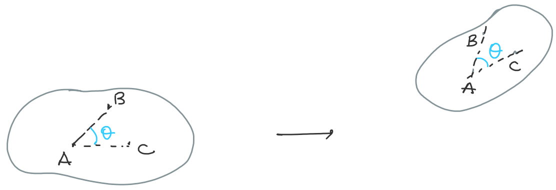
Cinematica del corpo rigido

martedì 19 novembre 2024 10:05

VINCOLO DI RIGIDITA' $\Rightarrow$

1) distanza tra due punti
2) orientamento relativo di due segmenti



$t = t_1$

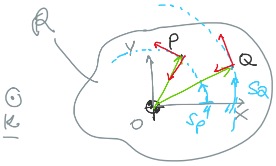
$B\hat{A}C = \theta$

$t = t_2$

$\overline{AB}, \overline{AC} = \text{cost}$

$B\hat{A}C = \theta$

MOTO ROTATORIO ATTORNO AD ASSE FISSO



$S = \{O; x, y, z\}$

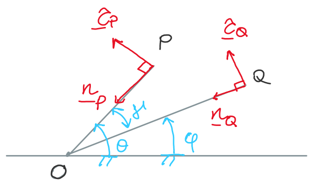
$P \Rightarrow$ traiettoria circolare $\Rightarrow r_P = \overline{OP}$

$Q \Rightarrow$ " " $\Rightarrow r_Q = \overline{OQ}$

$\Downarrow$

1 VDR) $\overline{OP}$ e $\overline{OQ}$ cost

$\overline{PQ}$ cost



$$\begin{cases} \underline{v}_P = |\overrightarrow{OP}|\,\dot{\theta}\,\underline{\tau}_P = r_P\,\dot{\theta}\,\underline{\tau}_P \\ \underline{a}_P = r_P\,\ddot{\theta}\,\underline{\tau}_P + r_P\,\dot{\theta}^2\,\underline{n}_P \end{cases}$$

$$\begin{cases} \underline{v}_Q = |\overrightarrow{OQ}|\,\dot{\varphi}\,\underline{\tau}_Q = r_Q\,\dot{\varphi}\,\underline{\tau}_Q \\ \underline{a}_Q = r_Q\,\ddot{\varphi}\,\underline{\tau}_Q + r_Q\,\dot{\varphi}^2\,\underline{n}_Q \end{cases}$$

$\theta \overset{?}{} \varphi \Rightarrow \gamma = \theta - \varphi = P\hat{O}Q = \text{cost}$

$\downarrow d/dt$

$0 = \dot{\theta} - \dot{\varphi}$

$\Downarrow$

$$\begin{cases} \dot{\theta} = \dot{\varphi} = \omega & \text{VELOCITA' ANGOLARE SCALARE} \\ \ddot{\theta} = \ddot{\varphi} = \dot{\omega} & \text{ACCELER. ANGOLA.} \end{cases}$$

$\Downarrow$

$$\boxed{\begin{cases} \underline{\omega} = \omega\,\underline{k} \\ \underline{\dot{\omega}} = \dot{\omega}\,\underline{k} \end{cases}}$$

NB GRANDEZZE CARATTERI DEL CORPO RIGIDO!!

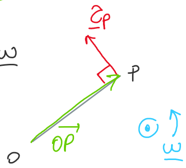
$\Downarrow$

$\underline{\omega}, \underline{\dot{\omega}} \parallel \underline{k}$

$\underline{v}_P = r_P\,\dot{\theta}\,\underline{\tau}_P$ — $\underline{\omega}$, $\overrightarrow{OP}$ — $\perp \overrightarrow{OP}, \perp \underline{\omega}$

r_P: $|\overrightarrow{OP}|$, $\overrightarrow{OP}$

$\dot{\theta}$: $\omega = \dot{\theta}$, $\underline{\omega} = \dot{\theta}\underline{k}$



$$\boxed{\underline{v}_P = \underline{\omega} \wedge \overrightarrow{OP}}$$

$\dot{\theta}$ $\quad r_P$

$\sin\alpha = 90°$

$\underline{v}_P = v_P\,\underline{\tau}_P \Rightarrow \underline{\underline{v_P = \dot{\theta}\,r_P}}$

$\underline{a}_P = r_P\,\ddot{\theta}\,\underline{\tau}_P + r_P\,\dot{\theta}^2\,\underline{n}_P \longrightarrow \underline{\omega}, \overrightarrow{OP}$

r_P: $|\overrightarrow{OP}|$; $\ddot{\theta}$: $\dot{\omega}$; $\underline{\tau}_P$: $\perp \underline{\dot{\omega}}$, $\perp \overrightarrow{OP}$

