第十三次

（13-1）波长为λ的平面电磁波垂直射入屏上的圆形小孔，设小孔半径为r_0 $(r_0 >> \lambda)$，求夫琅禾费衍射。

（13-2）（i）$u_x = \dfrac{u_x'\sqrt{1-v^2/c^2}}{1+\dfrac{v}{c^2}u_z'}$，$u_y = \dfrac{u_y'\sqrt{1-v^2/c^2}}{1+\dfrac{v}{c^2}u_z'}$，$u_z = \dfrac{u_z'+v}{1+\dfrac{v}{c^2}u_z'}$

若$u_x^2+u_y^2+u_z^2=c^2$，

即$\left(u_x'\sqrt{1-v^2/c^2}\right)^2+\left(u_y'\sqrt{1-v^2/c^2}\right)^2+\left(u_z'+v\right)^2=c^2\left(1+\dfrac{v}{c^2}u_z'\right)^2$

有：$u_x'^2+u_y'^2+u_z'^2=c^2$

（ii）$\dfrac{u^2}{c^2}=\dfrac{u_x^2+u_y^2+u_z^2}{c^2}=\dfrac{\left(u_x'\sqrt{1-v^2/c^2}\right)^2+\left(u_y'\sqrt{1-v^2/c^2}\right)^2+\left(u_z'+v\right)^2}{c^2\left(1+\dfrac{v}{c^2}u_z'\right)^2}=$

$$=\frac{\left(1-v^2/c^2\right)\left(u_x'^2+u_y'^2\right)+\left(u_z'+v\right)^2}{c^2\left(1+\dfrac{v}{c^2}u_z'\right)^2}$$

$$1-\frac{u^2}{c^2}=\frac{c^2\left(1+vu_z'/c^2\right)^2-\left(1-v^2/c^2\right)\left(u_x'^2+u_y'^2\right)-\left(u_z'+v\right)^2}{c^2\left(1+\dfrac{v}{c^2}u_z'\right)^2}$$

$$=\frac{c^2+\left(\dfrac{v}{c}u_z'\right)^2+2vu_z'-\left(1-v^2/c^2\right)\left(u_x'^2+u_y'^2\right)-\left(u_z'+v\right)^2}{c^2\left(1+\dfrac{v}{c^2}u_z'\right)^2}=\frac{c^2-\left(1-v^2/c^2\right)\left(u_x'^2+u_y'^2+u_z'^2\right)-v^2}{c^2\left(1+\dfrac{v}{c^2}u_z'\right)^2}$$

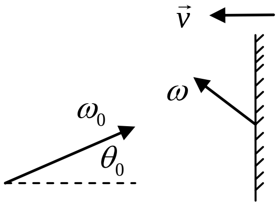
$$=\frac{1-\left(\dfrac{v}{c}\right)^2-\dfrac{u'^2}{\gamma^2c^2}}{\left(1+\dfrac{v}{c^2}u_z'\right)^2}=\frac{1-\dfrac{u'^2}{c^2}}{\gamma^2\left(1+\dfrac{v}{c^2}u_z'\right)^2}$$

即：$\dfrac{1}{(\gamma_u)^2}=\dfrac{1}{\gamma_{u'}^2\gamma^2\left(1+\dfrac{v}{c^2}u_z'\right)^2}$，　$\gamma_u=\gamma\cdot\gamma_{u'}\left(1+\dfrac{v}{c^2}u_z'\right)$

$$\gamma_u u_x=\gamma\cdot\gamma_{u'}\left(1+\frac{v}{c^2}u_z'\right)\cdot\frac{u_x'\sqrt{1-v^2/c^2}}{1+\dfrac{v}{c^2}u_z'}=\gamma_{u'}\cdot u_x'$$

第十四次

（14-1）光源 S 与接收器 R 相对静止，距离为l_0，S-R 装置浸泡在水里，已知静水的折射率为n，水流速度为$\vec{v}$，在流水方向平行和垂直于 S-R 连线的两种情况下，分别计算光源发出讯号到接收器收到讯号的时间。



