1 Introduction

1.1 Random Recursive Planar Trees

To build a random recursive planar tree,T, on n nodes one begins with a root vertex and adds $n-1$ vertices one by one. Let $V(T)$ be the vertex set of T. The descendants of any vertex $v \in V(T)$ are ordered (say from left to right), therefore if v has outdegree d there are $d+1$ distinct possible positions for a new vertex to attach to v. So if the outdegrees of every vertex are $d_1, d_2, d_3, \ldots, d_n$ then the number, N, of possible attachment points for a new vertex is

$$\begin{aligned} N &= (d_1+1)+(d_2+1)+\cdots+(d_n+1) \\ &= n+(d_1+d_2+\ldots d_n) \\ &= n+(n-1) = 2n-1. \end{aligned}$$

Each new vertex is attached to a vertex of T chosen uniformly at random from the $2n-1$ possiblities. To be even more explicit, given a random recursive planar tree T on n vertices then for chosen vertex v with outdegree d the probability, p that the new vertex will be attached to v is:

$$p = \frac{d+1}{\sum_{i=1}^{n} d_i}.$$

1.2 Barabasi-Albert Model

1.3 Barabasi-Albert Model for Trees

Given a tree T with vertices $V(T) = \{v_1, v_2, \ldots, v_n\}$ such that the degree of each vertex is $deg(v_i)$ for $i = 1, 2, \ldots n$ then we attach to node v with probability proportional to $deg(v)$. More specifically we choose node v with probability q where q is defined as follows:

$$q = \frac{deg(v)}{\sum_{v \in V(T)} deg(v)}.$$

Since q defined above and p from section 1.1 are equal there exists a corrospndence between the Barabasi-Albert prefferential attachment model for trees and random recursive planar trees. Furthermore, the limiting behaviour of these trees must be the same.

2 Degree Distribution

Let T be a random recursive planar tree on n nodes and X_{ni} be the number of vertices of outdegree $i \geq 0$ in a random plane recursive tree on n vertices.

Theorem 2.1. *Almost surely in the limit as* $n \to \infty n^{-1} Y_{ni} \to \frac{4}{(i+1)(i+2)(i+3)}$.

Proof. See [], Theorem 1.3. ∎

By the corrospondence between random recursive planar trees and Barabasi-Albert trees almost surely as $n \to \infty$ the degree distribution of a Barabasi-Albert tree tends to $\frac{4}{(i+1)(i+2)(i+3)}$.

References