MONASH
INFORMATION
TECHNOLOGY

FIT2004
Algorithms and Data Structures

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Referencing materials by
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Ready?

Agenda

- Hash Tables

Agenda

- Hash Tables
  - Cuckoo hashing

Let us begin…

Data Lookup

Getting what we stored…

- Very often, we store data/ information

Data Lookup

Getting what we stored…

- Very often, we store data/ information
- How do we retrieve it?

Data Lookup

Getting what we stored…

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- Very often, we store data/ information
- How do we retrieve it?
  - Look it up… AKA SEARCH

Data Lookup

Getting what we stored…

- Very often, we store data/ information
- How do we retrieve it?
  - Look it up… AKA SEARCH

- So how do we search?

Data Lookup

Getting what we stored…

- Very often, we store data/ information
- How do we retrieve it?
  - Look it up… AKA SEARCH

- So how do we search?
  - By the key

Data Lookup
Getting what we stored…

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- How do we retrieve it?
  - Look it up… AKA SEARCH

- So how do we search?
  - By the key
  - Key is unique
  - Data is then accessed

Data Lookup

Getting what we stored…

- Very often, we store data/ information
- How do we retrieve it?
  - Look it up… AKA SEARCH

- So how do we search?
  - By the key
  - Key is unique
    - Student ID
  - Data is then accessed
    - Name
    - Age
    - Address
    - Contact

Data Lookup
An array/ list

- Let say we store data in an array/ list

Data Lookup

An array/ list

- Let say we store data in an array/ list
- How fast can we find what we want?
  - Time complexity O(N) via linear search

Data Lookup

An array/ list

- Let say we store data in an array/ list
- How fast can we find what we want?
  - Time complexity O(N) via linear search
  - So can we make it faster?

Data Lookup

An sorted array/ ~~list~~

- If the data is sorted… Then we can use binary search!
  - Best case O(1)
  - Worst case O(log N)

Data Lookup

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- But we need to insert in order…
  - Giving us O(N) complexity still…

Data Lookup

An sorted array/ ~~list~~

- If the data is sorted… Then we can use binary search!
  - Best case O(1)
  - Worst case O(log N)
- But we need to insert in order…
  - Giving us O(N) complexity still…
- Likewise for delete, we need to shift
  - Giving us O(N) complexity still…

Questions?

Hash Table

Quick search!

- What if we have a way to know where we store the item?

Quick search!

- What if we have a way to know where we store the item?
- We need an array to access position in O(1) time

Hash Table

Quick search!

- What if we have a way to know where we store the item?
- We need an array to access position in O(1) time
  - Fixed size
  - O(1) random access

Hash Table

Direct-Addressing

- If we have N student
  - Student ID from 0 to N-1

Hash Table
Direct-Addressing

- If we have N student
  - Student ID from 0 to N-1
- We create an array N-size
  - StudentID = index

Hash Table

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- But what if our student ID is 8 digit?
  - Like Monash's?

Direct-Addressing

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- But what if our student ID is 8 digit?
  - Like Monash's?
  - Make a very big array?

- If we have N student
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  - StudentID = index

- But what if our student ID is 8 digit?
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  - Make a very big array?
- What if the student ID has alphabet?
  - Like Authcate?

Hash Table

Direct-Addressing

- If we have N student
  - Student ID from 0 to N-1
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  - StudentID = index
- But what if our student ID is 8 digit?
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  - Like Authcate?

Hash functions

Questions?

- A function to convert key into position in array
  - index = hash(key)

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  - Within the boundary of the array
    - % len(array)

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- Consider the following
  - StudentID = 322
  - len(array) = 10
  - Hash function = key % len(array)
  - What is the index?

Hash Functions

- A function to convert key into position in array
  - index = hash(key)
  - Within the boundary of the array
    - % len(array)

- Consider the following
  - StudentID = 322
  - len(array) = 10
  - Hash function = key % len(array)
  - Index = 2
  - Now we have another student, with the ID = 422
  - What is index?

- A function to convert key into position in array
  - index = hash(key)
  - Within the boundary of the array
    - % len(array)

- Consider the following
  - StudentID = 322
  - len(array) = 10
  - Hash function = key % len(array)
  - Index = 2
  - Now we have another student, with the ID = 422
  - What is index? = 2 as well! COLLISION

different key hash to the same index

Questions?

- So we see a collision…

- So we see a collision…
- Can we avoid it?

Hash Table
Avoiding Collisions

- So we see a collision…
- Can we avoid it?
  - Good hash function

Hash Table
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- So we see a collision…
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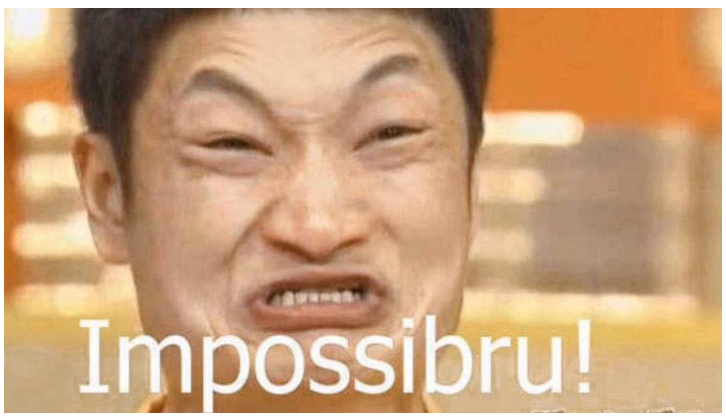


Understanding hash functions

- We want to use a hash table for a class of 50 students
- Assume that the hash function is based on birthdays (dd-mm), e.g., a student born on 01-Jan is hashed to index 1, 02-Jan on index 2, ..., 31-Dec on 365.
- How likely is that two students will be hashed to the same index, e.g., how likely is the collision?

$$Prob(no\ collision) = \frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \cdots \times \frac{(366-N)}{365}$$

$$Prob(no\ collision) = \frac{365!}{365^N (365-N)!}$$

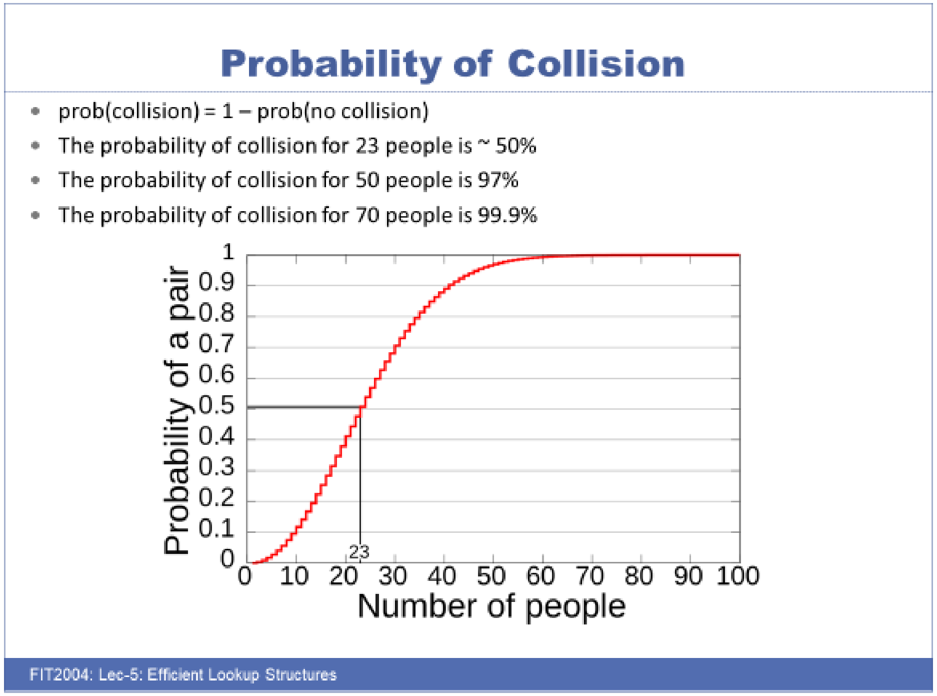
Visit https://pudding.cool/2018/04/birthday-paradox/ for an interactive explanation of the birthday paradox

FIT2004: Lec-5: Efficient Lookup Structures

Hash Table

Avoiding Collisions

- So we see a collision…
- Can we avoid it?
  - Good hash function
  - Kinda too hard…



