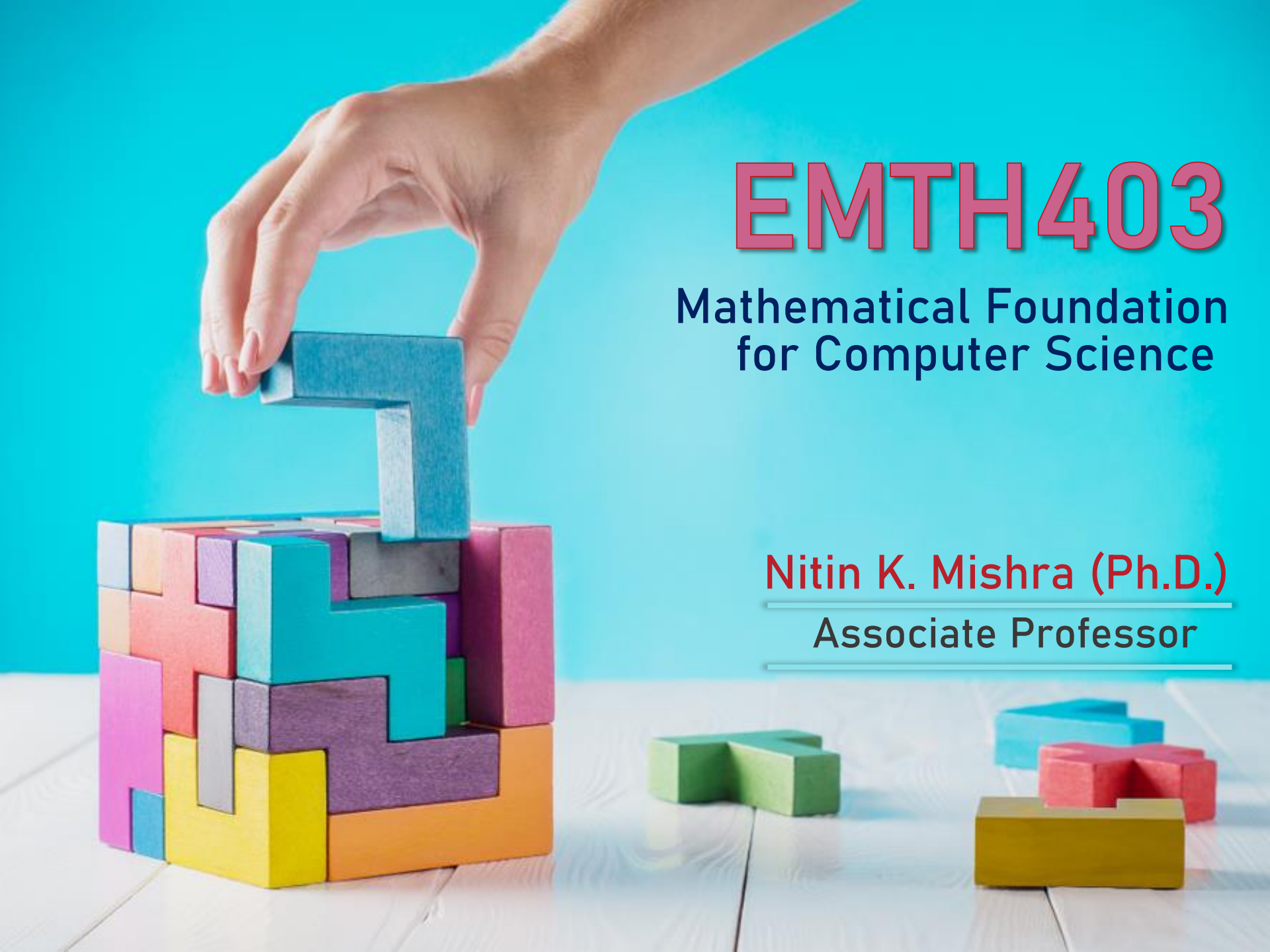


EMTH403

Mathematical Foundation for Computer Science

Nitin K. Mishra (Ph.D.)

