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LEVEL CROSSING RATE AND AVERAGE FADE DURATION OF NON-REGENERATIVE AND REGENERATIVE RELAYS IN MULTI-HOP RAYLEIGH FADING CHANNEL

Undergraduate Final Year Project

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DEDICATION

We would like to dedicate this thesis to:

- Our Almighty God
- Our parents
- Our fellow colleagues

APPROVAL AND CERTIFICATION

We confirm that this final year project has been done at the University of Rwanda, College of Science and Technology, School of Engineering, Department of Electrical and Electronics Engineering for the award of bachelor's degree in Electronics and Telecommunications Engineering under the supervision of Dr. Charles Kabiri. We also confirm that to the best of our knowledge, it is our original work. Contributions from other sources have been properly acknowledged.

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ABSTRACT

Multi-hop is a promising technology which enables high data rates resulting in high throughput, improves the system capacity and the performance over fading wireless channels. The multi-hop system can be enhanced using different protocols, including the DF (decode and forward) and AF (amplify and forward). In this study, we analyze the non-regenerative (AF) and regenerative (DF) relaying of the Multi-hop Rayleigh fading channel in terms of level crossing rate and average fade duration. We then investigate which type of relaying works better in terms of those two metrics.

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LIST OF ABBREVIATIONS, ACRONYMS AND SYMBOLS

LCR: Level Crossing Rate

AFD: Average Fade Duration

S: Source

D: Destination

R: Relay

$Y_1(t)$: The received signal at first relay

$\alpha_1(t)$: The fading amplitude between S and R_1

$N_1(t)$ and N_0: The noise (Additive White Gaussian Noise (AWGN))

f_m: The maximum Doppler frequency

ρ: The value of the specified level R normalized to the local rms amplitude of the fading envelope.

f_D: The maximum Doppler shift for the destination

f_S: The maximum Doppler shift for the source

f_i: The maximum Doppler shift for the relays

$P(R)$: The Cumulative Density Function (CDF) of Rayleigh channel

τ: Average Fade Duration

α_{S-R_1}: The amplitude between source S and the first relay R_1

n_{S-R_1}: The noise detected between source S and the first relay R_1

P_T: Power transmitted

1 INTRODUCTION

Transmitted radio signals in wireless communications experience multipath fading which later degrade the performance of the communication system due to envelope fluctuations. This is due to the obstruction of the tall buildings or mountains as many radio systems operate in urban areas, and leads to the loss of line of sight. Therefore, due to multiple reflections from various objects, the electromagnetic waves travel different paths before reaching the destination. It is important to characterize those envelope fluctuations in terms of fading metrics. Average Fade Duration and Level Crossing Rate are two metrics which are useful in characterizing the behavior of the envelope fluctuations and for designing diversity schemes used in mobile communications systems.

Multi-hop technology has emerged as a promising technique to increase the signal coverage area and it is categorized into regenerative and non-regenerative relays depending on the relay functionality. In multi-hop systems, nodes communicate with each other using wireless channels. It uses two or more hops to transmit information from source to destination. Nodes along the path can receive and forward packets wirelessly and as a result there is no need of common infrastructure. Therefore, the main components of the multi-hop communication are source, relays and destination nodes. Multi-hop presents several advantages including the extended coverage network, improved connectivity and high throughput.

We provide the performance analysis of regenerative relay known as “Direct-Forward” and non-regenerative relay known as “Amplify-Forward” systems in terms of Level Crossing Rate (LCR) and Average Fade Duration (AFD) dealing with the time varying nature of the Rayleigh Fading channel. The non-regenerative relay acts as a repeater which retransmits original signal after amplifying it, while regenerative relay decodes errors, regenerates the signal, and retransmits the copy of the received signal. The non-regenerative relay is classified into 2 categories: channel state information (CSI) assisted relays and fixed gain relays. In this paper we will emphasize on the fixed gain relays (blind/semi-blind relays).

We first provide the closed form expressions of the AFD and LCR for both regenerative and non-regenerative relays. We then proceed with analyzing their simulations in OriginPro 8 to determine which type of relays performs better.