Hash Table
Avoiding Collisions

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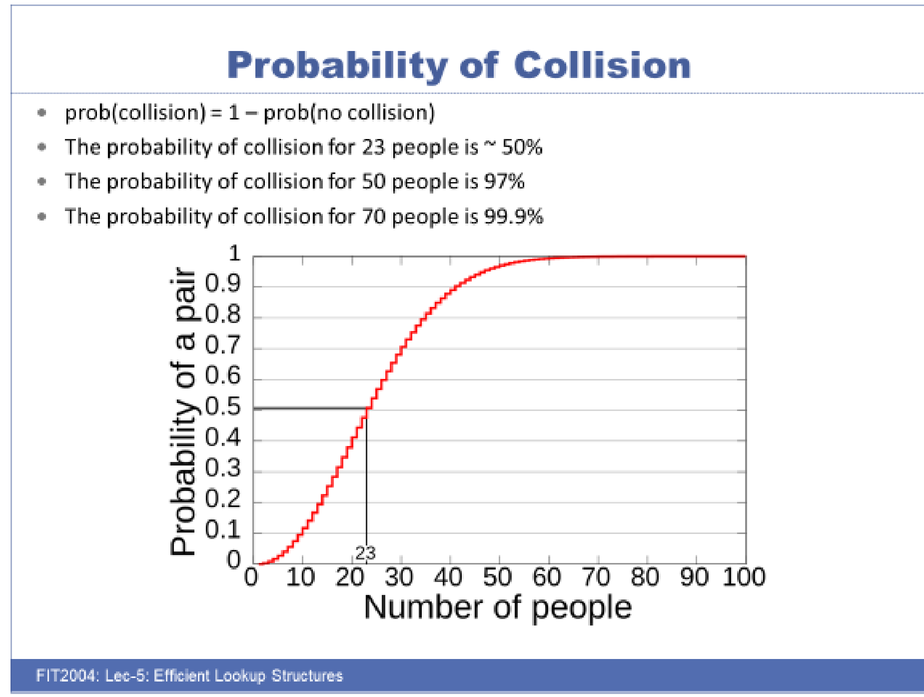

- So we see a collision…
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  - Good hash function
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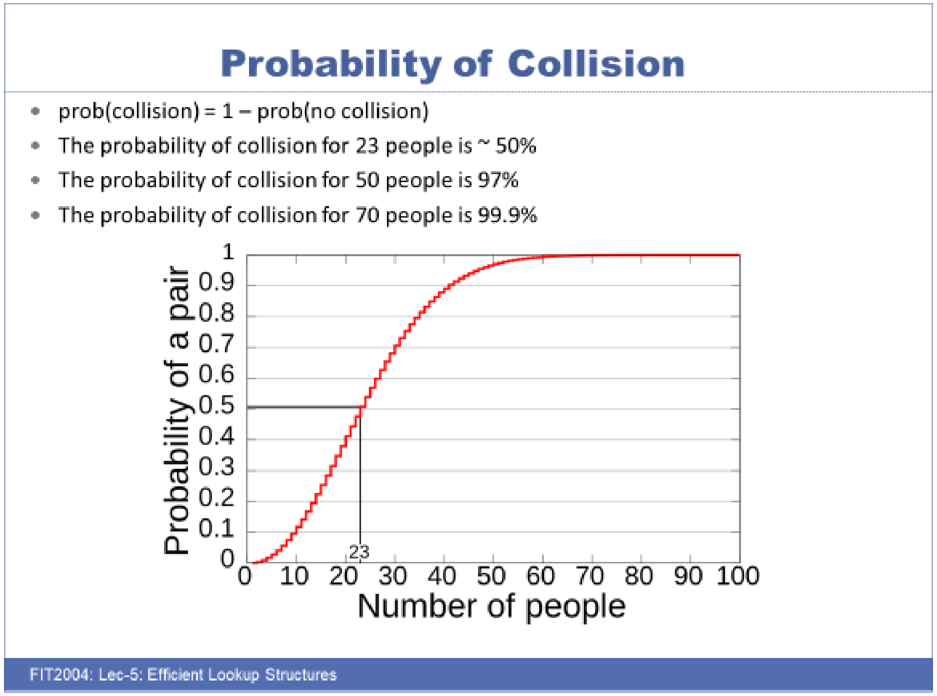
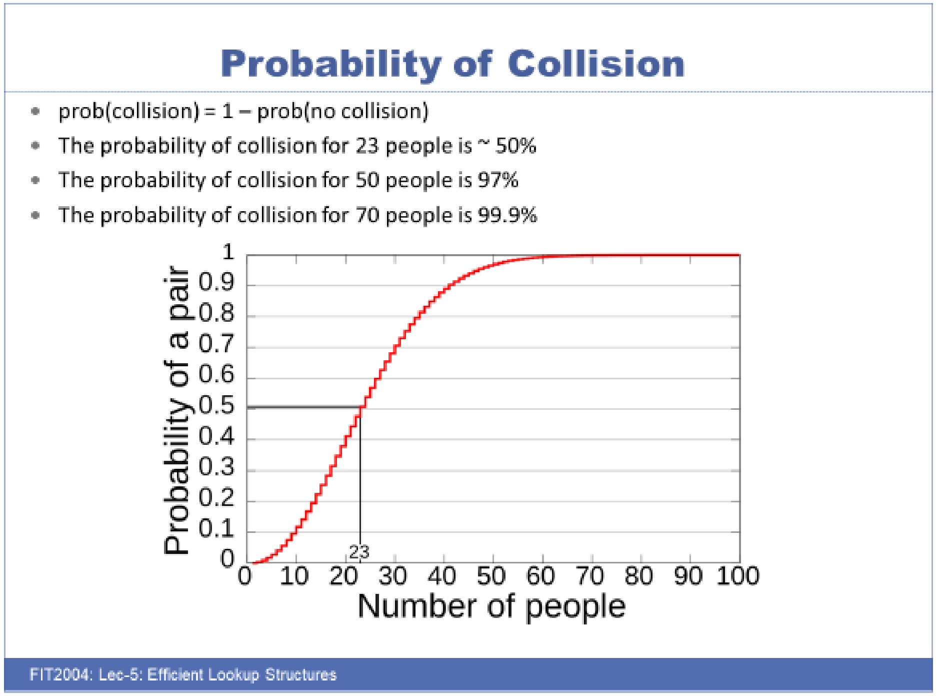
  - Unless our array is very very very very very big…



Hash Table
Avoiding Collisions

- So we see a collision…
- Can we avoid it?
  – Good hash function
  – Kinda too hard…
  – Unless our array is very very very very very big…
    M >> N
    size of arrya much bigger the size of input

Perfect hash
different key always give different index
need to all of items in advanced



Questions?

- We learnt from FIT1008 before

- Open addressing
- Separate chaining

- Open addressing
  - Linear probe
  - Quadratic probe
  - Double hashing
- Separate chaining

Hash Table

Collision resolution

- Open addressing aka closed hashing
  - Linear probe
  - Quadratic probe
  - Double hashing
- Separate chaining aka closed addressing

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Collision resolution

- Open addressing aka closed hashing
  - Linear probe
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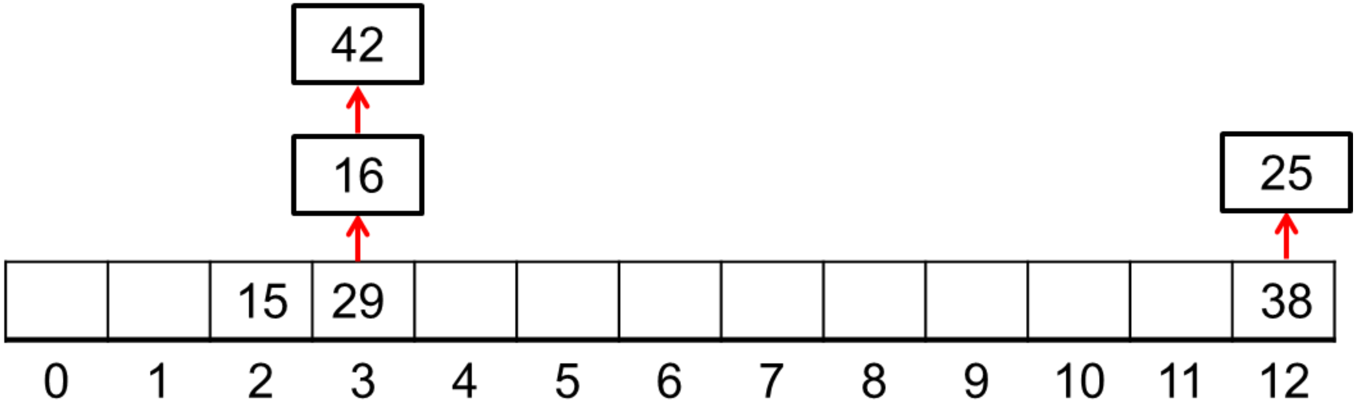
  - Double hashing
  - Cuckoo hashing
- Separate chaining aka closed addressing

- Just use **open addressing** and **separate chaining**

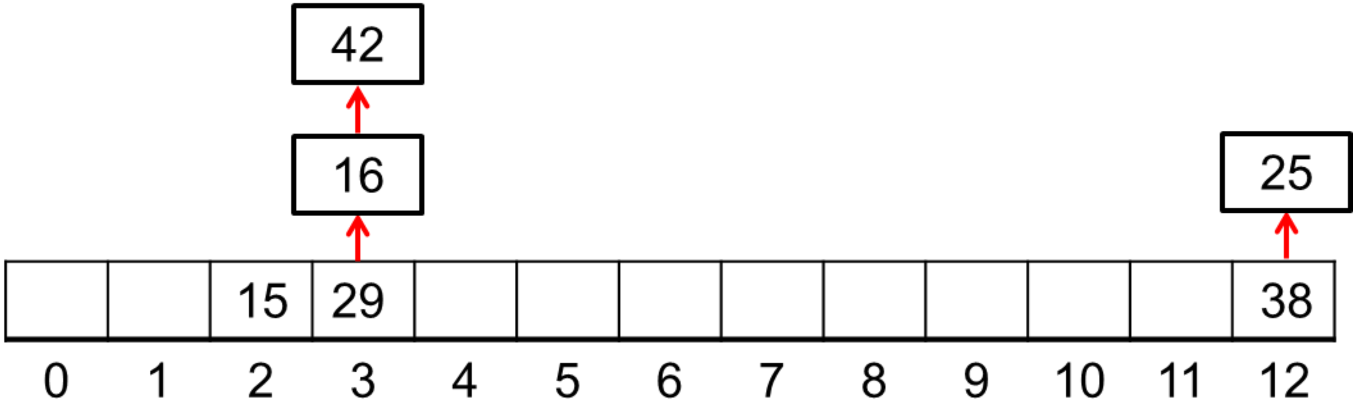
Questions?

