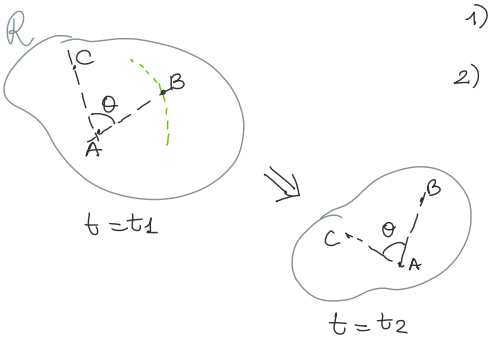
Statica del corpo rigido

venerdì 4 ottobre 2024 12:51

CORPO RIGIDO ⇒ VINCOLO DI RIGIDITÀ

1) $\overline{AB}$ è cost $\forall A, B \in R$

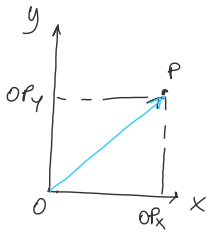
2) $\widehat{BAC} = \theta$ orientamento relativo di due segmenti



GDL ⇒ GRADI DI LIBERTÀ ⇒ n° param. indipen. ⇒ POSA → POSIZIONE, ORIENTAMENTO

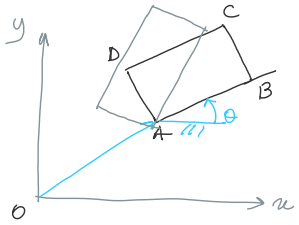
⇒ CASO PIANO +2D

* PUNTO MATERIALE +



$\overrightarrow{OP} \Rightarrow \begin{pmatrix} OP_x \\ OP_y \end{pmatrix} \Rightarrow$ 2 INFORM.

CORPO RIGIDO



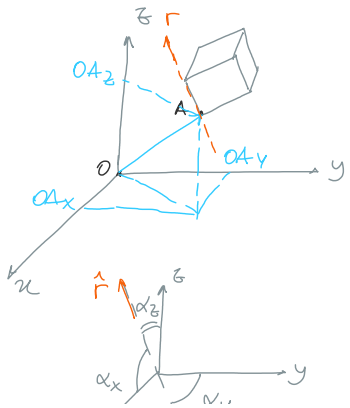
POSA → TRASLAZIONE IN 2 DIREZ ⊥
- a) POSIZIONE ⇒ POSIZ. PUNTO ∈ R. $\overrightarrow{OA} = \begin{pmatrix} OA_x \\ OA_y \end{pmatrix}$ (2)
- b) ORIENTAMENTO ⇒ θ (1)

→ ROTAZIONE $\underline{k}$

⇒ CASO 3D

TRASLAZIONI

a) POSIZIONE ⇒ $\overrightarrow{OA} = \begin{pmatrix} OA_x \\ OA_y \\ OA_z \end{pmatrix} \Rightarrow$ 3 GDL



b) ORIENTAMENTO ⇒ $\alpha_x, \alpha_y, \alpha_z$

COSENI DIRETTORI $\cos\alpha_x, \cos\alpha_y, \cos\alpha_z$

$\cos\alpha_x = \hat{r} \cdot \underline{i}$ $\cos\alpha_y = \hat{r} \cdot \underline{j}$ $\cos\alpha_z = \hat{r} \cdot \underline{k}$

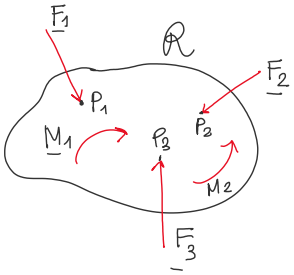
$\cos^2\alpha_x + \cos^2\alpha_y + \cos^2\alpha_z = 1$

3 GDL

3 ROTAZIONI

NB
- CORPO RIGIDO NEL 2D ⇒ (3) GDL
- CORPO RIGIDO NEL 2D ⇒ (6) GDL

– STATICA DEL CORPO RIGIDO –

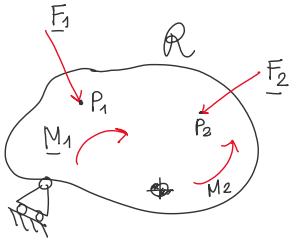


SUL CORPO R AGISCONO
- FORZE $(P_i, \underline{F}_i)$
- MOMENTI M_j

} → ESTERNE (ATTIVE, REATTIVE)

ATTIVE: applicate direttamente al corpo

REATTIVE: applicate da vincoli (esterni) → REAZIONI VINCOLARI



|| Eq.ni CARDINALI della STATICA ↓ || COND. NECESS. SUFFICIENTE

$$\Rightarrow \begin{cases} \underline{R}^{(ext)} = \underline{0} = \underline{R}^{(ext,a)} + \underline{R}^{(ext,r)} = \underline{0} \\ \underline{M}_O^{(ext)} = \underline{0} = \underline{M}_O^{(ext,a)} + \underline{M}_O^{(ext,r)} = \underline{0} \end{cases}$$

