EMTH403

Mathematical Foundation for Computer Science

Nitin K. Mishra (Ph.D.)

Associate Professor

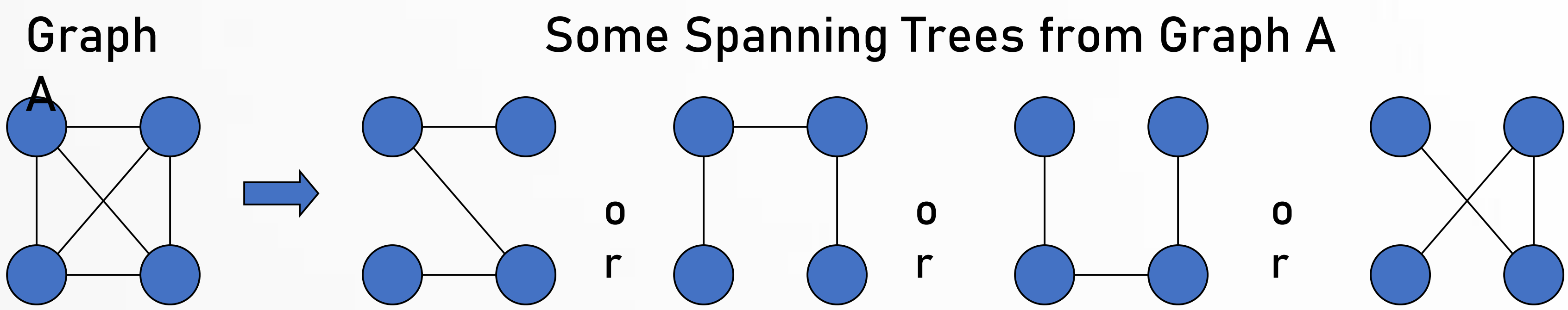
Lecture Outcomes

After this lecture, you will be able to

- understand what Spanning Trees
- understand what is Kruskal's Algorithm.

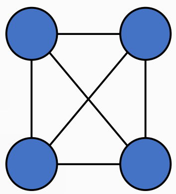
Spanning Trees

- A spanning tree of a graph is just a subgraph that contains all the vertices and is a tree.
- A graph may have many spanning trees.

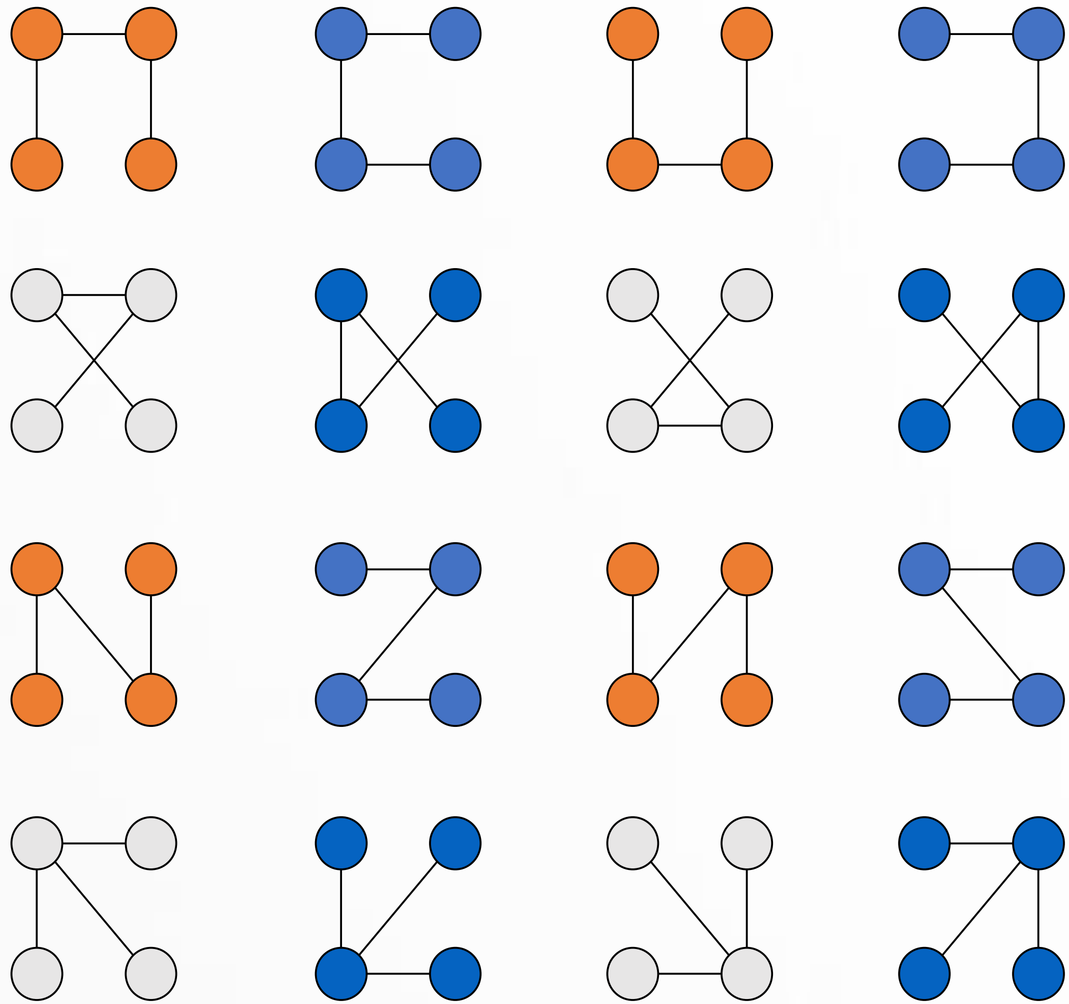


Spanning Trees

Complete Graph



All 16 of its Spanning Trees



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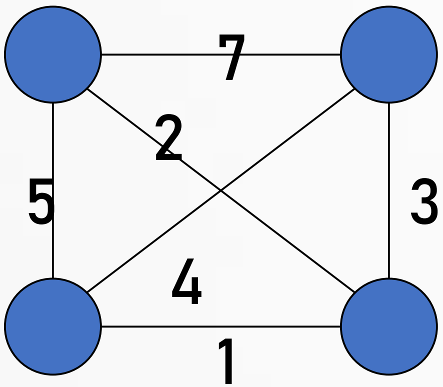
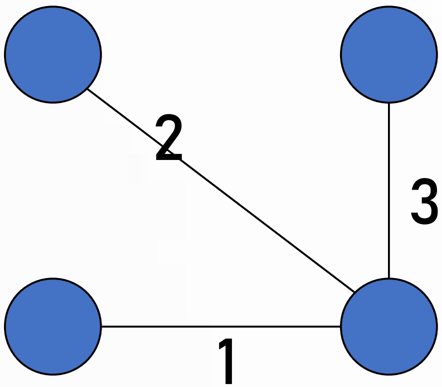
Minimum Spanning Trees

The Minimum Spanning Tree for a given graph is the Spanning Tree of minimum cost for that graph.

