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INFORMATION
TECHNOLOGY

FIT2004
Algorithms and Data Structures

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Referencing materials by
Nathan Companez, Aamir Cheema, Arun Konagurthu and Lloyd Allison

GROUP OF EIGHT AUSTRALIA

Faculty of Information Technology, Monash University

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Ready?

Agenda

- String retrieval

Agenda

- String retrieval
- Suffix array

Agenda

- String retrieval
- Suffix array
  - Very useful for a number of tasks…
  - Very popular question

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Let us begin…

Suffix Array

Sorted suffixes

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- Let say you have a word

Suffix Array
Sorted suffixes

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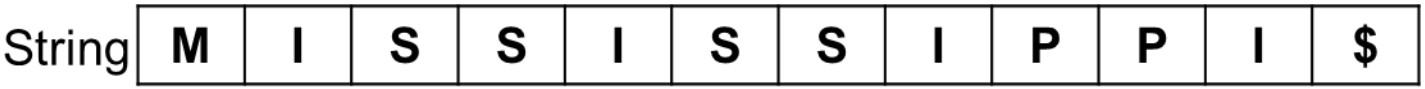
- Let say you have a word
  - List out the suffixes
  - Sort the suffixes

Suffix Array

Sorted suffixes



Suffix Array

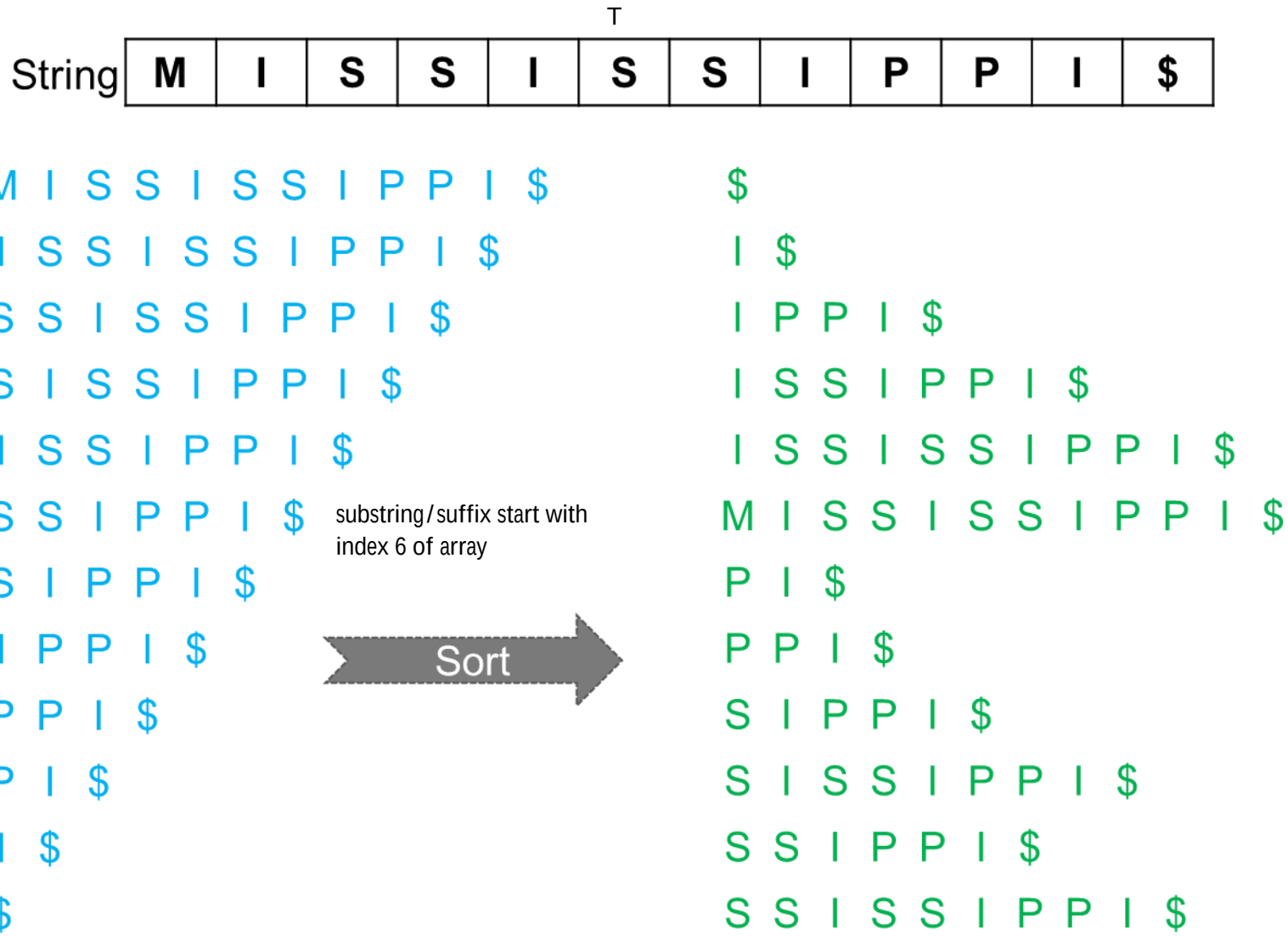
Sorted suffixes

String	M	I	S	S	I	S	S	I	P	P	I	$

1 M I S S I S S I P P I $
2 I S S I S S I P P I $
3 S S I S S I P P I $
4 S I S S I P P I $
5 I S S I P P I $
6 S S I P P I $
7 S I P P I $
8 I P P I $
9 P P I $
10 P I $
11 I $
12 $

Suffix Array

Sorted suffixes



Suffix Array

Sorted suffixes

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- Let say you have a word
  - List out the suffixes
  - Sort the suffixes
- What is the complexity of this?
  - N is the number of character

Suffix Array
Sorted suffixes

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- Let say you have a word
  - List out the suffixes
  - Sort the suffixes

- What is the complexity of this?
  - N is the number of character
  - O(N^2) to generate the suffixes
  - O(N^2 log N) to sort the suffixes
    - Note O(N) for string comparison

k * N log N k = N (comparison of the string) N log N is merge sort

quick sort or merge sort

Suffix Array
Sorted suffixes

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- Let say you have a word
  - List out the suffixes
  - Sort the suffixes
- What is the complexity of this?
  - N is the number of character
  - O(N^2) to generate the suffixes
  - O(N^2 log N) to sort the suffixes with merge sort
    - Note O(N) for string comparison

Suffix Array
Sorted suffixes

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- Let say you have a word
  - List out the suffixes
  - Sort the suffixes
- What is the complexity of this?
  - N is the number of character
  - O(N^2) to generate the suffixes
  - O(N^2 log N) to sort the suffixes with merge sort
    - Note O(N) for string comparison
    - We can reduce this to O(N^2) with radix sort
      - N passes (columns)

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Questions?

Suffix Array
Sorted suffixes

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- Usage?

Suffix Array
Sorted suffixes

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- Usage?
  - Search for substring
    - With binary search!

Suffix Array
Sorted suffixes

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- Usage?
  - Search for substring
    - With binary search!
    - Search for IPP

Suffix Array

Sorted suffixes

- Usage?
  - Search for substring (prefix of subfix)
    - With binary search!
    - Search for IPP

binary search (since it is in-order) on first character
then the second
until the end of prompt
then find whether there is terminal node

Suffix Array

Sorted suffixes

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- Usage?
  - Search for substring
    - With binary search!
    - Search for IPP
    - Complexity?
      - M is length of pattern

Suffix Array

Sorted suffixes

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- Usage?
  - Search for substring
    - With binary search!
    - Search for IPP O(M)
    - Complexity?
      - M is length of pattern
      - O(M log N) N for the original string length

Suffix Array
Sorted suffixes

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- Usage?
  - Search for substring
    - With binary search!
    - Search for IPP
    - Complexity?
      - M is length of pattern
      - O(M log N)
  - Finding longest repeated substring
    - Since it is in order, we can just compare one suffix with the next

Suffix Array
Sorted suffixes

- Usage?
  - Search for substring
    - With binary search!
    - Search for IPP
    - Complexity?
      - M is length of pattern
      - O(M log N)
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Suffix Array

Sorted suffixes

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- Usage?
  - Search for substring
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    - Search for IPP
    - Complexity
      - M is length of pattern
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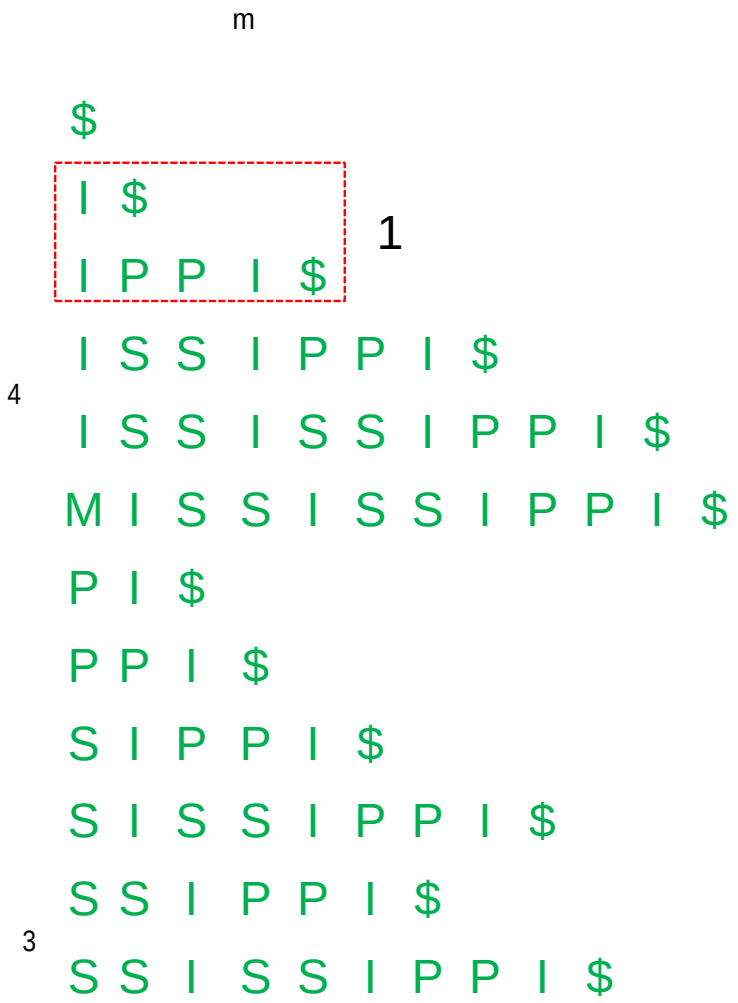

