Universitat Politècnica de Catalunya
Facultat de Matemàtiques i Estadística

Master in Advanced Mathematics and Mathematical Engineering

Master's thesis

Planng and Control of a Multiple-Quadcopter System Cooperatively Carrying a Slung Payload in Dynamical Environments

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Abstract

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Chapter 1

Introduction

1-1 Theme Relevance and Justification

1-1-1 Omni-wheeled robots

1-1-2 Model Predictive Control

1-2 Research Objective

The objective of the research included in this thesis is to:

> *Design a control algorithm for systems with an omni-wheeled robot and demonstrate it in simulated and real scenarios.*

1-3 Thesis Context

This thesis is a result of the collaboration between Facultat de Matemàtiques i Estadística (FME) and Institut de Robòtica i Informàtica Industrial (IRI). During the summer of 2018 I contacted Lluis Roslooking for a suitable project for my Master's Thesis and he proposed me this one.

1-4 Notation

Throughout this thesis the following notation will be used:

	Symbol	Definition
Scalars:	x	
Vectors:	$\boldsymbol{x}$	
Elements of vector:	$\boldsymbol{x}(i)$	i-th element of vector $\boldsymbol{x}$
Position vectors:	$\boldsymbol{p}_x$	
Coordinates of position vector:	x_x, y_x, z_x	x,y,z coordinates of position $\boldsymbol{p}_x$
Matrices:	$\boldsymbol{X}$	
Sets:	$\mathcal{X}$	
Time derivatives:	$\dot{x}$	$\frac{\partial x}{\partial t}$
Desired values:	$\bar{x}$	desired value of x
Predicted values:	$\hat{x}$	predicted value of x

Table 1-1: Thesis Notation

All vectors are column vectors and all positions are expressed in the East-North-Up (ENU) inertial frame unless otherwise stated.

Chapter 2

Problem Formulation

2-1 Omni-wheeled robot

The robot can be represented as a rigid body in a planar space. The location of the robot's Center Of Mass (COM) is defined as $\boldsymbol{p}_0 \in \mathbb{R}^2$ and the yaw of the robot is represeted as $\psi \in \mathbb{R}$.

Assuming the robot has $n \in \mathbb{Z}_{>0}$ wheels, we can define their

2-2 Dynamic Model

2-2-1 Motor Electro-dynamic Identification

The motors installed in the robot are IG-42CRGM, which are 24 V DC motors with a 1/49 reduction ratio. These motors are controlled by Voltage inputs. DC motors can be characterized by the following equations:

$$V = K\frac{\dot{\varphi}}{N} + Ri + L\frac{\partial i}{\partial t} \tag{2-1}$$

$$\tau_w = NKi \tag{2-2}$$

Where V and i are the total voltage and intensity through the motor, N,R,K and L are parameters of the motor, τ_w is the torque applied to the wheel and φ the wheel's angle. If we assume L to be negligible, we get the following expression:

$$\Gamma_w = \frac{NKV - K^2\dot{\varphi}}{R} \tag{2-3}$$

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Glossary

List of Acronyms

FME	Facultat de Matemàtiques i Estadística
IRI	Institut de Robòtica i Informàtica Industrial
MPC	Model Predictive Control
COM	Center Of Mass
ENU	East-North-Up

List of Symbols

Greek Symbols

$\phi_i \in \mathbb{R}$ Pitch of the i-th quadrotor
$\theta_i \in \mathbb{R}$ Roll of the i-th quadrotor
$\varphi_i \in \mathbb{R}$ Second payload angle of the i-th quadrotor
$\vartheta_i \in \mathbb{R}$ First payload angle of the i-th quadrotor

