

## 2017-2018 School Year Shanghai Huangpu District 7th Grade (I) Mid-term Mathematics Exam

One, multiple choice (Total 4 questions, 3 points per question, full score 12 points)

1. (3 points) Among the four expressions  $x^2y$ ,  $-\frac{1}{3}$ ,  $\frac{2x+3}{4}$ ,  $\frac{3}{n}$ , how many monomials are there?

( )

A. 1    B. 2    C. 3    D. 4

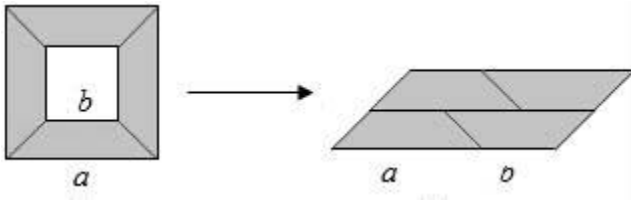
2. (3 points) The correct simplification is ( )

A.  $x^2 + x^3 = x^5$     B.  $(-a^3) \cdot a^3 = a^6$     C.  $(-x^3)^2 = x^6$     D.  $4a^2 - (2a)^2 = 2a^2$

3. (3 points) If the ones digit of a two-digit number is  $a$ , and the tens digit is  $b$ , then the value of this two-digit number is ( )

A.  $ba$     B.  $10b + a$     C.  $10a + b$     D.  $10(a + b)$

4. (3 points) 从边长为  $a$  的大正方形纸板中挖去一个边长为  $b$  的小正方形纸板后，将其裁成四个相同的等腰梯形（如图甲），然后拼成一个平行四边形（如图乙），那么通过计算两个图形阴影部分的面积，可以验证成立的公式为 ( )



- A.  $a^2 - b^2 = (a - b)^2$       B.  $(a + b)^2 = a^2 + 2ab + b^2$   
C.  $(a - b)^2 = a^2 - 2ab + b^2$       D.  $a^2 - b^2 = (a + b)(a - b)$

Two, fill in the blanks (Total 14 questions, 2 points per question, full score 28 points)

5. (2 points) Use an expression to express the reciprocal of the sum of x and y. \_\_\_\_\_
6. (2 points) Monomial  $-\frac{4a^3bc^2}{3}$  has a coefficient of \_\_\_\_\_, and its highest power is \_\_\_\_\_.
7. (2 points) Polynomial  $2a^2 - 3a + 4$  is a \_\_\_\_\_次\_\_\_\_\_项式.
8. (2 points) Take polynomial  $32x^3y - \frac{4}{5}y^2 + \frac{1}{3}xy - 12x^2$  and use the power of x to reorder: \_\_\_\_\_.
9. (2 points) If  $-2x^3y^m$  and  $3x^ny^2$  are like terms, then  $m + n =$  \_\_\_\_\_.
10. (2 points) Simplify:  $3a^2 - 6a^2 =$  \_\_\_\_\_.
11. (2 points) When  $x = -2$ , algebraic expression  $x^2 + 2x + 1$  equals \_\_\_\_\_.
12. (2 points) Simplify:  $(a - b) \cdot (b - a)^2 =$  \_\_\_\_\_ (Write answer in exponential form).
13. (2 points) Simplify:  $(-2x^2y) \cdot (-3x^2y^3) =$  \_\_\_\_\_.

14. (2 points) Use the scientific notation to write the simplified answer of  $(2 \times 10^9) \times (8 \times 10^3)$  \_\_\_\_\_.

15. (2 points) Simplify  $(\frac{2}{3})^{2016} \times (-\frac{3}{2})^{2017} =$  \_\_\_\_\_.

16. (2 points) Given that  $x - y = 2$ ,  $xy = 3$ , then  $x^2 + y^2$  equals \_\_\_\_\_.

17. (2 points) If  $2^m = 5$ ,  $2^n = 3$ , then  $2^{m+2n} =$  \_\_\_\_\_.

18. (2 points) If algebraic expression  $4y^2 - 2y + 5 = 7$ , then algebraic expression  $2y^2 - y + 5 =$  \_\_\_\_\_.

Three, calculation questions (Total 6 questions, questions 19, 20 are each 5 points, all other questions 6 points, full score 34 points)

19. (5 points) Simplify:  $(3x^2 - 2x + 1) - (x^2 - x + 3)$

20. (5 points) Use a multiplication formula to calculate:  $99.8^2 =$  \_\_\_\_\_.

