(2) Matterwaves

1 Last Time

- EM radiation is quantized in units of $E_\nu = h\nu$
- photons have momentum $p = E_\nu / c = h/\lambda$
- electron orbits have quantized angular momentum $L = n\hbar$

Announcements:

About office hours: follow the Staff link in the Stellar sidebar. There is a lot of pset help available Monday afternoon and evening. PG will post soon, just looking for a room.

Sign up for recitations!

Register your clickers!

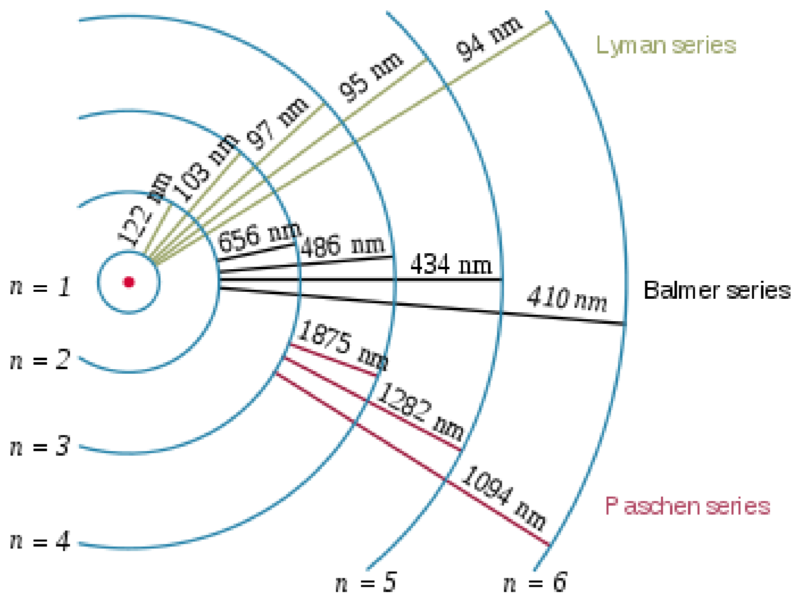
2 This Time

- electrons have a wavelength? $\lambda = h/p$???
- Dual-slit experiment with photons
- and with electrons!
- De Broglie and the end of the particle-wave distinction

3 Bohr and Matterwaves

- E&R 4.(3, 4), Sc 1.5, Ga 1.5

Ok, a quick recap of where we left off:



By quantizing angular momentum with $\hbar$ we saved the electron from its doom! With quantized orbits, it cannot spiral in! And we have determined the size of the hydrogen atom to boot!

To do get this Bohr:

- kept Newton, kept Coulomb
- ignored Maxwell
- and quantized $\vec{L}$, which is insane

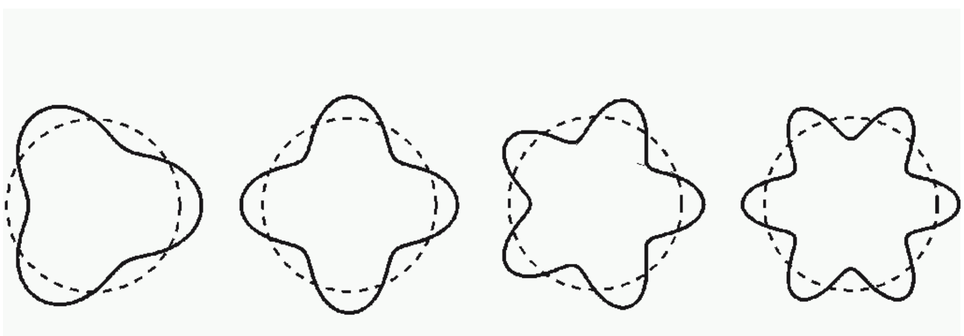
Though I assure you that by the end of this course you will know why quantizing L works.

Let's take this one step further before moving on. It just so happens that if you use Compton's equation for photon momentum and apply it without to much thought to our orbiting electrons you find a curious relationship.