- Separate chaining
  - Straight forward

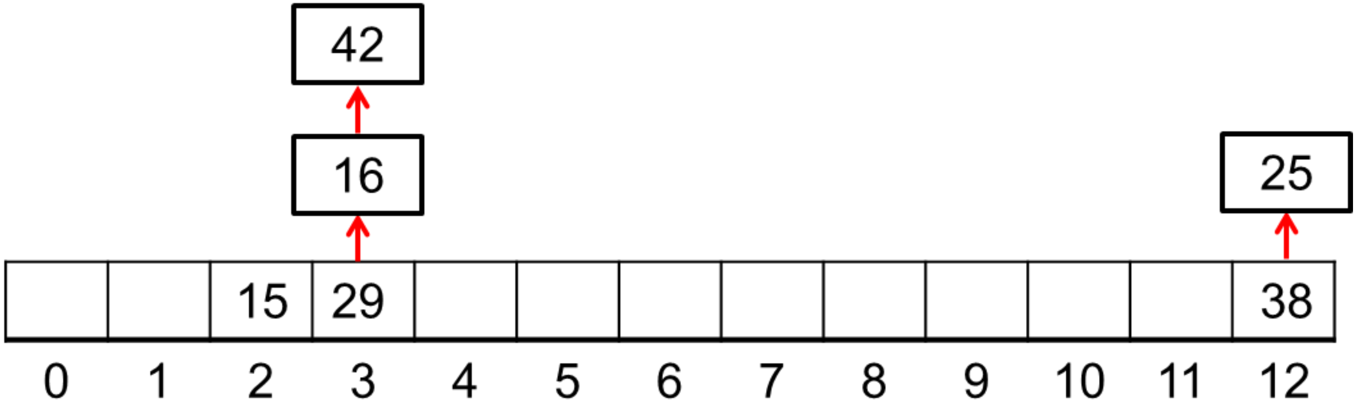


Hash Table
Collision resolution

- Separate chaining
  - Straight forward
  - Complexity to travel through the chain for operations

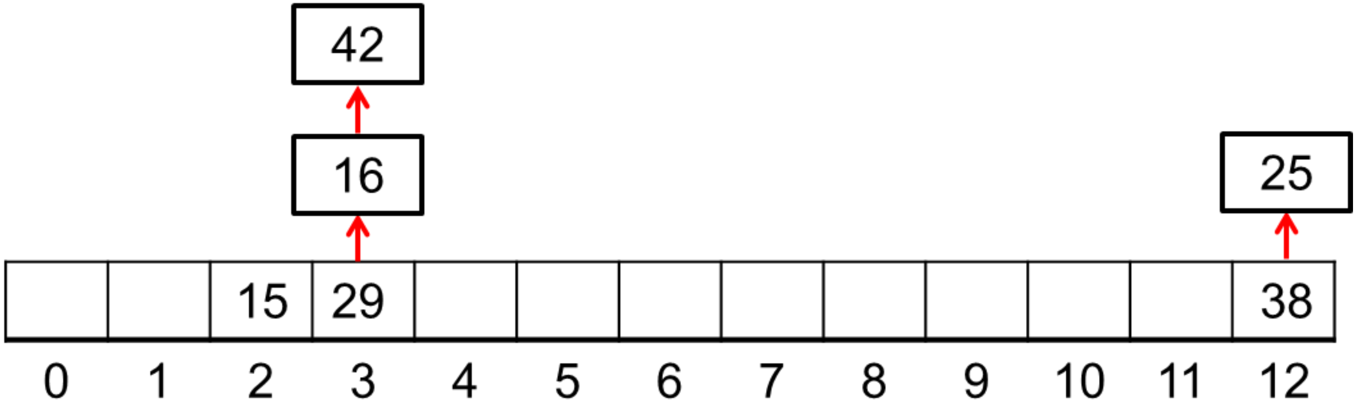


- Separate chaining
  - Straight forward
  - Complexity to travel through the chain for operations
  - The chain can be anything!



Hash Table
Collision resolution

- Separate chaining
  - Straight forward
  - Complexity to travel through the chain for operations
  - The chain can be anything!
    - List
    - Another hash table
    - Tree



Hash Table

Collision resolution

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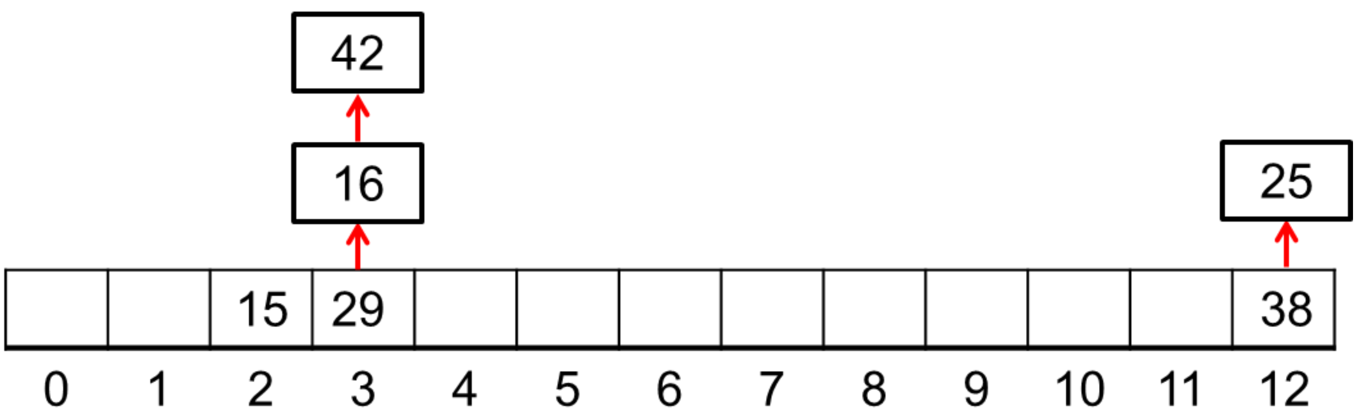
- Separate chaining
  - Straight forward
  - Complexity to travel through the chain for operations
  - The chain can be anything!
    - List
    - Another hash table
    - Tree tree insertion complexity ; O(N)

And many more!



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Questions?

- Open addressing
  - Linear probe
  - Quadratic probe

Hash Table
Collision resolution

- Open addressing
  - Linear probe
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  - Both from FIT1008

Hash Table

Collision resolution

- Open addressing
  - Linear probe
  - Quadratic probe
  - Both from FIT1008
  - Complexity is travelling through the probe length…
    - Primary clustering
    - Secondary clustering

- Open addressing
  - Linear probe
  - Quadratic probe
  - Both from FIT1008
  - Complexity is travelling through the probe length…
    - Primary clustering
    - Secondary clustering
    - Might need to resize
      (especially for quadratic probing as we might not find empty space)

- Open addressing
  - Linear probe +1, +2, +3
  - Quadratic probe
  - Double hashing +1. +4, +9 from original position
  - Complexity is travelling through the probe length…
    - Primary clustering
    - Secondary clustering
    - Might need to resize
      (especially for quadratic probing as we might not find empty space)

- Open addressing
  - Linear probe
  - Quadratic probe
  - Double hashing
    - 2nd hash function to determine the jumps/ probe
    - Instead of fixed
  - Complexity is travelling through the probe length…
    - Primary clustering
    - Secondary clustering
    - Might need to resize
      (especially for quadratic probing as we might not find empty space)

Questions?

- Now this is something new…

Hash Table

Cuckoo hashing

- Now this is something new…
  - In double hashing, we use a 2nd hash function

Cuckoo hashing

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  – In double hashing, we use a 2^{nd} hash function
  – For cuckoo hashing, we have a 2^{nd} hash table!