Polo "O" ARBITRARIO

	2D	3D
$\underline{R}^{(ext)} = \underline{0}$	$R_x^{(ext)} = 0$, $R_y^{(ext)} = 0$	$R_x^{(ext)} = 0$, $R_y^{(ext)} = 0$, $R_z^{(ext)} = 0$
$\underline{M}_O^{(ext)} = \underline{0}$	$M_{Oz}^{(ext)} = 0$	$M_{Ox}^{(ext)} = 0$, $M_{Oy}^{(ext)} = 0$, $M_{Oz}^{(ext)} = 0$
	3 Eq.ni SCALARI	6 Eq.ni SCALARI

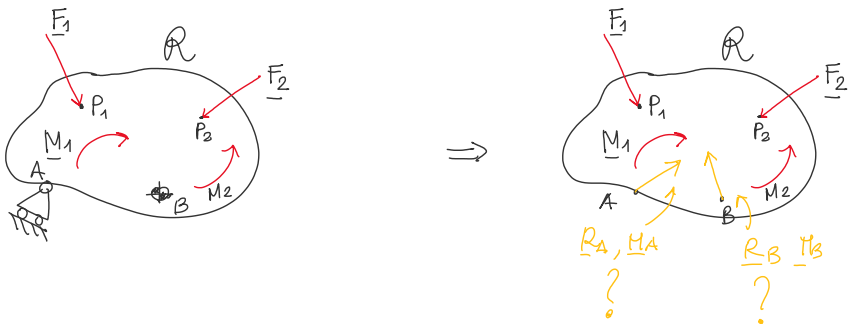
↓

3 INCOGNITE

PROBLEMA ISOSTATICO

(DCL)

DIAGRAMMA DI CORPO LIBERO



ANALISI GEOMETRICA ⟷ ANALISI FISICA

⇓

GDL TOLTI DAL VINCOLO

↓

CORPO PUO' MUOVERSI

CORPO LABILE?

⇓

Qual'è la reazione vincolare?

⇒ Quali sono le incognite?

↓

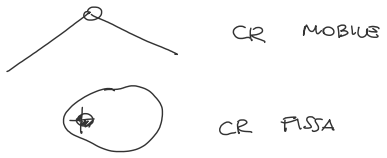
Quante sono?

⇓

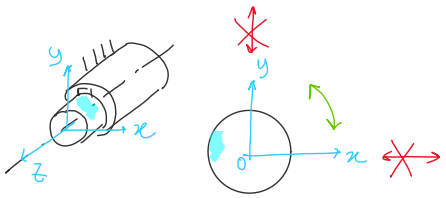
2D ⇒ 3 INCOG. + 3 Eq^{ni} ⇒ PROBLEMA RISOLVIBILE ISOSTATICO

ANALISI VINCOLI PIANI LISCI BILATERALI

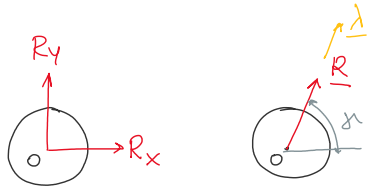
COPPIA ROTOIDALE O CERNIERA (CR)



AN. GEOMETRICA

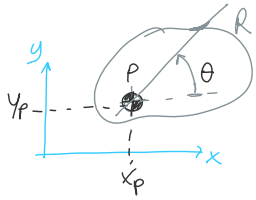


AN FISICA



BLOCCATO : T_x, T_y — 2 GDL ⇒ x_p = cost, y_p = cost

CONSENTITO : R_z + 1 GDL ⇒ $\theta(t)$

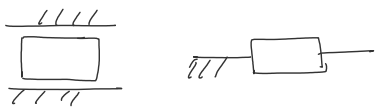


(O, R)

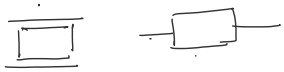
2 INCOGNITE, ⇒ R_x, R_y oppure |R|, direzione λ

TELAIO

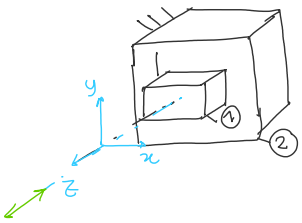
COPPIA PRISMATICA (CP)

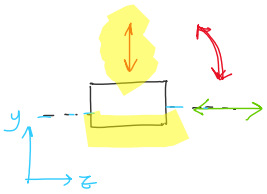


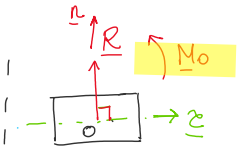
CP FISSA



CP MOBILE







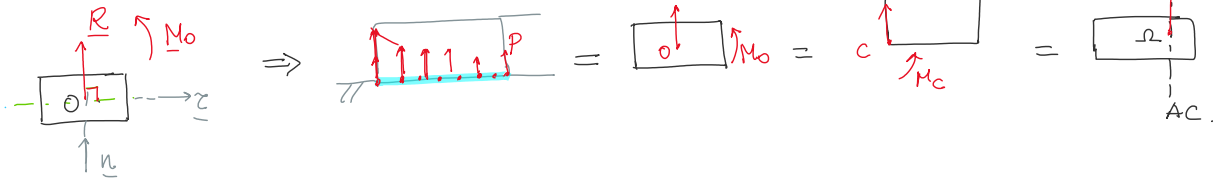
BLOCCARE: $T_y, R_x \Rightarrow -2$ GDL $\rightarrow \theta = cost$, $y_p = cost$

