(1) Introduction

1 Welcome to 8.04

- M. Evans lecturer
  - Astrophysics division
  - Work on gravitational wave detectors
  - I'm an experimentalist, so
  - I do math, but I'm not a mathematician
  - I'm very excited to be teaching 8.04!
- *Everything is on Stellar* *(show slide)*
  - Course schedule, psets, lecture notes, practice exams, ...
- Recitation instructors: Prof's Schechter, Stewart and Vuletic
  - R01 Prof. Schechter MW9 in 4-261
  - R02 Prof. Schechter MW10 in 4-261
  - R03 Prof. Stewart MW10 in 26-204
  - R04 Prof. Stewart MW11 in 26-204
  - R05 Prof. Vuletic MW1 in 26-302
  - R06 Prof. Vuletic MW2 in 26-302
  - yes, there are 2 at 10am. We are trying to avoid 8.044
  - for the first couple of weeks, feel free to change recitation sections (on Stellar)
  - after that, let the TA know if you need to change recitations
- TA: Paolo Glorioso
  - primary contact for questions about Pset grading

2 Goal of 8.04

- Understand why we **need** Quantum Mechanics
- Learn Quantum Mechanics!
  - The language of QM
  - How to solve QM problems
  - An intuition of QM phenomena

- QM is not hard
  - Everyone here can learn QM!
  - But is is not trivial - you will have to put in some effort
  - Solving problems is ESSENTIAL to developing an intuition for how QM works (useful in life, and on exams)
- My primary task is making QM so interesting that your curiosity drives you
- You will do your psets,
  - but the objective is not a finished pset
  - or the effect it has on your grade.
  - It is the skill and intuition you develop along the way.
- You will come to lecture and take notes,
  - but the objective is not the notes you take away
  - or the clicker questions you answer.
  - It is what you internalize by putting it in your own hand,
  - and the content added to the lecture by your questions.

3 Your Responsibilities

- To finish this semester with a working knowledge of QM
- Problem Sets
  - Due at 11am every Tuesday, in the usual boxes on the 3rd floor junction of Buildings 8 and 16
  - Collaboration is fine, but what you turn in must be written by you
  - To do well on exams, you should be able to do **all of the pset problems on your own**
  - Write your name, course number, pset number, and recitation number on the pset
  - Neatness is important! (staple, use scratch paper, don't compress, ...)
- Exams
  - 2 mid-term exams (tentatively March 6 and April 17)
  - 1 final exam (great!)

4 Grading

The score in the class will be computed according to

$$S_{\text{class}} = 0.75\, S_{\text{exams}} + 0.2\, S_{\text{pset}} + 0.05\, S_{\text{clicker}} \tag{1}$$

where

$$S_{\text{exam}} = 0.2\, S_{\text{mid}_1} + 0.3\, S_{\text{mid}_2} + 0.5\, S_{\text{final}} \tag{2}$$

- Pset average:
  - Late psets will not be accepted
  - Lowest pset score will be dropped (so missing one is ok)
  - Help your graders help you; put your answer in a box
- Clicker average: It matters THAT you answer, not HOW you answer

So, that is how your "score" is calculated, but what about your grade?

My goal is to give exams which test your understanding, such that 90% is an A, 80% a B, etc.

Since exam problems are new each year, I may misjudge their difficulty, so these grade boundaries are just a rule of thumb. I will set the grade boundaries at the end of the course such that, to the best of my ability, your grade represents the understanding you demonstrated.

This means that I don't know the numerical grade boundaries until after the final! (And thus I cannot tell you what score you need to get an A.)

