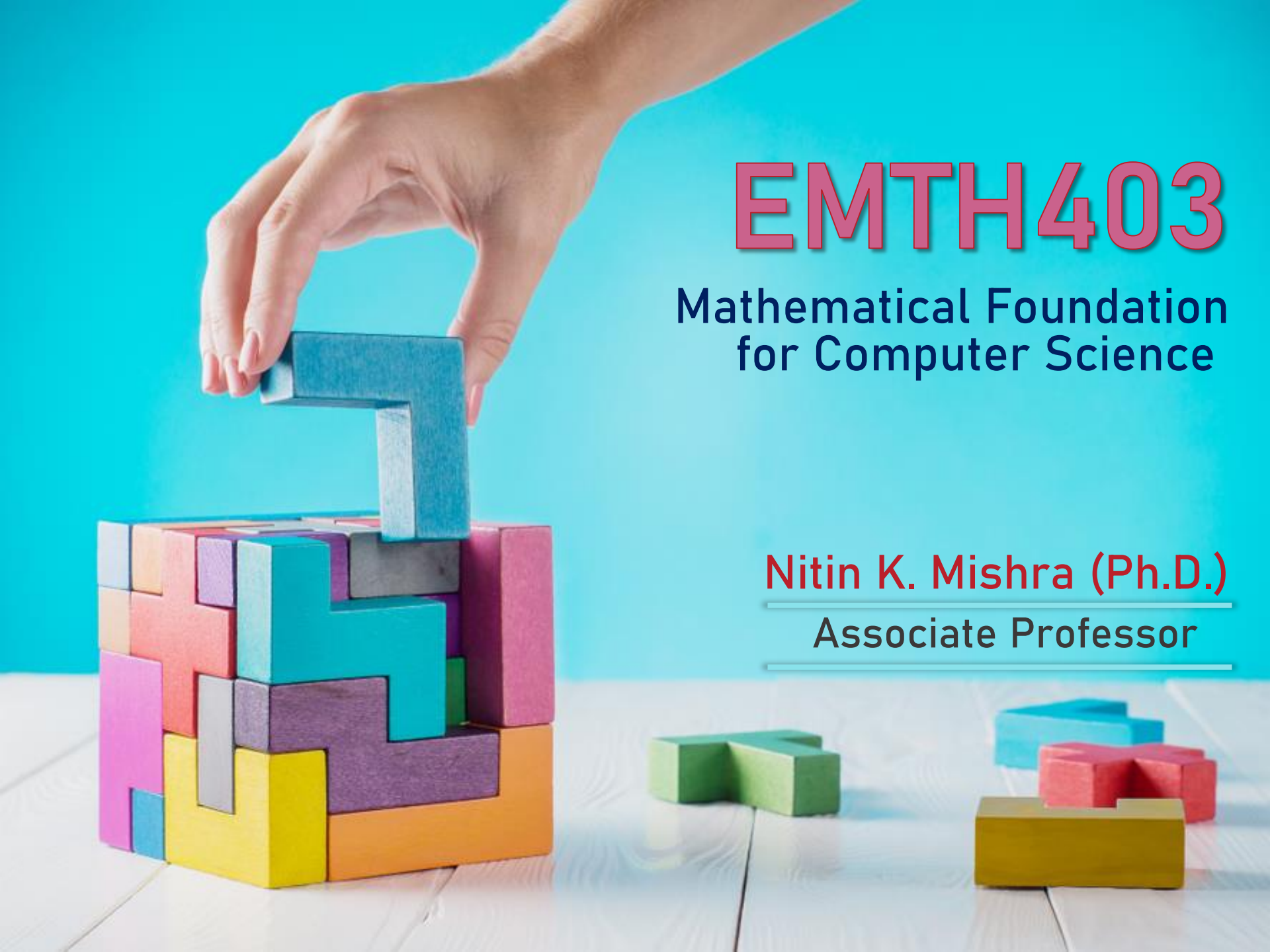


EMTH403

Mathematical Foundation for Computer Science

Nitin K. Mishra (Ph.D.)

