

#### MONASH INFORMATION TECHNOLOGY

# FIT2004 Algorithms and Data Structures

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Referencing materials by Rafael Dowsley, Nathan Companez, Aamir Cheema, Arun Konagurthu and Lloyd Allison





## Faculty of Information Technology, Monash University

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Ready?

## Agenda

- Circulation with Demands
- Circulation with Demands and Lower Bound
- Applications Example
  - Survey Design
  - Airline Scheduling





Let us begin...



- You have learnt Graph
- You have learnt Network Flow



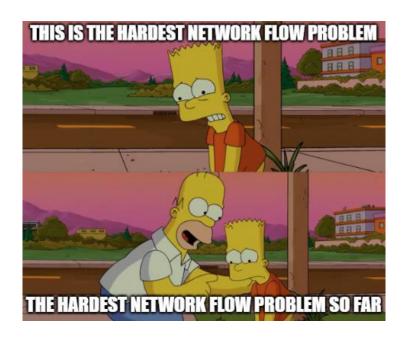
- You have learnt Graph
  - How many problems can be modelled as a Graph, then be solved
- You have learnt Network Flow



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  - Likewise, we have explore the simple Bipartite Matching problem.



- You have learnt Graph
  - How many problems can be modelled as a Graph, then be solved.
- You have learnt Network Flow
  - Likewise, we have explore the simple Bipartite Matching problem.
  - ... now let us push 1 step further!





## Questions?







Recall the 2 concepts from Network Flow



- Recall the 2 concepts from Network Flow
  - Capacity Constraint
    - Flow <= Capacity for an edge</p>
  - Flow conservation
    - Incoming flow to a vertex == outgoing flow from the vertex

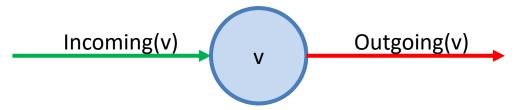


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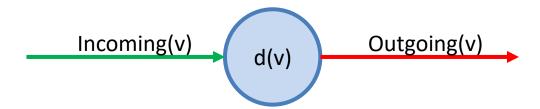
## A Feasibility Problem...



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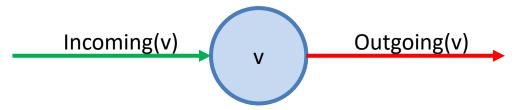
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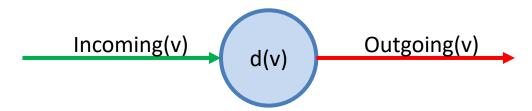


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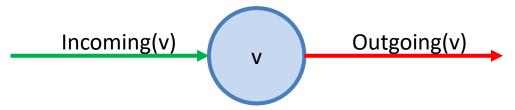


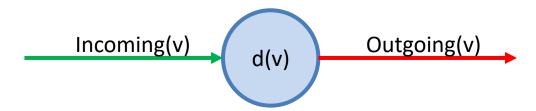


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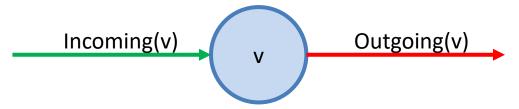


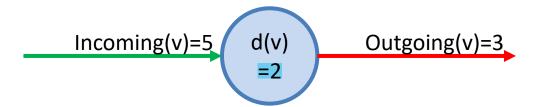


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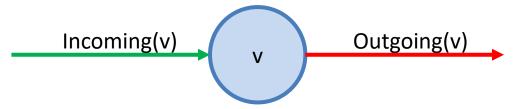


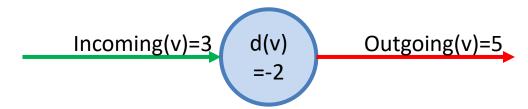


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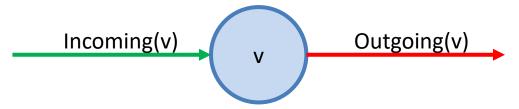


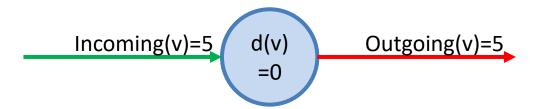


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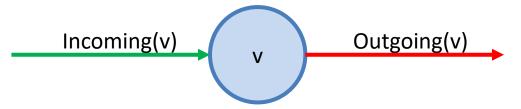


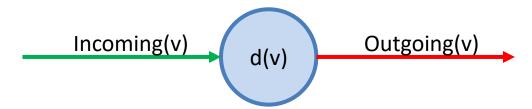


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## A Feasibility Problem...

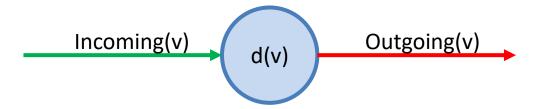
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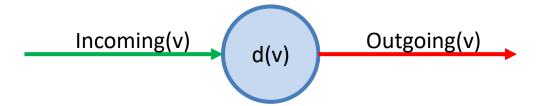
- Recall the 2 concepts from Network Flow
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    - Now what if we tweak this rule? incoming(v) outgoing(v) = demand(v)



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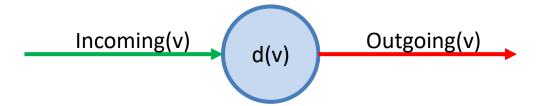


Circulation with Demands is a feasibility problem

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## A Feasibility Problem...

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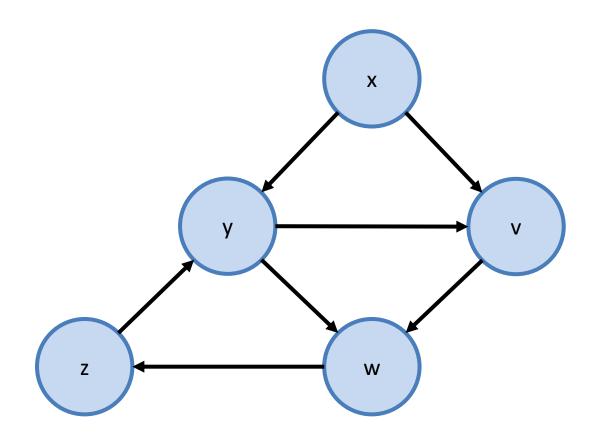
Circulation with Demands is a feasibility problem that satisfy both of the above!



## Questions?

## A Feasibility Problem...





## A Feasibility Problem...

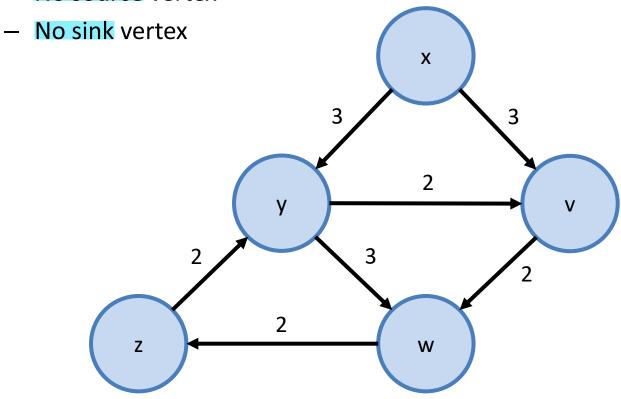


An example below...

 With the usual capacity Χ 2 W

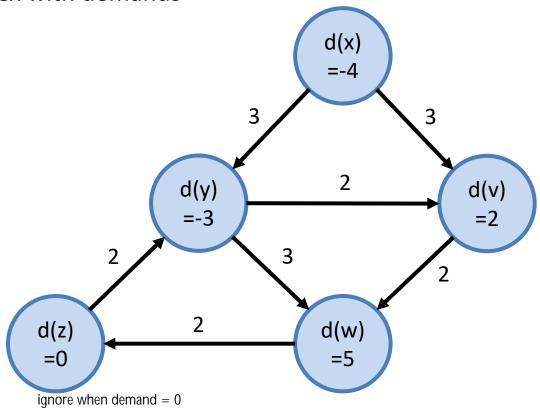


- An example below...
  - No source vertex



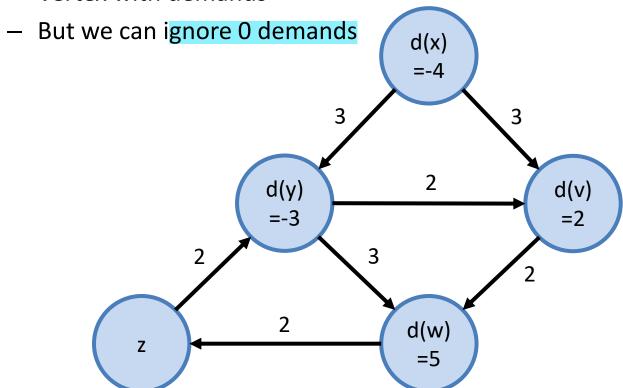


- An example below...
  - Vertex with demands



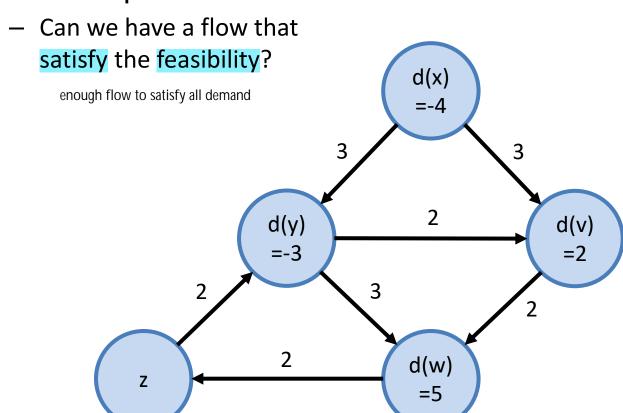


- An example below...
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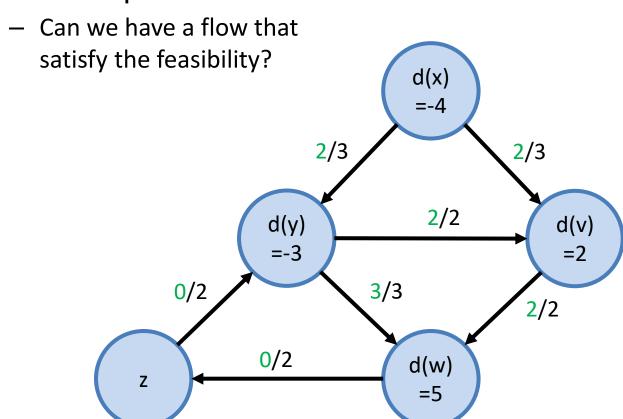
## A Feasibility Problem...





