10/09/23

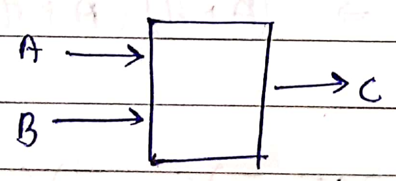
$f: \underbrace{(X+\bar{Z})}_{\text{clause}}(Y+\bar{Z})(\bar{X}+\bar{Y}+Z) = 1$ } POS form.
CNF (also called)
Conjuctive normal form

If the above expression satisfies 1.
all clauses will have to become 1.

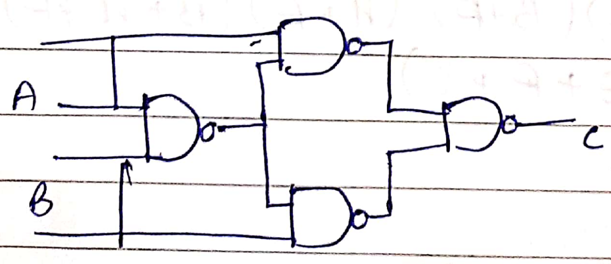
f: SAT problem (satisfiablity Problem)
Solution :-
$\exists\ x, y, z$ such that
$$\underbrace{(x+\bar{z})}_{C1}\ \underbrace{(y+\bar{z})}_{C2}\ \underbrace{(\bar{x}+\bar{y}+z)}_{C3} = 1$$

$C1 = 1$
$C2 = 1$ } all of it should be satisfied.
$C3 = 1$

We can do this by equivalence checking.
Equivalence checking : $f_1 \underset{?}{\equiv} f_2$



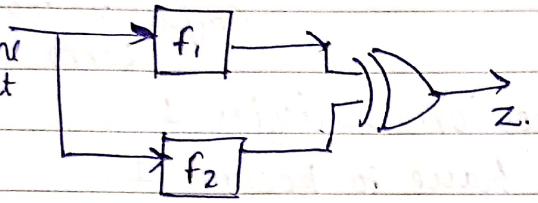
$C = \bar{A}B + A\bar{B}$
$f_1\ (A, B, C)$



$f_2\ (A, B, C)$

- now we have 2 function. f_1 & f_2

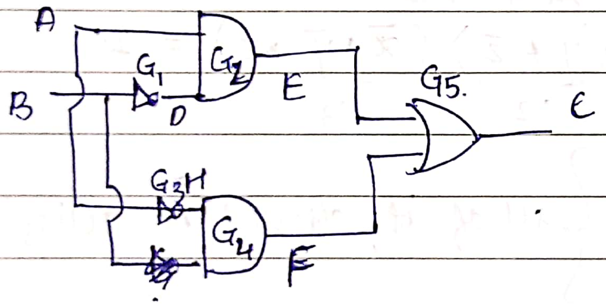
Same input



$Z = 0$ if $f_1 \equiv f_2$

$Z = 1$ otherwise

Example.



f_1

for gate G_1 (Behavior) $\Rightarrow (B+D)(\bar{B}+\bar{D})$

$G_2 \Rightarrow (A+\bar{E})(D+\bar{E})(\bar{A}+\bar{D}+E)$

$G_4 \Rightarrow (B+\bar{F})(H+\bar{F})(\bar{B}+\bar{F}+H)$

$G_5 \Rightarrow (\bar{E}+C)(\bar{F}+C)(F+E+\bar{C})$

$G_3 \Rightarrow (A+H)(\bar{A}+\bar{H})$

$G1 \cdot G2 \cdot G3 \cdot G4 \cdot G5$

$$F1 \cdot \left\{ \begin{array}{l} (B+D)(\bar{B}+\bar{D})(A+H)(\bar{A}+\bar{H})(A+\bar{E})(D+\bar{E}) \\ (\bar{A}+\bar{D}+E)(B+\bar{F})(H+\bar{F})(\bar{B}+\bar{H}+F)(\bar{E}+C) \\ (\bar{F}+C)(E+F+\bar{C}) \end{array} \right.$$

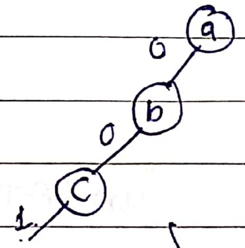
13 clauses

F2 will have 12 clauses.

Eg. We have expression
we need to check if its satisfyable.

$$(a+\bar{c})(b+\bar{c})(\bar{a}+\bar{b}+c) = 1$$

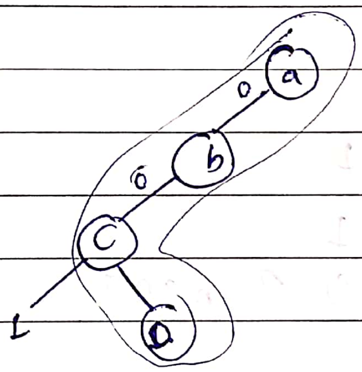
DPLL method., let $a > b > c$
MSB ... LSB.