5 Textbooks

No required text for this course. Two recommended texts (show slide)

- Grifiths, *Introduction of Quantum Mechanics*
  - students like this book
  - it has been used in 8.05 for many years
  - we will cover much of chapters 1-6
  - it does **NOT** cover any of the material you will see in lectures 1 and 2
- Scherrer, *Quantum Mechanics: An Accessible Introduction*
  - this book has a much softer start
  - chapter 1 is relevant to lectures 1 and 2
  - we will cover much of chapters 1-9

I will suggest relevant readings from from these texts each week along with the pset. There are many other relevant texts, all of which fail to cover all of 8.04 well, and yet can be very useful at times

- Gasiorowicz, *Quantum Physics*
- Shankar, *Principles of Quantum Mechanics*
- Eisberg & Resnick, *Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles*
- Liboff, *Introductory Quantum*

NB - There are 2 different languages used to describe QM: Wave mechanics (what we will use for most of this class) and Matrix mechanics (we may touch on this at the end, but covered in 8.05). Different texts emphasize one or the other depending on their style.

NB2 - Theoretical physicist are fond of using the unit system which suits them best, usually by making all of the constants relevant for their work equal to 1. To the best of my ability, we will stick with SI units in this class.

Questions?

6 Planck and the Black Body

- Sc 1.2, E&R 1-all, Ga 1-1

Enough! Lets talk about Physics! *(show slide)*

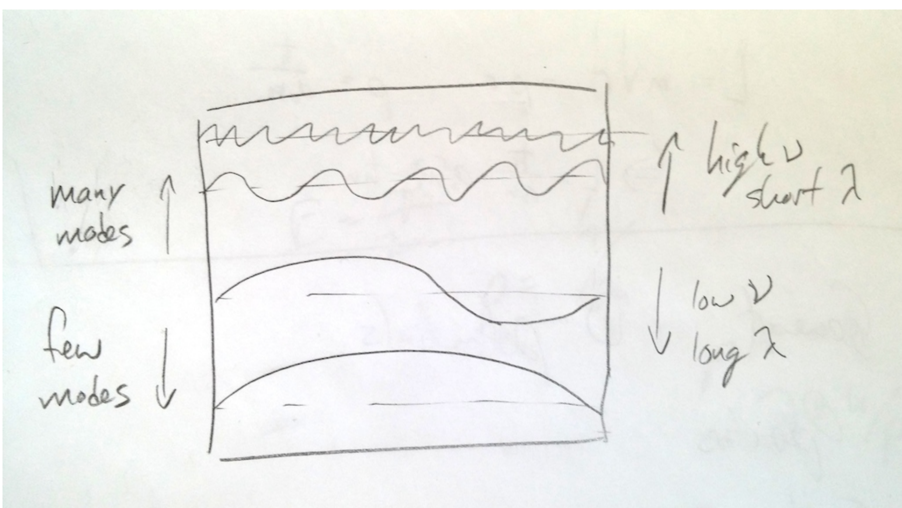
I'll start with a slightly embellished, and yet somewhat simplified version of history... It was 1900 and things were peachy in Physics. Maxwell's equations had put a pretty bow on the whole business of Electricity and Magnetism 40 years back, Kelvin and others had figured out statistical mechanics, and nothing much was left to be said. All we needed to do was understand why Michelson didn't detect the ether, and why the Sun is yellow.

The whole ether business led to General Relativity, which we will cover in its full quantum form next week. ;-)

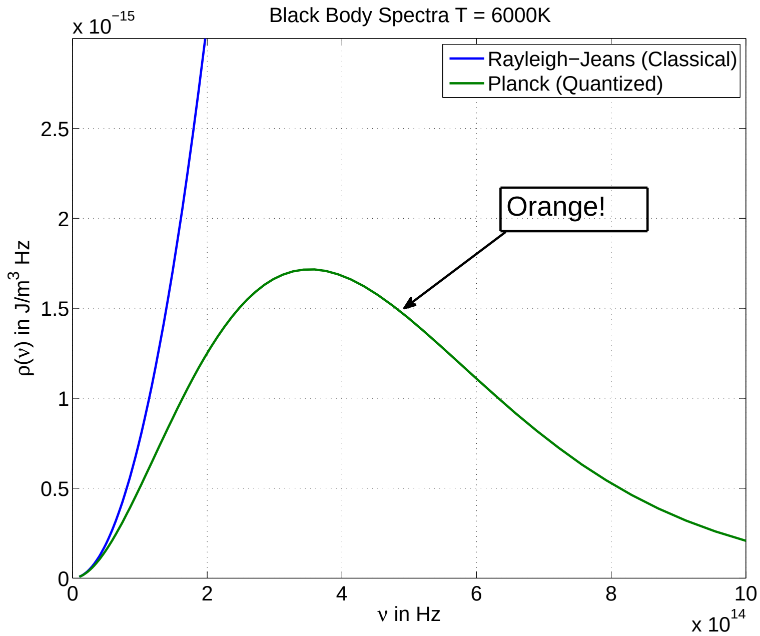
The problem of the Sun comes down to "black body radiation", or in plain English the radiation which comes off of something hot. A combination of E&M and StatMech predicts that the spectrum of radiation from a black body should have fixed amount of energy, $k_B T$, at each frequency.

