1000.tif (23.75KB , 709x1110像素) - 第13/22张

1.6. $v_r = \dot{r} = \lambda r, \quad v_\theta = r\dot{\theta} = \mu\theta$

$$a_r = \ddot{r} - r\dot{\theta}^2 = \lambda\dot{r} - \frac{(r\dot{\theta})^2}{r} = \underline{\lambda^2 r - \frac{(\mu\theta)^2}{r}}$$

$$a_\theta = \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta}) = \frac{1}{r}\frac{d}{dt}(\mu r\theta)$$

$$= \frac{\mu}{r}(\dot{r}\theta + r\dot{\theta}) = \frac{\mu}{r}(\lambda r\theta + \mu\theta)$$

$$= \underline{\mu\theta\left(\lambda + \frac{\mu}{r}\right)}$$

$$\vec{a} = a_r\hat{e}_r + a_\theta\hat{e}_\theta$$

1.8. $r = k(1+\cos\theta) = 2k\cos^2\frac{\theta}{2}$

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1.8.

$$r = k(1+\cos\theta) = 2k\cos^2\frac{\theta}{2}$$

$$\dot{r} = -k\sin\theta\cdot\dot{\theta}$$

$$v^2 = \dot{r}^2 + (r\dot{\theta})^2 = k^2\sin^2\theta\,\dot{\theta}^2 + k^2(1+\cos\theta)^2\dot{\theta}^2$$

$$= k^2\dot{\theta}^2(2+2\cos\theta) = 4k^2\dot{\theta}^2\cos^2\frac{\theta}{2}$$

$$\dot{\theta} = \frac{v}{2k\cos\frac{\theta}{2}}.$$

$$\dot{r} = -k\sin\theta\cdot\dot{\theta} = -v\sin\frac{\theta}{2}$$

$$r\dot{\theta} = 2k\cos^2\frac{\theta}{2}\cdot\frac{v}{2k\cos\frac{\theta}{2}} = v\cos\frac{\theta}{2}$$

$$\vec{v} = \vec{v}_r + \vec{v}_\theta = (-v\sin\frac{\theta}{2})\hat{e}_r + (v\cos\frac{\theta}{2})\hat{e}_\theta$$

$$r\dot{\theta}^2 = r\dot{\theta}\dot{\theta} = v\cos\frac{\theta}{2}\cdot\frac{v}{2k\cos\frac{\theta}{2}} = \frac{v^2}{2k}$$

$$\ddot{r} = -\frac{v}{2}\cos\frac{\theta}{2}\cdot\dot{\theta} = -\frac{v^2}{4k}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -\frac{3v^2}{4k}$$

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$$\dot r = -k\sin\theta\cdot\dot\theta = -v\sin\frac{\theta}{2}$$

$$r\dot\theta = 2k\cos^2\frac{\theta}{2}\cdot\frac{v}{2k\cos\frac{\theta}{2}} = v\cos\frac{\theta}{2}$$

$$\vec v = \vec v_r + \vec v_\theta = \left(-v\sin\frac{\theta}{2}\right)\hat e_r + \left(v\cos\frac{\theta}{2}\right)\hat e_\theta$$

$$r\dot\theta^2 = r\dot\theta\dot\theta = v\cos\frac{\theta}{2}\cdot\frac{v}{2k\cos\frac{\theta}{2}} = \frac{v^2}{2k}$$

$$\ddot r = -\frac{v}{2}\cos\frac{\theta}{2}\cdot\dot\theta = -\frac{v^2}{4k}$$

$$a_r = \ddot r - r\dot\theta^2 = -\frac{3v^2}{4k}$$

$$r^2\dot\theta = r\cdot r\dot\theta = 2kv\cos^3\frac{\theta}{2}$$

$$\frac{1}{r}\frac{d(r^2\dot\theta)}{dt} = \frac{1}{r}\cdot 2kv^2\left(3\cos^2\frac{\theta}{2}\right)\cdot\left(-\frac{1}{2}\sin\frac{\theta}{2}\right)\dot\theta$$

$$= \frac{1}{r}\cdot\left(-\frac{3rv^2}{2}\sin\frac{\theta}{2}\right)\dot\theta = -\frac{3v^2}{4k}\tan\frac{\theta}{2}$$

$$\vec a = \vec a_r + \vec a_\theta = -\frac{3v^2}{4k}\left(\hat e_r + \tan\frac{\theta}{2}\hat e_\theta\right)$$

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1001.tif (415.35KB , 745x570像素) - 第11/22张