1.1 Literature review

Since most cellular radio systems operate in urban areas with very long buildings, it is not easy to find the direct line of sight. As a result, the signal reaches the receiver through many paths resulting into multipath fading. Multipath fading is the problem to communication since the signal loses its strength on its way to the receiver, resulting in the degradation performance of the communications system.

It is therefore very necessary to discuss those random paths in terms of fading statistics so as to understand or avoid the effects of the signal fades. The two important second order statistics: level crossing rate and average fade duration can contribute in determining the effects of the signal fades. Moreover, these two parameters are of interest in the analysis of a multi-hop system. Multi-hop technology emerged as a technique to combat fading in wireless communications and it is based on the broadcast nature of the wireless medium. It allows exhausting or enjoying the benefits of the fact that the space is diverse by sharing physical resources through virtual relays.

In [1], Zoran, Nikola and George, studied the second order statistics of the multi-hop non-regenerative relay of a Rayleigh fading channel. They provided approximate solution for the average LCR at the output of multi-hop non-regenerative relay transmission system. Other works have studied the performance analysis of regenerative and non-regenerative systems in terms of bit error rate and outage probability [2]. On addition to that, Lydia et. al in [3] concluded that non-regenerative diversity systems outperform those using regenerative relays in terms of bit error rate (BER). Several other researches were also done on both relays like the development of decode-and-forward strategies for relay networks with many relays, antennas, sources and destination [4].

1.2 Problem Statement

LCR and AFD are two important second order statistic parameters which convey the useful information about the dynamic temporal behavior of multipath fading channels. Different works have been done on the analysis of regenerative and non-regenerative relays in terms of different parameters like Bit Error Rate (BER) and outage probability. However, none of them have touched together and come up with the performance of the time varying nature of the fading channels in terms of level crossing rate and average fade duration. It is important to analyze the second order statistics of a DF and an AF multi-hop Rayleigh fading channel so as to investigate which type of the relay outperforms the other in terms of those two metrics.

1.3 Objectives

The primary objective of this project is to provide the performance analysis of DF and AF systems in terms of Level Crossing Rate (LCR) and Average Fade Duration (AFD) dealing with the time varying nature of the Rayleigh Fading channel. In order to achieve this objective, we are going to first provide the expressions of the LCR and AFD for the regenerative and non-regenerative relays of the multi-hop Rayleigh fading channel.

Furthermore, we are going to come up with the OriginPro 8 simulations which will help in determining which type of the relays outperforms the other.

1.4 Methodology

In this project, we first derive the closed form expressions for the regenerative and non-regenerative relays of the multi-hop Rayleigh fading channel. We proceed with the computations in OriginPro 8 which contributed in determining the characteristics of all relays.

2 SYSTEM MODEL

The system model consists of the source S, relays in series and the destination D by assuming the Rayleigh channel. Here the source S transmits the information to the destination D through $N-1$ relays acting like intermediate stations from one hop to the next by receiving and transmitting information in order to enhance communication between the source and the destination as represented on the figure 1.

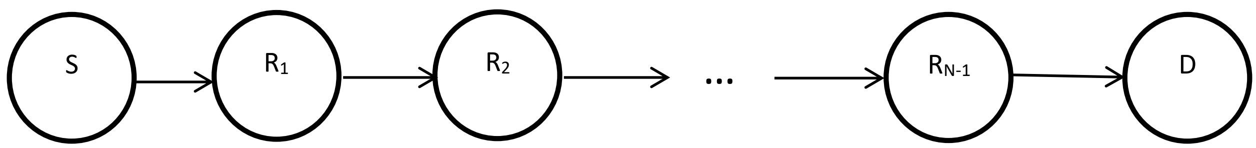

Figure 1- The system model of the amplify and forward multi-hop network

2.1 Rayleigh fading and second order statistics

Rayleigh fading channel is a statistical model of non-line of sight channels. In Rayleigh fading, the transmitter and the receiver can't see each other; there is no direct line of sight. Since there are many objects in the path to the destination, therefore the signal from the transmitter take different paths to reach the receiver. Signals at the receiver are expected to sum up. However, this is not the case since some signals will be in phase and others not due to the paths lengths. Therefore, Rayleigh fading is best used to analyze the situations like those and most importantly when there is no dominant signal or the line of sight.