Suffix Array
Sorted suffixes

- Usage?
  - Search for substring
    - With binary search!
    - Search for IPP
    - Complexity?
      - M is length of pattern
      - O(M log N)
  - Finding longest repeated substring
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Suffix Array

Sorted suffixes

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- Usage?
  - Search for substring
    - With binary search!
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    - Complexity?
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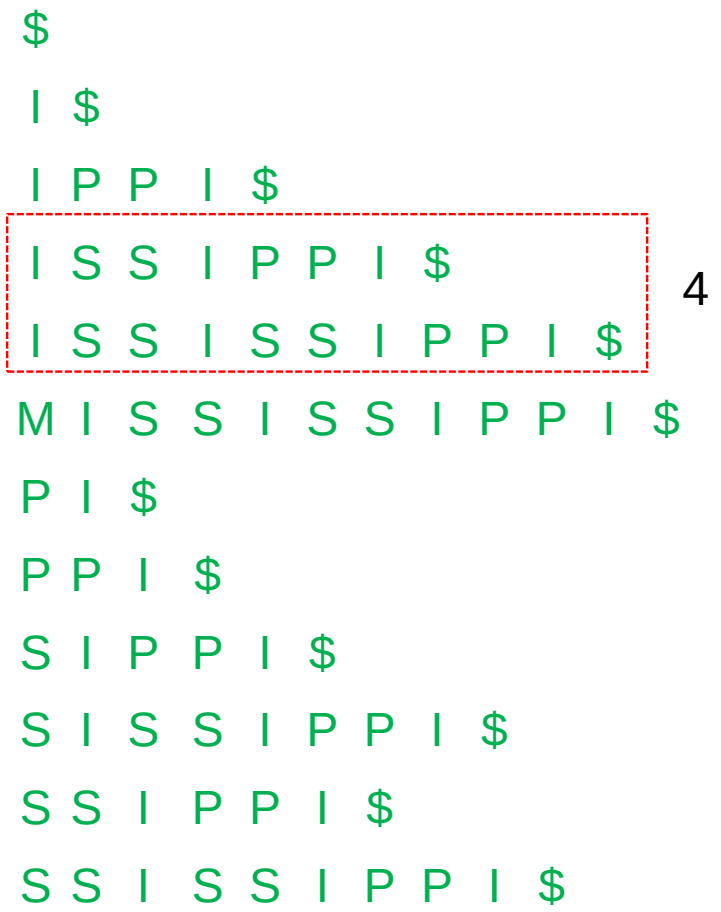

Suffix Array
Sorted suffixes

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- Usage?
  - Search for substring
    - With binary search!
    - Search for IPP
    - Complexity?
      - M is length of pattern
      - O(M log N)
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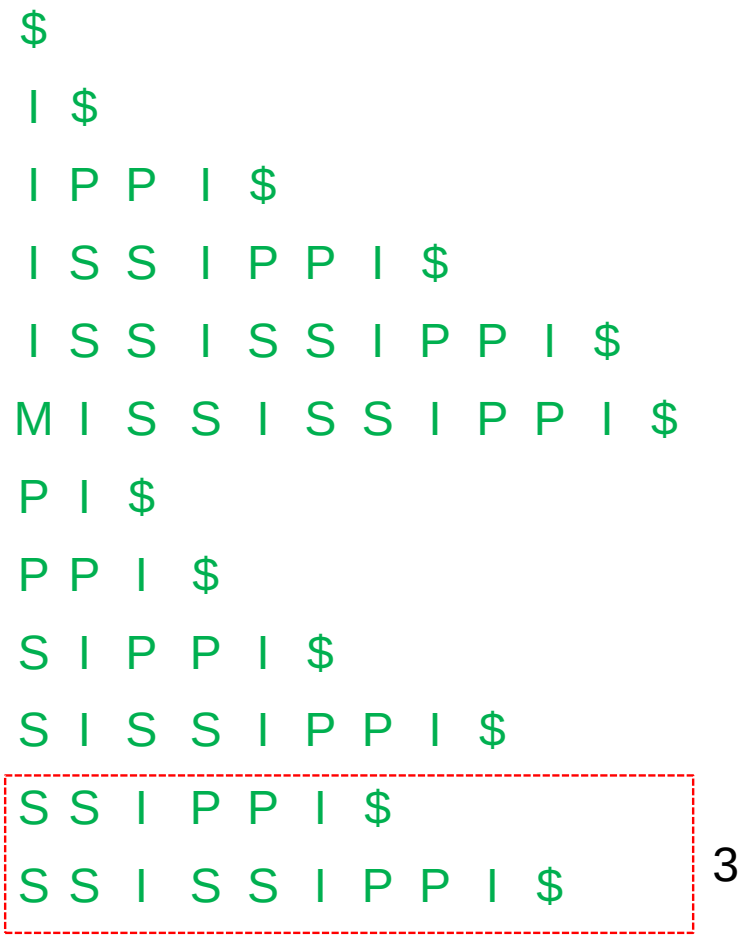
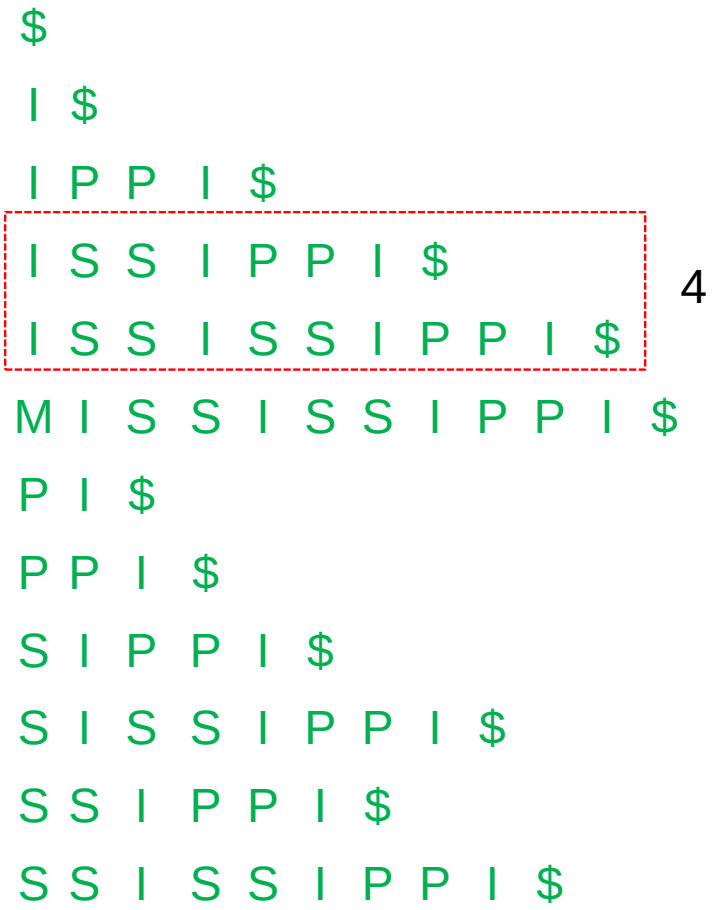


Suffix Array
Sorted suffixes

- Usage?
  - Search for substring
    - With binary search!
    - Search for IPP
    - Complexity?
      - M is length of pattern
      - O(M log N)
  - Finding longest repeated substring
    - Since it is in order, we can just compare one suffix with the next
    - This is the longest with length 4
      - ISSI
    - Complexity?



Suffix Array

Sorted suffixes

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- Usage?
  - Search for substring
    - With binary search!
    - Search for IPP
    - Complexity?
      - M is length of pattern
      - O(M log N)
  - Finding longest repeated substring
    - Since it is in order, we can just compare one suffix with the next
    - This is the longest with length 4
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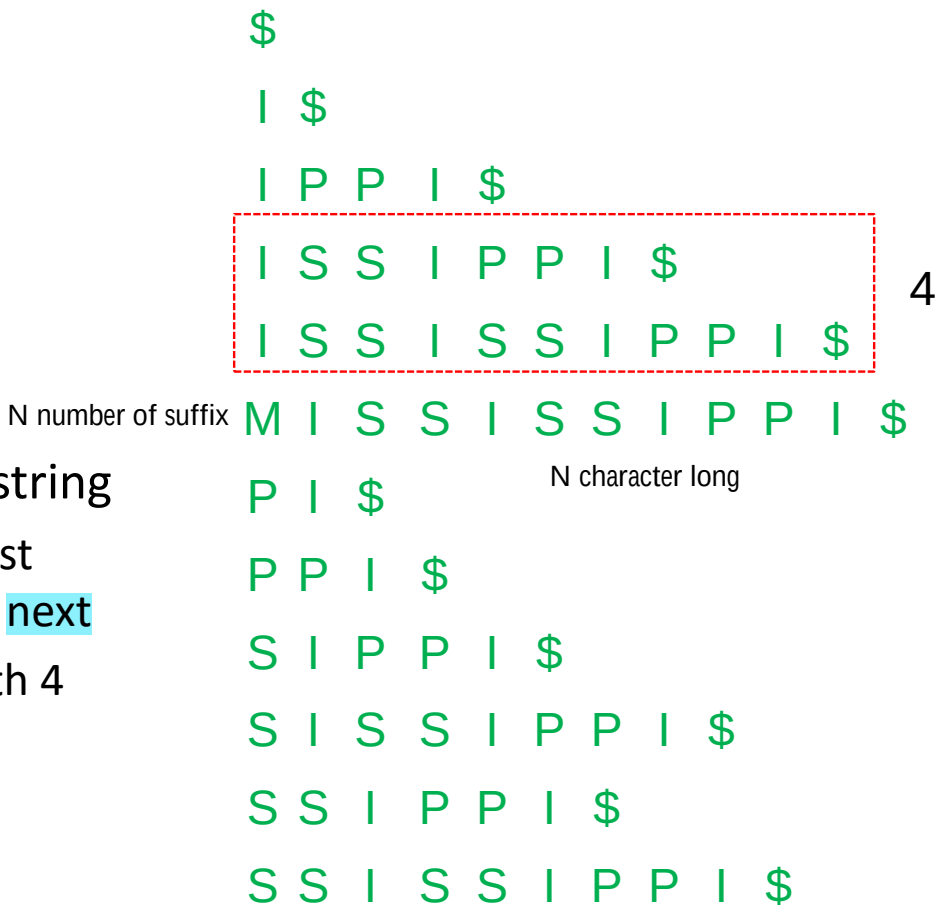
    - Complexity?
      - O(N^2)
      - Due to O(N) for char comparison



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Questions?