CONSENTO: $T_z \Rightarrow 1$ GDL $\rightarrow z(t)$

$(O, \underline{R}), \underline{M_O}$

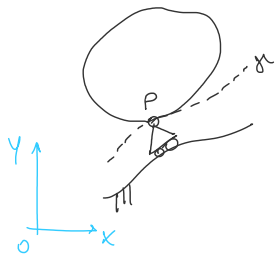
R_n | M_{Ox}

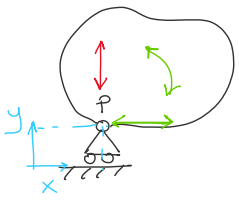
2 INCOGNITE

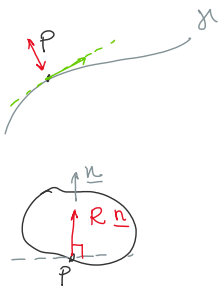


CARRELLO

$\Rightarrow$ OBBLIGA UN PUNTO DI $\mathcal{R}$ A PERCORRERE γ







BLOCCATA $T_y \Rightarrow y_p$ cost -1 GDL

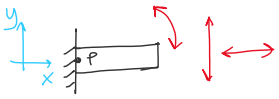
CONSENTITO $T_x, R_z \Rightarrow x_p(t)$, $\theta(t)$ $+2$ GDL

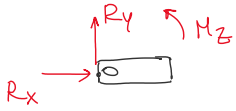
$(P, R\underline{n})$

$\downarrow$

1 INCOGNITA $\Rightarrow R$

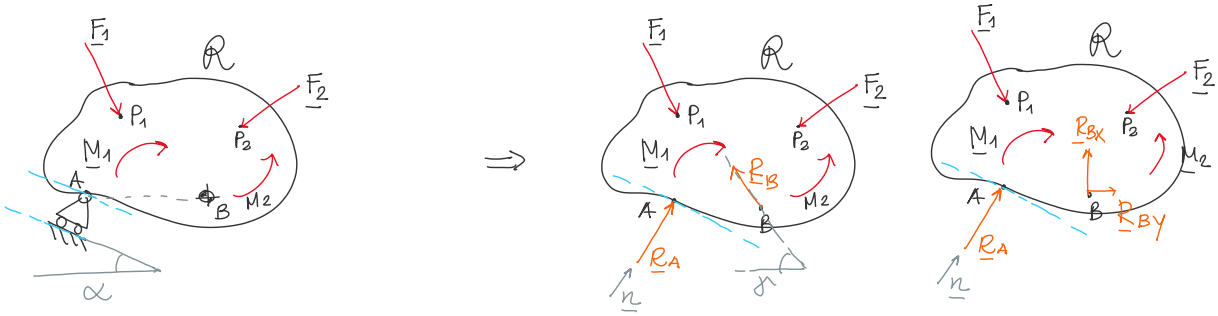
IN CASTRO





BLOCCATO T_x, T_y, R_z $\Rightarrow$ -3 GDL $\Rightarrow$ $(O, \underline{R})$, $\underline{M}_z$ $\Rightarrow$ 3 INCOGNITE

DIAGRAMMA DI CORPO LIBERO (DCL)

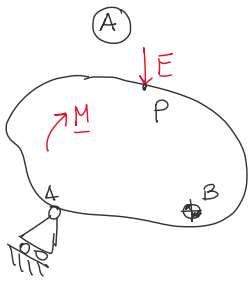


CARRELLO $\Rightarrow (A, \underline{R}_A \underline{n})$ $\Rightarrow$ R_A ?

CERNIERA $\Rightarrow (B, \underline{R}_B)$ $\Rightarrow$ R_B, α ?

R_{Bx}, R_{By} ?

PROBLEMI DI STATICA

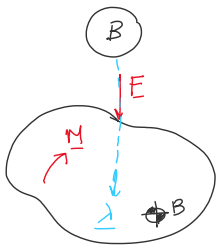


1) CORPO NON PUO' MUOVERSI $\Rightarrow$ 0 GDL

OBIETTIVO $\Rightarrow$ REAZ. VINCOLARI
X AVERE EQUILI. STATICO

$\Rightarrow$ NOTO, $\underbrace{\underline{R}^{(ext,a)}, \underline{M}_O^{(ext,a)}}_{\text{CORPO IN EQUILIBRIO } \forall \text{ SIST. AZIONI ATTIVE}}$ $\Rightarrow$ $\underline{M}, (P, \underline{F})$

VALUTARE $\Rightarrow$ LE REAZIONI VINCOLARI



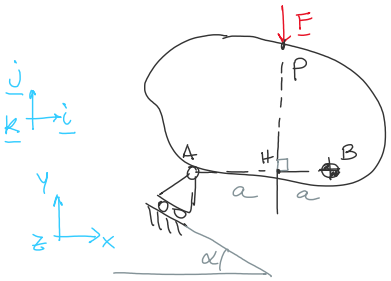
1) CORPO LABILE
$\Downarrow$
BLOCCARE GDL

$\Rightarrow$ NOTO $\underline{M}$, DIREZIONE di $\underline{F}$

$\Rightarrow$ OBIETTIVO $\Rightarrow$ QUANTO VALE F ?

