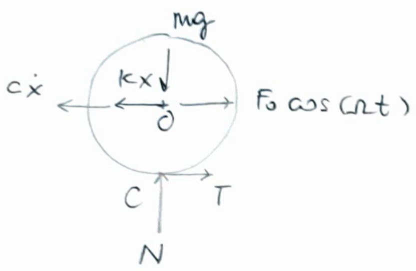
corpo 1 risulta scarico



Esercizio 3 - Dinamica



$$\begin{cases} x = r\theta \\ \dot{x} = r\dot{\theta} \\ \ddot{x} = r\ddot{\theta} \end{cases}$$



$\Rightarrow \quad \ddot{\theta}, N, T \quad \Rightarrow 3 \text{ INC} + 3 \text{ EQ}^{NI}$

$$C \circlearrowright \quad (kx + c\dot{x})\, r - F_0 \cos(\Omega t)\, r = M_C^{(ma)}$$

$$\underline{M}_C^{(ma)} = -J_C \ddot{\theta} \underline{k} + m \cancel{\overrightarrow{CG} \wedge \underline{a}_C}^{\,0} \qquad \text{con } G \equiv O$$

$$= -J_G \ddot{\theta} \underline{k} + m \overrightarrow{CG} \wedge \underline{a}_G$$

$$\underline{a}_G = \ddot{\theta} r \underline{i}$$

$$= -J_G \ddot{\theta} \underline{k} + m r \ddot{\theta} r (-\underline{k}) = -J_C \ddot{\theta} \underline{k}$$

$$J_C \ddot{\theta} + c r^2 \dot{\theta} + k r^2 \theta = F_0 \cos(\Omega t)\, r$$

$$\omega_n = \sqrt{\frac{K_{eq}}{M_{eq}}} = \sqrt{\frac{k r^2}{J_C}}$$

$$\xi = \frac{C_{eq}}{2 m_{eq} \omega_n} = \frac{c r^2}{2 J_C \omega_n} = \frac{c}{2\sqrt{km}} = \frac{c r^2}{2\sqrt{J_C k r^2}}$$

$$\theta(t) = \theta_{om}(t) + \theta_p(t)$$

$$\theta_p(t) = \theta_0 \cos(\Omega t + \varphi)$$

$$\tilde{\theta}_0 = \frac{(F_0/k)}{\sqrt{\left(1 - \left(\frac{\Omega}{\omega_n}\right)^2\right)^2 + \left(2\xi \frac{\Omega}{\omega_n}\right)^2}} =$$

$$\operatorname{tg} \tilde{\varphi} = \frac{2\xi\, \Omega/\omega_n}{1 - \left(\frac{\Omega}{\omega_n}\right)^2} =$$

$$\begin{cases} \omega_n = 0.544 \\ \xi = 0.02 \\ X_0 = 377° = 6.59 \text{ rad} \end{cases}$$