Boosting, Surrogate Losses, and Ensemble Methods

STAT5241 Section 2

Statistical Machine Learning

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Pros and cons for simple classifiers

- Typical simple/weak classifiers

  - Shallow decision tree, SVM, logistic regression, naive Bayes

- Don’t usually overfit

- Cannot solve complicated learning tasks

Can we make simple learners smarter?

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What about several simple model together?

- Input:

  - A dataset D

  - your top T favorite learners: $L_1, \ldots, L_T$

- Learning algorithm:

  - Estimate the error of learners: $L_1, \ldots, L_T$

  - Pick the best learner L^*

  - Train L^* on D and return results

Buckets of models

How to estimate the error?
Cross Validation!

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Pros and cons of a “bucket of models”

- Pros:

  - simple

  - not much worse than the best of the “base learners”

- Cons:

  - what if there’s not a single best learner?

Stack learners: first attempt

- Input:

  - A dataset D

  - your top T favorite learners: $L_1, \ldots, L_T$

- Learning algorithm:

  - Train $L_1, \ldots, L_T$ on dataset to get hypothesis $h_1, \ldots, h_T$

  - Create a new dataset D' containing (x',y'),...

    - x' is a vector of the T predictions $h_1(x), \ldots, h_T(x)$

    - y is the label for x

  - Train new classifier on D' to get h' -- which combines the predictions!

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Pro and cons of stacking

- Pros:

  - Fairly simple

  - Slow, but easy to parallelize

- Cons:

  - What if there's not a single best combination scheme?

  - E.g.: for movie recommendation sometimes L1 is best for users with many ratings and L2 is best for users with few ratings

Voting! (Ensemble methods)

Instead of learning a single (weak) classifier, learn **many weak classifiers** that are **good at different parts of the input space**

Output class: (Weighted) vote of each classifier

- Classifiers that are most “sure” will vote with more conviction
- Classifiers will be most “sure” about a particular part of the space
- On average, do better than single classifier!

But how do you ???

- force classifiers to learn about different parts of the input space?
- weigh the votes of different classifiers?

Boosting (Schapire, 1989)

- **Practically useful**
- **Theoretically interesting**

- Idea: given a weak learner, run it multiple times on (reweighted) training data, then let the learned classifiers vote
- On each iteration t:
  – weight each training example by how incorrectly it was classified
  – Learn a hypothesis – h_t
  – A strength for this hypothesis – s_t
- Final classifier:
  - A linear combination of the votes of the different classifiers weighted by their strength.

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Learning from weighted data

Sometimes not all data points are equal

- Some data points are more equal than others

Consider a weighted dataset

- D(i) – weight of *i* th training example ($\mathbf{x}^i$,y^i)
- Interpretations:
  - *i* th training example counts as D(i) examples
  - If I were to "resample" data, I would get more samples of "heavier" data points

Now, in all calculations, whenever used, *i* th training example counts as D(i) "examples"

- e.g., MLE for Naïve Bayes, redefine *Count(Y=y)* to be weighted count

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Ada Boost

Given: $(x_1, y_1), \ldots, (x_m, y_m)$ where $x_i \in X, y_i \in Y = \{-1, +1\}$
Initialize $D_1(i) = 1/m$.
For $t = 1, \ldots, T$:

- Train weak learner using distribution D_t.
- Getweak classifier $h_t : X \to \mathbb{R}$.
- Choose $\alpha_t \in \mathbb{R}$.
- Update:

$$D_{t+1}(i) = \frac{D_t(i)\exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

