In [1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

David Langus Rodriguez, Dean Ward, and Albert DiCroce worked as a group for this assignment.

Problem 1

1.

Overall yield is 8000 of 14000 parts or 57.14%

Problem 2

1.

Because the Verilog code executes in parallel, the value of x will be 1 (x = 1) and the value for y will be 2 (y = 2)

2.

Because C language executes sequentially the value for x will not be initialized with the (a + b) value. The variable x will be equal to 1 (x = 1) since x will be at the initialized value

3.

Since Verilog executes in parallel and C sequantially there is a difference.

Problem 3

1.

In [2]:
```
df = pd.read_csv('./fullFreqData/fpga1.csv', header=None)
```

In [3]:
```
df.shape
```

Out[3]:
```
(512, 100)
```

In [4]:
```
df_t = df.T
```

In [5]:
```
df_t
```

Out[5]:

	0	1	2	3	4	5	6	7	8	9
0	185.752	186.415	186.443	187.257	187.638	185.049	186.154	187.196	187.427	187.268
1	185.752	186.460	186.424	187.286	187.615	185.041	186.141	187.147	187.408	187.249
2	185.704	186.457	186.394	187.278	187.558	185.071	186.184	187.133	187.423	187.214
3	185.696	186.404	186.435	187.183	187.616	185.062	186.194	187.216	187.415	187.254
4	185.673	186.434	186.381	187.270	187.567	185.064	186.118	187.160	187.397	187.257
...	...	...	...	...	...	...	...	...	...	...
95	185.450	186.152	186.069	187.028	187.397	184.867	185.944	186.909	187.164	186.990
96	185.438	186.148	186.187	187.069	187.405	184.836	185.889	186.955	187.169	187.075
97	185.463	186.162	186.166	187.030	187.410	184.841	185.945	186.989	187.167	187.106
98	185.450	186.181	186.182	187.021	187.412	184.853	185.897	186.978	187.179	187.049
99	185.517	186.181	186.150	187.039	187.427	184.805	185.873	186.976	187.130	187.082

100 rows × 512 columns

2.

In [6]:
```
means = df_t.describe().T["mean"]
```

In [7]:
```
means_np = np.array(means)
```

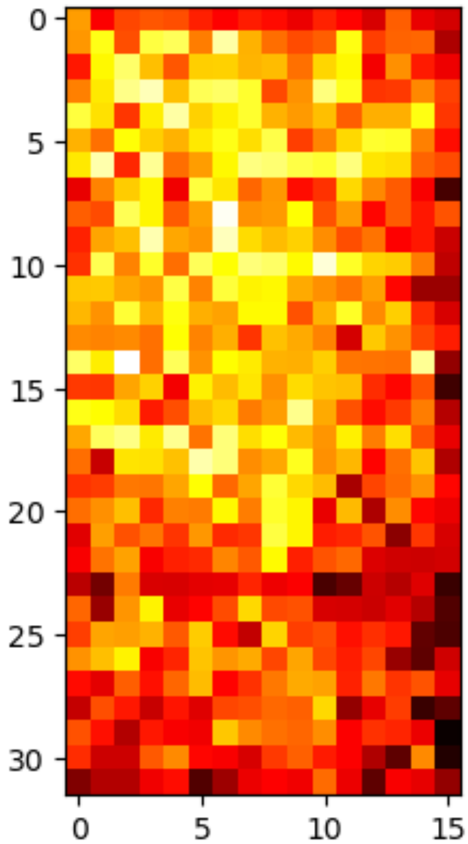
In [8]:
```
means_np_reshaped = np.flip(means_np.reshape(32, 16), axis=0)
```

In [9]:
```
means_np_reshaped.shape
```

Out[9]:
```
(32, 16)
```

3.

In [10]:
```
a = np.random.random((32, 16))
plt.imshow(means_np_reshaped, cmap='hot', interpolation='nearest')
plt.show()
```



4.

