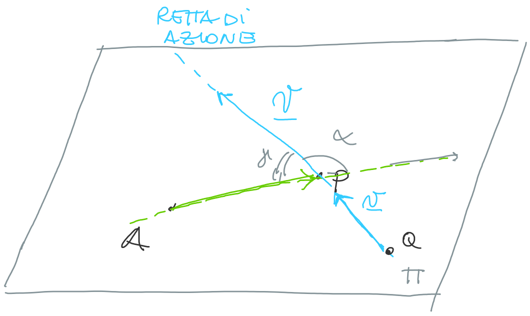
# Vettori Applicati

venerdì 27 settembre 2024 12:28

$(P, \vec{v})$

MOMENTO di $(P, \vec{v})$ < POLARE → VETTORE ; ASSIALE → SCALARE

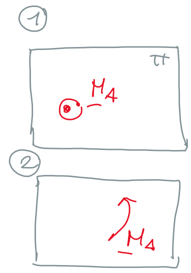


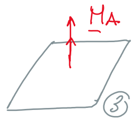

## MOMENTO POLARE

Scegliendo polo A $\qquad \underline{M}_A = \vec{AP} \wedge \underline{v}$

- •) VETTORE LIBERO!
- •) VETTORE $\perp \vec{AP}$, $\perp \vec{v}$ ⇒ DIREZ
- •) $|\underline{M}_A| = |\vec{AP}|\,|\vec{v}|\sin\alpha$ ⇒ MODULO
  $$\downarrow$$
  $$0 \leq \alpha \leq \pi$$
  $$\sin\alpha \geq 0$$
- •) VERSO ⇒ REGOLA MANO DX

$$\begin{cases} \vec{AP} \text{ e } \vec{v} \in \pi \\ \underline{M}_A \perp \pi \end{cases}$$



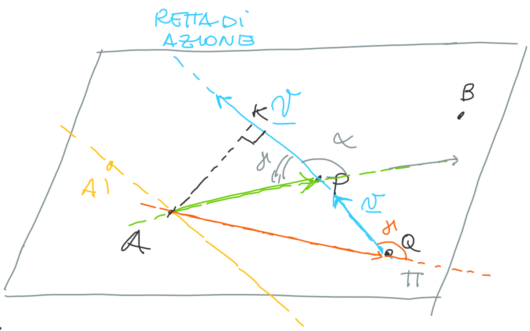

**PRINCIPIO DI TRASMISSIBILITA'**

⇒ possiamo spostare $(P, \vec{v})$ solo lungo la sua RDA

$(Q, \vec{v})$

$\downarrow$

$$\left.\begin{aligned} \underline{M}'_A &= \vec{AQ} \wedge \vec{v} \quad — \; |\vec{AQ}|\,|\vec{v}|\sin\alpha \\ \underline{M}_A &= \vec{AP} \wedge \vec{v} \quad — \; |\vec{AP}|\,|\vec{v}|\sin\gamma \end{aligned}\right) = ?$$




$$\overline{AQ}\sin\alpha = AP\sin\alpha = AK = \text{BRACCIO di } \vec{v}$$

$$\| \quad \text{distanza tra polo e RDA}$$

⇒ Noto $\underline{M}_A$, valutare $\underline{M}_B$

$$\underline{M}_B = \vec{BP} \wedge \vec{v}$$

$$\underline{M}_A = \vec{AP} \wedge \vec{v}$$

$$\vec{BP} = \vec{BA} + \vec{AP}$$

$$\underline{M}_B = \vec{BA} \wedge \vec{v} + \vec{AP} \wedge \vec{v}$$

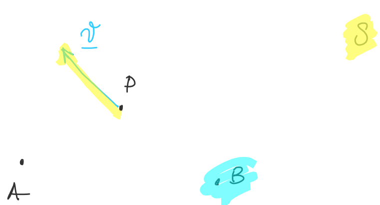
$$\boxed{\underline{M}_B = \underline{M}_A + \vec{BA} \wedge \vec{v}}$$

LEGGE DEL TRASPORTO DEL MOMENTO

⇒ posso spostare $(P, \vec{v})$, in un punto A ?

S $\quad \underline{M}_A = \vec{AP} \wedge \vec{v}$

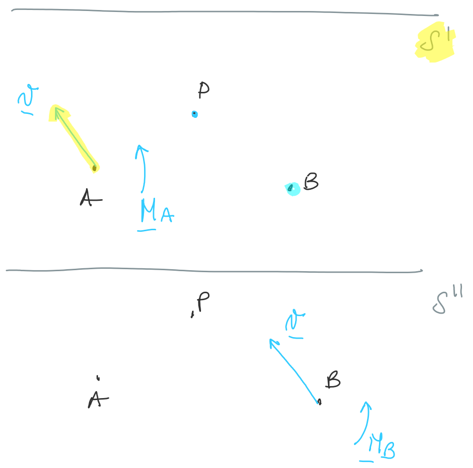
$\underline{M}_B = \vec{BP} \wedge \vec{v}$



$(S' \quad \underline{M}'_A = \underline{0})$

$S'' \quad \underline{M}'_A = \underline{M}_A$

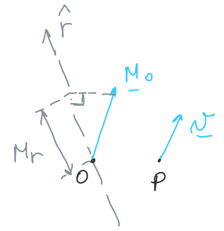
$$\boxed{\underline{M}'_B} = \underline{M}_A + \overrightarrow{BA} \wedge \underline{v} = \overrightarrow{AP} \wedge \underline{v} + \overrightarrow{BA} \wedge \underline{v} = \boxed{\overrightarrow{BP} \wedge \underline{v}}$$




