# Three-dimensional Reconstruction of Geological Solids Based on Section Topology Reasoning

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Abstract In order to solve the dynamic reconstruction and local updating problem of three-dimensional geological solids, topology reasoning is used for three-dimensional geological modeling. This can advance the level of the corresponding section automation in implementing the 3D geological solid dynamical reconstruction by the construction of and reasoning on topology on the 3D curved surface. This method has been successfully used in the Nanjing city geological modeling and the Zijin gold mine modeling. The results prove that this method adapts to coplanar section and noncoplanar section data, and improves the efficiency of 3D geological modeling.

Keywords geological section; topology reasoning; 3D geological solid reconstruction

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#### Introduction

Because most of the geological solids are covered up, they are invisible such that their survey data are often indirect and not self-contained. Qualitative data, from which much geological knowledge is based, could not be used for geological modeling in the past. The problem with three-dimensional geological solid reconstruction is always a difficult one <sup>[1,2,3]</sup>. Since the 1990s, many experts have thought that dynamical reconstruction based on the pure grid division method can suit simple spatial solids or regular spatial solids but not complex spatial solids. Much more geological knowledge and artificial intelligence reasoning are needed in the process of complex geological modeling as it can not implement dynamical reconstruction

automatically. So this research places emphasis on the fact that the dynamical reconstruction method of geological solids has changed from pure spatial modeling data structure and algorithms to geological knowledge expression and reasoning in the process of the dynamical reconstruction of geological solids.

In 1998, Chiaruttini C et al. studied the problem of applying spatial reasoning in the geological modeling procedure. They used a subsurface model and discussed the spatial restriction rule problem and the problem of diagnosing the geological model. Perrin M presented the geological syntax for the first time, and discussed geological consistency. In 1999, Roberto et al. discussed artificial intelligence reasoning algorithm in the process of explaining geology. In 2002, Schoniger et al. brought forward geological reasoning analysis under uncertain conditions, and

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In order to resolve the geological section correspondence problem and enable the three-dimensional geological solid model to be reconstructed and updated dynamically, this paper presents a dynamic modeling method based on noncoplanar geological section topology reasoning. By geological section topology relationship construction and reasoning on the three-dimensional curved surface, this method increases the dimensions of the geological section correspondence, and implements three-dimensional geological modeling dynamically. This method has been used in the Nanjing city geological modeling by implementation of GeoView software. The results indicate that this method is suitable for many kinds of section data, which include coplanar sections and noncoplanar sections, and improves the efficiency of 3D geological modeling.

# 1 Generation of topology on noncoplanar sections

The section data is the main source of 3D geological modeling. In existing modeling methods based on section data, the sections are mostly coplanar surfaces, and the surfaces of the 3D geologic solids often are generated by contour lines on the section in the proc-

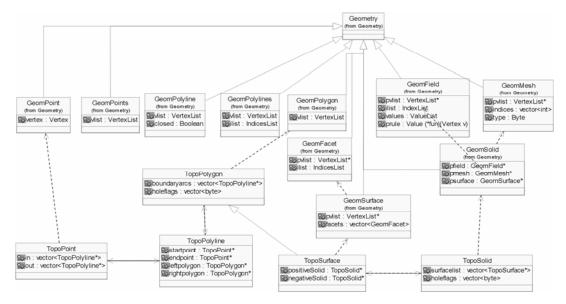
ess of modeling<sup>[5,7,8]</sup>. This kind of modeling method mainly includes four problems, i.e. the correspondence problem, tiling problem, branching problem, and fitting problem<sup>[12]</sup>. Studies have presented 3D geological modeling from topological cross-sections, but the topology on the sections is constructed manually<sup>[9,11]</sup>. Auto-generation of the topology on the noncoplanar section is still a key and difficult problem.

#### 1.1 Three-dimensional topology model

In vector spaces, the number of topology relationships between two linear spatial objects is 56; the number of topology relationships between two area spatial objects is 57; the number of topology relationships between one linear spatial object and one area spatial object is 97<sup>[4]</sup>. If we import the point spatial object and solid spatial object into the three-dimensional vector space, the topology relationships formed by them will be very complex. We thus need to predigest the topology model around the noncoplanar section. Fig.1 is the predigestion model of the three-dimensional topology. The object whose prefix is "geom" is the geometry object. The object whose prefix is "topo" is the topology object. We use TopoPoint, TopoPolyline, TopoPolygon, TopoSurface and TopoSolid to represent the 3D topology relationships. The TopoPolygon represents a simple polygon object, and the TopoSurface represents a curved surface with topology relationship. To some degree, a TopoSurface object is a noncoplanar section with topology relationship. We can also use many TopoSurface objects to represent a noncoplanar section with topology relationship if we wish.

