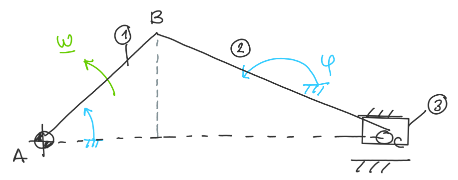
Esercit: An Cinem differenziale MDS

giovedì 21 novembre 2024 17:54



Noto : $\overline{AB} = r = 68.1$ mm

$\overline{BC} = \ell = 164$ mm

$\theta(\bar{t}) = 45°$

$\dot{\theta}(\bar{t}) = 6\,300$ rpm $\simeq 660$ rad/s

$\ddot{\theta}(\bar{t}) = 0$ rad/s²

$\underline{\omega} = \dot{\theta}\,\underline{k}$

OBIETTIVO : •) Valutare $\underline{v}$ di ogni punto

•) " $\underline{a}$ " "

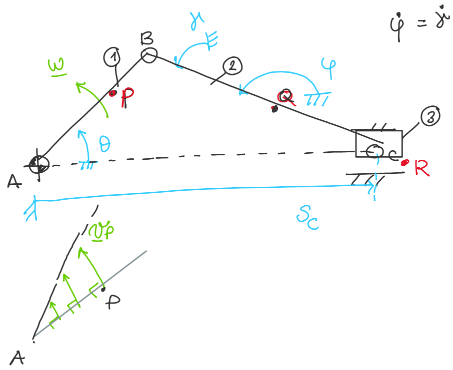
•) Cr del moto (assoluto) di ogni corpo

ANALISI VELOCITÀ

$$\underline{v}_{P①} = \cancel{\underline{v}_A}^{\,0} + \underline{\omega} \wedge \overrightarrow{AP} = \underline{\omega} \wedge \overrightarrow{AP} = \dot{\theta}\,\underline{k} \wedge \overrightarrow{AP}$$

$\varphi = \gamma + \pi/2$

$\dot{\varphi} = \dot{\gamma}$



$C_{r1} \equiv A$

$$\underline{v}_{Q②} = \underline{v}_B + \underline{\omega}_2 \wedge \overrightarrow{BQ} \qquad (1)$$

$\underline{\omega}_2 = \dot{\varphi}\,\underline{k}$

$\boxed{\underline{v}_{B②} = \underline{v}_{B①}}$

$$= \dot{\theta}\,\underline{k} \wedge \overrightarrow{AB} + \dot{\varphi}\,\underline{k} \wedge \overrightarrow{BQ}$$

?

$$\underline{v}_{R③} = \dot{s}_C\,\underline{i} \qquad (2)$$

?

$\underline{\omega}_3 = \underline{0}$

2 INCOGNITE $\dot{\varphi}$, $\dot{s}_C$ } 2 Eq^NI ?

$\boxed{\underline{v}_{C②} = \underline{v}_{C③}}$ Eq^NE CHIUSURA DELLE VELOCITÀ → 1 Eq^NE VETTOR. ⇒ 2 Eq^NI SCALARI

2 INCOG.

FFC (1) TRAIETTORIA (2)

$$\underline{v}_{C②} = \boxed{\dot{\theta}\,\underline{k} \wedge \overrightarrow{AB} + \dot{\varphi}\,\underline{k} \wedge \overrightarrow{BC} = \dot{s}_C\,\underline{i}} = \underline{v}_{C③}$$

$\dot{\varphi}$? $\dot{s}_C$?

1 noto 2 noto dir. 3 noto dir.

SOLUZ. GRAFICA

SOLUZ. GEOMETRICA

SOLUZ. ANALITICA

SOLUZ. NUMERICA

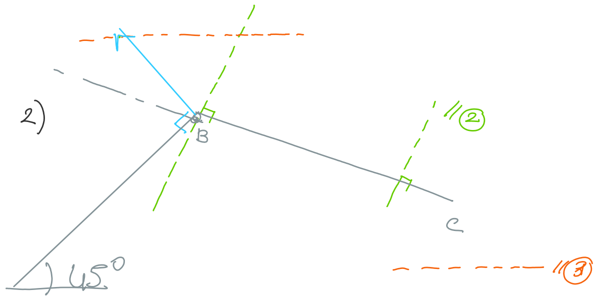
$\boxed{\text{SOLUZ. GRAFICA}}$

$\underbrace{①+②=③}$

TRIANGOLO DELLE VELOCITA'

