Review Materials for Probability and Statistical Inference

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This note is just a summary about the main materials covered in Probability and Statistical Inference. In this class, we will go through the details that are used in linear regression models.

I highly recommend this book for a review of probability and statistics:

All of Statistics by Larry Wasserman.

1 Probability

1.1 Discrete Type Random Variables

1. Binomial distribution $B(n, p)$.
2. Poisson distribution $\mathrm{Pois}(\lambda)$.
3. Multinomial distribution $\mathrm{Mul}(n; p_1, \ldots, p_k)$.
4. Geometric distribution $\mathrm{Geo}(p)$.
5. Negative Binomial distribution $\mathrm{NB}(r, p)$.

The Poisson distribution can be derived as a limiting case to the binomial distribution as the number of trials goes to infinity and the expected number of successes remains fixed, see here. Indeed, suppose $X_n \sim B(n, \frac{\lambda}{n})$, then for a fixed integer k,

$$
\begin{aligned}
\lim_{n\to\infty} \mathbb{P}\left[X_n = k\right] &= \lim_{n\to\infty} \frac{n!}{k!(n-k)!}\left(\frac{\lambda}{n}\right)^k \left(1-\frac{\lambda}{n}\right)^{n-k} \\
&= \frac{\lambda^k}{k!} \lim_{n\to\infty} \frac{n!}{n^k(n-k)!} \lim_{n\to\infty} \left(1-\frac{\lambda}{n}\right)^{n-k} \\
&= \frac{\lambda^k}{k!} \lim_{n\to\infty} \left(1-\frac{\lambda}{n}\right)^{n} \\
&= \frac{\lambda^k}{k!} e^{-\lambda},
\end{aligned}
$$

which is exactly the probability "density" function for a Poisson distributed random variable. Therefore, it can be used as an approximation of the binomial distribution if n is sufficiently large and p is sufficiently small. There is a rule of thumb stating that the Poisson distribution is a good approximation of the binomial distribution if n is at least 20 and p is smaller than or equal to 0.05, and an excellent approximation if $n \geq 100$ and $np \leq 10$.

1.2 Continuous Type Random Variables

1. Uniform distribution $U[a,b]$
2. Normal distribution/Gaussian distribution $N(\mu, \sigma^2)$.
3. Chi-squared distribution $\chi^2(n)$.
4. Gamma distribution $\text{Gamma}(\alpha, \beta)$.
5. Beta distribution $\text{Beta}(\alpha, \beta)$.

2 Statistical Inference Procedure

Materials for statistical inference can be found here. Generally speaking there a 3-step procedure for an inference, which is:

2.1 Build your model

In the univariate inference problem, you are always about to collect a set of independently and identically distributed (iid) samples, $X_1, X_2, \ldots, X_n$. Your task is to model the distribution, say $X_i \sim_{\text{i.i.d}} X \sim N(\mu, \sigma^2)$, $i = 1, \ldots, n$.

This is also how you *parametrize* the model: now instead of looking for a suitable pdf for X, you only need to find suitable values for μ and σ^2.

2.2 Estimation for model parameters

There are various way to estimate the model parameters. For example, in the above case, if we view μ and σ^2 as constant, then we can estimate them using ME, MLE. We also have various desired properties defined for the estimators, such as unbiasedness and consistency etc. Details can be found in Chapter 2-4 here.

On the other hand, we can also view μ and σ^2 as random variables thus introduce hierarchical structure of the estimation. See Chapter 5 here.

2.3 Understand the model

To begin with, you have the properties from the estimation of the parameters, so you know what you can expect and predict if the number of the samples from the same distribution. See Chapter 6-9 here. Moreover, you can do hypothesis testing on your assumptions. See Chapter 10 here.