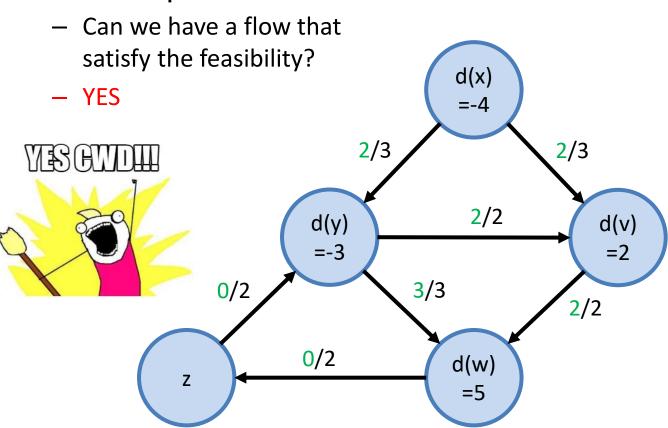
## A Feasibility Problem...





## A Feasibility Problem...





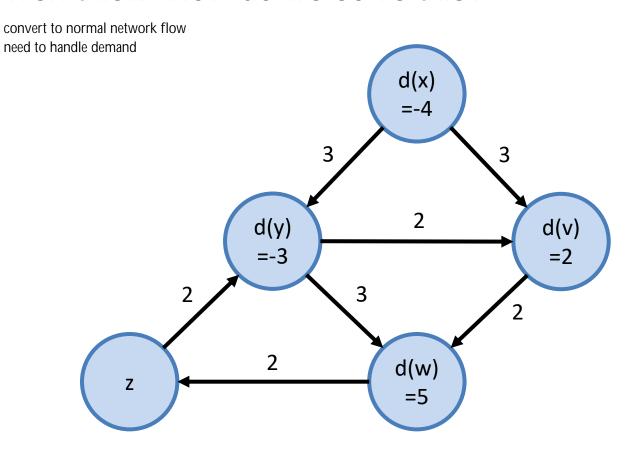


## Questions?

## How to Check Feasibility...



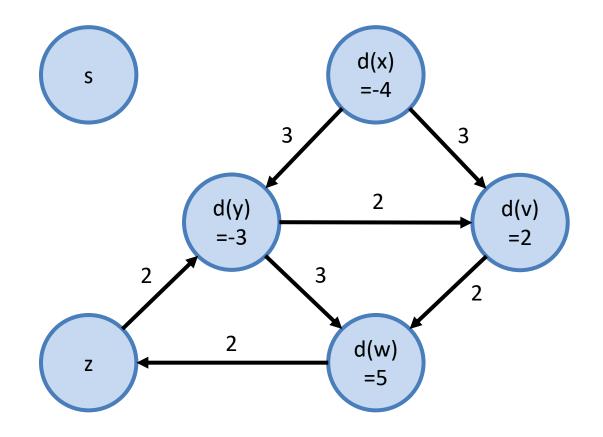
Given this... How do we solve this?



## How to Check Feasibility...



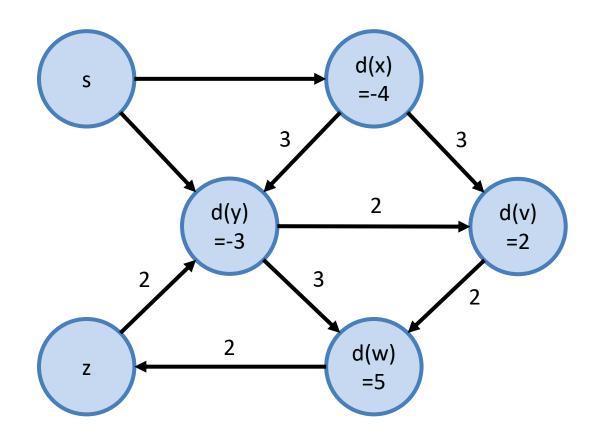
Make a source



### How to Check Feasibility...



Make a source, link to all negative demand



# How to Check Feasibility...



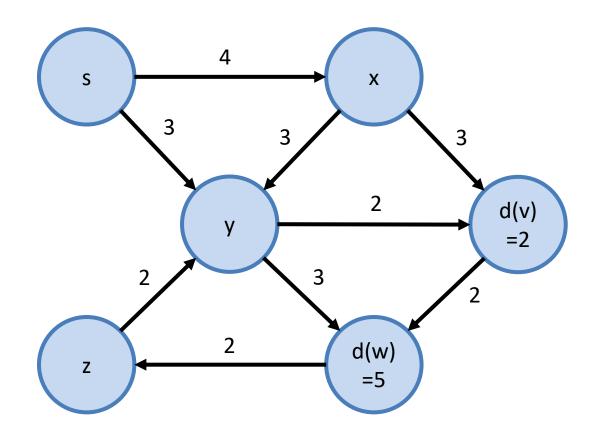
Make a source, link to all negative demand, weighted

imaginary source to provide demanded flow to vertices that has negative demand to neutralise the negative value 4 d(x)=-4 2 d(y) d(v)=-3 d(w)

### How to Check Feasibility...



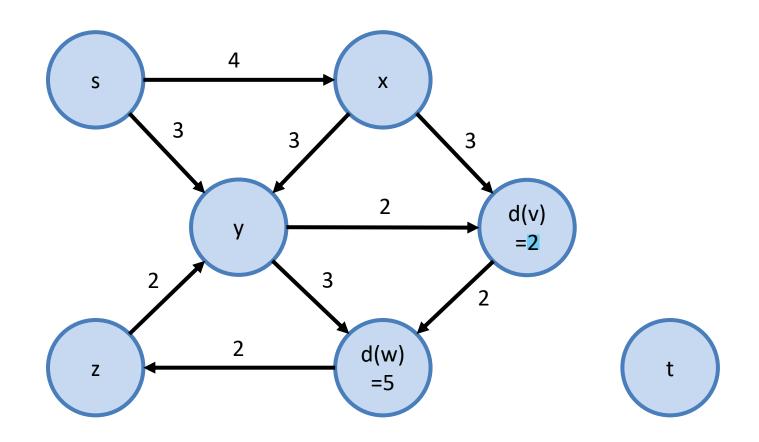
We are done! for source >.<</p>



## How to Check Feasibility...



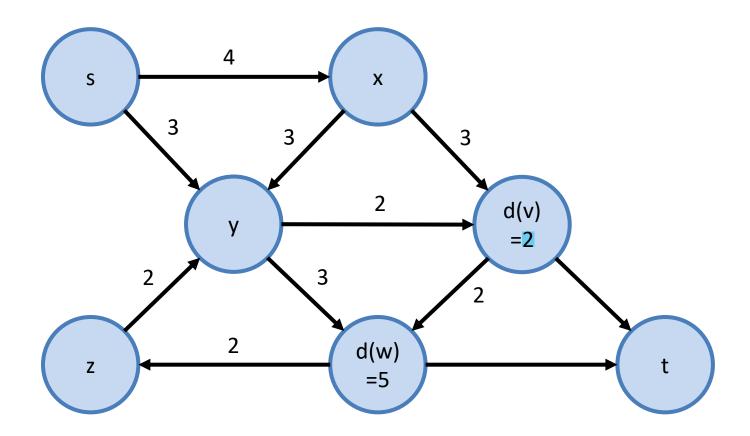
#### Make a sink



# How to Check Feasibility...



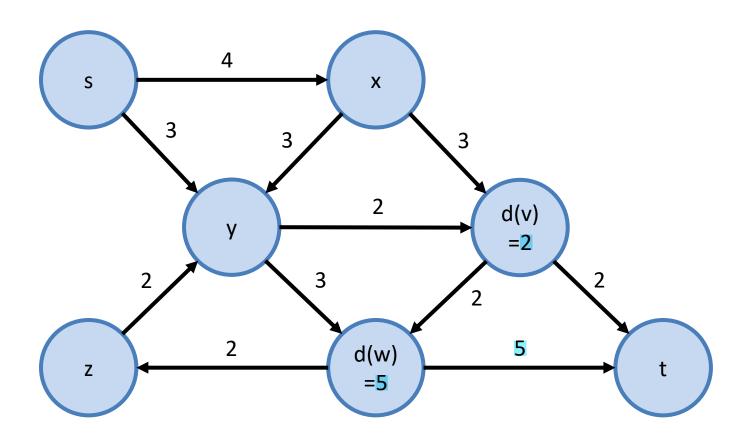
Make a sink, link from positive demand





## How to Check Feasibility...

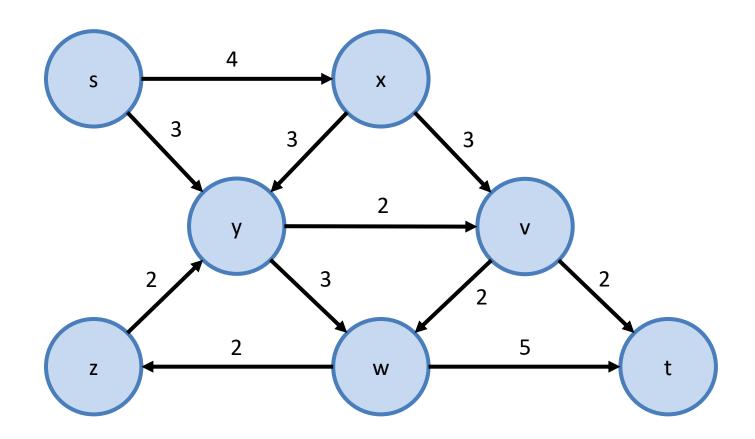
Make a sink, link from positive demand, weighted