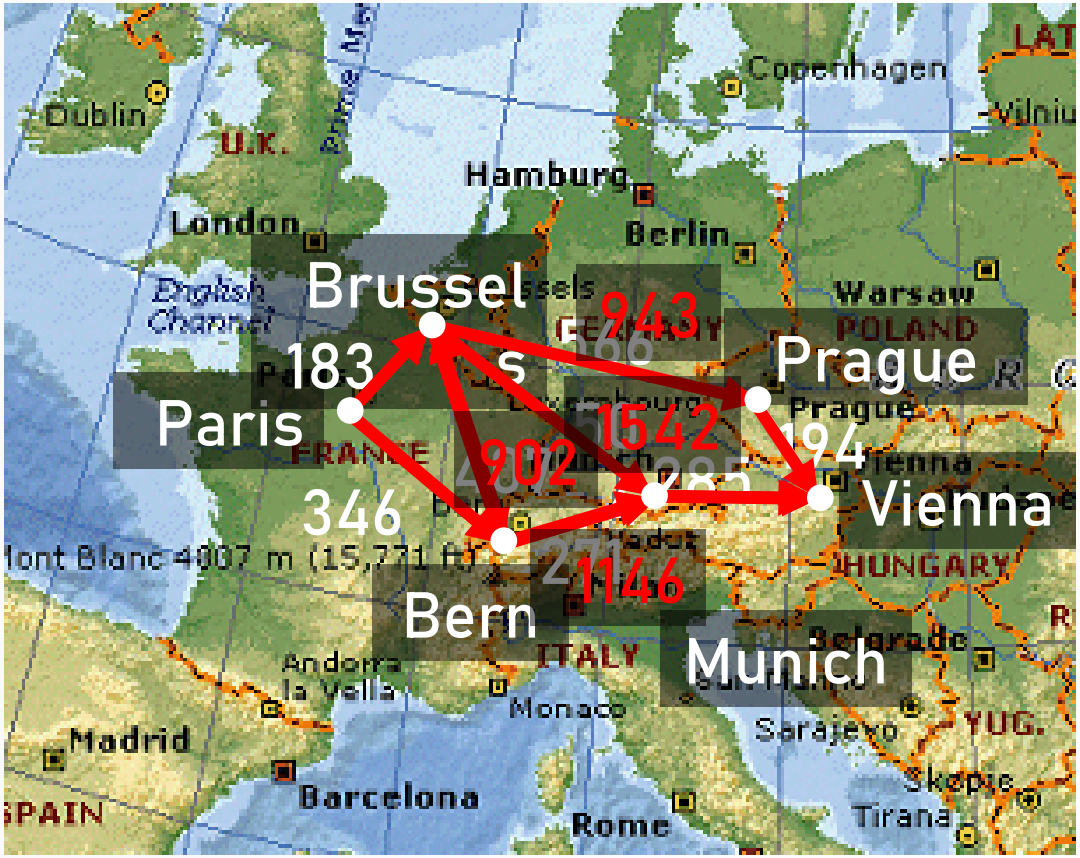
Associate Professor

Lecture Outcomes

After this lecture, you will be able to

- understand what are Shortest-Path Problems.
- understand what is a Shortest-Path Algorithm.
- understand what is Dijkstra's Algorithm.

Shortest Path Problem



Shortest Path Problem

- Many problems can be modeled using graphs with weights assigned to their edges.

- As an illustration, consider how an airline system can be modeled. We set up the basic graph model by representing cities by vertices and flights by edges.

Shortest Path Problem

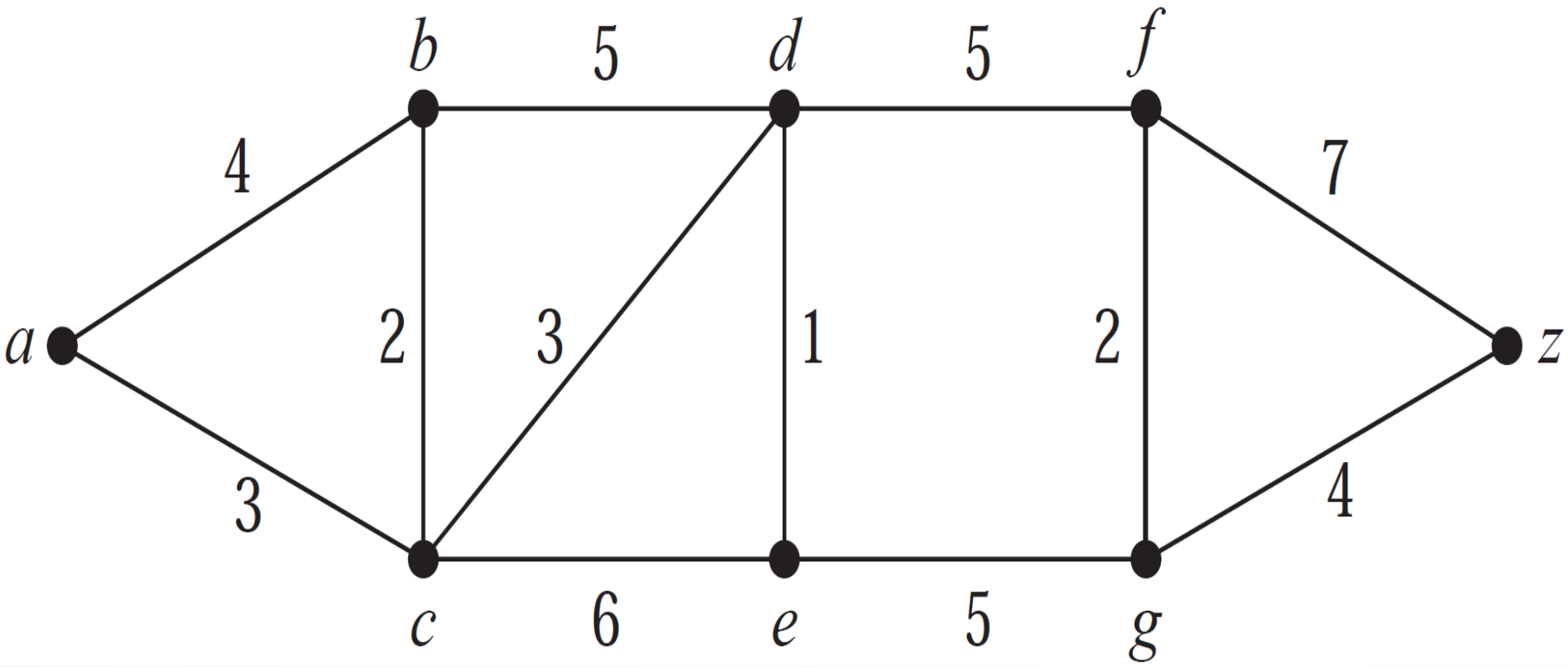
- Problems involving distances can be modeled by assigning distances between cities to the edges.

