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Question：已知 $\nabla\times\vec{A}=\nabla\times\vec{A}\Big|_{t'}+\nabla t'\times\dfrac{\partial\vec{A}}{\partial t'}$，

如何求：$\nabla\times\vec{A}\Big|_{t'}=\dfrac{q\vec{v}\times\vec{r}}{4\pi\varepsilon_0 c^2 r^3}$，$\nabla t'\times\dfrac{\partial\vec{A}}{\partial t'}=\dfrac{q\vec{n}\times\dot{\vec{v}}}{4\pi\varepsilon_0 c^3 r}$？

Answer：$\varphi(\vec{r},t)=\dfrac{q}{4\pi\varepsilon_0}\left[\dfrac{1}{\left(r-\vec{\beta}\cdot\vec{r}\right)}\right]_{ret}$，$\vec{A}(\vec{r},t)=\dfrac{q}{4\pi\varepsilon_0 c}\left[\dfrac{\vec{\beta}}{\left(r-\vec{\beta}\cdot\vec{r}\right)}\right]_{ret}=\dfrac{\vec{\beta}}{c}\varphi$，

ret 的意思是取 $t'=r-\dfrac{r(t')}{c}$ 时刻的值，$\vec{r}=r\vec{n}$，

$$\nabla\times\vec{A}\Big|_{t'}=\nabla_{t'}\times\left(\frac{\vec{\beta}\varphi}{c}\right)=\left(\nabla_{t'}\varphi\right)\times\frac{\vec{\beta}}{c}=-\frac{\vec{\beta}}{c}\times\left(\nabla_{t'}\varphi\right)=-\frac{\vec{\beta}}{c}\times\frac{q}{4\pi\varepsilon_0}\nabla_{t'}\frac{1}{\left(r-\vec{\beta}\cdot\vec{r}\right)},$$

$$=-\frac{\vec{\beta}}{c}\times\frac{q}{4\pi\varepsilon_0}\frac{-1}{\left(r-\vec{\beta}\cdot\vec{r}\right)^2}\nabla_{t'}\left(r-\vec{\beta}\cdot\vec{r}\right)=\frac{\vec{\beta}}{c}\times\frac{q}{4\pi\varepsilon_0}\frac{1}{\left(r-\vec{\beta}\cdot\vec{r}\right)^2}\left(\vec{n}-\vec{\beta}\right),$$

$$\approx\frac{\vec{\beta}}{c}\times\frac{q}{4\pi\varepsilon_0}\frac{\vec{n}}{r^2}=\frac{q\vec{v}\times\vec{r}}{4\pi\varepsilon_0 c^2 r^3}$$，（非相对论情况，取 $\beta\approx 0$）

$\dfrac{\partial t}{\partial t'}=\left(1-\vec{\beta}\cdot\vec{n}\right)$，$\nabla t'=\dfrac{\vec{n}}{c\left(1-\vec{\beta}\cdot\vec{n}\right)}$，$\dfrac{\partial t}{\partial t'}\nabla t'=\dfrac{\vec{n}}{c}$，，

$$\nabla t'\times\frac{\partial\vec{A}}{\partial t'}=\frac{\vec{n}}{c}\times\frac{\partial\vec{A}}{\partial t}=\frac{\vec{n}}{c}\times\frac{\partial}{\partial t}\left(\frac{\vec{\beta}\varphi}{c}\right)=\varphi\frac{\vec{n}}{c^2}\times\frac{\partial\vec{\beta}}{\partial t}=\frac{q}{4\pi\varepsilon_0 c^2}\left[\frac{\vec{n}}{\left(r-\vec{\beta}\cdot\vec{r}\right)}\right]_{ret}\times\frac{\dot{\vec{v}}}{c}\approx\frac{q\vec{n}\times\dot{\vec{v}}}{4\pi\varepsilon_0 c^3 r},$$