Simulating Blood Flow in Arteries Using Poiseuille's Law

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May 2025

Abstract

This project simulates blood flow in an artery using Poiseuille's law. Using python, the blood flow rate was analyzed as a function of the artery radii to understand how the size of the vessel influences circulation. The results were also fitted to a power law model to verify if the flow really follows the r^4 relationship from theory. Finally, error bars were added based on the uncertainty in the radius measurements, and the chi-squared value was calculated to evaluate how well the model aligns with the simulated data.

1 Introduction

Blood flow is very sensitive to the radius of an artery. When the arteries narrow as a result of plaque build-up, the amount of blood that can flow through them quickly drops. Poiseuille's law gives us a way to understand this by modeling the artery as a cylindrical tube. The equation is as follows:

$$Q = \frac{\pi r^4 \Delta P}{8\eta L} \tag{1}$$

Here, Q is the flow rate, r is the radius, ΔP is the pressure difference, η is the viscosity of the blood, and L is the length of the artery. This equation shows that the flow rate strongly depends on the radius raised to the fourth power.

2 Methods

A Python script was written to calculate the blood flow rate for artery radii ranging between 1 mm and 5 mm. I used values of:

- $\eta = 0.0035$ Pa·s (viscosity)
- $L = 0.5$ m (length)
- $\Delta P = 1333$ Pa (pressure difference, equal to 10 mmHg)

The flow was computed using Poiseuille's law. The *curve_fit* from SciPy was used to fit the data to a power law model ($Q = ar^b$) and the fitted exponent b was compared to the theoretical value of 4. An uncertainty of ± 0.1 mm in radius was assumed and propagated to estimate the resulting uncertainty in Q. Finally, a chi-squared test was conducted to assess how well the fitted model aligned with the simulated data.

3 Results

The flow rate increased very quickly as the radius increased, which agrees with the theory.

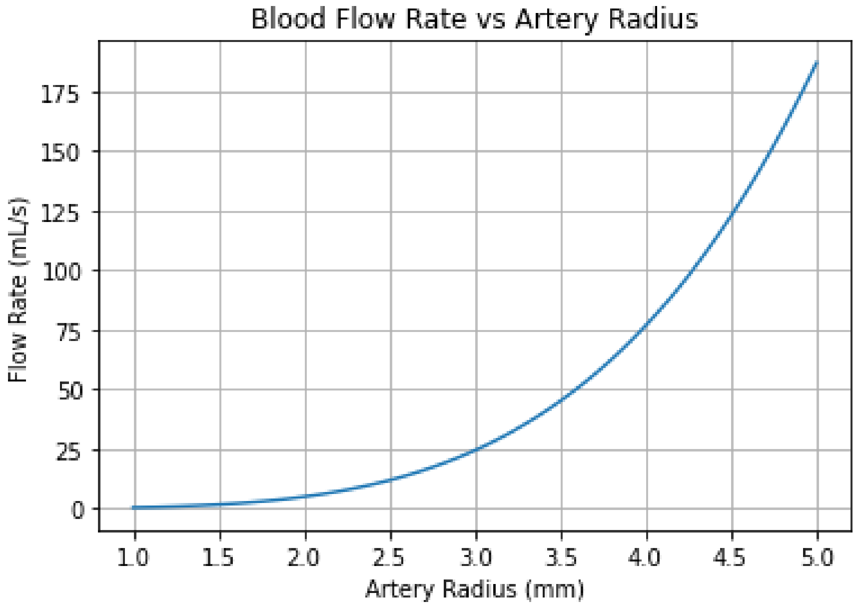

Figure 1: Simulated blood flow rate vs. artery radius. This Figure demonstrates the strong non-linear increase in flow rate with radius, reflecting the expected r^4 dependence. This highlights the physiological importance of small radius changes in arterial flow.

Next, the flow data were fitted to a power law model, revealing a best-fit exponent of $b = 4.00$, which is exactly what Poiseuille's Law predicts.

