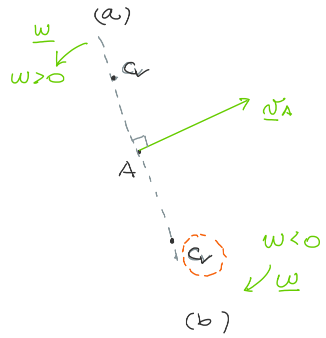
Centro delle velocità

martedì 19 novembre 2024 11:16

$\exists$ un punto di $\mathcal{R}$ o Π_m, C_v t.c. $\underline{v}_{C_v} = 0$ per $t = \bar{t}$, se $\underline{\omega} \neq \underline{0}$?

$$\underline{v}_{C_v} = \underline{0} = \underline{v}_A + \underline{\omega} \wedge \overrightarrow{AC_v} \qquad (1)$$

$$\underline{v}_A = -\underline{\omega} \wedge \overrightarrow{AC_v} \overset{!}{=} \underline{\omega} \wedge \overrightarrow{C_vA}$$



•) $\overline{AC_v} \perp \underline{v}_A, \perp \underline{\omega}$

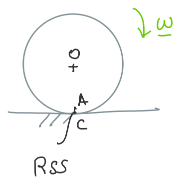
•) $\underline{v}_A \Rightarrow C_v$ sta su (b) per $\omega > 0$

•) $|\underline{v}_A| = |\underline{\omega}|\,|\overrightarrow{AC_v}|$

$$\overline{AC_v} = \frac{|\underline{v}_A|}{|\underline{\omega}|}$$

OSSERVAZIONI

1)

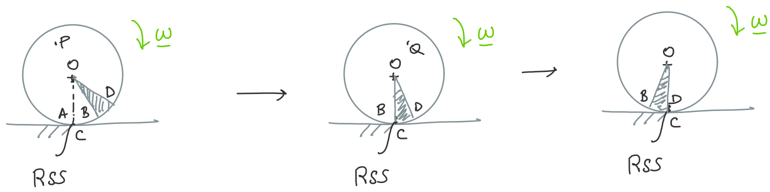


Qual'è il C_v?

$\underline{v}_{C_v} = \underline{0}$

$\underline{v}_A = \underline{v}_C = \underline{0}$ per RSS.

$C_v \equiv A \equiv C$



$C_v \equiv A$ $\quad$ $C_v \equiv B$ $\quad$ $C_v \equiv D$

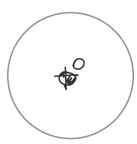
$\Rightarrow$ C_v "SI MUOVE" SUL CORPO RIGIDO
$\Downarrow$
E' UN PUNTO di $\mathcal{R}$ CHE CAMBIA NEL TEMPO

$$\underline{v}_P = \underline{v}_{C_v} + \underline{\omega} \wedge \overrightarrow{C_vP} \overset{!}{=} \underbrace{\underline{\omega} \wedge \overrightarrow{C_vP}}$$

$$\underline{v}_Q = \underbrace{\underline{\omega} \wedge \overrightarrow{C_vQ}}$$

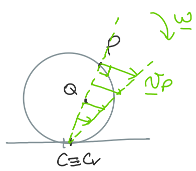
$$\underline{v}_P = \underset{\underline{0}}{\cancel{\underline{v}_{C_v}}} + \underline{v}_{PC_v}$$

2)



$O \equiv C_v \Rightarrow \forall t$, C_v E' FISSO

3) DISTRIBUZ. VELOCITA' Δ



$\underline{v}_P = \underline{\omega} \wedge \overrightarrow{C_vP} \Rightarrow |\underline{v}_P| = |\underline{\omega}|\, r_P$ $\quad$ ($r_P = \overline{CP}$)

$\underline{v}_Q = \underline{\omega} \wedge \overrightarrow{C_vQ} \Rightarrow |\underline{v}_Q| = |\underline{\omega}|\, r_Q$ $\quad$ ($r_Q = \overline{CQ}$)

$$\frac{|\underline{v}_P|}{|\underline{v}_Q|} = \frac{r_P}{r_Q}$$

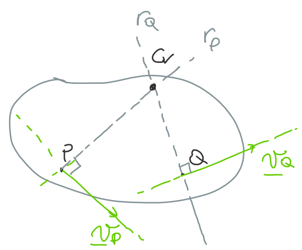
4) COME TROVIAMO IL C_v ?

