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INFORMATION
TECHNOLOGY

FIT2004
Algorithms and Data Structures

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Referencing materials by
Nathan Companez, Aamir Cheema, Arun Konagurthu and Lloyd Allison

GROUP OF EIGHT AUSTRALIA

Faculty of Information Technology, Monash University

Ready?

Agenda

- String retrieval

Agenda

- String retrieval
- Tries and suffix tries

Let us begin…

- String retrieval is one of the oldest retrieval task in the world…

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- Anything can be represented as a string

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- Anything can be represented as a string
  - DNA sequence
  - Images (RGB)
  - Keys
  - … and many more!

- So how can we search for string very fast?

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  - Sort the strings
  - Binary search for what you want

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  - What is our complexity?
    - N = number of strings
    - M = average length the string (instead of the longest)

- So how can we search for string very fast?
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    - O(MN log N) using merge sort… because O(M) for string comparison
    - O(MN) using radix sort
  - Binary search for what you want

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    - O(MN) using radix sort
  - Binary search for what you want
    - O(M log N)… again O(M) for string comparison
  - What is our complexity?
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Questions?

- When we search, we would need to go through every character of a string. Thus, we can use a special data structure that organise it according to characters…

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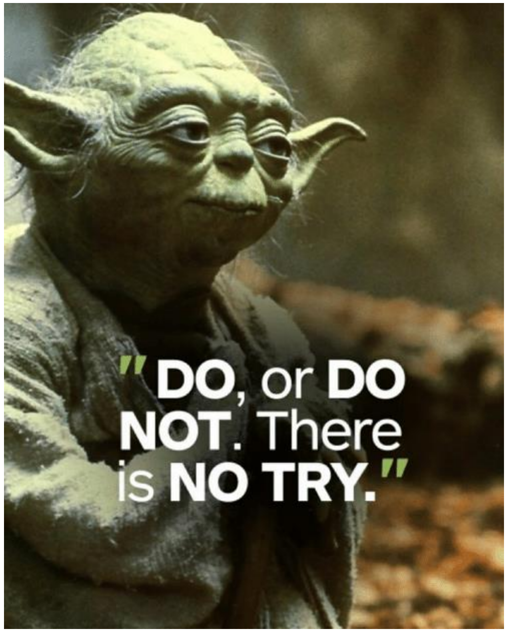
- We use reTRIEval tree

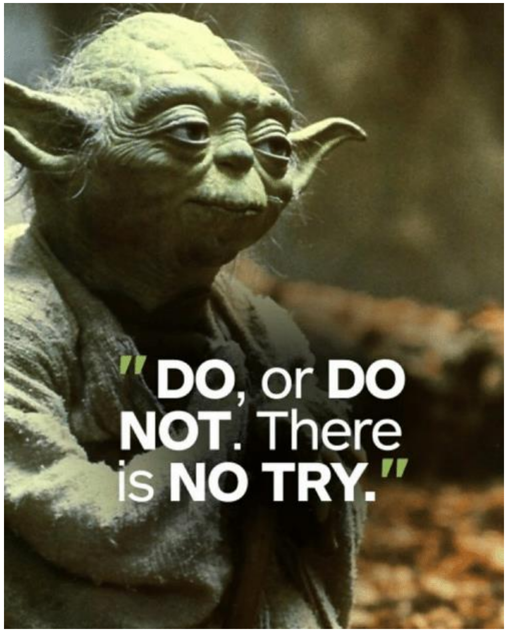


Tries

Efficient string retrieval

- When we search, we would need to go through every character of a string. Thus, we can use a special data structure that organise it according to characters…

- We use reTRIEval tree
  - A tree
  - M-child per node



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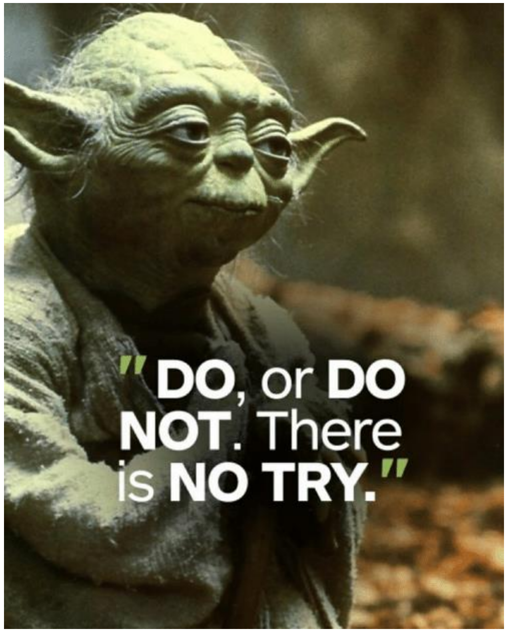
- We use reTRIEval tree
  – A tree
  – M-child per node 26 childs
    M = number of unique character



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  - Taco
  - Taro
  - Tarot
  - Coco
  - Chobo

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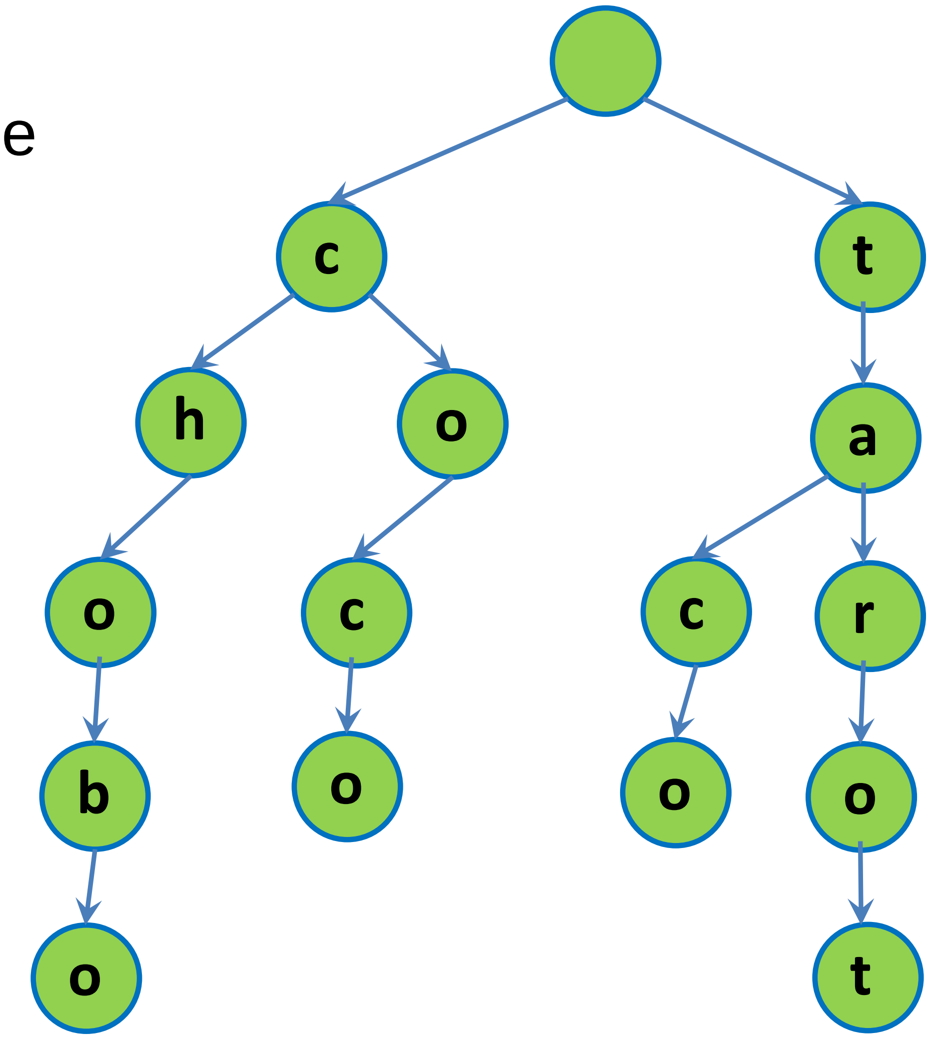
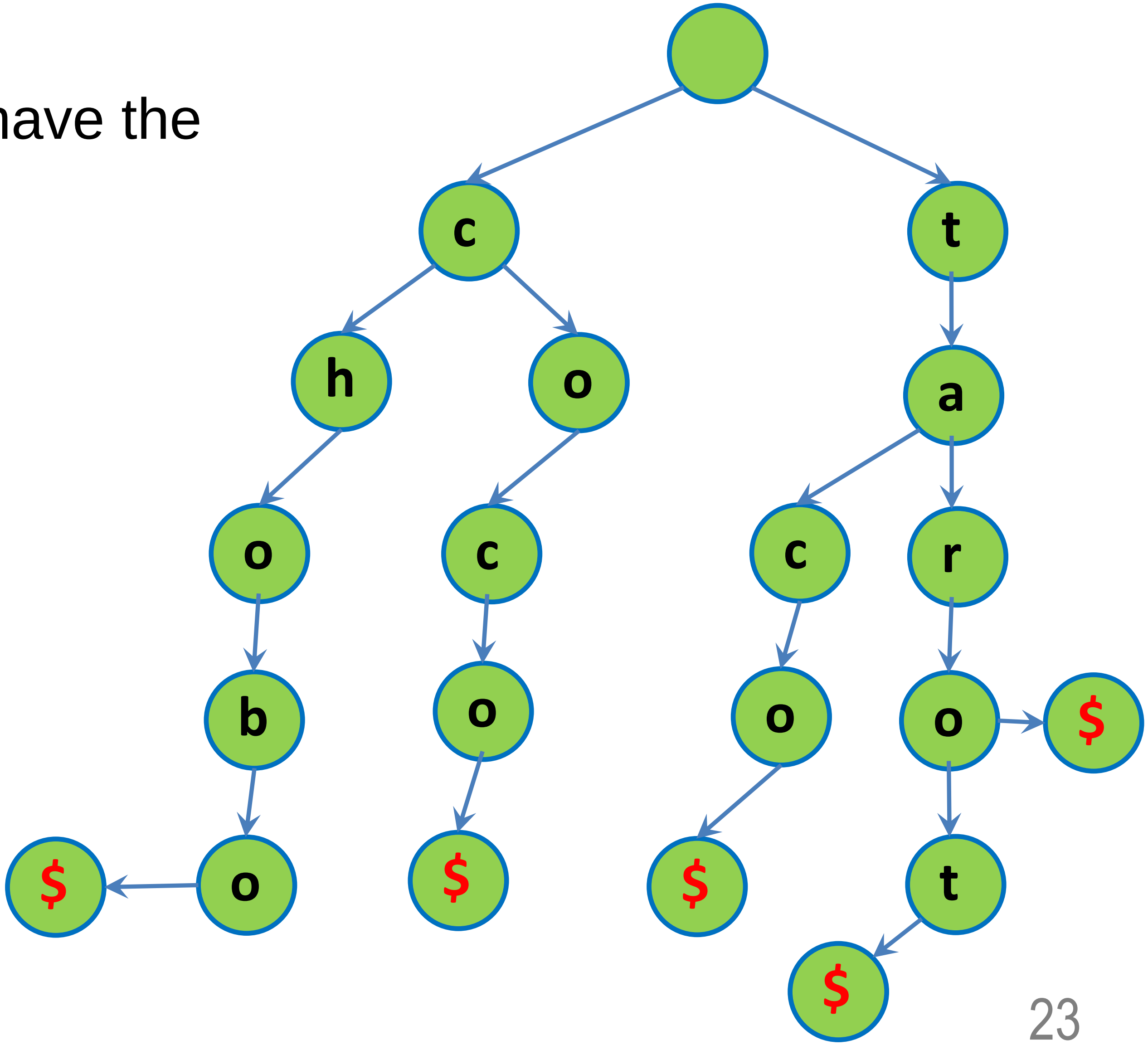


- Let assume we have the following words:
  - Taco$
  - Taro$
  - Tarot$ terminal character
  - Coco$
  - Chobo$
  - What is the trie?



Tries

Efficient string retrieval

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- Let assume we have the following words:
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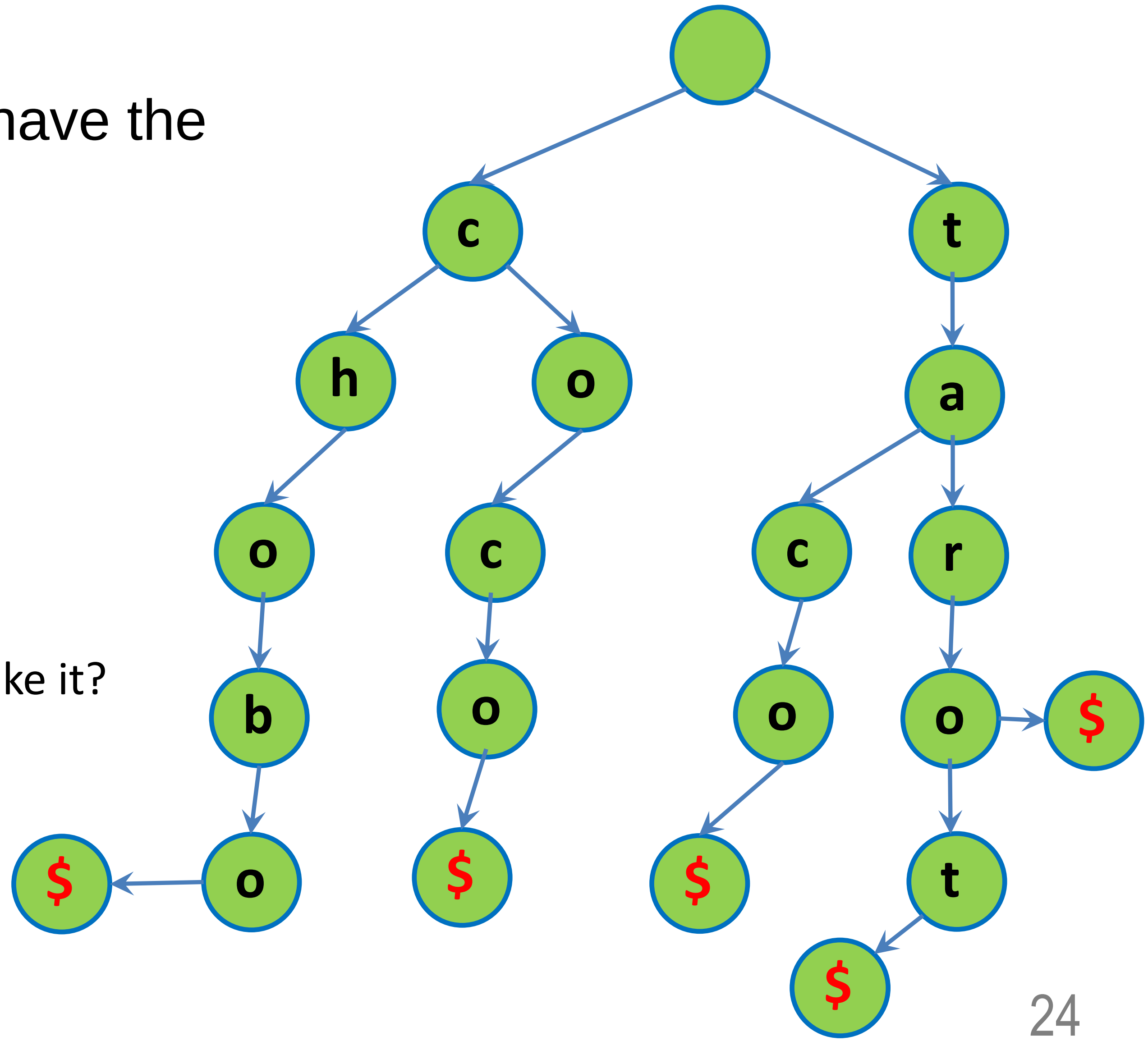
  - Chobo$
  - What is the trie?
  - So how do we make it?



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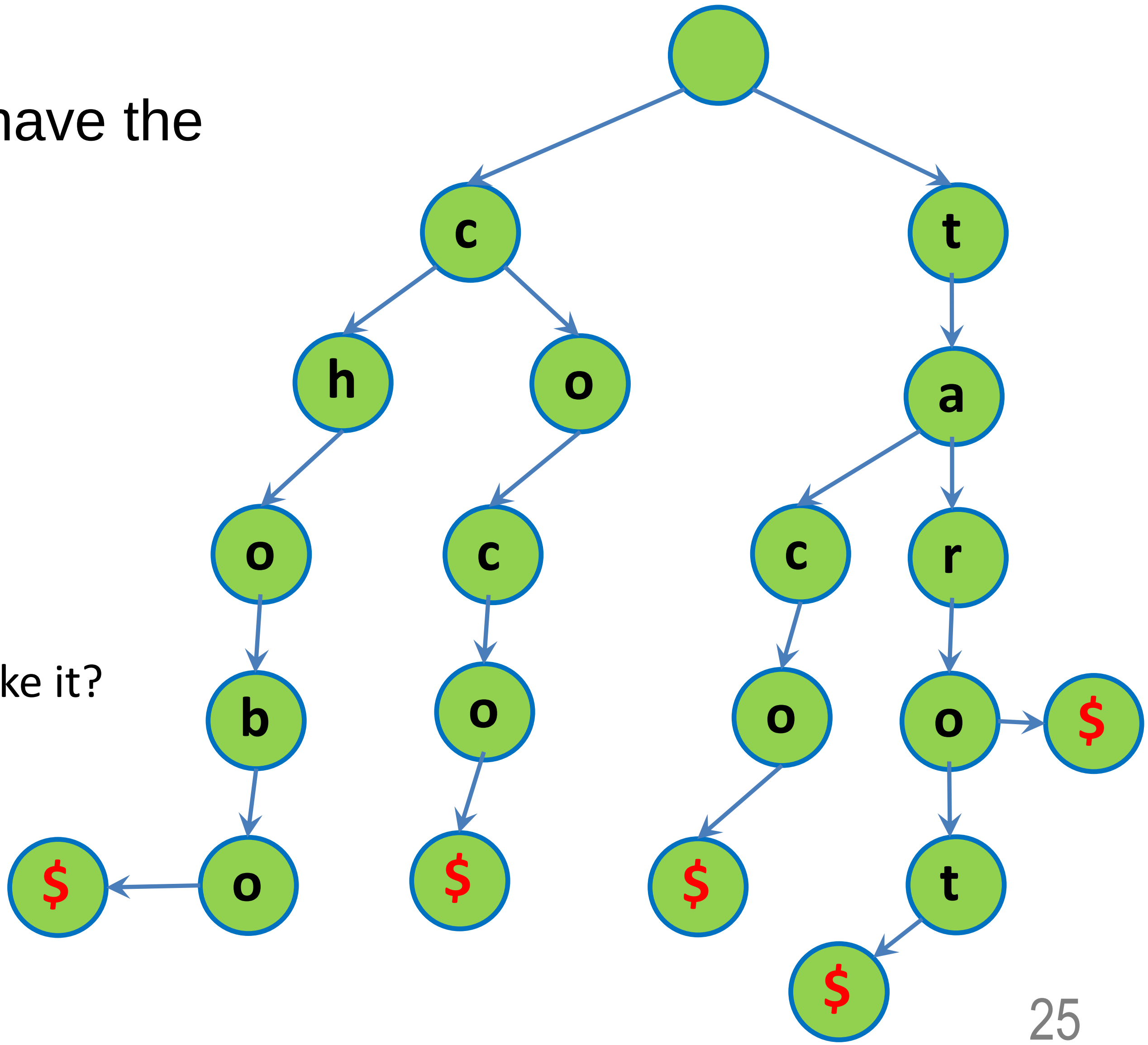

Tries

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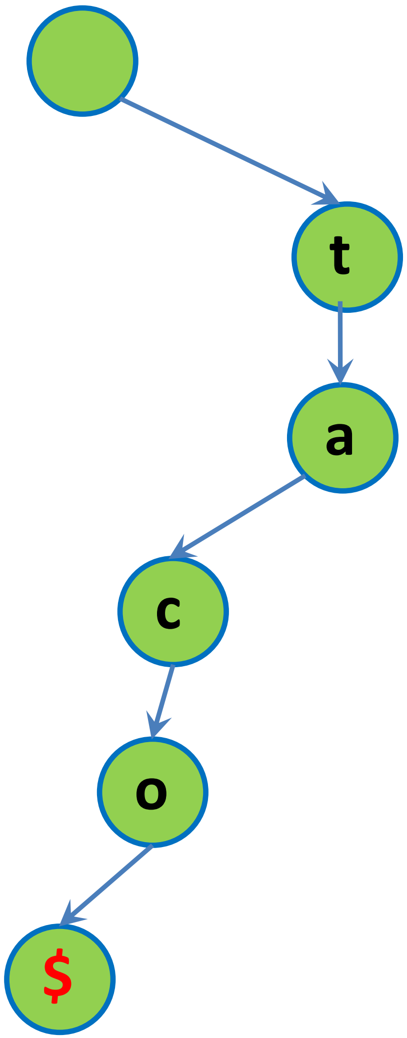
a..z
A..Z
52 characters
+ $
53 characters



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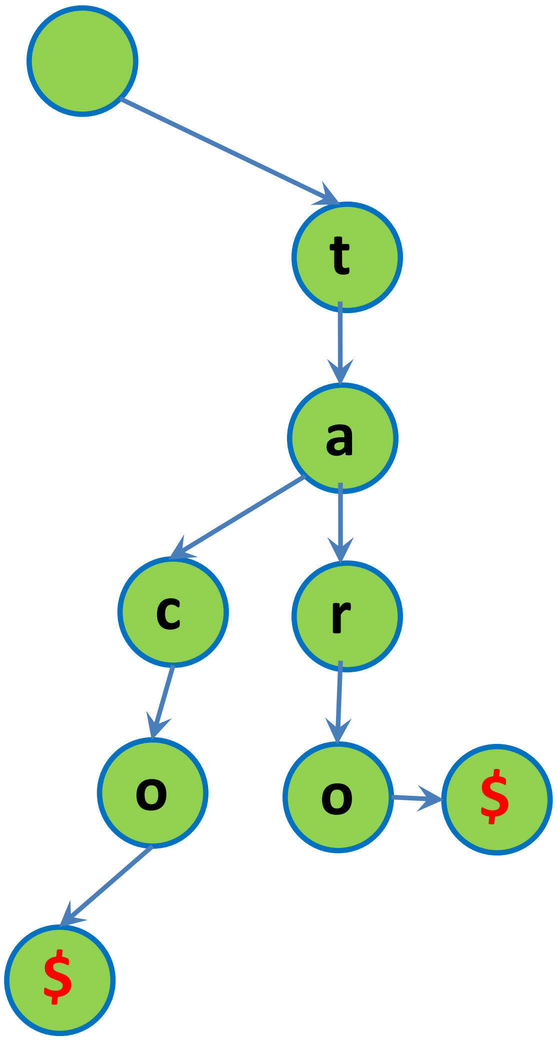

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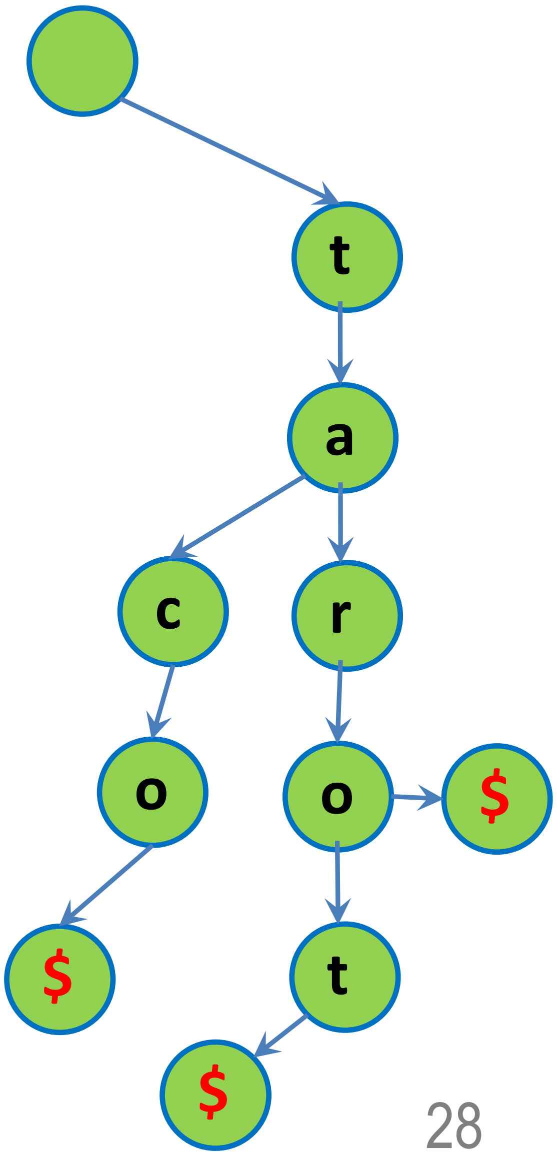

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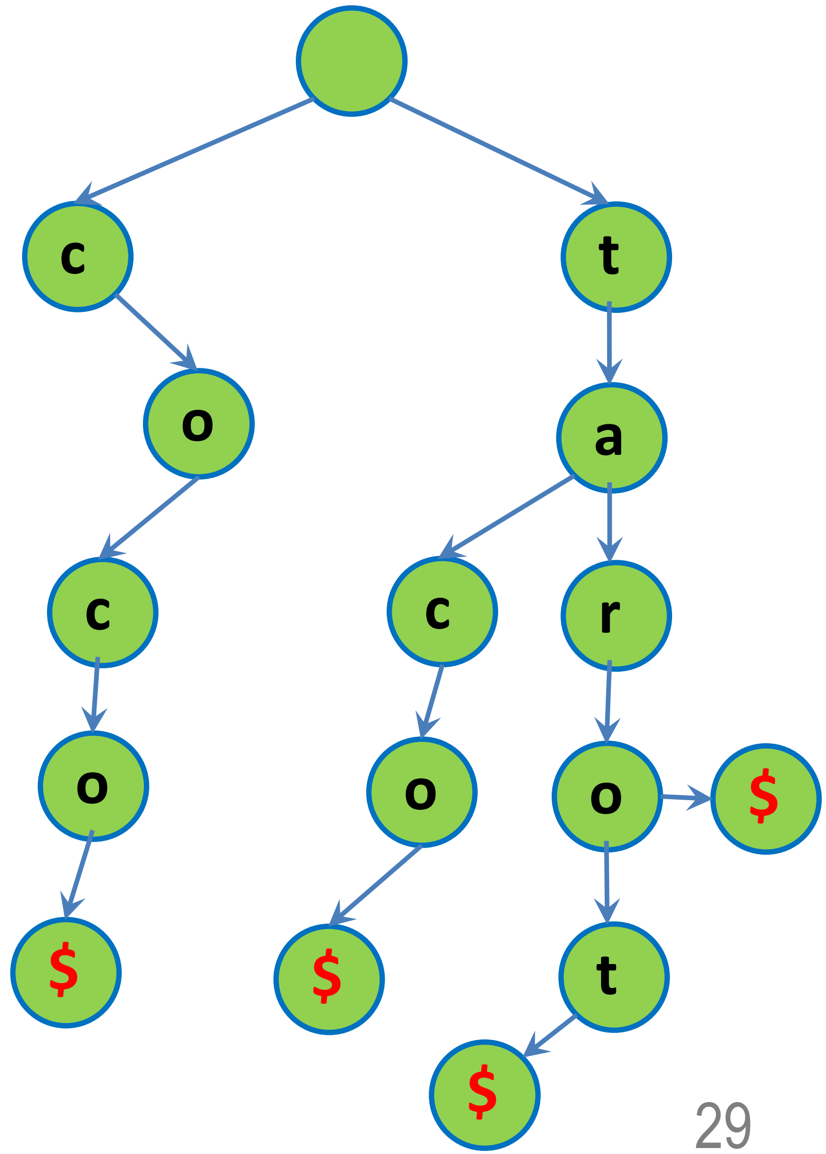