$$E_{@\nu} = k_B T \quad \text{for all modes with frequency } \nu \tag{3}$$

Consider a small conducting box. The modes it contains look like



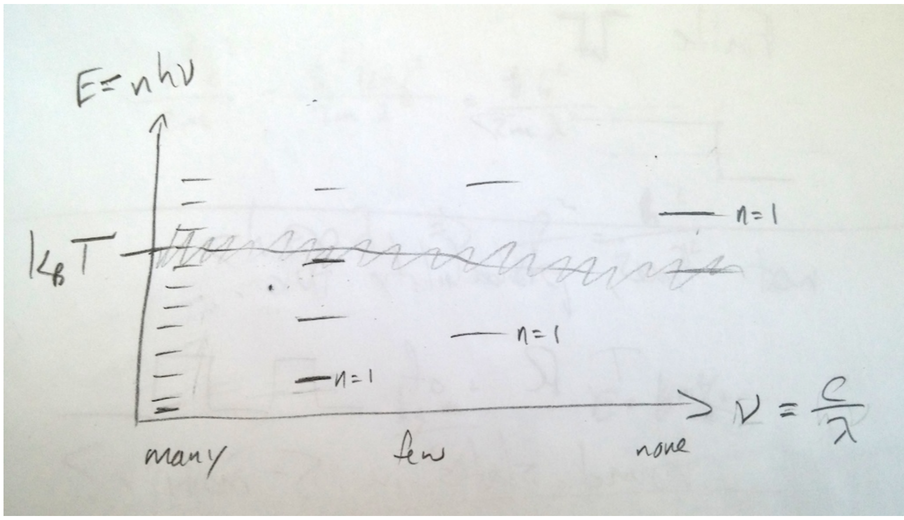
There are an infinity of frequencies accessible to EM radiation, with increasing density as the wavelength shrinks and the frequency increases, so the integrated power goes to infinity! Fortunately, this is not what is observed (or we would all be very cold).



To understand this, and to become famous, a fellow named Max Planck invented a reason why hot things could not produce very high frequency EM waves. He did this by asserting that the energy in each EM frequency was not a continuous value, but rather an integer multiple of some constant times that frequency:

$$E_{@\nu} = nh\nu \text{ , for and } n \in \mathbb{Z} \tag{4}$$

With the net result that for frequencies in which $h\nu \gg k_B T$, there is not enough thermal energy to push $n > 0$, so these modes are "frozen out" and contain *no energy*.



The probability of any given mode having energy E is given by the Boltzman distribution.

$$P(E) = \frac{e^{-E/k_BT}}{k_BT} \tag{5}$$

which gives a mean energy of k_BT.

Curve fitting gave Planck the value $h = 6.6 \times 10^{-34}\,\mathrm{J\,s}$. As far as anyone could tell, there was no reason to believe that EM radiation is quantized. It was just curve fitting and Planck's imagination.

