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INFORMATION
TECHNOLOGY

FIT2004 Algorithms and Data Structures

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Referencing materials by
Nathan Companez, Aamir Cheema, Arun Konagurthu and Lloyd Allison

GROUP OF EIGHT AUSTRALIA

Faculty of Information Technology, Monash University

Ready?

Agenda

- The Graph data structure
- Graph Traversal algorithms

Agenda

- The Graph data structure
  - Introduction
  - Representation
- Graph Traversal algorithms

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- The Graph data structure
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- Graph Traversal algorithms
  - Breadth First Search (BFS)
  - Depth First Search (DFS)
  - Dijkstra's shortest distance

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Basic for many graph-algorithms

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Let us begin…

- Master race of all data structure

Graph

Introduction

- Master race of all data structure
  - Everything can be represented as a graph

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Graph
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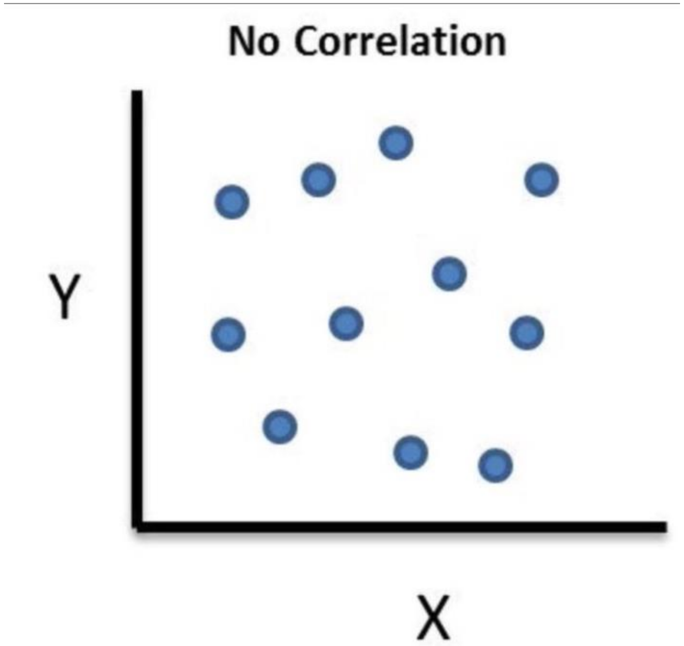

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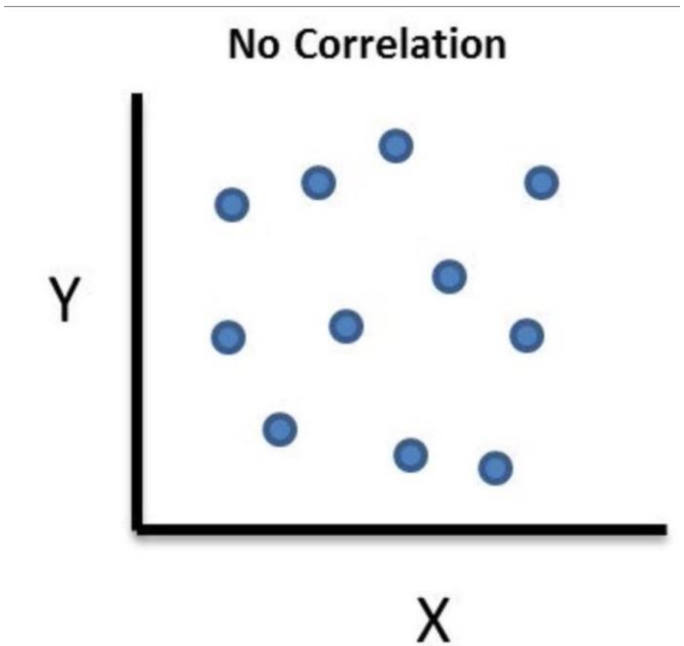

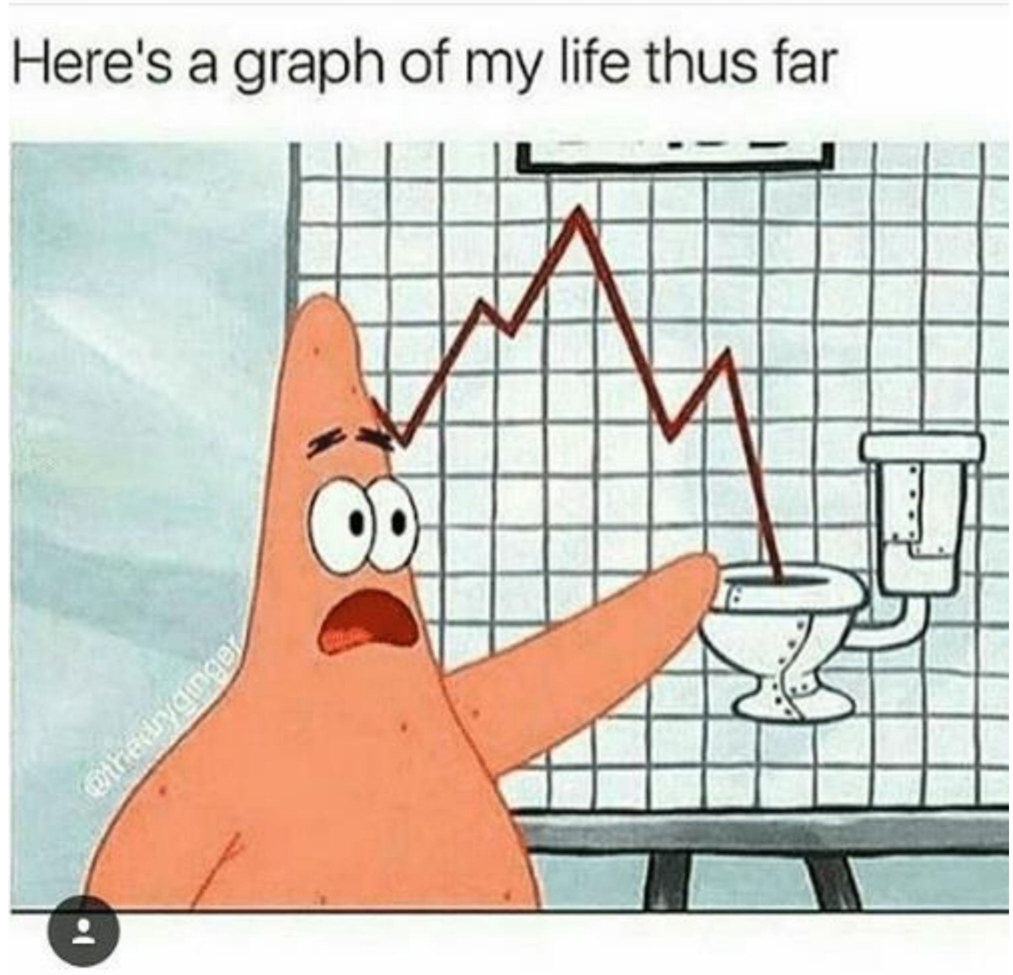

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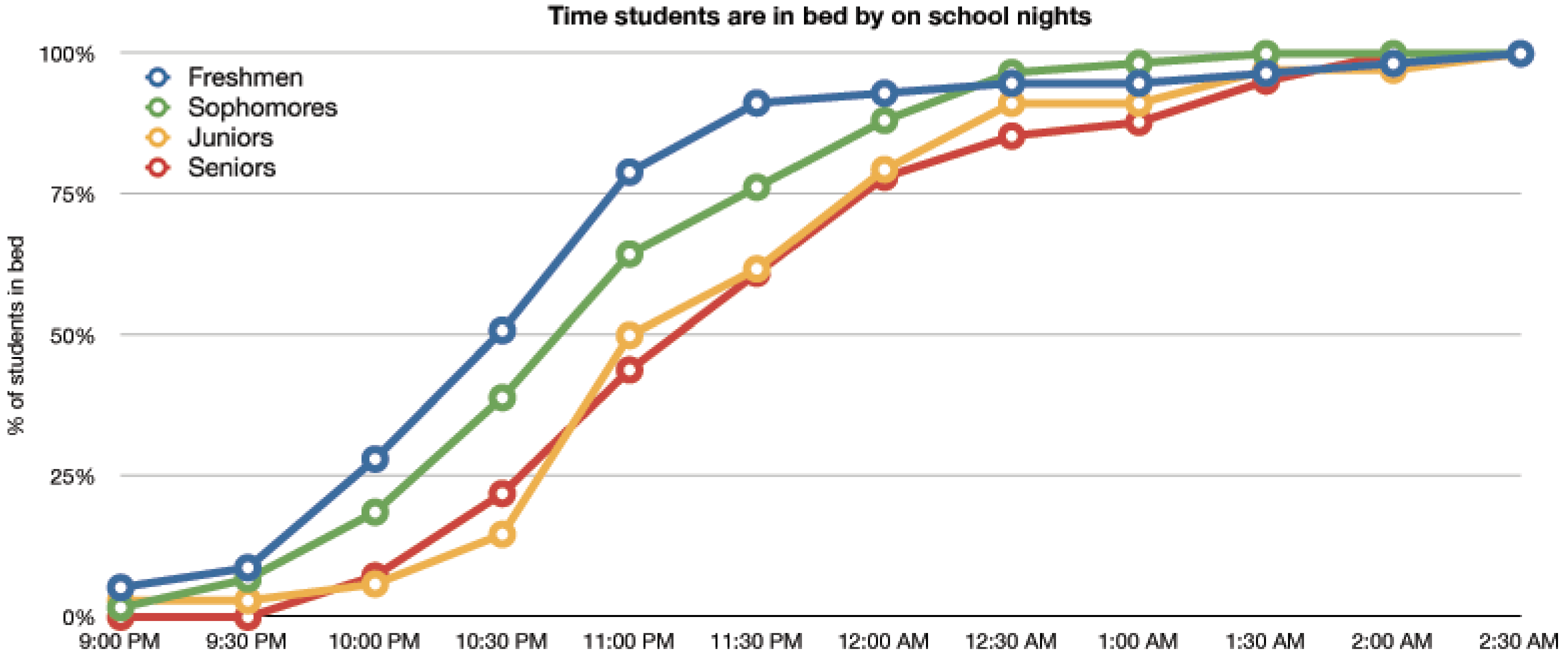

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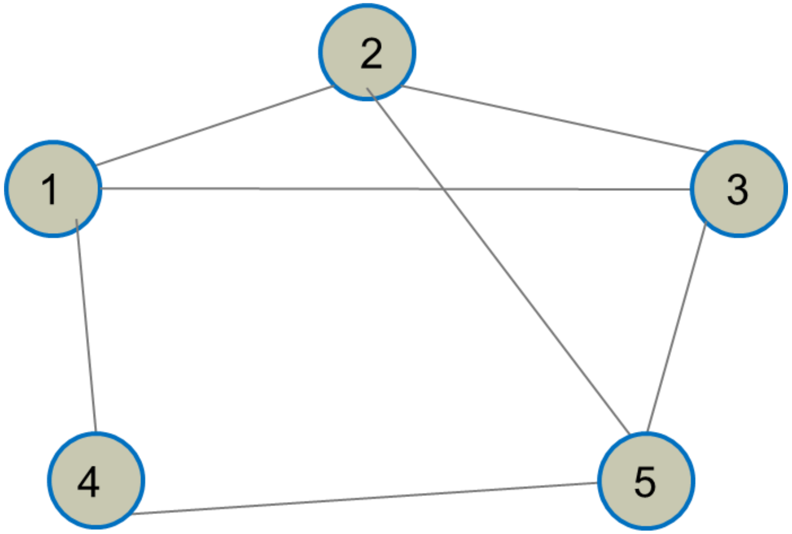

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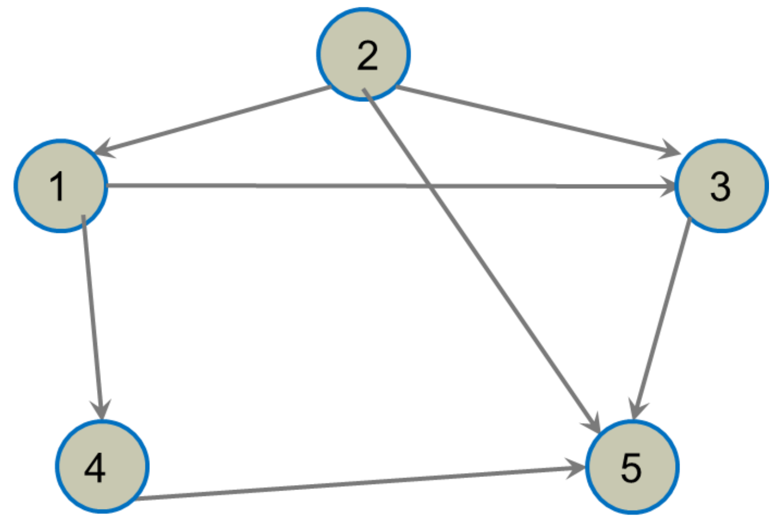

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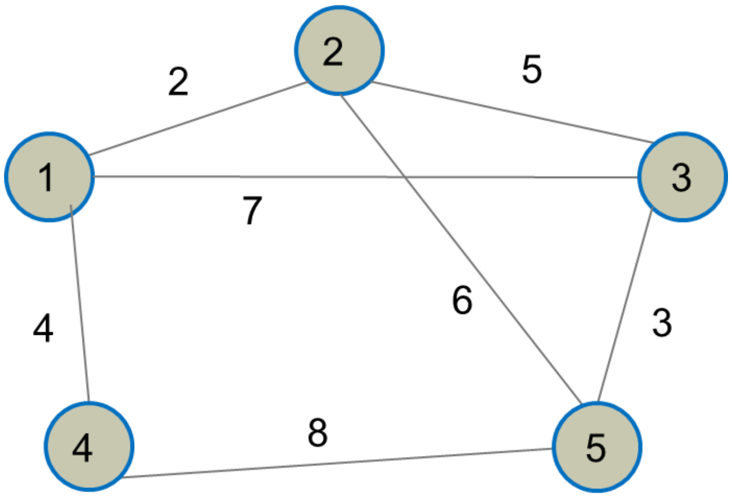
    - These links can be directed or undirected
    - These links can be weighted or unweighted



Questions?

- A graph, G=(V,E)
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Formal definitions

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- Simple graph
  - No self-edges arrow from itself to itself
  - No multi-edges between vertices
    no a to b, then b to a

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- Simple graph
  - No self-edges (known as loops also)
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  - Directed graph? V(V-1) = O(V^2)

    [V] * ([V] -1)

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  undirected graph [V] = [E]
- Maximum number of edges?
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- A graph is called sparse if $E \ll V^2$
- A graph is called dense if $E \approx V^2$

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Graph

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  - The World Wide Web (WWW) are in fact a graph
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    - Google's PageRank is a graph algorithm
      - Traversal through webpages and propagate authority
      - You can code it yourself, it is easy!

Questions?

Graph
Representation

- How do we represent graphs?

Graph
Representation

- How do we represent graphs? 2 possible way!

Graph
Representation

- How do we represent graphs? 2 possible way!
  - Adjacency matrix
  - Adjacency list prefer this a lot of time

- Adjacency matrix
  - Store edge information in a matrix

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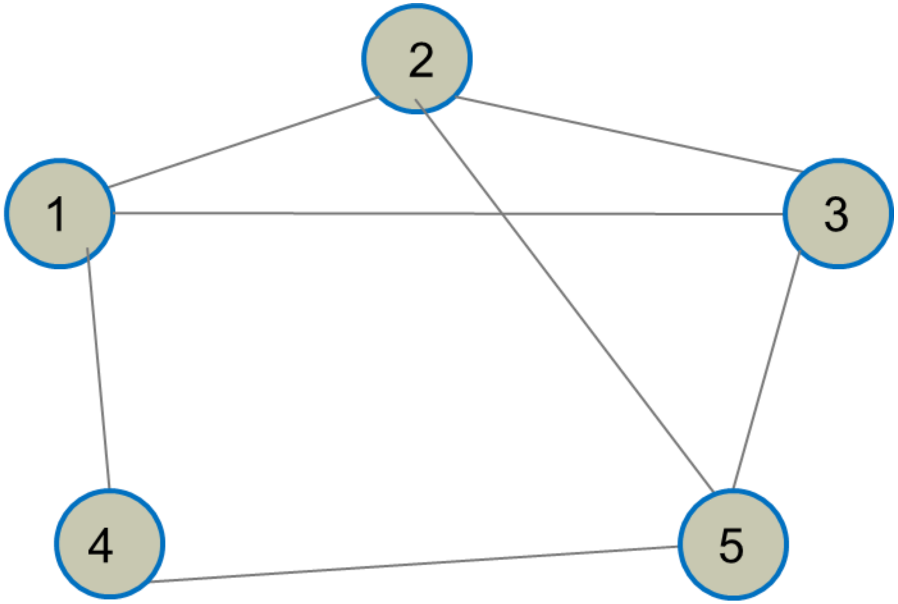
    - True/ False or 1/0 for unweighted

	1	2	3	4	5
1	F	T	T	T	F
2	T	F	T	F	T
3	T	T	F	F	T
4	T	F	F	F	T
5	F	T	T	T	F



Graph Representation

- Adjacency matrix
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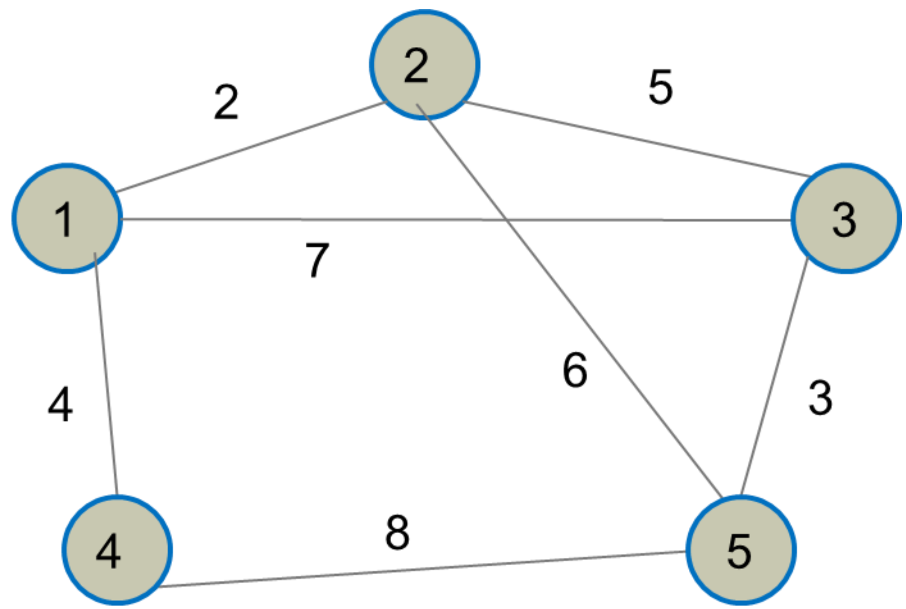
	1	2	3	4	5
1		2	7	4	
2	2		5		6
3	7	5			3
4	4				8
5		6	3	8	



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    good-dense graph, bad-sparse graph
    - O(V^2) as we need the matrix no matter what
  - Time complexity?
    - O(1) to check if an edge exist
    - O(V) to check/ traverse all adjacent vertices
      - Adjacent = neighbour

        have a edge

- Adjacency list

Graph
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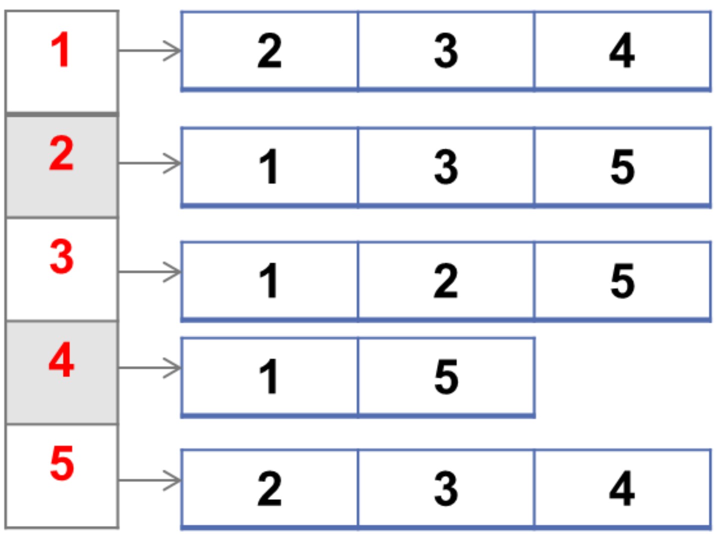
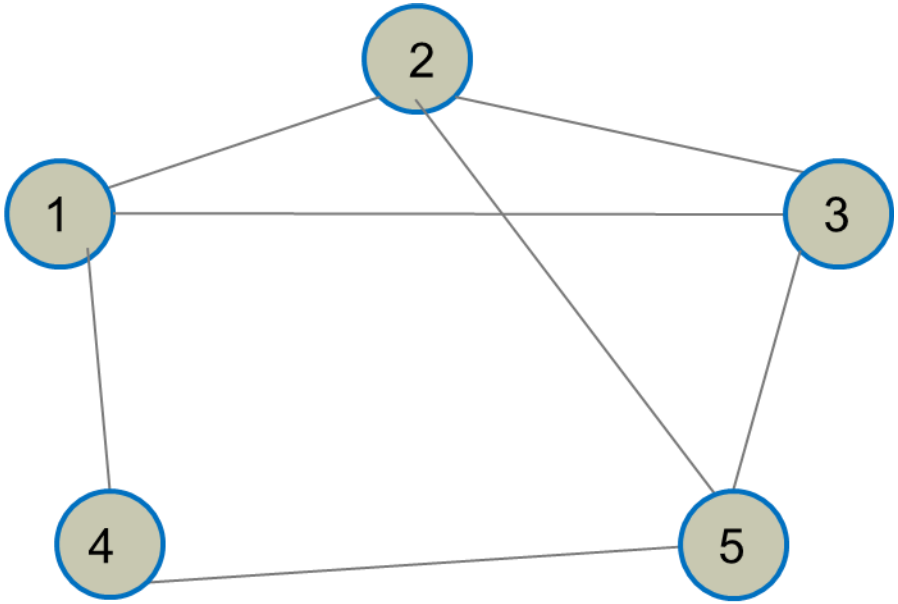
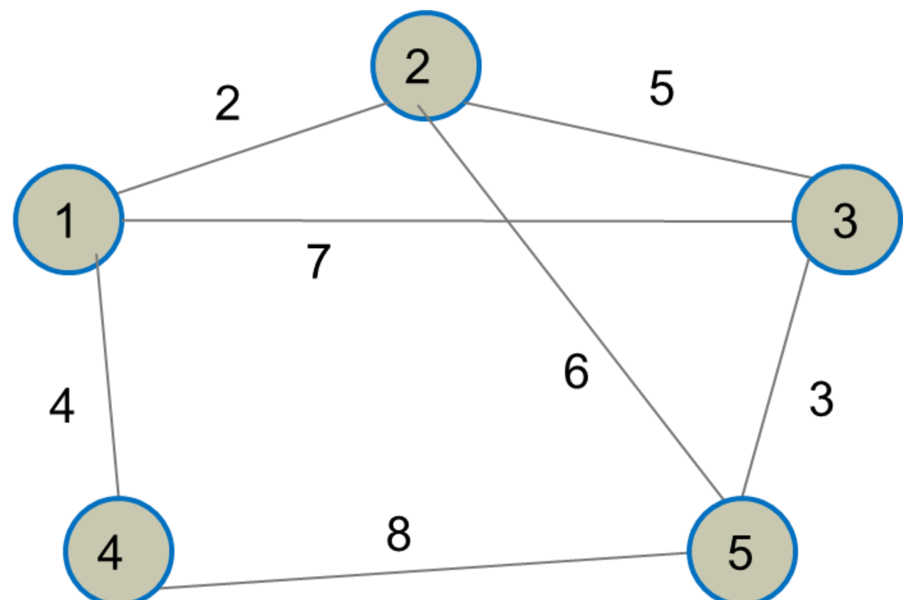




- Adjacency list
  - Array to store all Vertex object
  - Each vertex store a list of edges from it
    - With the weights

1	2 (2)	3 (7)	4 (4)
2	1 (2)	3 (5)	5 (6)
3	1 (7)	2 (5)	5 (3)
4	1 (4)	5 (8)	
5	2 (6)	3 (3)	4 (8)



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  - Space complexity?
    - O(V+E). Storing V vertices (as an array) and then total of E edges
      not O(VE) E = V[V-1), only link to every other vertices
  - Time complexity?

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    - O(V+E). Storing V vertices (as an array) and then total of E edges
  - Time complexity?

    binary search
    - O(log V) to check if an edge exist if the edges are sorted
    - O(X) to retrieve all of the adjacent vertices of a vertex

      when traverse use adjacency list

    since graph can be sparse, adjacency matrix would need to loop through until biggest vertex
    but Adjacency List has length of list, number of links to other vertices

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  - Array to store all Vertex object
  - Each vertex store a list of edges from it
    - With the weights
  - Space complexity?
    - O(V+E). Storing V vertices (as an array) and then total of E edges
  - Time complexity?
    - O(log V) to check if an edge exist if the edges are sorted
      - But you can't use binary search on linked list!
      - So this is still O(X) but you can terminate earlier once you reach a bigger vertex
    - O(X) to retrieve all of the adjacent vertices of a vertex
      - Where X = number of adjacent vertices (output-sensitive complexity)

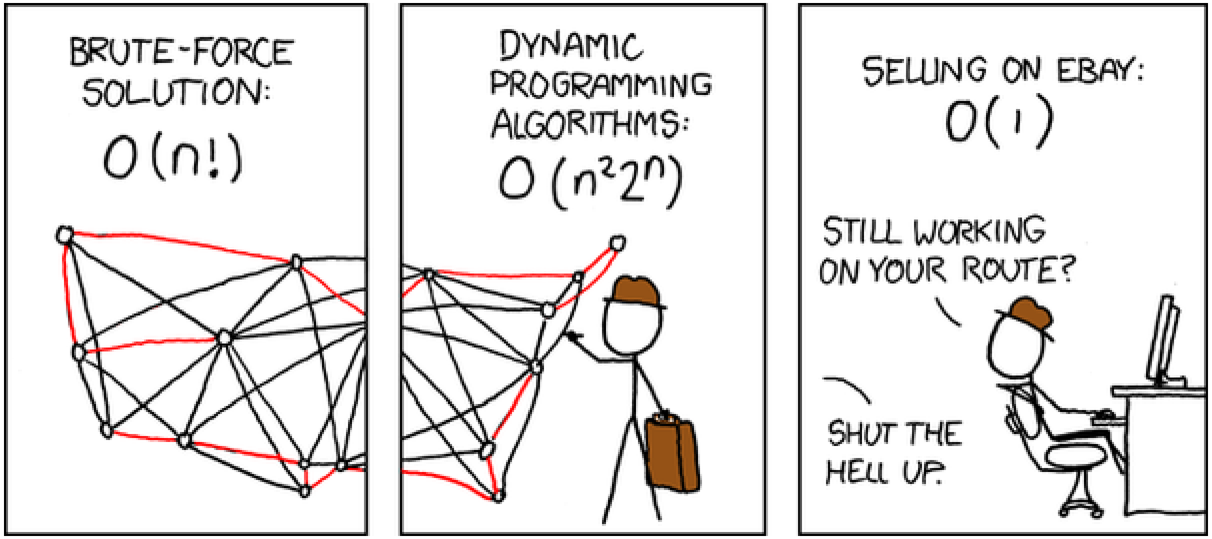
Questions?

- Going from a place to another

Graph
Traversal

- Going from a place (source vertex) to another

- Going from a place (source vertex) to another or everywhere!



