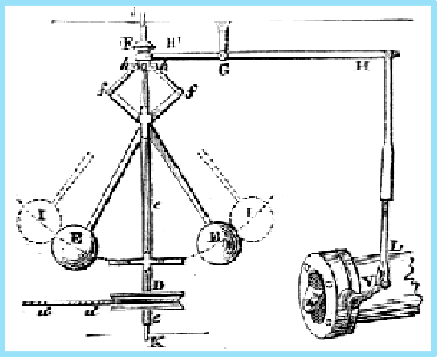
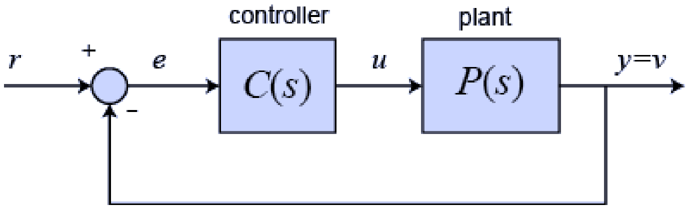


$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

$$\begin{bmatrix} B & AB & \dots & A^{k-1}B \end{bmatrix}$$

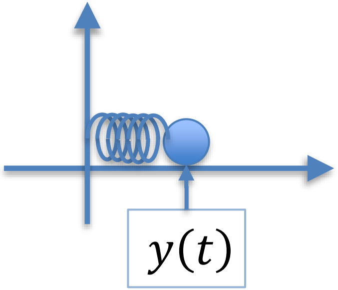
Teoria della stabilità di Lyapunov

Prof.ssa Lucia Pallottino



Corso di Laurea Magistrale in Ingegneria Robotica e dell'Automazione

System: armonic oscillator



$$\ddot{y}(t) + \omega^2 y(t) = u(t)$$

$$\begin{cases} x_1(t) = y(t) \\ x_2(t) = \dot{y}(t) \end{cases}$$

Position
Velocity

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -\omega^2 x_1(t) + u(t) \end{cases}$$

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -x_1(t) + u(t) \end{cases}$$

