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INFORMATION
TECHNOLOGY

FIT2004
Algorithms and Data Structures

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Referencing materials by
Nathan Companez, Aamir Cheema, Arun Konagurthu and Lloyd Allison

GROUP OF EIGHT AUSTRALIA

Faculty of Information Technology, Monash University

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Ready?

Agenda

- Sorting Algorithms
  - Comparison based
    - Selection
    - Insertion
  - Non-comparison based (the IMBA ones)
    - Counting
    - Radix

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Let us begin…

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- We can sort without comparing elements in a list!

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- We can sort without comparing elements in a list!
  - Counting sort
  - Radix sort

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Questions?

Counting Sort

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- Very simple concept
- I am sure we all know this…

- Now let us begin with a list

Counting Sort

MONASH University

- Very simple concept
- I am sure we all know this…

- Now let us begin with a list

- What is the maximum number?

Counting Sort

MONASH University

- Very simple concept
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- Now let us begin with a list

- What is the maximum number?
  - 5 but how do we know?

Counting Sort

MONASH University

- Very simple concept
- I am sure we all know this…

- Now let us begin with a list

- What is the maximum number?
  - 5 but how do we know? Loop through the list in O(N)

Counting Sort

MONASH University

- Our input

- We know max is 5

Counting Sort

MONASH University

- Our input

Anyone noticed
the list is crooked?
#OCDtrigger

- We know max is 5

Counting Sort

MONASH University

- Our input

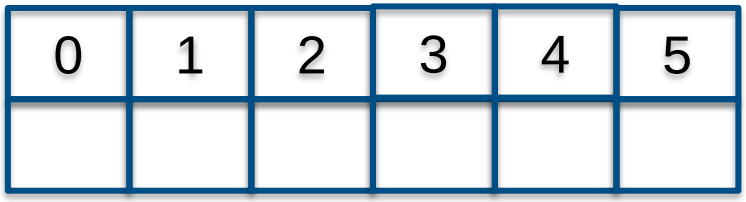
- We know max is 5

Counting Sort

MONASH University

- Our input

- We know max is 5



Counting Sort

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- Out input

4	2	1	3	1	4	5

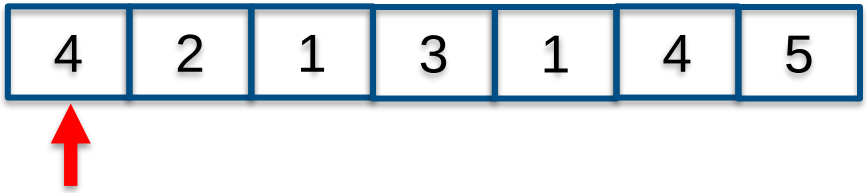
- We know max is 5

0	1	2	3	4	5
0	0	0	0	0	0

Counting Sort

MONASH University

- Our input



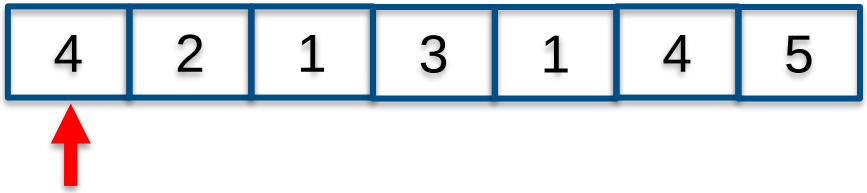
- We know max is 5

0	1	2	3	4	5
0	0	0	0	0	0

Counting Sort

MONASH University

- Our input



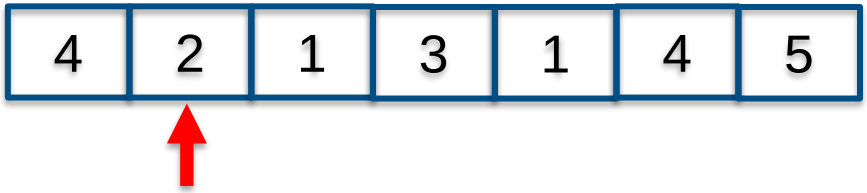
- We know max is 5

0	1	2	3	4	5
0	0	0	0	1	0

Counting Sort

MONASH University

- Our input



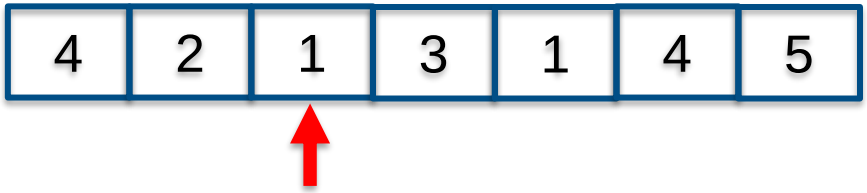
- We know max is 5

0	1	2	3	4	5
0	0	1	0	1	0

Counting Sort

MONASH University

- Out input



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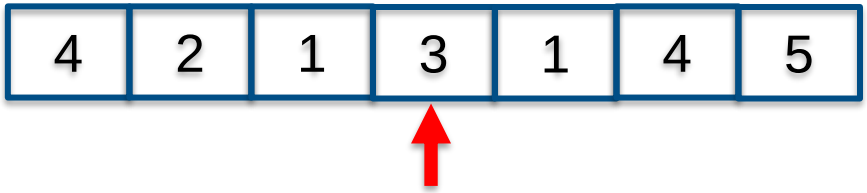
0	1	2	3	4	5
0	1	1	0	1	0

Counting Sort

MONASH University

- Our input



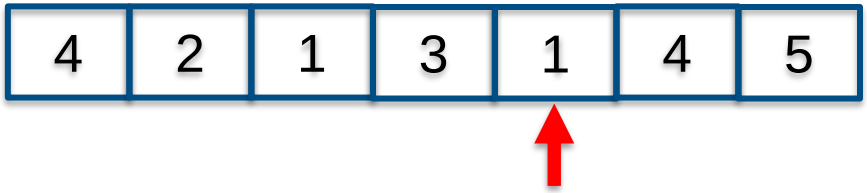
- We know max is 5

0	1	2	3	4	5
0	1	1	1	1	0

Counting Sort

MONASH University

- Our input



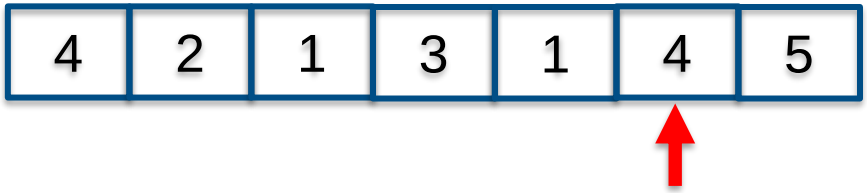
- We know max is 5

0	1	2	3	4	5
0	2	1	1	1	0

Counting Sort

MONASH University

- Our input



- We know max is 5

0	1	2	3	4	5
0	2	1	1	2	0

Counting Sort

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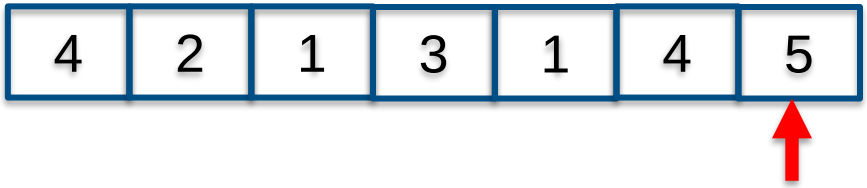
- Our input



- We know max is 5

0	1	2	3	4	5
0	2	1	1	2	1

Counting Sort

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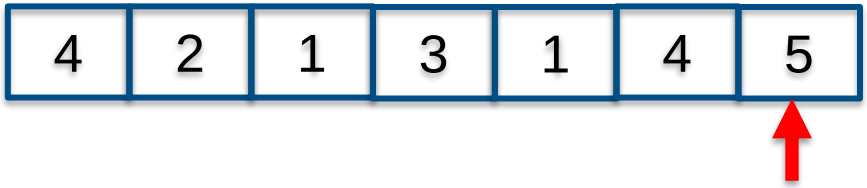
- Our input



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0	1	2	3	4	5
0	2	1	1	2	1

ItemID

Frequency

Counting Sort

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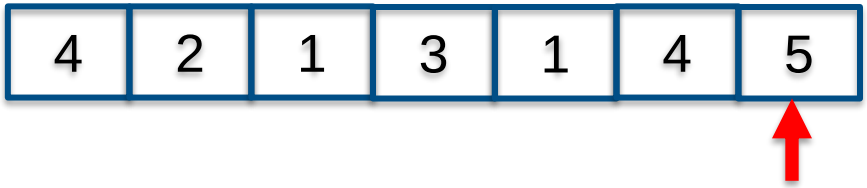
- Our input



- We know max is 5

0	1	2	3	4	5	ItemID
0	2	1	1	2	1	Frequency

- So how do we sort it now then?

Counting Sort

MONASH University

- Our input

- We know max is 5

0	1	2	3	4	5	ItemID
0	2	1	1	2	1	Frequency

- So how do we sort it now then?

Counting Sort

MONASH University

- Our input

- We know max is 5

0	1	2	3	4	5
0	2	1	1	2	1

ItemID

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Counting Sort

MONASH University

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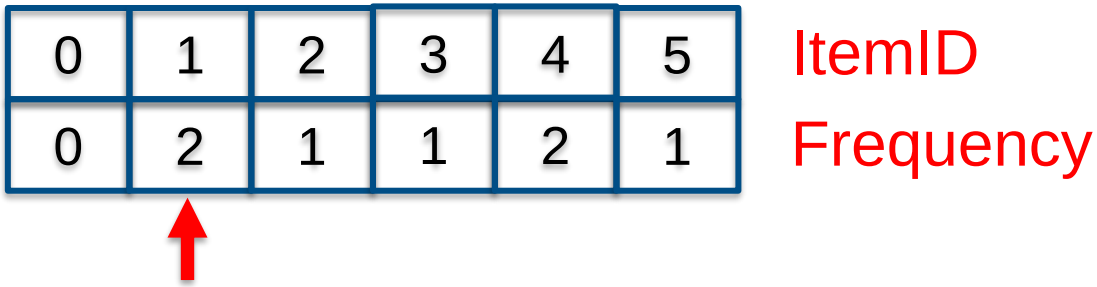
Counting Sort

MONASH University

- Our input

- We know max is 5



- So how do we sort it now then?

Counting Sort

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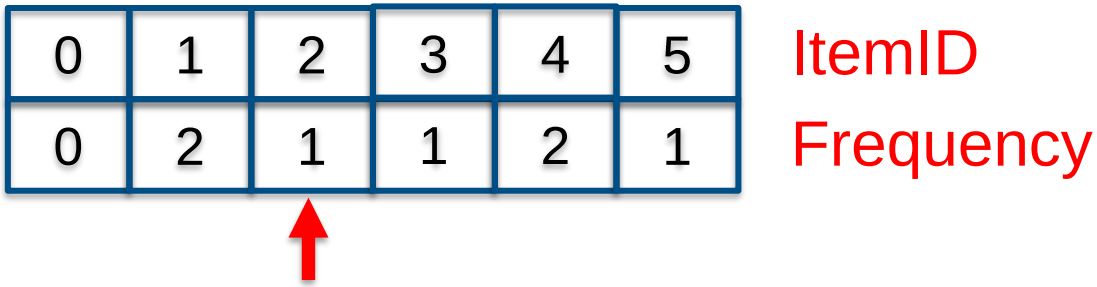
- Our input

- We know max is 5



- So how do we sort it now then?

Counting Sort

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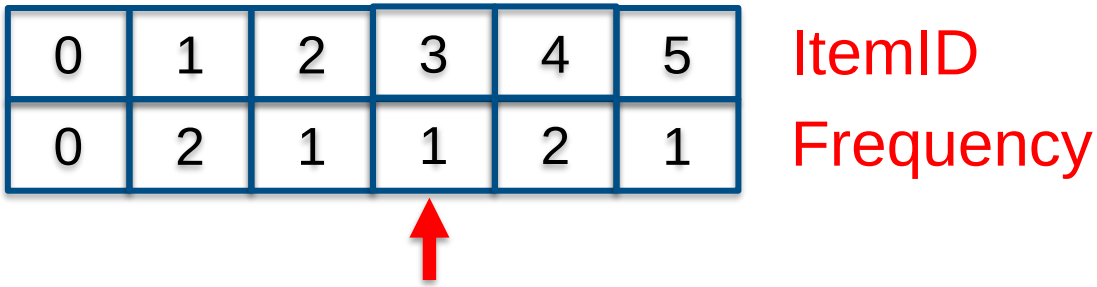
- Our input

- We know max is 5



- So how do we sort it now then?