ESEMPIO: CASO (A)

NOTO | RICHIESTO



$\alpha = 45°$

$(P, \underline{F})$ $\quad F = 100\ N$

$\overline{AB} = 2a$

$\overline{AH} = a$

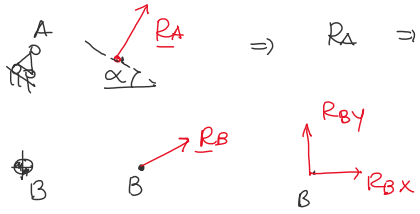
- Reazioni Vincolari
- DCL

1) SDR

2) AN. GEOMETRICA

$$n° GDL = 3 - \underset{CR}{2} - 1 = 0$$

3) AN FISICA



$\Rightarrow R_A \Rightarrow R_A \underline{u}$ (1)

$+$

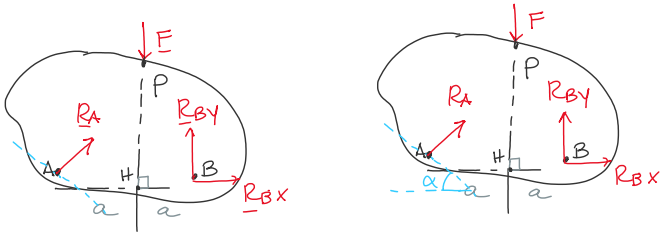
$\Rightarrow R_{BX}, R_{BY}$ (2) $= 3$ INCOGNITE

$\Rightarrow$ PROB. ISOSTATICO ✓ 3 INCOGNITE R_A, R_{BX}, R_{BY}

$=$

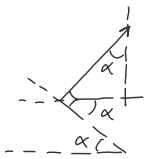
3 EQUAZ.

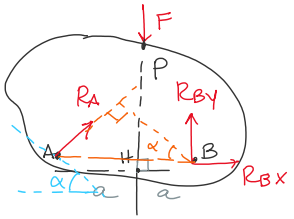
4) DCL PRELIMINARE



5) $E_p^{m.}$ CARDINALI STATICA

$$\begin{cases} \underline{R}^{(ext)} = \underline{0} = \underline{R}^{(ext,a)} + \underline{R}^{(ext,r)} \\ \qquad \underline{F} + \underline{R}_A + \underline{R}_B = \underline{0} \quad \rightarrow X, Y \\ \underline{M}_B^{(ext)} = \underline{0} = \underline{M}_B^{(ext,a)} + \underline{M}_B^{(ext,r)} \\ \qquad = \overline{BP} \wedge \underline{F} + \overline{BA} \wedge \underline{R}_A = \underline{0} \quad \rightarrow Z \end{cases}$$





X: $R_A \sin\alpha + R_{BX} = 0$

Y: $-F + R_{BY} + R_A \cos\alpha = 0$

Z: $-F \cdot a + R_A \cdot a \cos\alpha = 0$

B) [illegible]

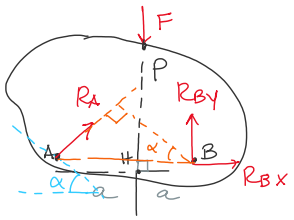
$$\begin{cases} R_A = \dfrac{F}{2\cos\alpha} = \dfrac{F}{2\frac{\sqrt{2}}{2}} \\ R_{Bx} = -R_A \sin\alpha = -\dfrac{F}{2\cos\alpha}\sin\alpha = -\dfrac{F}{2}\tan\alpha = -\dfrac{F}{2}\dfrac{\sqrt{2}}{2}\dfrac{2}{\sqrt{2}} = -\dfrac{F}{2} \\ R_{By} = -R_A\cos\alpha + F = -\dfrac{F}{2\cos\alpha}\cos\alpha + F = \dfrac{F}{2} \end{cases}$$

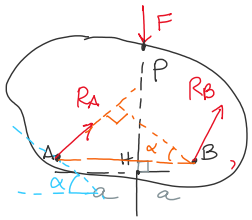
$R_{Bx} = R_{By}$

$\gamma = 45$

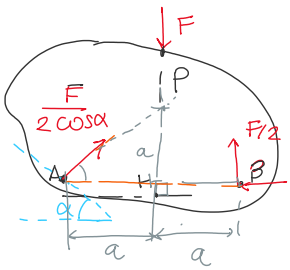
$$|\underline{R}_B| = R_B = \sqrt{R_{Bx}^2 + R_{By}^2} = \frac{F}{\sqrt{2}}$$

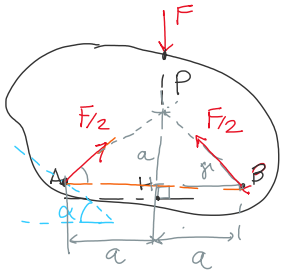
DCL PRELIM.

