

Deadline: 9th May 2024

Project

Part I:

Consider a 3 x 3 MIMO system. Let the channel matrix H of this MIMO channel be:

$$H = \begin{bmatrix} 3 & 0 & 8 \\ 0 & 1 & 0 \\ 4 & 0 & 6 \end{bmatrix}$$

Consider a transmit power of $P = -1.75$ dB and receiver noise power $\sigma_n^2 = 3$ dB.

(a) Derive the SVD of the channel matrix \mathbf{H} above (hand analysis)

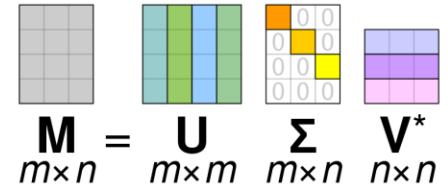
(b) Write a code that calculates the SVD of the channel matrix H and computes the singular values

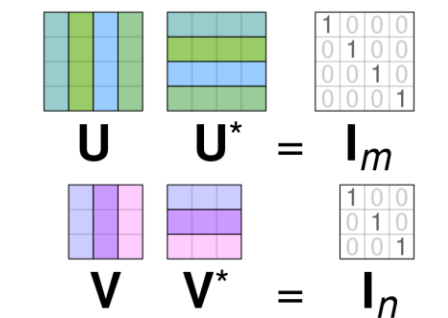
(c) Write a code for the water filling algorithm to find the optimal transmit powers that maximize the MIMO channel capacity.

(d) Compute the capacity of this MIMO channel in case of

(i) Multiplexing with and without channel knowledge.

(ii) Diversity with and without channel knowledge.



$$\mathbf{M}_{m \times n} = \mathbf{U}_{m \times m} \mathbf{\Sigma}_{m \times n} \mathbf{V}^*_{n \times n}$$


$$\mathbf{U}_{m \times m} \mathbf{U}^*_{m \times m} = \mathbf{I}_m$$

$$\mathbf{V}_{n \times n} \mathbf{V}^*_{n \times n} = \mathbf{I}_n$$

Note: Generalise the code so that it can work for any channel matrix H which could have different dimensions than the one provided and any given noise power and total transmit power.

Part II:

You are required to write a code that simulate a MIMO system (assume CSI is available at Rx side only):

Set m and n number of transmit and receive antenna

> Generate N_C (e.g., 100,000) realizations of the channel matrix $H(m \times n)$

> Each element $h_{ij} = |h_{ij}|e^{j\theta_{ij}}$

> $|h_{ij}|$ is Rayleigh $f(h) = \frac{h}{\sigma^2} e^{-(h/2\sigma^2)}$ with $2\sigma^2 = 1$ (unit power on each channel)

> θ_{ij} is uniform in the interval $[0, 2\pi]$

For each channel realization compute SVD to derive diagonal matrix D

Compute capacity based on Water Filling solution

$$> p_i = \max\left(0, \beta - \frac{\sigma^2}{\lambda_i}\right)$$

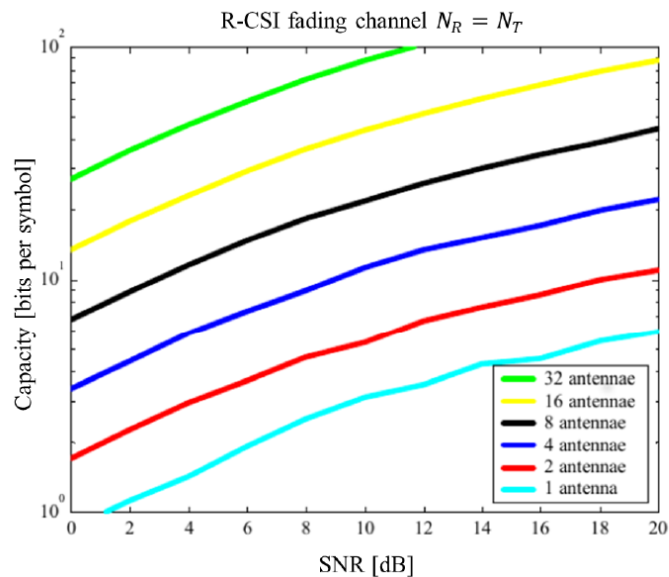
$$> P_T = \sum_{i=1}^k p_i$$

$$> C = \sum_{i=1}^k \log\left(1 + \frac{p_i \lambda_i}{\sigma^2}\right)$$

Average the capacity over all N_C channel realizations

(a) Calculates the Ergodic Rayleigh Fading channel capacity for a MIMO system.

(b) Plot the curves of the Ergodic capacity calculated in the previous step for different SNR values and different number of antennas as shown in the below figure (assume that the number of antennas at the transmitter and receiver are equal).



Project Requirements:

1. The required written hand analysis in Part I (a).
2. Commented codes for Part I (b), (c) and (d).
3. Display of the results for Part I (b), (c) and (d).
4. A written section that gives a brief introduction to the Rayleigh Fading channel in a MIMO system.
5. A written section that explains the Ergodic Rayleigh Fading channel capacity for a MIMO system.
6. A written section that explains in details the rules used to calculate the ergodic capacity.
7. Commented codes for Part II (a) and (b).

You should deliver a hard copy of the report in addition to sending an email with the codes and report attached to guc.course@gmail.com maximum by Thursday 9th of May.

4. You can work in a team that can consist of 3 students maximum.