Associate Professor

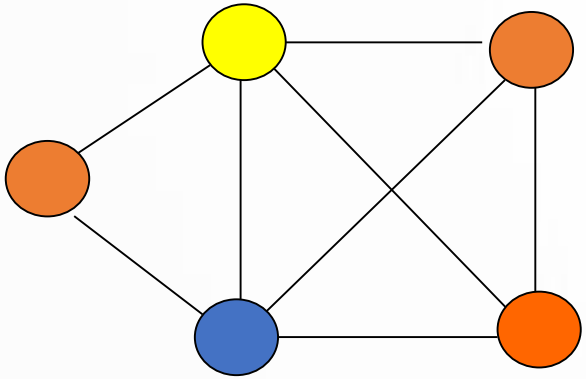
Lecture Outcomes

After this lecture, you will be able to

- understand what is graph coloring.
- understand what is a 4-Color Map Theorem.
- understand what is a chromatic number

Coloring Graphs

Definition: A graph has been colored if a color has been assigned to each vertex in such a way that adjacent vertices have different colors.



Definition: The chromatic number of a graph is the smallest number of colors with which it can be colored.

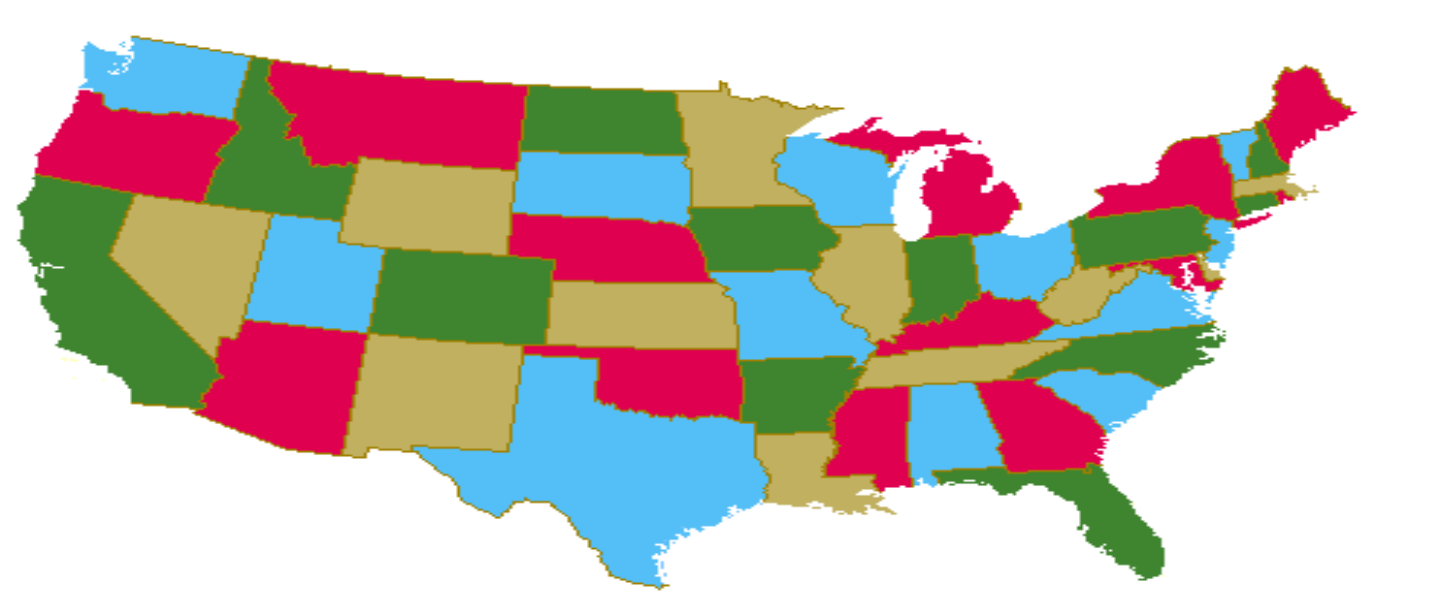
In the example above, the chromatic number is 4.

Coloring Graphs

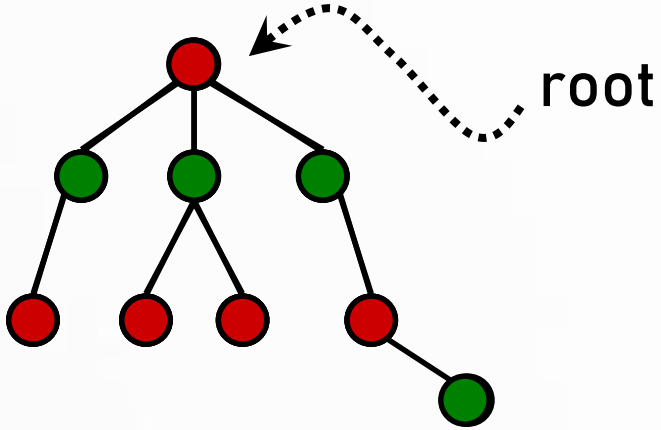
Four colors are sufficient to color a map of the contiguous United States.