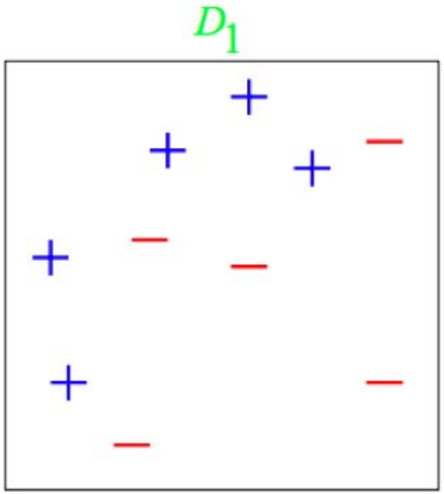
where Z_t is a normalization factor

$$Z_t = \sum_{i=1}^{m} D_t(i)\exp(-\alpha_t y_i h_t(x_i))$$

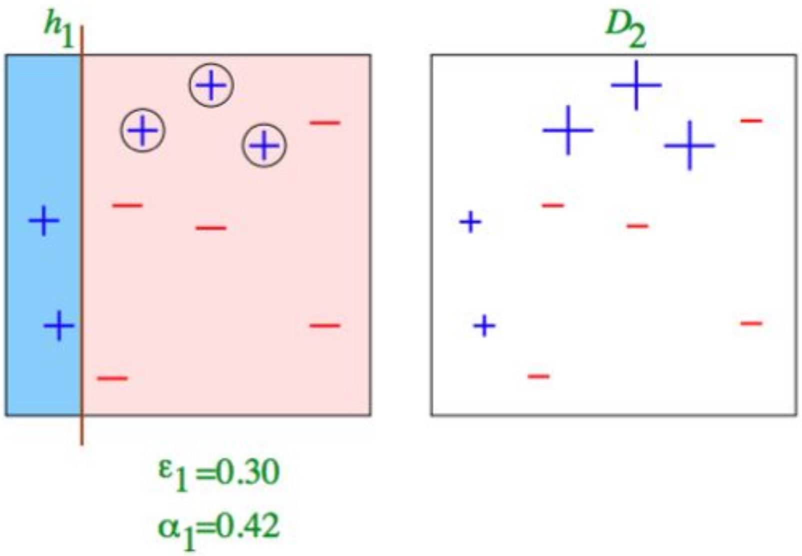
Output the final classifier:

$$H(x) = \operatorname{sign}\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right).$$

