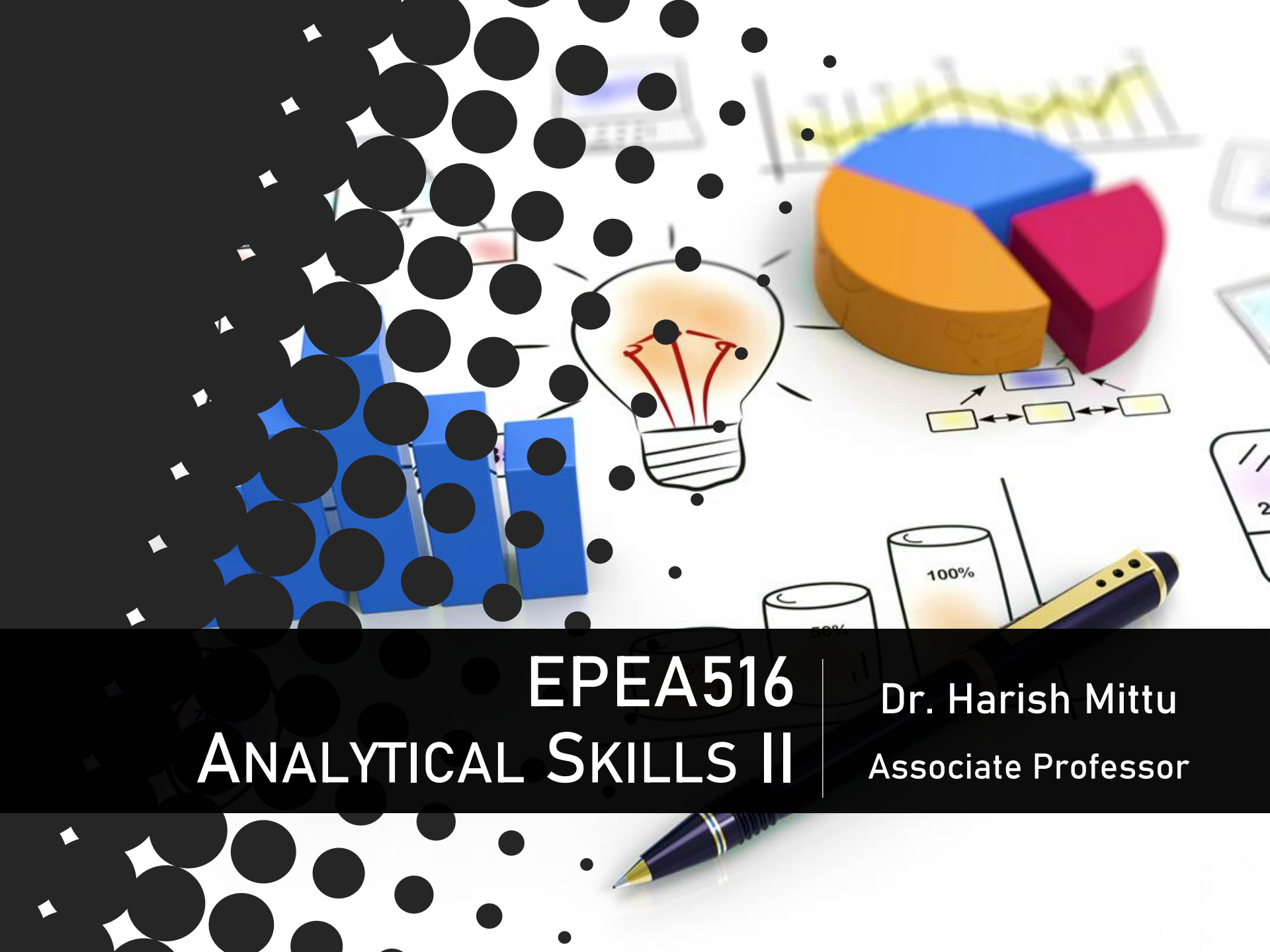

EPEA516
Analytical Skills II

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Learning Outcomes

After this lecture, you will be able to

- solve various problems relating to pipes and cisterns.

