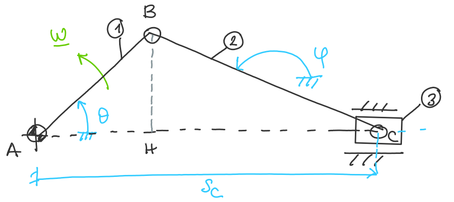
Eserc: An Cinem Posizione MDS

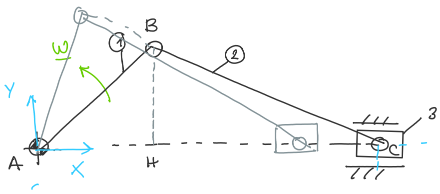
venerdì 22 novembre 2024 12:30

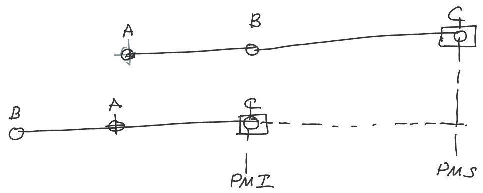


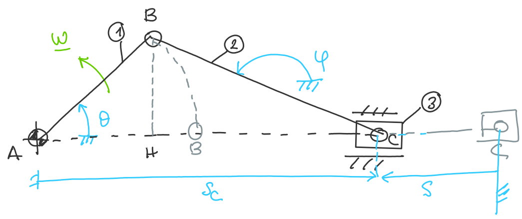
Noto: $\overline{AB} = r = 68.1$ mm
$\overline{BC} = \ell = 164$ mm

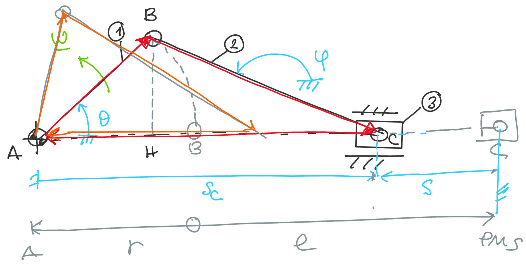
Noto $\theta(t)$, $\dot\theta(t)$, $\ddot\theta(t)$
↓ cin. posiz.
Trovare $\gamma(t)$, $\dot\gamma(t)$, $\ddot\gamma(t)$
$\underbrace{S_C(t),\ S_C'(t),\ S_C''(t)}$ → $t = \bar t$

$$\left[\begin{array}{l} \theta(\bar t) = 45^\circ \\ \dot\theta(\bar t) = 6\,300 \text{ rpm} \simeq 660 \text{ rad/s} \\ \ddot\theta(\bar t) = 0 \quad \text{rad}/s^2 \end{array}\right] \longrightarrow \begin{array}{l} \dot S_C(\bar t) \\ \ddot S_C(\bar t) \end{array}$$









Ep^{ne} chiusura ⇒ $\varphi(t)$, $s(t)$

$$\boxed{\overrightarrow{AB}(t) + \overrightarrow{BC}(t) + \overrightarrow{CA}(t) = \underline{0}}$$

$\overrightarrow{AB}(t) = (r\cos\underbrace{\theta}_{\theta(t)},\ r\sin\theta,\ 0)$

$\overrightarrow{BC}(t) = (-\ell\cos\varphi,\ -\ell\sin\underbrace{\varphi}_{\varphi(t)},\ 0)$

$\overrightarrow{CA}(t) = (-(r+\ell-\underbrace{s}_{s(t)}),\ 0,\ 0)$

$$\begin{bmatrix} r\cos\theta \\ r\sin\theta \\ 0 \end{bmatrix} + \begin{bmatrix} -\ell\cos\varphi \\ -\ell\sin\varphi \\ 0 \end{bmatrix} + \begin{bmatrix} -(r+\ell-s) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

2 INCOGNITE
$\begin{cases} \varphi(t) \\ s(t) \end{cases}$

$$\begin{array}{l} x: \\ y: \end{array} \begin{cases} r\cos\theta - \ell\cos\varphi - (r+\ell-s) = 0 \\ r\sin\theta - \ell\sin\varphi = 0 \end{cases}$$

$$\begin{cases} \sin\varphi = \frac{r}{\ell}\sin\theta = \lambda\sin\theta \qquad (1) \end{cases}$$

$\boxed{\lambda = \frac{r}{\ell}}$ < 0.1 – 1 (a)

$$s = r + \ell + \ell\cos\varphi - r\cos\theta \qquad (2)$$

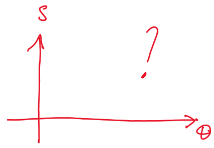
$< 0.1 \quad (b)$

$$\cos\varphi = -\sqrt{1-\sin^2\varphi} = -\sqrt{1-\lambda^2\sin^2\theta}$$

$$s(\theta) = s = r + \ell - \ell\sqrt{1-\lambda^2\sin^2\theta} - r\cos\theta$$

$$= r\left[1 - \cos\theta + \frac{1}{\lambda}\Big(1 - \underbrace{\sqrt{1-\lambda^2\sin^2\theta}}_{\substack{\text{linearizziamo} \\ -\cos\varphi}}\Big)\right]$$