21. (6 points) Simplify:  $(-a)^2 \cdot (-a^3) \cdot (-a) + (-a^2)^3 - (-a^3)^2$ .

22. (6 points) Simplify:  $(-2xy^2)^2 \cdot (\frac{1}{4}y^2 - \frac{1}{2}x^2 - \frac{3}{2}xy)$ .

23. (6 points) Simplify:  $(2x - 3)(x + 4) - (x - 1)(x + 1)$

24. (6 points) Simplify:  $(2a - b + c)(2a - b - c)$ .

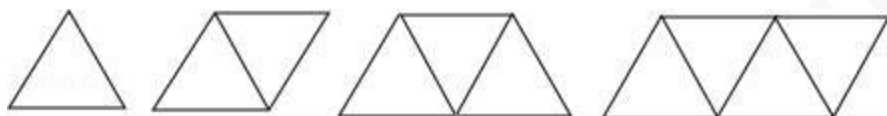
Four, short answer questions (total 4 questions, questions 25, 26 are 6 points each, all the rest are each 7 points, full score 26 points)

25. (6 points) First simplify and then calculate:  $(x - y)(y - x) - [x^2 - 2x(x + y)]$  .

$$x = \frac{1}{2}, y = -2$$

26. (6 points) Solve for x:  $2x(x + 1) - (3x - 2)x = 1 - x^2$  .

27. (7 points) Use 3 matches to create 1 triangle, then continue to create 2, 3, 4, ...



( 1 ) When there are 6 triangles, there are \_\_\_\_\_ matches.

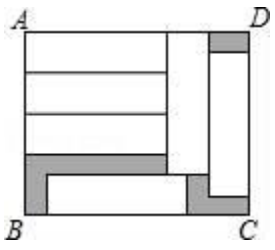
( 2 ) When there are n triangles, there are \_\_\_\_\_ matches.

( 3 ) If 2017 matches are used, then there are \_\_\_\_\_ triangles.

28. (7 points) As shown, in rectangle ABCD, there are 6 smaller congruent rectangles. It is given that the lengths and widths of the smaller rectangles are each a and b, and  $a > b$  .

( 1 ) Use the variables a 、 b to express the lengths of AD and AB ;

( 2 ) Use the variables a 、 b to express the area of the shaded parts.



## 2017-2018 School Year Shanghai Huangpu District 7th Grade (I) Mid-term Mathematics Exam

Answer Key

One, multiple choice (Total 4 questions, 3 points per question, full score 12 points)

1. (3 points) Among the expressions  $x^2y$ ,  $-\frac{1}{3}$ ,  $\frac{2x+3}{4}$ ,  $\frac{3}{n}$ , there are \_\_\_\_\_ monomials. ( )

A. 1    B. 2    C. 3    D. 4

【Solution】

Using the definition of monomials,

$\therefore$  Among the expressions  $x^2y$ ,  $-\frac{1}{3}$ ,  $\frac{2x+3}{4}$ ,  $\frac{3}{n}$ , there are  $x^2y$  and  $-\frac{1}{3}$  as monomials.

Correct answer: B

2. (3 points) The correct simplification is ( )

A.  $x^2+x^3=x^5$     B.  $(-a^3) \cdot a^3=a^6$     C.  $(-x^3)^2=x^6$     D.  $4a^2 - (2a)^2=2a^2$

【Solution】

A.  $x^2$  and  $x^3$  are not like terms, and cannot be combined; so, this option is incorrect;

B.  $(-a^3) \cdot a^3 = -a^{3+3} = -a^6$ ; so, this option is incorrect;

C、 $(-x^3)^2 = (-1)^2 \cdot (x^3)^2 = x^6$ ; so, this option is correct;

D、 $4a^2 - (2a)^2 = 4a^2 - 4a^2 = 0$ ; so, this option is incorrect.

