ASESMA-2023

Hands-on session - Phonons

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Outline

- 1. Introduction
- 2. Exercise 1: Phonons at Gamma in non-polar materials
- 3. Exercise 2: Phonon at Gamma in polar materials
- 4. Exercise 3: Phonons dispersion in non-polar materials
- 5. Exercise 4: Phonon dispersion in polar materials

Introduction: Phonons

Normal mode frequencies, ω , and eigenvectors, $\mathbf{U}_{s\alpha}$ are determined by the secular equation:

$$\sum_{\mathbf{s}',\beta} \tilde{D}_{\mathbf{s}\alpha,\mathbf{s}'\beta}(\mathbf{q}) \, \tilde{\mathbf{u}}_{\mathbf{s}'\beta}(\mathbf{q}) = \omega_{\mathbf{q}}^2 \, \tilde{\mathbf{u}}_{\mathbf{s}\alpha}(\mathbf{q})$$

where

$$ilde{D}_{slpha,s'eta}(\mathbf{q}) = rac{1}{\sqrt{M_s M_{s'}}} \sum_{\mathbf{R},\mathbf{R}'} \left| rac{\partial^2 E_{tot}}{\partial \mathbf{u}_{slpha}(\mathbf{R}) \partial \mathbf{u}_{s'eta}(\mathbf{R}')} e^{i\mathbf{q}(\mathbf{R}'-\mathbf{R})}
ight|$$

is the dynamical matrix.

This Matrix can be calculated from linear response and diagonalized to get phonon modes at **q**.

Interatomic force constants (IFC)

Go to the directory with the input files:

cd ~/ASESMA-2023/Day-5/Exercise1

In this directory you will find:

- *README.md* File describing how to do the exercise
- Si.scf.in Input file for the SCF ground-state calculation
- Si.ph.in Input file for the phonon calculation at Γ
- *Si.dynmat.in* Input file to impose the acoustic sum rule
- reference Directory with the reference results

```
&control
    calculation = 'scf'.
    restart mode = 'from scratch',
    prefix
                = 'si'.
    pseudo dir='../pseudo/',
    outdir='../tmp/',
&system
    ibrav=2,
    celldm(1)=10.2625,
    nat=2,
    ntyp=1,
    ecutwfc=60.0,
    ecutrho=720.0,
&electrons
    mixing beta=0.7,
    conv thr=1d-10,
ATOMIC SPECIES
Si 28.0855 Si.pbe-rrkj.UPF
ATOMIC POSITIONS (alat)
Si 0.00 0.00 0.00
Si 0.25 0.25 0.25
K POINTS automatic
444111
```

Step 1. Perform a Self-Consistent Field ground-state calculation for silicon at the equilibrium structure using the pw.x program.

Si.scf.in

Input file for the SCF calculations

mpirun -np 4 pw.x -inp Si.scf.in > Si.scf.out



Step 2. Perform a phonon calculation at Γ using the ph.x program.

Si.ph.in — Input file for the phonon calculations

```
Phonons at Gamma
&inputph
                                        The same prefix as in the SCF calculation
 prefix = 'Si',
                                        Threshold for self-consistency
 tr2 ph = 1.0d-14
 amass(1) = 28.0855,
                                        Atomic mass
                                        To calculate the dielectric tensor and effective charges
 epsil = .true.
 outdir = './tmp'
                                        Directory for temporary files
 fildyn = 'Si.dyn',
                                        File containing the dynamical matrix
                                        Coordinates of the q point (\Gamma) in units of 2*pi/a in the
0.0 0.0 0.0
                                        Cartesian reference system
```

mpirun -np 4 ph.x -inp Si.ph.in > Si.ph.out

Dynamical matrix file **Si.dyn**:

```
Dielectric Tensor:
         13.806378036722
                                   0.000000000000
                                                           -0.000000000000
         -0.000000000000
                                  13.806378036722
                                                            0.000000000000
         -0.000000000000
                                   0.000000000000
                                                           13.806378036722
Effective Charges E-U: Z {alpha}{s,beta}
     atom #
         -0.075668487215
                                   0.000000000000
                                                            0.000000000000
          0.000000000000
                                   -0.075668487215
                                                            0.000000000000
          0.000000000000
                                   0.000000000000
                                                           -0.075668487215
     atom #
         -0.075668487215
                                   0.000000000000
                                                            0.000000000000
          0.000000000000
                                  -0.075668487215
                                                           -0.000000000000
          0.000000000000
                                  -0.000000000000
                                                           -0.075668487215
     Diagonalizing the dynamical matrix
              0.000000000
                                           0.000000000 )
     q = (
                             0 000000000
```

Dielectric constant and Born effective charges (BECs)

```
0.078830 [THz] =
                                             2.629478 [cm-1]
 0.429342 -0.000000 -0.450832  0.000000 -0.335285  0.000000 )
 0.429342 -0.000000 -0.450832 0.000000 -0.335285 0.000000 )
                      0.078830 [THz] =
                                             2.629478 [cm-1]
  freq (
            2) =
          0.000000
                    0.386983 0.000000 0.195117 0.000000 )
 0.558725
          0.000000
                    0.386983 0.000000 0.195117 0.000000 )
  freq (
                      0.078830 [THz] =
                                             2.629478 [cm-1]
 0.059092
          0.000000 -0.383399 0.000000
                                       0.591197 0.000000 )
 0.059092
          0.000000 -0.383399 0.000000
                                       0.591197 0.000000 )
  freq (
                     15.292493 [THz] =
                                           510.102675 [cm-1]
-0.129158
          0.000000
                    0.093954 0.000000 0.688833 -0.000000 )
                     15.292493 [THz] =
                                           510.102675 [cm-1]
  freq (
          0.000000
                    0.626619 0.000000 -0.140928 0.000000 )
0.295782
          0.000000 -0.626619 0.000000 0.140928 0.000000 )
  freq (
                     15.292493 [THz] =
                                           510.102675 [cm-1]
-0.629151 0.000000 -0.313880 0.000000 -0.075155 0.000000 )
 0.629151 -0.000000 0.313880 -0.000000 0.075155 0.000000 )
```

Dynamical matrix file **Si.dyn**:

```
Diagonalizing the dynamical matrix
```

```
q = (0.000000000 0.00000000 0.000000000)
  freq (1) = 0.078830 [THz] = 2.629478 [cm-1]
(0.429342 - 0.000000 - 0.450832 \ 0.000000 - 0.335285 \ 0.000000)
(0.429342 - 0.000000 - 0.450832 0.000000 - 0.335285 0.000000)
  freq (2) = (0.078830 \, [THz] = (2.629478 \, [cm-1])
(0.558725 \ 0.000000 \ 0.386983 \ 0.000000 \ 0.195117 \ 0.000000)
(0.558725 \ 0.000000 \ 0.386983 \ 0.000000 \ 0.195117 \ 0.000000)
  freq (3) = (0.078830 \text{ [THz]}) = (2.629478 \text{ [cm-1]})
(0.059092\ 0.000000\ -0.383399\ 0.000000\ 0.591197\ 0.000000)
(0.059092 \ 0.000000 \ -0.383399 \ 0.000000 \ 0.591197 \ 0.000000)
  freq (4) = 15.292493 [THz] = 510.102675 [cm-1]
(-0.129158 \ 0.000000 \ 0.093954 \ 0.000000 \ 0.688833 \ -0.000000)
(0.129158\ 0.000000\ -0.093954\ 0.000000\ -0.688833\ 0.000000\ )
  freq (5) = (15.292493 \text{ [THz]}) = 510.102675 \text{ [cm-1]}
(-0.295782 \ 0.000000 \ 0.626619 \ 0.000000 \ -0.140928 \ 0.000000)
(0.295782\ 0.000000\ -0.626619\ 0.000000\ 0.140928\ 0.000000)
  freq (6) = (15.292493 \text{ [THz]}) = 510.102675 \text{ [cm-1]}
(-0.629151 \ 0.000000 \ -0.313880 \ 0.000000 \ -0.075155 \ 0.000000)
 0.629151 - 0.000000 \ 0.313880 - 0.000000 \ 0.075155 \ 0.000000 \ )
```

Acoustic modes



Optical modes



Acoustic sum rule at Γ

Problems with the frequency of the acoustic phonon mode at Γ and with effective charges.

Because of the numerical inaccuracies the interatomic force constants and effective charges do not strictly satisfy the acoustic sum rule (ASR).

ASR comes directly from the continous translational invariance of the crystal. If we translate the whole solid by a uniform displacement, the forces acting on the atoms must be zero.

As a consequence

For each
$$\alpha$$
, β and i :
$$\sum_{\mathbf{L},j} C_{\alpha i,\beta j}(\mathbf{R_L}) = 0 \qquad \sum_{j} Z^*_{j,\alpha \beta} = 0$$

As a consequence, the frequencies of the acoustic modes must be zero.

Acoustic sum rule at Γ

Reasons for numerical inaccuracies:

- Insufficiently accurate SCF thresholds (in pw.x and/or ph.x)
- > XC energy is computed in real space. More problematic for GGA than in LDA, For US pseudopotentials it could require large *ecutrho*.
- > K-points sampling not accurate enough (especially, BECs and dielectric constant required denser k-point sampling).

Acoustic sum rule at Γ

The acoustic sum rule (ASR) can however be imposed after the phonon calculations.

To do this we use the dynmat.x program that imposes the ASR on the elements of the dynamical matrix and diagonalize it.

The input file is Si.dynmat.in:

```
&input
fildyn = 'Si.dyn',
asr = 'simple',
/
A way to impose the acoustic sum rule (simple, crystal, one-dim, zero-dim)
```

mpirun -np 4 dynmat.x -inp Si.dynmat.in > Si.dynmat.out

The program dynmat.x produces the file dynmat.out which contains the new acoustic frequencies, which are exactly equal to zero.

```
q = 0.0000
                  0.0000
                            0.0000
 freq (1) = 0.000000 \text{ [THz]} = 0.000000 \text{ [cm-1]}
 0.000000 \ 0.000000 \ 0.707107 \ 0.000000 \ 0.000000 \ 0.000000 \ )
 0.000000 \ 0.000000 \ 0.707107 \ 0.000000 \ 0.000000 \ 0.000000
  freq (2) = 0.000000 [THz] 0.000000 [cm-1]
(-0.707107 \ 0.000000 \ 0.000000 \ 0.000000 \ 0.000000 \ 0.000000)
(-0.707107 \ 0.000000 \ 0.000000 \ 0.000000 \ 0.000000 \ 0.000000)
  freq (3) = 0.000000 \text{ [THz]} 0.000000 \text{ [cm-1]}
 0.000000 \quad 0.000000 \quad 0.000000 \quad 0.000000 \quad 0.707107 \quad 0.000000 \quad )
 0.000000 \quad 0.000000 \quad 0.000000 \quad 0.000000 \quad 0.707107 \quad 0.000000 \quad )
  freq (4) = 15.292290 \text{ [THz]} = 510.095902 \text{ [cm-1]}
 0.000000 \ 0.000000 \ 0.707107 \ 0.000000 \ 0.000000 \ 0.000000
 0.000000 \ 0.000000 \ -0.707107 \ 0.000000 \ 0.000000 \ 0.000000
  freq (5) = 15.292290 \text{ [THz]} = 510.095902 \text{ [cm-1]}
 0.000000 \ 0.000000 \ 0.000000 \ 0.000000 \ -0.707107 \ 0.000000 \ )
 0.000000 \quad 0.000000
                        0.000000 \quad 0.000000 \quad 0.707107 \quad 0.000000 \quad )
  freq (6) = 15.292290 \text{ [THz]} = 510.095902 \text{ [cm-1]}
(-0.707107 \ 0.000000 \ 0.000000 \ 0.000000 \ 0.000000 \ 0.000000)
 0.707107 0.000000
                        0.000000 \quad 0.000000
                                                0.000000 0.000000
```

Polar materials in the $\mathbf{q} = \mathbf{0}$ limit: a macroscopic electric field appears as a consequence of the long-range character of the Coulomb interaction (incompatible with Periodic Boundary Conditions).

A non-analytic term must be added to Interatomic Force Constants at q = 0:

$$ilde{C}_{m{s}lpha,m{s}'eta}(m{\mathsf{q}}) = ilde{C}_{m{s}lpha,m{s}'eta}^{\mathrm{analytic}}(m{\mathsf{q}}) + rac{4\pi}{\Omega} rac{(m{\mathsf{q}}\cdotm{Z}_{m{s}}^{\star})_{lpha}(m{\mathsf{q}}\cdotm{Z}_{m{s}'}^{\star})_{eta}}{m{\mathsf{q}}\cdotm{\epsilon}_{\infty}\cdotm{\mathsf{q}}}$$

Effective charges Z_s^* are related to polarization P induced by a lattice distortion: $Z_{s,\alpha\beta}^* = \Omega \frac{\partial P_\alpha}{\partial u_{s\beta}}$

Dielectric tensor $\epsilon_{\infty}^{\alpha\beta}$ is related to polarization **P** induced by an electric field **E** :

$$\epsilon_{\infty}^{\alpha\beta} = \delta_{\alpha\beta} + 4\pi \left. \frac{\partial P_{\alpha}}{\partial E_{\beta}} \right|_{u_{s}(q=0)=0}$$

All of the above can be calculated from (mixed) second order derivatives of the total energy.

Go to the directory with the input files:

cd ~/ASESMA-2023/Day-5/Exercise2

In this directory you will find:

- *README.md* File describing how to do the exercise
- AlAs.scf.in Input file for the SCF ground-state calculation
- AlAs.ph.in Input file for the phonon calculation at Γ
- AlAs.dynmat.in Input file to impose the acoustic sum rule
- reference Directory with the reference results

AlAs.ph.in

```
Phonons at Gamma
&inputph
 prefix = 'AlAs',
 tr2 ph = 1.0d-14,
 Amass(1) = 26.98,
 Amass(2) = 74.92,
 epsil = .true.
 outdir = './tmp'
 fildyn = 'AlAs.dyn',
0.0 0.0 0.0
```

Step 1. Perform a Self-Consistent Field groundstate calculation for the polar semiconductor AIAs.

mpirun -np 4 pw.x -inp AlAs.scf.in > AlAs.scf.out

Step 2. Perform a phonon calculation at Gamma for AlAs.

mpirun -np 4 ph.x -inp AlAs.ph.in > AlAs.ph.out

In the file ph.AlAs.out you will find information about the dielectric tensor and effective charges:

Dielectric constant in cartesian axis

Effective charges (d Force / dE) in cartesian axis

```
1 Al Mean Z*:
                             1.88275
  atom
Ex (
        1.88275
                   -0.00000
                               0.00000
Ey (
        0.00000
                    1.88275
                               0.00000)
       -0.00000
                   -0.00000
                                1.88275)
Ez (
  atom
          2 As Mean Z*:
                             -3.23400
        -3.23400
Ex (
                    0.00000
                               -0.00000)
Ey (
        0.00000
                   -3.23400
                               -0.00000
Ez (
        -0.00000
                    0.00000
                               -3.23400)
```

Diagonalizing the dynamical matrix

```
5.067777 [cm-1]
      1) =
             0.151928 [THz] =
freq (
      2) =
             0.151928 [THz] =
                               5.067777 [cm-1]
freq (
             0.151928 \, [THz] =
      3) =
                               5.067777 [cm-1]
freq (
freq (
      4) =
             11.258171 [THz] =
                               375.532153 [cm-1]
             11.258171 [THz] =
                               375.532153 [cm-1]
      5) =
freq (
                               375.532153 [cm-1]
             11.258171 [THz] =
```

No LO-TO splitting

Step 3. Impose Acoustic Sum Rule and add the non-analytic LO-TO splitting using the dynmat.x program.

```
&input
fildyn = 'AlAs.dyn',
amass(1) = 26.98,
amass(2) = 74.92,
asr = 'simple',
q(1) = 1.0, q(2) = 0.0, q(3) = 0.0
/
```

```
check the output file
```

Direction in the Brillouin zone along which we want to compute the LOTO splitting

Go to the directory with the input files:

cd ~/ASESMA-2023/Day-5/Exercise3

In this directory you will find:

- *README.md* File describing how to do the exercise
- Si.scf.in Input file for the SCF ground-state calculation
- Si.ph.in Input file for the phonon calculation at Γ
- Si.q2r.in Input file for calculation of Interatomic Force Constants
- Si.matdyn.in Input file for Fourier Interpolation for various q points
- Si.plotband.in Input file for plotting a phonon dispersion
- reference Directory with the reference results

Step 1. Perform a SCF calculation for silicon at the equilibrium structure using the pw.x program.

mpirun -np 4 pw.x -inp Si.scf. in > Si.scf.out

```
Phonons on uniform q-grid
&inputph
 prefix = 'Si',
 tr2 ph = 1.0d-14,
amass(1) = 28.0855,
 ldisp = .true.,
 nq1 = 4,
nq2 = 4,
nq3 = 4,
outdir = './tmp'
 fildyn = 'Si.dyn',
```

Step 2. Perform a phonon calculation on a uniform grid of q points using the ph.x program.

mpirun -np 4 ph.x -inp Si.ph.in > Si.ph.out

Option for the calculation on a grid Uniform grid of q points

$$\mathbf{q}_{ijk} = \frac{i-1}{\text{ng1}}\mathbf{G}_1 + \frac{j-1}{\text{ng2}}\mathbf{G}_2 + \frac{k-1}{\text{ng3}}\mathbf{G}_3$$

4x4x4 = 64 q-points => Use of symmetry => 8 non-equivalent q points The file Si.dyn0 contains a list of the non-equivalent q points (8, in this case).

```
(q-grid)
             (number of non-equivalent q-points)
0.00000000000000E+00
                        0.00000000000000E+00
                                                0.00000000000000E+00
-0.25000000000000E+00
                        0.25000000000000E+00
                                               -0.25000000000000E+00
0.50000000000000E+00
                       -0.50000000000000E+00
                                                0.50000000000000E+00
0.00000000000000E+00
                        0.50000000000000E+00
                                                0.00000000000000E+00
0.750000000000000E+00
                       -0.25000000000000E+00
                                                0.75000000000000E+00
0.50000000000000E+00
                        0.00000000000000E+00
                                                0.50000000000000E+00
0.00000000000000E+00
                                                0.00000000000000E+00
                       -0.10000000000000E+01
-0.50000000000000E+00
                       -0.10000000000000E+01
                                                0.00000000000000E+00
```

The phonon code ph.x generates a file for every non-equivalent q point (Si.dyn1, Si.dyn2, ..., Si.dyn8), which contain information about dynamical matrices, phonon frequencies and atomic displacements

Step 3. Calculation of the Interatomic Force Constants (IFC) using the q2r.x program.

Fourier transforms of IFC's:

$$ilde{C}_{slpha,s'eta}(\mathbf{q}_{\mathit{ijk}}) = rac{\partial^2 E_{tot}}{\partial ilde{\mathbf{u}}_{slpha}^*(\mathbf{q}_{\mathit{ijk}})\partial ilde{\mathbf{u}}_{s'eta}(\mathbf{q}_{\mathit{ijk}})}$$

 α, β are Cartesian components, and s, s' are atomic indices.

$$C_{slpha,s'eta}(\mathbf{R}_{lmn}) = rac{1}{N_{\mathbf{q}}} \sum_{i,j,k} ilde{C}_{slpha,s'eta}(\mathbf{q}_{ijk}) e^{i\mathbf{q}_{ijk}\cdot\mathbf{R}_{lmn}} \ egin{aligned} & C_{slpha,s'eta}(\mathbf{R}_{lmn}) \end{aligned}$$

Fourier transforms of IFC's on a grid of q points nq1 x nq2 x nq3 in reciprocal space

IFC's in a supercell nq1 x nq2 x nq3 in real space

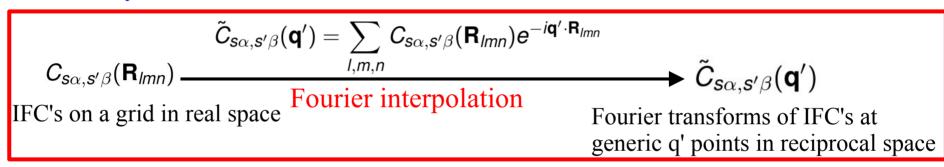
Input file Si.q2r.in:

The denser the grid of q points, the larger the vectors R for which the Interatomic Force Constants are calculated.

To perform the calculation:

mpirun -np 4 q2r.x -inp Si.q2r.in > Si.q2r.out

Step 4. Calculate phonons at generic q' points using IFC by means of the code matdyn.x

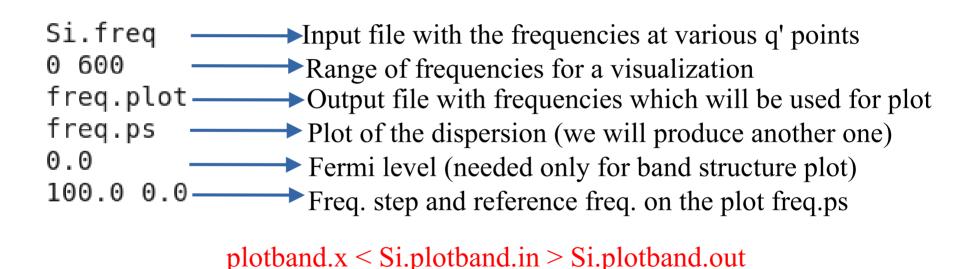


Input file Si.matdyn.in: mpirun -np 4 matdyn.x -i Si.matdyn.in > Si.matdyn.out

```
&input
 asr = 'simple',
                                                Acoustic sum rule
 amass(1) = 28.0855,
                                               Atomic mass
 flfrc = 'Si444.fc',
                                               ► File with IFC's
 flfrq = 'Si.freq'
                                              ➤ Output file with the frequencies
396
                                              Number of q points
  0.000000
             0.000000
                        0.000000
                                   0.000000
  0.012658
             0.000000
                        0.000000
                                   0.012658
                                              Coordinates of q points
```

Step 5. Plot the phonon dispersion using the plotband.x program and gnuplot.

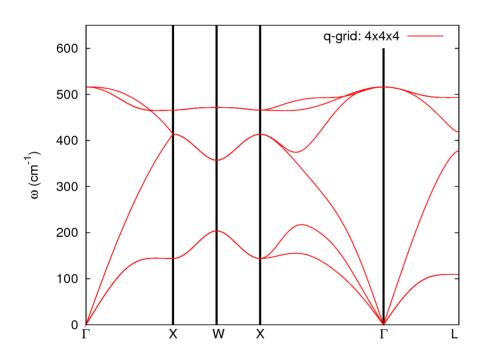
Input file Si.plotband.in:

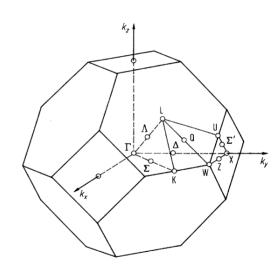


Use gnuplot and the file plot_dispersion.gp in order to plot the phonon dispersion of silicon (look at the file experimental_data.dat for the experimental reference).

You will get a postscript file phonon_dispersion.eps which you can visualize.

Phonon dispersion of silicon along some high-symmetry directions in the Brillouin zone (file phonon_dispersion.eps):





How to determine whether the quality of the Fourier interpolation is satisfactory? Compare with the direct calculation (no interpolation)!

Homework-1: Perform a direct phonon calculation (no interpolation) at several q' points and make a comparison with the phonon frequencies obtained from the Interpolation. Use exercise1 as an example. Some q' points along the Gamma-X high symmetry line are listed in the file reference/q_points_direct_calc.txt

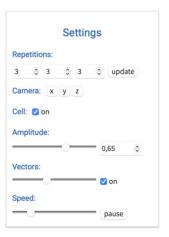
Homework-2: Perform a phonon dispersion calculation for several q-points grids (eg. 2x2x2, 4x4x4, and 6x6x6) and compare the dispersions. Do they converge?

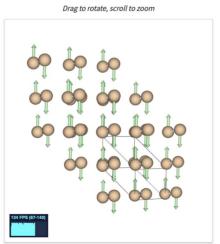
Phonon modes visualizer

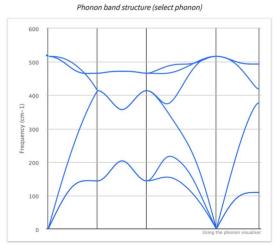
The phonon modes are not always easy to visualize, especially if we are not at Γ . An online phonon visualizer is very helpful in this regard.

https://www.materialscloud.org/work/tools/interactivephonon

Phonon dispersion: pw.Si.in & matdyn.modes







Go to the directory with the input files:

cd ~/ASESMA-2023/Day-5/Exercise4

In this directory you will find:

- *README.md* File describing how to do the exercise
- AlAs.scf.in Input file for the SCF ground-state calculation
- AlAs.ph.in Input file for the phonon calculation at Γ
- AlAs.q2r.in Input file for calculation of Interatomic Force Constants
- AlAs.matdyn.in Input file for Fourier Interpolation for various q points
- AlAs.plotband.in Input file for plotting a phonon dispersion
- reference Directory with the reference results

Step 1. Perform a SCF calculation for silicon at the equilibrium structure using the pw.x program.

mpirun -np 4 pw.x -inp AlAs.scf.in > AlAs.scf.out

Step 2. Perform a phonon calculation on a uniform grid of q points using the ph.x program.

mpirun -np 4 ph.x -inp AlAs.ph.in > AlAs.ph.out

Step 3. Calculation of the Interatomic Force Constants (IFC) using the q2r.x program.

mpirun -np 4 q2r.x -inp AlAs.q2r.in > AlAs.q2r.out

Step 4. Calculate phonons at generic q' points using IFC by means of the code matdyn.x

mpirun -np 4 matdyn.x -i AlAs.matdyn.in > AlAs.matdyn.out