## How to Check Feasibility...

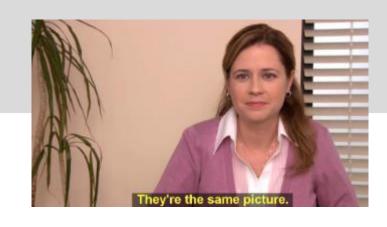


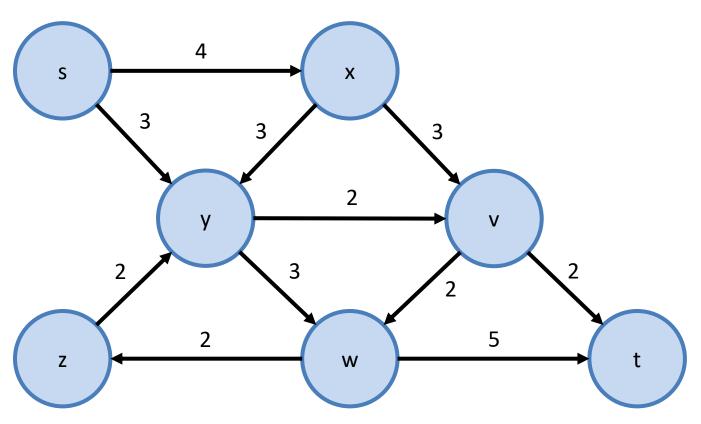
We are done! for sink >.<</p>



How to Check Feasibility...

Now same as network flow!





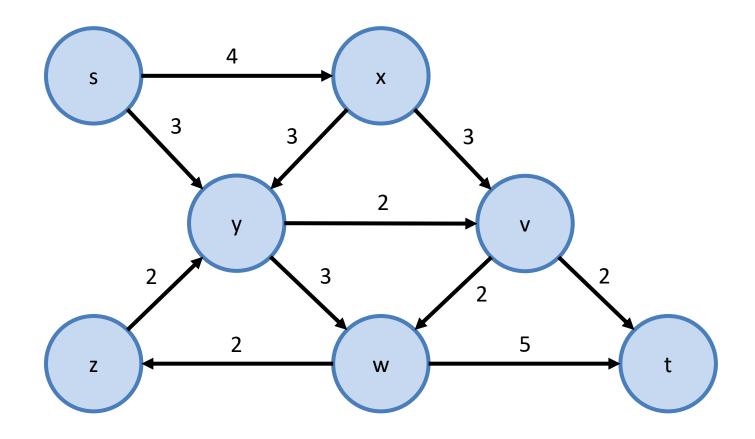


# Questions?

## How to Check Feasibility...



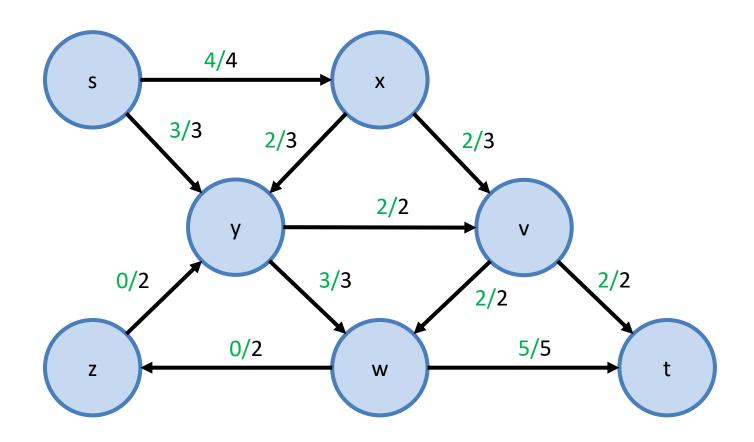
Then we run Ford-Fulkerson



### How to Check Feasibility...



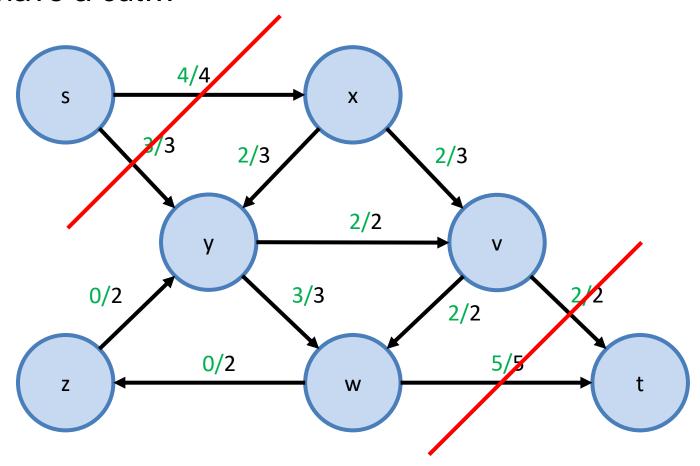
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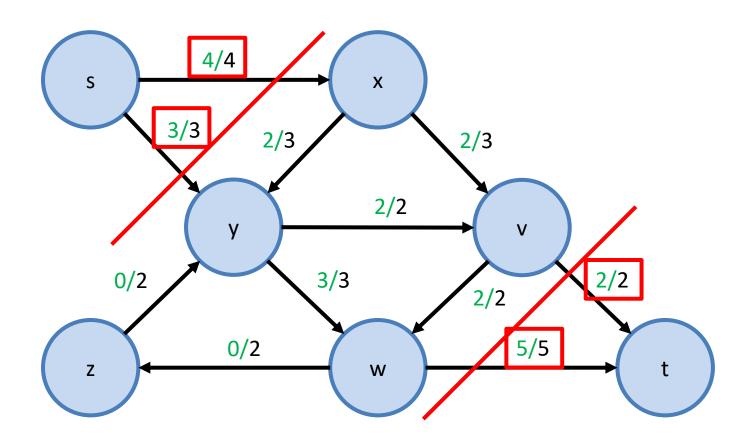
If we have a cut...



## How to Check Feasibility...



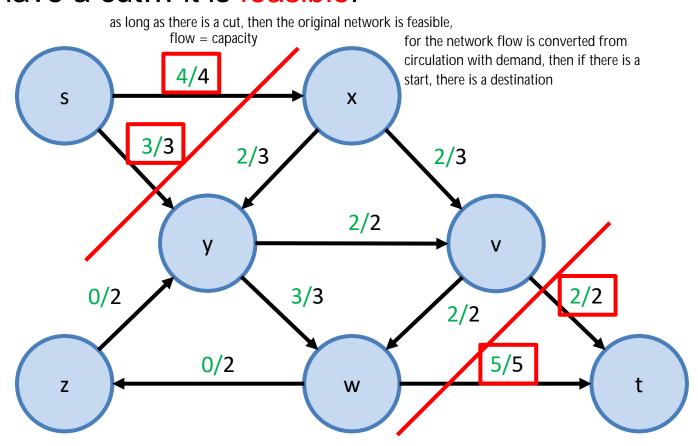
■ If we have a cut... find min-cut to get max-flow



# How to Check Feasibility...



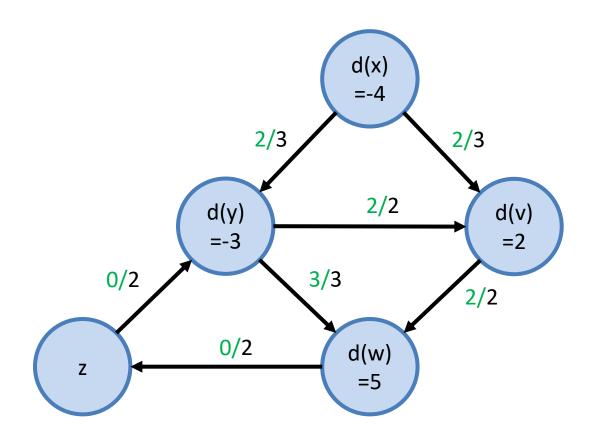
If we have a cut... it is feasible!



### How to Check Feasibility...



Then we just clean it up as the solution



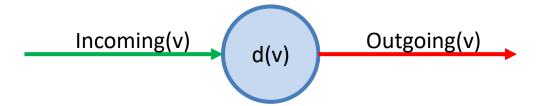


# Questions?



### A Feasibility Problem...

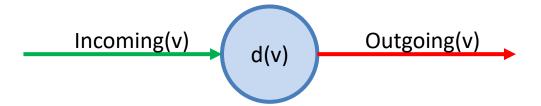
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 Circulation with Demands is a feasibility problem that satisfy both of the above!



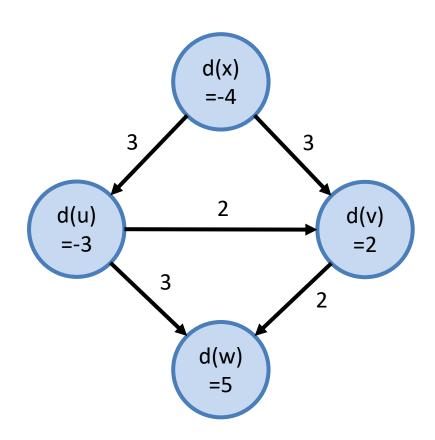
- Recall the 2 concepts from Circulation with Demands
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    - Lower bound for an edge <= Flow <= Capacity for an edge</p>
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 Circulation with Demands is a feasibility problem that satisfy both of the above!



Consider the following...