The heatmap below show a visual representation of the FPGA die and of which PUF freqencies are higher/low. The top center of the FPGA had the higher measured PUF clock frequencies. This is probably due to the less interconnect delays in that area of the chip and allow the RO to achieve a higher frequency

Problem 4

1 & 2.

In [11]:
```
array_of_pufs = []
for i in range(193):
    puf = np.zeros(256)
    df = pd.read_csv(f'./fullFreqData/fpga{i+1}.csv', header=None)
    df_t = df.T
    means = df_t.describe().T["mean"]
    means_np = np.array(means)
    means_np_reshaped = means_np.reshape(256, 2)
    for j in range(256):
        if means_np_reshaped[j, 0] >= means_np_reshaped[j, 1]:
            puf[j] = True
            # puf[j] = 1
        else:
            # puf[j] = 0
            puf[j] = False

    array_of_pufs.append(puf)
    #print(np.array(array_of_pufs)[i])
```

In [12]:
```
# Function to convert string of bit to a ByteArray
def bitstring_to_bytes(s):
    v = int(s, 2)
    b = bytearray()
    while v:
        b.append(v & 0xff)
        v >>= 8
    return bytes(b[::-1])
```

In [13]:
```
fpga_puf_1 = array_of_pufs[0].astype(int)
bit_string = np.array2string(fpga_puf_1).replace('\n', '').replace('[', '').

print(bitstring_to_bytes(bit_string).hex())
```

```
2db7bf6cc7461f85c35a0b794dd179d587ffffe97e0f37d01b0bdbaf155939ad
```

3.

In [14]:
```
# Calculate Hamming distances for all FPGAs pairs
unique_array_of_pufs = []
hamming = []
num_loops = 192 # Will stil loop through all 193 cause k=i+1
for i in range(num_loops):
    uniq_puf = np.zeros(256)
    puf1 = array_of_pufs[i].astype(bool)
    k = i+1
    while k <= num_loops:
        puf2 = array_of_pufs[k].astype(bool)
        for j in range(256):
            uniq_puf[j] = puf1[j] ^ puf2[j]
            # print(puf1)
            # print(puf2)
        # print(uniq_puf)
        hamming = np.sum(uniq_puf, 0)
        # print(hamming)
        k+=1
        unique_array_of_pufs.append(hamming)
```

In [15]:
```
len(unique_array_of_pufs) # Total number of Hamming distances 192*193/2
```

Out[15]:
```
18528
```

In [16]:
```
hamming_d = np.array(unique_array_of_pufs)
```

In [17]:
```
hamming_mean = np.mean(hamming_d) # Mean
hamming_mean
```

Out[17]:
```
118.73024611398964
```

In [18]:
```
np.median(hamming_d) # Median
```

Out[18]:
```
119.0
```

In [19]:
```
np.std(hamming_d) # Standard Div.
```

Out[19]:
```
8.090671702168963
```

4.

In [20]:
```
(hamming_mean / 256 * 100) # Uniqueness
```

Out[20]:
```
46.3790023882772
```

- 50% is the ideal uniqueness, our PUFs exibit good uniqueness values with respect to the ideal (46.37%)

In [21]:
```
# hamming_dist = np.sum(unique_array_of_pufs, 1)
hamming_mean = (np.sum(unique_array_of_pufs, None)/np.size(unique_array_of_p
print(hamming_mean)
print(np.median(unique_array_of_pufs, None))
print(np.std(unique_array_of_pufs, None))
# Number 4.4
print((hamming_mean/256) *100)
```

```
118.73024611398964
119.0
8.090671702168963
46.3790023882772
```

Problem 5

1 & 2.

