COMP 558 Assignment 2

Question 1

gaussian, sigma=4

gaussian, sigma=8

gaussian, sigma=12

gaussian, sigma=16

heat equation, sigma=4

heat equation, sigma=8

heat equation, sigma=12

heat equation, sigma=16

Total diffusion time is given by 0.5*sigma^2. For question 1, I used Matlab standard function imgaussfilt to do smoothing with gaussian. Then I implemented a separate function for heat equation where I calculate the gradients and perform numerical update in each iteration. I repeated for sigma values 4,8,12,16. (all with fixed iteration of 200 times.) The generated images are as above, we can see that for the same sigma value, heat equation produces blurrer images and also, in the images generated by heat equation, the outlining edges in the bottom left fades to dark as sigma increases. However, the bottom left corner, began to turns white as sigma increases.

I also took the difference between the result obtained with heat equation and the result obtained by the gaussian. The numerical values are as below: we can see that as sigma increases, the difference is growing larger.

Lastly, I changed the number of iterations. As we can see from below, as number of iterations increase, difference between results obtained by the two methods are getting smaller and smaller.

Based on the data I got, I think the discrepancies occurs between results obtained by the two methods because the number of iterations is not enough for the value update in the heat equation method. If we perform the value update n times with n goes to infinity, the difference will tends to 0.

```
>> Q1
Number of iterations:
    50

difference total for sigma 4 =
  814.3846

difference total for sigma 8 =
   1.2465e+03

difference total for sigma 12 =
   6.1293e+15

difference total for sigma 16 =
   7.3669e+32
```

```
>> Q1
Number of iterations:
   100

difference total for sigma 4 =
  811.7403

difference total for sigma 8 =
   1.2348e+03

difference total for sigma 12 =
   1.8678e+03

difference total for sigma 16 =
   1.7321e+21
```

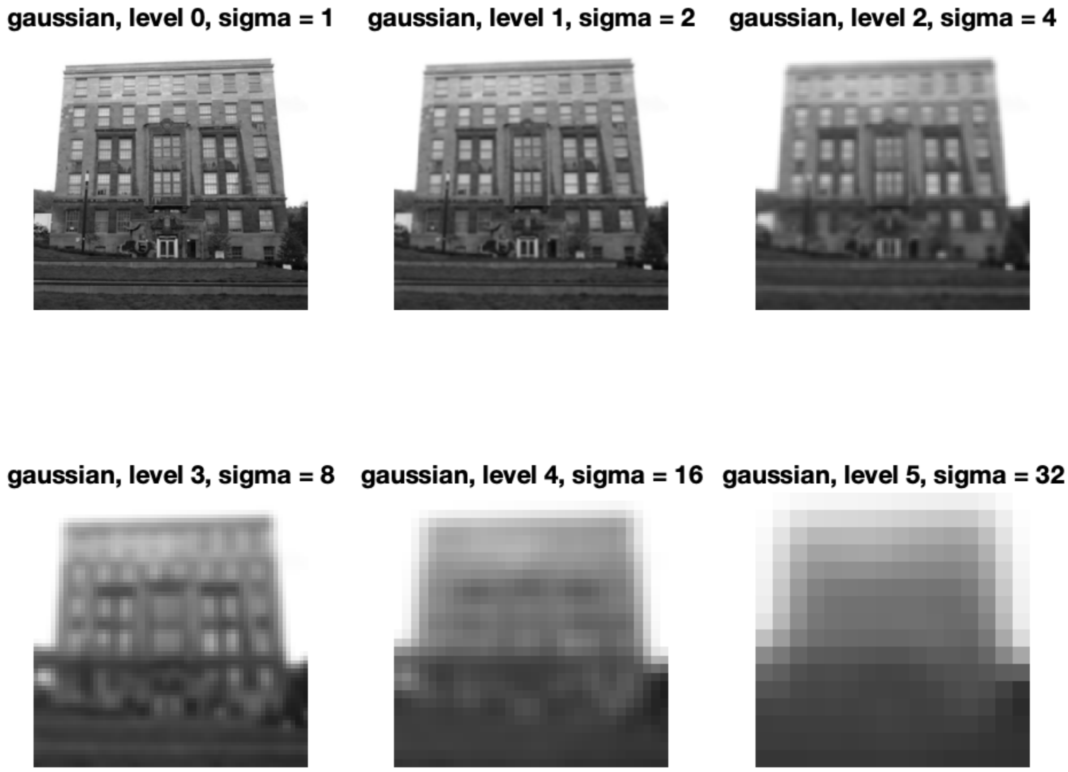
```
>> Q1
Number of iterations:
   200

difference total for sigma 4 =
  810.7769

difference total for sigma 8 =
   1.2292e+03

difference total for sigma 12 =
   1.8601e+03

difference total for sigma 16 =
   3.0021e+03
```

```
>> Q1
Number of iterations:
   400

difference total for sigma 4 =
  810.3787

difference total for sigma 8 =
   1.2265e+03

difference total for sigma 12 =
   1.8563e+03

difference total for sigma 16 =
   3.0006e+03
```

