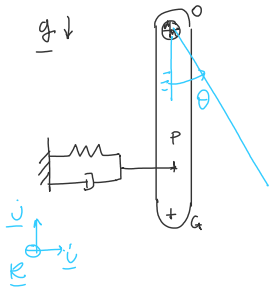
Esercitazione: oscillazioni libere smorzate

giovedì 12 dicembre 2024 17:27



Dati

$m = 2.5\ kg$

$J_G = 0.5\ kg\,m^2$

$a = \overline{OP} = 0.7\ m$

$b = \overline{PG} = 0.4\ m$

$k = 200\ N/m$

$c = 20\ Ns/m$

Richiesto

1) DCL preliminare

2) Eq.ne moto forma parametrica

3) $\omega_n, \xi \Rightarrow$ parametrica + numerica

↓ che tipo di oscillazioni?

4) legge oraria → numerica / disegnarla

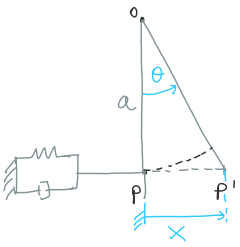
Hp: PICCOLE OSCILLAZIONI

C.I. ⇒ •) asta verticale

•) $\omega(0) = 1{,}5\ rad/s$

↳ $\theta(0) = 0$, $\dot\theta(0) = \omega_0$

•) $n_{gdl} = 3 - 2_{\text{CERNIERA}} = 1 \Rightarrow$ 1 COORDIN. $\Rightarrow \theta$

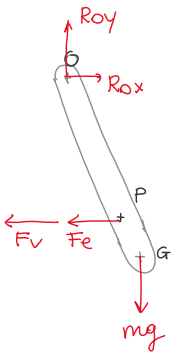


pp.ll. $\overline{OP}\sin\theta \simeq \overline{OP}\,\theta \Rightarrow x \simeq a\sin\theta \overset{!}{\simeq} a\theta$

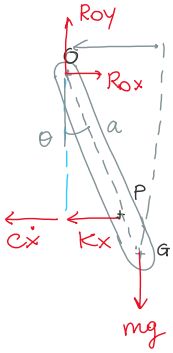
EQ.NI DI CONGRUENZA

$$\Rightarrow \begin{cases} x = a\theta \\ \dot x = a\dot\theta \\ \ddot x = a\ddot\theta \end{cases}$$

DCL



$\theta > 0,\ \dot\theta > 0 \Rightarrow x > 0,\ \dot x > 0$



INCOGNITE (3)

Rox, Roy, $\ddot\theta$

⇓

EQUAZ. < IICD (2) / IICD (1)

IICD O ↷

$$\underline{M}_O^e = \underline{M}_O^{(ma)} = J_O\,\ddot\theta\,\underline{k} + m\,\overrightarrow{OG} \wedge \underline{a}_O = J_O\,\ddot\theta\,\underline{k} \quad (1)$$

$$= J_G\,\ddot\theta\,\underline{k} + m\,\overrightarrow{OG} \wedge \underline{a}_G \quad (2)$$

$$\underline{a}_G = \ddot\theta\,\underline{k} \wedge \overrightarrow{OG} - \dot\theta^2\,\overrightarrow{OG}$$

$$(\underbrace{-Kx}_{a\theta} - \underbrace{c\dot x}_{a\dot\theta})\,a\cos\theta - mg(a+b)\underbrace{\sin\theta}_{\theta} = J_O\,\ddot\theta$$

$$-Ka^2\theta - ca^2\dot\theta - mg(a+b)\theta = J_O\,\ddot\theta$$

$$\boxed{J_O\,\ddot\theta + ca^2\dot\theta + [Ka^2 + mg(a+b)]\,\theta = 0}$$

J_{eq} c_{eq} k_{eq}

$$J_{eq}\,\ddot{\theta} + C_{eq}\,\dot{\theta} + K_{eq}\,\theta = 0$$

$$\updownarrow$$

$$\ddot{\theta} + 2\xi\omega_n\dot{\theta} + \omega_n^2\theta = 0$$

$$\omega_n = \sqrt{\frac{K_{eq}}{J_{eq}}} = \sqrt{\frac{ka^2 + mg(a+b)}{J_O}} = 5{,}95 \text{ rad/s}$$

$$J_O = J_G + m(a+b)^2$$

$$\xi = \frac{C_{eq}}{C_r} = 0{,}48 \Rightarrow \xi < 1 \Rightarrow \text{OSCILL. SOTTOSMOR. PERIODICHE}$$

$$\theta(t) = A\,e^{-\xi\omega_n t}\sin(\omega_s t + \varphi)$$

$$\downarrow$$

$$\omega_s = \omega_n\sqrt{1-\xi^2} = 5{,}24 \text{ rad/s}$$

$$\omega_s < \omega_n$$

A e φ dalle condiz. iniziali $\Rightarrow$ 1) asta verticale

2) $\omega(0) = 1{,}5$ rad/s $\Rightarrow$ $\theta(0) = 0$, $\dot{\theta}(0) = \omega_0$

$$\Rightarrow \begin{cases} \varphi = 0 \\ A = \dfrac{\omega_0}{\omega_s} = 0{,}29 \end{cases}$$

$$\theta(t) = 0{,}29\,e^{-2{,}48t}\sin(5{,}24\,t)$$

4 - 1