Problem 1

- Two pipes A and B can fill a cistern in 15 and 25 minutes. If both the pipes are opened simultaneously, how long will it take to fill the cistern?
- Let x = 15 and y = 25
- Both the pipes A & B together will fill the cistern in

$$= \frac{xy}{x + y} \text{ minutes}$$

$$= \frac{15 \times 25}{15 + 25} \text{ minutes}$$

Problem 1

$$= \frac{15 \times 25}{15 + 25} \text{ minutes}$$

$$= \frac{\overset{75}{\cancel{375}}}{\underset{8}{\cancel{40}}} \text{ minutes}$$

$$= \frac{75}{8} \text{ minutes}$$

Problem 1 – Alternative Method

- Two pipes A and B can fill a cistern in 15 and 25 minutes. If both the pipes are opened simultaneously, how long will it take to fill the cistern?
- Both the pipes A & B together will fill the cistern in

$$= \frac{\text{Total Cacity of Cistern = LCM of 15 and 25}}{\frac{\text{LCM of 15 and 25}}{15} + \frac{\text{LCM of 15 and 25}}{25}}$$

$$= \frac{75}{\frac{75}{15} + \frac{75}{25}} \quad 5 \quad 3$$

$$= \frac{75}{5+3} \text{ minutes} = \frac{75}{8} \text{ minutes}$$

Problem 2

- Two pipes A and B can fill a tank in 15 and 20 minutes respectively. If both the pipes are opened together and at the end of 5 minutes, the pipe A is closed, then how much extra time will B take to fill the tank?
- LCM of 15 and 20 = 60
- Part of the tank filled by both the pipes A & B in 5 minutes

$$= 5 \times \left(\frac{\text{LCM of 15 and 20}}{15} + \frac{\text{LCM of 15 and 20}}{20}\right) = 5 \times \left(\frac{\overset{4}{\cancel{60}}}{\cancel{15}} + \frac{\cancel{60}^{3}}{\cancel{20}}\right)$$

$$= 5 \times (4 + 3) = 5 \times 7 = 35$$

Problem 2

- Part of the tank filled by both the pipes A & B in 5 minutes

$= 35$

- Extra time taken by pipe B to fill the tank

$$= \frac{\text{LCM of 15 \& 20} - \text{Part of tank filled by both pipes in 5 min}}{\frac{\text{LCM of 15 and 20}}{20} \text{ or } \frac{60}{20} \text{ or } 3}$$

$$= \frac{60 - 35}{3}$$

$$= \frac{25}{3} \text{ minutes}$$

Problem 3

- Two pipes A and B can fill a tank in 20 and 50 minutes respectively. If both the pipes are opened together and after some time the pipe B was closed and the tank was filled in 15 minutes. Calculate the time after which the pipe A was closed.
- LCM of 20 and 50 = 100
- Part of tank filled by pipe B in 15 minutes

$$= 15 \times \left(\frac{\text{LCM of 20 and 50}}{50}\right)$$

Problem 3

- Part of tank filled by pipe B in 15 minutes

$$= 15 \times \left(\frac{\text{LCM of 20 and 50}}{50}\right)$$

$$= 15 \times \left(\frac{100}{50}\right)$$

$$= 30 \text{ units}$$

- Time after which pipe A was closed

$$= \left(\frac{\text{LCM of 20 and 50} - \text{Part of tank filled by pipe B in 15 mins.}}{\frac{\text{LCM of 20 and 50}}{20} \text{ or } \frac{100}{20}}\right)$$

$$= \left(\frac{100 - 30}{5}\right) \quad = \frac{70}{5}$$

$$= 14 \text{ minutes}$$

Problem 4

- Two pipes, A and B, can separately fill a cistern is 4 hours and 6 hours respectively, while a third pipe C can empty it in 3 hours. In what time will the cistern be full, if all the pipes are opened together?
- x = 4, y = 6 and z = - 3
- Three pipes together will fill the cistern

$$= \frac{xyz}{xy + yz + zx} \text{ hours}$$

$$= \frac{4 \times 6 \times (-3)}{4 \times 6 + 6 \times (-3) + (-3) \times 4} \text{ hours}$$