In [22]:
```
array_of_pufs = []

puf = np.zeros(256)
df = pd.read_csv(f'./fullFreqData/fpga1.csv', header=None)
df_t = df.T
#means = df_t.describe().T["mean"]
data = np.array(df_t)
data_re = data.reshape(100, 256, 2)
#print(data_re)
for i in range(100):
    for j in range(256):
        if data_re[i, j, 0] >= data_re[i, j, 1]:
            puf[j] = True
            # puf[j] = 1
        else:
            # puf[j] = 0
            puf[j] = False

    array_of_pufs.append(puf)
print(np.array(array_of_pufs)[i].shape)
```

```
(256,)
```

3.

In [23]:
```
# Calculate Hamming distances for all FPGAs pairs
unique_array_of_pufs = []
hamming = []
num_loops = 99 # Will stil loop through all 100 cause k=i+1
for i in range(num_loops):
    uniq_puf = np.zeros(256)
    puf1 = array_of_pufs[i].astype(bool)
    k = i+1
    while k <= num_loops:
        puf2 = array_of_pufs[k].astype(bool)
        for j in range(256):
            uniq_puf[j] = puf1[j] ^ puf2[j]
            # print(puf1)
            # print(puf2)
        # print(uniq_puf)
        hamming = np.sum(uniq_puf, 0)
        # print(hamming)
        k+=1
        unique_array_of_pufs.append(hamming)
```

In [24]:
```
len(unique_array_of_pufs) # Total number of Hamming distances 99*100/2
```

Out[24]:
```
4950
```

In [25]:
```
hamming_d = np.array(unique_array_of_pufs)
```

In [26]:
```
hamming_mean = np.mean(hamming_d) # Mean
hamming_mean
```

Out[26]:
```
0.0
```

In [27]:
```
np.median(hamming_d) # Median
```

Out[27]:
```
0.0
```

In [28]:
```
np.std(hamming_d) # Standard Div.
```

Out[28]:
```
0.0
```

4.

In [29]:
```
(hamming_mean / 256 * 100) # Reliableness
```

Out[29]:
```
0.0
```

- 0% is the ideal reliableness, our PUFs exibit good reliability values with respect to the ideal (0%)

Problem 6

1 & 2.

In [30]:
```
array_of_pufs = []
for i in range(193):
    puf = np.zeros(256)
    df = pd.read_csv(f'./fullFreqData/fpga{i+1}.csv', header=None)
    df_t = df.T
    means = df_t.describe().T["mean"]
    means_np = np.array(means)
    means_np_reshaped = means_np.reshape(256, 2)
    for j in range(256):
        if means_np_reshaped[j, 0] >= means_np_reshaped[j, 1]:
            puf[j] = True
            # puf[j] = 1
        else:
            # puf[j] = 0
            puf[j] = False

    array_of_pufs.append(puf)
print(np.array(array_of_pufs)[0])
```

```
[0. 0. 1. 0. 1. 1. 0. 1. 1. 0. 1. 1. 0. 1. 1. 1. 1. 0. 1. 1. 1. 1. 1. 1.
 0. 1. 1. 0. 1. 1. 0. 0. 1. 1. 0. 0. 0. 1. 1. 1. 0. 1. 0. 0. 0. 1. 1. 0.
 0. 0. 0. 1. 1. 1. 1. 1. 1. 0. 0. 0. 0. 1. 0. 1. 1. 1. 0. 0. 0. 0. 1. 1.
 0. 1. 0. 1. 1. 0. 1. 0. 0. 0. 0. 0. 1. 0. 1. 1. 0. 1. 1. 1. 1. 0. 0. 1.
 0. 1. 0. 0. 1. 1. 0. 1. 1. 1. 0. 1. 0. 0. 0. 1. 0. 1. 1. 1. 1. 0. 0. 1.
 1. 1. 0. 1. 0. 1. 0. 1. 1. 0. 0. 0. 0. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.
 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 0. 1. 0. 0. 1. 0. 1. 1. 1. 1. 1. 1. 0.
 0. 0. 0. 0. 1. 1. 1. 1. 0. 0. 1. 1. 0. 1. 1. 1. 1. 1. 0. 1. 0. 0. 0. 0.
 0. 0. 0. 1. 1. 0. 1. 1. 0. 0. 0. 0. 1. 0. 1. 1. 1. 1. 0. 1. 1. 0. 1. 1.
 1. 0. 1. 0. 1. 1. 1. 1. 0. 0. 0. 1. 0. 1. 0. 1. 0. 1. 0. 1. 1. 0. 0. 1.
 0. 0. 1. 1. 1. 0. 0. 1. 1. 0. 1. 0. 1. 1. 0. 1.]
```

3.

