CS6790: Geometry & Photometry-based Computer Vision
Assignment-2 (Finding Intrinsic parameters)

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1 Finding intrinsic parameters (Assuming full K matrix)

1.1 by using 5 perpendicularity relations between vanishing points

1.1.1 Procedure

1. The intrinsic parameters shall be determined using 5 sets of perpendicular vanishing points.

2. To get 5 sets of vanishing points, the given chess board images (5) are used. To get the vanishing vanishing point, the perpendicular pair of parallel lines in the chess board are used. refer to the Figure 1. Let the set of of points to deniote the parallel lines in one direction be $P_{l_1}, P_{l_2}, P_{l_3}, P_{l_4}$. Let the other set of points selected to denote the parallel lines in perpendicular direction be $P_{m_1}, P_{m_2}, P_{m_3}, P_{m_4}$.

3. The vanishing point is determined from the set of chosen points as follows:

   vanishing point 1,$VP_1 = (P_{l_1} \times P_{l_2}) \times (P_{l_3} \times P_{l_4})$
   vanishing point 2,$VP_2 = (P_{m_1} \times P_{m_2}) \times (P_{m_3} \times P_{m_4})$

4. Since the vanishing points VP_1, VP_2 are perpendicular, we can form an equation with $\Psi = K^{-T}K^{-1}$, with K being the matrix of camera intrinsic parameters as follows:

   $$(VP_1)^T \Psi (VP_2) = 0$$

   The K matrix has 5 free parameters as follows:

   $$\begin{bmatrix} f_x & s & u_x \\ 0 & f_y & u_y \\ 0 & 0 & 1 \end{bmatrix}$$

   where f_x, f_y = focal length in x, y direction respectively, u_x, u_y = camera center, s = skew parameter.

5. With 5 sets of perpendicular vanishing points, we can get 5 such equation as given in step 4 above.

   $$\begin{bmatrix} x_{l1}x_{m1} & (x_{l1}y_{m1} + y_{l1}x_{m1}) & (x_{l1}h_{m1} + h_{l1}x_{m1}) & y_{l1}y_{m1} & (y_{l1}h_{m1} + h_{l1}y_{m1}) & h_{l1}h_{m1} \\ x_{l2}x_{m2} & (x_{l2}y_{m2} + y_{l2}x_{m2}) & (x_{l2}h_{m2} + h_{l2}x_{m2}) & y_{l2}y_{m2} & (y_{l2}h_{m2} + h_{l2}y_{m2}) & h_{l2}h_{m2} \\ \vdots & & & & & \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = 0$$

   here,

   $$\Psi = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$$

6. The parameters shall be determined using the Singular value decomposition (SVD). i.e., the singular vector corresponding to the least singular value is the solution for Ψ.

7. Once Ψ is determined, we can use Cholesky decomposition to get the intrinsic camera paramerters K. The K matrix has 5 free parameters as follows:

$$\begin{bmatrix} f_x & s & u_x \\ 0 & f_y & u_y \\ 0 & 0 & 1 \end{bmatrix}$$

where f_x, f_y = focal length in x, y direction respectively, u_x, u_y = camera center, s = skew parameter.

8. **Result (chess board images of size** 600×400 **pixels)**:

$$K = \begin{bmatrix} f_x & s & u_x \\ 0 & f_y & u_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1191.2 & 8.2 & 326.2 \\ 0 & 811.3 & 153.6 \\ 0 & 0 & 1 \end{bmatrix}$$

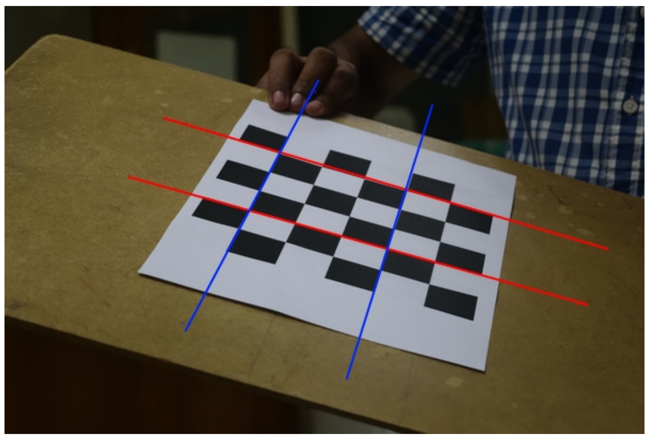

Figure 1: The perpendicular pair of parallel lines chosen to get the perpendicular rays from 2 vanishing points