Complete Graph



Minimum Spanning Tree



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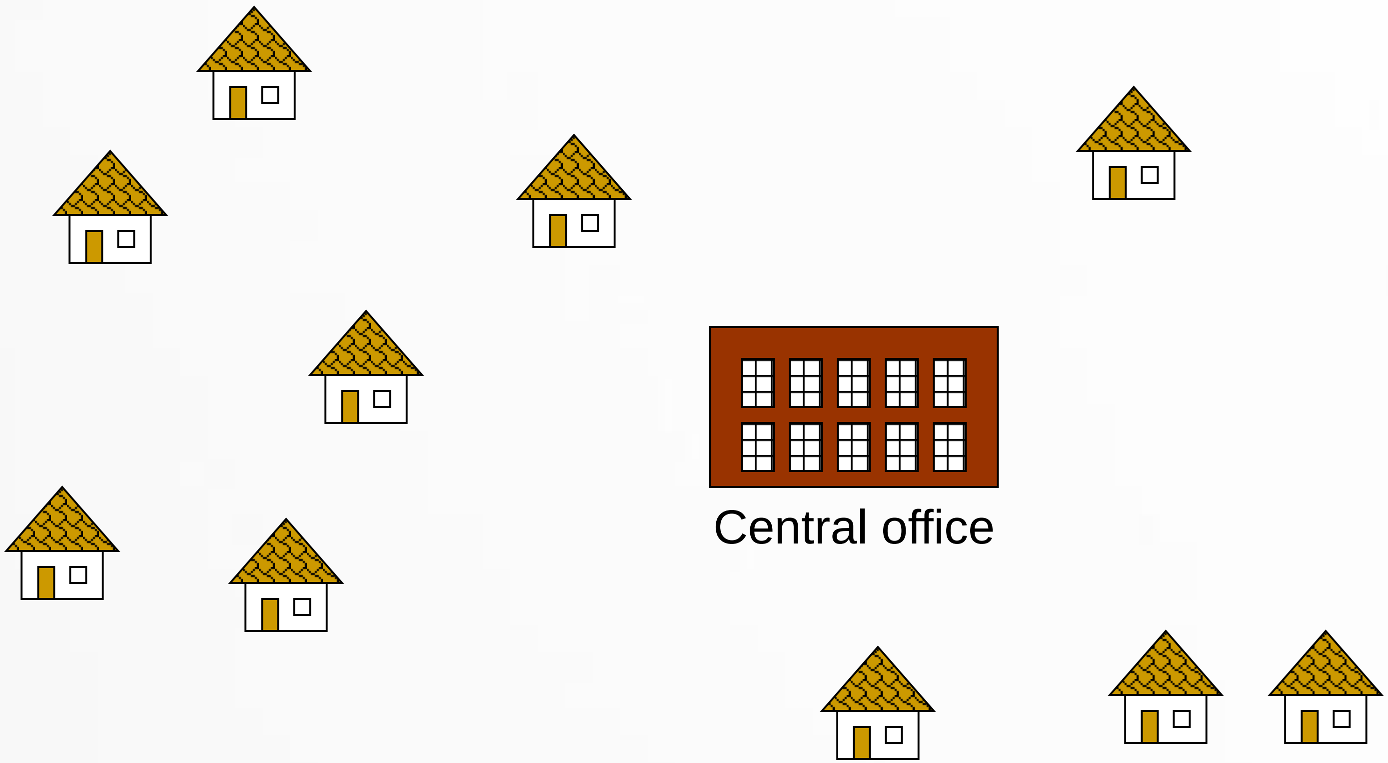
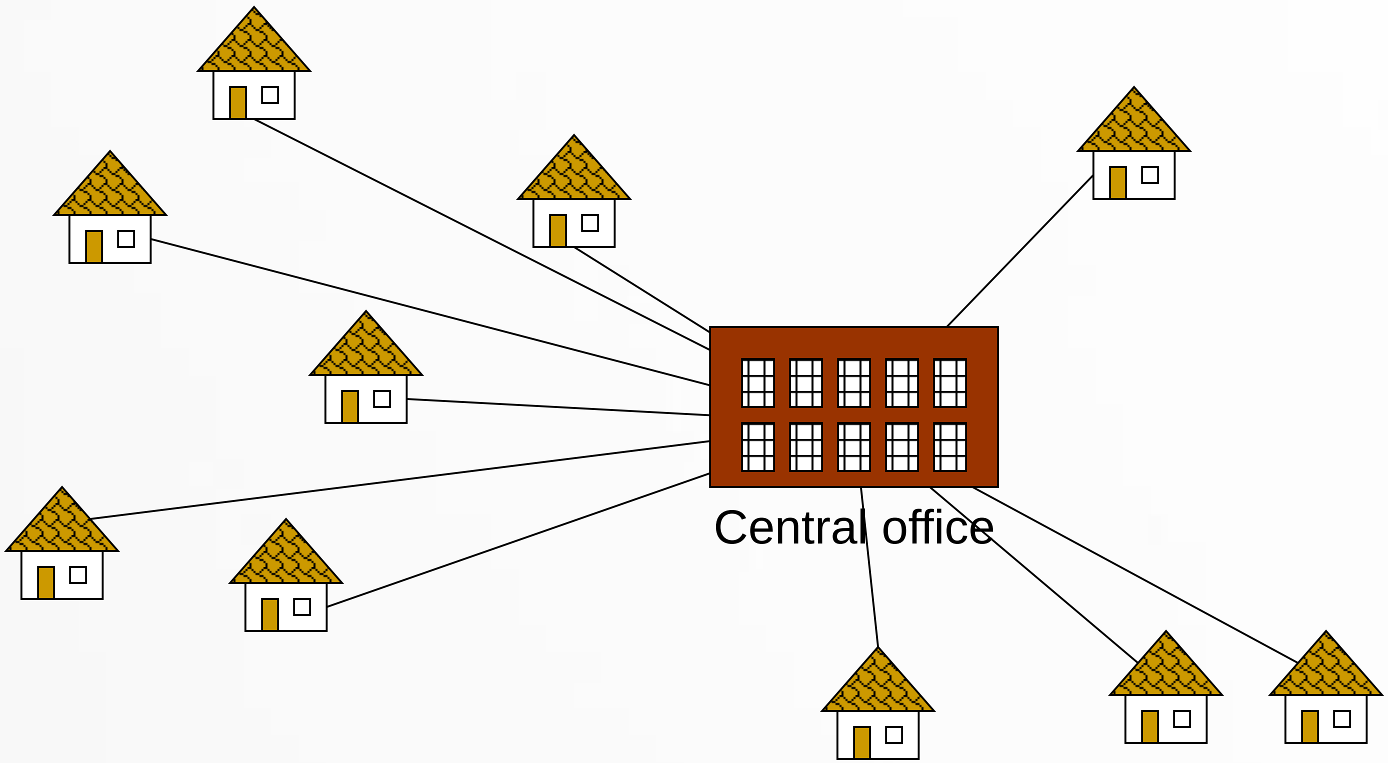
Spanning Trees



Spanning Trees



Expensive!

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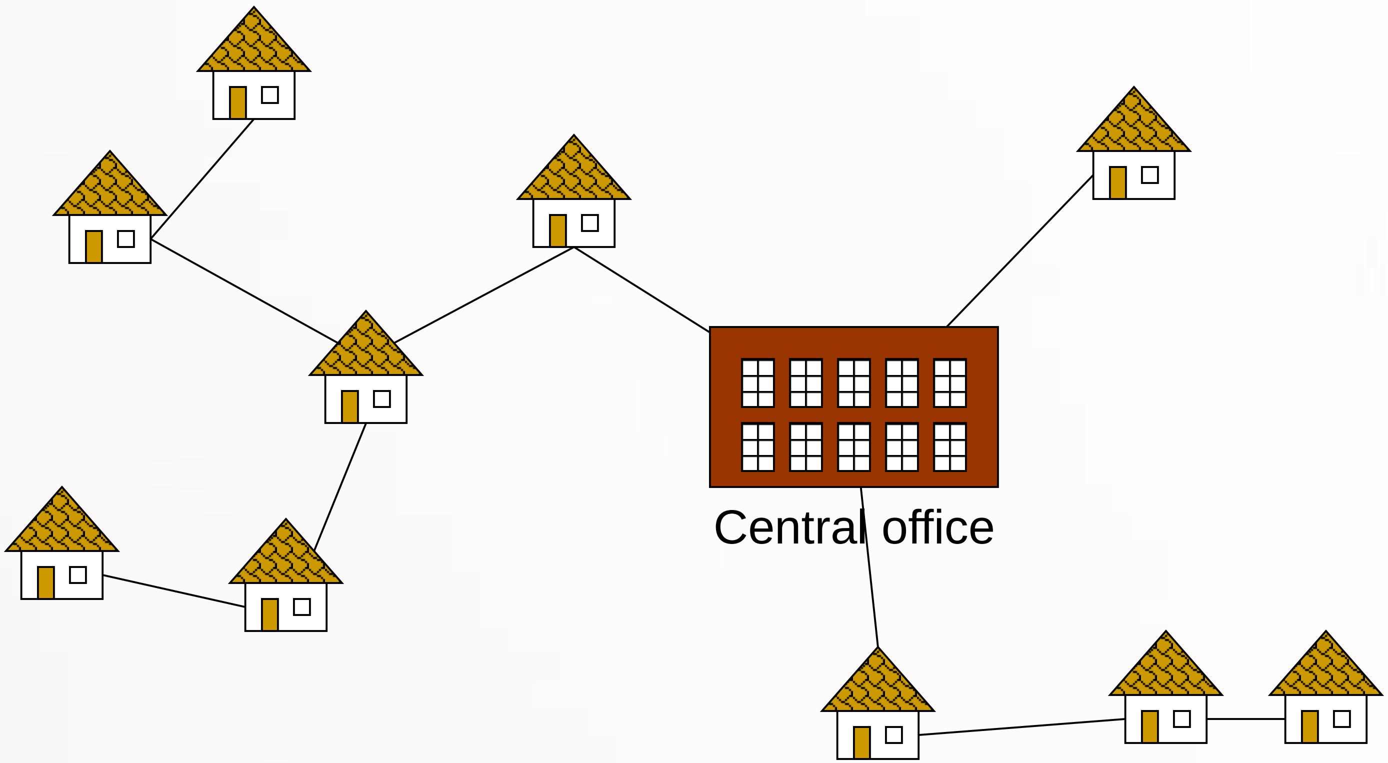
Spanning Trees



Minimize the total length of wire connecting the customers

Algorithms for Obtaining the Minimum Spanning Tree

Kruskal's Algorithm

Prim's Algorithm

Spanning Trees

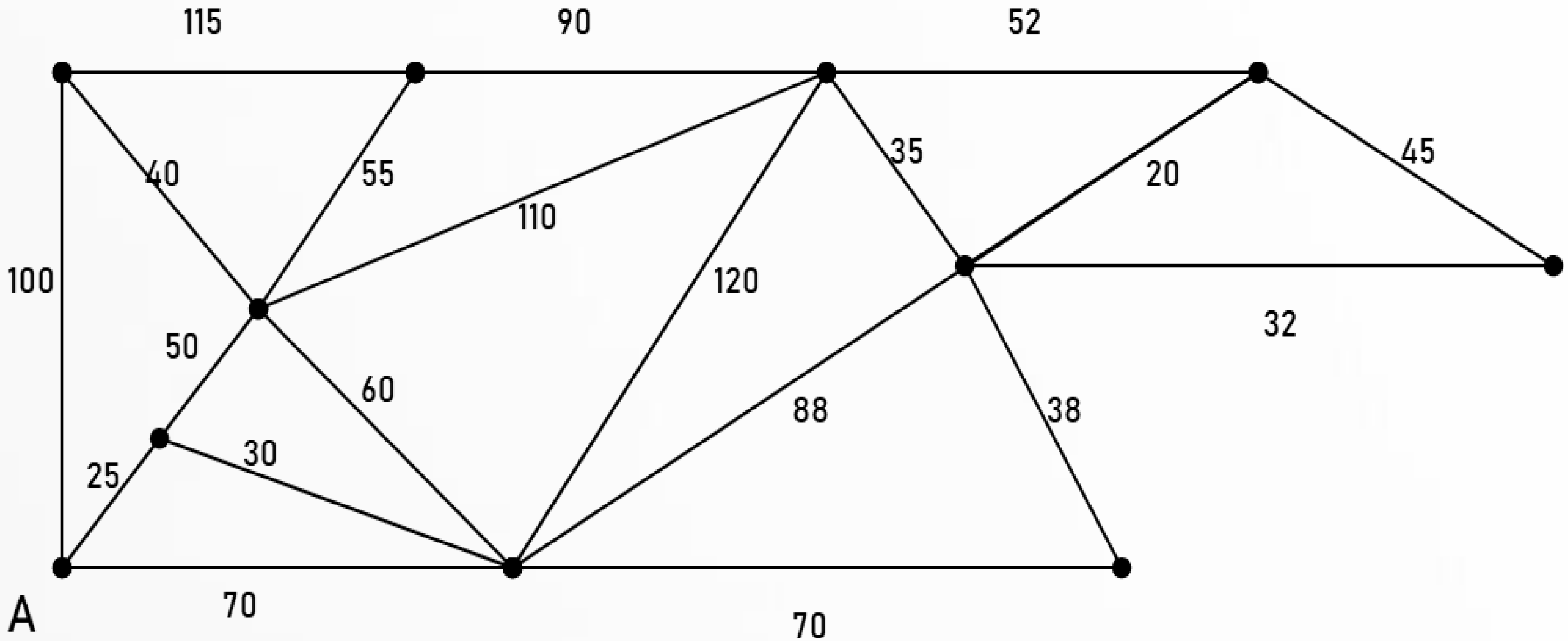
- There are two basic algorithms for finding minimum-cost spanning trees, and both are greedy algorithms
- Kruskal's algorithm: Start with no nodes or edges in the spanning tree, and repeatedly add the cheapest edge that does not create a cycle
    - Here, we consider the spanning tree to consist of edges only

