2.4)

a) $\dot{\vec{x}} = \underbrace{\begin{pmatrix} -3 & -4 \\ 2 & 1 \end{pmatrix}}_{A} \vec{x} + \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{B} u, \qquad \vec{x}(0) = \vec{x}_0$

$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \vec{x} \qquad D = 0$

$\tilde{\Phi} = ?$ mit Laplacetrans

$\tilde{\Phi}(s) = (sE - A)^{-1} \rightarrow \begin{pmatrix} s+3 & 4 \\ -2 & s-1 \end{pmatrix}^{-1} = \frac{1}{\det A} \cdot \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{21} \\ \tilde{a}_{12} & \tilde{a}_{22} \end{pmatrix}^T$

$\tilde{a}_{ij} = (-1)^{j+i} M_{ij}$ $\qquad \tilde{a}_{11} = s-1 \qquad \tilde{a}_{21} = 2$

$\tilde{a}_{21} = -4 \qquad \tilde{a}_{22} = s+3$

$\tilde{\Phi}(s) = \frac{1}{\underbrace{(s+3)(s-1)+8}} \begin{pmatrix} s-1 & -4 \\ 2 & s+3 \end{pmatrix} \rightarrow$

$s^2 - s + 3s - 3 + 8 = s^2 + 2s + 5$ $\rightarrow$ erweiterung auf vollständigen Quadrat

$\rightarrow (s+1)^2 + 4$

$$\frac{1}{(s+1)^2+4} \begin{pmatrix} s-1 & -4 \\ 2 & s+3 \end{pmatrix}$$

Verlauf von y(t): $\qquad Y(s) = G(s)\, X(s) \qquad Y(s) = C\, X(s) + D\, U(s)$

$X(s) = \cancel{(sE-A)^{-1} x_0} + (sE-A)^{-1} B\, U(s)$ $\qquad$ Angabe $x_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$X(s) = (sE-A)^{-1} B U(s)$

$Y(s) = C \cdot \Phi(s)\, B U(s) \Rightarrow (1 \;\; 0) \frac{1}{(s+1)^2+4} \begin{pmatrix} s-1 & -4 \\ 2 & s+3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} U(s)$

$Y(s) = \frac{1}{(s+1)^2+4} (s-1 \;\; -4) \begin{pmatrix} 1 \\ 0 \end{pmatrix} U(s) = \frac{1}{(s+1)^2+4} (s-1)\, U(s)$

Angabe: $u(t) = e^t$ $\circ\!\!-\!\!\bullet$ $\frac{1}{s-1}$ $\qquad Y(s) = \frac{\cancel{s-1}}{(s+1)^2+4} \frac{1}{\cancel{s-1}} = \frac{1}{(s+1)^2+4}$

laut Tabelle

Tabelle: $e^{at} \sin(bt) \circ\!\!-\!\!\bullet \frac{b}{(s-a)^2+b^2}$

$$\frac{1}{2}\,\frac{2}{(s+1)^2+2^2} \to \boxed{\frac{1}{2}\,e^{-t}\sin(2t) = y(t)}$$

$\underline{\Phi}(t) = \#$ TODO