7 Einstein and the Photoelectric Effect

- Sc 1.3, E&R 2-{1 2}, Ga 1-2

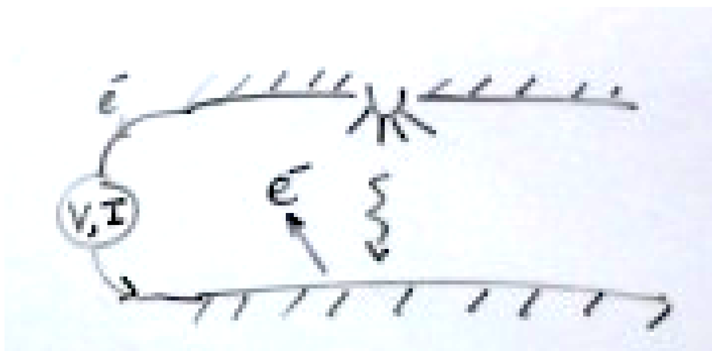
But in 1905 an ambitious student named Albert (age 26) explained the photoelectric effect in terms of quanta of light. The photoelectric effect, in which electrons are kicked out of a metal by an incident EM wave, had been known for some time, but its most interesting implications eluded physicists.

The interesting feature of the photoelectric effect is that the *energy* of the ejected electrons depends on the frequency of the EM wave, while the *rate* of electron ejection depends on the intensity.

Photoelectric Effect

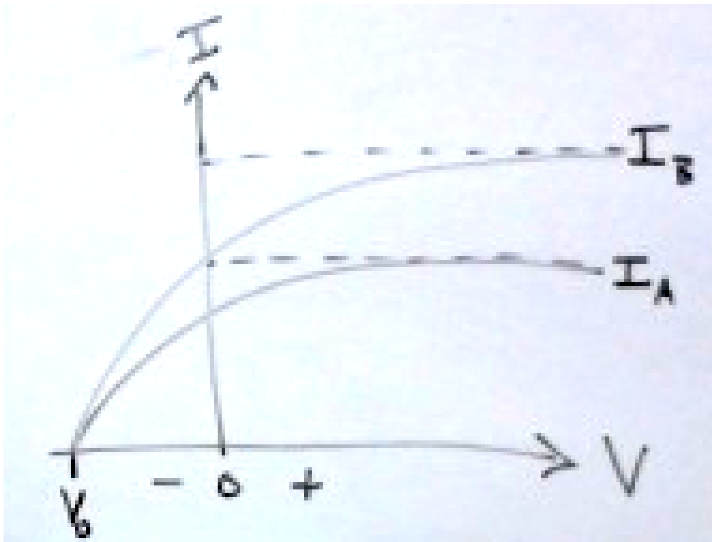
- Ejected e^- energy depends on EM frequency
- Ejection *rate* depends on EM intensity

An example experimental setup looks like this:



(show PHET http://phet.colorado.edu/en/simulation/photoelectric)

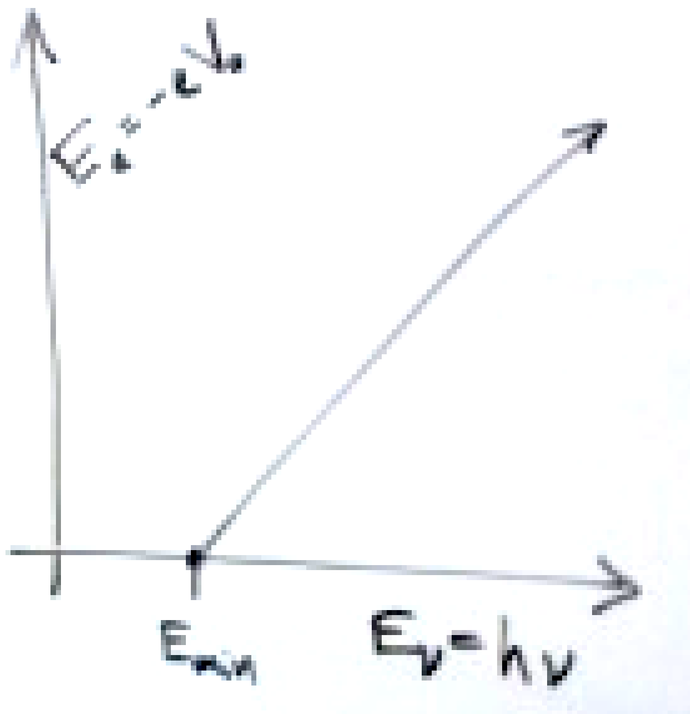
We can use the voltage in our setup to determine the energy of the electrons: is the voltage is working against them, they need a minimum energy to get across the gap. And we can use the current to determine the rate at which electrons are kicked out of the metal, with a voltage that pulls them across the gap no matter their energy. The plot shows current as a function of voltage for 2 different light intensities I_A and I_B.