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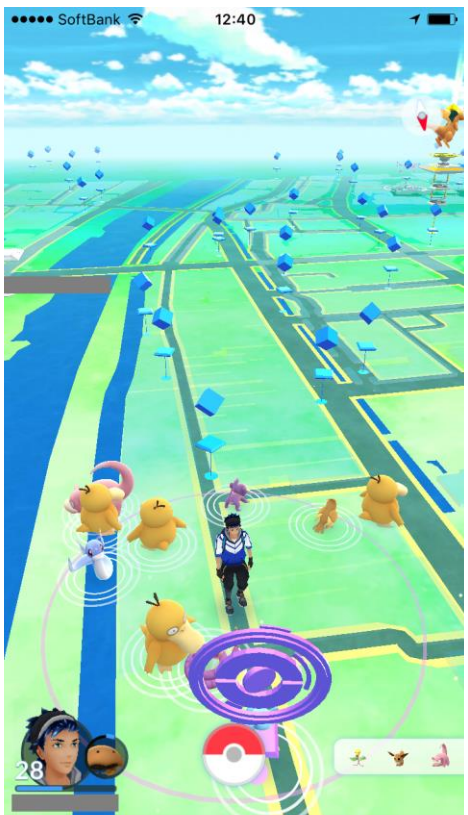

Graph
Traversal

- Breadth-First Search (BFS)
- Depth-First Search (DFS)

- Breadth-First Search (BFS)
  - Going wide
- Depth-First Search (DFS)
  - Going deep

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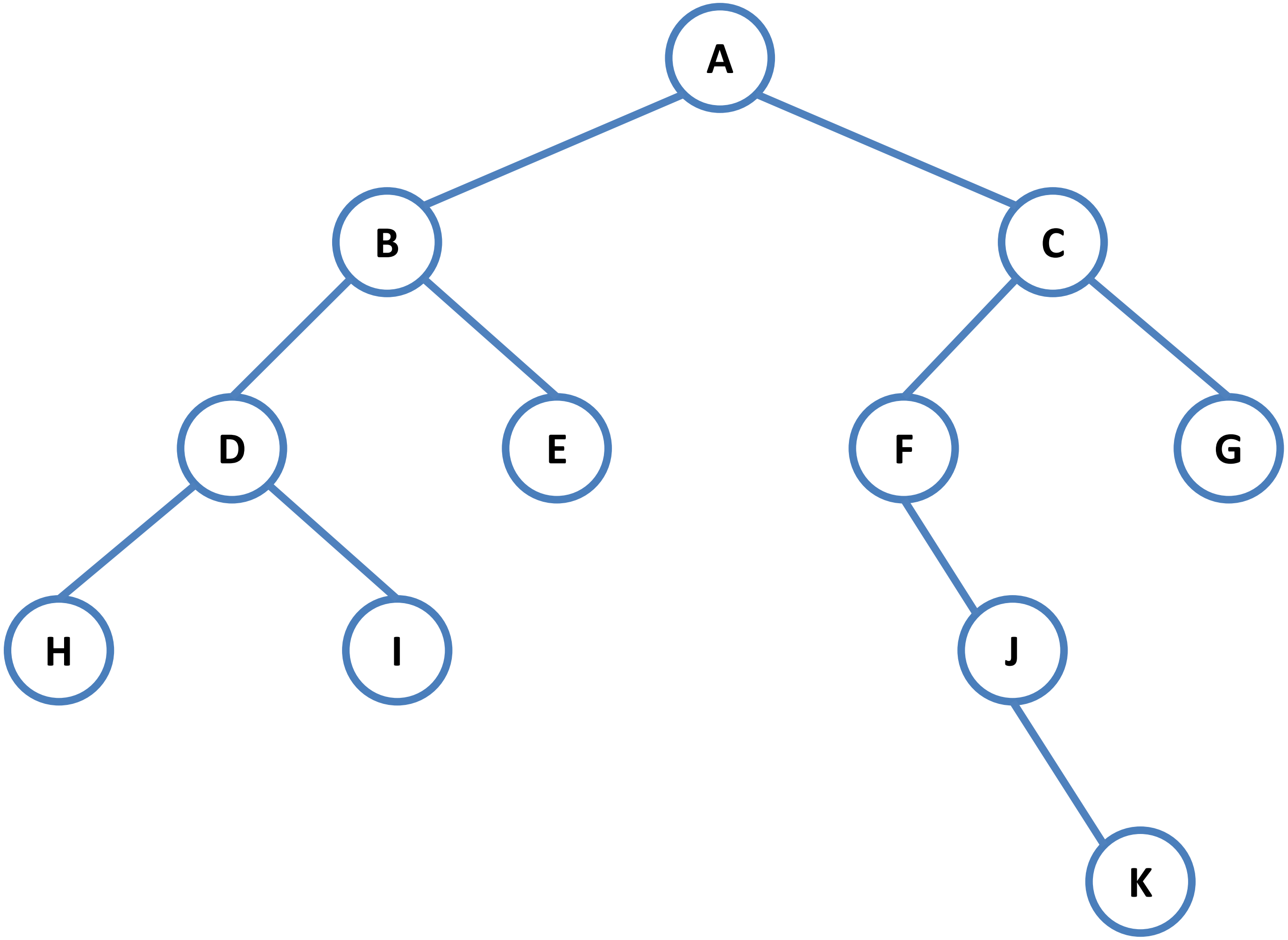
- Let us begin with a tree first

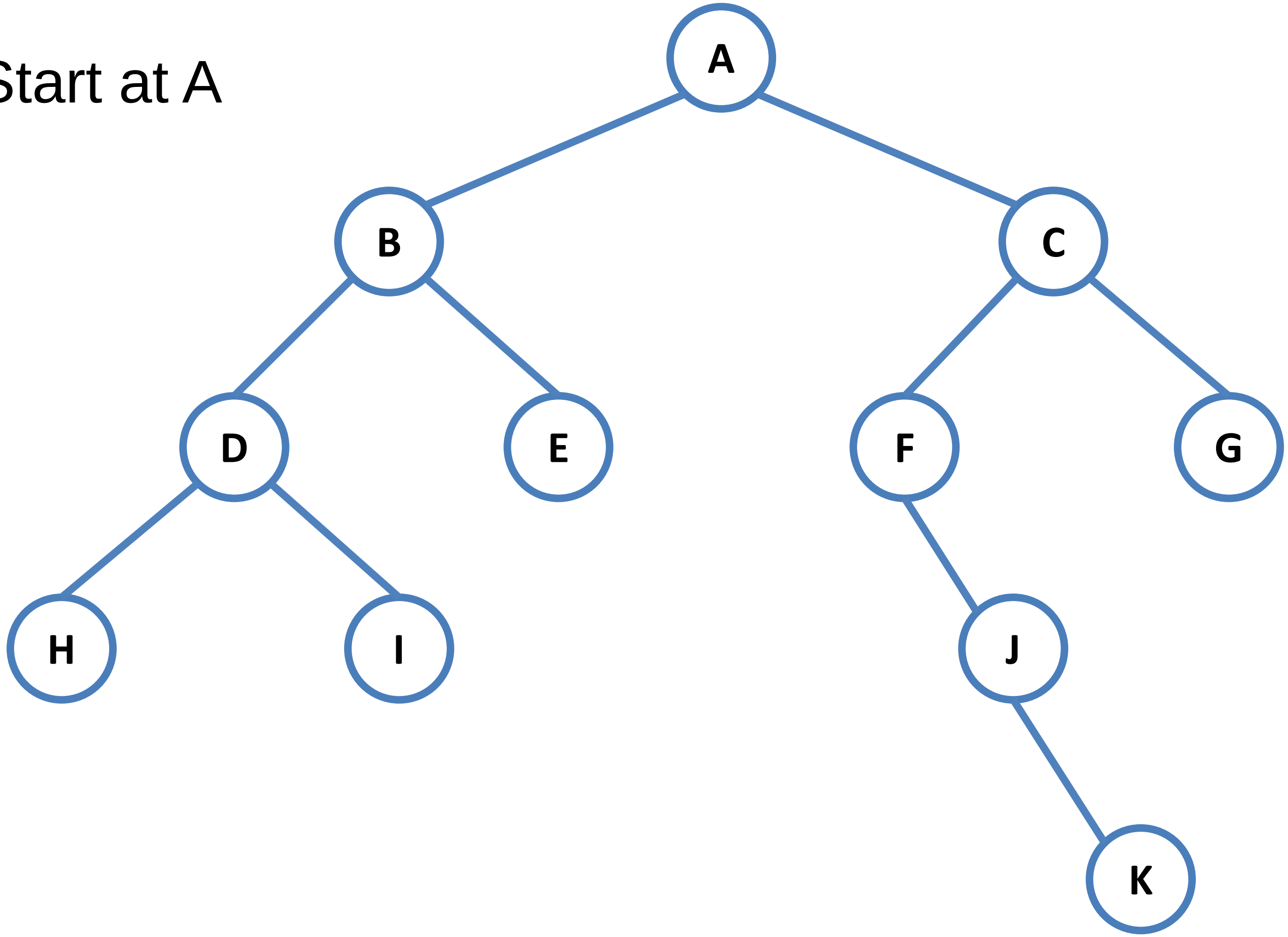
- Breadth-First Search (BFS)
  – Going wide
- Depth-First Search (DFS)
  – Going deep

- Let us begin with a tree first
  – Recall a tree is a graph without cycles

Graph
Traversal



- Start at A



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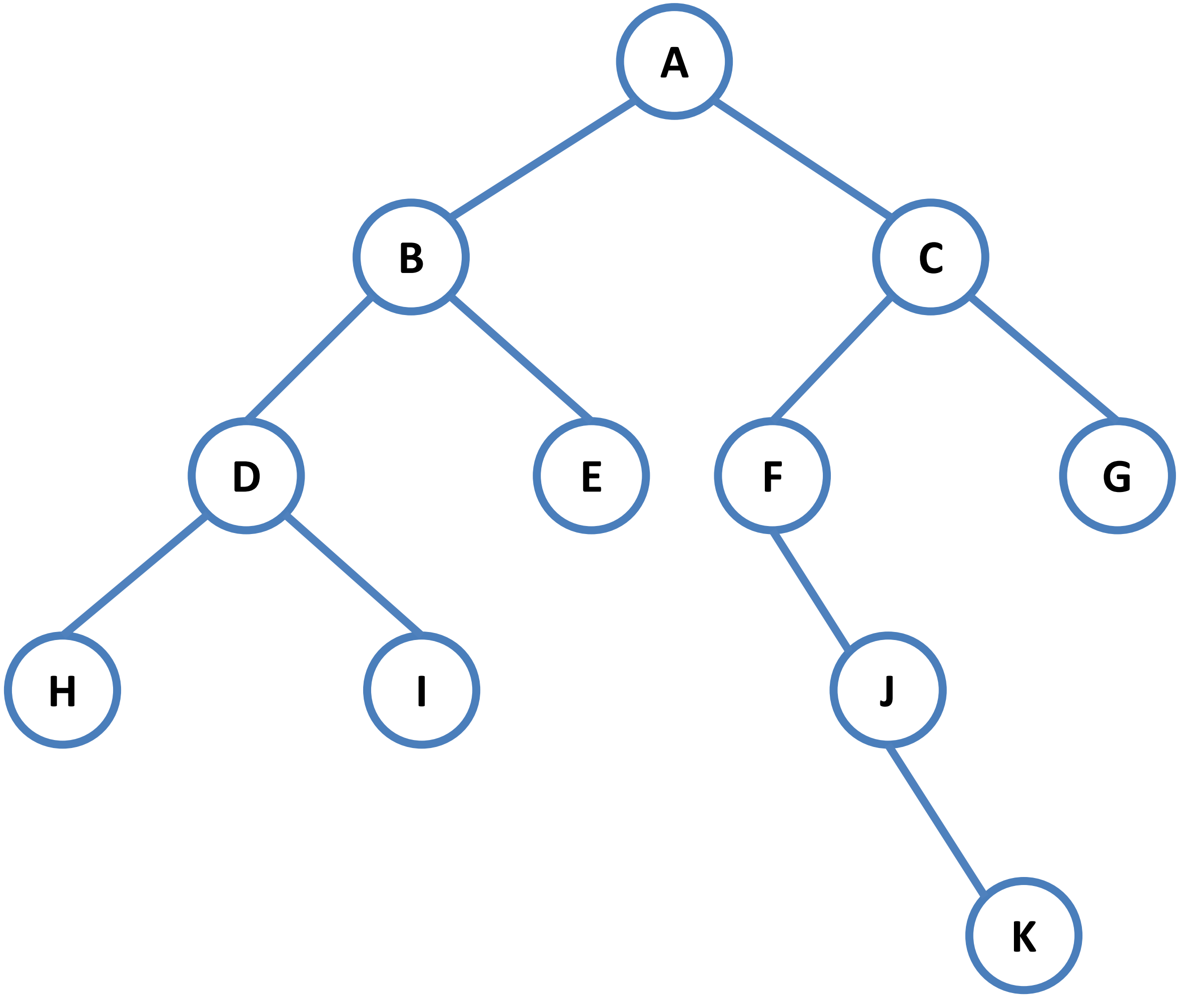
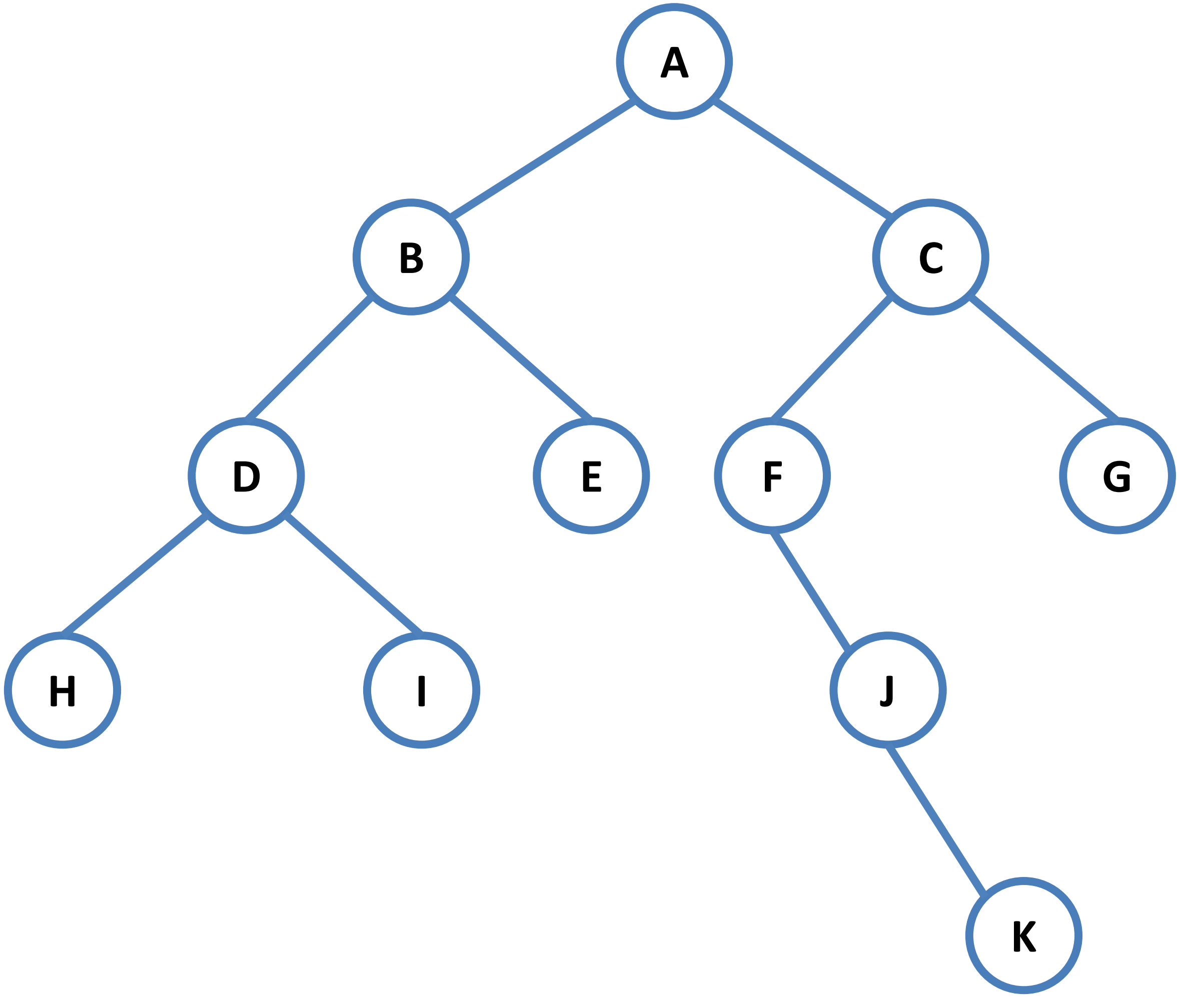
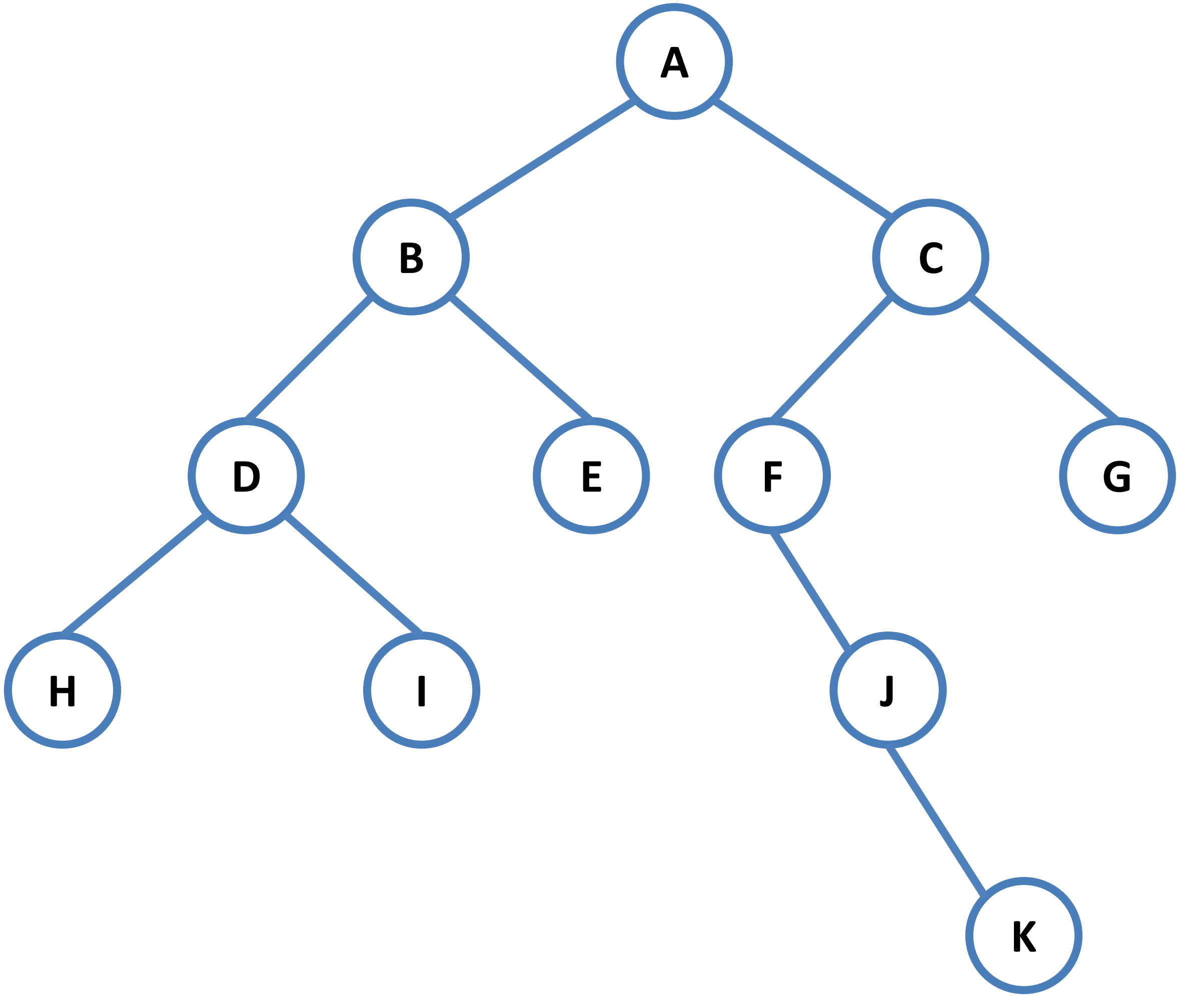


- Start at A, BFS



Graph
Traversal

- Start at A, BFS
- A



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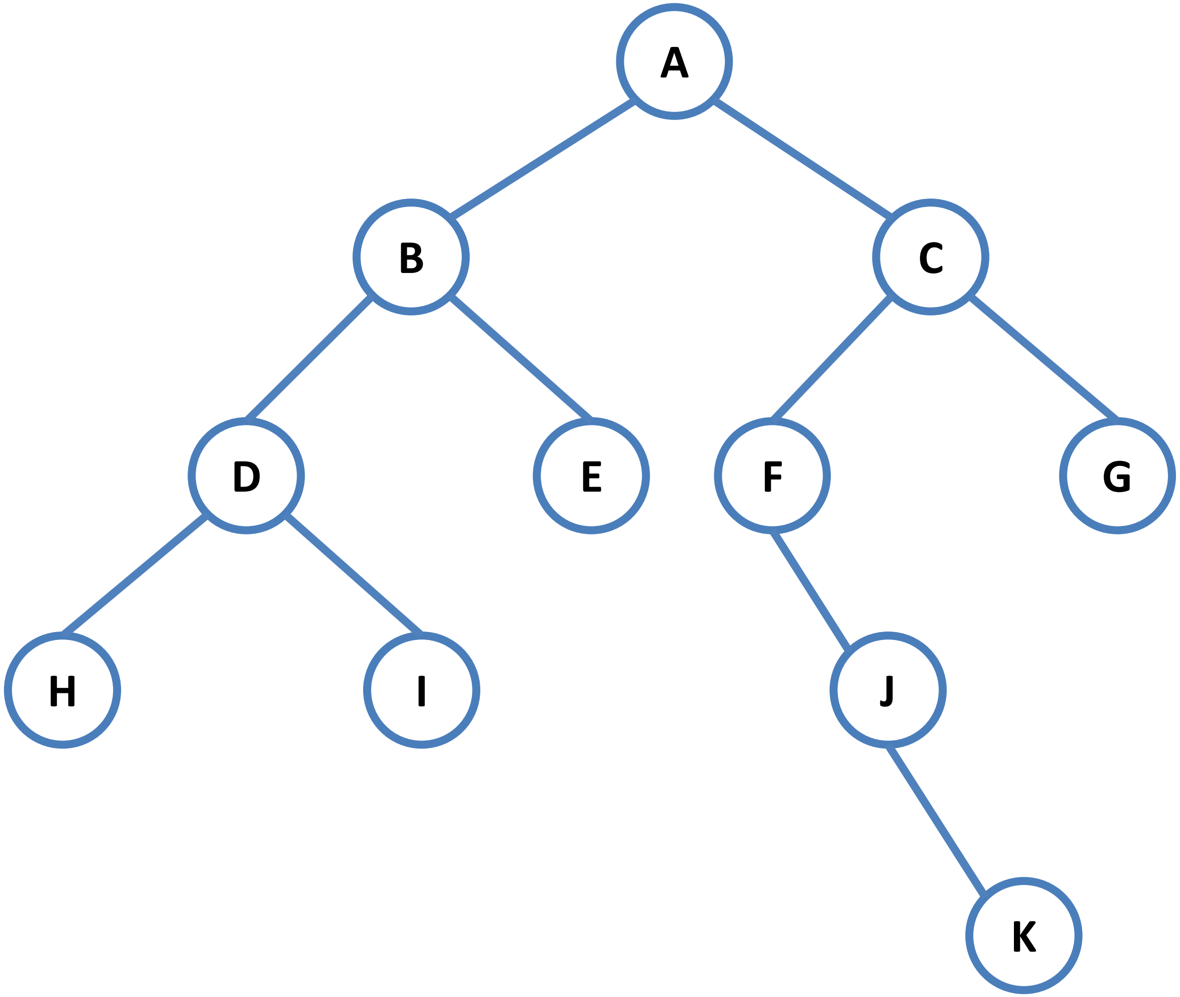
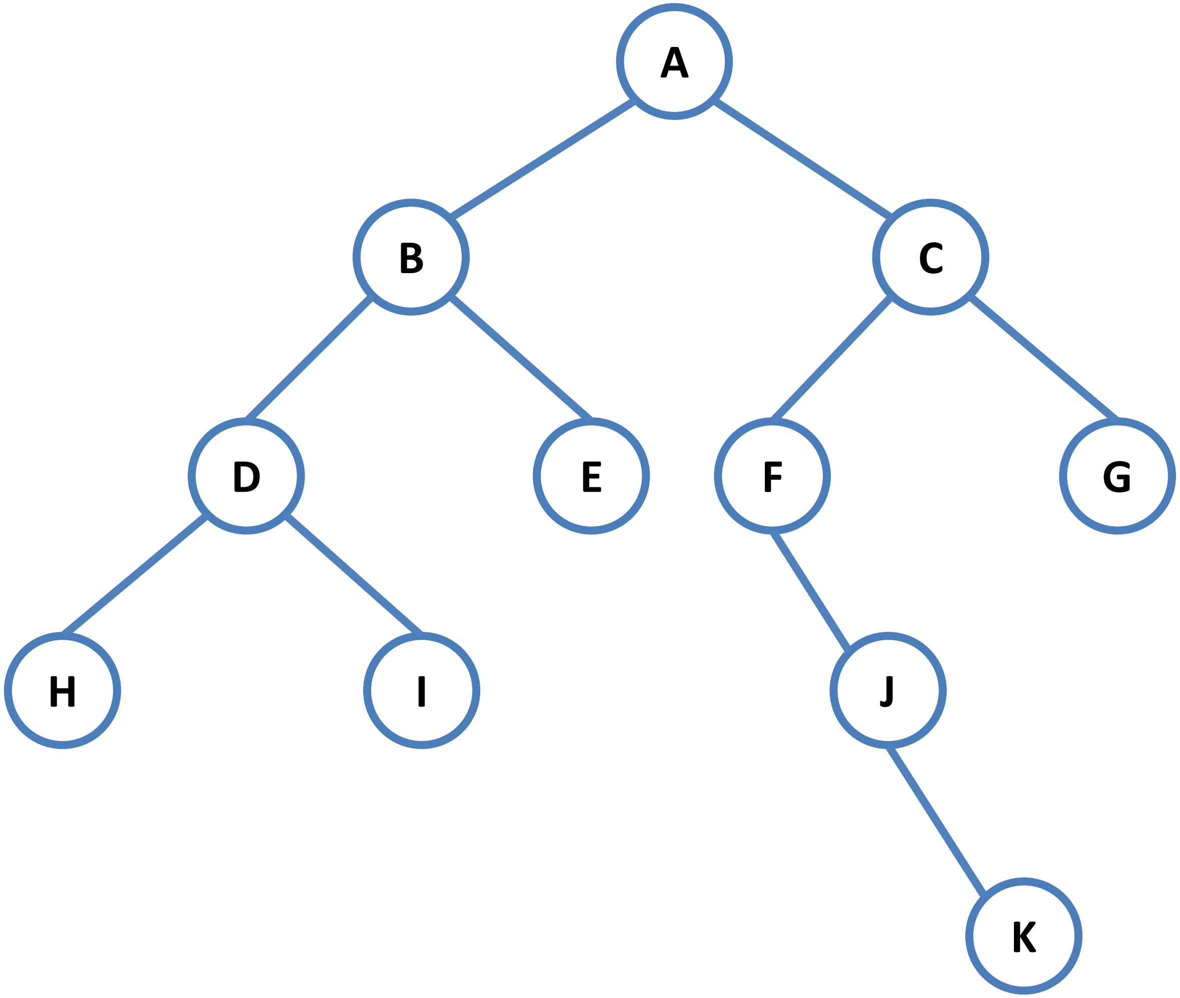
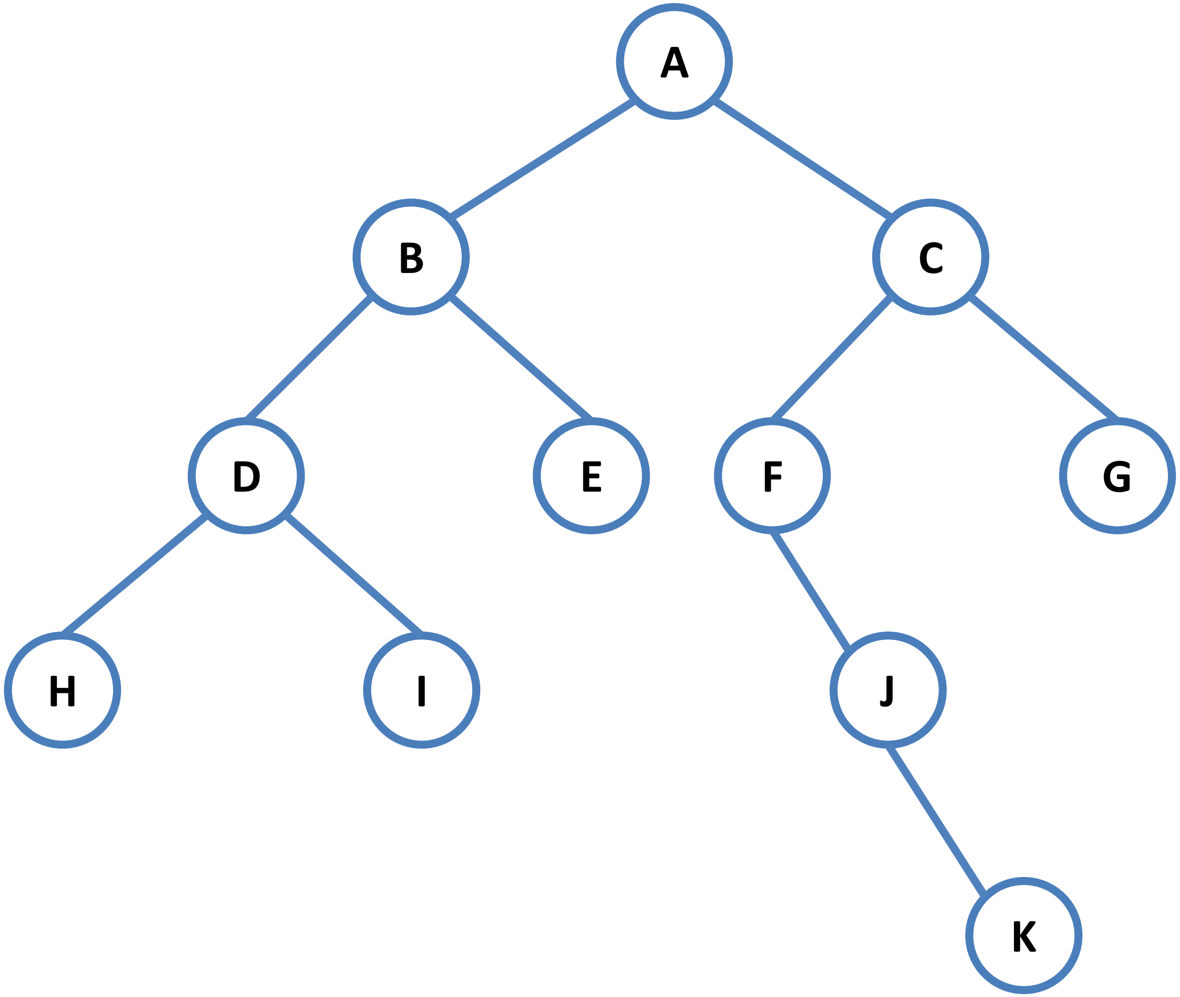
- Start at A, BFS
- A
- B