Source of map:

http://www.math.gatech.edu/~thomas/FC/fourcolor.html



Coloring Graphs - Trees



Pick any vertex as “root.”

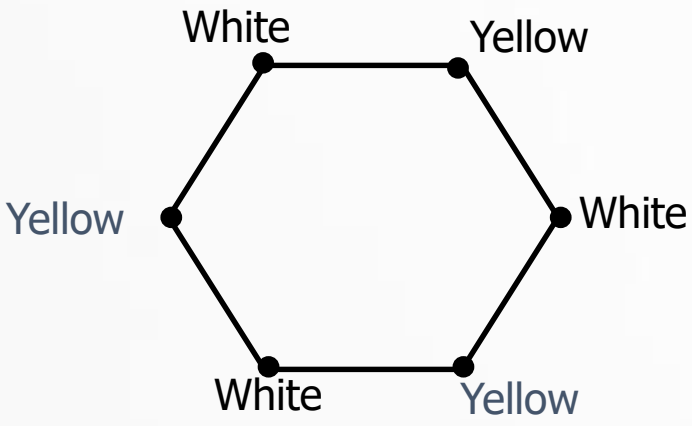
if (unique) path from root is

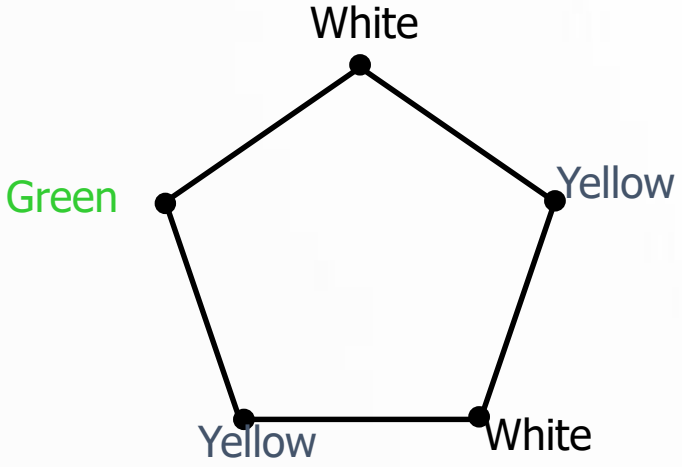
even length:

odd length:

Coloring Graphs - Trees

What is the chromatic number for each of the following graphs?



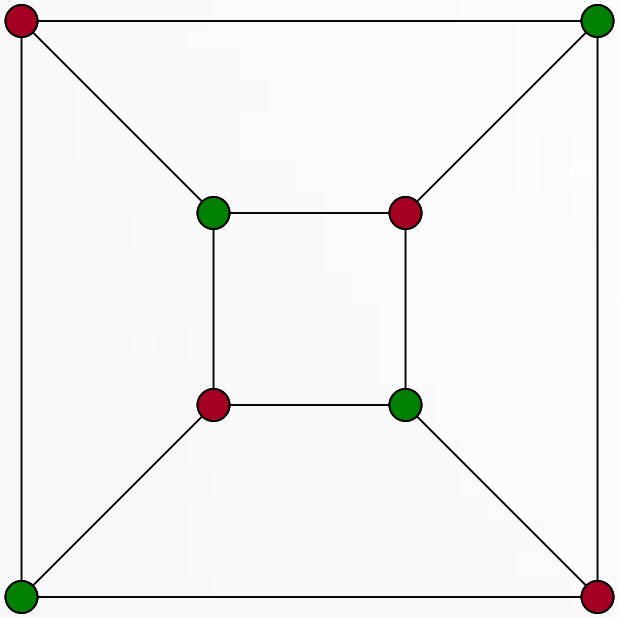


Chromatic number: 2

Chromatic number: 3

Coloring Graphs - 2-Colourable Graphs

When exactly is a graph 2-colourable?



This is 2-colourable.