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  - In double hashing, we use a 2nd hash function
  - For cuckoo hashing, we have a 2nd hash table!
  - Items are kicked between tables when there is collision
    - From 1st table to 2nd table
    - From 2nd table to 1st table

- Now this is something new…
  - In double hashing, we use a 2nd hash function
  - For cuckoo hashing, we have a 2nd hash table!
  - Items are kicked between tables when there is collision
    - From 1st table to 2nd table
    - From 2nd table to 1st table two table, two different size
    - Let us look at an example (more to come in the tutorial)

- Let us try to insert the following keys…

- Let us try to insert the following keys…
  - 23
  - 36
  - 114
  - 49

- Let us try to insert the following keys…
  - 23
  - 36
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  - 49
- Into the following tables…

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- Into the following tables…
  - Table 01, of size 13

- Let us try to insert the following keys…
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- Into the following tables…
  - Table 01, of size 13
  - Table 02, of size 7

prime number

- Let us try to insert the following keys…
  - 23
  - 36
  - 114
  - 49
- Into the following tables…
  - Table 01, of size 13
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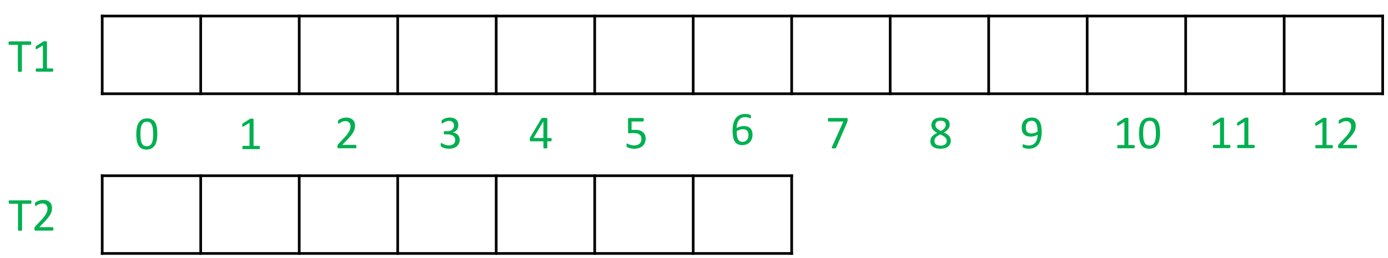

Hash Table

Cuckoo hashing

- Let us try to insert the following keys…
  - 23
  - 36
  - 114
  - 49
- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

T1

0	1	2	3	4	5	6	7	8	9	10	11	12

T2

Hash Table

Cuckoo hashing

- Insert 23
  - Hash1(23) =

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

T1													
	0	1	2	3	4	5	6	7	8	9	10	11	12
T2													

Hash Table
Cuckoo hashing

- Insert 23
  - Hash1(23) = 10 #QuikMath

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
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T1

0	1	2	3	4	5	6	7	8	9	10	11	12

T2

Hash Table

Cuckoo hashing

- Insert 23
  - Hash1(23) = 10

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1													
T2													

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  - Hash1(23) = 10

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
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	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2													

Hash Table
Cuckoo hashing

- Insert 36
  - Hash1(36) = 10

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2													

- Insert 36
  - Hash1(36) = 10 … we have collision

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
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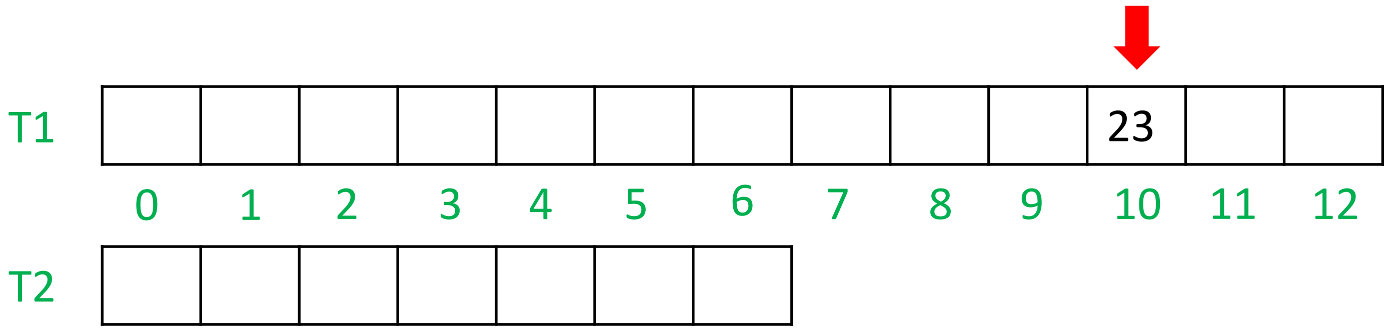
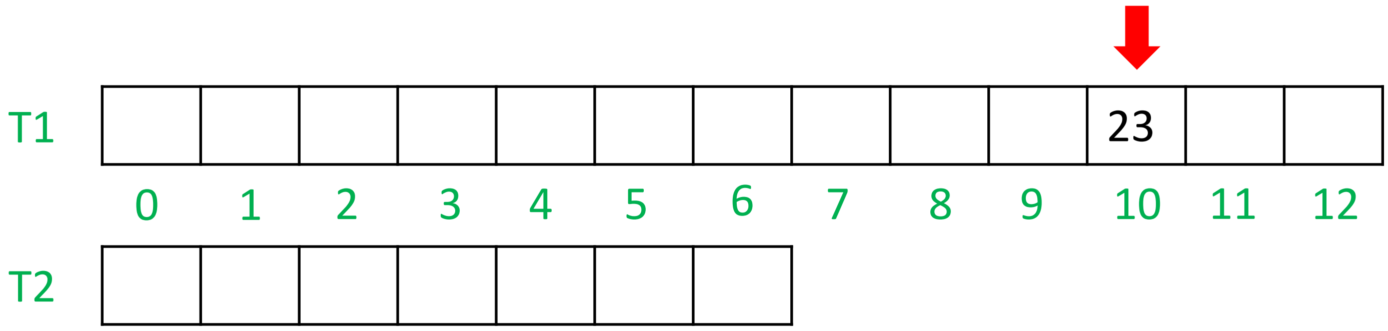


Hash Table
Cuckoo hashing

- Insert 36
  - Hash1(36) = 10 … we have collision

- Into the following tables…
  - Table 01, of size 13   . Hash1(key) = key % 13
  - Table 02, of size 7   . Hash2(key) = key % 7



- Insert 36
  - Hash1(36) = 10 … we have collision, so we kick 23 for 36

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

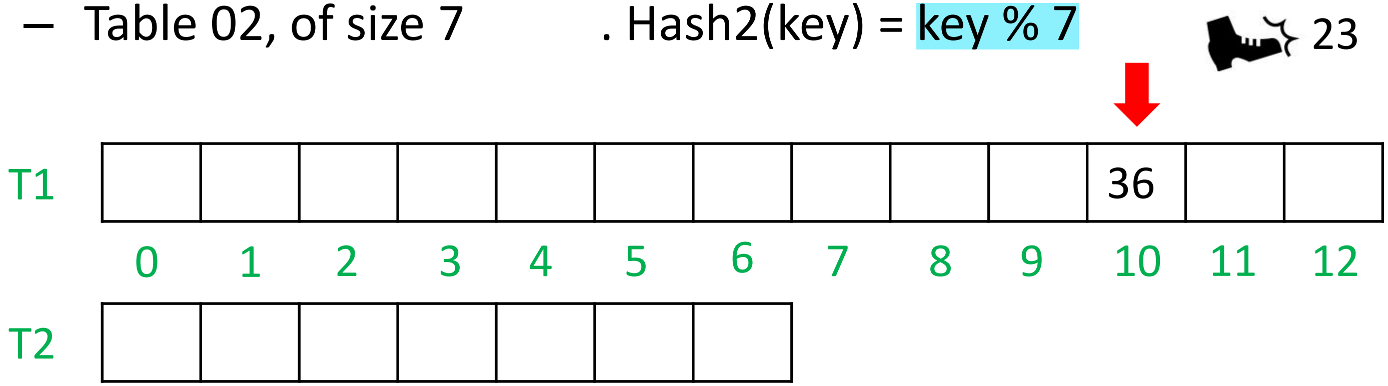
23

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											36		

T2							

- Insert 36
  - Hash1(36) = 10 … we have collision, so we kick 23 for 36
  - Hash2(23) = 2

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7



- Insert 36
  - Hash1(36) = 10 … we have collision, so we kick 23 for 36
  - Hash2(23) = 2