- Photon Momentum $p = E_\nu/c = h/\lambda$
- Electron Momentum $m_e v = h/\lambda_e$??
- Circumference of a Bohr orbit $d_n = 2\pi a_0 n^2$
- $\Rightarrow \lambda_e = \frac{h}{m_e v} = \frac{hr}{L} = \frac{h a_0 n^2}{n\hbar} = 2\pi a_0 n = \frac{d_n}{n}$
- $\Rightarrow d_n = n\lambda_e$

The circumference of the n^{th} Bohr orbit is n electron wavelengths. (show slide)

Bohr Orbits and Electron Waves



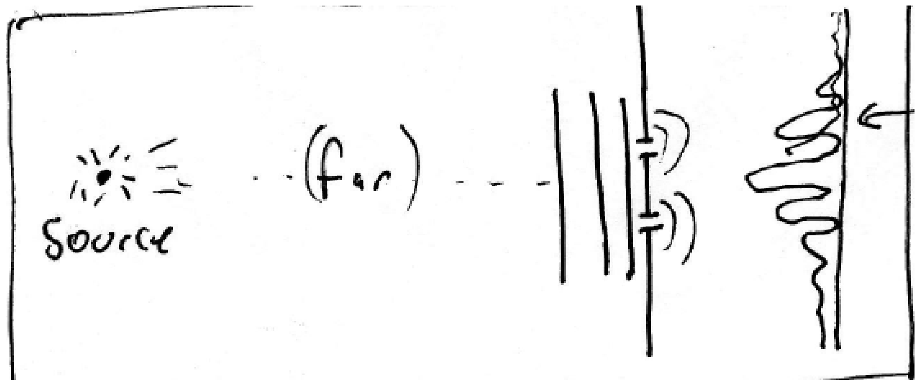
But these are just mindless math games; no better than an elite form of numerology... right?

4 Dual-Slit Experiment with Photons

- E&R 3.1, Sc 1.(3, 4), Ga (1.4, 2.1), Li 2.5

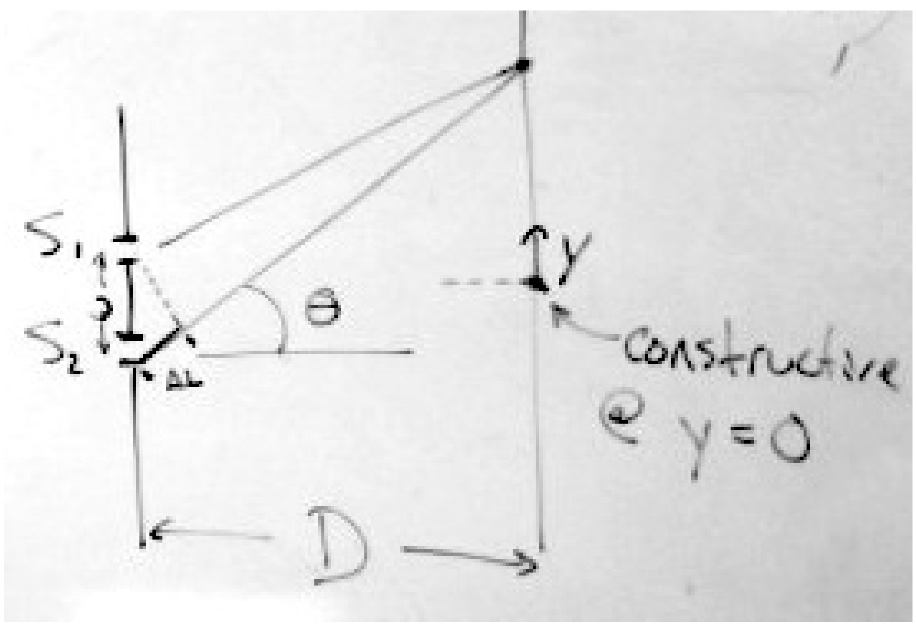
Enough of this funny business, we all know that light is a wave and there are experiments that prove it! This should be a friendly, familiar and relaxing experimental setup from 8.03:

Dual Slit Interference



Let's try to get some idea for the size of the interference pattern.

Dual Slit Interference Maxima



(show interference slide)

The incident plane wave leaking through the 2 slits can be treated as a pair of sources which oscillate in phase. If d is the distance between the slits, and D the distance to the screen, we can find the maxima in the interference pattern by finding the locations on the screen where the distance form the slit to the screen is the same for both slits.