NB $S, S', S'' \Rightarrow$ SONO EQUIVALENTI

MOMENTO ASSIALE

$M_r$ del vettore $(P, \underline{v})$, rispetto all'asse orientato $\hat{r}$



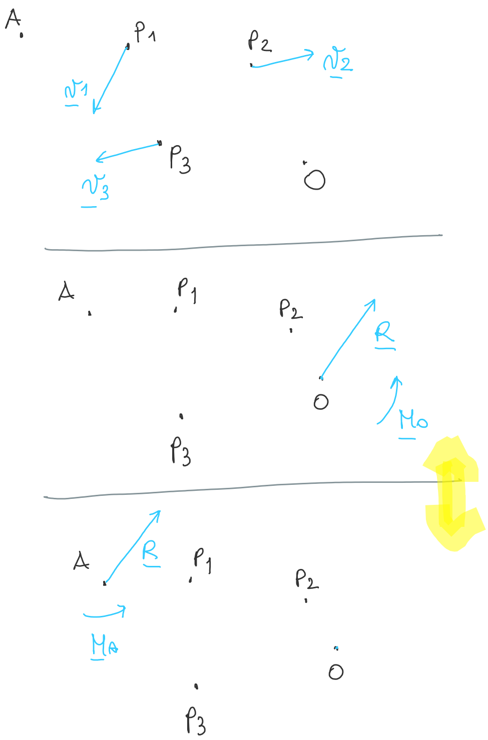

$O \in r \qquad M_r = \underline{M}_O \cdot \hat{r}$

↓

e' INDIPENDENTE DA "O"

SISTEMI DI VETTORI APPLICATI

$S \Rightarrow (P_i, \underline{v}_i)$ con $i = 1, 2 \dots n$




• RISULTANTE $\qquad \underline{R} = \sum_i \underline{v}_i$

• MOMENTO RISULTANTE rispetto ad "O" $\qquad \underline{M}_O = \sum_i \overrightarrow{OP_i} \wedge \underline{v}_i$

$\Downarrow$

SISTEMA EQUIVALENTE "RIDOTTO" RISPETTO AL POLO O

$(O, \underline{R}), \ \underline{M}_O$

$\updownarrow$

$(A, \underline{R}), \ \underline{M}_A$

$$\underline{M}_A = \sum_i \overrightarrow{AP_i} \wedge \underline{v}_i$$

LEGGE DEL TRASPORTO DEL MOMENTO RISULTANTE

$$\underline{M}_A = \underline{M}_O + \overrightarrow{AO} \wedge \underline{R} \quad ✓$$

$$\Sigma \ \overrightarrow{AP}_i \wedge \underline{v}_i$$

$$= \sum_i \vec{AP_i} \wedge \underline{v_i}$$

$$\vec{AP_i} = \vec{AO} + \vec{OP_i}$$

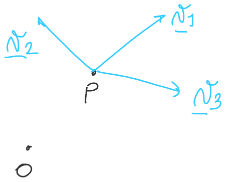
$$= \sum_i \vec{AO} \wedge \underline{v_i} + \sum_i \vec{OP_i} \wedge \underline{v_i}$$

$$= \vec{AO} \wedge \underline{R} + \underline{M_O}$$

CASI PARTICOLARI

1) SISTEMI VETTORI APPLICATI IN P

$S : (P, v_i)$

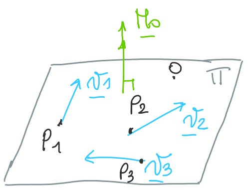



•) $(P, \underline{R})$

•) TEOREMA DI VARIGNON

$$\underline{M_O} = \sum \vec{OP} \wedge \underline{v_i} = \vec{OP} \wedge \underline{R}$$

2) SISTEMA DI VETTORI AP. PIANI

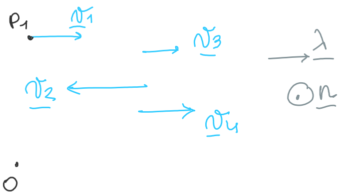



$(P_i, \underline{v_i})$ , $\underline{v_i} \in \pi$ $\quad i = 1, 2, \ldots n$

•) $\underline{R} \in \pi$

•) $\underline{M_O} \perp \pi$ $\quad$ con $\in \pi$

3) SISTEMA DI VETTORI PARALLELI

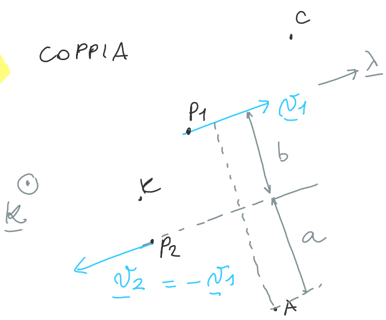



$(P_i, \underline{v_i})$ $\quad \underline{v_i} \parallel \underline{v_j}$

•) $\underline{R} \parallel \underline{\lambda}$

•) $\underline{M_O} \parallel \underline{n}$ , $\underline{M_O} \perp \underline{\lambda}$

4) COPPIA




$$\underline{R} = \overbrace{\underline{v_1}}^{v_1 \underline{\lambda} \text{ con } v_1 > 0} + \underline{v_2} = \underline{0}$$

•) $\underline{M_A} = \vec{AP_1} \wedge \underline{v_1} + \vec{AP_2} \wedge (-\underline{v_1})$ $\quad \boxed{\underline{M_A} \parallel \underline{k}}$

$$= [- v_1 (a+b) + v_1 (a)] \underline{k}$$

$$= - v_1 b \, \underline{k}$$

$$|\underline{M_A}| = v_1 \underbrace{b}_{\text{BRACCIO DELLA COPPIA}}$$

$$\underline{M_C} = \vec{CP_1} \wedge \underline{v_1} + \vec{CP_2} \wedge \underline{v_2}$$