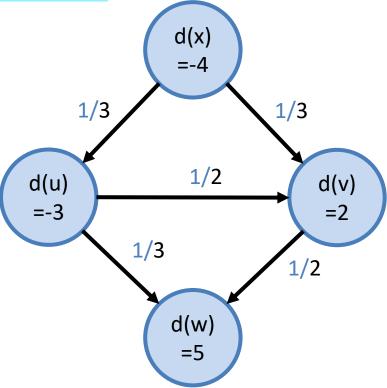




A Feasibility Problem...

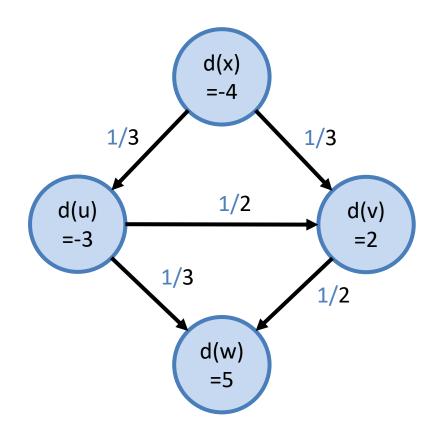
Consider the following...
 edges have lower bound

of 1



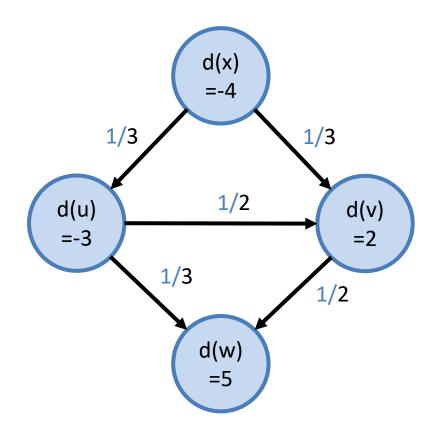


Can we find a feasible solution?





Can we find a feasible solution? Of course!

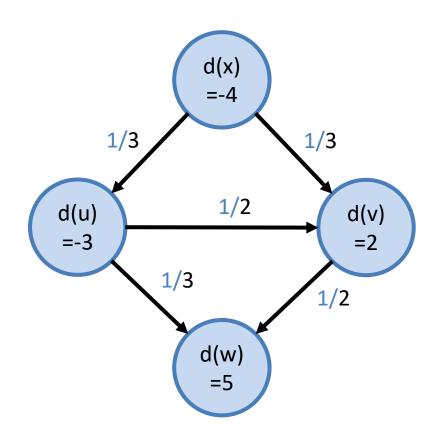




# Questions?

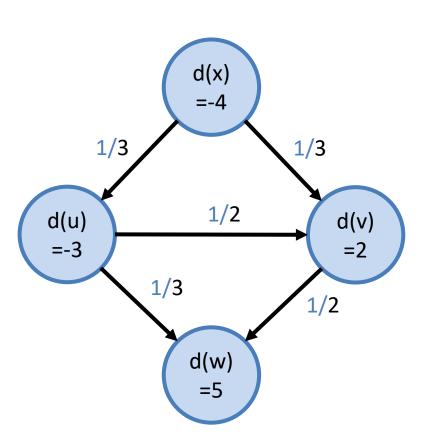


We will need to make some transformation...



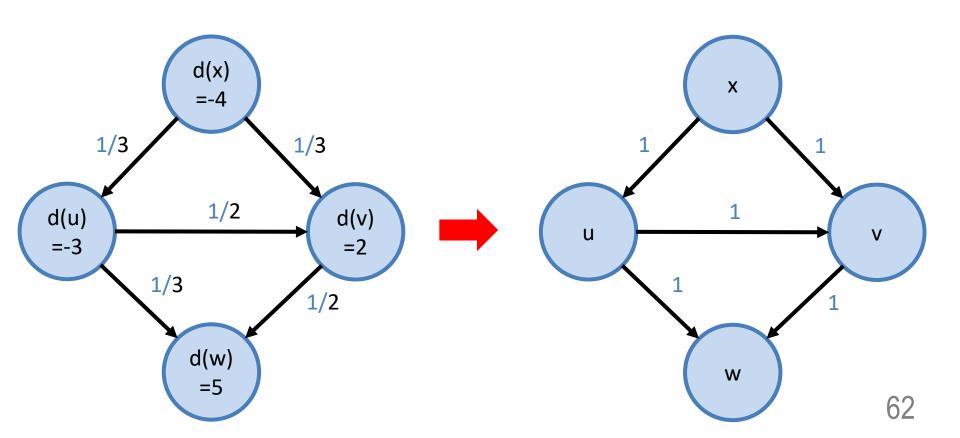


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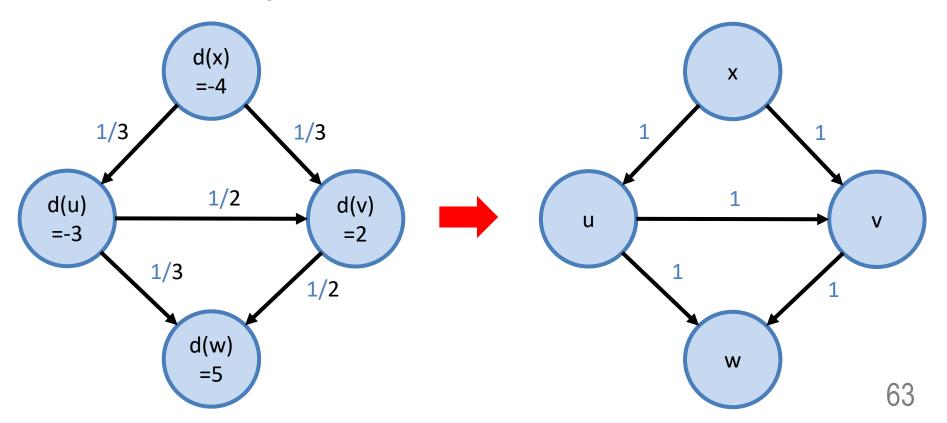


 We will need to make some transformation by removing the lower bound to a temp network...



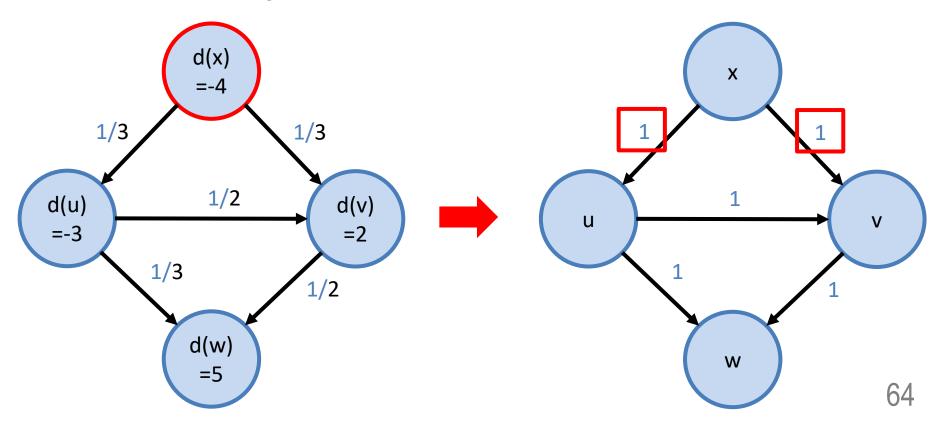


- We will need to make some transformation by removing the lower bound to a temp network...
- Thus, the original reduced...



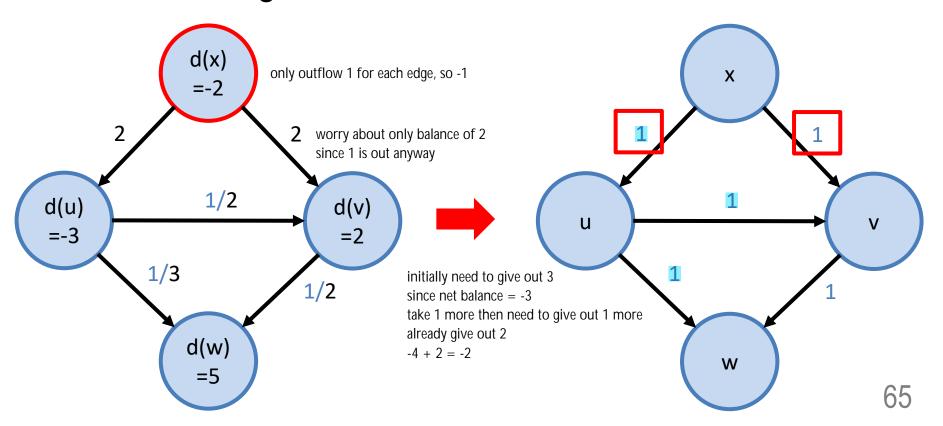


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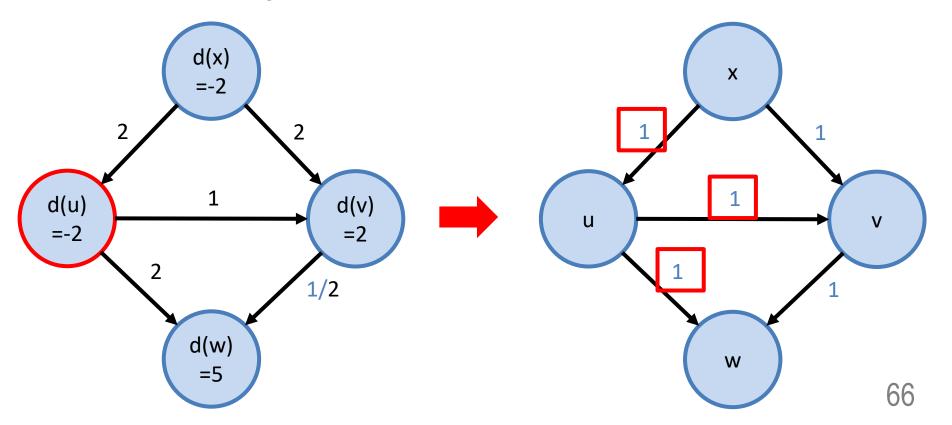