Counting Sort

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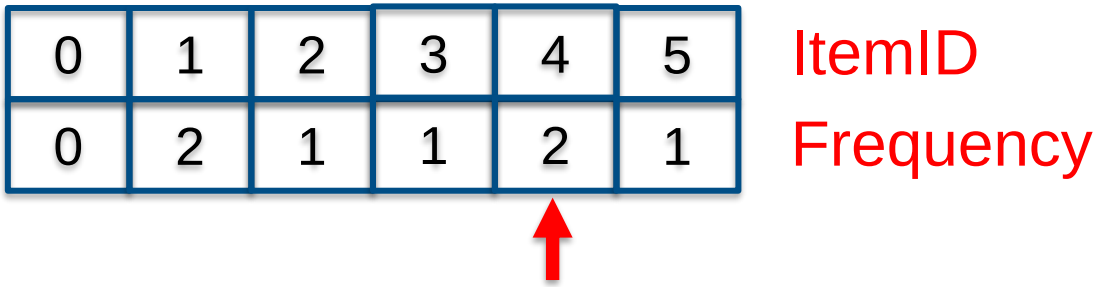
- Our input

- We know max is 5



- So how do we sort it now then?

Counting Sort

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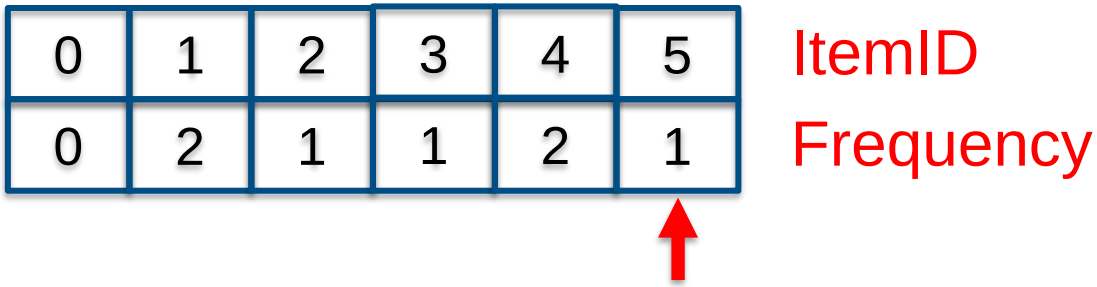
- Our input

- We know max is 5



- So how do we sort it now then?

Counting Sort

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- Our input

1	1	2	3	4	4	5

- We know max is 5

0	1	2	3	4	5	ItemID
0	2	1	1	2	1	Frequency

- So how do we sort it now then?

Counting Sort
Complexity

- Time?

Counting Sort
Complexity

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- Time?
  - Find the maximum O(N)

Counting Sort
Complexity

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- Time?
  - Find the maximum O(N)
  - Build the count-array O(M) where M is the max

Counting Sort
Complexity

MONASH University

- Time?
  - Find the maximum O(N)
  - Build the count-array O(M) where M is the max
  - Go through input list and update the count-array

Counting Sort
Complexity

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- Time?
  - Find the maximum O(N)
  - Build the count-array O(M) where M is the max
  - Go through input list and update the count-array
    - How to make it fast?

Counting Sort
Complexity

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- Time?
  - Find the maximum O(N)
  - Build the count-array O(M) where M is the max
  - Go through input list and update the count-array
    - How to make it fast?

0	1	2	3	4	5	Index
0	2	1	1	2	1	Frequency

Counting Sort
Complexity

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- Time?
  - Find the maximum O(N)
  - Build the count-array O(M) where M is the max
  - Go through input list and update the count-array
    - How to make it fast?
    - Therefore this is O(N) since we can have O(1) access to the count-array

Counting Sort
Complexity

MONASH University

- Time?
  - Find the maximum O(N)
  - Build the count-array O(M) where M is the max
  - Go through input list and update the count-array
    - How to make it fast?
    - Therefore this is O(N) since we can have O(1) access to the count-array
  - Loop through count-array to rebuild the original list O(M+N)

Counting Sort
Complexity

MONASH University

- Time?
  - Find the maximum O(N)
  - Build the count-array O(M) where M is the max
  - Go through input list and update the count-array
    - How to make it fast?
    - Therefore this is O(N) since we can have O(1) access to the count-array
  - Loop through count-array to rebuild the original list O(M)
  - Total = O(N + M + N + M + N) = O(N+M)

Counting Sort
Complexity

MONASH University

- Time?
  - Find the maximum O(N)
  - Build the count-array O(M) where M is the max
  - Go through input list and update the count-array
    - How to make it fast?
    - Therefore this is O(N) since we can have O(1) access to the count-array
  - Loop through count-array to rebuild the original list O(M)
  - Total = O(N + M + N + M + N) = O(N+M)

  - So we want M << N for this to be good
    - Else even N log N < M

Counting Sort
Complexity

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- Time?
  - Find the maximum O(N)
  - Build the count-array O(M) where M is the max
  - Go through input list and update the count-array
    - How to make it fast?
    - Therefore this is O(N) since we can have O(1) access to the count-array
  - Loop through count-array to rebuild the original list O(M)
  - Total = O(N + M + N + M + N) = O(N+M)

    last N to update the original list
  - So we want M << N for this to be good
  - If we are doing alphabets only, then the M = 26 for the 26 character (after ascii conversion + maths)

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Questions?

Counting Sort
Complexity

- Space?

Counting Sort
Complexity

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- Space?
  - Input list O(N)
  - Count-array O(M)

Counting Sort
Complexity

MONASH University

- Space?
  - Input list O(N)
  - Count-array O(M)

  - Total = O(N + M)
  - Auxiliary = O(M)

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Questions?

Counting Sort

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- Live programming session
- Let us try to code this since it is simple…

Counting Sort

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- Live programming session
- Let us try to code this since it is simple…

- I will start writing the first part
  - You try to add in your own codes and compare at each step

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Questions?

Counting Sort

Issue…

- Now imagine the following:

Counting Sort

Issue…

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- Now imagine the following:

  - What is my complexity?

Counting Sort

Issue…

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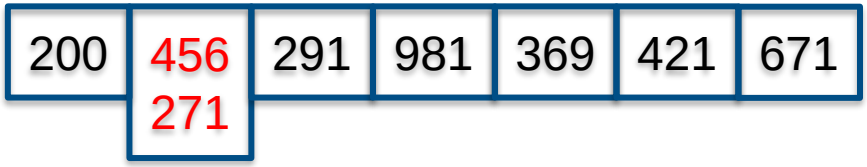
- Now imagine the following:

  - What is my complexity?
    - Time…
    - Space…

Counting Sort

Issue…

- Now imagine the following:



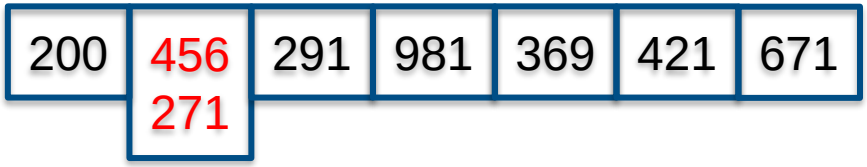
  - What is my complexity?
    - Time…
    - Space…

  - What if one of the value is LARGE

Counting Sort

Issue…

- Now imagine the following:



  - What is my complexity?
    - Time…
    - Space…

  - What if one of the value is LARGE

M is large!!!

Counting Sort

Issue…

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- Now imagine the following:

  - What is my complexity?
    - Time…
    - Space…
  - Let us leave it at it is first…

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Questions?

Counting Sort

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- Stable?

Counting Sort

MONASH University

- Stable?
  - No
  - We only remember the frequency

Counting Sort

MONASH University

- Stable?
  - No
  - We only remember the frequency
- But can we make it stable?

Counting Sort

MONASH University

- Stable? stability: maintain relative order for the equal values
  - No
  - We only remember the frequency
- But can we make it stable?
  - Yes but at the cost of memory

Counting Sort

second 4

4213145 second 4 must always behind first 4 to maintain stability

first 4

the 4a just to show it is first 4

4b to show it is second 4

they are just 4 actually

0	1	2	3	4	5
	1a	2	3	4a	5
	1b			4b	

Index

Frequency

ord("a") to convert character into integer, a = 97

count_array = [None] * (max_item+1)
for i in range(len(count_array)): count_array[i] = []
instantiate all sublists are distinctive (completely different to each other) in a list

Counting Sort

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- Stable?
  - No
  - We only remember the frequency
- But can we make it stable?
  - Yes but at the cost of memory
  - Similar to separate chaining

Counting Sort

MONASH University

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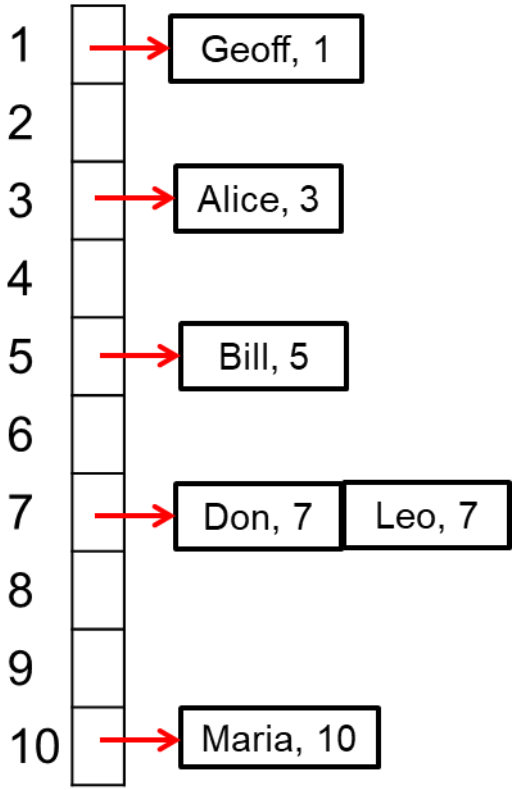

Marks	3	5	7	1	7	10
Name	Alice	Bill	Don	Geoff	Leo	Maria

Counting Sort

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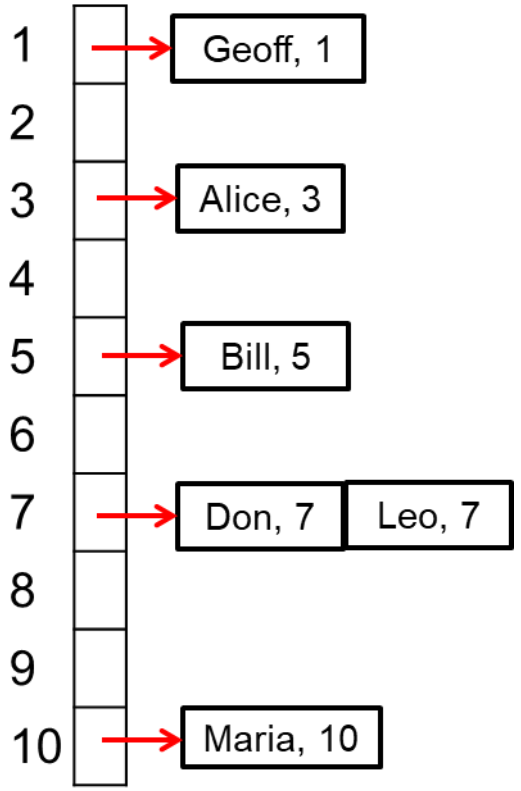
- Stable?
  - No
  - We only remember the frequency
- But can we make it stable?
  - Yes but at the cost of memory
  - Similar to separate chaining
  - At most we have N items only anyways
    - So it is O(M + N) space still



Marks	3	5	7	1	7	10
Name	Alice	Bill	Don	Geoff	Leo	Maria

Counting Sort

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  - At most we have N items only anyways
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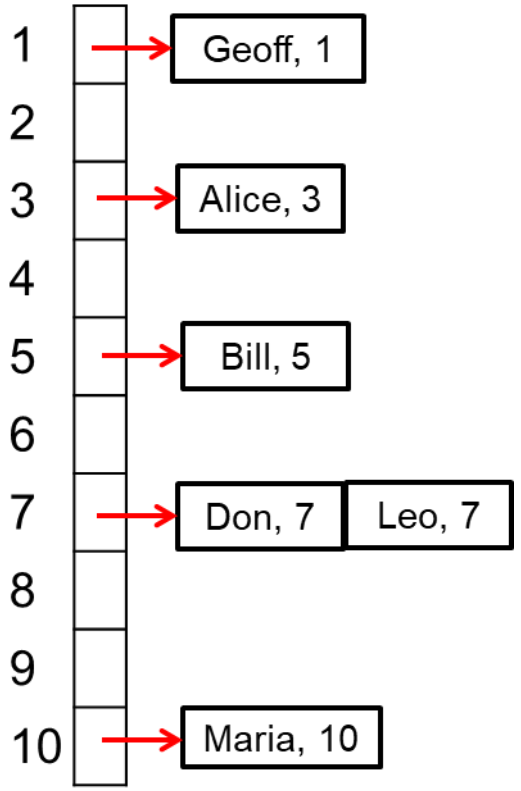
    - Can you see why?