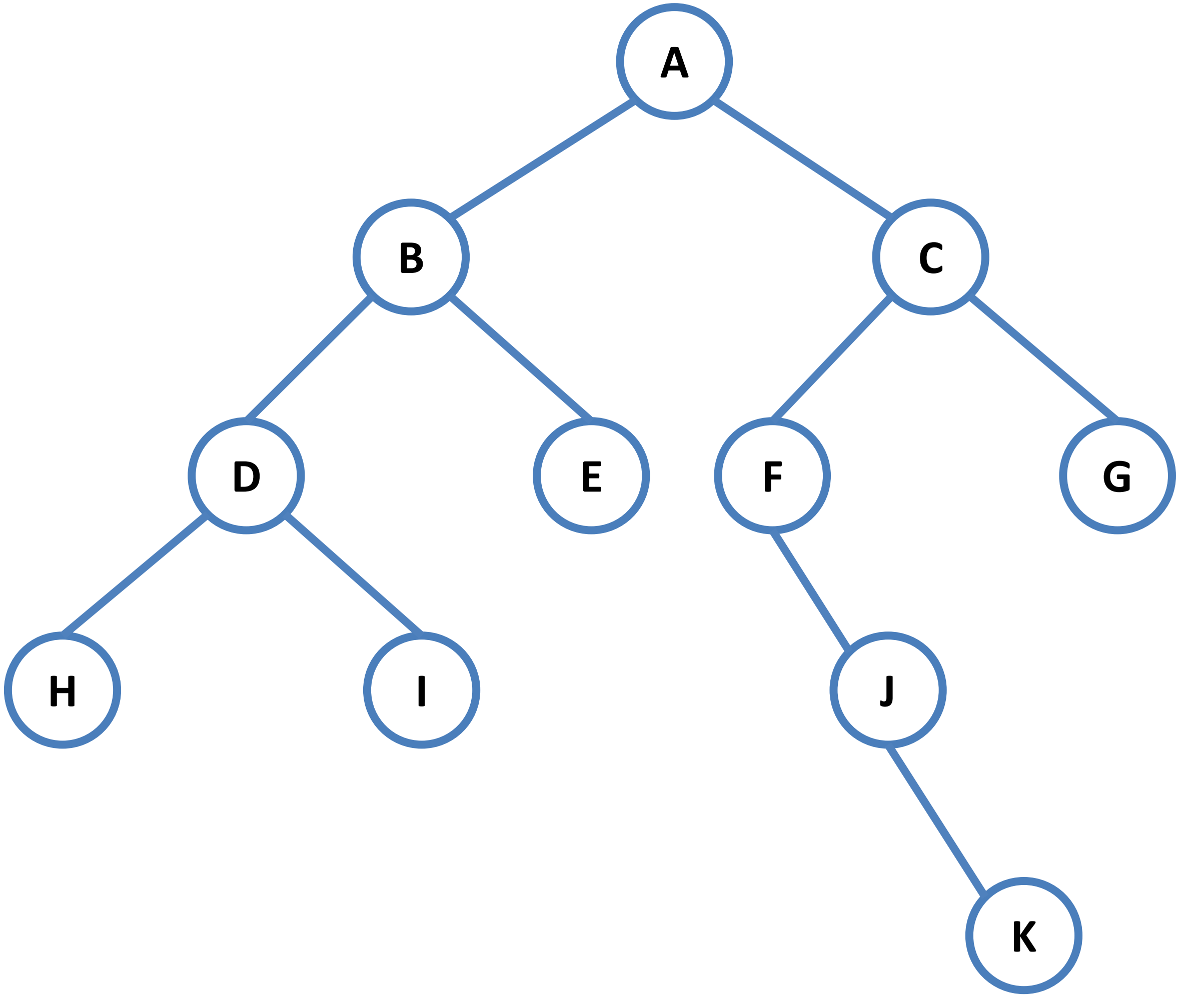
- Start at A, BFS
- A
- B
- C



- Start at A, BFS
- A
- B
- C
- D



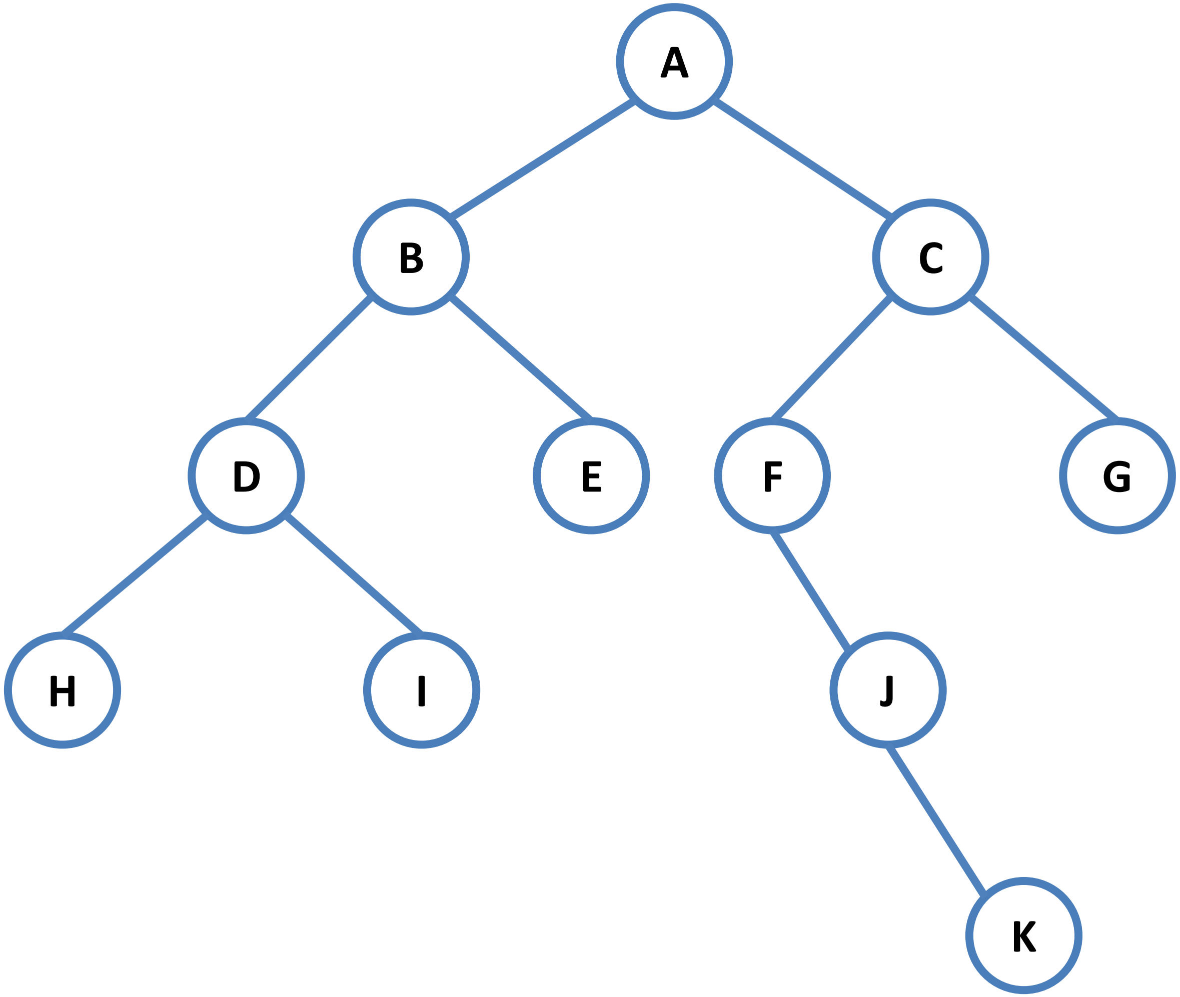
- Start at A, BFS
- A
- B
- C
- D
- E



Graph

Traversal

- Start at A, BFS
- A
- B
- C
- D
- E
- F



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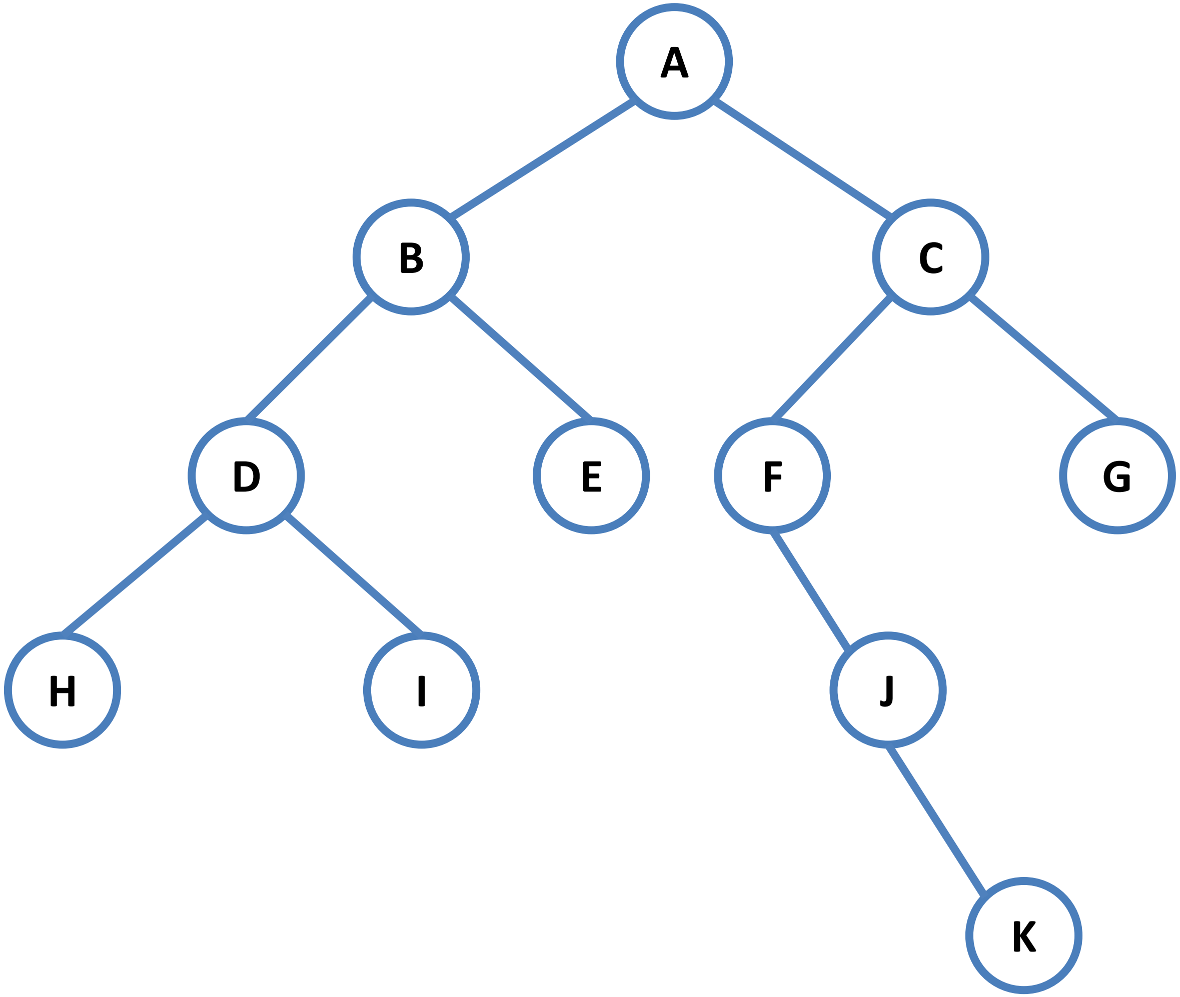
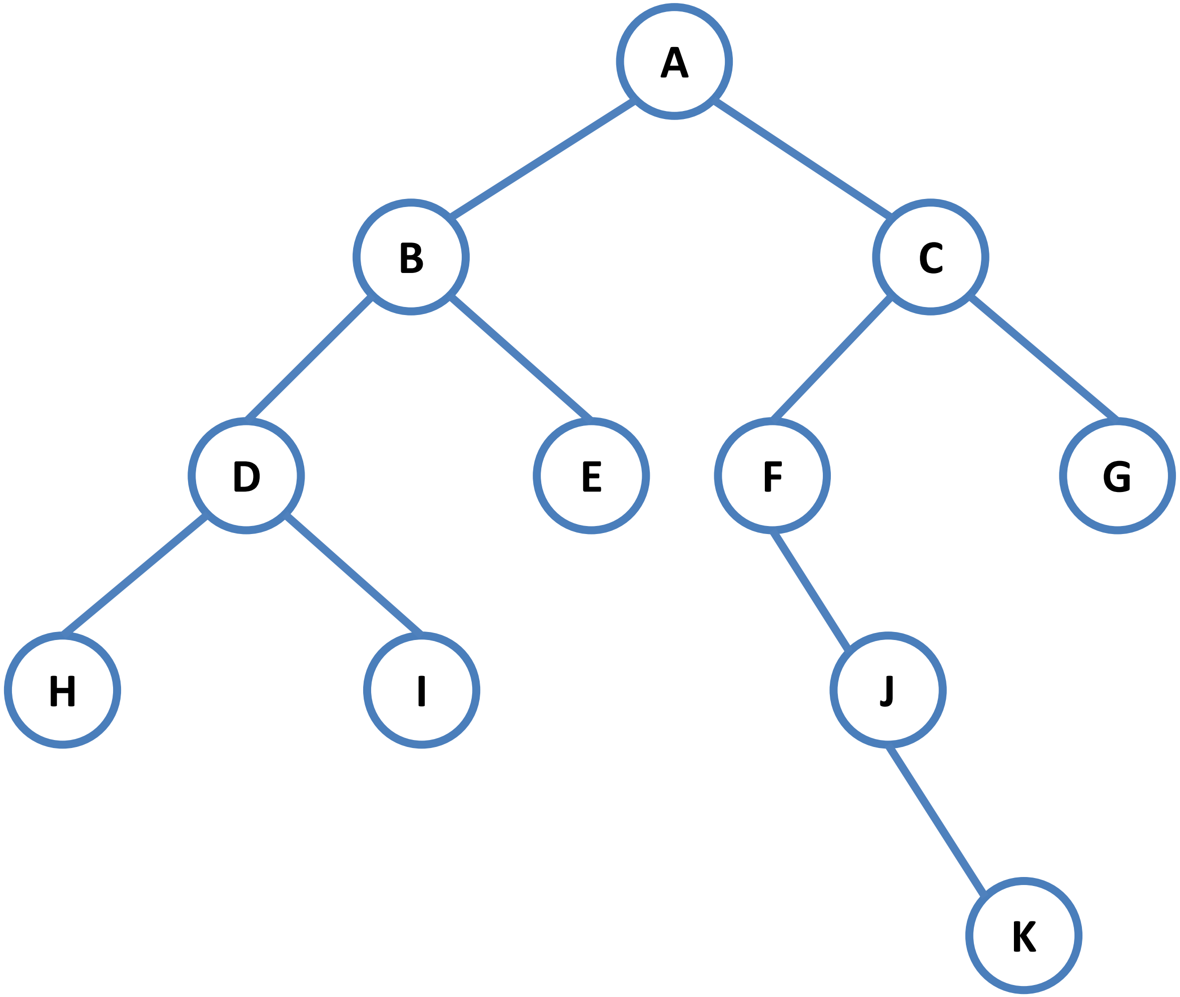
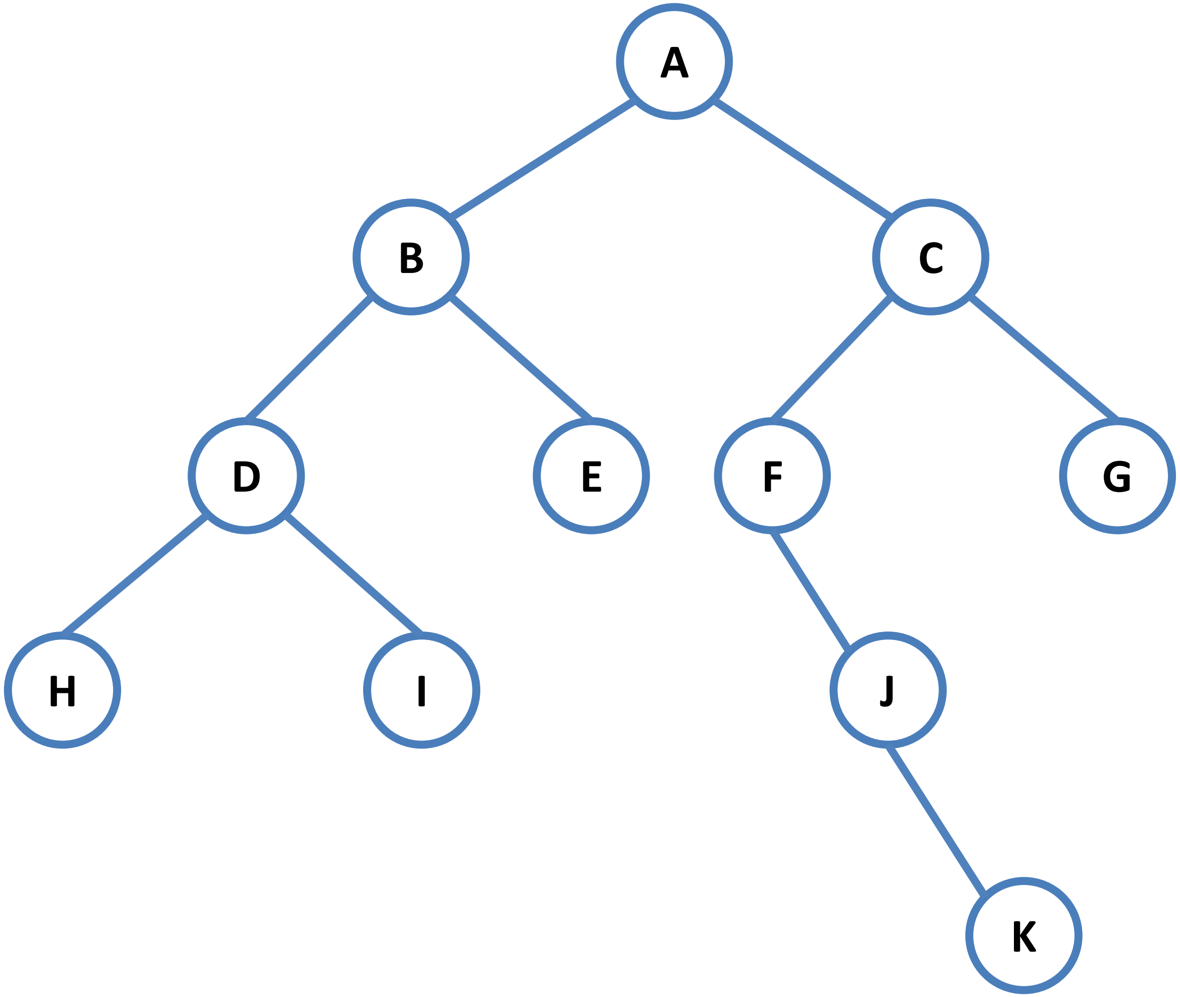
- A
- B
- C
- D
- E
- F
- G



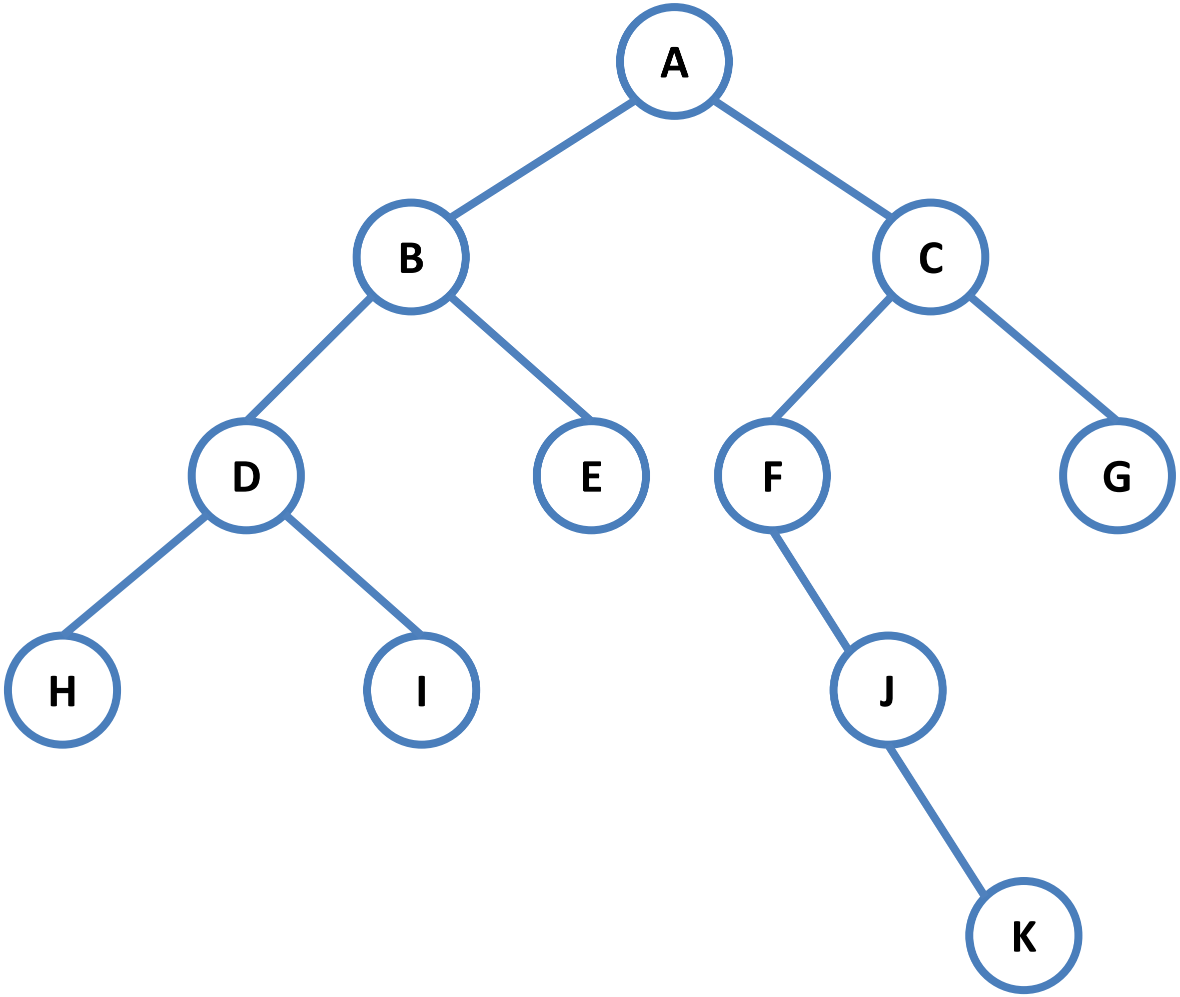
- Start at A, BFS
- A
- B
- C
- D
- E
- F
- G
- … and so on…



