

2017~2018 Mid-Term Quality Inspection of the First Semester in Hefei City, Anhui Province

Examination time: 120 minutes Full score: 150 points

Name: _____ Grade: _____

One, multiple choice (10 questions, 4 points per question, full score 40 points)

1 · -2017 has a reciprocal of ()

A · 2017 B · $\frac{1}{2017}$ C · $-\frac{1}{2017}$ D · ± 2017

2 · 482.2 billion written in scientific notation is ()

A · 4822×10^8 B · 4.822×10^{11} C · 48.22×10^{10} D · 0.4822×10^{12}

3 · Of the following groups of numbers, the equal group is ()

A · 2^3 and 3^2 B · 2^3 and $(-2)^3$ C · 3^2 and $(-3)^2$ D · -2^3 and -3^2

4 · In the following equation variants, which of the following is invalid? ()

A · If $a=b$, then $a+5=b+5$ B · If $a=b$, then $\frac{a}{-3} = \frac{b}{3}$

C · If $x+2=y+2$, then $x=y$ D · If $-3x=-3y$, then $x=y$

5 · Out of the following: abc · $2\pi R$ · $x+3y$ · 0 · $\frac{x-y}{2}$ · how many monomials are there? ()

A. 2 B. 3 C. 4 D. 5

6 · Which of the following equations has $x = \frac{5}{4}$ as a solution? ()

A. $-6x+2=1$ B. $-3x+4=3$ C. $\frac{2}{3}x+1=\frac{1}{3}x-2$ D. $2x+3=\frac{11}{2}$

7 · Given that $|a|=5$ · $b^3=-27$ · and $a > b$ · then the value of $a-b$ would be ()

A · 2 B · -2 or 8 C · 8 D · -2

8 · x , y are two rational numbers. Express “The reciprocal of the square of the sum of x and y ” using an algebraic expression. ()

A. $\frac{1}{x+y}$ B. $\frac{1}{x^2+y^2}$ C. $\frac{1}{(x+y)^2}$ D. None of the above

9 · If A is a three-degreed polynomial, and B is also a three-degreed polynomial, then $A + B$ has to be ()

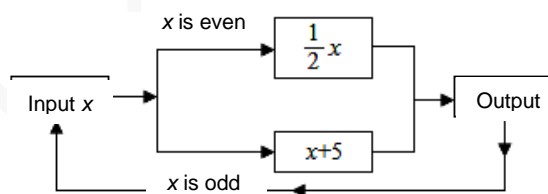
A. a six-degreed polynomial

B. a four-degreed polynomial

C. a polynomial (or monomial) that has a degree of no more than 3 D. three-degreed polynomial

10 · There is a machine that conducts certain input values into output values according to a certain function. If the input x value is 7, then the first output would be 12. Its second output would be 6. What would be the 2016th output? ()

A · 3 B · 8 C · 4 D · 2



Two, fill in the blanks (5 points per question, full score 20 points)

11 · The highest temperature today was 2°C, and the lowest was -8°C. Therefore, the difference between the highest and lowest temperatures today is ____°C.

12 · If $5a^3b^n - 8a^mb^2 = -3a^3b^2$, then $m = \underline{\hspace{1cm}}$ · $n = \underline{\hspace{1cm}}$ ·

13 · If a whole expression added with $x^2 - 2y^2$ equals $x^2 + y^2$, then this whole expression is ____.

14 · As shown below, the following figures are all made of identical sun-shaped icons according to certain patterns: the first pattern requires 2 suns, the second figure requires 4 suns, and the third figure requires 7 suns, ..., according to this pattern, the number of suns required for the fifth pattern is_____.



Three, (8 points per question, full score 16 points)

15. Calculate

(1) $-3 \times 2^3 - (-3 \times 2)^2 + 48 \div (-4)$

(2) $(\frac{2}{9} - \frac{1}{4} + \frac{1}{18}) \div (-\frac{1}{36})$

16 · Plot the following numbers on a number line, and then order them from greatest to least using the > symbol.