$a+\bar{c} = 0$ ✗
$b+\bar{c} = 0$ ✗
$\bar{a}+\bar{b}+c = 1$ ✓
failed.

∴ This isn't the correct representation

Back tracking.



$a+\bar{c} = 1$
$b+\bar{c} = 1$
$\bar{a}+\bar{b}+c = 1$
✓ correct assignment
SAT passed.

This is the DPLL method.

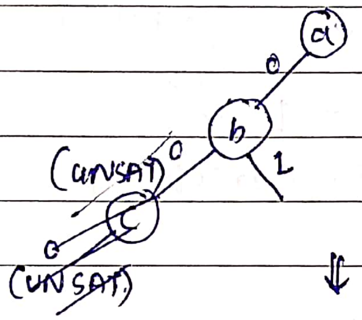
worst case scenario ⇒ no. of backtract = 2^N
N = no. of a, b, c. = 3.

∴ This algorith was very slow.

① Pure literal rule
② Unit ~~lit~~ clause rule
③ Boolean constraint propagation

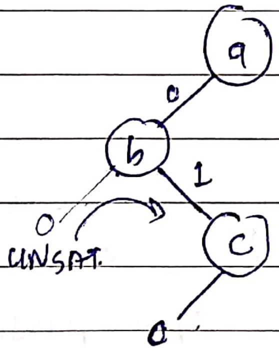
} These rules made it much faster.

Lets say we have
$(a+b)(a+\bar{c})(\bar{b}+c)$
C1 C2 C3.

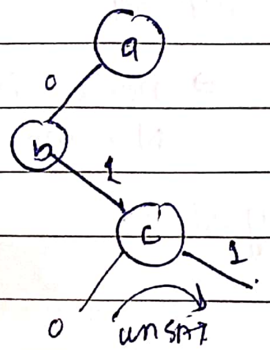


$a+b = 0$ (un SAT)

⇓



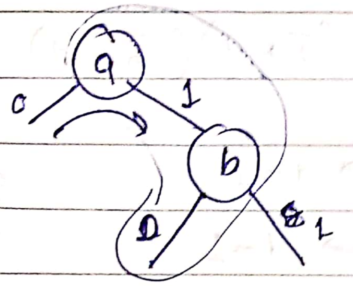
$(a+b) = 1$
$(a+\bar{c}) = 1$
$(\bar{b}+c) = 0$ } un SAT



$(a+b) = 1$
$(a+\bar{c}) = 0$ } un SAT
$(\bar{b}+c) = 1$

we explored all possibility of c & b
so will go to ⓐ

/ /



$b = 0 \qquad a = 1$

$$\left.\begin{array}{l} a + b = 1 \\ a + \bar{c} = 1 \\ \bar{b} + c = 1 \end{array}\right\} \text{all SAT}$$

$\therefore \; a = \cancel{0}\,1 \qquad b = 0$

lets say we have expression

$\bar{a}(a+\bar{c})(b+\bar{c})(\bar{a}+\bar{b}+c)(\bar{c}+e)(\bar{d}+e)(c+d+e)$

$\downarrow$ unit clause $\therefore \; \bar{a} = 1$, $\boxed{a = 0}$ } unit clause rule.

$\therefore \; \underset{1}{\underset{\downarrow}{\bar{a}}} \; \underset{\bar{c}}{\underset{\downarrow}{(a+\bar{c})}}(b+\bar{c}) \; \underset{1}{\underset{\downarrow}{(\bar{a}+\bar{b}+c)}}(\bar{c}+e)(\bar{d}+e)(c+d+e)$

$= \; \bar{c}(b+\bar{c})(\bar{c}+e)(\bar{d}+e)(c+d+e)$

$\downarrow$

$\bar{c} = 1 \qquad \boxed{c = 0}$

$\underset{1}{\underset{\downarrow}{\bar{c}}} \; \underset{1}{\underset{\downarrow}{(b+\bar{c})}} \; \underset{1}{\underset{\downarrow}{(\bar{c}+e)}} \; (\bar{d}+e) \; \underset{(d+e)}{\underset{\downarrow}{(c+d+e)}}$

$\rightarrow \; (\bar{d}+e)(d+e) \; \rightarrow \; \boxed{e = 1}$

Pure literal Rule

$\therefore \; a = 0 \quad c = 0 \quad e = 1$ } final SAT solution

$$f = (a+b)(\bar{a}+c)(\bar{c}+d)(\bar{a}+d)(\bar{b}+d)$$

$$\begin{cases} a=0 \Rightarrow b=1 \Rightarrow d=1 \\ a=1 \Rightarrow c=1 \Rightarrow d=1 \end{cases}$$

Stallmark's Method.

regardless of value of a, $d=1$

Necessary assignment.

Recursive learning method
local learning } see online.