Correct answer: C

3. (3 points) If the ones digit of a two-digit number is  $a$ , and the tens digit is  $b$ , then the value of this two-digit number is ( )

A.  $ba$  B.  $10b + a$  C.  $10a + b$  D.  $10(a + b)$

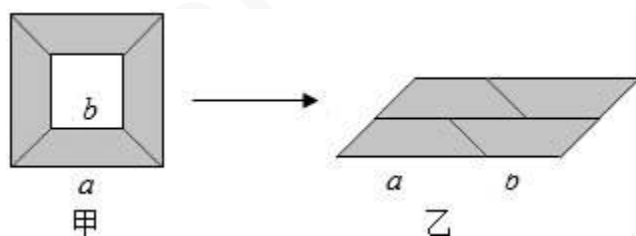
【Solution】

$\because$  the ones digit is  $a$ , and the tens digit is  $b$ ,

$\therefore$  this two-digit number can be expressed as  $10b + a$ .

Correct answer: B

4. (3 分) 从边长为  $a$  的大正方形纸板中挖去一个边长为  $b$  的小正方形纸板后，将其裁成四个相同的等腰梯形（如图甲），然后拼成一个平行四边形（如图乙）。那么通过计算两个图形阴影部分的面积，可以验证成立的公式为 ( )



A.  $a^2 - b^2 = (a - b)^2$  B.  $(a + b)^2 = a^2 + 2ab + b^2$

$$C \cdot (a - b)^2 = a^2 - 2ab + b^2 \quad D \cdot a^2 - b^2 = (a + b)(a - b)$$

【Solution】

由图 1 将小正方形一边向两方延长，得到两个梯形的高，两条高的和为  $a - b$ ，即平行四边形的高为  $a - b$ 。

∴两个图中的阴影部分的面积相等，即甲的面积  $= a^2 - b^2$ ，乙的面积  $= (a + b)(a - b)$ 。

$$\text{即：} a^2 - b^2 = (a + b)(a - b)。$$

所以验证成立的公式为： $a^2 - b^2 = (a + b)(a - b)$ 。

故选：D。

Two, fill in the blanks (Total 14 questions, 2 points per question, full score 28 points)

5. (2 points) the reciprocal of the sum of x and y can be expressed as  $\frac{1}{x+y}$ 。

【Solution】

According the question the answer is  $\frac{1}{x+y}$ 。

Correct answer:  $\frac{1}{x+y}$ 。

6. (2 points) Monomial  $-\frac{4a^3bc^2}{3}$  has a coefficient of  $-\frac{4}{3}$ ，次数是 6。

【Solution】



The coefficient is  $-\frac{4}{3}$ , 次数是 :  $2+1+3=6$  .

Correct answer:  $-\frac{4}{3}$  , 6 .

7 . ( 2 分 ) 多项式  $2a^2 - 3a + 4$  是  $a$  的 二 次 三 项式 .

【Solution】

多项式  $2a^2 - 3a + 4$  最高次项  $2a^2$  的次数为二 , 有三项 .

故答案为 : 二 , 三 .

8. (2 points) Take polynomial  $32x^3y - \frac{4}{5}y^2 + \frac{1}{3}xy - 12x^2$  and use the power of  $x$  to reorder:

$32x^3y - 12x^2 + \frac{1}{3}xy - \frac{4}{5}y^2$  .

【Solution】

Polynomial  $32x^3y - \frac{4}{5}y^2 + \frac{1}{3}xy - 12x^2$  ordered by the power of  $x$  is:  $32x^3y - 12x^2 + \frac{1}{3}xy - \frac{4}{5}y^2$  .

Correct answer:  $32x^3y - 12x^2 + \frac{1}{3}xy - \frac{4}{5}y^2$  .

9. (2 points) If  $-2x^3y^m$  and  $3x^n y^2$  are like terms, then  $m + n =$  5 .

【Solution】

$\because -2x^3y^m$  and  $3x^n y^2$  are like terms,

$\therefore n=3$  ,  $m=2$  .

$$\therefore m + n = 5$$

Correct answer: 5.

10. (2 points) Simplify:  $3a^2 - 6a^2 = \underline{\quad -3a^2 \quad}$ .

【Solution】

$$3a^2 - 6a^2 = -3a^2$$

Correct answer:  $-3a^2$ .

(2 points) When  $x = -2$ , algebraic expression  $x^2 + 2x + 1$  equals  $\underline{\quad 1 \quad}$ .

【Solution】

$$\text{Original Equation} = 4 - 4 + 1 = 1$$

Correct answer: 1

12. (2 points) Simplify:  $(a - b) \cdot (b - a)^2 = \underline{\quad (a - b)^3 \quad}$  (Write answer in exponential form).