$-3.5 \cdot 0 \cdot 2 \cdot \frac{2}{3} \cdot -2\frac{1}{3} \cdot 0.75 \cdot -1 \cdot$

Four, (8 points per question, full score 16 points)

17 · When $x=1$, ax^3+bx+4 has a value of 0. Find when the value of ax^3+bx+4 when $x=-1$.

18 · Given that $A=3a^2b+3ab^2+b^4$ · $B=a^2b+11ab^2+a^4$ · find $2A-B$ ·

Five, (2 questions, 10 points per question, full score 20 points)

19 · First simplify, then substitute: $3x^2-[7x-(4x-2x^2)]$; Among them $x=-2$.

20 · Kangbin bought 10 pens with 50 dollars and planned to sell them at a certain price. If the standard price of each pen is 6 dollars, the excess is recorded as a positive number, and the deficiency is recorded as a negative number, and the records are as follows: 0.5, 0.7 , -1, -1.5, 0.8, 1, -1.5, -2.1, 9, 0.9.

(1) What are the highest and lowest prices of these 10 pens?

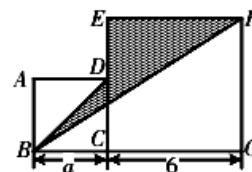
(2) Does Kangbin make a profit or a loss after selling the pen?

Six, (this question is worth **12 points**)

21 · The quadrilaterals ABCD and ECGF are both squares.

(1) Use algebra to express the area of the shaded part; (the result requires simplification)

(2) When $a=4$, find the shaded area.



Seven, (this question is worth **14 points**)

22 · Observe and think:

(1) When $a=3$ and $b=-1$, $a^2-b^2=$ ____ ; $(a+b)(a-b) =$ ____ ;

When $a=-5$ and $b=3$, $a^2-b^2=$ ____ ; $(a+b)(a-b) =$ ____ ;

(2) Think: Choose a set of values and substitute them into the calculation. What is the relationship between these two algebraic expressions?

(3) Based on your answers to the previous questions, can you quickly and accurately find the value of a^2-b^2 when $a=2016$ and $b=2017$?

Eight, (this question is worth 12 points)

23 · In order to find the value of $2+2^2+2^3\ldots+2^{n-1}+2^n$, Karen wrote down the following solution:

Let: $S=2+2^2+2^3\ldots+2^{n-1}+2^n$ ①

Multiply both sides by two to get: $2S=2^2+2^3\ldots+2^{n-1}+2^n+2^{n+1}$ ②

Subtract ②-①: $S=2^{n+1}-2$

(1) Conclusion: $2+2^2+2^3\ldots+2^{100}=\underline{\hspace{2cm}}$;

(2) Find the value of $4+4^2+4^3\ldots+4^{n-1}+4^n$;

(3) Karen designed a geometric figure as shown in the figure to represent the value of $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots + \frac{1}{2^n}$. Just like in Figure 1, the side length of the square in Figure 2 is also 1.

Split up the square in Figure 2 so that it models the expression $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots + \frac{1}{2^n}$ geometrically.

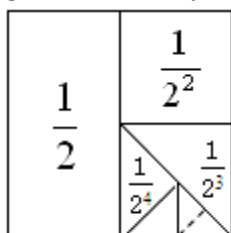


Figure 1

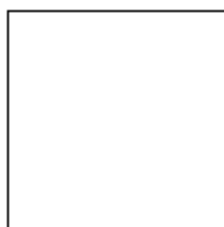


Figure 2

Answers and Solutions

One, multiple choice (10 questions, 4 points per question, full score 40 points)

C, B, C, B, B, D, C, B, C, D

Two, fill in the blanks (5 points per question, full score 20 points)

11 · The highest temperature today was 2°C , and the lowest was -8°C . Therefore, the difference between the highest and lowest temperatures today is 10 $^{\circ}\text{C}$.