The envelope of the sum of two quadrature Gaussian noise signals obeys a Rayleigh distribution. Rayleigh fading distribution characterizes the sum of the many contributions coming from different directions as a mobile moves over very small distances [5]. It is also used to describe statistical time varying nature of the envelope of a multipath component. The probability density function of Rayleigh distribution is given by (1) and figure 3. The Rayleigh distribution generates Rician distribution as the dominant line of sight dominates, i.e. Rayleigh fading is a special case of Rician fading when there is no direct line of sight signal.

A typical Rayleigh fading envelope appears as in figure 3. Therefore, using the speed, you are traveling at and the time plotted on the x-axis, you can determine the distance you have travelled from the transmitter or the base station.

$$P(r) = \begin{cases} \frac{r}{\sigma^2} e^{\left(-\frac{r^2}{2\sigma^2}\right)} & (0 \le r \le \infty) \\ 0 & (r < 0) \end{cases} \tag{1}$$

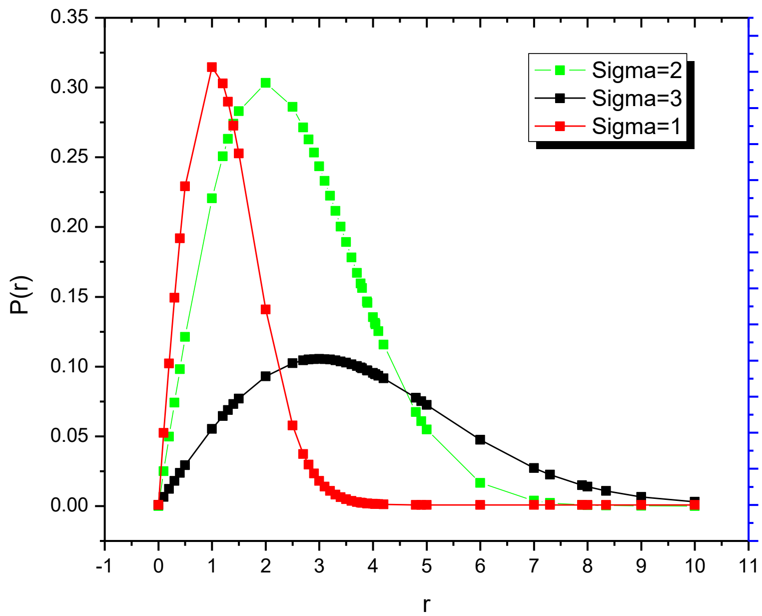

Figure 2- PDF of Rayleigh fading

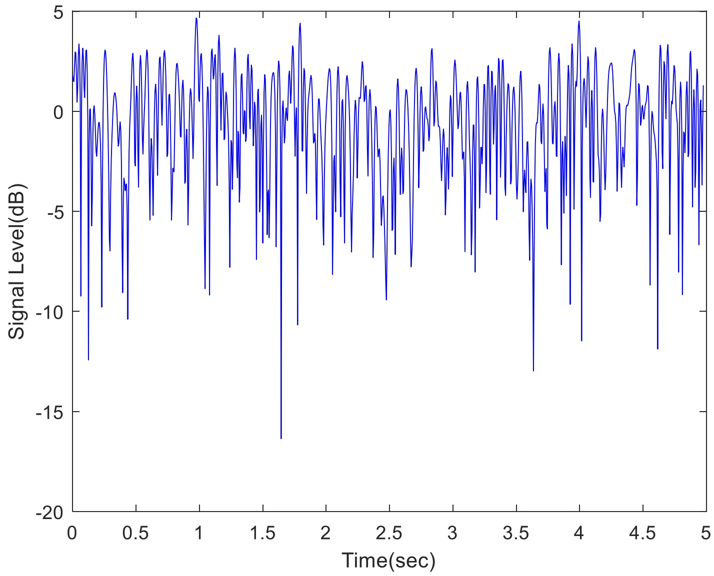

Figure 3- A typical Rayleigh envelope at a random frequency (20Hz)

