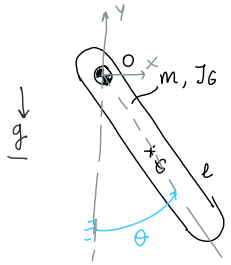
Esercizio: pendolo fisico

venerdì 6 dicembre 2024 11:38



Noto

⇒ barretta omogenea : m, G, $J_G = J_{zG}$

⇒ 1 GDL ⇒ θ: coordi. lagrangiana

⇒ $\theta = 0$ posizione EQUILIBRIO STATICO

$\theta(0) = \theta_0$ ⇒ comincia ad oscillare

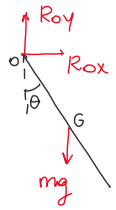
Trovare

⇒ Eq^{ne} moto ⇒ $f(\ddot\theta, \dot\theta, \theta) = 0$ ⇒ $\theta(t)$

$f(\ddot\theta, \dot\theta, \theta) = 0$ → Eq^{ne} differenziale $\ddot\theta, \dot\theta, \theta$

II CD scritto rispetto a: O (A), G (B)

DCL.



= SIST. EQUIV. = azioni di inerzia

$\underline{R}^{(e)} = m\underline{a}_G$

$\underline{M}_O^{(e)} = J_O\ddot\theta\underline{k} + \overrightarrow{OG} \wedge m\underline{a}_O$ (V2) (CASO A)

$= J_G\ddot\theta\underline{k} + \overrightarrow{OG} \wedge m\underline{a}_O$ (V1) (CASO C)

$\underline{M}_G^{(e)} = J_G\ddot\theta\underline{k}$ (V1 o V2 + polo G) (CASO B)

CASO A O↷

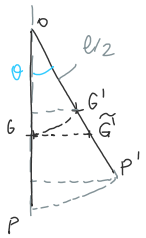
II CD ⇒ $\overrightarrow{OG} \wedge m\underline{g} = \underline{M}_O^{(e)} = J_O\ddot\theta\underline{k}$

$\overrightarrow{OG} \wedge m\underline{g} = \left(J_G + m\left(\frac{\ell}{2}\right)^2\right)\ddot\theta\underline{k}$

$-mg\frac{\ell}{2}\sin\theta = \left(J_G + \frac{m\ell^2}{4}\right)\ddot\theta$

$$\left(J_G + \frac{m\ell^2}{4}\right)\ddot\theta + mg\frac{\ell}{2}\sin\theta = 0$$

→ Eq^{ne} diff. 2° ORDINE + NON LINEARE



θ piccolo ⇒ $\widehat{GG'}$ ⇒ $\widetilde{GG'}$

$\begin{cases} \sin\theta \simeq \theta \\ \cos\theta \simeq 1 \end{cases}$ ⇒ $\overline{GG'} \simeq \frac{\ell}{2}\theta$

$$\left(J_G + \frac{m\ell^2}{4}\right)\ddot\theta + mg\frac{\ell}{2}\theta = 0$$

⇒ Eq^{ne} differ. LINEARE, del 2° ORDINE

CASO B G↷

II CD G↷ $\underline{M}_G^{(e)} = J_G\ddot\theta\underline{k}$

G↷ $\overrightarrow{GO} \wedge \underline{R}_O = J_G\ddot\theta\underline{k}$

DCL.

$$\left(-R_{ox}\frac{\ell}{2}\cos\theta - R_{oy}\frac{\ell}{2}\sin\theta\right)\underline{k} = J_G\ddot{\theta}\,\underline{k}$$

↑ ? ↑ ?

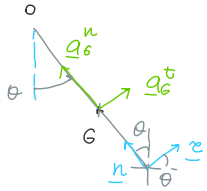
ICD

$$\underline{R}^{(\theta)} = \underline{R}_O + m\underline{g} = m\,\underline{a}_G$$

$$\underline{a}_G = \underline{a}_O + \underbrace{\dot{\underline{\omega}}\wedge\overrightarrow{OG}}_{\underline{a}_G^t} - \underbrace{\omega^2\overrightarrow{OG}}_{\underline{a}_G^n}$$

$$= \ddot{\theta}\,\underline{k}\wedge\overrightarrow{OG} - \dot{\theta}^2\overrightarrow{OG}$$

$$= \ddot{\theta}\frac{\ell}{2}\underbrace{\underline{\tau}}_{\underline{\tau}(\theta)} + \dot{\theta}^2\frac{\ell}{2}\underbrace{\underline{n}}_{\underline{n}(\theta)}$$



3 INCOGNITE
$R_{ox}, R_{oy}, \ddot{\theta}$

$$\begin{cases} x: \cdot\underline{i} & R_{ox} = m\underline{a}_G\cdot\underline{i} = m\left(\ddot{\theta}\frac{\ell}{2}\cos\theta - \dot{\theta}^2\frac{\ell}{2}\sin\theta\right) \\ y: \cdot\underline{j} & R_{oy} - mg = m\underline{a}_G\cdot\underline{j} = m\left(\ddot{\theta}\frac{\ell}{2}\sin\theta + \dot{\theta}^2\frac{\ell}{2}\cos\theta\right) \\ & -R_{ox}\frac{\ell}{2}\cos\theta - R_{oy}\frac{\ell}{2}\sin\theta = J_G\ddot{\theta} \end{cases}$$

$$-\frac{\ell}{2}\cos\theta\, m\left(\ddot{\theta}\frac{\ell}{2}\cos\theta - \dot{\theta}^2\frac{\ell}{2}\sin\theta\right) - \frac{\ell}{2}\sin\theta\left[mg + m\left(\ddot{\theta}\frac{\ell}{2}\sin\theta + \dot{\theta}^2\frac{\ell}{2}\cos\theta\right)\right] = J_G\ddot{\theta}$$

$$-\underbrace{m\left(\frac{\ell}{2}\right)^2\cos^2\theta}\,\ddot{\theta} + m\left(\frac{\ell}{2}\right)^2\dot{\theta}^2\sin\theta\cos\theta - mg\frac{\ell}{2}\sin\theta - \underbrace{m\left(\frac{\ell}{2}\right)^2\sin^2\theta}\,\ddot{\theta} - m\left(\frac{\ell}{2}\right)^2\dot{\theta}^2\cos\theta\sin\theta = J_G\ddot{\theta}$$

$$-m\left(\frac{\ell}{2}\right)^2\ddot{\theta} - mg\frac{\ell}{2}\sin\theta = J_G\ddot{\theta}$$

$$\left(J_G + m\left(\frac{\ell}{2}\right)^2\right)\ddot{\theta} + mg\frac{\ell}{2}\sin\theta = 0$$

CASO C ⇒ A CASA