Spanning Trees

- Prim's algorithm: Start with any one node in the spanning tree, and repeatedly add the cheapest edge, and the node it leads to, for which the node is not already in the spanning tree.
  - Here, we consider the spanning tree to consist of both nodes and edges

Kruskal's Algorithm

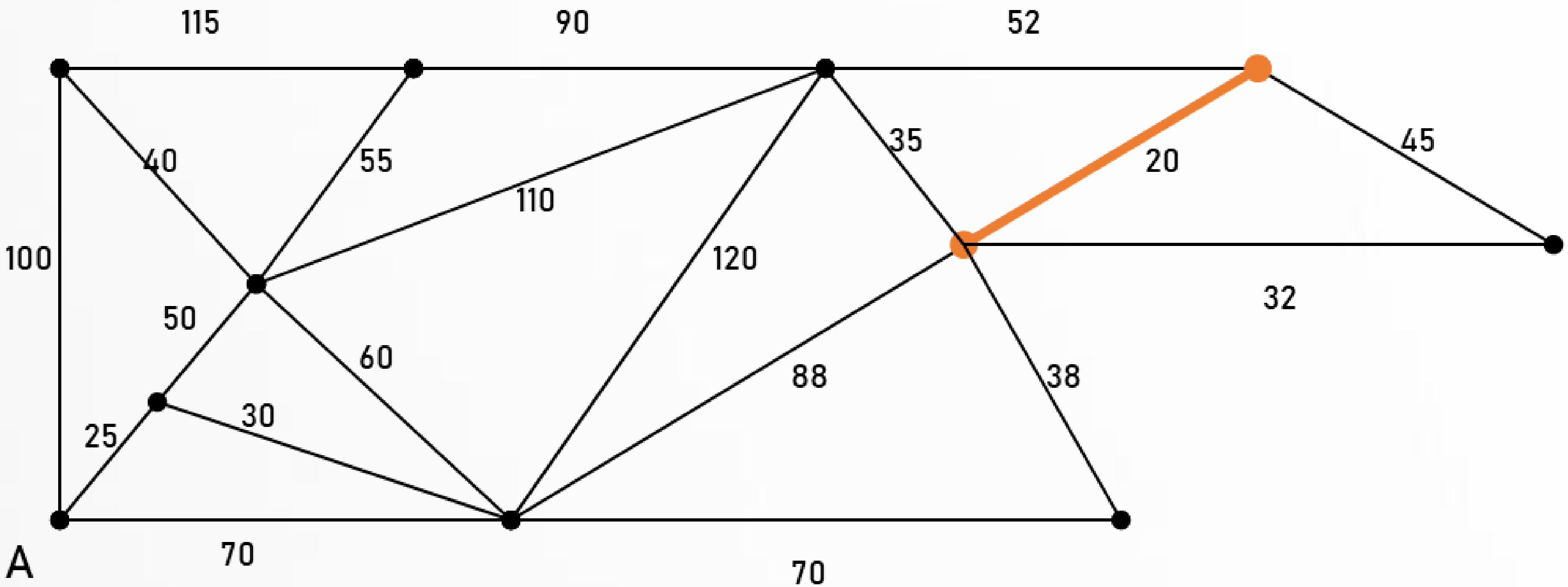
Find the minimum spanning tree using Kruskal's Algorithm.



List the edges in increasing order:

20, 25, 30, 32, 35, 38, 40, 45, 50, 52, 55, 60, 70, 70, 88, 90, 100, 110, 115, 120

Spanning Trees



Starting from the left, add the edge to the tree if it does not close up a circuit with the edges chosen up to that point: 20, 25, 30, 32, 35, 38, 40, 45, 50, 52, 55, 60, 70, 70, 88, 90, 100, 110, 115, 120

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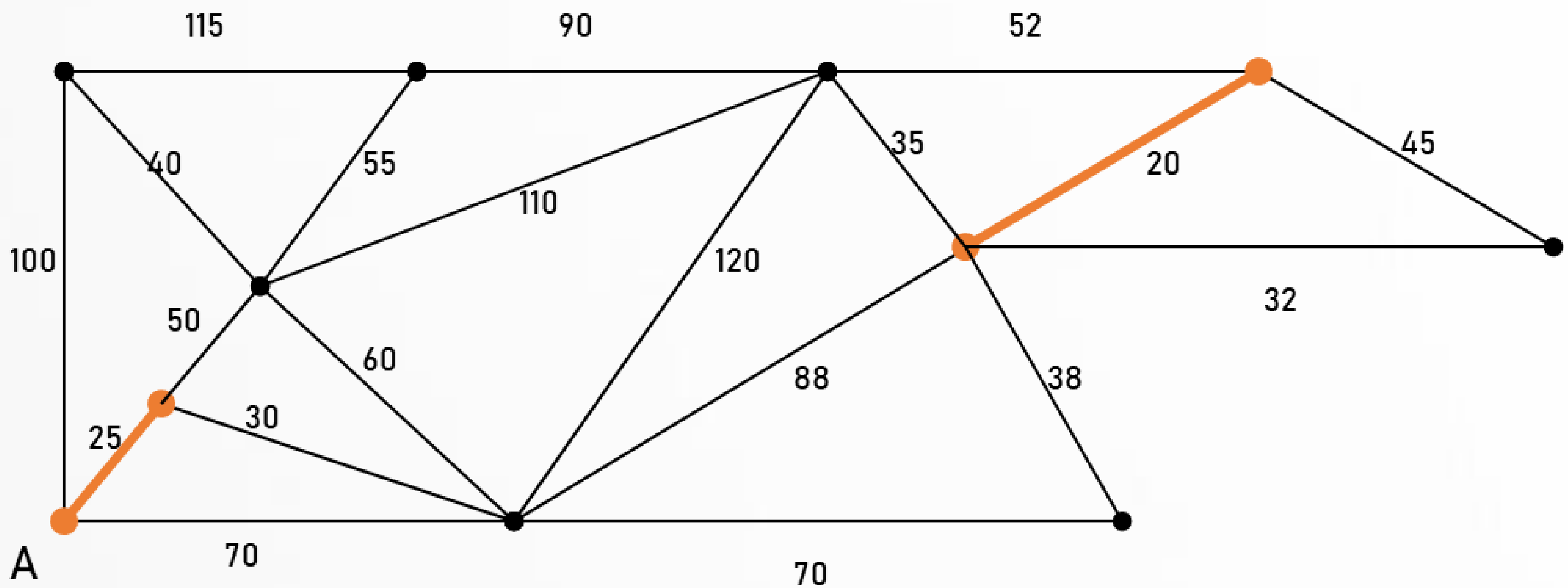
Kruskal's Algorithm



Add the next edge in the list to the tree if it does not close up a circuit with the edges chosen up to that point: 20, 25, 30, 32, 35, 38, 40, 45, 50, 52, 55, 60, 70, 70, 88, 90, 100, 110, 115, 120

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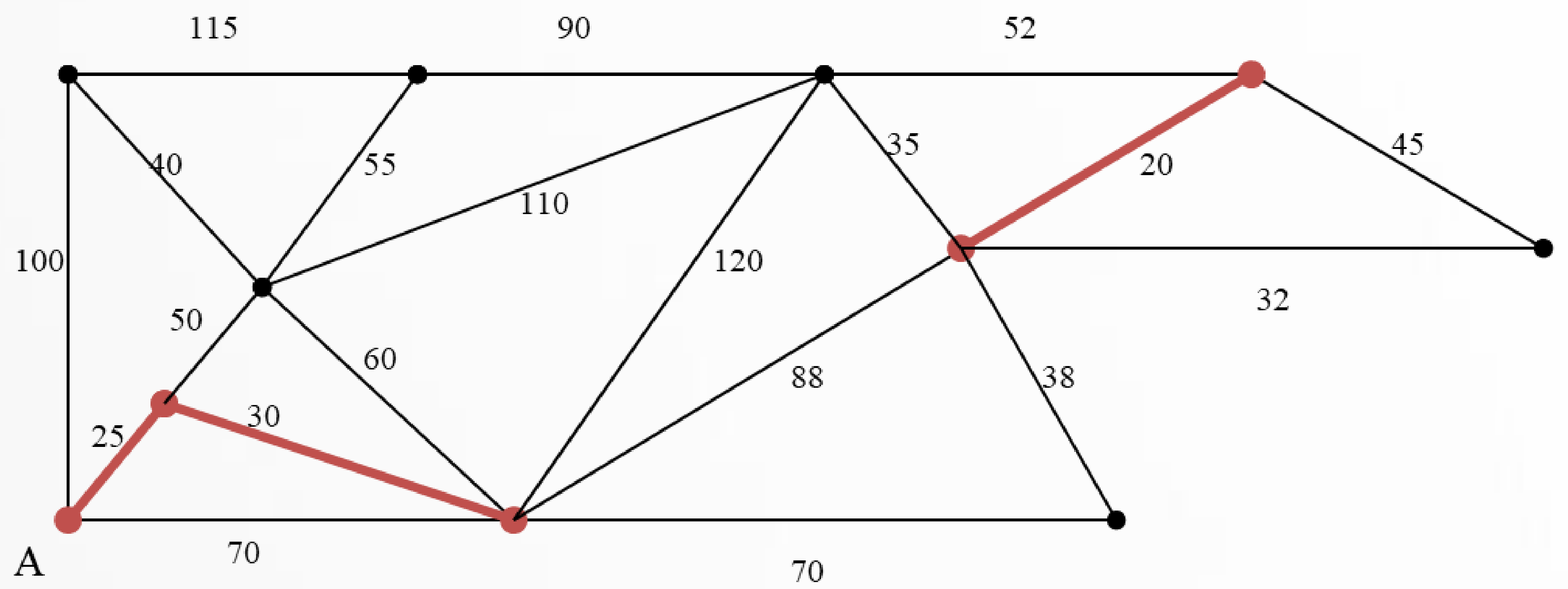
Kruskal's Algorithm