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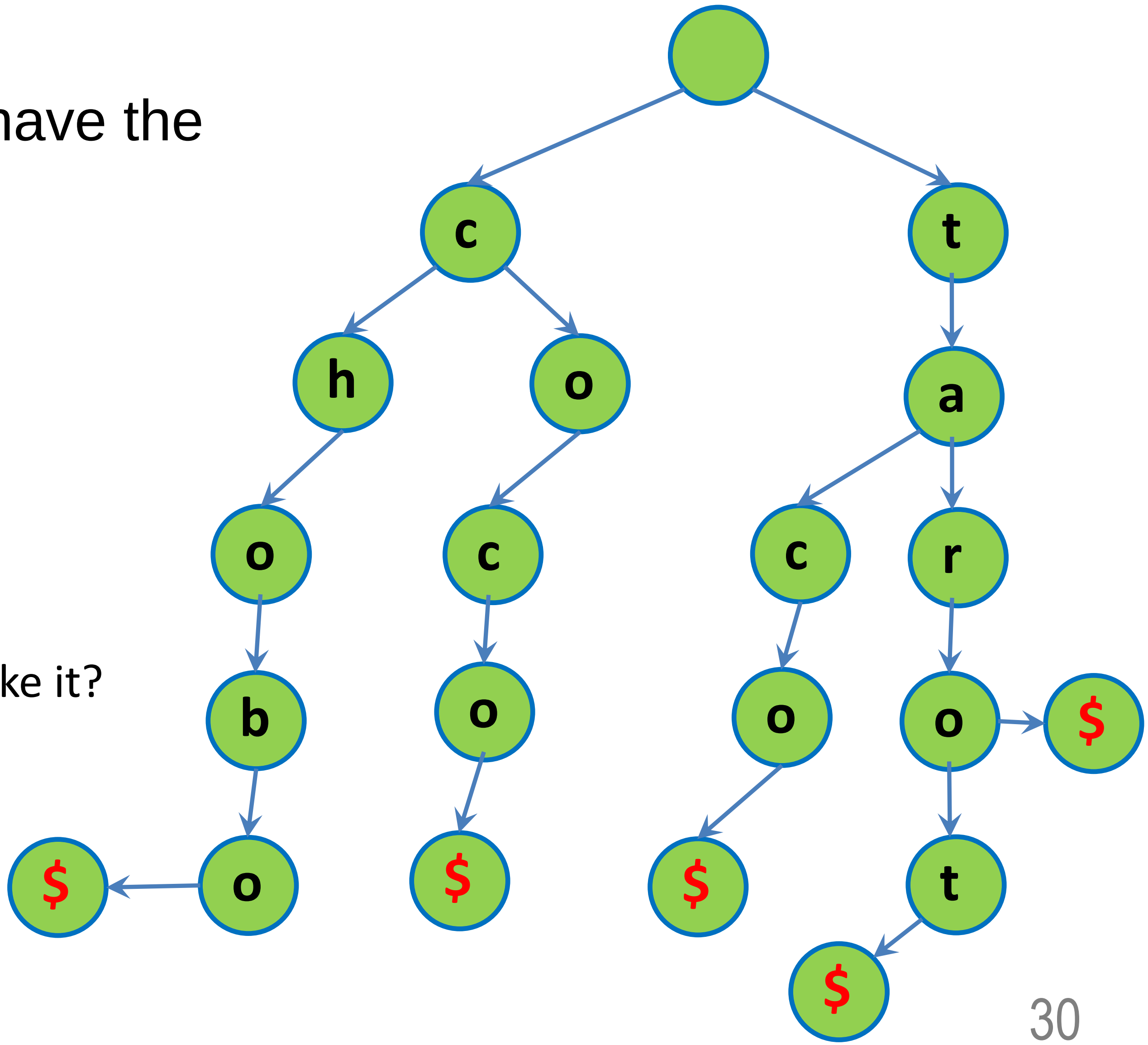
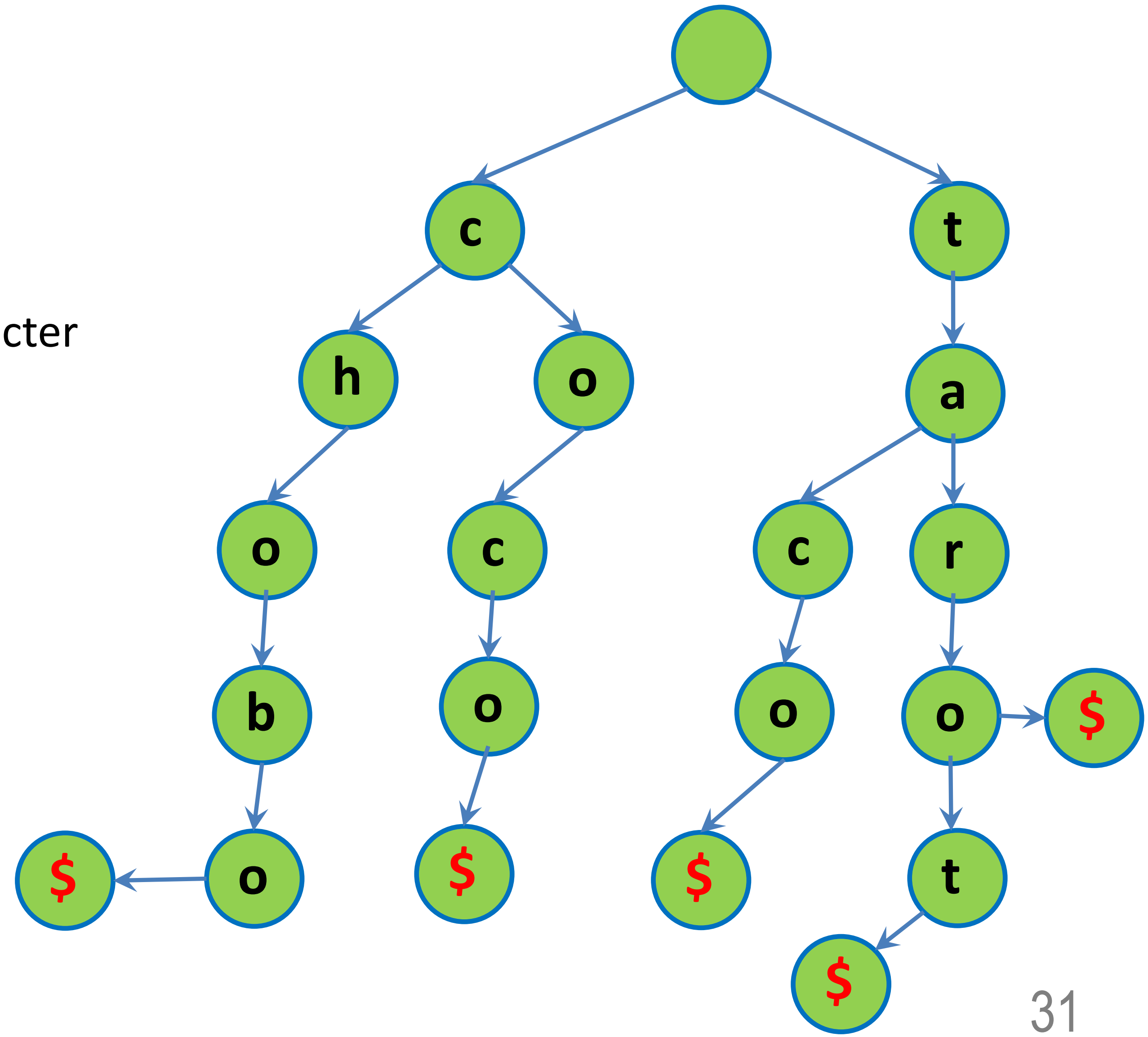
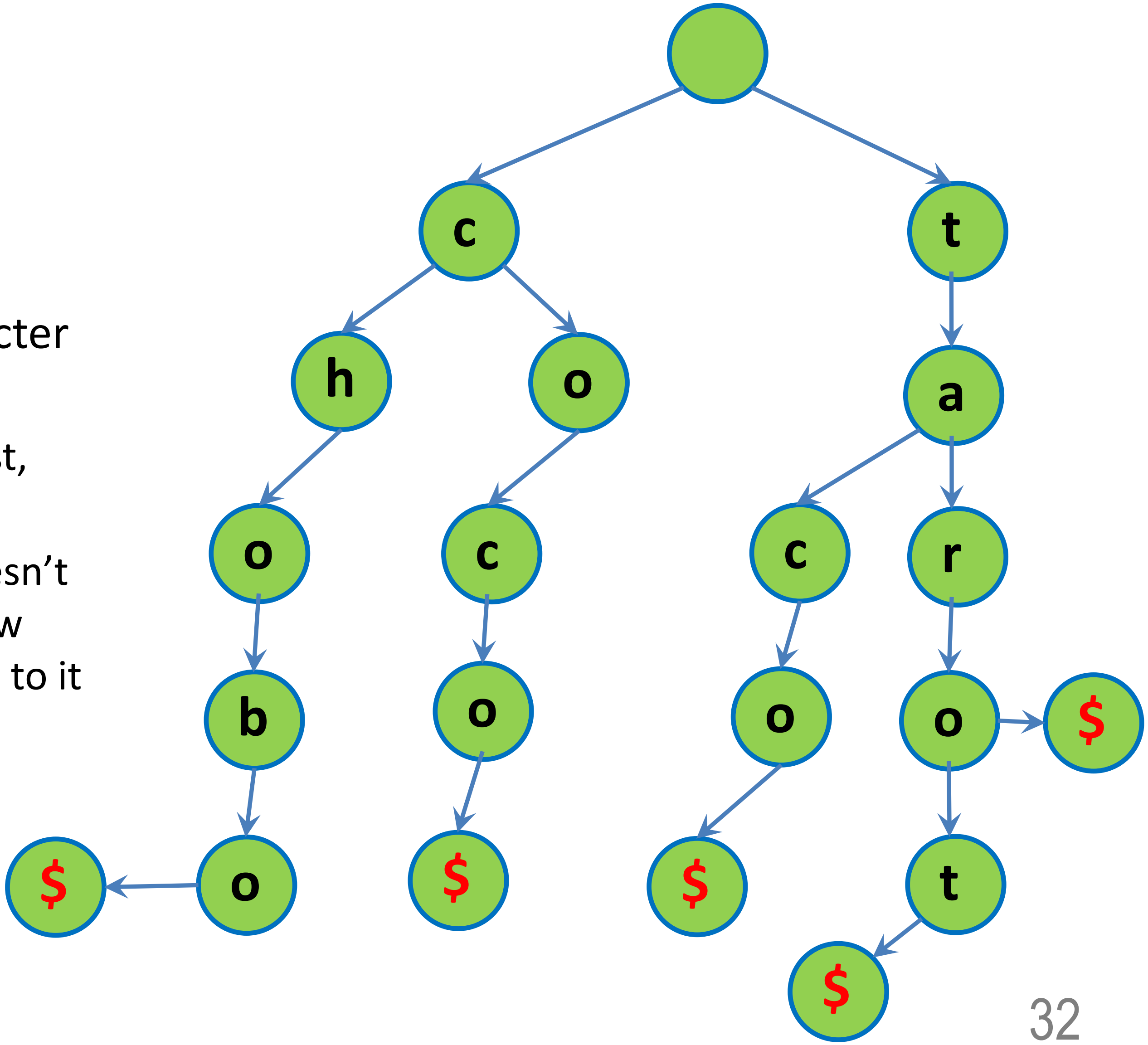


- So steps?
  - For each word
  - Start from root
  - Go through character by character



Tries
Efficient string retrieval

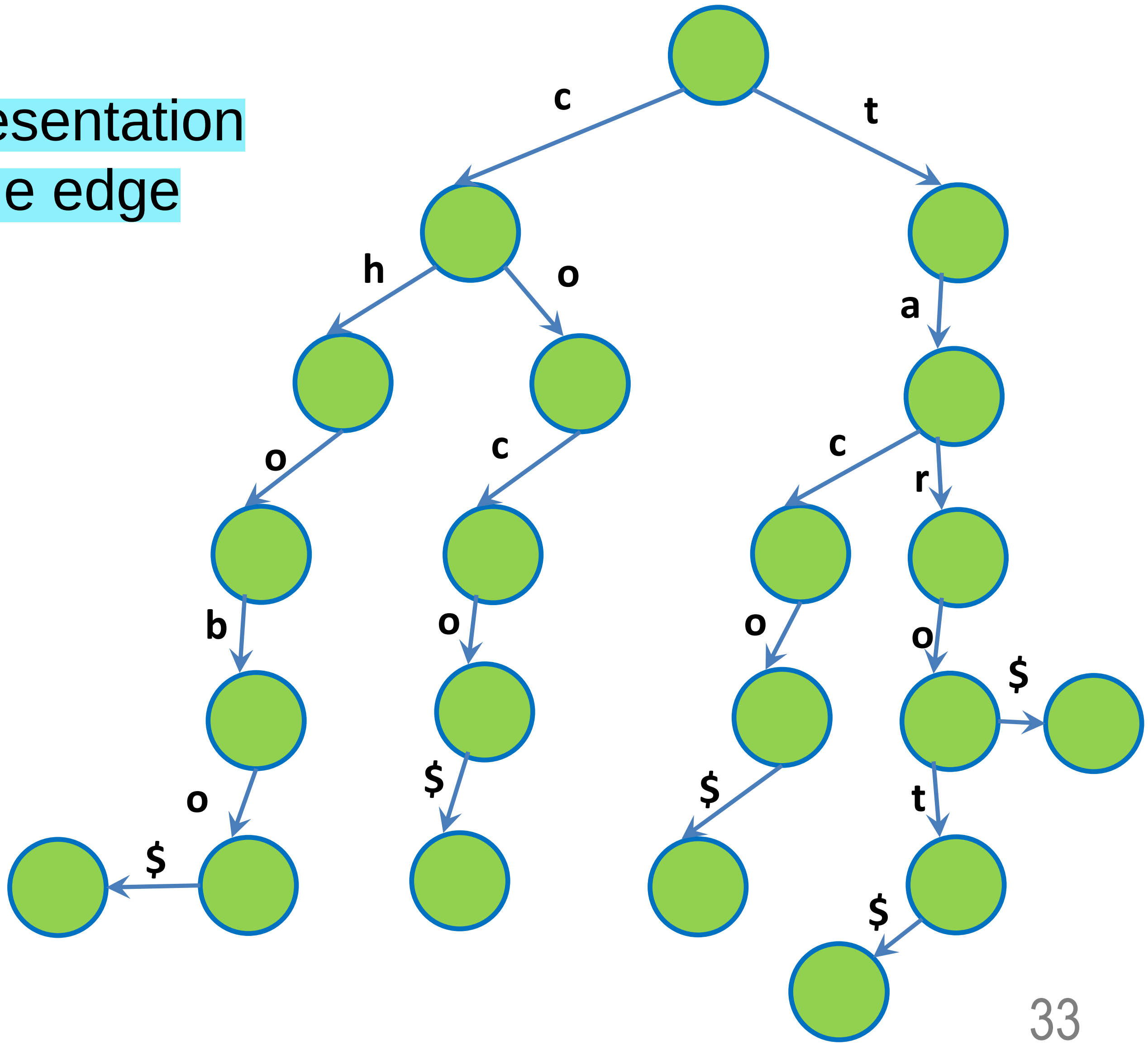
- So steps?
  - For each word
  - Start from root
  - Go through character by character
    - If character exist, follow through
    - If character doesn't exist, create new node and move to it



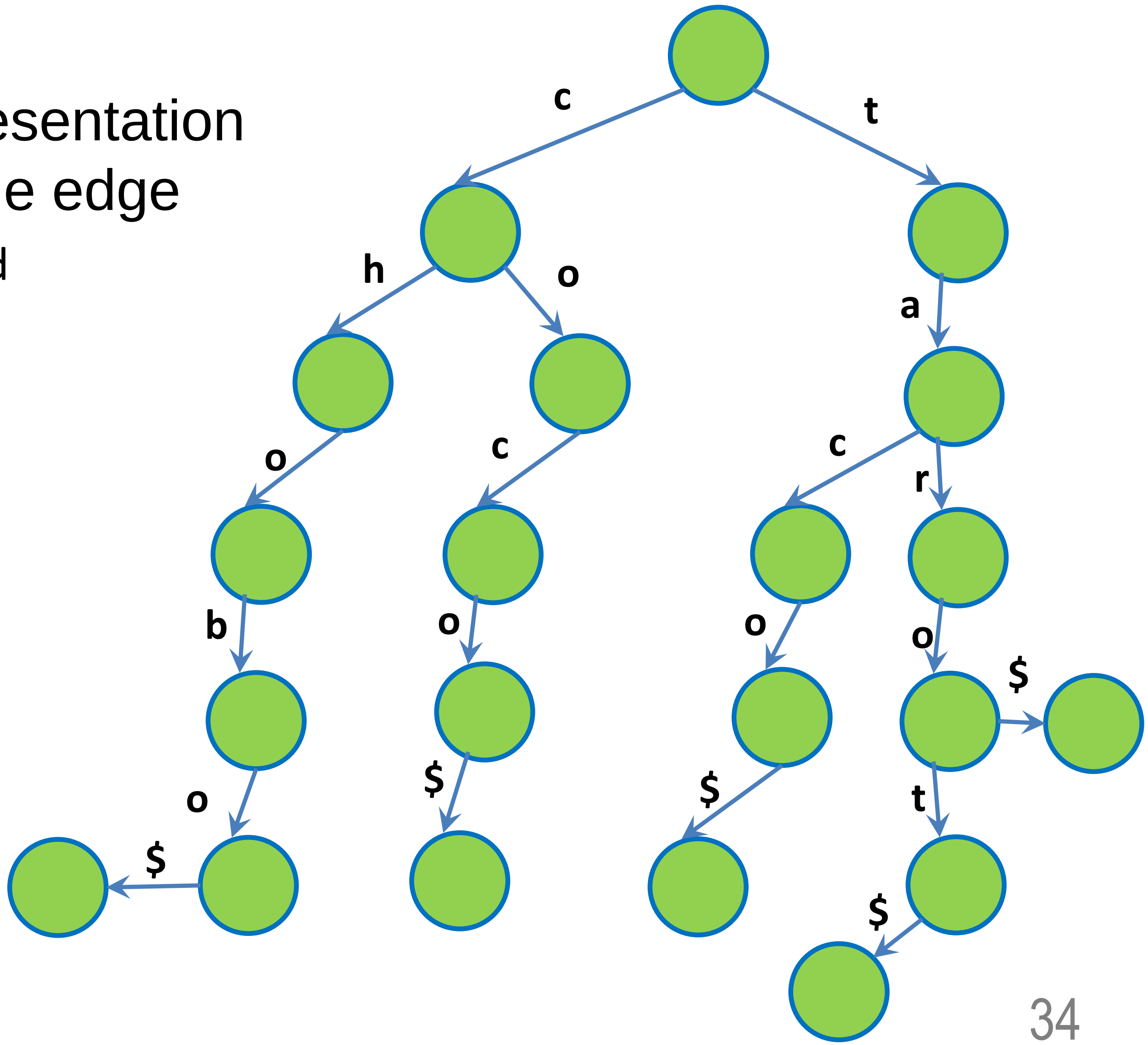
Tries

Efficient string retrieval

- The proper representation is character at the edge



- The proper representation is character at the edge
  - Both are accepted for your exam!



Tries
Efficient string retrieval

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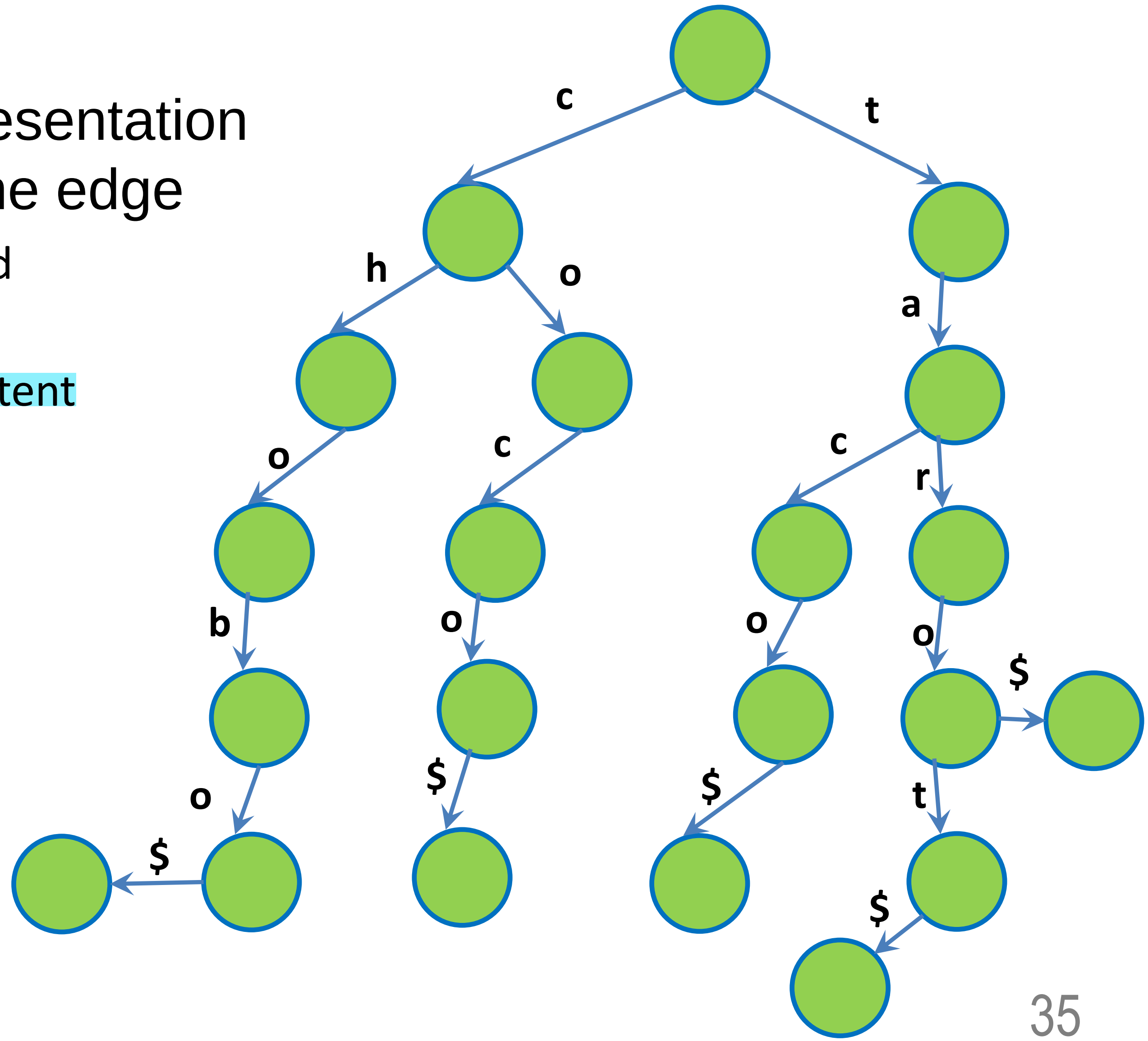
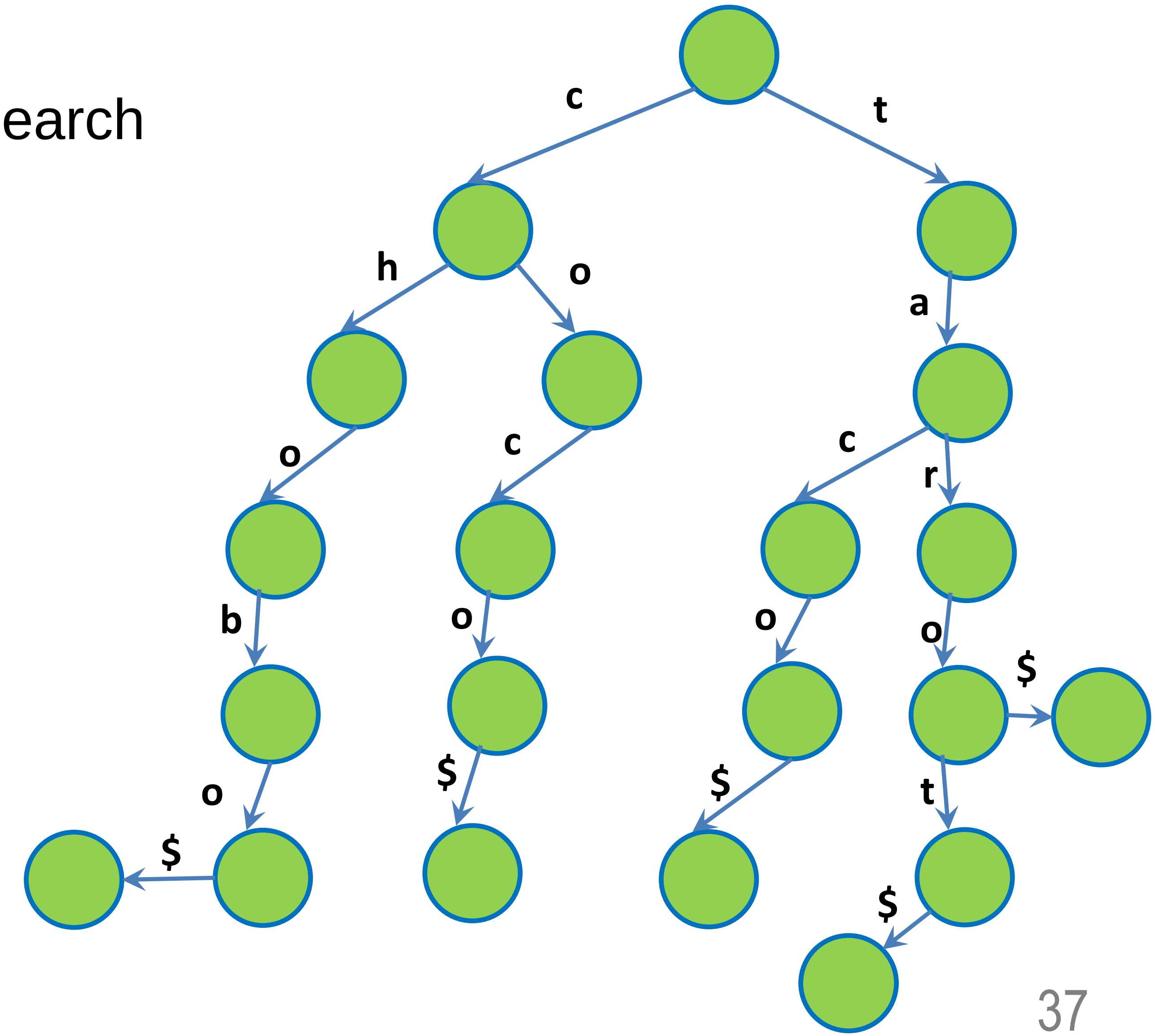
- The proper representation is character at the edge
  – Both are accepted for your exam!
  – This is also consistent with the graph representation



Questions?

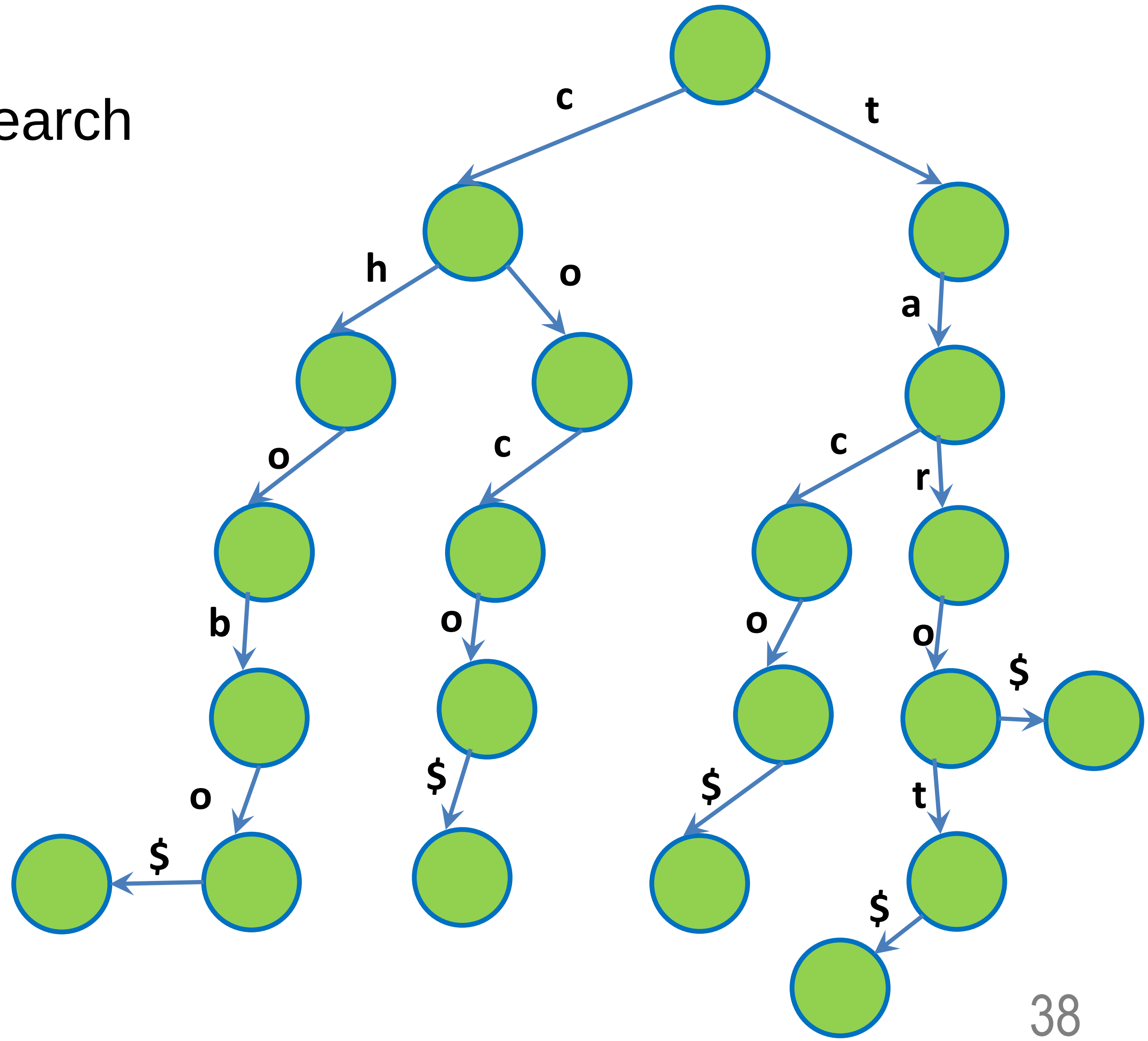
- So how do we search for retrieval?