Suffix Array
Sorted suffixes

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- Space complexity?

Suffix Array
Sorted suffixes

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- Space complexity?
  - O(N^2)

Suffix Array
Sorted suffixes

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- Space complexity?
  - O(N^2)
  - But can we do better?

Suffix Array

Sorted suffixes

index	1	2	3	4	5	6	7	8	9	10	11	12
String	M	I	S	S	I	S	S	I	P	P	I	$

Suffix Array

Sorted suffixes

index	1	2	3	4	5	6	7	8	9	10	11	12
String	M	I	S	S	I	S	S	I	P	P	I	$

1 M I S S I S S I P P I $
2 I S S I S S I P P I $
3 S S I S S I P P I $
4 S I S S I P P I $
5 I S S I P P I $
6 S S I P P I $
7 S I P P I $
8 I P P I $
9 P P I $
10 P I $
11 I $
12 $

Sort

12	$
11	I $
8	I P P I $
5	I S S I P P I $
2	I S S I S S I P P I $
1	M I S S I S S I P P I $
10	P I $
9	P P I $
7	S I P P I $
4	S I S S I P P I $
6	S S I P P I $
3	S S I S S I P P I $

Suffix Array:
Only stores IDs of suffixes.
The sorted suffices are shown just for illustration

Suffix Array

Sorted suffixes

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index	1	2	3	4	5	6	7	8	9	10	11	12
String	M	I	S	S	I	S	S	I	P	P	I	$

Suffix ID

1 M I S S I S S I P P I $
2 I S S I S S I P P I $
3 S S I S S I P P I $
4 S I S S I P P I $
5 I S S I P P I $
6 S S I P P I $
7 S I P P I $
8 I P P I $
9 P P I $
10 P I $
11 I $
12 $

Sort

Use rank to find

12	$
11	I $
8	I P P I $
5	I S S I P P I $
2	I S S I S S I P P I $
1	M I S S I S S I P P I $
10	P I $
9	P P I $
7	S I P P I $
4	S I S S I P P I $
6	S S I P P I $
3	S S I S S I P P I $

Suffix Array:
Only stores IDs of suffixes.
The sorted suffices are shown just for illustration

Suffix Array
Sorted suffixes

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- Space complexity?
  - O(N^2)
  - But can we do better?
  - O(N) if we use suffix ID and store the suffix ID (in order) only

Suffix Array
Sorted suffixes

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- Space complexity?
  - O(N^2)
  - But can we do better?
  - O(N) if we use suffix ID and store the suffix ID (in order) only
    - With suffix ID, we know where the suffix start
    - We can get the suffixes by doing suffix = string[id:]

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Questions?

Suffix Array

Sorted suffixes

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- Time complexity is O(N^2) with radix sort
- Space complexity is O(N) with suffix ID

Suffix Array

Sorted suffixes

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- Time complexity is O(N^2) with radix sort
  - Can we do better?
- Space complexity is O(N) with suffix ID

Suffix Array

Sorted suffixes

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- Time complexity is O(N^2) with radix sort
  - Can we do better?
  - Yes with prefix doubling!
- Space complexity is O(N) with suffix ID

Suffix Array
With prefix doubling

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- Let us see the example now

Constructing Suffix Array: Prefix Doubling

	1	2	3	4	5	6	7	8	9	10	11	12
String	M	I	S	S	I	S	S	I	P	P	I	$

Basic Idea:

- Generate suffixes
- Sort suffixes on their 1st characters

Rank	ID	
-	1	M I S S I S S I P P I $
-	2	I S S I S S I P P I $
-	3	S S I S S I P P I $
-	4	S I S S I P P I $
-	5	I S S I P P I $
-	6	S S I P P I $
-	7	S I P P I $
-	8	I P P I $
-	9	P P I $
-	10	P I $
-	11	I $
-	12	$

Constructing Suffix Array: Prefix Doubling

	1	2	3	4	5	6	7	8	9	10	11	12
String	M	I	S	S	I	S	S	I	P	P	I	$

Basic Idea:

- Generate suffixes
- Sort suffixes on their 1st characters
- Sort suffixes on first 2 characters

Rank	ID		
1	12	$	use radix sort to sort lexicographically at the same column get the Rank
2	2	I	S S I S S I P P I $
2	5	I	S S I P P I $
2	8	I	P P I $
2	11	I	$
6	1	M	I S S I S S I P P I $
7	9	P	P I $
7	10	P	I $
9	3	S	S I S S I P P I $
9	4	S	I S S I P P I $
9	6	S	S I P P I $
9	7	S	I P P I $

Constructing Suffix Array: Prefix Doubling

	1	2	3	4	5	6	7	8	9	10	11	12
String	M	I	S	S	I	S	S	I	P	P	I	$

Basic Idea:

- Generate suffixes
- Sort suffixes on their 1st characters
- Sort suffixes on first 2 characters
- Sort suffixes on first 4 characters

Rank	ID	
1	12	$
2	11	I $
3	8	I P P I $
4	2	I S S I S S I P P I $
4	5	I S S I P P I $
6	1	M I S S I S S I P P I $
7	10	P I $
8	9	P P I $
9	4	S I S S I P P I $
9	7	S I P P I $
11	3	S S I S S I P P I $
11	6	S S I P P I $

Constructing Suffix Array: Prefix Doubling

	1	2	3	4	5	6	7	8	9	10	11	12
String	M	I	S	S	I	S	S	I	P	P	I	$

Basic Idea:

- Generate suffixes
- Sort suffixes on their 1st characters
- Sort suffixes on first 2 characters
- Sort suffixes on first 4 characters
- …

Rank	ID	
1	12	$
2	11	I $
3	8	I P P I $
4	2	I S S I S S I P P I $
4	5	I S S I P P I $
6	1	M I S S I S S I P P I $
7	10	P I $
8	9	P P I $
9	7	S I P P I $
10	4	S I S S I P P I $
11	6	S S I P P I $
12	3	S S I S S I P P I $

Constructing Suffix Array: Prefix Doubling

	1	2	3	4	5	6	7	8	9	10	11	12
String	M	I	S	S	I	S	S	I	P	P	I	$

Basic Idea:

- Generate suffixes
- Sort suffixes on their 1st characters
- Sort suffixes on first 2 characters
- Sort suffixes on first 4 characters
- …
- Sort suffixes on all characters

Rank	ID	
1	12	$
2	11	I $
3	8	I P P I $
4	5	I S S I P P I $
5	2	I S S I S S I P P I $
6	1	M I S S I S S I P P I $
7	10	P I $
8	9	P P I $
9	7	S I P P I $
10	4	S I S S I P P I $
11	6	S S I P P I $
12	3	S S I S S I P P I $

Constructing Suffix Array: Prefix Doubling

	1	2	3	4	5	6	7	8	9	10	11	12
String	M	I	S	S	I	S	S	I	P	P	I	$

Basic Idea:

- Generate suffixes
- Sort suffixes on their 1st characters
- Sort suffixes on first 2 characters
- Sort suffixes on first 4 characters
- …
- Sort suffixes on all N characters

Time complexity:

- Cost of first sort (1 character)
  - 1.N log N
- Cost of second sort (2 characters)
  - 2.N log N
- Cost of i-th sort (2^{i-1} characters)
  - 2^{i-1} N log N
- Total cost:
- NlogN+2NlogN+4NlogN+…+N.NlogN
- (1 + 2 + 4 +… + N/2 + N)* N log N
  - (N+N/2 +N/4 + … + 1) → O(N)
- Total cost is still $O(N^2 \log N)$

Rank	ID
1	12
2	11
3	8
4	5
5	2
6	1
7	10
8	9
9	7
10	4
11	6
12	3

$
I $
I P P I $
I S S I P P I $
I S S I S S I P P I $
M I S S I S S I P P I $
P I $
P P I $
S I P P I $
S I S S I P P I $
S S I P P I $
S S I S S I P P I $

Suffix Array
With prefix doubling

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- Time complexity is O(N^2) with radix sort
  - Can we do better?
  - Yes with prefix doubling! it is useless
  - But complexity still the same O(N^2 log N), even slower than radix sort
- Space complexity is O(N) with suffix ID

Suffix Array
With prefix doubling

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- Time complexity is O(N^2) with radix sort
  - Can we do better?
  - Yes with prefix doubling!
  - But complexity still the same O(N^2 log N), even slower than radix sort
    - Due to the O(N) comparison
- Space complexity is O(N) with suffix ID

Suffix Array
With prefix doubling

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- Time complexity is O(N^2) with radix sort
  - Can we do better?
  - Yes with prefix doubling!
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Suffix Array
With prefix doubling

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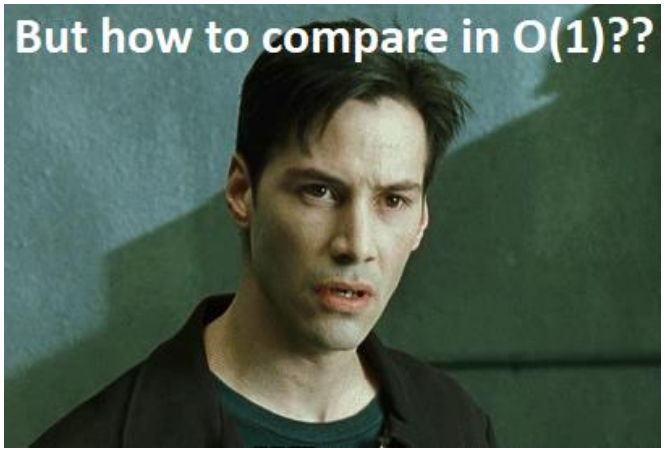

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Questions?