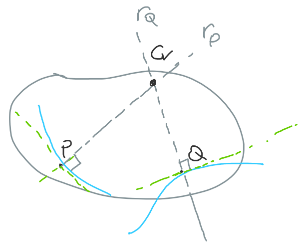
TEOREMA DI CHASLES

$r_P \cap r_Q \equiv C_v$

P e Q $\in \mathcal{R}$

- direzione $\underline{v}_P$ e $\underline{v}_Q$ $\Rightarrow$ $\cap$ delle dir $\perp$ $\underline{v}_P$ e $\underline{v}_Q$
- traiettorie di P e Q (γ_P, γ_Q) $\Rightarrow$ $\cap$ delle tg a γ_P e γ_Q





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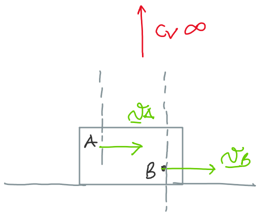

$\underline{v}_A = \underline{v}_B$

- $C_v \nexists \Rightarrow \underline{\omega} = \underline{0}$
- C_v VA' ALL'∞

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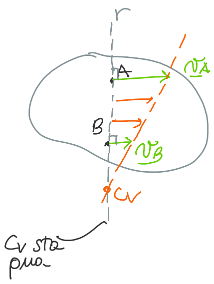
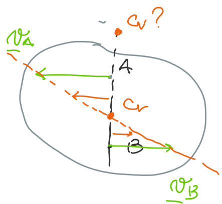
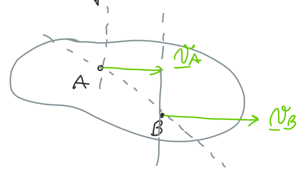


$\underline{v}_A \parallel \underline{v}_B$

$\underline{\omega} \neq \underline{0}$

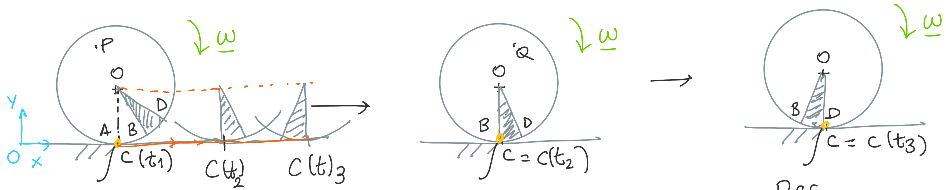
$\underline{AB} \perp \underline{v}_A$ e $\underline{v}_B$





$\nexists$ per CORPI RIGIDI !

$t = t_1$ $t = t_2$ $t = t_3$



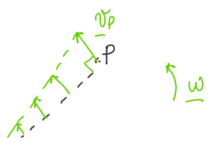
$C_v \equiv A$ $C_v \equiv B$ $C_v \equiv D$

$C_v \in \mathcal{R}$, TTM

$\Downarrow$

$\underline{v}_P = \underline{v}_{C_v} + \underline{\omega} \wedge \overrightarrow{C_vP}$

$= \cancel{0} + \underline{\omega} \wedge \overrightarrow{C_vP} \Rightarrow$



$t=t$

C_v

$t=t_1$ $\quad \underline{v}_P = \underline{v}_A + \underline{\omega} \wedge \vec{AP}$

$t=t_2$ $\quad \underline{v}_P = \underline{v}_B + \underline{\omega} \wedge \vec{BP}$

CENTRO DI ISTANTANEA ROTAZIONE (definito se $\underline{\omega} \neq \underline{0}$) $\quad C_{IR}(t) \Rightarrow$ PUNTO GEOMETRICO

$$\cancel{\underline{v}_P = \underbrace{\underline{v}_{C_{IR}}}_{\neq \underline{0}} + \underline{\omega} \wedge \vec{C_{IR}P}} \qquad \vec{C_{IR}P} = \vec{C_vP}$$

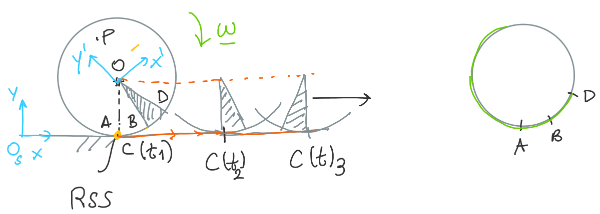
$C_{IR}(t) \Rightarrow$ SDR FISSO $\Rightarrow S=\{O_S;\, x,y\}$

$\Rightarrow$ TRAIETT. RETTIL.

