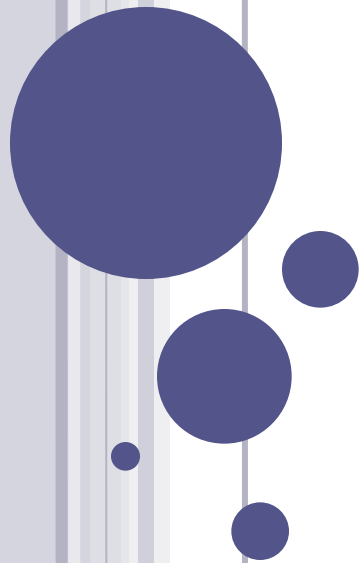


Linear data structure using sequential organization

Unit II

By

Pujashree Vidap

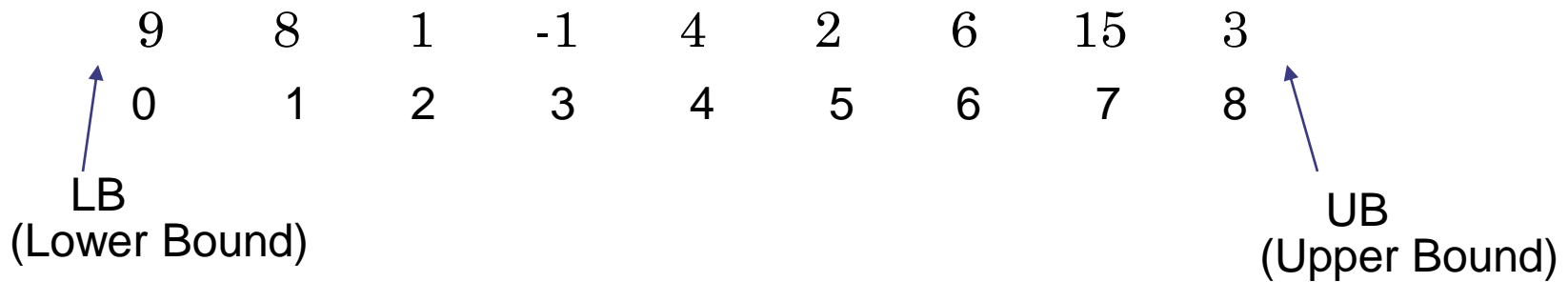


INTRODUCTION

- An Array is a **Data Structure** with which we can perform operations on a collection of similar data type such as simple list or tables of information.
- These structures are **composite or structured** data types.
- All elements in the array are of same type i.e. int, char, float etc.



- The individual elements within the array can be accessed by the integer called ***Index***.
- eg-: `int array[9]`

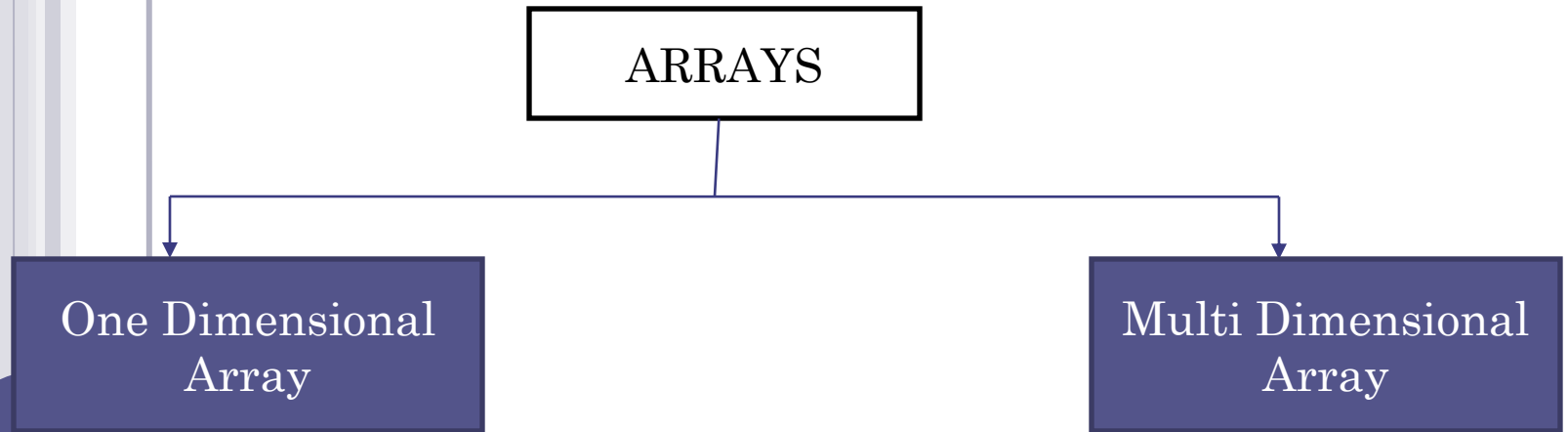


- $\text{Length of an Array} = \text{UB} - \text{LB} + 1$



- The zeroth element (9) in the list can be accessed by `array[0]`.
- An ***Index*** is also called a ***Subscript***.
- Therefore, individual elements of an Array are called ***subscripted*** variables eg. `array[0]`, `array[1]` etc.
- Thus, an Array can be defined as a finite ordered collection of items of same type.

Types of Array



(An Array whose elements are specified by a single subscript
Eg. A[0])

(An Array whose elements are specified by two or more than two subscripts
Eg. A[10][10] is a two dimensional array)

ARRAY AS AN ADT

```
abstract typedef<eltype,ub>ARRTYPE(ub,eltype);  
Condition type(ub)==int;
```

```
abstract eltype extract(a,i)  
ARRTYPE(ub,eltype) a;  
int i;  
precondition 0<=i<ub;  
postcondition extract ==ai;
```



```
abstract store(a,i,elt)
ARRTYPE(ub,eltype) a;
int i;
precondition 0<=i<ub;
postcondition a[i]=elt;
```



ARRAY AS AN ADT

Array is set pairs<index,value>

Class Generalarray

{

Generalarray(int j, Rangelist list, float
initvalue=defaultvalue);

// constructor produce new array of appropriate
type and size (produce j dimension array of float
with initialvalues)

float retrieve(index i);

void store(index i ,float x)

};



One Dimensional Array-:

- 1-D Arrays are suitable for processing lists of items of identical types.
- They are very useful for problems that require the same operation to be performed on a group of data items.