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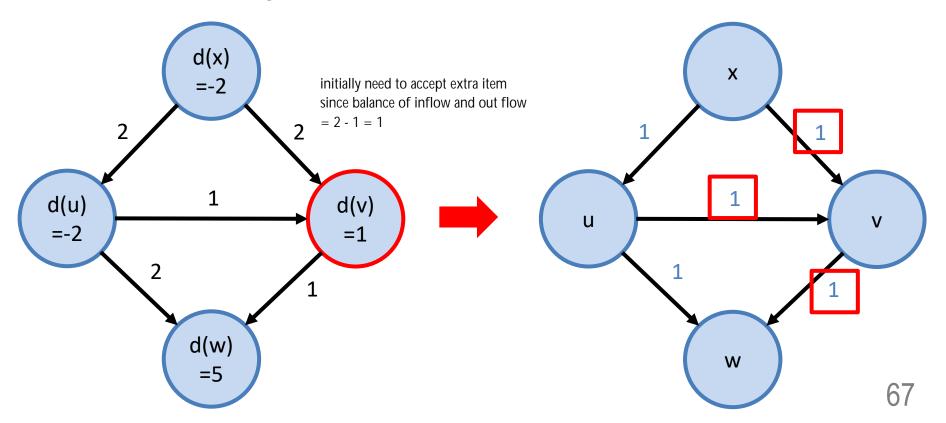


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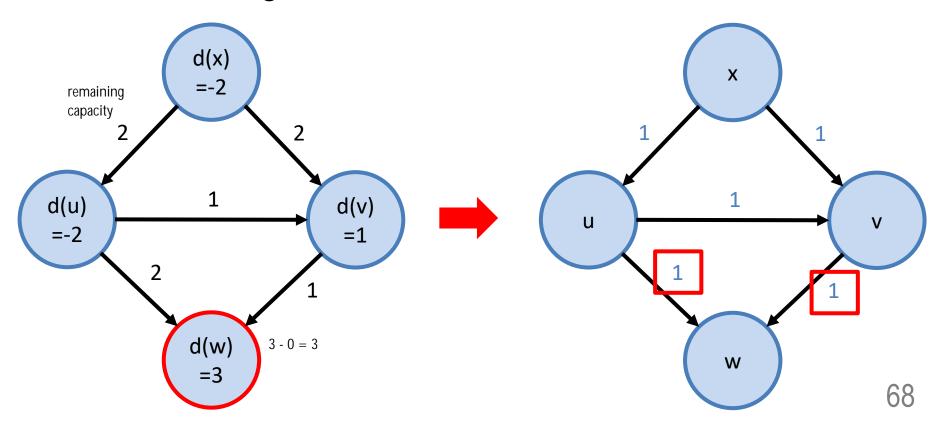


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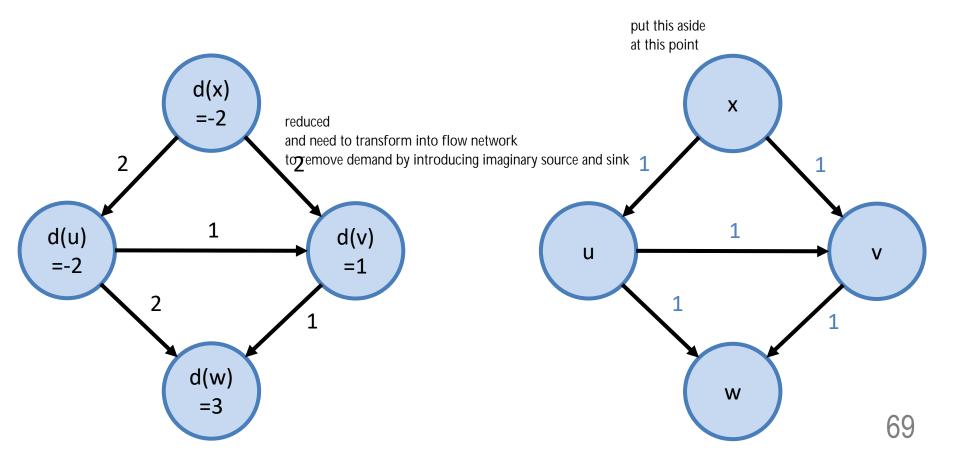
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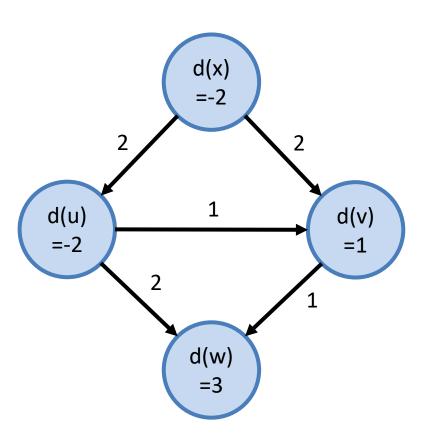
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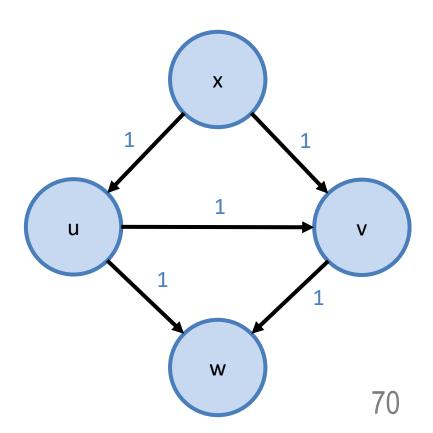




A Feasibility Problem...

Thus, the original reduced...





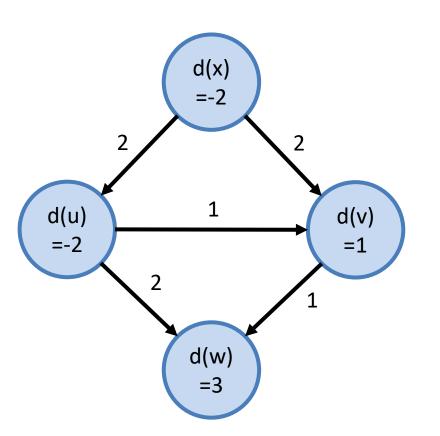


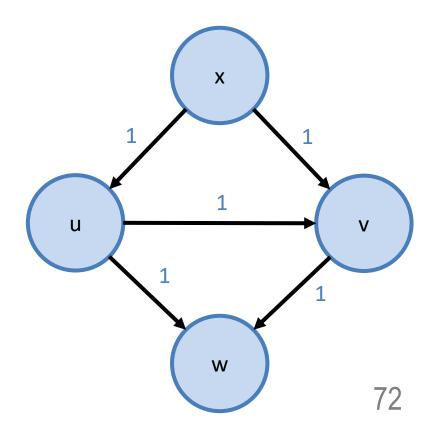
# Questions?



A Feasibility Problem...

 Then we follow the same Circulation with Demands as earlier for the reduced network...

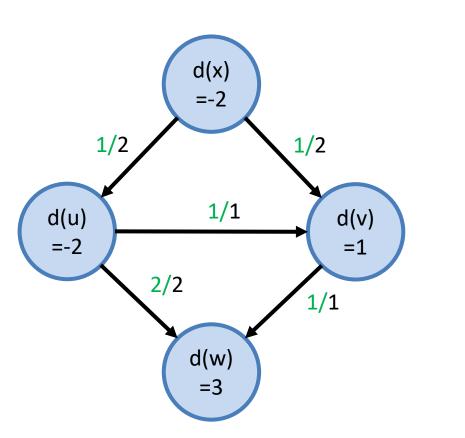


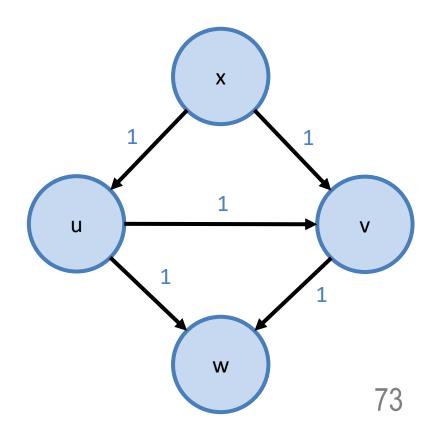




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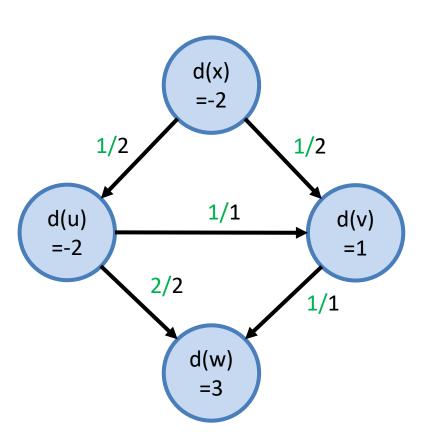
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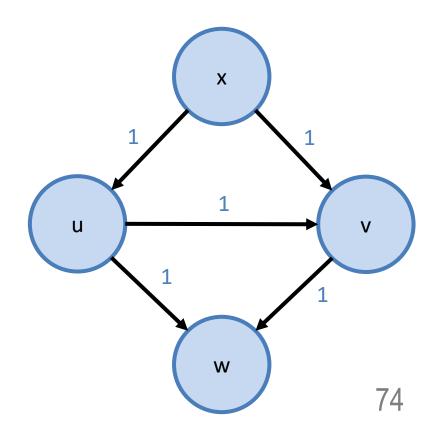




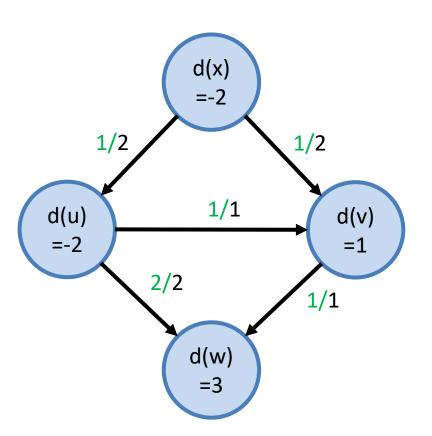


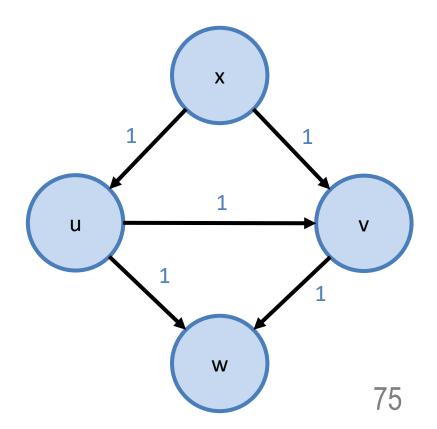
Then we follow the same Circulation with Demands as earlier for the reduced network... It is feasible!