Tries

Efficient string retrieval

- So how do we search for retrieval?
  - Search for “coco”



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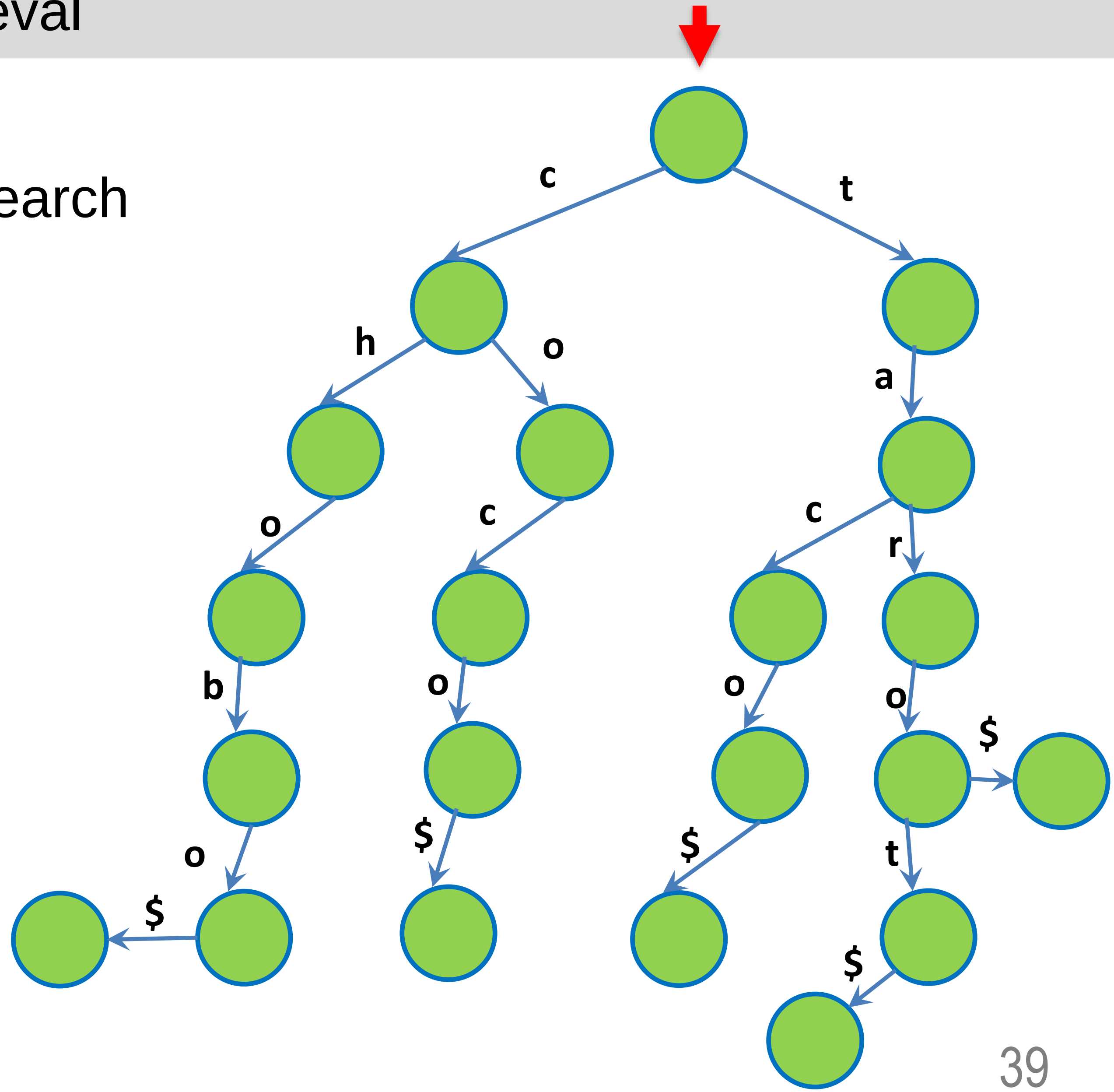

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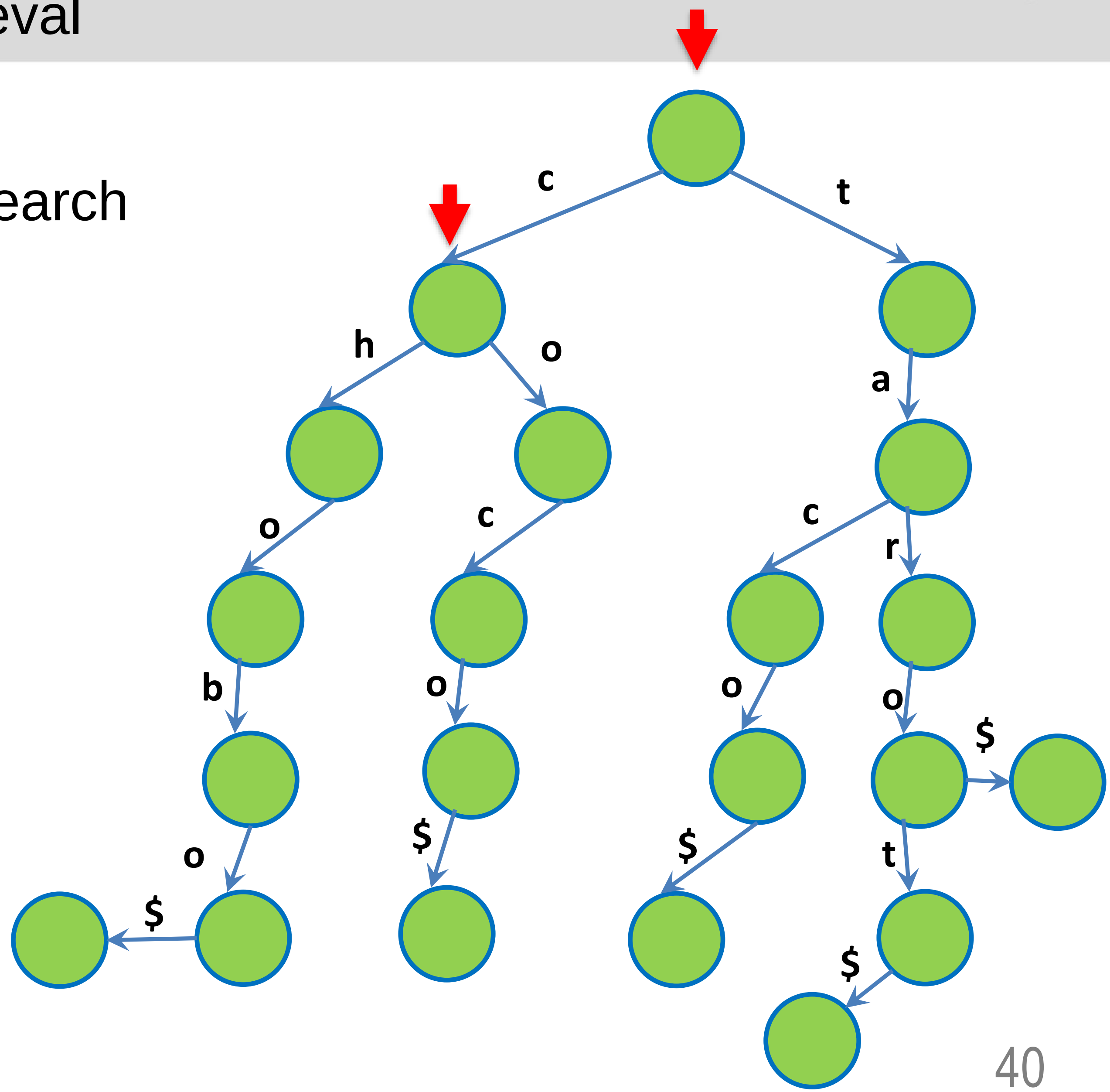

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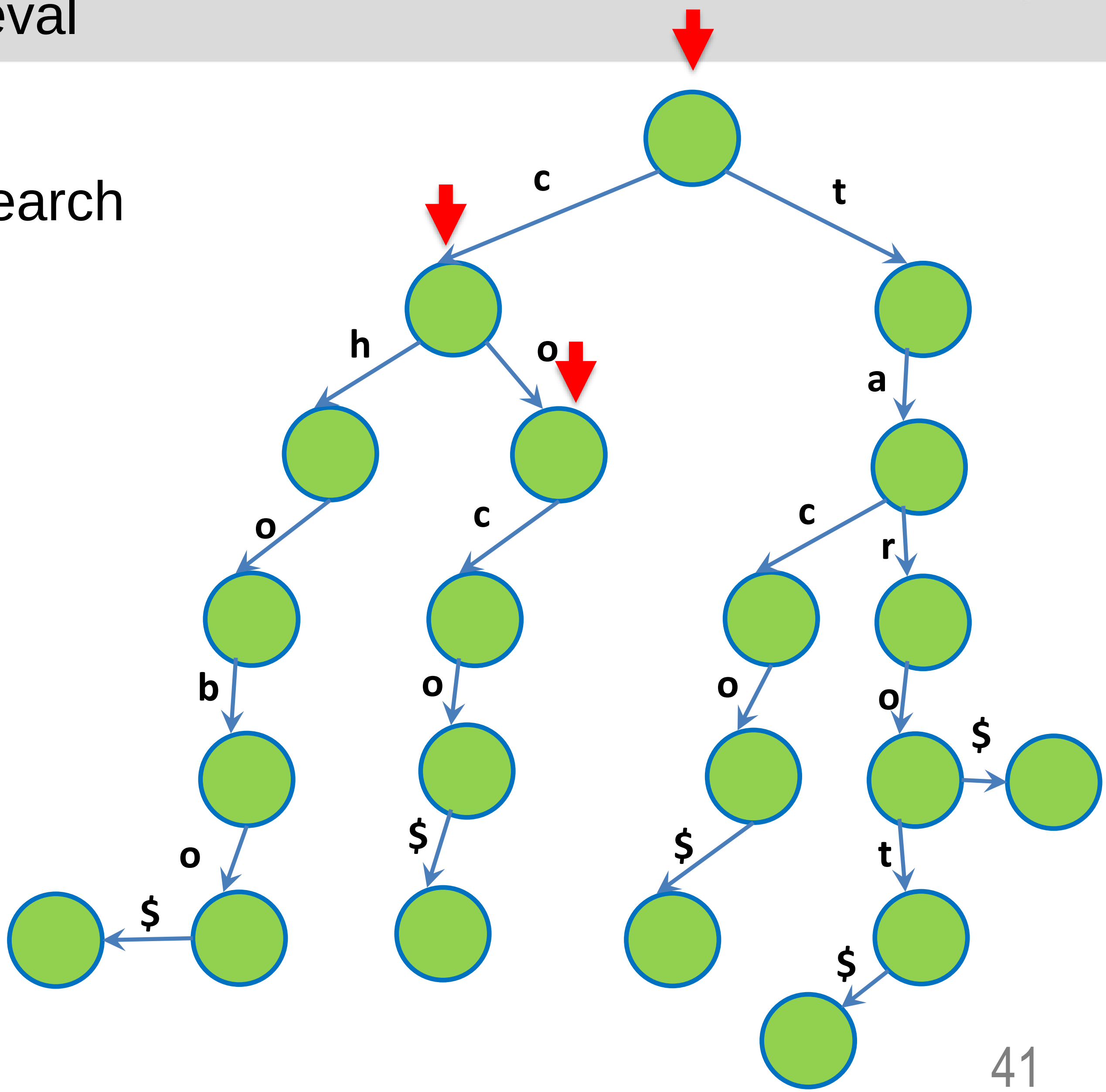

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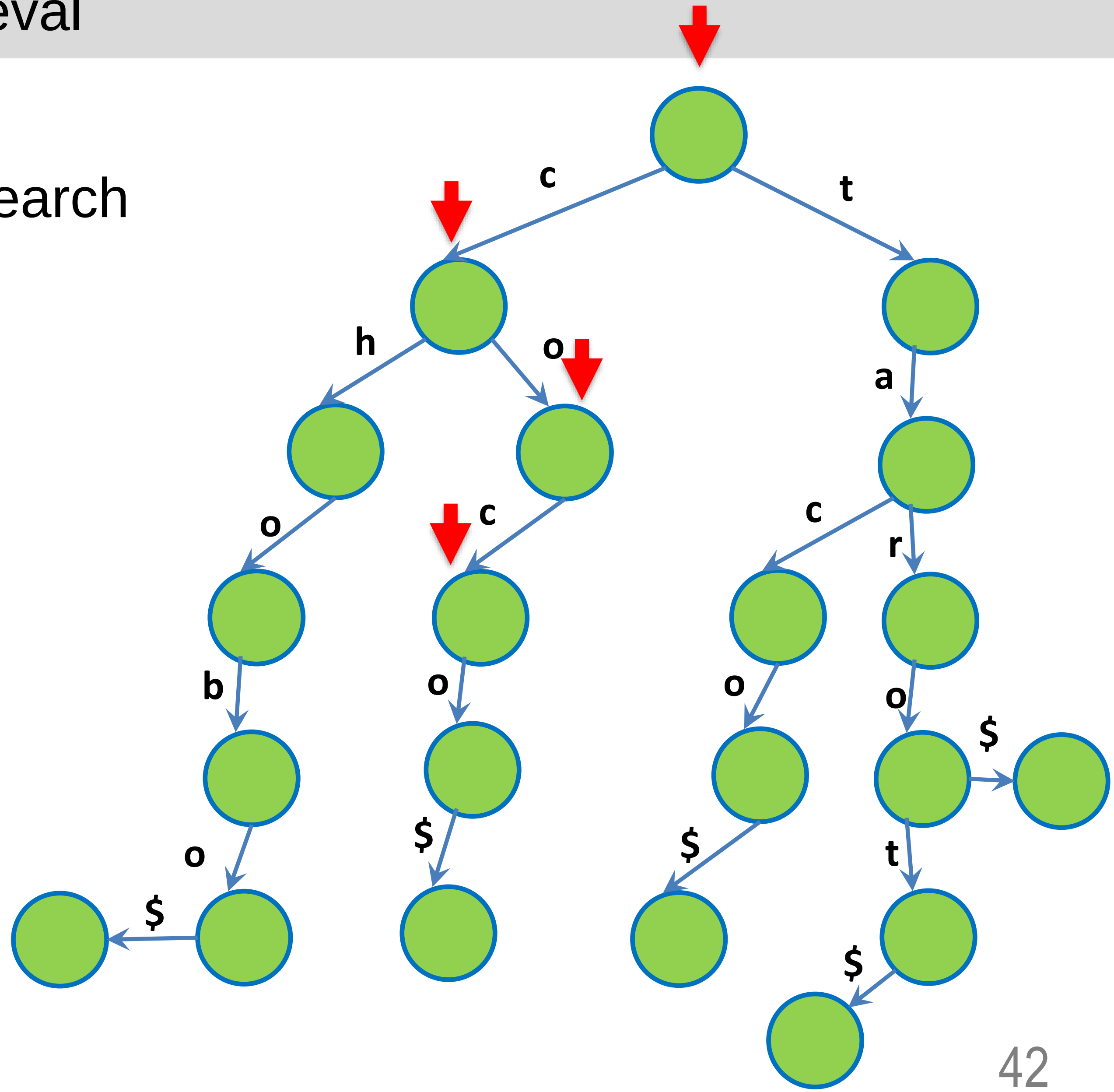
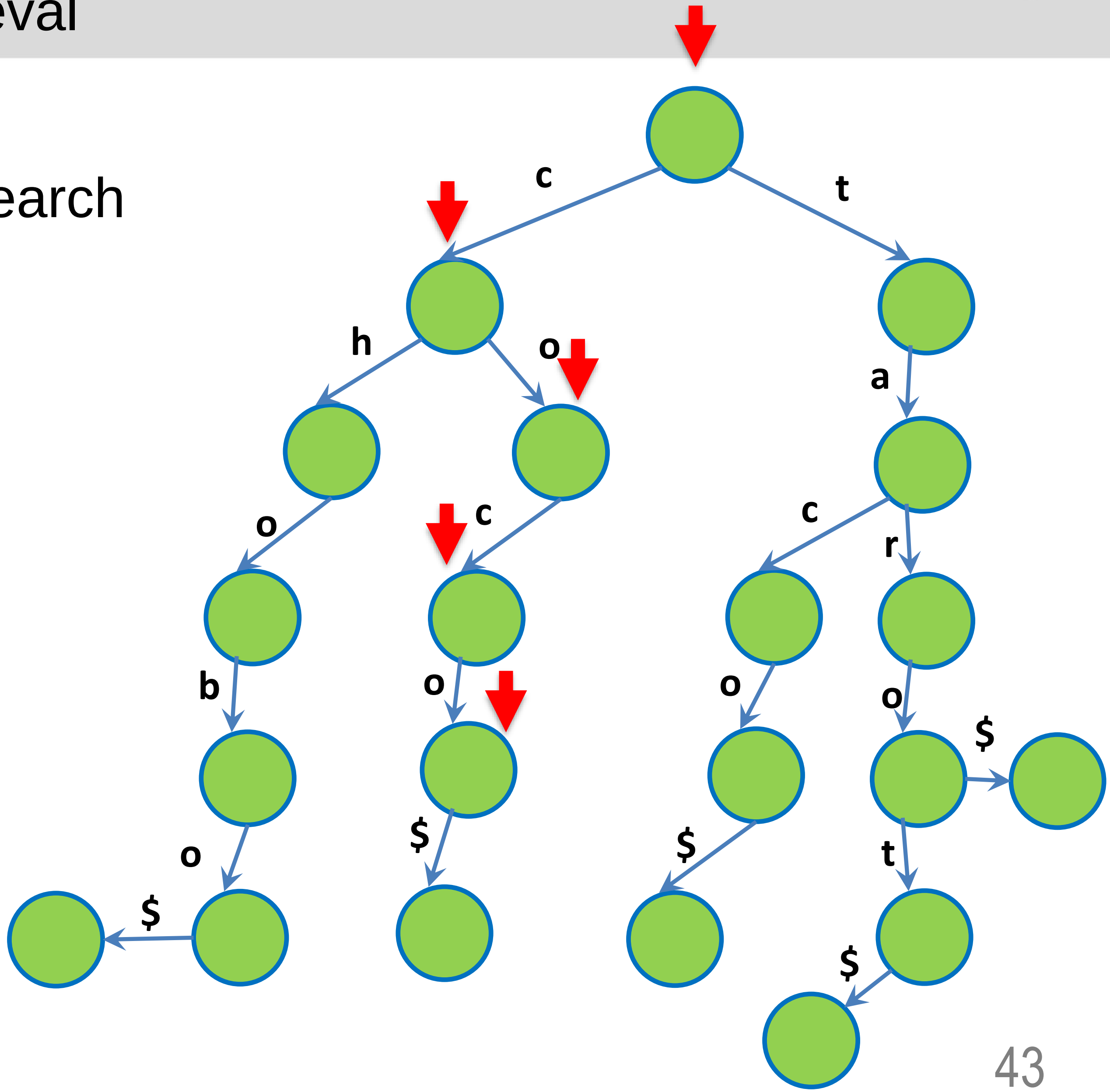
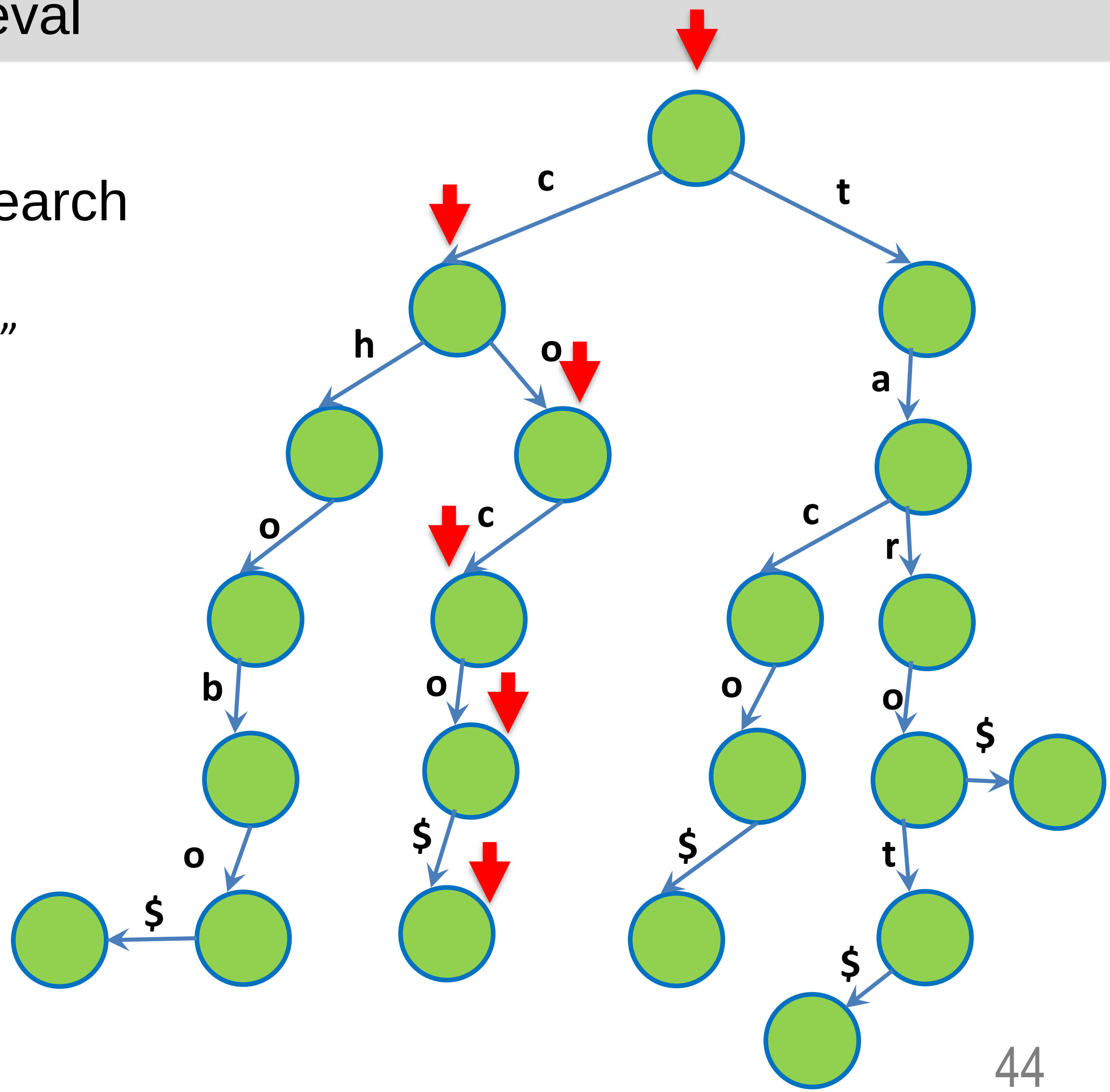


- So how do we search for retrieval?
  - Search for "coc<u>o</u>"

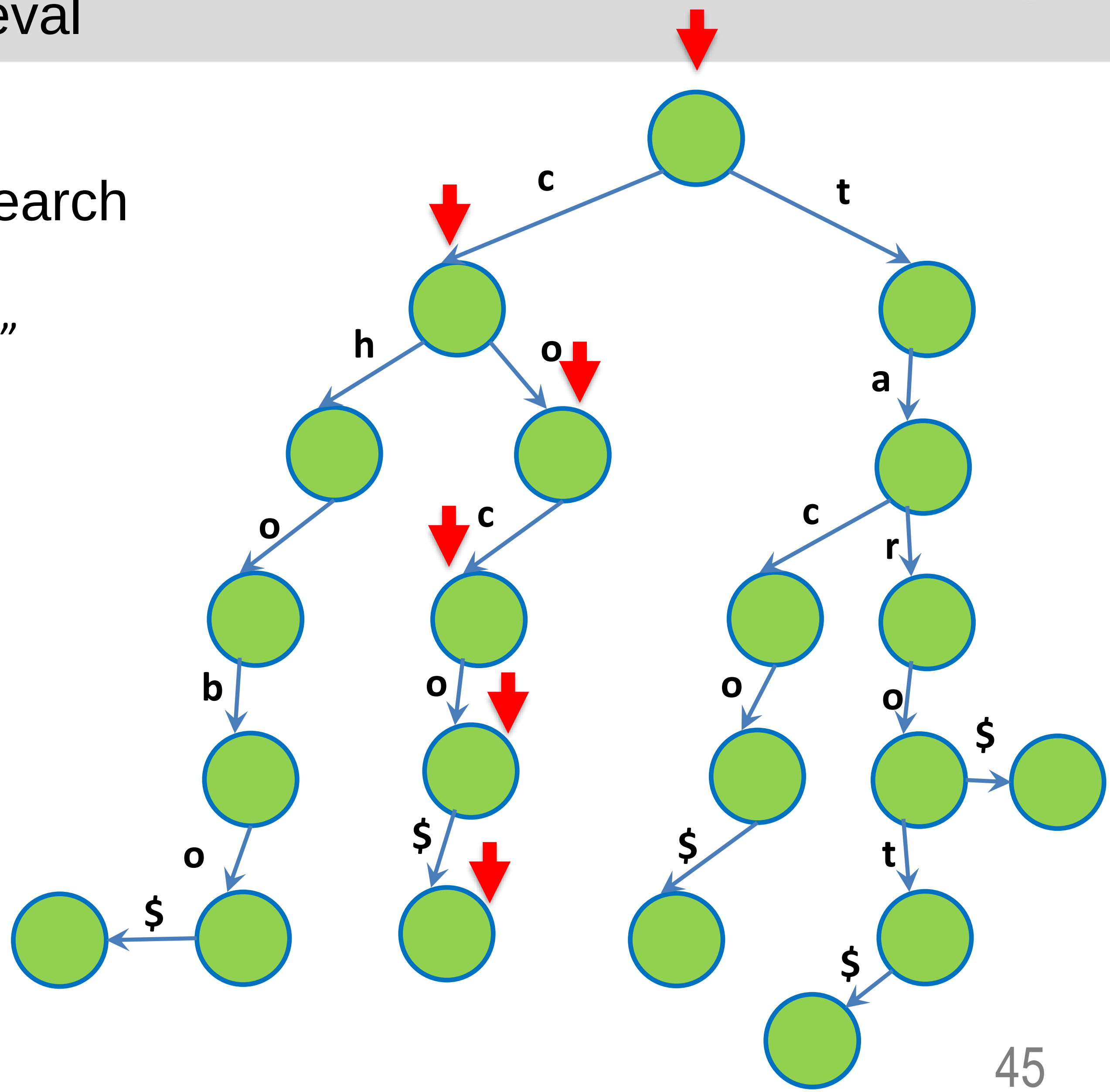


Tries
Efficient string retrieval

- So how do we search for retrieval?
  - Search for “coco$”



- So how do we search for retrieval?
  - Search for “coco$”
    so we found it!



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Questions?

Tries

Efficient string retrieval

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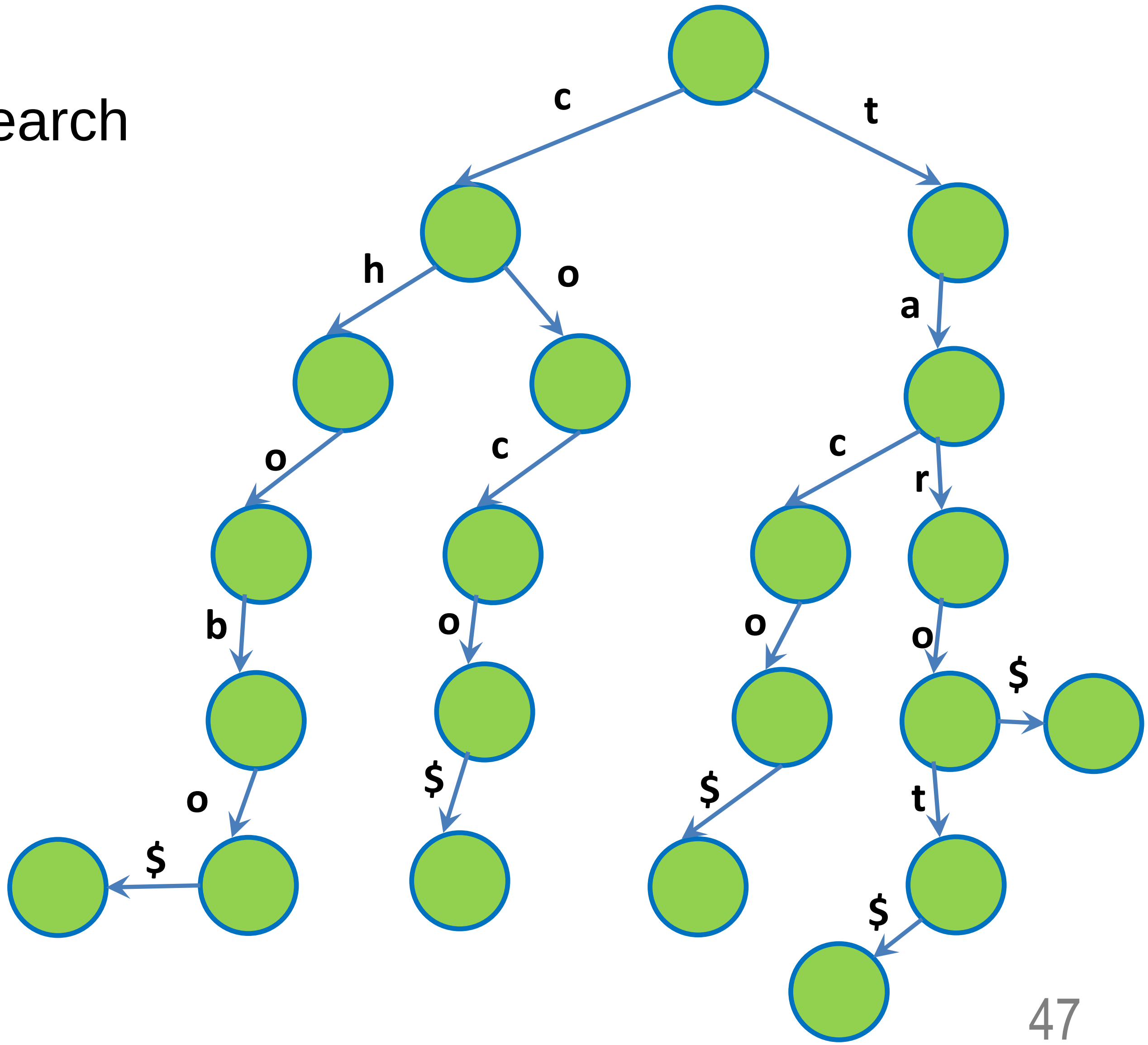
- So how do we search for retrieval?
  - Search for “tarot”



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Efficient string retrieval

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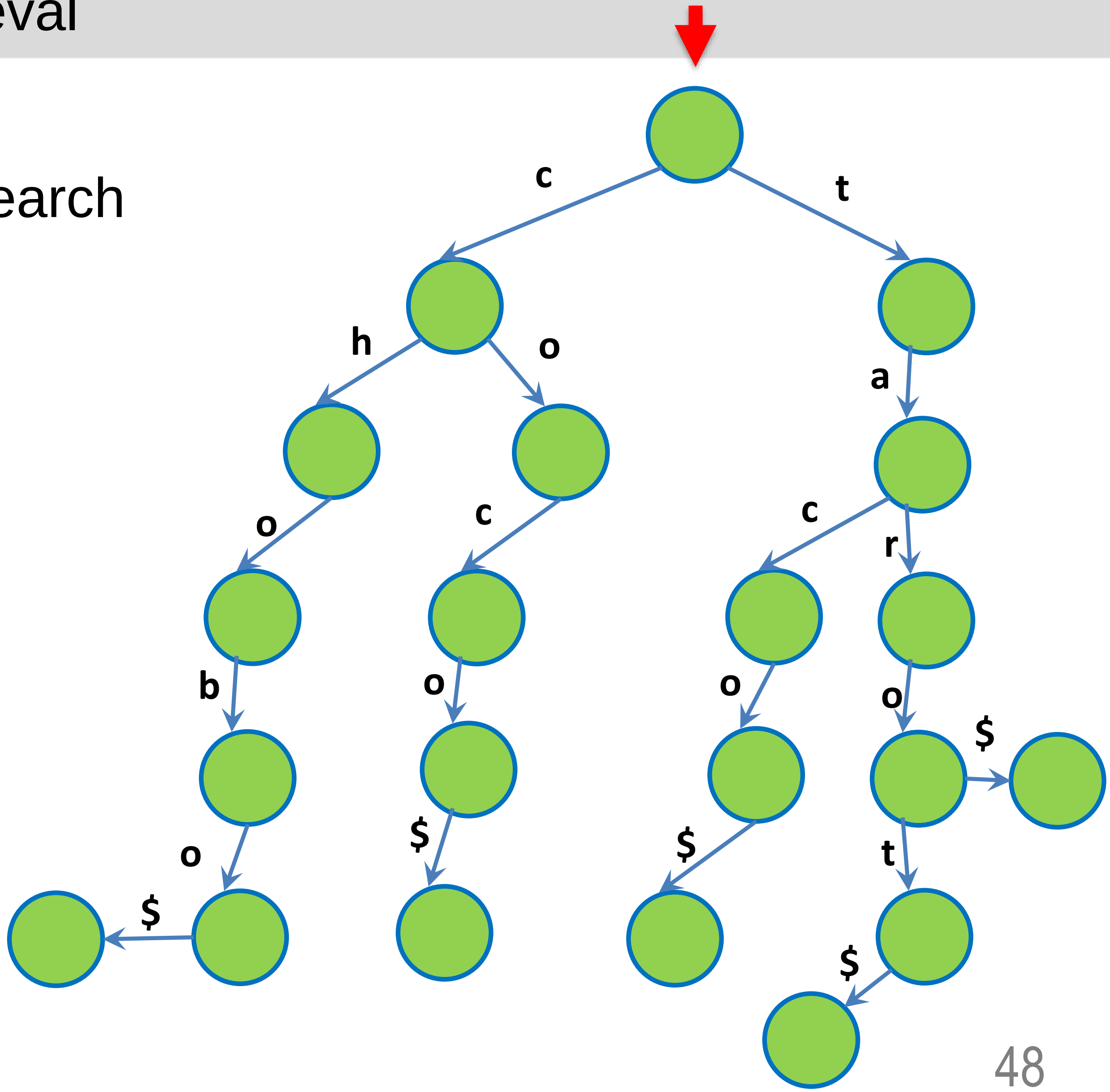
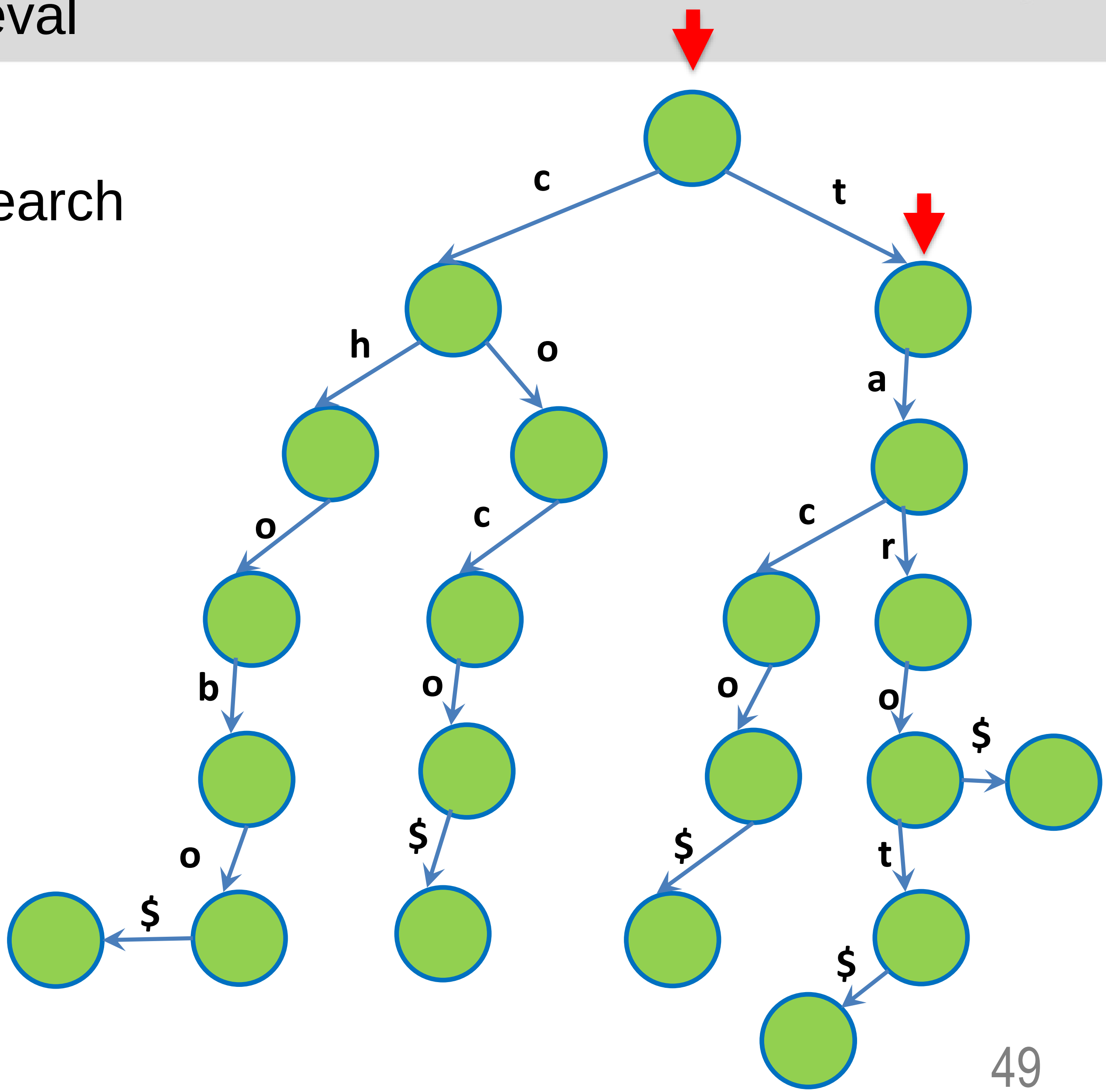