1.2 by computing the homography relation between metric co-ordinate system fixed to scene plane & image

1.2.1 Procedure

- The chess board images are used to find the homography between scene coordinates and the image coordinates.
- consider H as the homography matrix that transforms the scene coordinates to image coordinates.
- Consider transforming the circular points $I(1, +i, 0)$, $J(1, -i, 0)$ to the image co-ordinates. Since the points I, J which are lying in Absolute Conic of 3D space are transformed to the points on the Image of Absolute Conic (IAC) in the 2D image plane ((Ψ), each homography gives two contraints. $(h_1 \Psi h_1 = h_2 \Psi h_2,\ h_1^T \Psi h_2 = 0)$.
- Hence, 3 such homographies give 6 constraint equations which are used to solve for the parameters of $\Psi = K^{-T}K^{-1}$. here,

$$\Psi = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$$

- To find 3 homographies, the chess board images are considered such that the scene points $(0, 0, 1), (0, 1, 1), (1, 1, 1), (1, 0, 1)$ are transformed to image co-ordinates which are selected appropriately.
- The parameters shall be determined using the Singular value decomposition (SVD). i.e., the singular vector corresponding to the least singular value is the solution for Ψ.

- Once Ψ is determined, we can use Cholesky decomposition to get the intrinsic camera paramerters K. The K matrix has 5 free parameters as follows:

$$\begin{bmatrix} f_x & s & u_x \\ 0 & f_y & u_y \\ 0 & 0 & 1 \end{bmatrix}$$

where f_x, f_y = focal length in x, y direction respectively, u_x, u_y = camera center, s = skew parameter.

- **Result (chess board images of size** 600×400 **pixels)**:

$$K = \begin{bmatrix} f_x & s & u_x \\ 0 & f_y & u_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1459.45 & 4.2 & 458.2 \\ 0 & 834.35 & 220.5 \\ 0 & 0 & 1 \end{bmatrix}$$

The calculation was very sensitive to the selection of points. i.e., most of the times, it gives "matrix not positive definite" error during Cholesky factorization.

- **Result (triple box image of size** 890×738 **pixels)**:

$$K = \begin{bmatrix} f_x & s & u_x \\ 0 & f_y & u_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1738.5 & 4.6 & 562.2 \\ 0 & 1310.5 & 468.6 \\ 0 & 0 & 1 \end{bmatrix}$$

2 By assuming square pixels (i.e. skew = 0 and $fx = fy$)

By assuming the skew $= 0$ & $f_x = f_y$, the intrinsic camera parameters are reduced to 3. i.e., the number of constraint equations are reduced. The intrinsic camera parameter matrix can be defined as,

$$\begin{bmatrix} f & 0 & u_x \\ 0 & f & u_y \\ 0 & 0 & 1 \end{bmatrix}$$

2.1 by using 3 perpendicularity relations between vanishing points

2.1.1 Procedure

- The intrinsic parameters shall be determined using 3 sets of perpendicular vanishing points, as $f_x = f_y$ and $skew = 0$..
- To get 3 sets of vanishing points, the given chess board images (3) are used. To get the vanishing vanishing point, the perpendicular pair of parallel lines in the chess board are used. refer to the Figure 1. Let the set of of points to deniote the parallel lines in one direction be $P_{l_1}, P_{l_2}, P_{l_3}, P_{l_4}$. Let the other set of points selected to denote the parallel lines in perpendicular direction be $P_{m_1}, P_{m_2}, P_{m_3}, P_{m_4}$.
- The vanishing point is determined from the set of chosen points as follows:

$$\text{vanishing point 1}, VP_1 = (P_{l_1} \times P_{l_2}) \times (P_{l_3} \times P_{l_4})$$
$$\text{vanishing point 2}, VP_2 = (P_{m_1} \times P_{m_2}) \times (P_{m_3} \times P_{m_4})$$

- Since the vanishing points VP_1, VP_2 are perpendicular, we can form an equation with $\Psi = K^{-T}K^{-1}$, with K being the matrix of camera intrinsic parameters as follows:

$$(VP_1)^T \Psi (VP_2) = 0$$

The K matrix has 3 free parameters as follows:

$$\begin{bmatrix} f & 0 & u_x \\ 0 & f & u_y \\ 0 & 0 & 1 \end{bmatrix}$$

where f = focal length, u_x, u_y = camera center.

