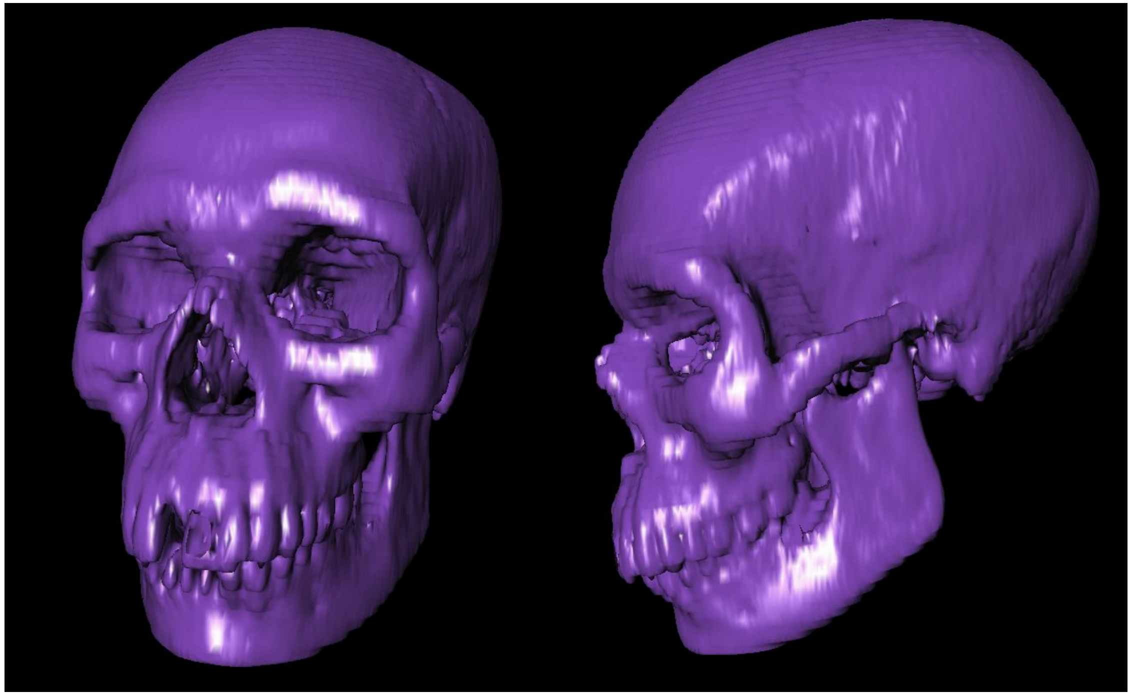
Marching Cubes: A High Resolution 3D Surface Construction Algorithm

Based on the Paper by William E. Lorensen and Harvey E. Cline

Introduction

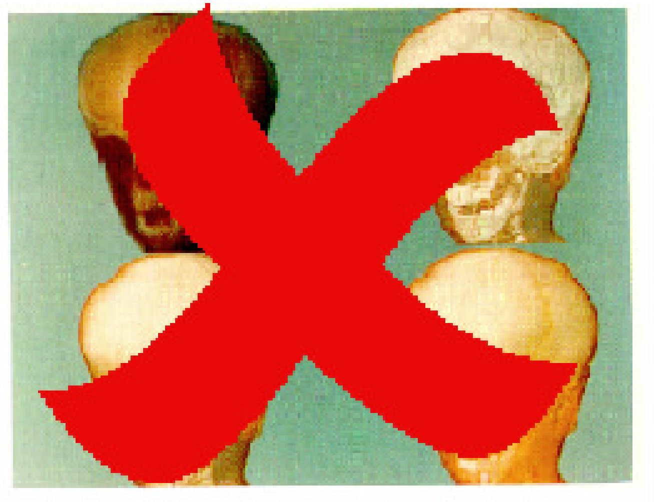
- The goal:

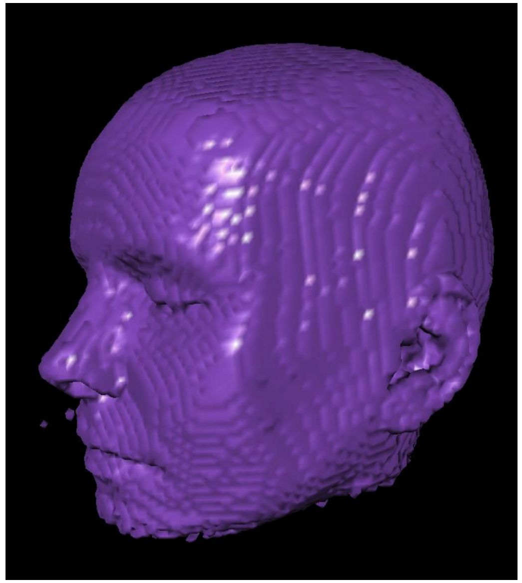
create a constant density surface from a 3D array of data