Tries

Efficient string retrieval

- So how do we search for retrieval?
  - Search for "t̲arot"



Tries

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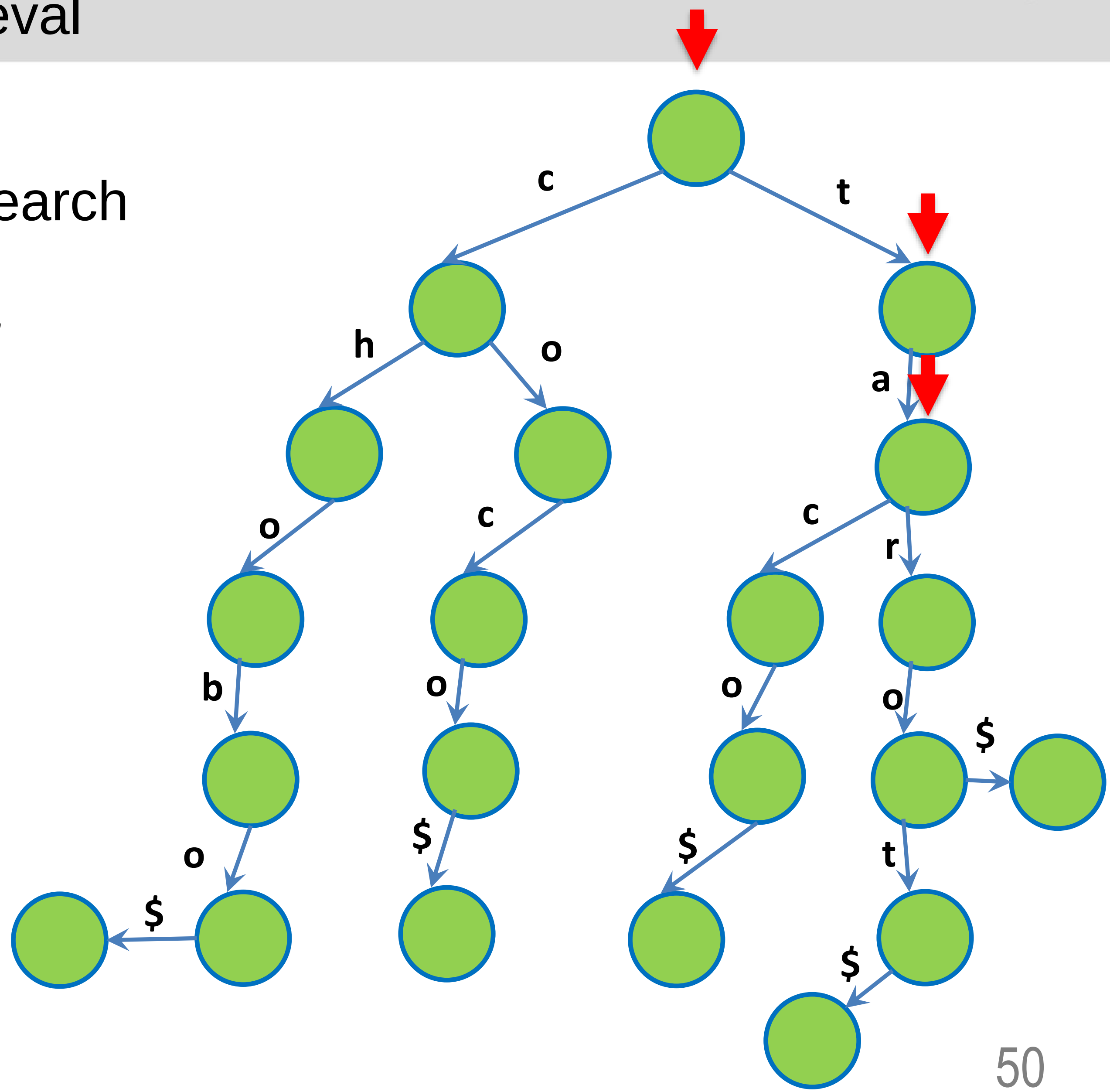

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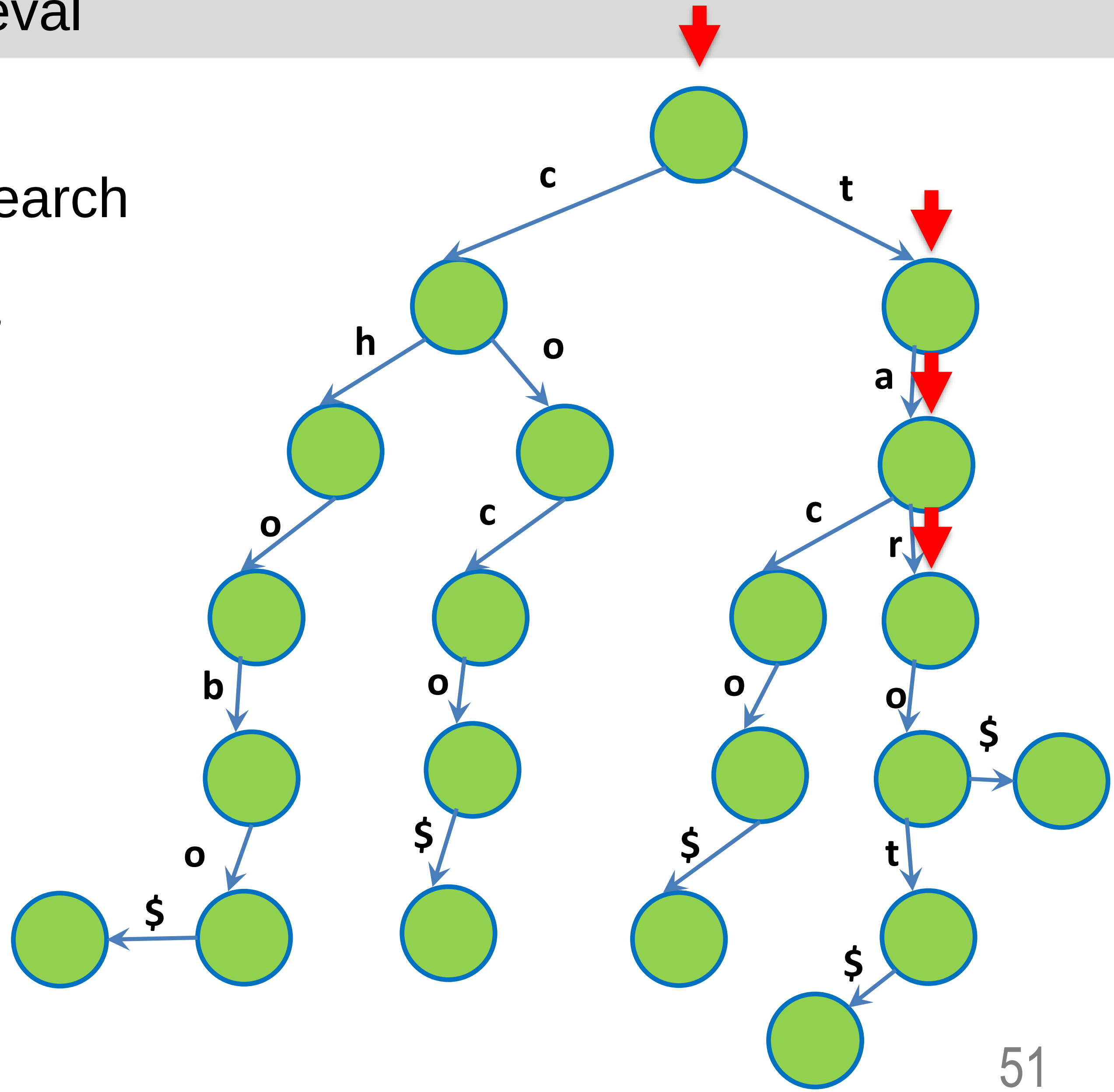

- So how do we search for retrieval?
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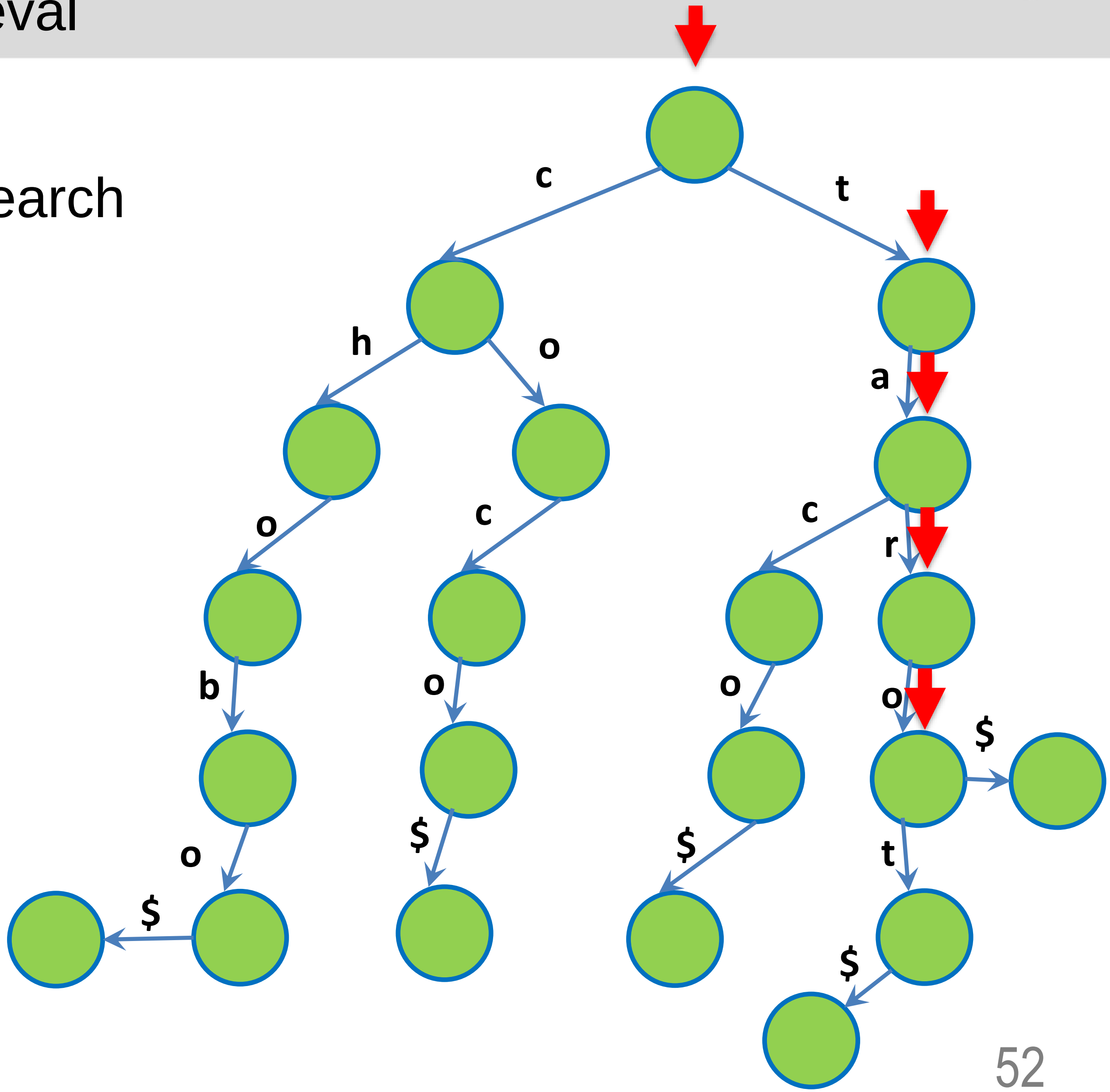
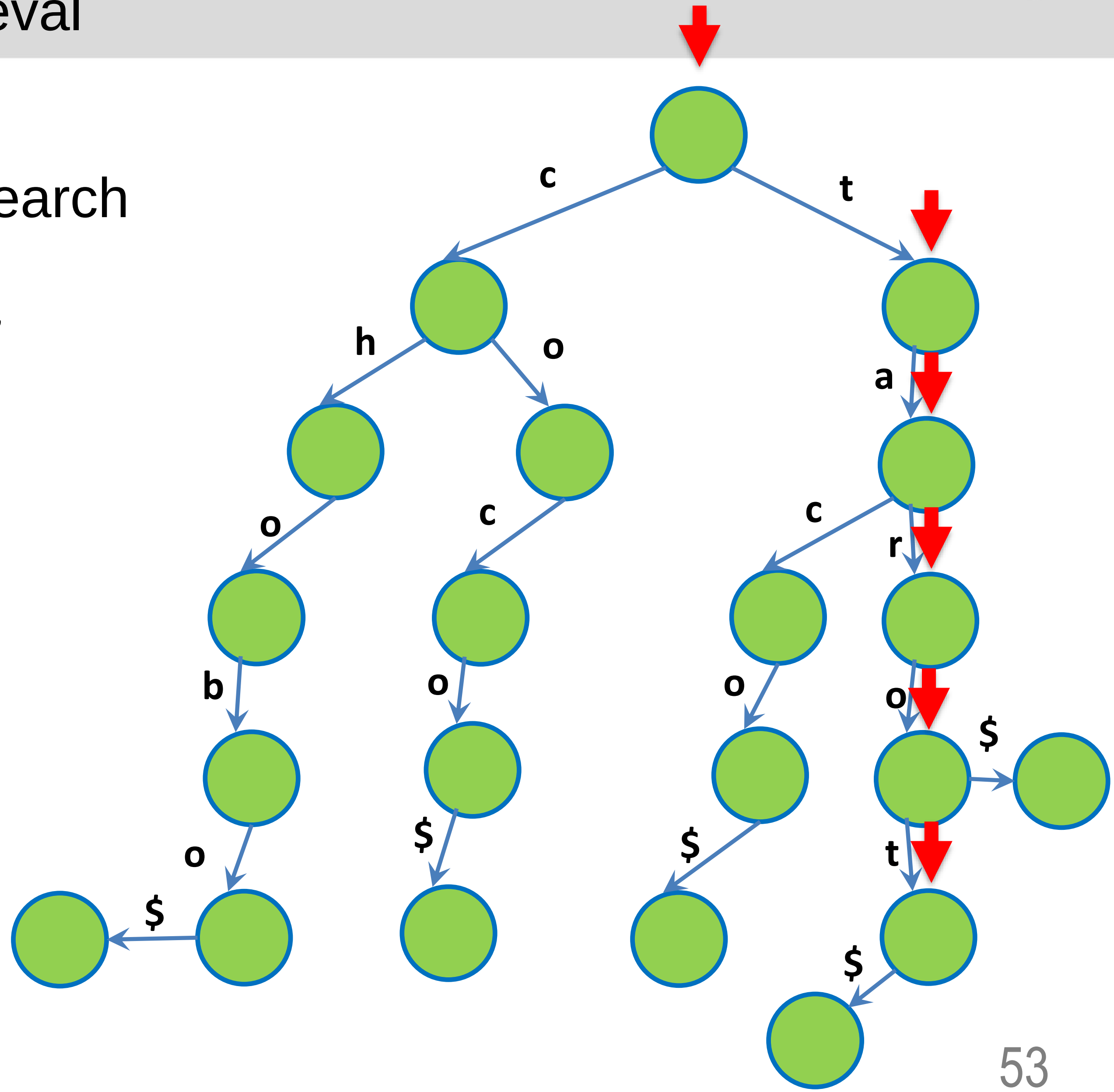


Tries

Efficient string retrieval

- So how do we search for retrieval?
  - Search for “taro<u>t</u>”



Tries

Efficient string retrieval

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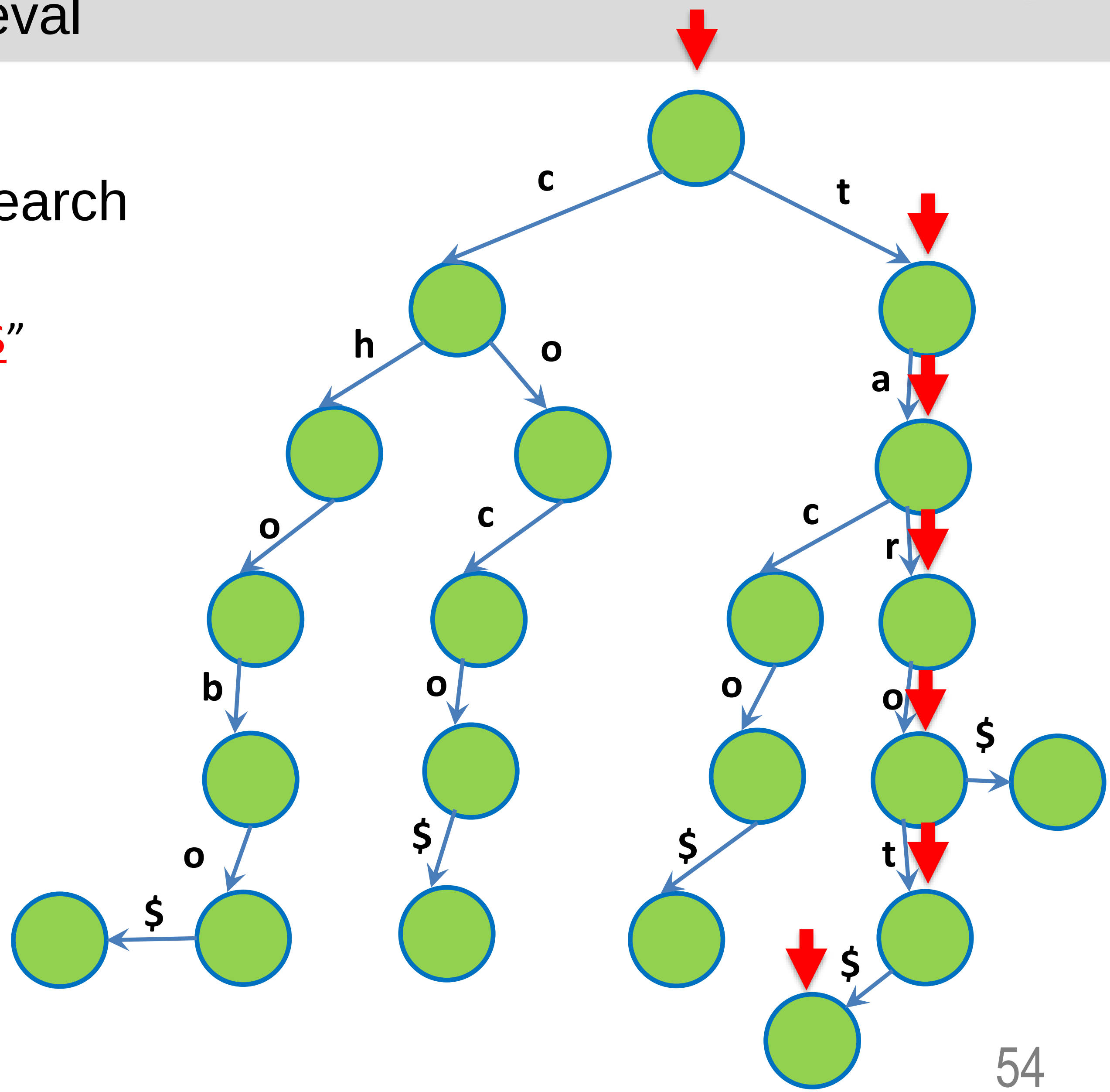
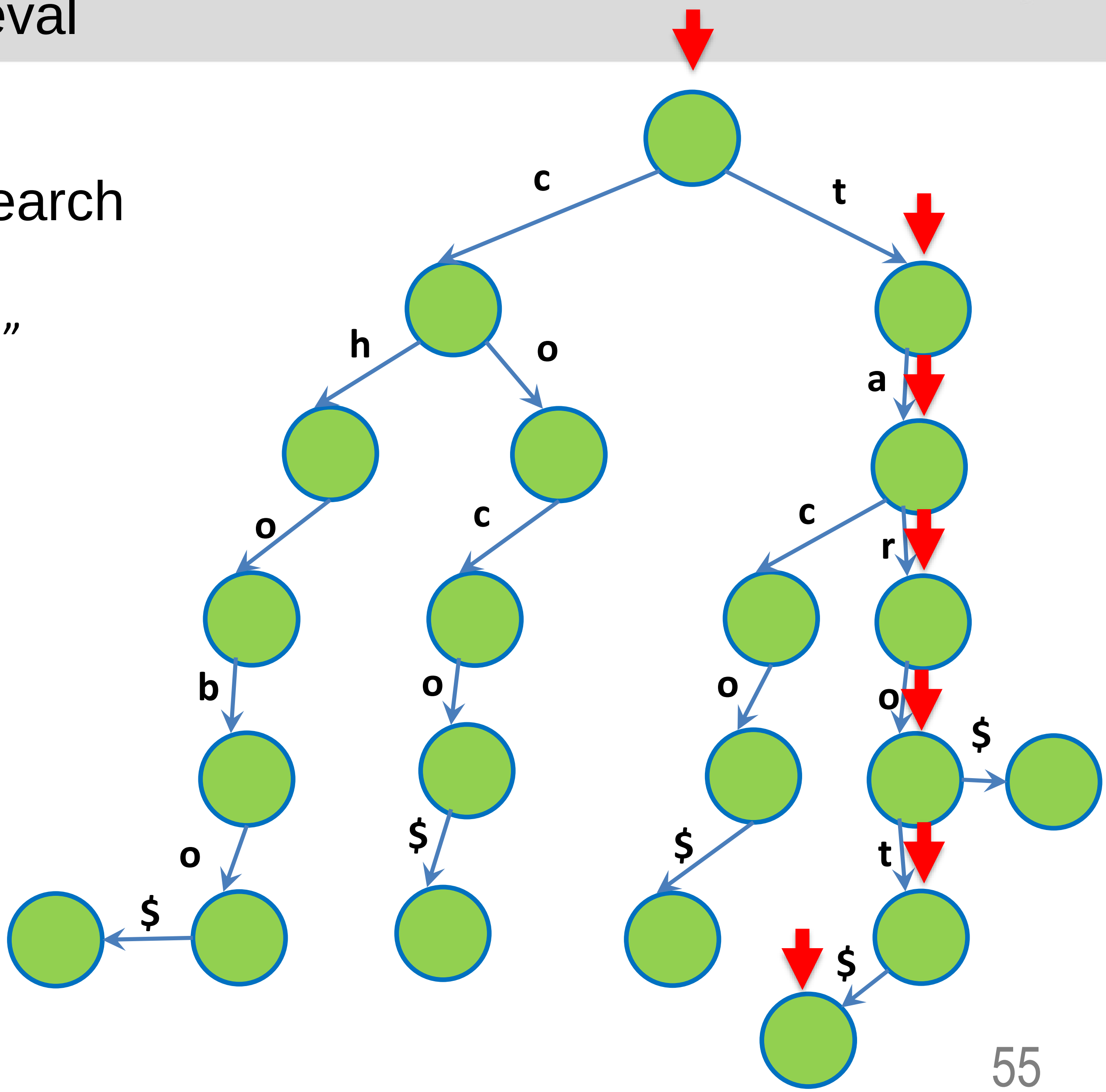
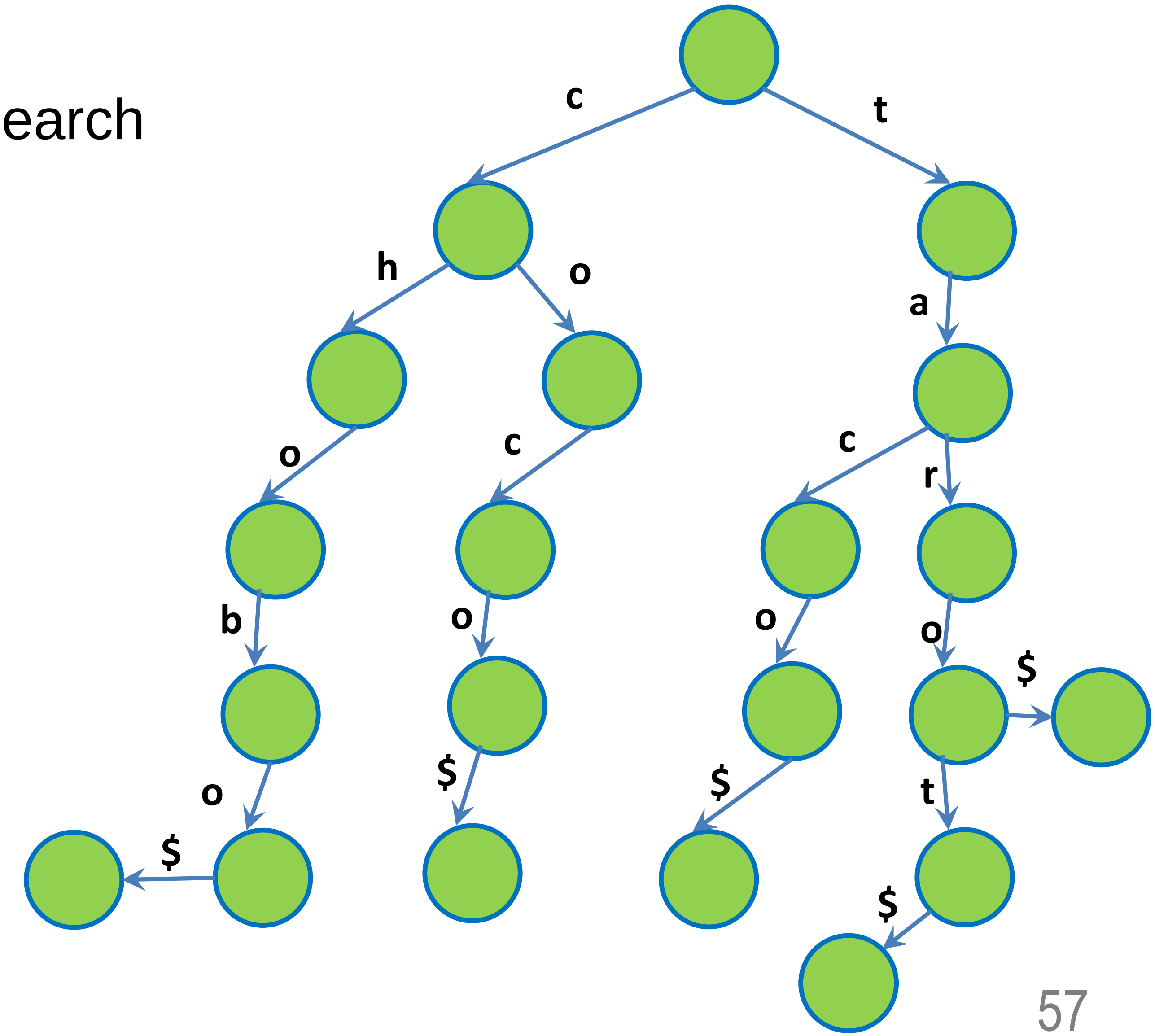


- So how do we search for retrieval?
  - Search for “tarot$”
    Found!



Questions?

