

Master Theorem

- $T(N) = a T(N/b) + f(N)$
 - Where $f(N) = O(N^k \log^p N)$
- $\log_b a > k$: $O(N^{\log_b a})$
- $\log_b a = k$
 - $p > -1$: $O(N^k \log^{p+1} N)$
 - $p = -1$: $O(N^k \log \log N)$
 - $p < -1$: $O(N^k \cdot 1)$
- $\log_b a < k$
 - $p > -1$: $O(N^k \log^p N)$
 - $p = -1$: $O(N^k \cdot 1)$
 - $p < -1$: $O(N^k \cdot 1)$
- Then what? Just **sub** it in ez!

Master Theorem Example 1

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| <ul style="list-style-type: none"> ▪ $T(N) = a T(N/b) + f(N)$ <ul style="list-style-type: none"> – Where $f(N) = O(N^k \log^p N)$ ▪ $\log_b a > k$: $O(N^{\log_b a})$ ▪ $\log_b a = k$ <ul style="list-style-type: none"> – $p > -1$: $O(N^k \log^{p+1} N)$ – $p = -1$: $O(N^k \log \log N)$ – $p < -1$: $O(N^k \cdot 1)$ ▪ $\log_b a < k$ <ul style="list-style-type: none"> – $p > -1$: $O(N^k \log^p N)$ – $p = -1$: $O(N^k \cdot 1)$ – $p < -1$: $O(N^k \cdot 1)$ | <ul style="list-style-type: none"> ▪ $T(N) = 1 * T(N/2) + c$ ▪ Where $f(N) = O(N^0 \log^0 N)$ ▪ $\log_b a = 0$ ▪ $k = 0$ ▪ $p = 0$ ▪ $O(N^0 \log^{0+1} N)$ ▪ $O(\log N)$ |
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- Handwritten notes: Red circles around $O(N^k \log^{p+1} N)$, $O(N^0 \log^{0+1} N)$, and $O(\log N)$. A red arrow points from the first circle to the second. The text "log N" is written in red next to the third circle.*

$\log^0(N) = \log(N)^0 = \log(N^0) = \log(1) = \log_b 1 = 0$ for $b = \text{any number}$

Master' theorem not work on $T(-N)$

- $T(N) = a T(N/b) + f(N)$
 - Where $f(N) = O(N^k * \log^p N)$
- $\log_b a > k$: $O(N^{\log_b a})$
- $\log_b a = k$
 - $p > -1$: $O(N^k * \log^{p+1} N)$
 - $p = -1$: $O(N^k * \log \log N)$
 - $p < -1$: $O(N^k * 1)$
- $\log_b a < k$
 - $p > -1$: $O(N^k * \log^p N)$
 - $p = -1$: $O(N^k * 1)$
 - $p < -1$: $O(N^k * 1)$
- $T(N) = 8 * T(N/2) + Nc$
- where $f(N) = O(N^1 * \log^0 N)$
- $\log_b a = 3$
- $k = 1$