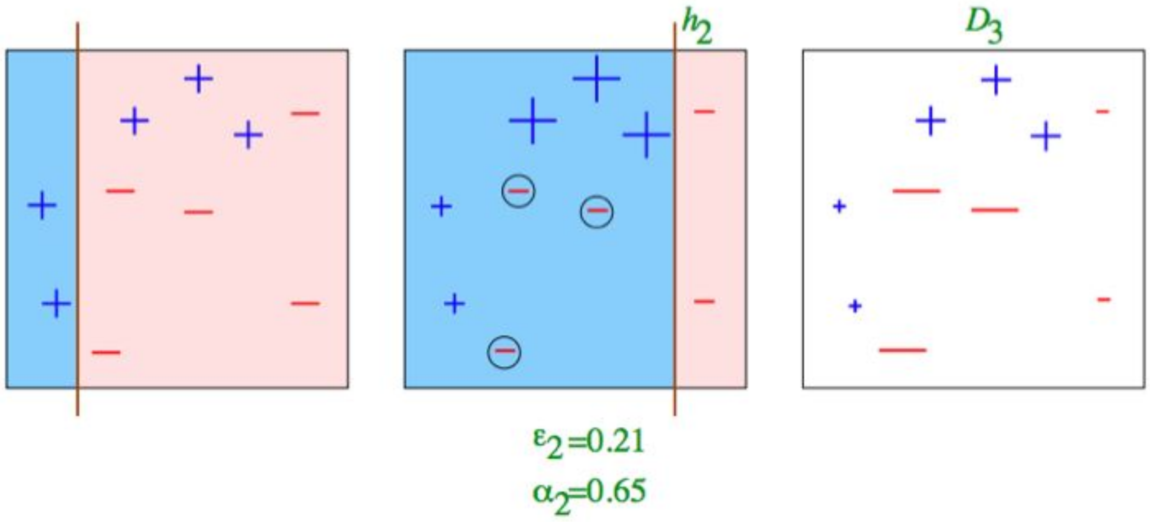
Toy example



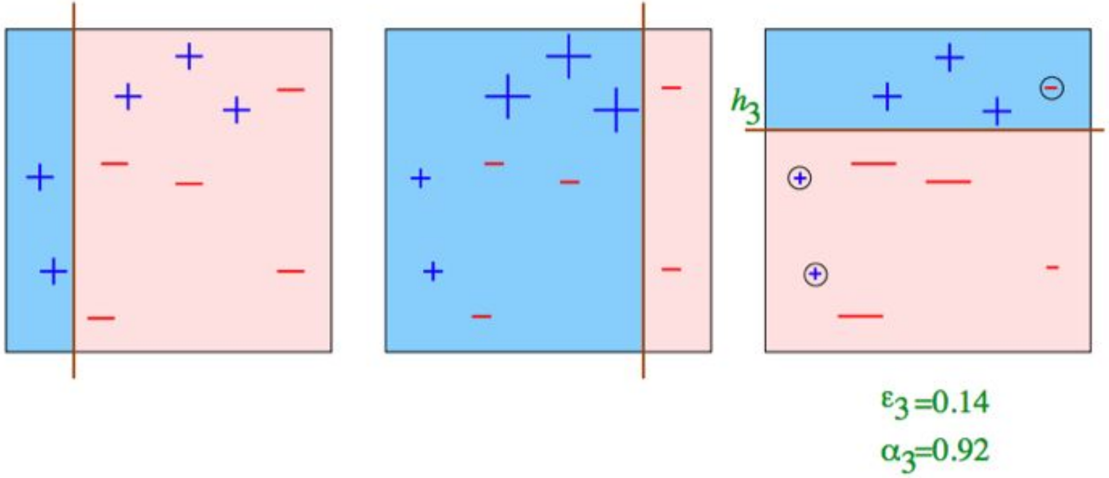
Toy example: Round 1



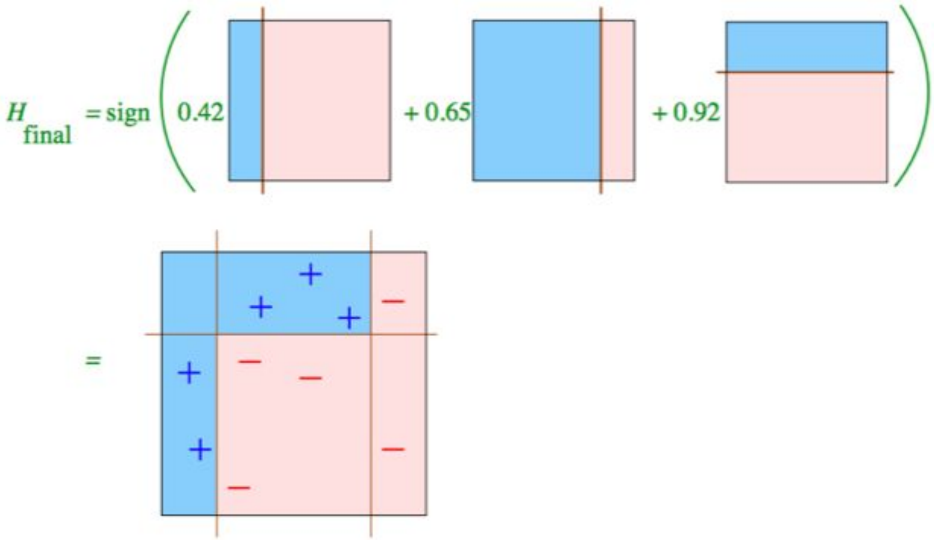
Toy example: Round 2



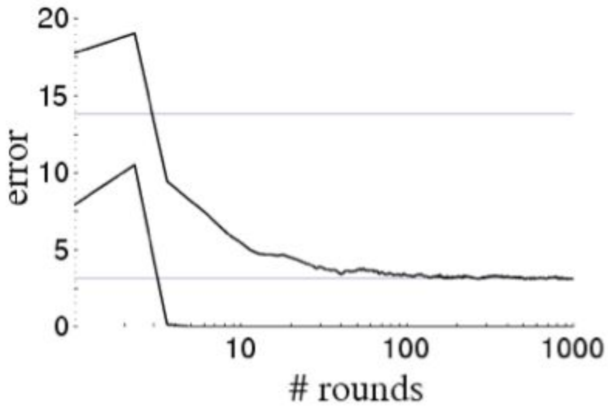
Toy example: Round 3



Toy example: Final classifier