Suffix Array

With prefix doubling with O(1) comparison

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- Time complexity is O(N^2) with radix sort
  - Can we do better?
  - Yes with prefix doubling!
  - But complexity still the same O(N^2 log N), even slower than radix sort
    - Due to the O(N) comparison
    - We use the rank table to get O(1) comparison!
- Space complexity is O(N) with suffix ID

Constructing Suffix Array: Prefix Doubling

	1	2	3	4	5	6	7	8	9	10	11	12
String	M	I	S	S	I	S	S	I	P	P	I	$

Basic Idea:

- Generate suffixes
- Sort suffixes on their 1st characters
- Sort suffixes on first 2 characters
- Sort suffixes on first 4 characters
- …
- Sort suffixes on all N characters

- **Suppose we could compare in O(1)**

logN time to sort

- logN sorts 1 -> 2 -> 4 -> 8
- O(NlogN) for each binary sort for N length
- O(NlogNlogN) = O(Nlog^{2}N)

Rank	ID
1	12
2	11
3	8
4	5
5	2
6	1
7	10
8	9
9	7
10	4
11	6
12	3

$
I $
I P P I $
I S S I P P I $
I S S I S S I P P I $
M I S S I S S I P P I $
P I $
P P I $
S I P P I $
S I S S I P P I $
S S I P P I $
S S I S S I P P I $

O(1) Comparison

	1	2	3	4	5	6	7	8	9	10	11	12
String	M	I	S	S	I	S	S	I	P	P	I	$

Comparing suffixes in O(1):

- Suppose already sorted on first k characters (2 in this example)
- Now sorting on 2k characters (4 in this example)

Observation 1:

- If current ranks are different, suffix with smaller rank is smaller (because its first k characters are smaller)
  - E.g., PPI$ < SSIP
  - Note comparison cost is O(1) By rank

	Rank	ID	
	1	12	$
	2	11	I $
	3	8	I P P I $
	4	2	I S S I S S I P P I $
	4	5	I S S I P P I $
	6	1	M I S S I S S I P P I $
	7	10	P I $
	8	9	P P I $
	9	4	S I S S I P P I $
	9	7	S I P P I $
11	11	3	S S I S S I P P I $
11	11	6	S S I P P I $

O(1) Comparison

	1	2	3	4	5	6	7	8	9	10	11	12
String	M	I	S	S	I	S	S	I	P	P	I	$

Observation 2:

If current ranks are the same

- First k characters must be the same
- The tie is to be broken on the next k characters, e.g.,

Rank	ID		k = 2, k characters that were sorted already
1	12	$	
2	11	I $	
3	8	I P P I $	
4	2	I S S I S S I P P I $	
4	5	I S S I P P I $	
6	1	M I S S I S S I P P I $	
7	10	P I $	
8	9	P P I $	
9	4	S I S S I P P I $	
9	7	S I P P I $	
11	3	S S I S S I P P I $	
11	6	S S I P P I $	

for first two character rank(4) = 9
to compare
need to get the 2 characters behind (prefix doubling)
substring = prefix of subfix string
if rank(4) = rank(7)
9 = 9
then compare rank(4 + k) ?= rank (7+k)
11 ?= 8
11 < 8
ID 7 before ID 4
since ID 4 + k is substring of ID 4
sine previous two characters were already shown

O(1) Comparison

	1	2	3	4	5	6	7	8	9	10	11	12
String	M	I	S	S	I	S	S	I	P	P	I	$

Observation 2:

If current ranks are the same

- First k characters must be the same
- The tie is to be broken on the next k characters, e.g.,
  - We need to compare “SSIPPI$” and “PPI$” on the first 2 characters

Rank	ID	
1	12	$
2	11	I $
3	8	I P P I $
4	2	I S S I S S I P P I $
4	5	I S S I P P I $
6	1	M I S S I S S I P P I $
7	10	P I $
8	9	P P I $
9	4	S I S S I P P I $
9	7	S I P P I $
11	3	S S I S S I P P I $
11	6	S S I P P I $

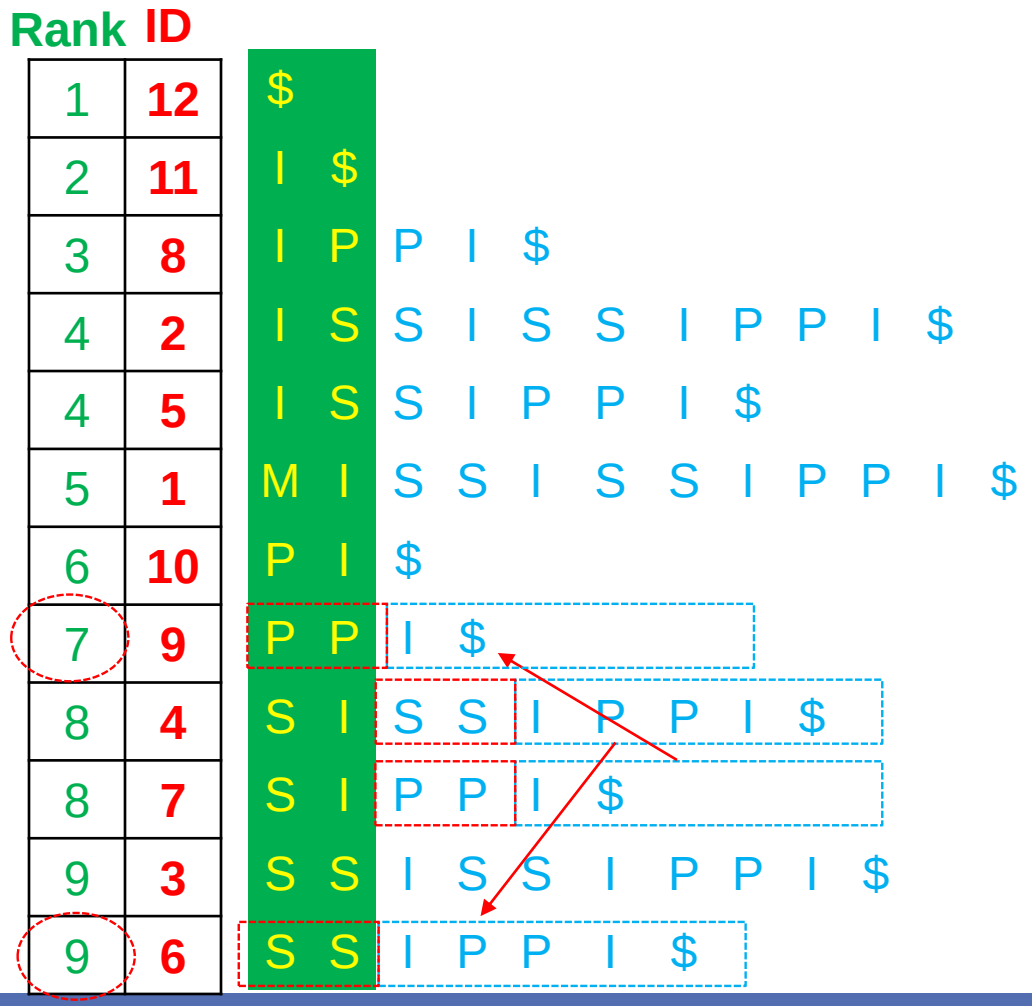
O(1) Comparison

	1	2	3	4	5	6	7	8	9	10	11	12
String	M	I	S	S	I	S	S	I	P	P	I	$

Observation 2:

If current ranks are the same

- First k characters must be the same
- The tie is to be broken on the next k characters, e.g.,
  - We need to compare “SSIPPI$” and “PPI$” on the first 2 characters
  - SSIPPI$ and PPI$ are suffixes and are already ranked on first 2 characters
    - E.g., PPI$ < SSIPPI$ because its rank is smaller
    - Therefore, suffix #7< suffix #4



Practice

	1	2	3	4	5	6	7	8	9	10	11	12
String	M	I	S	S	I	S	S	I	P	P	I	$

- **BUT WAIT!**
- **How did we do that quickly? Surely looking up the "second half" suffixes is O(N)?**

Rank	ID	
1	12	$
2	11	I $
3	8	I P P I $
4	2	I S S I S S I P P I $
4	5	I S S I P P I $
5	1	M I S S I S S I P P I $
6	10	P I $
7	9	P P I $
8	7	S I P P I $
9	4	S I S S I P P I $
10	6	S S I P P I $
11	3	S S I S S I P P I $

Practice

	1	2	3	4	5	6	7	8	9	10	11	12
String	M	I	S	S	I	S	S	I	P	P	I	$

- **BUT WAIT!**
- **How did we do that quickly? Surely looking up the "second half" suffixes is O(N)?**

- **USE SUFFIX IDs!**

Rank	ID	
1	12	$
2	11	I $
3	8	I P P I $
4	2	I S S I S S I P P I $
4	5	I S S I P P I $
5	1	M I S S I S S I P P I $
6	10	P I $
7	9	P P I $
8	7	S I P P I $
9	4	S I S S I P P I $
10	6	S S I P P I $
11	3	S S I S S I P P I $

Practice

	1	2	3	4	5	6	7	8	9	10	11	12
String	M	I	S	S	I	S	S	I	P	P	I	$

Suppose we are comparing suffix with ID 2 and 5:

- We need to compare SSIPPI$ and PPI$

Rank	ID	
1	12	$
2	11	I $
3	8	I P P I $
4	2	I S S I S S I P P I $
4	5	I S S I P P I $
6	1	M I S S I S S I P P I $
7	10	P I $
8	9	P P I $
9	7	S I P P I $
10	4	S I S S I P P I $
11	6	S S I P P I $
12	3	S S I S S I P P I $

Practice

	1	2	3	4	5	6	7	8	9	10	11	12
String	M	I	S	S	I	S	S	I	P	P	I	$

Suppose we are comparing suffix with ID 2 and 5:

- We need to compare SSIPPI$ and PPI$
- How do we find their ranks quickly?

Rank	ID	
1	12	$
2	11	I $
3	8	I P P I $
4	2	I S S I S S I P P I $
4	5	I S S I P P I $
6	1	M I S S I S S I P P I $
7	10	P I $
8	9	P P I $
9	7	S I P P I $
10	4	S I S S I P P I $
11	6	S S I P P I $
12	3	S S I S S I P P I $

Practice

	1	2	3	4	5	6	7	8	9	10	11	12
String	M	I	S	S	I	S	S	I	P	P	I	$

Suppose we are comparing suffix with ID 2 and 5:

- We need to compare SSIPPI$ and PPI$
- How do we find their ranks quickly?

- We want the ranks of suffixes: 2+k and 5+k
- I.e. suffixes 6 and 9

- This means we can calculate the IDs of the suffixes we want in O(1)

- Now we need to get from IDs to ranks in O(1)

Rank	ID	
1	12	$
2	11	I $
3	8	I P P I $
4	2	I S S I S S I P P I $
4	5	I S S I P P I $
6	1	M I S S I S S I P P I $
7	10	P I $
8	9	P P I $
9	7	S I P P I $
10	4	S I S S I P P I $
11	6	S S I P P I $
12	3	S S I S S I P P I $

Practice

	1	2	3	4	5	6	7	8	9	10	11	12
String	M	I	S	S	I	S	S	I	P	P	I	$

Suppose we are comparing suffix with ID 2 and 5:

- We need to compare SSIPPI$ and PPI$
- How do we find their ranks quickly?