2.1.1 Level Crossing Rate

The Level Crossing Rate (LCR) and the Average Fade Duration (AFD) are two important parameters of the multipath fading channels. The Level Crossing Rate (LCR) defines the expected rate at which the Rayleigh fading envelope crosses a specified positive level in a positive-going direction. In other words, from the figure 3, we are interested in knowing how many times per second do we get into the fade, and we need to determine the acceptable signal strength below which you say the signal is not usable, that's the threshold. In [2, Eq. 4.80], the number of level crossings per second is given by:

$$N_R = \int_0^{\infty} \dot{r}p(R,\dot{r})d\dot{r} = \sqrt{2\pi} f_m \rho e^{-\rho^2} \qquad (2)$$

Where $\dot{r}$ is the time derivative of $r(t)$,

$p(R,\dot{r})$ is the joint density function of r and $\dot{r}$ at $r = R$.

f_m is the maximum Doppler frequency and,

$\rho = \frac{R}{R_{rms}}$ is the specified level R normalized to the local rms amplitude of the fading envelope.

2.1.2 Average Fade Duration

On the other hand, the Average Fade Duration (AFD) defines the average period of time for which the received signal is below a specified level R, that's how much time we stay below the threshold. In [2, Eq. 4.84] for the Rayleigh fading, it is given in terms of ρ and f_m by (3). It determines the number of signaling bits that may be lost during a fade and decreases as the maximum Doppler frequency f_m increases.

$$\bar{\tau} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}} \qquad (3)$$

2.2 Non-regenerative and regenerative diversity systems

2.2.1 Non-regenerative diversity system

Non-regenerative relays (Amplify and forward) are the simplest form of relaying where the received signal at the relay will be amplified and then forwarded to the destination. The amplification is regarded as a multiplication with a multiplication factor. Comparing it to the "decode and forward," it is low-complexity solution as it doesn't require the decoding process at the relays. This protocol has got many advantages including: good diversity gain, high capacity when number of relays tends to infinity is increasing and others. The major drawback of this protocol is that the noise is also amplified as seen in (5). The use of both regenerative and non-regenerative relays is an advantage in multi-hop diversity system compared to the single hop since they are both chosen wisely [6].

Suppose that we are sending the signal $x(t)$ at first relay, R_1:

$$Y_1(t) = [\alpha_1(t) \times x(t)] + N_1(t) \qquad (4)$$

The received signal at R_2:

$$\mathrm{Y_2(t) = Y_1(t) \times G_1}$$

$$\mathrm{Y_2(t) = G_1 \times \alpha_2(t) \times [\alpha_1(t) \times x(t) + N_1(t)] + N_2(t)} \qquad (5)$$

The received signal at R_3:

$$\mathrm{Y_3(t) = Y_2(t) \times G_2}$$

Replacing the value of $\mathrm{Y_2(t)}$ in $\mathrm{Y_3(t)}$:

$\mathrm{Y_3(t) = [G_1 \times \alpha_2(t) \times \; G_2 \times \alpha_3(t) \times \; Y_2(t)] + N_3(t)}$

$\mathrm{\alpha(t) = \; G_1 \times \alpha_2(t) \times \; G_2 \times \alpha_3(t)}$ is the fading amplitude.

Therefore, the generalized total fading amplitude is given by:

$$\mathrm{\alpha(t) = \prod_{i=1}^{N} \alpha_i(t) \times G_{i-1}} \quad (6)$$

If $X_i(t)$ represents the Rayleigh envelope, the Rayleigh random process is presented in [1, eq.5]:

$$\prod_{i=1}^{N} X_i(t) \quad (7)$$

Comparing (6) and (7), we realized that the total fading amplitude is described by Rayleigh random process,

$$\alpha(t) = Y(t)$$

$$X_i(t) = \alpha_i(t) \times G_{i-1} \quad (8)$$

We define the Level Crossing Rate (LCR) as:

$$N(\rho) = \sqrt{2\pi} \times fm \times \rho \times e^{-\rho^2} \quad (9)$$

Given that we have two stations which are all mobile, we express fm from eq.9 according to [1, eq.4]: $fm = \sqrt{f_{mi}'^2 + f_{mi}''^2}$ (10)

Where:

- f_{mi}' : the maximum Doppler shift induced by the motion of the transmitting station.
- f_{mi}'': the maximum Doppler shift induced by the motion of the receiving station.

For the i[th] hop, the general equation is:

$$\sum_{i=0}^{N} f_i^2 = fm_s^2 + 2\sum_i^{N-1} fm_i^2 + fm_D^2 \quad (11)$$

Putting together (6) and (11) into (9),

We obtain the generalized formula of LCR in non-regenerative relays:

$$N(\alpha) = \sqrt{2\pi} \times (fm_s^2 + 2\sum_i^{N-1} fm_i^2 + fm_D^2) \times \alpha \times e^{-\alpha^2} \quad (12)$$

In order to obtain the AFD, we need to divide CDF of the Rayleigh channel by the LCR:

$$\tau = \frac{P(R)}{N} \quad (13)$$

$$\tau = \frac{e^{\alpha^2} - 1}{\sqrt{2\pi} \times \alpha \times (fm_S^2 + 2\sum_i^{N-1} fm_i^2 + fm_D^2)} \quad (14)$$

2.2.2 Regenerative diversity system

In the regenerative relays or in other words “decode and forward,” is complex compared to the “amplify and forward” where the relay decodes the received signal in order to get the original information. Next, the decoded information is retransmitted to the destination. Therefore, in the first slot when the transmitter sends the data to the relay station, it will be decoded and be checked to identify if there is no error in the decoded signal. If there is no error, the transmitter will send the signal immediately to the receiver. If there is an error, the relay will encode the signal and retransmit it to the destination (receiver), and as a result there will be the reduction of the error at the receiver. The complexity of “decode and forward” is due to its full processing capability. The “decode and forward” protocol requires the media access control layer which is not the case for the “amplify and forward” protocol [7].

Suppose the signal $s(t)$ is transmitted from the source in the “decode and forward” system with the same system model as in figure 1.

Therefore, the signal to reach at R_1 is given by:

$$Y(t) = [\alpha_{S-R_1} \times s(t)] + n_{S-R_1} \quad (15)$$

The signal $Y(t)$ at R_1 is the same as the signal at R_1 in “amplify and forward” as seen in (4)

The signal to reach to the next relay is given by:

$$D(t) = [\alpha_{R_1-R_2} \times Y'(t)] + n_{R_1-R_2} \quad (16)$$

$Y'(t)$ is the signal retransmitted by the relay (it is the information estimate).

The amplitudes α_{S-R_1} and $\alpha_{R_1-R_2}$ are Rayleigh distributed.

At the destination, D combines all the information sent by the relays together with the information that R_1 attempts to decode from the source S. It should be noted that all relays and the source S are transmitting at the same power.

Therefore, the Signal to Noise Ratio,

$$S = \frac{P_T}{N_o} \quad (17)$$

"Average Fade Duration and Level Crossing Rate" are related to the "Average Outage Rate (AOR)" and "Average Outage Duration (AOD) [8]" like:

The LCR and the AFD evaluated at threshold X_0 is equal to the AOR and AOD evaluated at threshold T_0 and their relationship is as follow:

$$X_0 = \sqrt{\frac{2^{2T_{0-1}}}{S}} \quad (18)$$

Replacing (18) into (9), we obtain the LCR:

$$N(X_0) = \sqrt{2\pi} \times fm \times X_0 \times e^{-{X_0}^2} \quad (19)$$

In order to obtain the AFD, we insert (18) into (19) to get:

$$\tau(X_0) = \frac{e^{{X_0}^2} - 1}{X_0 \times f_m \times \sqrt{2\pi}} \quad (20)$$

3 NUMERICAL RESULTS

We provide the illustrations of the LCR and AFD of the regenerative and non-regenerative. The expressions obtained above are validated under the computation program, OriginPro 8.