$\Downarrow$

POLARE FISSA $= \sigma_f$

$t=t_1$



$\Rightarrow$ SDR MOBILE $\Rightarrow S=\{O_M;\, x',y'\}$

$\Downarrow$

POLARE MOBILE $= \sigma_m$

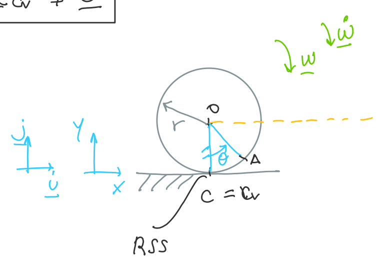
$\Rightarrow$ PROPR. FONDAMENTALE $\Rightarrow$ σ_m RSS su σ_f

$\Rightarrow$ toccano in ogni istante $\Rightarrow C_v / C_{IR}$

$\Rightarrow$ hanno tg in comune

$\Rightarrow$ descrivono la geome. del moto $\mathcal{R}$

$$\boxed{a_{C_v} \neq \underline{0}}$$



$\underline{\omega} = -\dot{\theta}\,\underline{k}$

$\underline{\dot{\omega}} = -\ddot{\theta}\,\underline{k}$

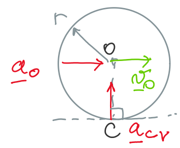
$\underline{v}_{C_v} = \underline{0}$

$\underline{a}_{C_v}$? TEOR. RIVALS $\Rightarrow$ $\underline{a}_{C_v} = \underline{a}_O + \underline{\dot{\omega}} \wedge \vec{OC_v} - \omega^2 \vec{OC_v}$

$$\underset{C_v}{\underline{a}_C} = \underline{a}_O + \underline{\omega} \wedge \vec{OC} - \omega^2 \vec{OC_v} \qquad (\neq)$$

$\underline{a}_O \longrightarrow$ $\underline{v}_O = \underline{\omega} \wedge \vec{CO} = \omega r \underline{i} = \dot{\theta} r \underline{i}$

$\underline{a}_O = \frac{d}{dt} \underline{v}_O = \ddot{\theta} r \underline{i}$

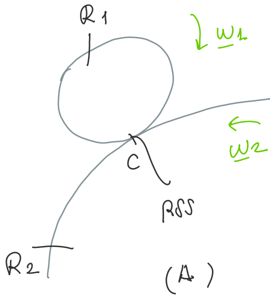


$$\underline{a}_{C_v} = \cancel{\ddot{\theta} r \underline{i}} + \underbrace{(-\ddot{\theta}\underline{k}) \wedge (-r\underline{j})} - \dot{\theta}^2(-r\underline{j})$$

$$= \dot\theta^2 r \underline{j} \qquad -\theta r \underline{u}$$

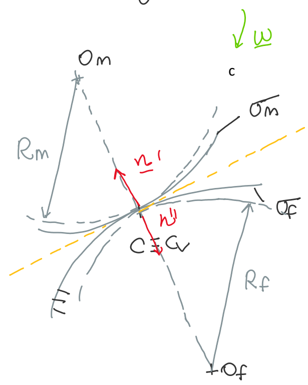
$$\boxed{\underline{a}_{C_V} = \underbrace{R_m}_{r} \underbrace{\omega^2}_{\dot\theta^2} \underbrace{\underline{n}}_{\underline{j}}}$$

CASO GENERALE



(A)

$\Rightarrow$



(B)

$$\boxed{\underline{a}_{C_V} = -D\,\omega^2\,\underline{n}}$$

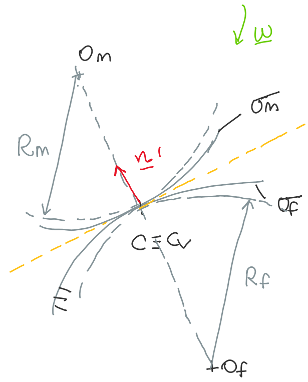
$$\underline{n}' = \frac{\overrightarrow{C_V O_m}}{|\overrightarrow{C_V O_m}|}$$

•) $\underline{n} \perp$ tg allo σ_m e σ_f $\Rightarrow$ possiamo sceglierlo

$$\underline{n}'' = \frac{\overrightarrow{C_V O_f}}{|\overrightarrow{C_V O_f}|}$$

•) A) Il moto RSS rispetto al TELAIO $\underline{\omega} = \underline{\omega}^{ASS}$