Introduction

- One surface at a time





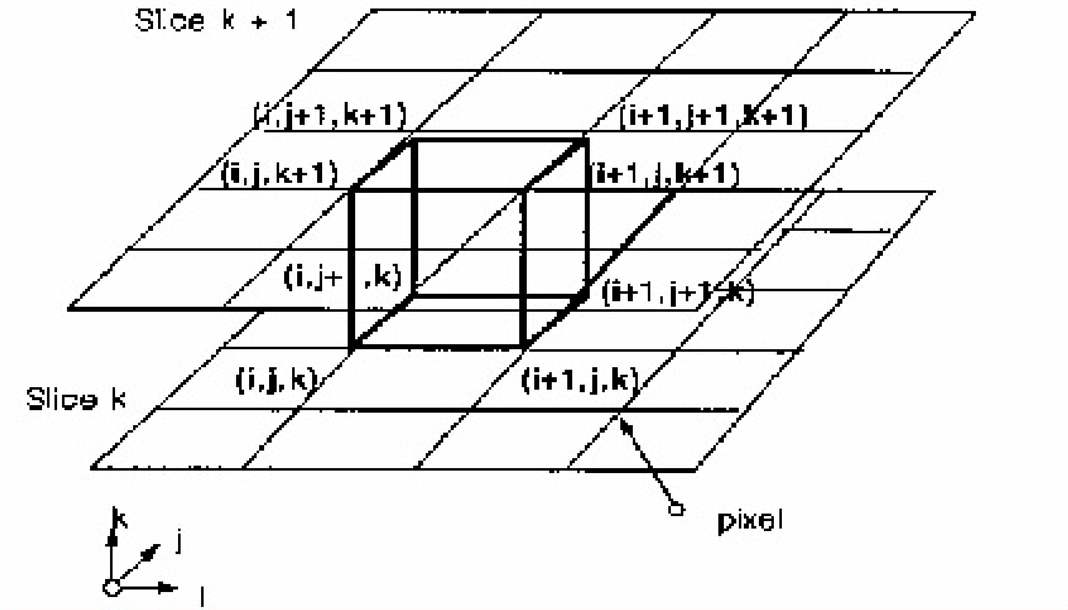
- For another surface must run again

Introduction

- Idea:
  - create a triangular mesh that will approximate the iso-surface
  - calculate the normals to the surface at each vertex of the triangle
- Algorithm:
  - locate the surface in a cube of eight pixels
  - calculate normals
  - march to the next cube

Surface intersection in a cube

- assign ZERO to vertex outside the surface
- assign ONE to vertex inside the surface



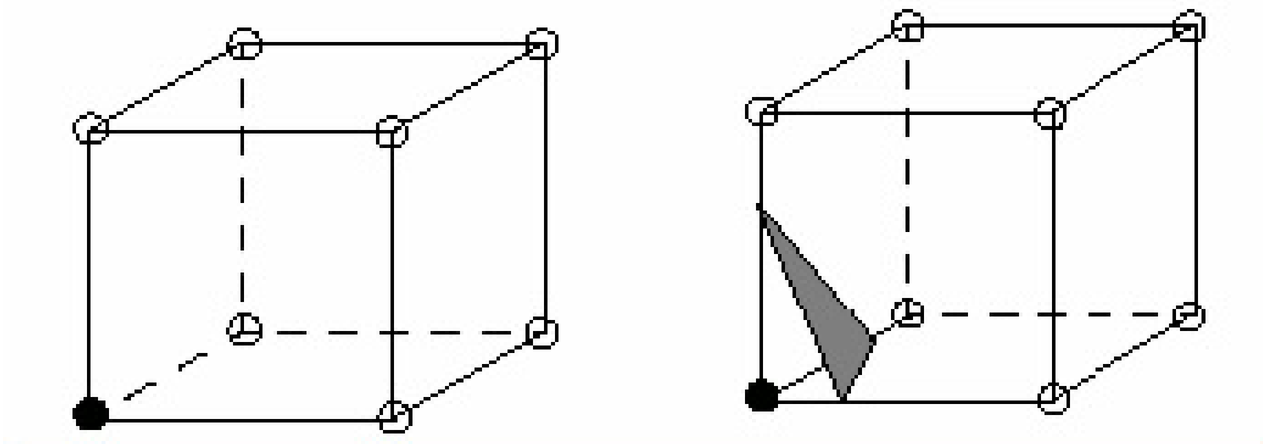
Note:

Surface intersects those cube edges where one vertex is outside and the other inside the surface