- So how do we search for retrieval?
  - Search for “cow”



Tries

Efficient string retrieval

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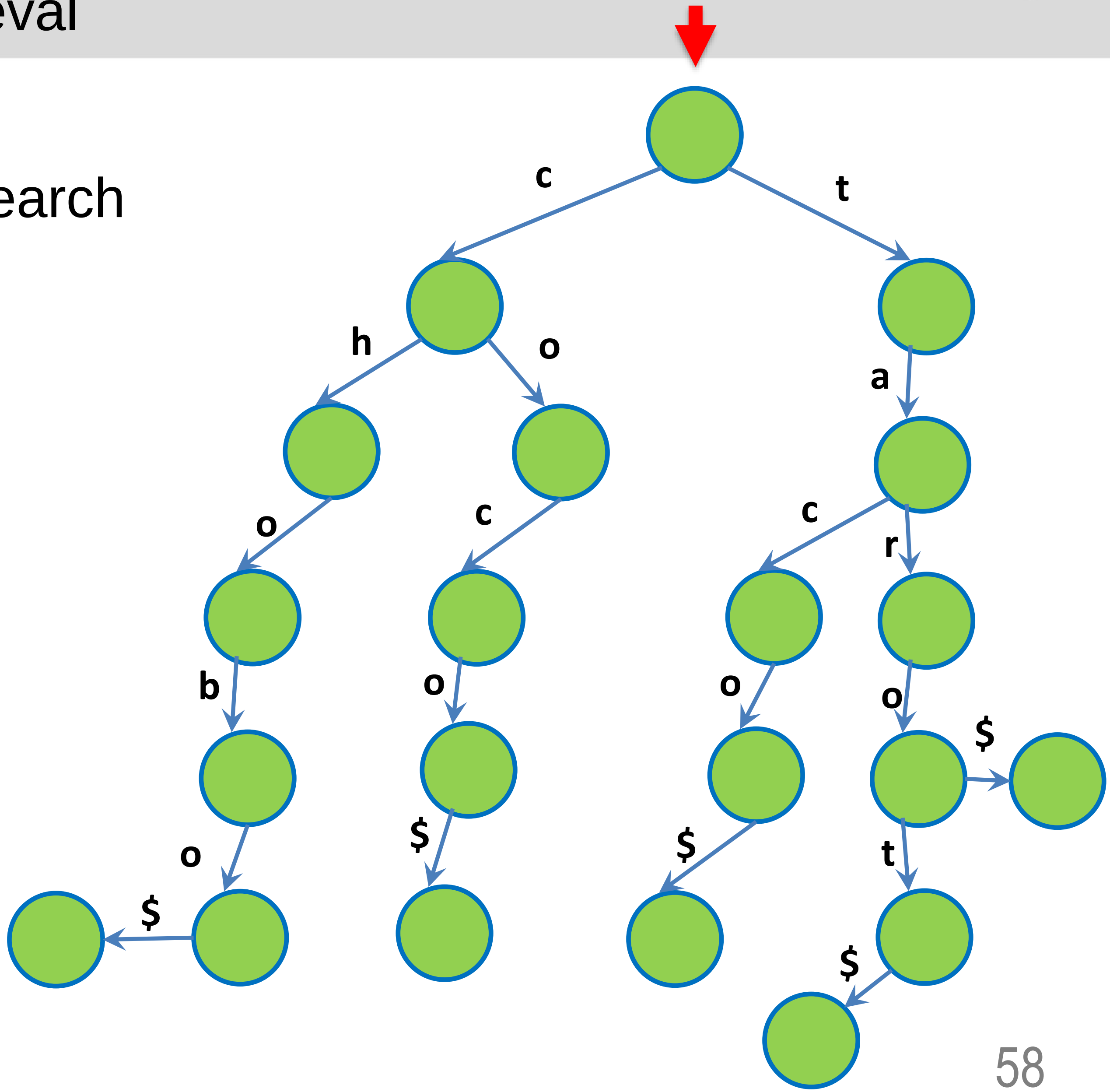

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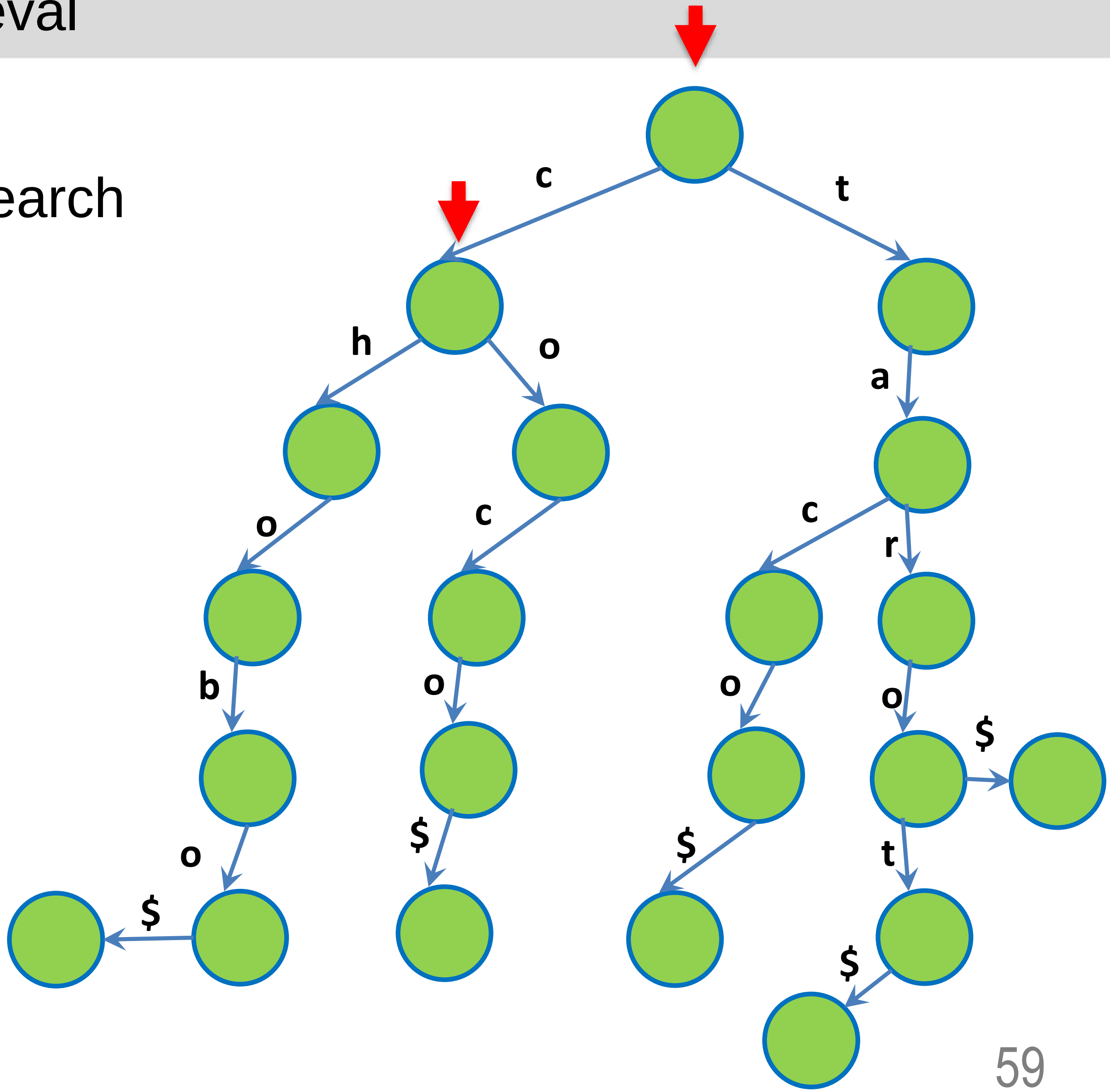

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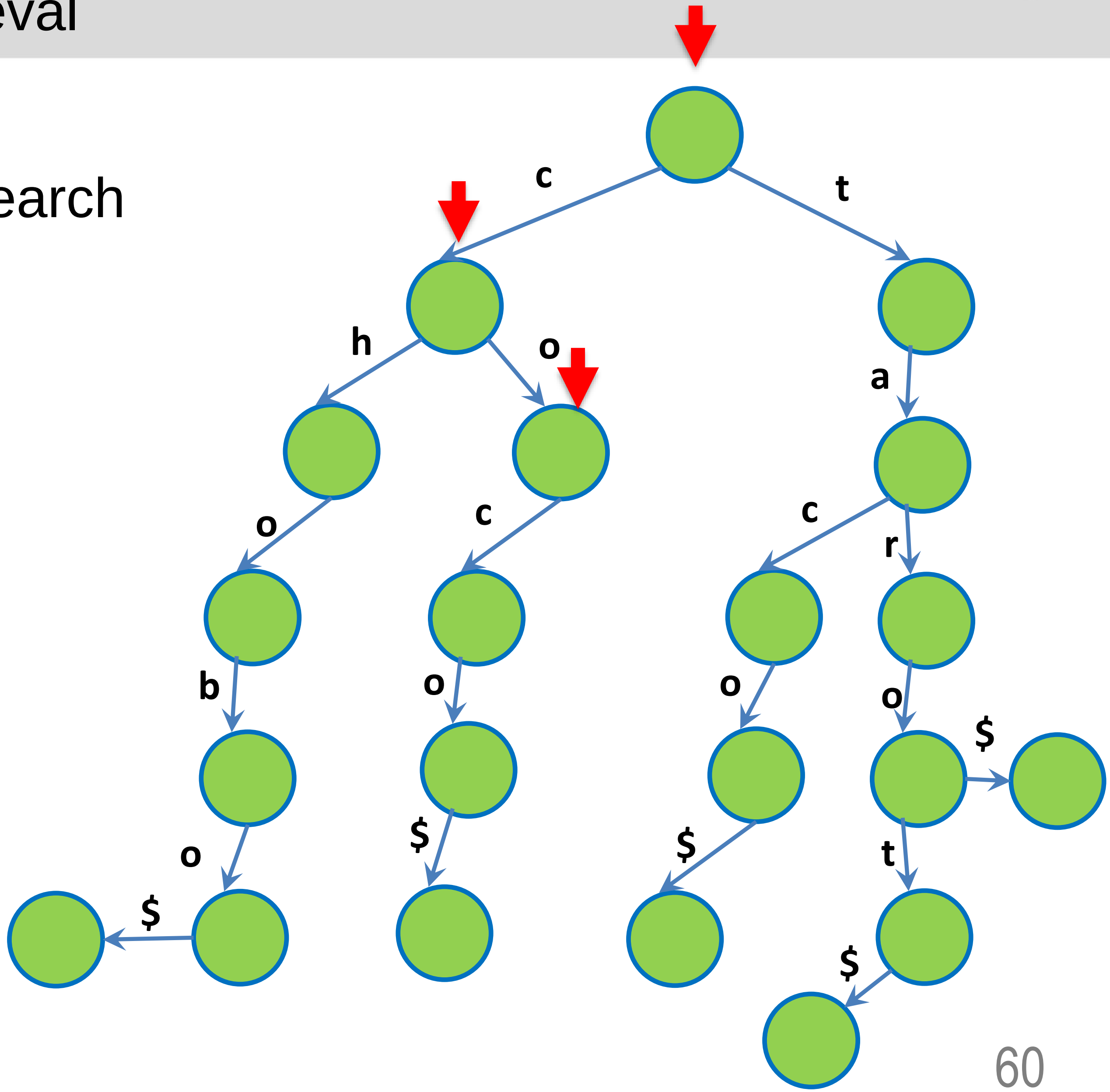

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Efficient string retrieval

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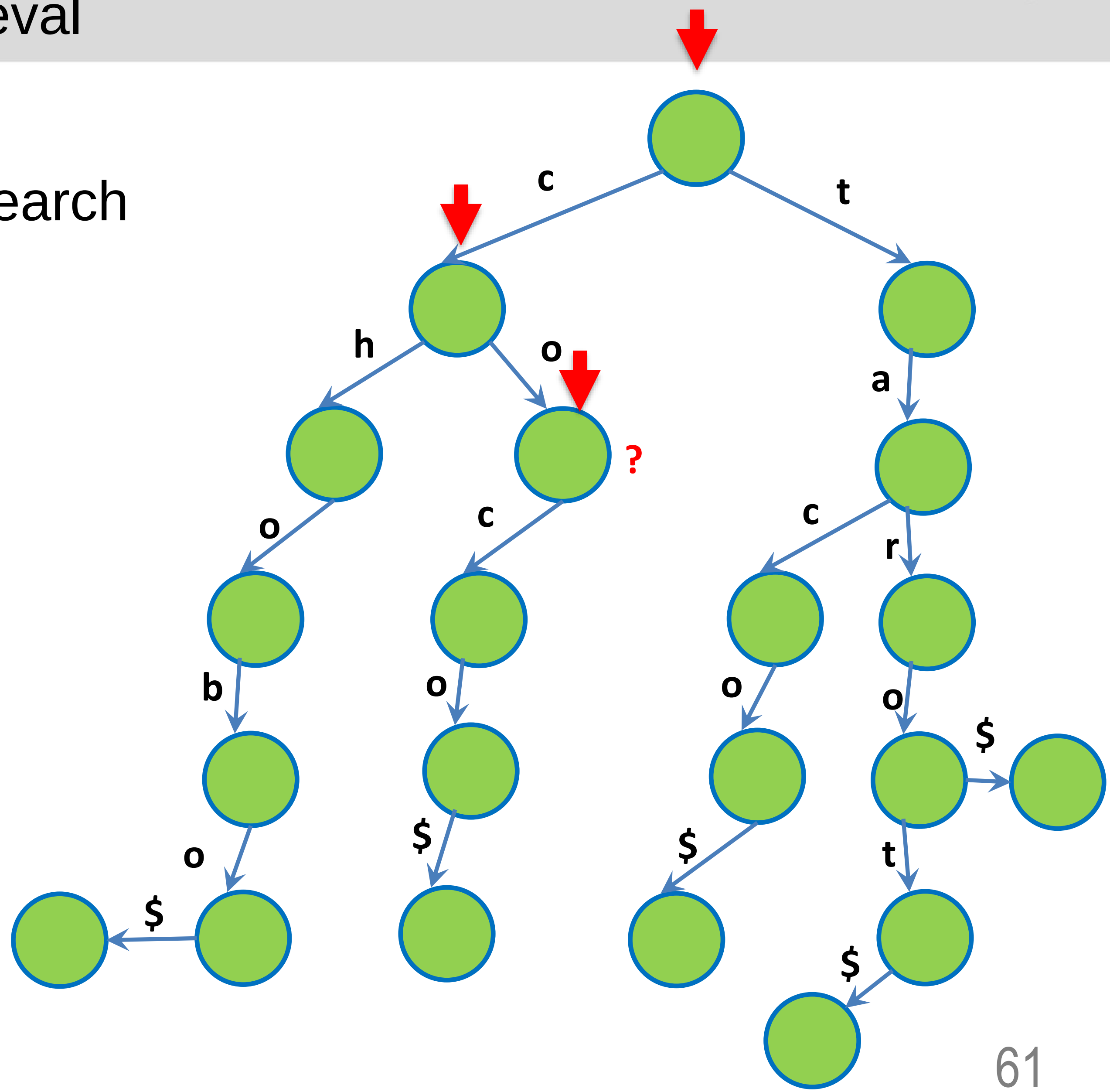
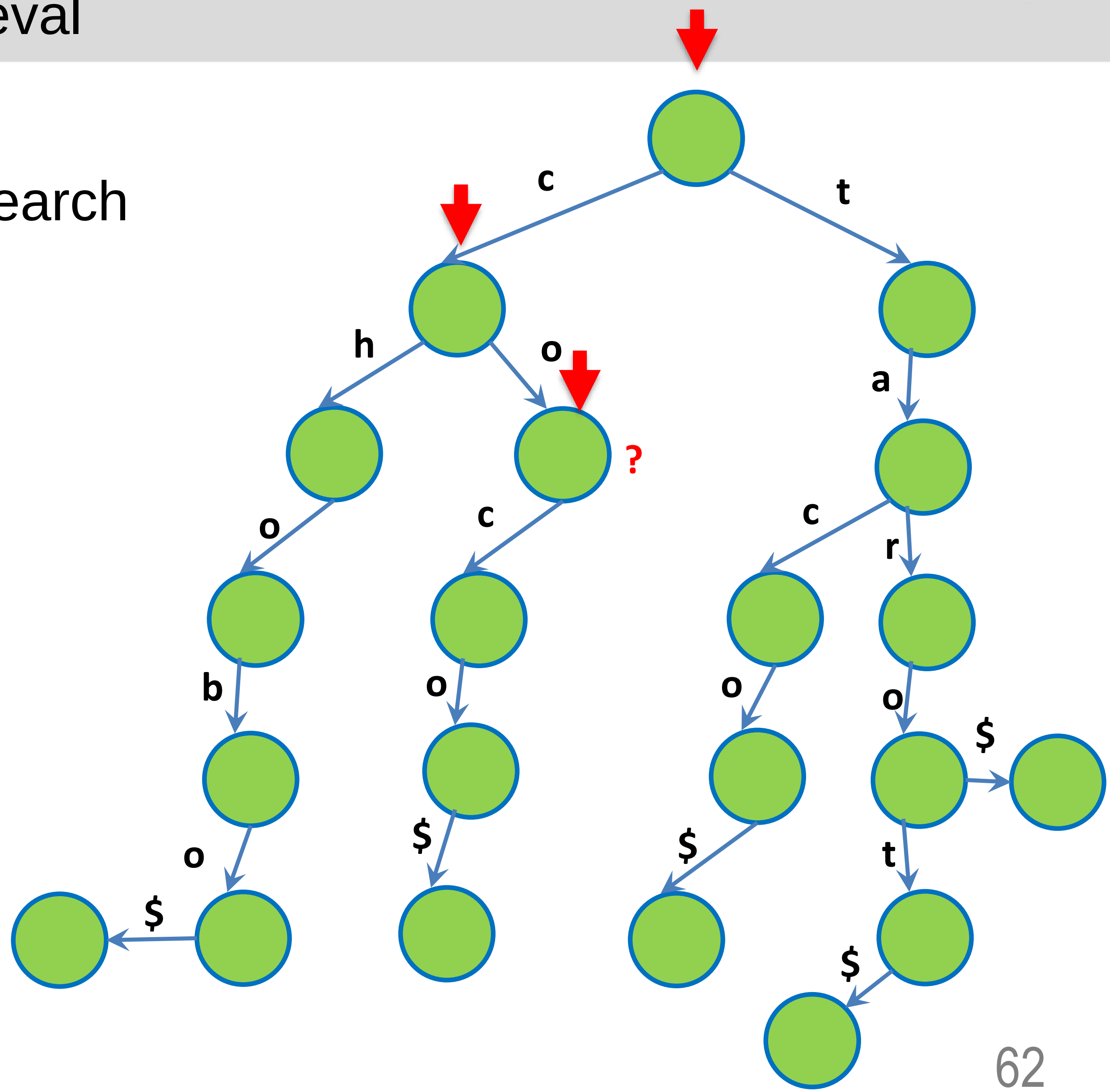


Tries

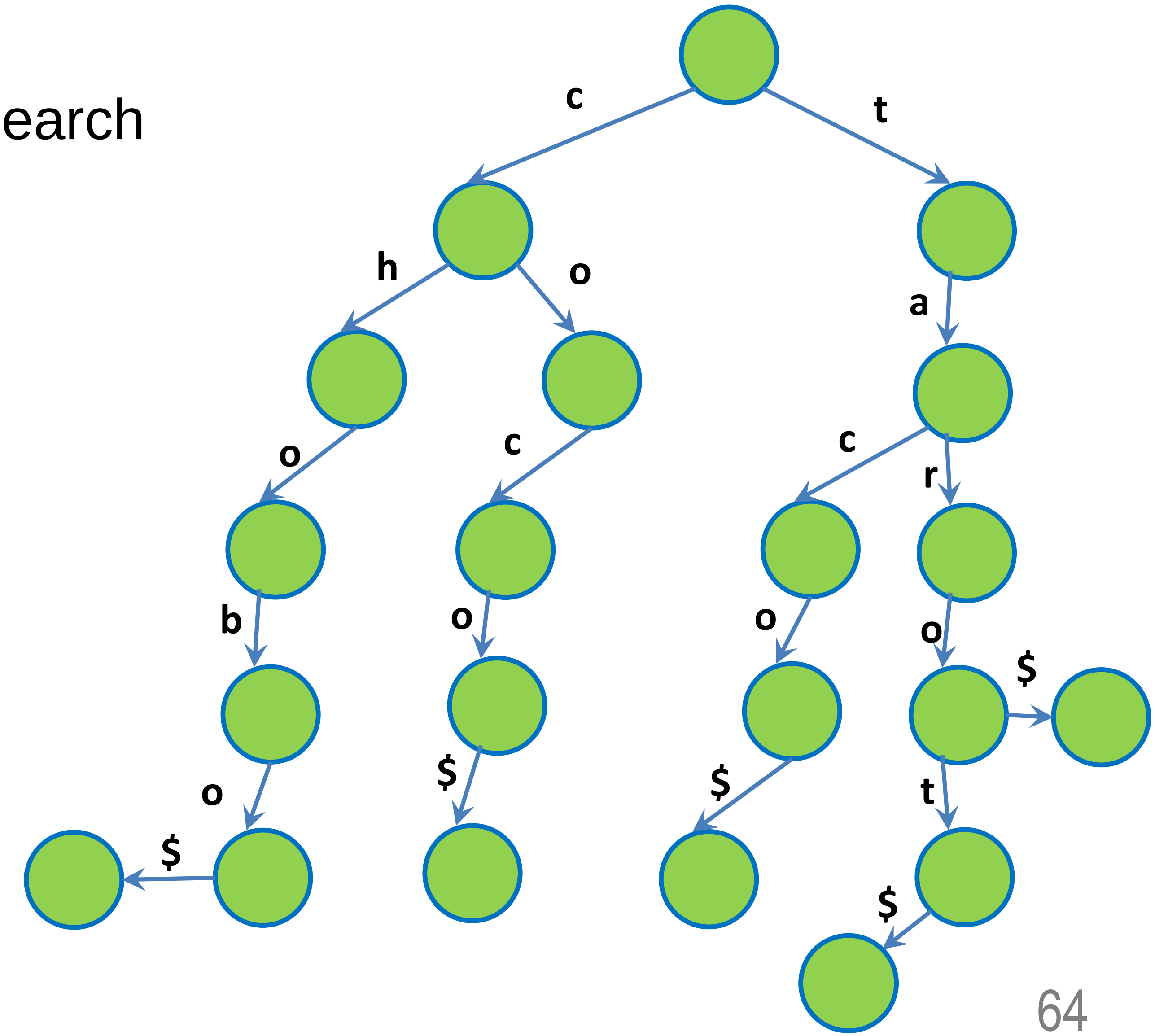
Efficient string retrieval

- So how do we search for retrieval?
  - Search for “cow”
    Not found T.T



Questions?

- So how do we search for retrieval?
  – Search for “tar”



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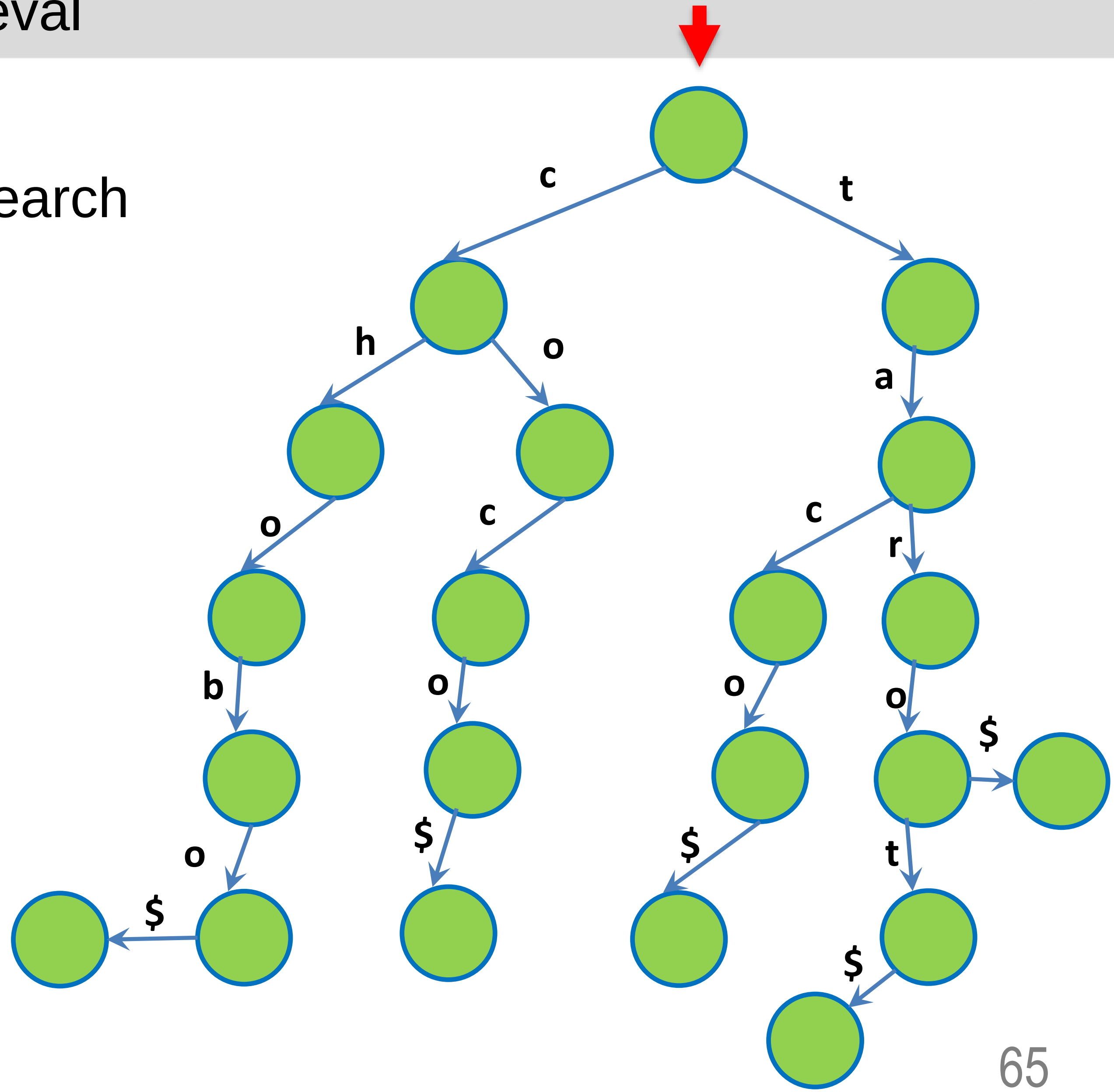

Tries
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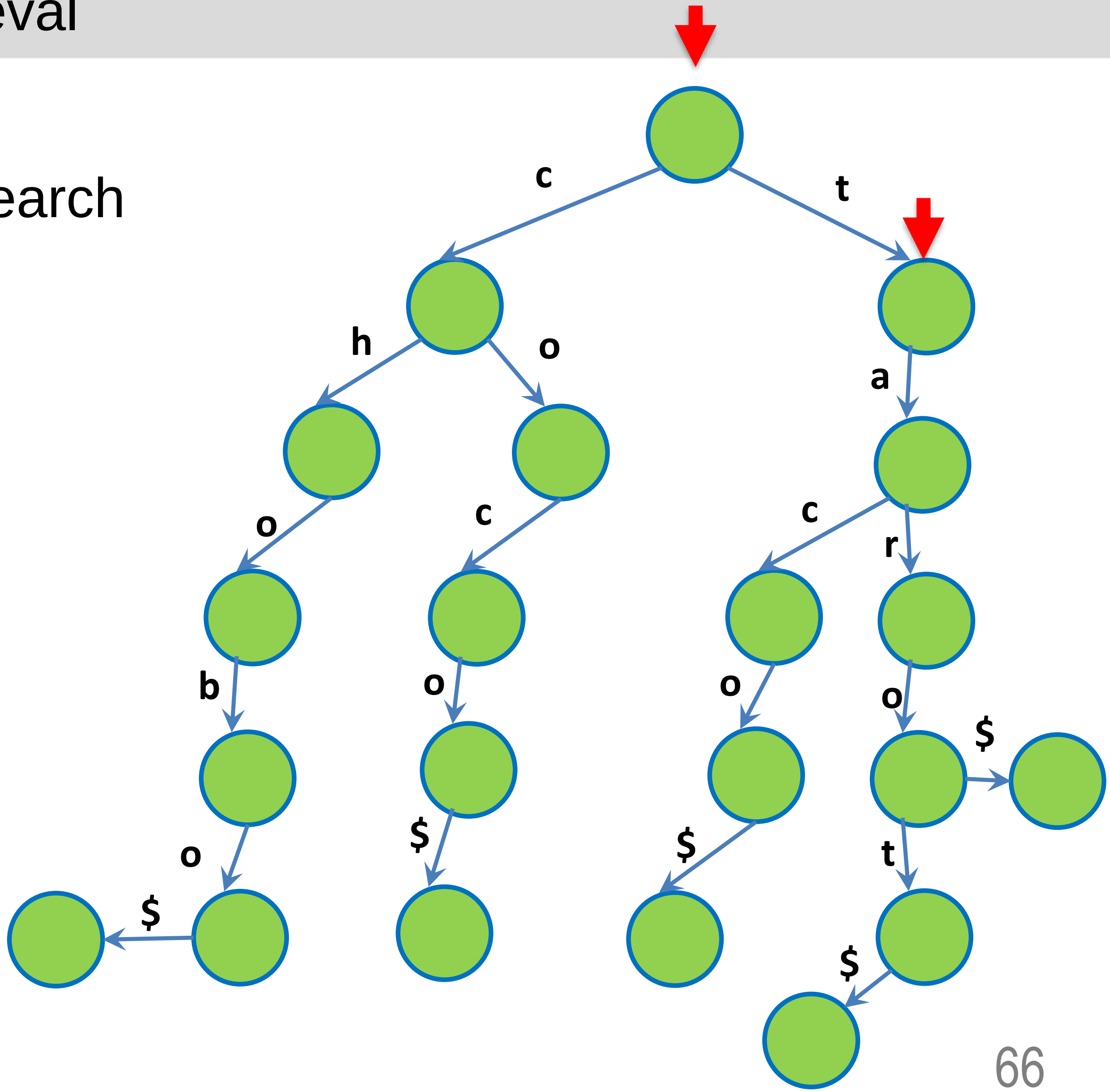
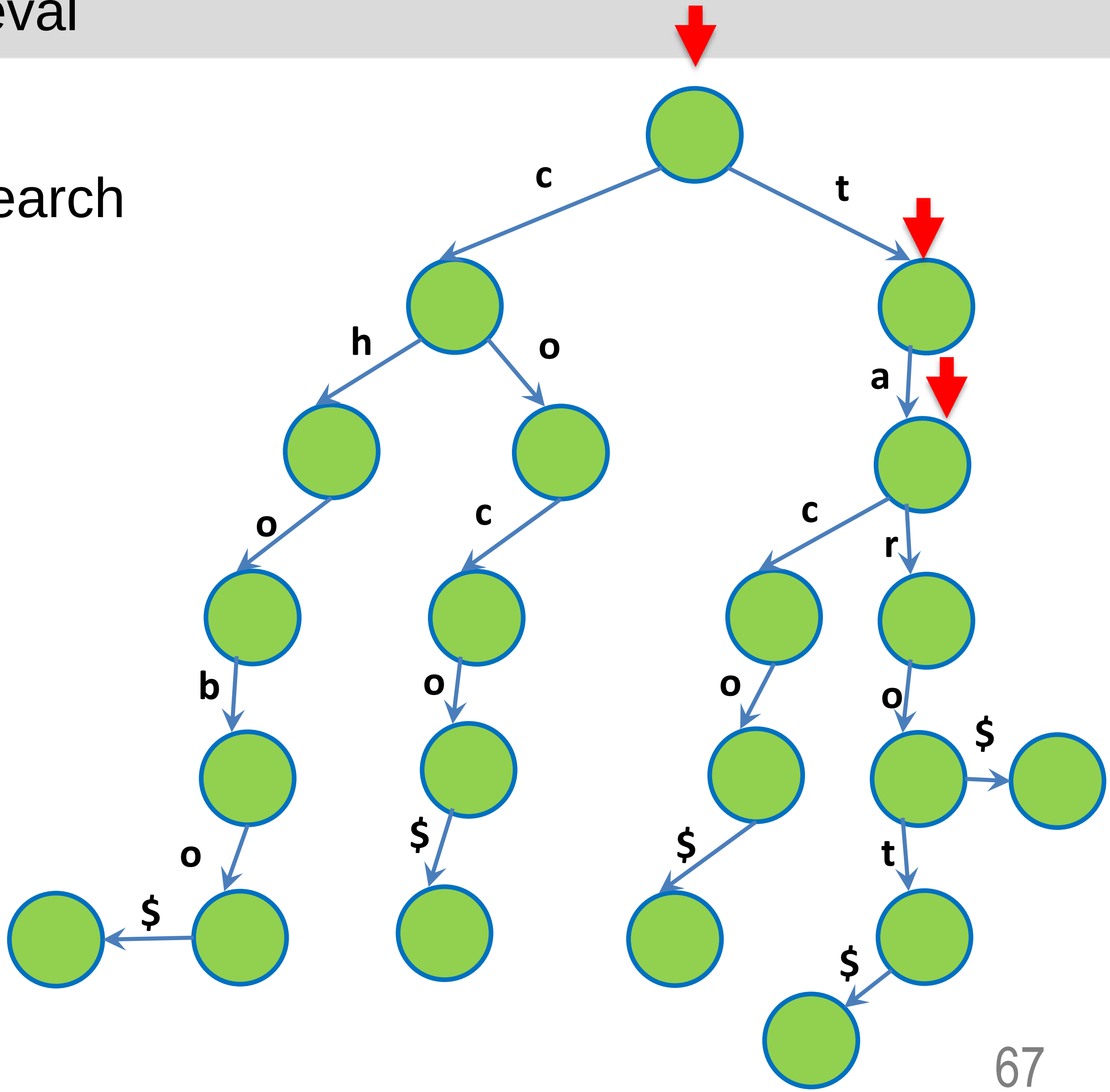