Problem 4

$$= \frac{4 \times 6 \times (-3)}{4 \times 6 + 6 \times (-3) + (-3) \times 4} \text{ hours}$$

$$= \frac{-72}{24 - 18 - 12} \text{ hours}$$

$$= \frac{-72}{-6} \text{ hours}$$

$$= 12 \text{ hours}$$

Alternative Method

LCM of 4, 6, and 3

= 12 (Total Capacity)

$\frac{12}{4} = 3$, $\frac{12}{6} = 2$, & $\frac{12}{-3} = -4$

Cistern will be filled in

$$= \frac{12}{3 + 2 - 4}$$

$$= \frac{12}{5 - 4}$$

= 12 hours

Problem 5

- Two taps A and B can fill a cistern in 60 minutes and 120 minutes, respectively. There is a third exhaust tap C at the bottom of the tank. If all the taps are opened at the same time, the cistern will be full in 90 minutes. In what time can exhaust tap C empty the cistern when it is full?
- x = 60, y = 120 and z = 90
- An exhaust tap C can empty the full cistern

$$= \frac{xyz}{yz + xz - xy} \text{ minutes}$$

Problem 5

- An exhaust tap C can empty the full cistern

$$= \frac{xyz}{yz + xz - xy} \text{ minutes}$$

$$= \frac{60 \times 120 \times 90}{120 \times 90 + 60 \times 90 - 60 \times 120} \text{ minutes}$$

$$= \frac{648000}{10800 + 5400 - 7200} \text{ minutes}$$

$$= \frac{\cancel{648000}^{72}}{\cancel{9000}} \text{ minutes}$$

$$= 72 \text{ minutes}$$

$$= 1 \text{ hour } 12 \text{ minutes}$$

Problem 6

- A pipe can fill a tank in 4 hours. Due to leakage at the bottom, it is filled in 8 hours. If the tank is full, how much time will the leak take to empty it?
- x = 4 and y = 8
- Amount of time in which the leak can empty the full tank $= \frac{xy}{y - x}$ hours

$= \frac{4 \times 8}{8 - 4}$ hours

Problem 6

- Amount of time in which leak can empty the full tank

$$= \frac{xy}{y - x} \text{ hours}$$

$$= \frac{4 \times 8}{8 - 4} \text{ hours}$$

$$= \frac{\overset{8}{\cancel{32}}}{\cancel{4}} \text{ hours}$$

$$= 8 \text{ hours}$$

Problem 7

- Two taps A and B can fill a tank in 3 hours and 6 hours, respectively. Due to a leakage at the bottom of the tank, tank is filled in 1 hour more. In how much time will the leak take to empty the tank?
- LCM of 3 and 6 = 6 (Total Capacity)
- $\frac{6}{3} = 2$ and $\frac{6}{6} = 1$
- Taps A and B fill the tank in $= \frac{6}{2+1} = \frac{6}{3} = 2$ hours
- Due to a leakage, tank is filled in = (2 + 1) hours = 3 hours

Problem 7

- Taps A and B fill the tank in $= \frac{6}{2+1} = \frac{\cancel{6}}{\cancel{3}} = 2$ hours
- Due to a leakage, tank is filled in = (2 + 1) hours = 3 hours
- Let time taken by leakage to empty the tank = x

LCM of 2 and x = 2x

$\frac{\cancel{2}x}{\cancel{2}} = x$ and $\frac{2\cancel{x}}{-\cancel{x}} = -2$

$$\frac{2x}{x-2} = 3$$

$$2x = 3(x-2)$$

$$2x = 3x - 12 \text{ or } 3x - 2x = 12$$

$$x = 12$$

Problem 8

- An inlet pipe fills water in a tank at the rate of 12 litres per minute. A leak at the bottom of a tank can empty the full tank in 18 hours. When the tank is full, the inlet is opened and due to leak, the tank is empty in 24 hours. Find out the capacity of the tank.
- x = 18 hours
- y = 12 = 12 x 60 litres per hours