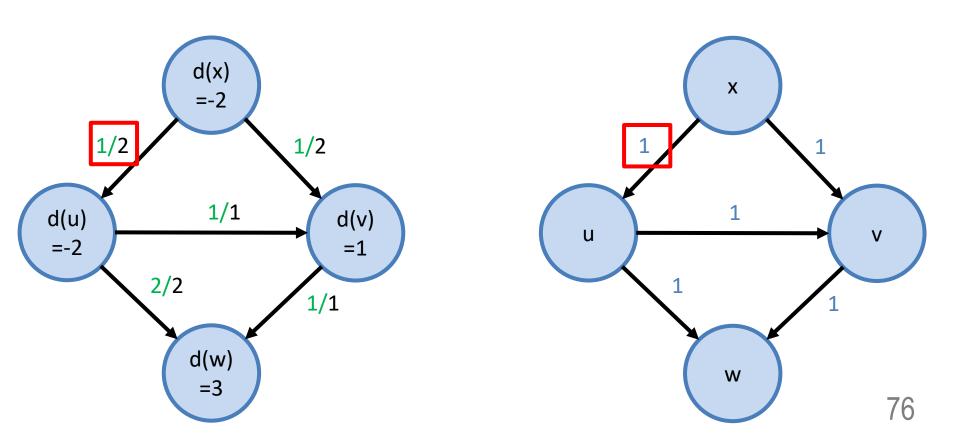




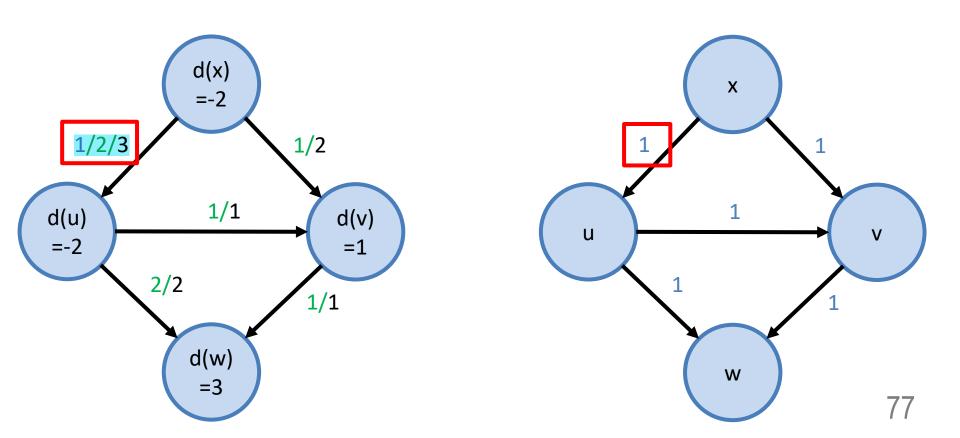




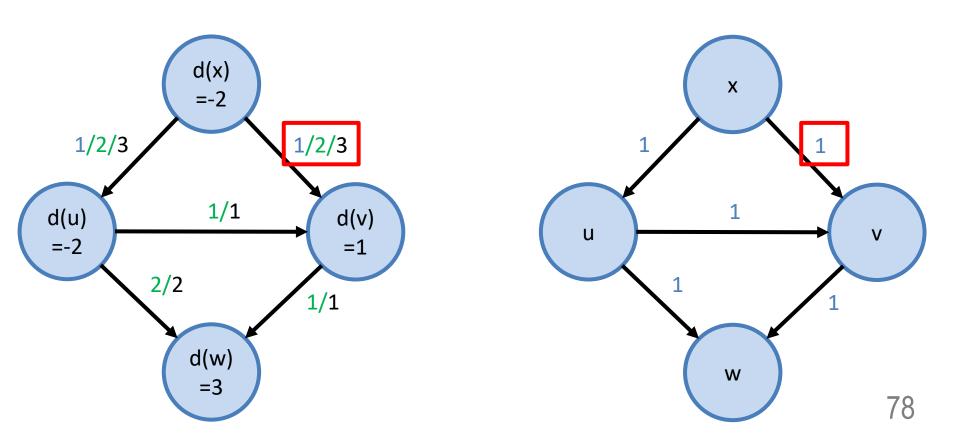




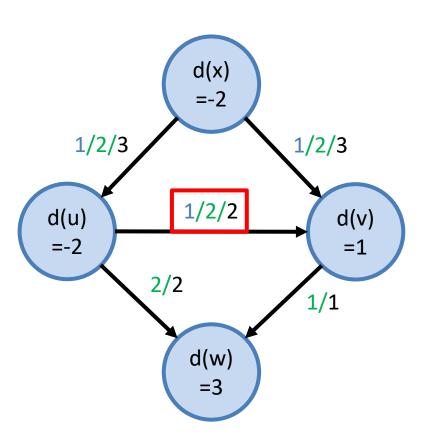


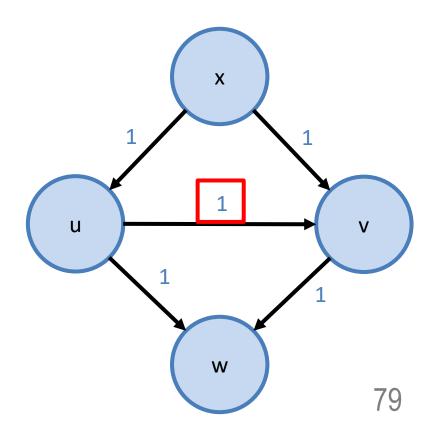




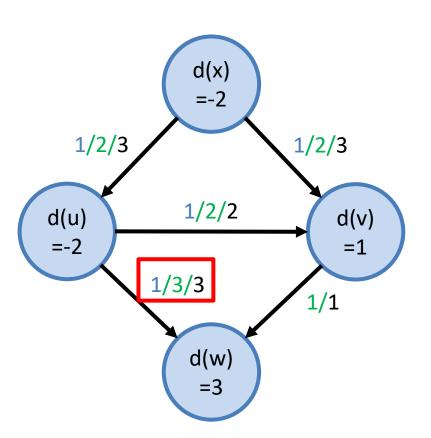


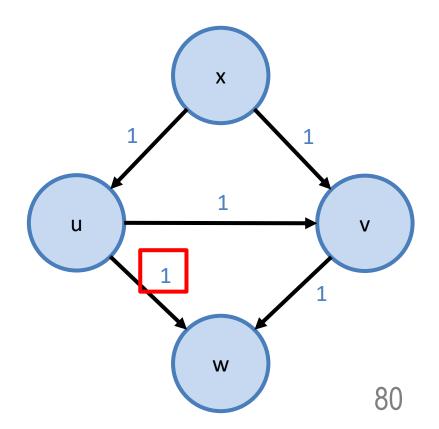




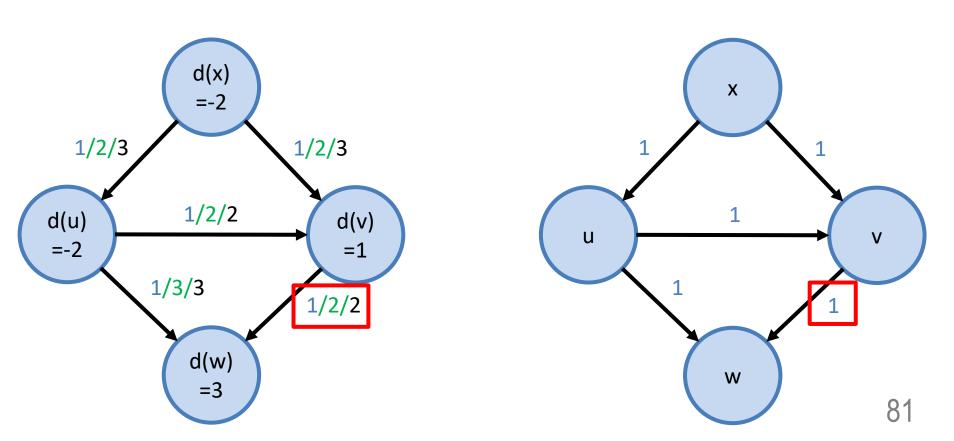






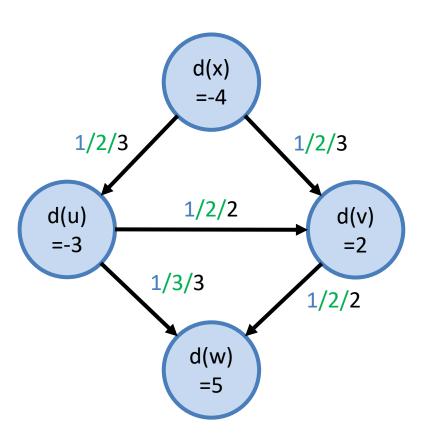


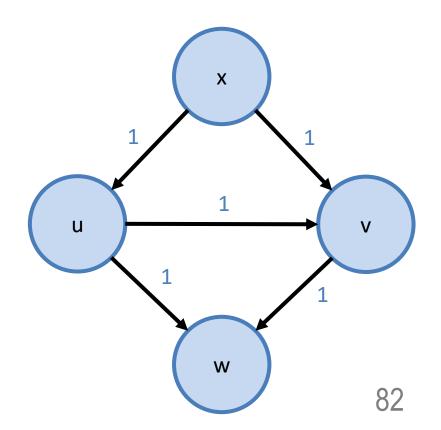






Don't forget the demand of the vertices as well!