12 · If $5a^3b^n - 8a^mb^2 = -3a^3b^2$, then $m = \underline{3}$ · $n = \underline{2}$ ·

13 · If a whole expression added with $x^2 - 2y^2$ equals $x^2 + y^2$, then this whole expression is $3y^2$.

14 · As shown below, the following figures are all made of identical sun-shaped icons according to certain patterns: the first pattern requires 2 suns, the second figure requires 4 suns, and the third figure requires 7 suns, ..., according to this pattern, the number of suns required for the fifth pattern is 21.



Three, (8 points per question, full score 16 points)

15. Calculate

$$(1) -3 \times 2^3 - (-3 \times 2)^2 + 48 \div (-4)$$

$$\begin{aligned} \text{S: Original Equation} &= -3 \times 8 - (-6)^2 - 12 \\ &= -24 - 36 - 12 \\ &= -72 \end{aligned}$$

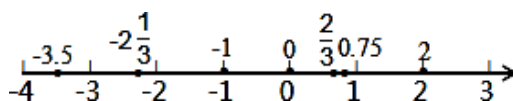
$$(2) \left(\frac{2}{9} - \frac{1}{4} + \frac{1}{18} \right) \div \left(-\frac{1}{36} \right)$$

$$\begin{aligned} \text{S: Original Equation} &= \left(\frac{2}{9} - \frac{1}{4} + \frac{1}{18} \right) \times (-36) \\ &= -8 + 9 - 2 \\ &= -1 \end{aligned}$$

16 · Plot the following numbers on a number line, and then order them from greatest to least using the $>$ symbol.

$$-3.5 \cdot 0 \cdot 2 \cdot \frac{2}{3} \cdot -2\frac{1}{3} \cdot 0.75 \cdot -1 \cdot$$

S: As shown:



Using " $>$ " to order: $2 > 0.75 > \frac{2}{3} > 0 > -1 > -2\frac{1}{3} > -3.5$.

Four, (8 points per question, full score 16 points)

17 · When $x=1$, ax^3+bx+4 has a value of 0. Find when the value of ax^3+bx+4 when $x=-1$.

S: \because When $x=1$, $ax^3+bx+4=0$.

\therefore When $x=1$, $ax^3+bx=-4$.

\therefore When $x=-1$ 时, $ax^3+bx=4$.

$\therefore ax^3+bx+4=4+4=8$.

18 · Given that $A=3a^2b+3ab^2+b^4$, $B=a^2b+11ab^2+a^4$, find $2A-B$.

$$\begin{aligned} S: 2A - B &= 2(3a^2b+3ab^2+b^4) - (a^2b+11ab^2+a^4) \\ &= 6a^2b+6ab^2+2b^4-a^2b-11ab^2-a^4 \\ &= +5a^2b-5ab^2+2b^4-a^4. \end{aligned}$$

Five, (2 questions, 10 points per question, full score 20 points)

19 · First simplify, then substitute: $3x^2-[7x-(4x-2x^2)]$; Among them $x=-2$.

S: Original equation $= 3x^2-(7x-4x+2x^2)$

$$= 3x^2-7x+4x-2x^2$$

$$= x^2-3x$$

When $x=-2$,

$$\text{Original equation} = (-2)^2-3 \times (-2) = 4-(-6) = 10$$

20 · Kangbin bought 10 pens with 50 dollars and planned to sell them at a certain price. If the standard price of each pen is 6 dollars, the excess is recorded as a positive number, and the

deficiency is recorded as a negative number, and the records are as follows: 0.5, 0.7, -1, -1.5, 0.8, 1, -1.5, -2.1, 9, 0.9.

- (1) What are the highest and lowest prices of these 10 pens?
- (2) Does Kangbin make a profit or a loss after selling the pens?

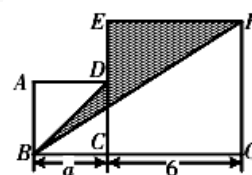
S: (1) The highest price is $6 + 1.9 = 7.9$ dollars, and the lowest price is $6 + (-2) = 4$ dollars;
 (2) $(6 + 0.5) + (6 + 0.7) + (6 - 1) + (6 - 1.5) + (6 + 0.8) + (6 + 1) + (6 - 1.5) + (6 - 2) + (6 + 1.9) + (6 + 0.9)$
 $= 59.8 > 50$.
 \therefore Kangbin did make a profit after selling the pens.

