Time series and Kalman Filters

Lecture 10

Xiaofei Shi

Learning objectives

- Learn how time series modeling extend from the regression model
- Learn how Dynamic Linear Models are related to time series framework

How do we model the data?

Why it is very hard to model time series data?

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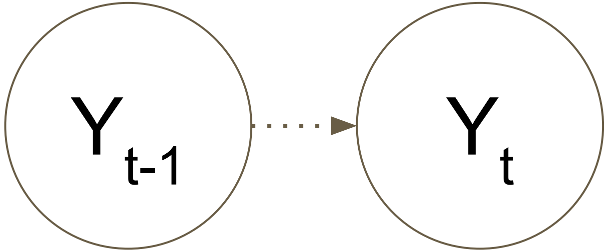
Traditional Time Series Setup

$$Y_t = Y_{t-1} + \epsilon_t$$



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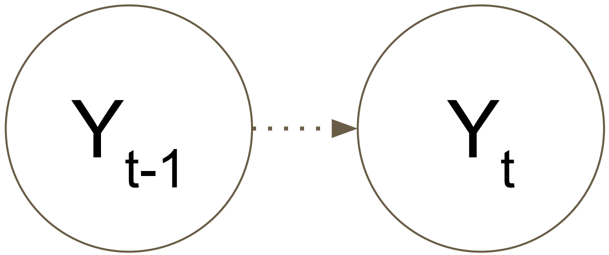


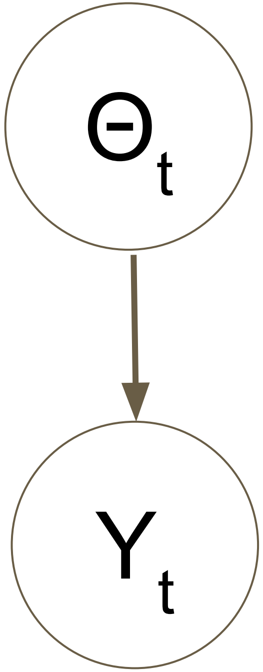
More complicated version:

- Autoregression Model: https://en.wikipedia.org/wiki/Autoregressive_model

- ARMA Model: https://en.wikipedia.org/wiki/Autoregressive–moving-average_model

Linear Regression Model - Data is a Noisy Measurement of the Process/State

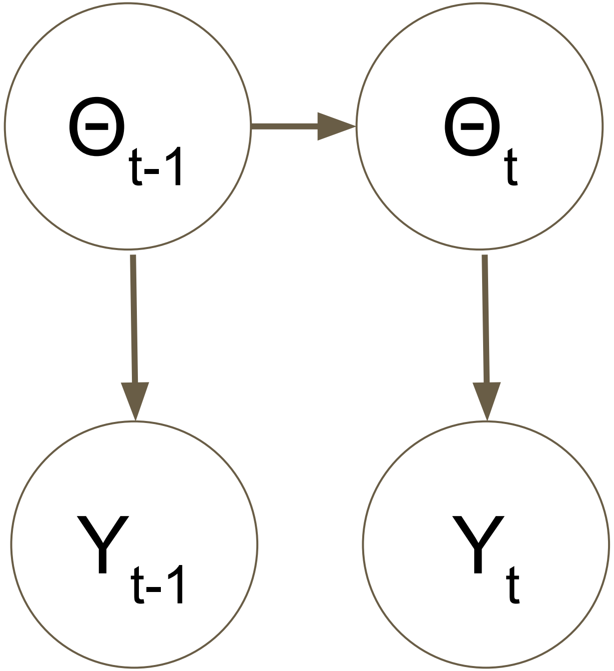
$$Y_t = \theta_t + \upsilon_t$$



To add temporal dependency, make the states depend on its previous one

$$Y_t = \theta_t + v_t$$

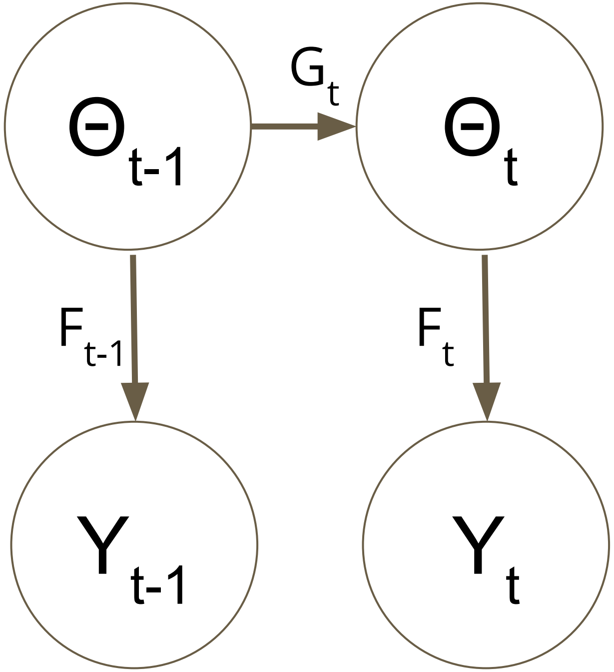
$$\theta_t = \theta_{t-1} + w_t$$



Dynamic Linear Model (State Space Models)

$$Y_t = F_t\theta_t + v_t$$

$$\theta_t = G_t\theta_{t-1} + w_t$$



Error terms can have different meanings

$$Y_t = F_t\theta_t + v_t \qquad v_t \sim N(0, V_t)$$

$$\theta_t = G_t\theta_{t-1} + w_t \qquad w_t \sim N(0, W_t)$$

- Random terms are all independent

Temporal models need to specify the “initial state”

$$Y_t = F_t\theta_t + v_t \qquad v_t \sim N(0, V_t)$$

$$\theta_t = G_t\theta_{t-1} + w_t \qquad w_t \sim N(0, W_t)$$

$$\theta_0 \sim N(m_0, C_0)$$

- Random terms are all independent

What are we interested in the model?

Bayesians always want the posterior!

$$P(\ ?\ | Y_1, \ldots, Y_{t-1})$$

Kalman filter is an algorithm to get the posterior expectation/covariance for "states" in the DLM

$$E(\theta_t | Y_1, \ldots, Y_{t-1})$$

$$Cov(\theta_t | Y_1, \ldots, Y_{t-1})$$

How to spot correlation across records?

If the correlation between Y_t and Y_(t-1) is not 0, they cannot be independent!

- What is the definition of the usual correlation?

Diagnostics via auto-correlation function

If the correlation between Y_t and Y_(t-1) is not 0, they cannot be independent!

$$\sum (Y_t - \bar{Y})(Y_{t-k} - \bar{Y})$$

Diagnostics via auto-correlation function

If the correlation between Y_t and Y_(t-1) is not 0, they cannot be independent!

$$\sum_{t=k-1}^{n} (Y_t - \bar{Y})(Y_{t-k} - \bar{Y})$$

Diagnostics via auto-correlation function

If the correlation between Y_t and Y_(t-1) is not 0, they cannot be independent!

$$\frac{1}{(n-k)} \sum_{t=k-1}^{n} (Y_t - \bar{Y})(Y_{t-k} - \bar{Y})$$

Diagnostics via auto-correlation function

If the correlation between Y_t and Y_(t-1) is not 0, they cannot be independent!

$$\frac{1}{(n-k)\sigma^2}\sum_{t=k-1}^{n}(Y_t-\bar{Y})(Y_{t-k}-\bar{Y})$$

Reading the autocorrelation function

acf() in R!