- We want the ranks of suffixes: 2+k and 5+k
- I.e. suffixes 6 and 9

- To have O(1) access to their ranks, we need an array indexed by ID which contains the ranks!

- In other words, the way the ranks are arranged on this slide is useless

Rank	ID	
1	12	$
2	11	I $
3	8	I P P I $
4	2	I S S I S S I P P I $
4	5	I S S I P P I $
6	1	M I S S I S S I P P I $
7	10	P I $
8	9	P P I $
9	7	S I P P I $
10	4	S I S S I P P I $
11	6	S S I P P I $
12	3	S S I S S I P P I $

O(1) Comparison

Index/ID	1	2	3	4	5	6	7	8	9	10	11	12
Rank	6	4	12	10	4	11	9	3	8	7	2	1
String	**M**	**I**	**S**	**S**	**I**	**S**	**S**	**I**	**P**	**P**	**I**	**$**

Note: The greyed out oldRank array has been left for reference, but does not exist in implementation

- If we want the rank of ID i, look at Rank[i]
- Going back to our example…

oldRank	ID
1	12
2	11
3	8
4	2
4	5
5	1
6	10
7	9
8	7
9	4
10	6
11	3

$
I $
I P P I $
I S S I S S I P P I $
I S S I P P I $
M I S S I S S I P P I $
P I $
P P I $
S I P P I $
S I S S I P P I $
S S I P P I $
S S I S S I P P I $

O(1) Comparison

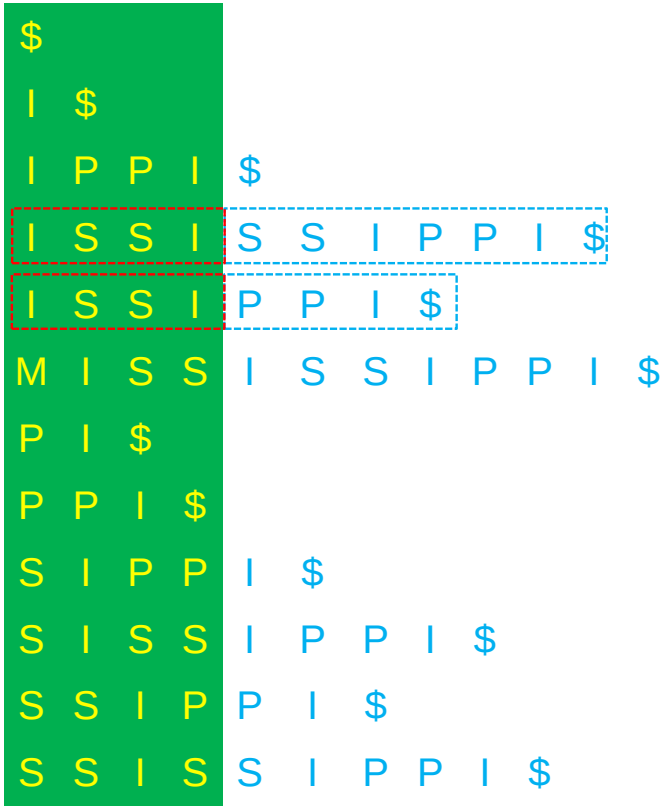
Index/ID	1	2	3	4	5	6	7	8	9	10	11	12
Rank	6	4	12	10	4	11	9	3	8	7	2	1
String	**M**	**I**	**S**	**S**	**I**	**S**	**S**	**I**	**P**	**P**	**I**	**$**

Note: The greyed out oldRank array has been left for reference, but does not exist in implementation

- If we want the rank of ID i, look at Rank[i]
- Going back to our example…

- We wanted to find the second parts of suffixes 2 and 5

oldRank	ID
1	12
2	11
3	8
4	2
4	5
5	1
6	10
7	9
8	7
9	4
10	6
11	3



O(1) Comparison

Index/ID	1	2	3	4	5	6	7	8	9	10	11	12
Rank	6	4	12	10	4	11	9	3	8	7	2	1
String	**M**	**I**	**S**	**S**	**I**	**S**	**S**	**I**	**P**	**P**	**I**	**$**

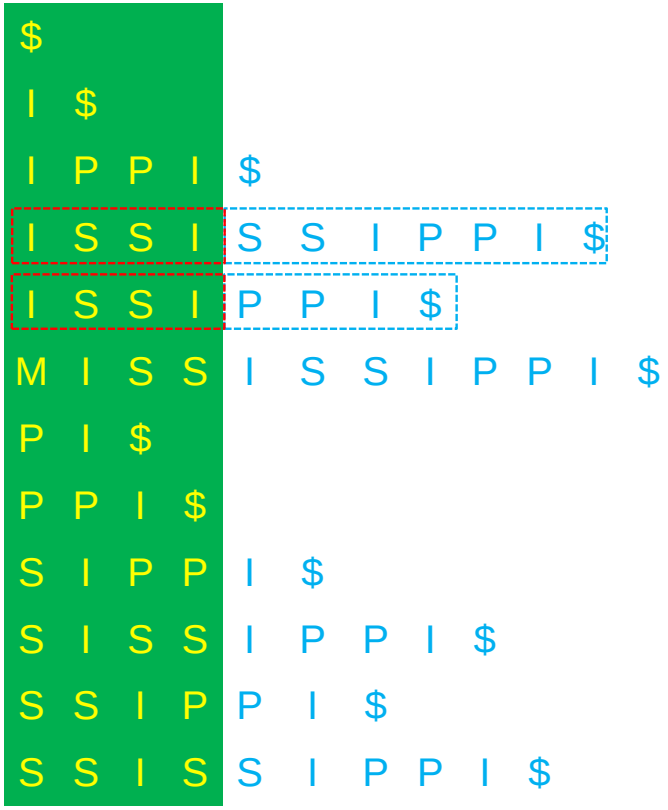
Note: The greyed out oldRank array has been left for reference, but does not exist in implementation

- If we want the rank of ID i, look at Rank[i]
- Going back to our example…

- We wanted to find the second parts of suffixes 2 and 5
- I.e. ID 6 and 9

- Rank[9] < rank[6]
- So ID 5 should come before ID 2 in the suffix array

oldRank	ID
1	12
2	11
3	8
4	2
4	5
5	1
6	10
7	9
8	7
9	4
10	6
11	3



Lets do an example (by hand in lecture)

ID	String	Rank
1	M	-
2	I	-
3	S	-
4	S	-
5	I	-
6	S	-
7	S	-
8	I	-
9	P	-
10	P	-
11	I	-
12	$	-

REMEMBER:
This rank array does not exist

It contains the same values as the other rank array, its just to make the algorithm easier to follow

Rank	SA											
-	1	M	I	S	S	I	S	S	I	P	P	I	$
-	2	I	S	S	I	S	S	I	P	P	I	$	
-	3	S	S	I	S	S	I	P	P	I	$		
-	4	S	I	S	S	I	P	P	I	$			
-	5	I	S	S	I	P	P	I	$				
-	6	S	S	I	P	P	I	$					
-	7	S	I	P	P	I	$						
-	8	I	P	P	I	$							
-	9	P	P	I	$								
-	10	P	I	$									
-	11	I	$										
-	12	$											

Lets do an example (by hand in lecture)

ID	String	Rank
1	M	-
2	I	-
3	S	-
4	S	-
5	I	-
6	S	-
7	S	-
8	I	-
9	P	-
10	P	-
11	I	-
12	$	-

Rank the first characters of each suffix

Rank	SA	
-	1	M I S S I S S I P P I $
-	2	I S S I S S I P P I $
-	3	S S I S S I P P I $
-	4	S I S S I P P I $
-	5	I S S I P P I $
-	6	S S I P P I $
-	7	S I P P I $
-	8	I P P I $
-	9	P P I $
-	10	P I $
-	11	I $
-	12	$

Lets do an example (by hand in lecture)

ID	String	Rank
1	M	3
2	I	2
3	S	5
4	S	5
5	I	2
6	S	5
7	S	5
8	I	2
9	P	4
10	P	4
11	I	2
12	$	1

Rank the first characters of each suffix

In practice we would do this using ord(), but since ranks are only comparative, the actual values don't matter, just their order. So we use numbers starting at 1

Rank	SA	
3	1	M I S S I S S I P P I $
2	2	I S S I S S I P P I $
5	3	S S I S S I P P I $
5	4	S I S S I P P I $
2	5	I S S I P P I $
5	6	S S I P P I $
5	7	S I P P I $
2	8	I P P I $
4	9	P P I $
4	10	P I $
2	11	I $
1	12	$

Lets do an example (by hand in lecture)

ID	String	Rank
1	M	3
2	I	2
3	S	5
4	S	5
5	I	2
6	S	5
7	S	5
8	I	2
9	P	4
10	P	4
11	I	2
12	$	1

Sort SA by ranks

Rank	SA	
3	1	M I S S I S S I P P I $
2	2	I S S I S S I P P I $
5	3	S S I S S I P P I $
5	4	S I S S I P P I $
2	5	I S S I P P I $
5	6	S S I P P I $
5	7	S I P P I $
2	8	I P P I $
4	9	P P I $
4	10	P I $
2	11	I $
1	12	$

Lets do an example (by hand in lecture)

ID	String	Rank
1	M	3
2	I	2
3	S	5
4	S	5
5	I	2
6	S	5
7	S	5
8	I	2
9	P	4
10	P	4
11	I	2
12	$	1

Sort SA by ranks

Note that this does not change the rank array, since IDs have kept the same ranks

We just rearranged the SA

Rank	SA
1	12
2	2
2	5
2	8
2	11
3	1
4	9
4	10
5	3
5	4
5	6
5	7

$										
I	S	S	I	S	S	I	P	P	I	$
I	S	S	I	P	P	I	$			
I	P	P	I	$						
I	$									
M	I	S	S	I	S	S	I	P	P	I	$
P	P	I	$							
P	I	$								
S	S	I	S	S	I	P	P	I	$	
S	I	S	S	I	P	P	I	$		
S	S	I	P	P	I	$				
S	I	P	P	I	$					

Lets do an example (by hand in lecture)

ID	String	Rank
1	M	3
2	I	2
3	S	5
4	S	5
5	I	2
6	S	5
7	S	5
8	I	2
9	P	4
10	P	4
11	I	2
12	$	1