Surface intersection in a cube

- There are $2^8=256$ ways the surface may intersect the cube

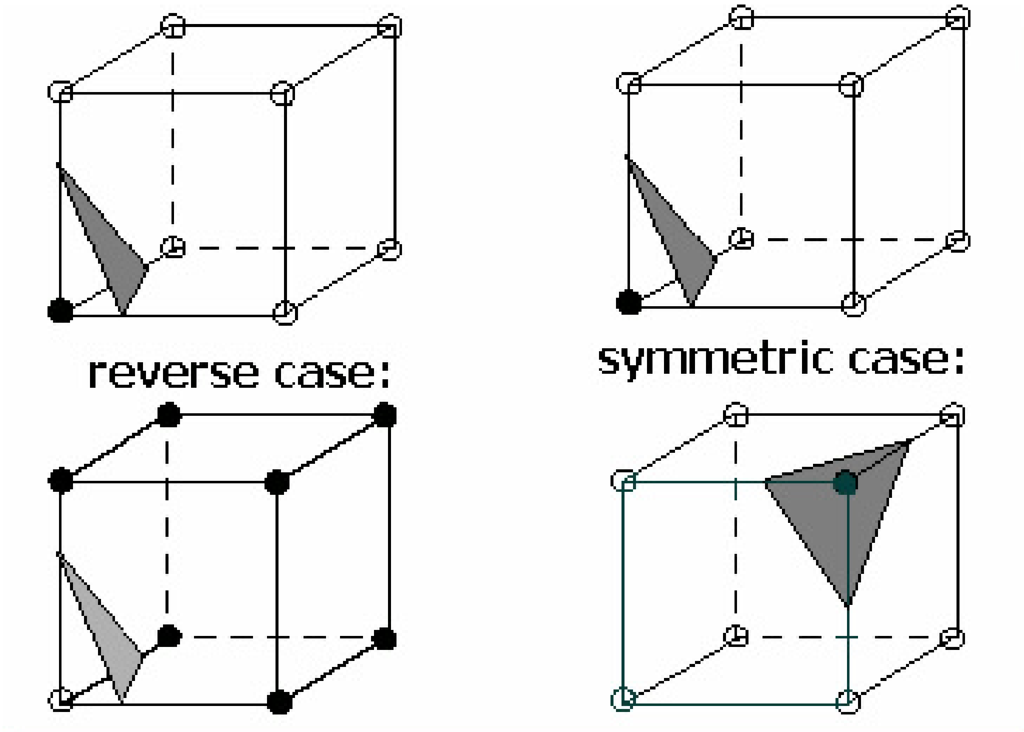
- Triangulate each case



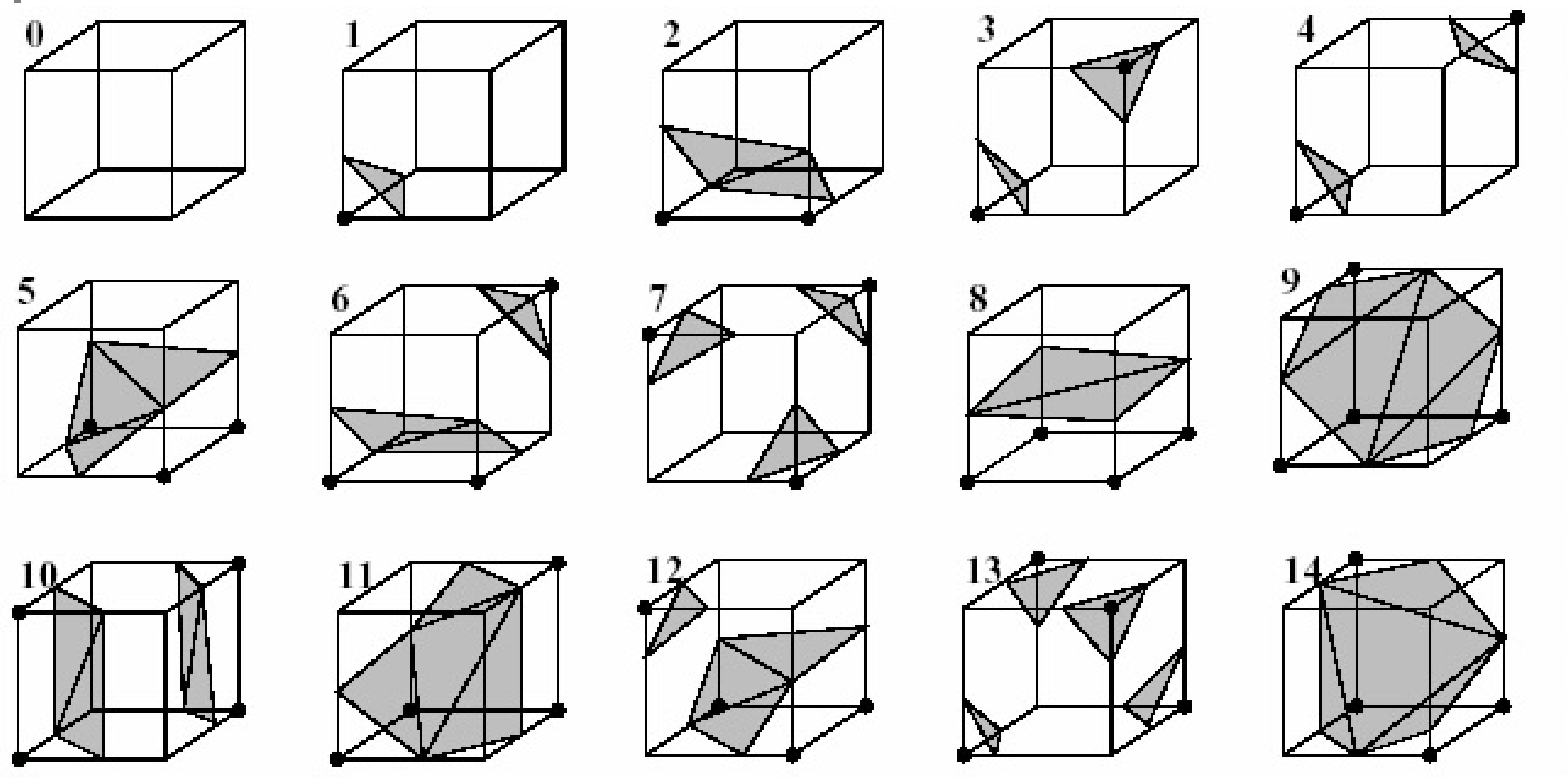
Patterns

- Note:

  using the symmetries reduces those 256 cases to 15 patterns

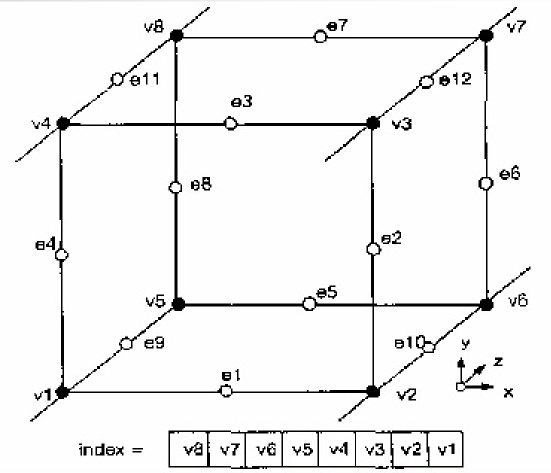


Patterns



Surface intersection in a cube

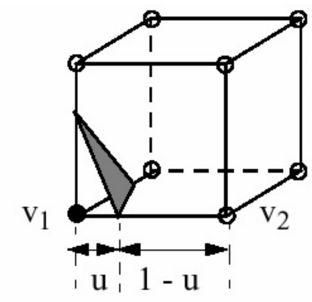
- Create an index for each case:



- Interpolate surface intersection along each edge

$$v_i = v_1 * (1 - u) + v_2 * u$$

$$u = \frac{v_1 - v_i}{v_1 - v_2}$$



Calculating normals

- Calculate normal for each cube vertex:

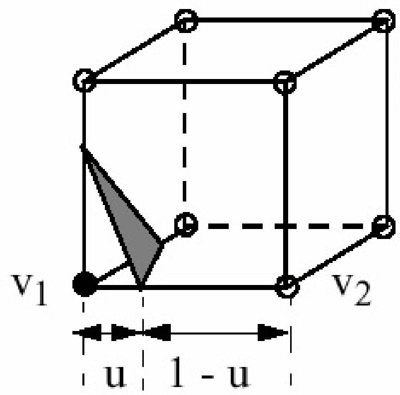
$$G_x(i,j,k) = \frac{D(i+1,j,k) - D(i-1,j,k)}{\Delta x}$$

$$G_y(i,j,k) = \frac{D(i,j+1,k) - D(i,j-1,k)}{\Delta y}$$

$$G_z(i,j,k) = \frac{D(i,j,k+1) - D(i,j,k-1)}{\Delta z}$$

- Interpolate the normals at the vertices of the triangles:

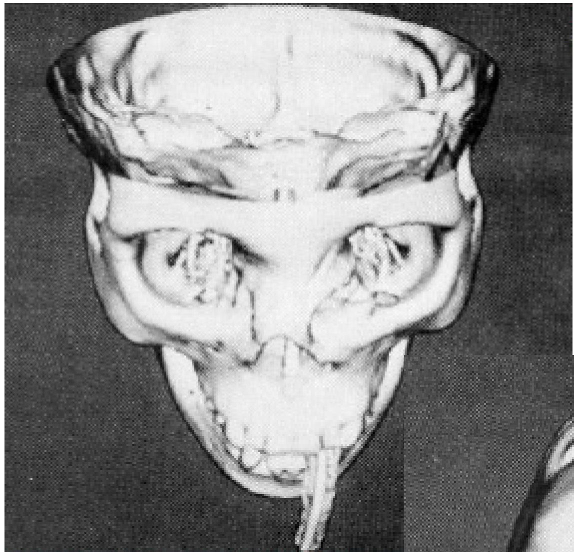
$$\overrightarrow{n_1} = u\overrightarrow{g_2} + (1-u)\overrightarrow{g_1}$$



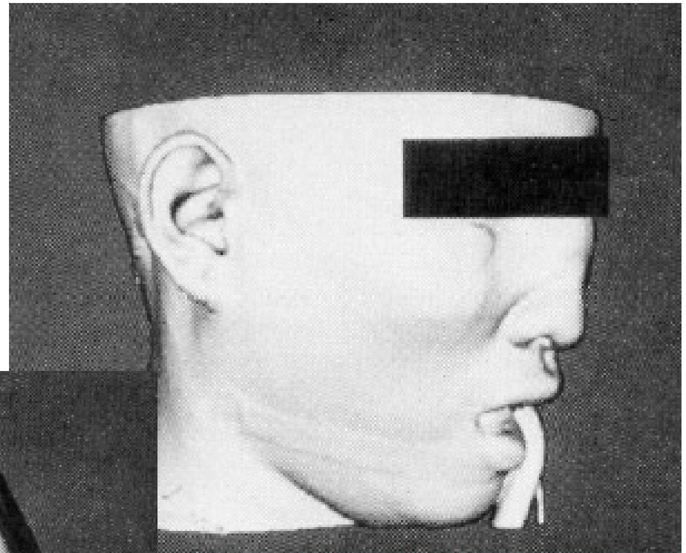
Summary

- Read four slices into memory
- Create a cube from four neighbors on one slice and four neighbors on the next slice
- Calculate an index for the cube
- Look up the list of edges from a pre-created table
- Find the surface intersection via linear interpolation
- Calculate a unit normal at each cube vertex and interpolate a normal to each triangle vertex
- Output the triangle vertices and vertex normals