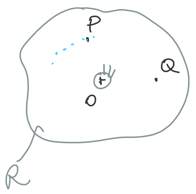
$\underbrace{|\overrightarrow{OP}|\ \omega^2}_{+\,\omega^2\,\overrightarrow{PO}}$ $\quad \underline{n}_P$: $\parallel \overrightarrow{PO}$

$\underline{a}_P = \underline{\dot{\omega}} \wedge \overrightarrow{OP} + \omega^2\,\overrightarrow{PO}$

$$\boxed{\underline{a}_P = \underline{\dot{\omega}} \wedge \overrightarrow{OP} - \omega^2\,\overrightarrow{OP}}$$

a_P^t $\quad a_P^n$

MOTO SFERICO – ATTORNO A PUNTO FISSO (3D)



$P \Rightarrow$ superficie sfera di raggio $|\overrightarrow{OP}| = r_P$

$\Downarrow$

$\underline{\omega} \not\parallel \underline{\dot{\omega}}$ possono essere qualsiasi

$$\boxed{\underline{v}_P = \underline{\omega} \wedge \overrightarrow{OP}}$$

$$\underline{a}_P = \frac{d\underline{v}_P}{dt}$$

$$= \underline{\dot{\omega}} \wedge \overrightarrow{OP} + \underline{\omega} \wedge \overbrace{\frac{d(\overrightarrow{OP})}{dt}}^{\underline{v}_P}$$

$$= \underline{\dot{\omega}} \wedge \overrightarrow{OP} + \underline{\omega} \wedge (\underline{\omega} \wedge \overrightarrow{OP})$$

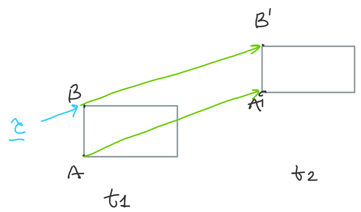
$$\boxed{= \underline{\dot{\omega}} \wedge \overrightarrow{OP} + (\underline{\omega} \cdot \cancel{\overrightarrow{OP}})\,\underline{\omega} - \underbrace{(\underline{\omega} \cdot \underline{\omega})}_{\omega^2}\,\overrightarrow{OP}}$$

ω

$\longrightarrow$ se $\underline{\omega} \parallel \underline{k}$, $\underline{\dot{\omega}} \parallel \underline{k}$ $\Rightarrow$ $\underline{a}_P = \underline{\dot{\omega}} \wedge \overrightarrow{OP} - \omega^2 \overrightarrow{OP}$ MOTO ROTAT. ASSE FISSO

MOTO TRASLATORIO

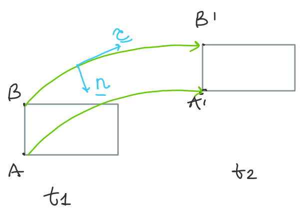
$\underline{\omega} = \underline{0} \quad \forall t \quad , \underline{\dot{\omega}} = \underline{0} \quad \Rightarrow$ orientamento del corpo cost.



•) punti con stesse traiettorie – RETTILINEO

•) $\underline{v}_B = \underline{v}_A = \underline{v} = \dot{s}\,\underline{t}$

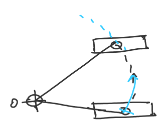
$\underline{a}_B = \underline{a}_A = \underline{a} = \ddot{s}\,\underline{t}$

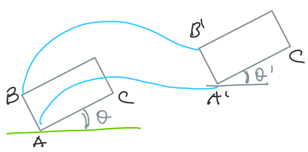


$\Rightarrow$ MOTO TRASL. CURVILINEO

$\underline{v} = \dot{s}\,\underline{t}$

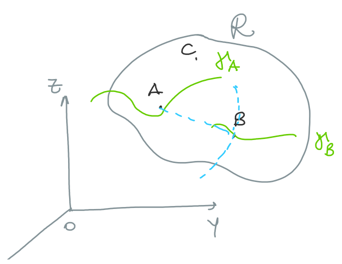
$\underline{a} = \ddot{s}\,\underline{t} + \frac{\dot{s}^2}{\rho}\,\underline{n}$





$\theta = \theta' = \theta(t)$ costante

MOTO ROTOTRASLATORIO



VINC. RIGID.

$$\begin{cases} \overline{AB} = \text{cost} \\ B\widehat{A}C = \text{cost} \end{cases} \quad \forall$$

$\Rightarrow \quad \underline{v}_A = \underline{v}_B + \underline{v}_{AB}$

$$\begin{bmatrix} \underline{v}_B = \underline{v}_A + \underline{v}_{BA} \\ \underline{a}_B = \underline{a}_A + \underline{a}_{BA} \end{bmatrix}$$

$\forall$ coppia di punti A e B ($\in R$)

$\downarrow$ + $\boxed{VPR}$ (*)

$\overline{AB}$ cost.