$$\omega = 1$$

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$

$$\lambda = \pm j$$

$$x(0) = \begin{pmatrix} \alpha \\ 0 \end{pmatrix}$$

$$\begin{cases} x_1(t) = \alpha\cos(t) \\ x_2(t) = -\alpha\sin(t) \end{cases}$$

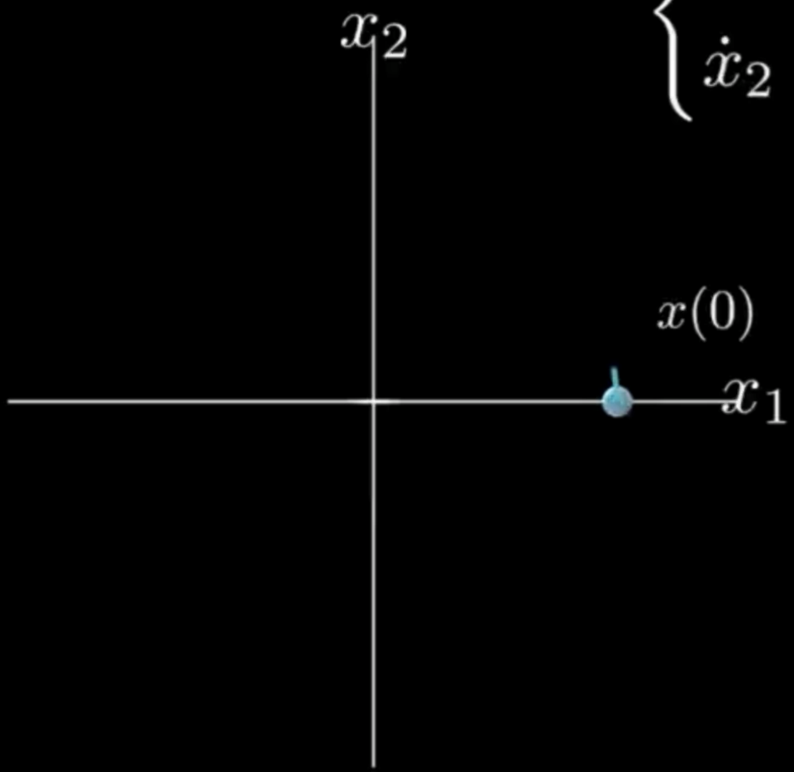
Solution

$$\begin{cases} x_1(t) = \cos(t) \\ x_2(t) = -\sin(t) \end{cases}$$

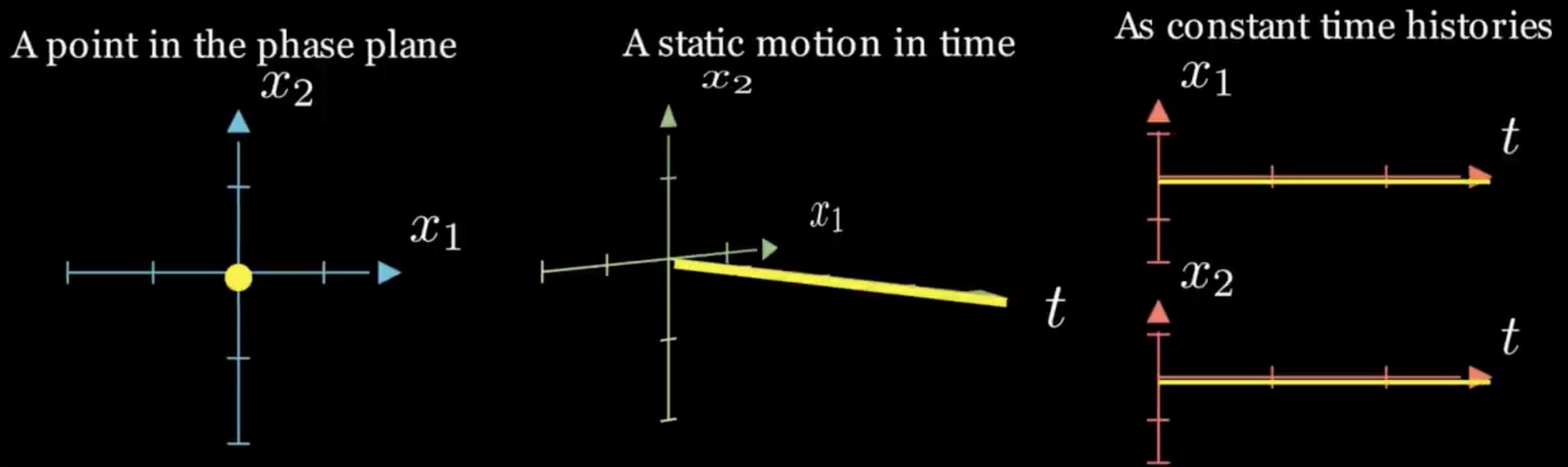
$$\alpha = 1$$

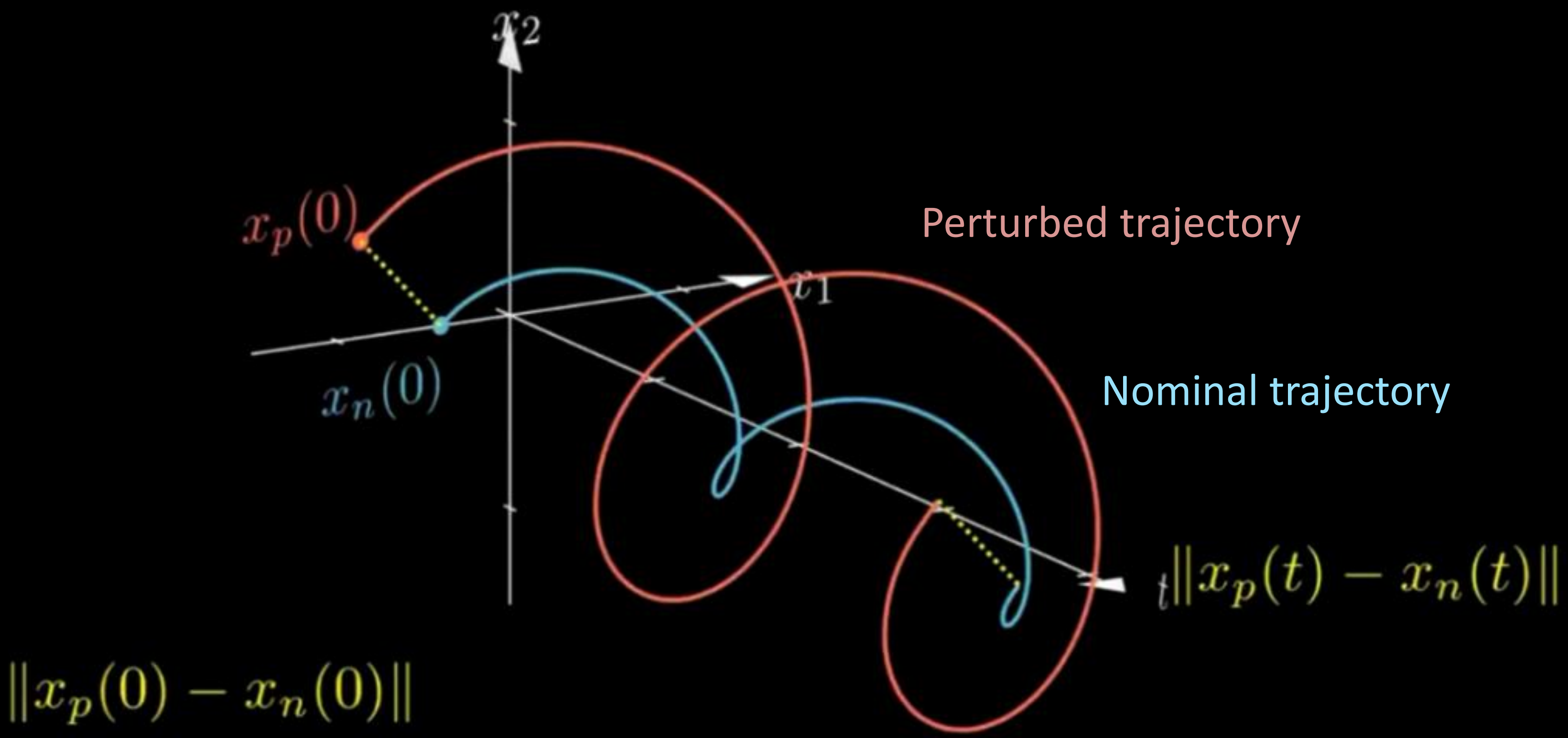
Solution

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 \end{cases}$$



Thus, the point (0,0) is an equilibrium for the harmonic oscillator.





x_2
x_1
t
$x_p(0)$
$x_n(0)$
Perturbed trajectory
Nominal trajectory
$\|x_p(t) - x_n(t)\|$
$\|x_p(0) - x_n(0)\|$

Video

Visual Explanation of Lyapunov Stability

Prof. Giordano Scarciotti (Imperial College)

https://www.youtube.com/watch?v=W8YpgG0KuOo