We considered a multi-hop system with source S, "4, 7 and 11" relays, and destination D for regenerative and non-regenerative relays. The fixed gain is assumed semi-blind and is equal to 1. Frequencies in the range of H_Z are chosen randomly for the source, hops and the destination for both regenerative and non-regenerative relays. The figures depict the received signal's LCR, $N(\alpha)$ or AFD, $\tau(\alpha)$ versus the threshold α for the non-regenerative relays. For the regenerative relays, the figures depict the received signal's LCR, $N(X_0)$ or AFD, $\tau(X_0)$ versus the threshold X_0.

From the figures 4 and 5, it is seen that, as we increase the threshold, the level crossing rate decreases exponentially for both regenerative and non-regenerative relays and are slightly equal for both relays. However, as we decrease the number of relays for the regenerative relays, we are approaching the zero level crossing rate as the threshold keeps increasing compared to the time when the number of relays is increased. And this phenomenon is opposite with the non-regenerative relays.

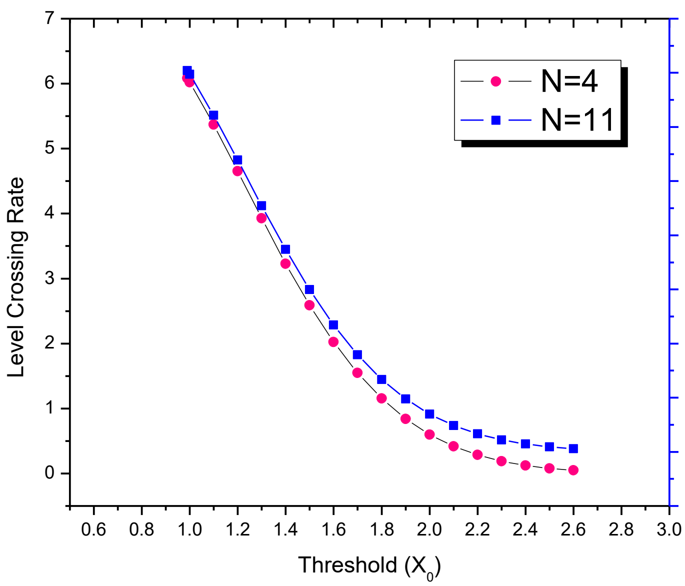

Figure 4- Level Crossing Rate with regenerative relays

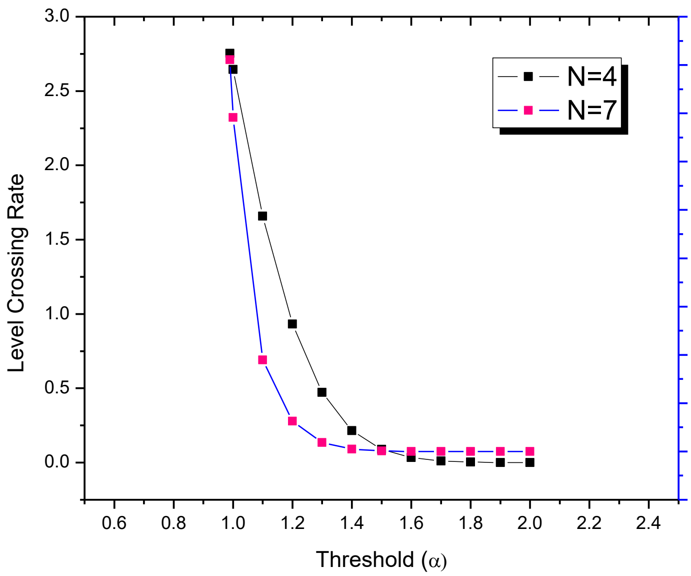

Figure 5- Level Crossing Rate with non-regenerative relays

On the other hand, under the same conditions, the average fade duration increases for the non-regenerative as the threshold increases. As the threshold keeps increasing and number of relays increases for the non-regenerative relays, the average fade duration also keeps increasing and it reaches a point where the average fade duration is equal for both number of relays as seen in figure 6.