Question 2

Part 1:

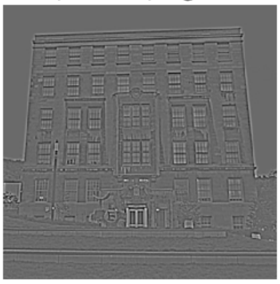
Below are result for the gaussian pyramid. I use imgaussfilt to perform filtering and then resize the image.



Part 2:

Below are results for the laplacian pyramid. I upsampled the successive level to the same size as the current level and then calculated the difference.

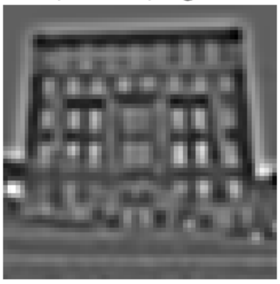
DoG, level 0, sigma = 1



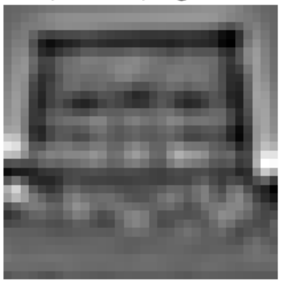
DoG, level 1, sigma = 2

DoG, level 2, sigma = 4

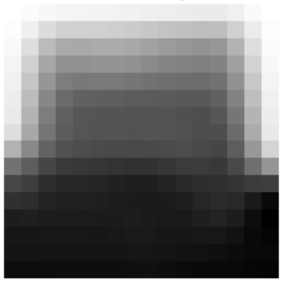
DoG, level 3, sigma = 8



DoG, level 4, sigma = 16

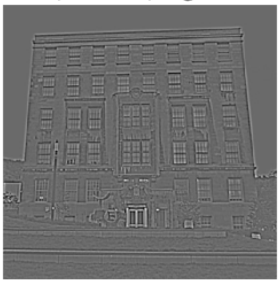


DoG, level 5, sigma = 32



Part 3:
Comparing the images formed by convolving with difference of gaussians and convolving with laplacian of gaussians directly, we can see that difference of gaussians produced slightly darker images but apart from that there is no other differences.