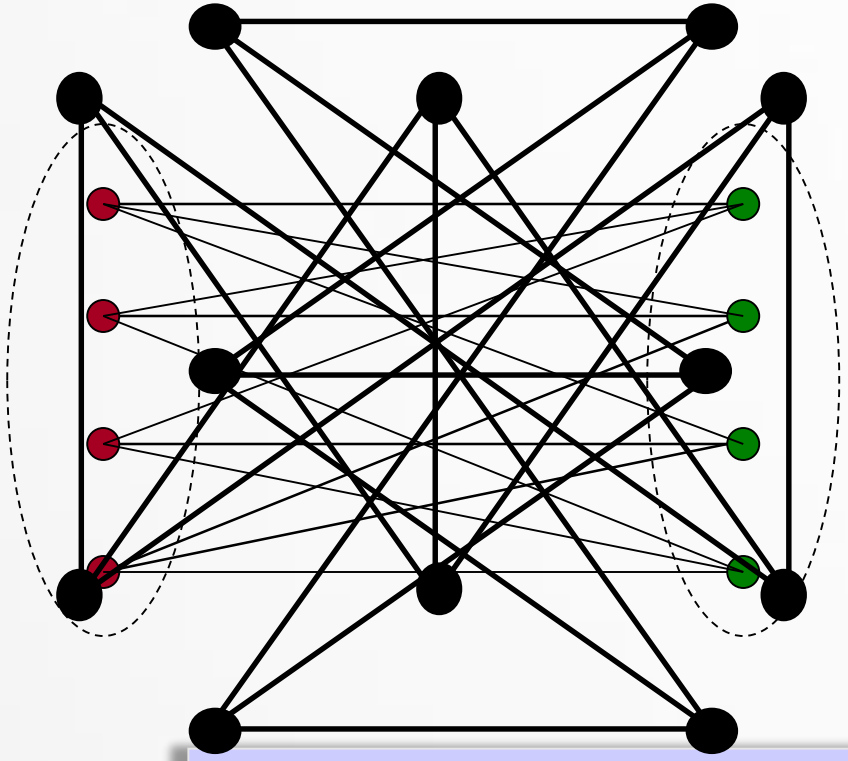
2 colourable: tree, even cycle, etc.

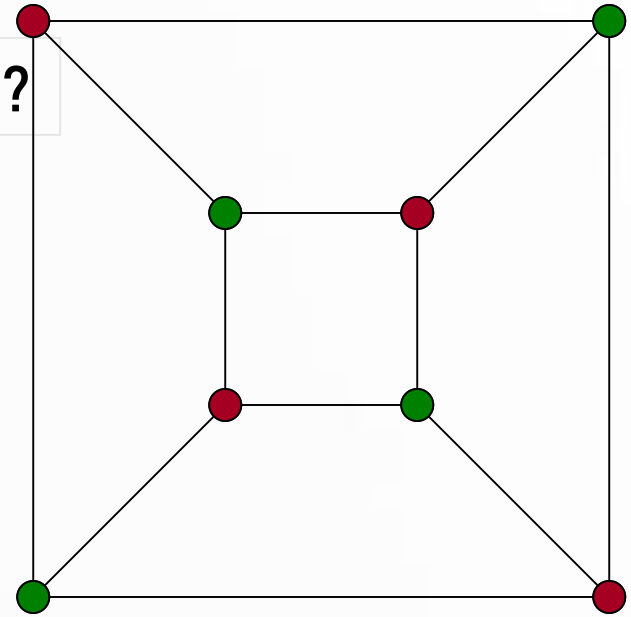
Not 2 colourable: triangle, odd cycle, etc.

Coloring Graphs - Bipartite Graphs

When exactly is a graph 2-colourable?

Is a bipartite graph 2-colourable?





Is a 2-colourable graph bipartite?

Fact. A graph is 2-colourable if and only if it is bipartite.

Coloring Graphs – Application in Exam Scheduling

Suppose that in a particular quarter there are students taking each of the following combinations of courses:

- Math, English, Biology, Chemistry ,
- Math, English, Computer Science, Geography
- Biology, Psychology, Geography, Spanish,
- Biology, Computer Science, History, French

Coloring Graphs – Application in Exam Scheduling

What is the minimum number of examination periods required for the exams in the ten courses specified so that students taking any of the given combinations of courses have no conflicts?