Now sort on first 2 characters

For each comparison, we use the trick outlined in the previous section

Rank	SA	
1	12	$
2	2	I S S I S S I P P I $
2	5	I S S I P P I $
2	8	I P P I $
2	11	I $
3	1	M I S S I S S I P P I $
4	9	P P I $
4	10	P I $
5	3	S S I S S I P P I $
5	4	S I S S I P P I $
5	6	S S I P P I $
5	7	S I P P I $

Lets do an example (by hand in lecture)

ID	String	Rank
1	M	3
2	I	2
3	S	5
4	S	5
5	I	2
6	S	5
7	S	5
8	I	2
9	P	4
10	P	4
11	I	2
12	$	1

Rank	SA	
1	12	$
2	11	I $
2	8	I P P I $
2	2	I S S I S S I P P I $
2	5	I S S I P P I $
3	1	M I S S I S S I P P I $
4	10	P I $
4	9	P P I $
5	4	S I S S I P P I $
5	7	S I P P I $
5	3	S S I S S I P P I $
5	6	S S I P P I $

Now we update the ranks

Lets do an example (by hand in lecture)

ID	String	Rank
1	M	3
2	I	2
3	S	5
4	S	5
5	I	2
6	S	5
7	S	5
8	I	2
9	P	4
10	P	4
11	I	2
12	$	1

Temp
1
1
1
1
1
1
1
1
1
1
1
1

Rank	SA	
1	12	$
2	11	I $
2	8	I P P I $
2	2	I S S I S S I P P I $
2	5	I S S I P P I $
3	1	M I S S I S S I P P I $
4	10	P I $
4	9	P P I $
5	4	S I S S I P P I $
5	7	S I P P I $
5	3	S S I S S I P P I $
5	6	S S I P P I $

Make an array, "Temp" to hold the new ranks

Lets do an example (by hand in lecture)

ID	String	Rank	Temp
1	M	3	1
2	I	2	1
3	S	5	1
4	S	5	1
5	I	2	1
6	S	5	1
7	S	5	1
8	I	2	1
9	P	4	1
10	P	4	1
11	I	2	1
12	$	1	1

Rank	SA	
1	12	$
2	11	I $
2	8	I P P I $
2	2	I S S I S S I P P I $
2	5	I S S I P P I $
3	1	M I S S I S S I P P I $
4	10	P I $
4	9	P P I $
5	4	S I S S I P P I $
5	7	S I P P I $
5	3	S S I S S I P P I $
5	6	S S I P P I $

For each pair of adjacent suffixes, compare them

Lets do an example (by hand in lecture)

ID	String	Rank
1	M	3
2	I	2
3	S	5
4	S	5
5	I	2
6	S	5
7	S	5
8	I	2
9	P	4
10	P	4
11	I	2
12	$	1

Temp
1
1
1
1
1
1
1
1
1
1
1
1

Rank	SA	
1	12	$
2	11	I $
2	8	I P P I $
2	2	I S S I S S I P P I $
2	5	I S S I P P I $
3	1	M I S S I S S I P P I $
4	10	P I $
4	9	P P I $
5	4	S I S S I P P I $
5	7	S I P P I $
5	3	S S I S S I P P I $
5	6	S S I P P I $

If they have different ranks already, then the second suffix is certainly larger

Lets do an example (by hand in lecture)

ID	String	Rank
1	M	3
2	I	2
3	S	5
4	S	5
5	I	2
6	S	5
7	S	5
8	I	2
9	P	4
10	P	4
11	I	2
12	$	1

Temp (new rank for index)	
1	
1	
1	
1	
1	
1	
1	
1	
1	
1	
2	2
1	1

Rank	SA	k =1
1	12	$
2	11	I $
2	8	I P P I $
2	2	I S S I S S I P P I $
2	5	I S S I P P I $
3	1	M I S S I S S I P P I $
4	10	P I $
4	9	P P I $
5	4	S I S S I P P I $
5	7	S I P P I $
5	3	S S I S S I P P I $
5	6	S S I P P I $

Set Temp[11] to Rank[12]+1

Lets do an example (by hand in lecture)

ID	String	Rank	Temp
1	M	3	1
2	I	2	1
3	S	5	1
4	S	5	1
5	I	2	1
6	S	5	1
7	S	5	1
8	I	2	1
9	P	4	1
10	P	4	1
11	I	2	2
12	$	1	1

Rank	SA	
1	12	$
2	11	I $
2	8	I P P I $
2	2	I S S I S S I P P I $
2	5	I S S I P P I $
3	1	M I S S I S S I P P I $
4	10	P I $
4	9	P P I $
5	4	S I S S I P P I $
5	7	S I P P I $
5	3	S S I S S I P P I $
5	6	S S I P P I $

If they have the same rank

Lets do an example (by hand in lecture)

ID	String	Rank	Temp
1	M	3	1
2	I	2	1
3	S	5	1
4	S	5	1
5	I	2	1
6	S	5	1
7	S	5	1
8	I	2	1
9	P	4	1
10	P	4	1
11	I	2	2
12	$	1	1

Rank	SA	
1	12	$
2	11	I $
2	8	I P P I $
2	2	I S S I S S I P P I $
2	5	I S S I P P I $
3	1	M I S S I S S I P P I $
4	10	P I $
4	9	P P I $
5	4	S I S S I P P I $
5	7	S I P P I $
5	3	S S I S S I P P I $
5	6	S S I P P I $

We need to use the O(1) trick

Lets do an example (by hand in lecture)

ID	String	Rank
1	M	3
2	I	2
3	S	5
4	S	5
5	I	2
6	S	5
7	S	5
8	I	2
9	P	4
10	P	4
11	I	2
12	$	1

Temp
1
1
1
1
1
1
1
1
1
1
2
1

Rank	SA	
1	12	$
2	11	I $
2	8	I P P I $
2	2	I S S I S S I P P I $
2	5	I S S I P P I $
3	1	M I S S I S S I P P I $
4	10	P I $
4	9	P P I $
5	4	S I S S I P P I $
5	7	S I P P I $
5	3	S S I S S I P P I $
5	6	S S I P P I $

Sometimes, one or both suffixes will not have 2k chars (like I$)

Lets do an example (by hand in lecture)

ID	String	Rank
1	M	3
2	I	2
3	S	5
4	S	5
5	I	2
6	S	5
7	S	5
8	I	2
9	P	4
10	P	4
11	I	2
12	$	1

Temp
1
1
1
1
1
1
1
1
1
1
2
1

Rank	SA
1	12
2	11
2	8
2	2
2	5
3	1
4	10
4	9
5	4
5	7
5	3
5	6

$										
I	$									
I	P	P	I	$						
I	S	S	I	S	S	I	P	P	I	$
I	S	S	I	P	P	I	$			
M	I	S	S	I	S	S	I	P	P	I	$
P	I	$								
P	P	I	$							
S	I	S	S	I	P	P	I	$		
S	I	P	P	I	$					
S	S	I	S	S	I	P	P	I	$	
S	S	I	P	P	I	$				

In such a case, the shorter one comes earlier!

Lets do an example (by hand in lecture)

ID	String	Rank	Temp
1	M	3	1
2	I	2	1
3	S	5	1
4	S	5	1
5	I	2	1
6	S	5	1
7	S	5	1
8	I	2	3
9	P	4	1
10	P	4	1
11	I	2	2
12	$	1	1

k + 1 for the one that has the same rank

rank(8 +1) > rank(11+1)

Rank	SA	k + 1
1	12	$
2	11	I $
2	8	I P P I $
2	2	I S S I S S I P P I $
2	5	I S S I P P I $
3	1	M I S S I S S I P P I $
4	10	P I $
4	9	P P I $
5	4	S I S S I P P I $
5	7	S I P P I $
5	3	S S I S S I P P I $
5	6	S S I P P I $

Set Rank[8] = Rank[11] + 1

Lets do an example (by hand in lecture)

ID	String	Rank	Temp
1	M	3	1
2	I	2	4
3	S	5	1
4	S	5	1
5	I	2	1
6	S	5	1
7	S	5	1
8	I	2	3
9	P	4	1
10	P	4	1
11	I	2	2
12	$	1	1

Rank	SA	
1	12	$
2	11	I $
2	8	I P P I $
2	2	I S S I S S I P P I $
2	5	I S S I P P I $
3	1	M I S S I S S I P P I $
4	10	P I $
4	9	P P I $
5	4	S I S S I P P I $
5	7	S I P P I $
5	3	S S I S S I P P I $
5	6	S S I P P I $

Continue in this way

Lets do an example (by hand in lecture)

ID	String	Rank	Temp
1	M	3	1
2	I	2	4
3	S	5	1
4	S	5	1
5	I	2	4
6	S	5	1
7	S	5	1
8	I	2	3
9	P	4	1
10	P	4	1
11	I	2	2
12	$	1	1

Rank	SA	
1	12	$
2	11	I $
2	8	I P P I $
2	2	I S S I S S I P P I $
2	5	I S S I P P I $
3	1	M I S S I S S I P P I $
4	10	P I $
4	9	P P I $
5	4	S I S S I P P I $
5	7	S I P P I $
5	3	S S I S S I P P I $
5	6	S S I P P I $

Continue in this way

Lets do an example (by hand in lecture)

ID	String	Rank
1	M	3
2	I	2
3	S	5
4	S	5
5	I	2
6	S	5
7	S	5
8	I	2
9	P	4
10	P	4
11	I	2
12	$	1

Temp
5
4
1
1
4
1
1
3
1
1
2
1

Rank	ID SA
1	12
2	11
2	8
2	2
2	5
3	1
4	10
4	9
5	4
5	7
5	3
5	6

$										
I	$									
I	P	P	I	$						
I	S	S	I	S	S	I	P	P	I	$
I	S	S	I	P	P	I	$			
M	I	S	S	I	S	S	I	P	P	I	$
P	I	$								
P	P	I	$							
S	I	S	S	I	P	P	I	$		
S	I	P	P	I	$					
S	S	I	S	S	I	P	P	I	$	
S	S	I	P	P	I	$				

Continue in this way

Lets do an example (by hand in lecture)

ID	String	Rank
1	M	3
2	I	2
3	S	5
4	S	5
5	I	2
6	S	5
7	S	5
8	I	2
9	P	4
10	P	4
11	I	2
12	$	1

Temp
5
4
1
1
4
1
1
3
1
6
2
1

if x = 4, y = 4

Rank	SA
1	12
2	11
2	8
2	2
2	5
3	1
4	10
4	9
5	4
5	7
5	3
5	6

$										
I	$									
I	P	P	I	$						
I	S	S	I	S	S	I	P	P	I	$
I	S	S	I	P	P	I	$			
M	I	S	S	I	S	S	I	P	P	I	$
P	I	$								
P	P	I	$							
S	I	S	S	I	P	P	I	$		
S	I	P	P	I	$					
S	S	I	S	S	I	P	P	I	$	
S	S	I	P	P	I	$				