$\Downarrow$

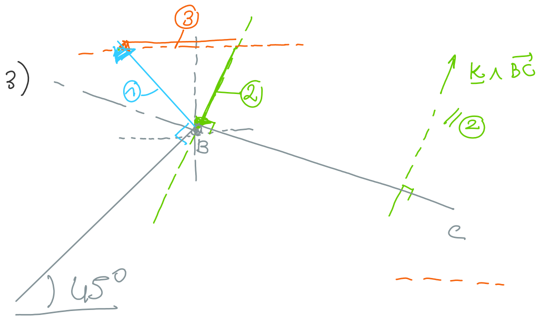
SOLUZIONE SU SCALA



1) disegno il vettore noto

2) riporto i vettori di direzione nota sulle estremità del vettore noto

3) chiudo il Δ



4) valutare il segno delle incognite

$\dot{\varphi} >< 0$?

$\dot{s}_c ><0$

③ $\dot{s}_c \, \underline{i} \Rightarrow \dot{s}_c < 0$

② $\dot{\varphi}(\underline{k} \wedge \vec{BC}) \Rightarrow \dot{\varphi} < 0$

$\downarrow$

$\underline{k} \wedge \vec{BC}$

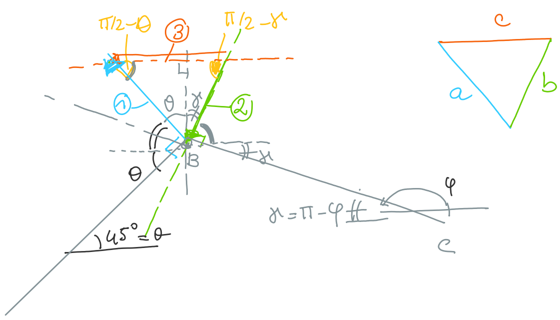
$\boxed{\text{SOLUZIONE GEOMETRICA}}$

$\hookrightarrow \quad |\dot{s}_c|, \ |\dot{\varphi}|$

$a = |\dot{\theta}| \, \overline{AB} = \dot{\theta} r$

$b = |\dot{\varphi}| \, \overline{BC} = |\dot{\varphi}| \, l$

$c = |\dot{s}_c|$



$$\boxed{\dot{\theta}\underline{k} \wedge \vec{AB} + \dot{\varphi}\underline{k} \wedge \vec{BC} = \dot{s}_c \underline{i}}$$

$$\frac{a}{\sin(\pi/2-\gamma)} = \frac{b}{\sin(\pi/2-\theta)} = \frac{c}{\sin(\theta+\gamma)}$$

$$|\dot{\varphi}| = -\frac{\dot{\theta} r}{l} \frac{\cos\theta}{\underbrace{\cos\varphi}_{<0}} > 0 \Rightarrow \dot{\varphi} = -|\dot{\varphi}| = \frac{\dot{\theta} r}{l} \frac{\cos\theta}{\cos\varphi} = -202{,}7 \text{ rad/s}$$

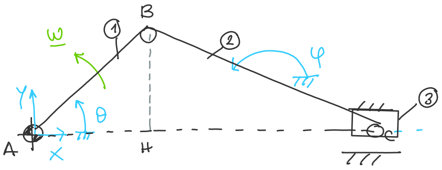
$$|\dot{s}_c| = \frac{\dot{\theta} r}{\cos\varphi} \sin(\theta-\varphi) > 0 \Rightarrow \dot{s}_c = -|\dot{s}_c| = -\frac{\dot{\theta} r}{\cos\varphi}\sin(\theta-\varphi) = -41{,}5 \text{ m/s}$$