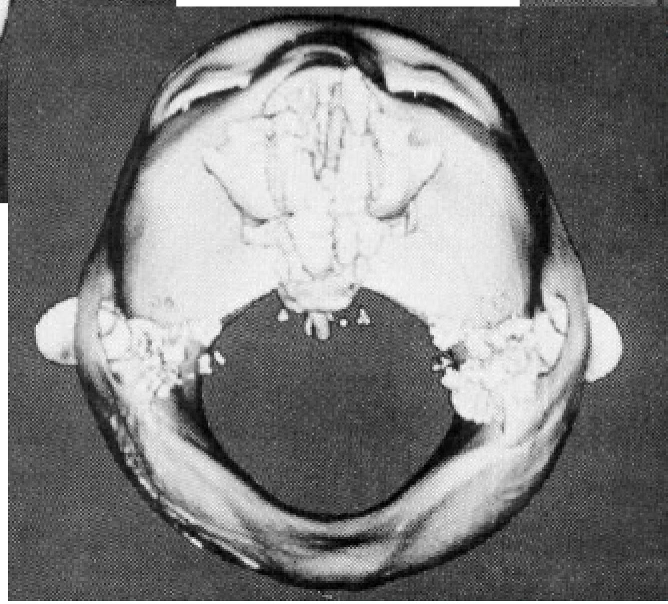
Results



Bone surface

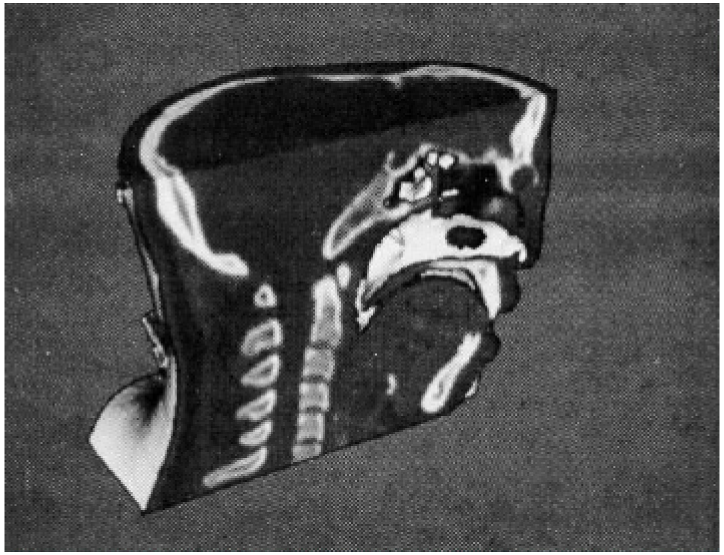


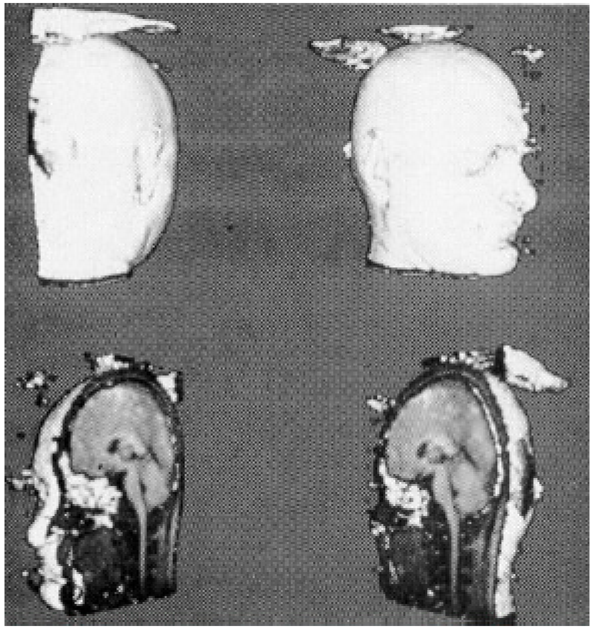
Soft tissue surface



Soft tissue – top view

Additional features





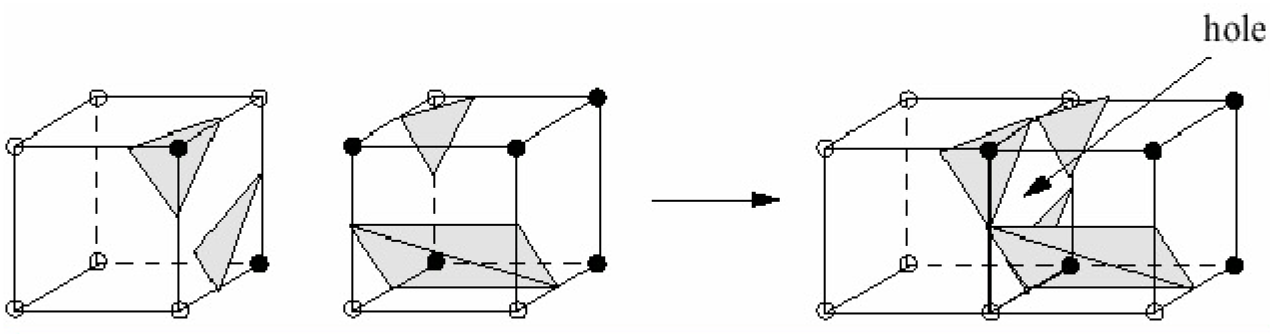
- By using solid modeling and texture mapping

Pros and Cons

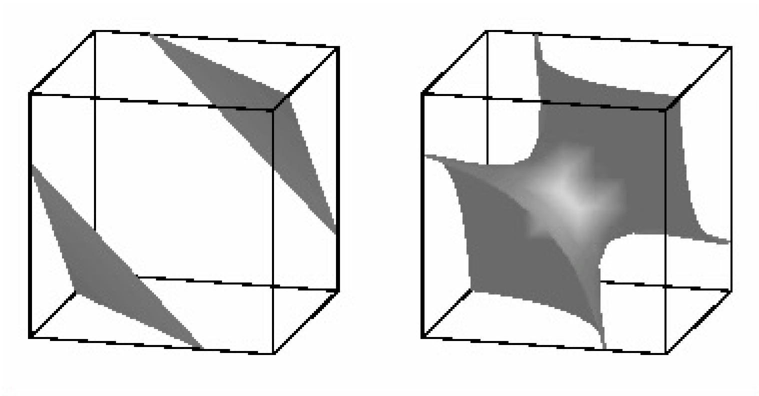
- Pros:
  - Simple rendering and manipulation
  - High resolution
- Cons:
  - Possible holes in the model
  - Model complexity

Holes

- Holes:

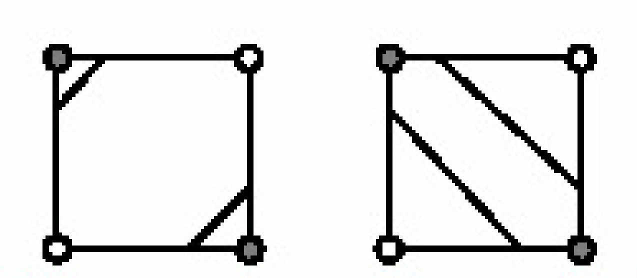


- Wrong surface:



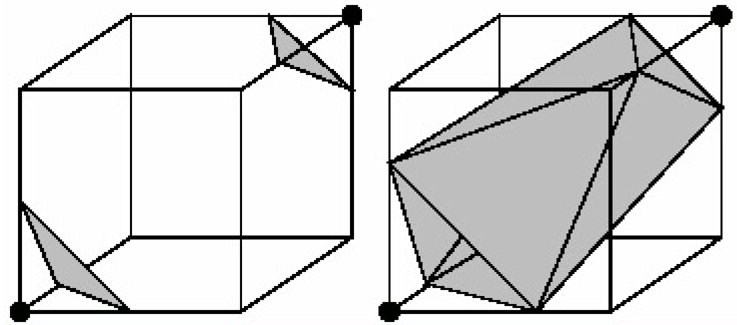
Holes

- The reason – “ambiguous face”



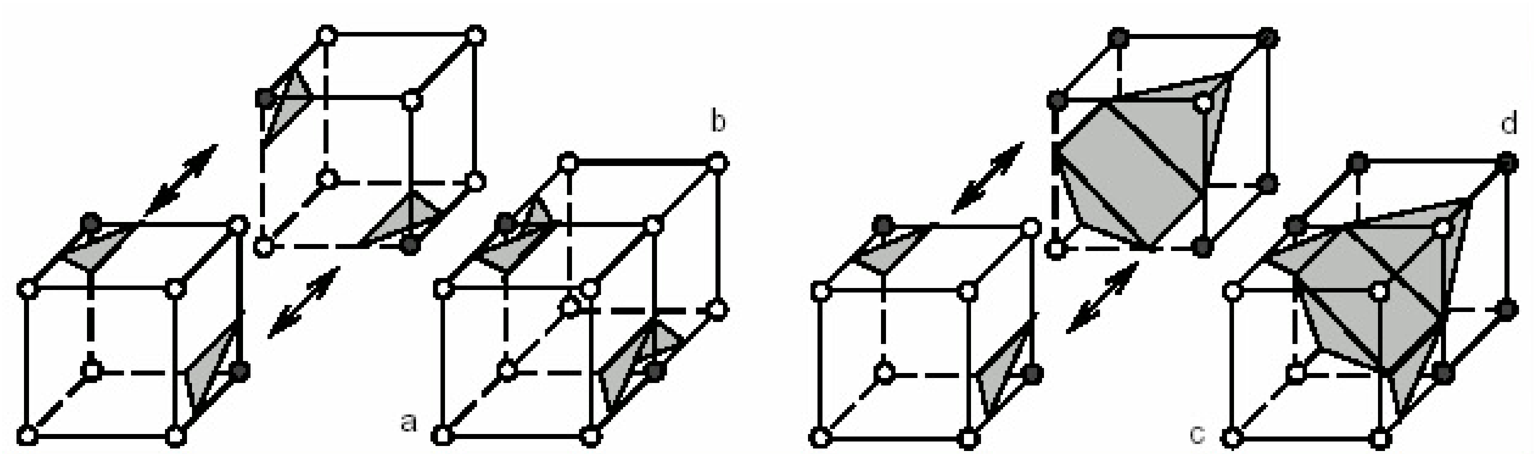
in 2D

in 3D



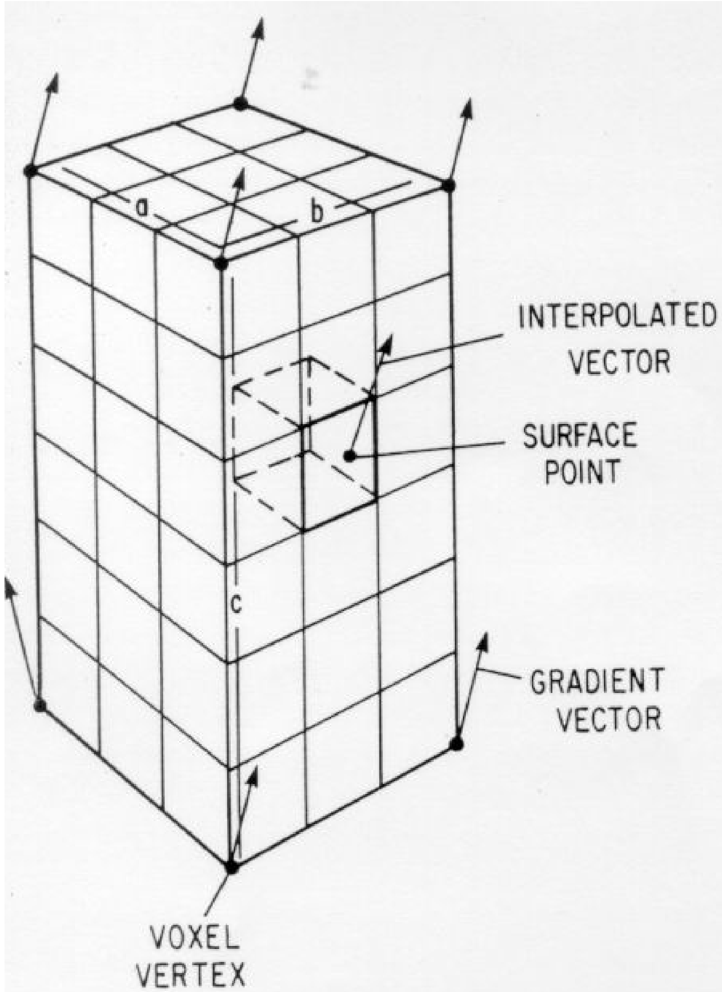
Resolving ambiguities

- Face adjacency



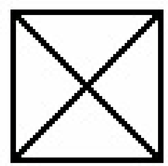
Resolving ambiguities

- Subdivision