Coloring Graphs – Application in Scheduling

Twelve faculty members in a mathematics department serve on the following committees:

Undergraduate education: Sineman, Limitson, Axiomus, Functionini

Graduate Education: Graphian, Vectorades, Functionini, Infinitescu

Colloquium: Lemmeau, Randomov, Proofizaki

Library: Van Sum, Sineman, Lemmeau

Staffing: Graphian, Randomov, Vectorades, Limitson

Promotion: Vectorades, Van Sum, Parabolton

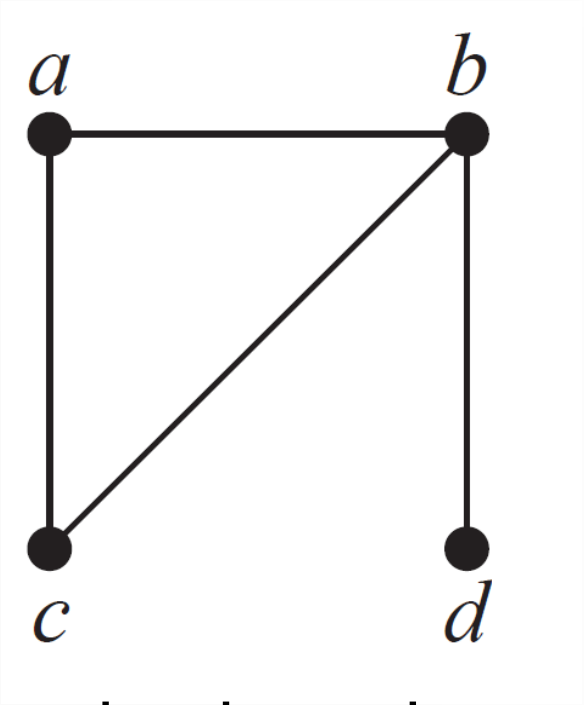
Coloring Graphs – Application in Scheduling

The committees must all meet during the first week of classes, but there are only three time slots available.

Find a schedule that will allow all faculty members to attend the meetings of all committees on which they serve.

Coloring Graphs – Chromatic Number

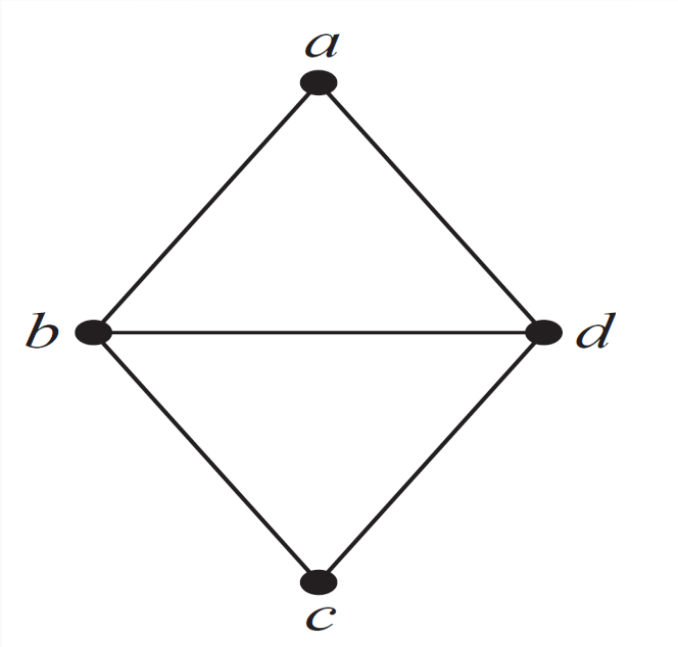
Find the chromatic number of the given graph.



Since there is a triangle, at least 3 colors are needed. Clearly 3 colors suffice, since we can color *a* and *d* the same color.

Coloring Graphs – Chromatic Number

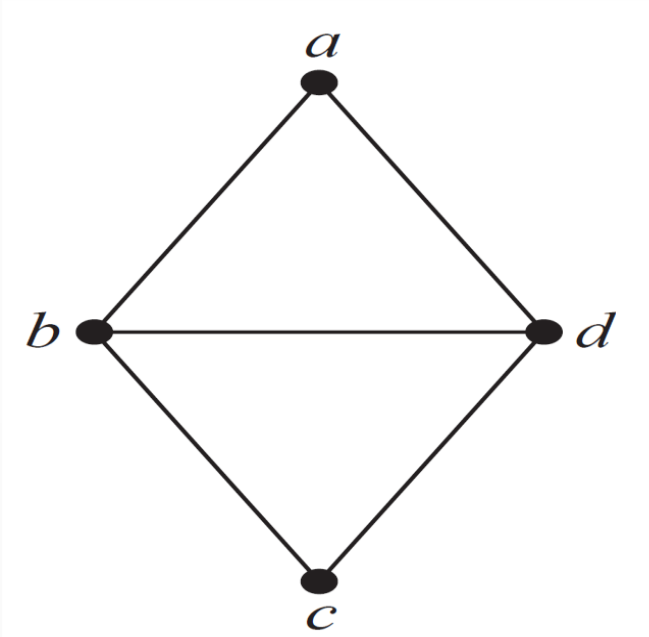
Find the chromatic number of the given graph.