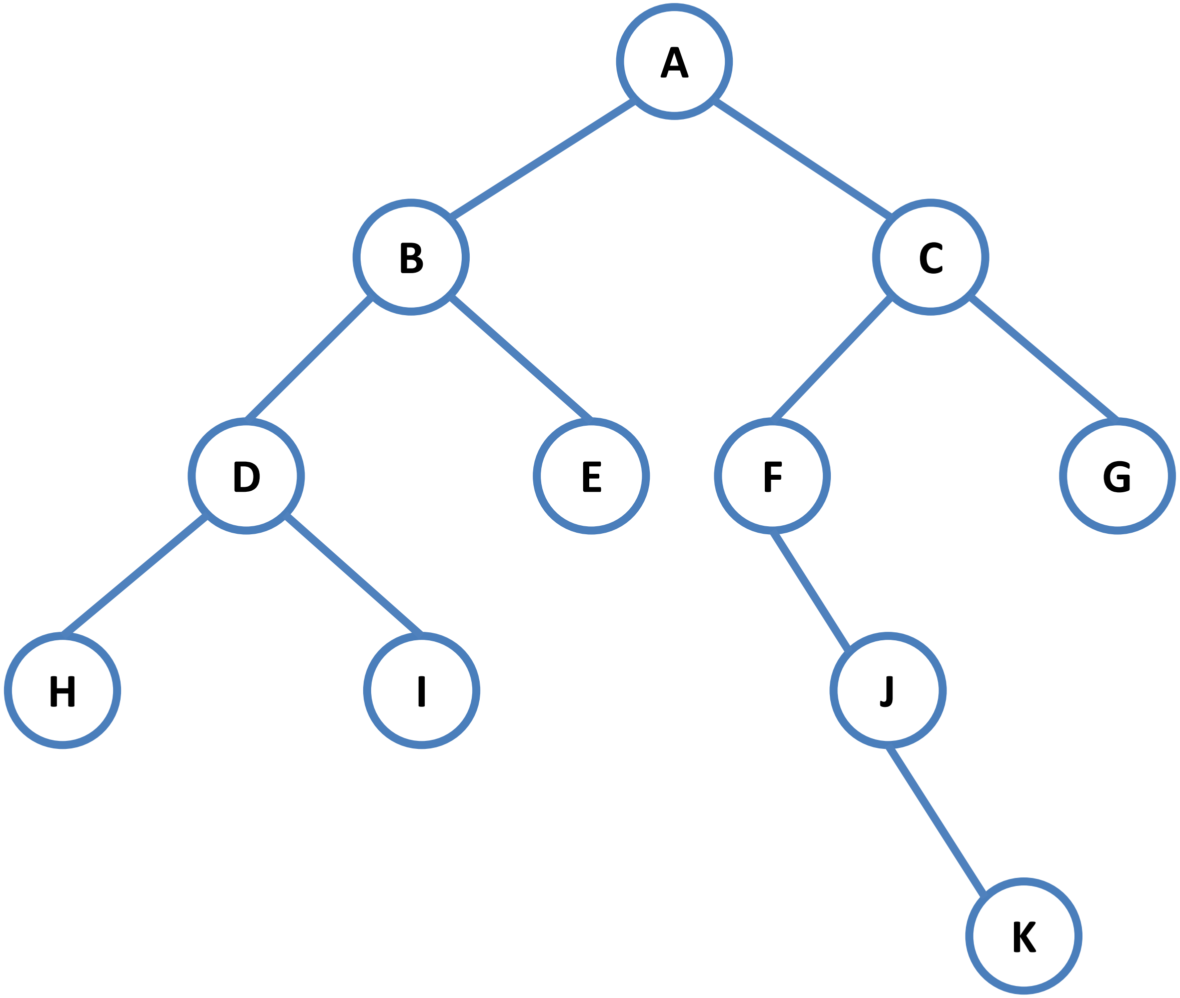
- Start at A



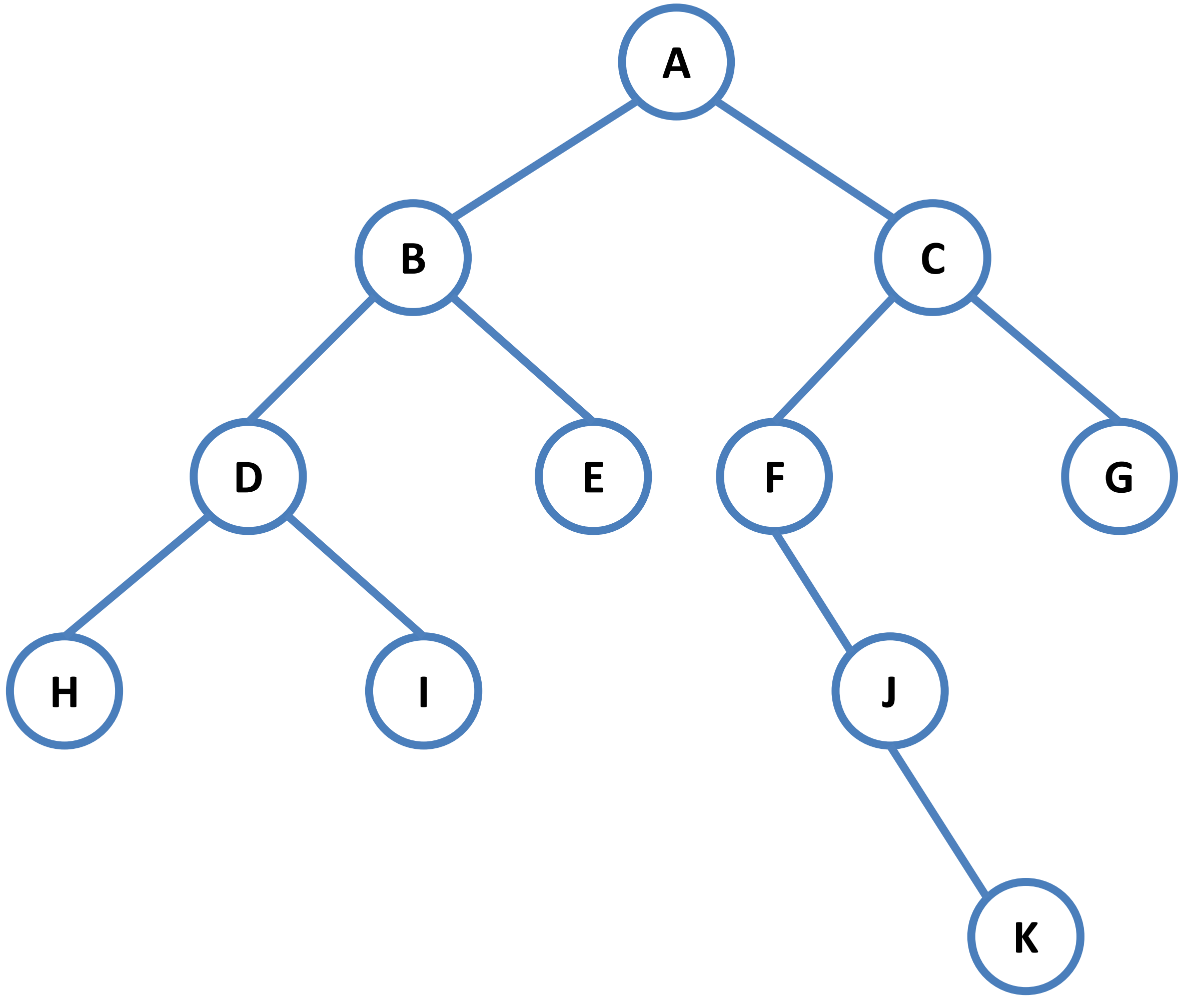
- Start at A, DFS



- Start at A, DFS
- A

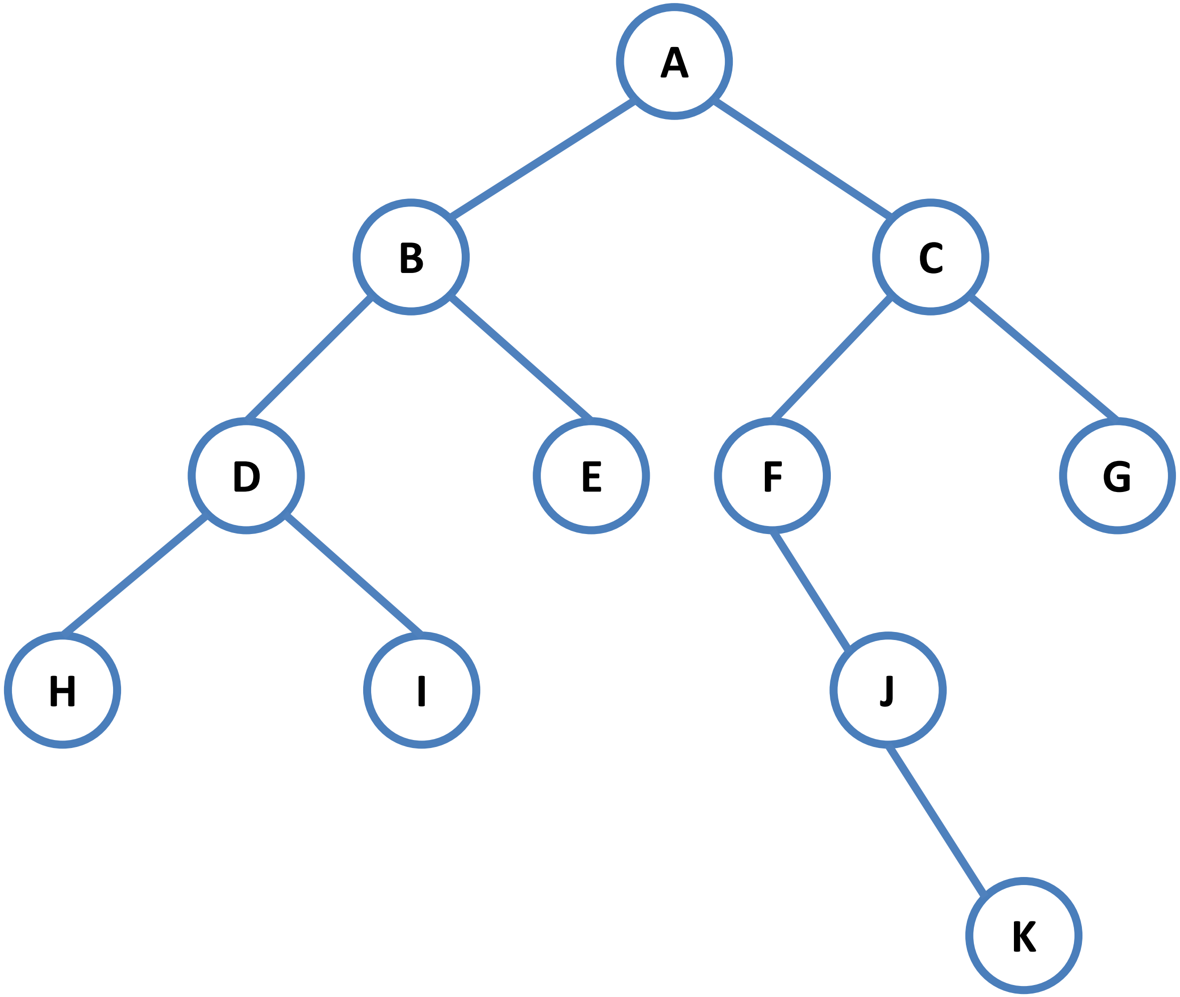


- Start at A, DFS
- A
- B

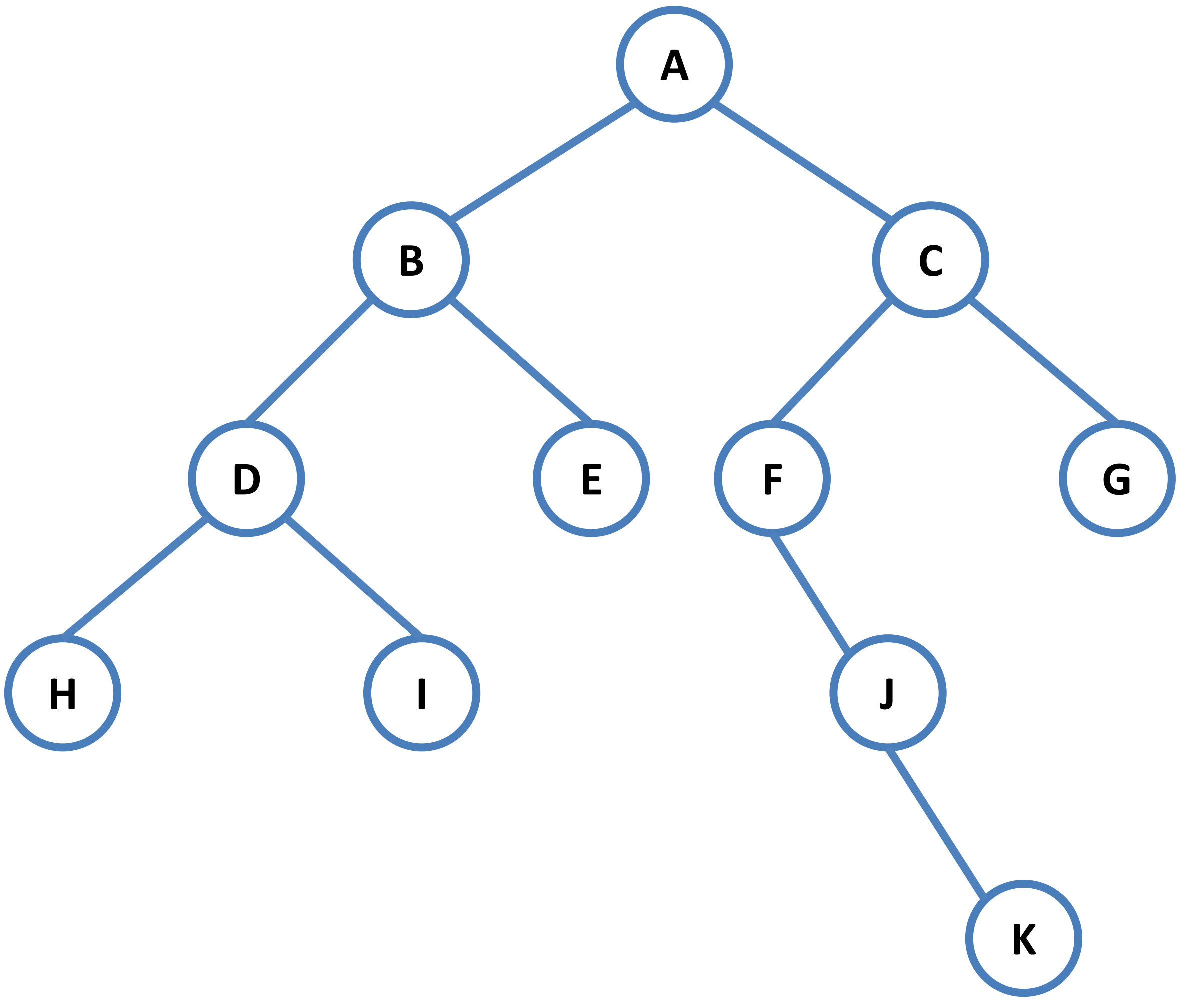


Graph
Traversal

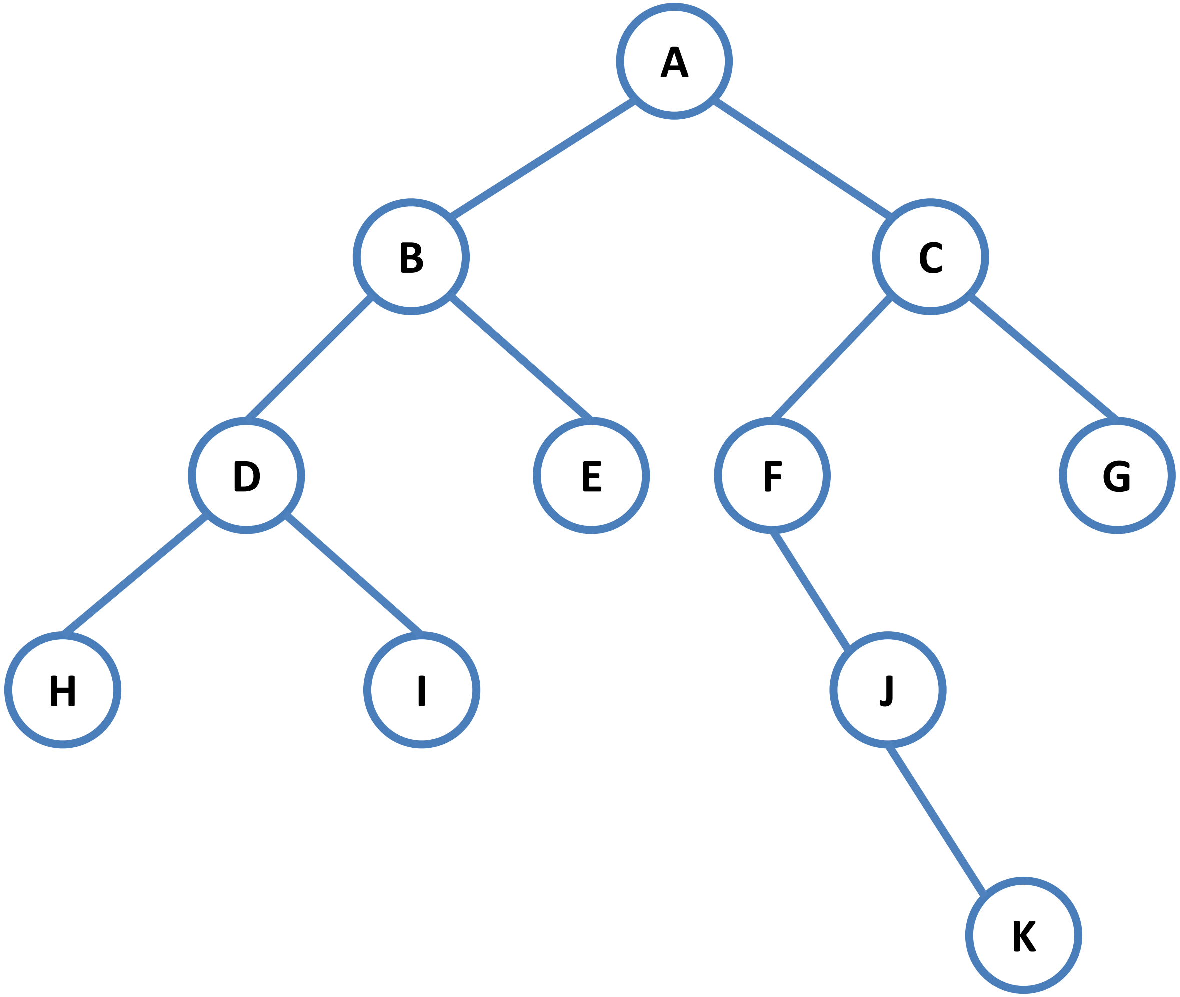
- Start at A, DFS
- A
- B
- D, go deep!