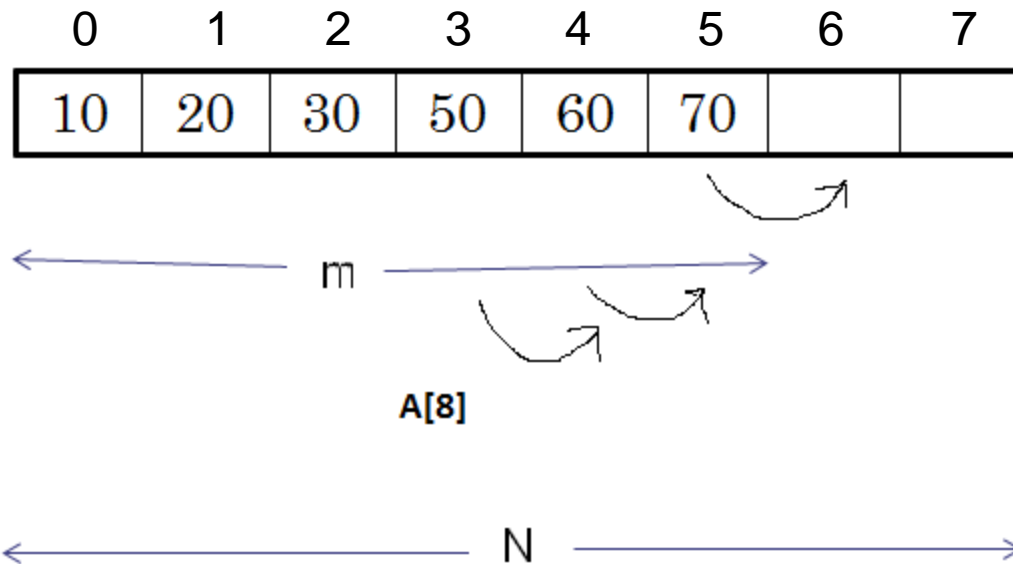
Algorithm Traversal (A,N)

```
{  
  // Description:- A is an array of size N.  
  1. Read N  
  2. for( i=0 to N-1)  
      2.1 Read A[i]  
      2.2 write A[i]  
}
```

Time Complexity:- ???



2. INSERTION of an element-:



Insert 40 at location 4th



After Insertion-:

0	1	2	3	4	5	6	7
10	20	30	40	50	60	70	

↑
Inserted

← $m+1$ →

← N →



5. for $i=m-1$ to $loc-1$ do

5.1 $A[i+1] = A[i]$

6. $A[loc-1] = val$

7. For $i=0$ to m do

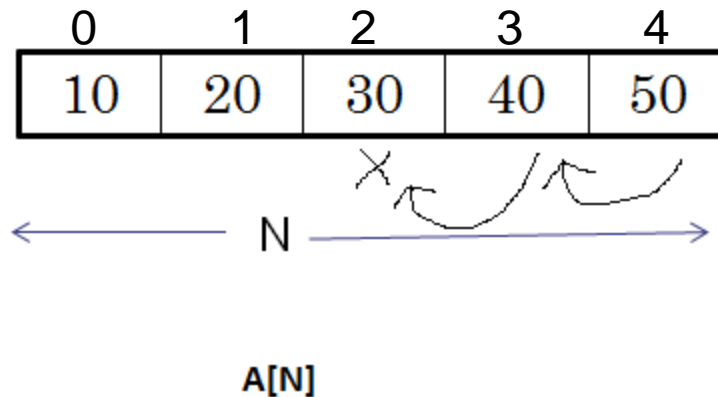
7.1 Print $A[i]$

}

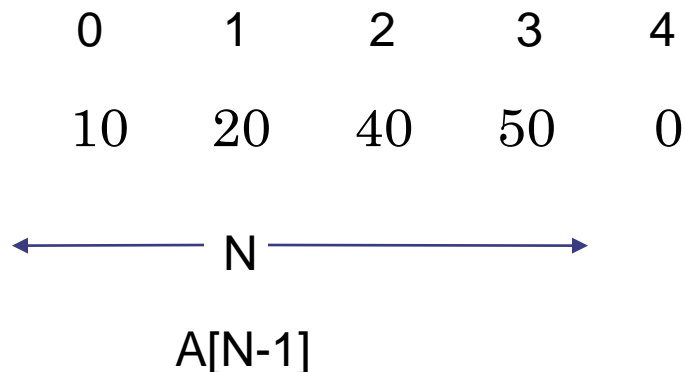
Time Complexity-:???



3. Deletion of an element from an array:-



Delete an element at location 3rd



Algorithm Deletion(A,N, loc)

{

 Description-: A is an Array of size N

 loc is the location of deletion.

1. Enter N

2. for (i=0 to N-1)

 2.1 Enter A[i]

3. Enter loc

4. for (i=loc to N-1)

 4.1 $A[i-1] = A[i]$

5. for (i=0 to N-2)

 5.1 Print A[i]

}

Time Complexity-: ????



ADDRESS CALCULATION:-

○ One Dimensional Array:-

0	1	2	3	4
A	B	C	D	E

List[5]

Logical View:-

100	A	0
100+1	B	1
100+2	C	2
100+3	D	3
100+4	E	4
Physical Address	E	Logical Address

- The array elements are stored in contiguous memory locations by sequential allocation techniques.
- The Address of *ith* element of the array can be obtained if we know-:
 1. The starting address i.e. the address of the first element called **Base Address** denoted by **B**
 2. The size of the element in the array denoted by **W**

$$\text{Address of List [i]} = B + (i - LB) * W$$

where LB is Lower Bound of the array

$$\text{List}[5] = \text{List}[0---4] ; LB=0$$



ARRAYS AS PARAMETERS

Pass by value

Pass by reference

Returning an array from function.

```
sum(a,ub);//calling
```

```
void sum(int a[ ],int size);
```



SEQUENTIAL MEMORY ORGANIZATION

- Advantages and disadvantages ??



PROBLEM

- Write a program that accepts an array of n numbers and returns the median of numbers in the array.
- Consider the linear array AAA [5:50], BBB [-5:10] and CCC [18]
 - a) Find the number of elements in each array.
 - b) Suppose Base (AAA) = 300 and $w=4$ word per memory cell for AAA. Find the address of AAA [15], AAA [35] and AAA [55].



Write a program that reads name of the students, age, roll no, Phone of 4 students and perform the following operation

1. Search a record of particular student.
2. Insert a particular record at specific position.
3. Delete a particular record from specific position.

Write a program to merge two sorted array in ascending order.



MERGE-SORT: MERGE EXAMPLE

c:



A:



↑
i=0

b:



↑
j=0



MERGE-SORT: MERGE EXAMPLE

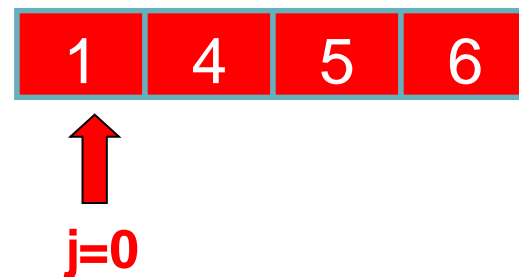
c:



A:

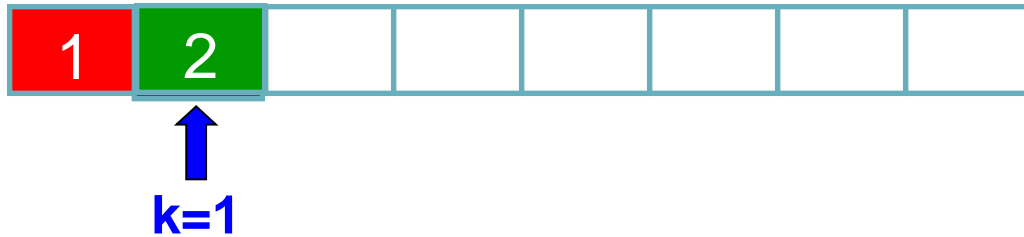


b:



MERGE-SORT: MERGE EXAMPLE

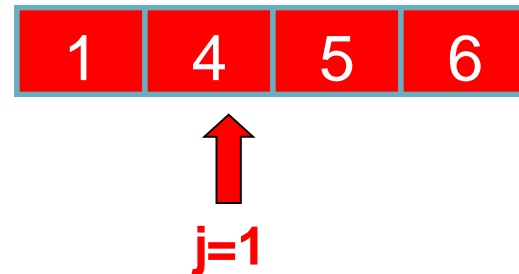
c:



A:



b:

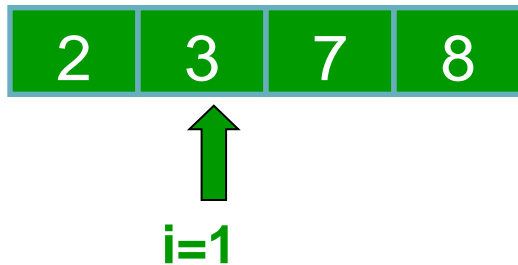


MERGE-SORT: MERGE EXAMPLE

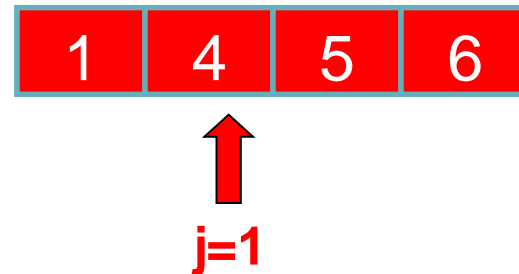
c:



A:

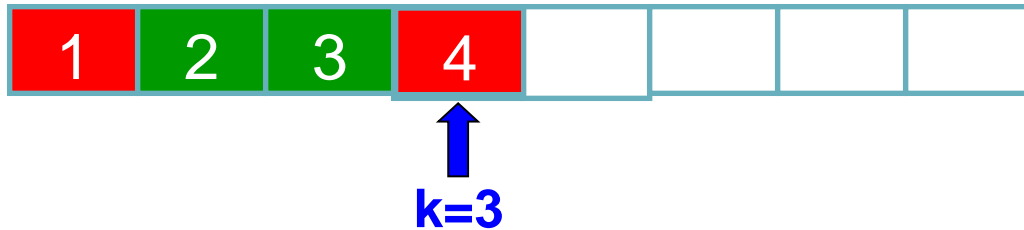


b:

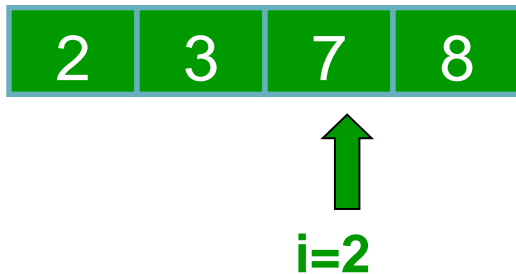


MERGE-SORT: MERGE EXAMPLE

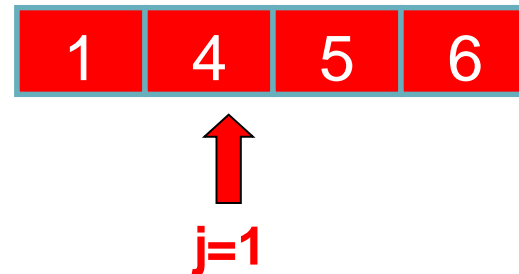
c:



A:

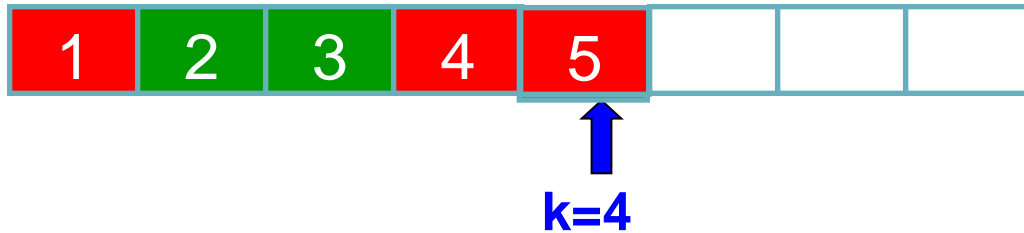


b:

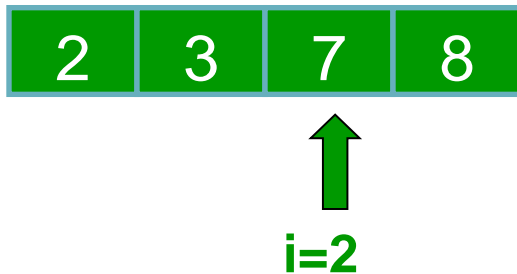


MERGE-SORT: MERGE EXAMPLE

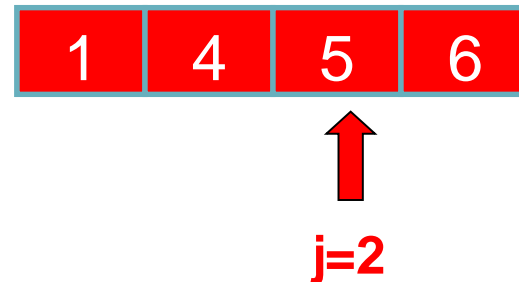
c:



A:

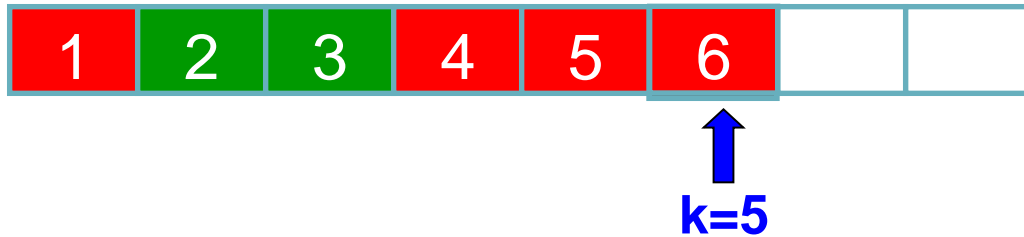


b:

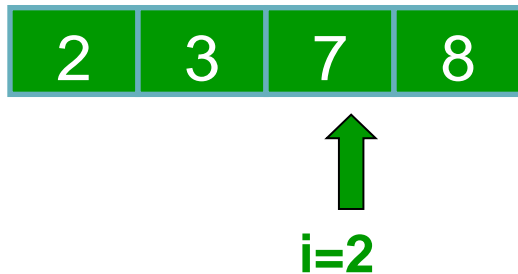


MERGE-SORT: MERGE EXAMPLE

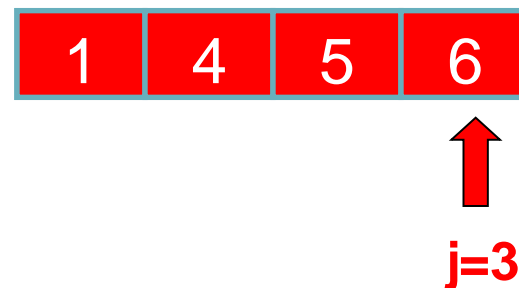
c:



A:

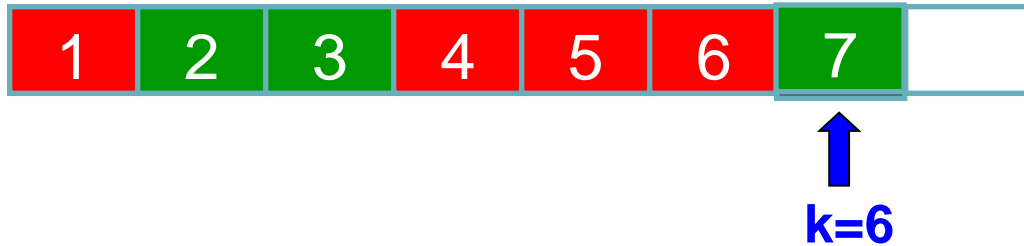


b:

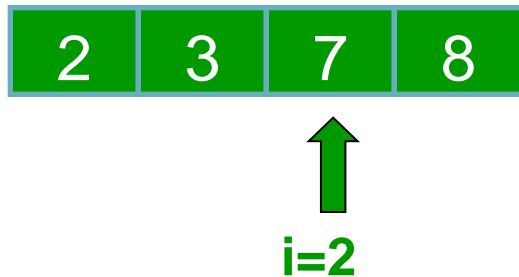


MERGE-SORT: MERGE EXAMPLE

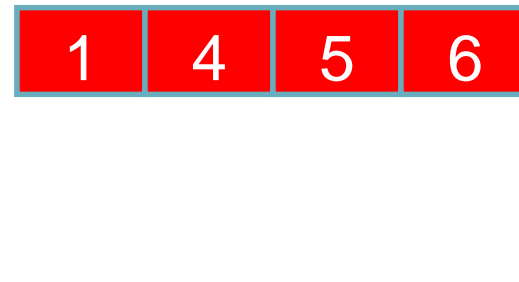
c:



A:

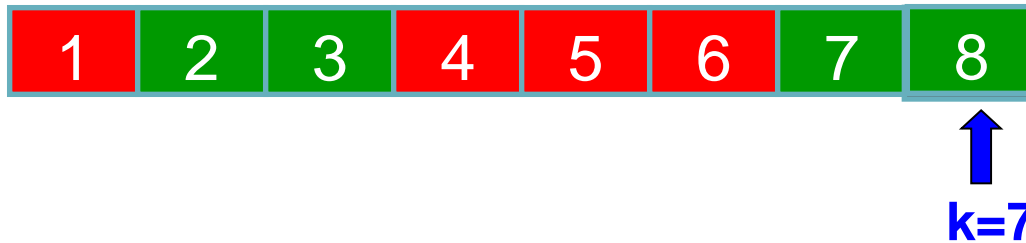


b:

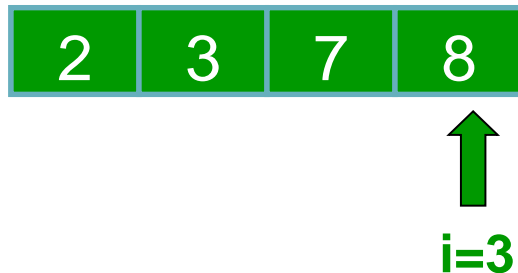


MERGE-SORT: MERGE EXAMPLE

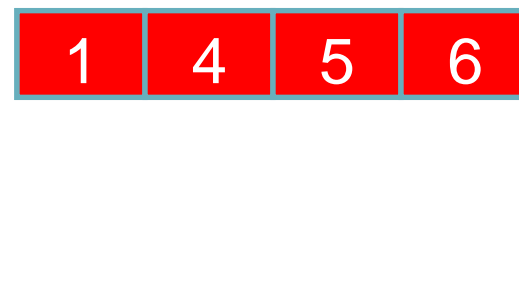
c:



A:



b:



MERGE-SORT: MERGE EXAMPLE

c:



↑
k=8

A:



↑
i=4

b:



↑
j=4



Algorithm Merge (A ,b, c, n, m)

{

//A is an array of n elements, b, is an array of m,c is the final merged array

1. i := 0;

2. j:= 0;

3. k := 0;

4. Do

4.1 If(A [i] <= b [j])

4.1.1 c [k] := A [i]

4.1.2 i := i + 1

4.1.3 k: = k +1

4.2 else

4.2.1 c [k] := b[j]

4.2.2 j := j +1

4.2.3 k := k +1

while (i <= n && j <= m);



6. While ($j \leq m$)

6.1 $c[k] := b[j]$

6.2 $j := j + 1$

6.3 $k := k + 1$

7. While($i \leq n$)

7.1 $c[k] := A[i]$

7.2 $i := i + 1$

7.3 $k := k + 1$

}



○ Two Dimensional Array-:

A [m] [n] represents an array A where m is no.of rows and n is no.of columns.

Eg-:

		0	1	2
A[2][3]	0	4	5	6
	1	7	8	9

m*n
1*2



Row Major Order-:

100	4	0,0
100+1	5	0,1
100+2	6	0,2
100+3	7	1,0
100+4	8	1,1
100+5	9	1,2

In row-major storage, a multidimensional array in linear memory is accessed such that rows are stored one after the other.



Address of A [i] [j]= $B + [(i-LB_R)*n + (j-LB_C)]*W$

where :

- LB_R is the Lower Bound of Rows
- LB_C is the Lower Bound of Columns
- W is the size of element
- n is the no.of columns
- Eg-: $A[1][2]; m=2; n=3; LB_R=0; LB_C=0$
 $A[1][2] = 100 + ((1-0) * 3 + (2-0)) * 1$
 $= 105$



Column Major Order-:

100	4	0,0
100+1		1,0
100+2	7	0,1
100+3	5	1,1
100+4	8	0,2
100+5	6	1,2
	9	

In column-major storage, a multidimensional array in linear memory is accessed such that columns are stored one after the other.



$$\text{Address of A [i] [j]} = B + [(i - \text{LB}_R) + (j - \text{LB}_C) * m] * W$$

where

- LB_R is the Lower Bound of Rows
- LB_C is the Lower Bound of Columns
- W is the size of element
- m is the no. of rows



SOLVE

- Consider the 25×4 matrix array SCORE.
Suppose $\text{base}(\text{SCORE}) = 200$ and there are $w = 4$ words per memory cell. Suppose the programming language stores two dimensional arrays using row major order .Find the address of $\text{SCORE}[12,3]$.
- Also solve considering the elements are stored in column major order.