TRINOMIO INVARIANTE $\tau$

$$\tau = \underline{M_O} \cdot \underline{R}$$

↓ SDR

TRINOMIO $\mathscr{T} = M_{Ox} R_x + M_{Oy} R_y + M_{Oz} R_z$

INVARIANTE: $\mathscr{T} = \underline{M}_O \cdot \underline{R} = \underline{M}_A \cdot \underline{R}$ INDIPEND. DAL POLO

$$= (\underbrace{\cancel{\vec{OA} \wedge \underline{R}}}_{\perp \underline{R}} + \underline{M}_A)\cdot \underline{R}$$

$$= \cancel{(OA \wedge \underline{R})\cdot \underline{R}} + \underline{M}_A \cdot \underline{R}$$

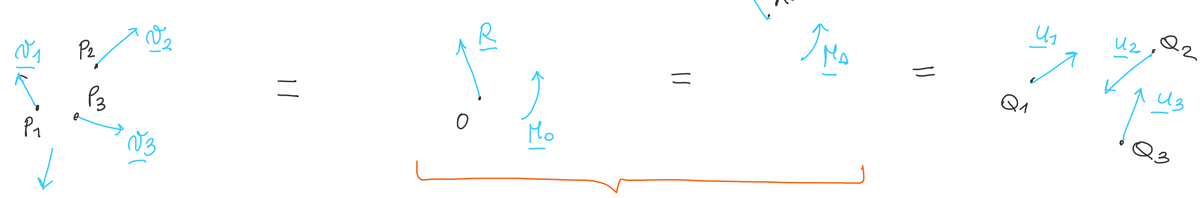
$$\boxed{\mathscr{T} = 0 \Rightarrow \underline{M}_O \perp \underline{R}}$$

$\Rightarrow \underline{M}_O = \underline{0}$

$\Rightarrow \underline{R} = \underline{0}$

$S = (P_i, \underline{v}_i) \quad i = 1,2 \ldots N$

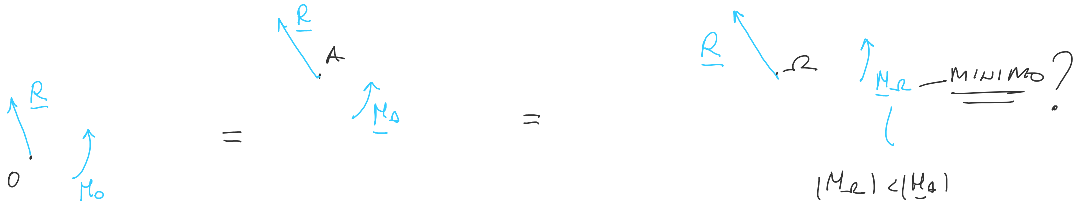
$S' = (Q_j, \underline{u}_j) \quad j = 1,2,\ldots M$




$\Rightarrow$ CONDIZIONE EQUIVA. $\begin{cases} \underline{R} = \underline{R}' \\ \underline{M}_O = \underline{M}_O' \end{cases}$

$\Rightarrow$ ? $\exists$ UN POLO COMODO T.C. SE RIDOTTO È MINIMO

$\Downarrow$ $\Omega$ $\quad\Updownarrow$ MINIMO $\underline{M}_\Omega$




$|M_\Omega| < |\underline{M}_A|$

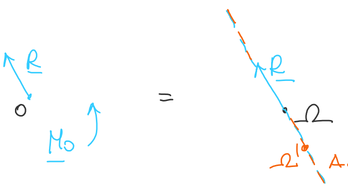
e

$\underline{M}_\Omega < |\underline{M}_O|$

$\boxed{\mathscr{T} = 0}$

↑

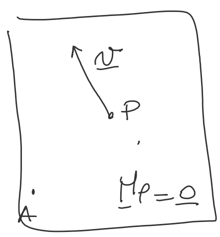
$\underline{M}_O \cdot \underline{R} = 0$




ASSE CENTRALE $\Rightarrow$ RDA della RISULTANTE

$\underline{M}_\Omega = \underline{0} = \vec{\Omega O} \wedge \underline{R} + \underline{M}_O \quad (1)$

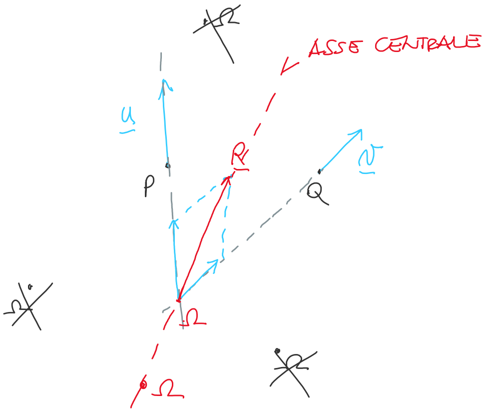
$\vec{\Omega O} \wedge \underline{R} = - \underline{M}_O$



$\underline{M}_O \perp \underline{R}$

$\downarrow$

$\underline{M}_O \cdot \underline{R} = 0$ ✓ $\Omega \ \exists$




SISTEMA EQUIV. MINIMO = $\underline{u}$ e $\underline{v}$ complanari

$\downarrow$

$(\Omega, \underline{R})$, $\underline{M}_\Omega = 0$

·) $\underline{R} = \underline{u} + \underline{v}$

·) $\Omega$ ?

ANALITICAMENTE RISOLVIAMO .