For the auto-generation topology relationship on the noncoplanar section, we use the extended 3D node (*TopoPoint*), 3D Arc (*TopoPolyline*), and 3D face (*TopoPolygon* and *TopoSurface*) to represent the topology on the noncoplanar section. *TopoPoint* is a kind of special 3D point; it can be the start node or the end node of the *TopoPolyline*. *TopoPolyline* is a 3D directional curve; it can only intersect with other *TopoPolylines* on the 3D node. The *TopoPolygon* is a coplanar polygon in 3D space, whose boundary is composed of some 3D arcs. The *TopoSurface* is a set of *TopoPolygons*. It can be noncoplanar. The *TopoSolid* is composed of one or many *TopoSurfaces* and a

mesh in the solid space. The construction of the topology relationship on the curved surface or on the noncoplanar section is based on the frontal four topology objects. This model resolves the compatibility problem between the 2D topology model and the 3D topology model.



Three-dimensional topology data structure model

## Auto-generation algorithm of topology relationship on a curved surface

At present, the auto-generation algorithms of 2D topology are ripe<sup>[13-17]</sup>, but there is no algorithm implemented for the auto-generation of 3D topology. This paper thus presents an algorithm, called the auto-generation of topology on the 3D curved surface based on projection (AGTP), to solve this problem. The basic principle is to set up the projection relationship between the 3D section and the 2D section and ensure the topology is not changed by projection. The main steps are as follows.

- 1) S is the origin section. By polyline intersection and cut calculation, we obtain the node table and the arc table. By this step, for example, the section shown in the Fig.2 will generate Tables 1 and 2.
  - 2) Handle micro-arcs and hanging arcs.
- 3) Select the projection plan. The selection of the projection plan must ensure that the topology relationships have not been changed after the projection

Table 1 Node list

Node name	In-arc	Out-arc
$P_1$	$L_1, L_2$	$L_7$
$P_2$	$L_3, L_4$	$L_1$
$P_3$	$L_5, L_7$	$L_4, L_6$
$P_4$	$L_6$	$L_5, L_8$
$P_5$	$L_8$	$L_2, L_3$

Table 2 Arc list

Arc name	Start node	End node	Left polygon	Right polygon
$L_1$	$P_2$	$P_1$	NULL	NULL
$L_2$	$P_5$	$P_1$	NULL	NULL
$L_3$	$P_5$	$P_2$	NULL	NULL
$L_4$	$P_3$	$P_2$	NULL	NULL
$L_5$	$P_4$	$P_3$	NULL	NULL
$L_6$	$P_3$	$P_4$	NULL	NULL
$L_7$	$P_1$	$P_3$	NULL	NULL
$L_8$	$P_4$	$P_5$	NULL	NULL

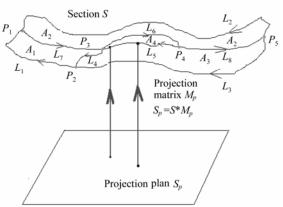


Fig.2 Noncoplanar section and projection plan

is executed. If the coplanar plan  $S_p$  is the projection plan of section S, any vertical line of  $S_p$  has only one intersection point with section S. This is very important. The essential part of this step is to set up a projection matrix  $M_p$ . The  $M_p$  must have a converse matrix, noted as  $M_r$ , for a successful conversion operation.

4) Projection transformation. Transform the arcs and nodes by the projection matrix, with the projection plan being  $S_p$ . If we want to use an existing 2D topology reconstruction algorithm, we can add a rotation and translation matrix, called  $M_t$  (the matrix must have A converse matrix), which can make  $S_p * M_t$  parallel to the XOY, or XOZ, or YOZ plan. The nodes transformed by  $M_p * M_t$  are called  $PP_i$ , corresponding to  $P_i$ . The arcs transformed by  $M_p * M_t$  are called  $LP_i$ , corresponding to  $L_i$ . This operation can transform the noncoplanar section into a coplanar section (Fig.3).