In [31]:
```
uniformity_per_puf = []
for puf in np.array(array_of_pufs):
    unique, counts = np.unique(puf, return_counts=True)
    uniformity_per_puf.append(counts[0] / 256 * 100)
```

In [32]:
```
np.array(uniformity_per_puf) # Uniformity per PUF with respect to the zero (
```

Out[32]:
```
array([41.796875, 50.      , 49.609375, 50.78125 , 55.859375, 48.828125,
       38.671875, 51.5625  , 51.953125, 43.359375, 45.3125  , 47.65625 ,
       49.609375, 46.875   , 45.703125, 41.796875, 47.65625 , 48.4375  ,
       45.3125  , 51.171875, 48.4375  , 47.65625 , 48.828125, 50.      ,
       47.65625 , 41.796875, 50.78125 , 42.96875 , 53.90625 , 50.78125 ,
       47.65625 , 47.65625 , 51.953125, 51.953125, 47.265625, 50.      ,
       46.09375 , 45.3125  , 53.515625, 51.171875, 40.234375, 51.5625  ,
       50.390625, 44.921875, 53.125   , 48.828125, 45.3125  , 53.515625,
       42.96875 , 54.296875, 46.484375, 41.015625, 48.046875, 49.21875 ,
       54.296875, 50.390625, 53.125   , 46.875   , 42.578125, 43.75    ,
       55.078125, 50.      , 46.484375, 50.78125 , 46.875   , 51.171875,
       50.390625, 50.      , 47.65625 , 46.09375 , 49.21875 , 47.65625 ,
       50.      , 51.5625  , 46.09375 , 48.046875, 51.5625  , 46.484375,
       45.3125  , 44.921875, 47.265625, 50.      , 45.3125  , 47.265625,
       50.390625, 48.046875, 46.09375 , 50.390625, 48.046875, 46.09375 ,
       43.75    , 42.96875 , 49.609375, 40.625   , 41.796875, 51.953125,
       52.34375 , 44.921875, 48.046875, 54.296875, 44.921875, 48.046875,
       49.609375, 50.      , 49.21875 , 44.53125 , 47.65625 , 50.      ,
       43.359375, 46.875   , 55.078125, 50.78125 , 51.171875, 48.828125,
       53.125   , 42.1875  , 56.25    , 46.484375, 46.09375 , 41.015625,
       44.921875, 50.78125 , 43.75    , 49.21875 , 41.796875, 36.71875 ,
       44.53125 , 55.46875 , 49.609375, 53.515625, 49.21875 , 42.96875 ,
       50.390625, 46.484375, 49.21875 , 44.53125 , 48.4375  , 46.484375,
       40.625   , 47.65625 , 48.828125, 51.5625  , 42.578125, 47.65625 ,
       46.875   , 45.703125, 44.140625, 49.21875 , 43.359375, 53.90625 ,
       51.953125, 45.703125, 44.921875, 44.140625, 48.4375  , 51.171875,
       46.875   , 42.1875  , 44.921875, 46.484375, 53.125   , 51.5625  ,
       51.5625  , 45.703125, 49.21875 , 45.3125  , 50.390625, 46.09375 ,
       41.796875, 42.1875  , 39.453125, 45.703125, 45.703125, 46.875   ,
       52.34375 , 48.828125, 49.609375, 48.828125, 46.09375 , 48.046875,
       51.953125, 42.96875 , 55.859375, 45.3125  , 42.96875 , 48.828125,
       42.96875 , 45.3125  , 57.8125  , 45.3125  , 48.046875, 40.234375,
       51.5625  ])
```