Classically, there is no reason why the energy of the electrons should not depend on the intensity of the EM wave: greater field amplitude means more force means faster electrons, right?

Einstein took Planck's assertion seriously, and showed that the energy of the electrons ejected in the photoelectric effect could be explained if he assumed that the EM wave was actually a collection of quanta with energy $h\nu$ (these quanta later became known as photons). Higher intensity means more photons, but the energy transfer happens between one photon and one electron and if that energy is not enough to pull an electron out of the metal that it lives in, then no electrons are ejected. *Regardless of intensity.*

This can be seen clearly by changing the frequency of the light source. If Planck was right, and the energy of each photon is fixed by $E_\nu = h\nu$, then we should find that higher frequency light results in higher energy electrons, and thus a more negative stopping voltage V_0.



This is exactly what is observed.

8 Compton Effect

- Sc 1.3, E&R 2-4, Ga 1-3

A few years later, in 1923, Arthur Compton (age 31) took this whole business one step further, and demonstrated experimentally that not only is light quantized into photons with energy $h\nu$, but that these little bits of light carry a fixed amount of momentum,

Photon Momentum

$$p = mv = (E/c^2)c = E/c = h\nu/c = h/\lambda \tag{6}$$

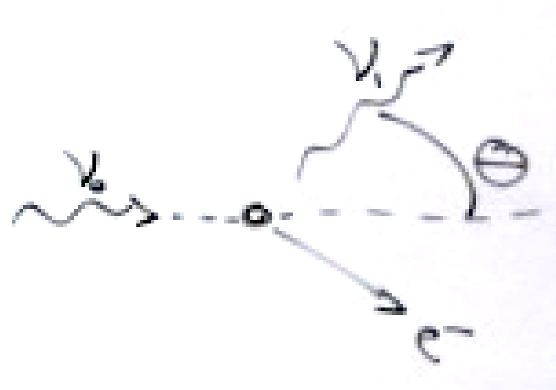
$$\text{where } \lambda \text{ is wave length, with } \lambda = c/\nu \tag{7}$$

He did this by showing that when a photon scatters off of an electron, the wavelength of the photon changes. Classically, there is no reason for this. (The electron should move at the driving frequency and thereby change EM wave amplitude, but not its frequency.)

Recall how scattering works from 8.01:

add a normal 2 mass scattering drawing, labeled "8.01"

add a EM wave exciting an electron drawing, labeled "8.02"



use λ, not ν here, and label "8.04"

where, as usual, we need to conserve energy and momentum in the collision. Doing so leads to

Compton equation

$$\lambda_1 - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta) \tag{8}$$

which is, as before, exactly what is observed.

This brings us *unavoidably* to the conclusion that photons exist and carry momentum; they can be treated like little soccer balls bouncing off of electrons.

Ok... we have seen that Classical Physics is in trouble: EM radiation appears to behave both like waves and like particles. Next, we will learn another way in which Classical Physics predicts the end of the world!

Questions?

For the record (but not for lecture)...