2-D ARRAY

- Declaration:

```
int a[2][3]={1,2,3,4,5,6}
```

```
int a[][3]={1,2,3,4,5,6}
```

```
int x[3][4] = {0, 1 ,2 ,3 ,4 , 5 , 6 , 7 , 8 , 9 , 10 , 11}
```

```
int x[3][4] = {{0,1,2,3}, {4,5,6,7}, {8,9,10,11}};
```

Second dimension must be specified

- Accessing array elements:

```
cout<<a[i][j];
```

- Inputting array elements:

```
cin>>a[i][j];
```



- Passing 2D arrays to functions

Calling: `print(a,row,col);`

Prototyping: `void print(int[][],int,int);`

Defination:`void print(int x[][5],int r,int c);`

Input in 2-d matrix

```
for(i=0;i<r;i++)  
{for(j=0;j<c;j++)  
{cin>>a[i][j];  
}
```



RETURNING 2D ARRAY

```
○ int main()
{
    int x[3][3], y[3][3];
    int (*a)[3];
    int i,j;
    printf("Enter the matrix1:
    \n");
    for(i = 0; i < 3; i++)
        for(j = 0; j < 3; j++)
            scanf("%d",&x[i][j]);
    printf("Enter the
    matrix2:");
    for(i = 0; i < 3; i++)
```

```
        for(j = 0; j < 3; j++)
            scanf("%d",&y[i][j]);
    a = Matrix_sum(x,y);
    printf("The sum of the
    matrix is: \n");
    for(i = 0; i < 3; i++){
        for(j = 0; j < 3; j++){
            printf("%d",a[i][j]);
            printf("\t");
        }
        printf("\n");
    }
    return 0;
}
```



- ```
int (*(Matrix_sum)(int matrix1[][3], int
matrix2[][3]))[3]{
 int i, j;
 static int m3[3][3];
 for(i = 0; i < 3; i++){
 for(j = 0; j < 3; j++){
 m3[i][j] = matrix1[i][j] + matrix2[i][j];
 }
 }
 return m3;
 }
```



# CONSTRUCTING TWO DIMENSIONAL ARRAY

```
class Test
{
int **p;
int row,col;
public:
Test(int,int);
void getdata();
void display();
};
```


```
void Test:: getdata()
{
for(int i=0;i<row;i++)
{
for(int j=0;j<col;j++)
{
cin>>p[i][j];
}
}
}
```



# CONSTRUCTING TWO DIMENSIONAL ARRAY

```
void Test :: display()
{
for(int i=0;i<row;i++)
{for(int j=0;j<col;j++)
{cout<<p[i][j];}
cout<<"\n";
}}
Test ::Test(int x,int y)
{
row=x;
col=y;
p=new int *[row];
```

```
for(int i=0;i<row;i++)
{
p[i]=new int[col];
}
}
void main()
{
int m,n;
cout<<" Enter the no. of rows
and columns ";
cin>>m>>n;
Test t(m,n);
cout<<"enter array";
t.getdata(m,n);
t.display(m,n);
}
```



# OUTPUT

**Enter the no. of rows and**

**Columns 2 2**

**Enter array**

**2**

**2**

**2**

**2**

**2 2**

**2 2**



## Assignment on 2D array

- Matrix operations like addition, subtraction, multiplication, transpose etc.
- Operations on square matrix like symmetric matrix, diagonal matrix, upper triangular, lower triangular and identity matrix etc.
- Print the matrix along with row and column sum
- Generating magic square matrix or checking given square matrix is magic square



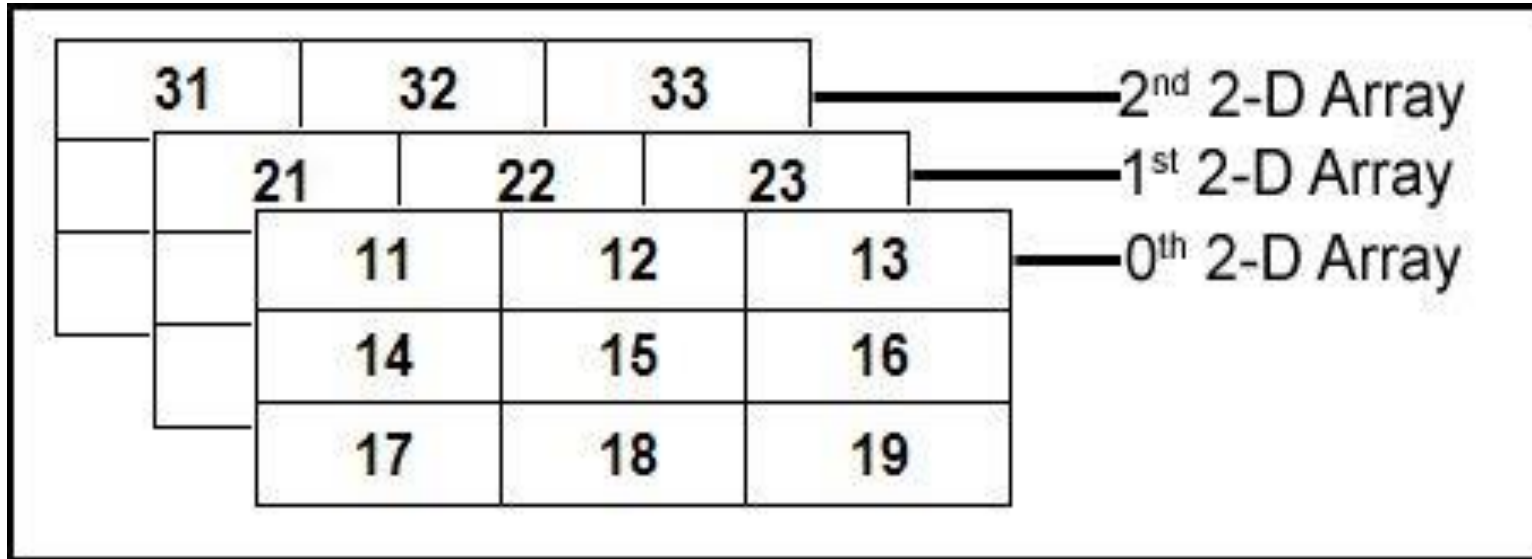
# MULTI-DIMENSIONAL ARRAY

- We can have three or more dimensions of an array.
- `int a[3][3][3]`
- No of elements will be:  $3*3*3=27$