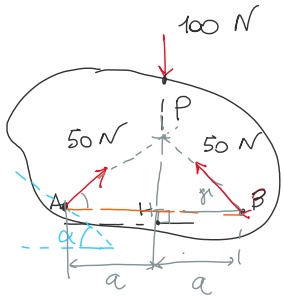




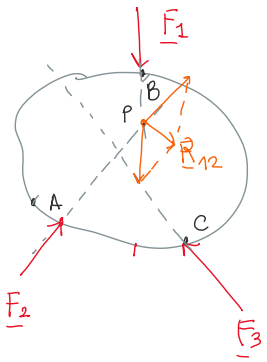
DCL DEFINITIVO







CASO PARTICOLARE

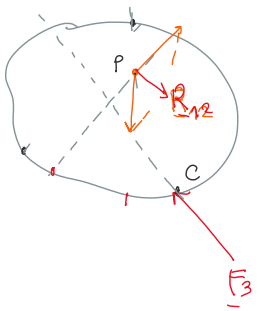


3 FORZE COMPLANARI SU R

$$\begin{cases} \underline{R}^{(ext)} = \underline{0} \\ \underline{M}_O^{(ext)} = \underline{0} \end{cases}$$

$$\rightarrow \quad \underline{F}_1 + \underline{F}_2 = \underline{R}_{12}$$

$$\underline{R}^{(ext)} = \underline{R}_{12} + \underline{F}_3 = \underline{0}$$



$$\rightarrow \quad \underline{M}_P^{(ext)} \neq \underline{0}$$

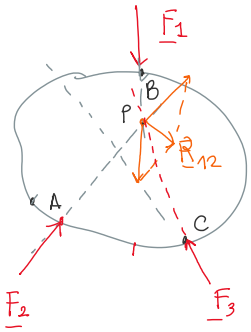
$$\underline{M}_C^{(ext)} \neq \underline{0}$$

$\Rightarrow$ EQUIL. ALLA ROTAZIONE $\rightarrow$ $\underline{M}_O^{(ext)} = \underline{0}$

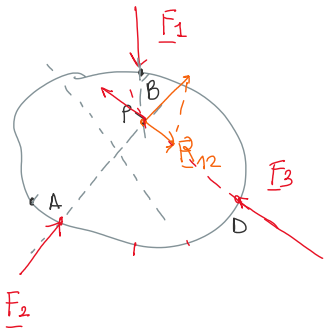
$\Downarrow$

LE 3 FORZE DEVONO ESSE CONCORRENTI $\subset$ IN UNO STESSO PUNTO

C N ~~S~~



$$\begin{cases} \underline{M}_P^{(ext)} = \underline{0} \\ \underline{R}^{(ext)} = ? \end{cases}$$

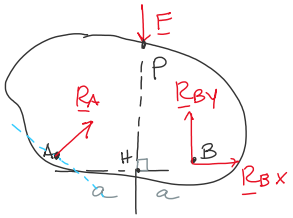


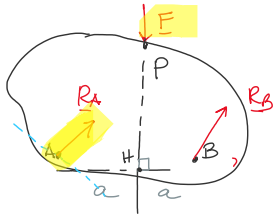
$$\underline{R}_{12} + \underline{F}_3 = \underline{F}_1 + \underline{F}_2 + \underline{F}_3 = \underline{0}$$

$$\Downarrow$$

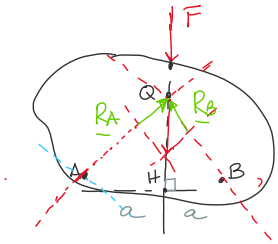
$$\underline{R}^{(ext)} = \underline{0}$$

ESEMPIO PRECEDENTE





SOLUZIONE GRAFICA

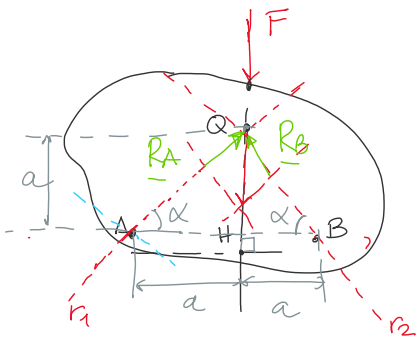


$\underline{R}_A \Rightarrow$ NOTA DIREZ

$\underline{R}_B \Rightarrow$ PASSA PER B (APPLICATA IN B)

$\downarrow$

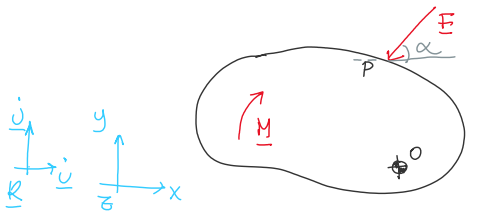
3 FORZE $\Rightarrow$ CONCOR. $\Rightarrow$ PASSA PER Q



$$\Rightarrow \quad \underline{F} + \underline{R}_A + \underline{R}_B = \underline{0}$$

$$\underline{F} = -(\underline{R}_A + \underline{R}_B)$$

ESEMPIO : CASO (B)



NOTO : $\underline{M}$, direzione di $\underline{F}$ (α)

VALUTARE : $|E|$?