We should conserve both momentum and energy, with the twist that we must use the relativistically correct energy $E = \sqrt{m^2c^4 + p^2c^2}$ in which the photon has no mass so $E_\nu = p_\nu c$. Conservation of momentum gives:

$$\begin{aligned}
\vec{p}_{\nu 0} &= \vec{p}_e + \vec{p}_{\nu 1} \\
\Rightarrow |\vec{p}_e|^2 &= |\vec{p}_{\nu 0}|^2 + |\vec{p}_{\nu 1}|^2 - 2\ \vec{p}_{\nu 0} \cdot \vec{p}_{\nu 1} \\
\Rightarrow p_e^2 c^2 = E_{e1}^2 - (m_e c^2)^2 &= E_{\nu 0}^2 + E_{\nu 1}^2 - 2\ E_{\nu 0} E_{\nu 1} \cos\theta
\end{aligned}$$

Conservation of energy gives us

$$\begin{aligned}
E_{\nu 0} + m_e c^2 &= E_{e1} + E_{\nu 1} \\
\Rightarrow E_{e1}^2 + (m_e c^2)^2 - 2E_{e1} m_e c^2 &= E_{\nu 0}^2 + E_{\nu 1}^2 - 2\ E_{\nu 0} E_{\nu 1}
\end{aligned}$$

and

$$\begin{aligned}
E_{e1} &= E_{\nu 1} - E_{\nu 0} + m_e c^2 \\
\Rightarrow E_{e1} &= h(\nu_1 - \nu_0) + m_e c^2
\end{aligned}$$

We can combine these to find

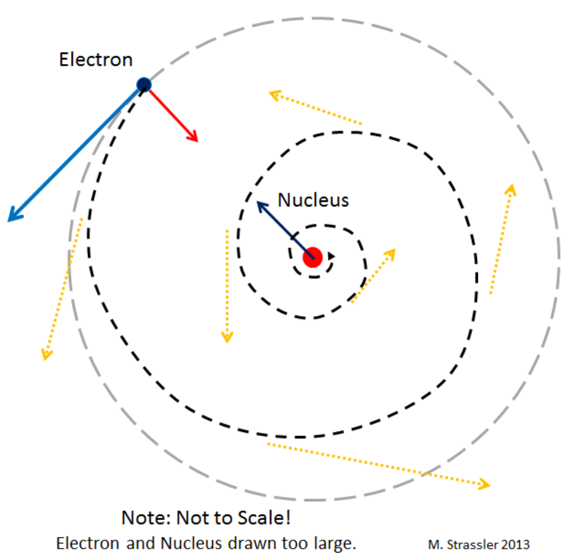
$$\begin{aligned}
(E_{e1} - m_e c^2) m_e c^2 &= E_{\nu 0} E_{\nu 1} (1 - \cos\theta) \\
\Rightarrow h(\nu_1 - \nu_0) m_e c^2 &= h^2 \nu_0 \nu_1 (1 - \cos\theta) \\
\Rightarrow \frac{\nu_1 - \nu_0}{\nu_0 \nu_1} &= \frac{h}{m_e c^2} (1 - \cos\theta) \\
\Rightarrow \lambda_1 - \lambda_0 &= \frac{h}{m_e c} (1 - \cos\theta)
\end{aligned}$$

9 Atomic Stability and Spectra

- Sc 1.5, E&R 4-{3, 4}, Ga 1-5

We all know now that atoms are made of electrons going around a nucleus of protons and neutrons, but this was big news back in 1911. Once Rutherford forced us to this conclusion with his scattering experiments, we were faced with a little problem: an electron going around a proton is an accelerating charge, and so according to classical E&M should radiate away its energy and crash into the proton. If Classical E&M were right, this would take less than a pico-second and it would be the death of us all, not to mention the end of everything interesting in the universe.

Once again, we can be thankful that everything you learned in 8.02 is wrong. To be fair, everything in 8.01 is also wrong. In fact, all physical theories to date are wrong, intractable, or both, in some limit. Remember, in physics we are in the business of developing models with demonstrable predictive power, not declaring truth, so our models only need to work in the region of parameter space which is of interest to us.



