Universitat Politècnica de Catalunya
Facultat de Matemàtiques i Estadística

Master in Advanced Mathematics and Mathematical Engineering

Master's thesis

Planng and Control of a Multiple-Quadcopter System Cooperatively Carrying a Slung Payload in Dynamical Environments

Iñigo Moreno Caireta

Supervised by Lluis Ros

and Directed by Mercè Ollé

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Abstract

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Chapter 1

Introduction

1-1 Theme Relevance and Justification

1-1-1 Omni-wheeled robots

1-1-2 Model Predictive Control

1-2 Research Objective

The objective of the research included in this thesis is to:

> *Design a control algorithm for systems with an omni-wheeled robot and demonstrate it in simulated and real scenarios.*

1-3 Thesis Context

This thesis is a result of the collaboration between Facultat de Matemàtiques i Estadística (FME) and Institut de Robòtica i Informàtica Industrial (IRI). During the summer of 2018 I contacted Lluis Roslooking for a suitable project for my Master's Thesis and he proposed me this one.

1-4 Notation

Throughout this thesis the following notation will be used:

	Symbol	Definition
Scalars:	x	
Vectors:	$\boldsymbol{x}$	
Elements of vector:	$\boldsymbol{x}(i)$	i-th element of vector $\boldsymbol{x}$
Position vectors:	$\boldsymbol{p}_x$	
Coordinates of position vector:	x_x, y_x, z_x	x,y,z coordinates of position $\boldsymbol{p}_x$
Matrices:	$\boldsymbol{X}$	
Sets:	$\mathcal{X}$	
Time derivatives:	$\dot{x}$	$\frac{\partial x}{\partial t}$
Desired values:	$\bar{x}$	desired value of x
Predicted values:	$\hat{x}$	predicted value of x

Table 1-1: Thesis Notation

All vectors are column vectors and all positions are expressed in the East-North-Up (ENU) inertial frame unless otherwise stated.

Chapter 2

Problem Formulation

2-1 Omni-wheeled robot

The robot can be represented as a rigid body in a planar space. The location of the robot's Center Of Mass (COM) is defined as $\boldsymbol{p}_0 \in \mathbb{R}^2$ and the yaw of the robot is represeted as $\psi \in \mathbb{R}$.

Assuming the robot has $n \in \mathbb{Z}_{>0}$ wheels, we can define their

2-2 Dynamic Model

2-2-1 Motor Dynamics Identification

The motors in our robot are controlled by velocity inputs. We will represent the inputs of the i-th motor as $u_i \in [-1, 1]$. The motors are also equipped with encoders from which we can deduce the angle of each wheel, denoted as $theta_i \in \mathbb{R}$.

We can try to identify the model using process model identification. A tool that, given some identification data and a transfer function with unknown parameters, estimates the value of the parameters. We will assume that the relationship between input velocity and the real velocity follow a linear transfer function:

$$\frac{\dot{\theta}_i(s)}{u_i(s)} = \frac{K_i}{1+\tau_i s} \rightarrow \frac{\theta_i(s)}{u_i(s)} = \frac{K_i}{s(1+\tau_i s)} \qquad 2\text{-}1$$

Which corresponds to the following set of ODEs:

$$\dot{\theta}_i + \tau \ddot{\theta}_i = u\, K \qquad 2\text{-}2$$

To obtain the identification data we can make the arduino generate a square wave and record the data from the encoder. This will give us data discretized with different time-intervals as the time-interval will depend on the loop size. To get data with equal time-intervals we can set a higher sampling frequency and interpolate the values between each sample.

When identifying the model we saw that the values of K were different for varying amplitudes as we can see in figure 2-1.