（14-2）

$k_\mu=\left(k_0\cos\theta_0,k_0\sin\theta_0,0,i\omega_0/c\right)$， $k_0=\omega_0/c$，变换到镜子系，，，，

$$k'_\mu=a_{\mu\nu}k_\nu=\begin{pmatrix}\gamma & 0 & 0 & -i\beta\gamma\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ i\beta\gamma & 0 & 0 & \gamma\end{pmatrix}\begin{pmatrix}k_0\cos\theta_0\\ k_0\sin\theta_0\\ 0\\ i\omega_0/c\end{pmatrix},$$

得 $k'_\mu=\left(\gamma k_0\left(\cos\theta_0+\beta\right),k_0\sin\theta_0,0,i\omega_0\gamma\left(1+\beta\cos\theta_0\right)/c\right)$，反射后， $k''_x=-k'_x$，，

$k''_\mu=\left(-\gamma k_0\left(\cos\theta_0+\beta\right),k_0\sin\theta_0,0,i\omega_0\gamma\left(1+\beta\cos\theta_0\right)/c\right)$，变换回到实验室系，，

$$k'''_\mu=\tilde{a}_{\mu\nu}k''_\nu=\begin{pmatrix}\gamma & 0 & 0 & i\beta\gamma\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ -i\beta\gamma & 0 & 0 & \gamma\end{pmatrix}\begin{pmatrix}-\gamma k_0\left(\cos\theta_0+\beta\right)\\ k_0\sin\theta_0\\ 0\\ i\omega_0\gamma\left(1+\beta\cos\theta_0\right)/c\end{pmatrix}=\begin{pmatrix}-\gamma^2k_0\left(\cos\theta_0+\beta\right)-\omega_0\gamma^2\beta\left(1+\beta\cos\theta_0\right)/c\\ k_0\sin\theta_0\\ 0\\ i\gamma^2\beta k_0\left(\cos\theta_0+\beta\right)+i\omega_0\gamma^2\left(1+\beta\cos\theta_0\right)/c\end{pmatrix}$$

，反射光波的频率：

$\omega=\omega_0\gamma^2\left(1+\beta\cos\theta_0\right)+\gamma^2\beta ck_0\left(\cos\theta_0+\beta\right)=\omega_0\gamma^2\left[\left(1+\beta\cos\theta_0\right)+\beta\left(\beta+\cos\theta_0\right)\right]$，，

反射角： $tg\theta=\left|\dfrac{k'''_y}{k'''_x}\right|=\dfrac{\sin\theta_0}{\gamma^2\left[\beta\left(1+\beta\cos\theta_0\right)+\left(\cos\theta_0+\beta\right)\right]}$

特别地，当垂直入射时， $\theta_0=0$， $\omega=\omega_0\gamma^2\left(1+\beta\right)^2$，，，，，

第十五次

（15-1）$\partial_1F_{23}+\partial_3F_{12}+\partial_2F_{31}=\partial_xB_x+\partial_zB_z+\partial_yB_y=\nabla\cdot\vec{B}=0$，

$\partial_1F_{34}+\partial_3F_{41}+\partial_4F_{13}=\partial_x\left(-i\dfrac{E_z}{c}\right)+\partial_z\left(i\dfrac{E_x}{c}\right)-\dfrac{\partial}{\partial ict}B_2=\dfrac{i}{c}\left(-\dfrac{\partial E_z}{\partial x}+\dfrac{\partial E_x}{\partial z}+\dfrac{\partial B_2}{\partial t}\right)=0$，

$$\partial_2 F_{34}+\partial_3 F_{42}+\partial_4 F_{23}=\partial_y\left(-i\frac{E_z}{c}\right)+\partial_z\left(i\frac{E_y}{c}\right)+\frac{\partial}{\partial ict}B_1=\frac{i}{c}\left(-\frac{\partial E_z}{\partial y}+\frac{\partial E_y}{\partial z}-\frac{\partial B_1}{\partial t}\right)=0\text{ , , , ,}$$

$$\partial_2 F_{14}+\partial_1 F_{42}+\partial_4 F_{21}=\partial_y\left(-i\frac{E_x}{c}\right)+\partial_x\left(i\frac{E_y}{c}\right)-\frac{\partial}{\partial ict}B_z=\frac{i}{c}\left(-\frac{\partial E_x}{\partial y}+\frac{\partial E_y}{\partial x}+\frac{\partial B_z}{\partial t}\right)=0$$