Positions of Interference of Maxima

$$\Delta L = d \sin\theta = n\lambda \tag{1}$$

$$\sin\theta \simeq \frac{y}{D} \text{ for } \theta \ll 1 \tag{2}$$

$$\Rightarrow n\lambda \simeq \frac{d\,y}{D} \qquad \Rightarrow y \simeq \frac{D}{d} n\lambda \tag{3}$$

We can take this one step further and really compute the intensity at each point on the screen. For this we'll need to do a little complex math, but it will be good practice for the future. Let's start with some electric field amplitudes (thinking of Huygen's principal)...

Wave Amplitudes

$$E_1 = \frac{A}{r_1}\, e^{i(kr_1 - \omega t)} \tag{4}$$

$$E_2 = \frac{A}{r_2}\, e^{i(kr_2 - \omega t)} \tag{5}$$

$$\text{where } k = 2\pi/\lambda = 2\pi\nu/c = \omega/c \tag{6}$$

where k is the wave number, ω the angular frequency and r_n the propagation distance. Each wave has a magnitude and a phase.

Propagation Distance

$$
\begin{aligned}
r_1 &= \sqrt{D^2 + (y - d/2)^2} = D\sqrt{1 + \frac{(y-d/2)^2}{D^2}} &(7)\\
&\simeq D\left(1 + \frac{(y-d/2)^2}{2D^2}\right) = D + \frac{(y-d/2)^2}{2D} &(8)\\
&\text{the best Taylor series ever!}\\
r_2 &= \sqrt{D^2 + (y + d/2)^2} \simeq D + \frac{(y+d/2)^2}{2D} &(9)
\end{aligned}
$$

Ok... I'm going to do a little more math on the board than I like, but I want you to see the kind of math that I hope feels obvious to you.

Interference Pattern - Amplitude

$$
\begin{aligned}
E &= E_1 + E_2 \simeq \frac{A}{R}\left(e^{i(kr_1 - \omega t)} + e^{i(kr_2 - \omega t)}\right)\\
&\quad \text{where} \quad R = \frac{r_1 + r_2}{2} &(10)\\
&= \frac{A}{R}e^{-i\omega t}\left(e^{ikr_1} + e^{ikr_2}\right) &(11)\\
&= \frac{A}{R}e^{-i\omega t}e^{ik(r_1+r_2)/2}\left(e^{ik(r_1-r_2)/2} + e^{-ik(r_1-r_2)/2}\right)\\
&= \frac{A}{R}e^{i(kR-\omega t)} \cdot 2\cos(k(r_1 - r_2)/2)\\
&= 2\frac{A}{R}e^{i(kR-\omega t)}\cos\left(k\frac{(y-d/2)^2 - (y+d/2)^2}{4D}\right)\\
&= 2\frac{A}{R}e^{i(kR-\omega t)}\cos\left(\frac{kyd}{2D}\right) &(12)
\end{aligned}
$$

And finally, the intensity on the screen is proportional to the amplitude squared:

Interference Pattern - Intenisty

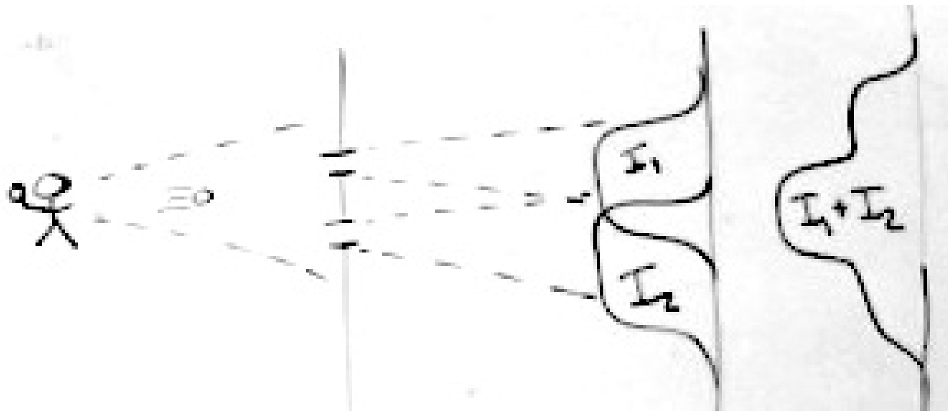
$$
\begin{aligned}
I &= \frac{c\epsilon_0}{2}\left|E\right|^2 = \frac{c\epsilon_0}{2}(E\ E^*) &(13)\\
&= 2c\epsilon_0\frac{A^2}{R^2}\left|e^{i(kR-\omega t)}\right|^2\cos\left(\frac{kyd}{2D}\right)^2 &(14)\\
&= 2c\epsilon_0\frac{A^2}{R^2}\cos\left(\frac{kyd}{2D}\right)^2 &(15)\\
&= c\epsilon_0\frac{A^2}{R^2}\left(1+\cos\left(\frac{2\pi\ yd}{\lambda\ D}\right)\right) &(16)\\
&\Rightarrow \text{ maxima at } y = \frac{D}{d}n\lambda &(17)
\end{aligned}
$$