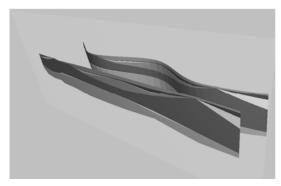


Fig.3 Noncoplanar section and projection section

- 5) Select  $LP_i$ , obtain its start node and end node, and call the start node  $P_s$ .
- 6) If the left polygon of  $LP_i$  is empty, left polygon searching will be done. Get the end node firstly, get all the arcs connected with this node secondly, then eliminate the arcs positioned at the right of  $LP_i$ ; select the arc called  $LP_t$ , which ensures that the inclination between  $LP_i$  and  $LP_t$  is minimal.
- 7) Let  $LP_t$ = $LP_t$ , repeat step 6 until the end node of  $LP_t$  (if the directions of the two adjacency arcs are face to face, it will be the start node of  $LP_t$ ) is equal to  $P_s$ , save all the passed arcs, and construct the polygon  $AP_t$  using the arcs. Then set the left polygon or left polygon of the arcs to be equal to  $AP_t$ .
- 8) If the right polygon of  $LP_i$  is empty, right polygon searching will be done. Get the end node firstly, then get all the arcs connected with this node, then eliminate the arcs positioned at the left of  $LP_i$ ; select the arc called  $LP_t$ , which ensures that the inclination between  $LP_i$  and  $LP_t$  is minimal.
- 9) Let  $LP_i=LP_i$ ; repeat step 8 until the end node of  $LP_t$  (if the directions of the two adjacency arcs are face to face, it will be the start node of  $LP_t$ ) is equal to  $P_s$ , store all the passed arcs and construct the

polygon  $AP_l$  using the arcs, then set the left polygon or left polygon of the arcs to be equal to  $AP_l$ .

- 10) Repeat steps 5 to 9 for all the arcs, and the topology will be constructed. The polygon set we obtained is called ASP,  $APS = \{AP_i | 0 \le i \le m, m \text{ is the number of the polygons}\}$ .
- 11) Replace  $\{LP_j\}$  with  $\{L_j\}$ , and reconstruct the polygon  $A_i$ ; the  $\{LP_j\}$  is the arc list of  $AP_i$ . Do this operation for all the polygons on the projection plan and set up the 3D topology relationship model. If the reverse matrix of  $M_p*M_t$  is valid, the geometric information of the topology polygons can be received by the reverse transformation.

By all of the steps above, we can construct the topology relationship on the 3D curved surface. In this paper, the 3D curved surface is in the noncoplanar section. The following topology reasoning for modeling is based on it.

## 2 Topo-reasoning for modeling

In this paper, the geology section topology reasoning adopts a production system. A production system (or production rule system) is a computer program typically used to provide some form of artificial intelligence, which consists primarily of a set of rules about behavior. These rules, termed productions, are a basic representation found useful in AI planning, expert systems and action selection. A production system provides the mechanism necessary to execute productions in order to achieve some goal for the system. Productions consist of two parts: a sensory precondition (or "IF" statement) and an action (or "THEN"). If a production's precondition matches the current state of the world, then the production is said to be triggered. If a production's action is executed, it is said to have fired. A production system also contains a database, sometimes called working memory, which maintains data about the current state or knowledge, and a rule interpreter. The rule interpreter must provide a mechanism for prioritizing productions when more than one is triggered.

#### 2.1 The rules of reasoning for 3D modeling

For the sake of a compact and clear statement, we bring out the following definitions.

**Definition 1** The unique property, which is used to judge whether topology polygon A and B are of the same type or not, is called the key type property. For instance, if topology polygon A's stratum lithology is Q, it is represented as KeyTypeProperty(A,Q).

**Definition 2** The adjacent key type property set is a set composed of all the key type properties of topology polygon A's adjacent topology polygons. This operation is called AdjacentKTPSet.

**Definition 3** If polygon A on section SA and polygon B on section SB are both part of the same geological solid, and the two polygons corresponded with each other, then we say that A matches B, represented as Corresponding (A, B).

**Definition 4** If sections SA and SB are adjacent to each other, polygon A is on section SA, noted as Existing (SA, A); but if the polygon matching A on section B does not exist, it is noted as *Inexisting* (SB, A); then A is annihilating on section SB, represented as Annihilating(SA, A, SB).

**Definition 5** If sections SA and SB are adjacent to each other, the polygon A is on the section SA, and the key type property is O; there are two polygons  $B_1$  and  $B_2$  on section SB, and both the two polygons' key type property are Q; then A has two branches on section SB, noted as Bifurcating  $(A, B_1, B_2)$ .