- Start at A, DFS
- A
- B
- D
- H

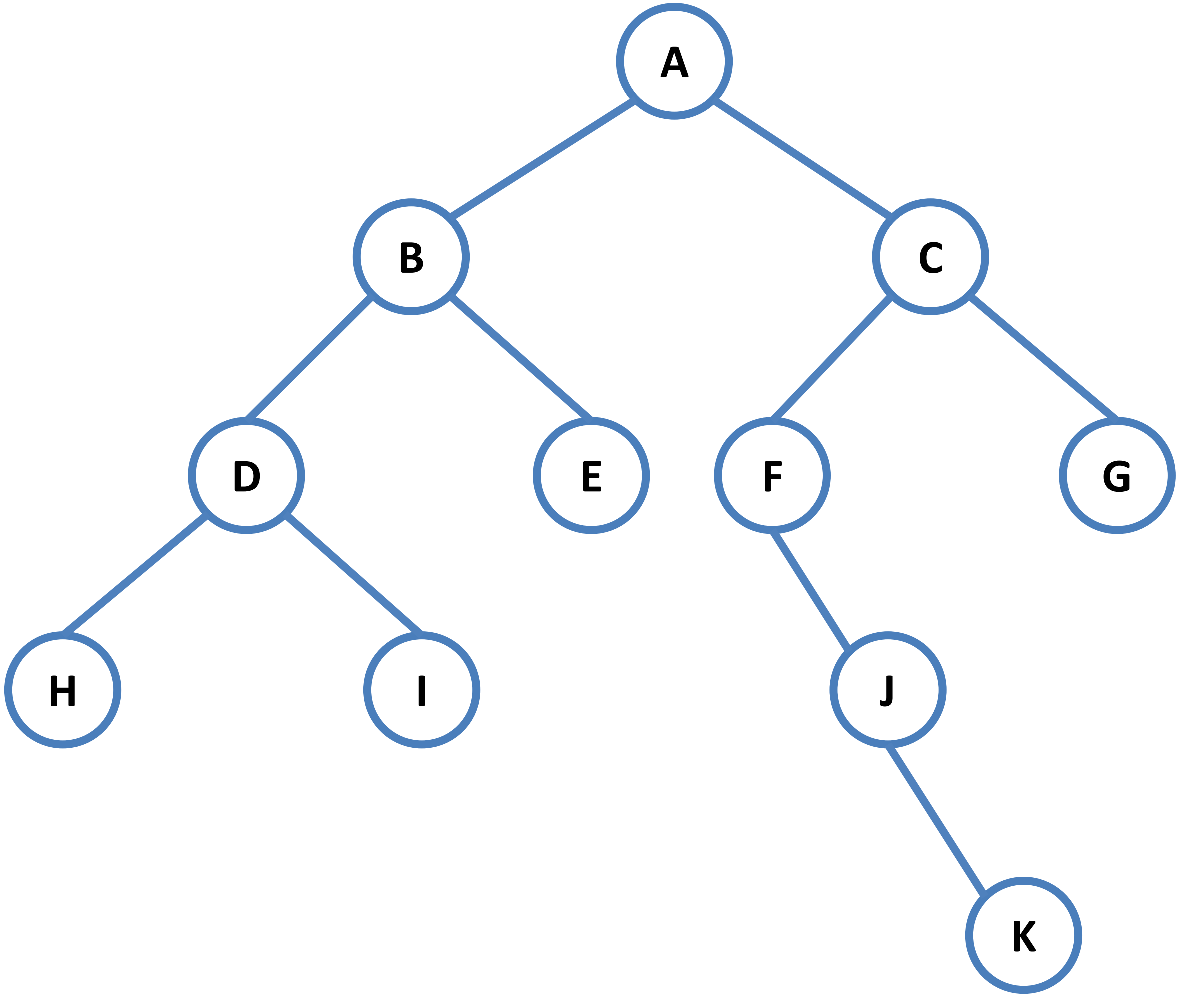


- Start at A, DFS
- A
- B
- D
- H
- can't go deeper

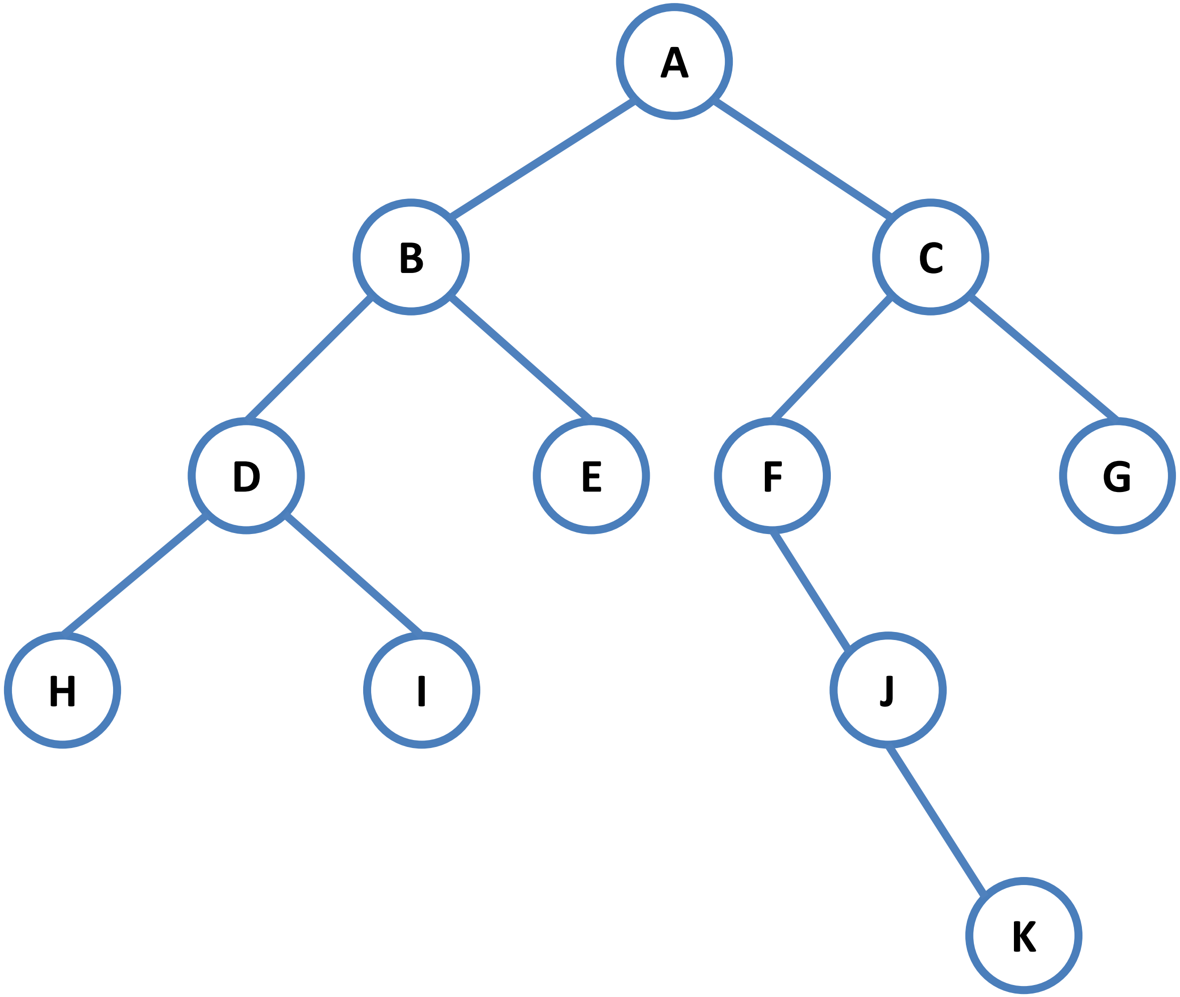


Graph
Traversal

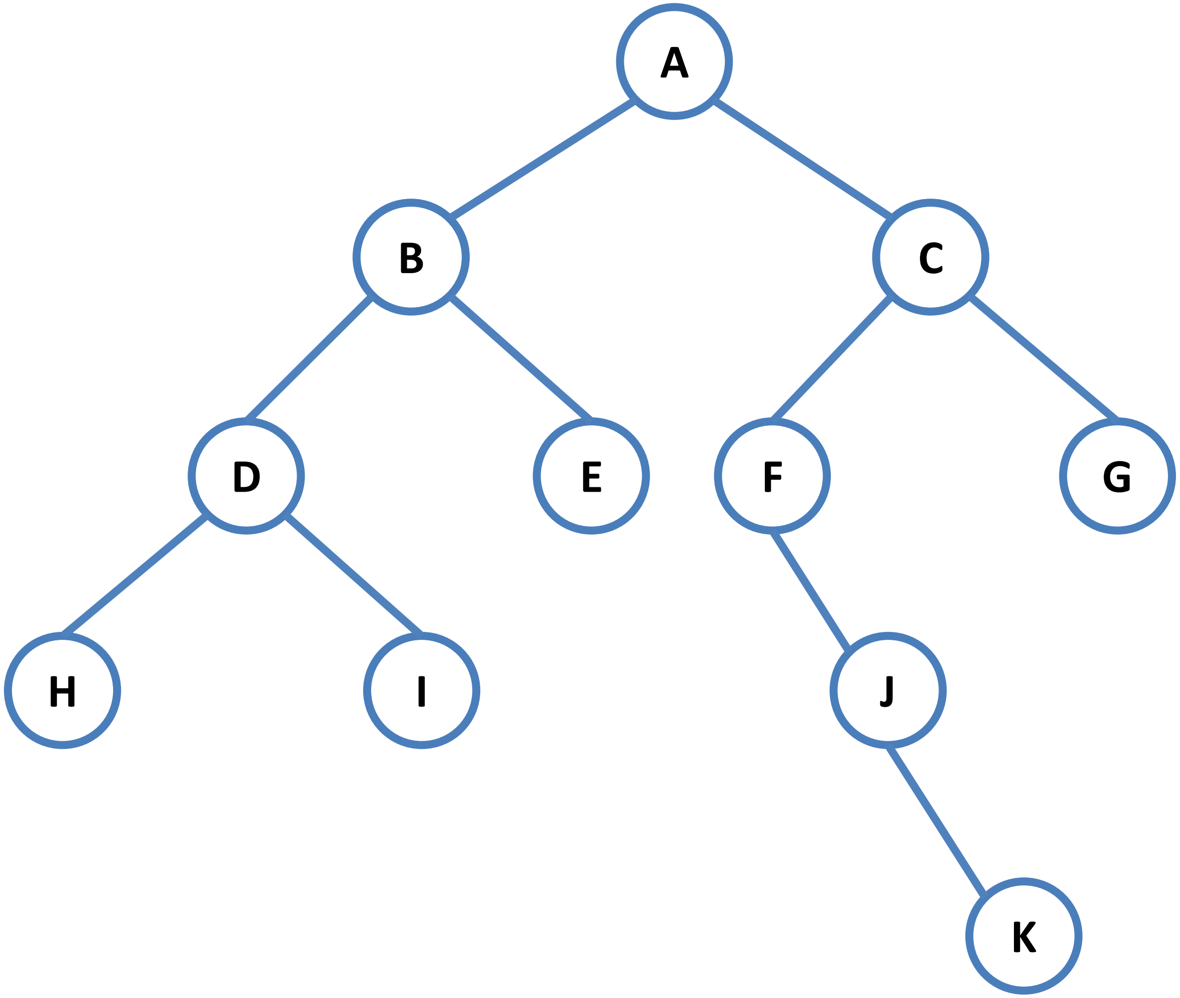
- Start at A, DFS
- A
- B
- D
- H
- I



- Start at A, DFS
- A
- B
- D
- H
- I
- E

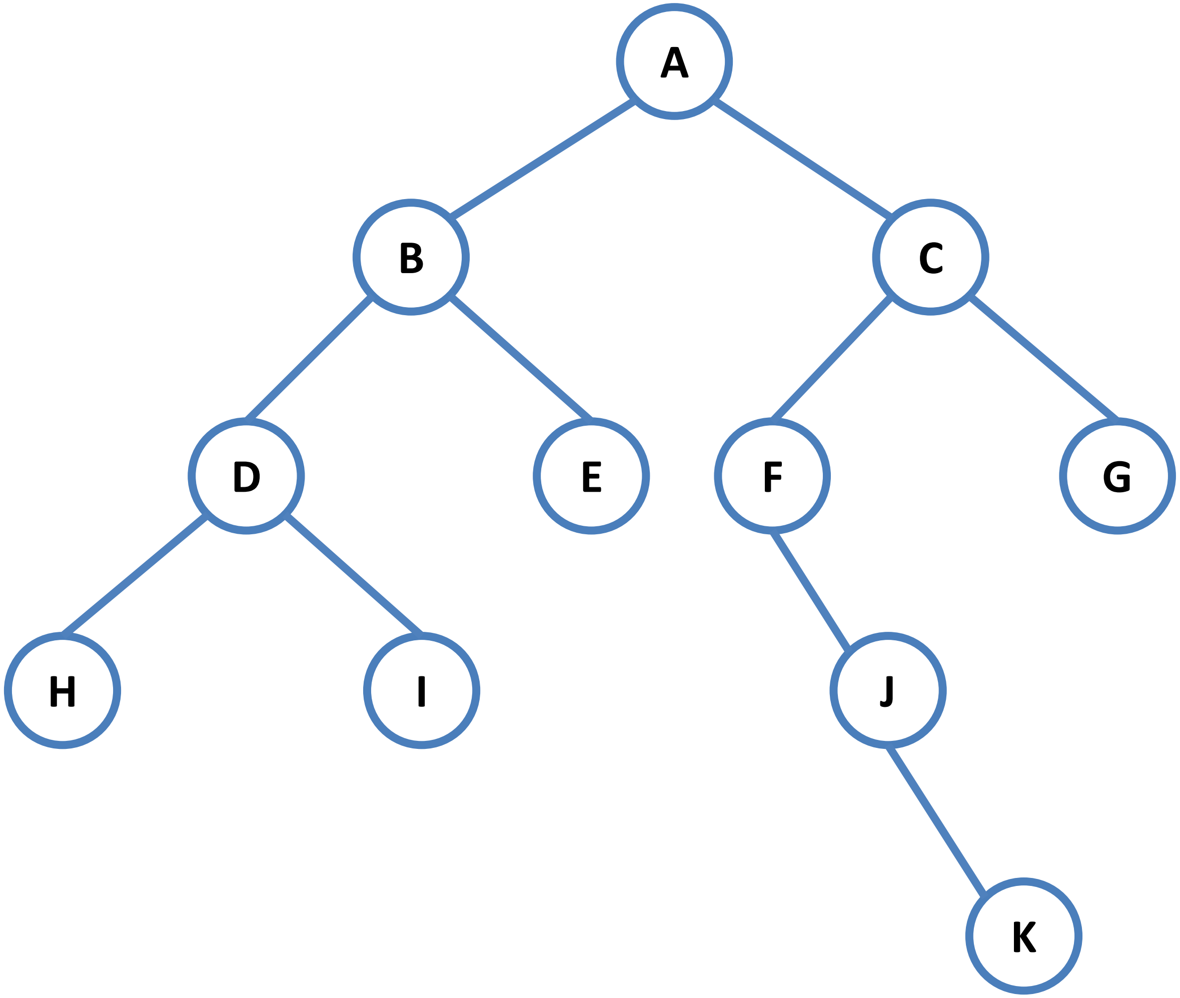


- Start at A, DFS
- A
- B
- D
- H
- I
- E
- C

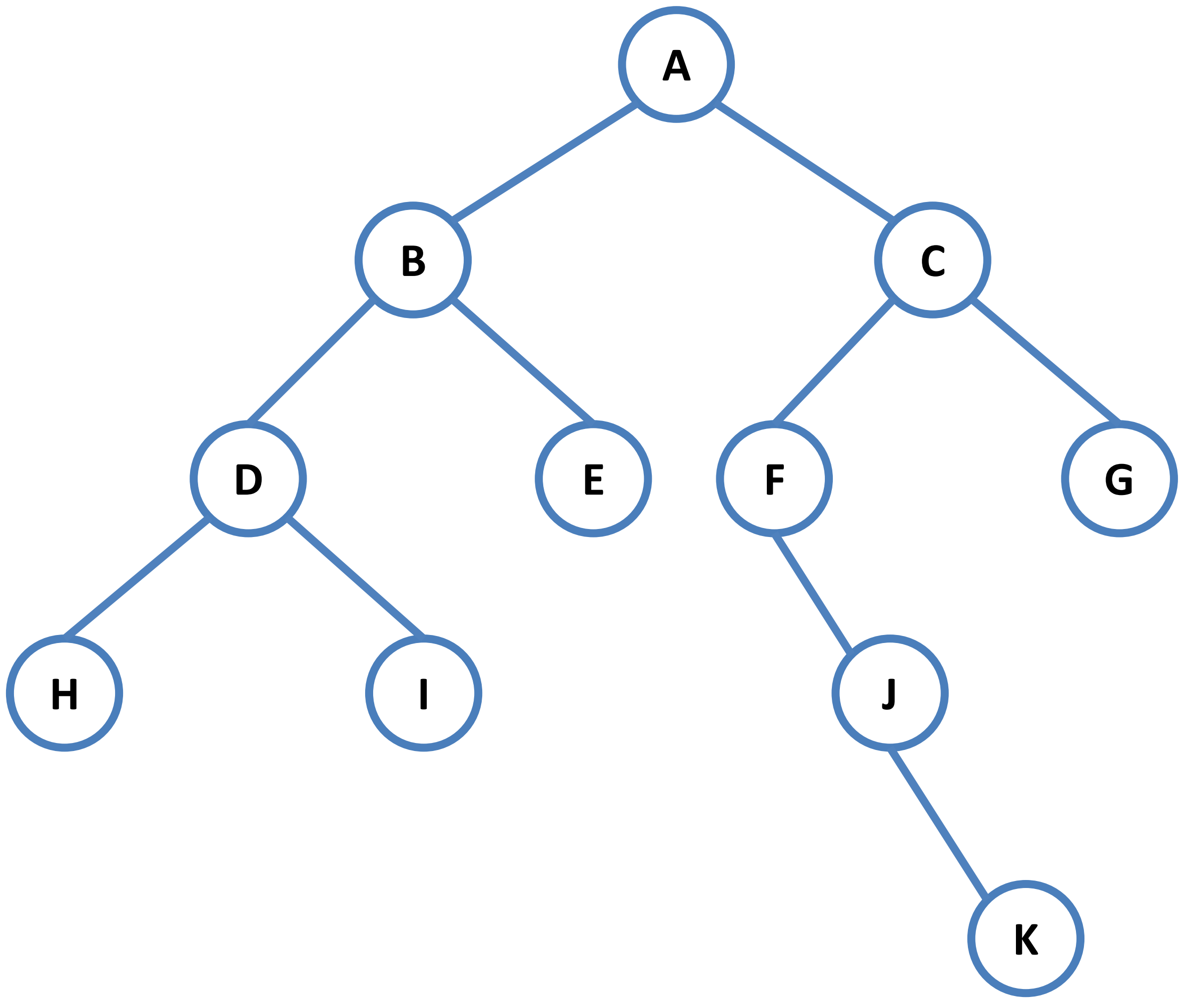


Graph
Traversal

- Start at A, DFS
- A
- B
- D
- H
- I
- E
- C
- F

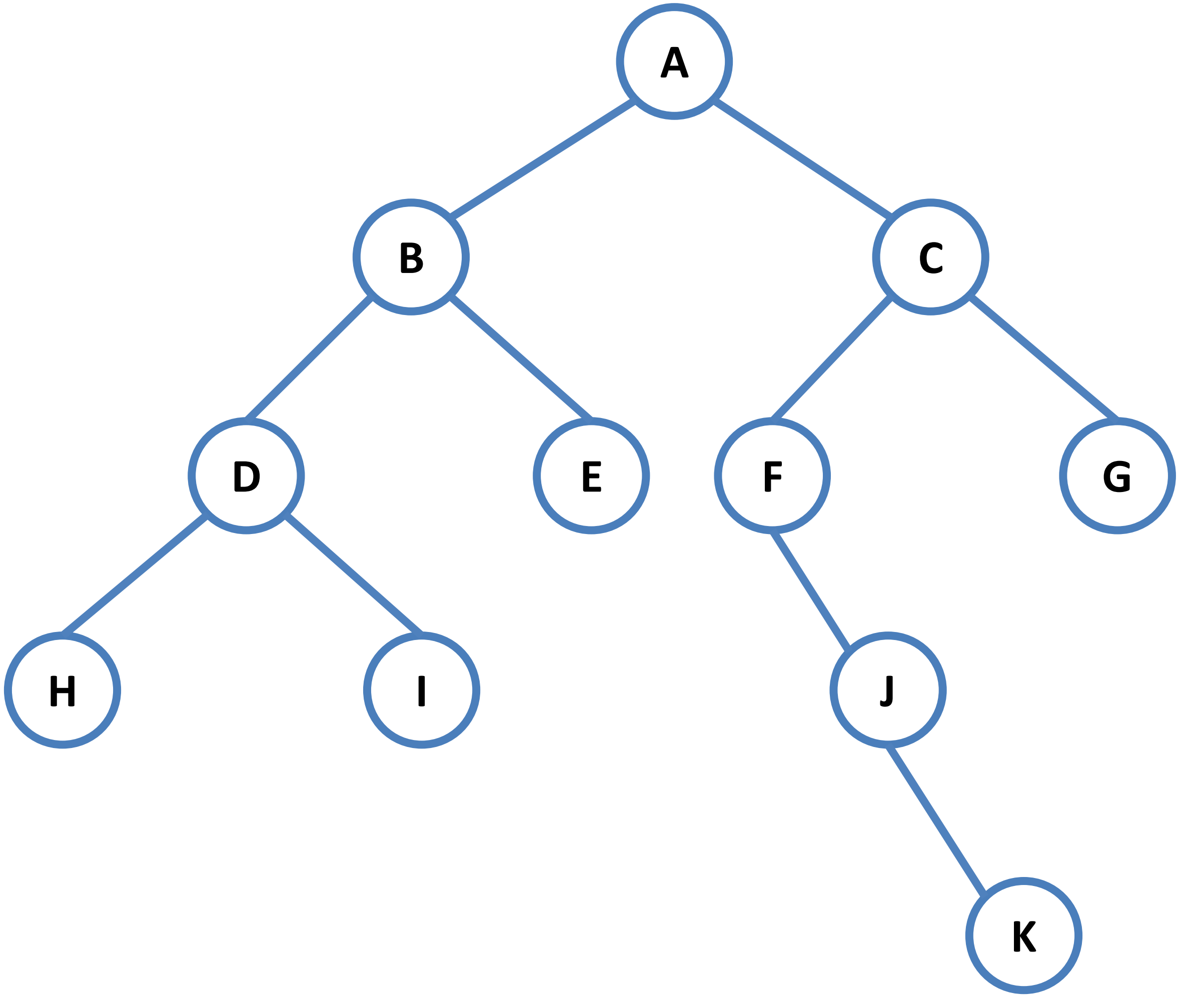


- Start at A, DFS
- A
- B
- D
- H
- I
- E
- C
- F
- J



Graph
Traversal

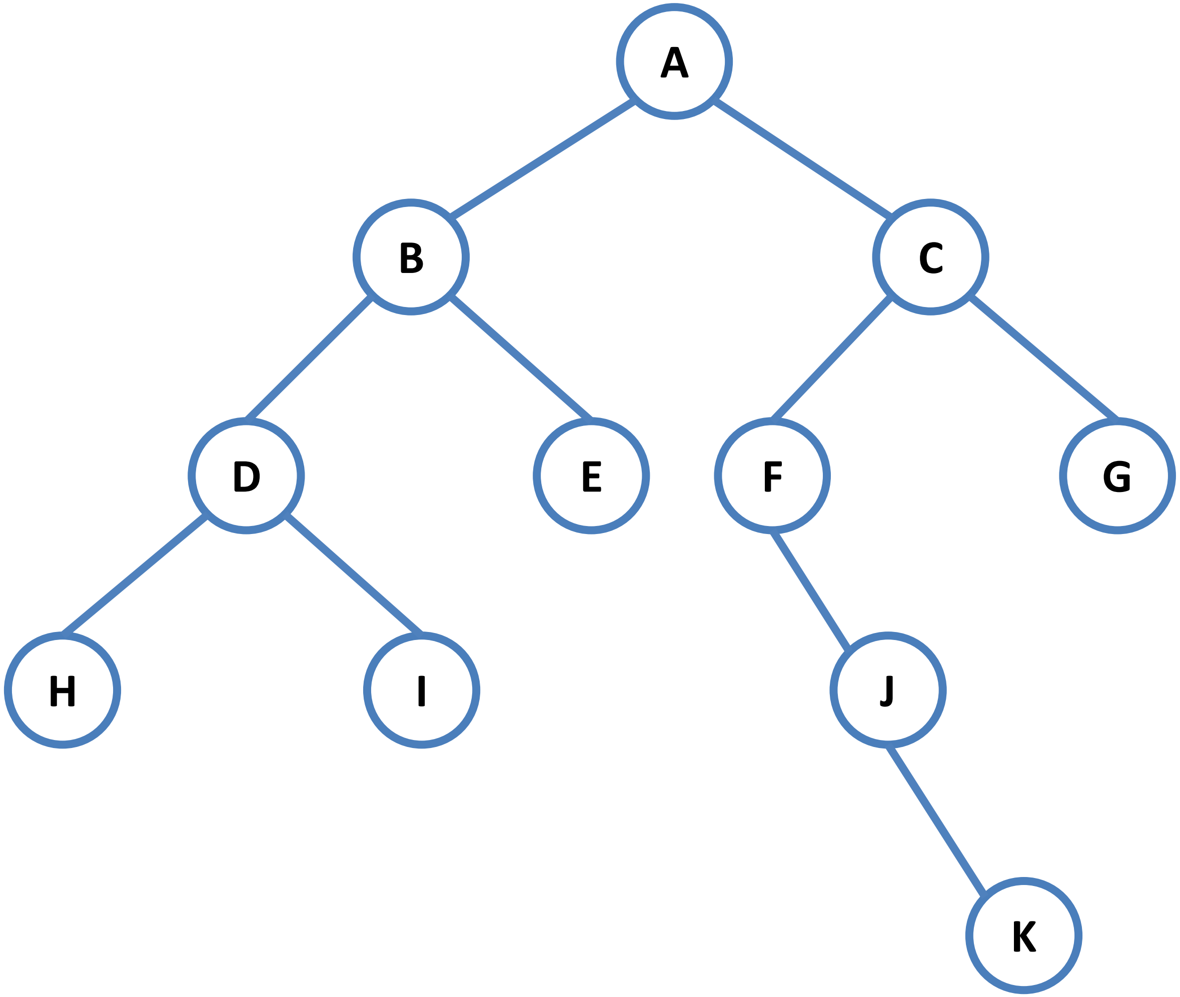
- Start at A, DFS
- A
- B
- D
- H
- I
- E
- C
- F
- J
- K



Graph

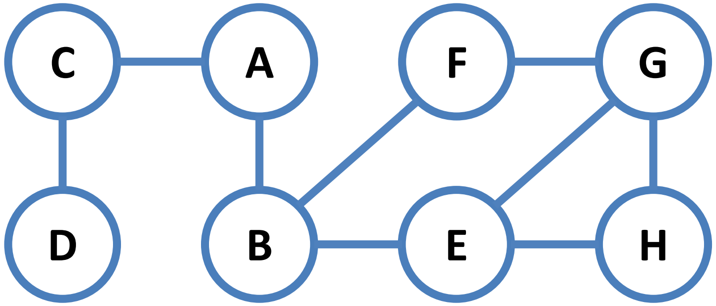
Traversal

- Start at A, DFS
- B
- D
- H
- I
- E
- C
- F
- J
- K
- G, finally

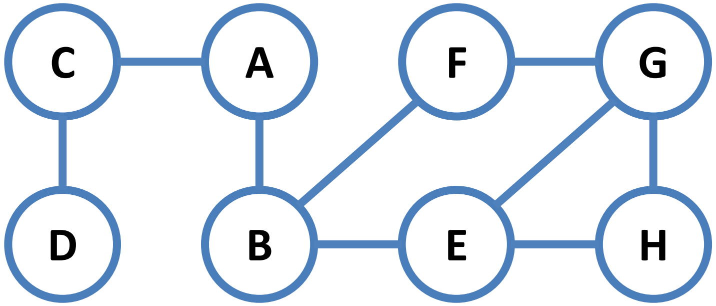


Questions?

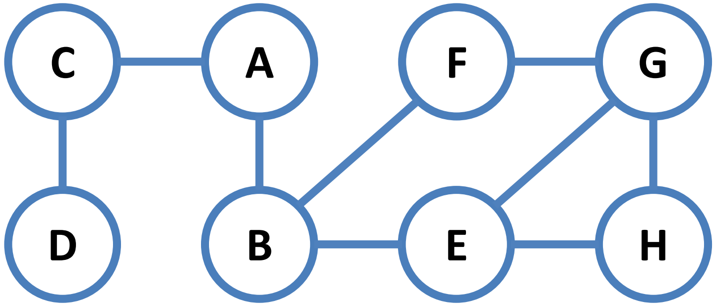
- How would you implement it?



- How would you implement it?
  - Let say we begin from vertex A

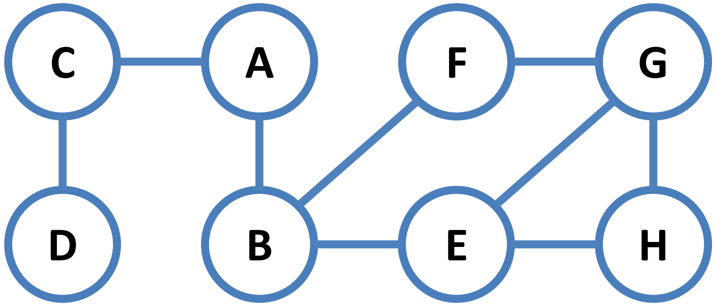


- How would you implement it?
  - Let say we begin from vertex A
  - What is our traversal?



Graph
BFS Implementation

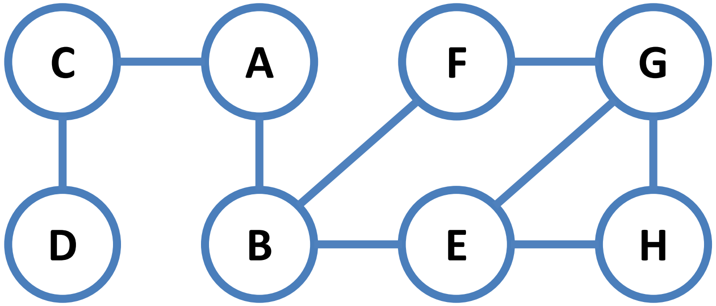
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered



Discovered										
Visited										

Graph
BFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Start with A

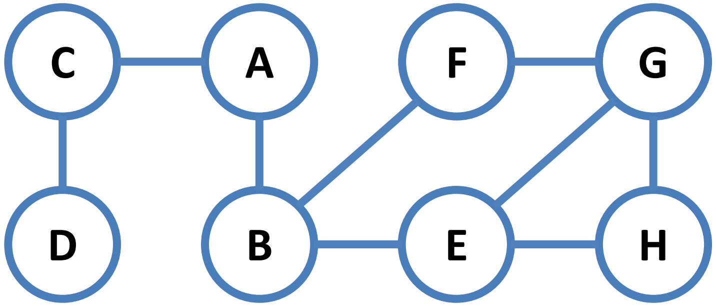


Discovered										
Visited										

Graph

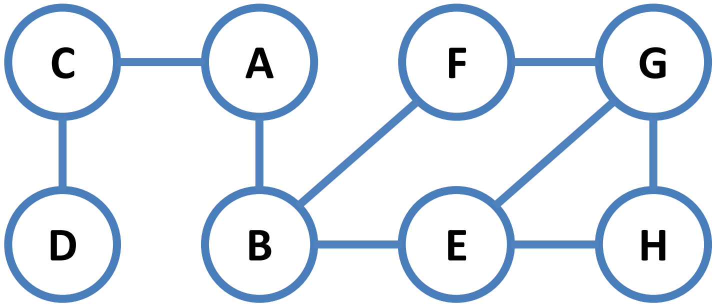
BFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it



Discovered	A									
Visited										

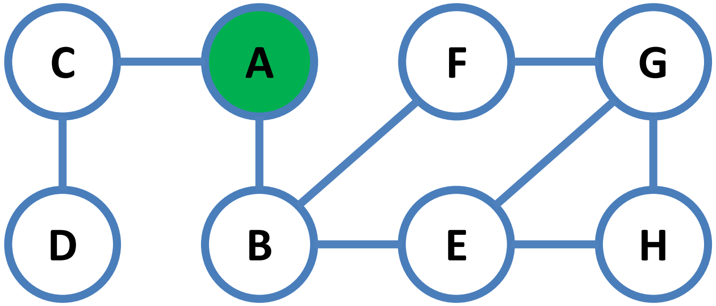
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty



Discovered	A									
Visited										

Graph
BFS Implementation

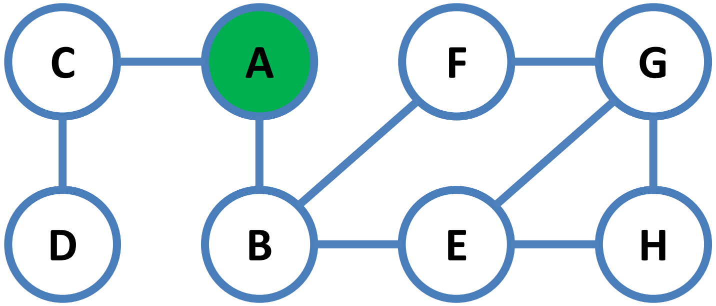
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered



Discovered	A									
Visited										

Graph
BFS Implementation

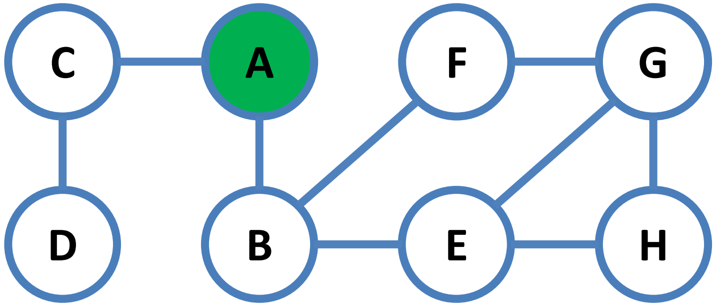
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited



Discovered										
Visited	A									

Graph
BFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served



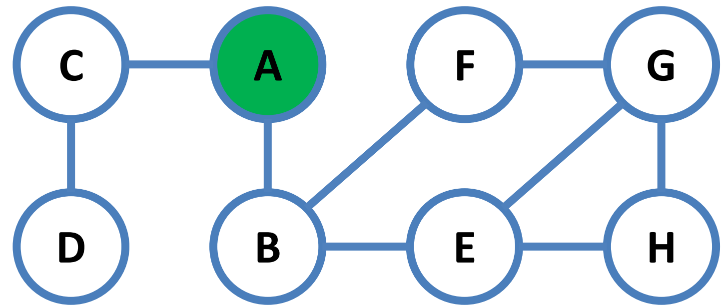
Discovered										
Visited	A									

Graph

BFS Implementation

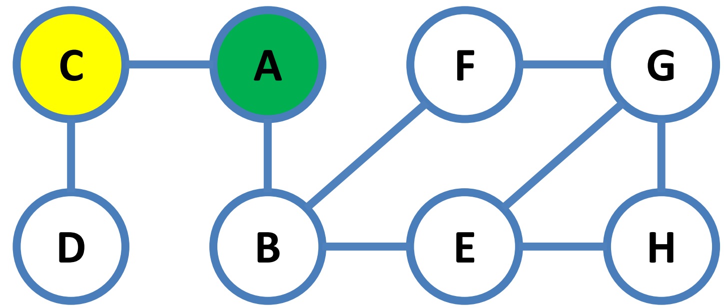
MONASH University

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



Discovered										
Visited	A									

Graph
BFS Implementation



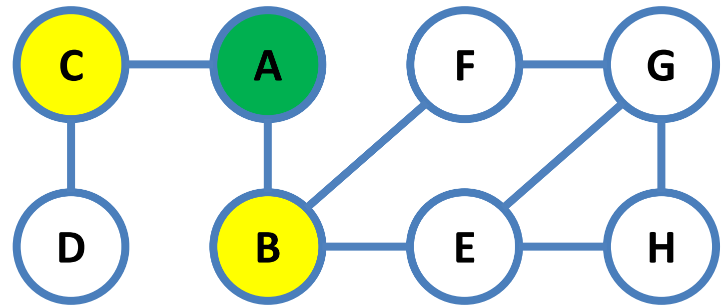
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue

Discovered	C									
Visited	A									

Graph

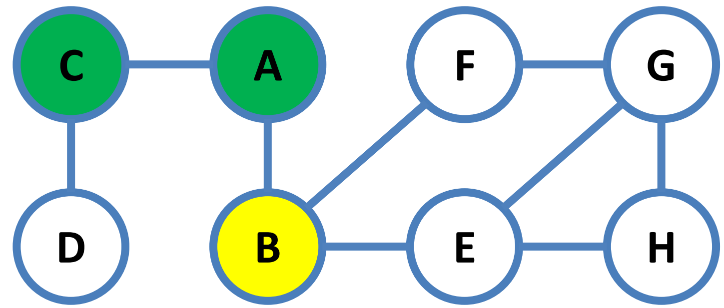
BFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



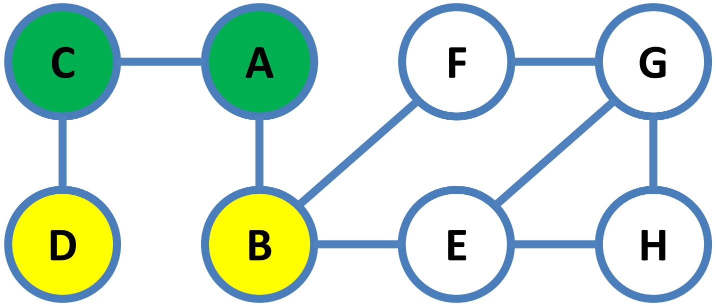
Discovered	C	B								
Visited	A									

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



Discovered		B								
Visited	A	C								

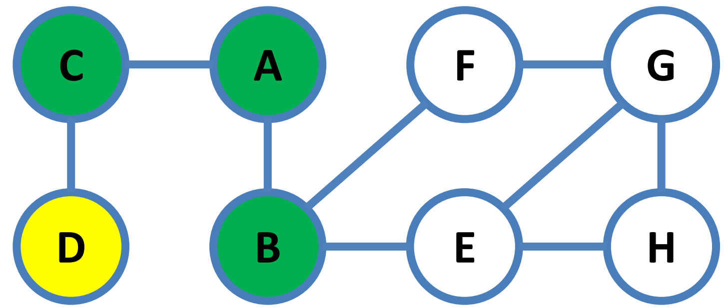
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



Discovered		B	D							
Visited	A	C								

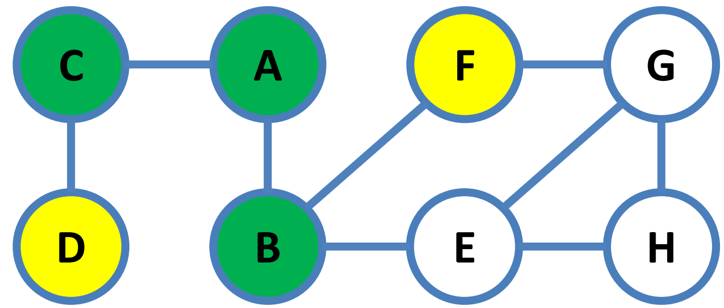
Graph
BFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



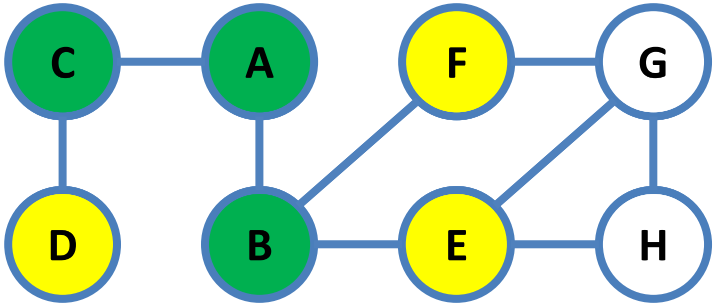
Discovered			D							
Visited	A	C	B							

Graph
BFS Implementation



- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue

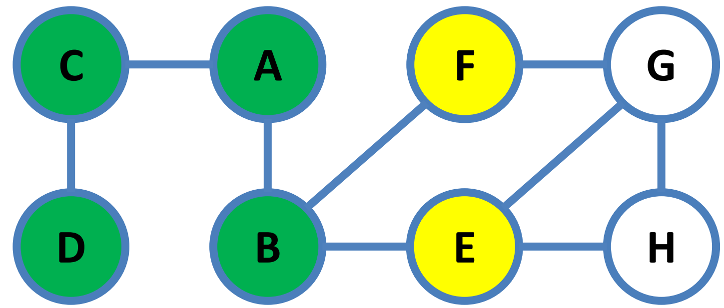
Discovered			**D**	**F**						
Visited	A	C	B							



- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue

Discovered			**D**	**F**	**E**					
Visited	A	C	B							

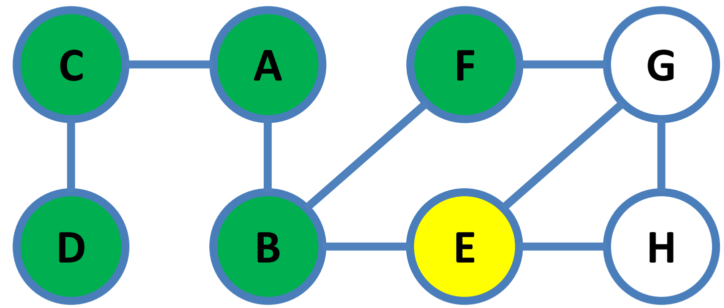
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



Discovered				**F**	**E**					
Visited	A	C	B	D						

Graph
BFS Implementation

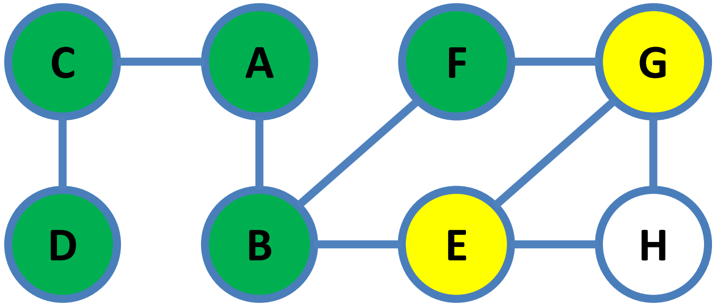
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



Discovered					**E**					
Visited	A	C	B	D	F					

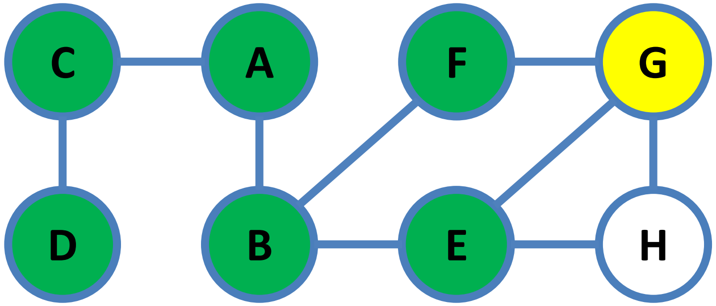
Graph
BFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



Discovered					E	G				
Visited	A	C	B	D	F					

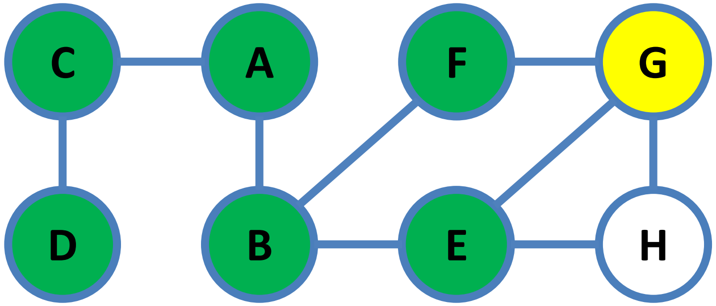
Graph
BFS Implementation



- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue

Discovered						G				
Visited	A	C	B	D	F	E				

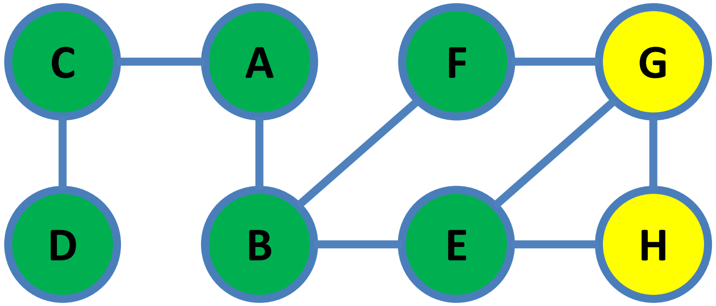
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



Discovered						G	G?			
Visited	A	C	B	D	F	E				

Graph
BFS Implementation

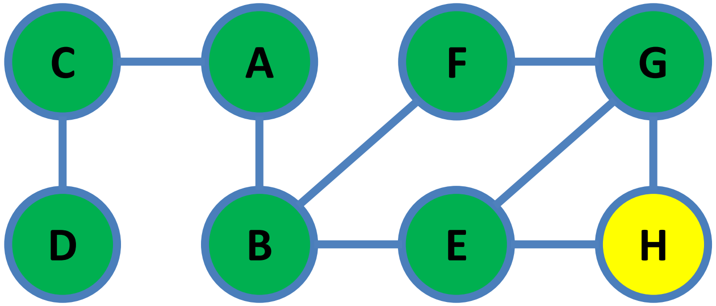
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



Discovered						G	H			
Visited	A	C	B	D	F	E				

Graph
BFS Implementation

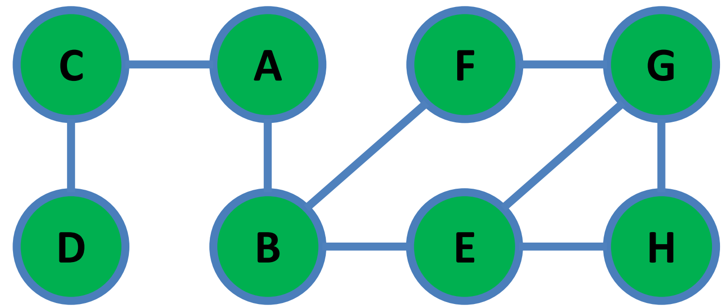
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



Discovered							**H**			
Visited	A	C	B	D	F	E	G			

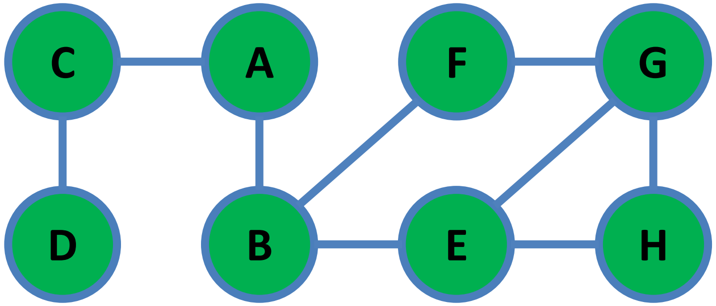
Graph
BFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



Discovered										
Visited	A	C	B	D	F	E	G	H		

Graph
BFS Implementation



- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue
- The traversal answer is not unique

Discovered										
Visited	A	C	B	D	F	E	G	H		

- Complexity?