$\begin{cases} 2D \Rightarrow \text{MOTO RELAT.} \Rightarrow \text{MOTO CIRCOLA.} \\ 3D \Rightarrow \text{MOTO RELAT} \Rightarrow \text{MOTO SFERICO} \end{cases}$

$\underline{v}_{BA} = \underline{\omega} \wedge \overrightarrow{AB}$ < $\underline{\omega} \perp$ piano 2D ; $\underline{\omega}$ qualsiasi 3D

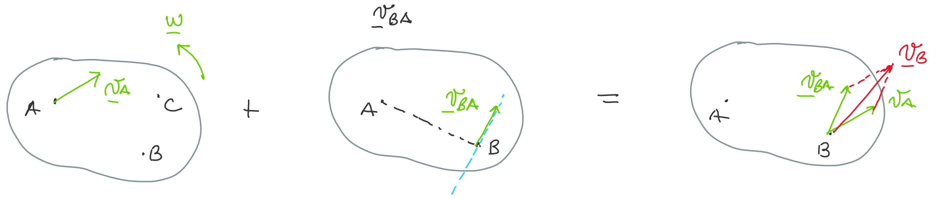
FFC $\boxed{\underline{v}_B = \underline{v}_A + \underline{\omega} \wedge \overrightarrow{AB}}$ FORMULA FONDAMENTALE CINEMAT. PER $\mathcal{R}$ < 2D, 3D

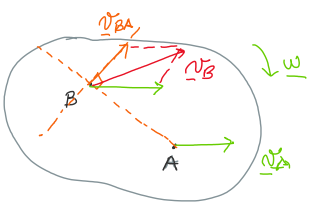
— LEGGE DI DISTRIBUZIONE DELLE VELOCITÀ

$\Rightarrow A \in \mathcal{R}$

$\Rightarrow \underline{v}_A, \underline{\omega}$ $\rightarrow$ FFC $\underline{v}_P$ $P \in \mathcal{R}$.

$$\underline{v}_A + \underbrace{\underline{\omega} \wedge \overrightarrow{AB}}_{\underline{v}_{BA}} = \underline{v}_B$$





$\underline{v}_B = \underline{v}_A + \underline{\omega} \wedge \overrightarrow{AB}$

$\underline{a}_{BA} = $ < 2D – MOT-ROT ASSE FISSO $= \dot{\underline{\omega}} \wedge \overrightarrow{AB} - \omega^2 \overrightarrow{AB}$; 3D – MOTO SFERICO $= \dot{\underline{\omega}} \wedge \overrightarrow{AB} + (\underline{\omega} \cdot \overrightarrow{AB})\,\underline{\omega} - \omega^2 \overrightarrow{AB}$

$\underline{a}_B = \underline{a}_A + \underline{a}_{BA}$

3D: $\boxed{\underline{a}_B = \underline{a}_A + \dot{\underline{\omega}} \wedge \overrightarrow{AB} + (\underline{\omega} \cdot \overrightarrow{AB})\,\underline{\omega} - \omega^2 \overrightarrow{AB}}$ LEGGE DI DISTRIBUZ. DELLE ACCEL.

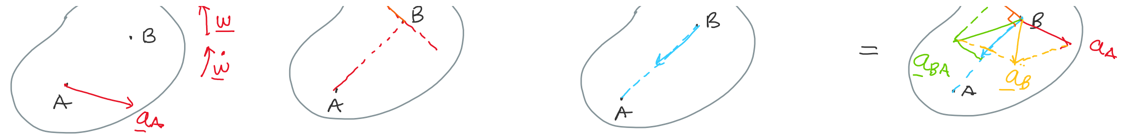
$\downarrow$

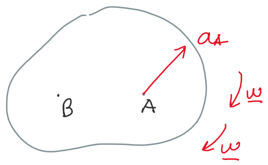
2D

$\underline{\omega} \perp \overrightarrow{AB}$

$\boxed{\underline{a}_B = \underline{a}_A + \dot{\underline{\omega}} \wedge \overrightarrow{AB} - \omega^2 \overrightarrow{AB}}$ TEOREMA DI RIVALS – TR

$$\underline{a}_A + \underset{\underline{\dot\omega} \wedge \overrightarrow{AB}}{\overset{\underline{a}^t_{BA}}{\parallel}} + \underset{-\omega^2 \overrightarrow{AB}}{\overset{\underline{a}^n_{BA}}{\parallel}} = \underline{a}_B$$





MOTO RIGIDO PIANO $\Leftarrow$ NOSTRO PROBLEMA $\Rightarrow$ $\boxed{2D}$ < FFC / TR

$\mathcal{R} \Rightarrow \Pi_m$ PIANO MOBILE SOLIDALE AL CORPO RIGIDO

$\Downarrow$

MOTO di Π_M rispetto ad un piano fisso