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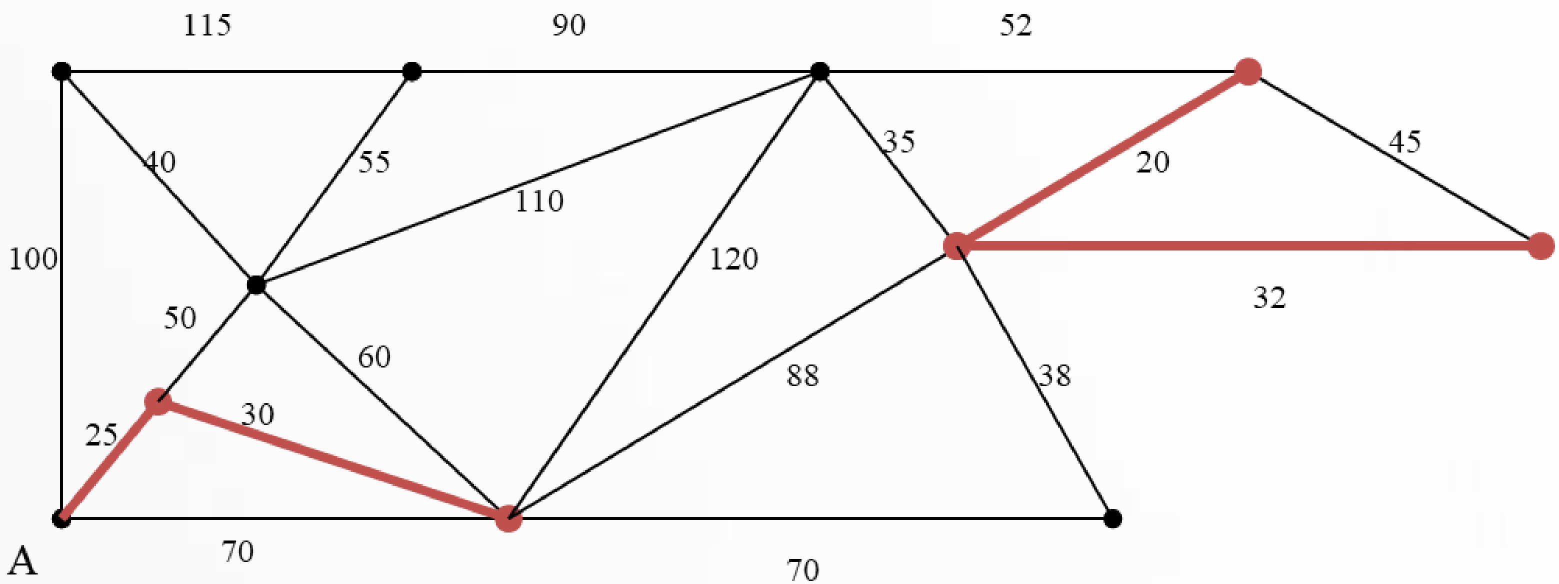
Kruskal's Algorithm



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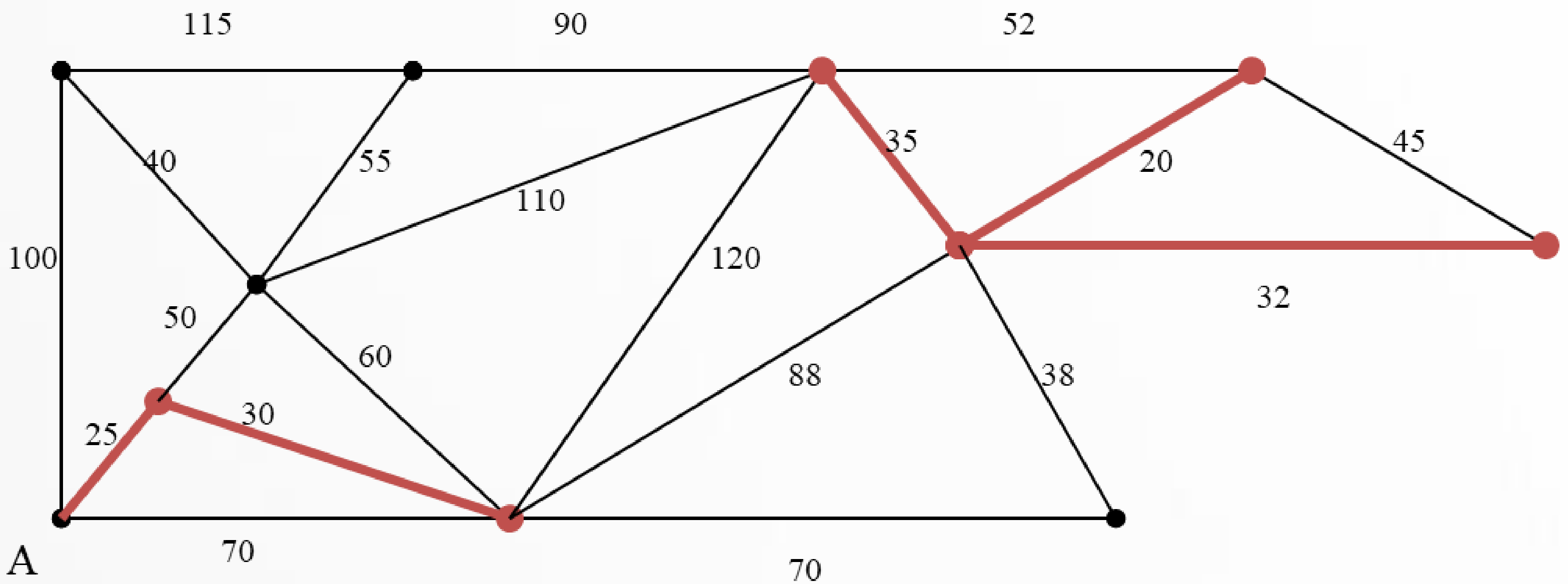
Kruskal's Algorithm



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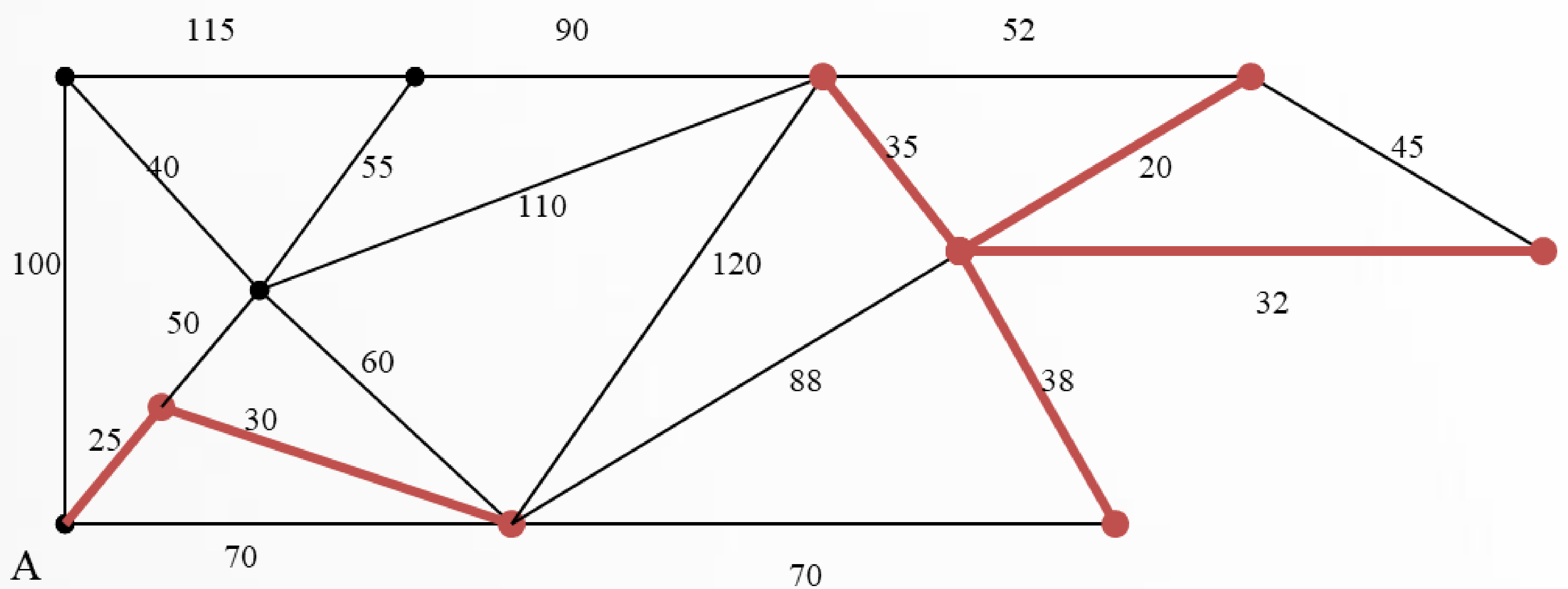
Kruskal's Algorithm



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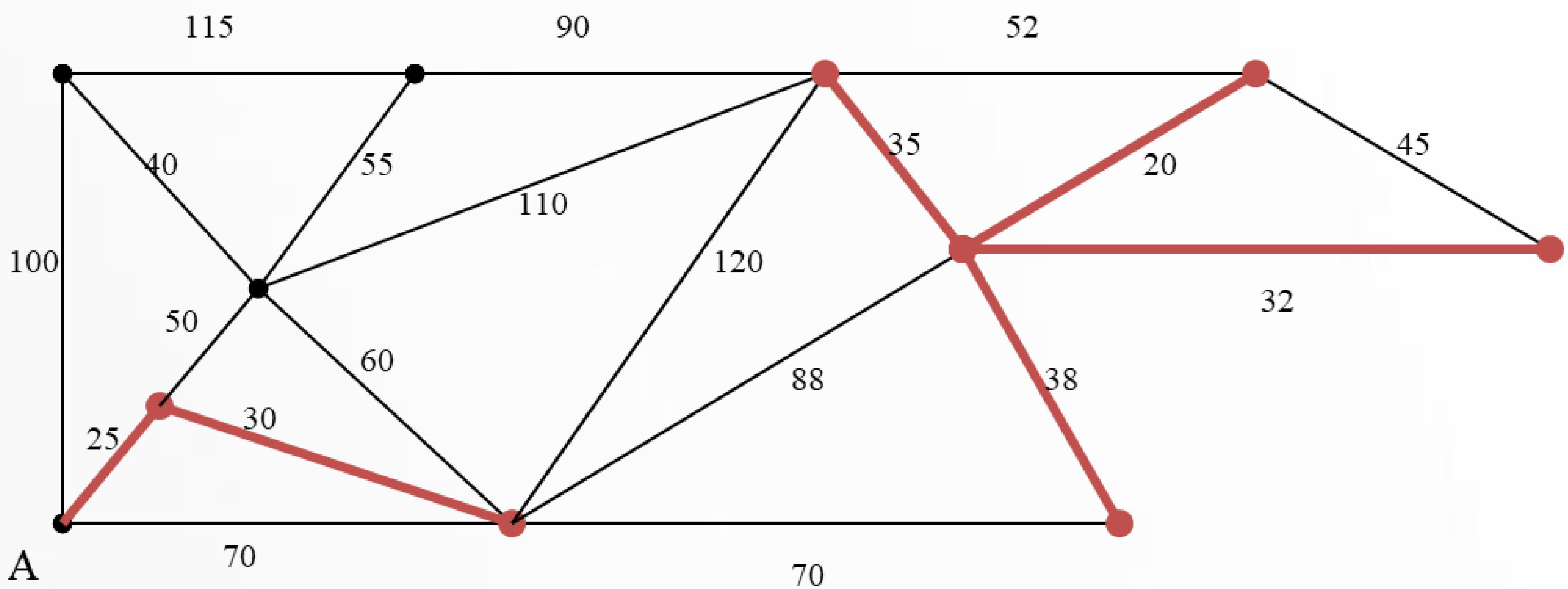
Kruskal's Algorithm



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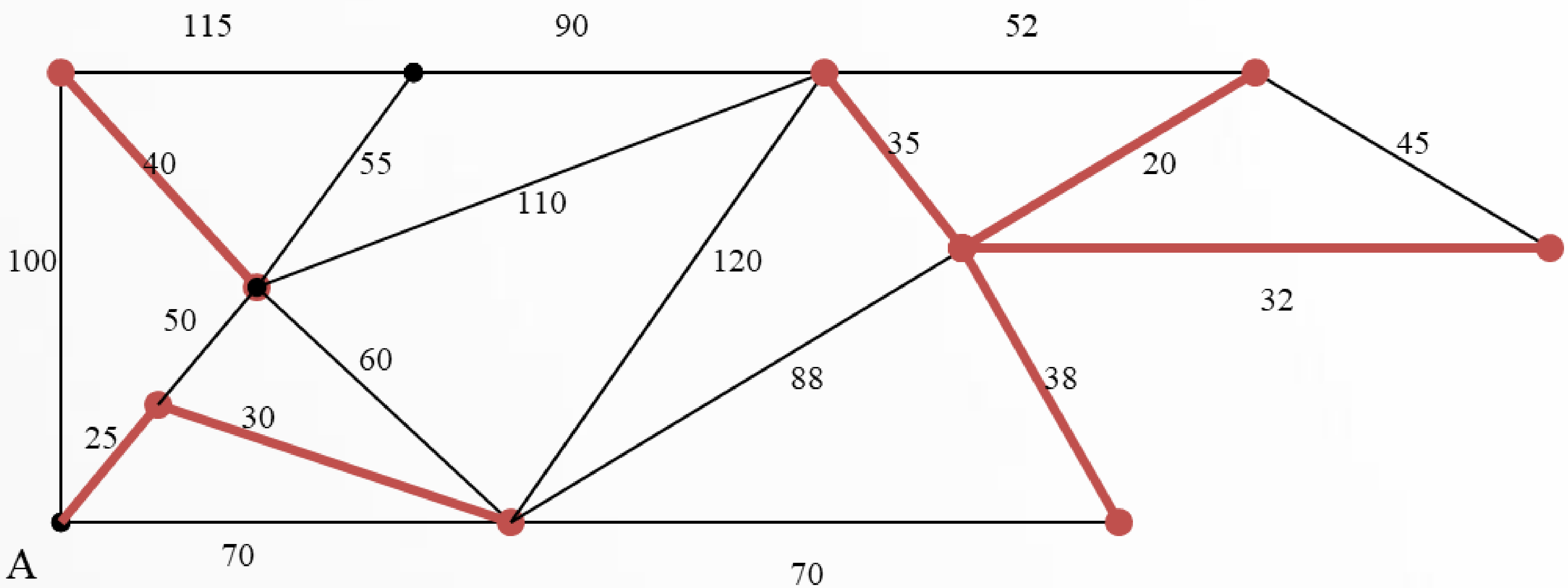
Kruskal's Algorithm



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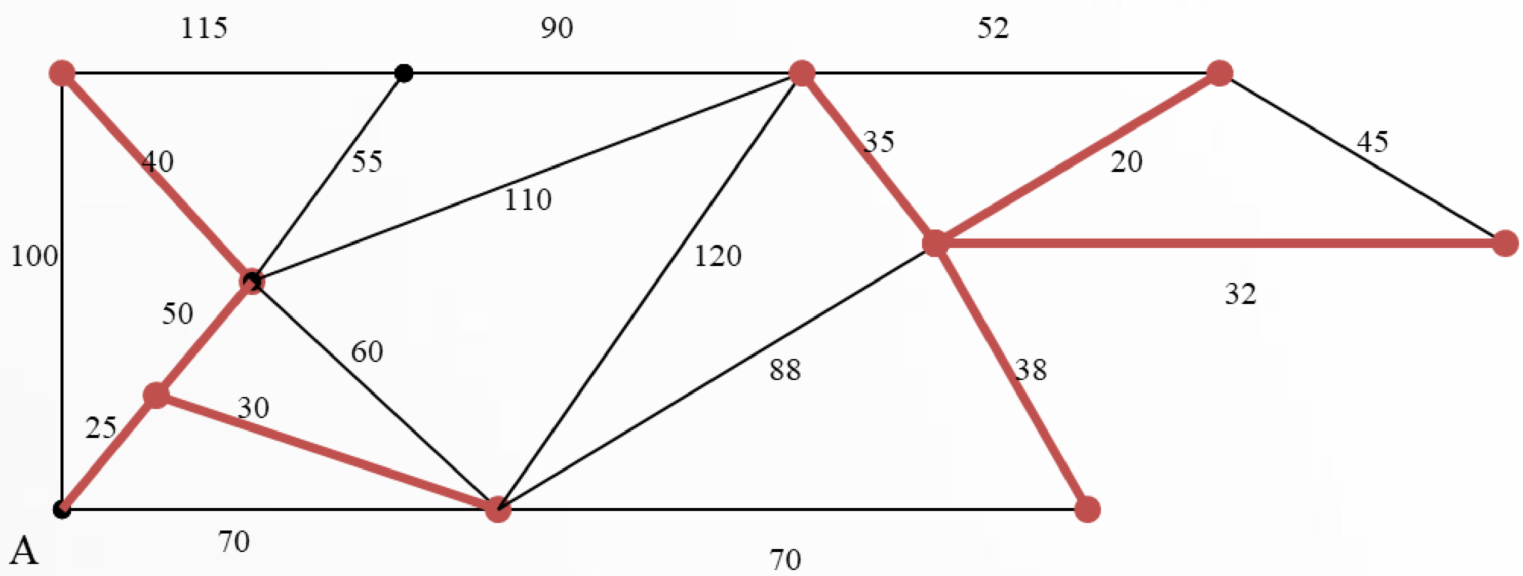
Kruskal's Algorithm



Add the next edge in the list to the tree if it does not close up a circuit with the edges chosen up to that point. Notice that the edge of weight 45 would close a circuit, so we skip it.

20, 25, 30, 32, 35, 38, 40, 45, 50, 52, 55, 60, 70, 70, 88, 90, 100, 110, 115, 120

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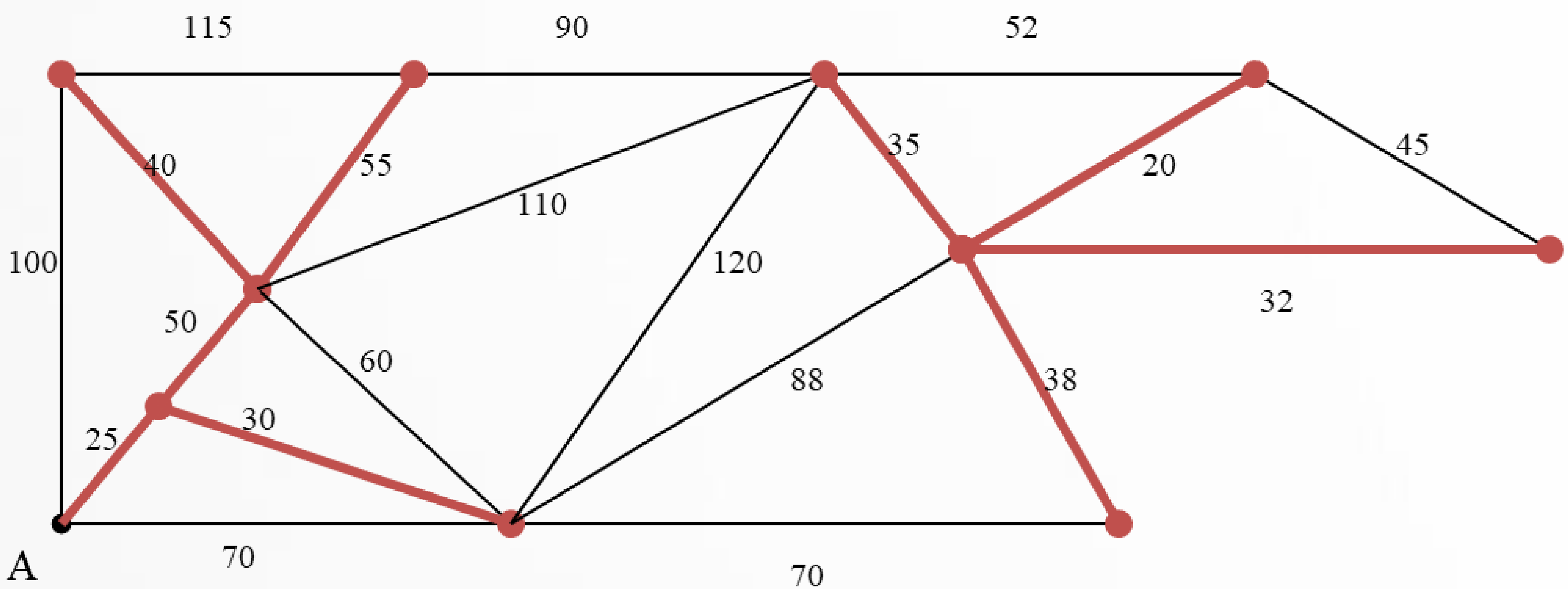
Kruskal's Algorithm



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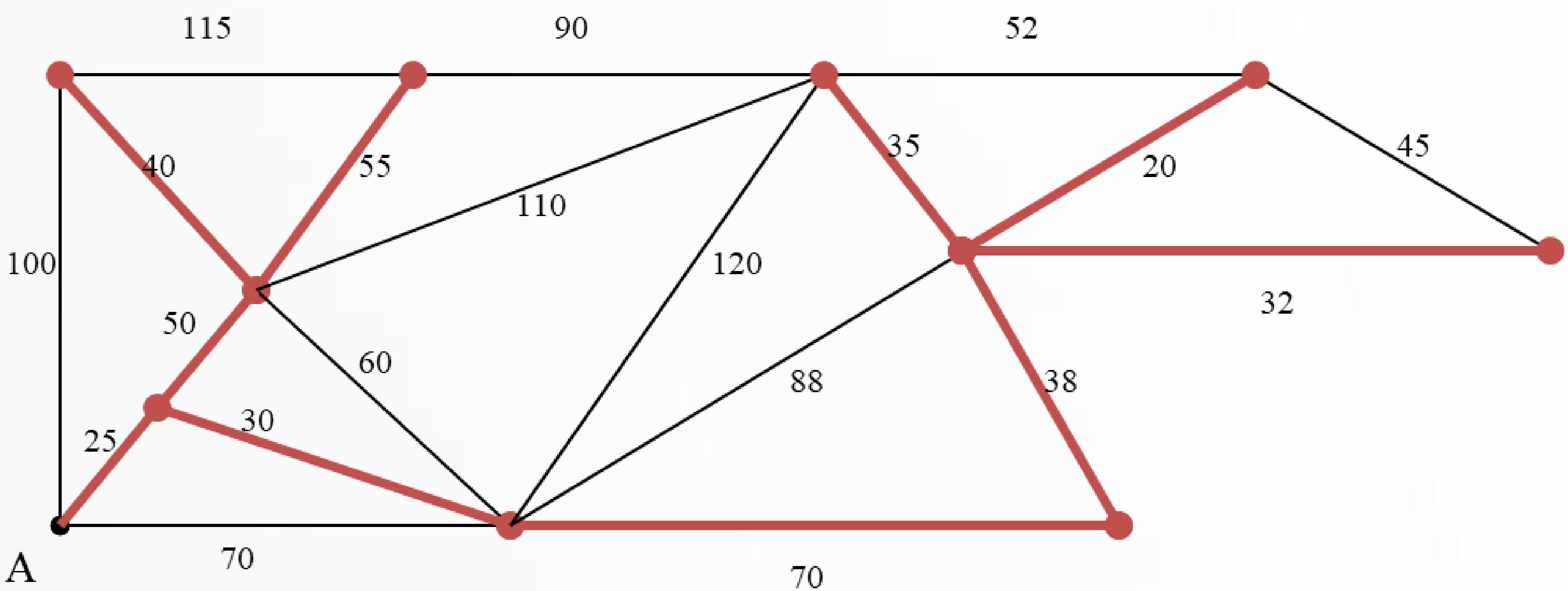
Kruskal's Algorithm



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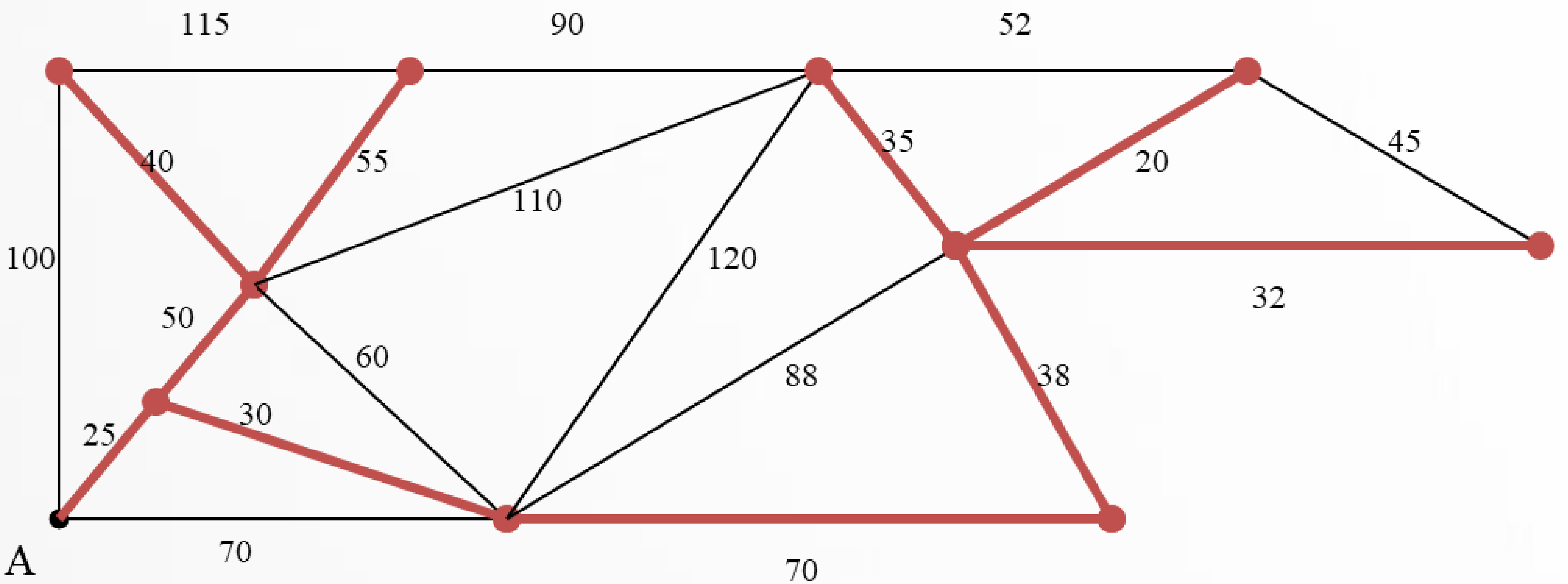
Kruskal's Algorithm

Done!



The tree contains every vertex, so it is a spanning tree. The total weight is 395

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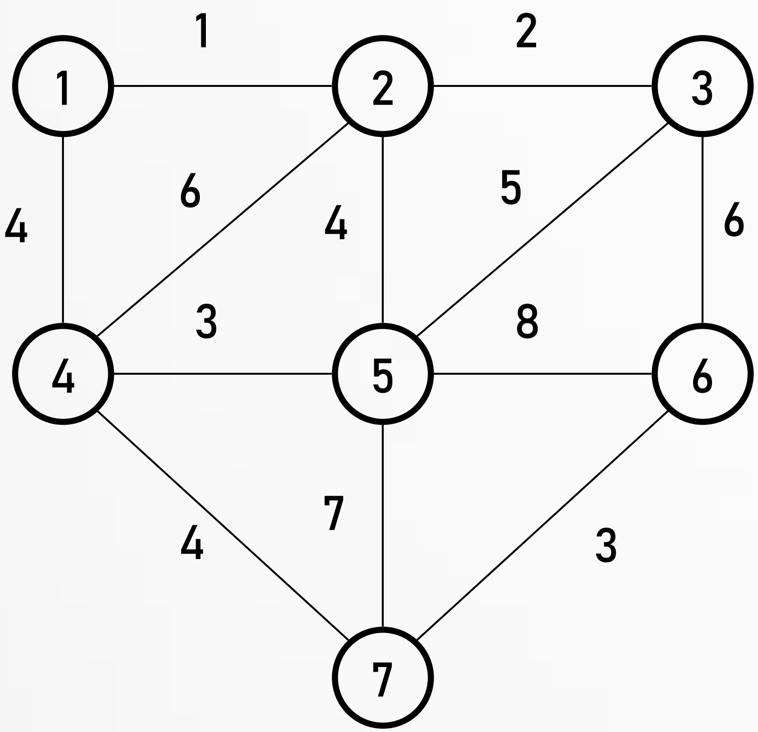
Kruskal's Algorithm