Reazioni vincolari ?

1) SDR

2) AN. GEOMETRICA

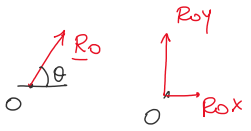
$\Phi O \Rightarrow -2$ gdl

$\Downarrow$

n° gdl = $3 - 2 = +1$ (CR)

↓

CABILE



AN. FISICA

2 INCOGNITE REATTIVE < $|\underline{R}_0|$, θ ; R_{0x}, R_{0y}

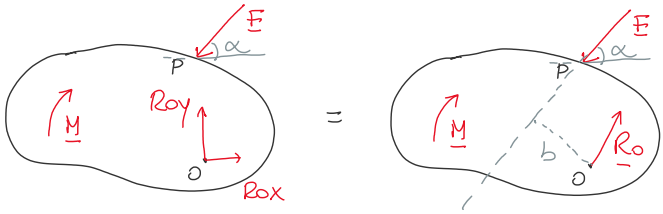
\+

1 INCOG. ATTIVA → F

$\Downarrow$

3 INCOG. + 3 EQUAZ. } $\Rightarrow$ PROB. ISOSTATICO

3) DCL PRELIMINARE



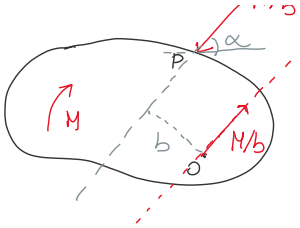
4) EQ.NI CARDINALI

$$O\begin{cases} \underline{R}^{(ext)} = \underline{F} + \underline{R}_0 = \underline{0} \\ \underline{M}_O^{(ext)} = \underline{M} + \overrightarrow{OP} \wedge \underline{F} = \underline{0} \end{cases}$$

$$\begin{cases} x: & -F\cos\alpha + R_{0x} = 0 \\ y: & -F\sin\alpha + R_{0y} = 0 \\ z: & -M + Fb = 0 \end{cases}$$

$$\begin{cases} F = \dfrac{M}{b} \\ R_{0x} = F\cos\alpha = \dfrac{M}{b}\cos\alpha \\ R_{0y} = F\sin\alpha = \dfrac{M}{b}\sin\alpha \end{cases} \quad R_0 = \sqrt{R_{0x}^2 + R_{0y}^2} = \frac{M}{b}$$

DCL DEFINITIVO



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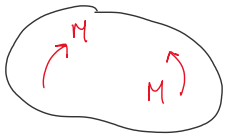
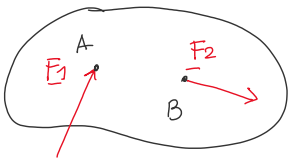
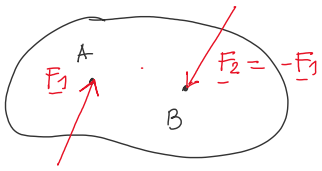


$M = Fb$

CASO PARTICOLARE

CORPO RIGIDO SU CUI AGISCONO 2 FORZE

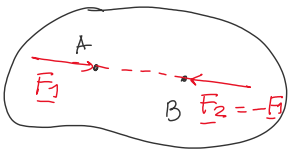




$\underline{F}_1 = -\underline{F}_2 \Rightarrow \underline{R}^{(ext)} = \underline{F}_1 + \underline{F}_2 = \underline{0}$

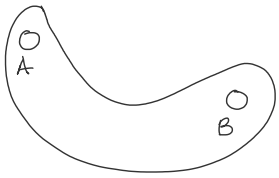
2 FORZE ⇓ CON. NECES. SUFF. EQ. STATICO

N.B.

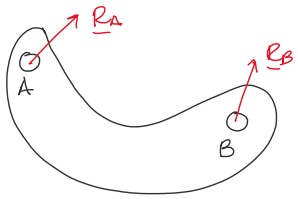


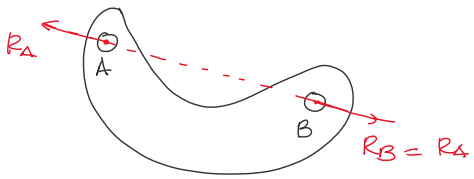
$\underline{M}_A = \underline{0} \Rightarrow$ $\underline{F}_1$ e $\underline{F}_2$ ⇒ COPPIA A BRACCIO NULLO

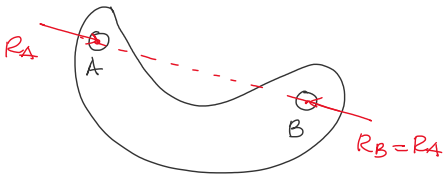
CORPO R SCARICO



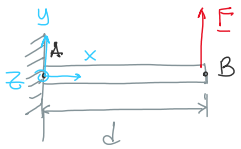
DCL PRELIM.