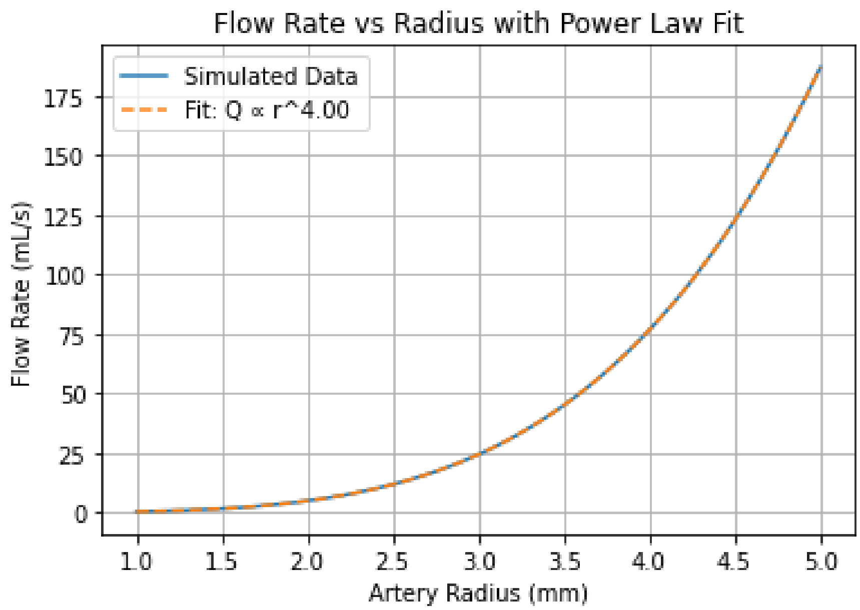

Figure 2: Shows the fitted power-law curve. The exponent b = 4.00 confirms agreement with the theoretical model.

To include uncertainty, error bars were added based on the radius uncertainty.

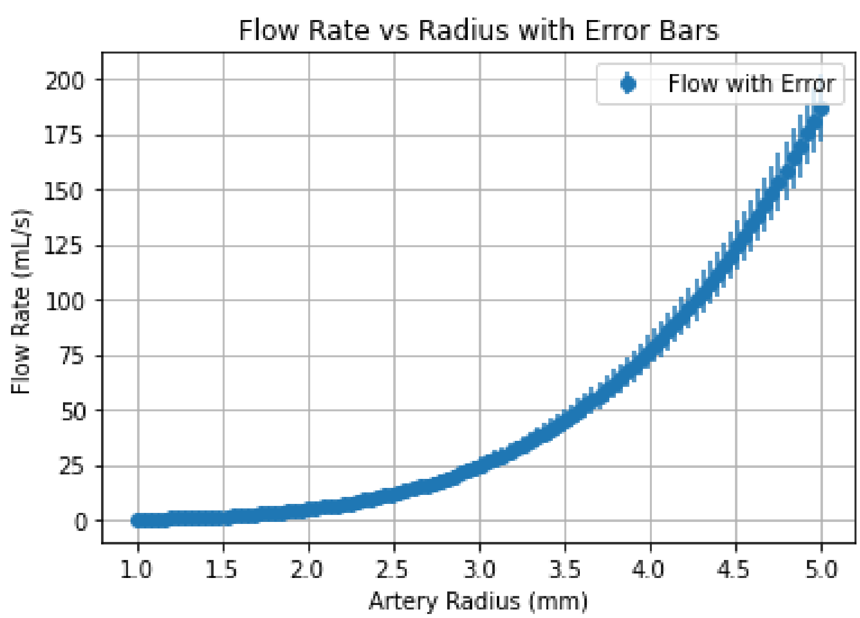

Figure 3: Flow rate with propagated uncertainty from radius. Includes error bars based on ± 0.1 mm uncertainty in radius. As radius increases, error bars grow larger due to sensitivity of flow rate to radius at higher values.

The chi-squared value of 98.21 indicates a good match between the model and the simulated data.

4 Conclusion

This project confirmed that Poiseuille's Law accurately describes how blood flows through arteries. The flow rate follows the r^4 pattern very closely, which means even small changes in radius can have a big effect on circulation. Using Python helped simulate the system, visualize the results, and perform a fit with uncertainty analysis.

5 References

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