- So how do we search for retrieval?
  - Search for "t<u>a</u>r"



Tries
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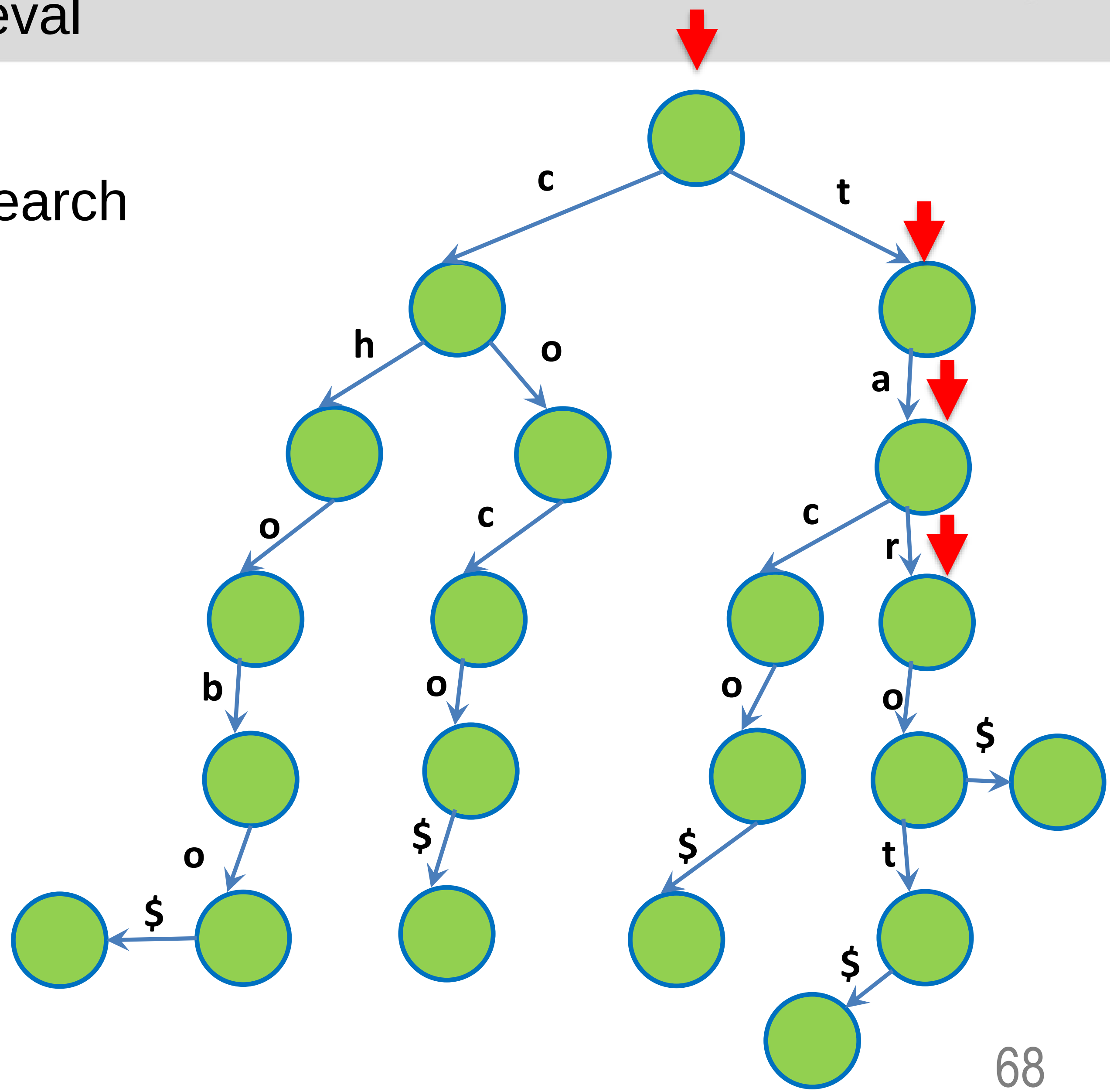
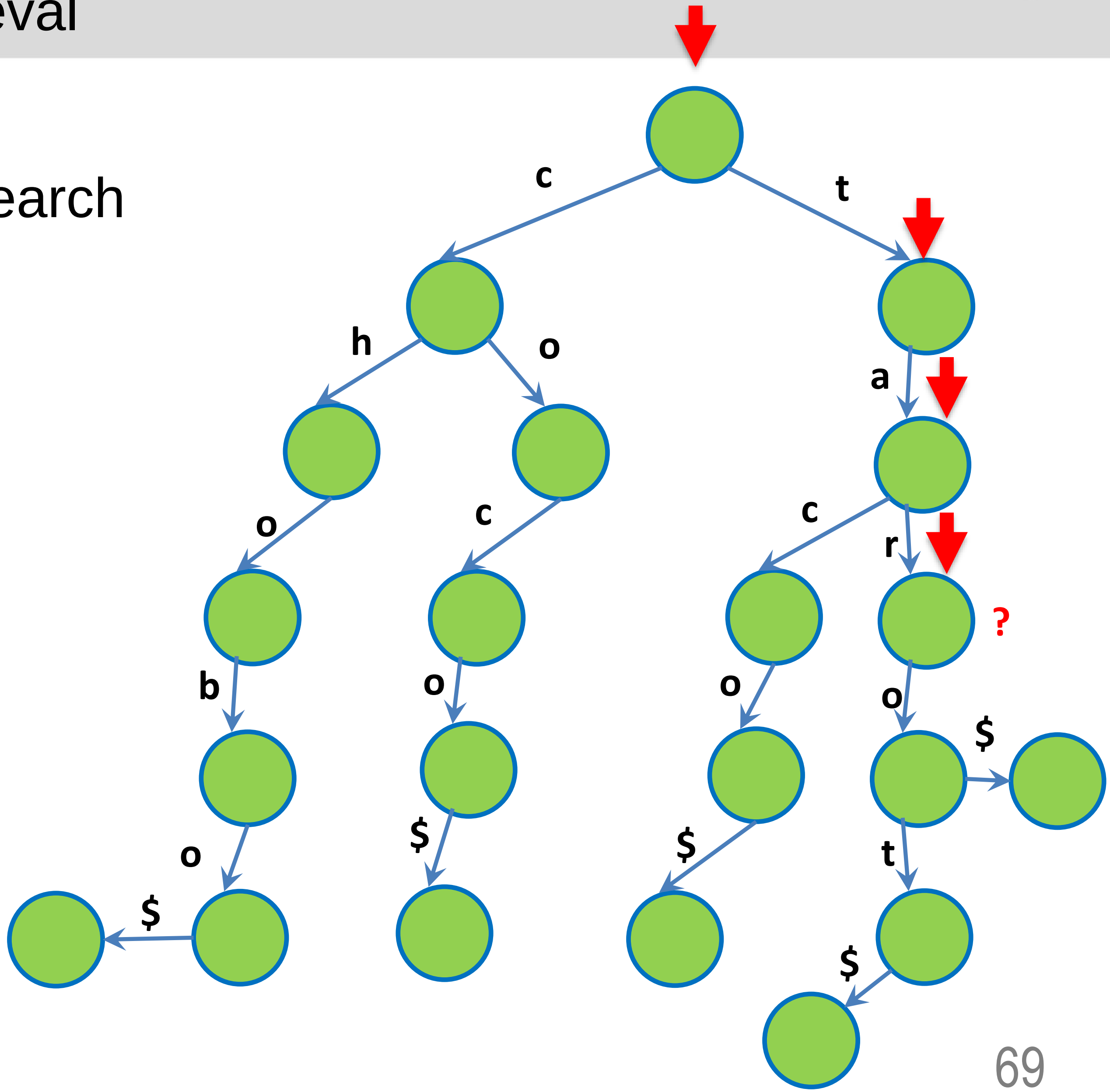


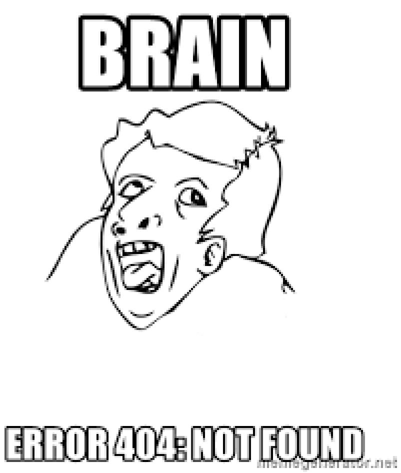
- So how do we search for retrieval?
  - Search for “tar$”

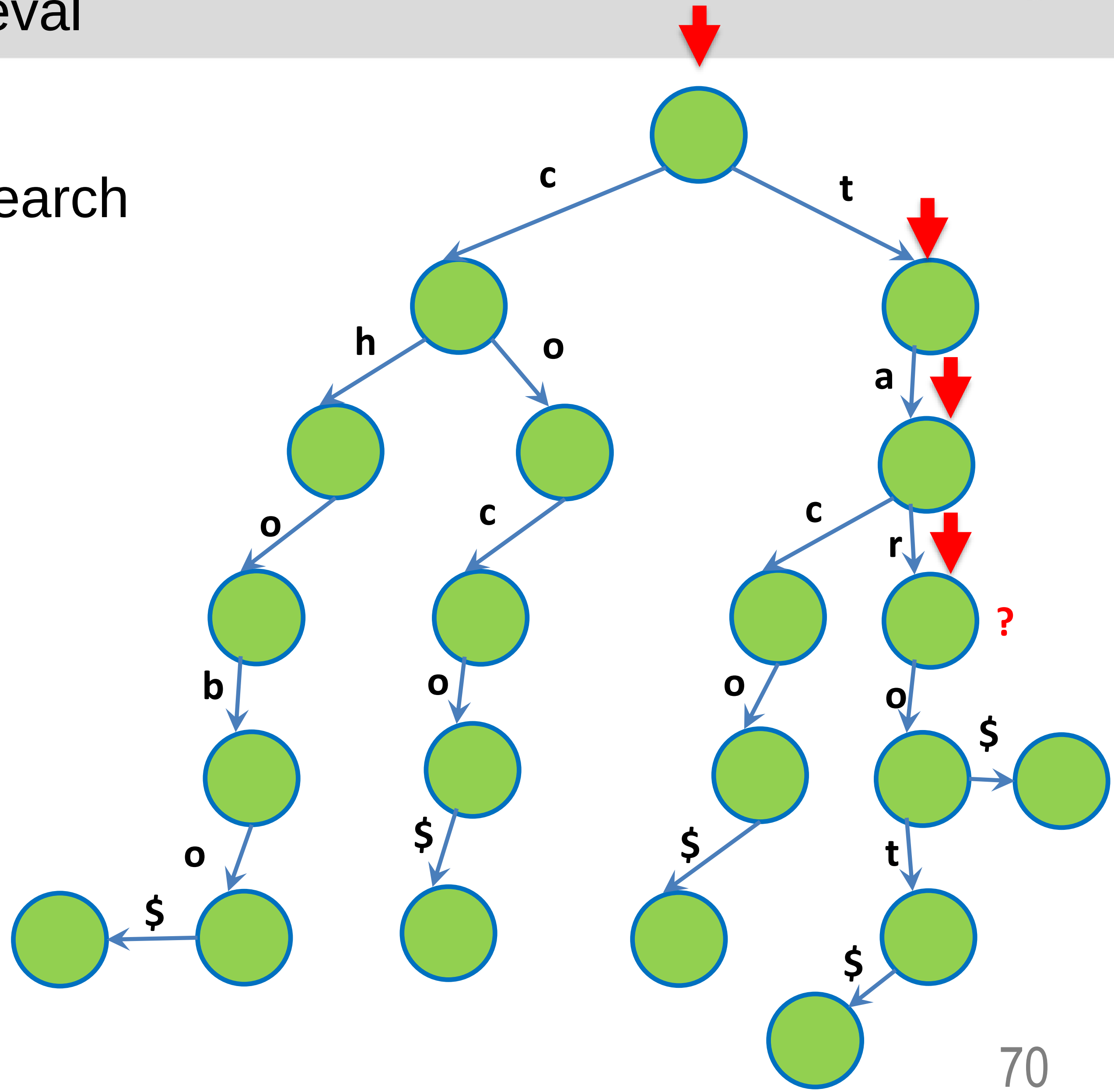


Tries

Efficient string retrieval

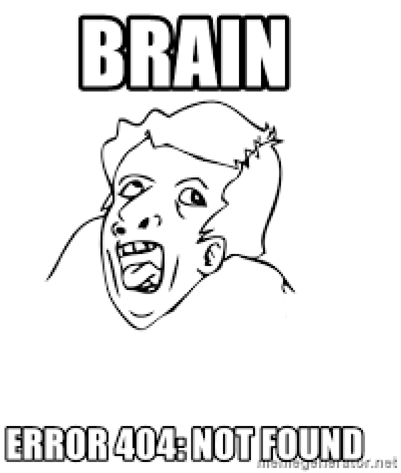
- So how do we search for retrieval?
  - Search for "tar$"
    Not found =(

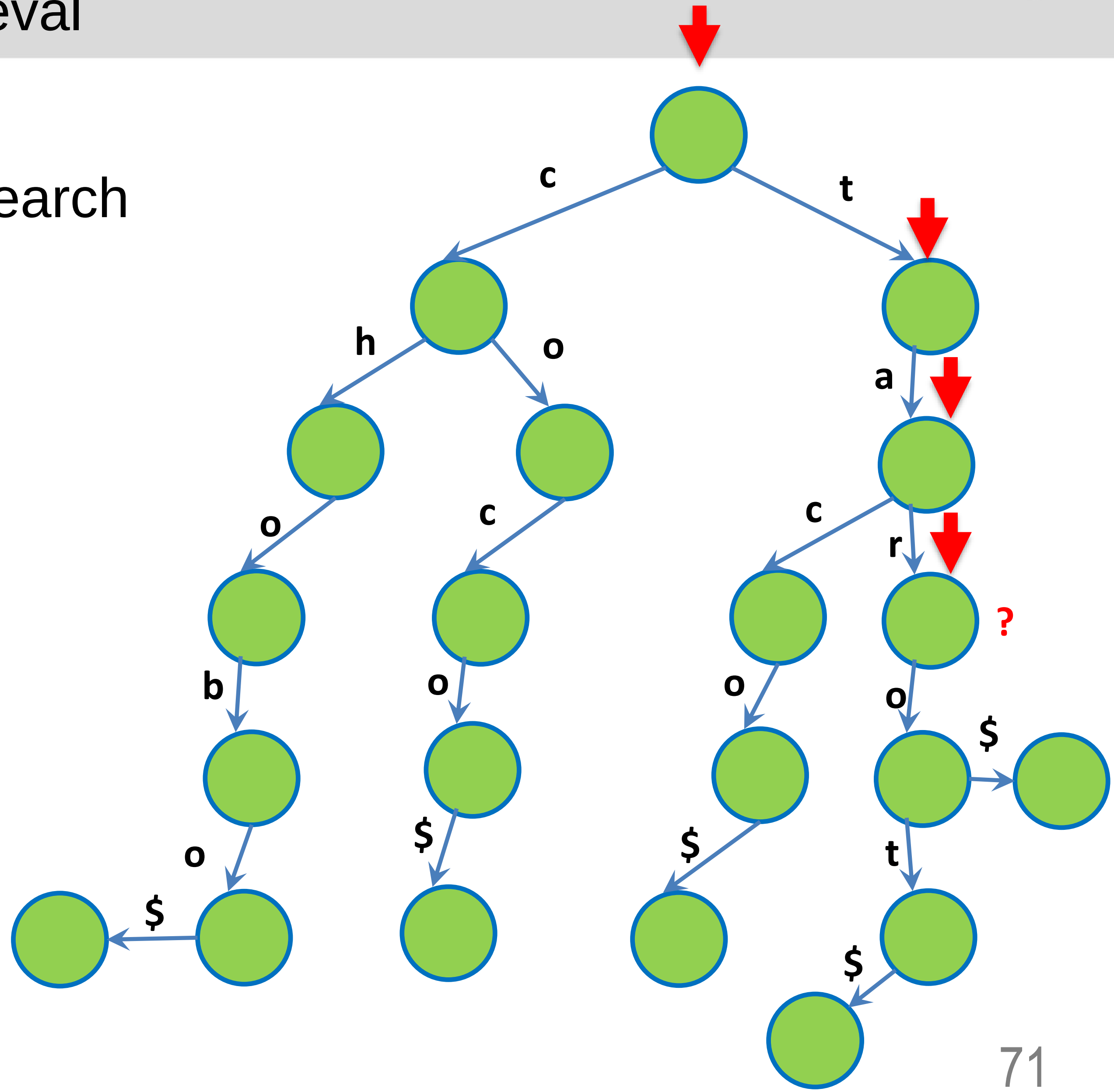




Tries
Efficient string retrieval

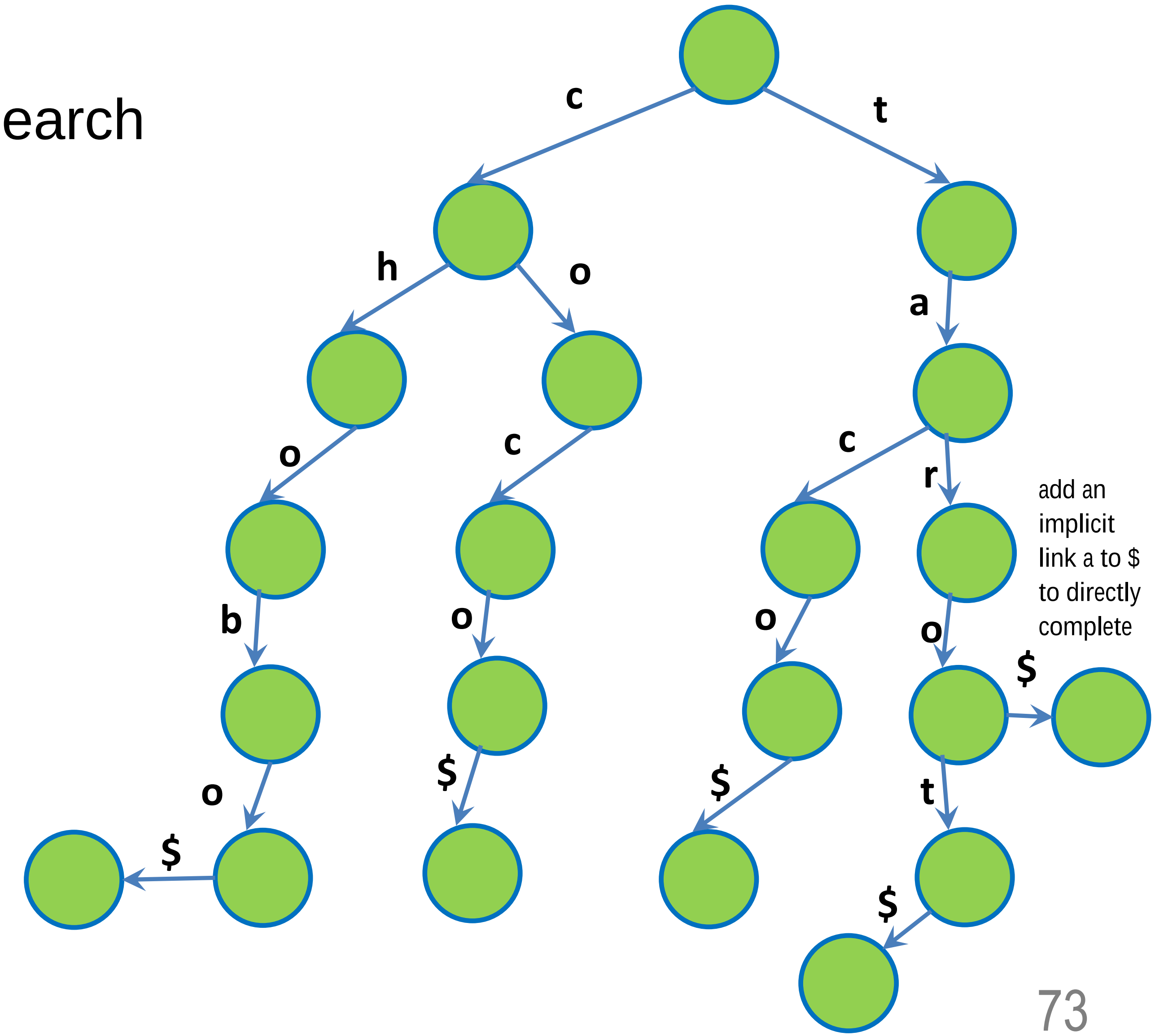
- So how do we search for retrieval?
  – Search for “tar$”
    Not found =(
    Need those $$$



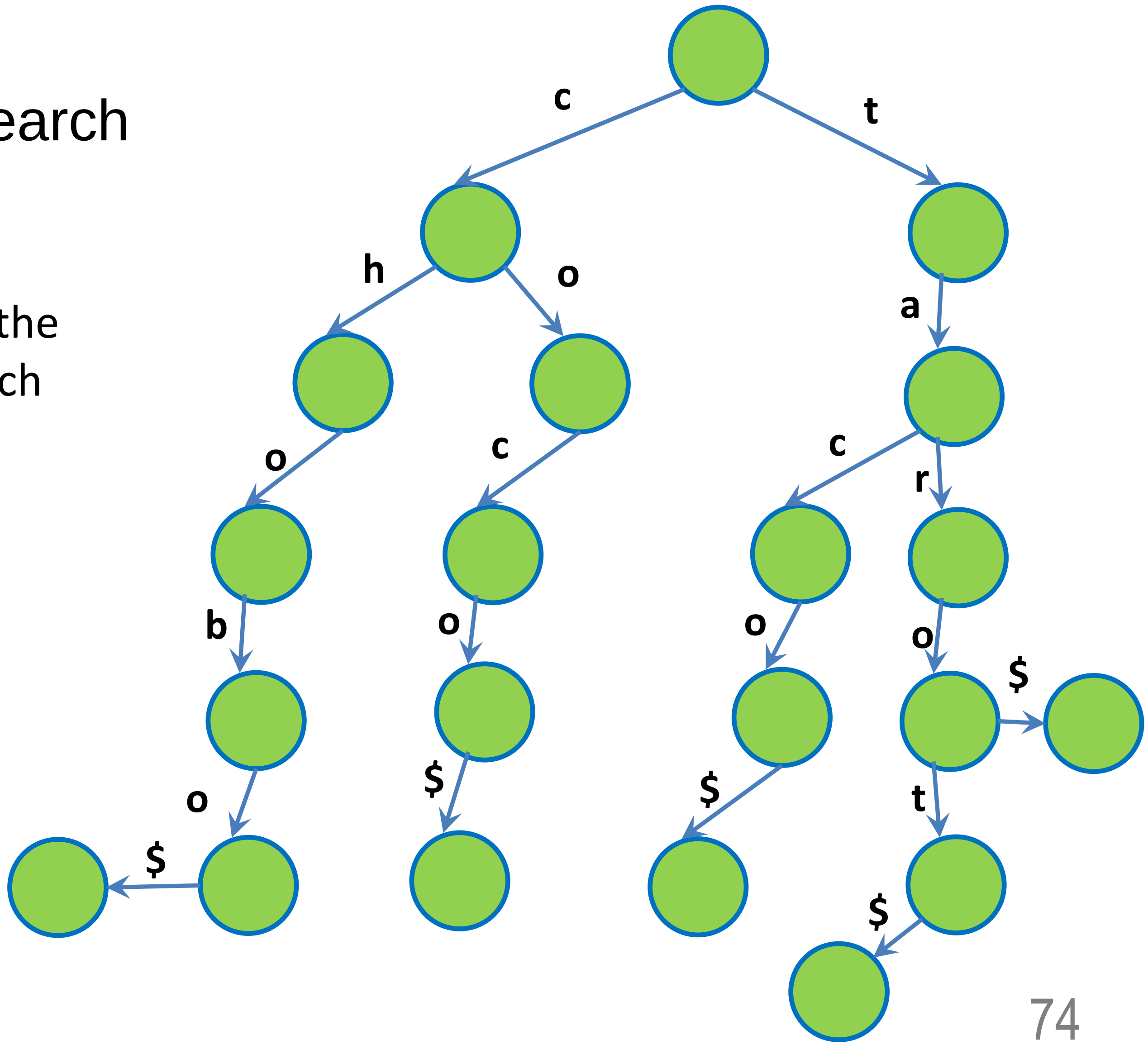


Questions?

- So how do we search for retrieval?
  - Complexity?



- So how do we search for retrieval?
  - Complexity?
    O(M) where M is the length of the search string…



Tries

Efficient string retrieval

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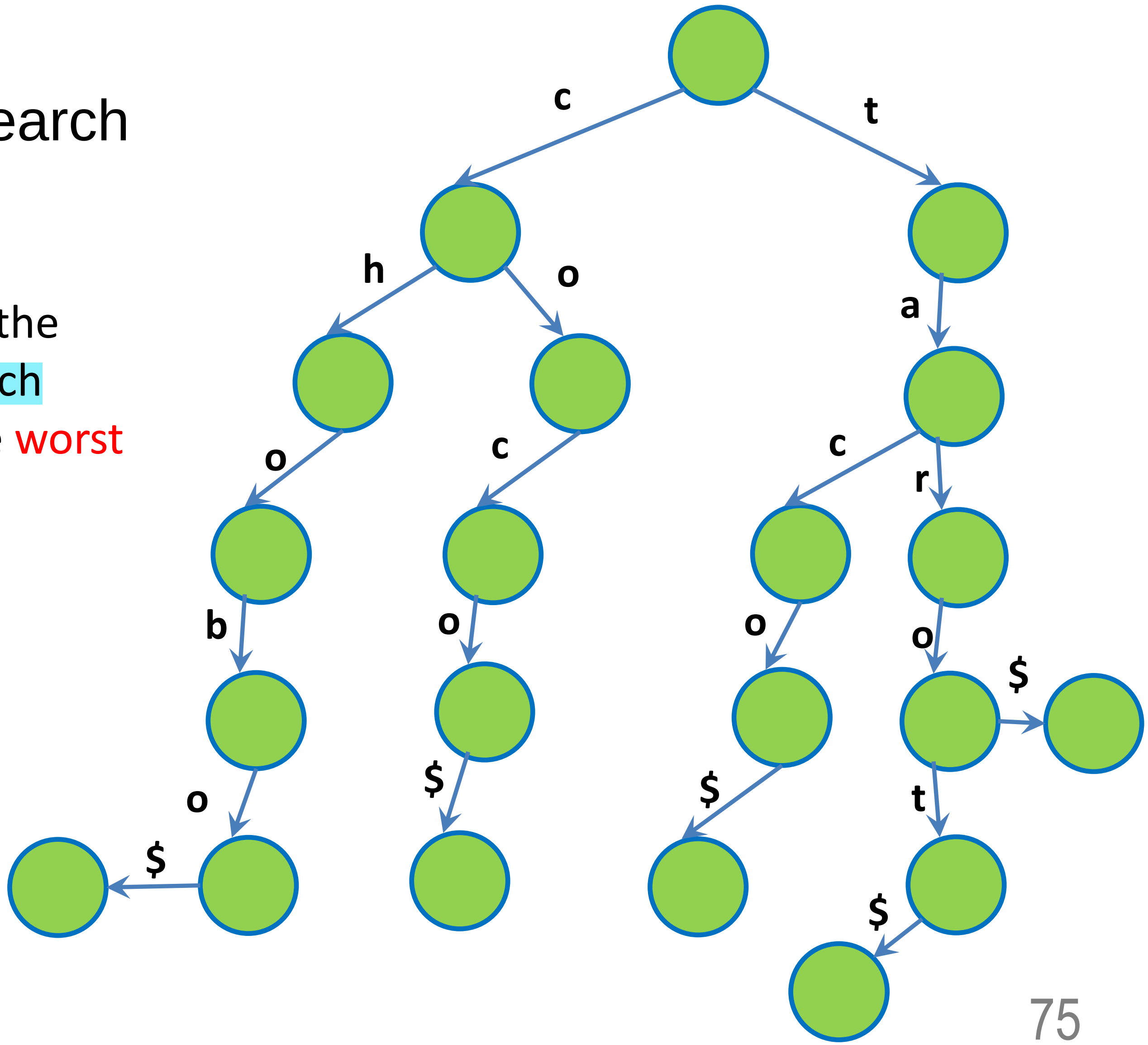
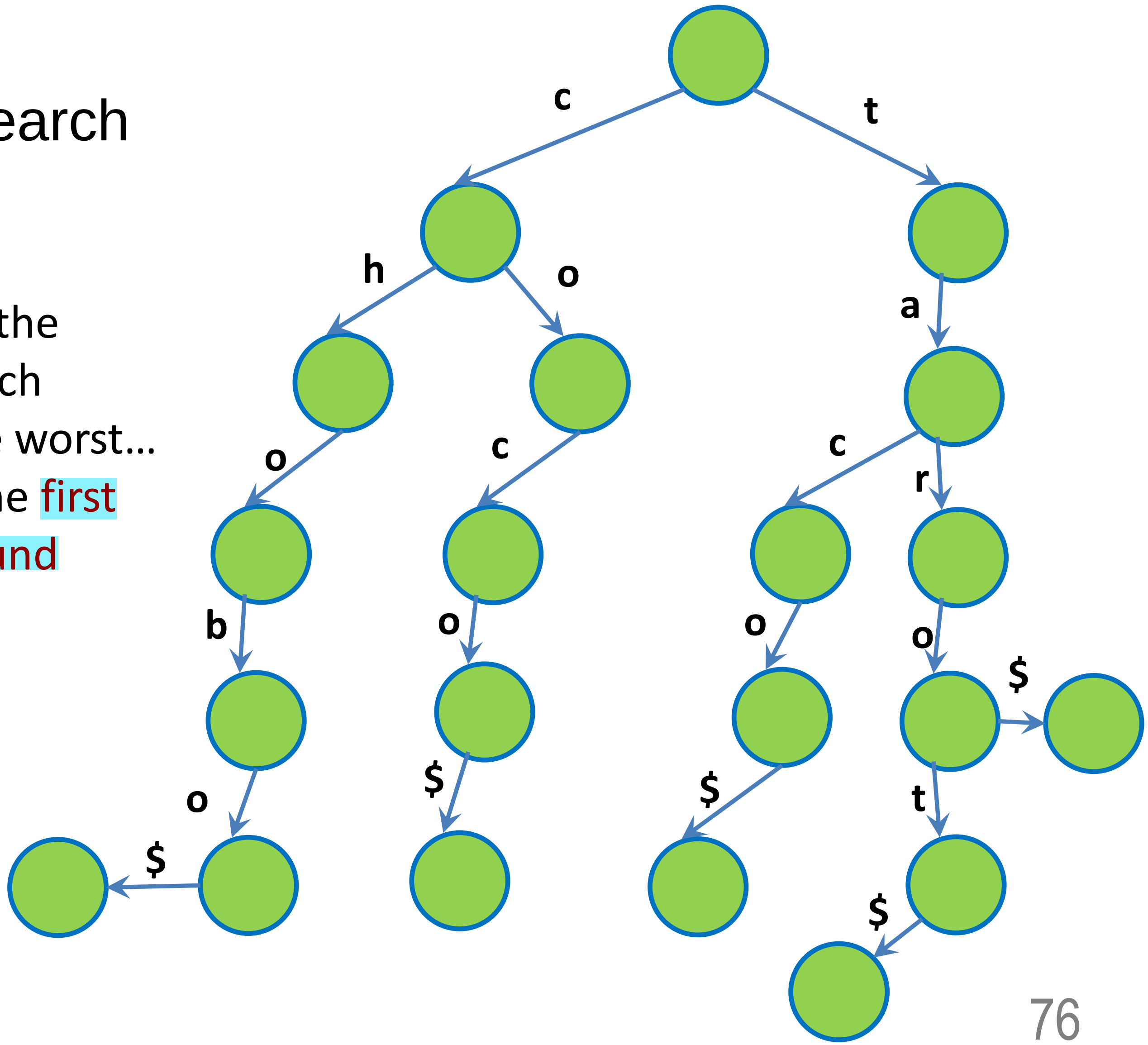


Tries
Efficient string retrieval

- So how do we search for retrieval?
  - Complexity?
    O(M) where M is the length of the search string… This is the worst…
    O(1) best when the first character isn't found

add implicit
link from one node
to the terminal node
by comparing
frequency
selected and
move alone the
correspond path



Questions?

