3.1) a) G(s) berechnen

$$\dot{\vec{x}}_1 = \begin{pmatrix}-3 & 2 & -2\\ -6 & 0 & 5\\ -6 & 2 & 3\end{pmatrix}\vec{x}_1 + \begin{pmatrix}0\\1\\0\end{pmatrix}u_1$$

$$y_1 = \begin{pmatrix}2 & 0 & 0\end{pmatrix}\vec{x}_1$$

Step 1: $\Phi(s) = (sE - A)^{-1}$

$$\begin{pmatrix}s+3 & -2 & 2\\ 6 & s & -5\\ 6 & -2 & s-3\end{pmatrix}^{-1}$$

Inverse mit adj(A):

$$A^{-1} = \frac{\operatorname{adj} A}{\det A} \qquad \operatorname{adj} A = \begin{pmatrix}\tilde{a}_{11} & \tilde{a}_{12} & \cdots\\ \tilde{a}_{21} & a_{22} & \cdots\\ \vdots & & \ddots\end{pmatrix}^T \qquad \tilde{a}_{ij} = (-1)^{i+j} M_{ij}$$

$\tilde{a}_{11} = s\cdot(s-3) - 10$ ✓ $\quad \tilde{a}_{12} = -(6(s-3)+36)$ ✓ $\quad \tilde{a}_{13} = -12 - 6s$ ✓ $\quad \tilde{a}_{21} = -(-2(s-3)+4)$

$\tilde{a}_{22} = (s+3)(s-3) - 12$ ✓ $\quad \tilde{a}_{23} = -(-2(s+3)+12)$ $\quad \tilde{a}_{31} = 10 - 2s$ ✓ $\quad \tilde{a}_{32} = -(5(s+3)-12)$

$\tilde{a}_{33} = s(s+3) + 12$ ✓

$$\begin{aligned}\det A &= (s+3)\,s(s-3) + 60 - 24 - 12s + 12(s-3) - 10(s+3)\\ &= s^3 - 9s - 12s + 12s - 36 - 10s - 30 + 60 - 24\\ &= \underline{s^3 - 19s - 30}\end{aligned}$$

NR: $60 - 24 - 36 - 30 = -30$

$-9 - 12 + 12 - 10 = -19$

$$\operatorname{adj} A = \begin{pmatrix}s^2 - 3s - 10 & 2s - 10 & -2s + 10\\ -6s - 18 & s^2 - 3^2 & -5s - 3\\ -6s - 12 & 2s + 18 & s^2 + 3s + 12\end{pmatrix}$$

$$\boxed{\Phi(s) = \frac{1}{s^3 - 19s - 30}\begin{pmatrix}s^2 - 3s - 10 & 2s - 10 & -2s + 10\\ -6s - 18 & s^2 - 3^2 & -5s - 3\\ -6s - 12 & 2s + 18 & s^2 + 3s + 12\end{pmatrix}}$$

Step 2: $G(s) = C\,\Phi(s)\,B + \cancel{D}$

$$G(s) = \frac{1}{(s^3 - 19s - 30)}\begin{pmatrix}2 & 0 & 0\end{pmatrix}\begin{pmatrix}s^2 - 3s - 10 & 2s - 10 & -2s + 10\\ -6s - 18 & s^2 - 3^2 & -5s - 3\\ -6s - 12 & 2s + 18 & s^2 + 3s + 12\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}$$

$$G_1(s) = \frac{2(2s-10)}{(s^3-19s-30)}$$

Teilerfremd?

$$\frac{4(s-5)}{s^3-19s-30} \qquad z(s) \overset{!}{=} 0 \qquad s = 5$$

$n(5) = 5^3 - 19 \cdot 5 - 3 = \underline{0}$

<u>nicht TF</u>

viel Effizienter: $G(s) = C\,\Phi(s)\,B$

$$(2\ 0\ 0)\begin{pmatrix} s+3 & -2 & 2 \\ 6 & s & -5 \\ 6 & -2 & s-3 \end{pmatrix}^{-1}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$\rightarrow 2\,\Phi_{12} \qquad \Phi_{12} = \frac{1}{\det A} \cdot \tilde{a}_{21} = 4(s-5)$

$$G_1(s) = \frac{4(s-5)}{s^3-19s-30}$$

Polynomdiv:

$s^3 + 0s^2 - 19s - 30 : s-5 = s^2 + 5s + 6 = \underline{(s+3)(s+2)}$

$-(s^3 - 5s^2)$

$\quad 5s^2 - 19s - 30$

$\quad -5s^2 + 25s$

$\qquad 6s - 30$

$\qquad -6s + 30$

$\qquad \underline{\underline{0}}$

Linearfaktorzerlegung

	1	5	6
-3		-3	-6
	1	2	0

$$G_1(s) = \frac{4}{(s+3)(s+2)}$$

Sprungfähigkeit: $\lim_{s\to\infty} \frac{4}{(s+3)(s+2)} = 0 \rightarrow$ nicht sprungfähig

BIBO: $\mathrm{Re}(s_i) < 0 \qquad s_1 = -2\ ;\ s_2 = -3 \rightarrow$ BIBO stabil

b) $\dot{\vec{x}}_2 = \begin{pmatrix} 8 & -15 \\ 6 & -10 \end{pmatrix}\vec{x}_2 + \begin{pmatrix} 1 \\ 4 \end{pmatrix} u_2$

$y_2 = (2\quad 0)\vec{x}_2 + 4u_2$

Step 1: $\Phi(s) = (sE-A)^{-1} = M^{-1}$

$$= \begin{pmatrix} s-8 & +15 \\ -6 & s+10 \end{pmatrix}^{-1} = \frac{1}{\det M}\begin{pmatrix} m_{22} & -m_{12} \\ -m_{21} & m_{11} \end{pmatrix}$$

$$= \frac{1}{\underbrace{(s-8)(s+10)+6\cdot 15}}\begin{pmatrix} s+10 & -15 \\ 6 & s-8 \end{pmatrix} = \underline{\frac{1}{s^2+2s+10}\begin{pmatrix} s+10 & -15 \\ 6 & s-8 \end{pmatrix}}$$

$s^2 - 8s + 10s - 80 + 90 = s^2 + 2s + 10$

Step 2: $G(s) = C\,\Phi(s)^{-1}B + D = \frac{1}{s^2+2s+10}\begin{pmatrix}2 & 0\end{pmatrix}\begin{pmatrix}s+10 & -15\\ 6 & s-8\end{pmatrix}\begin{pmatrix}1\\4\end{pmatrix} + 4 =$

$= \frac{2}{s^2+2s+10}\begin{pmatrix}s+10 & -15\end{pmatrix}\begin{pmatrix}1\\4\end{pmatrix} + 4 = \frac{2}{s^2+2s+10}\left(s+10-60\right) + 4$

$$\frac{2s-100}{s^2+2s+10} + \frac{(s^2+2s+10)4}{s^2+2s+10} = \frac{4s^2+8s+2s+40-100}{s^2+2s+10} = \underline{\frac{4s^2+10s-60}{s^2+2s+10}}$$

Pole: $s^2+2s+10$ $\quad {}_1s_2 = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} = -\frac{2}{2} \pm \sqrt{\left(\frac{2}{2}\right)^2 - 10} = \underline{-1 \pm j\,9}$

Sprungfähigkeit: $\lim\limits_{s\to\infty} \frac{4s^2+10s-60}{s^2+2s+10} = 4 \neq 0 \rightarrow$ Sprungfähig

BIBO: $Re(s_i) < 0 \quad Re(s_1) < 0\,;\ Re(s_2) < 0 \rightarrow$ stabil