

A hand is shown placing a blue L-shaped block onto a colorful geometric structure made of various blocks. The structure is composed of blocks in shades of blue, red, yellow, green, and purple, arranged in a complex, stepped pattern. The background is a solid light blue, and the surface the blocks are on is a light-colored wooden table. Several other L-shaped blocks in different colors (green, blue, red, yellow) are scattered on the table in the foreground.

EMTH403

Mathematical Foundation
for Computer Science

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Lecture Outcomes



After this lecture, you will be able to

- understand what are Planar Graphs
- understand what is Euler's Formulae and how $K_{3,3}$ is a non-planar Graph.

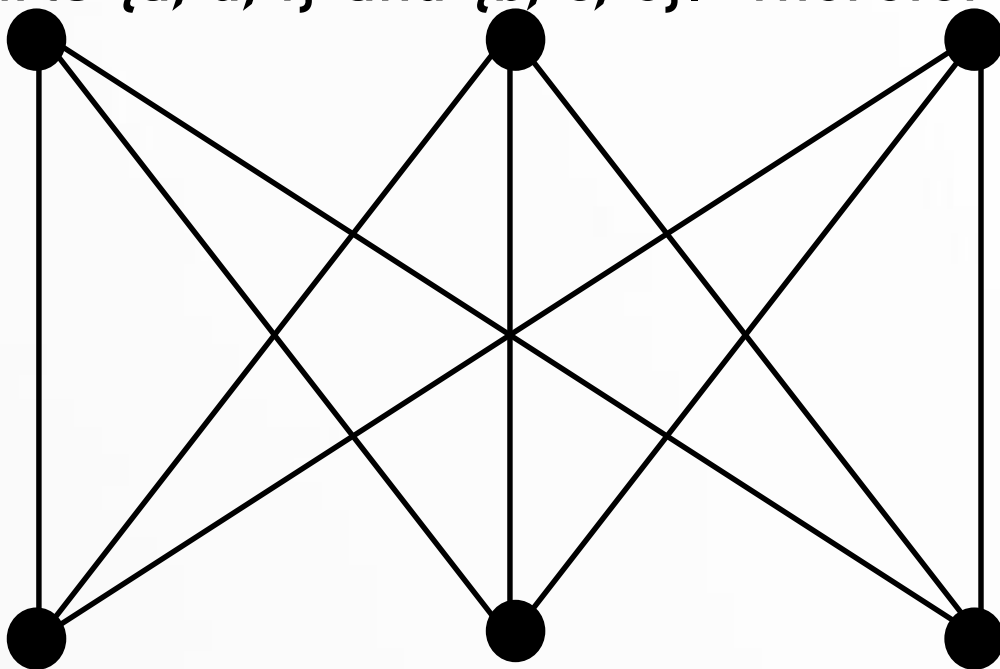
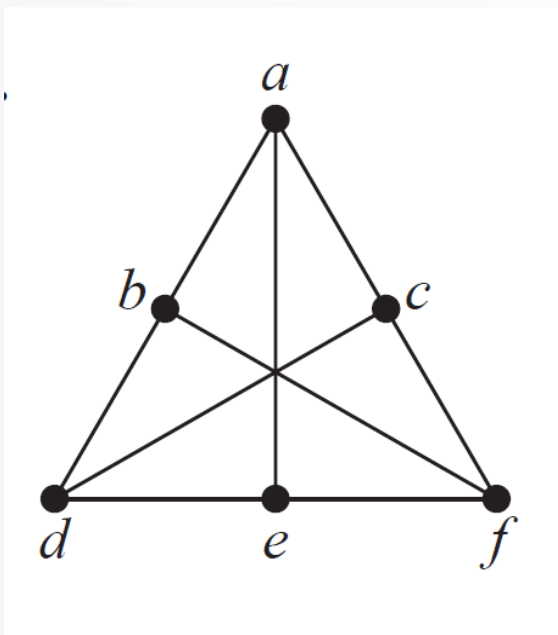
Theorem 1 (Kuratowski, 1930)

A graph is planar if and only if it does not contain a subgraph that is a K_5 or $K_{3,3}$ configuration.

Planar Graphs

Determine whether the given graph is planar. If so, draw it so that no edges cross.

- This is $K_{3,3}$, with parts $\{a, d, f\}$ and $\{b, c, e\}$. Therefore it

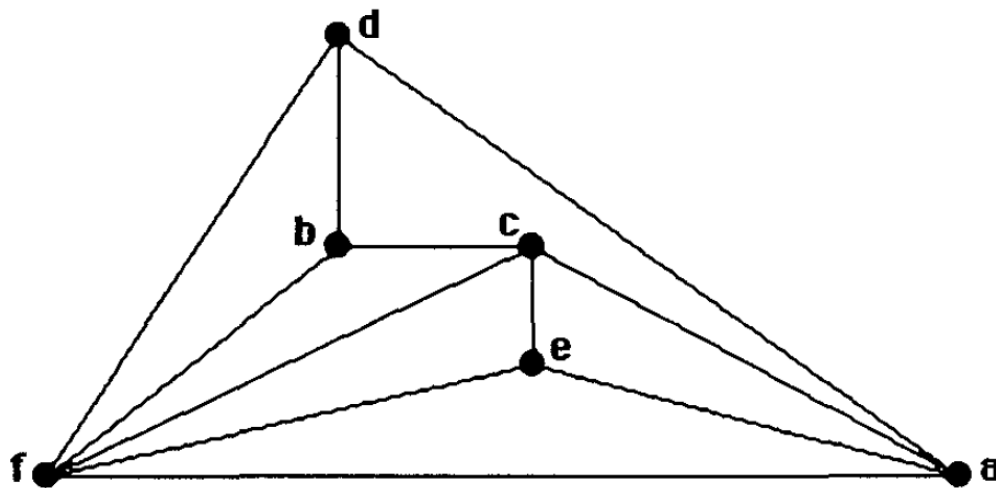
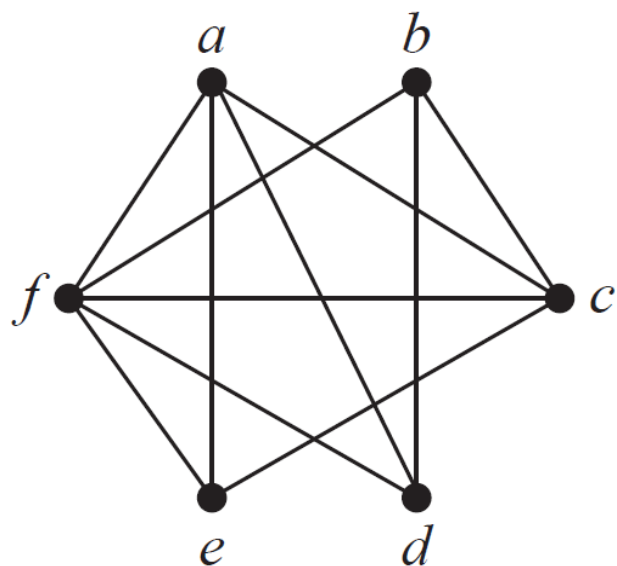


$K_{3,3}$

Planar Graphs

Ques:- Determine whether the given graph is planar. If so, draw it so that no edges cross.

Sol:- This graph can be untangled if we play with it long enough. The following picture gives a planar representation



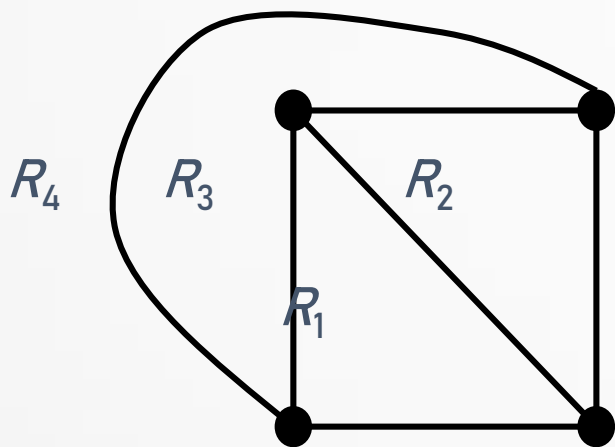
Euler's Formula

- Euler devised a formula for expressing the relationship between the number of vertices, edges, and regions of a planar graph.
- These may help us determine if a graph can be planar or not.

Euler's Formula

Let G be a connected planar simple graph with e edges and v vertices. Let r be the number of regions in a planar representation of G . Then $r = e - v + 2$.

Or $2 = v - e + r$.



of edges, $e = 6$

of vertices, $v = 4$

of regions, $r = e - v + 2 = 4$

Euler's Formula

Euler characteristic (simple form):

$2 = \text{number of vertices} - \text{number of edges} + \text{number of faces}$

Or in short-hand,

$$2 = |V| - |E| + |F|$$

where V = set of vertices

E = set of edges

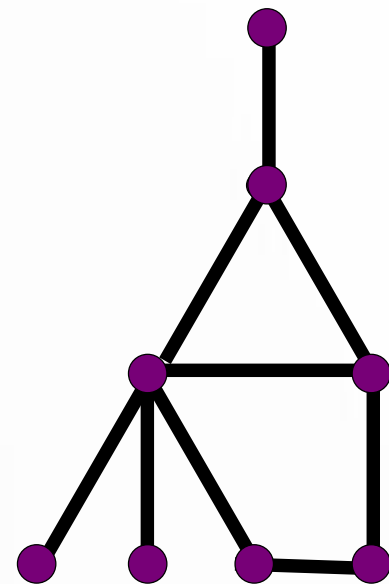
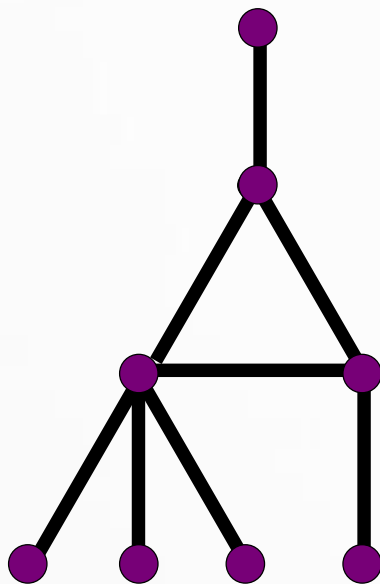
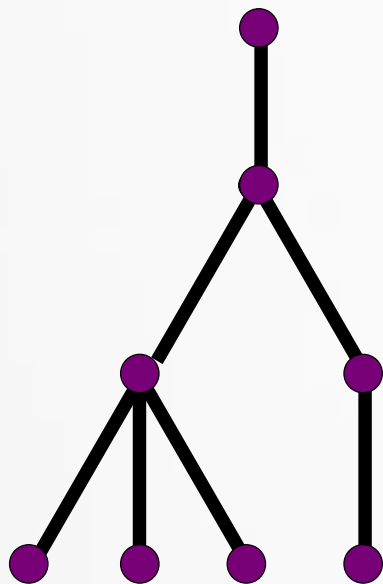
F = set of faces = set of regions

For a planar connected graph $|V| - |E| + |F| = 2$

Euler's Formula

Defⁿ: A tree is a connected graph that does not contain a cycle.

A forest is a graph whose components are trees.



Lemma 2.1: Any tree with n vertices has $n-1$ edges.

Euler's Formula

$$2 = |V| - |E| + |F|$$



$$\chi = 1 - 0 + 1 = 2$$

Euler's Formula

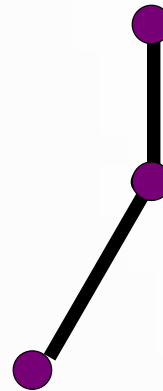
$$2 = |V| - |E| + |F|$$



$$\chi = 2 - 1 + 1 = 2$$

Euler's Formula

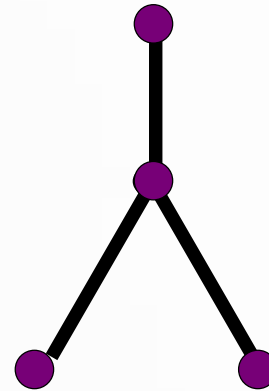
$$2 = |V| - |E| + |F|$$



$$\chi = 3 - 2 + 1 = 2$$

Euler's Formula

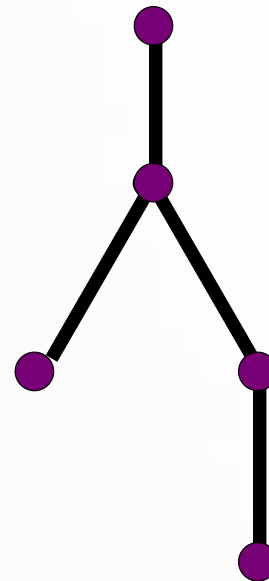
$$2 = |V| - |E| + |F|$$



$$\chi = 4 - 3 + 1 = 2$$

Euler's Formula

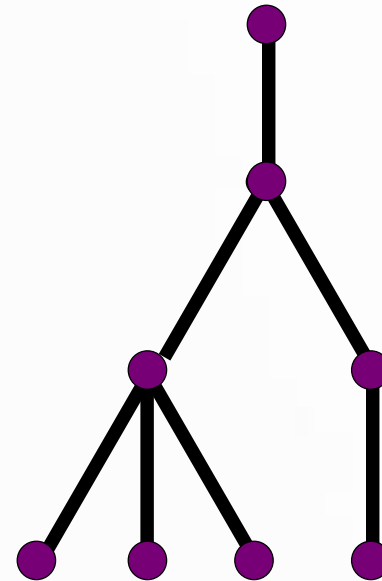
$$2 = |V| - |E| + |F|$$



$$\chi = 5 - 4 + 1 = 2$$

Euler's Formula

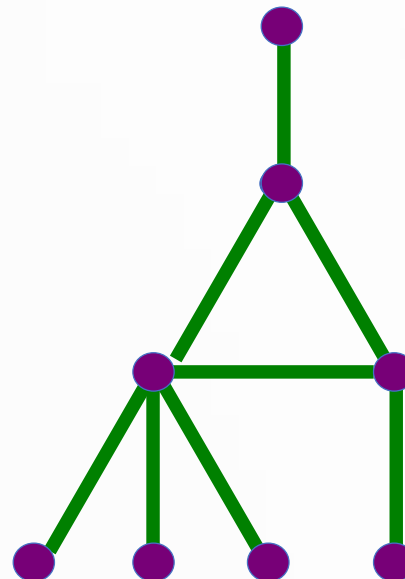
$$2 = |V| - |E| + |F|$$



$$\chi = 8 - 7 + 1 = 2$$

Euler's Formula

$$2 = |V| - |E| + |F|$$

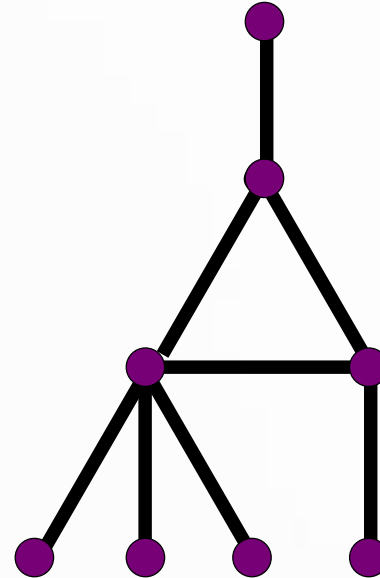


$$\chi = 8 - 8 + 2 = 2$$

Not a tree.

Euler's Formula

$$2 = |V| - |E| + |F|$$

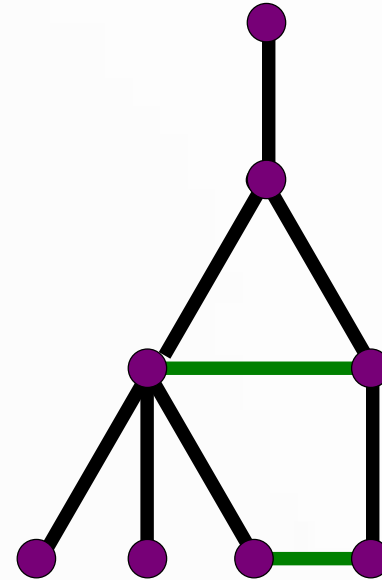


$$\chi = 8 - 8 + 2 = 2$$

Not a tree.

Euler's Formula

$$2 = |V| - |E| + |F|$$



$$\chi = 8 - 9 + 3 = 2$$

Not a tree.

Euler's Formula

Ques:- Suppose that a connected planar graph has six vertices, each of degree four. Into how many regions is the plane divided by a planar representation of this graph?

Sol:- By Euler's formula $r = e - v + 2$. Given $v = 6$.

Given that each vertex has degree 4,

Then sum of the degrees is $6 \cdot 4 = 24$.

By the handshaking theorem there are $e = 24/2 = 12$ edges

Solving $r = 12 - 6 + 2 = 8$, $r = 8$.

Answer: $c(8)$.

Euler's Formula

Ques:- How many regions would be in a plane graph with 10 vertices each of degree 3?

Sol:- By Euler's formula $r = e - v + 2$. Given $v = 10$.

Given that each vertex has degree 4,

Then sum of the degrees is $10 \cdot 3 = 30$.

By the handshaking theorem there are $e = 30/2 = 15$ edges

Solving $r = 15 - 10 + 2 = 5 + 2$, $r = 7$.

Answer: c(7).

That's all for now...