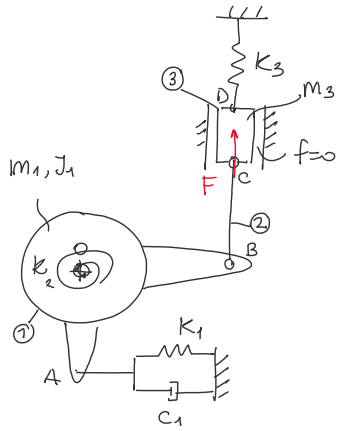
Esercizio oscillazioni forzate

venerdì 13 dicembre 2024 12:41



Noto:

$\overline{OA} = a = 5\,cm$

$\overline{OB} = b = 10\,cm$

$J_1 = J_O = 0{,}036\ kg\,m^2$

$O \equiv G_1$

$m_3 = 2\,kg$

$K_1 = 2\,N/m$

$k_2 = 0{,}05\ Nm$

$K_3 = 1\,N/m$

$c = 8{,}96\ Ns/m$

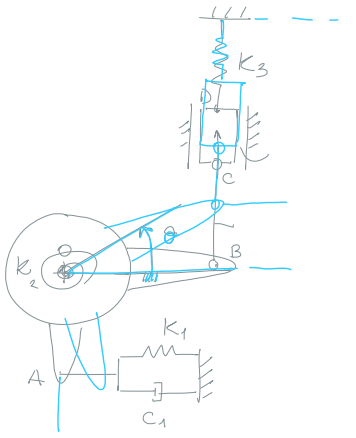
$F = F_0 \cos(\Omega t)$ $\quad F_0 = 0.03\,N$, $\Omega = 1\,rad/s$

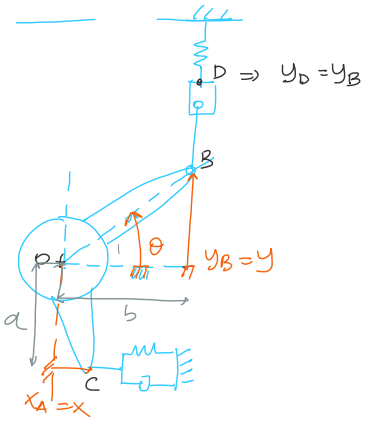
Hp piccole oscillazioni

Valutare

1) Definire le coordinate + eq^ni congruenza
2) Eq^ne moto
3) ω_n, ξ
4) legge oraria $\Rightarrow$ diagramma ampiezza e fase
   ↓
   A REGIME

1) PROBLEMA A 1 GDL





$x = a\sin\theta \overset{\cdot}{\simeq} a\theta$

$y = b\sin\theta \overset{\cdot}{\simeq} b\theta$

$\theta > 0,\ x > 0,\ y > 0$

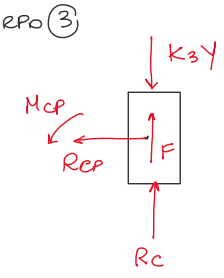
$\begin{cases} x = a\theta \\ \dot{x} = a\dot{\theta} \\ \ddot{x} = a\ddot{\theta} \end{cases}$ $\quad \begin{cases} y = b\theta \\ \dot{y} = b\dot{\theta} \\ \ddot{y} = b\ddot{\theta} \end{cases}$

DCL

ASTA ② SCARICA

$R_C = R_B$

CORPO ③

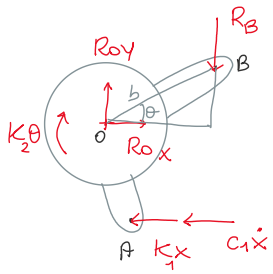


$x: \quad R_{cp} = 0$

$\circlearrowleft: \quad M_{cp} = 0$

$-K_3 y + R_c + F = m\ddot{y}$ (Eq 12)

CORPO ①



$R_{OY}, R_{OX}, R_B, \ddot{\theta}$

4 INCOG.

↓ ? ↓ ?

2 INCOGNITE

$R_C, \ddot{y}$

II CD O $\quad -R_B\, b\cos\theta - (K_1x + C_1\dot{x})\, a\cos\theta - K_2\theta = J_O\ddot{\theta}$

$R_B = R_C$

Ep(2)

$$\begin{cases} -K_3y + R_C + F = m_3\ddot{y} & \text{Ep(1)} \\ -R_C\, b\cos\theta - (K_1x + C_1\dot{x})\, a\cos\theta - K_2\theta = J_O\ddot{\theta} & \text{Ep(2)} \end{cases}$$

$R_C, \ddot{y}$

$$\begin{cases} R_C = m_3\ddot{y} + K_3y - F \\ -(m_3\ddot{y} + K_3y - F)\, b - (K_1x + C_1\dot{x})\, a - K_2\theta = J_O\ddot{\theta} \end{cases}$$

$y = b\theta$ $\qquad$ $a\theta$

$$(J_O + m_3b^2)\ddot{\theta} + \underbrace{Ca^2}_{C_{eq}}\dot{\theta} + \underbrace{(K_2 + K_1a^2 + K_3b^2)}_{K_{eq}}\theta = \underbrace{bF_0}_{M_0}\cos(\omega t)$$

J_{eq}

$$J_{eq}\ddot{\theta} + C_{eq}\dot{\theta} + K_{eq}\theta = M_0\cos(\omega t)$$

$$\omega_n = \sqrt{\frac{K_{eq}}{J_{eq}}} = 1{,}077 \text{ rad/s} \quad > \Omega$$

$$\xi = \frac{C_{eq}}{2J_{eq}\omega_n} = 0{,}186 \quad < 1 \Rightarrow \text{OSCIL. SOTTOSMOR. PERIODICHE}$$

$$\theta(t) = \theta_{om}(t) + \theta_p(t)$$

$$\theta_p(t) = \theta_0\cos(\omega t - \varphi)$$

$$\begin{cases} \theta_0 = 0{,}124 \text{ rad} \\ \varphi = 68° \end{cases} \Rightarrow \theta_p(t) = 0{,}124\cos(t - 68°)$$