CS6730: Probabilistic Graphical Models
Homework-1

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Solutions

Q-1

Consider a variant of the Monty hall problem, where the protocol is as follows. There are three doors, behind one of which is a car, and behind two of which are goats. **You first choose a door. The host then randomly picks one of the two remaining doors. It is observed to be a goat.** What is the probability that the car is behind the door you have already chosen? What would be a good Bayesian network for modelling this problem?

answer:

Let F be the random variable to denote the first choice, H be the random variable that denotes the choice of the host. Also, let $C(car), G(goat)$ are the discrete states of random variables $F\&H$.

$$\begin{aligned}
P(F = C|H = G) &= \frac{P(F = C, H = G)}{P(H = G)} \\
&= \frac{P(F = C)P(H = G|F = C)}{\sum_{i=\{G,C\}} P(F = i, H = G)} \\
&= \frac{\frac{1}{3} * 1}{P(F = C)P(H = G|F = C) + P(F = G)P(H = G|F = G)} \\
&= \frac{\frac{1}{3} * 1}{(\frac{1}{3} * 1) + (\frac{2}{3} * \frac{1}{2})} \\
&= \frac{1}{2}
\end{aligned} \tag{1}$$

Q-2

answer:

Conditional independencies for Graphs

(I) $C \perp\!\!\!\perp D|A, B$
$A \perp\!\!\!\perp B$

(II) $A \perp\!\!\!\perp B, D$
$C \perp\!\!\!\perp B, D$

(III) $A \perp\!\!\!\perp B, C, D$
$B \perp\!\!\!\perp C, D$
$C \perp\!\!\!\perp D$

Conditional independencies for Tables

(I) $P(A=0)=0.5, P(A=1)=0.5,$
$P(B=0)=0.5, P(B=1)=0.5$
$P(C=0)=0.5, P(C=1)=0.5$
$P(D=0)=0.5, P(D=1)=0.5$
$P(A=0,B=0)=0.5, P(A=0,B=1)=0, P(A=1,B=0)=0, P(A=1,B=1)=0.5$
$P(A=0,C=0)=0.5, P(A=0,C=1)=0, P(A=1,C=0)=0, P(A=1,C=1)=0.5$
$P(A=0,D=0)=0.5, P(A=0,D=1)=0, P(A=1,D=0)=0, P(A=1,D=1)=0.5$
$P(B=0,C=0)=0.5, P(B=0,C=1)=0, P(B=1,C=0)=0, P(B=1,C=1)=0.5$
$P(B=0,D=0)=0.5, P(B=0,D=1)=0, P(B=1,D=0)=0, P(B=1,D=1)=0.5$
$P(C=0,D=0)=0.5, P(C=0,D=1)=0, P(C=1,D=0)=0, P(C=1,D=1)=0.5$

Conditional independencies:

- From the table as well as the probabilities calculated above, we can notice that the probability is non-zero when A,B,C,D = (0,0,0,0) or (1,1,1,1). So, itseems that all the random variables are depending on one particular random variable. But we could not find out which variable it is. So, No conclusion about conditional independencies can be made.

(II) $P(A=0)=0.5, P(A=1)=0.5,$
$P(B=0)=0.5, P(B=1)=0.5$
$P(C=0)=0.5, P(C=1)=0.5$
$P(D=0)=0.6, P(D=1)=0.4$
$P(A=0,B=0)=0.25, P(A=0,B=1)=0.25, P(A=1,B=0)=0.25, P(A=1,B=1)=0.25$
$P(A=0,C=0)=0.15, P(A=0,C=1)=0.35, P(A=1,C=0)=0.25, P(A=1,C=1)=0.25$
$P(A=0,D=0)=0.20, P(A=0,D=1)=0.30, P(A=1,D=0)=0.40, P(A=1,D=1)=0.10$
$P(B=0,C=0)=0.20, P(B=0,C=1)=0.30, P(B=1,C=0)=0.30, P(B=1,C=1)=0.20$
$P(B=0,D=0)=0.25, P(B=0,D=1)=0.25, P(B=1,D=0)=0.25, P(B=1,D=1)=0.25$
$P(C=0,D=0)=0.35, P(C=0,D=1)=0.15, P(C=1,D=0)=0.25, P(C=1,D=1)=0.25$

Conditional independencies:

- $A \perp\!\!\!\perp B$
- $C \perp\!\!\!\perp D | A, B$

$$
\begin{aligned}
P(C=0,D=0|A=0,B=0) &= \frac{P(A=0,B=0,C=0,D=0)}{P(A=0,B=0)} = \frac{0.015}{0.25} = \frac{3}{50} \\
&= P(C=0|A=0,B=0).P(D=0|A=0,B=0) \\
&= \frac{\sum_D P(A=0,B=0,C=0,D).\sum_C P(A=0,B=0,C,D=0)}{P(A=0,B=0)*P(A=0,B=0)} \\
&= \frac{0.05*0.075}{0.25*0.25} = \frac{3}{50},
\end{aligned}
$$

and so on

(III) $P(A=0)=1, P(A=1)=0,$
$P(B=0)=0.25, P(B=1)=0.75$
$P(C=0)=0.20, P(C=1)=0.80$
$P(D=0)=0.40, P(D=1)=0.60$
$P(A=0,B=0)=0.25, P(A=0,B=1)=0.75, P(A=1,B=0)=0, P(A=1,B=1)=0$
$P(A=0,C=0)=0.20, P(A=0,C=1)=0.80, P(A=1,C=0)=0, P(A=1,C=1)=0$
$P(A=0,D=0)=0.40, P(A=0,D=1)=0.60, P(A=1,D=0)=0, P(A=1,D=1)=0$
$P(B=0,C=0)=0.05, P(B=0,C=1)=0.20, P(B=1,C=0)=0.15, P(B=1,C=1)=0.60$
$P(B=0,D=0)=0.10, P(B=0,D=1)=0.15, P(B=1,D=0)=0.30, P(B=1,D=1)=0.45$

$P(C=0, D=0) = 0.08, P(C=0, D=1) = 0.12, P(C=1, D=0) = 0.32, P(C=1, D=1) = 0.48$

Conditional independencies:

- $A \perp\!\!\!\perp B, C, D$ (i.e., $P(A=i, B=j) = P(A=i).P(B=j)$ and so on)
- $B \perp\!\!\!\perp C, D$
- $C \perp\!\!\!\perp D$

Graphs vs. Table distributions

- Distribution (I) is not conforming with any of the graphs given
- Distribution (II) is conforming with the graph (I)
- Distribution (III) is conforming with graph (III)

Q-3

answer:

Distribution tables:

(a)

	C=0,D=0	C=0,D=1	C=1,D=0	C=1,D=1
A=0,B=0	0	0	0	0
A=0,B=1	0	0.5	0.5	0
A=1,B=0	0	0	0	0
A=1,B=1	0	0	0	0

(b)

	C=0,D=0	C=0,D=1	C=1,D=0	C=1,D=1
A=0,B=0	0.25	0	0	0
A=0,B=1	0	0.25	0	0
A=1,B=0	0	0	0.25	0
A=1,B=1	0	0	0	0.25

(c) $(A \perp\!\!\!\perp C|B, D), (B \perp\!\!\!\perp D|A, C)$ - Not possible to be represented by Bayesian network

Bayesian networks:

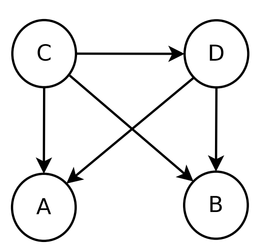

Figure 1: (a) $A \perp\!\!\!\perp B|C, D$

(c) $(A \perp\!\!\!\perp C|B, D), (B \perp\!\!\!\perp D|A, C)$ - Not possible to be represented by Bayesian network, as it introduces cycle in the network

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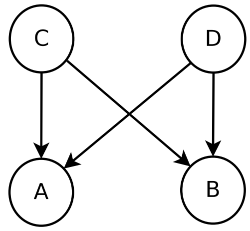

Figure 2: (b)(A $\perp\!\!\!\perp B|C, D), (C \perp\!\!\!\perp D)$

Q-4

Consider the Bayesian network given below:

Consider a distribution over 8 random variables $X_1, X_2, \ldots, X_8$ given by the Bayesian network to the left.

1. Give the largest set of random variables that is independent of X_3.

   answer:

   $X_3 \perp\!\!\!\perp X_2, X_5, X_7$

2. Give the largest set of random variables that is independent of X_3, conditioned on X_1.

   answer:

   $X_3 \perp\!\!\!\perp X_2, X_4, X_5, X_7, X_8 | X_4$

3. Give the largest set of random variables that is independent of X_3, conditioned on X_1 and X_4.

   answer:

   $X_3 \perp\!\!\!\perp X_2, X_4, X_5, X_7, X_8 | X_1, X_4$