【Solution】

$$(a - b) \cdot (b - a)^2 = (a - b) \cdot (a - b)^2 = (a - b)^3$$

Correct answer:  $(a - b)^3$ .

13. (2 points) Simplify :  $(-2x^2y) \cdot (-3x^2y^3) = \underline{6x^4y^4}$  .

【Solution】

$$(-2x^2y) \cdot (-3x^2y^3) = 6x^4y^4 .$$

Correct answer:  $6x^4y^4$  .

14. (2 points) Use the scientific notation to write the simplified answer of  $(2 \times 10^9) \times (8 \times 10^3)$

$1.6 \times 10^{13}$  .

【Solution】

$$(2 \times 10^9) \times (8 \times 10^3) = 1.6 \times 10^{13} .$$

Correct answer:  $1.6 \times 10^{13}$

15. (2 points) Simplify  $(\frac{2}{3})^{2016} \times (-\frac{3}{2})^{2017} = \underline{-\frac{3}{2}}$  .

【Solution】

$$\begin{aligned} & (\frac{2}{3})^{2016} \times (-\frac{3}{2})^{2017} \\ &= (\frac{2}{3})^{2016} \times (-\frac{3}{2})^{2016} \times (-\frac{3}{2}) \\ &= (\frac{2}{3} \times \frac{3}{2})^{2016} \times (-\frac{3}{2}) \\ &= -\frac{3}{2} . \end{aligned}$$

Correct answer:  $-\frac{3}{2}$  .

16. (2 points) Given that  $x - y = 2$ ,  $xy=3$ , then  $x^2+y^2$  equals 10 .

【Solution】

$$x^2+y^2= (x - y )^2+2xy .$$

Substituting  $x - y = 2$  and  $xy = 3$ , we get:  $(x - y )^2+2xy=4+6=10$  .

Simplified:  $x^2+y^2=10$  .

Correct answer:10

17. (2 points) If  $2^m=5$ ,  $2^n=3$ , then  $2^{m+2n}=$  45 .

【Solution】

$$2^{m+2n}=2^m \cdot 2^{2n}=5 \times 9=45 .$$

Correct answer 45

18. (2 points) If algebraic expression  $4y^2 - 2y+5 = 7$ , then algebraic expression  $2y^2 - y+5 =$  6 .

【Solution】

$$\because 4y^2 - 2y+5=7 .$$

$$4y^2 - 2y=2 .$$

$$\therefore 2y^2 - y = 1$$

Then original equation =  $1 + 5 = 6$ ,

Correct answer: 6

Three, calculation questions (Total 6 questions, questions 19, 20 are each 5 points, all other questions 6 points, full score 34 points)

19. (5 points) Simplify:  $(3x^2 - 2x + 1) - (x^2 - x + 3)$

【Solution】

$$\text{Original Equation} = 3x^2 - 2x + 1 - x^2 + x - 3$$

$$= 2x^2 - x - 2$$

20. (5 points) Use a multiplication formula to calculate:  $99.8^2 = \underline{\hspace{2cm}}$ .

【Solution】

$$99.8^2$$

$$= (100 - 0.2)^2$$

$$= 100^2 - 2 \times 100 \times 0.2 + 0.2^2$$

$$= 9960.04$$

21. (6 points) Simplify:  $(-a)^2 \cdot (-a^3) \cdot (-a) + (-a^2)^3 - (-a^3)^2$ .

【Solution】

$$\text{Original Equation} = -a^2 \cdot (-a^3) \cdot (-a) + (-a^6) - a^6$$

$$= a^6 - a^6 - a^6$$

$$= -a^6$$

22. (6 points) Simplify:  $(-2xy^2)^2 \cdot (\frac{1}{4}y^2 - \frac{1}{2}x^2 - \frac{3}{2}xy)$ .

【Solution】

$$\text{Original equation} = 4x^2y^4 (\frac{1}{4}y^2 - \frac{1}{2}x^2 - \frac{3}{2}xy)$$

$$= x^2y^6 - 2x^4y^4 - 6x^3y^5$$

23. (6 points) Simplify:  $(2x - 3)(x + 4) - (x - 1)(x + 1)$

【Solution】

$$\text{Original Equation} = 2x^2 + 8x - 3x - 12 - (x^2 - 1)$$

$$= 2x^2 + 8x - 3x - 12 - x^2 + 1$$

$$= x^2 + 5x - 11$$

24. (6 points) Simplify:  $(2a - b + c)(2a - b - c)$ .

