2.4)

a)

$$\dot{\vec{x}} = \begin{pmatrix} -3 & -4 \\ 2 & 1 \end{pmatrix} \vec{x} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u, \qquad \vec{x}(0) = \vec{x}_0$$

Over the matrices: A and B

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \vec{x} \qquad\qquad D = 0$$

C

$\vec{\underline{\Phi}} = ?$ mit Laplacetrans

$$\vec{\underline{\Phi}}(s) = (sE - A)^{-1} \;\to\; \begin{pmatrix} s+3 & 4 \\ -2 & s-1 \end{pmatrix}^{-1} = \frac{1}{\det A} \cdot \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{21} \\ \tilde{a}_{12} & \tilde{a}_{22} \end{pmatrix}^T$$

$$\tilde{a}_{ij} = (-1)^{j+i} M_{ij} \qquad \begin{array}{ll} \tilde{a}_{11} = s-1 & \tilde{a}_{21} = 2 \\ \tilde{a}_{21} = -4 & \tilde{a}_{22} = s+3 \end{array}$$

$$\vec{\underline{\Phi}}(s) = \frac{1}{(s+3)(s-1)+8} \begin{pmatrix} s-1 & -4 \\ 2 & s+3 \end{pmatrix} \;\to\;$$

$$s^2 - s + 3s - 3 + 8 = s^2 + 2s + 5 \qquad \to \text{erweiterung auf vollständiger Quadrat}$$
$$\to (s+1)^2 + 4$$

$$\frac{1}{(s+1)^2+4} \begin{pmatrix} s-1 & -4 \\ 2 & s+3 \end{pmatrix}$$

Verlauf von y(t): $Y(s) = G(s) X(s)$ $\qquad Y(s) = C X(s) + D U(s)$

$$X(s) = (sE-A)^{-1} \cancel{X_0} + (sE-A)^{-1} B U(s) \qquad \text{Angabe } x_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$X(s) = (sE-A)^{-1} B U(s)$$

$$Y(s) = C \cdot \underline{\Phi}(s) B U(s) \;\to\; \begin{pmatrix} 1 & 0 \end{pmatrix} \frac{1}{(s+1)^2+4} \begin{pmatrix} s-1 & -4 \\ 2 & s+3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} U(s)$$

$$Y(s) = \frac{1}{(s+1)^2+4} \begin{pmatrix} s-1 & -4 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} U(s) = \frac{1}{(s+1)^2+4} (s-1) U(s)$$

Angabe: $u(t) = e^t \circ\!\!-\!\!\bullet \dfrac{1}{s-1}$ laut Tabelle

$$Y(s) = \frac{s-1}{(s+1)^2+4} \cdot \frac{1}{s-1} = \frac{1}{(s+1)^2+4}$$

Tabelle: $e^{at} \sin(bt) \circ\!\!-\!\!\bullet \dfrac{b}{(s-a)^2+b^2}$

$$\frac{1}{2} \cdot \frac{2}{(s+1)^2 + 2^2} \quad \rightarrow \quad \boxed{\frac{1}{2} e^{-t} \sin(2t) = y(t)}$$

$\underline{\Phi(t)}$: $\quad \Phi_{11}(s) = \dfrac{s-1}{(s+1)^2 + 4}$

$\Rightarrow \dfrac{s}{N} - \boxed{\dfrac{1}{N}}$ haben wir schon

$\dfrac{s}{N}$: $\dfrac{s}{(s+1)^2 + 4}$ $\qquad$ s ist in Zeitbereich $\dfrac{d}{dt}$

$\dfrac{d}{dt}\left(\dfrac{1}{2} e^{-t} \sin(2t)\right) \rightarrow \dfrac{1}{2}\left(e^{-t} 2\cos(2t) - e^{-t}\sin(2t)\right) - \dfrac{1}{2} e^{-t} \sin(2t) =$

$$= \quad e^{-t}\cos(2t) - e^{-t}\sin(2t)$$

$\Phi_{21}(s)$: $\dfrac{2}{(s+1)^2+4}$ $\quad \circ\!\!-\!\!\bullet \quad \not{2} \cdot \dfrac{1}{\not{2}} e^{-t} \sin(2t)$

$\Phi_{12}(s)$: $\dfrac{-4}{(s+1)^2+4}$ $\quad \circ\!\!-\!\!\bullet \quad -2e^{-t}\sin(2t)$

$\Phi_{22}(s)$: $\dfrac{s+3}{(s+1)^2+4}$ $\quad \circ\!\!-\!\!\bullet \quad \dfrac{s}{N} + \dfrac{3}{N} = \dfrac{1}{2}\left(e^{-t}2\cos(2t) - e^{-t}\sin(2t)\right) + \dfrac{3}{2} e^{-t}\sin(2t)$

$$= \quad e^{-t}\cos(2t) + e^{-t}\sin(2t)$$

$$\underline{\Phi(t)} = e^{-t}\begin{pmatrix} \cos(2t) - \sin(2t) & -2\sin(2t) \\ \sin(2t) & \cos(2t) + \sin(2t) \end{pmatrix}$$