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Counting Sort

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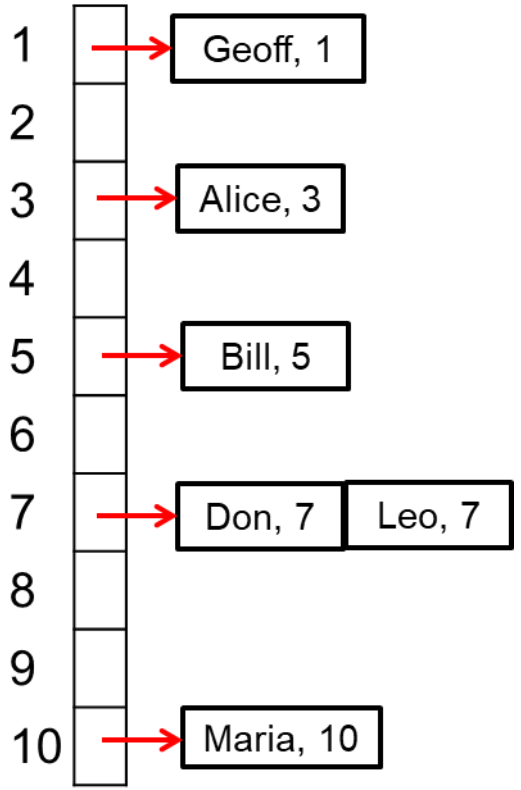

Marks	3	5	7	1	7	10
Name	Alice	Bill	Don	Geoff	Leo	Maria

N items

Counting Sort

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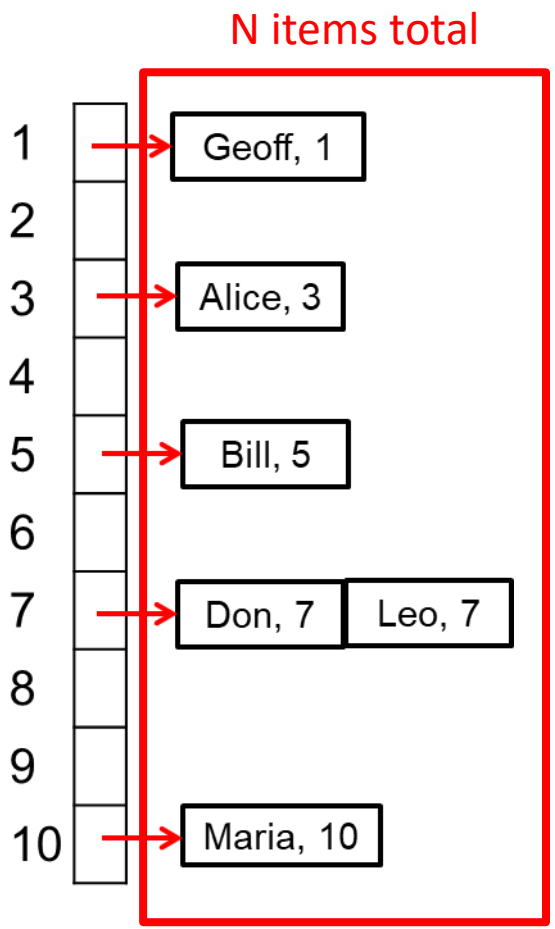

Marks	3	5	7	1	7	10
Name	Alice	Bill	Don	Geoff	Leo	Maria

N items

Counting Sort

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Not O(N*M)

N items total

- Stable?
  - No
  - We only remember the frequency
- But can we make it stable?
  - Yes but at the cost of memory
  - Similar to separate chaining
  - At most we have N items only anyways
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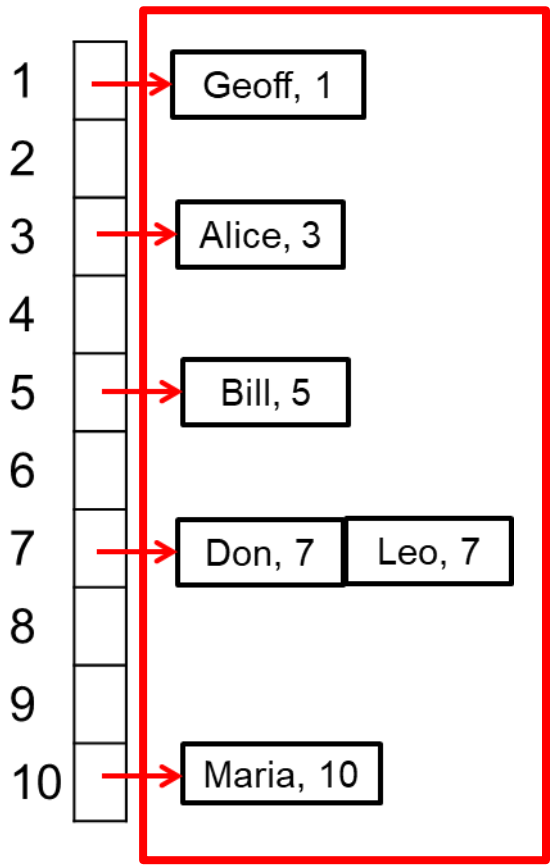

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Name	Alice	Bill	Don	Geoff	Leo	Maria

N items

Counting Sort

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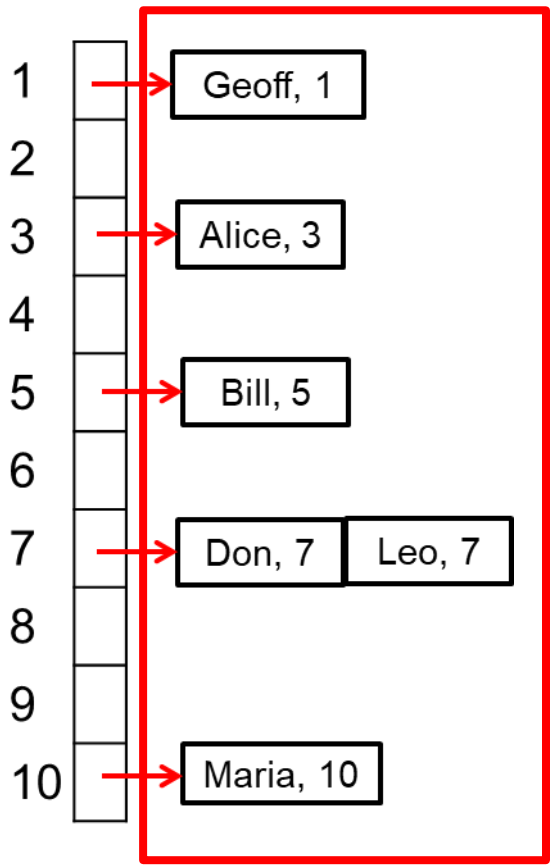
Not O(N*M)

N items total

- Stable?
  - No
  - We only remember the frequency

VERY COMMON MISCONCEPTION



- But can we make it stable?
  - Yes but at the cost of memory
  - Similar to separate chaining
  - At most we have N items only anyways
    - So it is O(M + N) space still
    - Can you see why? auxiliary space different list to different positions values from input list

the elements from input list spread out in the count_array
so their lists of values by values are from input list so just O(N)

Marks	3	5	7	1	7	10
Name	Alice	Bill	Don	Geoff	Leo	Maria

N items

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Questions?

Counting Sort

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- Stable?
  - No
  - We only remember the frequency
- But can we make it stable?
  - Yes but at the cost of memory
  - Similar to separate chaining
  - There is another way, refer to Nathan's amazing slide

Stable Counting Sort (Method 1)

Input

(3,a)	(1,p)	(3,c)	(7,f)	(5,g)	(3,b)	(7,d)	(8,w)

Construct count:

- For each key in input,
- count[key] += 1

count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

Output

1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	(1,p)	(3,c)	(7,f)	(5,g)	(3,b)	(7,d)	(8,w)

Construct count:

- For each key in input,
- count[key] += 1

Construct position:

- Initialise first position as a 1

count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	1
2	0
3	0
4	0
5	0
6	0
7	0
8	0

Output

1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	(1,p)	(3,c)	(7,f)	(5,g)	(3,b)	(7,d)	(8,w)

Construct count:

- For each key in input,
- count[key] += 1

Construct position:

- Initialise first position as a 1
- position[i] = position[i-1] + count[i-1]

count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	1
2	2
3	0
4	0
5	0
6	0
7	0
8	0

Output

1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

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3	3
4	0
5	1
6	0
7	2
8	1

position

1	1
2	2
3	2
4	0
5	0
6	0
7	0
8	0

Output

1	2	3	4	5	6	7	8

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4	0
5	1
6	0
7	2
8	1

position

1	1
2	2
3	2
4	5
5	0
6	0
7	0
8	0

Output

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4	0
5	1
6	0
7	2
8	1

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3	2
4	5
5	5
6	0
7	0
8	0

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6	0
7	2
8	1

position

1	1
2	2
3	2
4	5
5	5
6	6
7	0
8	0

Output

1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

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2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	1
2	2
3	2
4	5
5	5
6	6
7	6
8	0

Output

1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

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3	3
4	0
5	1
6	0
7	2
8	1

position

1	1
2	2
3	2
4	5
5	5
6	6
7	6
8	8

Output

1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

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Construct output

- Go through input, looking at each (key, val)
- Set output[position[key]] to the (key, val) pair from input
- Increment position[key]

count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	1
2	2
3	2
4	5
5	5
6	6
7	6
8	8

Output

1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	(1,p)	(3,c)	(7,f)	(5,g)	(3,b)	(7,d)	(8,w)

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count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	1
2	2
3	2
4	5
5	5
6	6
7	6
8	8

Output

1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	(1,p)	(3,c)	(7,f)	(5,g)	(3,b)	(7,d)	(8,w)

Construct count:

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count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	1
2	2
3	3
4	5
5	5
6	6
7	6
8	8

Output

	(3,a)						
1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	(1,p)	(3,c)	(7,f)	(5,g)	(3,b)	(7,d)	(8,w)

Construct count:

- For each key in input,
- count[key] += 1

Construct position:

- Initialise first position as a 1
- position[i] = position[i-1] + count[i-1]

Construct output

- Go through input, looking at each (key, val)
- Set output[position[key]] to the (key, val) pair from input
- Increment position[key]

count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	2
2	2
3	3
4	5
5	5
6	6
7	6
8	8

Output

(1,p)	(3,a)						
1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	(1,p)	(3,c)	(7,f)	(5,g)	(3,b)	(7,d)	(8,w)

Construct count:

- For each key in input,
- count[key] += 1

Construct position:

- Initialise first position as a 1
- position[i] = position[i-1] + count[i-1]

Construct output

- Go through input, looking at each (key, val)
- Set output[position[key]] to the (key, val) pair from input
- Increment position[key]

count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	2
2	2
3	4
4	5
5	5
6	6
7	6
8	8

Output

(1,p)	(3,a)	(3,c)					
1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	(1,p)	(3,c)	(7,f)	(5,g)	(3,b)	(7,d)	(8,w)

Construct count:

- For each key in input,
- count[key] += 1

Construct position:

- Initialise first position as a 1
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Construct output

- Go through input, looking at each (key, val)
- Set output[position[key]] to the (key, val) pair from input
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count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	2
2	2
3	4
4	5
5	5
6	7
7	6
8	8

Output

(1,p)	(3,a)	(3,c)			(7,f)		
1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	(1,p)	(3,c)	(7,f)	(5,g)	(3,b)	(7,d)	(8,w)

Construct count:

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Construct position:

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1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	2
2	2
3	4
4	5
5	6
6	7
7	6
8	8

Output

(1,p)	(3,a)	(3,c)		(5,g)	(7,f)		
1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	(1,p)	(3,c)	(7,f)	(5,g)	(3,b)	(7,d)	(8,w)

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count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	2
2	2
3	5
4	5
5	6
6	7
7	6
8	8

Output

(1,p)	(3,a)	(3,c)	(3,b)	(5,g)	(7,f)		
1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

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count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	2
2	2
3	5
4	5
5	6
6	7
7	7
8	8

Output

(1,p)	(3,a)	(3,c)	(3,b)	(5,g)	(7,f)	(7,d)	
1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	(1,p)	(3,c)	(7,f)	(5,g)	(3,b)	(7,d)	(8,w)

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count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	2
2	2
3	5
4	5
5	6
6	7
7	7
8	9

Output

(1,p)	(3,a)	(3,c)	(3,b)	(5,g)	(7,f)	(7,d)	(8,w)
1	2	3	4	5	6	7	8

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Questions?

Counting Sort

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- Stable?
  - No
  - We only remember the frequency
- But can we make it stable?
  - Yes but at the cost of memory
  - Similar to separate chaining
  - There is another way, refer to Nathan's amazing slide
  - Are the complexity the same?

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Questions?

MONASH University

Have a break again!

Counting Sort

Remember this issue…

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- Now imagine the following:

  - What is my complexity?
    - Time…
    - Space…
  - Let us leave it at it is first…

Counting Sort

Remember this issue…

MONASH University

- Now imagine the following:

  - What is my complexity?
    - Time…
    - Space…
  - Let us leave it at it is first… We shall resolve this now…

Counting Sort

Remember this issue…

MONASH University

- Now imagine the following:

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Questions?