Six, (this question is worth 12 points)

21 · The quadrilaterals ABCD and ECFG are both squares.

(1) Use algebra to express the area of the shaded part; (the result requires simplification)

(2) When $a = 4$, find the shaded area.



S: As shown, we can set the equation: $S_{\text{shaded}} = S_{ABCD} + S_{CEFG} - S_{\triangle ABD} - S_{\triangle BGF}$.

\therefore The side length of ABCD is a , and the side length of ECFG is 6 .

$$\therefore S_{ABCD} = a^2, S_{CEFG} = 6^2, S_{\triangle ABD} = \frac{1}{2} a^2, S_{\triangle BGF} = \frac{1}{2} \times (a + 6) \times 6.$$

$$\therefore S_{\text{shaded}} = a^2 + 6^2 - \frac{1}{2} a^2 - \frac{1}{2} \times (a + 6) \times 6 = \frac{1}{2} a^2 - 3a + 18.$$

$$(2) \text{ When } a = 4, S_{\text{shaded}} = \frac{1}{2} \times 4^2 - 3 \times 4 + 18 = 14.$$

Seven, (this question is worth 14 points)

22 · Observe and think:

(1) When $a = 3$ and $b = -1$, $a^2 - b^2 = \underline{8}$; $(a + b)(a - b) = \underline{8}$;

When $a = -5$ and $b = 3$, $a^2 - b^2 = \underline{16}$; $(a + b)(a - b) = \underline{16}$;

(2) Think: Choose a set of values and substitute them into the calculation. What is the relationship between these two algebraic expressions?

(3) Based on your answers to the previous questions, can you quickly and accurately find the value of a^2-b^2 when $a=2016$ and $b=2017$?

S: (2) When $a=3$ and $b=2$, $a^2-b^2=3^2-2^2=9-4=5$
 $(a+b)(a-b)=(3+2) \times (3-2)=1 \times 5=5$

(3) $a=2016$, $b=2017$,
 $a^2-b^2=(a+b)(a-b)=(2016+2017) \times (2016-2017)=4033 \times (-1)=-4033$

Eight, (this question is worth 12 points)

23 · In order to find the value of $2+2^2+2^3 \dots +2^{n-1}+2^n$, Karen wrote down the following solution:

Let: $S=2+2^2+2^3 \dots +2^{n-1}+2^n$ ①

Multiply both sides by two to get: $2S=2^2+2^3 \dots +2^{n-1}+2^n+2^{n+1}$ ②

Subtract ②-①: $S=2^{n+1}-2$

(1) Conclusion: $2+2^2+2^3 \dots +2^{100}=\underline{\hspace{2cm}}$;

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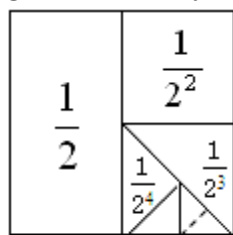


Figure 1

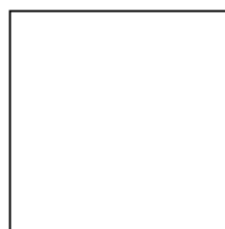


Figure 2

S: (1) Set $S=1+2+2^2+2^3+\dots+2^{100}$ ①

Then $2S=2+2^2+2^3+\dots+2^{100}+2^{101}$, ②

②-① gets us $S=2^{101}-1$.

(2) Set $S = 4 + 4^2 + 4^3 \dots + 4^{n-1} + 4^n$; ①

Therefore $4S = 4^2 + 4^3 \dots + 4^{n-1} + 4^n + 4^{n+1}$; ②

② - ① gives us $3S = 4^{n+1} - 4$,

Therefore $S = \frac{4^{n+1} - 4}{3}$

(3) As shown:

