QBio202 - SS 2023 - Exercises 1

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1a) $\frac{d}{dx}\left(\frac{1}{1-x}\right)$

$= \frac{1}{(1-x)^2}$

$= \left(\frac{1}{1-x}\right)^2$

Therefore, $y = \frac{1}{1-x}$ solves $y' = y^2$.

1b) $\frac{dy}{dx}$

$= \frac{d}{dx}\left(e^{3x} - \frac{e^x}{2}\right)$

$= 3e^{3x} - \frac{e^x}{2}$

$= 3\left(e^{3x} - \frac{e^x}{2}\right) + e^x$

$= 3y + e^x$

Therefore, $y = e^{3x} - \frac{e^x}{2}$ solves $y' = 3y + e^x$.

1c) $\frac{dy}{dx}$

$= \frac{d}{dx}(4 + \ln(x))$

$= \frac{1}{x}$

$\Rightarrow xy' = \frac{x}{x} = 1$

Therefore, $y = 4 + \ln(x)$ solves $xy' = 1$.

1d) $\frac{dy}{dx}$

$= \frac{d}{dx}\left(2e^x - x - 1\right)$

$= 2e^x - 1$

$= x + \left(2e^x - x - 1\right)$

$= x + y$

Therefore, $y = 2e^x - x - 1$ solves $y' = x + y$.

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2a) $\frac{dy}{dt} = -y$

$$\Rightarrow \frac{1}{y}dy = -dt$$

$$\Rightarrow \int \frac{1}{y}dy = -\int dt$$

$$\Rightarrow \ln(y) = -t + C$$

$$\Rightarrow y = e^{-t+C} = e^{C} \cdot e^{-t} = k \cdot e^{-t}$$

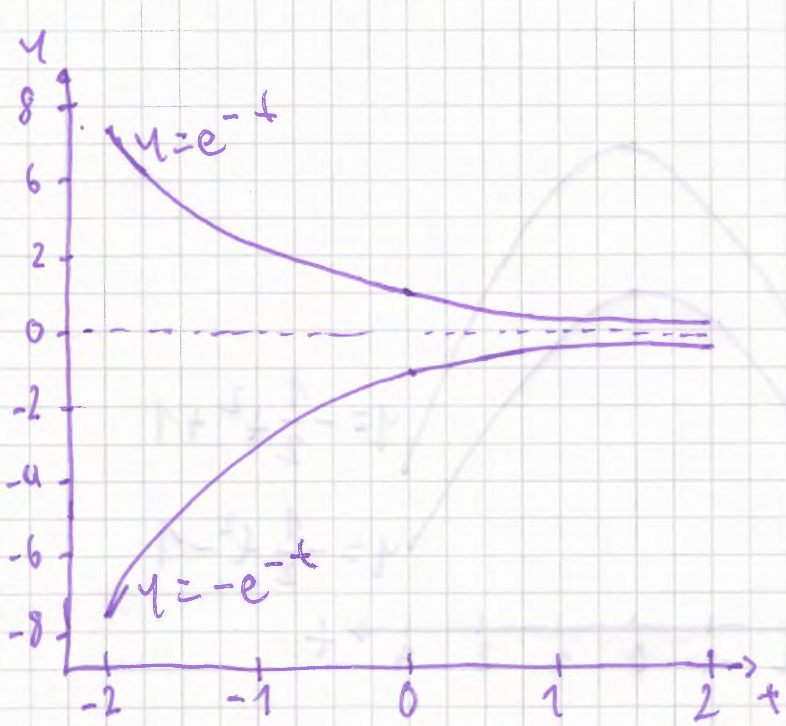
where C is the constant of integration, $k = e^{C}$.

<u>Initial value $y(0) = 1$:</u>

$$k \cdot e^{0} = 1 \Rightarrow k = 1$$

<u>Initial value $y(0) = -1$:</u>

$$k \cdot e^{0} = -1 \Rightarrow k = -1$$



2b) $\frac{dy}{dt} = -t$

$\Rightarrow dy = -t\,dt$

$\Rightarrow \int dy = -\int t\,dt$

$\Rightarrow y = -\frac{1}{2}t^2 + C$

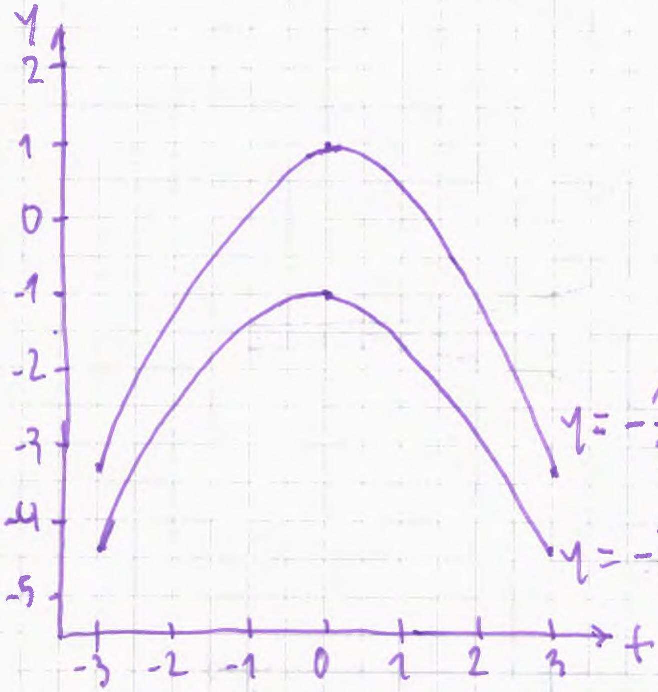
where C is the constant of integration

Initial value $y(0)=1$:

$-\frac{1}{2}0^2 + C = 1 \Rightarrow C = 1$

Initial value $y(0)=-1$:

$-\frac{1}{2}0^2 + C = -1 \Rightarrow C = -1$



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2c) $\frac{dy}{dt} = e^{-t}$

$\Rightarrow dy = e^{-t} dt$

$\Rightarrow \int dy = \int e^{-t} dt$

$\Rightarrow y = -e^{-t} + C$

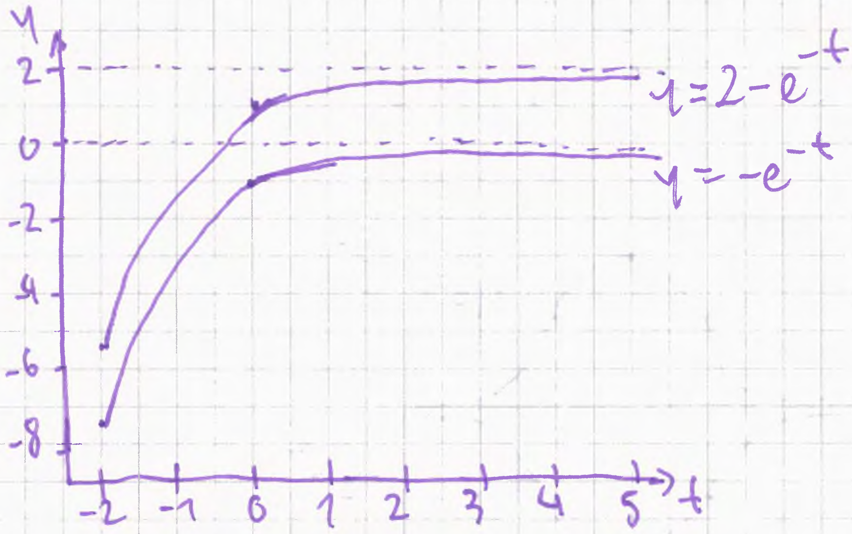
where C is the constant of integration

Initial value $y(0) = 1$:

$-e^{-0} + C = 1 \Rightarrow C = 2$

Initial value $y(0) = -1$:

$-e^{-0} + C = -1 \Rightarrow C = 0$



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2 d) $\frac{dy}{dt} = -yt$

$\Rightarrow \frac{1}{y} dy = -t\,dt$

$\Rightarrow \int \frac{1}{y} dy = -\int t\,dt$

$\Rightarrow \ln(y) = -\frac{1}{2}t^2 + C$

$\Rightarrow y = e^{-\frac{1}{2}t^2 + C} = e^{C} \cdot e^{-\frac{1}{2}t^2} = C^* \cdot e^{-\frac{1}{2}t^2}$

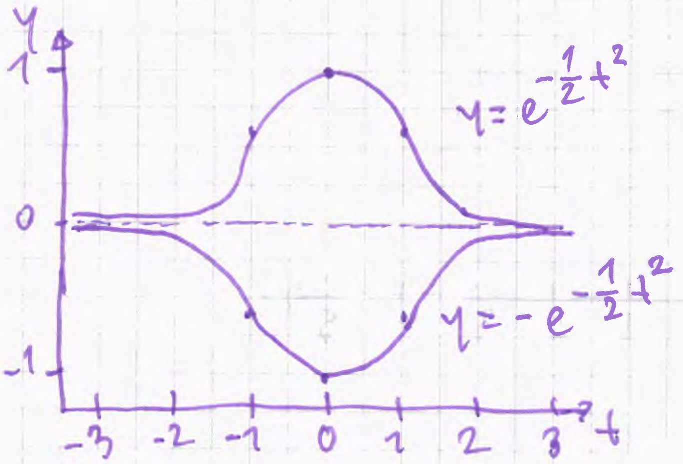
where C is the constant of integration, $C^* = e^C$.

Initial value $y(0) = 1$:

$C^* \cdot e^{-\frac{1}{2}0^2} = 1 \Rightarrow C^* = 1$

Initial value $y(0) = -1$:

$C^* \cdot e^{-\frac{1}{2}0^2} = -1 \Rightarrow C^* = -1$



4a) The half-life of the drug is 2 h

$\Rightarrow$ after 6 hours the concentration is

$$2^{-\left(\frac{6}{2}\right)} = 2^{-3} = \frac{1}{8} = 0{,}125$$

of the initial concentration.

4b) Let

- $y(t)$ be the ~~con~~ percentage of the initial drug concentration after t hours
- $y(0) = 1$ the initial drug concentration at time $t = 0$ be 100%

Then

$$\frac{dy}{dt} = cy$$

$$\Rightarrow \frac{1}{y}\, dy = c \cdot dt$$

$$\Rightarrow \int \frac{1}{y}\, dy = c \cdot \int dt$$

$$\Rightarrow \ln(y) = c \cdot t + C$$

$$\Rightarrow y = e^{C} \cdot e^{ct} = C^{*} \cdot e^{ct}$$

when C is the constant of integration, $C^{*} = e^{C}$.

Initial value $y(0) = 1$

$$\Rightarrow C^{*} \cdot e^{c \cdot 0} = 1$$

$$\Rightarrow C^{*} = 1$$

Therefore, $y(t) = e^{ct}$ for some constant c

The half-life of the drug is 2 hours

$$\Rightarrow y(2) = \frac{1}{2} \Rightarrow e^{c \cdot 2} = \frac{1}{2} \Rightarrow 2 \cdot c = \ln\left(\frac{1}{2}\right)$$

$$\Rightarrow \boxed{c = -\frac{\ln(2)}{2}}$$

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