T

T

T

T

T

T	F	F	T	F	F		F
F	F	T	T	T	T		F
T	F	T	T	T	T		F
F	T	F	F	T	F		F
T	T	F	T	T	T		
F	T	T	T	T	T		T
T	T	T	T	T	T		T

A	B	C	$(\neg A \vee \neg B) \wedge \neg(\neg C \vee \neg B) \wedge (\neg A \vee C) \equiv \neg A \wedge B \wedge C$
F	F	F	F
F	F	T	F
F	T	F	F
F	T	T	T
T	F	F	F
T	F	T	F
T	T	F	F
T	T	T	F

a) $\alpha \vdash \beta$ iff $\alpha \Rightarrow \beta$ is valid, which by implication elimination is valid iff $\neg\alpha \vee \beta$ is valid. Note that $(\neg\alpha \vee \beta) \equiv \neg(\alpha \wedge \neg\beta)$ by De Morgan laws. Thus, $\neg\alpha \vee \beta$ is valid iff $\alpha \wedge \neg\beta$ false over all models, that is, $\alpha \wedge \neg\beta$ is unsatisfiable.

IGNORE THIS PART

~~Exercise 4/~~

~~a) $(A \vee C) \wedge (B \vee C) \Rightarrow (B \wedge C)$~~

A	B	C	$(A \vee C)$	$(B \vee C)$	$(A \vee C) \wedge (B \vee C)$	$(B \wedge C)$ [illegible]
F	F	F	F			
F	F	T	T			
F	T	F				
F	T	T				
T	T	F				
T	T	F				
F	T	T				
T	T	T				

José Dixon
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Assignment 3 – COSC 470
Exercise 1: Prove each of the following

a) True $\models a$ iff for all models in which True is true a is true (def of entails). Thus we have, True $\models a$ iff a is true in all models; which is the definition of a valid sentence.

b) False $\models a$ iff for all models in which False is true, a is true (def of entails). Thus, the statement is vacuously true, since there are no models in which False is true.

c) $a \models B$ iff for all models in which a is true, $(\Rightarrow)$ Therefore, if $a \models B$ then $a \Rightarrow B$ is true for all models where $a =$ True, and it is also true for all models where $a =$ False by definition of implication. So if $a \models B$ then $a \Rightarrow B$ is valid. $(\Leftarrow)$ If $a \Rightarrow B$ is valid, then if a is true and B is true, and we have $a \models B$, and if a is false, then $a \models B$ by part (b). Thus, if $a \Rightarrow B$ is valid then $a \models B$. Thus, $a \models B$ iff $a \Rightarrow B$ is valid.

d) $(a \equiv B) \equiv (a \models B) \wedge (B \models a)$. Thus, by the deduction theorem (i.e. part (c)) $a \equiv B$ iff $a \Rightarrow B$ and $B \Rightarrow a$ are valid. These are both valid iff $(a \Rightarrow B) \wedge (B \Rightarrow a)$ is valid, which is equivalent to $a \Leftrightarrow B$ by biconditional elimination.

F	T	T
T	F	T
T	T	T
F	F	T
F	T	T
T	F	T
T	T	T