CS 6730 Probabilistic Graphical Models *Due:* Feb 6, 2018

Assignment 1

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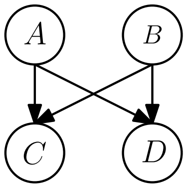
1. Consider a variant of the Monty hall problem, where the protocol is as follows. There are three doors, behind one of which is a car, and behind two of which are goats. You first choose a door. **The host then randomly picks one of the two remaining doors. It is observed to be a goat.** What is the probability that the car is behind the door you have already chosen? What would be a good Bayesian network for modelling this problem?

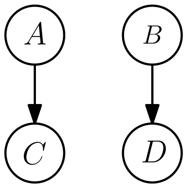
2. Consider the directed graphs on the left and probability distributions on the right. A, B, C, D are all binary random variables. The tables on the right give joint probabilities of all the 16 configurations. For each graph on the left, list all possible conditional independence statements implied by it. For each distribution on the right, list all conditional independences satisfied by it. Finally, for each graph on the left, list all distributions on the right that conform to it. Give reasons.



(I)

	$C, D = 0, 0$	$C, D = 0, 1$	$C, D = 1, 0$	$C, D = 1, 1$
$A, B = 0, 0$	0.5	0	0	0
$A, B = 0, 1$	0	0	0	0
$A, B = 1, 0$	0	0	0	0
$A, B = 1, 1$	0	0	0	0.5

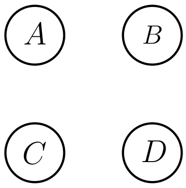
(I)



(II)

	$C, D = 0, 0$	$C, D = 0, 1$	$C, D = 1, 0$	$C, D = 1, 1$
$A, B = 0, 0$	0.015	0.035	0.06	0.14
$A, B = 0, 1$	0.05	0.05	0.075	0.075
$A, B = 1, 0$	0.105	0.045	0.07	0.03
$A, B = 1, 1$	0.18	0.02	0.045	0.005

(II)



	$C, D = 0, 0$	$C, D = 0, 1$	$C, D = 1, 0$	$C, D = 1, 1$
$A, B = 0, 0$	0.02	0.03	0.08	0.12
$A, B = 0, 1$	0.06	0.09	0.24	0.36
$A, B = 1, 0$	0	0	0	0
$A, B = 1, 1$	0	0	0	0

(III)

(III)

Note: There is no need to give conditional independences which can be implied. For example there is no need to give both $A \perp B, C$ and $A \perp B$, just the first one is enough as the second can be implied from it.

3. For each item on the list of conditional independence statements below:

   (a) Give a distribution over the binary random variables A, B, C, D such that they satisfy only the conditional independence statements and no other. Give the solution in the form of a 4×4 joint distribution as in the previous problem. If no solution exists, prove it.

(b) Give a Bayesian network, for which the set of implied conditional independence statements match exactly with the item on the list. If no solution exists, prove it.

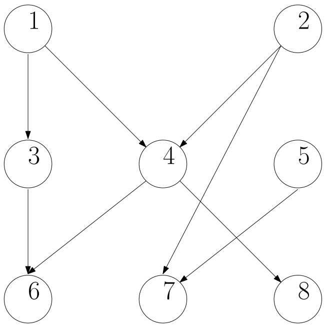
Note: Two conditional independence statements, written differently but meaning the same will be considered as identical. e.g. $A \perp B|C, D$ is the same as $A \perp B, C|C, D$.

(a) $(A \perp B|C, D)$,

(b) $(A \perp B|C, D), (C \perp D)$

(c) $(A \perp C|B, D), (B \perp D|A, C)$

4. Consider the Bayesian network given below:



Consider a distribution over 8 random variables $X_1, \ldots, X_8$ given by the Bayesian network to the left.

(a) Give the largest set of random variables that is independent of X_3.

(b) Give the largest set of random variables that is independent of X_3, conditioned on X_1.

(c) Give the largest set of random variables that is independent of X_3, conditioned on X_1 and X_4.

5. Write and submit a python code that gives samples from any directed graphical model given its parameters. A template code that defines the format in which the graph and the parameters are represented, and conventions to be followed is given as a supplementary file.