Since there is a triangle, at least 3 colors are needed. Clearly 3 colors suffice, since we can color a and c the same color.

Coloring Graphs – Chromatic Number

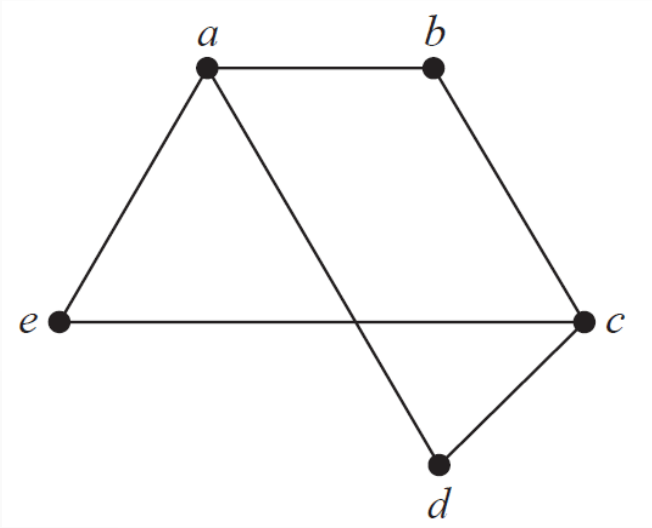
Find the chromatic number of the given graph.



Since there is a triangle, at least 3 colors are needed. Clearly 3 colors suffice, since we can color a and c the same color.

Coloring Graphs – Chromatic Number

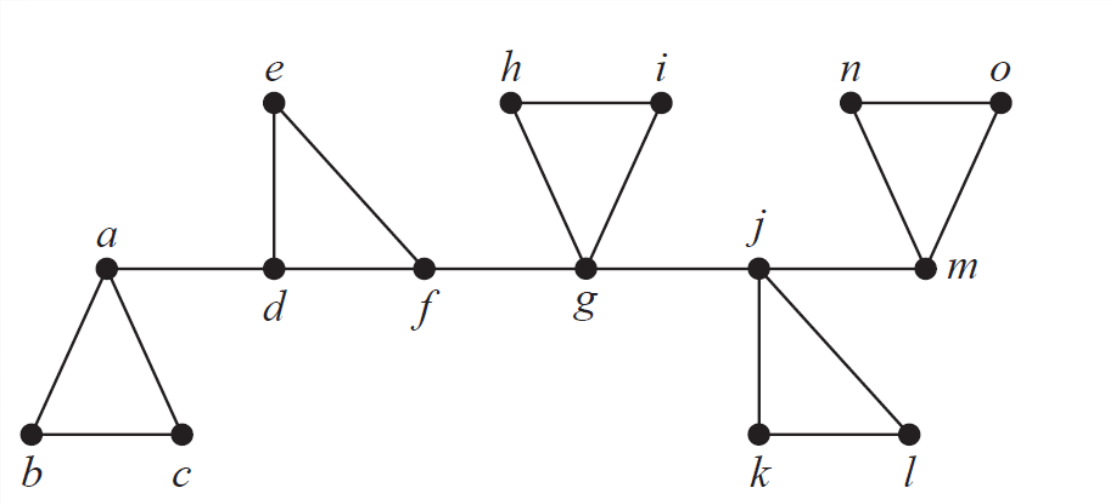
Find the chromatic number of the given graph.



Since there is an edge, at least 2 colors are needed. The coloring in which b, d, and e are red and a and c blue shows that 2 colors suffice.

Coloring Graphs – Chromatic Number

Find the chromatic number of the given graph.



Since there is a triangle, at least 3 colors are needed. It is not hard to construct a 3-coloring. We can let a, f, h, j, and n be blue; let b, d, g, k, and m be green; and let c, e, i, l, and o be yellow.

That's all for now…