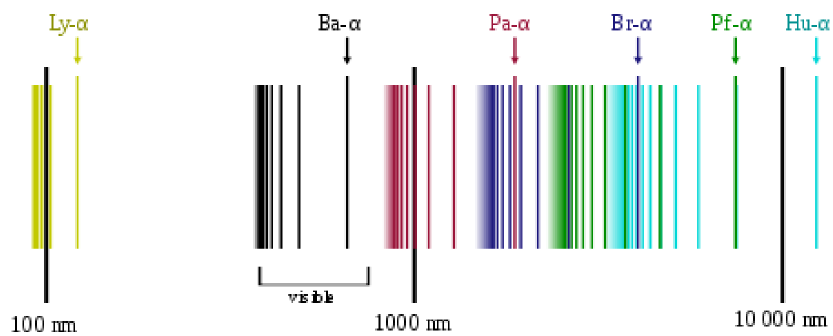
There were, however several clues to the funny business that was going on inside atoms. In particular, it had been observed that various species of gasses only interacted with (absorbing or emitting) specific wavelengths of light.

(show NEON slide, and spectrograph slide)

Hydrogen, being the simplest of atoms with only one electron was the favored system. Various experimenters found collections of emission lines of hydrogen, and they stuck their names on them, but they didn't now how to predict the wavelengths or frequencies of these lines.

(show H2 spectrograph slide)

From this we have:



- Lyman series (n = 1)
- Balmer series (n = 2)
- Paschen series (n = 3)
- Brackett series (n = 4)
- Pfund series (n = 5)
- Humphreys series (n = 6)

Rydberg managed to unify all of these with his equation describing the frequency of each of these lines (basically by cheating off of Balmer):

Rydberg Equation

$$\frac{1}{\lambda} = \frac{\nu}{c} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \qquad \text{where } R = 1.1 \times 10^7\,\mathrm{m}^{-1} \quad (9)$$

in which R is a ***fit parameter with no physical motivation****.*

If we combine this with Planck's constant, we can find the energy of the photons associated with each spectral line

$$E_{2\to 1} = h\nu = R\,h\,c\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \tag{10}$$

$$\text{where} \quad R\,h\,c = 2.2 \times 10^{-18}\,\text{J}. \tag{11}$$

Great, but what about the electron spiraling to its demise?

Development was rapid in those years, and in 1913 a new model was proposed which seemed to work well. Niels Bohr (age 27), connecting the quantization in EM waves to the quantized spectral of hydrogen, took a great leap. How about quantizing the angular momentum of the electron? After all, *Planck's constant already has units of angular momentum.*

Bohr's Great Guess

$$E_\nu = h\nu \quad : \quad [J] = h[1/s]$$

$$\Rightarrow h = E_\nu/\nu \quad : \quad h = [J\ s] = \left[\frac{kg\ m^2}{s^2} s\right] = [kg\ m^2/s]$$

$$and$$

$$\vec{L} = \vec{r} \times \vec{p} \quad : \quad L = [m][kg\ m/s] = [kg\ m^2/s]$$

This sounds nuts, but it seems to work!

If we require that $L = nh/2\pi$ for the electron in orbit around a proton in hydrogen, we get orbits only at specific radii. We start with the basics ingredients

Coulomb) $\vec{F} = k_e \frac{q_1 q_2}{r^2}\hat{r} = -k_e\frac{e^2}{r^2}\hat{r}$

with $k_e = \dfrac{1}{4\pi\epsilon_0}$ in MKS units

Circular Orbit) $\vec{a} = -\frac{v^2}{r}\hat{r}$

Newton) $m_e\frac{v^2}{r} = k_e\frac{e^2}{r^2}$

At this point we use Bohr's "postulate" (read "lucky guess") to make a substitution and solve for the radii of the Bohr orbits

$$
\begin{aligned}
L = n\hbar \qquad &\text{where} \quad \hbar \equiv \frac{h}{2\pi} \\
L = r m_e v = n\hbar \qquad &\Rightarrow \qquad v = \frac{n\hbar}{r m_e} \\
m_e \frac{v^2}{r} = \frac{n^2\hbar^2}{m_e r^3} = k_e \frac{e^2}{r^2} \qquad &\Rightarrow \qquad r = \frac{\hbar^2}{m_e k_e e^2} n^2 \\
r = a_0 n^2 \qquad &\text{where} \quad a_0 = 53\,\text{pm} \;\; \text{“Bohr radius”}
\end{aligned}
$$