  = 720 litres per hours
- z = 24 hours

Problem 8

- x = 18, y = 720, and z = 24
- Capacity of cistern $= \frac{xyz}{z - x}$ liters

$= \frac{18 \times 720 \times 24}{24 - 18}$ liters

$= \frac{18 \times 720 \times \cancel{24}^{4}}{\cancel{6}}$ liters

= 51840 liters

Problem 8 - Alternative Method

- x = 18 hours, z = 24 hours
- y = 12 litres/min. = 12 x 60 litres/hrs = 720 litres/hrs
- LCM of 18 and 24 = 72 (Total Capacity)

$$\frac{\overset{4}{\cancel{72}}}{\cancel{18}} = -4 \quad \& \quad \frac{\overset{3}{\cancel{72}}}{\cancel{24}} = -3$$

- Capacity of cistern = 72y = 72 (720)

  = 51840 litres

Problem 9

- An inlet A fills a cistern 5 times faster than the second inlet B. If inlet B can fill a cistern in 42 minutes, then find the time when the cistern will be full if both the inlets are opened together.
- $m = 5$
- $x = 42$ minutes
- If both the inlets A & B are opened together then the time taken to fill the cistern $= \frac{x}{m + 1}$ hours

Problem 9

- m = 5 and x = 42 minutes
- If both the inlets A & B are opened together then the time taken to fill the cistern $= \frac{x}{m + 1}$ minutes

$$= \frac{42}{5 + 1} \text{ minutes}$$

$$= \frac{\overset{7}{\cancel{42}}}{\cancel{6}} \text{ minutes}$$

$$= 7 \text{ minutes}$$

Problem 10

- An inlet A fills a cistern 5 times faster than the second inlet B. If inlet A can fill a cistern in 42 minutes, then find the time when the cistern will be full if both the inlets are opened together.
- m = 5
- y = 42 minutes
- If both the inlets A & B are opened together then the time taken to fill the cistern $= \{\frac{m}{m+1}\}y$ hours

Problem 10

- m = 5 and y = 42 minutes
- If both the inlets A & B are opened together then the time taken to fill the cistern $= \frac{m \times y}{m + 1}$ hours

$$= \frac{5 \times 42}{5 + 1} \text{ minutes}$$

$$= \frac{5 \times \cancel{42}^{7}}{\cancel{6}} \text{ minutes}$$

$$= 35 \text{ minutes}$$

Problem 11

- An inlet A fills a cistern 5 times faster than the second inlet B. If inlet A takes 48 minutes less than the inlet B to fill a cistern. Find the time to fill the cistern completely if both the inlets are opened together.
- m = 5
- x = 48 minutes
- If both the inlets A & B are opened together then the time taken to fill the cistern $= \frac{mx}{(m-1)^2}$ hours

Problem 11

- m = 5 and x = 48 minutes
- If both the inlets A & B are opened together then the time taken to fill the cistern $= \frac{mx}{(m-1)^2}$ hours

$$= \frac{5 \times 48}{(5-1)^2} \text{ minutes} = \frac{5 \times 48}{(4)^2} \text{ minutes}$$

$$= \frac{5 \times \cancel{48}^{\,3}}{\cancel{16}} \text{ minutes}$$

$$= 15 \text{ minutes}$$

Problem 12

- An inlet A fills a cistern 5 times faster than the second inlet B. If inlet A takes 48 minutes less than the inlet B to fill a cistern. Find the time taken by inlet A to fill the cistern completely.
- m = 5
- x = 48 minutes
- Time taken by inlet A to fill the cistern completely

$$= \frac{x}{m - 1} \text{ minutes}$$

Problem 12

- m = 5 and x = 48 minutes
- Time taken by inlet A to fill the cistern completely