Radix Sort

A different outlook…

MONASH University

- With this input…

Radix Sort

A different outlook…

MONASH University

- With this input…

  - What if we view it differently?

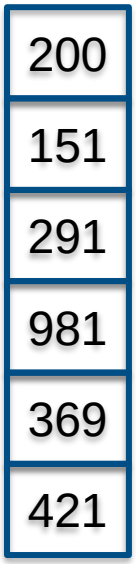
Radix Sort

A different outlook…

MONASH University

- With this input…

  - What if we view it differently?



Radix Sort

A different outlook…

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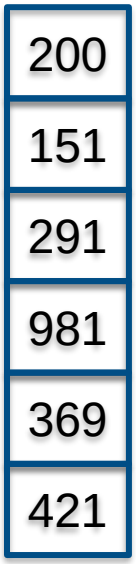
- With this input…

  - What if we view it differently? How would we sort it?



Radix Sort

A different outlook…

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- With this input…
  - What if we view it differently? How would we sort it?

Radix Sort

A different outlook…

MONASH University

- With this input…
  - What if we view it differently? How would we sort it?
    - Right most digit = least significant
    - Left most digit = most significant

200
151
291
981
369
421
671

Radix Sort

A different outlook…

MONASH University

- With this input…
  - What if we view it differently? How would we sort it?
    - Right most digit = least significant
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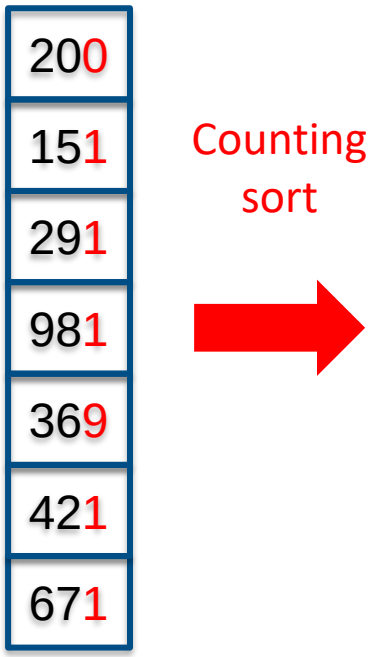

Counting sort

Radix Sort

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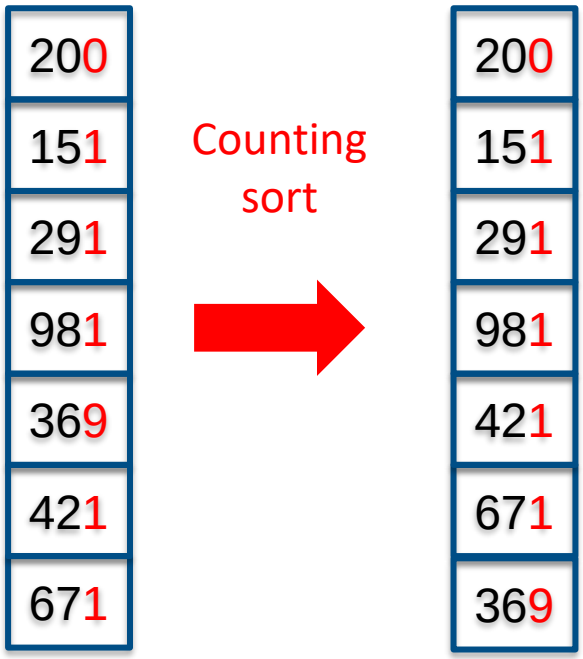

Radix Sort

A different outlook…

MONASH University

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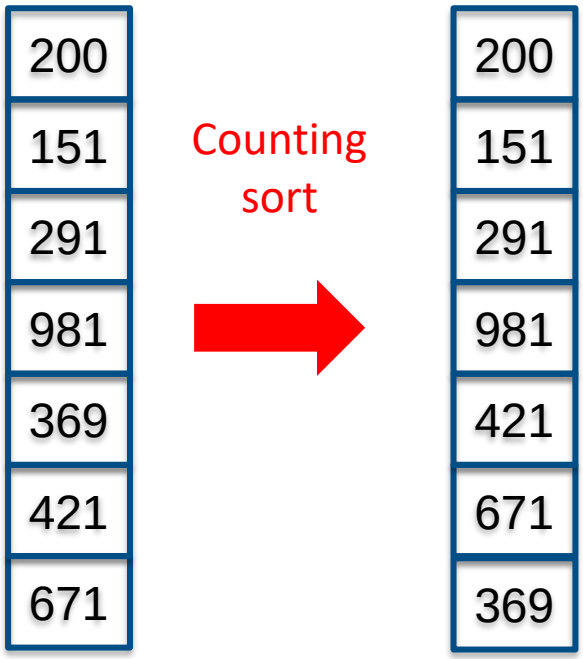

Radix Sort

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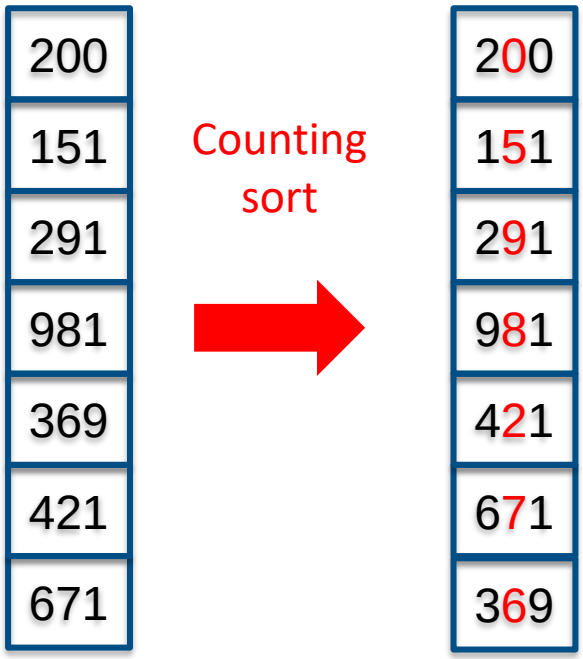

Radix Sort

A different outlook…

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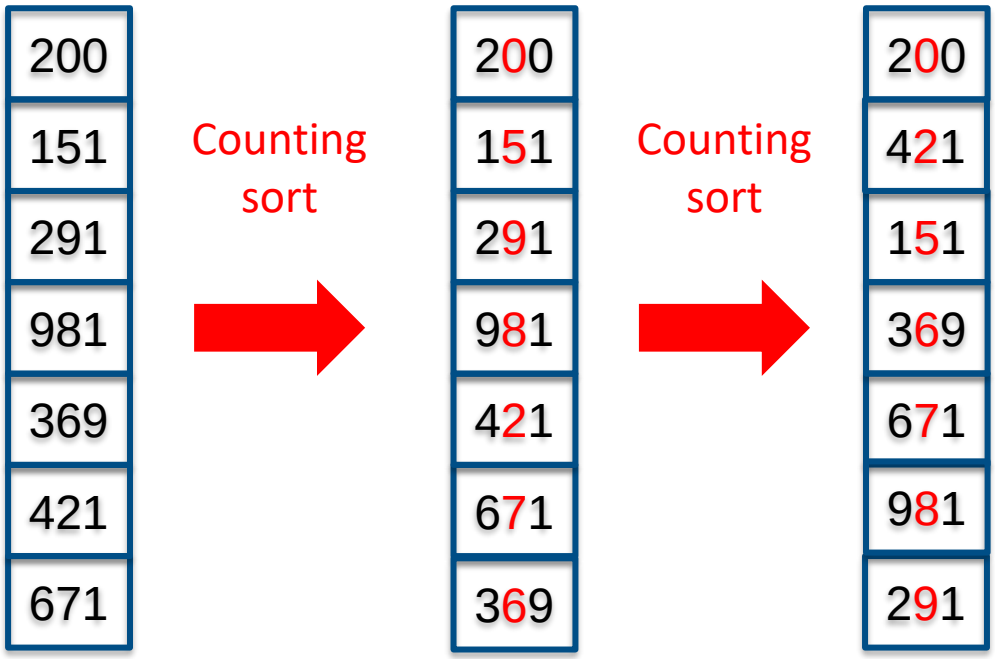

Radix Sort

A different outlook…

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  - What if we view it differently? How would we sort it?
    - Right most digit = least significant
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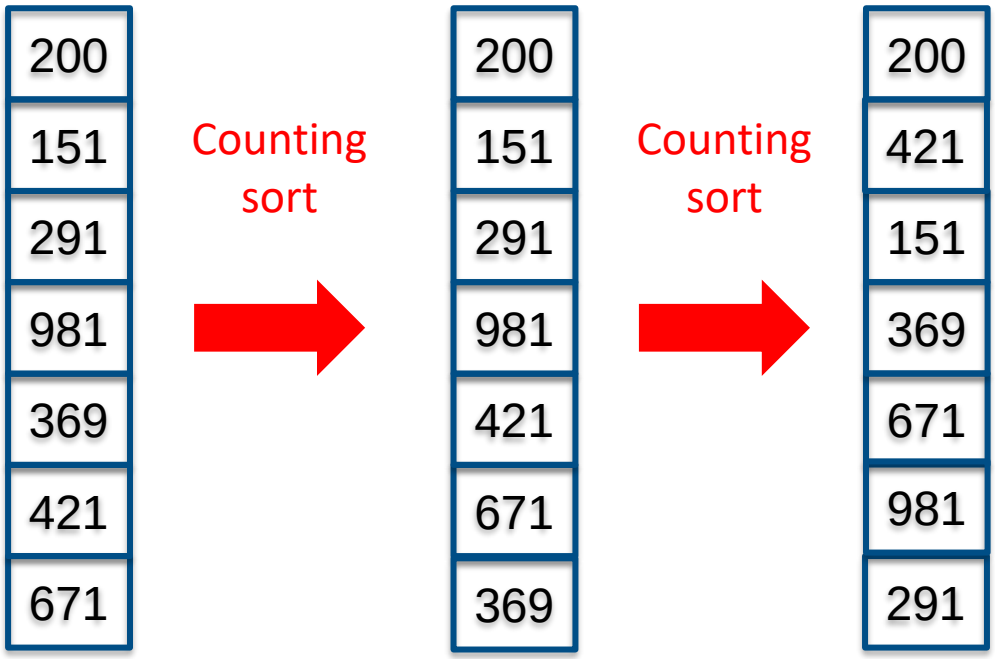

Radix Sort

A different outlook…

- With this input…
  - What if we view it differently? How would we sort it?
    - Right most digit = least significant
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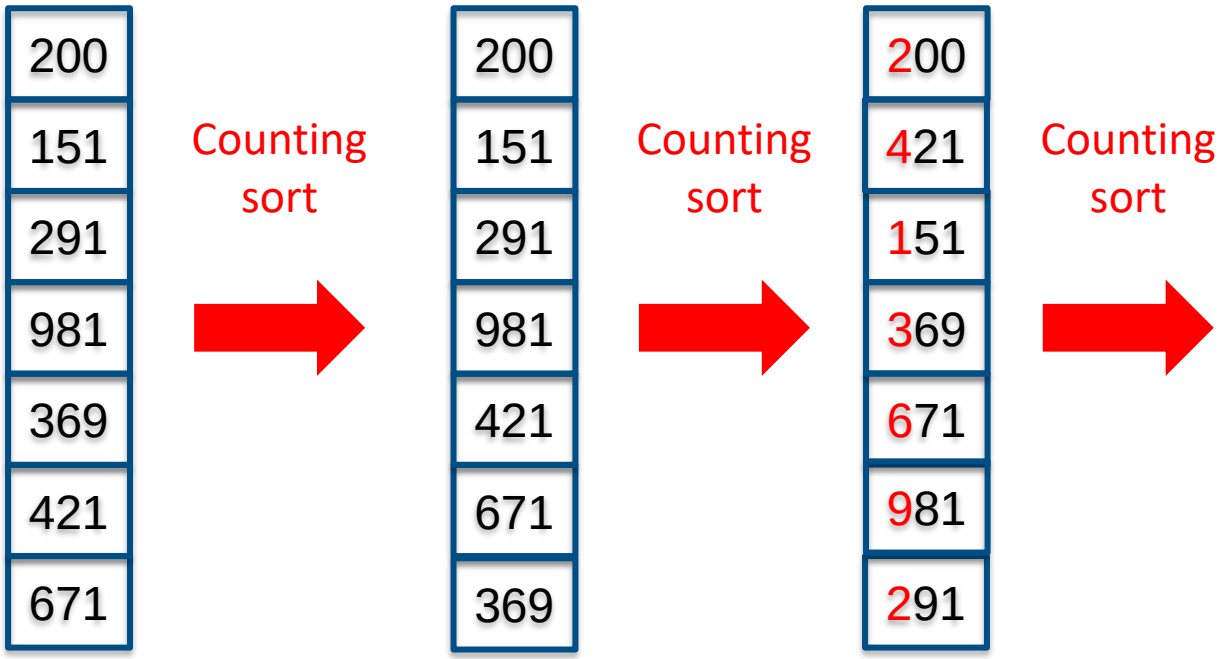