$\vec{\Omega O} \wedge \underline{R} = -\underline{M}_O$

$\boxed{\vec{O\Omega} \wedge \underline{R} = \underline{M}_O}$ ⇒ 3 Eq.ni SCALARI

$\downarrow$

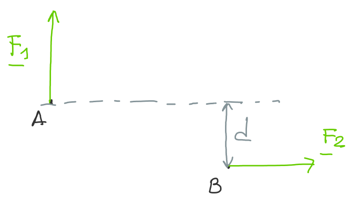
2 Eq.ni SCALARI INDIPENDENTI

$\downarrow$

Eq.ne DI UNA RETTA

? $\begin{pmatrix} O\Omega_x \\ O\Omega_y \\ O\Omega_z \end{pmatrix}$

ESERCIZIO




Noto :

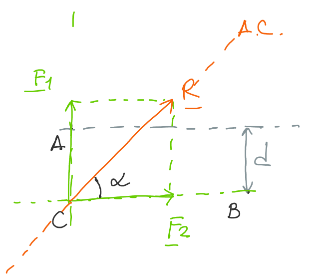
$S$ : $(A, \underline{F}_1)$ $(B, \underline{F}_2)$

$|\underline{F}_1| = |\underline{F}_2| = 10\ N$

$d = 0.5\ m$

Trovare : – ASSE CENTRALE – SEM } ⟨ (A) SOL. GRAFICA / SOL. ANALIT. (B)

(A) SOL. GRAFICA




SEM: $(C, \underline{R})$

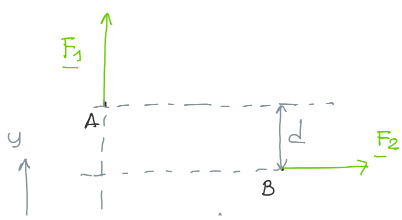
$\alpha = 45°$

Direzione di $\underline{R}$ nota

(B) SOL. ANALITICA

Scegliamo il polo A :

SE RIDOTTO RISPETTO AD "A"



$\underline{R} = \underline{F}_1 + \underline{F}_2$


$\underline{M}_A = \overrightarrow{AB} \wedge \underline{F}_2$

$$\underline{R} = \begin{pmatrix} 0 \\ F_1 \\ 0 \end{pmatrix} + \begin{pmatrix} F_2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} F_2 \\ F_1 \\ 0 \end{pmatrix} = \begin{pmatrix} 10 \\ 10 \\ 0 \end{pmatrix} N$$

$$\underline{M}_A = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ AB_x & AB_y & 0 \\ F_2 & 0 & 0 \end{vmatrix} = -F_2\, AB_y\, \underline{k} = +F_2\, d\, \underline{k} = 5\, \underline{k} \ (Nm)$$

$$\underline{M}_\Omega = \underline{0} = \overrightarrow{\Omega A} \wedge \underline{R} + \underline{M}_A$$

$$\overrightarrow{A\Omega} \wedge \underline{R} = \underline{M}_A \quad \Rightarrow \quad \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ A\Omega_x & A\Omega_y & A\Omega_z \\ F_2 & F_1 & 0 \end{vmatrix} = F_2\, d\, \underline{k}$$

$\begin{pmatrix} A\Omega_x \\ A\Omega_y \\ A\Omega_z \end{pmatrix}$ ?

$$\underline{j}\, F_2\, A\Omega_z + A\Omega_x F_1\, \underline{k} - F_2\, A\Omega_y\, \underline{k} - F_1\, A\Omega_z\, \underline{i} = F_2\, d\, \underline{k}$$

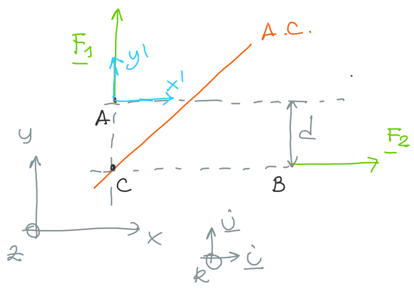
$$\begin{array}{ll} \cdot\, \underline{i} & x \\ \cdot\, \underline{j} & y \\ \cdot\, \underline{k} & z \end{array} \left\{ \begin{array}{l} -F_1\, A\Omega_z = 0 \\ F_2\, A\Omega_z = 0 \\ A\Omega_x F_1 - A\Omega_y F_2 = F_2\, d \end{array} \right.$$

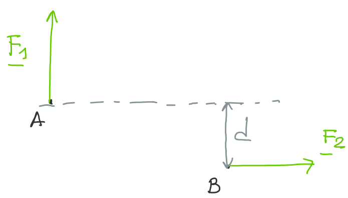
$\Rightarrow A\Omega_z = 0 \Rightarrow$ siamo nel piano $x, y$

dipendenti

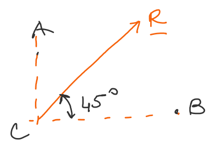
$$A\Omega_x - A\Omega_y = d$$

$$\boxed{A\Omega_y = A\Omega_x - d} \Leftrightarrow y' = x' - 0.5$$



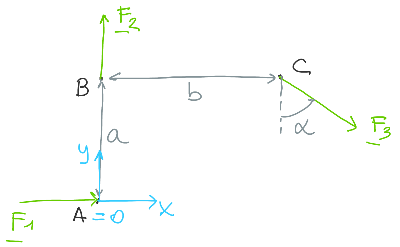

=




ESERCIZIO

Noto

$S: \ (A, \underline{F}_1), \ (B, \underline{F}_2), \ (C, \underline{F}_3)$




$|F_1| = 100 N = |F_1| = F$

$|F_3| = 100\sqrt{2}\, N = F\sqrt{2}$

$\alpha = 45°$

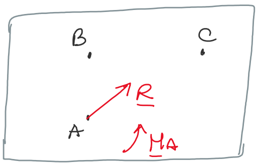
$\overline{AB} = a = 1\, m$

$\overline{BC} = b = 2\, m$

## Valutare

1) SE RIDOTTO RISPETTO AD "A"
2) " " "C"
3) SE M $\Rightarrow$ ASSE CENTRALE

1) $(A, \underline{R})$ , $\underline{M}_A$

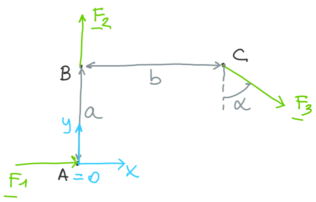



$$\underline{R} = \underline{F}_1 + \underline{F}_2 + \underline{F}_3 = \begin{pmatrix} F_1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ F_2 \\ 0 \end{pmatrix} + \begin{pmatrix} F_3 \sin\alpha \\ -F_3 \cos\alpha \\ 0 \end{pmatrix} = \begin{pmatrix} F + F\sqrt{2}\, \sqrt{2}/2 \\ F - F\sqrt{2}\, \sqrt{2}/2 \\ 0 \end{pmatrix} = F \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = (200, 0, 0)\ N$$