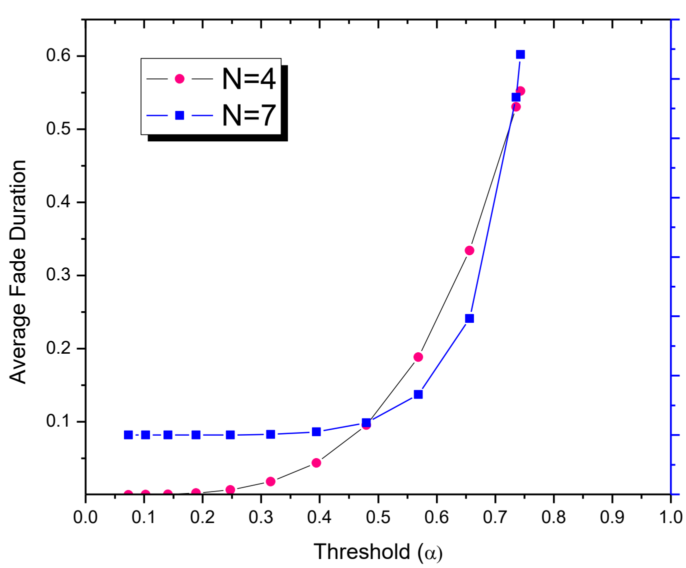

Figure 6- Average Fade Duration with non-regenerative relays

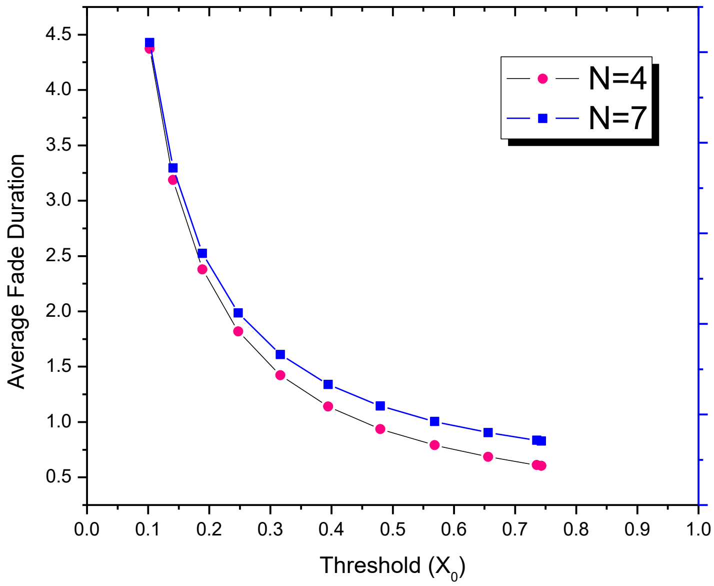

Figure 7- Average Fade Duration with regenerative relays

Whereas for regenerative relays as seen in figure 7, as the number of relays decreases, the fade duration tends to be zero faster than when you increase the number of relays, even though for both cases the decrease is exponential. The increase of the average fade duration is not a good scenario. This is because, if for example, you are transmitting a $1000bps$ and the average fade duration is $20ms$, you realize that $20\ bits$ are lost, and as you keep increasing the fade duration, the number of bits lost increases.

4 CONCLUSION

In this study, the expressions of the level crossing rate and average fade duration for both regenerative and non-regenerative relays in multi-hop Rayleigh fading channel have been developed. The expressions given are valid and were analyzed basing on the available research literatures and the knowledge from class. They were all used to analyze the performance of the multi-hop system in Rayleigh fading channel basing on the two metrics. It was noticed that the behavior of the level crossing rate is quite the same for both regenerative and non-regenerative relays. However, the average fade duration decreases for the regenerative relays as the threshold increases. We conclude that under the same conditions, the regenerative relays outperform the non-regenerative relays in terms of average fade duration whereas in terms of level crossing rate, the behavior is the same.

5 RECOMMENDATIONS

There are different fading distributions which are all the results of the way the rays behave from the receiver to the destination including the Rician and Nakagami-m distributions. Though they all have the relationship with the Rayleigh distribution channel, they all deserve to be studied alone and analyzed with the two metrics, “level crossing rate and average fade duration” but also with other parameters like outage probability and bit error rate.

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