**Notice** Definition 5 just states cases 1 to 2. The other bifurcating cases are complex, but they all can be brought out case by case.

**Definition 6** The basic rule in the process of topology reasoning for modeling based on geological sections is as follows.

If topology polygon A is on the section  $S_1$ , and the topology polygon B is on section  $S_2$ , they can fit the four conditions:

- (1) A and B have the same stratum lithology Q;
- (2) A and B have the same topology node numbers;
- (3) A and B have the same topology arc numbers;
- (4) A and B have the same adjacent key type prop-

Then we say that *B* is the extension of *A* on section  $S_2$ . If so, polygon A and B will be connected to tiling. We can list all the facts of the relationships among the sections, topology polygons, arcs, and notes, and then do reasoning based on the facts and the rules. In this way, the section corresponding problem is transformed to the topology reasoning problem. In the next content, we will discuss the topology reasoning in the following four cases.

### 2.2 Topology reasoning and modeling for the case which has no topology change

Shown in Fig.4 is the common simple geological section series wherein the adjacent two sections have the same topology relationship set. We can thus build the fact database and rule database for reasoning and the polygon corresponding query.

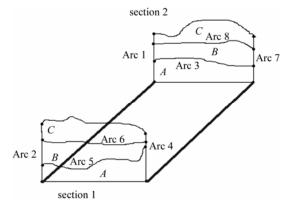


Fig.4 Simple section series

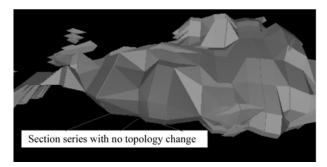


Fig.5 Mine solid constructed by a simple section series

The corresponding polygon is the key to implement the dynamic reconstruction of the geology solid based on the sections. The code above implements the corresponding reasoning of the two polygons on different sections. The problems left for the dynamical reconstruction is the tiling and fitting. The references have presented the solutions for these two problems<sup>[8]</sup>, and we will not discuss them again in this paper. In the next context we will often refer to this method about topology reasoning and modeling for the case in which no topology change occurs. We briefly call it the "Basic Method". Fig. 5 shows a part of an actual mine solid constructed by this method.

#### 2.3 Topology reasoning and modeling for the case which has stratum annihilation

Geologic actions such as faults, magma inrush, etc., may result in stratum annihilation. Stratum *B* appeared in section 1, but not in section 2, as shown in Fig.3. In this condition, the annihilation stratum *B* can not find the corresponding stratum on the adjacent section. So we need to insert a dummy section named section1-2 between section 1 and section 2 by the half-distance rule. In the process of mutual modeling, the operator can judge which cases have stratum annihilation, but the computer can not because only the operator can reason on the information he obtains using his/her brain. In order to make the computer have the capability to reason on this case, a reasoning model based on the annihilation stratum case must be constructed firstly.

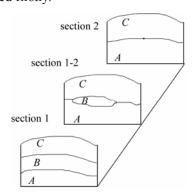


Fig.6 Section series with stratum annihilation

The rule for judging stratum annihilation among the adjacent sections is that if stratum *A* appears on section *SA* but not on section *SB*, then stratum *A* annihilates on section *SB*. In this way, we can let the computer judge the stratum annihilation. In the case that has stratum annihilation, the procedure of modeling is no different from the case stated in the "Basic Method" except for the judging and reasoning on stratum annihilation additionally. Otherwise, definition 6 must erase condition (2), condition (3) and condition (4) for the geological solid reconstruction in this case. Fig.7 shows an actual mine solid model for this case.

# 2.4 Topology reasoning and modeling for the case that has stratum bifurcation

In the actual geological environment, there is another instance called the stratum bifurcation which

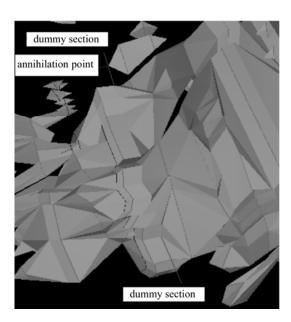


Fig.7 Mine solid with annihilation case

does not include stratum annihilation. Section 1 shows a mine solid contour and a wall rock contour, but the mine solid contour is divided into two contours in section 2 in Fig.8. In this case, the mine solid contour polygon can not find its corresponding polygon on section 2 directly. So we must insert a dummy section named section1-2 between section1 and section 2 by the half-distance rule if stratum bifurcation appears.