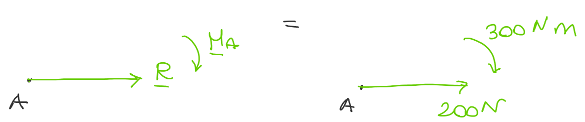
$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}^T = (v_x \ v_y \ v_z) =$$

$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = (v_x, v_y, v_z)$$

$$\underline{M}_A = \vec{AC} \wedge \underline{F}_3 = -F\sqrt{2}\,(a+b)\,\underline{k} = -300\ Nm\ \underline{k} = -M_A \underline{k}$$



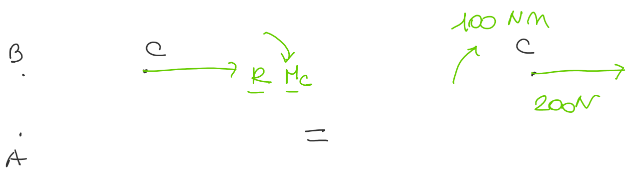

=




||

2) $\underline{R}$

$$\underline{M}_C = \vec{CB} \wedge \underline{F}_2 + \vec{CA} \wedge \underline{F}_1 = -100\ Nm$$




3) SEM $\quad \underline{M}_\Omega = \underline{0} = \vec{\Omega A} \wedge \underline{R} + \underline{M}_A$

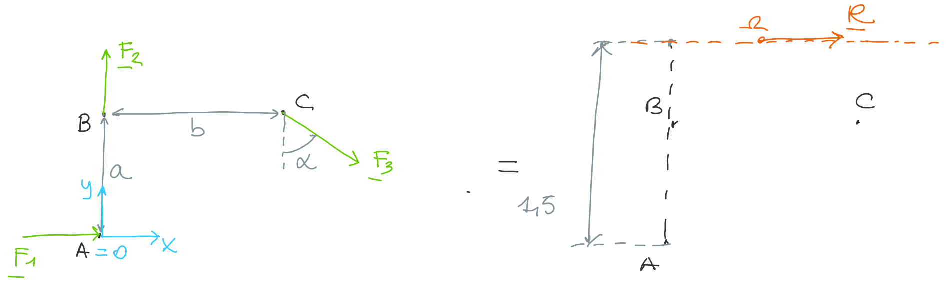
$$\vec{A\Omega} \wedge \underline{R} = \underline{M}_A$$

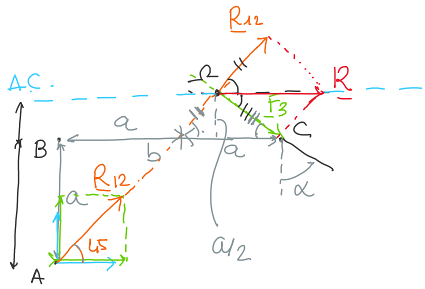
$$A\Omega_z\, |\underline{R}|\, \underline{j} - A\Omega_y\, |\underline{R}|\, \underline{k} = -M_A\, \underline{k}$$

$$\begin{matrix} \underline{i} \\ \underline{j} \end{matrix} \begin{cases} 0 = 0 \\ A\Omega_z = 0 \end{cases}$$

$$\underset{\sim}{R} \Big| \; -A\Omega_y \, |R| = -M_A \quad \Rightarrow \quad A\Omega_y = \frac{M_A}{|R|} = 1,5 \text{ m}$$

$$y = 1.5 \text{ m}$$





$b = 2a$

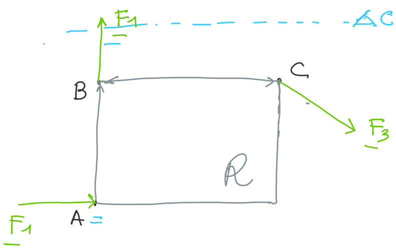
$|F_1| = |F_2| = 100$ N

$R_{12} = F_1 + F_2 \qquad |R_{12}| = \sqrt{2}\,100$ N $= |F_3|$

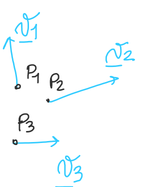
$R_{123} = R_{12} + F_3 = R \parallel \underline{i}$

$$\Omega A_y = a + \frac{a}{2} = \frac{3}{2}a = 1.5 \text{ m}$$

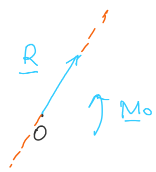
$$\left[ \begin{array}{l} \Omega \in AC \\ A.C. \parallel R \end{array} \right]$$



ASSE CENTRALE DI SISTEMI DI VETTORI APPLICATI CON $\underset{\sim}{R} \neq 0$



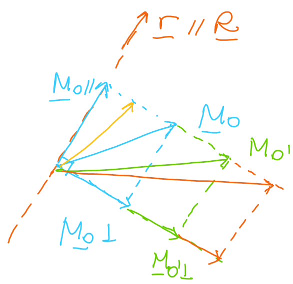
$=$



$\Rightarrow \exists \Omega$ t.c. $M_\Omega$ è MINIMO?