1.14 选用直角坐标系，

质点运动方程：

$$m\ddot{x} = -mk\dot{x}$$

$$m\ddot{z} = -mg - mk\dot{z}$$

积分，得：

$$\dot{x} = v_{0x} e^{-kt}$$

$$\dot{z} = v_{0z} e^{-kt} + \frac{g}{k}\left(e^{-kt} - 1\right)$$

其中 $\tan\alpha = \frac{v_{0z}}{v_{0x}}$，

当 $t = T$ 时，$\tan(-\alpha) = \frac{\dot{z}}{\dot{x}}$

即：$-\frac{v_{0z}}{v_{0x}} = \frac{v_{0z} e^{-kT} + \frac{g}{k}\left(e^{-kT} - 1\right)}{v_{0x} e^{-kT}}$

$$2 v_{0z} e^{-kT} + \frac{g}{k}\left(e^{-kT} - 1\right) = 0.$$

$$e^{kT} = \frac{g/k + 2v_{0z}}{g/k} = 1 + \frac{2k v_{0z}}{g} = 1 + \frac{2k v_0 \sin\alpha}{g}$$

$$T = \frac{1}{k} \ln\left(1 + \frac{2k v_0}{g} \sin\alpha\right)$$

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image002.tif (3.60MB , 4800x6278像素) - 第8/22张

导论作业（理论力学部分）

(2) $V(x) = V_0\left(\frac{a}{x} + \frac{x}{a}\right)$,

$\frac{dV}{dx} = V_0\left(-\frac{a}{x^2} + \frac{1}{a}\right) = 0$, $x = \pm a$, 取 $x = a$,

$\frac{d^2V}{dx^2} = 2aV_0x^{-3}$, $\left.\frac{d^2V}{dx^2}\right|_{x=a} = \frac{2V_0}{a^2}$

$V(x) = V(a) + V'(a)\cdot(x-a) + \frac{V''(a)}{2}(x-a)^2 = V(a) + \frac{V''(a)}{2}(x-a)^2$

$\vec{F} = -\frac{dV}{dx}\hat{e}_x = -V''(a)\cdot(x-a)\,\hat{e}_x = -\frac{2V_0}{a^2}(x-a)\cdot\hat{e}_x$

$m\ddot{x} = F = -\frac{2V_0}{a^2}(x-a)$. 令 $\xi = x - a$

$m\ddot{\xi} + \frac{2V_0}{a^2}\xi = 0$. $\omega = \sqrt{\frac{2V_0}{ma^2}}$

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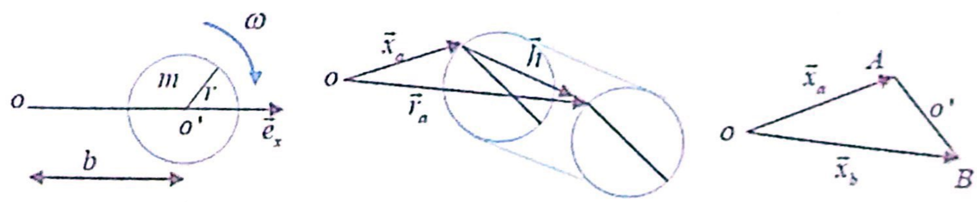
(2-9)



1）选过圆心直线上的两对称点，$\vec{v}_a=-\vec{v}_b$，$\vec{r}_a=\vec{x}_a+\vec{h}$，....

$$\sum_{a,b}\vec{r}_i\times m_i\vec{v}_i=\left(\vec{x}_a+\vec{h}\right)\times m\vec{v}_a+\left(\vec{x}_b+\vec{h}\right)\times m\vec{v}_b=\left(\vec{x}_a-\vec{x}_b\right)\times m\vec{v}_a+m\vec{h}\times\left(\vec{v}_a+\vec{v}_b\right)$$

$$=\left(\vec{x}_a-\vec{x}_b\right)\times m\vec{v}_a=\overline{AB}\times m\vec{v}_a=2mr\cdot r\omega\vec{e}_z$$

$$\vec{L}=\sum_i\vec{r}_i\times m_i\vec{v}_i=\iint 2r^2\omega\cdot\rho r dr d\theta\vec{e}_z=2\omega\rho\int_0^a r^3dr\int_0^\pi d\theta\int_0^H dh\vec{e}_z=\frac{\omega\rho a^4\pi H}{2}\vec{e}_z=\frac{Ma^2}{2}\omega\vec{e}_z$$