Do experiments for higher values of amplitude (when lab is empty)

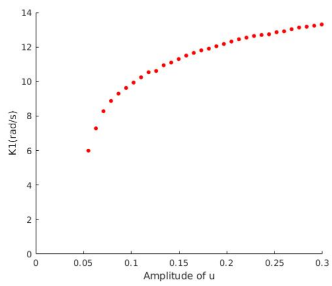

Figure 2-1: Relationship between K_1 and the amplitude of u

As the relationship looks like it has a vertical and a horizontal asymptote, we will try to model it with a rational function of the form:

$$K_i = \frac{a_i\, u_i + b_i}{u_i + c_i} \qquad 2\text{-}3$$

see if τ depends on u

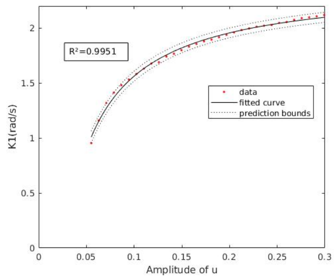

Figure 2-2: Predicted relationship between K_1 and the amplitude of u

2-2-2 Aerodynamic Drag

We will use the same model for aerodynamic drag model as in [6]. The drag on the payload, denoted as $\boldsymbol{F}_{Dl} \in \mathbb{R}^3$, is modeled using a quadratic drag model, while the drag on each quadrotor, denoted as $\boldsymbol{F}_{Di} \in \mathbb{R}^3$, is modeled using a linear drag model.

$$\boldsymbol{F}_{Dl} = -k_{Dl}\|\dot{\boldsymbol{p}}_l\|^2 \frac{\dot{\boldsymbol{p}}_l}{\|\dot{\boldsymbol{p}}_l\|} = -k_{Dl}\|\dot{\boldsymbol{p}}_l\|\dot{\boldsymbol{p}}_l \tag{2-4}$$

$$\boldsymbol{F}_{Di} = -k_{Di}\|\dot{\boldsymbol{p}}_i\| \frac{\dot{\boldsymbol{p}}_i}{\|\dot{\boldsymbol{p}}_i\|} = -k_{Di}\dot{\boldsymbol{p}}_i \quad \forall i \in \{1 \dots n\} \tag{2-5}$$

Where $k_{Dl} \in \mathbb{R}$ and $k_{Di} \in \mathbb{R}$ are the drag constants identified in [6].

Drag Inclusion in Equations of Motion (EOMs)

To use include drag in our problem we need to include the drag force calculated in equations 2-4 and 2-5 in the EOMs declared in equations **??** and **??**.

The drag force on the drones $\boldsymbol{F}_{Di}$ can be added to the input force $\boldsymbol{F}_{ui}$ (we will denote this sum as $\boldsymbol{F}_{uDi}$). The drag force on the payload can be included by subtracting $\frac{\boldsymbol{F}_{Dl}}{m_l}$ from all appearances of $\ddot{\boldsymbol{p}}_l$ in the EOMs, in the same way that $g\boldsymbol{e}_3$ is subtracted. The resulting EOMs from these changes are:

$$\dot{\boldsymbol{q}}_i = \boldsymbol{w}_i \times \boldsymbol{q}_i \quad \forall i \in \{1 \dots n\} \tag{2-6}$$

$$\boldsymbol{M}_q(\ddot{\boldsymbol{p}}_l - g\boldsymbol{e}_3 - \frac{\boldsymbol{F}_{Dl}}{m_l}) = \sum_{i=1}^{n}(-m_i l_i \|\boldsymbol{w}_i\|^2 \boldsymbol{q}_i + \boldsymbol{F}_{uDi}^{\parallel}) \tag{2-7}$$

$$\dot{\boldsymbol{w}}_i = \frac{1}{l_i}\boldsymbol{q}_i \times (\ddot{\boldsymbol{p}}_l - g\boldsymbol{e}_3 - \frac{\boldsymbol{F}_{Dl}}{m_l}) - \frac{1}{m_i l_i}(\boldsymbol{q}_i \times \boldsymbol{F}_{uDi}^{\perp}) \quad \forall i \in \{1 \dots n\} \tag{2-8}$$

