Cinematica del punto materiale

venerdì 15 novembre 2024 12:39

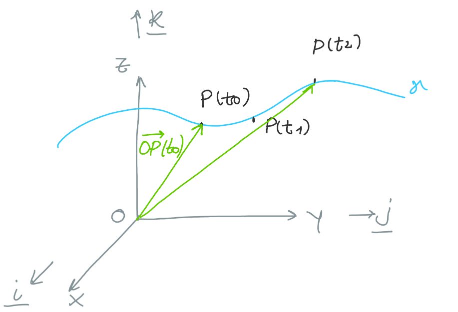
MOTO → SDR < SPAZIO / TEMPO

CINEMATICA < PUNTO MATERIALE / CORPO RIGIDO / SISTEMI DI CORPI RIGIDI

– CINEMATICA PUNTO MATERIALE –

PUNTO P

SDR $S = \{O; x, y, z\}$, SDR FISSO



TRAIETTORIA DI $P \Rightarrow \gamma$

$\Rightarrow$ POSIZIONE DI "P"

•) VETTORE POSIZIONE $\quad \overrightarrow{OP}(t) = P(t) - O$

$\overrightarrow{OP}$

Eq^ne VETTORIALE DEL MOTO DI P

•) VETTORE VELOCITA' $\quad \underline{v}_P(t) = \dfrac{d\,\overrightarrow{OP}(t)}{dt}$

•) VETTORE ACCELER. $\quad \underline{a}_P(t) = \dfrac{d^2\,\overrightarrow{OP}(t)}{dt^2} = \dfrac{d\,\underline{v}_P(t)}{dt}$

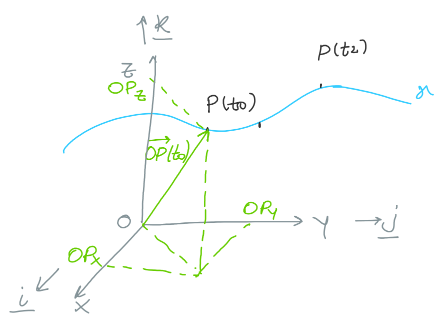
$\Rightarrow$ RAPPRESENTAZIONE CARTESIANA

$$\overrightarrow{OP}(t) = \begin{bmatrix} OP_x(t) \\ OP_y(t) \\ OP_z(t) \end{bmatrix} = \overbrace{OP_x(t)\,\underline{i}}^{\text{IL COMPONENTE}} + OP_y(t)\,\underline{j} + OP_z(t)\,\underline{k} = \begin{bmatrix} x_p(t) \\ y_p(t) \\ z_p(t) \end{bmatrix}$$

LA COMPONENTE

$$\vec{v}_p(t) = \begin{bmatrix} \dot{x}_p(t) \\ \dot{y}_p(t) \\ \dot{z}_p(t) \end{bmatrix}$$

$$\vec{a}_p(t) = \begin{bmatrix} \ddot{x}_p(t) \\ \ddot{y}_p(t) \\ \ddot{z}_p(t) \end{bmatrix}$$

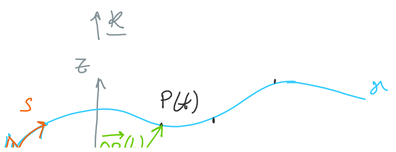


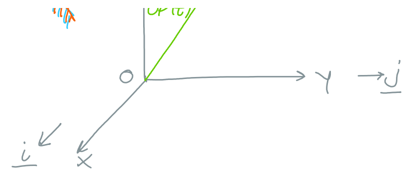
MOTO DEL PUNTO MAT LUNGO TRAIETTORIA NOTA

•) s concorde con il verso percorrenza di P su γ

•) $s(t)$

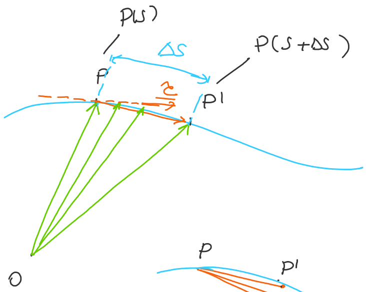
$\underline{v}_P(s)$





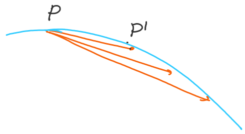
$OP(t) \neq OP(s)$, $a_p(s)$

$$v_P(s) = \frac{d\,\vec{OP}(t)}{dt} = \frac{d\,\vec{OP}(s)}{dt} = \underbrace{\frac{d\,\vec{OP}(s)}{ds}}_{?}\;\overbrace{\frac{ds}{dt}}^{\dot{s}} = \dot{s}$$



$$\frac{d\vec{OP}}{ds} = \lim_{\Delta s\to 0}\frac{\vec{OP}(s+\Delta s) - \vec{OP}(s)}{\Delta s}$$

$$= \lim_{\Delta s\to 0}\frac{\vec{OP'} - \vec{OP}}{\Delta s} = \lim_{\Delta s\to 0}\frac{\vec{PP'}}{\Delta s}$$