- Complexity?
  - Time is O(V+E)

- Complexity?
  - Time is O(V+E)
    - Each vertex is visited once

- Complexity?
  - Time is O(V+E)
    - Each vertex is visited once
    - Each edge is visited twice

- Complexity?
  - Time is O(V+E)
    - Each vertex is visited once
    - Each edge is visited twice
      - For each <u,v> we visit from u and also from v

        directed only visit once

- Complexity?
  - Time is O(V+E)
    - Each vertex is visited once
    - Each edge is visited twice
      - For each <u,v> we visit from u and also from v
  - Space is O(V+E)

- Complexity?
  - Time is O(V+E)
    - Each vertex is visited once
    - Each edge is visited twice
      - For each <u,v> we visit from u and also from v
  - Space is O(V+E)
    - V maximum for the discovered queue

- Complexity?
  - Time is O(V+E)
    - Each vertex is visited once
    - Each edge is visited twice
      - For each <u,v> we visit from u and also from v
  - Space is O(V+E)
    - V maximum for the discovered queue
    - E to stored all of the edges (adjacency list)

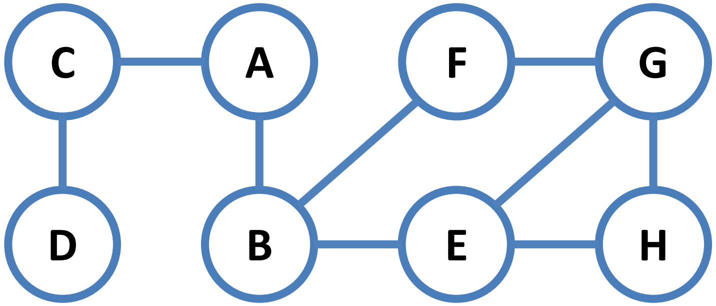
- Complexity?
  - Time is O(V+E)
    - Each vertex is visited once
    - Each edge is visited twice
      - For each <u,v> we visit from u and also from v
  - Space is O(V+E)
    - V maximum for the discovered queue
    - E to stored all of the edges (adjacency list)
  - But don’t we need to check the discovered queue for each vertex v?
    - O(V) search through the queue?

- Complexity?
  - Time is O(V+E)
    - Each vertex is visited once
    - Each edge is visited twice
      - For each <u,v> we visit from u and also from v
  - Space is O(V+E)
    - V maximum for the discovered queue
    - E to stored all of the edges (adjacency list)
  - But don’t we need to check the discovered queue for each vertex v?
    - O(V) search through the queue?
    - NO! Implement a Node class with self.discovered = True/ False

      in individual vertex

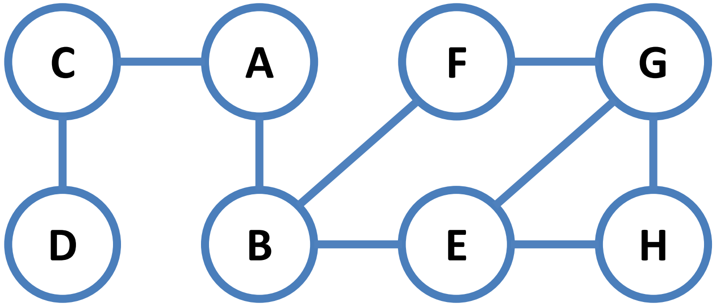
Questions?

- How would you implement it?
  - Let say we begin from vertex A
  - What is our DFS traversal?



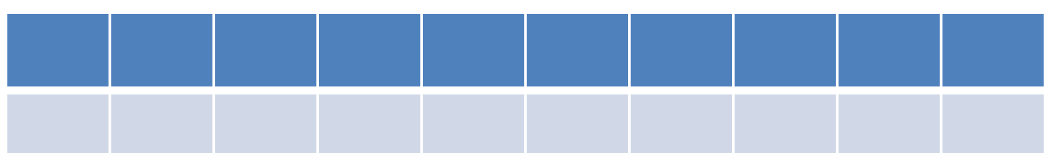
Graph
DFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered



Discovered

Visited



DFS Implementation

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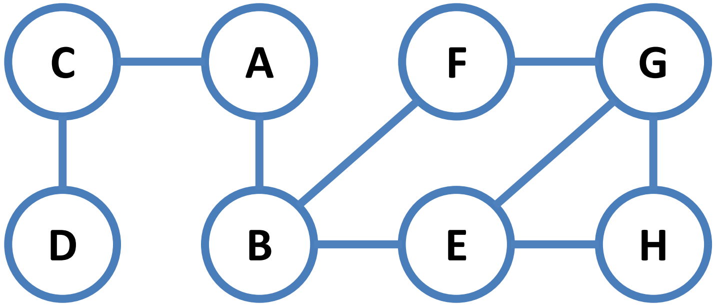
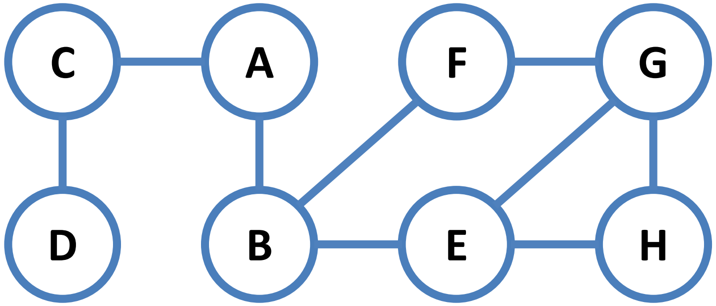
  - Let say we begin from vertex A
  - Have a ~~queue~~ stack for discovered



Discovered										
Visited										

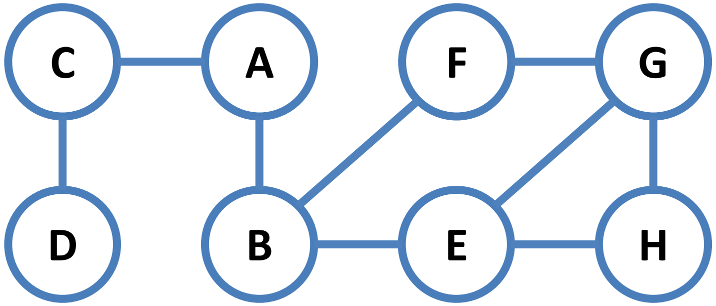
DFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a stack for discovered
    - Push source (A) into it



Discovered	A									
Visited										

Graph
DFS Implementation



- How would you implement it?
  - Let say we begin from vertex A
  - Have a stack for discovered
    - Push source (A) into it
  - While discovered is not empty
    - Pop from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue

Discovered	A									
Visited										

Graph
DFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
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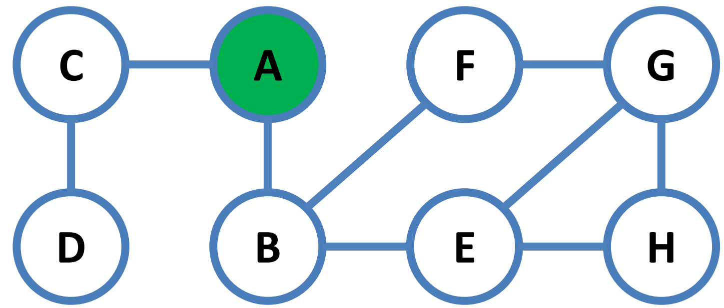

Discovered										
Visited	A									

Graph
DFS Implementation

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  - Let say we begin from vertex A
  - Have a stack for discovered
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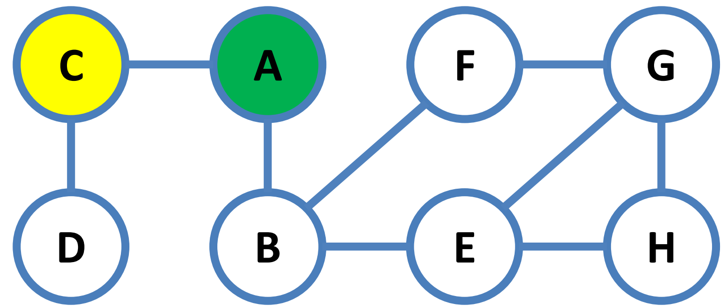

Discovered	C									
Visited	A									

Graph
DFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a stack for discovered
    - Push source (A) into it
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    - Pop from discovered, to visited
    - For each edge <u,v> where u is the served
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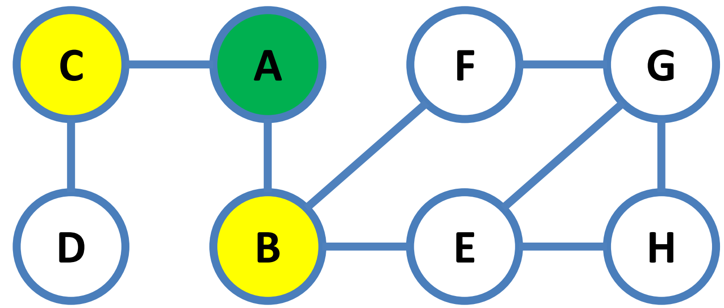

Discovered	C	B								
Visited	A									

Graph
DFS Implementation

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  - Let say we begin from vertex A
  - Have a stack for discovered
    - Push source (A) into it
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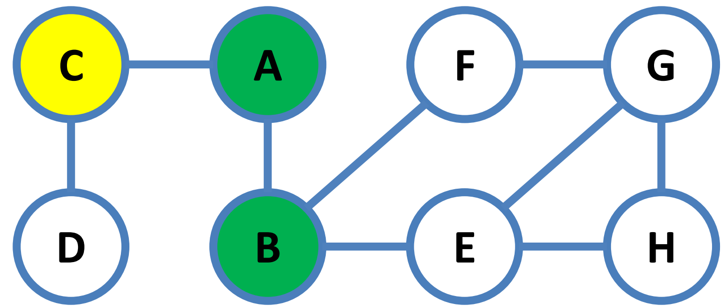

Discovered	C									
Visited	A	B								

Graph
DFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
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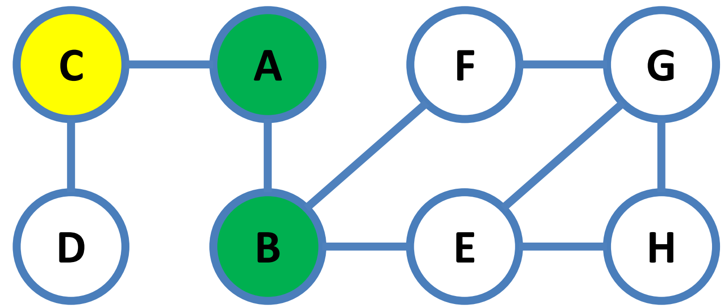
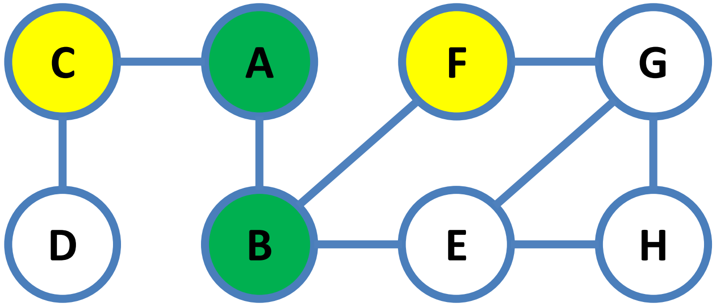


Discovered	**C**	**A?**								
Visited	A	B								

DFS Implementation



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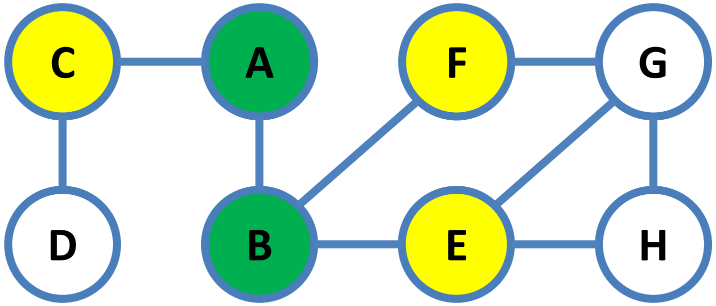
Discovered	C	F								
Visited	A	B								

Graph
DFS Implementation



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DFS pop B and push to visited (last in first out)
BFS serve C and append to visited (first in first out)

Discovered	C	F	E							
Visited	A	B								

Graph

DFS Implementation

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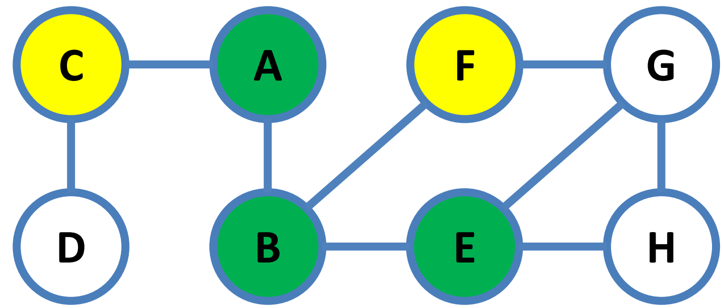
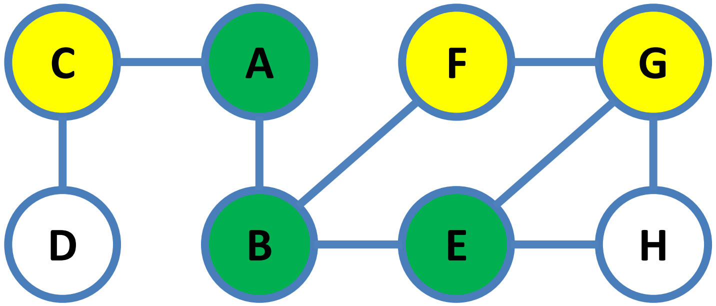


Discovered	**C**	**F**								
Visited	A	B	E							

Graph
DFS Implementation



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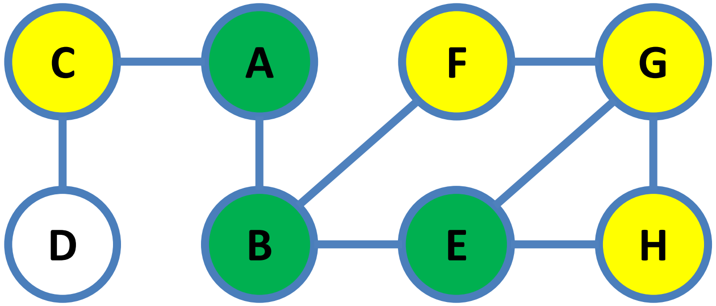
Discovered	**C**	**F**	**G**							
Visited	A	B	E							



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Discovered	C	F	G	H						
Visited	A	B	E							

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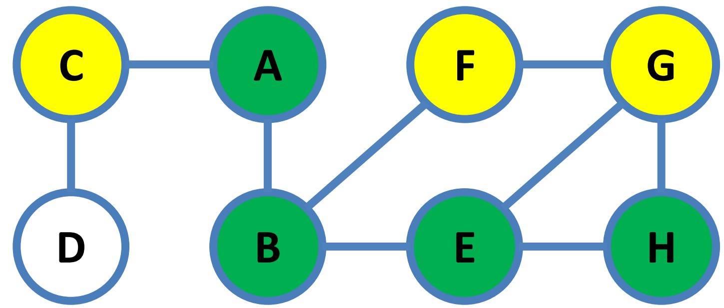
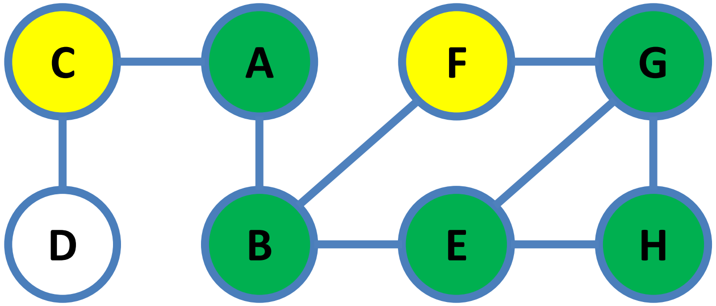


Discovered	C	F	G							
Visited	A	B	E	H						



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Discovered	**C**	**F**								
Visited	A	B	E	H	G					

Graph
DFS Implementation

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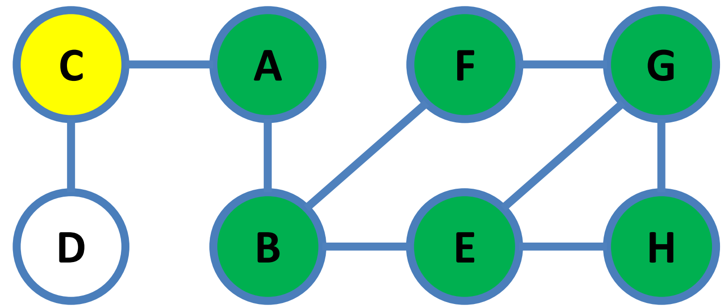

Discovered	C									
Visited	A	B	E	H	G	F				

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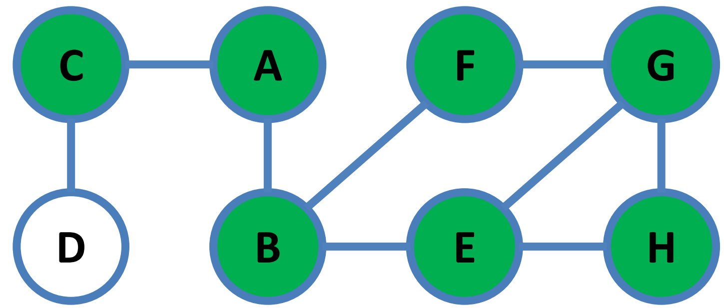

Discovered										
Visited	A	B	E	H	G	F	C			

Graph
DFS Implementation

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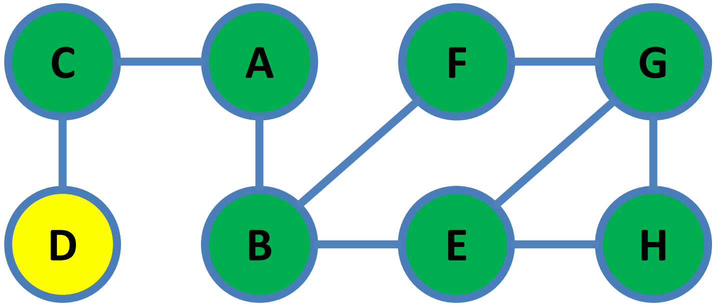

Discovered	D									
Visited	A	B	E	H	G	F	C			

DFS Implementation

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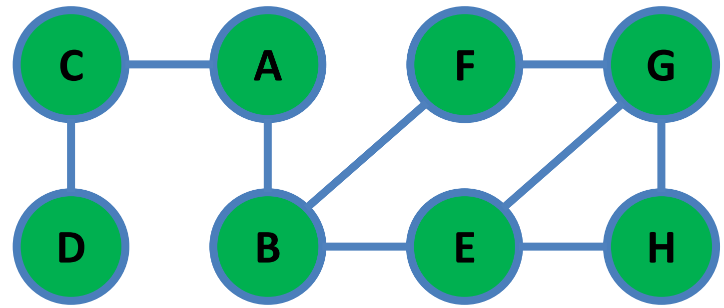

Discovered										
Visited	A	B	E	H	G	F	C	D		

- Complexity?

DFS Implementation

- Complexity?
  - Time is O(V+E)
    - Explanation same as BFS

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  - Time is O(V+E)
    - Explanation same as BFS
  - Space is O(V+E)
    - Explanation same as DFS

DFS Implementation

- Can you think of another way to implement DFS?

DFS Implementation

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- Let us just write them all out as a live coding session!

Questions?

- Can you think of another way to implement DFS?
- Recursion!
  - Best way to implement any form of traversal
    - Just like when you implemented tree/ trie traversal

      Recursion stack is the discovered stack in DFS

```
def dfs(current_vertex):
    current_vertex.visited = True
    for next_vertex in current_vertex.adjacent:
        if next_vertex.visited == False:
            dfs(next_vertex)

source_vertex = A
dfs(source_vertex)
```

DFS Implementation

- Can you think of another way to implement DFS?
- Recursion!
  - Best way to implement any form of traversal
    - Just like when you implemented tree/ trie traversal
  - Make sense because we are going depth-first like how recursion does it!

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    current_vertex.visited = True
    for next_vertex in current_vertex.adjacent:
        if next_vertex.visited == False:
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```

Questions?

DFS and BFS application?

- As mentioned earlier, it is the basic algorithm for many more complex algorithm… the application include:

DFS and BFS application?

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  - Reachability
  - Finding all connected components
  - Finding cycles
  - Shortest path (brute force)
  - Shortest path (non-brute force) on unweighted graph
  - Topological sort (later on)
  - … and many more!

DFS and BFS application?

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  - Topological sort (later on)
  - … and many more!

  - We will see more in unit notes and tutorials

Questions?

Break!

- Classical problem

- Classical problem
  - Given a set of locations
  - Given routes between locations
  - Given the distance between locations

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- Classical problem
  - Given a set of locations as vertices V
  - Given routes between locations as edges E
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  - Can you find the shortest distance from a source to a destination?
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- If the graph is unweighted?
  - Use BFS from the source!

Graph

Shortest distance and path

- Classical problem
  - Given a set of locations as vertices V
  - Given routes between locations as edges E
  - Given the distance between locations as weights W
  - Can you find the shortest distance from a source to a destination?
  - What is the path? This would require backtracking

- If the graph is unweighted?
  - Use BFS from the source!
  - Look back our BFS example

    tree going down level by level
    so give shortest distance

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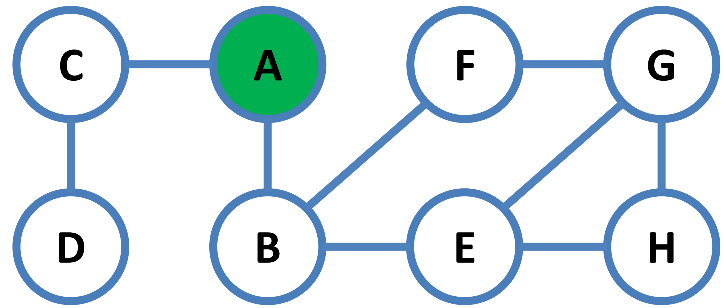

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it with a distance 0
  - While discovered is not empty
    - Serve from discovered, to visited
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Discovered	A,0									
Visited										

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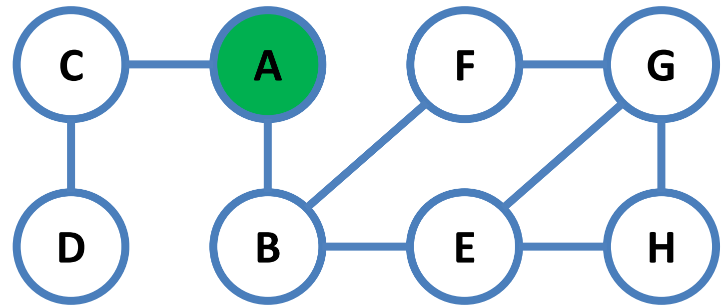
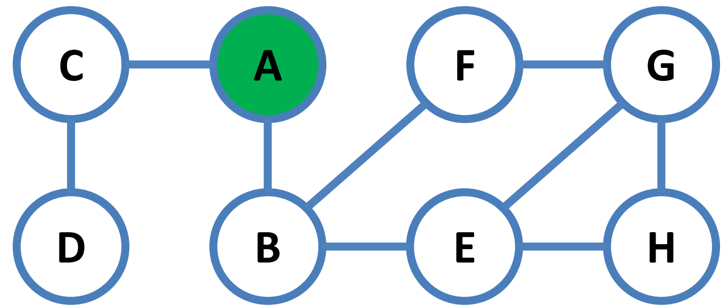


Discovered										
Visited	A,0									

Graph

Shortest distance with BFS



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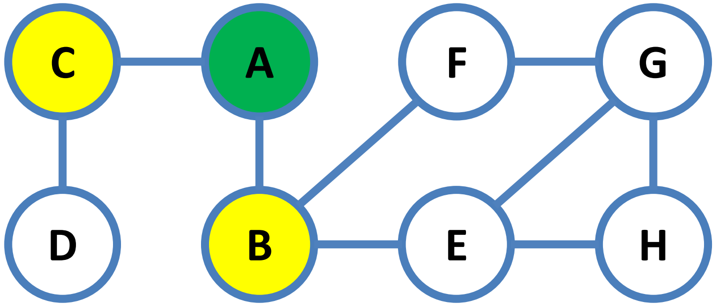
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue
      - v.distance = u.distance + 1

Discovered										
Visited	A,0									

Shortest distance with BFS



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Discovered	**C,1**	**B,1**								
Visited	A,0									

Graph

Shortest distance with BFS

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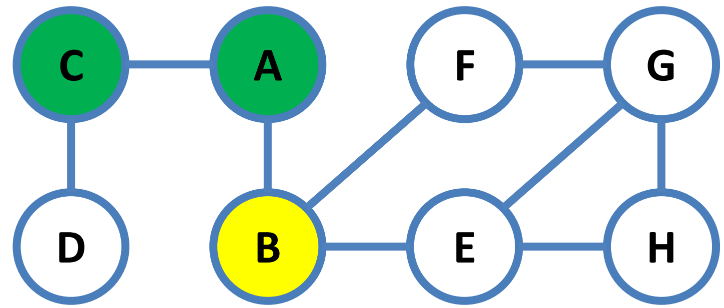

Discovered		**B,1**								
Visited	A,0	C,1								

Graph

Shortest distance with BFS

MONASH University

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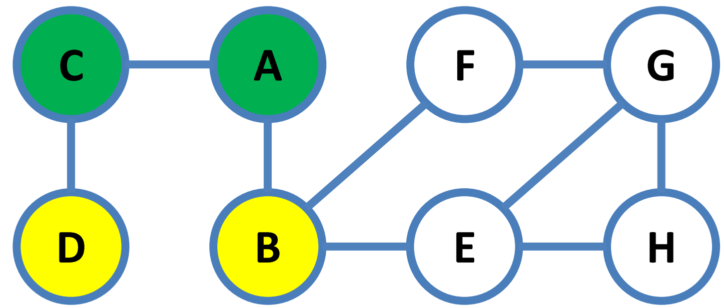

Discovered		**B,1**	**D,2**							
Visited	A,0	C,1								

- How would you implement it?
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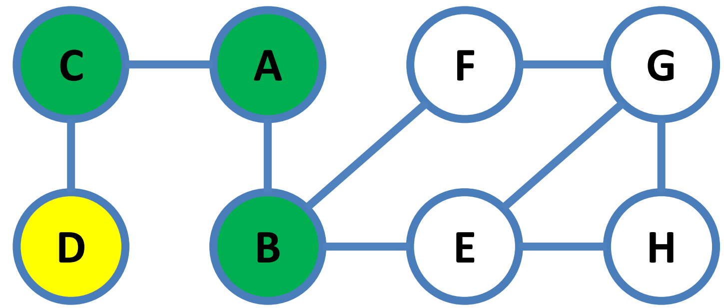
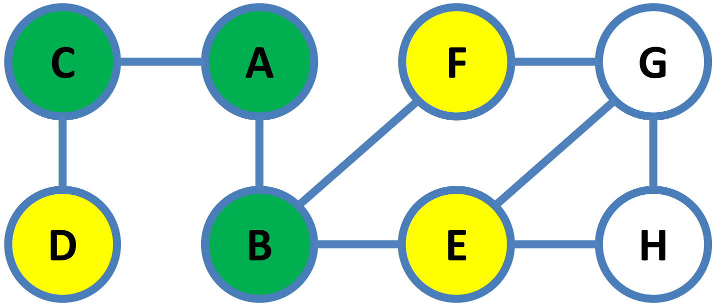


Discovered			**D,2**							
Visited	A,0	C,1	B,1							

Graph

Shortest distance with BFS



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Discovered			**D,2**	**F,2**	**E,2**					
Visited	A,0	C,1	B,1							

Graph

Shortest distance with BFS

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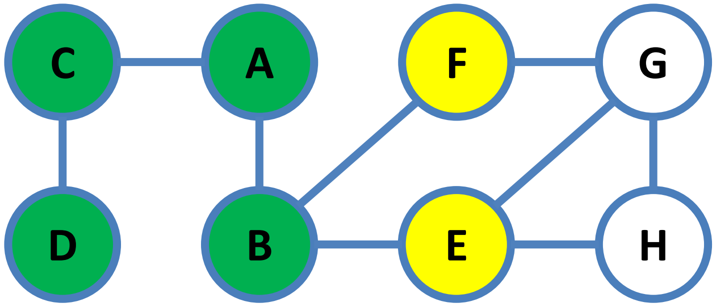

Discovered				**F,2**	**E,2**					
Visited	A,0	C,1	B,1	D,2						

Shortest distance with BFS

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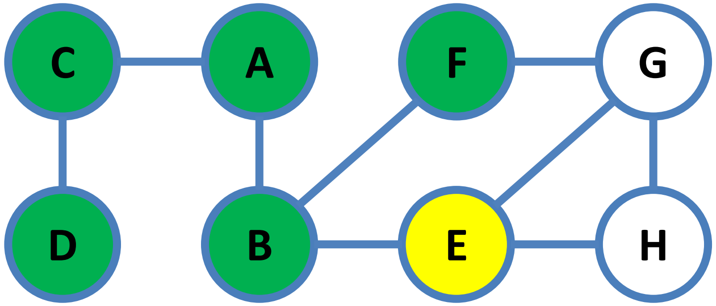
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue
      - v.distance = u.distance + 1



Discovered					E,2					
Visited	A,0	C,1	B,1	D,2	F,2					

Graph

Shortest distance with BFS

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it with a distance 0
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
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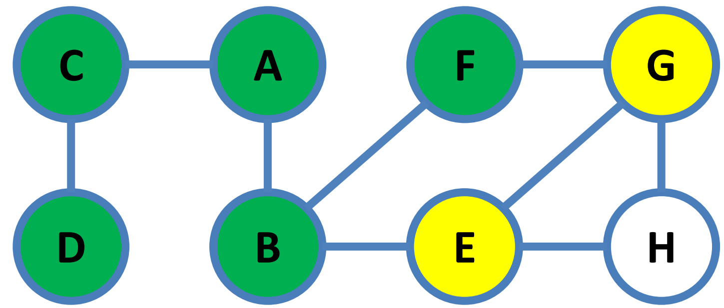
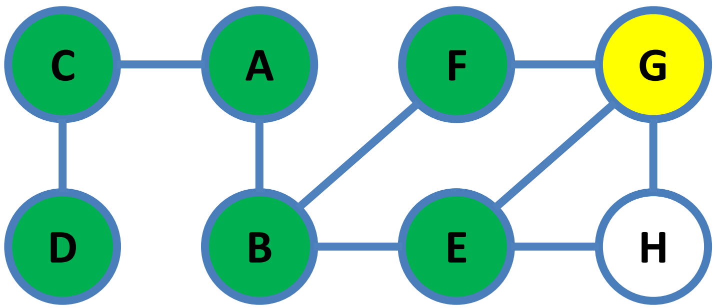


Discovered					**E,2**	**G,3**				
Visited	A,0	C,1	B,1	D,2	F,2					

Graph

Shortest distance with BFS



- How would you implement it?
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Discovered						**G,3**				
Visited	A,0	C,1	B,1	D,2	F,2	E,2				

Graph
Shortest distance with BFS

- How would you implement it?
  - Let say we begin from vertex A
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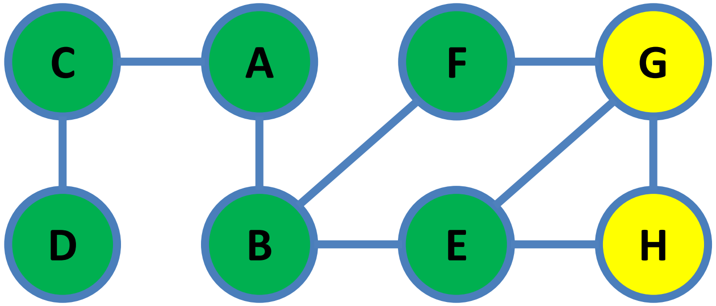

Discovered						G,3	H,3			
Visited	A,0	C,1	B,1	D,2	F,2	E,2				

Graph
Shortest distance with BFS

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it with a distance 0
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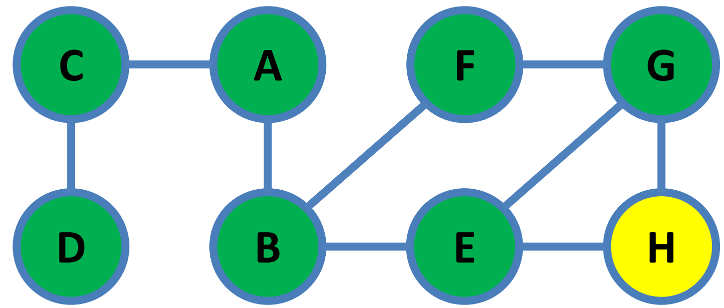
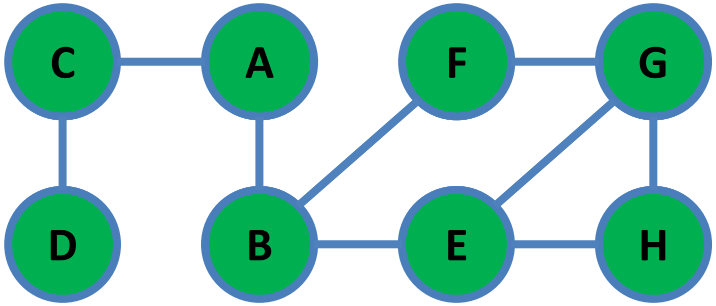


Discovered							H,3			
Visited	A,0	C,1	B,1	D,2	F,2	E,2	G,3			

Graph

Shortest distance with BFS



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      - v.distance = u.distance + 1

        if v = E (v = target destination)
        break (to stop early)

Discovered										
Visited	A,0	C,1	B,1	D,2	F,2	E,2	G,3	H,3		

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Questions?

