Submit questions 1–4 over ILIAS, and attempt to solve question 5 in preparation for the tutorial on Thursday, April 13.

Questions

1. Verify that the given function is a solution to the given differential equation (y' is short-hand for $\frac{dy}{dx}$):
   (a) $y = \frac{1}{1-x}$ solves $y' = y^2$
   (b) $y = e^{3x} - \frac{e^x}{2}$ solves $y' = 3y + e^x$
   (c) $y = 4 + \ln x$ solves $xy' = 1$
   (d) $y = 2e^x - x - 1$ solves $y' = x + y$

2. Solve the following initial value problems starting from $y(t = 0) = 1$ and $y(t = 0) = -1$. Draw both solutions on the same graph.
   (a) $\frac{dy}{dt} = -y$
   (b) $\frac{dy}{dt} = -t$
   (c) $\frac{dy}{dt} = e^{-t}$
   (d) $\frac{dy}{dt} = -y \cdot t$

3. Which functions $y(x)$ fulfil the following conditions:
   For every $x > 0$, the tangent on the point $(x|y)$ of the function graph intersects with the x-axis at $(-x|0)$.

4. Most drugs in the bloodstream decay according to the equation $\frac{\mathrm{d}y}{\mathrm{d}t} = cy$, where y is the concentration of the drug in the bloodstream.
   (a) If the half-life of a drug is 2 hours, what fraction of the initial dose remains after 6 hours?
   (b) For a half-life of 2 hours, which value has the constant c?
   (c) A drug is administered intravenously to a patient at a rate r (in $\mathrm{mg\,h^{-1}}$) and is cleared from the body at a rate proportional to the amount of drug still present in the body. Set up and solve the differential equation, assuming there is no drug initially present in the body.

5. Numerical simulations using python.
   (a) Install a python distribution.
   (b) The `scipy` package provides various functions to numerically integrate differential equations. Among these are `odeint`, `solve_ivp` and `ode`. Read the docs and find out how to simulate the an initial value problem consisting of the differential equation
   $$\frac{\mathrm{d}x}{\mathrm{d}t} = rx\left(1 - \frac{x}{K}\right)$$
   and the initial condition
   $$x(0) = x_0.$$
   (c) The `matplotlib.pyplot` package provides a lot of functions to generate plots. Find out how to plot the result of the numerical integration of the initial value problem stated above. Use $r = 0.1$, $K = 10$ and $x_0 = 0.1$ and plot the simulated function $x(t)$. Label the axis.
   (d) Now systematically vary the initial condition between $x_0 = 0$ and $x_0 = 15$ in 50 steps and plot all simulated time courses on one plot.
   (e) Save all your code in an executable and documented `jupyter` notebook.

Submit your solutions (questions 1–4) by **Wednesday, April 12, 2023**.