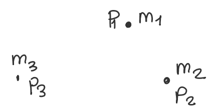
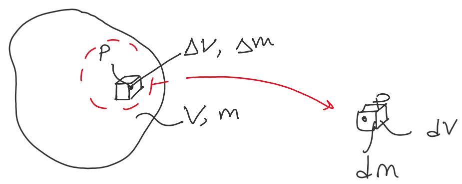
Massa e Baricentro

venerdì 29 novembre 2024 12:50

SISTEMI
- DISCRETI
- CONTINUI





— MASSA —

S.DIS $\quad m = \sum_i m_i \quad$ MASSA TOTALE

S. CON $\quad V \longleftrightarrow m \quad$ MASSA TOTALE

DENSITA' MEDIA $\quad \bar{\rho} = \dfrac{\Delta m}{\Delta V}$

DENSITA' LOCALE $\quad \rho = \lim_{\Delta V \to P} \dfrac{\Delta m}{\Delta V} = \rho(P)$

- SIST. OMOGENEO $\quad \rho = \text{cost}$
- SIST NON OMOGENEO $\quad \rho(P)$

MASSA ELEM. INFINITESIMO $\quad dm = \rho(P)\, dV$

$\quad\quad \Big| \int$

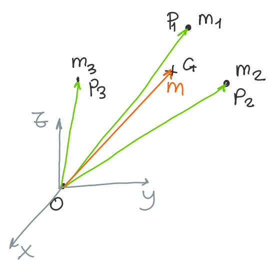
MASSA TOTALE $\quad m = \int_V \rho(P)\, dV$

— BARICENTRO G —

S DIS $\quad$ consideriamo "O" arbitrario

$$\vec{OG} = \frac{1}{m} \sum_i \vec{OP_i}\, m_i$$

G e' INDIPENDENTE DA "O"



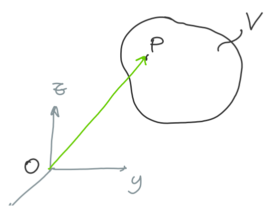
→ se introduciamo SDR cartesiano

$$\begin{cases} x_G = \dfrac{1}{m} \sum_i m_i\, x_{P_i} \\ y_G = \dfrac{1}{m} \sum_i m_i\, y_{P_i} \\ z_G = \dfrac{1}{m} \sum_i m_i\, z_{P_i} \end{cases}$$

S CON

$$\vec{OG} = \frac{1}{m} \int \vec{OP}\, dm$$

$$= \frac{1}{m} \int_V \vec{OP}\, \rho\, dV$$

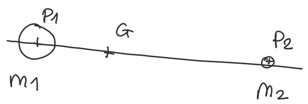


in " " $\rho(P)$

$$\begin{cases} x_G = \frac{1}{m}\int x_P\,\rho\,dV \\ y_G = \frac{1}{m}\int y_P\,\rho\,dV \\ z_G = \frac{1}{m}\int z_P\,\rho\,dV \end{cases} \xrightarrow[\substack{\text{CORPO} \\ \text{OMOGENEO} \\ \downarrow \\ \rho \text{ cost}}]{} \begin{cases} x_G = \frac{1}{m}\underset{m/V}{\rho}\int x_P\,dV = \frac{1}{V}\int x_P\,dV \\ y_G = \frac{1}{V}\int y_P\,dV \\ z_G = \frac{1}{V}\int z_P\,dV \end{cases}$$

PROPRIETA' BARICENTRI

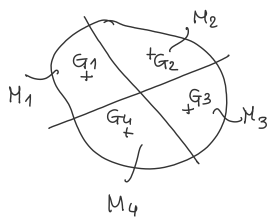
a)