## 3D REPRESENTATION OF AN WITH ITS MEMORY ALLOCATION



| <----- 0 <sup>th</sup> 2D Array -----> |      |      |      |      |      |      |      |      | <----- 1 <sup>st</sup> 2D Array -----> |      |      |      |      |      |      |      |      | <----- 2 <sup>nd</sup> 2D Array -----> |      |      |      |      |      |      |      |      |
|----------------------------------------|------|------|------|------|------|------|------|------|----------------------------------------|------|------|------|------|------|------|------|------|----------------------------------------|------|------|------|------|------|------|------|------|
| 11                                     | 12   | 13   | 14   | 15   | 16   | 17   | 18   | 19   | 21                                     | 22   | 23   | 24   | 25   | 26   | 27   | 28   | 29   | 31                                     | 32   | 33   | 34   | 35   | 36   | 37   | 38   | 39   |
| 1000                                   | 1002 | 1004 | 1006 | 1008 | 1010 | 1012 | 1014 | 1016 | 1018                                   | 1020 | 1024 | 1026 | 1028 | 1030 | 1032 | 1034 | 1036 | 1038                                   | 1040 | 1042 | 1044 | 1046 | 1048 | 1050 | 1052 | 1054 |

# USING MULTI-DIMENSIONAL ARRAYS

```
void main()
{
int a[2][2][2],i,j,k;
for(i=0;i<2;i++)
 for(j=0;j<2;j++)
 for(k=0;k<2;k++)
 cin>>a[i][j][k];
```

|   |   |
|---|---|
| 0 | 1 |
| 1 | 2 |

|   |   |
|---|---|
| 1 | 2 |
| 2 | 3 |



- Row major printing

```
for(i=0;i<2;i++)
 for(j=0;j<2;j++)
 for(k=0;k<2;k++)
 cout<<a[i][j][k]
 cout<<"\n"
```

### Output

```
0(0,0,0) 1(0,0,1)
1(0,1,0) 2(0,1,1)
1(1,0,0) 2(1,0,1)
2 (1,1,0) 3(1,1,1)
```

- Column major

```
for(i=0;i<2;i++)
 for(j=0;j<2;j++)
 for(k=0;k<2;k++)
 cout<<a[k][j][i];
 cout<<"\n"
```

### Output

```
0(0,0,0) 1(1,0,0)
1(0,1,0) 2(1,1,0)
1(0,0,1) 2(1,0,1)
2 (0,1,1) 3(1,1,1)
```



# ADDRESS CALCULATIONS

- Row major

- $A[i][j][k] = b + [(i - lb_i) * d2 * d3 + (j - lb_j) * d3 + (k - lb_k)] * W$

Where  $b$  = base address

$lb$  = lower bound

$d2$  = no of rows

$d3$  = no of cols

Where  $i$  is plane

$j$  is rows,  $k$  is columns



# STRING MANIPULATION USING ARRAY

- String?
- Write a program to check whether a string is palindrome or not?
- Write a program to compare two string.
- Write a program to find substring from a given string.



# CONCEPT OF ORDERED LIST

- Arrays can be used to implement other data structure like ordered or linear list ex. List of Days of week, values in a deck of cards etc.
- Linear list of elements arranged in particular order.
- List of no. arranged in particular order.
- Can be represented using arrays
- Operations on ordered list : find length, read list(display), retrieve  $i^{\text{th}}$  element
  - ✓ searching
  - ✓ insertion
  - ✓ deletion



# ADT POLYNOMIAL

```
class polynomial
```

```
{ //A(x)=a0xe0+.....anxn, a set of ordered pair of <ei,ai>
 where ai is non zero float coefficient and ei is a non
 negative integer exponent.
```

```
 public:
```

```
 polynomial(); //construct the polynomial p(x)=0.
```

```
 polynomial add(polynomial poly);
```

```
 //return sum of polynomials *this and poly.
```

```
 polynomial mul(polynomial poly);
```

```
 //return multiplication of polynomials *this and poly.
```

```
 float eval(float f);
```

```
 //evaluate the polynomial *this at f and return result.
```

```
 }
```



# SINGLE VARIABLE POLYNOMIAL

- Polynomial of the form

$$A(x) = C_{m-1}x^{m-1} + C_{m-2}x^{m-2} + \dots + C_0x^0$$

- Example:  $8x^3 + 3x + 6$  (array representation)

## Representation 1:

- Polynomial representation as an array where indices represent the exponent and array element represent corresponding coefficients

|   |   |   |   |
|---|---|---|---|
| 6 | 3 | 0 | 8 |
| 0 | 1 | 2 | 3 |



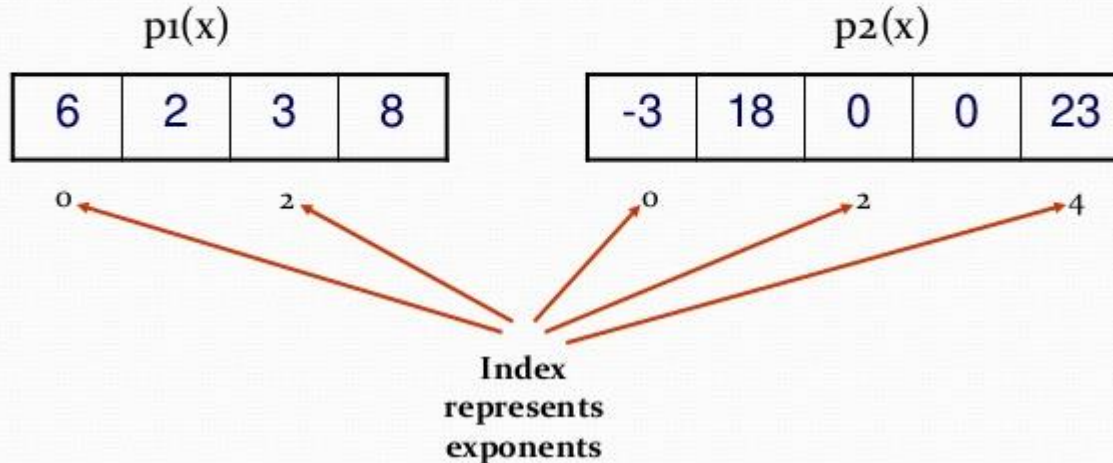


# Representation 1:

- Array Implementation:

- $p_1(x) = 8x^3 + 3x^2 + 2x + 6$

- $p_2(x) = 23x^4 + 18x - 3$



# DISADVANTAGE

- If exponent is large ,scanning the large array will be time consuming
- In case of sparse polynomial, lot of wastage will be there.
- Array size needs to be pre-defined.