即 $\nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t}$

（15-2）由 $F'_{\mu\nu}=a_{\mu\lambda}a_{\nu\tau}F_{\lambda\tau}$，验证电磁场的变换公式。

，$a_{\mu\nu}=\begin{pmatrix}\gamma & 0 & 0 & i\beta\gamma\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ -i\beta\gamma & 0 & 0 & \gamma\end{pmatrix}$，$\tilde{a}=a^{-1}=\begin{pmatrix}\gamma & 0 & 0 & -i\beta\gamma\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ i\beta\gamma & 0 & 0 & \gamma\end{pmatrix}$，

$$F_{\mu\nu}=\begin{pmatrix}0 & B_3 & -B_2 & -iE_1/c\\ -B_3 & 0 & B_1 & -iE_2/c\\ B_2 & -B_1 & 0 & -iE_3/c\\ iE_1/c & iE_2/c & iE_3/c & 0\end{pmatrix}$$

$$\begin{pmatrix}0 & B'_3 & -B'_2 & -iE'_1/c\\ -B'_3 & 0 & B'_1 & -iE'_2/c\\ B'_2 & -B'_1 & 0 & -iE'_3/c\\ iE'_1/c & iE'_2/c & iE'_3/c & 0\end{pmatrix}=\begin{pmatrix}\gamma & 0 & 0 & -i\beta\gamma\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ i\beta\gamma & 0 & 0 & \gamma\end{pmatrix}\begin{pmatrix}0 & B_3 & -B_2 & -iE_1/c\\ -B_3 & 0 & B_1 & -iE_2/c\\ B_2 & -B_1 & 0 & -iE_3/c\\ iE_1/c & iE_2/c & iE_3/c & 0\end{pmatrix}\begin{pmatrix}\gamma & 0 & 0 & i\beta\gamma\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ -i\beta\gamma & 0 & 0 & \gamma\end{pmatrix}$$

$$=\begin{pmatrix}E_1\gamma\beta/c & \gamma B_3+E_2\gamma\beta/c & -\gamma B_2+\gamma\beta E_3/c & -iE_1\gamma/c\\ -B_3 & 0 & B_1 & -iE_2/c\\ B_2 & -B_1 & 0 & -iE_3/c\\ iE_1\gamma/c & i\beta\gamma B_3+iE_2\gamma/c & -i\beta\gamma B_2+iE_3\gamma/c & E_1\beta\gamma/c\end{pmatrix}\begin{pmatrix}\gamma & 0 & 0 & i\beta\gamma\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ -i\beta\gamma & 0 & 0 & \gamma\end{pmatrix},,,$$

$$=\begin{pmatrix}0 & \gamma B_3+E_2\gamma\beta/c & -\gamma B_2+\gamma\beta E_3/c & -iE_1/c\\ -\gamma B_3-E_2\gamma\beta/c & 0 & B_1 & -i\beta\gamma B_3-iE_2\gamma/c\\ \gamma B_2-E_3\gamma\beta/c & -B_1 & 0 & i\beta\gamma B_2-iE_3\gamma/c\\ iE_1/c & i\beta\gamma B_3+iE_2\gamma/c & -i\beta\gamma B_2+iE_3\gamma/c & 0\end{pmatrix}$$

对比，得：

$E'_1=E_1$，$B'_1=B_1$，，，，，

$B'_2=\gamma\left(B_2-E_3\beta/c\right)$，$B'_3=\gamma\left(B_3+E_2\beta/c\right)$，$E'_2=\gamma\left(\beta B_3c+E_2\right)$，$E'_3=\gamma\left(-\beta B_2c+E_3\right)$，