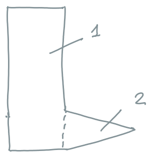
$$\frac{\overline{GP_1}}{m_1} = \frac{m_2}{\overline{GP_2}}$$

b) se i punti $P_i \in \pi(r)$, anche $G \in \pi(r)$

c) PROP. DISTRIBUTIVA

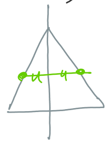


$$\vec{OG} = \frac{1}{\sum M_i}\sum \vec{OG_i}\,M_i$$

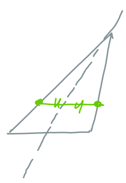


d) se il sistema ha assi di simmetria, il G si trova su questi assi

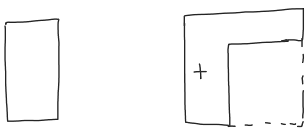
SIMME. ORTOGONALE

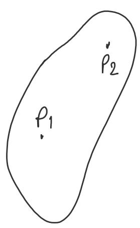


SIMM. OBLIQUA



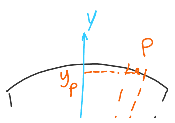
e) baricentro sta "dentro"





ESEMPI CALCOLO BARICENTRO

▶ ARCO DI CIRCONFERENZA

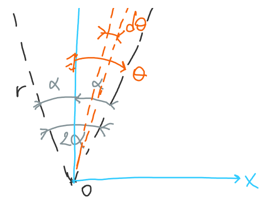


$L = r\,2\alpha$

1) scegliamo un SDR comodo

2) "O" ⇒ $\vec{OG}$

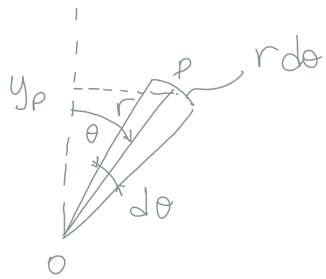
3) ... $x_G = 0$ ⇒ $G \in Y$



3) per simmetria $x_G = \dots$

$\Downarrow$

y_G?



SOLUZIONE

(3D) $$y_G = \frac{1}{V}\int y_p \, dV$$

(1D) $$y_G = \frac{1}{\underbrace{L}_{r2\alpha}}\int y_p \, \underbrace{dL}_{r\,d\theta}$$

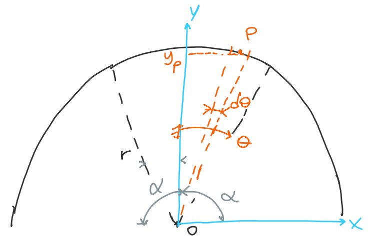
$$y_G = \frac{1}{r2\alpha}\int_{-\alpha}^{\alpha} y_p \, r \, d\theta$$

$$= \frac{1}{r2\alpha}\int_{-\alpha}^{\alpha} r\cos\theta \, r \, d\theta$$

$$= \frac{r^2}{r2\alpha}\sin\theta \Big|_{-\alpha}^{\alpha} = \frac{r\sin\alpha}{\alpha}$$

$$[\overrightarrow{OG}] = \begin{bmatrix} 0 \\ \frac{r\sin\alpha}{\alpha} \end{bmatrix}$$

▶ SEMICIRCONFERENZA

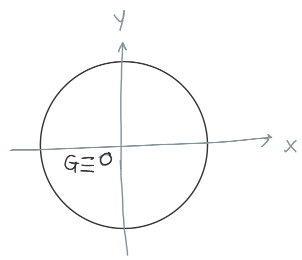


$\alpha = \pi/2$

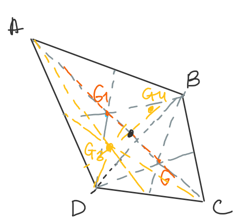
$$y_G = \frac{r\sin(\pi/2)}{\pi/2} = \frac{2r}{\pi}$$

▶ CIRCONFERENZA

$\alpha = \pi \implies \begin{cases} y_G = 0 \\ x_G = 0 \end{cases}$



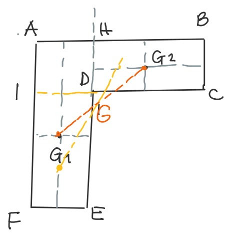
▶ BARICENTRO DI UN QUADRILATERO



$G \Rightarrow \left.\begin{matrix} G_1 \, , \, \overset{\triangle}{ABD} \\ G_2 \, , \, \overset{\triangle}{BCD} \end{matrix}\right\}$ retta G_1 e G_2 - - -

$G \Rightarrow \left.\begin{matrix} G_3 \, , \, \overset{\triangle}{ACD} \\ G_4 \, , \, \overset{\triangle}{ACB} \end{matrix}\right\}$ retta per G_3 e G_4 - - -