By quantizing angular momentum with $\hbar$ we saved the electron from its doom! With quantized orbits, it cannot spiral in! And we have determined the size of the hydrogen atom to boot!

Fine, fine, now we have a radius based on Planck's constant, but how do we compare it with experiment?

We can make the connection by computing the energy of each orbit. Classically, this is just

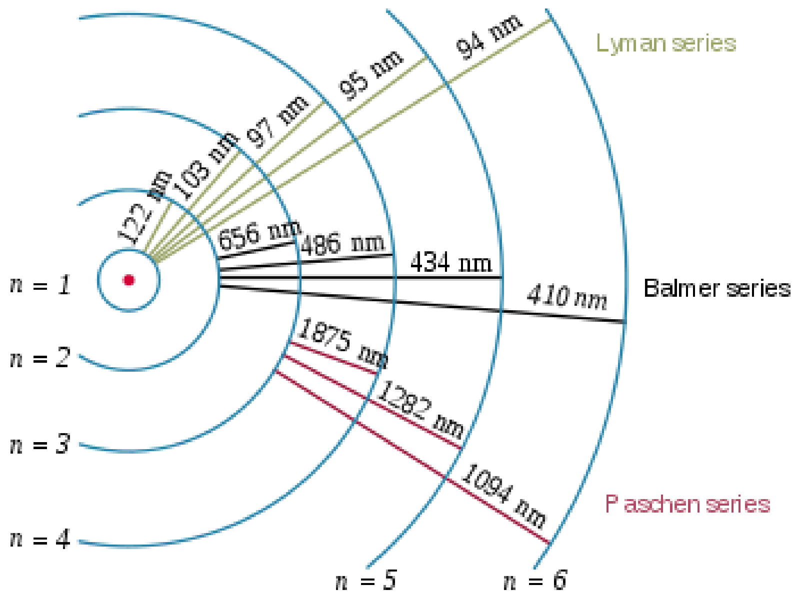
Energy of n^{th} Bohr Orbit

$$
\begin{aligned}
E_n &= KE + U = \tfrac{1}{2} m v^2 - \frac{k_e e^2}{r} = \tfrac{1}{2}\frac{k_e e^2}{r} - \frac{k_e e^2}{r} = -\frac{k_e e^2}{2r} \\
&= \frac{-k_e e^2}{2 a_0 n^2} = \frac{-m_e (k_e e^2)^2}{2\hbar^2 n^2} = \frac{-2.2 \times 10^{-18}\,\text{J}}{n^2}
\end{aligned}
$$

I would bet real money that Bohr spent a lot of time trying to find a combination of fundamental constants that gave that number, the Rydberg energy, before he did all the math we just did. But with this model in hand, and with EM radiation quantized in units of $h\nu = \hbar\omega$, the Rydberg equation just comes out as conservation of energy. As the electron moves from one orbit to another, it emits or absorbs a photon with the necessary energy.

Energy of photon emitted moving from n_1 to n_2

$$E_\nu = h\nu = E_{n1} - E_{n2} = 2.2 \times 10^{-18}\,\mathrm{J}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) \quad (12)$$



Ok, let's back up and think about this. We just did something dramatic. We just quantized *the angular momentum of an electron*. That is very different from quantizing the energy of an EM wave to get photons. We are quantizing the movement of *matter*. And yet, at this point, in 1913, we are still just bumping around in the dark. We have no idea *why* angular momentum should be quantized. (Though I assure you that by the end of this course you will know why.)

Next time we will continue to pull on this thread and see how far down the rabbit hole we end up.