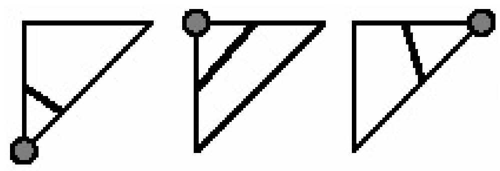


Resolving ambiguities

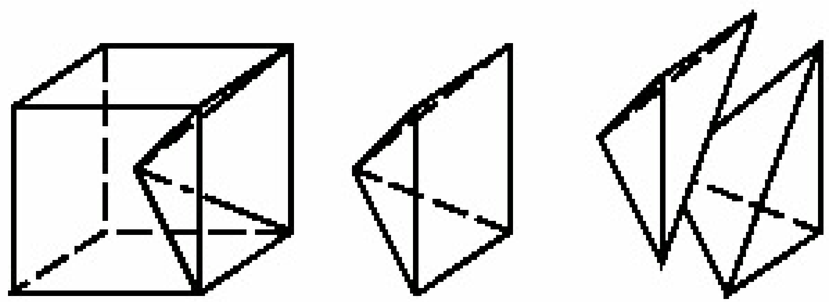
- Simplex decomposition
  - in 2D:



patterns:

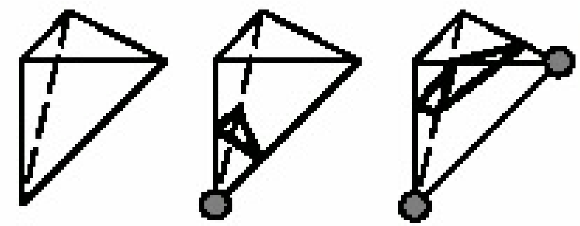


  - in 3D:



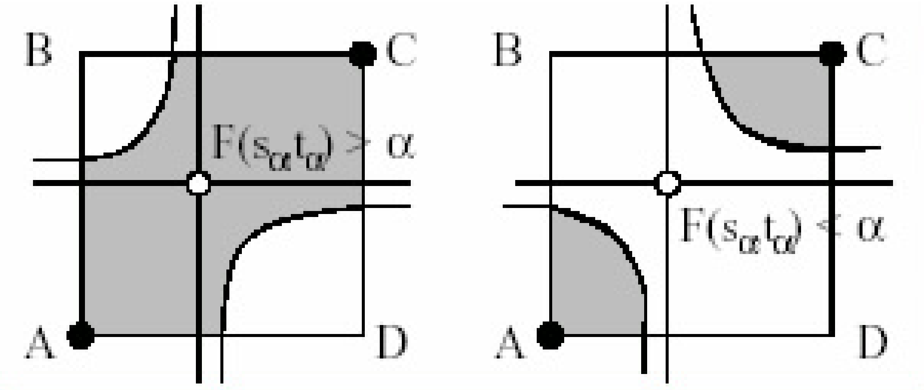
12 tetrahedra in the cube

patterns:



Resolving ambiguities

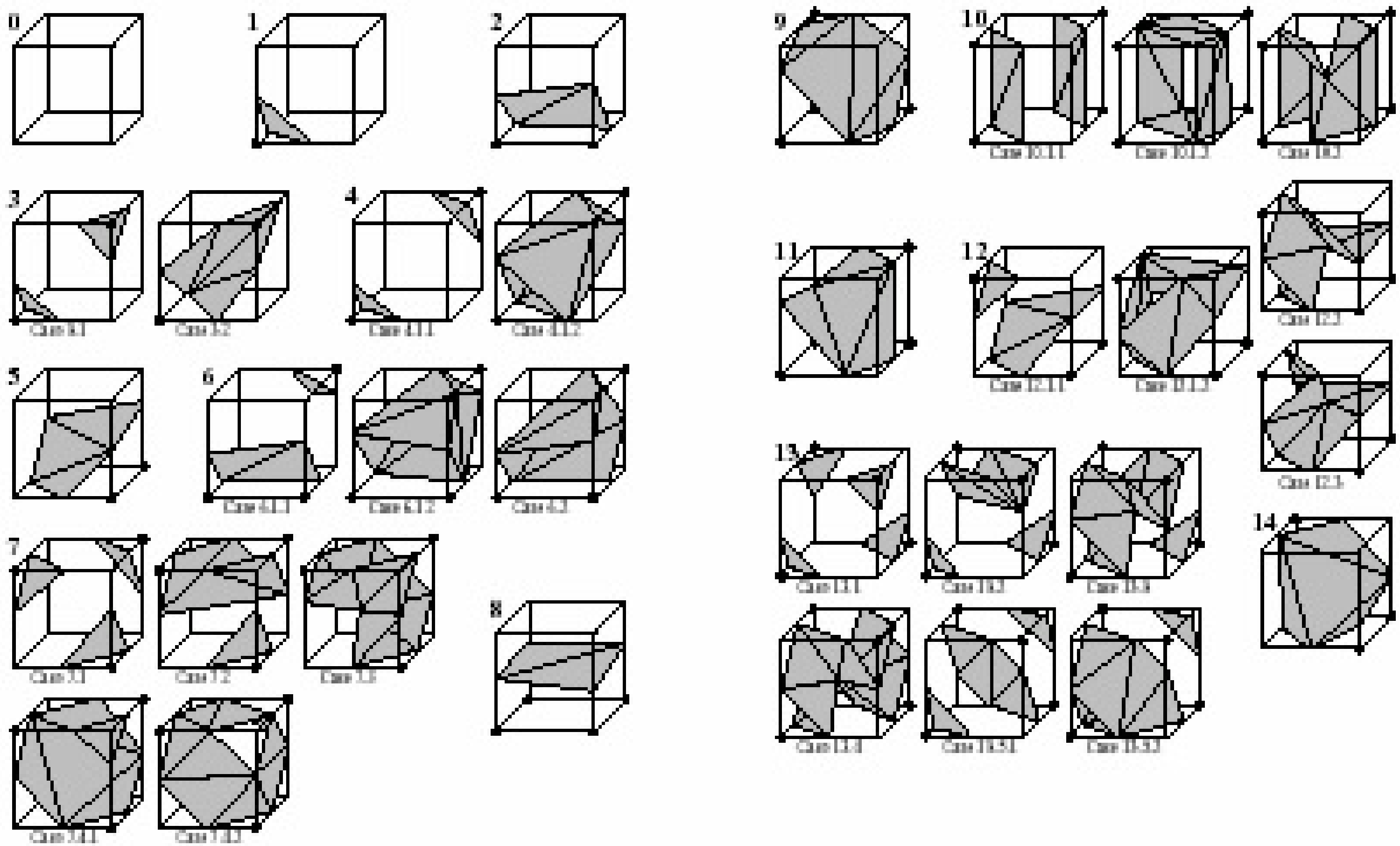
- Bilinear contours



$$F(s_\alpha, t_\alpha) = \frac{AC - BD}{A + C - B - D}$$

Resolving ambiguities

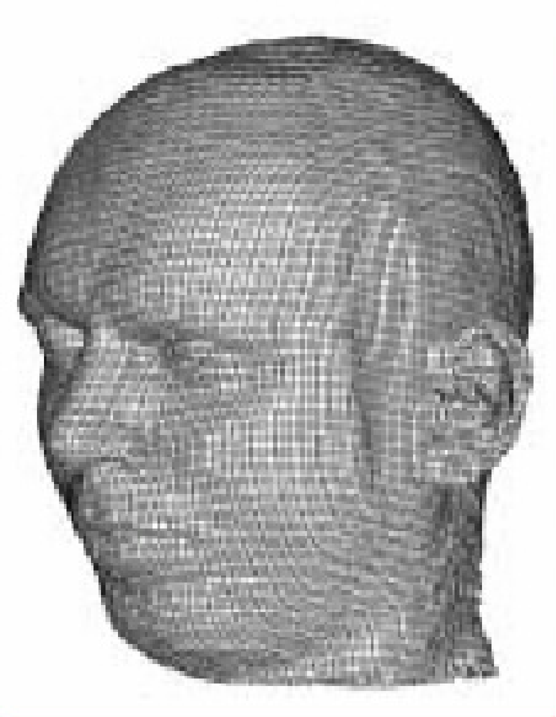
- Marching cubes 33 by Evgeni Chernyaev



Model simplification

- Example:

samples	triangles
032x032x016	~3,000
064x064x032	~18,000
128x128x064	~100,000
256x256x128	~820,000
512x512x128	…

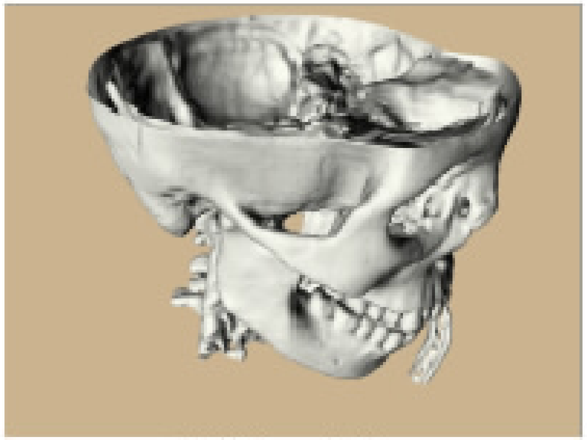


Model simplification

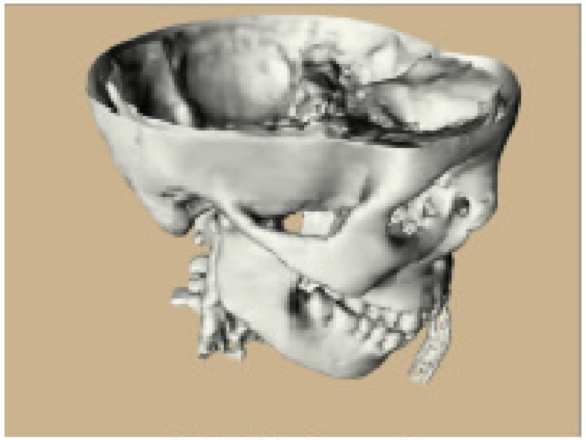
- Splitting box algorithm

Model simplification

- Mesh simplification algorithms



Full Resolution
(569K Gouraud shaded triangles)



75% decimated
(142K Gouraud shaded triangles)

References

- Marching Cubes: A High Resolution 3D Surface Construction Algorithm / William E. Lorensen, Harvey E.Cline – SIG '87
- Two Algorithms for the Tree-Dimensional Reconstruction of Tomograms / H. E. Cline, W. E. Lorenson, S. Ludke, C. R. Crawford, B.C. Teeter - Medical Physics, '88, pp 320-327.
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- An Evaluation of Implicit Surface Tilers / P. Ning, J. Bloomenthal – Computer Graphics and Applications November'93, 13(6):33-41