Given some infinite collection of objects, X, if any $x \in X$ almost surely has property $\mathcal{P}$ what can we say about X? To clarify what I mean consider interested in the following example.

A *random recursive tree* (RRT), T_n, with vertices $V(T) = \{v_1, \ldots, v_n\}$ is a labelled, rooted tree obtained by assigning a root vertex v_1 then adding $n-1$ vertices one by one such that each new vertex is joined by an edge to a randomly and uniformly chosen existing vertex.

Local scenario: Let X_{ni} be the number of vertices of degree $i \geq 1$ in a RRT on n vertices. Janson [] proves that as $n \to \infty$ almost surely:

$$\frac{X_{ni}}{n} \to 2^{-i}$$

Global scenario: in the limit as $n \to \infty$ then $\frac{Y_{ni}}{(n-1)!} \to ?$. So my first hurdle is to appropriately define Y_{ni}. Two possible versions of Y_{ni}:

(i) Y_{ni} is the number of RRTs on n vertices such that $\frac{X_{ni}}{n} = 2^{-i}$

(ii) Y_{ni} is the number of RRTs on n vertices that are in some appropriately defined neighbourhood (dependent on n and possibly i) of $\frac{X_{ni}}{n} = 2^{-i}$.

I initially posed the question using definition (i), however this seems far to restrictive.

Finally, I appear to be attempting to find an analogue of the expectation of an event, i.e. given that for each infinite tree T we know that T having the degree distribution 2^{-i} is a measure 1 event but what proportion of trees have this property?