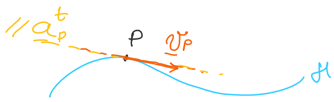


$$= \underbrace{1}\,\underline{\tau} = \underline{\tau}$$

$$\lim_{\Delta s\to 0}\frac{|\vec{PP'}|}{\Delta s} = \lim_{\Delta s\to 0}\frac{\text{corda}}{\text{arco}} = 1$$

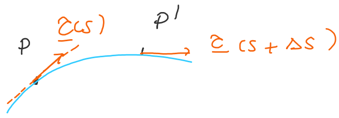
$$\boxed{v_P(s) = \dot{s}\,\underline{\tau}}$$ tg alla traiettoria

$$a_p(s) = \frac{d}{dt}v_P(s) = \frac{d(\dot{s}\,\underline{\tau})}{dt} = \underbrace{\ddot{s}\,\underline{\tau}}_{\substack{\parallel\\ \text{ACCEL.}\\ \text{TANGENZIALE}\\ \underline{a}_P^t}} + \underbrace{\dot{s}\left(\frac{d\underline{\tau}}{dt}\right)}_{\dot{s}^2\left(\frac{d\underline{\tau}}{ds}\right)}$$

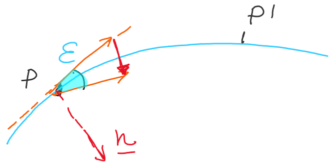


$$\frac{d\underline{\tau}}{dt} = \frac{d\underline{\tau}}{ds}\,\frac{ds}{dt} = \dot{s}\,\frac{d\underline{\tau}}{ds}$$

$$\frac{d\underline{\tau}}{ds} = \lim_{\Delta s\to 0}\frac{\underline{\tau}(s+\Delta s) - \underline{\tau}(s)}{\Delta s}$$

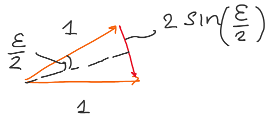


$$= \quad \underline{n}$$



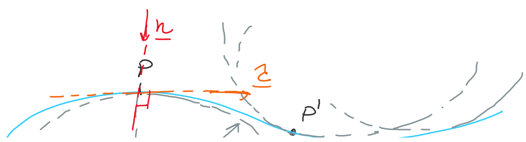
$$\left|\frac{d\underline{\tau}}{ds}\right| = \lim_{\Delta s\to 0}\frac{2\sin\left(\frac{\varepsilon}{2}\right)}{\Delta s} =$$

$$= \lim_{\Delta s\to 0}\underbrace{\left(\frac{2}{\varepsilon}\,\sin\left(\frac{\varepsilon}{2}\right)\right)}_{1}\cdot\frac{\varepsilon}{\Delta s} = C$$

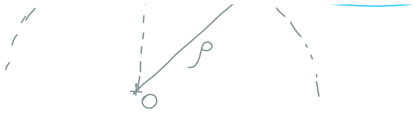


ε = ANGOLO DI CONTINGENZA

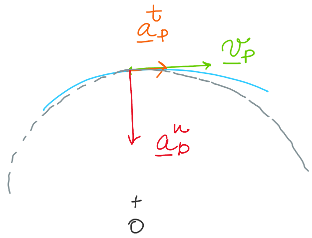
$$\lim_{\Delta s\to 0}\frac{\varepsilon}{\Delta s} = C$$ CURVATURA DI γ IN P



$$\rho = \frac{1}{\;}$$ RAGGIO DI CURVATURA



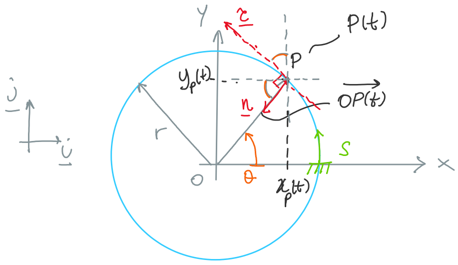
$$\underline{a}_p(s) = \frac{d}{dt}\underline{v}_p(s) = \frac{d(\dot{s}\,\underline{\tau})}{dt} = \underbrace{\ddot{s}\,\underline{\tau}}_{\substack{\parallel \\ \text{ACCEL. TANGENZIALE} \\ \underline{a}_p^t}} + \underbrace{\frac{\dot{s}^2}{\rho}\,\underline{n}}_{\substack{\parallel \\ \text{ACC. NORMALE} \\ \underline{a}_p^n}}$$



$\underline{a}_p^t$: $\frac{\Delta}{\Delta t}$ DEL MODULO DELLA $\underline{v}_p$

$\underline{a}_p^n$: $\frac{\Delta}{\Delta t}$ DELLA DIREZIONE DELLA $\underline{v}_p$

MOTO LUNGO TRAIETTORIA CIRCOLARE

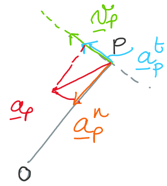


PARAMETRIZZARE s con la coor. angolare θ

$$\begin{cases} s = r\theta \\ \dot{s} = r\dot{\theta} \\ \ddot{s} = r\ddot{\theta} \end{cases}$$

$$\underline{v}_p = \dot{s}\,\underline{\tau} = r\dot{\theta}\,\underline{\tau} = r\dot{\theta}\,(-\sin\theta\,\underline{i} + \cos\theta\,\underline{j})$$

