1.3)

a) $5\ddot{y} - \frac{1}{10}\dot{y}y = 7{,}5tu$

Linearität: ist es $\dot{\vec{x}} = A\vec{x} + B\vec{u}$, $\vec{y} = C\vec{x} + D\vec{u}$?

→ Zustände einführen $x_1 = y$; $x_2 = \dot{y}$ → Zustandsvektor

$$\dot{\vec{x}} = \frac{d}{dt}\vec{x} = \frac{d}{dt}\begin{pmatrix}x_1\\x_2\end{pmatrix}$$

$$\frac{d}{dt}\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}x_2\\\ddot{y}\end{pmatrix} = \begin{pmatrix}x_2\\(7{,}5tu + \frac{1}{10}x_2x_1)\frac{1}{5}\end{pmatrix}$$

$\begin{pmatrix}x_2\\(7{,}5tu + \frac{1}{10}x_2x_1)\frac{1}{5}\end{pmatrix}$ kann das in $A\vec{x} + B\vec{u}$ überführt werden?

weil nur ein Eigang

$$\begin{pmatrix}x_2\\(7{,}5tu + \frac{1}{10}x_2x_1)\frac{1}{5}\end{pmatrix} = \begin{pmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} + \begin{pmatrix}b_{11} & b_{12}\\b_{21} & b_{22}\end{pmatrix}u$$

$$\begin{pmatrix}x_2\\(7{,}5tu + \frac{1}{10}x_2x_1)\frac{1}{5}\end{pmatrix} = \begin{pmatrix}a_{11}x_1 + a_{12}x_2\\a_{21}x_1 + a_{22}x_2\end{pmatrix} + \begin{pmatrix}b_1 u\\b_2 u\end{pmatrix}$$ ← geht für $a_{11} = b_1 = 0$

$1{,}5tu + \frac{1}{50}x_1x_2 = a_{21}x_1 + a_{22}x_2 + b_2 u$ nicht linear weil $\underline{\underline{x_1x_2}}$

b) $\frac{1}{2}y''' - 10y'' - \frac{y}{1+t} = \int_0^t \sqrt{2}\,u(\tau)\,d\tau + \frac{1}{3}u'$ → $y''' = \left(\sqrt{2}\int_0^t u(\tau)\,d\tau + \frac{1}{3}u' + 10y'' + \frac{y}{1+t}\right)\cdot 2$

$x_1 = y$, $x_2 = y'$, $x_3 = y''$, $x_4 = \int_0^t u(\tau)\,d\tau$, $x_5 = u$, $\dot{u} = v$

$$\vec{x} = \begin{pmatrix}x_1\\x_2\\x_3\\\int_0^t u(\tau)\,d\tau\\x_5\end{pmatrix} \qquad \vec{u} = v \qquad \dot{\vec{x}} = \begin{pmatrix}x_2\\x_3\\y'''\\u\\v\end{pmatrix}$$

$$\dot{\vec{x}} = \begin{pmatrix} x_2 \\ x_3 \\ \left(\int_0^t \sqrt{2}\, u(\tilde{\tau})\, d\tilde{\tau} + \frac{1}{3} u' + 10 y'' + \frac{y}{1+t}\right) \cdot 2 \\ u \\ v \end{pmatrix}$$

jetzt schauen ob es in Matrix form geht:

$$\begin{pmatrix} x_2 \\ x_3 \\ \left(\int_0^t \sqrt{2}\, u(\tilde{\tau})\, d\tilde{\tau} + \frac{1}{3} v + 10 x_3 + \frac{x_1}{1+t}\right) \cdot 2 \\ u \\ v \end{pmatrix} = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 + a_{35}x_5 \\ a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 + a_{45}x_5 \\ a_{51}x_1 + a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{pmatrix} v$$

$$= \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \frac{2}{1+t} & 0 & 20 & 2\sqrt{2} & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 2/3 \\ 0 \\ 1 \end{pmatrix}$$

Linear, aber Zeitvariant weil t explizit

c) $\cos\left(\frac{4}{5}\pi\right)\ddot{y} + 3y = \frac{7}{10} u \qquad x_1 = y, \; x_2 = \dot{y} \qquad \cos\left(\frac{4}{5}\pi\right) = k$

$$\dot{\vec{x}} = \begin{pmatrix} x_2 \\ -\frac{3x_1}{k} + \frac{7}{10k} u \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} u$$

$$\begin{pmatrix} 0 & 1 \\ -\frac{3}{k} & 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ \frac{7}{10k} \end{pmatrix}$$

Linear & zeitinv.