## Representaion 1:

private:

int degree; //degree <=maxdegree

float coef[maxdegree+1];

// if a.degree=n

//a.coef[i]=a<sub>n-i</sub> 0<=i<=n coefficients are stored in  
order of decreasing exponents



## REPRESENTATION 2 : Using Dynamic array based on degree of polynomial

- private :

```
int degree;
```

```
float *coef;
```

```
Polynomial::polynomial(int d)
```

```
{
```

```
degree =d;
```

```
coef=new float[degree+1];
```

```
}
```

```
//here degree much less than Maxdegree so
disadvantage of representation 1 and 2 are
overcome
```



Representation 3 :POLYNOMIAL AS  
ARRAY OF STRUCTURES (Store only  
nonzero terms to overcome problems  
with representation 1 and 2

- Example:  $8x^3+3x+6$

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 3 | 8 | 1 | 3 | 0 | 6 |
| 0 | 1 | 2 | 3 | 4 | 5 |



# POLYNOMIAL AS ARRAY OF STRUCTURES


```
class term {
friend polynomial;
private:
float coef;
int exp; };
class polynomial
{//initializing capacity with appropriate initial value
 and term with 0
private:
term *termarray;//array of nonzero terms
int capacity;// size of termarray
int terms; //number of nonzero terms
};
```



## ADDING TWO POLYNOMIAL: $O(m+n)$

```
polynomial polynomial ::add(polynomial b)
{ polynomial c;
 int apos=0;bpos=0;
 while((apos<terms)&&(bpos<b.terms)) {
 if(termarray[apos].exp==b.termarray[b.pos].exp) {
 float t=termarray[apos].coef + b.termarray[b.pos].coef;
 If(t) c.newterm(t,termarray[apos].exp);
 apos++,bpos++;
 }
 else if (termarray[apos].exp<b.termarray[b.pos].exp){

 c.newterm(b.termarray[bpos].coef,b.termarray[bpos].exp);
 bpos++;
 }
 }
```



```
else
{
 c.newterm(termarray[apos].coef,termarray[apos].exp);
 apos++;}
}
// add remaining terms of *this
for(;apos<terms;apos++)
c.newterm(termarray[apos].coef,termarray[apos].exp);
for(;bpos<b.terms;bpos++)
c.newterm(b.termarray[bpos].coef,b.termarray[bpos].exp
return c;
}
```



## Adding new term and doubling array size when necessary

```
void polynomial::newterm(float coeff,int exp)
{
 if(term==capacity){
 capacity=capacity*2;
 term*temp=new term[capacity];
 Copy(temp, termarray);
 delete[] termarray;
 termarray=temp;
 }
 termarray[terms].coef=coeff;
 termarray[terms].exp=exp;
 terms=terms+1;
}
```





Analysis:

$O(n+m)$  where  $n, m$  is the number of terms in  $A, B$ .



# POLYNOMIAL MULTIPLICATION

- $A(x)=5x^3+2x^2+1$
- $B(x)=2x^3+x$



# SPARSE MATRIX

- A sparse matrix is the matrix in which max elements are 0;
- **sparse ...** many elements are zero
- **dense ...** few elements are zero

Example:

0 0 3 0 4

0 0 5 7 0

0 0 0 0 0

0 2 6 0 0



# APPLICATION

- Airline flight matrix.
  - airports are numbered 1 through  $n$
  - $\text{flight}(i,j)$  = list of nonstop flights from airport  $i$  to airport  $j$
  - $n = 1000$  (say)
  - $n \times n$  array of list references  $\Rightarrow$  4 million bytes
  - total number of flights = 20,000 (say)
  - need at most 20,000 list references  $\Rightarrow$  at most 80,000 bytes
- Diagonal matrix is sparse matrix



# PROBLEM

- Not space efficient
- Retrieval time is more

So we can do representation of these matrix as:

- 1) Rows
- 2) Column
- 3) Value



# SPARSE MATRIX REPRESENTATION

A= 0 0 3 0 4  
0 0 5 7 0  
0 0 0 0 0  
0 2 6 0 0

Can be represented as <row,col,value>

| 4 | 5 | 6 |
|---|---|---|
| 0 | 2 | 3 |
| 0 | 4 | 4 |
| 1 | 2 | 5 |
| 1 | 3 | 7 |
| 3 | 1 | 2 |
| 3 | 2 | 6 |



```
class sparsematrix;
class Matrixterm
{
friend class sparsematrix;
private:
int row,col,value;
};
```

```
In class sparsematrix
Private:
int trows,tcols,terms,capacity;
Matrixterm *as;
```



# ADT OF SPARSE MATRIX

class sparsematrix

{//a set of triples,  $\langle \text{row}, \text{column}, \text{value} \rangle$ , where *row* and *column* are integers and form a unique combination, and *value* comes from the set *item.t*

*Public:*

*Sparsematrix(int r,int c,int t);* // constructor used to create sparse matrix

*Sparsematrix transpose();* // return the sparse matrix obtained by interchanging row and column value of every triple in *\*this*.

*Sparsematrix add(Sparsematrix b);* // if the dimentions of *\*this* and *b* are same then addition is returned.

*Sparsematrix multiply(Sparsematrix b);* // if the no of   
 // column in *\*this* is equal to no. of rows *b* then   
 // multiplication is returned  $d[i][j] = \sum (a[i][k].b[k][j])$

};





# Transpose a Matrix

(1) for each **row**  $i$

take element  $\langle i, j, \text{value} \rangle$  and store it in element  $\langle j, i, \text{value} \rangle$  of the transpose.

difficulty: where to put  $\langle j, i, \text{value} \rangle$

$(0, 0, 15) \implies (0, 0, 15)$

$(0, 3, 22) \implies (3, 0, 22)$

$(0, 5, -15) \implies (5, 0, -15)$

$(1, 1, 11) \implies (1, 1, 11)$

Move elements down very often.

(2) For all elements in **column**  $j$ ,

place element  $\langle i, j, \text{value} \rangle$  in element  $\langle j, i, \text{value} \rangle$



(1) Represented by a two-dimensional array.

(2) Each element is characterized by **<row, col, value>**.

|      |   |   |     |           |      |   |   |     |
|------|---|---|-----|-----------|------|---|---|-----|
| a[0] | 6 | 6 | 8   |           | b[0] | 6 | 6 | 8   |
| [1]  | 0 | 0 | 15  |           | [1]  | 0 | 0 | 15  |
| [2]  | 0 | 3 | 22  |           | [2]  | 0 | 4 | 91  |
| [3]  | 0 | 5 | -15 |           | [3]  | 1 | 1 | 11  |
| [4]  | 1 | 1 | 11  |           | [4]  | 2 | 1 | 3   |
| [5]  | 1 | 2 | 3   | transpose | [5]  | 2 | 5 | 28  |
| [6]  | 2 | 3 | -6  | →         | [6]  | 3 | 0 | 22  |
| [7]  | 4 | 0 | 91  |           | [7]  | 3 | 2 | -6  |
| [8]  | 5 | 2 | 28  |           | [8]  | 5 | 0 | -15 |
| (a)  |   |   |     |           | (b)  |   |   |     |

**row, column in ascending order**

Sparse matrix and its transpose stored as triples



```
algorithm transpose (matrixterm a[], matrixterm b[])
 /* b is set to the transpose of a */
```

```
{
 n := a[0].value;
 b[0].row := a[0].col;
 b[0].col := a[0].row;
 b[0].value := n;
 if (n > 0) {
 currentb := 1;
 for i := 0 to i < a[0].col step 1
 for j := 1 to j <= n step 1
 if (a[j].col == i) {
 b[currentb].row = a[j].col;
 b[currentb].col = a[j].row;
 b[currentb].value = a[j].value;
 currentb++
 }
 }
 }
}
```



Scan the array “columns” times. The array has “elements” elements.