$$= \frac{x}{m - 1} \text{ minutes}$$

$$= \frac{48}{5 - 1} \text{ minutes}$$

$$= \frac{\overset{12}{\cancel{48}}}{\cancel{4}} \text{ minutes}$$

$$= 12 \text{ minutes}$$

Problem 13

- An inlet A fills a cistern 5 times faster than the second inlet B. If inlet A takes 48 minutes less than the inlet B to fill a cistern. Find the time taken by inlet B to fill the cistern completely.
- m = 5
- x = 48 minutes
- Time taken by inlet B to fill the cistern completely

$$= \frac{mx}{m-1} \text{ minutes}$$

Problem 13

- m = 5 and x = 48 minutes
- Time taken by inlet B to fill the cistern completely

$$= \frac{mx}{m-1} \text{ minutes}$$

$$= \frac{5 \times 48}{5-1} \text{ minutes}$$

$$= \frac{5 \times \overset{12}{\cancel{48}}}{\cancel{4}} \text{ minutes}$$

$$= 60 \text{ minutes}$$

Problem 14

- Two inlets A and B can fill a tank in 16 and 24 minutes respectively. If both the pipes are opened simultaneously, then find the time after which inlet B should be closed, so that the tank is full in 8 minutes.
- Time after which inlet B should be closed, so that the tank is full in t minutes i.e., $T = y \left\{ 1 - \frac{t}{x} \right\}$ minutes
- x = 16, y = 24, and t = 8

Problem 14

- Time after which inlet B should be closed, so that the tank is full in t minutes i.e., $T = y\left\{1 - \frac{t}{x}\right\}$ minutes

$x = 16,\ y = 24$, and $t = 8$

$$T = 24\left\{1 - \frac{8}{16}\right\}$$

$$T = 24\left\{\frac{16 - 8}{16}\right\}$$

$$T = \overset{12}{\cancel{24}}\left\{\frac{\cancel{8}}{\cancel{16}_{2}}\right\}$$

$T = 12$ minutes

Problem 15

- Two inlets A and B can fill a tank in 16 and 24 minutes respectively. They are opened alternatively for 1 min. each, beginning with inlet A. In how much time will the tank be filled completely?
- LCM of 16 and 24 = 48 (Total Capacity)
- Two inlets A and B opened alternatively for 1 min.
- Units filled in tank in 2 mins. $= \frac{\overset{3}{\cancel{48}}}{\cancel{16}} + \frac{\overset{2}{\cancel{48}}}{\cancel{24}}$

$= 3 + 2 = 5$

Problem 15

- LCM of 16 and 24 = 48 (Total Capacity)
- Units filled in tank in 2 mins. = 5

Minutes	Tank Filled
2	5
2 x 9 = 18	5 x 9 = 45

- Remaining Part = 48 – 45 = 3
- One unit is filled by inlet A = $\frac{16}{48} = \frac{1}{3}$ minutes
- Tank will be filled completely = $18 + \frac{1}{3}$

$$= \frac{54 + 1}{3} = \frac{55}{3} \text{ minutes}$$

Conclusion

- Capacity of cistern $= \frac{xyz}{z - x}$ liters
- Time
  - Both pipes A & B together will fill/empty cistern in

$$= \frac{xy}{x + y} \text{ hours}$$

  - Three pipes together will fill the cistern

$$= \frac{xyz}{xy + yz + zx} \text{ hours}$$

Conclusion

- Time
  - An exhaust tap C can empty the full cistern

$$= \frac{xyz}{yz + xz - xy} \text{ mins.}$$

  - Amount of time in which leak can empty the full tank

$$= \frac{xy}{y - x} \text{ hrs}$$

  - Time after which inlet B should be closed, so that the tank is full in t minutes i.e., $T = y \left\{ 1 - \frac{t}{x} \right\}$ minutes

Summary

- Pipes & Cisterns
  - Capacity of Cistern/Tank
  - Time – Fill/Empty

That’s all for now…