Continue in this way

Lets do an example (by hand in lecture)

ID	String	Rank	Temp
1	M	3	5
2	I	2	4
3	S	5	1
4	S	5	1
5	I	2	4
6	S	5	1
7	S	5	1
8	I	2	3
9	P	4	7
10	P	4	6
11	I	2	2
12	$	1	1

Rank	SA	
1	12	$
2	11	I $
2	8	I P P I $
2	2	I S S I S S I P P I $
2	5	I S S I P P I $
3	1	M I S S I S S I P P I $
4	10	P I $
4	9	P P I $
5	4	S I S S I P P I $
5	7	S I P P I $
5	3	S S I S S I P P I $
5	6	S S I P P I $

Continue in this way

Lets do an example (by hand in lecture)

ID	String	Rank	Temp
1	M	3	5
2	I	2	4
3	S	5	1
4	S	5	8
5	I	2	4
6	S	5	1
7	S	5	1
8	I	2	3
9	P	4	7
10	P	4	6
11	I	2	2
12	$	1	1

Rank	SA	
1	12	$
2	11	I $
2	8	I P P I $
2	2	I S S I S S I P P I $
2	5	I S S I P P I $
3	1	M I S S I S S I P P I $
4	10	P I $
4	9	P P I $
5	4	S I S S I P P I $
5	7	S I P P I $
5	3	S S I S S I P P I $
5	6	S S I P P I $

Continue in this way

Lets do an example (by hand in lecture)

ID	String	Rank	Temp
1	M	3	5
2	I	2	4
3	S	5	1
4	S	5	8
5	I	2	4
6	S	5	1
7	S	5	8
8	I	2	3
9	P	4	7
10	P	4	6
11	I	2	2
12	$	1	1

Rank	SA	
1	12	$
2	11	I $
2	8	I P P I $
2	2	I S S I S S I P P I $
2	5	I S S I P P I $
3	1	M I S S I S S I P P I $
4	10	P I $
4	9	P P I $
5	4	S I S S I P P I $
5	7	S I P P I $
5	3	S S I S S I P P I $
5	6	S S I P P I $

Continue in this way

Lets do an example (by hand in lecture)

ID	String	Rank	Temp
1	M	3	5
2	I	2	4
3	S	5	9
4	S	5	8
5	I	2	4
6	S	5	1
7	S	5	8
8	I	2	3
9	P	4	7
10	P	4	6
11	I	2	2
12	$	1	1

Rank	SA	
1	12	$
2	11	I $
2	8	I P P I $
2	2	I S S I S S I P P I $
2	5	I S S I P P I $
3	1	M I S S I S S I P P I $
4	10	P I $
4	9	P P I $
5	4	S I S S I P P I $
5	7	S I P P I $
5	3	S S I S S I P P I $
5	6	S S I P P I $

Continue in this way

Lets do an example (by hand in lecture)

ID	String	Rank	Temp
1	M	3	5
2	I	2	4
3	S	5	9
4	S	5	8
5	I	2	4
6	S	5	9
7	S	5	8
8	I	2	3
9	P	4	7
10	P	4	6
11	I	2	2
12	$	1	1

Rank	SA	
1	12	$
2	11	I $
2	8	I P P I $
2	2	I S S I S S I P P I $
2	5	I S S I P P I $
3	1	M I S S I S S I P P I $
4	10	P I $
4	9	P P I $
5	4	S I S S I P P I $
5	7	S I P P I $
5	3	S S I S S I P P I $
5	6	S S I P P I $

Continue in this way

Lets do an example (by hand in lecture)

ID	String	Rank	Temp
1	M	3	5
2	I	2	4
3	S	5	9
4	S	5	8
5	I	2	4
6	S	5	9
7	S	5	8
8	I	2	3
9	P	4	7
10	P	4	6
11	I	2	2
12	$	1	1

Rank	SA	
1	12	$
2	11	I $
2	8	I P P I $
2	2	I S S I S S I P P I $
2	5	I S S I P P I $
3	1	M I S S I S S I P P I $
4	10	P I $
4	9	P P I $
5	4	S I S S I P P I $
5	7	S I P P I $
5	3	S S I S S I P P I $
5	6	S S I P P I $

This is our new Rank array, so overwrite the old one

Lets do an example (by hand in lecture)

ID	String	Rank
1	M	5
2	I	4
3	S	9
4	S	8
5	I	4
6	S	9
7	S	8
8	I	3
9	P	7
10	P	6
11	I	2
12	$	1

Temp
5
4
9
8
4
9
8
3
7
6
2
1

Rank	SA
1	12
2	11
3	8
4	2
4	5
5	1
6	10
7	9
8	4
8	7
9	3
9	6

$

I $

I P P I $

I S S I S S I P P I $

I S S I P P I $

M I S S I S S I P P I $

P I $

P P I $

S I S S I P P I $

S I P P I $

S S I S S I P P I $

S S I P P I $

This is our new Rank array, so overwrite the old one

Lets do an example (by hand in lecture)

ID	String	Rank
1	M	5
2	I	4
3	S	9
4	S	8
5	I	4
6	S	9
7	S	8
8	I	3
9	P	7
10	P	6
11	I	2
12	$	1

Rank	SA	k = 2
1	12	$
2	11	I $
3	8	I P P I $
4	2	I S S I S S I P P I $
4	5	I S S I P P I $
5	1	M I S S I S S I P P I $
6	10	P I $
7	9	P P I $
8	4	S I S S I P P I $
8	7	S I P P I $
9	3	S S I S S I P P I $
9	6	S S I P P I $

Now we have the suffixes sorted by the first 2. Go for 4!

Lets do an example (by hand in lecture)

ID	String	Rank
1	M	5
2	I	4
3	S	11
4	S	9
5	I	4
6	S	10
7	S	8
8	I	3
9	P	7
10	P	6
11	I	2
12	$	1

Rank	ID
1	12
2	11
3	8
4	2
4	5
5	1
6	10
7	9
8	7
9	4
10	6
11	3

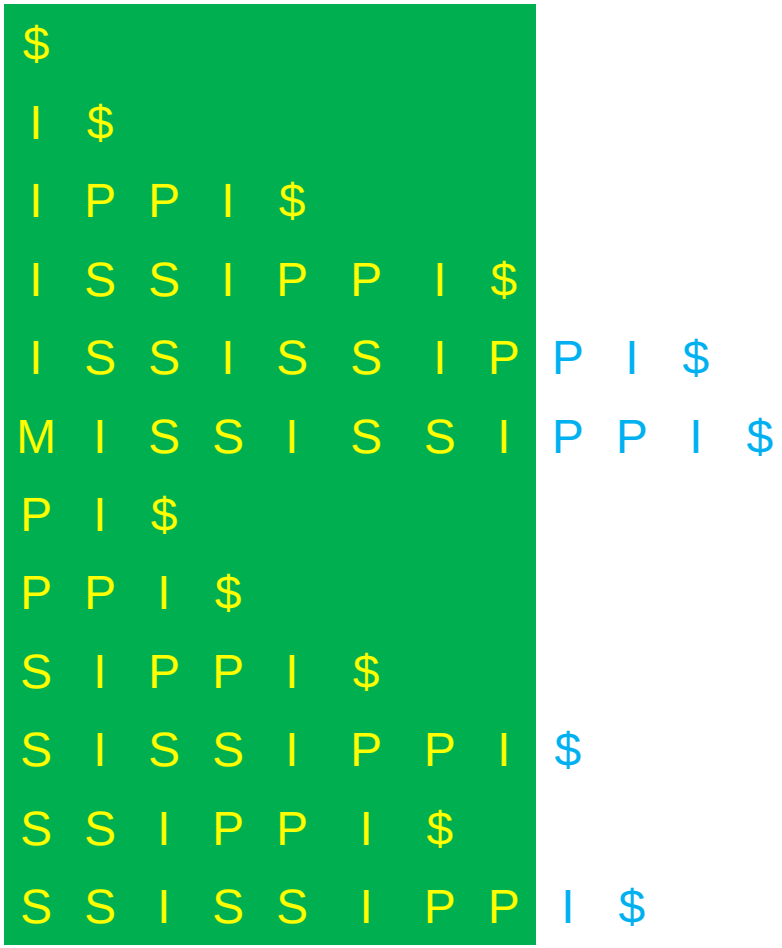
k = 4, compare rank(id + k)

$										
I	$									
I	P	P	I	$						
I	S	S	I	S	S	I	P	P	I	$
I	S	S	I	P	P	I	$			
M	I	S	S	I	S	S	I	P	P	I	$
P	I	$								
P	P	I	$							
S	I	P	P	I	$					
S	I	S	S	I	P	P	I	$		
S	S	I	P	P	I	$				
S	S	I	S	S	I	P	P	I	$	

Lets do an example (by hand in lecture)

ID	String	Rank
1	M	6
2	I	5
3	S	12
4	S	10
5	I	4
6	S	11
7	S	9
8	I	3
9	P	8
10	P	7
11	I	2
12	$	1

Rank	ID
1	12
2	11
3	8
4	5
5	2
6	1
7	10
8	9
9	7
10	4
11	6
12	3



Lets do an example (by hand in lecture)

ID	String	Rank
1	M	6
2	I	5
3	S	12
4	S	10
5	I	4
6	S	11
7	S	9
8	I	3
9	P	8
10	P	7
11	I	2
12	$	1

Rank	ID
1	12
2	11
3	8
4	5
5	2
6	1
7	10
8	9
9	7
10	4
11	6
12	3

$
I $
I P P I $
I S S I P P I $
I S S I S S I P P I $
M I S S I S S I P P I $
P I $
P P I $
S I P P I $
S I S S I P P I $
S S I P P I $
S S I S S I P P I $

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Questions?

Suffix Array

With prefix doubling with O(1) comparison

- Let say we have a suffix “SSISSIPPI$”

Suffix Array

With prefix doubling with O(1) comparison

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- Let say we have a suffix “SSISSIPPI$”
  - It has the suffix ID of 3
  - What is the suffix ID of “PPI$”?

Suffix Array

With prefix doubling with O(1) comparison

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- Let say we have a suffix “SSISSIPPI$”
  - It has the suffix ID of 3
  - What is the suffix ID of “PPI$”?

