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Project

1)

$$H = \begin{bmatrix} 3 & 0 & 8 \\ 0 & 1 & 0 \\ 4 & 0 & 6 \end{bmatrix} \qquad P_T = -1{,}75\ dB \qquad \sigma_n^2 = 3dB$$

*to find U get $\det(H \times H^T - \lambda I) = 0$

$$\begin{bmatrix} 73-\lambda & 0 & 60 \\ 0 & 1-\lambda & 0 \\ 60 & 0 & 52-\lambda \end{bmatrix}$$

$$(73-\lambda)\left[(1-\lambda)(52-\lambda)\right] + 60\left[-60(1-\lambda)\right] = 0$$
$$(73-\lambda)(52-53\lambda+\lambda^2) - 3600 + 3600\lambda = 0$$
$$3796 - 3869\lambda + 73\lambda^2 - 52\lambda + 53\lambda^2 - \lambda^3 - 3600 + 3600\lambda = 0$$
$$-\lambda^3 + 126\lambda^2 - 321\lambda + 196 = 0 \quad \#$$
$$\lambda_1 = 123{,}4118 \quad \lambda_2 = 1{,}5882 \quad \lambda_3 = 1 \quad \# \rightarrow \text{eigen values}$$

*using $\lambda_1 = 123{,}4118$

$$\begin{bmatrix} -50{,}4118 & 0 & 60 \\ 0 & -122{,}4118 & 0 \\ 60 & 0 & -71{,}4118 \end{bmatrix} \begin{bmatrix} U_{11} \\ U_{12} \\ U_{13} \end{bmatrix} = 0$$

$-50{,}4118\, U_{11} + 60\, U_{13} = 0 \rightarrow U_{11} = 1{,}19\, U_{13}$

$-122{,}4118\, U_{12} = 0 \rightarrow U_{12} = 0$

$$U_1 = \begin{bmatrix} 1{,}19 \\ 0 \\ 1 \end{bmatrix} \#$$

*using $\lambda_2 = 1{,}5882$

$$\begin{bmatrix} 71{,}4118 & 0 & 60 \\ 0 & -0{,}5882 & 0 \\ 60 & 0 & 50{,}4118 \end{bmatrix} \begin{bmatrix} U_{21} \\ U_{22} \\ U_{23} \end{bmatrix} = 0$$

$71{,}4118\, U_{21} + 60\, U_{23} = 0 \rightarrow U_{23} = -1{,}19\, U_{21}$

$-0{,}5882\, U_{22} = 0 \rightarrow U_{22} = 0$

$$U_2 = \begin{bmatrix} 1 \\ 0 \\ -1{,}19 \end{bmatrix} \#$$

*using $\lambda_3 = 1$

$$\begin{bmatrix} 72 & 0 & 60 \\ 0 & 0 & 0 \\ 60 & 0 & 51 \end{bmatrix} \begin{bmatrix} U_{31} \\ U_{32} \\ U_{33} \end{bmatrix} = 0$$

$72\, U_{31} + 60\, U_{33} = 0 \rightarrow U_{33} = -1{,}2\, U_{31}$

$$U_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \#$$

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$$\overline{U_1} = \begin{bmatrix} -0.7656 \\ 0 \\ -0.6433 \end{bmatrix} \qquad \overline{U_2} = \begin{bmatrix} -0.6433 \\ 0 \\ 0.7656 \end{bmatrix} \qquad \overline{U_3} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

* get D-matrix $\rightarrow \begin{bmatrix} \sqrt{\lambda_1} & 0 & 0 \\ 0 & \sqrt{\lambda_2} & 0 \\ 0 & 0 & \sqrt{\lambda_3} \end{bmatrix}$ $D = \begin{bmatrix} 11.109 & 0 & 0 \\ 0 & 1.26 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

* to get V & use $V_i = \frac{1}{\sigma_i} A^T u_i$

$$V_1 = \frac{1}{11.109} \begin{bmatrix} 3 & 0 & 4 \\ 0 & 1 & 0 \\ 8 & 0 & 6 \end{bmatrix} \begin{bmatrix} -0.7656 \\ 0 \\ -0.6433 \end{bmatrix} = \begin{bmatrix} -0.4383 \\ 0 \\ -0.8988 \end{bmatrix} \#$$

$$V_2 = \frac{1}{1.26} \begin{bmatrix} 3 & 0 & 4 \\ 0 & 1 & 0 \\ 8 & 0 & 6 \end{bmatrix} \begin{bmatrix} -0.6433 \\ 0 \\ 0.7656 \end{bmatrix} = \begin{bmatrix} 0.8988 \\ 0 \\ -0.4387 \end{bmatrix} \#$$

$$V_3 = \frac{1}{1} \begin{bmatrix} 3 & 0 & 4 \\ 0 & 1 & 0 \\ 8 & 0 & 6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \#$$

$$U = \begin{bmatrix} -0.7656 & -0.6433 & 0 \\ 0 & 0 & 1 \\ -0.6433 & 0.7656 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} -0.4383 & 0.8988 & 0 \\ 0 & 0 & 1 \\ -0.8988 & -0.4387 & 0 \end{bmatrix}$$

multiplexing: ① with channel knowledge $C = \log_2 \left(1 + \frac{P_T \lambda_{max}}{\sigma_n^2}\right)$

$$C = \log_2 \left(1 + \frac{10^{-0.175} \times 123.4118}{10^{0.13}}\right) = 5.4039$$

② without channel knowledge $C = \sum \log_2 \left(1 + \frac{P_T \lambda_i}{m \sigma_n^2}\right)$

$$C = \log_2 \left(1 + \frac{10^{-0.175} \times 123.4118}{3 \times 10^{0.13}}\right) + \log_2 \left(1 + \frac{10^{-0.175} \times 1.5882}{3 \times 10^{0.13}}\right)$$

$$+ \log_2 \left(1 + \frac{10^{-0.175} \times 1}{3 \times 10^{0.13}}\right) = 4.27376$$

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diversity: ① with channel knowledge $C = \log_2 \left(1 + \frac{P_T \lambda_{max}}{\sigma_n^2}\right)$

$$C = \log_2 \left(1 + \frac{10^{-0.175} \times 123.4118}{10^{0.13}}\right) = 5.4039$$

② without channel knowledge $C = \log_2 \left(1 + \frac{P_T}{m\sigma_n^2} \sum \lambda_i\right)$

$$C = \log_2 \left(1 + \frac{10^{-0.175}}{3 \times 10^{0.13}} \left[123.4118 + 1.5882 + 1\right]\right) = 3.9135$$

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