Boosting for handwritten digit recognition

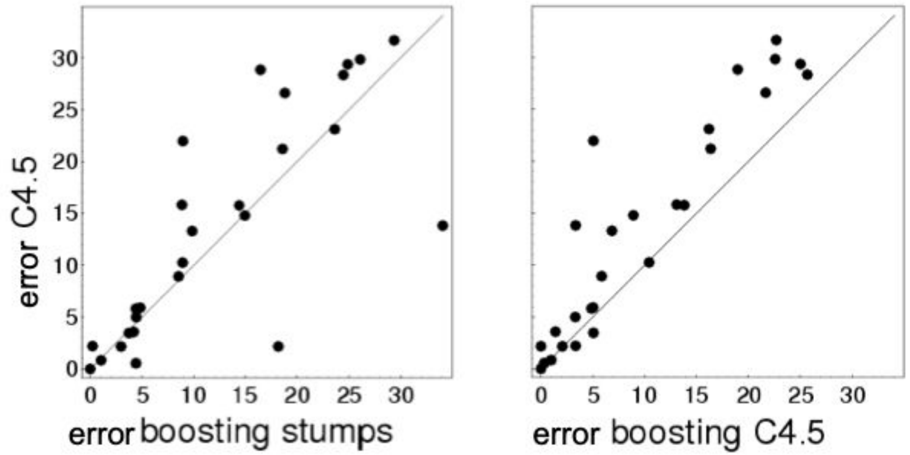


Boosting often

– Robust to overfitting
– Test set error decreases even after training error is zero

Experimental results

Comparison of C4.5, Boosting C4.5, Boosting decision stumps (depth 1 trees), 27 benchmark datasets