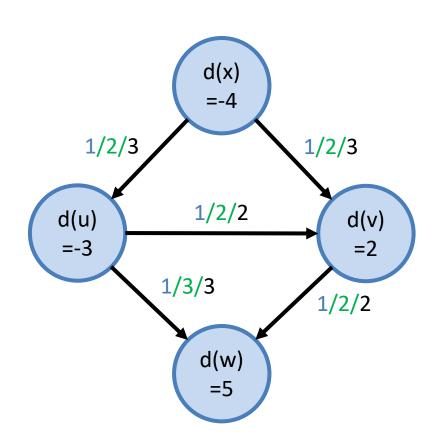






A Feasibility Problem...

And we are done!



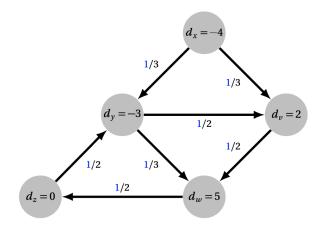


# Questions?



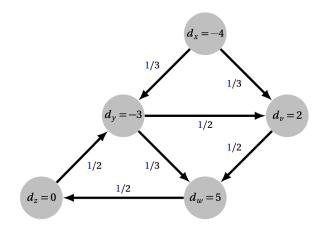
A Feasibility Problem...

 The following is an example from the Clayton campus



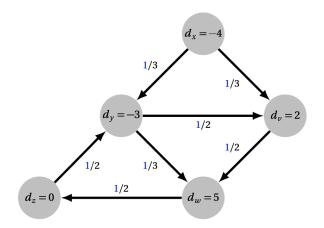


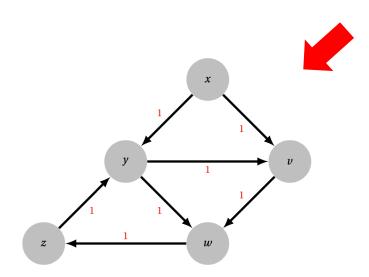
- The following is an example from the Clayton campus
- Work it out on your own to see if it is feasible before we discuss in class





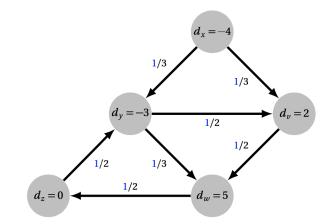
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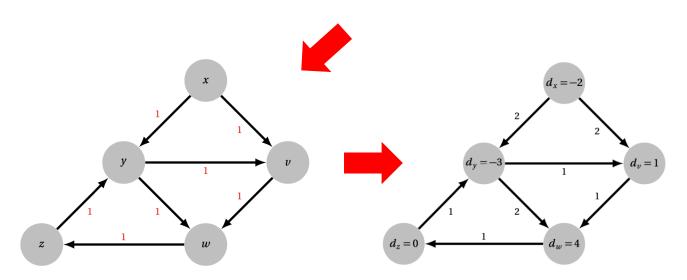






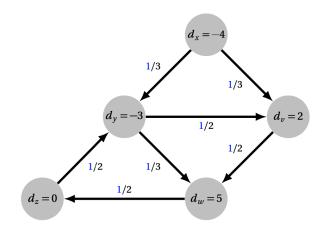
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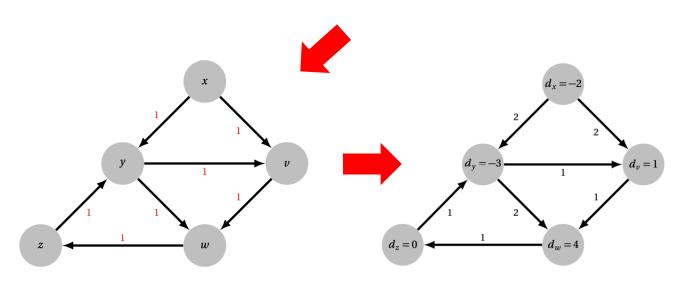






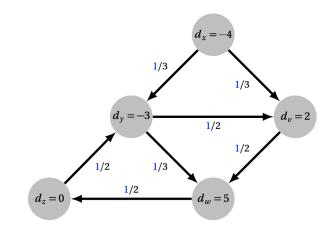
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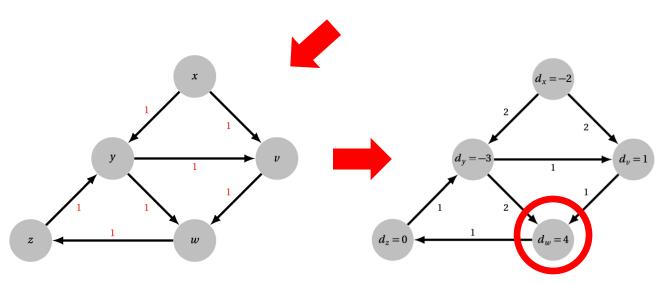






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# Questions?



- We know what is a flow network.
- We know how to design flow network for bipartite matching.
- We know how to design flow network for circulation with demands and lower bound?



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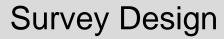
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- We know how to design flow network for circulation with demands and lower bound? We shall see them now...
- Not that we only deal with integers, to make it simpler



# Questions?







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- You have P products



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  - You do not want to ask the customer for too many reviews.
  - You do want to ask the customer for at least some reviews.
  - Each product needs to have at least some reviews.
  - Each product do not require more than some reviews.



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- Let us add some notations



- You have C customers who have used the product,  $c_1, c_2, c_3, ..., c_n$
- You have P products,  $p_1, p_2, p_3, ..., p_m$
- You want to conduct a survey, but...
  - You do not want to ask the customer  $c_i$  for too many reviews  $c_i^+$ .
  - You do want to ask the customer  $c_i$  for at least some reviews  $c_i$ .
  - Each product  $p_j$  needs to have at least some reviews  $p_j^-$ .
  - Each product  $p_i$  do not require more than some reviews  $p_i^+$ .
  - Of course, each customer can only give a review per product.
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# **Applications with Network Flow**Survey Design

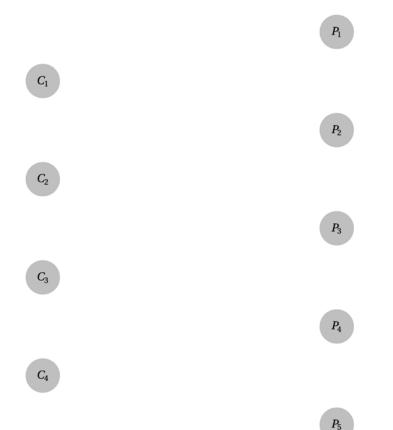


Now let us go through 1 by 1

# Applications with Network Flow Survey Design



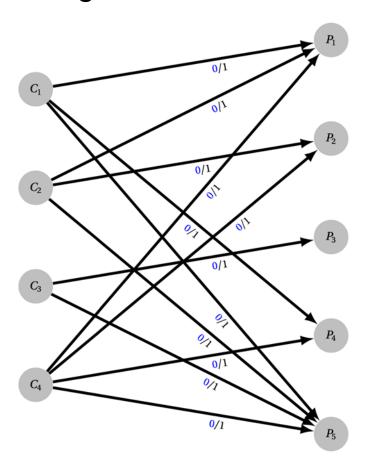
The customer and the product.



## Survey Design



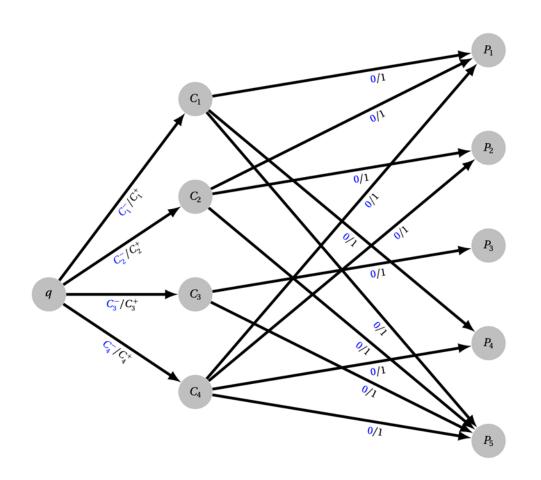
Each customer can give 0 or 1 review for the product



## Survey Design



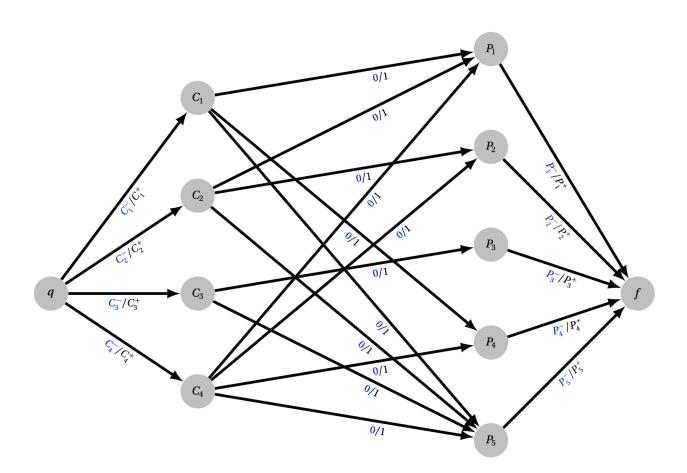
There is range of review expected from the customer