Noto φ da: $\quad r \sin\theta = l \sin\varphi$

$$\varphi = \arcsen\left(\frac{r}{l}\sin\theta\right)$$

SOLUZIONE ANALITICA

$$\boxed{\dot\theta\,\underline{k} \wedge \vec{AB} + \dot\varphi\,\underline{k} \wedge \vec{BC} = \dot{s}_C\,\underline{i}}$$



$$\vec{AB} = r\begin{bmatrix}\cos\theta\\ \sin\theta\\ 0\end{bmatrix}$$

$$\vec{BC} = \ell\begin{bmatrix}-\cos\varphi\\ -\sin\varphi\\ 0\end{bmatrix}$$

$$\begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ 0 & 0 & \dot\theta\\ r c\theta & r s\theta & 0\end{vmatrix} + \begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ 0 & 0 & \dot\varphi\\ -\ell c\varphi & -\ell s\varphi & 0\end{vmatrix} = \begin{bmatrix}\dot{s}_C\\ 0\\ 0\end{bmatrix}$$

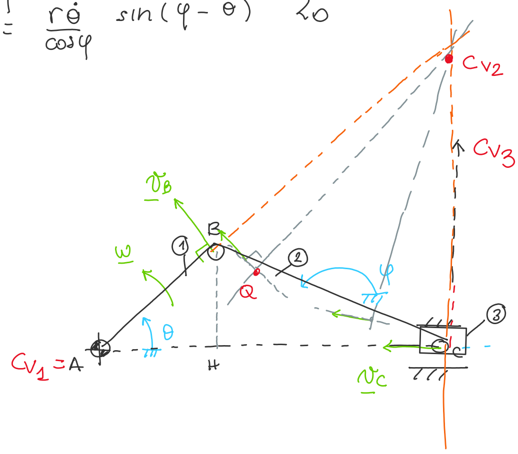
$$r\dot\theta\begin{bmatrix}-\sin\theta\\ \cos\theta\\ 0\end{bmatrix} + \ell\dot\varphi\begin{bmatrix}\sin\varphi\\ -\cos\varphi\\ 0\end{bmatrix} = \begin{bmatrix}\dot{s}_C\\ 0\\ 0\end{bmatrix}$$

$$\begin{array}{ll}\cdot\,\underline{i} & x\\ \cdot\,\underline{j} & y\end{array}\begin{cases}-r\dot\theta\sin\theta + \ell\dot\varphi\sin\varphi = \dot{s}_C\\ r\dot\theta\cos\theta - \ell\dot\varphi\cos\varphi = 0\end{cases} \rightarrow 2\,\text{Eq}^{\text{ni}}\ \text{SCALARI}:\ \dot\varphi,\ \dot{s}_C$$

$$\dot\varphi = \frac{r\dot\theta\cos\theta}{\ell\cos\varphi} < 0$$

$$\dot{s}_C = -r\dot\theta\sin\theta + \cancel{\ell}\sin\varphi\,\frac{r\dot\theta\cos\theta}{\cancel{\ell}\cos\varphi} = \frac{r\dot\theta}{\cos\varphi}\left(\sin\theta\cos\varphi + \sin\varphi\cos\theta\right) = \frac{r\dot\theta}{\cos\varphi}\sin(\varphi-\theta) < 0$$

CV MOTO ASSOLUTO



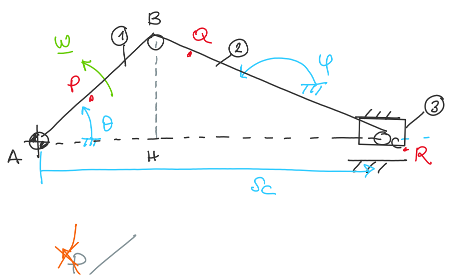
$C_{V_1} \equiv A$

$C_{V_3}\ \nexists$

C_{V_2}

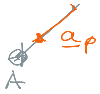
$\hookrightarrow\ \vec{v}_{B(2)},\ \vec{v}_{C(2)} \Rightarrow \vec{v}_{Q(2)} = \underline{\omega}_2 \wedge \vec{C_{V_2}Q} = \dot\varphi\,\underline{k} \wedge \vec{C_{V_2}Q}$