- With 3 sets of perpendicular vanishing points, we can get 3 such equation as given in step 4 above.

$$\begin{bmatrix} x_{l1}x_{m1} + y_{l1}y_{m1} & (x_{l1}h_{m1} + h_{l1}x_{m1}) & (y_{l1}h_{m1} + h_{l1}y_{m1}) & h_{l1}h_{m1} \\ x_{l2}x_{m2} + y_{l2}y_{m2} & (x_{l2}h_{m2} + h_{l2}x_{m2}) & (y_{l2}h_{m2} + h_{l2}y_{m2}) & h_{l2}h_{m2} \\ \vdots & & & \end{bmatrix} \begin{bmatrix} a \\ c \\ e \\ f \end{bmatrix} = 0$$

here,

$$\Psi = \begin{bmatrix} a & 0 & c \\ 0 & a & e \\ c & e & f \end{bmatrix}$$

- The parameters shall be determined using the Singular value decomposition (SVD). i.e., the singular vector corresponding to the least singular value is the solution for Ψ.
- Once Ψ is determined, we can use Cholesky decomposition to get the intrinsic camera paramerters K.
- **Result**:

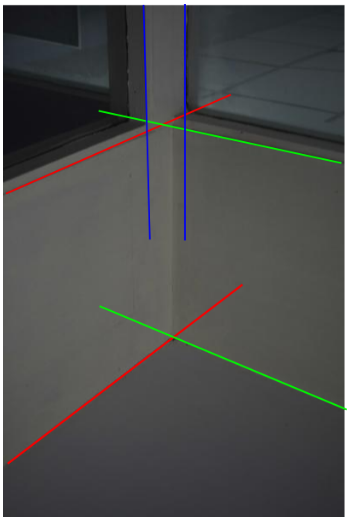

Figure 2: The perpendicular pair of parallel lines chosen to get the perpendicular rays from 3 vanishing points $(400 \times 600 image)$

$$K = \begin{bmatrix} f & 0 & u_x \\ 0 & f & u_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 708.3 & 0 & 188.3 \\ 0 & 708.3 & 452.2 \\ 0 & 0 & 1 \end{bmatrix}$$

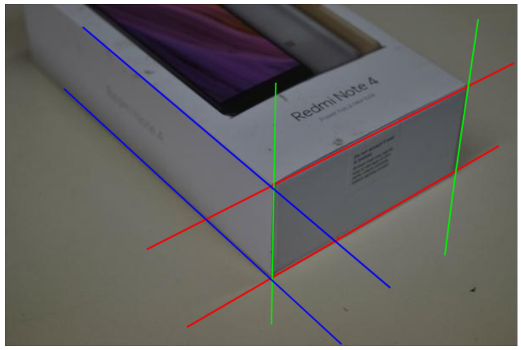

Figure 3: The perpendicular pair of parallel lines chosen to get the perpendicular rays from 3 vanishing points ($600 \times 400 image$)

$$K = \begin{bmatrix} f & 0 & u_x \\ 0 & f & u_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1333.36 & 0 & 137.8 \\ 0 & 1333.36 & 389.2 \\ 0 & 0 & 1 \end{bmatrix}$$

2.2 by computing the homography relation between metric co-ordinate system fixed to scene plane & image

2.2.1 Procedure

- As we have seen in the question 1, one homography calculation gives us 2 constraints. Hence, 2 such homographies give 4 constraint equations which are used to solve for the parameters of $\Psi = K^{-T}K^{-1}$. here,

$$\Psi = \begin{bmatrix} a & 0 & c \\ 0 & a & e \\ c & e & f \end{bmatrix}$$

- The parameters shall be determined using the Singular value decomposition (SVD). i.e., the singular vector corresponding to the least singular value is the solution for Ψ.

- Once Ψ is determined, we can use Cholesky decomposition to get the intrinsic camera paramerters K. The K matrix has 3 free parameters as follows:

$$\begin{bmatrix} f & 0 & u_x \\ 0 & f & u_y \\ 0 & 0 & 1 \end{bmatrix}$$

  where f = focal length, u_x, u_y = camera center.

- **Result (chess board images of size 600×400 pixels)**:

$$K = \begin{bmatrix} f & 0 & u_x \\ 0 & f & u_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1382.8 & 0 & 358.2 \\ 0 & 1382.8 & 160.5 \\ 0 & 0 & 1 \end{bmatrix}$$

- **Result (triple box image of size 890×738 pixels)**:

$$K = \begin{bmatrix} f & 0 & u_x \\ 0 & f & u_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1315.5 & 0 & 495.6 \\ 0 & 1315.5 & 486.9 \\ 0 & 0 & 1 \end{bmatrix}$$