（ 15-3 ） $E_i^2-p_i^2c^2=E_f^2-p_f^2c^2$ ， 即 ： $m^2c^4=\left(E_1+E_2\right)^2-\left(\vec{p}_1+\vec{p}_2\right)^2c^2$ ，

$$m^2c^4=\left(E_1^2-p_1^2c^2\right)+\left(E_2^2-p_2^2c^2\right)+2E_1E_2-2\left(\vec{p}_1\cdot\vec{p}_2\right)c^2,$$

$$=m_1^2c^4+m_2^2c^4+2\sqrt{\left(m_1^2c^4+p_1^2c^2\right)\left(m_2^2c^4+p_2^2c^2\right)}-2p_1p_2\cos\theta c^2,,,,$$

$$m^2=m_1^2+m_2^2+\frac{2}{c^2}\left[\sqrt{\left(m_1^2c^2+p_1^2\right)\left(m_2^2c^2+p_2^2\right)}-p_1p_2\cos\theta\right]$$

（15-4）（1）实验室系Σ，总动量 $p=\frac{1}{c}\sqrt{\left(E_1^2-m_1^2c^4\right)}$，总能量 $E=E_1+m_2c^2$，变换到质心系 $p'=\gamma_c\left(p-\beta_c E/c\right)=0$，$\beta_c=\frac{pc}{E}$，

（2）记 $M^2c^4=E^2-p^2c^2=\left(E_1+m_2c^2\right)^2-\left(E_1^2-m_2^2c^4\right)=m_1^2c^4+m_2^2c^4+2E_1m_2c^2$，

$\gamma_c=\frac{1}{\sqrt{1-\beta_c^2}}=\frac{E_1+m_2c^2}{Mc^2}$，$p_2'=m_2c\beta_c\gamma_c=m_2c\frac{pc}{E}\cdot\frac{E_1+m_2c^2}{Mc^2}=\frac{m_2\sqrt{E_1^2-m_1^2c^4}}{Mc^2}$，

$\vec{p}_2'=-\vec{p}_1'$，$p_2'=p_1'=\frac{m_2\sqrt{E_1^2-m_1^2c^4}}{Mc^2}$，$E_1'^2=m_1^2c^4+p_1'^2c^2=m_1^2c^4+\frac{m_2^2\left(E_1^2-m_1^2c^4\right)}{M^2}$，

$E_1'=\frac{m_1^2c^2+m_2E_1}{M}$，$E_2'=\frac{m_2^2c^2+m_2E_1}{M}$，总能量 $E_1'+E_2'=Mc^2$

（3）$E_1'=\frac{m_1^2c^2+m_2E_1}{M}$，$m_1=m_2=m$，$E_1\gg mc^2$，$M\approx\frac{1}{c}\sqrt{2mE_1}$，$E_1'\approx\frac{mE_1c}{\sqrt{2mE_1}}$，

$$E_1\approx\frac{2E_1'^2}{mc^2}=\frac{2\times2.2^2}{0.511\times10^{-3}}=1.9\times10^5GeV$$

（15-5）在Σ系，$\vec{E}=E\vec{e}_x$，$\vec{B}=B\vec{e}_y$，变换到Σ'系，$\vec{E}_\perp'=\gamma_u\left(\vec{E}+\vec{u}\times\vec{B}\right)_\perp$，

$\vec{B}_\perp'=\gamma_u\left(\vec{B}-\frac{\vec{u}}{c^2}\times\vec{E}\right)_\perp$，选 $\vec{u}=\frac{c^2B}{E}\vec{e}_z$，$\gamma_u=\frac{1}{\sqrt{1-\frac{c^2B^2}{E^2}}}$，则

$\vec{E}_\perp'=\gamma_u\left(\vec{E}+\vec{u}\times\vec{B}\right)_\perp=\gamma_uE\left(1-\frac{c^2B^2}{E^2}\right)\vec{e}_x=\frac{E}{\gamma_u}\vec{e}_x$，$\vec{B}_\perp'=0$，

运动方程 $\frac{dp_x'}{dt'}=\frac{dm\gamma v_x'}{dt'}=eE'=\frac{eE}{\gamma_u}$，$\frac{dp_y'}{dt'}=\frac{dp_z'}{dt'}=0$，，初始条件：$v_x'=v_y'=v_z'=0$，

$x'=y'=z'=0$，

解得：$\gamma v_x' = \dfrac{v_x'}{\sqrt{1-\left(v_x'/c\right)^2}} = \dfrac{eEt'}{m\gamma_u}$，即：$v_x' = \dfrac{1}{\sqrt{1+\left(eEt'/mc\gamma_u\right)^2}}\dfrac{eEt'}{m\gamma_u}$，