- How to implement it?

- How to implement it? With OOP!

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  - Node class

```python
class Trie:
    def __init__(self):
        self.root = Node()

class Node:
    def __init__(self, data=None):
        self.data = data
        self.links = [None] * 27
```

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```

root has from index 0 to 26
in total 27 characters
each element point to another node
that has a link with multiple element in it
repeat

$ in the link has index 0

26 characters + $

  - Then we need to code the traversal from the root
    - If a link exist, travel through it
      This is O(1) due to the array data structure

Questions?

- Benefits?

Tries

Efficient string retrieval

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  - Better for string search than BST/ AVL
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  - Can sort very quickly by traversing the string
    - The edges/ links are in-order (from a to z)
    - This is O(MN)
- Disadvantage?
  - At times can be slower than hash table
  - Wasted space if the self.link array is left empty most of the time

waste about 26 memory in link since storing 26 characters in order

Questions?

Usage?

- Height of the trie = length of the longest string

Tries

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- Complexity is based on the length of the string we are inserting/ deleting/ searching
- We can search for the prefix of strings!
  - Useful for auto correct/ auto complete
  - And many other applications!

Questions?

Suffix Trie
For suffixes

- Same as a trie
- But for suffixes

Suffix Trie
For suffixes

- Can you make a suffix trie for apple?

Suffix Trie

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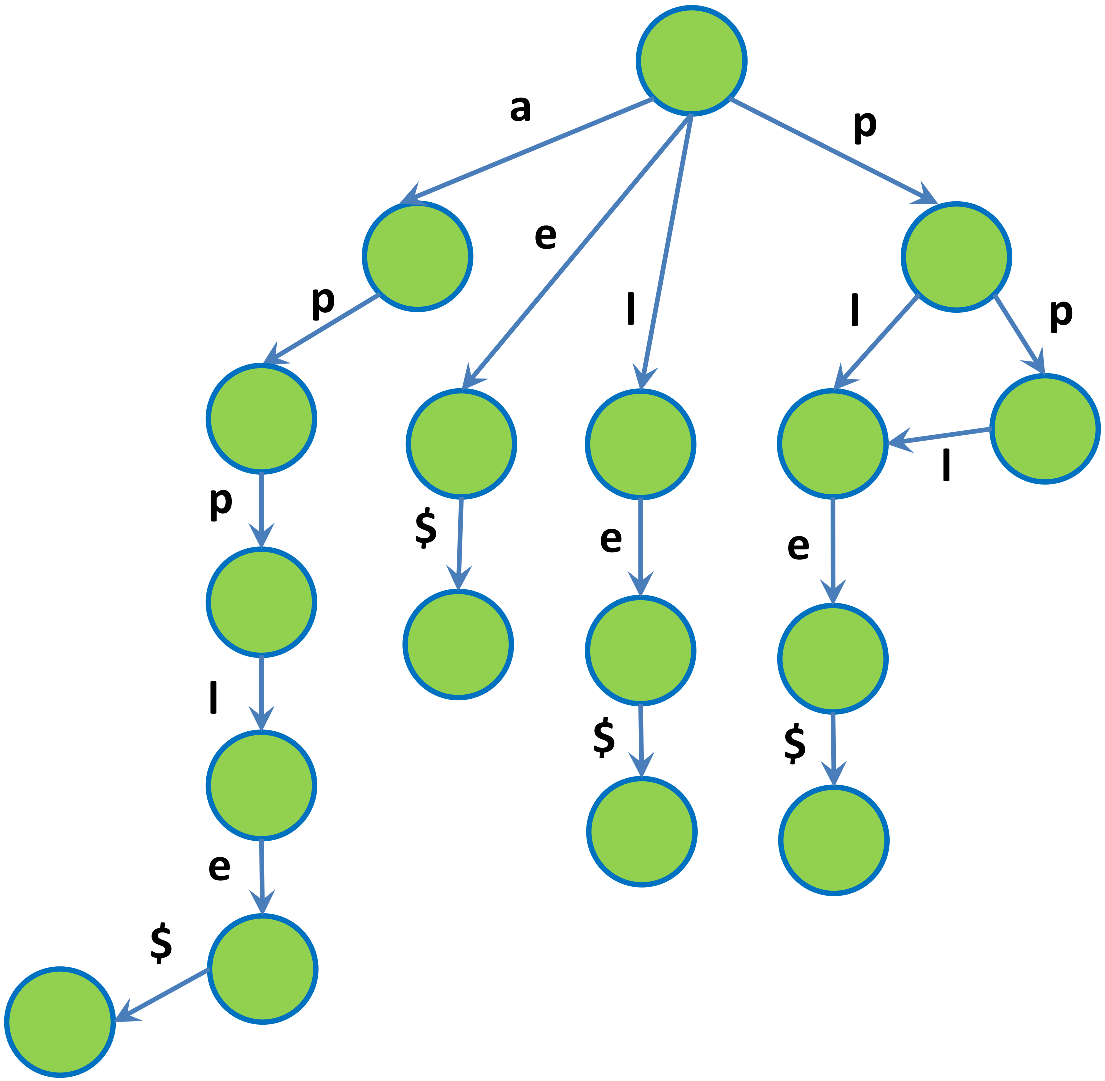

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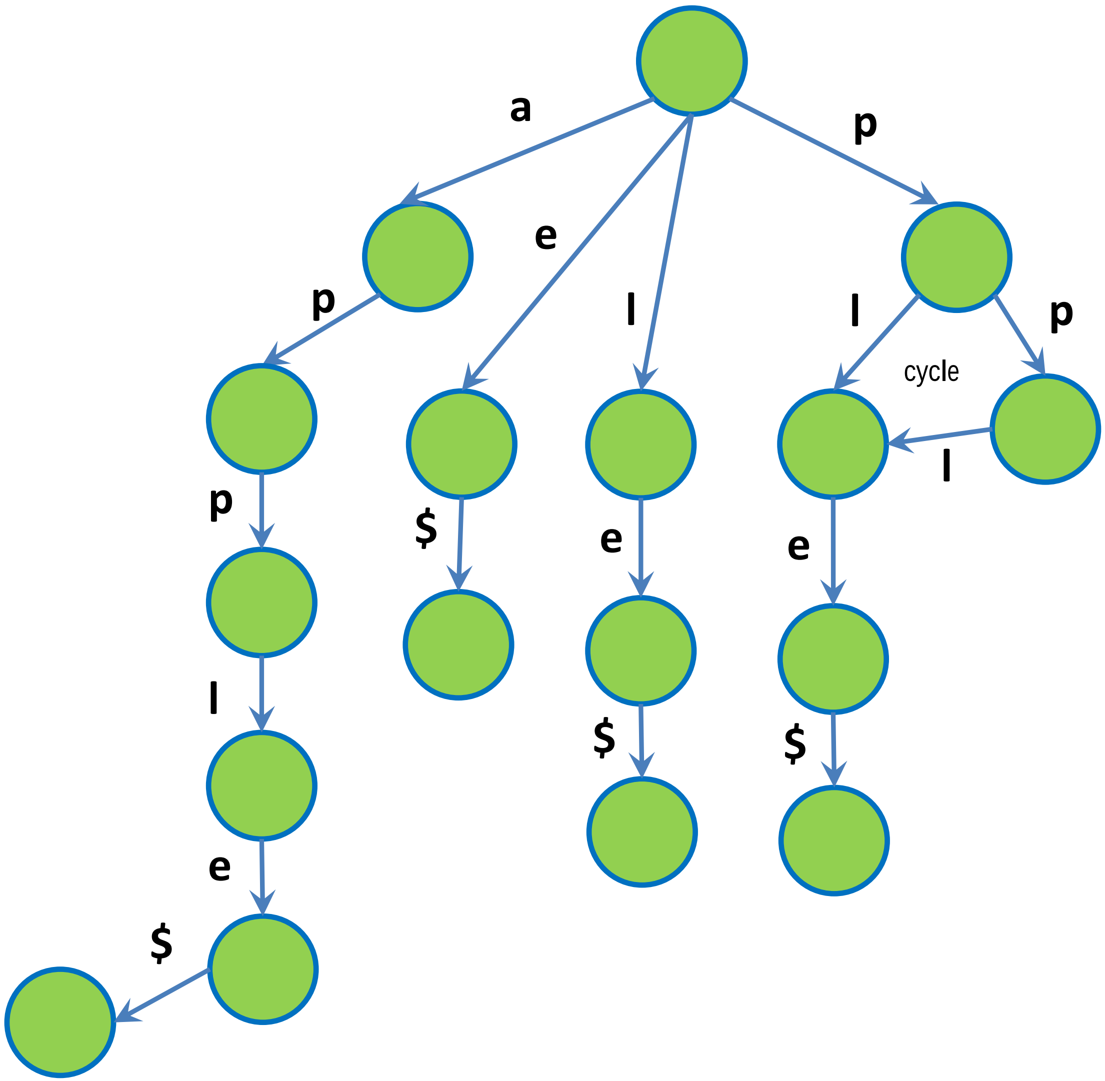
  NO! CYCLE!
  so this is wrong…



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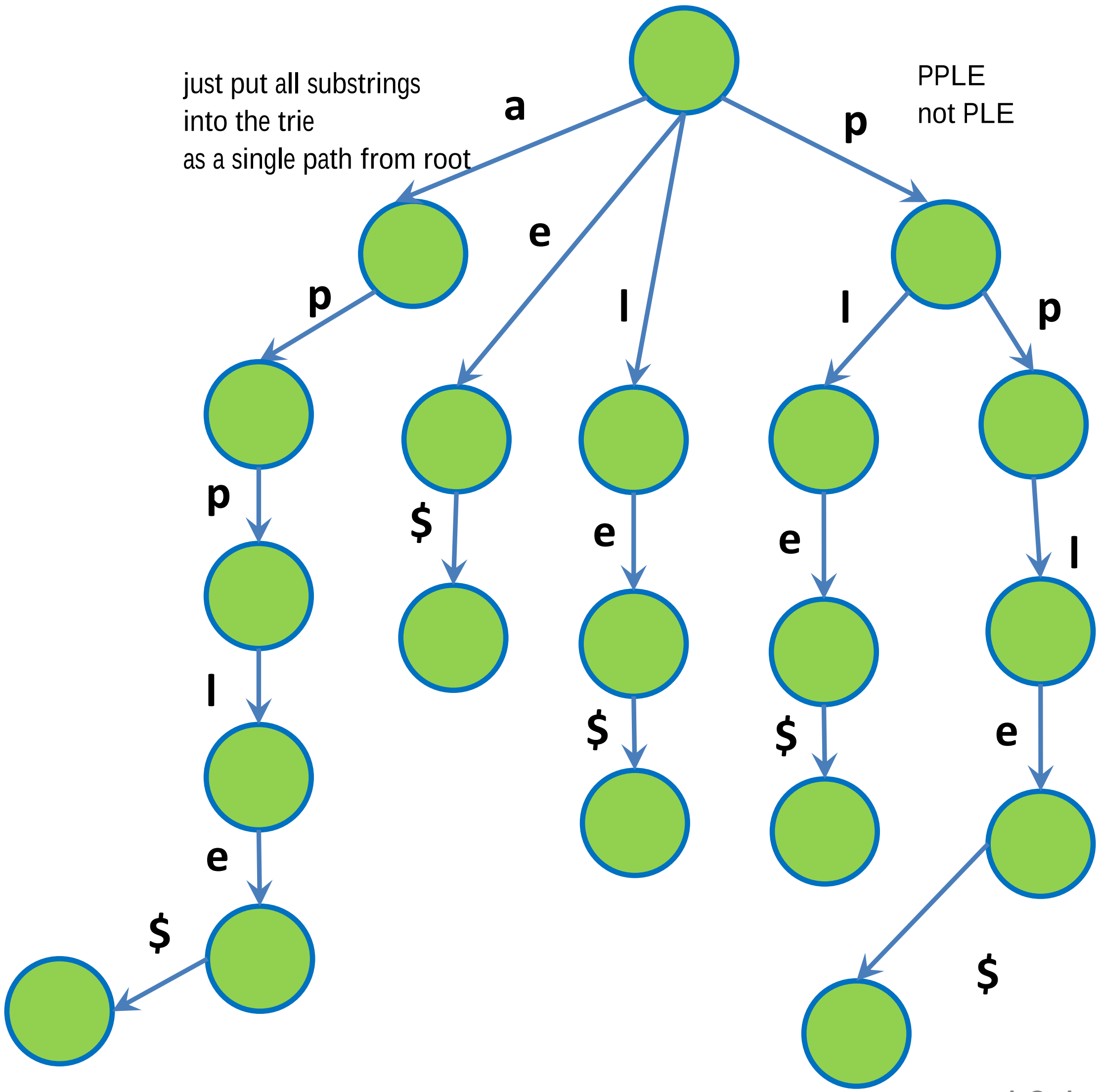

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Applications?

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Applications?

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  – $O(N^2)$

Suffix Trie

Applications?

- Space complexity?
  - O(N^2)
  - N suffixes, longest suffix is N character

Suffix Trie
Applications?

- Space complexity?
  - O(N^2)
  - N suffixes, longest suffix is N character
  - Have N number of leaves!

Questions?

- What is a suffix tree?

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Suffix Trie Suffix Tree

- What is a suffix tree?
  - Using our same example

Suffix Tree

A tree, not a trie

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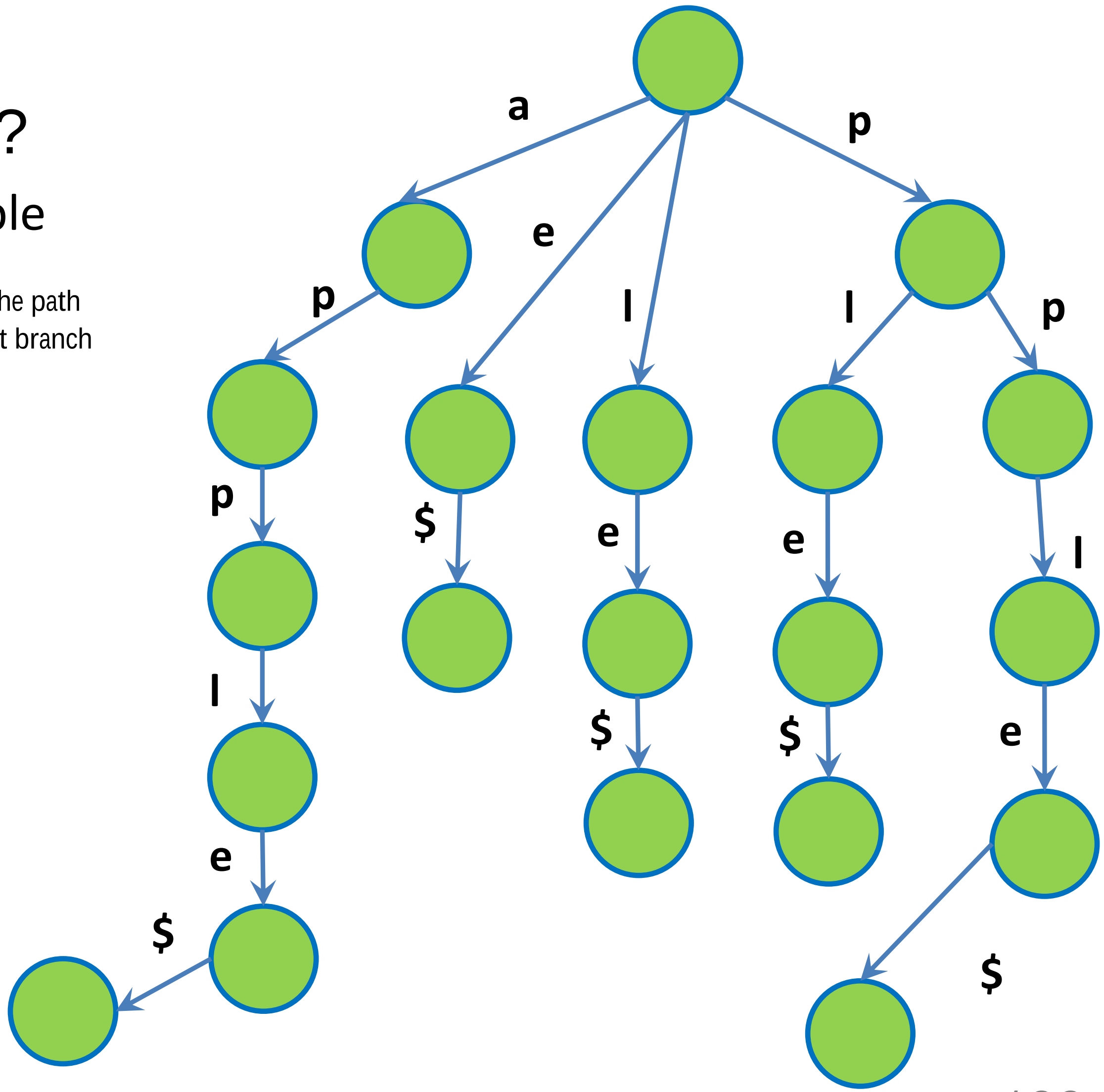
compress the path
that has not branch



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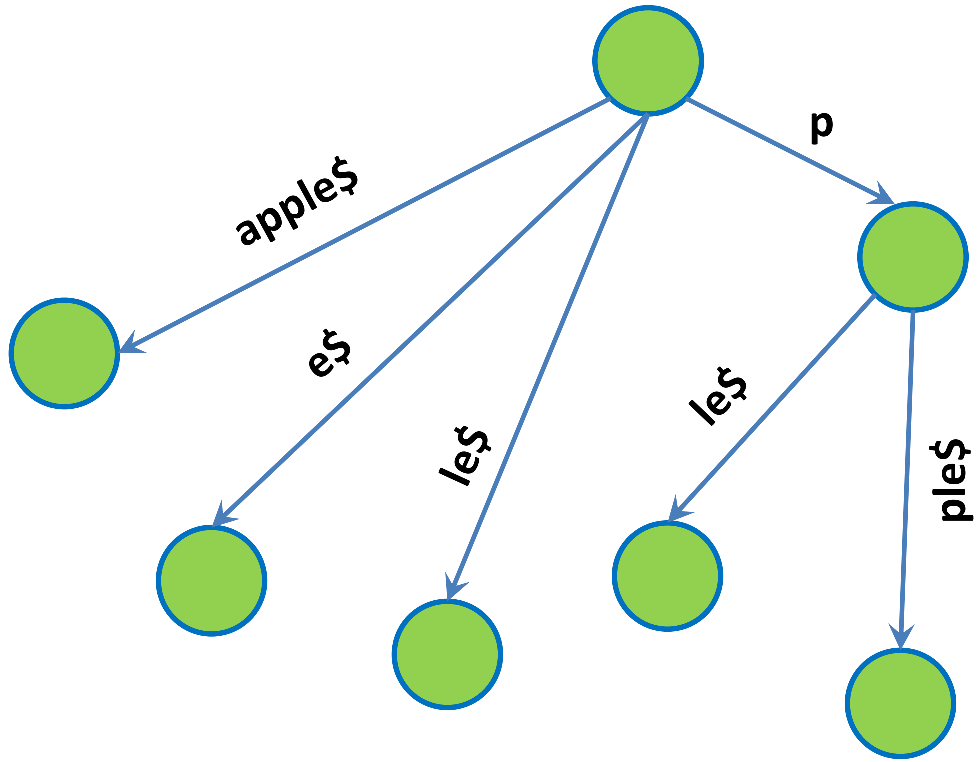

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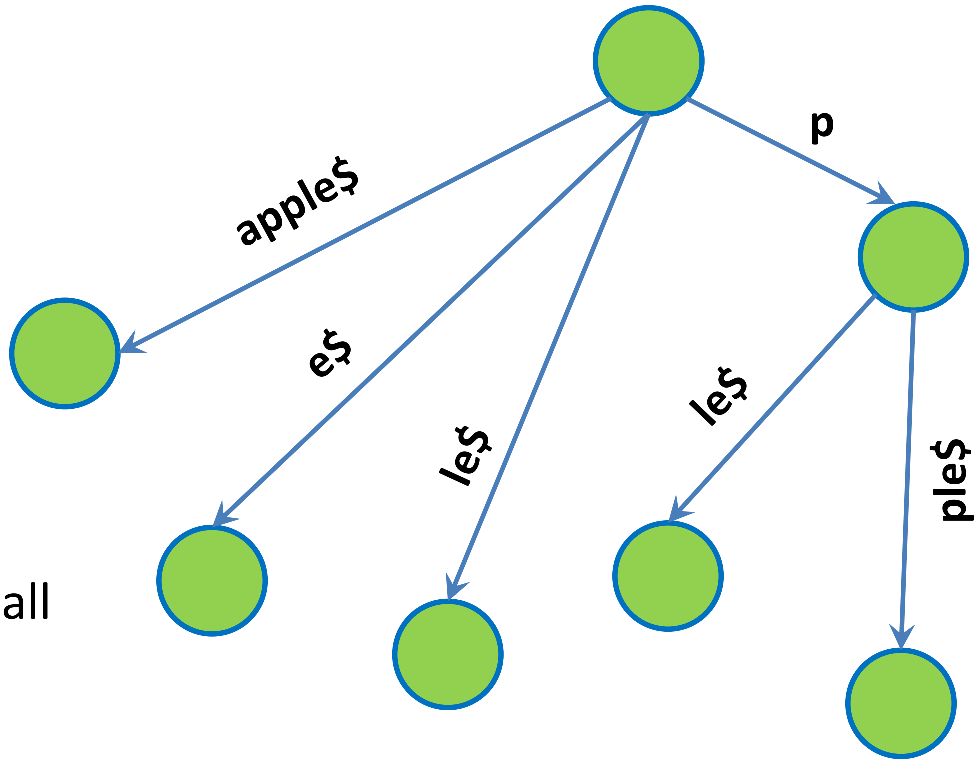

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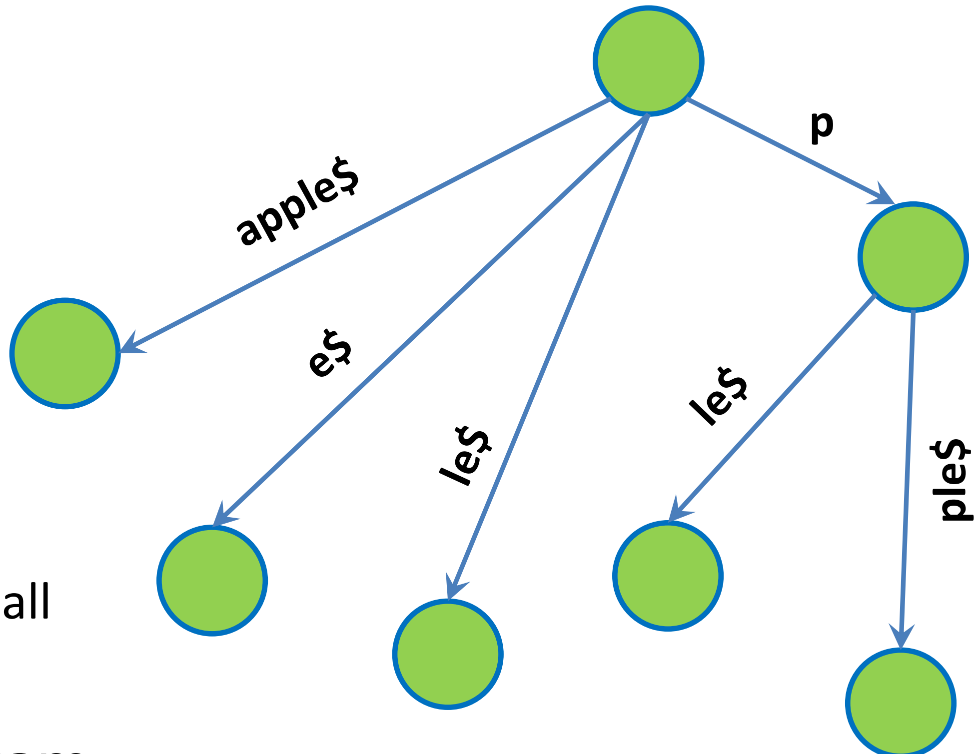

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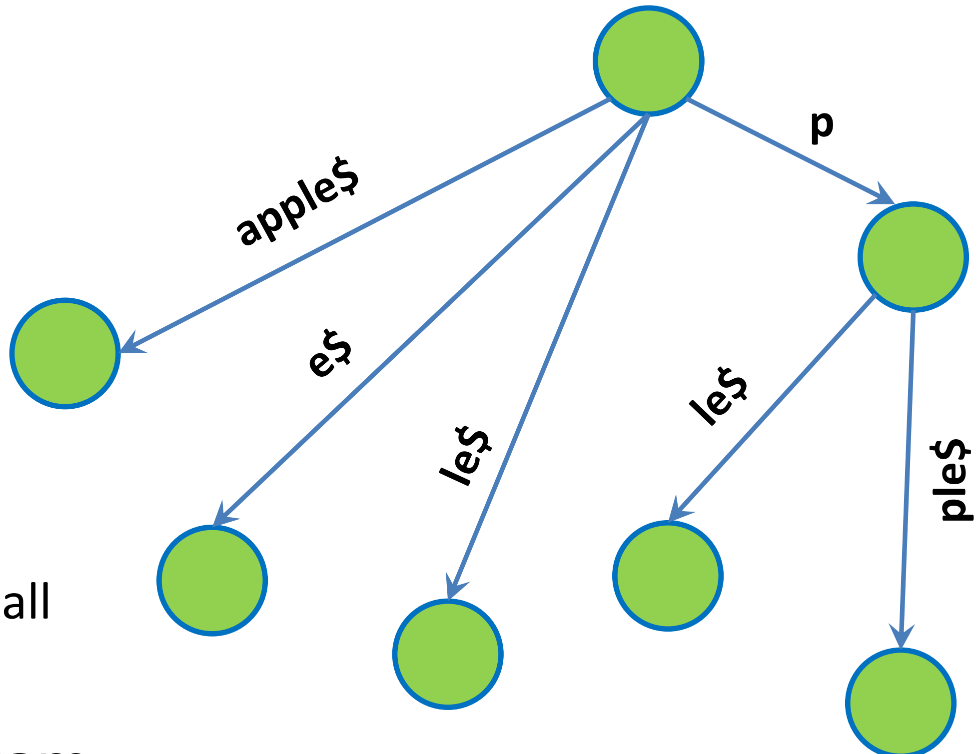
- Note: Some like to separate out the $ node



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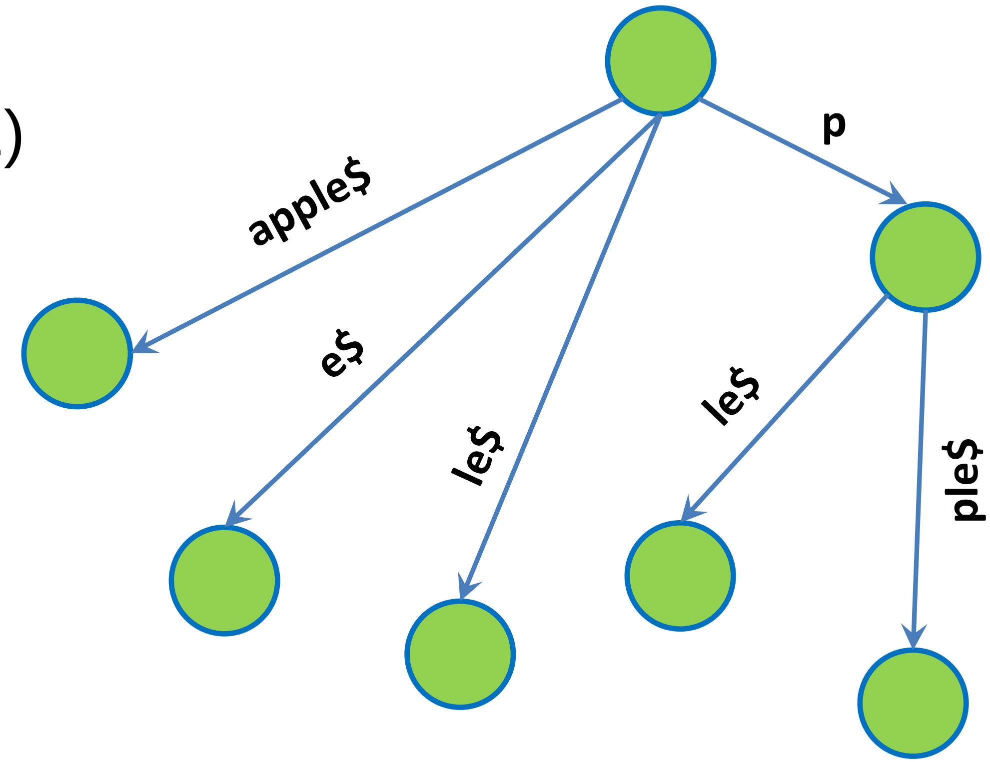