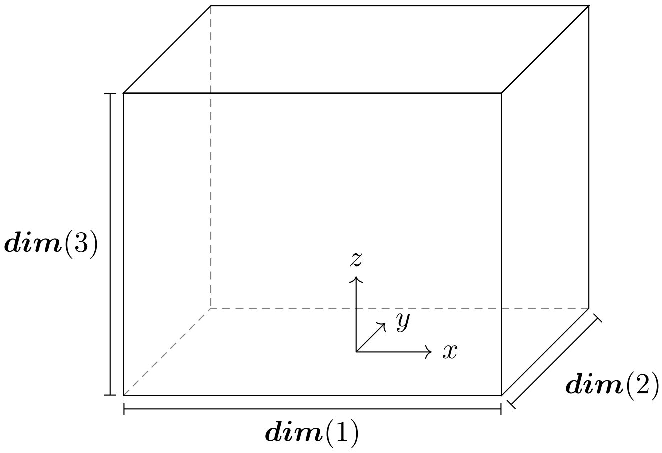

Figure 2-3: Environment Schematic

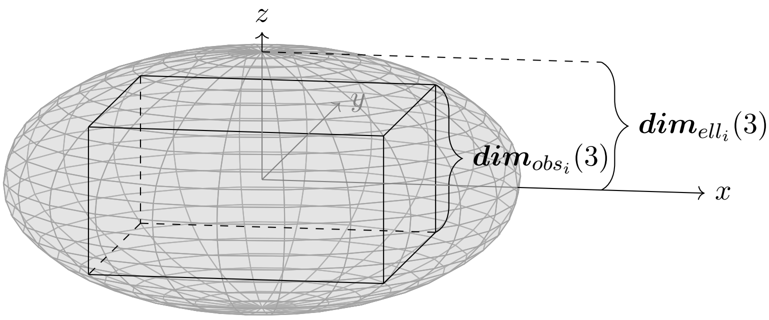

Figure 2-4: Obstacle dimensions

2-2-3 Environment Definition

The environment is defined as a 3 dimensional box with the origin in the floor's center. We denote the box's dimensions as $\boldsymbol{dim} \in \mathbb{R}^3_{>0}$. A schematic of the environment can be seen in figure 2-3.

2-2-4 Obstacle Modeling

Obstacles are modeled as three dimensional boxes and their behavior is modeled as linear movement using a Kalman Filter. This is required so that the planner takes into account future movement of the obstacles. The number of obstacles is denoted by $n_{obs} \in \mathbb{Z}_{\geq 0}$, their center position by $\boldsymbol{p}_{obs_i} \in \mathbb{R}^3$ and their dimension by $\boldsymbol{dim}_{obs_i} \in \mathbb{R}^3_{>0}$.

Often we will use the smallest ellipsoid that contains the box. The equations to find the radial dimensions of the ellipsoid (denoted as $\boldsymbol{dim}_{ell_i} \in \mathbb{R}^3$) are:

$$\boldsymbol{dim}_{ell_i} = \frac{\sqrt{3}}{2}\boldsymbol{dim}_{obs_i} \quad \forall i \in \{1 \dots n_{obs}\} \tag{2-9}$$

A schematic of an obstacle and its dimensions can be seen in figure 2-4. For further explanation refer to his thesis[6].

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Glossary

List of Acronyms

FME	Facultat de Matemàtiques i Estadística
IRI	Institut de Robòtica i Informàtica Industrial
MPC	Model Predictive Control
COM	Center Of Mass
ENU	East-North-Up
EOMs	Equations of Motion

List of Symbols

Greek Symbols

$\phi_i \in \mathbb{R}$	Pitch of the i-th quadrotor
$\theta_i \in \mathbb{R}$	Roll of the i-th quadrotor
$\varphi_i \in \mathbb{R}$	Second payload angle of the i-th quadrotor
$\vartheta_i \in \mathbb{R}$	First payload angle of the i-th quadrotor