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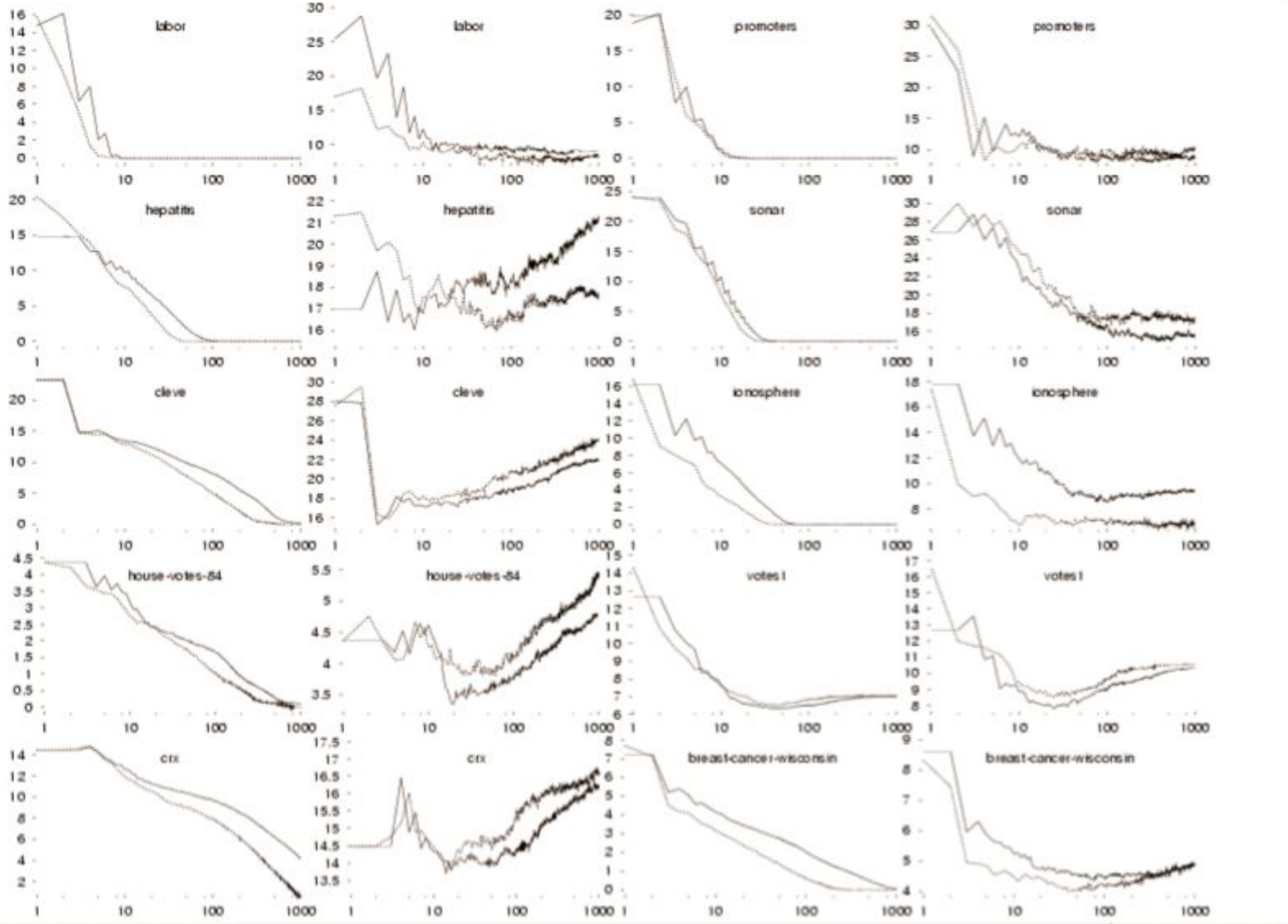
AdaBoost and AdaBoost.MH on Train (left) and Test (right) data from Irvine repository. [Schapire and Singer, ML 1999]



Boosting and logistic regression

Logistic regression assumes:

$$P(Y=1|X) = \frac{1}{1+\exp(f(x))}$$

And tries to maximize data likelihood:

$$P(\mathcal{D}|H) = \prod_{i=1}^{m} \frac{1}{1+\exp(-y_i f(x_i))}$$

Equivalent to minimizing log loss

$$\sum_{i=1}^{m} \ln(1+\exp(-y_i f(x_i)))$$

Boosting minimizes similar loss function!!

$$\frac{1}{m}\sum_i \exp(-y_i f(x_i)) = \prod_t Z_t$$

Both smooth approximations of 0/1 loss!

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Logistic regression:

- Minimize loss fn

$$\sum_{i=1}^{m} \ln(1 + \exp(-y_i f(x_i)))$$

- Define

$$f(x) = \sum_j w_j x_j$$

where x_j predefined

Boosting:

- Minimize loss fn

$$\sum_{i=1}^{m} \exp(-y_i f(x_i))$$

- Define

$$f(x) = \sum_t \alpha_t h_t(x)$$

where $h_t(x_i)$ defined dynamically to fit data

(not a linear classifier)

- Weights α_j learned incrementally

Takeaways

- Combine weak classifiers to obtain very strong classifier
  - Weak classifier – slightly better than random on training data
  - Resulting very strong classifier – can eventually provide zero training error
- AdaBoost algorithm
- Most popular application of Boosting:
  - Boosted shallow decision trees!
  - Very simple to implement, very effective classifier

References

- Trevor Hastie, Robert Tibshirani, Jerome Friedman: The Elements of Statistical Learning: Data Mining, Inference and Prediction, Chapter 10
- Ziv Bar-Joseph, Tom Mitchell, Pradeep Ravikumar and Aarti Singh: CMU 10-701

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