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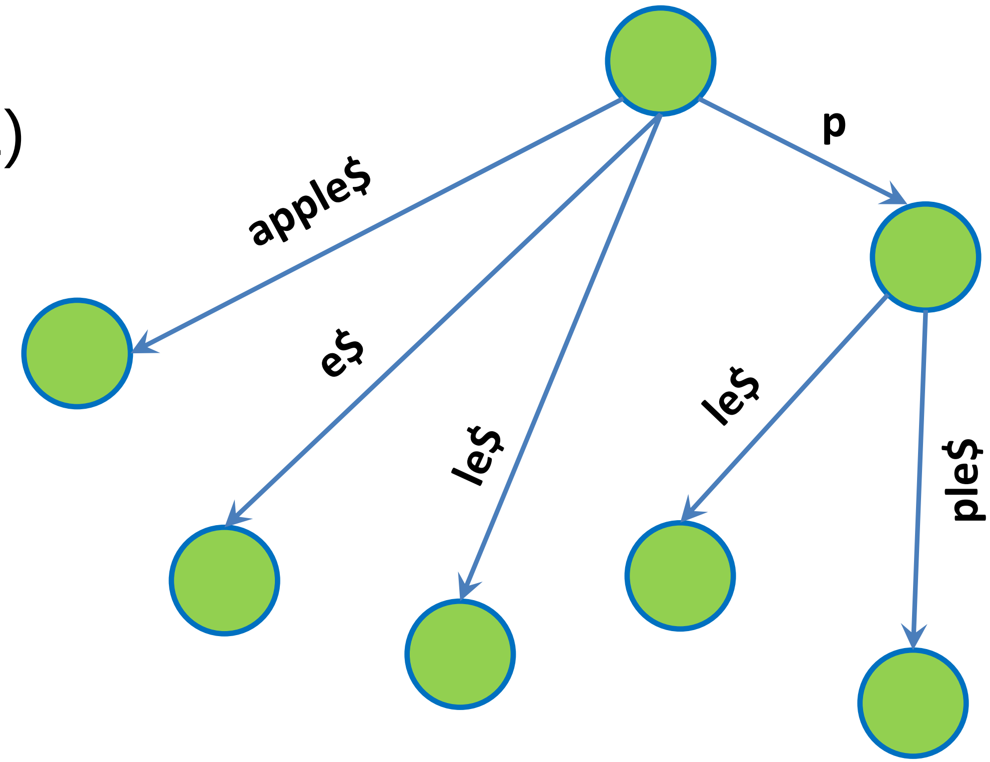

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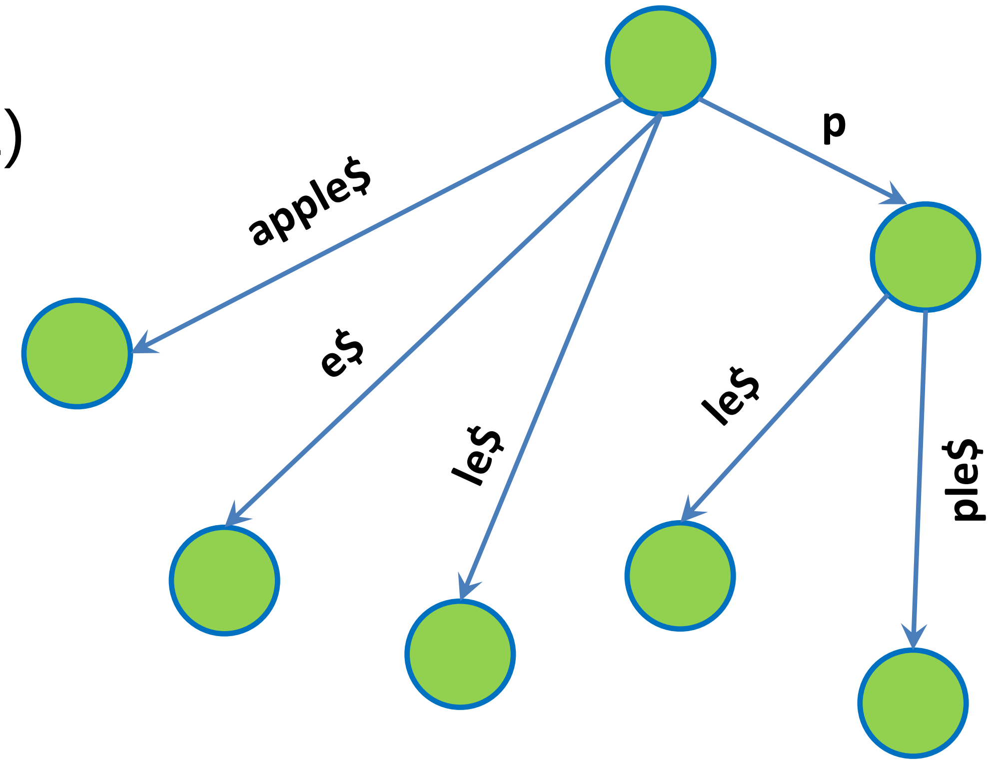

a	p	p	l	e	$
0	1	2	3	4	5

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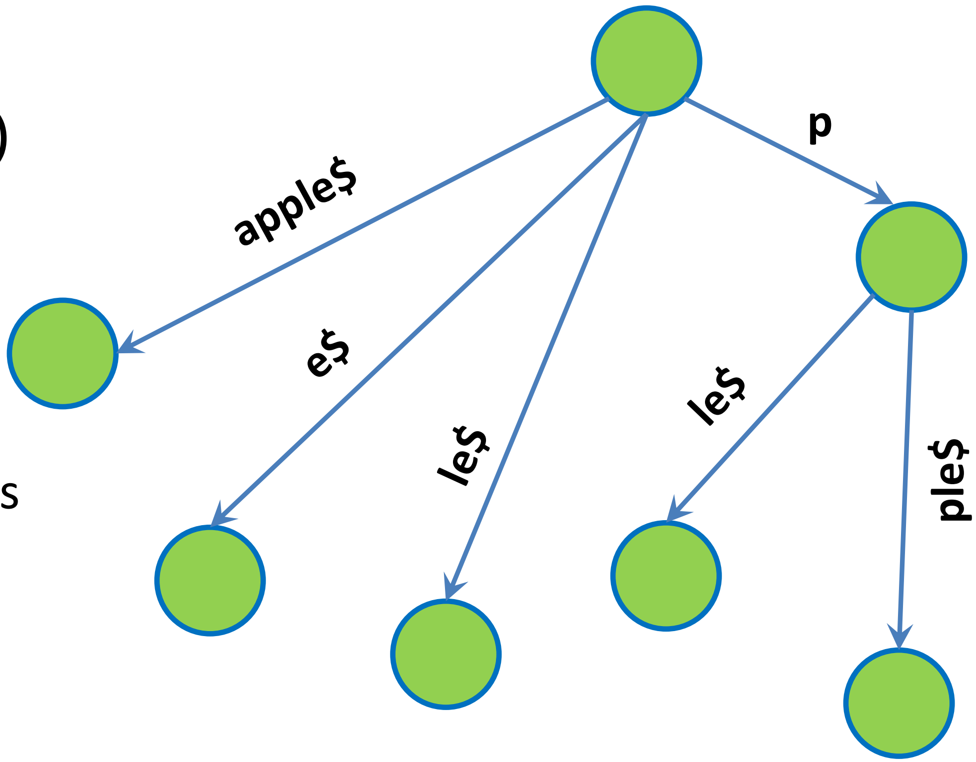

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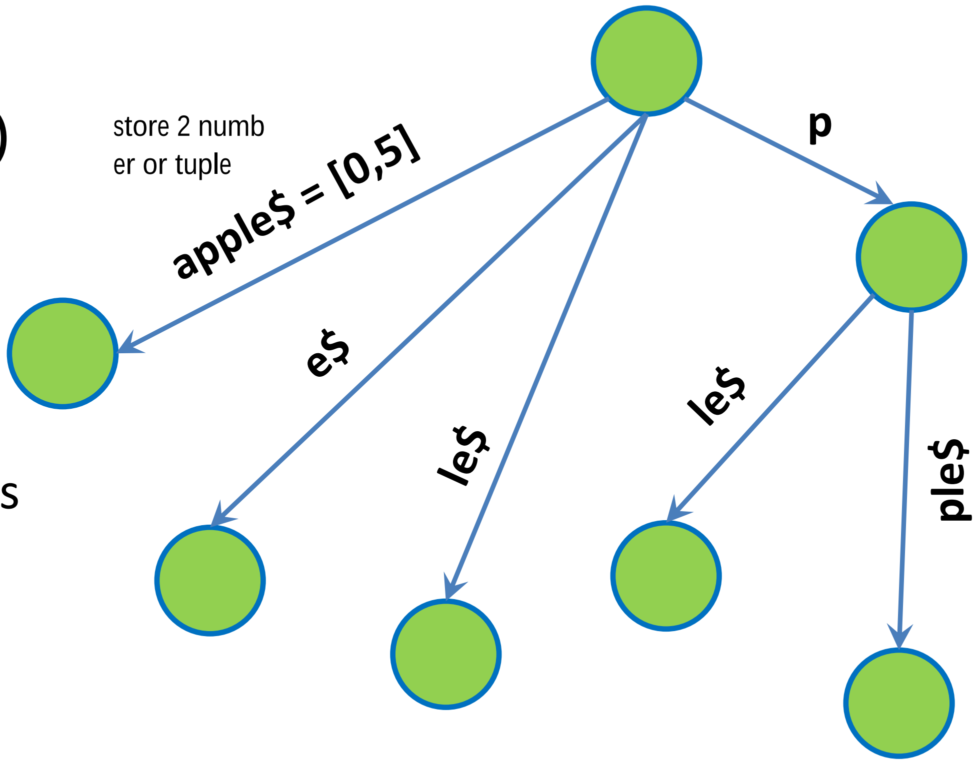

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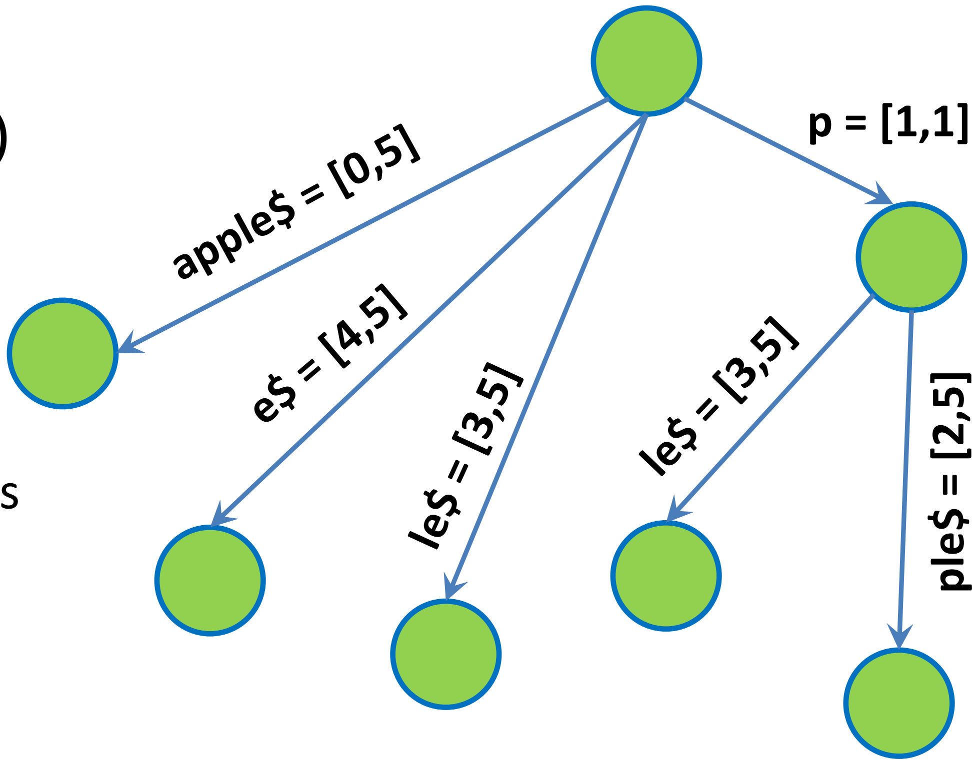

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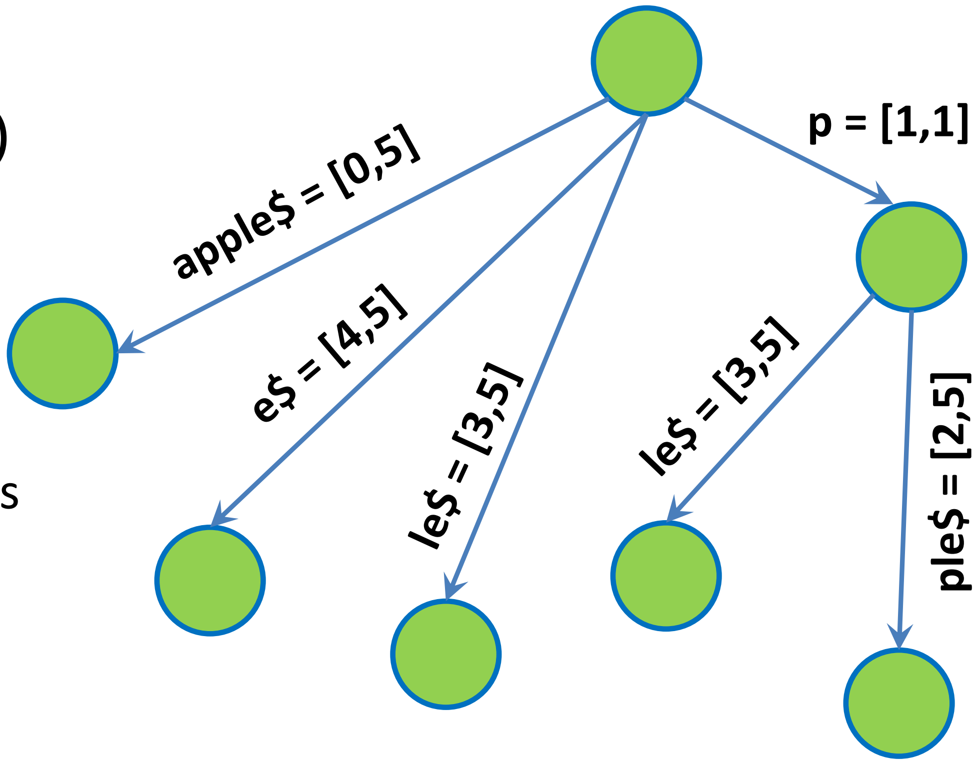

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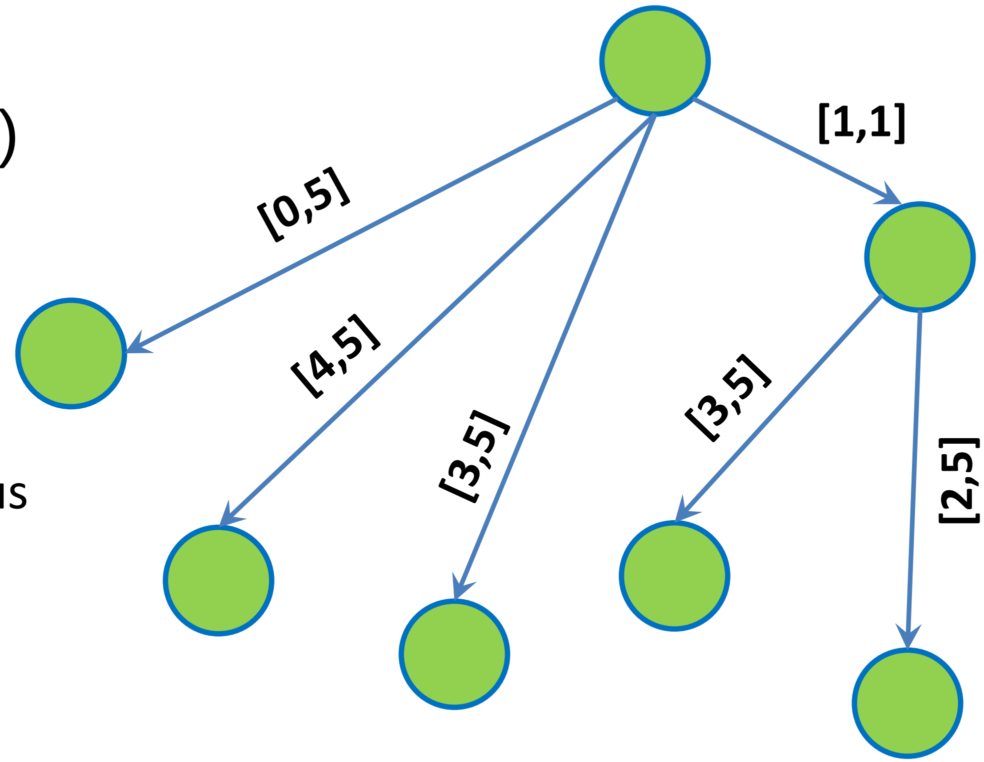

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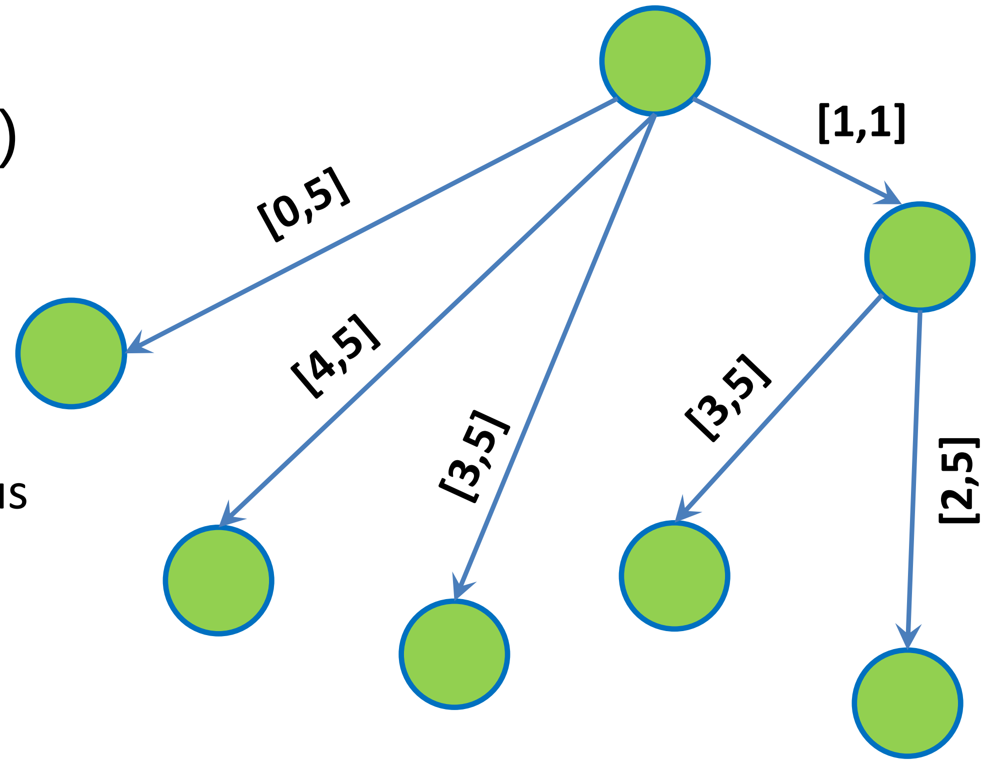

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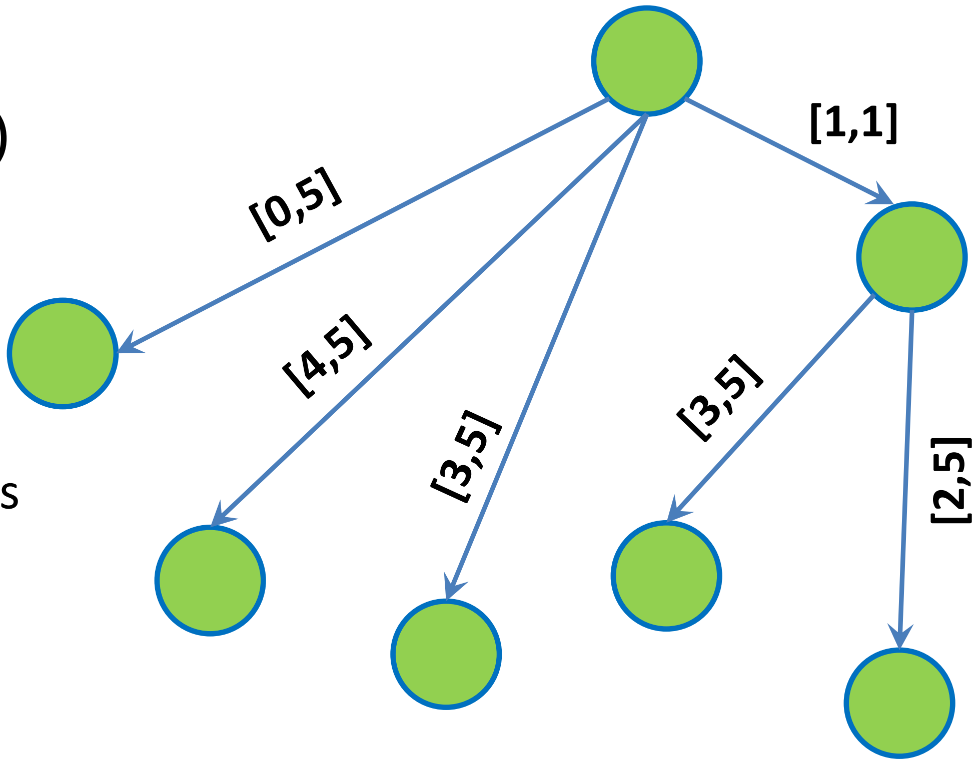

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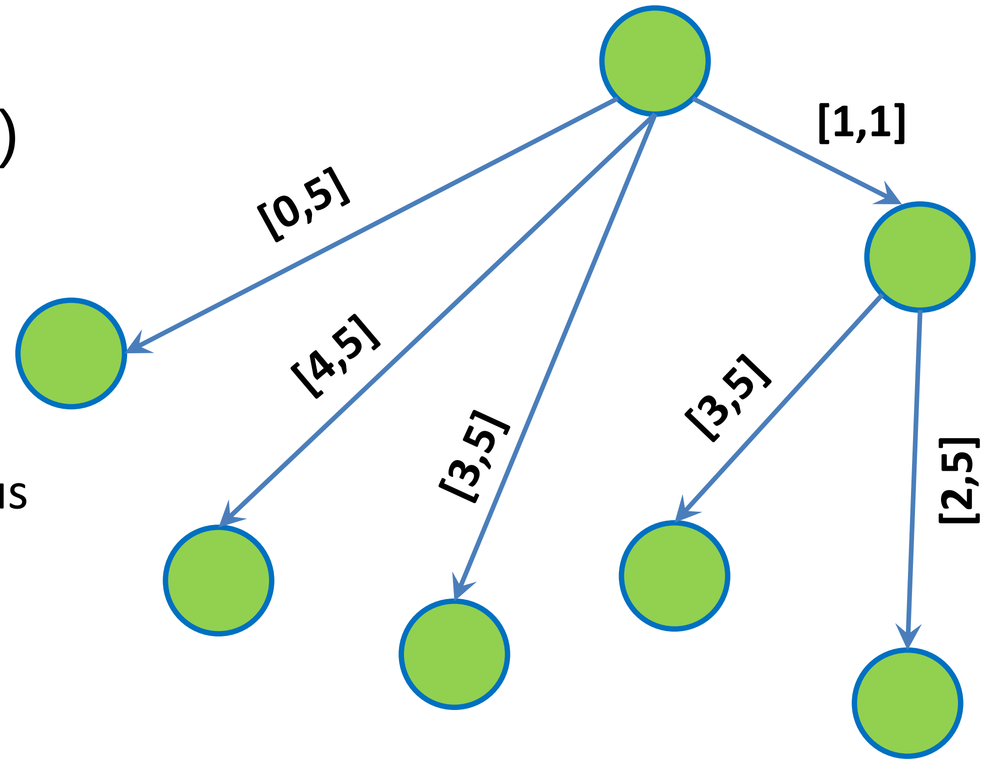

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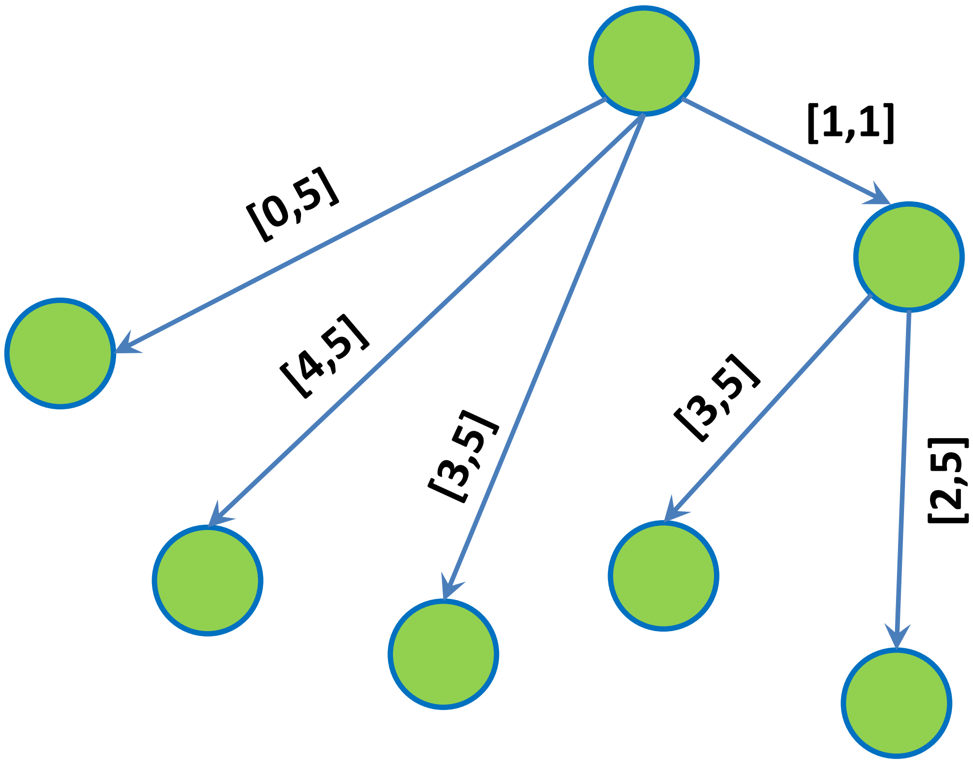

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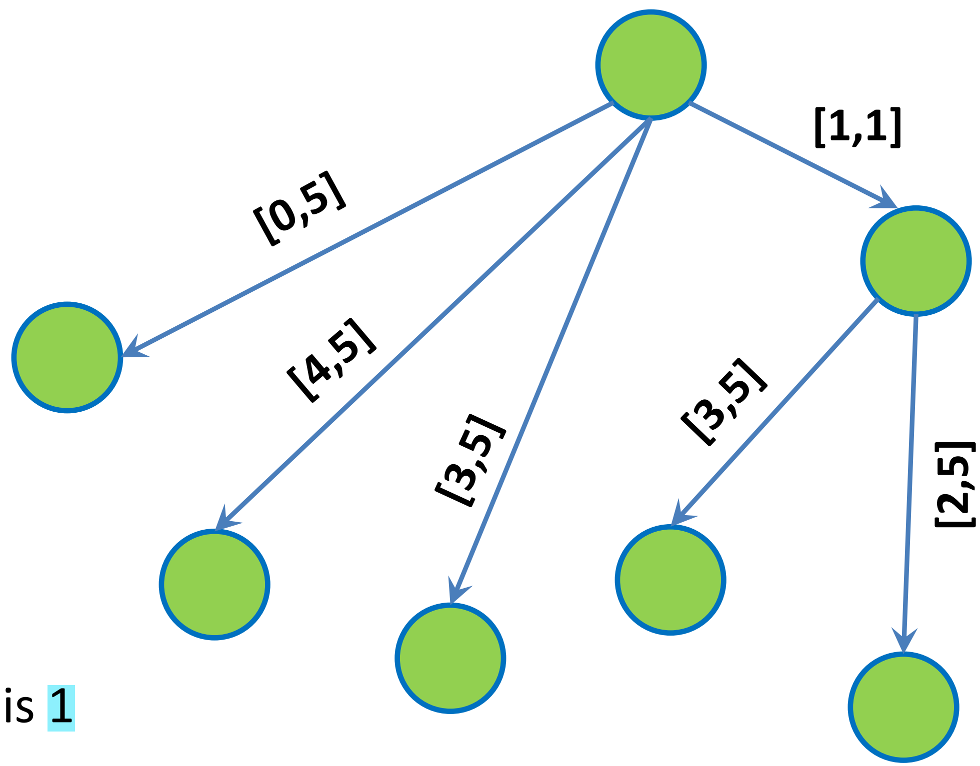
    * cause we go till root which is 1 from leaves



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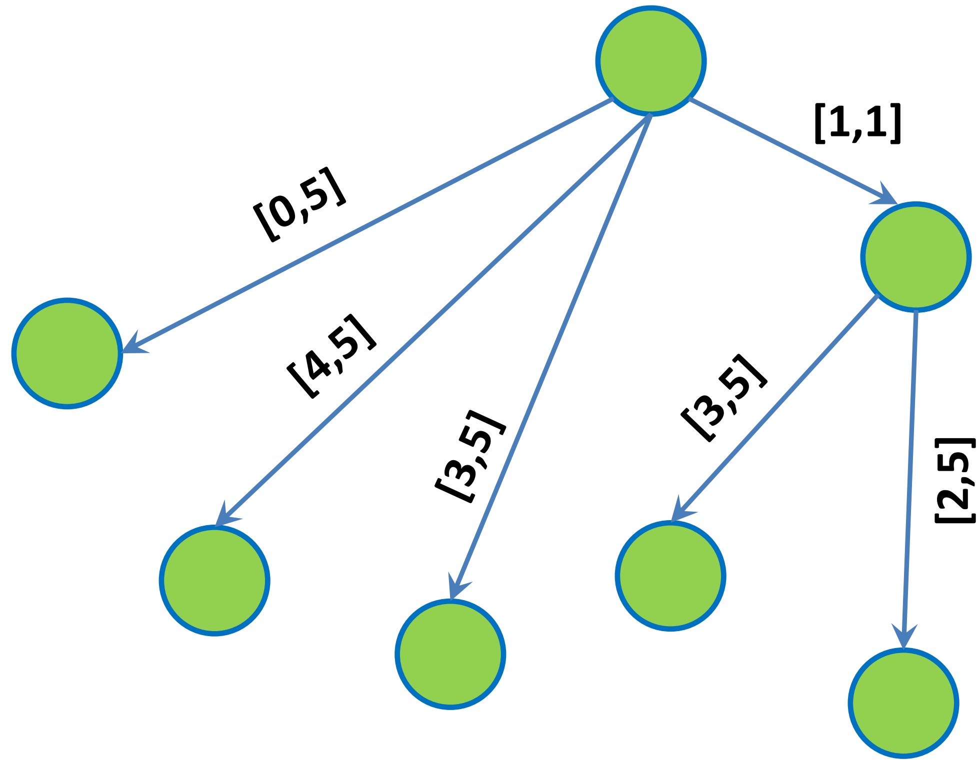

[i,j] or (i,j)

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We learn the hax called Ukkonen’s algorithm (1995) in FIT3155 to do in O(N)

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- … and some of the same functions

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- Let us try to implement it!

- As a class activity
- … and some of the same functions
  - Better than you searching online and not understanding what is happening
  - But 2 implementation
    - Iterative
    - Recursive (efficient)

Questions?

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Thank You