- Into the following tables…
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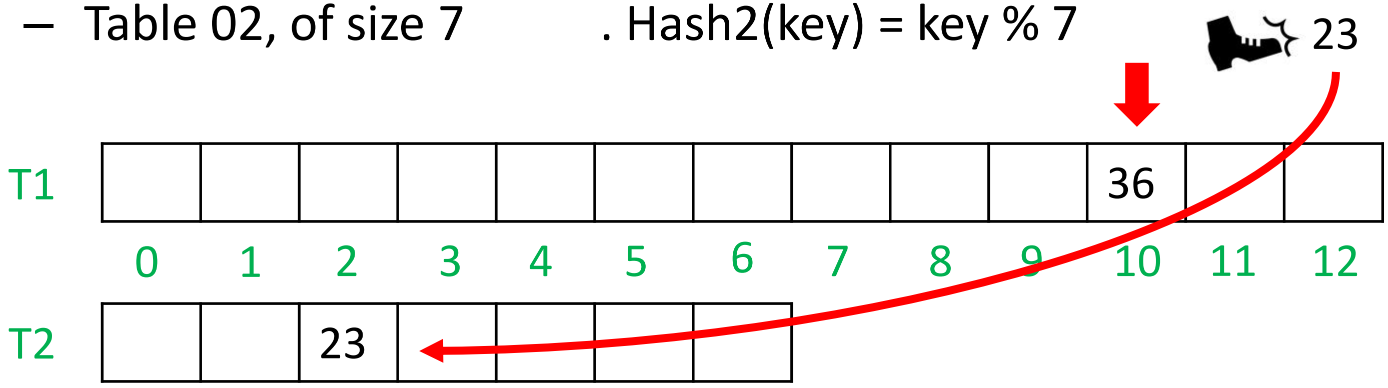
  - Table 02, of size 7 . Hash2(key) = key % 7



Hash Table

Cuckoo hashing

- Insert 36
  - Hash1(36) = 10 … we have collision, so we kick 23 for 36
  - Hash2(23) = 2

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

T1										36			
	0	1	2	3	4	5	6	7	8	9	10	11	12
T2			23										

- Insert 114
  - Hash1(114) = 10 … we have collision

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

T1										36			
	0	1	2	3	4	5	6	7	8	9	10	11	12
T2			23										

Hash Table

Cuckoo hashing

- Insert 114
  - Hash1(114) = 10 … we have collision, so we kick 36 for 114

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

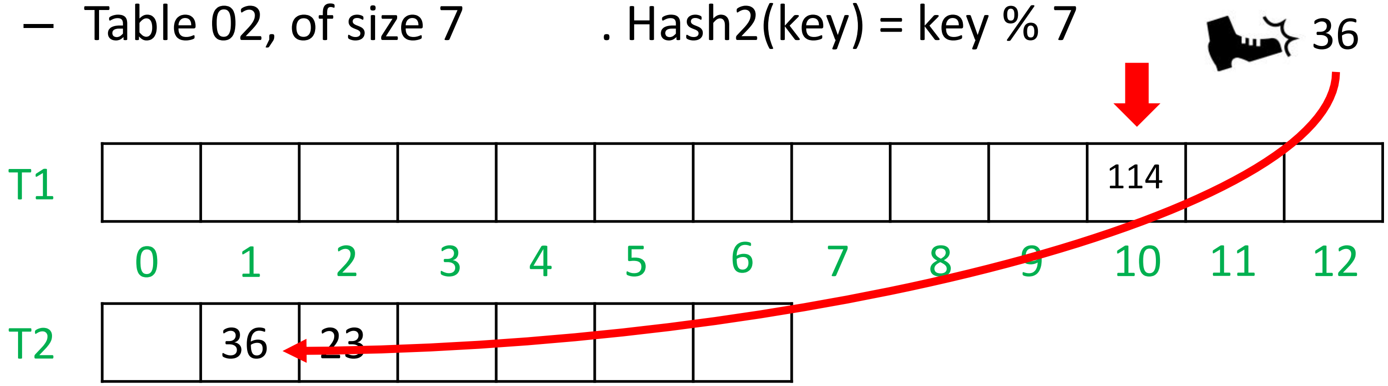

36

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											114		

	0	1	2	3	4	5	6
T2			23				

- Insert 114
  - Hash1(114) = 10 … we have collision, so we kick 36 for 114
  - Hash2(36) = 1

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7



Hash Table

Cuckoo hashing

- Insert 114
  - Hash1(114) = 10 … we have collision, so we kick 36 for 114
  - Hash2(36) = 1

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

T1										114			
	0	1	2	3	4	5	6	7	8	9	10	11	12
T2		36	23										

Cuckoo hashing

- Insert 49
  - Hash1(49) = 10

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

T1

0	1	2	3	4	5	6	7	8	9	10	11	12
										114		

T2

0	1	2	3	4	5	6
	36	23				

- Insert 49
  - Hash1(49) = 10… collision!

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

T1											114		
	0	1	2	3	4	5	6	7	8	9	10	11	12
T2		36	23										

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- Insert 49
  - Hash1(49) = 10… collision! Kick 114 for 49

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											114		
T2		36	23										

Hash Table

Cuckoo hashing

- Insert 49
  - Hash1(49) = 10… collision! Kick 114 for 49

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											49		
T2		36	23										

Hash Table
Cuckoo hashing

- Insert 49
  - Hash1(49) = 10… collision! Kick 114 for 49
  - Hash2(114) = 2

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

114

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											49		
T2		36	23										

Hash Table
Cuckoo hashing

- Insert 49
  - Hash1(49) = 10… collision! Kick 114 for 49
  - Hash2(114) = 2… collision!

- Into the following tables…
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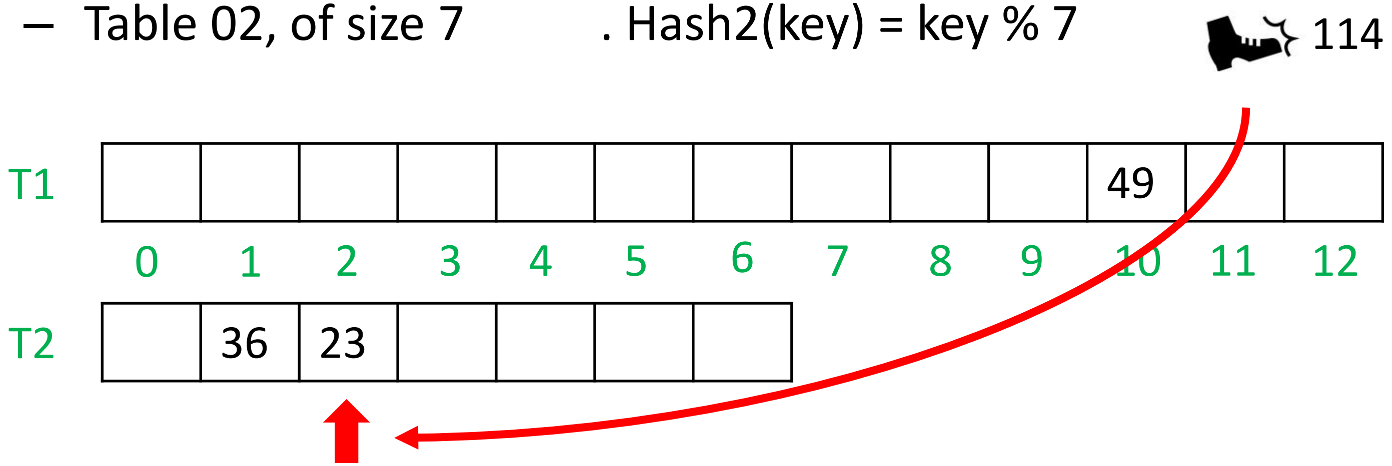
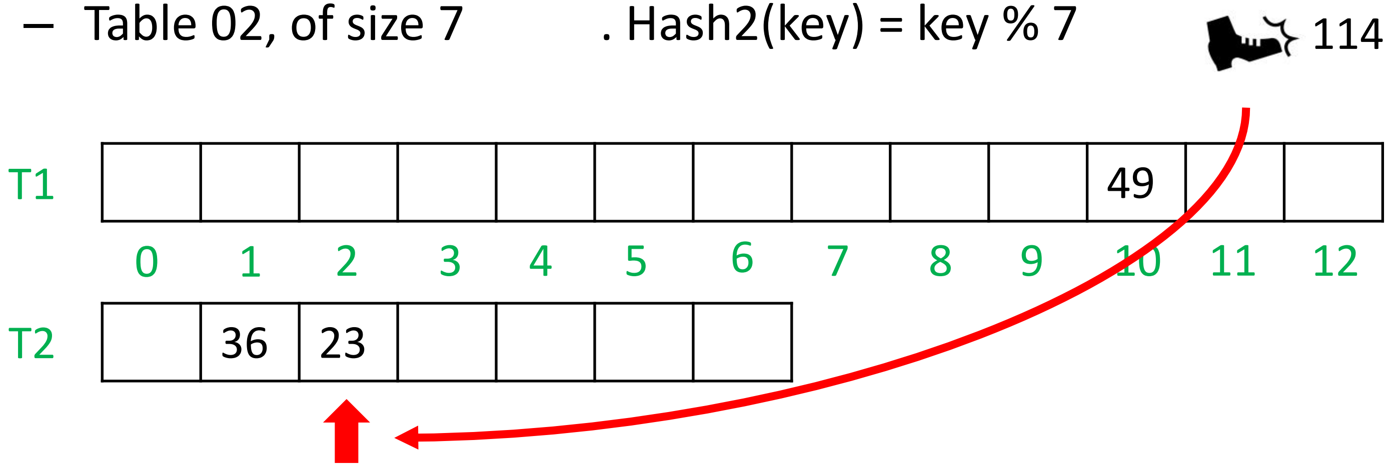
  - Table 02, of size 7 . Hash2(key) = key % 7