Radix Sort

A different outlook…

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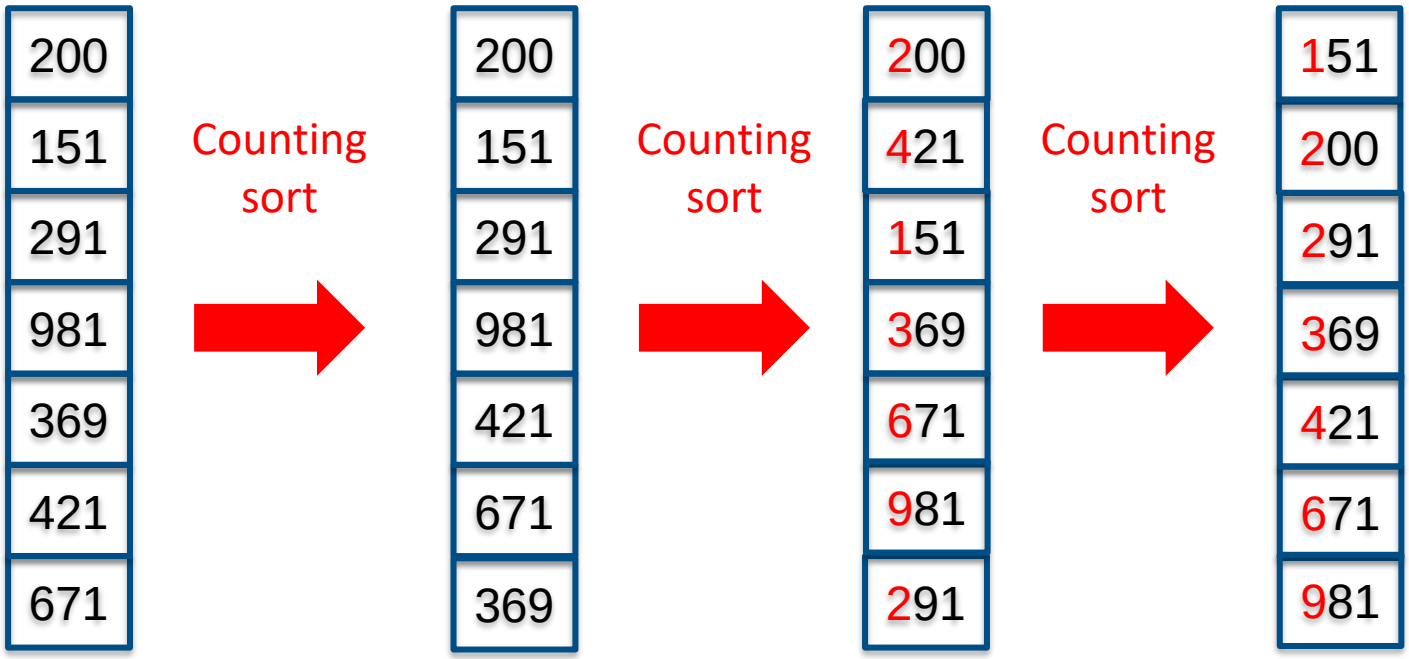
    - Left most digit = most significant



Radix Sort

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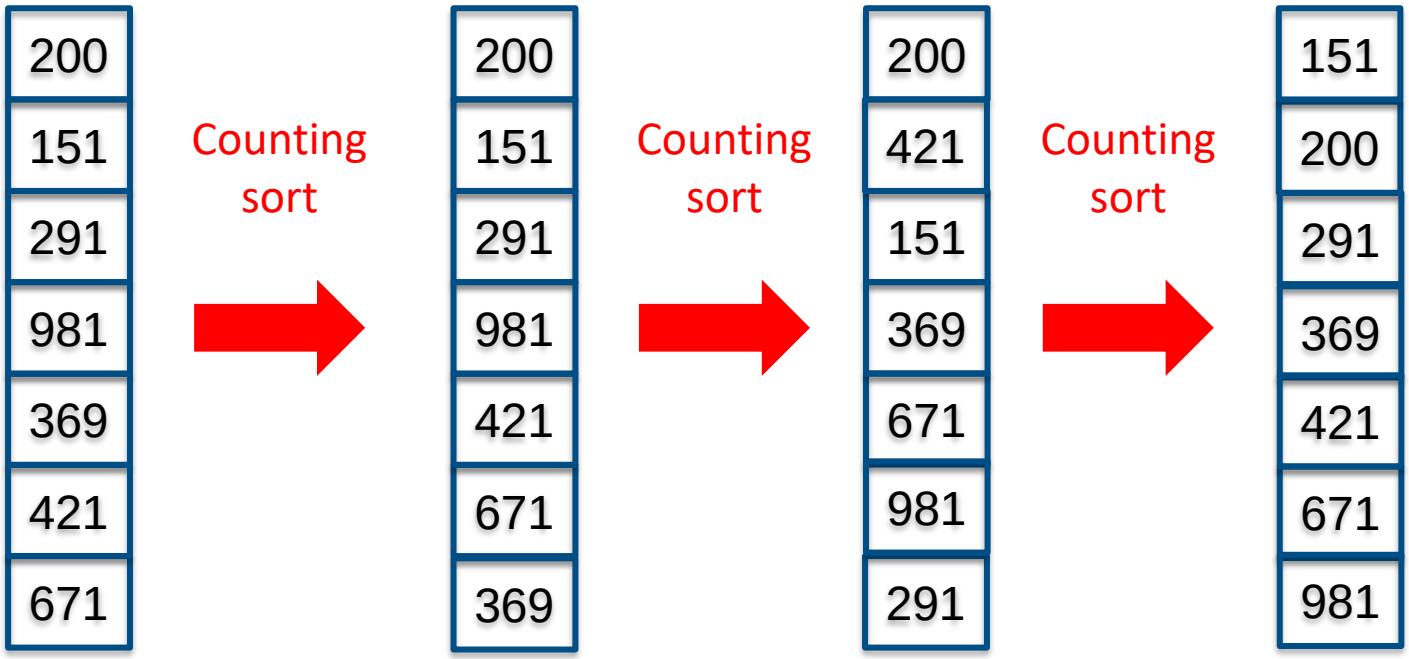

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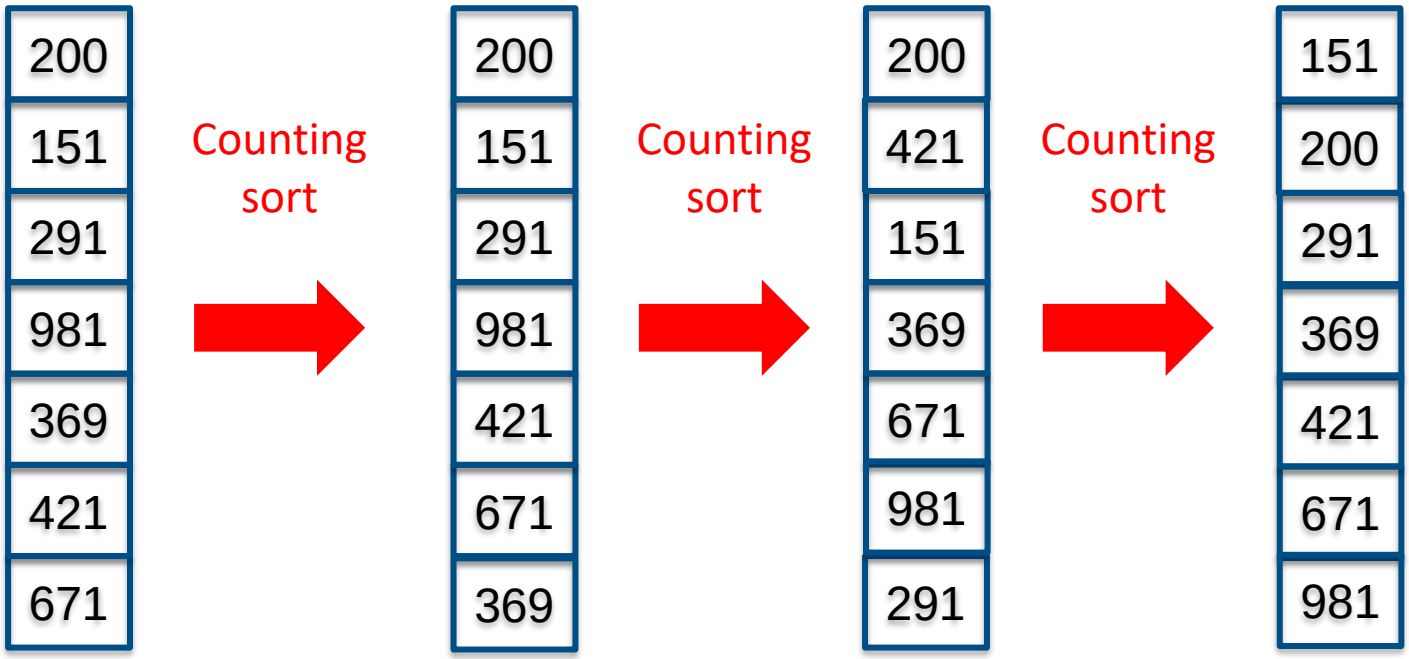
Questions?

Radix Sort

A different outlook…

- With this input…
  - What if we view it differently? How would we sort it?
  - But the sorting need to be stable



Radix Sort

A different outlook…

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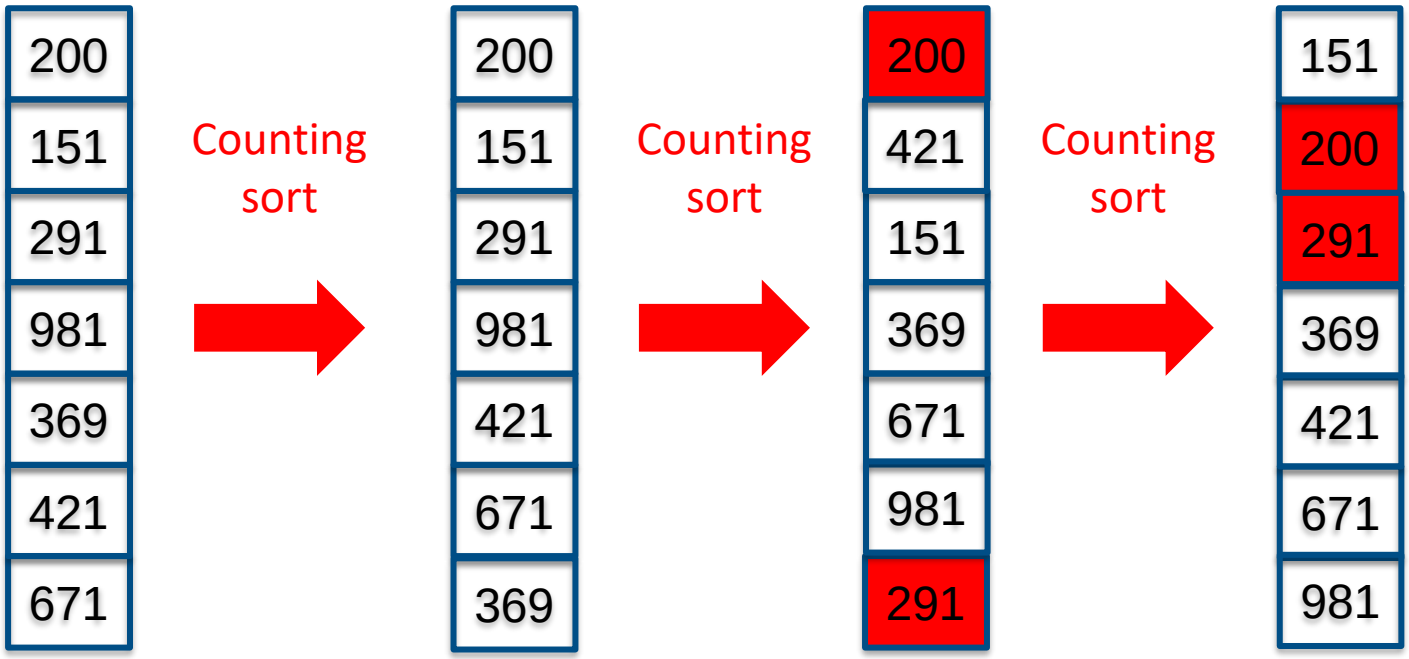
- With this input…
  - What if we view it differently? How would we sort it?
  - But the sorting need to be stable