$\underline{\tau}(t)$, $\underline{\tau}(\theta) = \begin{bmatrix} -\sin\theta \\ +\cos\theta \end{bmatrix}$



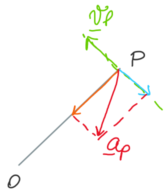
$$\begin{cases} \dot{\theta} > 0 \\ \ddot{\theta} > 0 \end{cases}$$

$$\underset{\sim}{\underline{a}_p} = \ddot{s}\,\underline{\tau} + \frac{\dot{s}^2}{\rho}\,\underline{n}$$

$$= r\ddot{\theta}\,\underline{\tau} + \frac{(r\dot{\theta})^2}{r}\,\underline{n}$$

$\underline{n}(\theta) = \begin{bmatrix} -\cos\theta \\ -\sin\theta \end{bmatrix}$

$$= \underset{\underline{a}_p^t}{r\ddot{\theta}\,\underline{\tau}} + \underset{\underline{a}_p^n}{r\dot{\theta}^2\,\underline{n}}$$



$$\begin{cases} \dot{\theta} > 0 \\ \ddot{\theta} < 0 \end{cases}$$

MOTO CIRCOLARE UNIFORME

$\dot{s} = \text{cost} \Rightarrow \dot{\theta} = \text{cost}$, $\ddot{\theta} = 0$

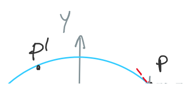
$\Rightarrow$ MOTO PERIODICO

VELOCITA' ANGOLARE $= \omega = \dot{\theta}$

$$T = \frac{2\pi}{|\omega|}$$

Le eq^{ne} CARTESIANE DEL MOTO:

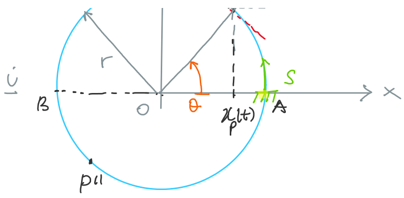
$$\begin{cases} x_p(t) = r\cos(\omega t + \theta_0) \\ y_p(t) = r\sin(\omega t + \theta_0) \end{cases}$$



MOTO ARMONICO

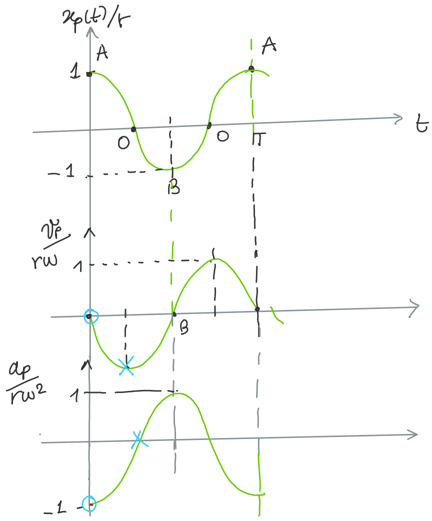
$\Rightarrow$ moto della proiezione di P su uno degli assi con P avente moto circ. uniforme

$\Rightarrow$ moto periodo $T = \frac{2\pi}{|\omega|}$



$$\begin{cases} x_p(t) = r\cos(\omega t + \theta_0) & (1) \\ \dot{x}_p(t) = -r\omega \sin(\omega t + \theta_0) & (2) \\ \ddot{x}_p(t) = -r\omega^2 \cos(\omega t + \theta_0) & (3) \end{cases}$$

se Hp $\theta_0 = 0 \Rightarrow P(0) \equiv A$

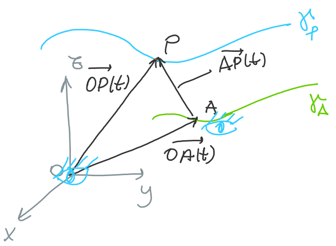


$(1) \to (3) \Rightarrow \quad \omega^2 x_p + \ddot{x}_p = 0$

$$\boxed{\ddot{x}_p + \omega^2 x_p = 0}$$

Eq^{ne} MOTO ARMONICO

MOTO RELATIVO DI UN PUNTO P



$$v_P = \frac{d\overrightarrow{OP}(t)}{dt} = \frac{d}{dt}(P - O)$$

$\boxed{\overrightarrow{OP}(t)}$ $\rightarrow$ $v_P \leftrightarrow v_A$? $a_P \leftrightarrow a_A$?

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$$

$\downarrow d/dt$

$$\frac{d\overrightarrow{OP}}{dt} = \frac{d\overrightarrow{OA}}{dt} + \frac{d\overrightarrow{AP}}{dt}$$

$\frac{d}{dt}(P - A) = v_P - v_A = v_{PA}$ VELOCITA' RELATIVA DI P RISPETTO AD A

$$\boxed{v_P = v_A + v_{PA}}$$

VEL RELATIVA

$\Downarrow d/dt$

$$\boxed{a_P = a_A + a_{PA}}$$

ACC. RELATIVA