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- How would you modify the following to find the path?
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it with a distance 0
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  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue
      - v.distance = u.distance + 1
      - v.previous = u # enable backtracking

Questions?

distance calculated from source

- What if the graph is weighted?

- What if the graph is weighted?
  – BFS is not able to do it anymore

- What if the graph is weighted?
  - BFS is not able to do it anymore since need Vertex object to store additional info self.weight
  - Sooo, we call in Dijkstra

- What if the graph is weighted?
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- What if the graph is weighted?
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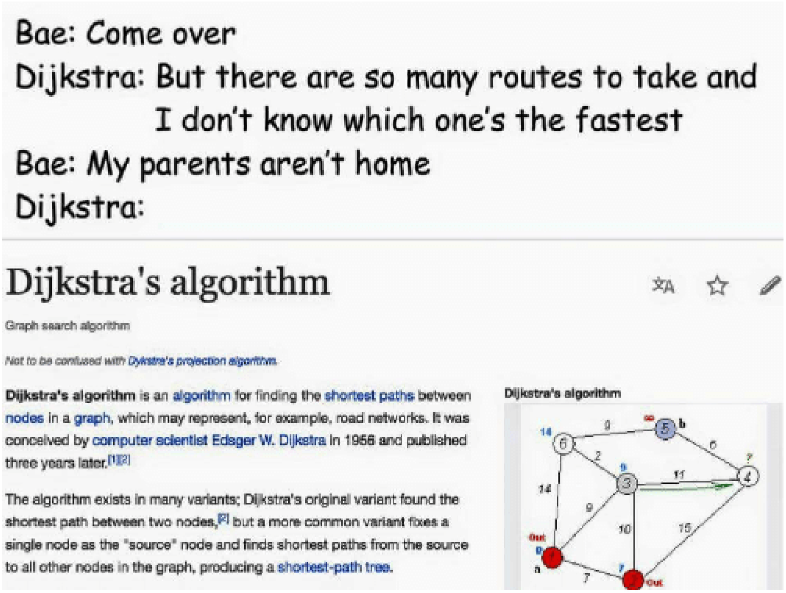
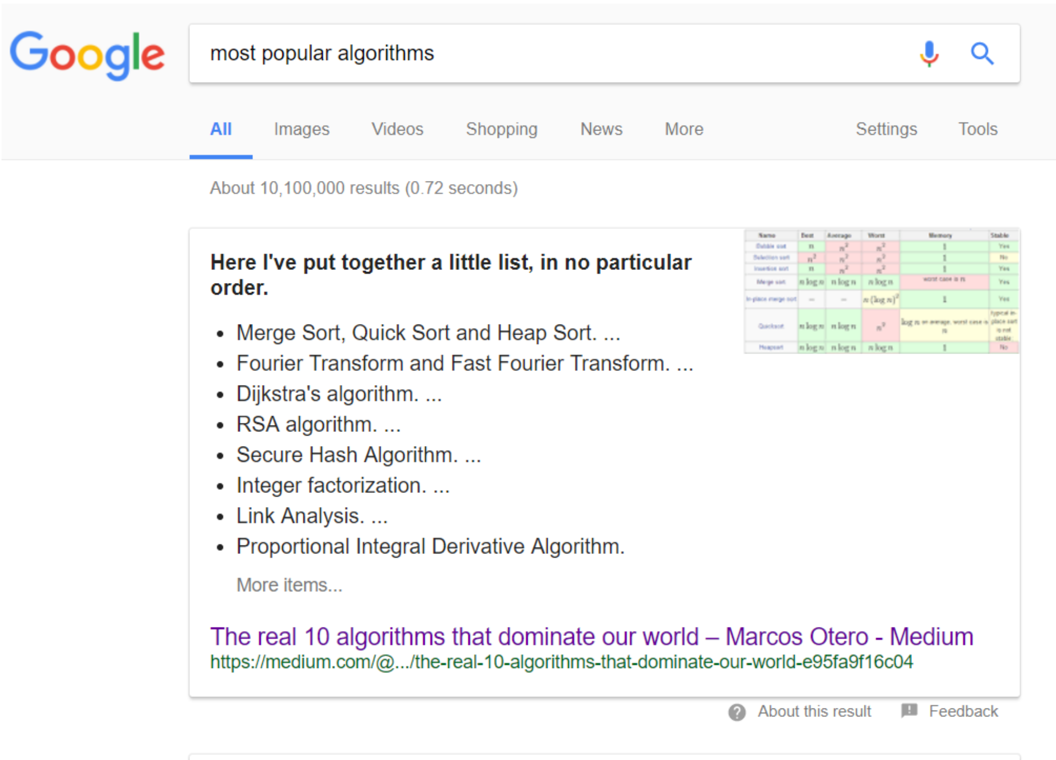
- So Dijkstra came up with the shortest distance algorithm
  - Recall we can backtrack (previous) to get the path

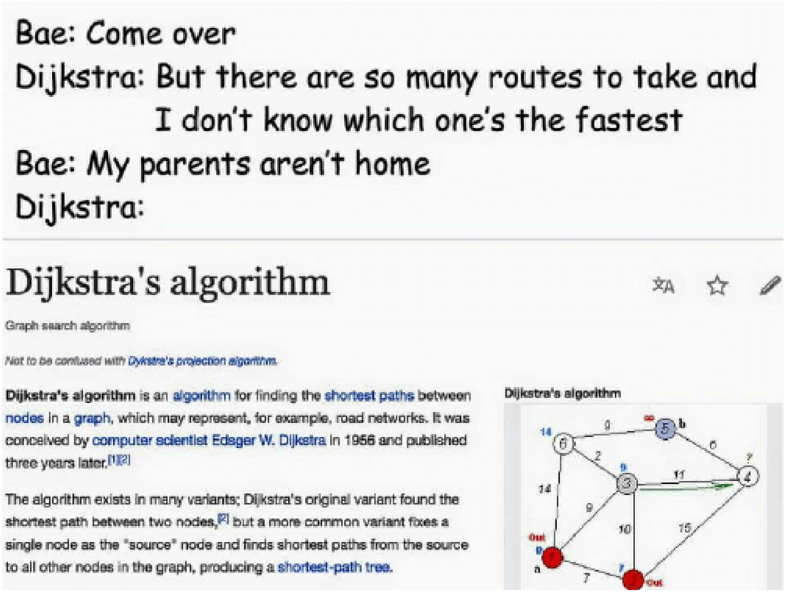


How Dijkstra came up with his algorithm

Graph

Shortest path with Dijkstra





How Dijkstra came up with his algorithm

- It is a combination of 2 algorithms

- It is a combination of 2 algorithms
  - Dynamic programming
  - Greedy

change queue in BFS into Heap (re-arrange itself with priority)
to become Dijkstra

- It is a combination of 2 algorithms
  - Dynamic programming
    The minimum distance from A to C can be the minimum of A to B (which we know) and minimum of B to C (which we know as well).
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    If I am at A, I can reach B and C. B is the closest, so I go to B and this is the shortest from A to B. I do not need to check if A to C then C to B (A->C->B) is the shortest anymore.

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  - Greedy
    If I am at A, I can reach B and C. B is the closest, so I go to B and this is the shortest from A to B. I do not need to check if A to C then C to B (A->C->B) is the shortest anymore. since no checking, don't know the C ot B combine with A to C can have shorter distance than A to B directly

    but GREED IS NOT GOOD… when will this fail? When C to B is negative!

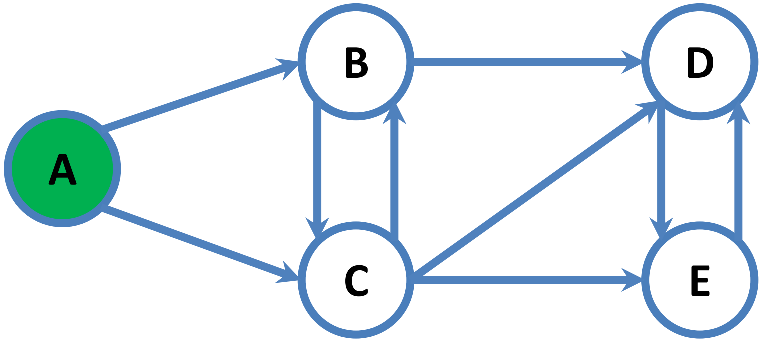
  - Thus, Dijkstra doesn't work for negative edges
    Note: might work at times when the negative edge isn't part of a cycle

Questions?

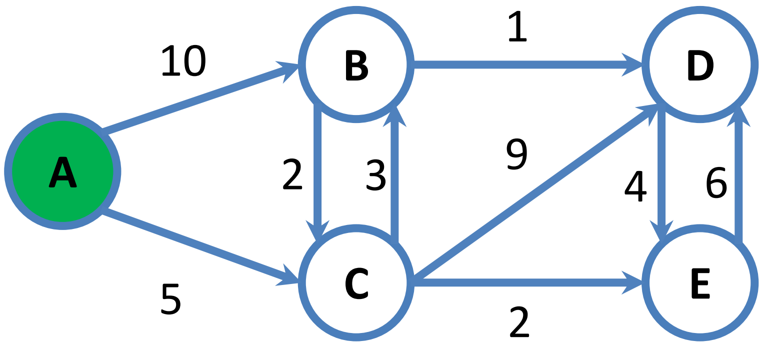
- So how does Dijkstra work?

- So how does Dijkstra work?
  - Consider the following directed graph

distance of visited vertex can not be changed



- So how does Dijkstra work?
  - Consider the following directed graph
    - Graph is weighted



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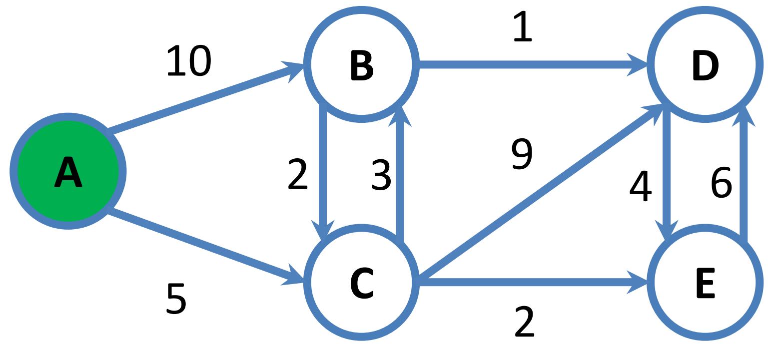
  - So let us begin the algorithm…



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  - Consider the following directed graph
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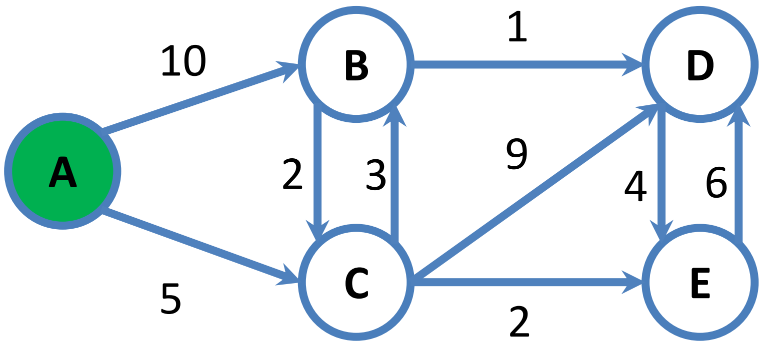
  - So let us begin the algorithm…
  - We are at A (source), and



Graph
Shortest path with Dijkstra

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  - So let us begin the algorithm…
  - So what happen is we will slowly wander to the closest point (from A)
    - A = 0
    - B = infinity
    - C = infinity
    - D = infinity
    - E = infinity

- So how does Dijkstra work?
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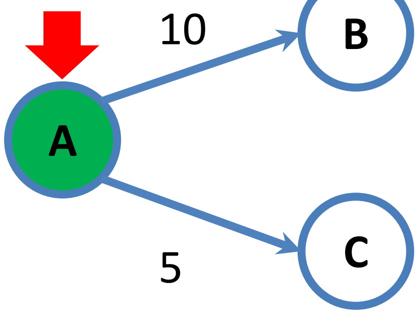

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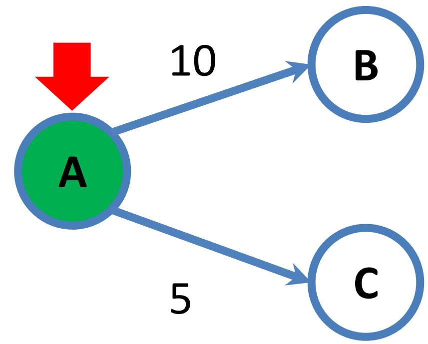

  - So let us begin the algorithm…
  - So what happen is we will slowly wander to the closest point (from A)
    - A = 0, from here, we can see B and C (edges from A). Update distance
    - B = 10
    - C = 5
    - D = infinity
    - E = infinity

- So how does Dijkstra work?
  - Consider the following directed graph
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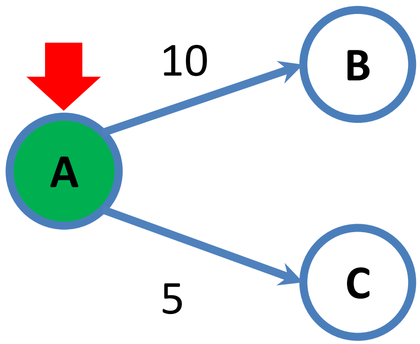

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    - D = infinity
    - E = infinity
    - Closest is C, so we move to C

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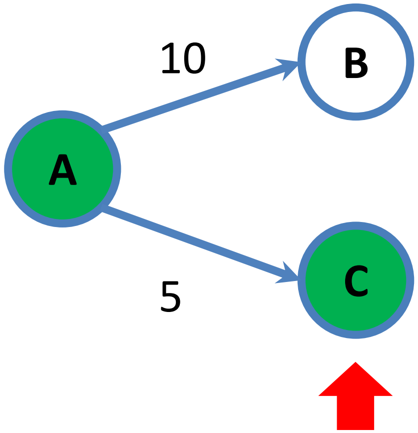

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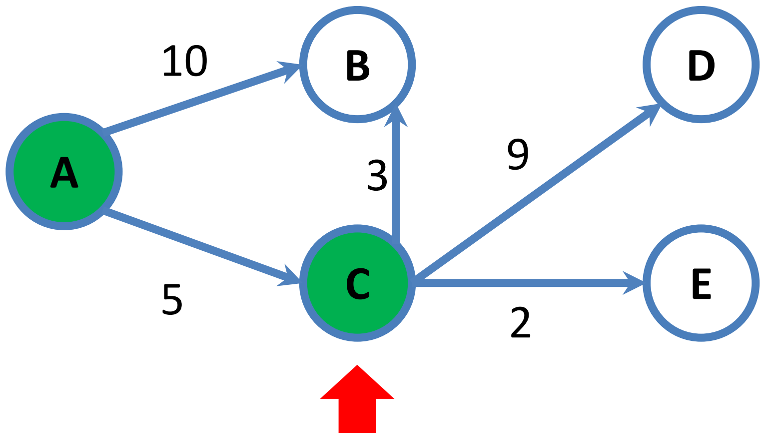

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    - C = 5, from here, we can see B, D and E
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Graph
Shortest path with Dijkstra

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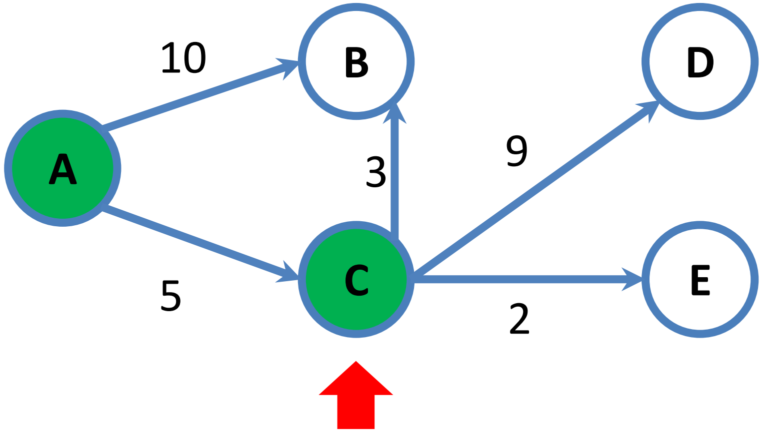

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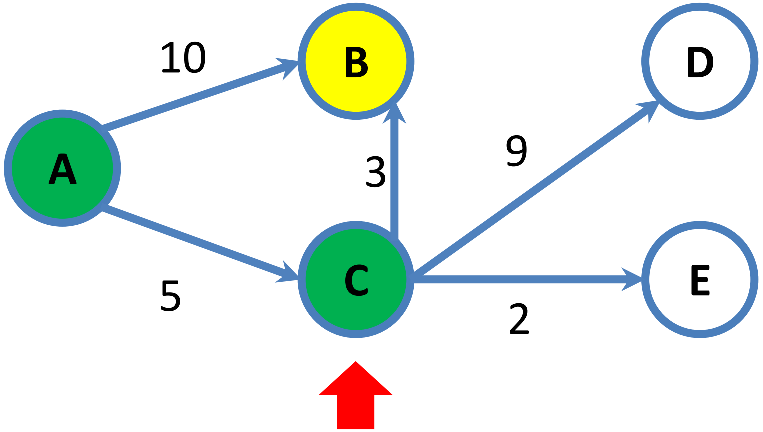

  - So let us begin the algorithm…
  - So what happen is we will slowly wander to the closest point (from A)
    - A = 0, from here, we can see B and C (edges from A). Update distance
    - B = 10 (A->B) vs 8 (A->C->B)
    - C = 5, from here, we can see B, D and E. Update the distance
    - D = infinity
    - E = infinity

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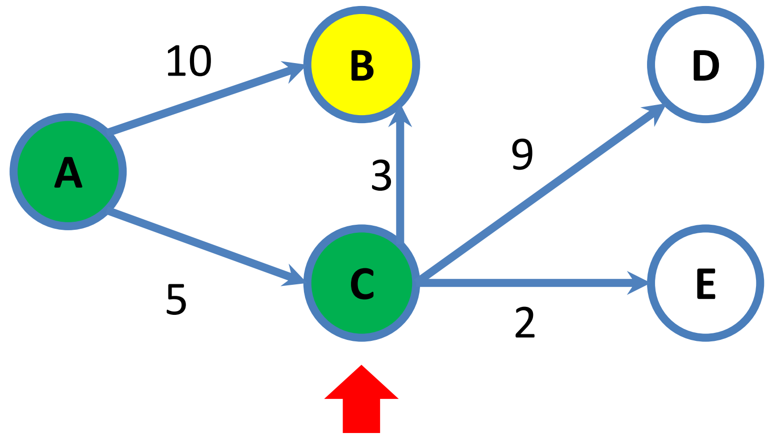

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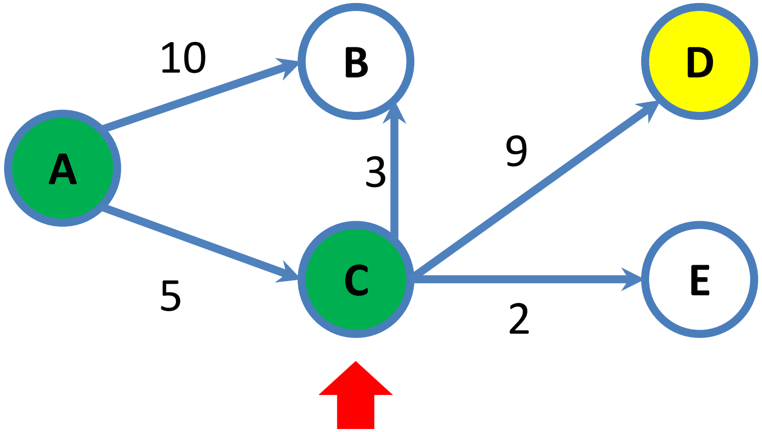

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    - B = 8
    - C = 5, from here, we can see B, D and E. Update the distance
    - D = 9?
    - E = infinity

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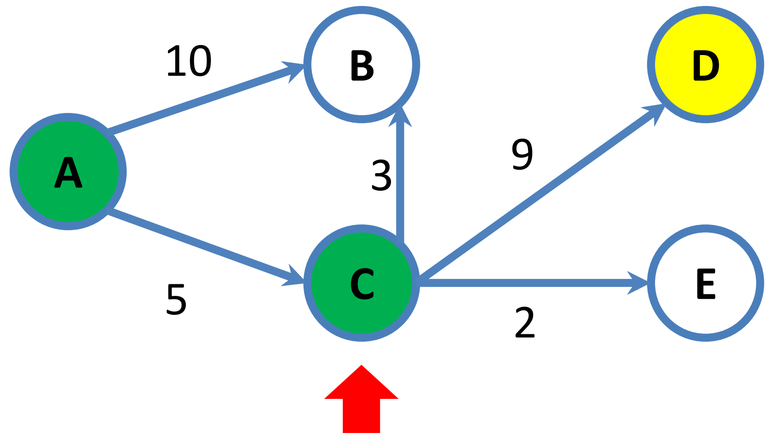

  - So let us begin the algorithm…
  - So what happen is we will slowly wander to the closest point (from A)
    - A = 0, from here, we can see B and C (edges from A). Update distance
    - B = 8
    - C = 5, from here, we can see B, D and E. Update the distance
    - D = 14 because distance is from A
    - E = infinity comparing with infinity

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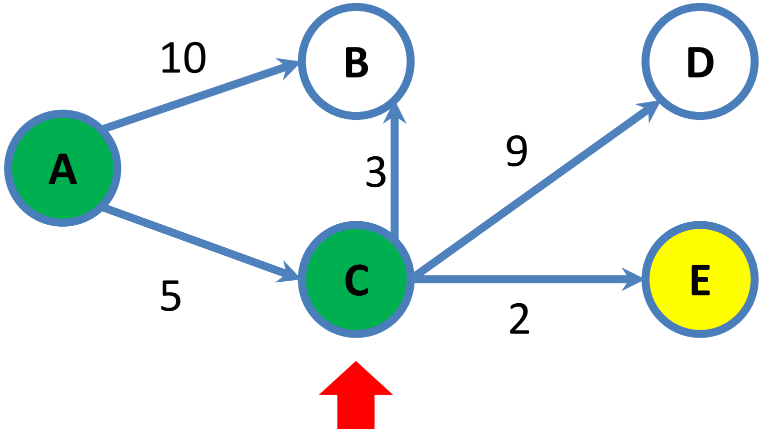

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  - So what happen is we will slowly wander to the closest point (from A)
    - A = 0, from here, we can see B and C (edges from A). Update distance
    - B = 8
    - C = 5, from here, we can see B, D and E. Update the distance
    - D = 14
    - E = 7

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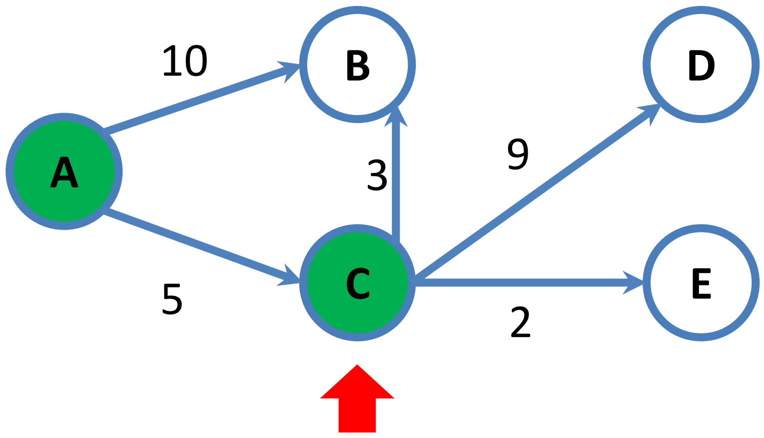

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    - B = 8
    - C = 5, from here, we can see B, D and E. Update the distance
    - D = 14
    - E = 7
    - Closest is E, so we go E

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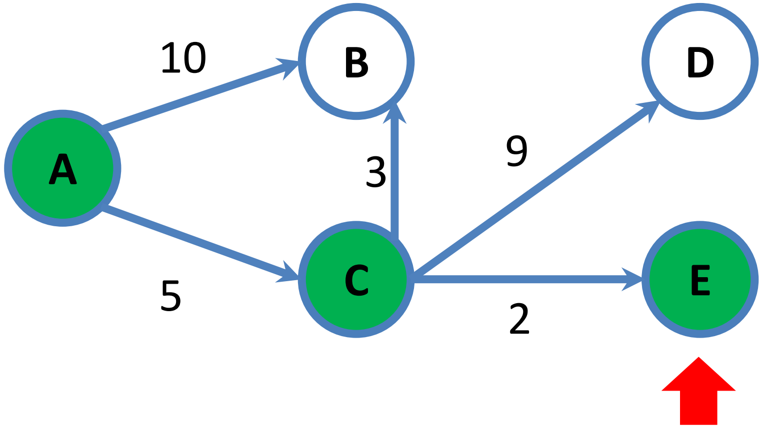

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    - C = 5, from here, we can see B, D and E. Update the distance
    - D = 14
    - E = 7
    - Closest is E, so we go E

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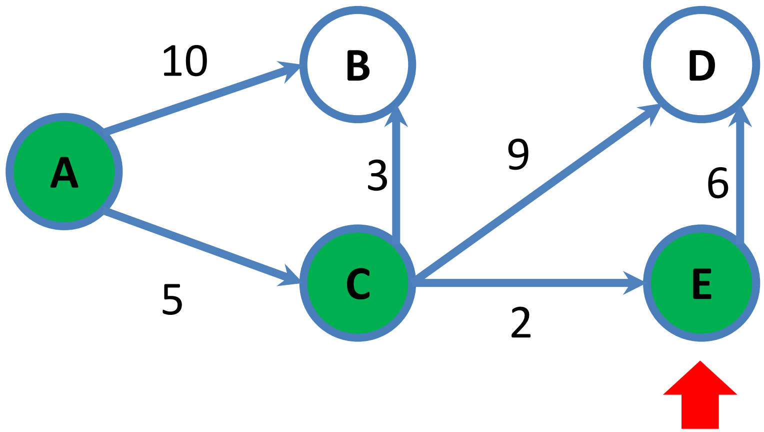

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    - B = 8
    - C = 5, from here, we can see B, D and E. Update the distance
    - D = 14
    - E = 7, from here, we can see D.