Radix Sort

A different outlook…

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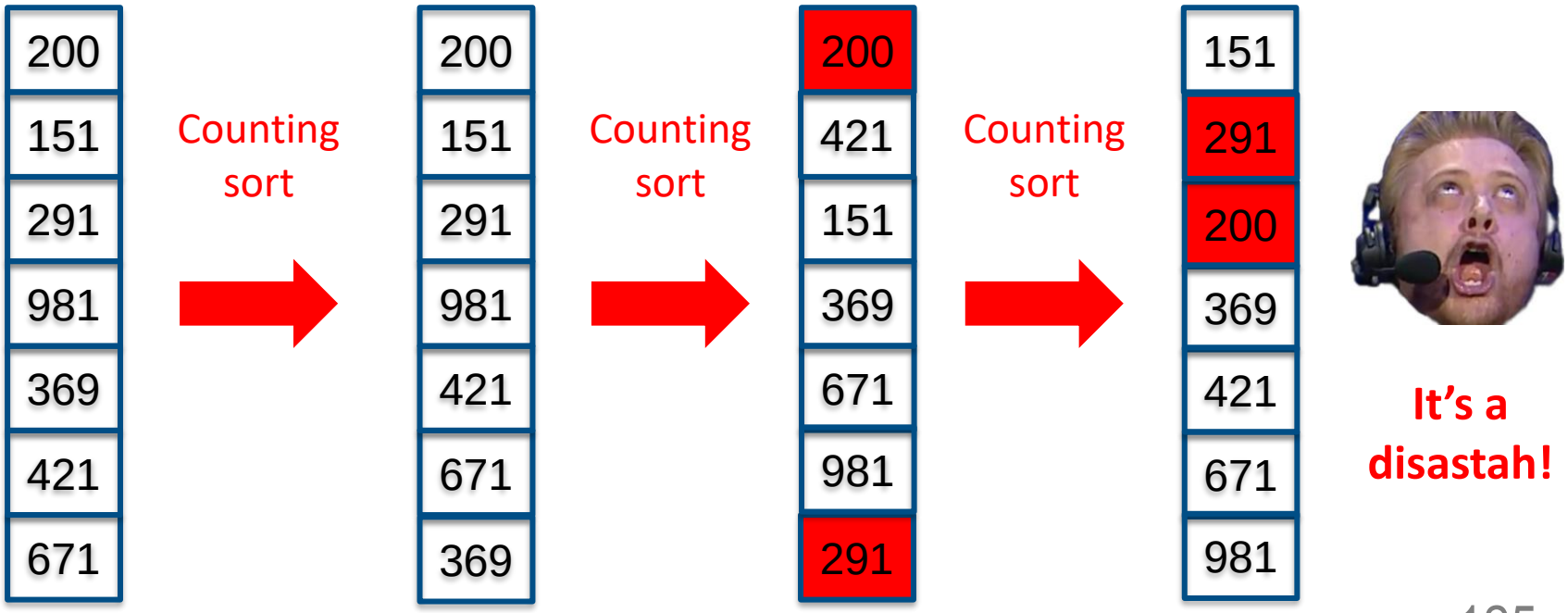
- With this input…
  - What if we view it differently? How would we sort it?
  - But the sorting need to be stable, if not…



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Questions?

Radix Sort
Complexity

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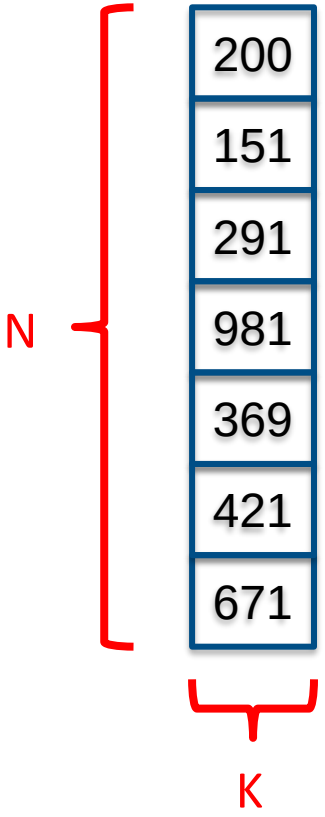
- What is the complexity?
  - Time
  - Space

Radix Sort
Complexity

- What is the complexity?
  - Time
  - Space



Radix Sort
Complexity

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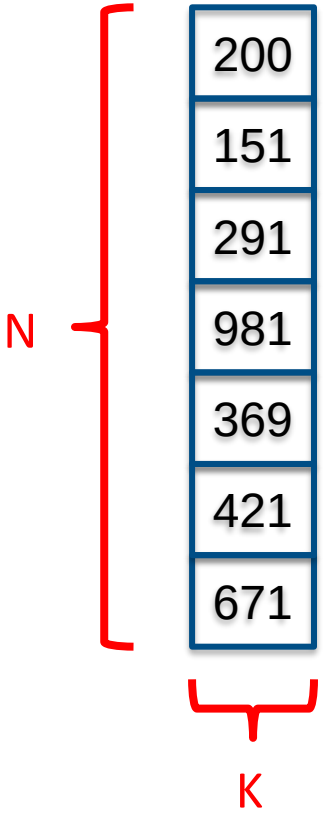
- What is the complexity?
  - Time
    - O(KN)?
  - Space



Radix Sort
Complexity

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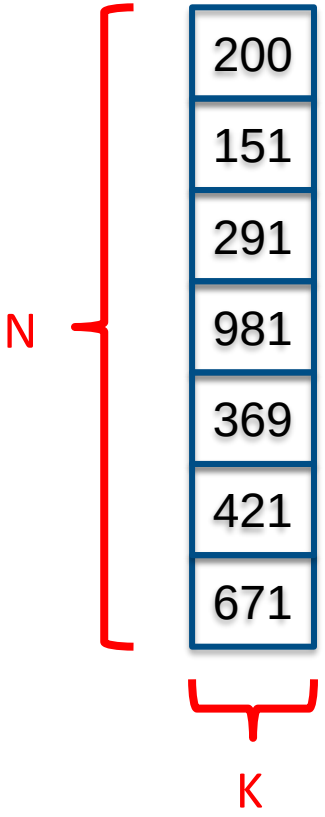
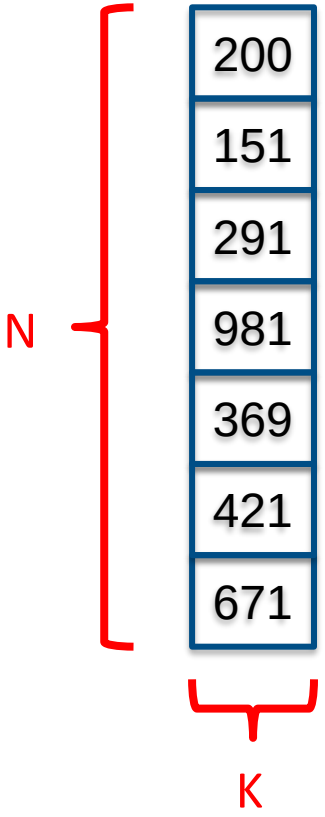
- What is the complexity?
  - Time
    - O(KN) + O(KM)
      where M is the number of unique characters
  - Space



Radix Sort
Complexity

MONASH University

- What is the complexity?
  - Time
    - O(KN) + O(KM)
      where M is the number of unique characters
    - Why? Recall counting sort, we account for the max
  - Space



Radix Sort
Complexity

MONASH University

- What is the complexity?
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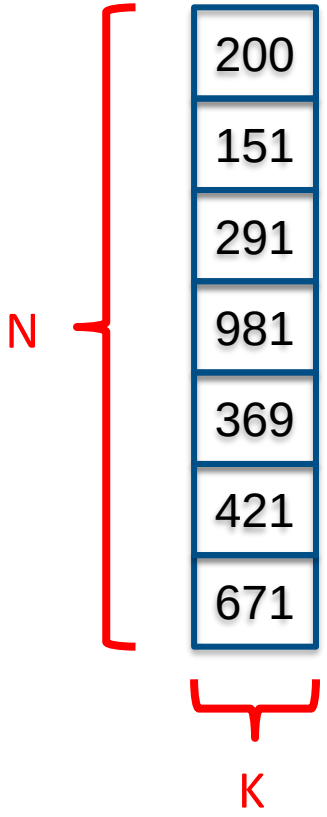
    - Why? Recall counting sort, we account for the max giving us O(N+M)
  - Space



Radix Sort
Complexity

MONASH University

- What is the complexity?
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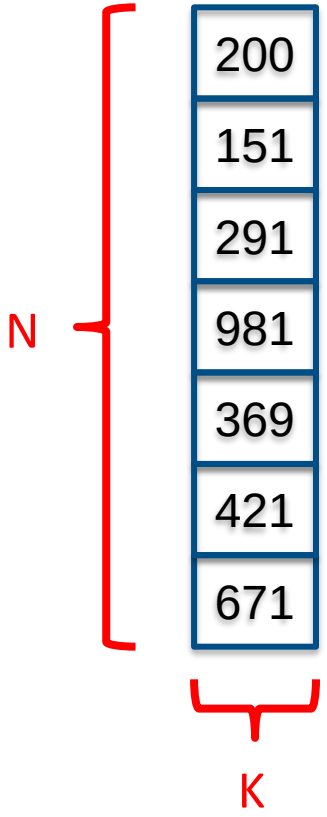
    - Why? Recall counting sort, we account for the max giving us O(N+M)
    - Then we have K columns
  - Space



Radix Sort
Complexity

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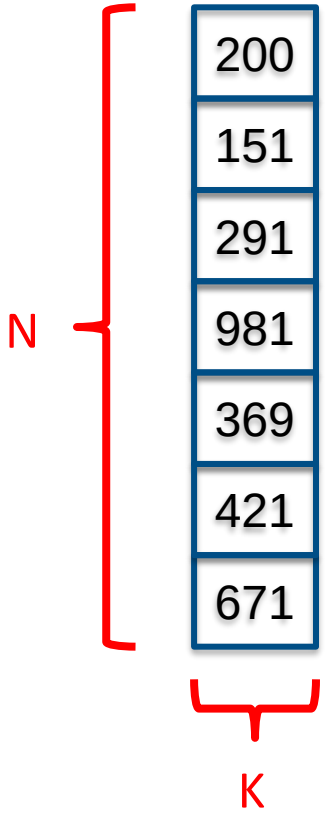

Radix Sort
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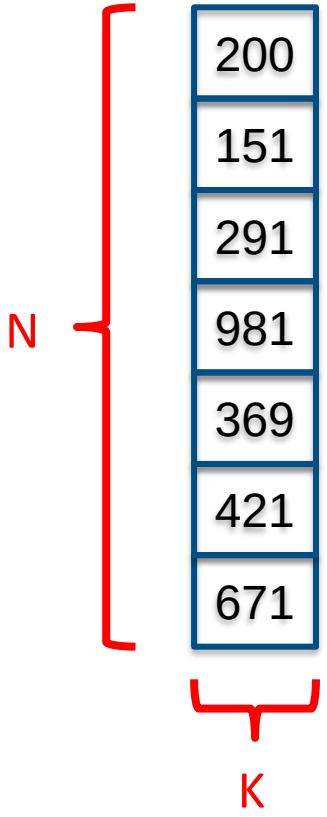

Radix Sort
Complexity

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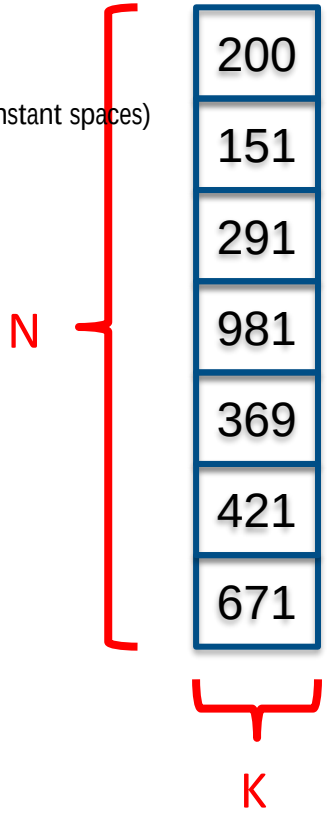
- What is the complexity?
  - Time
    - O(KN + KM) where M is the number of unique characters
    - Why? Recall counting sort, we account for the max giving us O(N+M)
    - Then we have K columns giving us O(K) * O(N+M)
  - Space
    - Input is O(KN)
    - Each counting sort needs O(M+N)

three columns, O(KM)

M depends on biggest number in single digit: 9 (0~9, 10 spaces, constant spaces)

maximum count_array

loop through to count frequencies for each count_array



Radix Sort
Complexity

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    - Input is O(KN)
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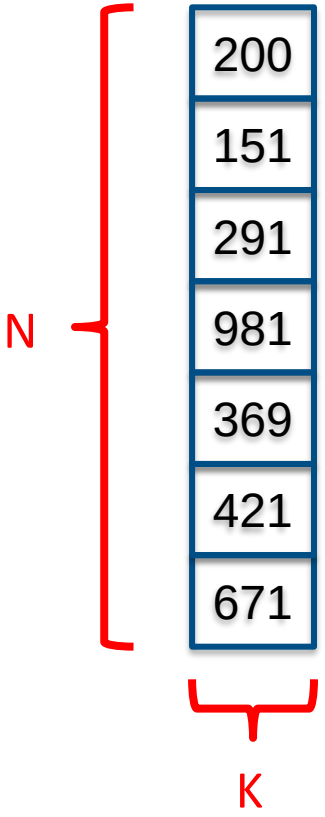