$O(\text{columns} * \text{elements})$



# Comparison with 2-D array representation

$O(\text{columns} * \text{elements})$  vs.  $O(\text{columns} * \text{rows})$   
elements  $\rightarrow$  columns \* rows when non sparse =  
 $O(\text{columns} * \text{columns} * \text{rows})$

**Problem:** Scan the array “columns” times.

**Solution: TO REDUCE COMPLEXITY**

Determine **the number of elements** in each column of the original matrix.

==

Determine the **starting positions of each row** in the transpose matrix.



|      |   |   |     |
|------|---|---|-----|
| a[0] | 6 | 6 | 8   |
| a[1] | 0 | 0 | 15  |
| a[2] | 0 | 3 | 22  |
| a[3] | 0 | 5 | -15 |
| a[4] | 1 | 1 | 11  |
| a[5] | 1 | 2 | 3   |
| a[6] | 2 | 3 | -6  |
| a[7] | 4 | 0 | 91  |
| a[8] | 5 | 2 | 28  |

|                |     |     |     |     |     |     |
|----------------|-----|-----|-----|-----|-----|-----|
|                | [0] | [1] | [2] | [3] | [4] | [5] |
| ROW_TERMS =    | 2   | 1   | 2   | 2   | 0   | 1   |
| STARTING_POS = | 1   | 3   | 4   | 6   | 8   | 8   |



```
algorithm fast_transpose(term a[], term b[])
```

```
{
 row_terms[MAX_COL],
 starting_pos[MAX_COL];
 num_cols = a[0].col,
 num_terms = a[0].value;
 b[0].row = num_cols; b[0].col = a[0].row;
 b[0].value = num_terms;
 if (num_terms > 0)
 for (i = 0; i < num_cols; i++)
 row_terms[i] = 0;
 for (i = 1; i <= num_terms; i++)
 row_term [a[i].col]++
 starting_pos[0] = 1;
 for (i = 1; i < num_cols; i++)
 starting_pos[i] = starting_pos[i-1] + row_terms [i-1];
```



```
for (i=1; i <= num_terms, i++) {
 j = starting_pos[a[i].col]++;
 b[j].row = a[i].col;
 b[j].col = a[i].row;
 b[j].value = a[i].value;
}
}
}
```



# ANALYSIS

Compared with previous algorithm.

$O(\text{columns} + \text{elements})$  vs.  $O(\text{columns} * \text{element})$

Cost: Additional `row_terms` and `starting_pos` arrays are required.



# SPARSE MATRIX ADDITION



○ Algorithm `add_sp_mat(sp1[][3],sp2[][3],sp3[][3])`

{1. if( `sp1[0][0] != sp2[0][0] || sp1[0][1] != sp2[0][1]` )

1.1 `write("Invalid matrix size ");`

1.2 `exit(0);`

2. `tot1 = sp1[0][2]; tot2 = sp2[0][2]; k1 = k2 = k3 = 1;`

3. `while ( k1 <= tot1 && k2 <= tot2)`

3.1 if ( `sp1[k1][0] < sp2[k2][0]` )

{ `sp3[k3][0] = sp1[k1][0];`

`sp3[k3][1] = sp1[k1][1];`

`sp3[k3][2] = sp1[k1][2];`

`k3++;k1++; }`

3.1 else if ( `sp1[k1][0] > sp2[k2][0]` )

{ `sp3[k3][0] = sp2[k2][0];`

`sp3[k3][1] = sp2[k2][1];`

`sp3[k3][2] = sp2[k2][2];`

`k3++;k2++; }`



```
3.1 else if (sp1[k1][0] == sp2[k2][0])
 3.1.1 if (sp1[k1][1] < sp2[k2][1])
 { sp3[k3][0] = sp1[k1][0];
 sp3[k3][1] = sp1[k1][1];
 sp3[k3][2] = sp1[k1][2];
 k3++;k1++; }
 3.1.1else if (sp1[k1][1] > sp2[k2][1])
 { sp3[k3][0] = sp2[k2][0];
 sp3[k3][1] = sp2[k2][1];
 sp3[k3][2] = sp2[k2][2];
 k3++;k2++; }
 3.1.1. else
 { sp3[k3][0] = sp2[k2][0];
 sp3[k3][1] = sp2[k2][1];
 sp3[k3][2] = sp1[k1][2] + sp2[k2][2];
 k3++;k2++;k1++; }
 }
```



```
4.while (k1 <=tot1)
 { sp3[k3][0] = sp1[k1][0];
 sp3[k3][1] = sp1[k1][1];
 sp3[k3][2] = sp1[k1][2];
 k3++;k1++;
 }
```

```
5 while (k2 <= tot2)
 { sp3[k3][0] = sp2[k2][0];
 sp3[k3][1] = sp2[k2][1];
 sp3[k3][2] = sp2[k2][2];
 k3++;k2++; }
```

```
6.sp3[0][0] = sp1[0][0];
```

```
7.sp3[0][1] = sp1[0][1];
```

```
8.sp3[0][2] = k3-1;
```

```
}
```



# COMPLEXITY

- $O(m+n)$  where  $m$  and  $n$  are the no. of non zero terms in sparse matrix 1 and sparse matrix 2.



# CASE STUDY

Use of sparse matrix in social network and maps

- Connection between the people can be shown with the help of sparse matrix
- Direct Route between the cities can be represented as a sparse matrix.
- Represent it with examples!!!



# Case studies from Syllabus

- Study use of sparse matrix in Social Networks and Maps.
- Study how Economists use polynomials to model economic growth patterns.
- How medical researchers use them to describe the behavior of Covid-19 virus.





# Introduction of Sparse Matrices for Machine Learning

- Large sparse matrices are common in general and especially in applied machine learning
- The interest in sparsity arises because its exploitation can lead to enormous computational savings and because many large matrix problems that occur in practice are sparse.
- $\text{sparsity} = \text{count zero elements} / \text{total elements}$
- Data:
  - Whether or not a user has watched a movie in a movie catalog.
  - Whether or not a user has purchased a product in a product catalog.
  - Count of the number of listens of a song in a song catalog.



- Some areas of study within machine learning must develop specialized methods to address sparsity directly as the input data is almost always sparse. Examples are:
  - Natural language processing for working with documents of text.
  - Recommender systems for working with product usage within a catalog.
  - Computer vision when working with images that contain lots of black pixels.



```

from numpy import array
from scipy.sparse import csr_matrix

create dense matrix
A = array([[1, 0, 0, 1, 0, 0], [0, 0, 2, 0, 0, 1], [0, 0,
0, 2, 0, 0]])
print(A)

convert to sparse matrix (CSR method)
S = csr_matrix(A)
print(S)

reconstruct dense matrix
B = S.todense()
print(B)

sparsity = 1.0 - count_nonzero(A) / A.size

```

```

6 [[1 0 0 1 0 0]
7 [0 0 2 0 0 1]
8 [0 0 0 2 0 0]]
9 (0, 0) 1
1 (0, 3) 1
0 (1, 2) 2
1 (1, 5) 1
1 (2, 3) 2
1
2 [[1 0 0 1 0 0]
1 [0 0 2 0 0 1]
3 [0 0 0 2 0 0]]

0.722222222222

```



**THANK YOU....  
ANY QUESTIONS????**