SE RIDOTTO SU "O"

$\downarrow$



$$\underbrace{M_O = \overset{M_\parallel}{M_{O\parallel}} + M_{O\perp}}$$

$$r = \frac{R}{|R|}$$

CAMBIA CON IL POLO

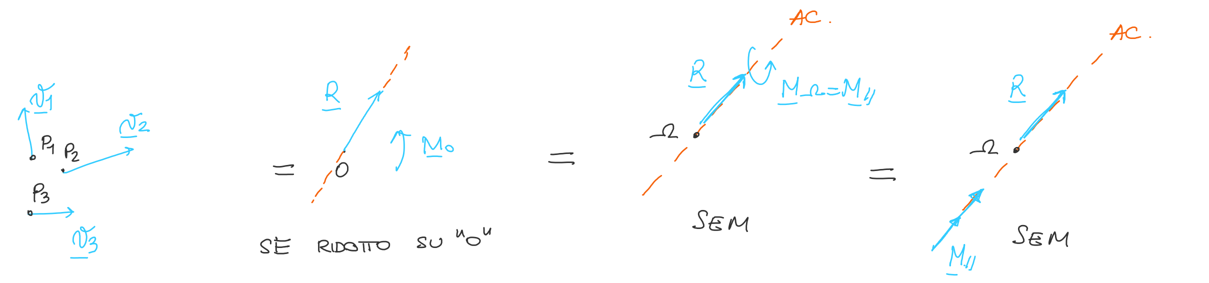
$$M_{O\parallel} = \left( M_O \cdot \frac{R}{|R|} \right) \frac{R}{|R|} = (M_O \cdot r)\, r$$

$$= \underbrace{(M_O \cdot R)}_{\tau} \frac{R}{|R|^2} = \boxed{\tau \frac{R}{|R|^2}} = M_\parallel \quad \text{uguale } \forall\, O$$

$$M_{O\perp} \Rightarrow \exists\, \Omega \text{ t.c. } M_{\Omega\perp} = 0 \Rightarrow M_\Omega = M_\parallel = \frac{\tau R}{|R|^2}$$

$\downarrow$

$\Omega \in$ ASSE CENTRALE




$(\Omega, R)$, $M_\parallel$
- WRENCH
- $w > 0$ $R$ e $M_\parallel$ stesso verso
- $w < 0$ $R$ e $M_\parallel$ verso opposto

CERCHIAMO $\Omega \in$ A.C.

$$M_\Omega \parallel R \Rightarrow M_\Omega \wedge R = 0$$

$\downarrow$ LEG. DEL TRASP.

$$(\vec{\Omega O} \wedge R + M_O) \wedge R = 0$$

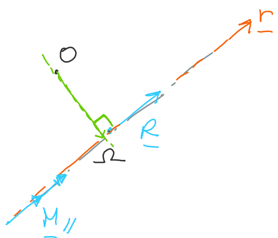
$$(\vec{\Omega O} \wedge R) \wedge R + M_O \wedge R = 0$$

$$(R \wedge \vec{O\Omega}) \wedge R + M_O \wedge R = 0$$

Noto $R, M_O$ $\Rightarrow$ $\boxed{|R|^2 \vec{O\Omega} - R(R \cdot \vec{O\Omega}) + M_O \wedge R = 0}$

Eq.ne DETERMIN. DELL'ASSE CENTRALE
$\downarrow$
2 Eq.ni INDIP.
$\downarrow$
$\tau \neq 0$

— DETERMINAZIONE DI UN PUNTO DELL'AC — $\tau = 0$ / $\tau \neq 0$




cerchiamo $\Omega \in$ A.C. t.c. $\vec{O\Omega} \perp$ A.C., $\perp r$

$$\vec{O\Omega} \cdot R = 0 \leftrightarrow \vec{O\Omega} \cdot r = 0$$

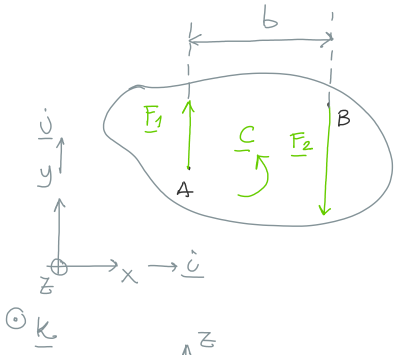
$$|R|^2 \vec{O\Omega} - R\,(\cancel{R \cdot \vec{O\Omega}}) + M_O \wedge R = 0$$

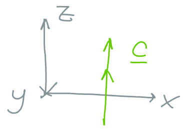
$\emptyset$

$$\boxed{\overrightarrow{O\Omega} = \frac{\underline{R} \wedge \underline{M}_O}{|\underline{R}|^2}}$$

$\Omega \in A.C.$

ESERCIZIO




Noto :

$|\underline{F}_1| = 5000\,N = F_1$

$|\underline{F}_2| = 8000\,N = F_2$

$|\underline{C}| = 25000\,Nm = C$

$b = 4m$

$S : (A, \underline{F}_1), (B, \underline{F}_2), \underline{C}$

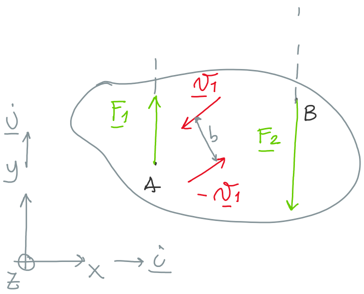
$\underline{F}_1 = F_1\,\underline{j}$

$\underline{F}_2 = -F_2\,\underline{j}$

$\underline{C} = C\,\underline{k}$

1) $\tau = 0$ o $\tau \neq 0$?

$$\tau = \underbrace{\underline{M}_A}_{//\underline{k}} \cdot \underbrace{\underline{R}}_{//\underline{j}} = 0$$



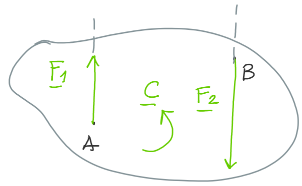

$C = v_1\, b$

2) SEM $\Rightarrow$ AC.