Radix Sort
Complexity

MONASH University

- What is the complexity?
  - But we know M = 10 for 0, 1, …, 9
  - Time
    - O(KN + KM)
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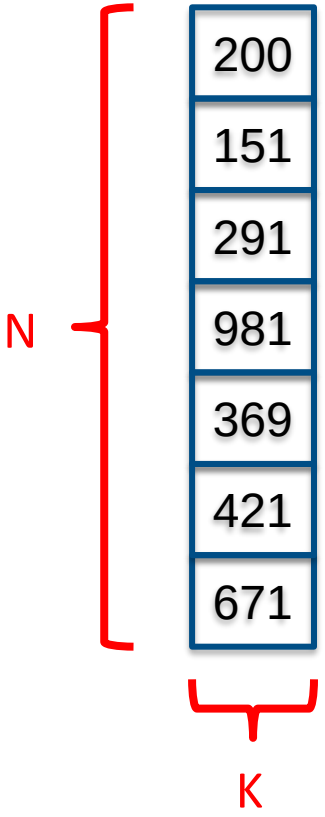

Radix Sort
Complexity

MONASH University

- What is the complexity?
  - But we know M = 10 for 0, 1, …, 9
  - Time
    - O(KN + KM) ≈ O(KN)
      where M is the number of unique characters
    - Why? Recall counting sort, we account for the max giving us O(N+M)
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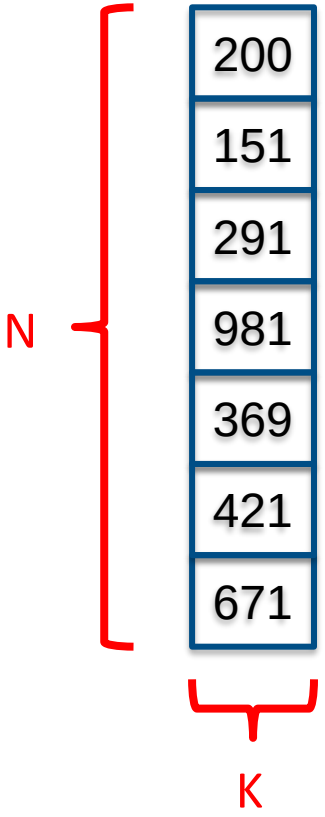
      giving us O(K) * O(N+M)
  - Space
    - Input is O(KN)
    - Each counting sort needs O(M+N)
    - Total is O(KN + M + N) ≈ O(KN)
    - Auxiliary is O(M + N) ≈ O(N)



Radix Sort
Complexity

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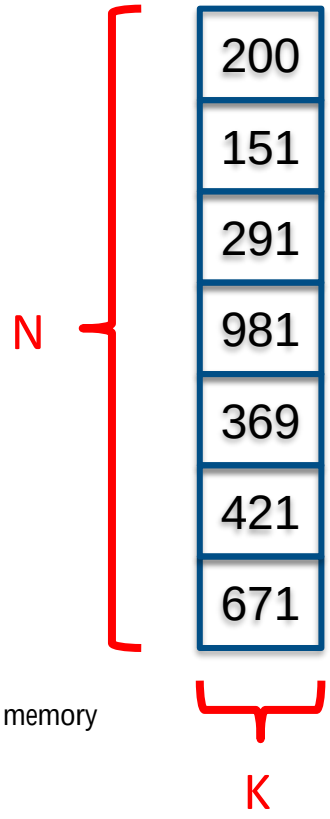
- What is the complexity?
  - Better than merge sort O(k N log N)!
  - But we know M = 10 for 0, 1, …, 9
  - Time
    - O(KN + KM) ≈ O(KN) M is constant, 10 for digits, 26 for alphabets
      where M is the number of unique characters
    - Why? Recall counting sort, we account for the max giving us O(N+M)
    - Then we have K columns running K times of counting sort
      giving us O(K) * O(N+M)
  - Space
    - Input is O(KN) finish counting sort for each column, count_array can be removed from memory
    - Each counting sort needs O(M+N)
    - Total is O(KN + M + N) ≈ O(KN)
    - Auxiliary is O(M + N) ≈ O(N) when doing counting sort, all value would be copied but the pointer in the array is changed



Radix Sort
Complexity

MONASH University

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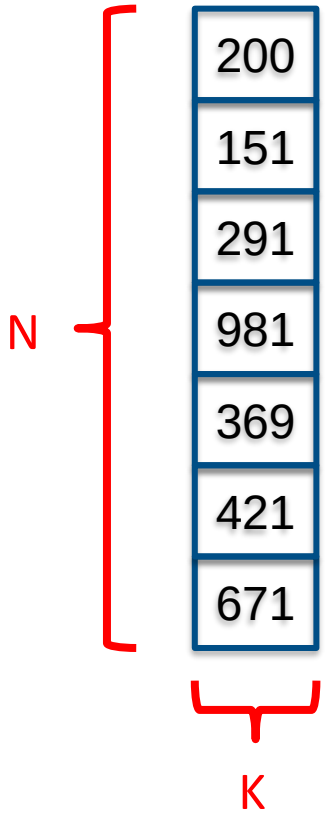
  - Space
    - Input is O(KN)
    - Each counting sort needs O(M+N)
    - Total is O(KN + M + N) ≈ O(KN)
    - Auxiliary is O(M + N) ≈ O(N) <- why no K? Come ask me if interested…



mips array just save memory address of the elements

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Questions?

Radix Sort
Complexity

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- What is the complexity?
  - What if k is bigger?
  - But we know M = 10 for 0, 1, …, 9
  - Time
    - O(KN + KM) ≈ O(KN)
      where M is the number of unique characters
    - Why? Recall counting sort, we account for the max giving us O(N+M)
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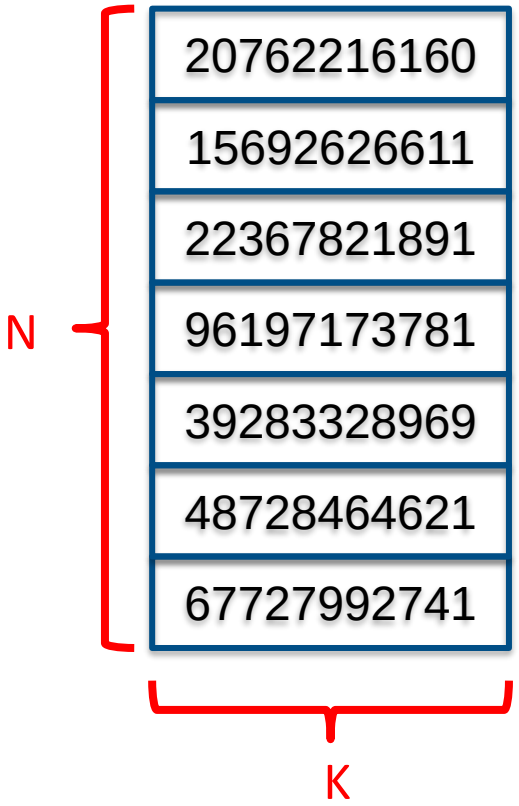
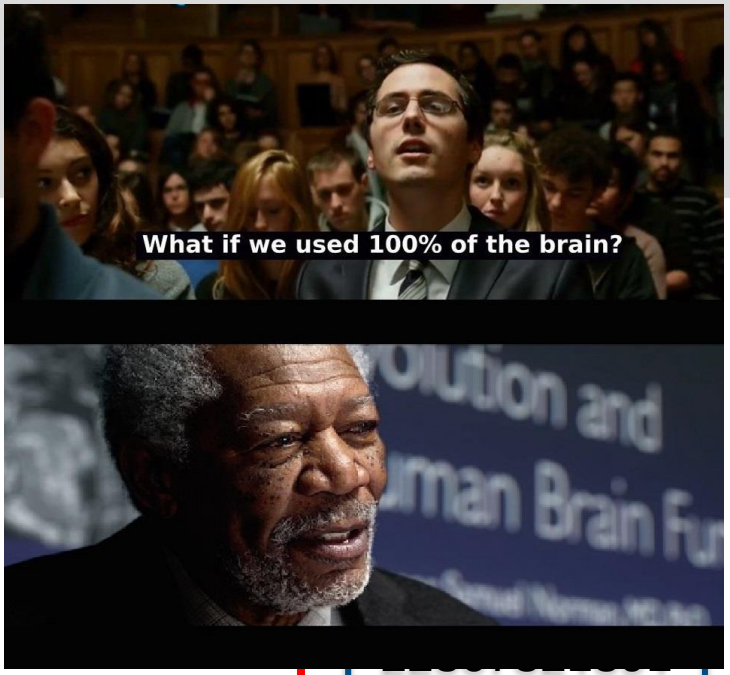


Radix Sort
Complexity



- What is the complexity?
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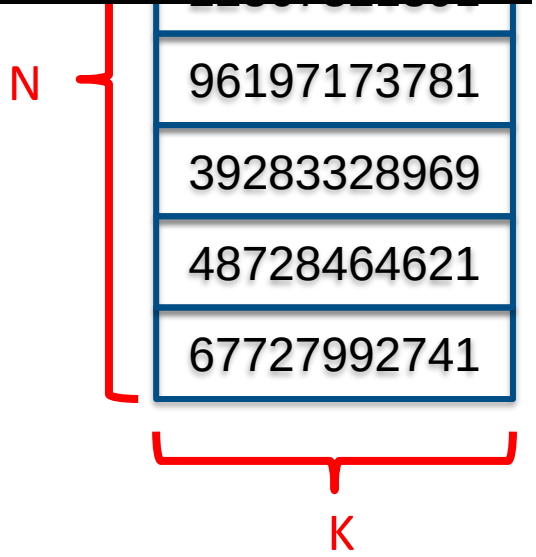

Radix Sort
Complexity

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- What is the complexity?
  - What if k is bigger? two column together if 100, meaning comparing two column together, K/2
  - We increase M = 100 for 0, 1, …, 99 K = log_M 10^(number of digits)
  - Time
    - O(KN + KM) ≈ O(KN) M is constant, 10 for digits, 26 for alphabets where M is the number of unique characters
    - Why? Recall counting sort, we account for the max giving us O(N+M)
    - Then we have K columns running K times of counting sort giving us O(K) * O(N+M) finish counting sort for each column, count_array can be removed from memory
  - Space
    - Input is O(KN)
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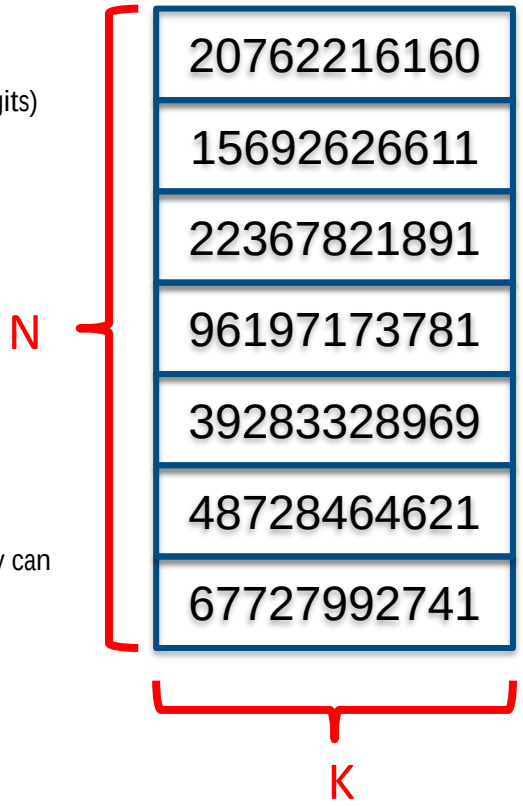

Radix Sort
Complexity

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- Time complexity is O(KN +KM)
- Space complexity is O(KN + M + N)

Radix Sort
Complexity

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- Time complexity is O(KN +KM)
- Space complexity is O(KN + M + N)

- M is the base

M is 2 = binary (base 2)
M is 10 = Decimal (base 10)

Radix Sort
Complexity

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- Time complexity is O(KN +KM)
- Space complexity is O(KN + M + N)

- M is the base
  - For decimal numbers, it is 10 from 0 to 10

Radix Sort
Complexity

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- Time complexity is O(KN +KM)
- Space complexity is O(KN + M + N)

- M is the base
  - For decimal numbers, it is 10 from 0 to 10
  - For binary numbers,

Radix Sort
Complexity

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- Time complexity is O(KN +KM)
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- M is the base
  - For decimal numbers, it is 10 from 0 to 10
  - For binary numbers, it is 2 from 0 to 1

Radix Sort
Complexity

MONASH University

- Time complexity is O(KN +KM)
- Space complexity is O(KN + M + N)

- M is the base
  - For decimal numbers, it is 10 from 0 to 10
  - For binary numbers, it is 2 from 0 to 1
  - We can increase the M, to reduce the K?