- Insert 49
  - Hash1(49) = 10... collision! Kick 114 for 49
  - Hash2(114) = 2... collision! Kick 23 for 114

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7



Hash Table

Cuckoo hashing

- Insert 49
  - Hash1(49) = 10… collision! Kick 114 for 49
  - Hash2(114) = 2… collision! Kick 23 for 114

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

T1

										49		
0	1	2	3	4	5	6	7	8	9	10	11	12

T2

	36	114				

23

Hash Table
Cuckoo hashing

- Insert 49
  - Hash1(49) = 10… collision! Kick 114 for 49
  - Hash2(114) = 2… collision! Kick 23 for 114
  - Hash1(23) = 10

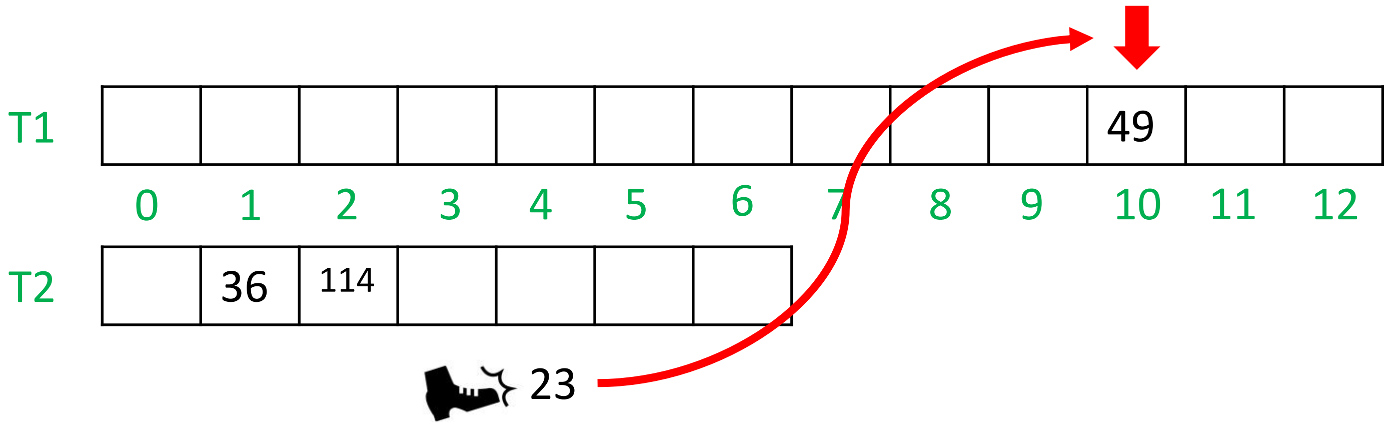
- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											49		
T2		36	114										

23 go to table1

- Insert 49
  - Hash1(49) = 10… collision! Kick 114 for 49
  - Hash2(114) = 2… collision! Kick 23 for 114
  - Hash1(23) = 10… collision!

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

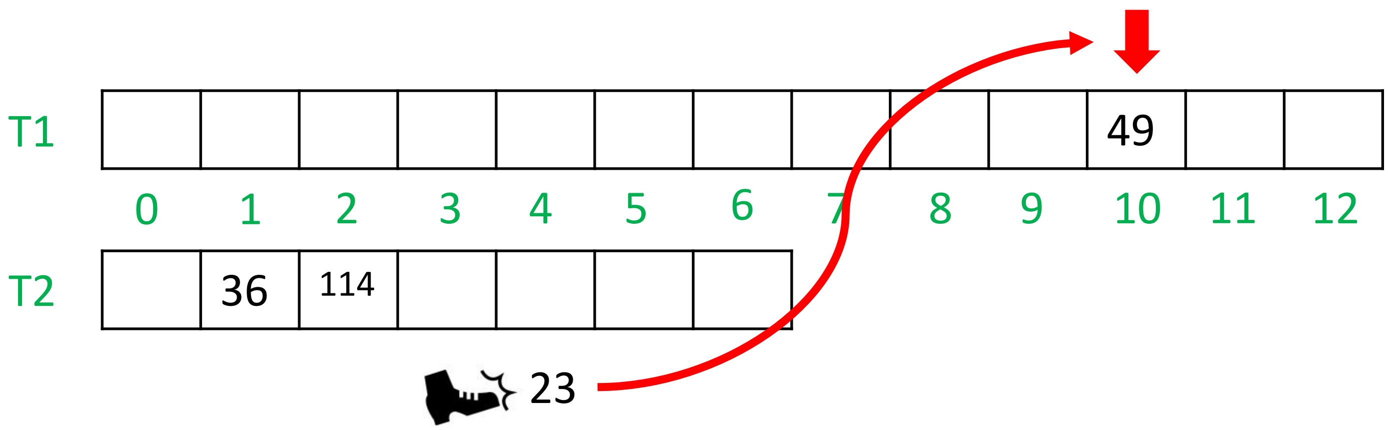


Hash Table

Cuckoo hashing

- Insert 49
  - Hash1(49) = 10… collision! Kick 114 for 49
  - Hash2(114) = 2… collision! Kick 23 for 114
  - Hash1(23) = 10… collision! Kick 49 for 23

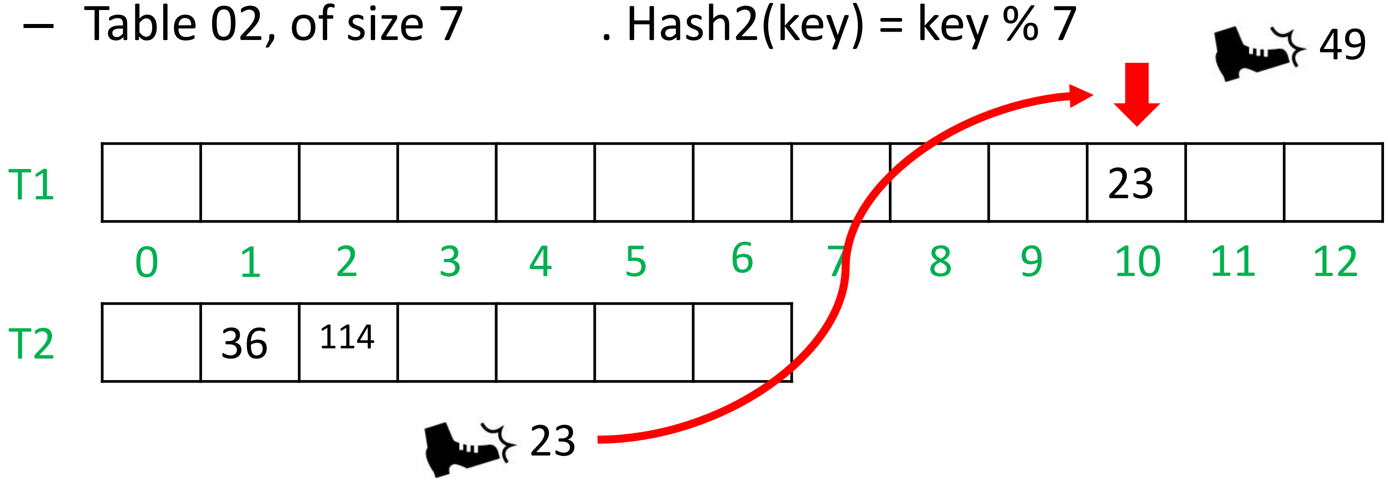
- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7



Hash Table
Cuckoo hashing

- Insert 49
  - Hash1(49) = 10… collision! Kick 114 for 49
  - Hash2(114) = 2… collision! Kick 23 for 114
  - Hash1(23) = 10… collision! Kick 49 for 23

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7



- Insert 49
  - Hash1(49) = 10… collision! Kick 114 for 49
  - Hash2(114) = 2… collision! Kick 23 for 114
  - Hash1(23) = 10… collision! Kick 49 for 23