ESERCIZIO



Noto: $(B, \underline{F}) \Rightarrow |\underline{F}| = 1000\,N$

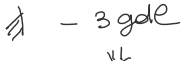
$d = \overline{AB} = 4\,m$

Valutare: 1) Reazione vincolare all'incastro

2) SE delle reaz. vincolari rispetto "B"

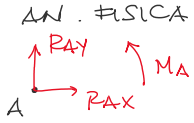
•) SDR

•) AN. GEOM.



—

AN. FISICA



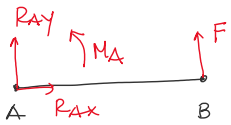
$(A, \underline{R}_A)$, M_A

⇓

$ngdl = 3 - 3 = 0$

3 INCOGNITE + 3 EQ^{NI} CAR. STAT. } ISOSTAT.

1) DCL PRELIMINARE



$\underline{R}_A = -\underline{F}$

I e II CARD. STATICA

$\underline{R}^{(ext)} = \underline{0} = \underline{F} + \underline{R}_A = \underline{0}$

$A \uparrow \quad \underline{M}_A^{(ext)} = \underline{0} = \overrightarrow{AB} \wedge \underline{F} + \underline{M}_A = \underline{0}$

ICS

$x: R_{Ax} = 0$

$y: R_{Ay} + F = 0 \rightarrow R_{Ay} = -F < 0$

$z: F\,b + M_A = 0 \quad M_A = -Fb < 0$

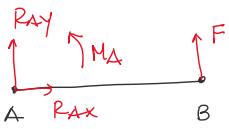
$R_{Ay} = -1000\ N$

$M_A = -4000\ Nm$

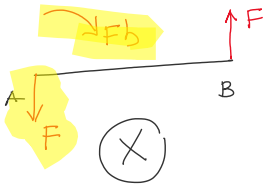
$\underline{R}_A = R_{Ay}\,\underline{j} = -F\,\underline{j}$

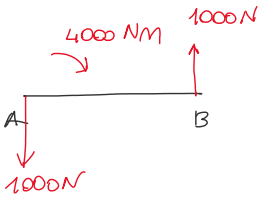
$\underline{M}_A = M_A\,\underline{k} = -Fb\,\underline{k}$

DCL PRELIMINARE



DCL DEFINITIVO





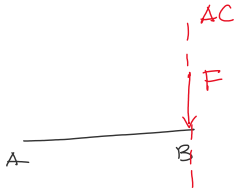
2) Dato $(A, \underline{R}_A)$, $\underline{M}_A \Rightarrow$ SE $\Rightarrow (B, \underline{R}_A), \underline{M}_B$

$\underline{M}_B = \overrightarrow{BA} \wedge \underline{R}_A + \underline{M}_A = +b\,F\,\underline{k} + (-Fb\,\underline{k}) = \underline{0}$

↳ SE $(B, \underline{R}_A) \Rightarrow (B, -F\underline{j})$

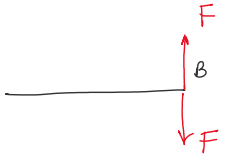
AC della REAZIONE VINCOLARE !!!

SE =



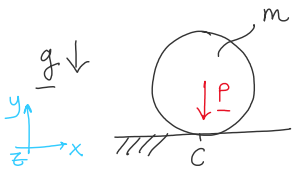
↓

DCL DEF

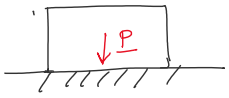


VINCOLI NEL PIANO, UNILATERALI

VINCOLO DI APPOGGIO SEMPLICE (SENZA ATTRITO)



CONTATTO PUNTIFORME
NONCONFORME



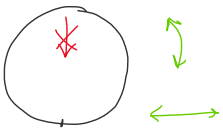
CONTATTO CONFORME
O ESTESO

UNILATERALE ⇒ CONSENTITO IL DISTACCO del CORPO dalla superficie

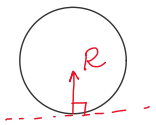
AN. GEOM.

BLOCCA : 1 TRASLAZ ⇒ y ⇒ T_y } — 1 gdl

CONSENTE : 1 TRAS + ROTAZ = T_x, R_z



AN. FISICA



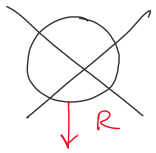
⇒ $(C, \underline{R}) \Rightarrow (C, \underline{R}_n \underline{j})$

$R > 0$

NOTA DIREZIONE + VERSO = ENTRANTE NEL CORPO!

