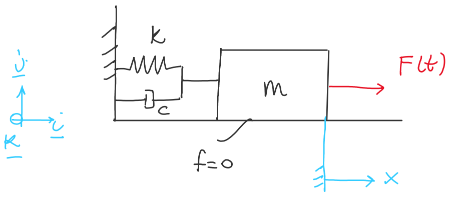
Oscillazioni forzate

venerdì 13 dicembre 2024 10:40



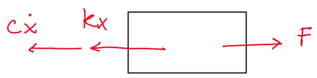
$F(t) = F_0 \cos(\Omega t)$

Ω è LA PULSAZIONE DELLA FORZANTE

COORDINATA $\Rightarrow$ x — c.i. $x=0$, $\dot{x}=0$ $\Rightarrow$ MOLLA e SMORZ. SONO IN CONDIZIONI DI RIPOSO

OBIETTIVO $\boxed{x(t)}$

DCL $\underline{i}$



ICD $\underline{i}$: $-c\dot{x} - kx + F = m\ddot{x}$

$$\boxed{m\ddot{x} + c\dot{x} + kx = F_0 \cos(\Omega t)}$$

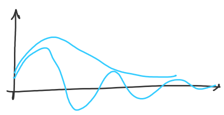
Eq^{ne} MOTO – OSCILL. FORZATE

Eq^{ne} DIFFERENZIALE
2° ORDINE – LINEARE
A COEF. COSTANTI – NON OMOGENEA

$x(t) = x_{om}(t) + x_p(t)$

SOLUZIONE OMOGENEA

$\xi < 1$ OSCIL. PERIO.
$\xi > 1$ OSCIL. APERIO



$t \to \infty \Rightarrow x_{om}(t) \to t$

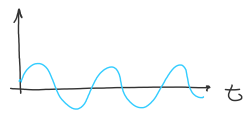
$\Downarrow$

FASE TRANSITORIA
$\Rightarrow$ dipende dalle c.i.

SOLUZIONE PARTICOLARE = SOLUZIONE A REGIME

$$\boxed{x_p(t) = x_0 \cos(\Omega t - \varphi)}$$

– ξ, ω_n
– F_0, Ω



$\Downarrow$

FASE REGIME

$$\boxed{m\ddot{x} + c\dot{x} + kx = F_0 \cos(\Omega t)}$$

$$\boxed{x_p(t) = x_0 \cos(\Omega t - \varphi)}$$

$\dot{x}_p(t) = -x_0 \Omega \sin(\Omega t - \varphi)$

$\ddot{x}_p(t) = -x_0 \Omega^2 \cos(\Omega t - \varphi)$

$$\begin{cases} -m x_0 \Omega^2 \cos(\Omega t - \varphi) - c x_0 \Omega \sin(\Omega t - \varphi) + k x_0 \cos(\Omega t - \varphi) = F_0 \cos(\Omega t) \\ \quad \hookrightarrow x_0 ? \\ \quad \hookrightarrow \varphi ? \end{cases}$$

$\cos(\Omega t - \varphi) = \cos(\Omega t)\cos\varphi + \sin(\Omega t)\sin\varphi$ *

$\sin(\Omega t - \varphi) = \sin(\Omega t)\cos\varphi - \cos(\Omega t)\sin\varphi$

$$-m x_0 \Omega^2 \left[\cos(\Omega t)\cos\varphi + \sin(\Omega t)\sin\varphi \right] - c x_0 \Omega \left[\sin(\Omega t)\cos\varphi - \cos(\Omega t)\sin\varphi \right] +$$

$$+ k \cdot x_0 \left[\cos(\Omega t)\cos\varphi + \sin(\Omega t)\sin\varphi \right] = F_0 \cos(\Omega t)$$

$$\Rightarrow \boxed{A\cos(\Omega t) + B\sin(\Omega t) = F_0\cos(\Omega t)} \Rightarrow \begin{cases} A = F_0 \\ B = 0 \end{cases}$$

$$\overbrace{\left[(k - m\Omega^2)\cos\varphi + c\Omega\sin\varphi \right] x_0}^{A} \cos\Omega t \; +$$

$$+ \underbrace{\left[(k - m\Omega^2)\sin\varphi - c\Omega\cos\varphi \right] x_0}_{B} \sin\Omega t = F_0 \cos(\Omega t)$$

$$\begin{cases} \left[(k - m\Omega^2)\cos\varphi + c\Omega\sin\varphi \right] x_0 = F_0 & (1) \\ (k - m\Omega^2)\sin\varphi - c\Omega\cos\varphi = 0 & (2) \end{cases}$$

ho semplificato $x_0 \neq 0$

$$(2) \rightarrow \frac{\sin\varphi}{\cos\varphi} = \boxed{tg\,\varphi = \frac{c\Omega}{k - m\Omega^2} = \frac{2\xi(\Omega/\omega_n)}{1 - \left(\frac{\Omega}{\omega_n}\right)^2}} \Rightarrow$$