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$$\vec{L}=\sum_i \vec{r}_i\times m_i\vec{v}_i=\iint 2r^2\omega\cdot\rho r dr d\theta\vec{e}_z=2\omega\rho\int_0^a r^3dr\int_0^\pi d\theta\int_0^H dh\vec{e}_z=\frac{\omega\rho a^4\pi H}{2}\vec{e}_z=\frac{Ma^2}{2}\omega\vec{e}_z$$

2）$\vec{r}_i=b\vec{e}_x+\vec{r}_i{}'$，$\vec{v}_i=\vec{v}_i{}'$，

$$\vec{L}=\sum_i \vec{r}_i\times m_i\vec{v}_i=\sum_i\left(b\vec{e}_x+\vec{r}_i{}'\right)\times m_i\vec{v}_i{}'=b\sum_i\vec{e}_x\times m_i\vec{v}_i{}'+\sum_i\vec{r}_i{}'\times m_i\vec{v}_i{}'，$$

由对称性，$\sum_i\vec{e}_x\times m_i\vec{v}_i{}'=0$，$\vec{L}=\sum_i\vec{r}_i{}'\times m_i\vec{v}_i{}'=\vec{L}'$，

或者，$\vec{v}_c=0$，$\vec{L}=\vec{r}_c\times M\vec{v}_c+\vec{L}'=\vec{L}'=I\omega\vec{e}_z$，

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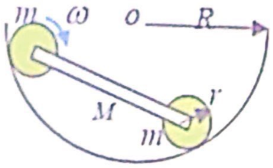

(2-10)

(a) $OM=\sqrt{(R-r)^2-(l/2)^2}$，$(R-r)\dot{\varphi}=r\omega$，$v_c=OM\cdot\dot{\varphi}=r\omega\sqrt{1-\frac{l^2}{4(R-r)^2}}$，

(b) 圆柱体：动量　$\vec{P}=m\vec{v}_{1c}=mr\omega\vec{e}_\theta$，

相对 o 点的角动量：

$$\vec{L}=\vec{r}_{1c}\times m\vec{v}_{1c}+\vec{L}'=(R-r)\vec{e}_r\times mr\omega\vec{e}_\theta-I_m\omega(\vec{e}_r\times\vec{e}_\theta)=[mr(R-r)-I_m]\omega(\vec{e}_r\times\vec{e}_\theta)$$

动能　$T=\frac{1}{2}m\vec{v}_{1c}^2+T'=\frac{1}{2}mr^2\omega^2+\frac{1}{2}I_m\omega^2$

刚性杆：动量　$\vec{P}=M\vec{v}_c=Mr\omega\sqrt{1-\frac{l^2}{4(R-r)^2}}\vec{e}_\theta$，

相对 o 点的角动量：$\vec{L}=\vec{r}_c\times M\vec{v}_c+\vec{L}'$

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$$\vec{L}=\vec{r}_{1c}\times m\vec{v}_{1c}+\vec{L}'=(R-r)\vec{e}_r\times mr\omega\vec{e}_\theta-I_m\omega(\vec{e}_r\times\vec{e}_\theta)=\left[mr(R-r)-I_m\right]\omega(\vec{e}_r\times\vec{e}_\theta)$$

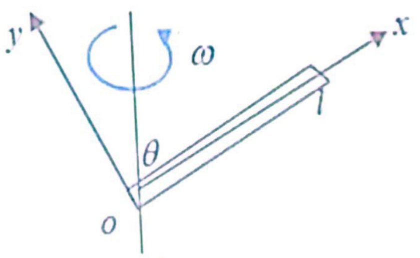
动能 $T=\frac{1}{2}m\vec{v}_{1c}^2+T'=\frac{1}{2}mr^2\omega^2+\frac{1}{2}I_m\omega^2$

刚性杆：动量 $\vec{P}=M\vec{v}_c=Mr\omega\sqrt{1-\frac{l^2}{4(R-r)^2}}\vec{e}_\theta$.

相对 o 点的角动量：$\vec{L}=\vec{r}_c\times M\vec{v}_c+\vec{L}'$

$$=Mr\omega\left[(R-r)-\frac{l^2}{4(R-r)}\right](\vec{e}_r\times\vec{e}_\theta)+I_M\frac{r\omega}{R-r}(\vec{e}_r\times\vec{e}_\theta)$$

动能 $T=\frac{1}{2}M\vec{v}_c^2+T'=\frac{1}{2}Mr^2\omega^2\left[1-\frac{l^2}{4(R-r)^2}\right]+\frac{1}{2}I_M\left(\frac{r\omega}{R-r}\right)^2$



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