Make a disjoint set for each vertex

{1}{2}{3}{4}{5}{6}{7}

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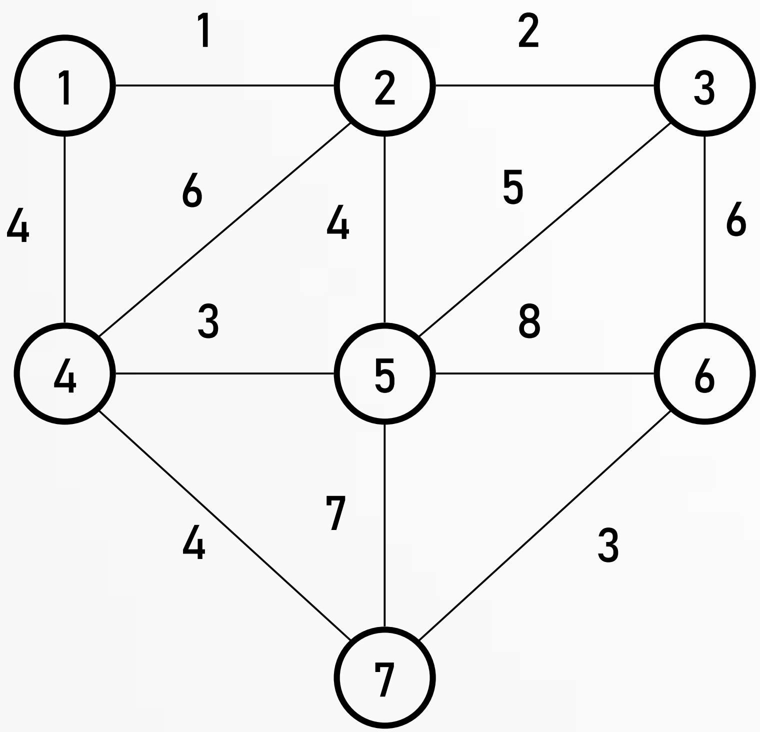
Kruskal's Algorithm



Sort edges by weight

1: {1,2}   {1}{2}{3}{4}{5}{6}{7}
2: {2,3}
3: {4,5}
3: {6,7}
4: {1,4}
4: {2,5}
4: {4,7}
5: {3,5}
⋮

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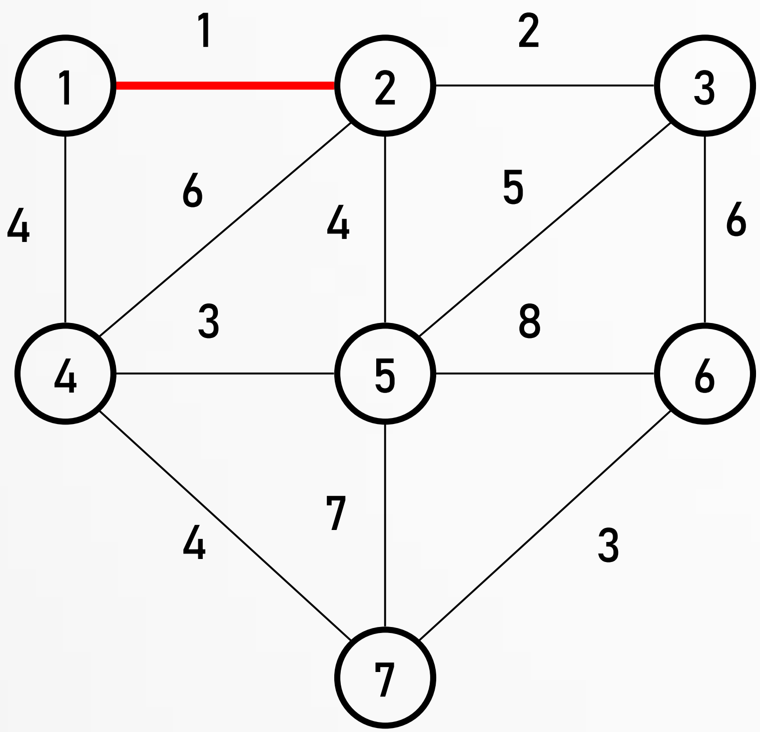
Kruskal's Algorithm



Add first edge to X if no cycle created

1: {1,2} {1}{2}{3}{4}{5}{6}{7}
2: {2,3}
3: {4,5}
3: {6,7}
4: {1,4}
4: {2,5}
4: {4,7}
5: {3,5}
⋮

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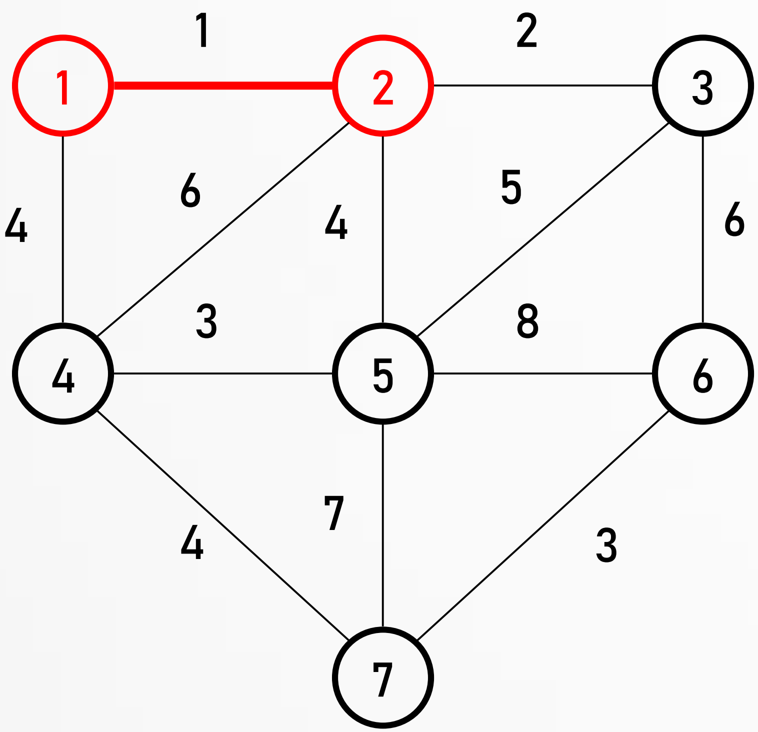
Kruskal's Algorithm



Merge vertices in added edges

1: {1,2}   {1,2}{3}{4}{5}{6}{7}
2: {2,3}
3: {4,5}
3: {6,7}
4: {1,4}
4: {2,5}
4: {4,7}
5: {3,5}
⋮

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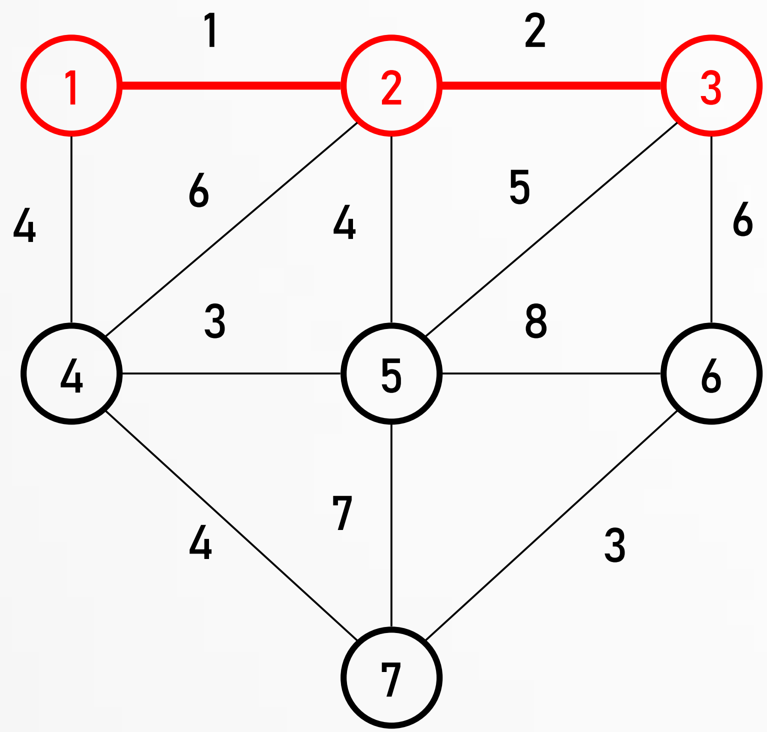
Kruskal's Algorithm



Process each edge in order

1: {1,2} {1,2}{3}{4}{5}{6}{7}
2: {2,3} {1,2,3}{4}{5}{6}{7}
3: {4,5}
3: {6,7}
4: {1,4}
4: {2,5}
4: {4,7}
5: {3,5}
⋮