Radix Sort
Complexity

MONASH University

- Time complexity is O(KN +KM)
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- M is the base
  - For decimal numbers, it is 10 from 0 to 10
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  - If we deal with the English alphabet, this would be 26 from a to z

Radix Sort
Complexity

MONASH University

- Time complexity is O(KN +KM)
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- M is the base
  - For decimal numbers, it is 10 from 0 to 10
  - For binary numbers, it is 2 from 0 to 1
  - We can increase the M, to reduce the K?
  - If we deal with the English alphabet, this would be 26 from a to z
  - Nathan did a good analysis on it

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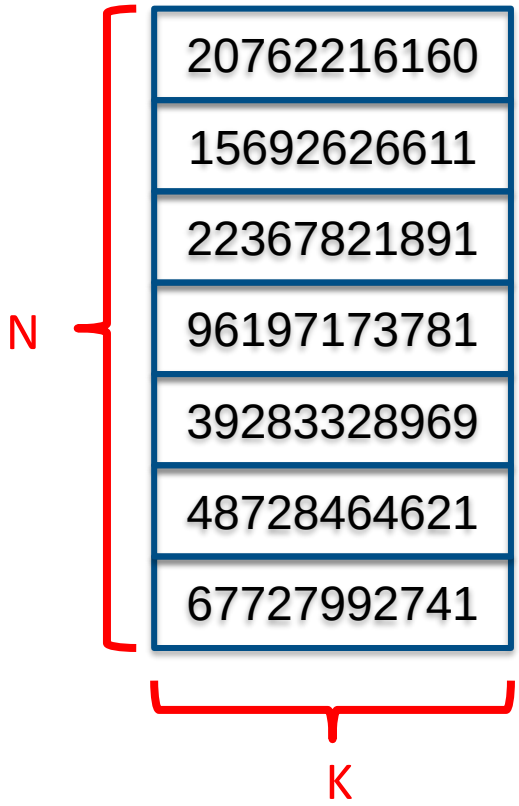
Questions?

Radix Sort

TL;DR

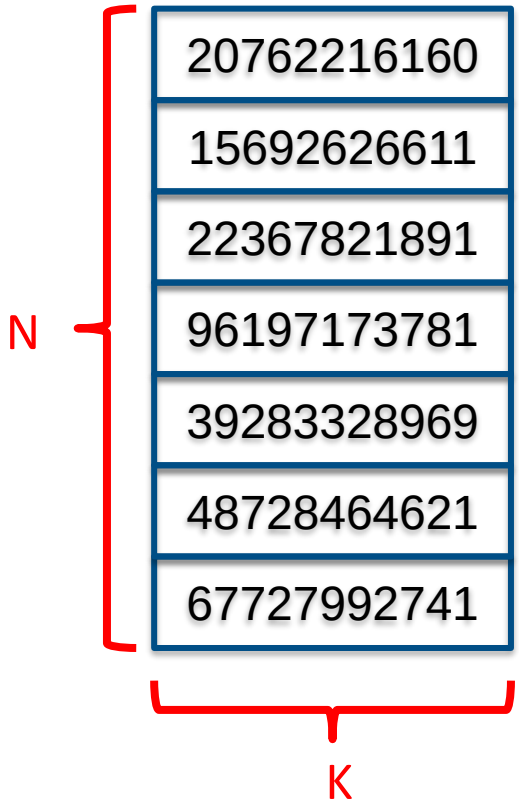
- So you know radix sort
- What have you notice?



Radix Sort

TL;DR

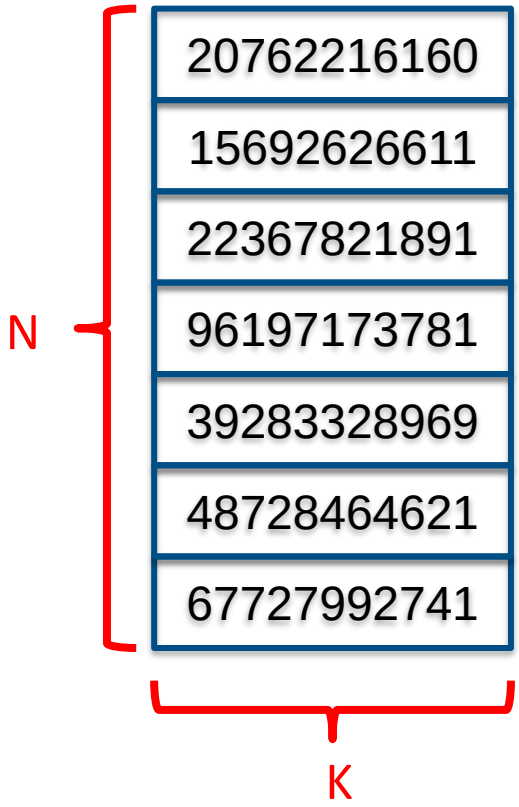
- So you know radix sort
- What have you notice?
  - It is counting sort really, done multiple times



Radix Sort

TL;DR

- So you know radix sort
- What have you notice?
  - It is counting sort really, done multiple times
  - Usually least significant (right) to most significant (left)



Radix Sort
TL;DR

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    - We can reduce this by increasing the base
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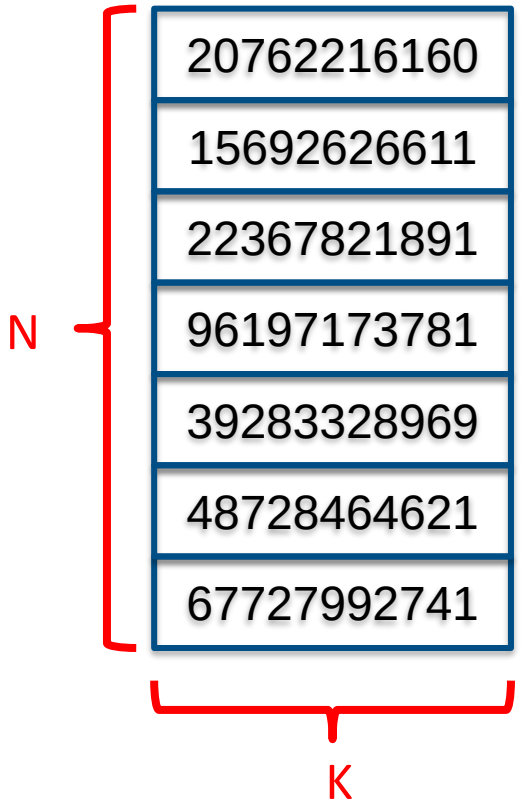
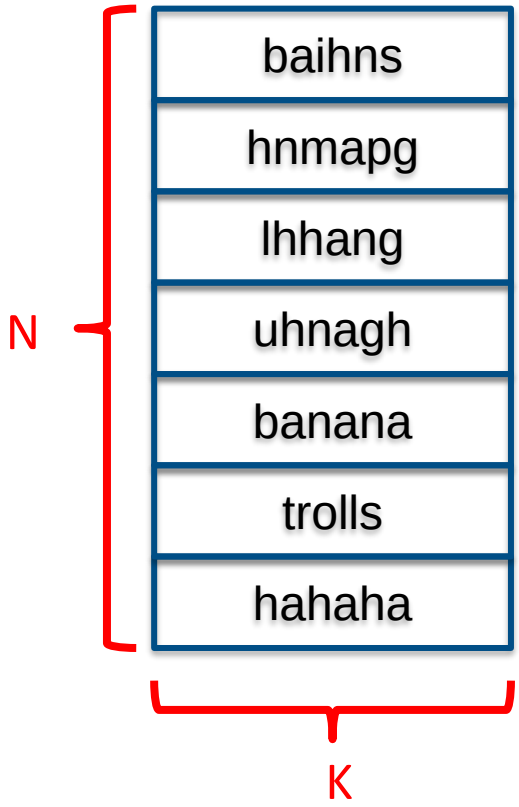


Radix Sort
TL;DR

- So you know radix sort
- What have you notice?
  - It is counting sort really, done multiple times
    - We can reduce this by increasing the base
    - Works well for characters as well
  - Usually least significant (right) to most significant (left)



Radix Sort
TL;DR

MONASH University

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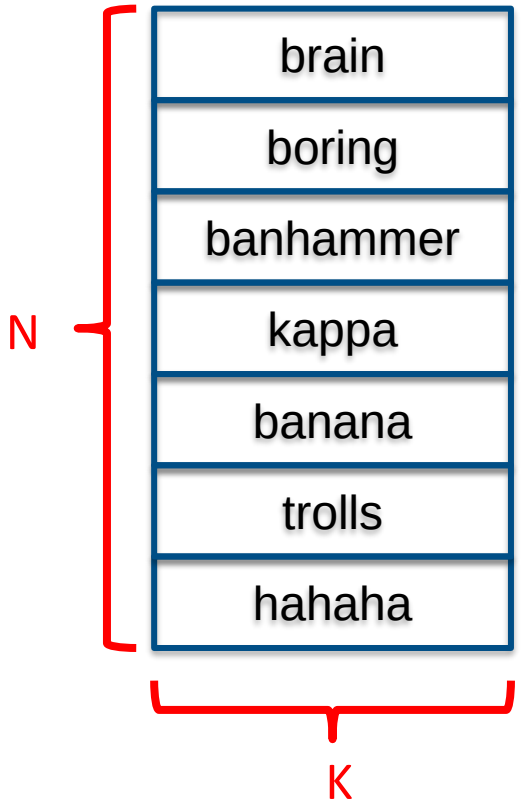
  - Usually least significant (right) to most significant (left)
- But what if they are not the same length?



Radix Sort
TL;DR

MONASH University

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  - It is counting sort really, done multiple times
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    - Works well for characters as well
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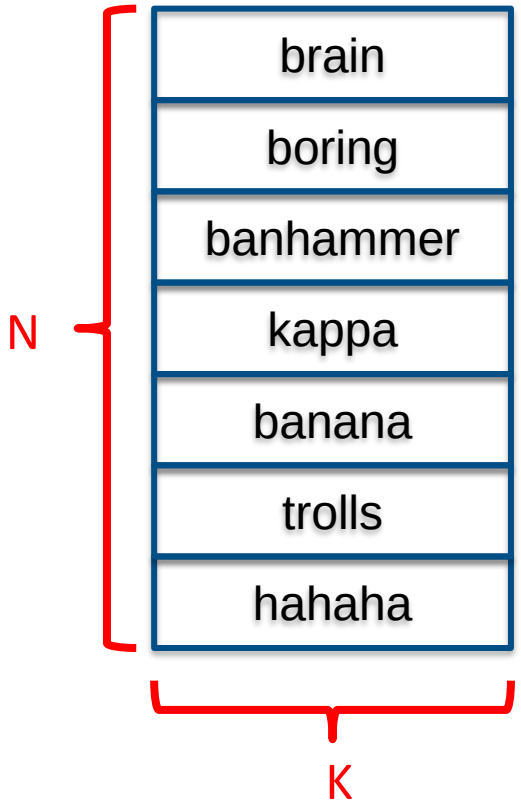
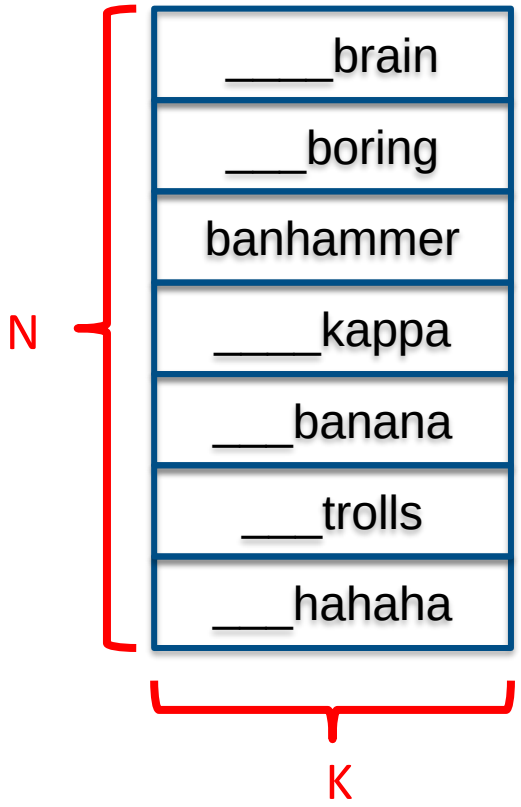
- But what if they are not the same length?
  - Left-aligned?
  - Right-aligned?



Radix Sort
TL;DR

MONASH University

- So you know radix sort
- What have you notice?
  - It is counting sort really, done multiple times
    - We can reduce this by increasing the base
    - Works well for characters as well
  - Usually least significant (right) to most significant (left)
- But what if they are not the same length? Add spaces!
  - Left-aligned?
  - Right-aligned?



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Questions?

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Thank You