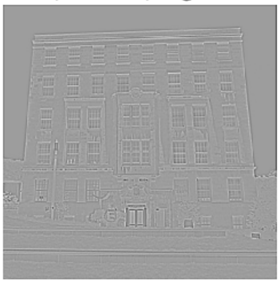
DoG, level 0, sigma = 1



DoG, level 1, sigma = 2

DoG, level 2, sigma = 4

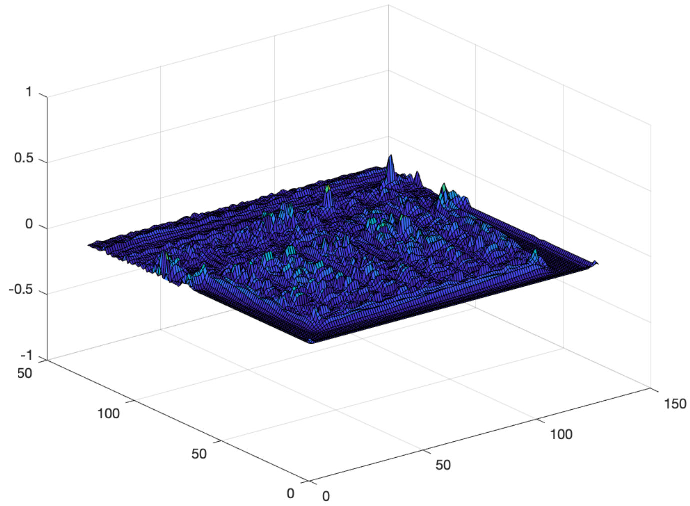
LoG, level 0, sigma = 1

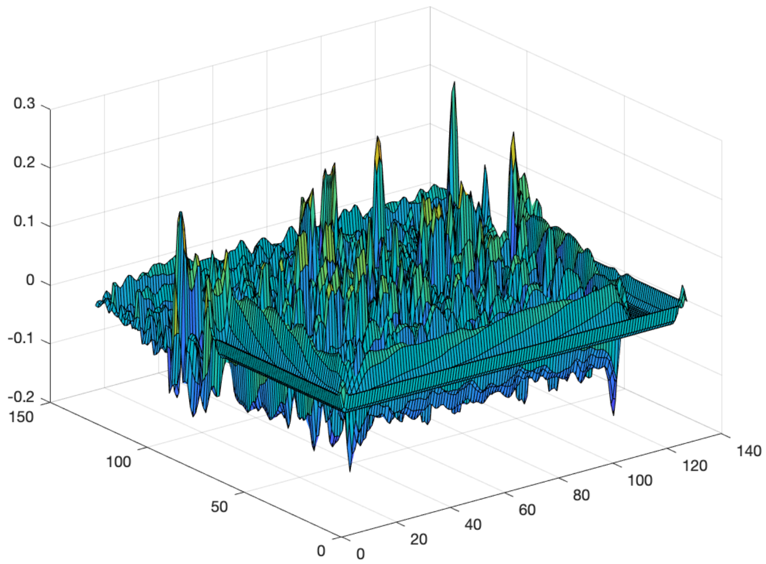


LoG, level 1, sigma = 2

LoG, level 2, sigma = 4

And from the visualization graph formed by taking difference and the absolute difference between DOG and LOG, we can see that there is barely any difference between the two(approaches zero).





Quantitative explanation:

The relationship between laplacian of gaussian, $\sigma^2\nabla^2 G$ and difference of gaussian D, can be derived from the heat diffusion equation:

$$\frac{\partial G}{\partial \sigma} = \sigma \nabla^2 G.$$

Thus, $\sigma\nabla^2 G = \frac{\partial G}{\partial \sigma} \approx \frac{G(x,y,k\sigma) - G(x,y,\sigma)}{k\sigma - \sigma}$

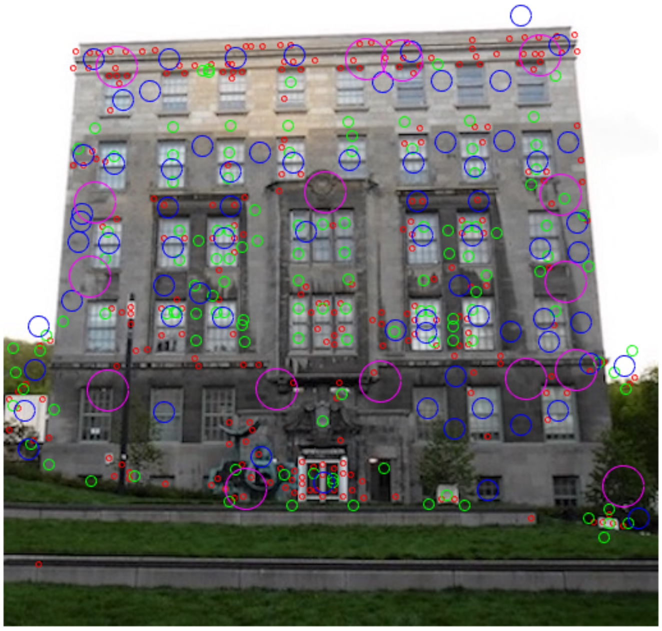
$$\Rightarrow G(x,y,k\sigma) - G(x,y,\sigma) \approx (k-1)\sigma^2\nabla^2 G$$

In this question, we use $k=2$

$$\Rightarrow G(x,y,2\sigma) - G(x,y,\sigma) \approx \sigma^2\nabla^2 G.$$

Part 4:
For this part I first save all the levels of DoG into a 3d matrix. Then I iterate through the inner levels. In each level, I iterate through each pixel and save all the points in the 3*3 neighborhood above and below into an array(26 in total for each pixel). Then I choose the point if it is not equal to zero and either it is larger than the max of the array, exceeds the mean of array by a certain threshold, or it is smaller than the min of the array, below the mean of array by a certain threshold. And finally, I overlay those points on the original image in different colors and radii based on the sigma value associated with the level it comes from.

I also rotate the image by 20 degrees and resize it to 1024*1024. And run the algorithm again. The results are approximately the same.



Question 3:
My idea in deciding the inliers in this question is that: firstly, I represent the line model with the equation R = x*cos(theta) + y*sin(theta). I calculate R based on the random generated point E. Then for every other edge point E*, I calculate the value of its R* and comparing it to R, if there absolute difference is smaller than a certain threshold, and the absolute difference between their gradient orientation is smaller than a certain threshold, then I choose this point.

The inliers plot I generated is as below: