학번: 2018111373 이름: 최인렬

<Assignment 2>

$\{\epsilon_t\}$가 백색잡음과정 $WN(0, \sigma_\epsilon^2)$을 따를 때, 다음과 같은 확률과정 $\{Z_t\}$를 생각해 보자.

$$Z_t = \mu + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2}, \qquad t = 1, 2, \ldots$$

확률과정 $\{Z_t\}$에 대하여 $k = 0,1,2,3$일 때 γ_k를 구하시오.

$$Z_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$$

$$\varepsilon_t \sim WN(0, \sigma^2)$$

$$\gamma_0 = Var(Z_t) = Var(\mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2})$$

$$= Var(\varepsilon_t) + \theta_1^2 Var(\varepsilon_{t-1}) + \theta_2^2 Var(\varepsilon_{t-2})$$

$$= \sigma^2 (1 + \theta_1^2 + \theta_2^2)$$

$$\gamma_1 = Cov(Z_t, Z_{t-1}) = E[(Z_t - \mu)(Z_{t-1} - \mu)]$$

$$(Z_t - \mu)(Z_{t-1} - \mu) = (\varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2})(\varepsilon_{t-1} + \theta_1 \varepsilon_{t-2} + \theta_2 \varepsilon_{t-3})$$

$$= \varepsilon_t \varepsilon_{t-1} + \varepsilon_t \theta_1 \varepsilon_{t-2} + \varepsilon_t \theta_1 \varepsilon_{t-3} +$$
$$\theta_1 \varepsilon_{t-1} \varepsilon_{t-1} + \theta_1 \varepsilon_{t-1} \theta_1 \varepsilon_{t-2} + \theta_1 \varepsilon_{t-1} \theta_1 \varepsilon_{t-3} +$$
$$\theta_2 \varepsilon_{t-2} \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} \theta_1 \varepsilon_{t-2} + \theta_2 \varepsilon_{t-2} \theta_1 \varepsilon_{t-3}$$

백색잡음 과정의 성질을 보면, $E(Z_t) = 0$, $Cov(Z_t, Z_{t+k}) = 0$

i) $E(Z_t) = E(\varepsilon_t) = 0$

$$Cov(Z_t, Z_{t+k}) = Cov(\varepsilon_t, \varepsilon_{t+k}) = 0$$

$$= E(\varepsilon_t \varepsilon_{t+k}) - E(\varepsilon_t) E(\varepsilon_{t+k})$$

$$E(Z_t, Z_{t+k}) = E(\varepsilon_t, \varepsilon_{t+k}) = 0$$

ii) $Var(Z_t) = E(Z_t^2) - E(Z_t)^2 = E(Z_t^2) - 0$

$$\therefore E(Z_t^2) = Var(Z_t)$$

$$E[(Z_t-\mu)(Z_{t-1}-\mu)] = E[\theta_1 \varepsilon_{t-1}\varepsilon_{t-1}] + E[\theta_1\theta_2 \varepsilon_{t-2}\varepsilon_{t-2}]$$

$$= \theta_1 E(\varepsilon_{t-1}^2) + \theta_1\theta_2 E(\varepsilon_{t-2}^2) = \theta_1 \sigma^2 (1+\theta_2)$$

* $\varepsilon_{t+a}\varepsilon_{t+b}$ 인 항은 어차피 기댓값이 0 이므로 계산에서 제외.

$$\therefore \gamma_1 = \theta_1 \sigma^2 (1+\theta_2)$$

$$\gamma_2 = Cov(Z_t, Z_{t-2}) = E[(Z_t-\mu)(Z_{t-2}-\mu)]$$

$$(Z_t-\mu)(Z_{t-2}-\mu) = (\varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2})(\varepsilon_{t-2} + \theta_1\varepsilon_{t-3} + \theta_2\varepsilon_{t-4})$$

$$\begin{aligned} = \; & \varepsilon_t\varepsilon_{t-2} + \varepsilon_t\theta_1\varepsilon_{t-3} + \varepsilon_t\theta_2\varepsilon_{t-4} + \\ & \theta_1\varepsilon_{t-1}\varepsilon_{t-2} + \theta_1\varepsilon_{t-1}\theta_1\varepsilon_{t-3} + \theta_1\varepsilon_{t-1}\theta_2\varepsilon_{t-4} + \\ & \theta_2\varepsilon_{t-2}\varepsilon_{t-2} + \theta_2\varepsilon_{t-2}\theta_1\varepsilon_{t-3} + \theta_2\varepsilon_{t-2}\theta_2\varepsilon_{t-4} \end{aligned}$$

$$\therefore \gamma_2 = E[\theta_2 \varepsilon_{t-2}^2] = \theta_2\sigma^2$$

$$\gamma_3 = Cov(Z_t, Z_{t-3}) = E[(Z_t-\mu)(Z_{t-3}-\mu)]$$

$$(Z_t-\mu)(Z_{t-3}-\mu) = (\varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2})(\varepsilon_{t-3} + \theta_1\varepsilon_{t-4} + \theta_2\varepsilon_{t-5})$$

$$\begin{aligned} = \; & \varepsilon_t\varepsilon_{t-3} + \varepsilon_t\theta_1\varepsilon_{t-4} + \varepsilon_t\theta_2\varepsilon_{t-5} + \\ & \theta_1\varepsilon_{t-1}\varepsilon_{t-3} + \theta_1\varepsilon_{t-1}\theta_1\varepsilon_{t-4} + \theta_1\varepsilon_{t-1}\theta_2\varepsilon_{t-5} + \\ & \theta_2\varepsilon_{t-2}\varepsilon_{t-3} + \theta_2\varepsilon_{t-2}\theta_1\varepsilon_{t-4} + \theta_2\varepsilon_{t-2}\theta_2\varepsilon_{t-5} \end{aligned}$$

$$\therefore \gamma_3 = 0.$$

$$\begin{aligned} \therefore\; \gamma_0 &= \sigma^2(1+\theta_1^2+\theta_2^2) \\ \gamma_1 &= \theta_1\sigma^2(1+\theta_2) \\ \gamma_2 &= \theta_2\sigma^2 \\ \gamma_3 &= 0 \end{aligned}$$