Master theorem, Divide and Conquer

1. Calculate the time complexity of the following recurrence relations.
   [Any method is acceptable as long as steps are shown]

   A. $T(n) = 2T(n/2) + 1/n$
   B. $T(n) = 2T(n/3) + n$
   C. $T(n) = T(n/2) + T(n/5) + n$
   D. $T(n) = 2T(n/4) + n^2$

2. You have been asked to sort the following array of integers in ascending order.

Index	0	1	2	3	4	5	6	7
Number	23	21	19	15	12	11	5	3

You decided to use Quick sort using the first element as pivot for the task. However your teacher says " Your approach won't be efficient in this case".

   A. Why do you think your teacher says so?
   B. Write the recurrence relation of your approach and calculate the time complexity. You have to show the steps and proper mathematical logic.

3. Inspired by the Karatsuba algorithm, a curious CSE student Benjamin decided to modify the algorithm in his own way. For a n digit number, instead of using subproblems of size n/2, Benjamin used subproblems of size n/3. He believes with this modification, he can get a faster algorithm than Karatsuba's. So, for finding product of two n digit numbers A and B,
Benjamin splitted A into 3 subproblems (A_1 , A_2 , A_3) each with n/3 digits and B into 3 subproblems (B_1 , B_2 , B_3) each with n/3 digits. Benjamin wants to write a divide and conquer algorithm for finding the product of A and B from these smaller subproblems. (Assume n is a power of 3)

   A. Write A in terms of A_1 , A_2 , A_3 and n. Write B in terms of B_1 , B_2 , B_3 and n
   B. Calculate the product AB from your answers in (A)
   C. Help Benjamin write the pseudocode of the divide and conquer algorithm
   D. Calculate the time complexity of your algorithm and validate whether Benjamin's claim of getting a faster algorithm is true or not.