Soluzione esatta della posizione di "c"



$$\dot{s} = \frac{ds}{dt}$$

$$\ddot{s} = \frac{d\dot{s}}{dt}$$

$\boxed{(a) \quad \lambda = 0{,}1 - 1}$

$$x = -\lambda^2\sin^2\theta \qquad x \to 0$$

$$f(x) = \sqrt{1+x} = (1+x)^{1/2} \longleftrightarrow f'(\theta) = \sqrt{1-\lambda^2\sin^2\theta}$$

$x = x_0 = 0 \rightarrow$ sviluppo serie Taylor

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)(x-x_0)^2}{2!} + o((x-x_0)^2)$$

$$f(x) = f(0) + f'(0)\,x + o(x)$$

$$f'(x) = \frac{1}{2}(1+x)^{1-1/2}$$

$$f(x) = 1 + \frac{1}{2}x + o(x)$$

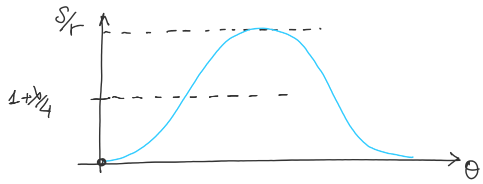
$$f(\theta) = 1 - \frac{1}{2}\lambda^2\sin^2\theta$$

$$s \simeq r\left[1 - \cos\theta + \frac{1}{\lambda}\left(1 - 1 + \frac{1}{2}\lambda^2\sin^2\theta\right)\right]$$

$$\simeq r\left[1 - \cos\theta + \frac{\lambda}{2}\sin^2\theta\right] \qquad \sin^2\theta = \frac{1-\cos(2\theta)}{2}$$

$$\simeq r\Big[\underbrace{1 + \frac{\lambda}{4}}_{\substack{\text{valor} \\ \text{medio}}} \underbrace{-\cos\theta}_{T_1 = 2\pi} \underbrace{-\frac{\lambda}{4}\cos(2\theta)}_{T_1 = \pi}\Big]$$

Soluz. approssimata al 2° ordine



$$\dot{s} = r\dot{\theta}\left[\sin\theta + \frac{\lambda}{2}\sin(2\theta)\right]$$

$$\ddot{s} = r\ddot{\theta}\left[\sin\theta + \frac{\lambda}{2}\sin(2\theta)\right] + r\dot{\theta}^2\left[\cos\theta + \lambda\cos(2\theta)\right]$$

$$\ddot{s}\,(\ddot{\theta} = 0) = r\dot{\theta}^2\left[\cos\theta + \lambda\cos(2\theta)\right]$$

$\boxed{(b) \quad \lambda < 0.1}$

•) SVI. SER. TAYLOR $\Rightarrow$ $f(x) \simeq f(0)$; $\cos x = -1$

$\lambda < 0.1$

$\dfrac{r}{e} < 0.1$

$s = r + \ell + \ell\cos\varphi - r\cos\theta$ ($\simeq \ell$)

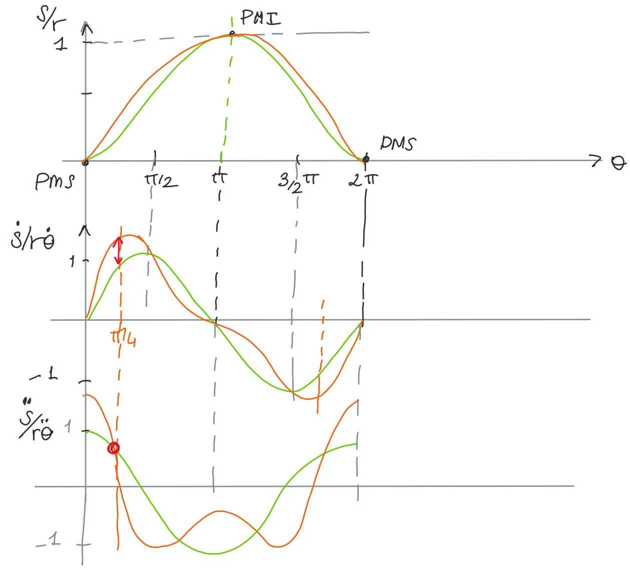
$\simeq r - r\cos\theta$

$$\begin{cases} s(\theta) = \boxed{r[1-\cos\theta]} \\ \dot{s}(\theta) = r\dot{\theta}\sin\theta \\ \ddot{s}(\theta) = r\dot{\theta}^2\cos\theta \end{cases}$$

SOLUZ. APPROSSIMATA AL 1° ORDINE

$\lambda = 0.5$

$\ddot{\theta} = 0$



— 1° ORDINE

— 2° ORDINE ≃ ESATTA

$\theta = 45° \leftarrow \boxed{t = \bar{t}}$

$\equiv \pi/4$

	SOLUZ. ESATTA	SOLUZ. II ORD.	SOLUZ I ORDIN.
s (m)	0,0272	0,027 (0.7%)	0.0199 (27%)
$\dot{s}$ (m/s)	41,527	41,097 (1%)	31,769 (23%)
$\ddot{s}$ (m/s²)	21566,4	20959 (3%)	20959 (3%)