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2		36	114										

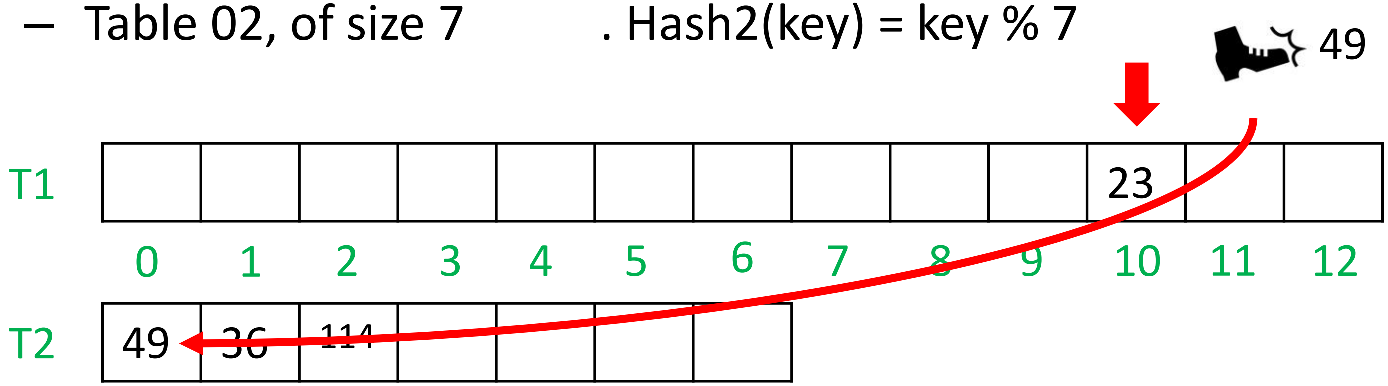
Hash Table
Cuckoo hashing

- Insert 49
  - Hash1(49) = 10… collision! Kick 114 for 49
  - Hash2(114) = 2… collision! Kick 23 for 114
  - Hash1(23) = 10… collision! Kick 49 for 23
  - Hash1(49) = 0
- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2		36	114										

Hash Table
Cuckoo hashing

- Insert 49
  - Hash1(49) = 10… collision! Kick 114 for 49
  - Hash2(114) = 2… collision! Kick 23 for 114
  - Hash1(23) = 10… collision! Kick 49 for 23
  - Hash1(49) = 0
- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7



Hash Table
Cuckoo hashing

- Insert 49
  - Hash1(49) = 10… collision! Kick 114 for 49
  - Hash2(114) = 2… collision! Kick 23 for 114
  - Hash1(23) = 10… collision! Kick 49 for 23
  - Hash1(49) = 0 we r finished!
- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2	49	36	114										

Hash Table
Cuckoo hashing

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- We have inserted all items in…
  - 23
  - 36
  - 114
  - 49
- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2	49	36	114										

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Questions?

Hash Table

Cuckoo hashing

- Now let us try and insert 140

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

T1										23			
	0	1	2	3	4	5	6	7	8	9	10	11	12
T2	49	36	114										

- Now let us try and insert 140
  - Hash1(140) = 10… so we kick 23 out to T2

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											140		
T2	49	36	114										

Hash Table

Cuckoo hashing

- Now let us try and insert 140
  - Hash1(140) = 10… so we kick 23 out to T2
  - Hash2(23) = 2… so we kick 114 out to T1

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											140		
T2	49	36	23										

- Now let us try and insert 140
  - Hash1(140) = 10… so we kick 23 out to T2
  - Hash2(23) = 2… so we kick 114 out to T1
  - Hash1(114) = 10… so we kick 140 out to T2

- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

T1											114		
	0	1	2	3	4	5	6	7	8	9	10	11	12
T2	49	36	23										

Hash Table

Cuckoo hashing

- Now let us try and insert 140
  - Hash1(140) = 10… so we kick 23 out to T2
  - Hash2(23) = 2… so we kick 114 out to T1
  - Hash1(114) = 10… so we kick 140 out to T2
  - Hash2(140) = 0… so we kick 49 out to T1
- Into the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											114		
T2	140	36	23										

- Now let us try and insert 140
  - Hash1(140) = 10… so we kick 23 out to T2
  - Hash2(23) = 2… so we kick 114 out to T1
  - Hash1(114) = 10… so we kick 140 out to T2
  - Hash2(140) = 0… so we kick 49 out to T1
  - Hash1(49) = 10… so we kick 114 out to T2

T1											49		
	0	1	2	3	4	5	6	7	8	9	10	11	12
T2	140	36	23										

Hash Table
Cuckoo hashing

- Now let us try and insert 140
  - Hash1(140) = 10… so we kick 23 out to T2
  - Hash2(23) = 2… so we kick 114 out to T1
  - Hash1(114) = 10… so we kick 140 out to T2
  - Hash2(140) = 0… so we kick 49 out to T1
  - Hash1(49) = 10… so we kick 114 out to T2
  - Hash2(114) = 2… so we kick 23 out to T1

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											49		
T2	140	36	114										

Hash Table

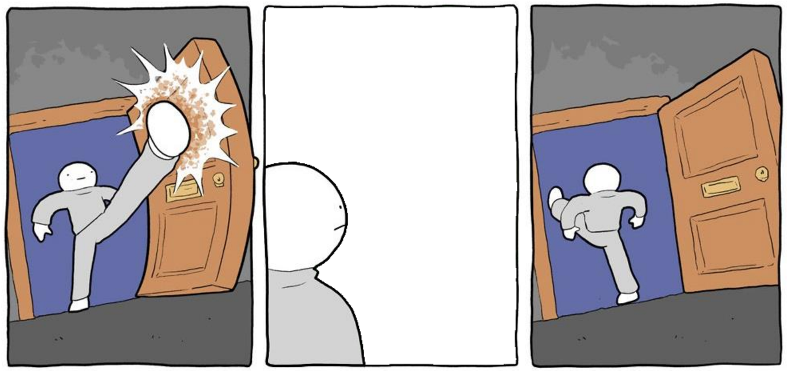
Cuckoo hashing

- Now let us try and insert 140
  - Hash1(140) = 10… so we kick 23 out to T2
  - Hash2(23) = 2… so we kick 114 out to T1
  - Hash1(114) = 10… so we kick 140 out to T2
  - Hash2(140) = 0… so we kick 49 out to T1
  - Hash1(49) = 10… so we kick 114 out to T2
  - Hash2(114) = 2… so we kick 23 out to T1
  - Hash1(23) = 10… so we kick….

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											49		
T2	140	36	114										

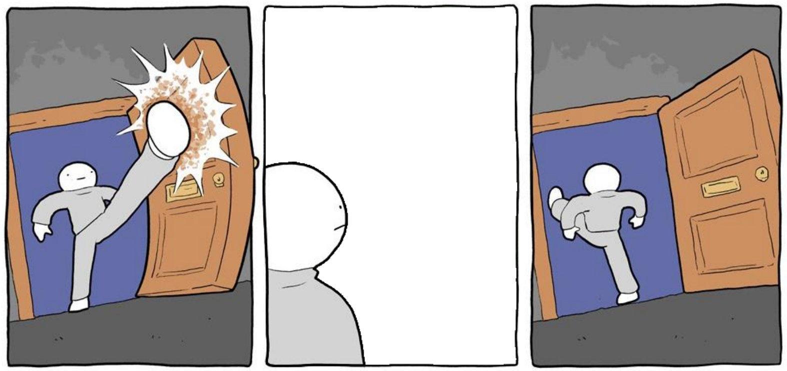
Cuckoo hashing

- It can be a lot of kicking…



T1										49			
	0	1	2	3	4	5	6	7	8	9	10	11	12
T2	140	36	114										

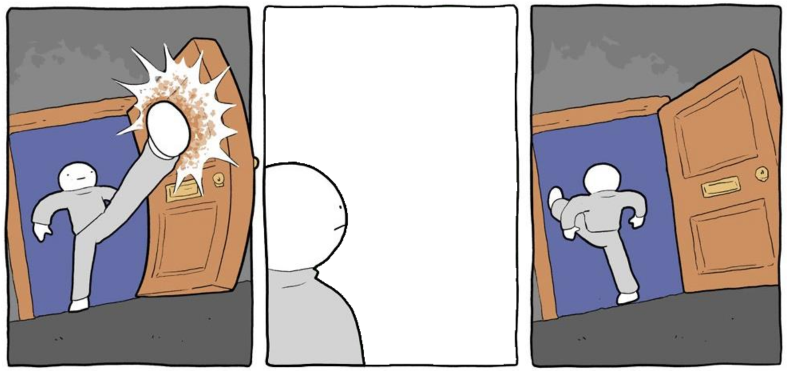
Hash Table
Cuckoo hashing



- It can be a lot of kicking…
  - Can even be infinite!

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											49		
T2	140	36	114										

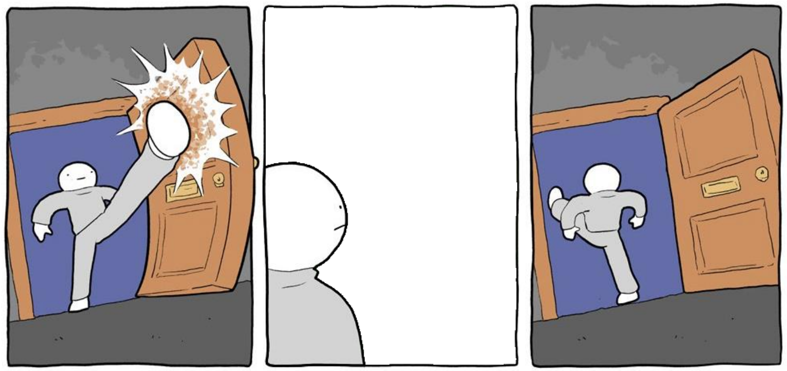
- It can be a lot of kicking…
  - Can even be infinite!
  - So how?



