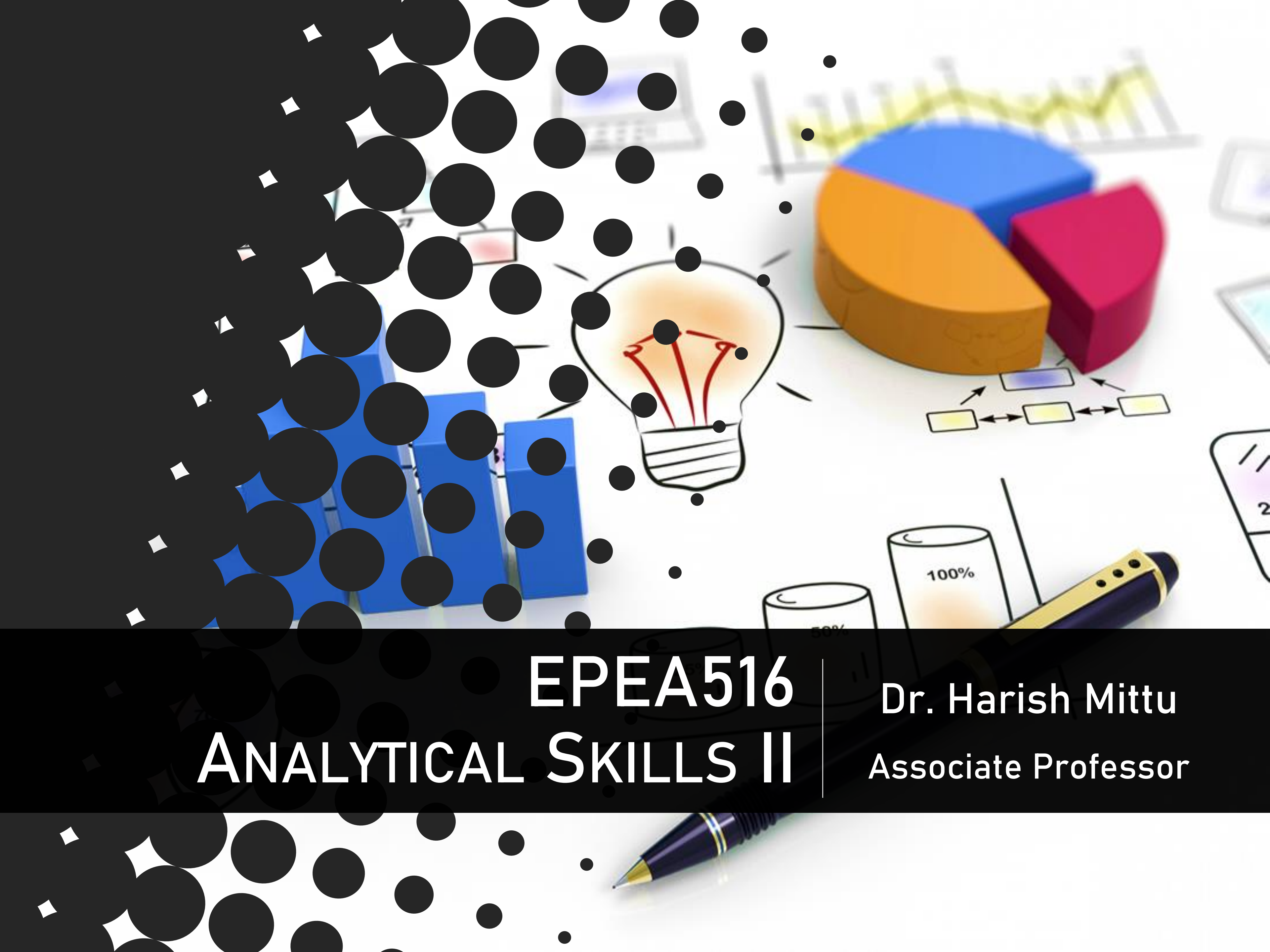

EPEA516
Analytical Skills II

Dr. Harish Mittu
Associate Professor

Learning Outcomes

After this lecture, you will be able to

- differentiate between arithmetic and geometric progression,
- compute general term and sum of ‘n’ terms of arithmetic and geometric progression,
- analyze important points of arithmetic and geometric progression.

Sequence

- Set of Numbers
- Definite Order/Rule
- $\{a_{\mathrm{n}}\} = a_1, a_2, a_3, a_4, \ldots\ldots, a_{\mathrm{n}}$
- Finite - 1, 3, 5, 7, ……, 21
- Infinite - 2, 4, 6, 8, 10, …..

Progression

- Terms of a Sequence
- Pattern
- Examples
  - 3, 5, 7, 9, ..., 21
  - 1, 4, 9, 16, ...
  - 8, 5, 2, – 1, – 4, ...

Series

- Sequence - Numbers
- Numbers – Terms - Predefined Rule
- Specific Pattern
- Adding/Subtracting - Terms of a Sequence
- Examples
  - $3 + 5 + 7 + 9 + \ldots + 21$
  - $1 + 4 + 9 + 16 + \ldots$
  - $8 + 5 + 2 + (-1) + \ldots$

Arithmetic Progression (A. P.)

- Terms of Sequence
- Increase/Decrease - Fixed Number
- Fixed Number - Common Difference
- Let First term = a

  Common Difference = d

  A.P. - a, a + d, a + 2d, ..., a + (n - 1) d, ..

  T_n = a + (n - 1) d

  d = $T_n - T_{n-1}$

General Term – A. P.

- Let a and d be the first term and common difference.
- A.P. - a, a + d, a + 2d, ..., a + (n - 1) d
- T_1 = a = a + (1 - 1) d
- T_2 = a + d = a + (2 - 1) d
- T_3 = a + 2d = a + (3 - 1) d
- ……………………………………………
- T_n = a + (n - 1) d

Sum of 'n' Terms– A. P.

- Let a and d be the first term and common difference.
- n = Terms
- A.P. – a, a + d, a + 2d, ..., a + (n – 1) d
- S_n = a + (a + d) + (a + 2d) + ... + (a + (n – 1) d) .... (1)
- S_n = (a + (n – 1) d) + ... (a + 2d) + (a + d) + a .... (2)
- $2S_n$ = n{2a + (n – 1) d}
- $S_n = \frac{n}{2}\{2a + (n - 1)\, d\}$

Important Points – A. P.

- T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = …. = T_{n} - $T_{\mathrm{n}-1}$ = Constant

  T_1, T_2, T_3, T_4, …., $T_{\mathrm{n}-1}$, and T_{n} - A. P.

- Three numbers a, b, c are in A.P.

  iff b – a = c – b

  iff a + c = 2b

- a – d, a, and a + d
- a – 3d, a – d, a + d, and a + 3d
- a – 2d, a – d, a, a + d, and a + 2d

Important Points – A. P.

- T_n or l = a + (n - 1) d
- First Term = a, Common Difference = d, and Terms = m
- nth term from the end = (m - n + 1)th term from beginning
- T_{m-n+1} = a + (m – n + $\cancel{1}$ - $\cancel{1}$) d
- nth term from the end = T_{m-n+1} = a + (m - n) d

Important Points – A. P.

- $T_n = S_n - S_{n-1}$
- $S_n = \frac{n}{2}\{2a + (n - 1)d\}$
  - $S_n = \frac{n}{2}\{a + [a + (n - 1)d]\}$
  - $S_n = \frac{n}{2}\{a + l\}$

(Because T_n or $l = a + (n - 1)d$)

  - $S_n = \frac{n}{2}\{a + l\}$

Important Points – A. P.

- $S_n = \frac{n}{2}\{a + l\}$
- $S_n = \frac{n}{2}\{a + \underline{(n - 1)\,d - (n - 1)\,d} + l\}$
- $S_n = \frac{n}{2}\{l - (n - 1)\,d + l\}$

(Because $l = a + (n - 1)\,d$)

- $S_n = \frac{n}{2}[2l - (n - 1)\,d]$

Geometeric Progression (G. P.)