SE RIDOTTO su "A"

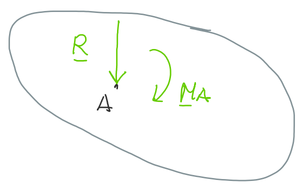
$\underline{R} = \underline{F}_1 + \underline{F}_2 = +F_1\,\underline{j} - F_2\,\underline{j} = (5000 - 8000)\,\underline{j}\ N = -3000\,\underline{j}\ N$

$$\underline{M}_A = \underbrace{\overrightarrow{AB} \wedge \underline{F}_2}_{-b\,F_2\,\underline{k}} + \underbrace{\underline{C}}_{C\,\underline{k}} = (-32000 + 25000)\,\underline{k}\,Nm = -7000\,\underline{k}\ Nm$$




||

SE. RIDOTTO "A"




MOMENTO RISULTANTE DEL SISTEMA.

$$\underline{M}_O = \sum_i \overrightarrow{OP_i} \wedge \underline{v}_i + \sum_j \underline{C}_j$$

$(P_i, \underline{v}_i)$ , $\underline{C}_j$

$i = 1, 2 \dots N$

$j = 1, 2 \dots M$

$$\underline{M}_\Omega = \underline{0} = \vec{\Omega A} \wedge \underline{R} + \underline{M}_A$$

$$\vec{A\Omega} \wedge \underline{R} = \underline{M}_A$$

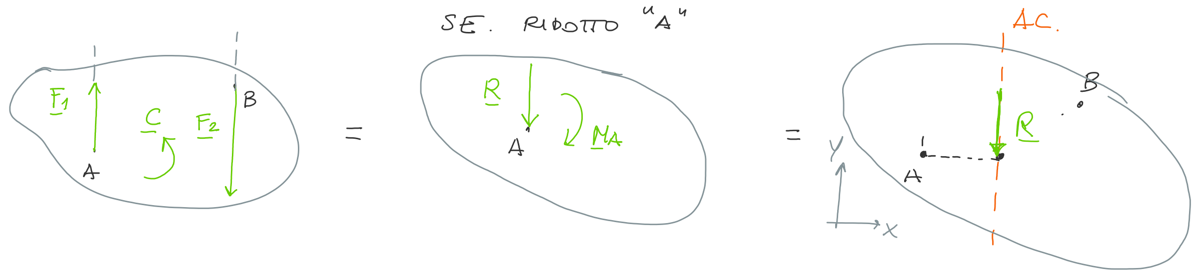
$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ A\Omega_x & A\Omega_y & 0 \\ 0 & -|\underline{R}| & 0 \end{vmatrix} = \boxed{-|\underline{R}|\, A\Omega_x\, \underline{k} = -|\underline{M}_A|\, \underline{k}}$$

$Ep^{ne}$ vett. × A.C.

$$\begin{cases} x: & 0 = 0 \\ y: & 0 = 0 \\ z: & +|\underline{R}|\, A\Omega_x = +|\underline{M}_A| \end{cases}$$

$$\left\| \; A\Omega_x = \frac{|\underline{M}_A|}{|\underline{R}|} = 2.33 \text{ m} \; \right\|$$

S.E. RIDOTTO "A"



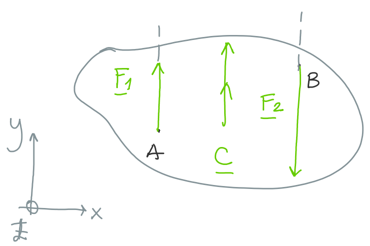

$\Rightarrow$ Non abbiamo osservato $\tau = 0$

$$\boxed{\vec{O\Omega} = \frac{\underline{R} \wedge \underline{M}_O}{|\underline{R}|^2}} \longrightarrow \quad \vec{A\Omega} = \frac{\underline{R} \wedge \underline{M}_A}{|\underline{R}|^2}$$

$$= \frac{(-|\underline{R}|\underline{j}) \wedge (-|\underline{M}_A|\, \underline{k})}{|\underline{R}|^2} = \frac{|\underline{M}_A|}{|\underline{R}|}\, \underline{i}$$

$$\vec{A\Omega} = A\Omega_x\, \underline{i}$$

## ESERCIZIO




$\Rightarrow$ stessi valori numerici dell'es. precedente

$\Downarrow$

CAMBIO DIREZIONE COPPIA

$\Rightarrow$ Trovare: SE MINIMO

$\downarrow$

$\tau \neq 0$

$\downarrow$

$\Omega \in AC$ t.c. $(\Omega, \underline{R})$, $|\underline{M}_{//}|$

$$\underline{M}_{//} = \tau \frac{\underline{R}}{|\underline{R}|^2}$$

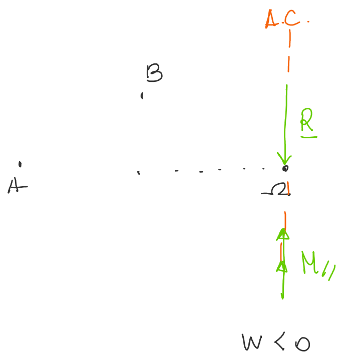
$$\underline{R} = \underline{F}_1 + \underline{F}_2 = -|\underline{R}|\,\underline{j} \qquad \text{(NON CAMBIA!)}$$

$$\underline{M}_A = \vec{AB} \wedge \underline{F}_2 + \underline{C} = -|\underline{F}_2|\, b\, \underline{k} + C\,\underline{j}$$

$$\tau \neq 0 \Rightarrow \quad \vec{A\Omega} = \frac{\underline{R} \wedge \underline{M}_A}{|\underline{R}|^2} = \frac{1}{|\underline{R}|^2} \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & -|\underline{R}| & 0 \\ 0 & C & -|\underline{F}_2|\, b \end{vmatrix} = \frac{1}{|\underline{R}|^{\cancel{2}}} \left(\cancel{|\underline{R}|}\, |\underline{F}_2|\, b\right) \underline{i}$$