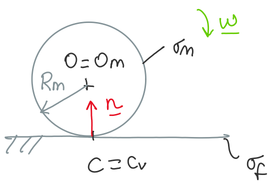
B) Il moto RSS rispetto al CORPO IN MOTO $\underline{\omega} = \underline{\omega}^{REL} = \underline{\omega}_2 - \underline{\omega}_1$, con $R \to \underline{\omega}_1$, corpo in moto $\to \underline{\omega}_2$



•) $$\frac{1}{D} = \frac{1}{\tilde R_f} - \frac{1}{\tilde R_m}$$

$\tilde R_f \neq R_f$, $\quad \tilde R_m \neq R_m$

$$\left.\begin{aligned} \tilde R_f &= \overrightarrow{C_V O_f} \cdot \underline{n} \\ \tilde R_m &= \overrightarrow{C_V O_m} \cdot \underline{n} \end{aligned}\right\} \text{RAGGI DI CURVATURA CON SEGNO}$$



$R_f = \infty$

$$\underline{a}_{C_V} = -D\,\omega^2\,\underline{n}$$

$$\frac{1}{D} = \cancel{\frac{1}{\tilde R_f}} - \frac{1}{\tilde R_m}$$

$$\tilde R_f = \overrightarrow{C_V O_f} \cdot \underline{n} = \infty$$

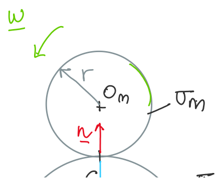
$$\tilde R_m = \overrightarrow{C_V O_m} \cdot \underline{n} = (R_m \underline{n}) \cdot \underline{n} = R_m > 0$$

$$\frac{1}{D} = -\frac{1}{R_m} \Rightarrow D = -R_m$$

$$\Downarrow$$

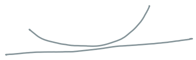
$$\boxed{\underline{a}_{C_V} = R_m \omega^2 \underline{n}}$$

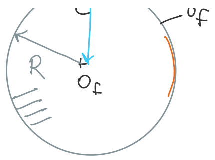
POLARI CIRCOLARI



$$\underline{a}_{C_V} = -D\omega^2\underline{n}$$

$$\tilde R_f = \overrightarrow{C_V O_f} \cdot \underline{n} = -R$$

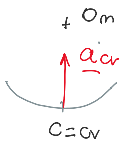




$$\tilde{R}_m = \overrightarrow{C_vO_m} \cdot \underline{n} = r$$

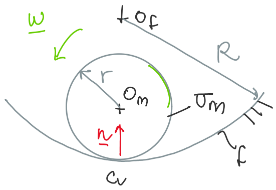
$$\frac{1}{D} = \frac{1}{\tilde{R}_f} - \frac{1}{\tilde{R}_m} = -\frac{1}{R} - \frac{1}{r}$$

$$\frac{1}{D} = -\frac{r+R}{Rr}$$



$$D = -\frac{Rr}{r+R} \qquad D < 0$$

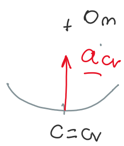
$$\underline{a}_{cv} = \frac{Rr}{r+R}\,\omega^2\,\underline{n}$$



$$\tilde{R}_f = \overrightarrow{C_vO_f} \cdot \underline{n} = R$$

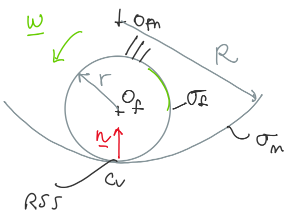
$$\tilde{R}_m = \overrightarrow{C_vO_m} \cdot \underline{n} = r$$

$$\frac{1}{D} = \frac{1}{R} - \frac{1}{r} = \frac{r-R}{Rr}$$



$$D = \frac{Rr}{r-R} \qquad < 0$$

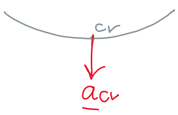
$$\underline{a}_{cv} = \frac{Rr}{r-R}\,\omega^2\,\underline{n}$$



$$\tilde{R}_f = \overrightarrow{C_vO_f} \cdot \underline{n} = r$$

$$\tilde{R}_m = \overrightarrow{C_vO_m} \cdot \underline{n} = R.$$

$$\frac{1}{D} = \frac{1}{r} - \frac{1}{R} = \frac{R-r}{rR}$$



$$D = \frac{rR}{R-r} \qquad > 0$$

$$\underline{a}_{cv} = -\frac{rR}{R-r}\,\omega^2\,\underline{n}$$