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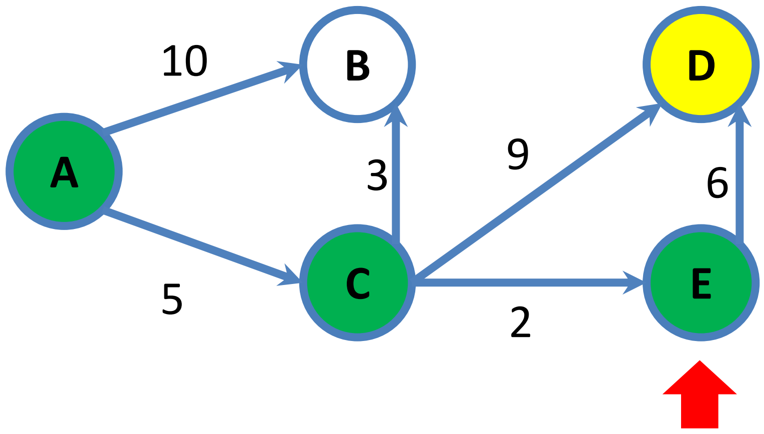

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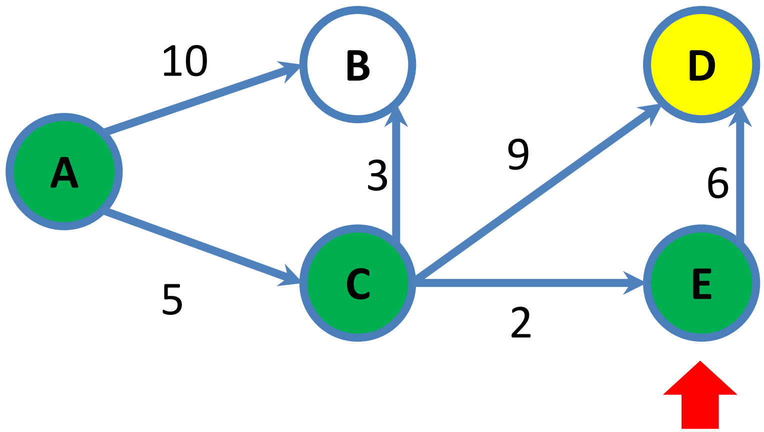
    - B = 8
    - C = 5, from here, we can see B, D and E. Update the distance
    - D = 14 vs 7+6 (A->E->D)
    - E = 7, from here, we can see D. Update the distance



- So how does Dijkstra work?
  - Consider the following directed graph
    - Graph is weighted
  - So let us begin the algorithm…
  - So what happen is we will slowly wander to the closest point (from A)
    - A = 0, from here, we can see B and C (edges from A). Update distance
    - B = 8
    - C = 5, from here, we can see B, D and E. Update the distance
    - D = 13
    - E = 7, from here, we can see D. Update the distance

- So how does Dijkstra work?
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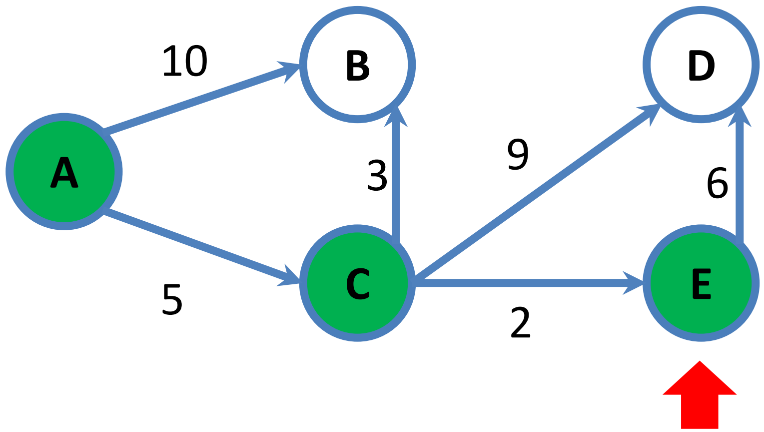

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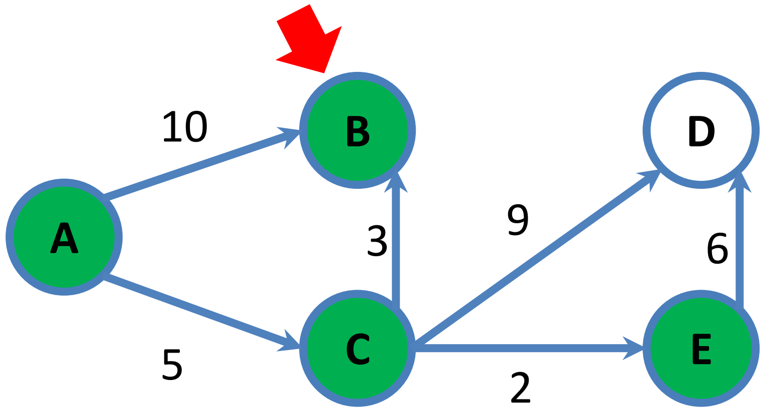
    - A = 0, from here, we can see B and C (edges from A). Update distance
    - B = 8
    - C = 5, from here, we can see B, D and E. Update the distance
    - D = 13
    - E = 7, from here, we can see D. Update the distance
    - Closest is B, so we go B

Shortest path with Dijkstra

- So how does Dijkstra work?
  - Consider the following directed graph
    - Graph is weighted



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    - B = 8
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    - D = 13
    - E = 7, from here, we can see D. Update the distance
    - Closest is B, so we go B

- So how does Dijkstra work?
  - Consider the following directed graph
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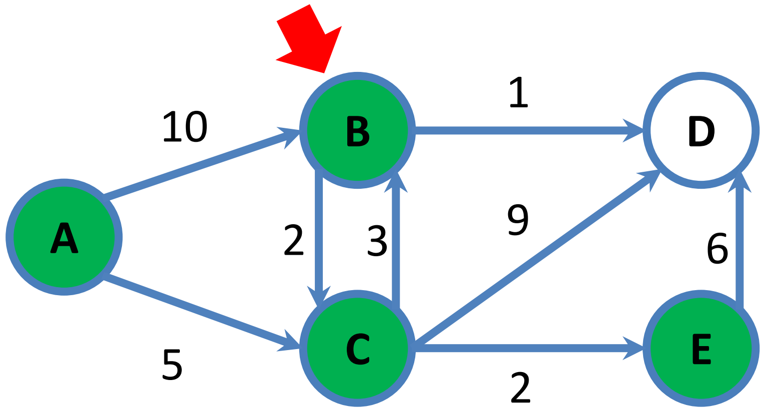

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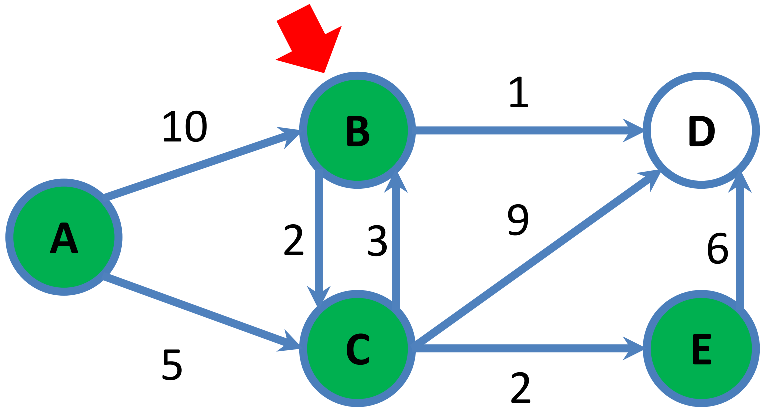

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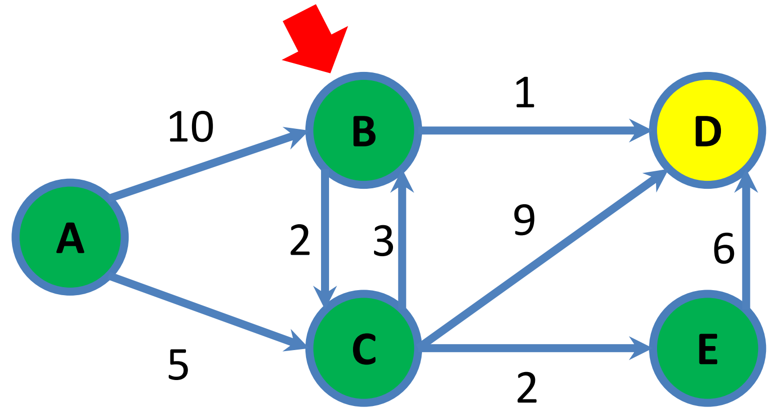

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Graph
Shortest path with Dijkstra

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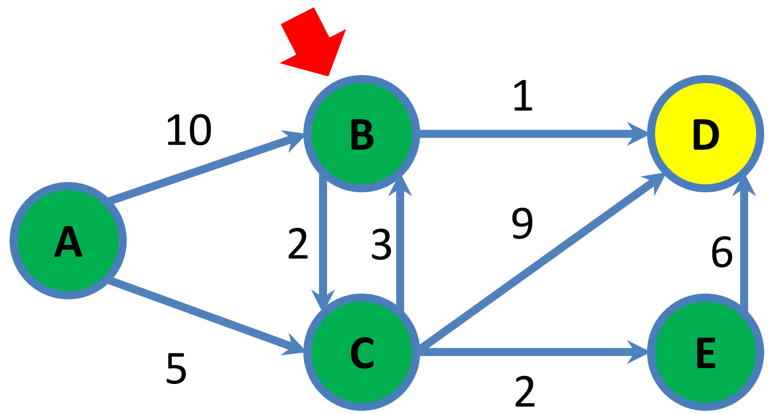

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    - B = 8, from here, we can see C and D. Update distance for D.
    - C = 5, from here, we can see B, D and E. Update the distance
    - D = 9 (8+1 via A->B->D)
    - E = 7, from here, we can see D. Update the distance

update distance from source A to the discovered vertex
in A, compare distance to B and C, choose the smallest distance among the discovered vertices to visit
the visited vertex can not change its distance
the visited vertex can not change its distance

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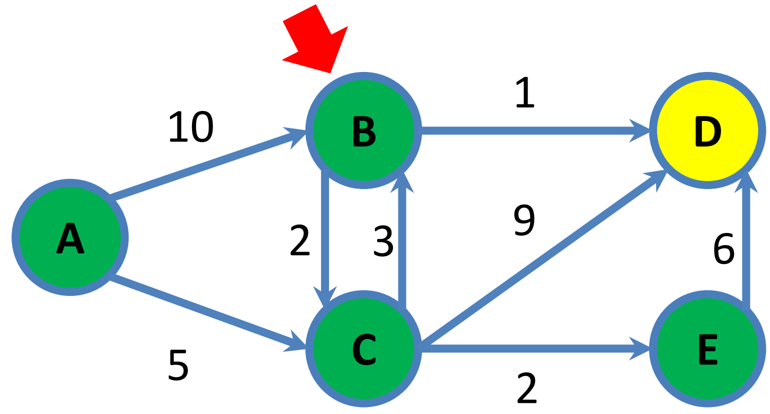
    - Graph is weighted



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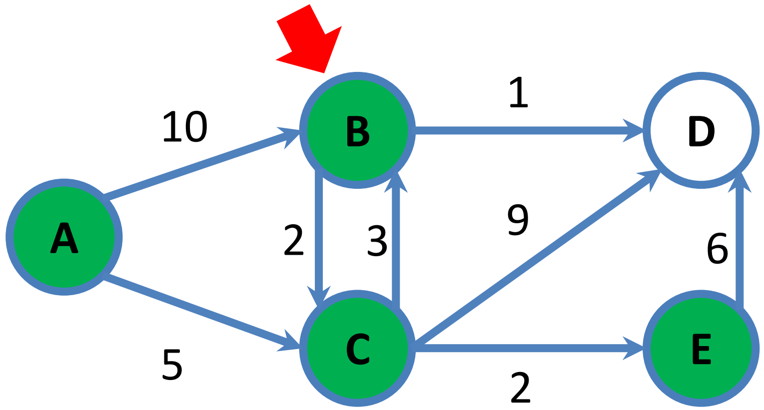

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    - Closest is D, so we move there

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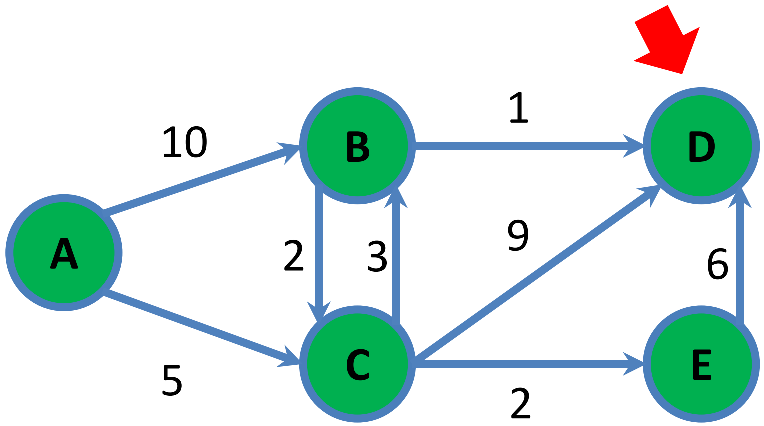

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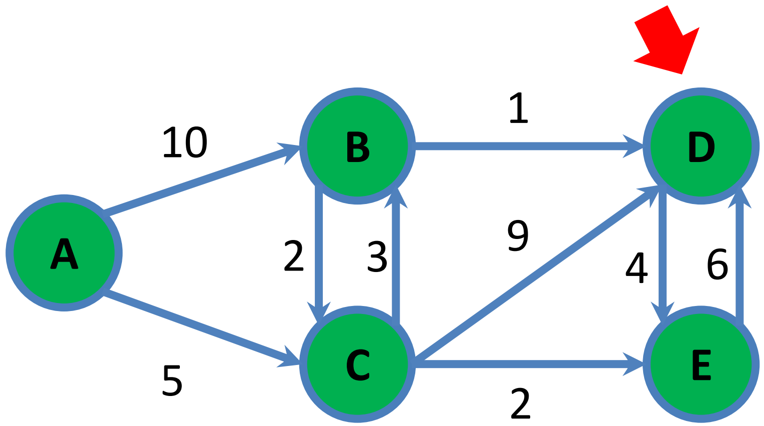

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    - C = 5, from here, we can see B, D and E. Update the distance
    - D = 9, from here, we can see E but E is already finalized
    - E = 7, from here, we can see D. Update the distance
    - Closest is D, so we move there

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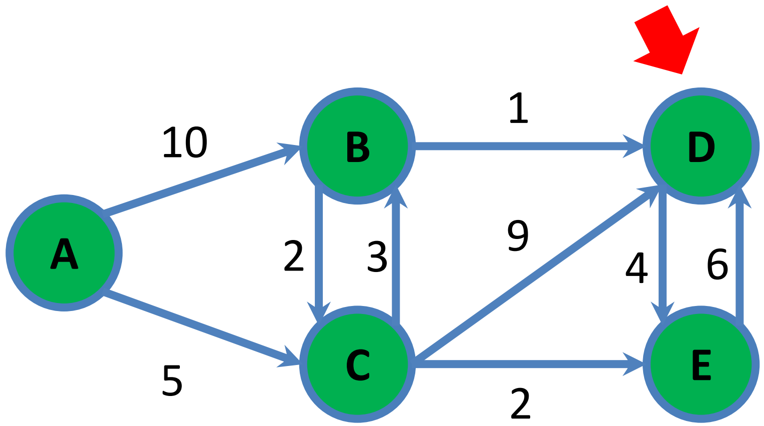
  - Consider the following directed graph
    - Graph is weighted



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    - D = 9, from here, we can see E but E is already finalized
    - E = 7, from here, we can see D. Update the distance
    - And we are done!

Questions?

- Algorithm?

- Algorithm? Very similar to the BFS except…

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  - Priority queue instead of a normal queue
    - Serve the closest vertex (not finalized)

- Algorithm? Very similar to the BFS except…
  - Priority queue instead of a normal queue
    - Serve the closest vertex (not finalized)
  - Update the distance if the neighbour vertex is visited but not finalized
    - To the shorter one

- This is the BFS algorithm, we change to Dijkstra now
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it with a distance 0
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue
      - v.distance = u.distance + 1

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- Try to modify this as part of the in-class activity use MinHeap()

- This is the BFS algorithm, we change to Dijkstra now
  - Let say we begin from vertex A
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    - Put source (A) into it with a distance 0
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v,w> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue
        - » Set v.distance = u.distance + w   v.previous = u, for the first path from the source to v has v.distance
      - If vertex v is discovered but not visited and v.distance > u.distance + w
        - » Update v.distance = u.distance + w   if new path found with lower distance from source to v, then v.previous = new u, to switch to second path that has lower v.distance

edge realisation

discovered = MinHeap()
discovered.append(source.distance, source)   (key, data)
discovered Heap always have lowest distance element at the beginning

- This is the BFS algorithm, we change to Dijkstra now
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  - We use a min-heap for our priority queue!

- This is the BFS algorithm, we change to Dijkstra now
  - Let say we begin from vertex A
    like Minheap, for discovered, to add in vertex with distance, Minheap woud prioritise the one with lowest distance
  - Have a priority queue for discovered
    - Put source (A) into it with a distance 0
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v,w> where u is the served
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      - If vertex v is discovered but not visited and v.distance > u.distance + w
        - » Update v.distance = u.distance + w
  - We use a min-heap for our priority queue!
    - Note that we need a pointer to the nodes to update distance in O(1)

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- Algorithm can be as follows (might differ):

```
discover_queue = MinHeap()
discover_queue.append([source,0])

while discover_queue is not empty:
    u = discover_queue.serve()
    u.visited = True
    for each <u,v,w> in u.edges:
        if v.visited = True:
            pass
        else:
            if v.discovered = False:
                discover_queue.append([v, u.distance+w])
                v.discovered = True
            else:
                if v.distance > u.distance+w:
                    discover_queue.update(v, u.distance+w)
                    v.discovered = True
```

Questions?

- Complexity?

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Graph

Shortest path with Dijkstra

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```

O(V)

discovered-queue

Graph

Shortest path with Dijkstra

O(V)

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```

Serve: O(log V)

if dense graph,maximum number of edges from each vertex
= V (total number of vertices) -1

upheap() method in MinHEAP

O(V)

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```

Serve: O(log V)

how many V, how many

edges of u only not entire graph

O(V)

bounded by V - 1
u can connect to any other vertices

Graph

Shortest path with Dijkstra

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O(V)

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```

Serve: O(log V)

O(V)

Update: O(log V)

- Time Complexity?

MinHeap like Tree

O(V)

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```

Serve: O(log V)

O(V)

search the vertex by key in tree

Update: O(log V)

Graph

Shortest path with Dijkstra

- Time Complexity? O(V^2 log V)

O(V)

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```

Serve: O(log V)

O(V)

Update: O(log V)

Graph
Shortest path with Dijkstra

- Time Complexity? O(V^2 log V) = O(E log V)

O(V)

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Serve: O(log V)

O(V)

Update: O(log V)

Graph

Shortest path with Dijkstra

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- Time Complexity? O(V^2 log V) = O(E log V)
  - Recall for dense graph, E ≈ V^2

O(V)

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```

downHeap tree, so log(V)

Serve: O(log V)

O(V)

Update: O(log V)

V^2

- Time Complexity? $O(V^2 \log V) = O(E \log V)$
  - Recall for dense graph, $E \approx V^2$

- Note that with Fibonacci heap instead of your binary heap, we can reduce the complexity further to $O(E + V \log V)$

- Time Complexity? O(V^2 log V) = O(E log V)
  - Recall for dense graph, E ≈ V^2

- Note that with Fibonacci heap instead of your binary heap, we can reduce the complexity further to O(E + V log V) = O(V^2 + V log V) = O(V^2)
  - For dense graph

Questions?

- What if we have a single source
  - As usual
- But single target?

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- What if we have a single source
  - As usual
- But single target?
- We can terminate after we have move the target vertex to the visited portion!
  - We would have the shortest distance
  - We can backtrack for the shortest path
    - Via vertex.previous attribute

Questions?

- Why does Dijkstra work?

- Why does Dijkstra work?
  - Let us use Nathan’s slides

Proof of Correctness

Claim: For every vertex v which has been removed from the queue, dist[v] is correct

- Notation:
  - V is the set of vertices
  - Q is the set of vertices in the queue
  - S = V / Q = the set of vertices who have been removed from the queue

Base Case

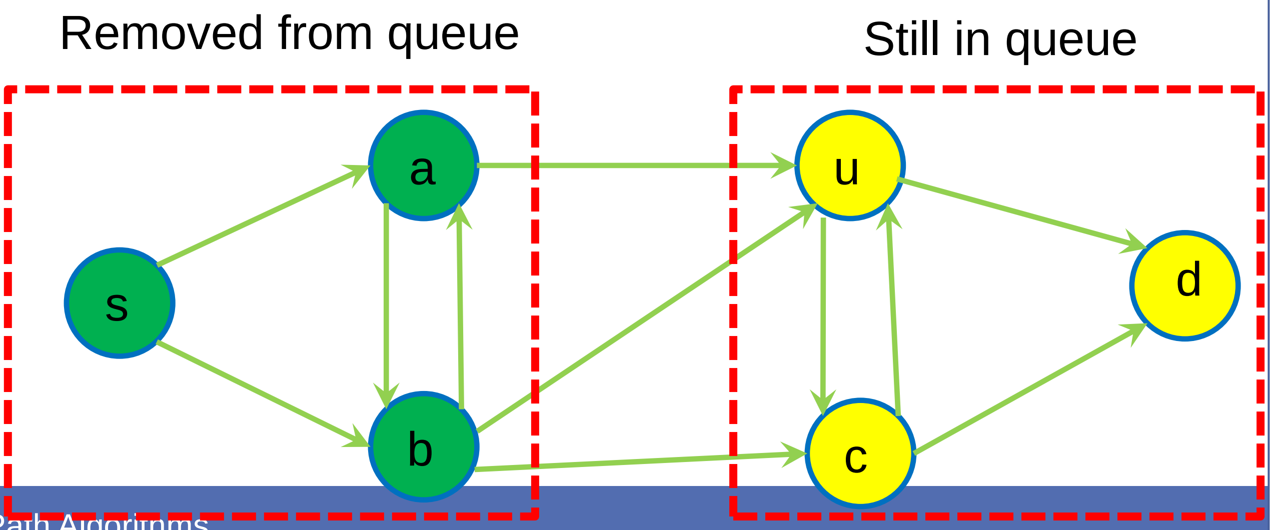
- Dist[s] is initialised to 0, which is the shortest distance from s to s (since there are no negative weights)

Proof of Correctness

Claim: For every vertex v which has been removed from the queue, dist[v] is correct

Inductive Step:

- Assume that the claim holds for all vertices which have been removed from the queue (S)
- Let u be the next vertex which is removed from the queue
- We will show that dist[u] is correct



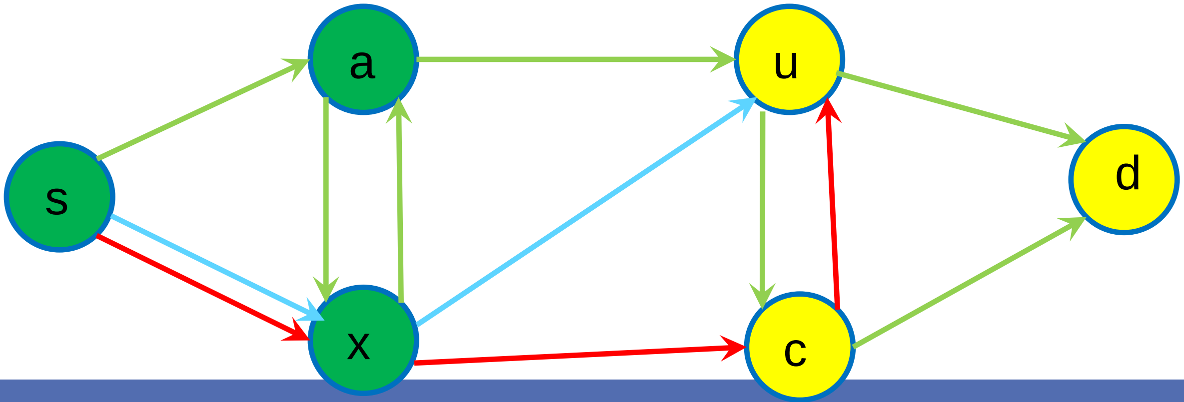
Proof of Correctness

Claim: For every vertex v which has been removed from the queue, dist[v] is correct

Inductive Step:

- Suppose (for contradiction) there is a shorter path P, s⇝u with len(P) < dist[u]
- Let x be the furthest vertex on P which is in S (i.e. has been finalised)
- By the inductive hypothesis, dist[x] is correct (since it is in S)

Current path
Assumed shorter path (P)



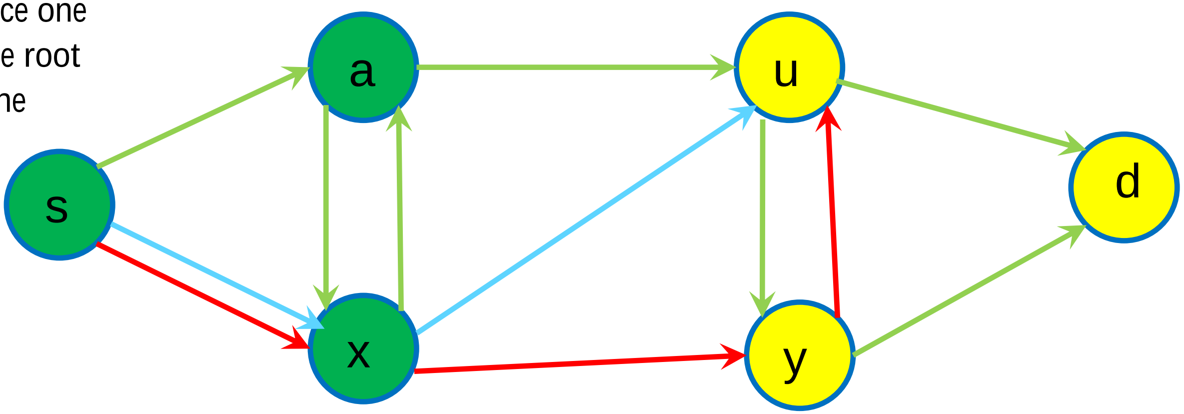
Proof of Correctness

Claim: For every vertex v which has been removed from the queue, dist[v] is correct

Inductive Step:

- By the inductive hypothesis, dist[x] is correct (since it is in S)
- Let y be the next vertex on P after x
- Len(P) < dist[u] (by assumption)
- Edge weights are non-negative
- Len($s \rightsquigarrow y$) <= len(P) < dist[u]

MinHeap() only serve the minuum distance one
so, MinHeap() would serve the one on the root
and compare its children roots and put one
to be root, so O(log(V))

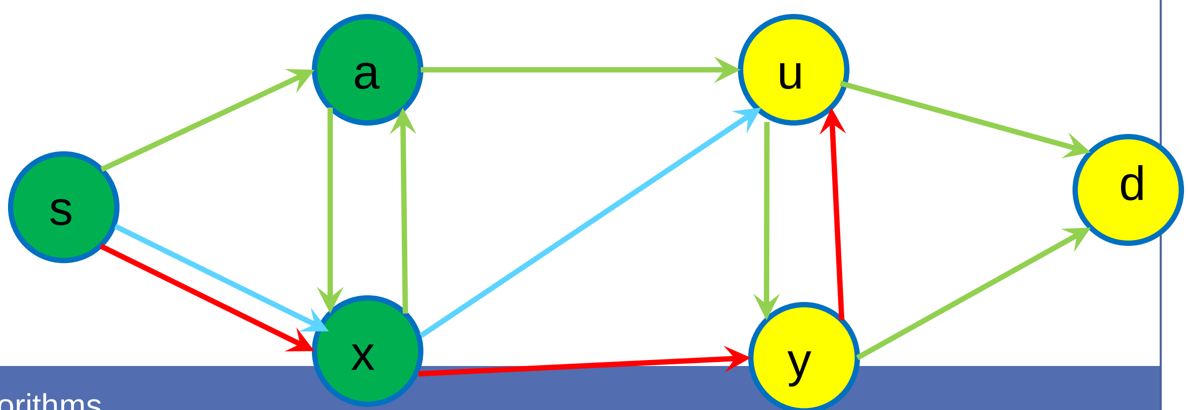


Proof of Correctness

Claim: For every vertex v which has been removed from the queue, dist[v] is correct

Inductive Step:

- Len(s⇝y) <= len(P) < dist[u]
- Since we said that P (via x and y) is a shortest path…
- dist[y] = len(s⇝y) < dist[u]
- So dist[y] < dist[u]…
- If y ≠ u, why didn't y get removed before u???
- If y = u, how can dist[y] <dist[u]???



- Why does Dijkstra work?
  - Let us use Nathan's slides
  - Or let me just explain it on the whiteboard…
  - Via proof by contradiction!

Questions?

Graph

Other shortest path?

- Bellman-Ford not work for negative distances
- Floyd-Warshall all pair

Other shortest path?

- Bellman-Ford
- Floyd-Warshall
  - With transitive closure

Other shortest path?

- Bellman-Ford
- Floyd-Warshall
  - With transitive closure

- We see it later in next lectures

Other shortest path?

- Bellman-Ford
  - Single source
  - Can know negative edges
- Floyd-Warshall
  - With transitive closure
  - Single or more sources
  - Can know negative edges

- We see it later in next lectures

Questions?

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Thank You