- Sequence – Non-zero Numbers
- Every Term (except the first one)
- Common/Constant Ratio - Preceding Term
- Let First term = a

  Common/Constant Ratio = r

  G.P. - a, ar, ar^2, ar^3, …, ar^{n-1}, ..

  T_n = ar^{n-1}

  $r = \frac{T_n}{T_{n-1}}$

General Term – G. P.

- Let a and r be the first term and common ratio.
- G.P. - a, ar, ar^2, ar^3, …, ar^{n-1}
- T_1 = a = ar^0 = ar^{1-1}
- T_2 = ar^1 = ar^{2-1}
- T_3 = ar^2 = ar^{3-1}
- ……………………………………………
- T_n = ar^{n-1}

Sum of 'n' Terms– G. P.

- Let a and r be the first term and common ratio. n = Terms
- G.P. - a, ar, ar^2, ar^3, ..., ar^{n-1}
- $S_n = a + ar + ar^2 + ar^3 + \ldots + ar^{n-1}$ .... (1)
- $rS_n = ar + ar^2 + ar^3 + \ldots + ar^{n-1} + ar^n$ .... (2)
- $rS_n - S_n = ar^n - a$
- $(r - 1)\, S_n = a(r^n - 1)$
- $S_n = \dfrac{a(r^n - 1)}{(r - 1)}$

Important Points – G. P.

- $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = ... = \frac{T_n}{T_{n-1}}$ = Constant = r

  $T_1, T_2, T_3, T_4, ..., T_{n-1}$, and T_n - G. P.

- Three numbers a, b, c are in G.P.

  iff $\frac{b}{a} = \frac{c}{b}$

  iff $b^2 = ac$

- $\frac{a}{r}$, a, and ar
- $\frac{a}{r^3}$, $\frac{a}{r}$, ar, and ar^3
- $\frac{a}{r^2}$, $\frac{a}{r}$, a, ar, and ar^2

Important Points – G. P.

- T_n or l = ar^{n-1}
- First Term = a, Common Ratio = r, and Terms = m
- nth term from the end = (m - n + 1)th term from beginning
- T_{m-n+1} = $ar^{m-n\cancel{+1}\cancel{-1}}$
- nth term from the end = T_{m-n+1} = ar^{m-n}
- nth term from the end in terms of last term 'l' & common

  ratio 'r' = $\dfrac{l}{r^{n-1}}$

Important Points – G. P.

- $S_n = \dfrac{a(r^n - 1)}{(r - 1)}$ when r > 1
- $S_n = \dfrac{a(1 - r^n)}{(1 - r)}$ when r < 1
- S_n = na when r = 1
- $S_n = \dfrac{lr - a}{r - a}$, where l = last term & r ≠ 1
- $S_\infty = \dfrac{a}{(1 - r)}$ when | r | < 1 i.e., – 1 < r < 1

Conclusion

- Progression
  - Terms of a Sequence (Pattern)
- Arithmetic Progression
  - Terms of Sequence
  - Increase/Decrease - Fixed Number
- Geometric Progression
  - Sequence – Non-zero Numbers
  - Every Term (except the first one)
  - Common/Constant Ratio - Preceding Term

Conclusion

- Arithmetic Progression
  - d = T_{n} - $T_{\mathrm{n}-1}$
  - T_{n} = a + (n - 1) d
  - S_{n} = $\frac{n}{2}$ {2a + (n - 1) d}

Conclusion

- Geometric Progression
  - r = $\frac{T_{\mathrm{n}}}{T_{\mathrm{n}-1}}$
  - T_{n} = ar$^{\mathrm{n}-1}$
  - S_{n} = $\frac{a(r^n - 1)}{(r - 1)}$ when r > 1
  - S_{n} = $\frac{a(1 - r^n)}{(1 - r)}$ when r < 1
  - S_{n} = na when r = 1
  - S_{∞} = $\frac{a}{(1 - r)}$ when | r | < 1 i.e., - 1 < r < 1

Summary

- Progression
- Arithmetic Progression
- Geometric Progression

That's all for now…