NON DIPENDE DA F_0

$$\xi = \frac{c}{2m\omega_n}$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$(1)^2 + (2)^2$$

$$\begin{cases} \left[(k - m\Omega^2)\cos\varphi + c\Omega\sin\varphi \right]^2 = (F_0/x_0)^2 \\ + \qquad\qquad\qquad\qquad + \\ \left((k - m\Omega^2)\sin\varphi - c\Omega\cos\varphi \right)^2 = 0 \end{cases}$$

$$(k - m\Omega^2)^2 \underbrace{(\cos^2\varphi + \sin^2\varphi)}_{=1} + (c\Omega)^2 \underbrace{(\sin^2\varphi + \cos^2\varphi)}_{=1} + 2c\Omega(\cancel{k - m\Omega^2}) +$$

$$- 2c\Omega(\cancel{k - m\Omega^2}) = \left(\frac{F_0}{x_0}\right)^2$$

$$(k - m\Omega^2)^2 + (c\Omega)^2 = \left(\frac{F_0}{x_0}\right)^2$$

$$\boxed{x_0 = \frac{F_0}{\sqrt{(k - m\Omega^2)^2 + (c\Omega)^2}} = \frac{F_0/k}{\sqrt{\left(1 - \left(\frac{\Omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\Omega}{\omega_n}\right)^2}}}$$

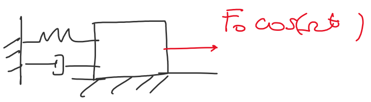
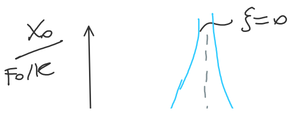
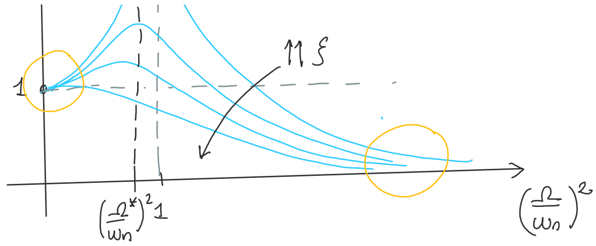


DIAGRAMMA DI AMPIEZZA



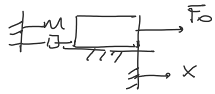


$$\frac{X_0}{F_0/K} = \frac{1}{\sqrt{\left(1-\left(\frac{\Omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\Omega}{\omega_n}\right)^2}}$$

1) se $\Omega = 0 \Rightarrow F = F_0$

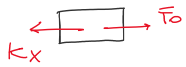
⇓

FORZANTE COSTANTE



$Kx = F_0$

$X = F_0/K$



2) se $\Omega = \omega_n \Rightarrow \frac{\Omega}{\omega_n} = 1$

$$\frac{X_0}{F_0/K} = \frac{1}{2\xi}$$

se $\xi \to 0 \Rightarrow X_0 \to \infty$ CONDIZIONE DI RISONANZA

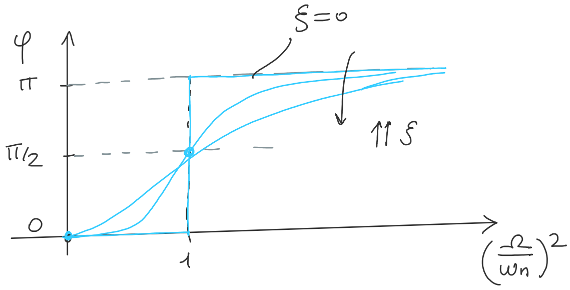
⇓

ROTTURA

3) se $\Omega \gg \omega_n$

RISPOSTA MAX $\Rightarrow \Omega^* = \omega_n\sqrt{1-2\xi^2}$

DIAGRAMMA DI FASE

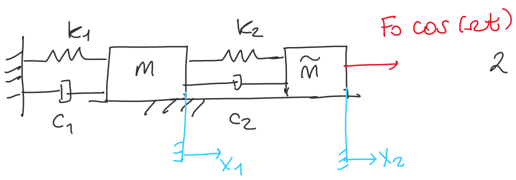


$$\tan\varphi = \frac{2\xi\,\Omega/\omega_n}{1-\left(\frac{\Omega}{\omega_n}\right)^2}$$

1) se $\Omega = 0 \rightarrow \tan\varphi = 0 \rightarrow \varphi = 0$

2) se $\Omega = \omega_n \Rightarrow \tan\varphi \longrightarrow \infty \Rightarrow \varphi = \pi/2$

3) se $\Omega \gg \omega_n \Rightarrow \tan\varphi \to 0^- \Rightarrow \varphi = \pi$



2 gdl $\Rightarrow \omega_{n1}, \omega_{n2}$