ANALISI DELLE ACCELERAZIONI



$$\underline{a}_{P(1)} = \cancel{\underline{a}_A} + \dot{\underline{\omega}} \wedge \vec{AP} - \omega^2\vec{AP} = \underbrace{\ddot\theta\,\underline{k} \wedge \vec{AP}}_{\ddot\theta = 0} - \dot\theta^2\vec{AP} = -\dot\theta^2\vec{AP}$$

(r)



$$a_{Q(2)} = a_B + \dot{\underline{\omega}}_2 \wedge \vec{BQ} - \omega_2^2 \vec{BQ}$$

$$a_{B(2)} = a_{B(1)}$$

$$= -\dot{\theta}^2 \vec{AB} + \ddot{\varphi}\,\underline{k} \wedge \vec{BQ} - \dot{\varphi}^2 \vec{BQ}$$

$\ddot{\varphi}$?

$\dot{\varphi}$: TROVATA RISOLVENDO PROB. VELOC.

$$a_{R(3)} = \frac{d}{dt}\left(\dot{s}_c\, \underline{i}\right) = \ddot{s}_c\, \underline{i}$$

$\ddot{s}_c$?

2 INCOGNITE $\ddot{\varphi}$, $\ddot{s}_c$

$$\boxed{\underset{TR}{a_{C(2)}} = \underset{TRASETT.}{a_{C(3)}}}$$

EQ^{NE} CHIUSURA delle ACCELERAZIONI

(m/s²) 2,96 10^4 ... 6,7 10^3

$$\boxed{-\dot{\theta}^2 \vec{AB} + \ddot{\varphi}\,\underline{k} \wedge \vec{BC} - \dot{\varphi}^2 \vec{BC} = \ddot{s}_c\, \underline{i}}$$

$\ddot{\varphi}$? $\ddot{s}_c$?

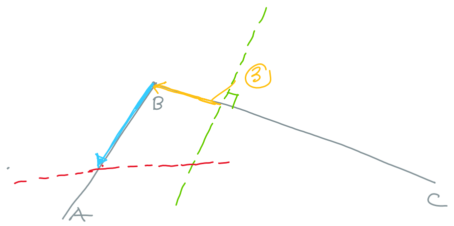
noto (1) — noto dir (2) — noto (3) — noto dir. (4)

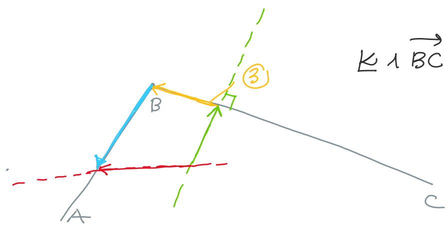
SOLUZIONE GRAFICA

⇒ POLIGONO DELLE ACCELERAZIONI

(1) + (3) + (2) = (4)

$$\begin{cases} \ddot{s}_c < 0 \\ \ddot{\varphi} > 0 \end{cases}$$





$\underline{k} \wedge \vec{BC}$

SOLUZIONE ANALITICA

$$\boxed{-\dot{\theta}^2 \vec{AB} + \ddot{\varphi}\,\underline{k} \wedge \vec{BC} - \dot{\varphi}^2 \vec{BC} = \ddot{s}_c\, \underline{i}}$$

$$-\dot{\theta}^2 r \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} + \ddot{\varphi}\,\ell \begin{bmatrix} \sin\varphi \\ -\cos\varphi \\ 0 \end{bmatrix} - \dot{\varphi}^2 \begin{bmatrix} -\cos\varphi \\ -\sin\varphi \\ 0 \end{bmatrix} = \ddot{s}_c \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$x: \quad \ddot{\varphi} = \frac{\ell\,\dot{\varphi}^2 \sin\varphi - r\dot{\theta}^2 \sin\theta}{\ell \cos\varphi}$$

Y: $\ddot{s}_c = -\dot{\theta}^2 r\cos\theta + \dot{\varphi}^2 \ell\cos\varphi + \mathrm{tg}\,\varphi\,(\ell\,\varphi\,\sin\varphi -$ [illegible] $)$

$$\begin{cases} \ddot{\varphi} = 1{,}218\ 10^5\ \text{rad/s}^2 & >0 \\ \ddot{s}_c = -\ 2{,}156\ 10^4\ \text{m/s}^2 & <0 \end{cases}$$