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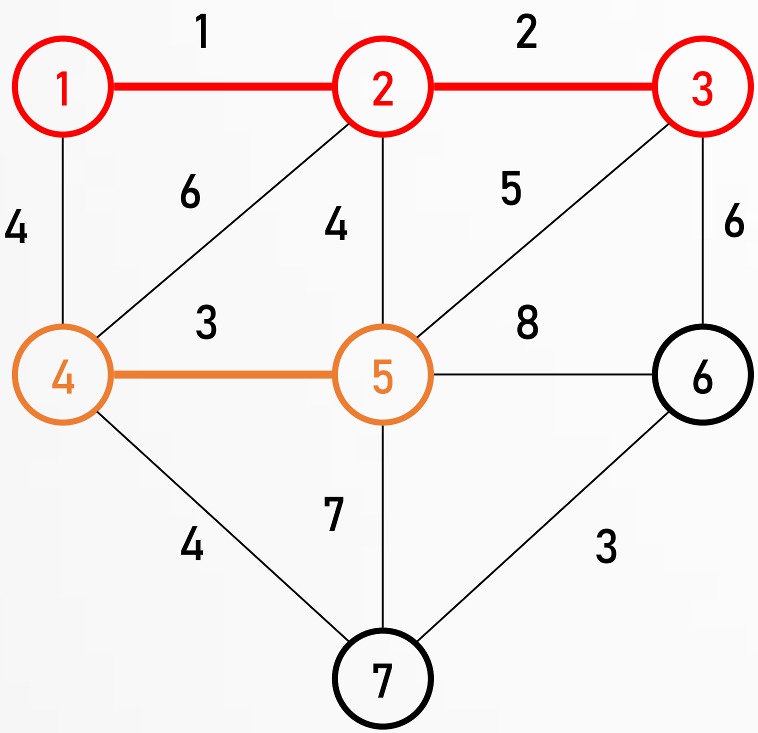
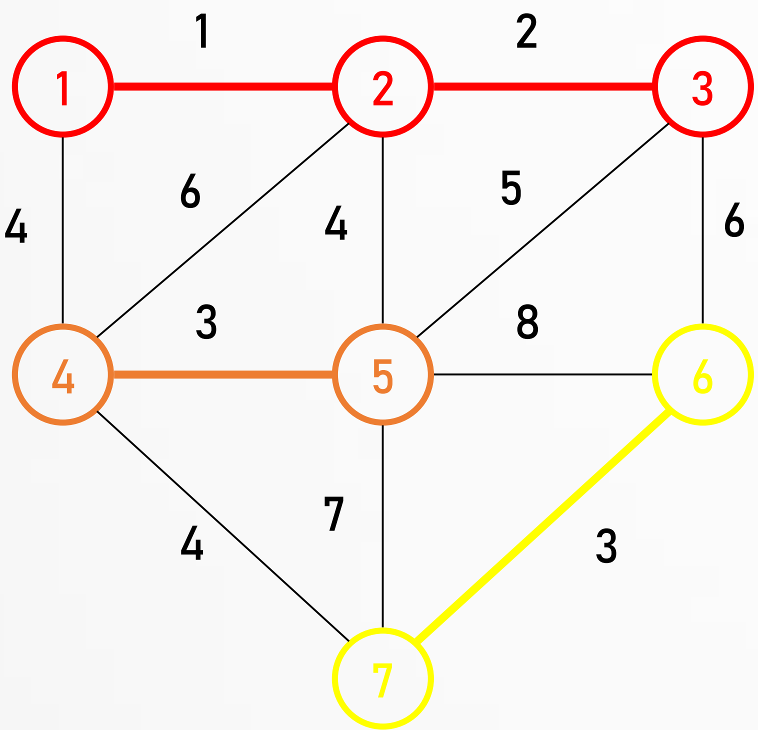


1: {1,2} {1,2}{3}{4}{5}{6}{7}
2: {2,3} {1,2,3}{4}{5}{6}{7}
3: {4,5} {1,2,3}{4,5}{6}{7}
3: {6,7}
4: {1,4}
4: {2,5}
4: {4,7}
5: {3,5}
⋮

Note that each set is a connected component of *G*

Kruskal's Algorithm



1: {1,2} {1,2}{3}{4}{5}{6}{7}
2: {2,3} {1,2,3}{4}{5}{6}{7}
3: {4,5} {1,2,3}{4,5}{6}{7}
3: {6,7} {1,2,3}{4,5}{6,7}
4: {1,4}
4: {2,5}
4: {4,7}
5: {3,5}
⋮

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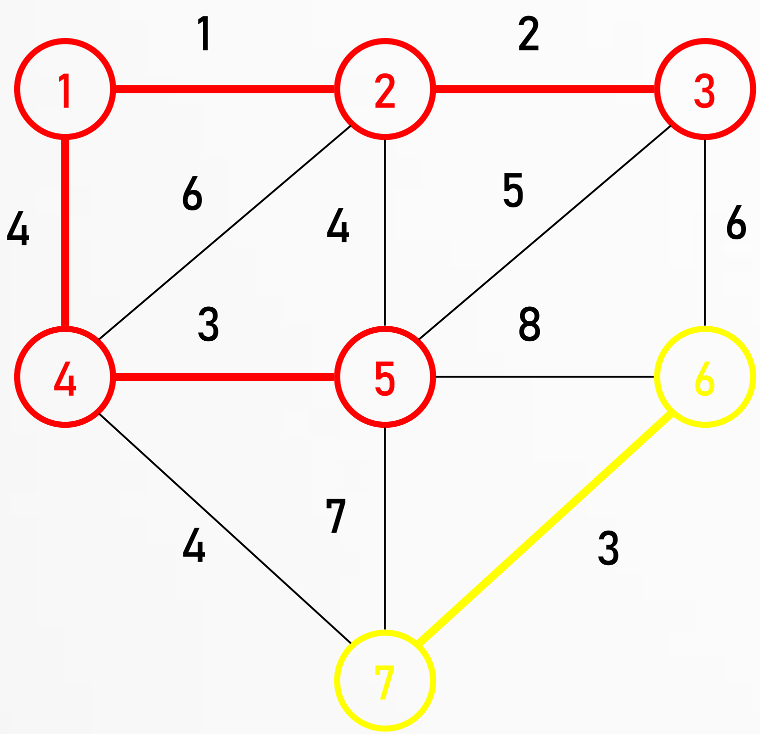
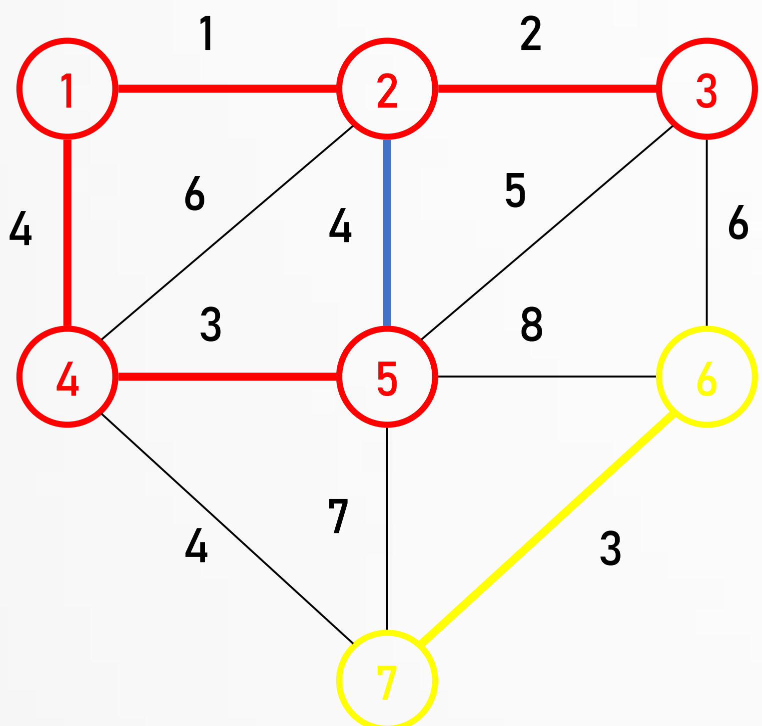


1: {1,2} {1,2}{3}{4}{5}{6}{7}
2: {2,3} {1,2,3}{4}{5}{6}{7}
3: {4,5} {1,2,3}{4,5}{6}{7}
3: {6,7} {1,2,3}{4,5}{6,7}
4: {1,4} {1,2,3,4,5}{6,7}
4: {2,5}
4: {4,7}
5: {3,5}
⋮

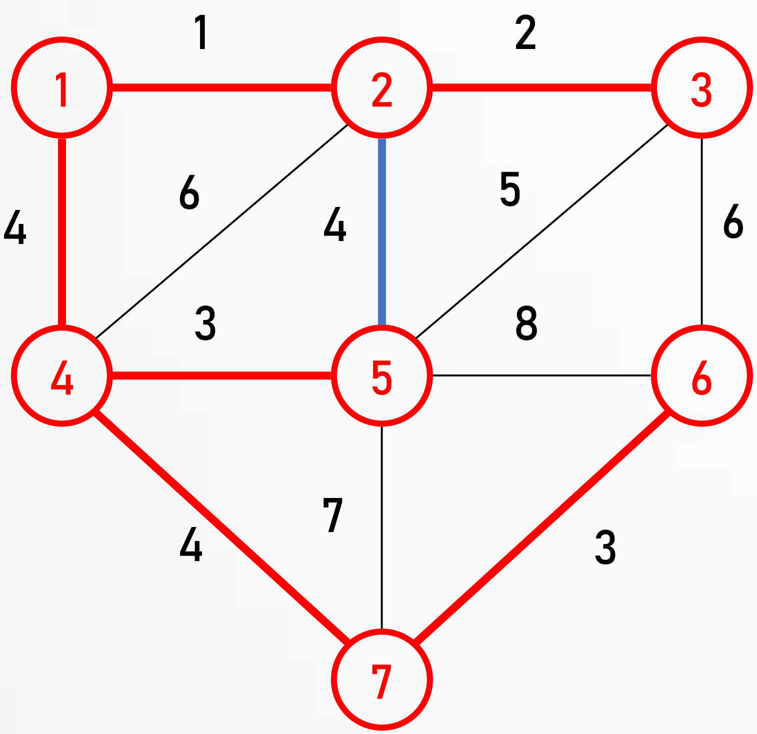
Kruskal's Algorithm



Must join separate components

1: {1,2} {1,2}{3}{4}{5}{6}{7}
2: {2,3} {1,2,3}{4}{5}{6}{7}
3: {4,5} {1,2,3}{4,5}{6}{7}
3: {6,7} {1,2,3}{4,5}{6,7}
4: {1,4} {1,2,3,4,5}{6,7}
4: {2,5} rejected
4: {4,7}
5: {3,5}
⋮

Kruskal's Algorithm



Done when all vertices in one set
Then they are all connected
Exactly |V| - 1 edges

1: {1,2} {1,2}{3}{4}{5}{6}{7}
2: {2,3} {1,2,3}{4}{5}{6}{7}
3: {4,5} {1,2,3}{4,5}{6}{7}
3: {6,7} {1,2,3}{4,5}{6,7}
4: {1,4} {1,2,3,4,5}{6,7}
4: {2,5} rejected
4: {4,7} {1,2,3,4,5,6,7} done
5: {3,5}
⋮

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That's all for now...