## Survey Design



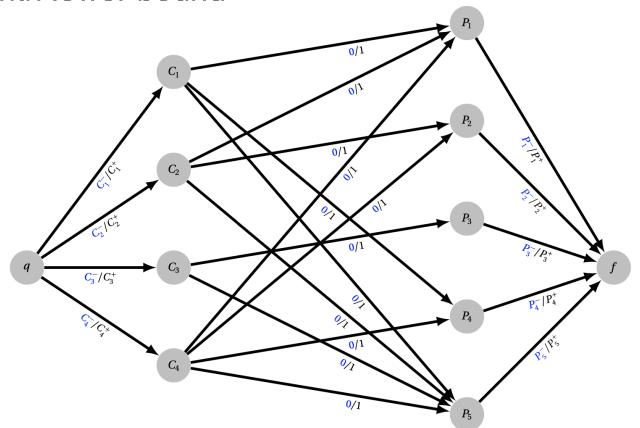
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## Survey Design



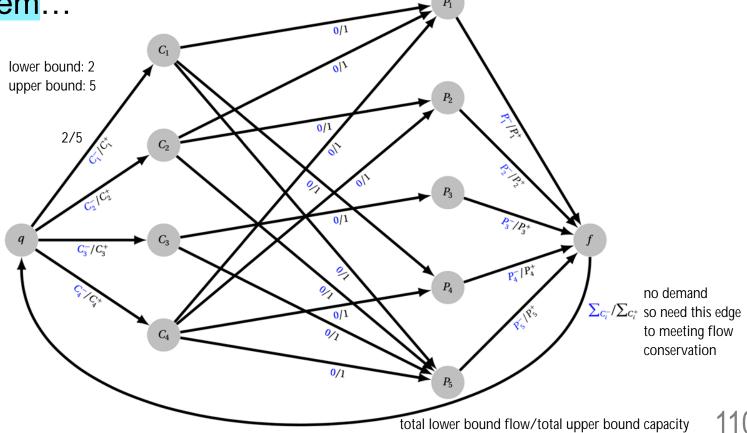
 In a way, it do look like a bipartite matching problem but with lower bound



## Survey Design



In a way, it do look like a bipartite matching problem but with lower bound, but since it is a circulation problem...





# Questions?



## Airline Scheduling



You have a collection of airplanes



- You have a collection of airplanes
- You have a list of routes
- Some of the routes are very profitable, thus you want to fly the routes
  - Departure location
  - Departure time
  - Arrival location
  - Arrival time



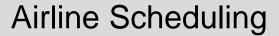
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- The airplanes can start flying from any location



- You have a collection of airplanes, k
- You have a list of routes, r<sub>1</sub>, r<sub>2</sub>,..., r<sub>n</sub>
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- Imagine you have the following routes:
  - Route 1: SYD 6am MEL 7am
  - Route 2: CBR 8am SYD 9am
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- Can you cover these 4 vital routes, using only 2 planes?

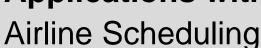


## Airline Scheduling

First, we list down the routes imagine the x-axis as time...





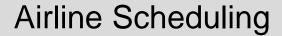




- First, we list down the routes imagine the x-axis as time...
  - Since they are vital flights, we want to always fly and thus lower-bound is set to 1.

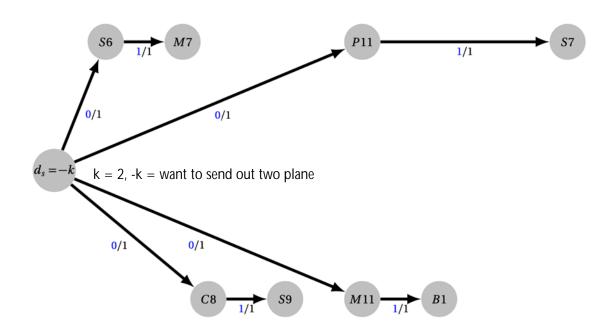


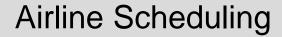






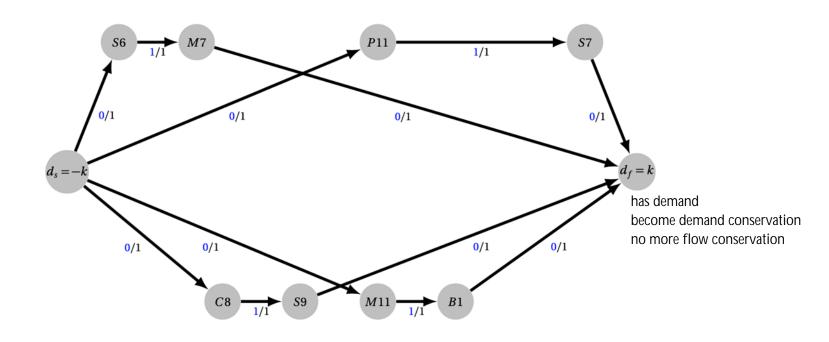
- Then we add a source, which we can place our planes from in any of the locations.
  - Lower bound is 0 because there is not requirement to be placed at which location

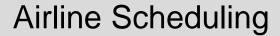






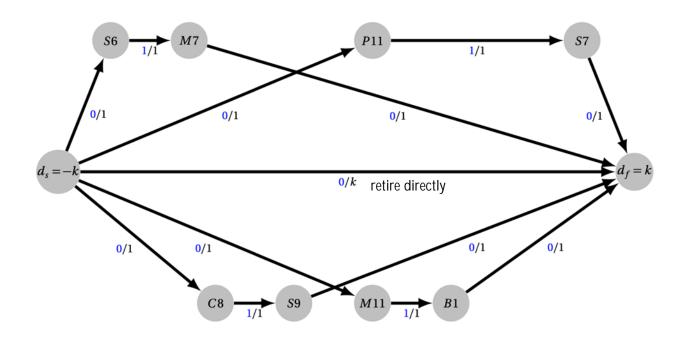
- Next, we add a sink which we can retire our planes at any of the locations.
  - No requirement for the planes to retire from any location

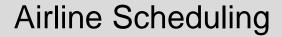






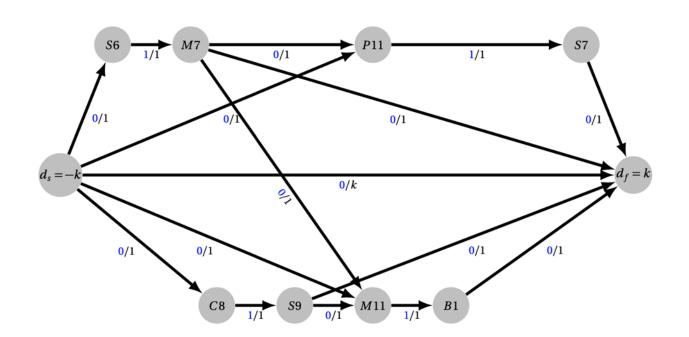
- But what if we don't need all our planes to cover all vital routes?
  - Thus, they can go from start to retire directly







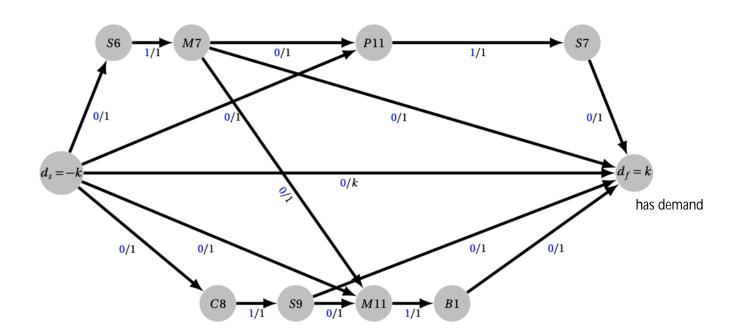
- Since it is possible for a plane to follow a route, then go to another route instead of retiring...
  - We add the edge, again it is optional



## Airline Scheduling

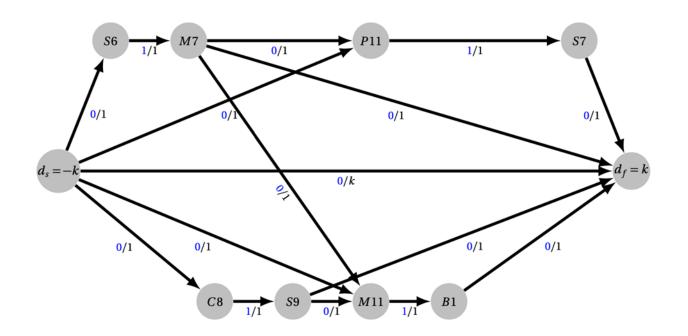


Then we just solve this as it is!





- Then we just solve this as it is!
- Answer is yes with 2 planes
  - Plane1: Route1 (S6->M7), then Route4 (P11->S7)
  - Plane2: Route2 (C8->S9), then Route3 (M11->B1)





# Questions?

Other Examples?



# Applications with Network Flow Other Examples?



Several examples in the studio

## Other Examples?



- Several examples in the studio
  - Choosing profitable projects
  - Determining if teams/ players can progress in a tournament

#### Other Examples?



#### Several examples in the studio

- Choosing profitable projects
- Determining if teams/ players can progress in a tournament

#### ... and many more

- Open-pit mining
- Image segmentation (e.g., background/foreground segmentation)
- Network connectivity
- Data mining
- Distributed computing
- Network intrusion detection
- Edge-disjoint paths in graphs
- Network reliability
- Multi-camera scene reconstruction
- Gene function prediction



# Questions?



## Thank You