So, what did we learn here:

- Add amplitudes and square them to get intensities
- Overall phases don't matter, relative phases do
- This wave calculation matches experiment, so light is a wave!

After all, if light were made of chunks of stuff the picture would look very different.

Dual Slit with BBs



Q: Where did the BB hit the screen?

A: A point on the screen

Q: How do you get an "intensity"?

A: By throwing many BBs: $I(y) = P(y \text{ to } y + dy)$

Q: Do you see interference?

A: No, we just add intensities

Q: Is light a wave, or a bunch of chunks?

A: A wave!

Q: Didn't we just get done showing that it is chunky with black body spectra, the photoelectric effect and Compton scattering? And what happens if we make the energy very low, so that there is less than one photon in the there at a time?

A: Chunky, with interference (show slide)

The plot thickens. It seems that light is sometimes best modeled as a wave, and other times as a particle, and sometimes as both in the same experiment. And what about our wayward electrons? They must behave like chunks, right?

(show Hitachi video)

Both light and matter can behave like a particle or a wave, with $\lambda = h/p$ (de Broglie (d'Broy), 1924)

And you might have noticed, that our nice deterministic world of Classical mechanics has been infected with randomness. How is the location of a photon's arrival on the screen determined?

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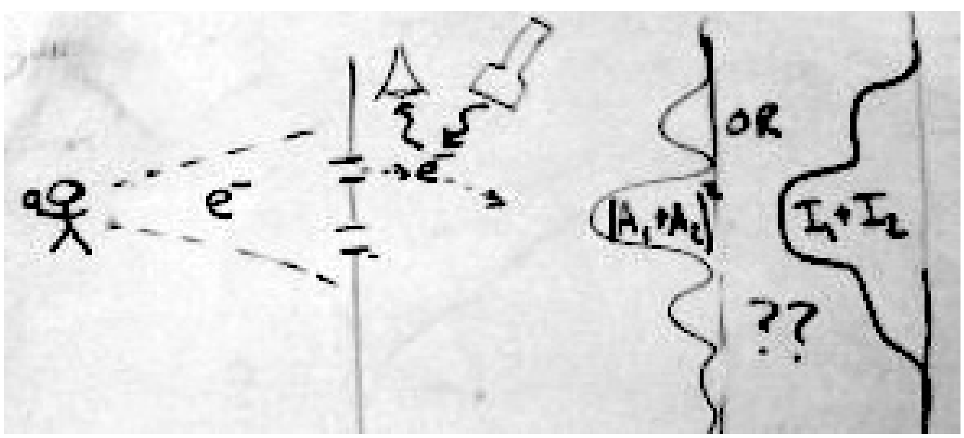
5 The Birth of Uncertainty

- E&R 3.3

Let's find these electrons. Which way are they going? Top slit or bottom slit? Electrons are particles and we can see them by bouncing light off of them, just like anything else.

Dual Slit Uncertainty



In order to distinguish between the top slit and the bottom slit, we need $\lambda < d$. Classically, we could set the light intensity low enough that it couldn't disturb the electrons, but since we know we need at least $h\nu$ to make a photon, we have to risk some disturbance.

Finding Electrons

$$\Delta p_e \sim p_\nu = h/\lambda_\nu \tag{18}$$

$$\Delta \theta \sim \Delta p_e/p_e \tag{19}$$

$$\Delta y \sim D\Delta p_e/p_e \sim D\lambda_e/\lambda_\nu \tag{20}$$

$$\text{recall} \quad y_1 - y_0 = D\lambda_e/d \tag{21}$$

$$\Rightarrow \Delta y > y_1 - y_0 \quad \text{for } \lambda_\nu < d \tag{22}$$

$\Rightarrow$ if you can find the electron, you destroy the interference!

Enough bumping around in the dark. Classical mechanics is broken. Next time we will talk about how to bring this all together into something that works.