Latin Symbols

$\Delta t \in \mathbb{R}_+$	Time between stages in the Model Predictive Control (MPC) formulation
$\boldsymbol{e}_3 \in \mathbb{R}^3$	Down vector, $[0, 0, -1]^T$
$\boldsymbol{F}_{Di} \in \mathbb{R}^3$	Aerodynamic drag on the i-th drone
$\boldsymbol{F}_{Dl} \in \mathbb{R}^3$	Aerodynamic drag on the payload
$\boldsymbol{F}_{uDi}$	Addition of $\boldsymbol{F}_{ui}$ and $\boldsymbol{F}_{Di}$
$\boldsymbol{F}_{ui} \in \mathbb{R}^3$	Input force of the i-th quadrotor
$\boldsymbol{F}_{ui}^{\parallel} \in \mathbb{R}^3$	Orthogonal projection of $\boldsymbol{F}_{ui}$ along $\boldsymbol{q}_i$
$\boldsymbol{F}_{ui}^{\perp} \in \mathbb{R}^3$	Orthogonal projection of $\boldsymbol{F}_{ui}$ to the plane normal to $\boldsymbol{q}_i$
$\boldsymbol{M}_q \in \mathcal{S}_+^3$	Mass Matrix
$\boldsymbol{p}_i \in \mathbb{R}^3$	Position of the i-th quadrotor
$\boldsymbol{p}_l \in \mathbb{R}^3$	Payload's location
$\boldsymbol{q}_i \in \mathsf{S}^2$	Direction of the i-th link
$\boldsymbol{T}_i \in \mathbb{R}^3$	Tension exerted on the i-th drone
$\boldsymbol{w}_i \in \mathbb{R}^3$	Rotation of the i-th link's direction
$\boldsymbol{z} \in \mathbb{R}^{3n+2+n_{var}}$	Asd
$\boldsymbol{dim} \in \mathbb{R}^3_{>0}$	Environment dimensions
$\boldsymbol{dim}_{ell_i} \in \mathbb{R}^3$	Ellipsoid dimensions of the i-th obstacle
$\boldsymbol{dim}_{obs_i} \in \mathbb{R}^3_{>0}$	Dimension of the i-th obstacle
$\boldsymbol{p}_{obs_i} \in \mathbb{R}^3$	Position of the i-th obstacle
$\boldsymbol{ss}_i \in \mathbb{R}^6$	System states for the i-th quadrotor
$\boldsymbol{ss}_{\dot{z}i} \in \mathbb{R}^2$	System states of $h_{\dot{z}}$ for the i-th quadrotor
$\boldsymbol{ss}_{\phi i} \in \mathbb{R}^2$	System states of h_ϕ for the i-th quadrotor
$\boldsymbol{ss}_{\theta i} \in \mathbb{R}^2$	System states of h_θ for the i-th quadrotor
C_x	Cost of objective x
$h_{\dot{z}}$	Identified system for $\dot{z}$
h_ϕ	Identified system for ϕ
h_θ	Identified system for θ
$k \in \mathbb{R}_+$	Maximum iteration factor for external planner
$k_{Di} \in \mathbb{R}$	Drag constant of the i-th quadrotor
$k_{Dl} \in \mathbb{R}$	Drag constant of the payload
$l_i \in \mathbb{R}_{>0}$	Length of the i-th link

$m_i \in \mathbb{R}_{>0}$	Mass of the i-th quadrotor
$m_l \in \mathbb{R}_{>0}$	Payload's mass
$N \in \mathbb{Z}_+$	Number of stages hin the MPC formulation
$n \in \mathbb{Z}_{>0}$	Number of quadrotors
$n_{maxit} \in \mathbb{Z}_+$	Maximum number of iterations of the MPC solver
$n_{obs} \in \mathbb{Z}_{\geq 0}$	Number of obstacles
$n_{var} \in \mathbb{Z}_{>0}$	Number of variables in the state vector
$slack \in \mathbb{R}_{\geq 0}$	First slack variable
$slack_{env} \in \mathbb{R}$	Second slack variable
$t_{horizon} \in \mathbb{R}_+$	Planning time horizon
t_{it}	Time per iteration
$t_{MPC} \in \mathbb{R}_+$	MPC solve time
$t_{step} \in \mathbb{R}_+$	Control loop time
W_x	Weight of objective x

Other

$\mathsf{S}^2 \subset \mathbb{R}^3$	Set of unit vectors in $\mathbb{R}^3$
$\boldsymbol{slacks} \in \mathbb{R}^2$	Slack variables
$\boldsymbol{u}_i \in \mathbb{R}^3$	Inputs of the i-th quadrotor
$\boldsymbol{u}_t \in \mathbb{R}^{3n}$	Inputs of all quadrotors
$\boldsymbol{x} \in \mathbb{R}^{n_{var}}$	State vector