In [33]:
```python
np.mean(np.array(uniformity_per_puf)) # Uniformity mean %
```

Out[33]:
```
47.785783678756474
```

In [34]:
```python
np.median(np.array(uniformity_per_puf)) # Uniformity median %
```

Out[34]:
```
47.65625
```

In [35]:
```python
np.std(np.array(uniformity_per_puf)) # Uniformity Standard Div. %
```

Out[35]:
```
3.8453372087750086
```

4.

- 50% is the ideal uniformity, our PUFs exibit good uniformity values with respect to the ideal (47.79%)

5.

In [36]:
```
array_of_pufs = []
for i in range(193):
    puf = np.zeros(256)
    df = pd.read_csv(f'./fullFreqData/fpga{i+1}.csv', header=None)
    df_t = df.T
    means = df_t.describe().T["mean"]
    means_np = np.array(means)
    means_re = means_np.reshape(32, 16).T.flatten()
    means_np_reshaped = means_re.reshape(256, 2)
    for j in range(256):
        if means_np_reshaped[j, 0] >= means_np_reshaped[j, 1]:
            puf[j] = True
            # puf[j] = 1
        else:
            # puf[j] = 0
            puf[j] = False

    array_of_pufs.append(puf)
print(np.array(array_of_pufs)[0])
```

```
[0. 1. 0. 0. 0. 0. 0. 0. 0. 0. 1. 0. 0. 0. 1. 0. 0. 0. 0. 1. 0. 1. 1. 1.
 0. 0. 0. 1. 0. 0. 0. 1. 0. 0. 0. 1. 0. 0. 0. 1. 0. 0. 1. 0. 1. 1. 1. 1.
 0. 1. 1. 0. 0. 1. 0. 1. 1. 0. 0. 1. 0. 0. 1. 1. 0. 0. 1. 1. 0. 0. 0. 1.
 0. 0. 1. 1. 0. 0. 1. 1. 0. 1. 0. 1. 0. 1. 0. 0. 1. 0. 0. 0. 1. 1. 1. 1.
 0. 1. 0. 0. 0. 0. 0. 1. 0. 1. 1. 0. 0. 1. 1. 1. 1. 1. 0. 0. 0. 0. 1. 1.
 0. 0. 0. 1. 0. 0. 1. 1. 0. 1. 1. 1. 0. 1. 1. 1. 0. 0. 0. 1. 0. 1. 0. 1.
 0. 1. 1. 0. 0. 1. 0. 0. 1. 1. 0. 0. 0. 0. 1. 1. 1. 0. 1. 1. 0. 1. 1. 0.
 0. 0. 0. 1. 0. 0. 1. 1. 0. 1. 1. 1. 0. 0. 0. 1. 1. 0. 0. 0. 0. 1. 1. 1.
 0. 1. 1. 1. 0. 1. 1. 1. 0. 1. 0. 1. 0. 1. 1. 1. 1. 0. 1. 1. 0. 0. 0. 1.
 0. 0. 0. 0. 0. 1. 0. 1. 0. 1. 1. 0. 1. 1. 0. 0. 0. 1. 0. 1. 0. 0. 1. 1.
 1. 0. 1. 0. 0. 0. 1. 1. 0. 1. 0. 0. 0. 0. 1. 0.]
```

In [37]:
```
uniformity_per_puf = []
for puf in np.array(array_of_pufs):
    unique, counts = np.unique(puf, return_counts=True)
    uniformity_per_puf.append(counts[0] / 256 * 100)
```

In [38]:
```
np.array(uniformity_per_puf) # Uniformity per PUF with respect to the zero (
```

Out[38]:
```
array([54.6875  , 52.34375 , 53.125   , 54.296875, 52.734375, 58.984375,
       55.46875 , 57.03125 , 53.125   , 52.34375 , 49.21875 , 50.390625,
       53.125   , 48.4375  , 56.640625, 54.6875  , 53.90625 , 50.78125 ,
       55.078125, 54.296875, 50.      , 54.296875, 57.8125  , 50.390625,
       56.25    , 48.046875, 52.34375 , 55.46875 , 55.46875 , 53.125   ,
       53.90625 , 54.296875, 56.640625, 45.703125, 58.984375, 55.078125,
       60.15625 , 60.546875, 55.46875 , 61.71875 , 53.90625 , 54.6875  ,
       55.859375, 48.046875, 55.46875 , 49.609375, 53.90625 , 58.203125,
       50.390625, 49.21875 , 52.734375, 55.078125, 45.703125, 54.296875,
       46.484375, 53.90625 , 57.03125 , 56.640625, 47.65625 , 53.515625,
       55.078125, 52.734375, 58.203125, 55.078125, 59.375   , 53.515625,
       57.8125  , 54.6875  , 49.609375, 55.078125, 57.03125 , 54.6875  ,
       52.34375 , 56.640625, 57.8125  , 55.859375, 59.765625, 56.25    ,
       53.125   , 51.953125, 51.5625  , 57.8125  , 53.515625, 46.09375 ,
       52.734375, 52.34375 , 55.46875 , 49.21875 , 59.375   , 50.390625,
       54.296875, 52.34375 , 55.859375, 59.375   , 53.90625 , 53.90625 ,
       55.078125, 52.734375, 53.125   , 55.859375, 55.078125, 55.46875 ,
       50.78125 , 55.859375, 57.03125 , 53.90625 , 57.03125 , 53.125   ,
       55.859375, 48.046875, 55.859375, 50.      , 47.65625 , 53.515625,
       54.296875, 54.296875, 50.78125 , 55.078125, 52.34375 , 52.34375 ,
       51.5625  , 52.34375 , 50.390625, 53.515625, 55.46875 , 52.734375,
       55.859375, 51.5625  , 53.125   , 48.4375  , 52.34375 , 53.515625,
       58.984375, 53.125   , 58.984375, 53.125   , 53.515625, 53.90625 ,
       53.125   , 53.90625 , 52.34375 , 51.5625  , 48.4375  , 58.203125,
       62.109375, 56.25    , 57.8125  , 51.5625  , 57.8125  , 52.734375,
       55.46875 , 52.34375 , 56.640625, 53.125   , 44.53125 , 50.390625,
       54.296875, 52.734375, 51.953125, 56.25    , 53.125   , 50.78125 ,
       50.390625, 57.8125  , 51.5625  , 56.640625, 55.078125, 51.953125,
       56.640625, 58.984375, 51.171875, 57.421875, 53.515625, 53.125   ,
       53.90625 , 53.515625, 50.390625, 50.      , 56.640625, 56.25    ,
       54.296875, 56.25    , 51.5625  , 51.953125, 56.25    , 48.4375  ,
       58.984375, 53.90625 , 52.734375, 52.34375 , 53.90625 , 50.78125 ,
       56.25    ])
```

In [39]:
```python
np.mean(np.array(uniformity_per_puf)) # Uniformity mean %
```

Out[39]:
```
53.84148316062176
```

In [40]:
```python
np.median(np.array(uniformity_per_puf)) # Uniformity median %
```

Out[40]:
```
53.90625
```

In [41]:
```python
np.std(np.array(uniformity_per_puf)) # Uniformity Standard Div. %
```

Out[41]:
```
3.1690515478293086
```

- 50% is the ideal uniformity, our PUFs exibit good uniformity values with respect to the ideal (53.84%)

- If we were responsible for selecting a PUF based on uniformity, we would choose row pair because it is closer to the ideal 50% uniformity