Suffix Array

With prefix doubling with O(1) comparison

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- Let say we have a suffix “SSISSIPPI$”
  - It has the suffix ID of 3
  - What is the suffix ID of “PPI$”?
    - 3 + 6 = 9
    - #ez

Suffix Array

With prefix doubling with O(1) comparison

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- Let say we have a suffix “SSISSIPPI$”
  - It has the suffix ID of 3
  - What is the suffix ID of “PPI$”?
    - 3 + 6 = 9
      - 3 is the original ID
      - 6 is the number of characters from start
    - #ez

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Questions?

Suffix Array

With prefix doubling with O(1) comparison

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- Time complexity is O(N^2) with radix sort
  - Can we do better?
  - Yes with prefix doubling!
  - But complexity still the same O(N^2 log N), even slower than radix sort
    - Due to the O(N) comparison
    - We use the rank table to get O(1) comparison!
    - So we can use this to sort very quickly within the same rank
- Space complexity is O(N) with suffix ID

O(1) Comparison

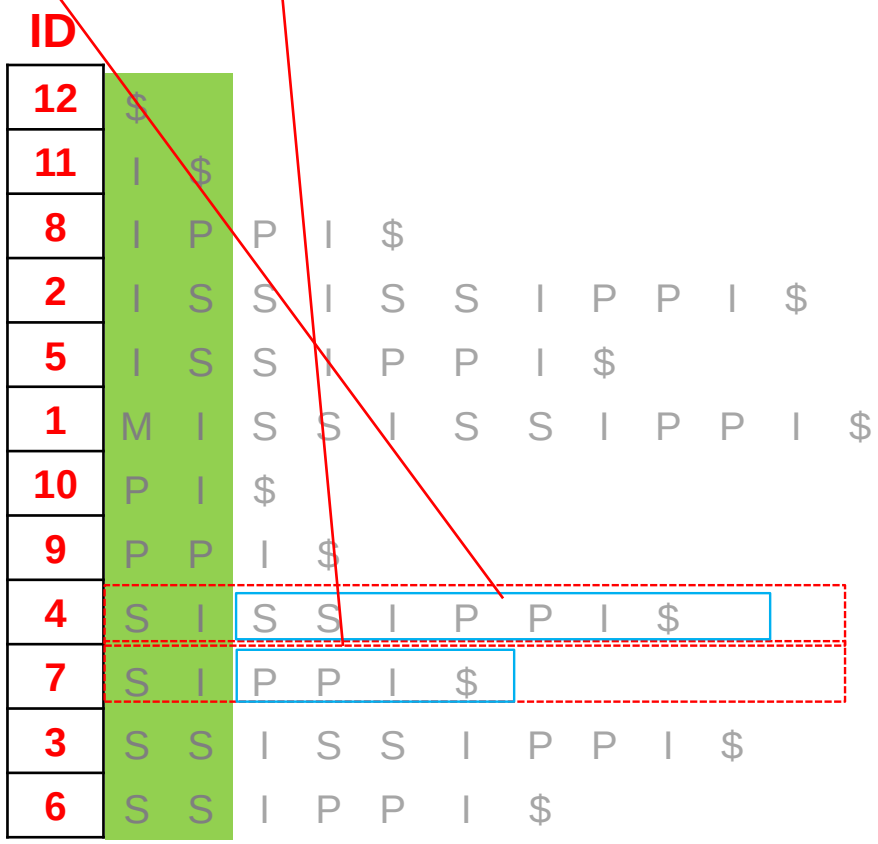
Index/ID	1	2	3	4	5	6	7	8	9	10	11	12
Rank	6	4	11	9	4	11	9	3	8	7	2	1
String	M	I	S	S	I	S	S	I	P	P	I	$

Note: We don't need to store the suffixes (shown grey) – we only need Suffix IDs.

Comparing Suffix with IDs 4 and 7:

- Their ranks are equal.
  - **Rank[4]** = **Rank [7]**
  - E.g., they are the same on first 2 characters
- We need to compare them on the next 2 characters
- Compare ranks of suffixes 4+2=6 and 7+2=9
  - **Rank[6]** > **Rank[9]**
  - So, Suffix #7 is smaller than #4

Note: Comparison takes O(1) and we do not need to store all suffixes – space used is O(N)



O(1) Comparison

Index/ID	1	2	3	4	5	6	7	8	9	10	11	12
Rank	6	4	12	10	4	11	9	3	8	7	2	1
String	M	I	S	S	I	S	S	I	P	P	I	$

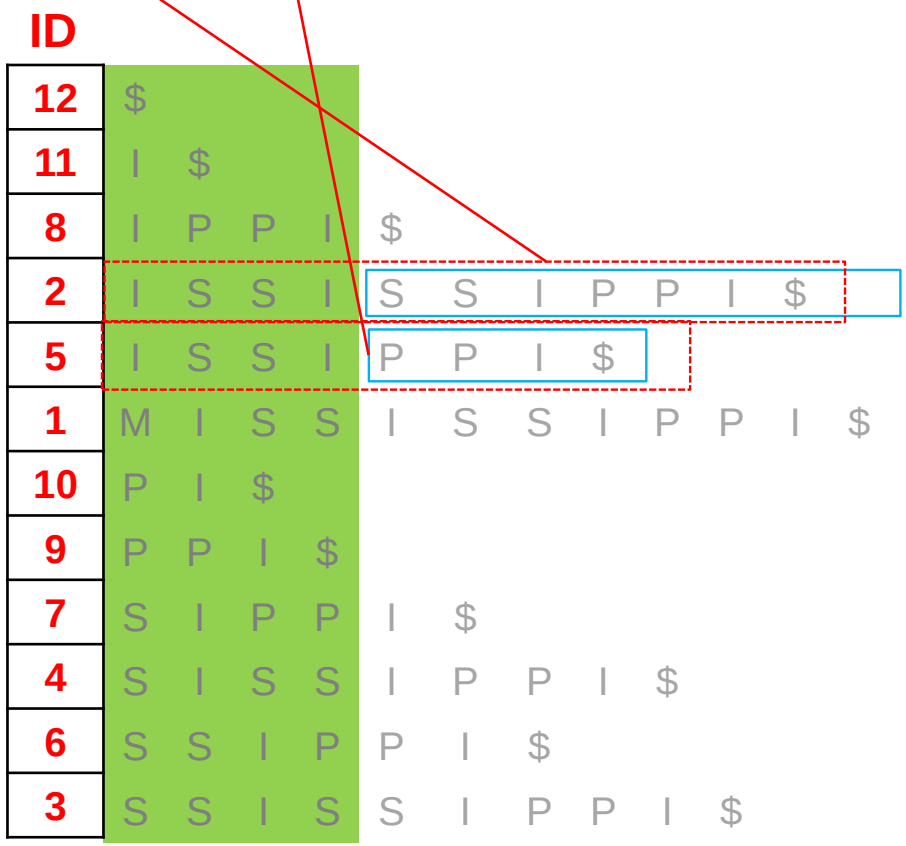
Note: We don't need to store the suffixes (shown grey) – we only need Suffix IDs.

Suppose array has been sorted on first 4 characters.

Comparing Suffix with IDs 2 and 5:

- Their ranks are equal.
  - **Rank[2] = Rank [5]**
  - E.g., they are the same on first 4 characters
- We need to compare them on the next 4 characters
  - Should we instead compare them on all remaining characters???
    - No, array is sorted on first 4 only
- Compare ranks of suffixes 2+4=6 and 5+4=9
  - **Rank[6] > Rank[9]**
  - So, Suffix #5 is smaller than #2

Note: Each comparison takes O(1) and we do not need to store all suffixes – space used is O(N)



Construction Cost of Suffix Array

Index/ID	1	2	3	4	5	6	7	8	9	10	11	12
Rank	6	5	12	10	4	11	9	3	8	7	2	1
String	**M**	**I**	**S**	**S**	**I**	**S**	**S**	**I**	**P**	**P**	**I**	**$**

Time Complexity (prefix doubling):

- We need to sort O(log N) times
  - Sort on 1 characters
  - Sort on 2 characters
  - …
  - Sort on N/2 characters
  - Sort on N characters
- Each sorting requires O(N log N) comparisons
- Each comparison takes O(1)
- Total cost: $O(N \log^2 N)$

Space Complexity:

- O(N)

O(N) prefix doubling

N downward for every suffix

O(1) comparison
since just compare rank(id + k)
however, id + k would results logN
complexity at most
k = 1, 2 , 4,8
may be possible to have
have repetitive comparison time
up to log(N) N is the length of string

12	$
11	I $
8	I P P I $
5	I S S I P P I $
2	I S S I S S I P P I $
1	M I S S I S S I P P I $
10	P I $
9	P P I $
7	S I P P I $
4	S I S S I P P I $
6	S S I P P I $
3	S S I S S I P P I $

Suffix Array

With prefix doubling with O(1) comparison

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- Time complexity is O(N^2) with radix sort
  - Can we do better?
  - Yes with prefix doubling!
    - K character is sorted
    - So when we calculate the next K character (total of 2K)
      - We reuse the sorted first K
      - This is possible since they are suffixes of the same string!

      first k appear before which is sorted just use it
  - But complexity still the same O(N^2 log N), even slower than radix sort
    - Due to the O(N) comparison
    - We use the rank table to get O(1) comparison!
    - So we can use this to sort very quickly within the same rank
- Space complexity is O(N) with suffix ID

Suffix Array

With prefix doubling with O(1) comparison

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- Time complexity is O(N^2) with radix sort
  - Can we do better?
  - Yes with prefix doubling!
    - K character is sorted
    - So when we calculate the next K character (total of 2K)
      - We reuse the sorted first K
      - This is possible since they are suffixes of the same string!
  - But complexity still the same O(N^2 log N), even slower than radix sort
    - Due to the O(N) comparison
    - We use the rank table to get O(1) comparison!
    - So we can use this to sort very quickly within the same rank
    - So complexity now is O(N log^2 N) with prefix doubling that has O(1) comparison
- Space complexity is O(N) with suffix ID

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Questions?

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- We can build
  - Suffix trie
  - Suffix tree
  - Suffix array

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- We can build
  - Suffix trie
  - Suffix tree
  - Suffix array
- For anything we can treat as a string
  - Including long text (as a single string!)

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- We can build
  - Suffix trie
  - Suffix tree
  - Suffix array
- For anything we can treat as a string
  - Including long text (as a single string!)
- Next week, we learn Burrows-Wheeler Transform (BWT)
  - Which is even better!

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Questions?

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Thank You