Latin Symbols

$\Delta t \in \mathbb{R}_+$ Time between stages in the Model Predictive Control (MPC) formulation
$\boldsymbol{e}_3 \in \mathbb{R}^3$ Down vector, $[0, 0, -1]^T$
$\boldsymbol{F}_{Di} \in \mathbb{R}^3$ Aerodynamic drag on the i-th drone
$\boldsymbol{F}_{Dl} \in \mathbb{R}^3$ Aerodynamic drag on the payload
$\boldsymbol{F}_{uDi}$ Addition of $\boldsymbol{F}_{ui}$ and $\boldsymbol{F}_{Di}$
$\boldsymbol{F}_{ui} \in \mathbb{R}^3$ Input force of the i-th quadrotor
$\boldsymbol{F}_{ui}^{\|} \in \mathbb{R}^3$ Orthogonal projection of $\boldsymbol{F}_{ui}$ along $\boldsymbol{q}_i$
$\boldsymbol{F}_{ui}^{\perp} \in \mathbb{R}^3$ Orthogonal projection of $\boldsymbol{F}_{ui}$ to the plane normal to $\boldsymbol{q}_i$
$\boldsymbol{M}_q \in \mathcal{S}_+^3$ Mass Matrix
$\boldsymbol{p}_i \in \mathbb{R}^3$ Position of the i-th quadrotor
$\boldsymbol{p}_l \in \mathbb{R}^3$ Payload's location
$\boldsymbol{q}_i \in \mathsf{S}^2$ Direction of the i-th link
$\boldsymbol{T}_i \in \mathbb{R}^3$ Tension exerted on the i-th drone
$\boldsymbol{w}_i \in \mathbb{R}^3$ Rotation of the i-th link's direction
$\boldsymbol{z} \in \mathbb{R}^{3n+2+n_{var}}$ Asd
$\boldsymbol{dim} \in \mathbb{R}_{>0}^3$ Environment dimensions
$\boldsymbol{dim}_{ell_i} \in \mathbb{R}^3$ Ellipsoid dimensions of the i-th obstacle
$\boldsymbol{dim}_{obs_i} \in \mathbb{R}_{>0}^3$ Dimension of the i-th obstacle
$\boldsymbol{p}_{obs_i} \in \mathbb{R}^3$ Position of the i-th obstacle
$\boldsymbol{ss}_i \in \mathbb{R}^6$ System states for the i-th quadrotor
$\boldsymbol{ss}_{\dot{z}i} \in \mathbb{R}^2$ System states of $h_{\dot{z}}$ for the i-th quadrotor
$\boldsymbol{ss}_{\phi i} \in \mathbb{R}^2$ System states of h_ϕ for the i-th quadrotor
$\boldsymbol{ss}_{\theta i} \in \mathbb{R}^2$ System states of h_θ for the i-th quadrotor
C_x Cost of objective x
$h_{\dot{z}}$ Identified system for $\dot{z}$
h_ϕ Identified system for ϕ
h_θ Identified system for θ
$k \in \mathbb{R}_+$ Maximum iteration factor for external planner
$k_{Di} \in \mathbb{R}$ Drag constant of the i-th quadrotor
$k_{Dl} \in \mathbb{R}$ Drag constant of the payload
$l_i \in \mathbb{R}_{>0}$ Length of the i-th link

$m_i \in \mathbb{R}_{>0}$	Mass of the i-th quadrotor
$m_l \in \mathbb{R}_{>0}$	Payload's mass
$N \in \mathbb{Z}_+$	Number of stages hin the MPC formulation
$n \in \mathbb{Z}_{>0}$	Number of quadrotors
$n_{maxit} \in \mathbb{Z}_+$	Maximum number of iterations of the MPC solver
$n_{obs} \in \mathbb{Z}_{\geq 0}$	Number of obstacles
$n_{var} \in \mathbb{Z}_{>0}$	Number of variables in the state vector
$slack \in \mathbb{R}_{\geq 0}$	First slack variable
$slack_{env} \in \mathbb{R}$	Second slack variable
$t_{horizon} \in \mathbb{R}_+$	Planning time horizon
t_{it}	Time per iteration
$t_{MPC} \in \mathbb{R}_+$	MPC solve time
$t_{step} \in \mathbb{R}_+$	Control loop time
W_x	Weight of objective x

Other

$\mathsf{S}^2 \subset \mathbb{R}^3$	Set of unit vectors in $\mathbb{R}^3$
$\boldsymbol{slacks} \in \mathbb{R}^2$	Slack variables
$\boldsymbol{u}_i \in \mathbb{R}^3$	Inputs of the i-th quadrotor
$\boldsymbol{u}_t \in \mathbb{R}^{3n}$	Inputs of all quadrotors
$\boldsymbol{x} \in \mathbb{R}^{n_{var}}$	State vector