	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											49		
T2	140	36	114										

Hash Table
Cuckoo hashing

- It can be a lot of kicking…
  - Can even be infinite!
  - So how?



- Set a counter…
  - If count_kick >= max_kick: resize!

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											49		
T2	140	36	114										

Hash Table

Cuckoo hashing

- Complexity?

resize

13 * 2 =26
next prime = 29

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											49		

	0	1	2	3	4	5	6
T2	140	36	114				

7 * 2 = 14, next prime = 17

Hash Table
Cuckoo hashing

- Complexity?
  - Insert can be O(1) when the T1[Hash1(key)] is empty

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											49		
T2	140	36	114										

- Complexity?
  - Insert can be O(1) when the T1[Hash1(key)] is empty
  - Else we need to keep kicking…

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											49		
T2	140	36	114										

Hash Table
Cuckoo hashing

- Complexity?
  – Insert can be O(1) when the T1[Hash1(key)] is empty
  – Else we need to keep kicking…
    - Up to (K) where K is the maximum kick possible

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											49		
T2	140	36	114										

Hash Table
Cuckoo hashing

- Complexity?
  - Insert can be O(1) when the T1[Hash1(key)] is empty
  - Else we need to keep kicking…
    - Up to (K) where K is the maximum kick possible
  - If after K-kicks, still can't put key…
    - Resize!

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											49		
T2	140	36	114										

Hash Table
Cuckoo hashing

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- Complexity?
  - Insert can be O(1) when the T1[Hash1(key)] is empty
  - Else we need to keep kicking…
    - Up to (K) where K is the maximum kick possible
  - If after K-kicks, still can’t put key…
    - Resize!
    - O(N) to resize one of the tables
    - Where N is the number of keys in the selected table

can resize both table, if memory small, resize smaller table

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											49		
T2	140	36	114										

Questions?

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- Searching

- Using the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2	49	36	114										

Hash Table
Cuckoo hashing

- Searching
  - The item can only appear in either T1 or T2…

Linear Probing O(N), N = probe length

- Using the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2	49	36	114										

Hash Table
Cuckoo hashing

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- Searching
  - The item can only appear in either T1 or T2…
  - So we only need to hash each item a maximum of 2-times
    - Hash for T1 if not linear probing, just Cuckoo hashing seaching
    - If not in T1, hash for T2
- Using the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2	49	36	114										

Hash Table

Cuckoo hashing

- Searching for key 36

- Using the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2	49	36	114										

Hash Table

Cuckoo hashing

- Searching for key 36
  - Hash1(36) = 10

- Using the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

T1											23		
	0	1	2	3	4	5	6	7	8	9	10	11	12
T2	49	36	114										

- Searching for key 36
  - Hash1(36) = 10, T1[10] is not 36

- Using the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2	49	36	114										

- Searching for key 36
  - Hash1(36) = 10, T1[10] is not 36
  - Hash2(36) = 1

- Using the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2	49	36	114										

Hash Table
Cuckoo hashing

- Searching for key 36
  - Hash1(36) = 10, T1[10] is not 36
  - Hash2(36) = 1, T1[1] is 36… FOUND!

- Using the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2	49	36	114										

Hash Table
Cuckoo hashing

- Searching for key 10

- Using the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2	49	36	114										

- Searching for key 10
  - Hash1(10) = 10

- Using the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2	49	36	114										

- Searching for key 10
  - Hash1(10) = 10, T1[10] is not 10

- Using the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2	49	36	114										

- Searching for key 10
  - Hash1(10) = 10, T1[10] is not 10
  - Hash2(10) = 3

- Using the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2	49	36	114										

Hash Table
Cuckoo hashing

- Searching for key 10
  - Hash1(10) = 10, T1[10] is not 10
  - Hash2(10) = 3, T2[3] is not 10

- Using the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2	49	36	114										

- Searching for key 10
  - Hash1(10) = 10, T1[10] is not 10
  - Hash2(10) = 3, T2[3] is not 10
  - So key 10 doesn't exist because it is not in T1 or T2

- Using the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2	49	36	114										

Hash Table

Cuckoo hashing

- Guaranteed O(1) complexity!
  - For search

- Using the following tables…
  - Table 01, of size 13 . Hash1(key) = key % 13
  - Table 02, of size 7 . Hash2(key) = key % 7

T1										23			
	0	1	2	3	4	5	6	7	8	9	10	11	12
T2	49	36	114										

Questions?

Hash Table

Cuckoo hashing

- What if we want to delete?

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2	49	36	114										

- What if we want to delete?
  - Same as search to look for key

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2	49	36	114										

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- What if we want to delete?
  - Same as search to look for key
  - If key is found, we delete

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2	49	36	114										

- What if we want to delete?
  - Same as search to look for key
  - If key is found, we delete
    - Removing the item but always search both tables

T1										23			
	0	1	2	3	4	5	6	7	8	9	10	11	12
T2	49	36	114										

Hash Table
Cuckoo hashing

- What if we want to delete?
  - Same as search to look for key
  - If key is found, we delete
    - Removing the key but always search both tables
    - Replace key with FLAG and might not need to search both tables

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2	49	36	114										

Hash Table
Cuckoo hashing

- What if we want to delete?
  - Same as search to look for key
  - If key is found, we delete
    - Removing the key but always search both tables
    - Replace key with FLAG and might not need to search both tables
    - Mainly just implementation decision

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2	49	36	114										

Hash Table
Cuckoo hashing

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- What if we want to delete?
  - Same as search to look for key
  - If key is found, we delete
    - Removing the key but always search both tables
    - Replace key with FLAG and might not need to search both tables
    - Mainly just implementation decision

	0	1	2	3	4	5	6	7	8	9	10	11	12
T1											23		
T2	49	36	114										

Questions?

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- Now this is something new…
  - In double hashing, we use a 2nd hash function
  - For cuckoo hashing, we have a 2nd hash table!
  - Items are kicked between tables when there is collision
    - From 1st table to 2nd table
    - From 2nd table to 1st table

Hash Table
Cuckoo hashing

- Now this is something new…
  - In double hashing, we use a 2nd hash function
  - For cuckoo hashing, we have a 2nd hash table!
  - Items are kicked between tables when there is collision
    - From 1st table to 2nd table
    - From 2nd table to 1st table

  - Ideally, our complexity would be
    - O(1) for insert with 2 good hash functions (as to not resize often)
    - O(1) for search always!
    - O(1) for delete using flags

Questions?

- Hash tables are awesome…

Hash Table

Are great but…

- Hash tables are awesome… but…

Hash Table

Are great but…

- Hash tables are awesome… but…
  - Hard to produce good hash function need to know all items in advanced
    - Even more perfect hash is not possible

Hash Table

Are great but…

- Hash tables are awesome… but…
  - Hard to produce good hash function
    - Even more perfect hash is not possible
  - Data are not ordered
    - So some operations are costly

Hash Table

Are great but…

- Hash tables are awesome… but…
  - Hard to produce good hash function
    - Even more perfect hash is not possible
  - Data are not ordered
    - So some operations are costly
    - What is max?
    - What is min?

Hash Table
Are great but…

- Hash tables are awesome… but…
  - Hard to produce good hash function
    - Even more perfect hash is not possible
  - Data are not ordered Hash and Dictionary are not ordered
    - So some operations are costly
    - What is max?
    - What is min?
- In reality, time is O(1)
  - But we use a lot of space
  - Sparse table for this to happen!

Hash Table

Are great but…

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- Hash tables are awesome… but…
  - Hard to produce good hash function
    - Even more perfect hash is not possible
  - Data are not ordered
    - So some operations are costly not recommended for big company
      tree is recommended
    - What is max?
    - What is min?
- In reality, time is O(1)
  - But we use a lot of space
  - Sparse table for this to happen!
    - Resize when load factor is reached only 33% to 67% then need to resize
    - This resize cost of O(N*loadfactor) is added to the insert cost; but armotized to O(1)… Why?

Complexity of resizing - intuition

- Imagine spreading out the resize work over the insertions
- This concept is called "amortized analysis" (not examinable)

Table size	Total work for insertion	Total work for resize
m	m/2	2m + m/2
2m	m	4m + m
4m	2m	8m + 2m
…	…	…
2^im	2^{i-1}m	2^{i+1}m + (2^{i-1}m)

- The amortized cost of each insert is O(1), even though most of the work occurs on one specific insert (the one which triggers a resize)

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- In reality, time is O(1)
  - But we use a lot of space
  - Sparse table for this to happen!
    - Resize when load factor is reached
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Hash Table

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  - Everyday eat instant noodle $1, then once a week eat fancy $50

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    - Rinse and Repeat!

Questions?

Thank You