【Solution】

Original equation =  $[(2a - b) + c][(2a - b) - c]$ .

=  $(2a - b)^2 - c^2$ .

=  $4a^2 - 4ab + b^2 - c^2$ .

Four, short answer questions (total 4 questions, questions 25, 26 are 6 points each, all the rest are each 7 points, full score 26 points)

25. (6 points) First simplify and then calculate:  $(x - y)(y - x) - [x^2 - 2x(x + y)]$ .

$x = \frac{1}{2}$ ,  $y = -2$ .

【Solution】

$(x - y)(y - x) - [x^2 - 2x(x + y)]$

=  $-x^2 + 2xy - y^2 - x^2 + 2x^2 + 2xy$

=  $4xy - y^2$ .

When  $x = \frac{1}{2}$ ,  $y = -2$ , original equation =  $4 \times \frac{1}{2} \times (-2) - (-2)^2 = -4 - 4 = -8$ .

26. (6 points) Solve for x:  $2x(x + 1) - (3x - 2)x = 1 - x^2$ .

【Solution】

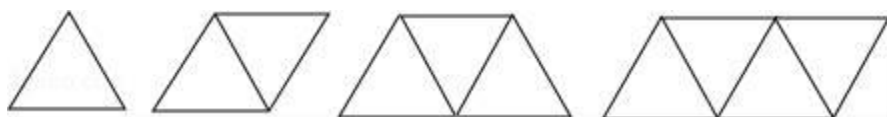
$$2x(x+1) - (3x-2)x = 1 - x^2$$

Removing the brackets:  $2x^2+2x - 3x^2+2x = 1 - x^2$

Simplified:  $4x = 1$

Correct answer:  $x = \frac{1}{4}$

27. (7 points) Use 3 matches to create 1 triangle, then continue to create 2, 3, 4, ...



( 1 ) When there are 6 triangles, there are 13 matches.

( 2 ) When there are  $n$  triangles, there are  $2n+1$  matches.

( 3 ) If 2017 matches are used, then there are 1008 triangles.

### 【Solution】

( 1 ) From the pattern:

When there is 1 triangle, 3 matchsticks are needed;

When there are 2 triangles, 5 matchsticks are needed;

When there are 3 triangles, 7 matchsticks are needed;

When there are 4 triangles, 9 matchsticks are needed;



When there are 5 triangles, 11 matchsticks are needed;

When there are 6 triangles, 13 matchsticks are needed;

...

The pattern is  $3 + 2(n - 1) = 2n + 1$ .

( 2 ) When there are  $n$  triangles,  $3 + 2(n - 1) = 2n + 1$  matchsticks are needed.

( 3 )  $2n + 1 = 2017$ .

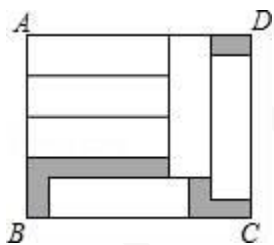
$\therefore n = 1008$

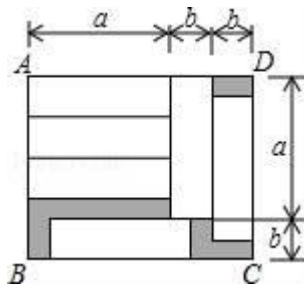
Correct answer:  $9 \cdot 2n + 1 \cdot 1008$ .

28. (7 points) As shown, in rectangle ABCD, there are 6 smaller congruent rectangles. It is given that the lengths and widths of the smaller rectangles are each  $a$  and  $b$ , and  $a > b$ .

( 1 ) Use the variables  $a$  and  $b$  to express the lengths of AD and AB ;

( 2 ) Use the variables  $a$  and  $b$  to express the area of the shaded parts.





【Solution】

( 1 ) From the picture:  $AD=a+2b$  ·  $AB=a+b$  ;

( 2 )  $S_{\text{shaded portions}} = (a+b)(a+2b) - 6ab$

$$=a^2+2ab+ab+2b^2 - 6ab$$

$$=a^2 - 3ab+2b^2 .$$