积分：

$$x' = \int_0^{t'} \frac{eEt'}{m\gamma_u}\frac{1}{\sqrt{1+\left(eEt'/mc\gamma_u\right)^2}}dt' = \frac{m\gamma_u c^2}{2eE}\int_0^{t'}\frac{1}{\sqrt{1+\left(eEt'/mc\gamma_u\right)^2}}d\left[1+\left(eEt'/mc\gamma_u\right)^2\right]$$

$$= \frac{m\gamma_u c^2}{eE}\left[\sqrt{1+\left(eEt'/mc\gamma_u\right)^2}-1\right],$$

变换回到 Σ 系 $t=\gamma_u t'$，$x = x' = \dfrac{m\gamma_u c^2}{eE}\left[\sqrt{1+\left(eEt/mc\gamma_u^2\right)^2}-1\right]$

（15-6）碰撞前，总能量：$\hbar\omega + m_0c^2$，总动量：$\hbar\vec{k}$, , , , ,

碰撞后，总能量：$\hbar\omega' + \sqrt{m_0^2c^4 + p_e^2c^2}$，总动量：$\hbar\vec{k}' + \vec{p}_e$, , ,

能量守恒：$\hbar\omega + m_0c^2 = \hbar\omega' + \sqrt{m_0^2c^4 + p_e^2c^2}$

动量守恒：$\hbar\vec{k} = \hbar\vec{k}' + \vec{p}_e$

解出：$\omega - \omega' = \dfrac{2\hbar}{m_0c^2}\omega\omega'\sin^2\dfrac{\theta}{2}$

（ 15-7 ）（ ） $L' = L + \dfrac{d\Lambda}{dt}$ ，$\dfrac{d\Lambda}{dt} = \dfrac{\partial\Lambda}{\partial t} + \vec{v}\cdot\nabla\Lambda$ ，L-eq: $\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial\dot{q}_i}\right) - \dfrac{\partial L}{\partial q_i} = 0$ ，

$\dfrac{\partial L'}{\partial\dot{q}_i} = \dfrac{\partial L}{\partial\dot{q}_i} + \dfrac{\partial}{\partial\dot{q}_i}\left(\dfrac{d\Lambda}{dt}\right) = \dfrac{\partial L}{\partial\dot{q}_i} + \dfrac{\partial\Lambda}{\partial q_i}$ ，$\dfrac{\partial L'}{\partial q_i} = \dfrac{\partial L}{\partial q_i} + \dfrac{\partial}{\partial q_i}\left(\dfrac{d\Lambda}{dt}\right) = \dfrac{\partial L}{\partial q_i} + \dfrac{d}{dt}\left(\dfrac{\partial\Lambda}{\partial q_i}\right)$ ，

$$\frac{d}{dt}\left(\frac{\partial L'}{\partial\dot{q}_i}\right) - \frac{\partial L'}{\partial q_i} = \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{q}_i}\right) + \frac{d}{dt}\left(\frac{\partial\Lambda}{\partial q_i}\right) - \frac{\partial L}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial\Lambda}{\partial q_i}\right) = \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0,$$

（ ） $L = -m_0 c^2\sqrt{1\ \dfrac{v^2}{c^2}}\ -(\varphi\ -\vec{v}\ \cdot$ ，规 范 变 换 ：$\vec{A}' = \vec{A} + \nabla\Lambda$ ，$\varphi' = \varphi - \dfrac{\partial\Lambda}{\partial t}$ ，

$$L' = -m_0c^2\sqrt{1-\frac{v^2}{c^2}} - q(\varphi' - \vec{v}\cdot\vec{A}') = -m_0c^2\sqrt{1-\frac{v^2}{c^2}} - q(\varphi - \vec{v}\cdot\vec{A}) + q\left(\frac{\partial\Lambda}{\partial t} + \vec{v}\cdot\nabla\Lambda\right), , ,$$

$$= L + q\left(\frac{\partial\Lambda}{\partial t} + \vec{v}\cdot\nabla\Lambda\right) = L + q\frac{d\Lambda}{dt}$$