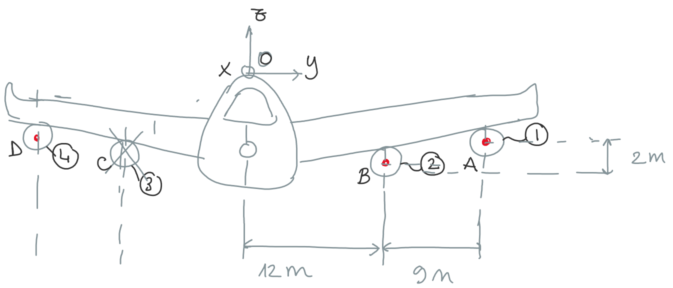
$$= \frac{|\underline{F}_2|\, b}{|\underline{R}|}\, \underline{i} = 10{,}67\ \underline{i}\ (m)$$

SE MINIMO




$$\underline{M}_{//} = \tau \frac{\underline{R}}{|\underline{R}|^2} = (\underline{R} \cdot \underline{M}_A) \frac{\underline{R}}{|\underline{R}|^2} = \frac{|\underline{R}|\,|\underline{C}|\,\underline{R}}{|\underline{R}|^2} = |\underline{C}|\,\underline{j} = 25\,000\ \underline{j}\ (Nm)$$

## ESERCIZIO



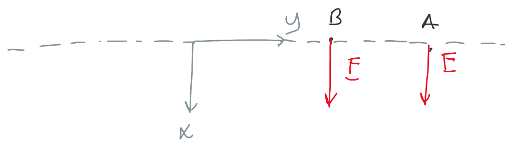

Noto : forza di spinta

$|\underline{F}| = 50$ KN $\rightarrow \underline{F} = |\underline{F}|\,\underline{i}$

motore 3 spento

$(A, \underline{F}), (B, \underline{F}), (D, \underline{F})$

Trovare : SE MINIMO

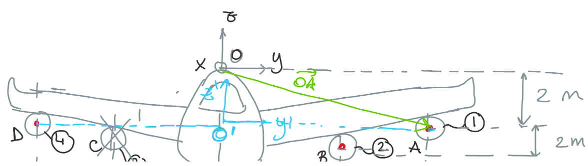



1) Quanto vale $\tau$? $\quad \tau = \underline{M}_O \cdot \underline{R}$

$$\underline{R} = 3\,\underline{F} = 3\,|\underline{F}|\,\underline{i} = 150\ KN\ \underline{i}$$

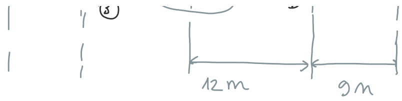
$$\underline{M}_O = \vec{OB} \wedge \underline{F} + \underbrace{\vec{OA} \wedge \underline{F}} + \vec{OD} \wedge \underline{F}$$

$$\vec{OA} \wedge \underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & OA_y & OA_z \end{vmatrix} =$$



$$\begin{vmatrix} |F| & 0 & 0 \end{vmatrix}$$

(A) : $OA_z |F| \underline{j} - OA_y |F| \underline{k}$




$OA_y = 12 + 9 = 21 \text{ m}$

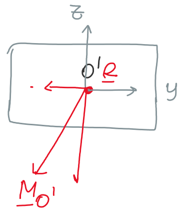
$OA_z = -2 \text{ m}$

$$\underline{M}_{O'} = \overrightarrow{O'B} \wedge \underline{F} + \overrightarrow{O'A} \wedge \underline{F} + \overrightarrow{O'D} \wedge \underline{F}$$

$$= O'B_z |F| \underline{j} - O'_B |F| \underline{j} \; \underbrace{- \cancel{O'A_y |F| \underline{k}}} \; \underbrace{+ \cancel{O'D_y |F| \underline{k}}}$$

$$= -100 \, k \, \underline{j} - 600 \, k \, \underline{k} \quad (N)$$

$$= \begin{pmatrix} 0 \\ -100k \\ -600k \end{pmatrix} N$$




$$\tau = \underline{M}_{O'} \cdot \underline{R} = 0$$

$$\Rightarrow \quad \underline{M}_\Omega = \underline{0} = \overrightarrow{\Omega O'} \wedge \underline{R} + \underline{M}_{O'} = \underline{0}$$

$$\overrightarrow{O'\Omega} \wedge \underline{R} = \underline{M}_{O'}$$

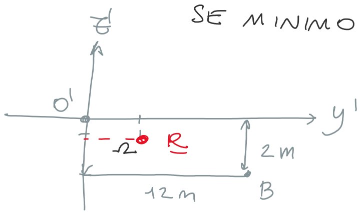
$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ O'\Omega_x & O'\Omega_y & O'\Omega_z \\ |\underline{R}| & 0 & 0 \end{vmatrix} = \begin{pmatrix} 0 \\ M_{oy} \\ M_{oz} \end{pmatrix}$$

$$\| \; O'\Omega_z |\underline{R}| \underline{j} - O'\Omega_y |\underline{R}| \underline{k} = M_{oy} \underline{j} + M_{oz} \underline{k} \; \|$$ 2 Eq^ni SCALARI

$$\begin{array}{ll} y \; \underline{j} & \\ z \; \underline{k} & \end{array} \begin{cases} O'\Omega_z |\underline{R}| = M_{oy} \\ -O'\Omega_y |\underline{R}| = M_{oz} \end{cases}$$

$$\begin{cases} O'\Omega_z = \dfrac{M_{oy}}{|\underline{R}|} = \dfrac{-100k}{150k} = -0.667 \text{ m} \\ O'\Omega_y = -\dfrac{M_{oz}}{|\underline{R}|} = \dfrac{600k}{150k} = 4 \text{ m} \end{cases}$$

SE MINIMO




$\Rightarrow$ A.C. RETTA // ASSE X

PASSANTE PER $\Omega$

$$\Rightarrow \quad \overrightarrow{O'\Omega} = \frac{\underline{R} \wedge \underline{M}_{O'}}{|\underline{R}|^2} = \frac{1}{|\underline{R}|^2} \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ |\underline{R}| & 0 & 0 \\ 0 & M_{oy} & M_{oz} \end{vmatrix} = 4\underline{j} - 0.667 \, \underline{k}$$