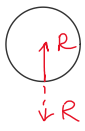
↓

L'APPOGGIO NON TIRA



APP. SEMPLICE VS CARRELLO

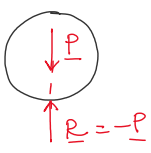
$R > 0$

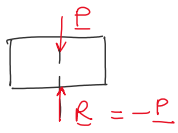


$R \gtrless 0$

⇒ ESEMPIO

DCL





ELEMENTI FLESSIBILI : FUNI, CAVE

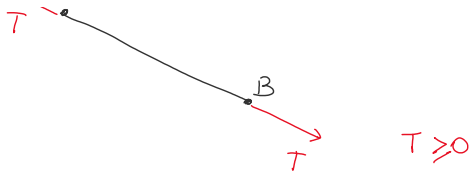
IL … INESTENSIBILE …

2) PRIVA DI MASSA

3) NO ATTRITO

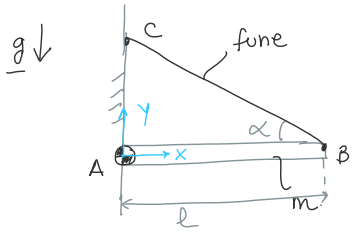
⇓

SOLO TRAZIONE



AN. GEOMG. — 1 GDL ⇒ AN. FISICA ⇒ T

ESEMPIO



NOTO : m, ℓ

Valutare : – tensione fune
– DCL BARN. della trave

°) AN. GEOMET.

⊕ — 2 gdl

FUNE — 1 gdl

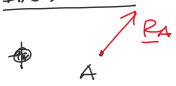
— 3 gdl

+ 3 CORPI

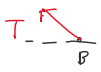
∅ GDL ⇒ CORPO BLOCCATO

3 INCOG. + 3 Eq^{ni} ⇒ CASO ISOSTAT.

AN. FISICA

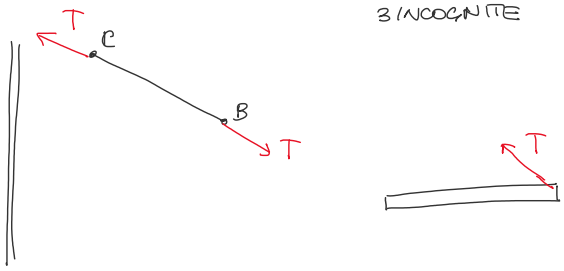


(A, $\underline{R_A}$) → 2 INCOGNITE

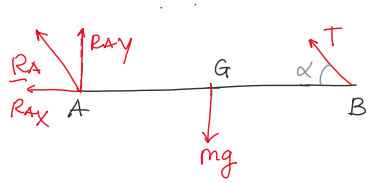


(B, $\underline{T}$), con direzione nota (α) + verso noto

3 INCOGNITE



°) DCL PRELIMINARE

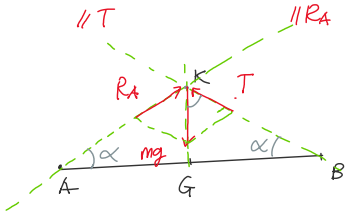


°) EQ CARDINALI STATICA

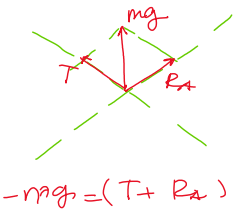
$\underline{R}^{ext} = \underline{0} = \underline{R_A} + m\underline{g} + \underline{T} = \underline{0}$

$\underline{M}_A^{(ext)} = \underline{0} = \overrightarrow{AG} \wedge m\underline{g} + \overrightarrow{AB} \wedge \underline{T} = \underline{0}$

SOLUZ GRAFICA ⇒ 3 forze CONCOR. IN UN PUNTO



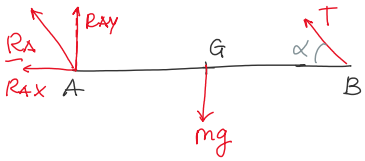
$m\underline{g} = -(\underline{T} + \underline{R_A})$



$-m\underline{g} = (\underline{T} + \underline{R_A})$

SOLUZ ANALITICA

$$\begin{cases} x: & -R_{Ax} - T\cos\alpha = 0 \\ y: & R_{Ay} + T\sin\alpha - mg = 0 \\ z: & -mg\frac{\ell}{2} + T\sin\alpha\,\ell = 0 \end{cases}$$



$$T = \frac{mg}{2\sin\alpha}$$

$$R_{Ax} = -T\cos\alpha = -\frac{mg}{2\sin\alpha}\cos\alpha = -\frac{mg}{2\tg\alpha} < 0$$

$$R_{Ay} = mg - T\sin\alpha = mg - \frac{mg}{2} = \frac{mg}{2}$$

$$R_A = \sqrt{\left(\frac{mg}{2}\right)^2 + \left(\frac{mg}{2\tg\alpha}\right)^2} = \frac{mg}{2}\sqrt{1 + \frac{1}{\tg^2\alpha}} = \frac{mg}{2}\sqrt{\frac{\sin^2\alpha + \cos^2\alpha}{\sin^2\alpha}} = \frac{mg}{2\sin\alpha}$$

$$\boxed{R_A = T}$$

DCL DEF