▶ BARICENTRO FIGURA "L"

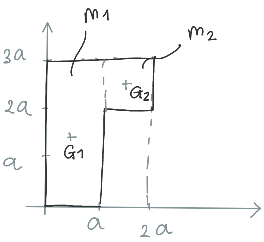


$G_1 \Rightarrow \triangle HEF$ ___

$G_2 \Rightarrow$ H B C D

$G_3 \Rightarrow$ A B C I ___

$G_4 \Rightarrow$ F I D E



$$\rho \Rightarrow \begin{cases} m_1 = \rho A_1 = \rho\, 3a^2 \\ m_2 = \rho A_2 = \rho a^2 \end{cases} \quad m = m_1 + m_2 = \rho\, 4a^2$$

$$\vec{OG_1} = \left(\frac{a}{2} ; \frac{3}{2}a\right)$$

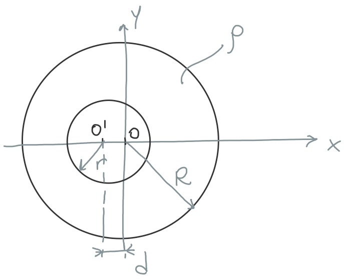
$$\vec{OG_2} = \left(\frac{3}{2}a ; \frac{5}{2}a\right)$$

$$\longrightarrow \vec{OG} = \frac{1}{m}\left(m_1 \vec{OG_1} + m_2 \vec{OG_2}\right)$$

$$[\vec{OG}] = \frac{1}{4\cancel{a^2}\cancel{\rho}}\left(\cancel{\rho}\, 3\cancel{a^2}\begin{bmatrix} a/2 \\ 3/2\, a \end{bmatrix} + \cancel{\rho}\,\cancel{a^2}\begin{bmatrix} 3/2\, a \\ 5/2\, a \end{bmatrix}\right)$$

$$= \frac{1}{4}\begin{bmatrix} \frac{3}{2}a + \frac{3}{2}a \\ \frac{9}{2}a + \frac{5}{2}a \end{bmatrix} = \begin{bmatrix} 3/4\, a \\ 7/4\, a \end{bmatrix}$$

▷ CERCHIO FORATO



cerchio "PIENO" $\Rightarrow \tilde{M} = \pi R^2 \rho$

foro $\Rightarrow m = \pi r^2 \rho$

$$M = \tilde{M} - m = \pi\rho\,(R^2 - r^2)$$

$$\Rightarrow M\,\vec{OG} = \tilde{M}\,\cancel{\vec{OO}} - m\,\vec{OO'}$$

$$\quad\quad\;\; \emptyset$$

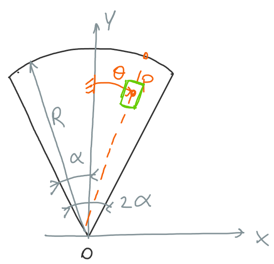
$$M\vec{OG} = -m\,\vec{OO'}$$

Per simmetria: $y_G = 0$

$$\Rightarrow M x_G = -m\,(-d)$$

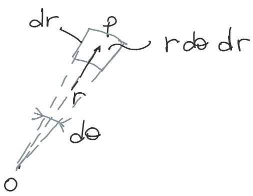
$$x_G = \frac{m d}{M} = \frac{r^2}{R^2 - r^2}\, d$$

▷ SETTORE CIRCOLARE



•) per simmetria $\quad x_G = 0$

•) $\quad y_G = \dfrac{1}{A}\displaystyle\int y_P\, dA$



$\Downarrow$

coor. polari (r, θ)

$$y_G = \frac{1}{R^2\alpha}\int_{-\alpha}^{\alpha}\int_0^R (r\cos\theta)\, r\, d\theta\, dr$$

$$= \frac{1}{R^2\alpha}\int_0^R r^2 dr \int_{-\alpha}^{\alpha}\cos\theta\, d\theta$$

$$= \frac{1}{R^2\alpha}\left(\frac{r^3}{3}\Big|_0^R \;\sin\theta\Big|_{-\alpha}^{\alpha}\right)$$

$$= \frac{2}{3}\,\frac{\sin\alpha}{\alpha}\, R$$