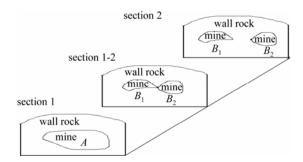


Fig.8 Section series with stratum bifurcation

In the procedure of mutual modeling, the operator can directly judge which case has stratum bifurcation, but the computer can not. So the most important thing is to make the computer have the capability to reason on the case that has stratum bifurcation.

We note the mine solid contour polygon on section 1 as A, and the mine solid contour polygons on section 2 as  $B_1$  and  $B_2$ . The key type property of the mine solid is Q. In the case that has stratum bifurcation, the procedure of modeling has no difference with the case stated in 2.3 except for the judging and reasoning for the stratum bifurcation additionally. It must be de-

noted that this paper aims just to study the case with 1 to 2 branches; the other bifurcating cases are complex, but they all can be concluded by this simple case. Fig.9 shows an actual mine solid model for this case.



Mine solid bifurcating case

#### 2.5 Topology reasoning and modeling for the case that has faults

Because of the influence of faults, the touch relationships among the strata may change. Stratum break and slippage will happen with fault action. It will result in the arc set of the fault, and the polygons connected to these arcs change. All of these will make the polygon corresponding by the algorithm in 2.1 abort. In order to state the situation clearly, we insert a dummy assistant section named section 1-2 between section1 and section 2, shown in Fig.10.

In section 1, there are three geological solids originally, but in the presence of faults, the three geological solids are divided into nine geological monocases. They are  $B_1$ ,  $B_2$ , and  $B_3$ , which have the same lithologies B,  $A_1$ ,  $A_2$ ,  $A_3$ , and  $C_1$ ,  $C_2$ ,  $C_3$ , which have the same lithology C. There are still two faults named  $F_1$ and  $F_2$  in section 1. The fault  $F_1$  is composed of 5 arcs, the wide line arc named  $Arc F_1 A$  is special because the left and right polygons' lithology are both A. In the same way, there are still two special arcs named  $Arc F_1 B$  and  $Arc F_1 C$ . This type of arc, wherein the key type properties of the left polygon and right polygon are the same, is called the "connatural arc". The properties of the connatural arcs are named "connatural arc properties".

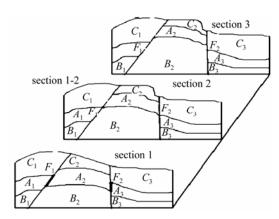
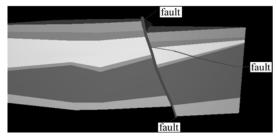


Fig.10 Section series with faults

When the two adjacent sections with the faults can fit the following two conditions: ① the numbers of the connatural arcs included by the corresponding fault pair are equal to each other; ② the connatural arc property set of the connatural arcs included by the corresponding faults are equal to each other. We consider that there are no topology changes between the two adjacent sections. In this case, we can use the method in the "Basic Method" for modeling.



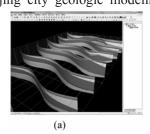
Geological solid with faults

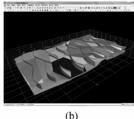
If any one of the two conditions is false, there are topology changes happening between the two adjacent sections. This is the rule used by the computer to judge whether there is a need to insert a dummy section or not. Except for the judging and reasoning on the fault and topology, the procedure of modeling is the same as in 2.1, but the conditions of definition 6 must be changed. The change is that all the arcs in the faults in the section should not be counted in arc operations. Fig.11 shows an actual geological solid model for this case.

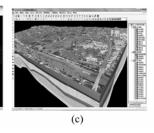
#### Results and conclusions

In order to solve the dynamic reconstruction and part updating problem of three-dimensional geological solids, topology reasoning is imported to three-

mine modeling successfully. Fig.12 shows some results. Fig.12(a) shows the imported noncoplanar sections; Fig.12(b) shows the geological solid constructed by these noncoplanar sections; Fig.12(c) shows the integration geological solid model created by this method; Fig.12(d) is the cutting model of mode C in Fig.12(c). This proves that this method is fit for coplanar section and noncoplanar section data, and improves the efficiency of 3D geological modeling.







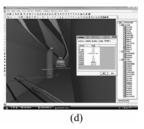


Fig.12 Some results implemented by this method in GeoView

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