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13.1 From the definition of conditional probability, we have

$$P(X|Y) = \frac{P(X \wedge Y)}{P(Y)}$$

In this question, $P(a|b\wedge a) = \frac{P(a\wedge(b\wedge a))}{P(b\wedge a)} = \frac{P(b\wedge a)}{P(b\wedge a)}$

$= 1$

$\square$

13.3 a. True.

$$P(a|b,c) = \frac{P(a\wedge b\wedge c)}{P(b\wedge c)} \qquad P(b|a,c) = \frac{P(b\wedge a\wedge c)}{P(a\wedge c)}$$

$P(a|b,c) = P(b|a,c)$

$P(a\wedge b\wedge c) = P(b\wedge a\wedge c)$

$\Rightarrow P(b\wedge c) = P(a\wedge c)$

$\Rightarrow \frac{P(b\wedge c)}{P(c)} = \frac{P(a\wedge c)}{P(c)} \Rightarrow P(a|c) = P(b|c)$

$\square$

b. False. Because $P(a|b,c) = P(a)$ only means

a is independent from b, c. Still we know nothing

about relation between b, c.

Counter example: if a, b record results of 2 independent coins,

$c=b$, then $p(a|b,c) = p(a)$. a and b, c are irrelevant

$P(b|c)=1$. $P(b) = \frac{1}{2}$ $P(b|c) \neq P(b)$

c. False. Because a is independent of b cannot imply

a is independent of b and c.

Counter example: if a and b record 2 independent coins,

$c = a$ XOR b, then $P(a|b) = P(a)$

But $P(a|b,c) = 0$. $P(a|c) = P(\neg b) = \frac{1}{2}$

$P(a|b,c) \neq P(a|c)$

13.4 $P(A) = 0.4$ $P(B) = 0.3$ $P(A \vee B) = 0.5$

$P(A \wedge B) = P(A) + P(B) - P(A \vee B) = 0.2$

$P(A \wedge \neg B) = P(A) - P(A \wedge B) = 0.2$

$P(B \wedge \neg A) = P(B) - P(B \wedge A) = 0.1$

$P(\neg B \wedge \neg A) = 1 - P(A \wedge B) - P(A \wedge \neg B) - P(B \wedge \neg A) = 0.5$

	A	¬A
B	0.2	0.1
¬B	0.2	0.5

It is rational.. Agent 1 can lose or win..

A1		A2		outcomes and payoffs to A1			
				$a \wedge b$	$a \wedge \neg b$	$\neg a \wedge b$	$\neg a \wedge \neg b$
a	0.4	a	4 to 6	-6	-6	6	4
b	0.3	b	3 to 7	-7	3	-7	3
$a \vee b$	0.5	$\neg(a \vee b)$	5 to 5	5	5	5	-5
				-8	2	4	2

A1		A2		Outcomes and payoffs to A1			
				$a \wedge b$	$a \wedge \neg b$	$\neg a \wedge b$	$\neg a \wedge \neg b$
a	0.4	a	4 to 6	-6	-6	4	4
b	0.3	b	3 to 7	-7	3	-7	3
$a \vee b$	0.7	$\neg(a \vee b)$	3 to 7	3	3	3	-7
				-10	0	0	0

$$P(A \wedge B) = P(A) + P(B) - P(A \vee B) = 0.3 + 0.4 - 0.7 = 0.$$

So it is rational because there is no possibility that agent 1 is always loss. Because a and b both true will never happen.

13.8

a $P(toothache) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2$

b. $P(cavity) = 0.108 + 0.012 + 0.072 + 0.008 = 0.2.$

$P(\neg cavity) = 1 - 0.2 = 0.8$

$P(Cavity) = \langle 0.2, 0.8 \rangle$

c. $P(toothache \mid cavity) = \frac{0.108 + 0.012}{0.2} = 0.6$

$P(\neg toothache \mid cavity) = \frac{0.072 + 0.008}{0.2} = 0.4$

$P(Toothache \mid cavity) = \langle 0.6, 0.4 \rangle$

d. $P(Cavity \mid toothache \vee catch)$

$P(toothache \vee catch) = 1 - (0.008 + 0.576) = 0.416$

$P(cavity \mid toothache \vee catch) = \frac{0.108 + 0.012 + 0.072}{0.416} = 0.4615$

$P(\neg cavity \mid toothache \vee catch) = \frac{0.016 + 0.064 + 0.144}{0.416} = 0.5384$

$P(Cavity \mid toothache \vee catch) = \langle 0.4615, 0.5384 \rangle$