2.2)

$$\dot{\vec{x}} = \underbrace{\begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{d}{m} \end{pmatrix}}_{A} \vec{x} + \underbrace{\begin{pmatrix} 0 \\ \frac{1}{m} \end{pmatrix}}_{B} u, \qquad \vec{x}(0) = \vec{x}_0$$

$$y = \underbrace{\begin{pmatrix} 1 & 0 \end{pmatrix}}_{c^T} \vec{x}$$

$m=2,\ d=4,\ k=6$

$$A = \begin{pmatrix} 0 & 1 \\ -3 & -2 \end{pmatrix}$$

veg. Zustandstrans. : $\vec{x} = V\vec{z}$? System transf.

Step 1: EW von A $\quad \det(A - \lambda E) = \begin{vmatrix} -\lambda & 1 \\ -\frac{k}{m} & -\frac{d}{m} - \lambda \end{vmatrix} = \frac{d\lambda}{m} + \lambda^2 + \frac{k}{m} \overset{!}{=} 0$

$$\lambda^2 + \frac{d\lambda}{m} + \frac{k}{m} \overset{!}{=} 0$$

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow \frac{-\frac{d}{m} \pm \sqrt{\frac{d^2}{m^2} - 4\frac{k}{m}}}{2} = \frac{-2 \pm \sqrt{8 - 12}}{2} \rightarrow \begin{cases} -1 + \sqrt{2}j \\ -1 - \sqrt{2}j \end{cases}$$

Step 2: EV:

$\lambda_1 = -1 + \sqrt{2}j$: $(A - \lambda_1 E)\vec{v} = 0$

$$\begin{pmatrix} 1 - i\sqrt{2} & 1 \\ -3 & -2 + 1 - \sqrt{2}i \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$$

$$= \begin{pmatrix} 1 - \sqrt{2}i & 1 \\ -3 & -1 - \sqrt{2}i \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \quad \xrightarrow{\text{II} \cdot -\frac{1}{3}(1 - \sqrt{2}i)}$$

NR $\quad \frac{(1 - \sqrt{2}i)(1 + \sqrt{2}i)}{3} = \frac{1 + 2}{3}$

$$\begin{pmatrix} 1 - \sqrt{2}i & 1 \\ 1 - \sqrt{2}i & 1 \end{pmatrix} \xrightarrow{\text{II} - \text{I}} \begin{pmatrix} 1 - \sqrt{2}i & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$v_2 = t$

$v_1(1 - \sqrt{2}i) + t = 0$

$$v_1 = \frac{-t}{1 - \sqrt{2}i} \rightarrow -\frac{1 + \sqrt{2}i}{3} t$$

$$\vec{v}_1 = \begin{pmatrix} -\frac{1 + \sqrt{2}i}{3} t \\ t \end{pmatrix} \overset{t=-3}{=} \boxed{\begin{pmatrix} 1 + \sqrt{2}i \\ -3 \end{pmatrix}}$$

$\lambda_2 = -1 - \sqrt{2}j$: $\begin{pmatrix} 1 + i\sqrt{2} & 1 \\ -3 & -2 + 1 + \sqrt{2}i \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \quad \begin{pmatrix} 1 + i\sqrt{2} & 1 \\ -3 & -1 + \sqrt{2}i \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \xrightarrow{\text{II} \cdot \frac{1 + i\sqrt{2}}{-3}}$

$$\rightarrow \left(\begin{array}{cc|c} 1 + i\sqrt{2} & 1 & 0 \\ 1 + i\sqrt{2} & -\frac{(1 - \sqrt{2}i)(1 + \sqrt{2}i)}{-3} & 0 \end{array}\right) \xrightarrow{\text{II} - \text{I}} \left(\begin{array}{cc|c} 1 + i\sqrt{2} & 1 & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow v_2 = t \quad v_1 = \frac{-t}{1 + \sqrt{2}i}$$

$\vec{V}_2 = \begin{pmatrix} \frac{-(1-\sqrt{2}i)}{3} t \\ t \end{pmatrix} \quad t = -3 \quad \vec{V}_2 = \boxed{\begin{pmatrix} 1-\sqrt{2}i \\ -3 \end{pmatrix}}$

Trick: Wenn EV konj. kompl. sind:

$\tilde{A} = T^{-1} \bar{V}^{-1} A \bar{V} T \qquad \bar{V} = (\vec{V}_1 \; \vec{V}_1^*) = \begin{pmatrix} 1+\sqrt{2}i & 1-\sqrt{2}i \\ -3 & -3 \end{pmatrix}$

$\Delta = \begin{pmatrix} \alpha_1 + i\beta_1 & 0 \\ 0 & \alpha_1 - i\beta_1 \end{pmatrix} = \begin{pmatrix} 1+\sqrt{2}i & 0 \\ 0 & 1-\sqrt{2}i \end{pmatrix} \qquad T = \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} \quad T^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$

$T^{-1} = \frac{1}{-2i}\begin{pmatrix} i & -1 \\ i & 1 \end{pmatrix}$

$\tilde{A} = \frac{1}{-2i}\begin{pmatrix} i & -1 \\ i & 1 \end{pmatrix}\begin{pmatrix} 1+\sqrt{2}i & 0 \\ 0 & 1-\sqrt{2}i \end{pmatrix}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} = \qquad V = (\mathrm{Re}\, v_1 \;\; \mathrm{Im}\, v_1)$

$= \begin{pmatrix} \alpha_1 & \beta_1 \\ -\beta_1 & \alpha_1 \end{pmatrix} = \underline{\begin{pmatrix} -1 & \sqrt{2} \\ -\sqrt{2} & -1 \end{pmatrix}}$ gleich $\qquad = \begin{pmatrix} 1 & \sqrt{2} \\ -3 & 0 \end{pmatrix}$

V^{-1} mit adj Methode

$\tilde{A} = V^{-1} A V = \frac{1}{3\sqrt{2}}\begin{pmatrix} 0 & -\sqrt{2} \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} 1 & \sqrt{2} \\ -3 & 0 \end{pmatrix}$

$V^{-1} = \frac{\mathrm{adj}\, V}{\det V} \qquad \mathrm{adj}\, V = \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} \\ \tilde{a}_{21} & \tilde{a}_{22} \end{pmatrix}^T$

$\Rightarrow \frac{1}{3\sqrt{2}}\begin{pmatrix} 3\sqrt{2} & 2\sqrt{2} \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 1 & \sqrt{2} \\ -3 & 0 \end{pmatrix} =$

$\tilde{a}_{ij} = (-1)^{i+j} M_{ij}$

$\begin{pmatrix} -6\sqrt{2} + 3\sqrt{2} & \cancel{6} \\ -\cancel{6} & -3\cancel{\sqrt{2}} \end{pmatrix} \cdot \frac{1}{3\sqrt{2}} = \underline{\boxed{\begin{pmatrix} -1 & \sqrt{2} \\ -\sqrt{2} & -1 \end{pmatrix}}}$

$V^{-1} = \frac{1}{3\sqrt{2}}\begin{pmatrix} a_{22} & -a_{21} \\ -a_{12} & a_{11} \end{pmatrix}^T$

$V^{-1} = \frac{1}{3\sqrt{2}}\begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix} = \frac{1}{3\sqrt{2}}\begin{pmatrix} 0 & -\sqrt{2} \\ 3 & 1 \end{pmatrix}$

$\tilde{A} = D + N$

$\tilde{\Phi} = e^{\tilde{A}t} = e^{Dt} e^{Nt} \qquad \begin{pmatrix} & \sqrt{2} \\ -\sqrt{2} & \end{pmatrix}\begin{pmatrix} & \sqrt{2} \\ -\sqrt{2} & \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 0 & \sqrt{2} \\ -\sqrt{2} & 0 \end{pmatrix} \neq 0$

$e^{Nt} \rightarrow$ keine Ahnung was das ist

Trick: $\tilde{\Phi} = \begin{pmatrix} e^{\alpha_1 t}\cos\beta t & e^{\alpha_1 t}\sin\beta t \\ -e^{\alpha_1 t}\sin\beta t & e^{\alpha_1 t}\cos \end{pmatrix} \rightarrow \underline{\boxed{e^{-t}\begin{pmatrix} \cos(\sqrt{2}t) & \sin(\sqrt{2}t) \\ -\sin(\sqrt{2}t) & \cos(\sqrt{2}t) \end{pmatrix}}}$

das Transformierte System:

$\vec{x} = V\vec{z}$

$$\dot{\vec{z}} = \begin{pmatrix} -1 & \sqrt{2} \\ -\sqrt{2} & -1 \end{pmatrix}$$