- Problems involving flight time can be modeled by assigning flight times to edges.

- Problems involving fares can be modeled by assigning fares to the edges.

Shortest Path Problem

Graphs that have a number assigned to each edge are called weighted graphs.



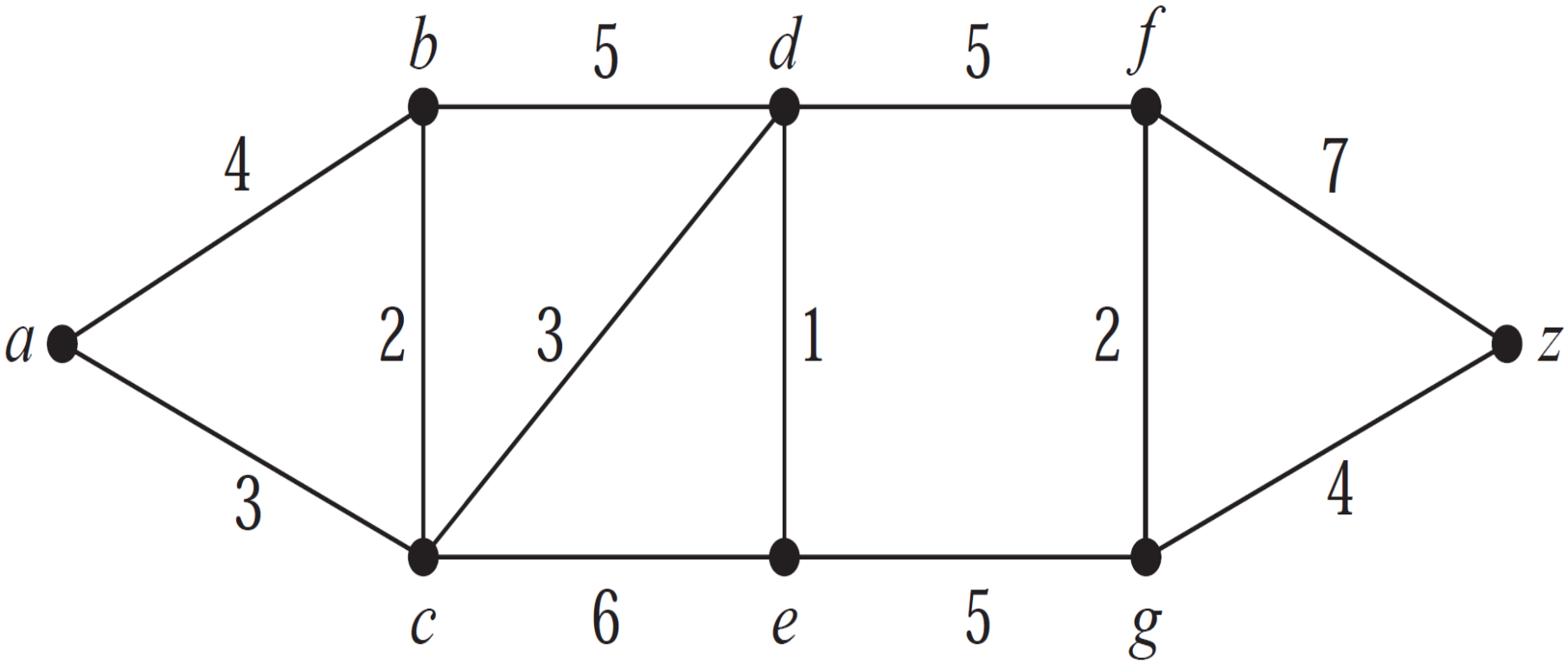
Shortest Path Problem

- There are several different algorithms that find a shortest path between two vertices in a weighted graph.

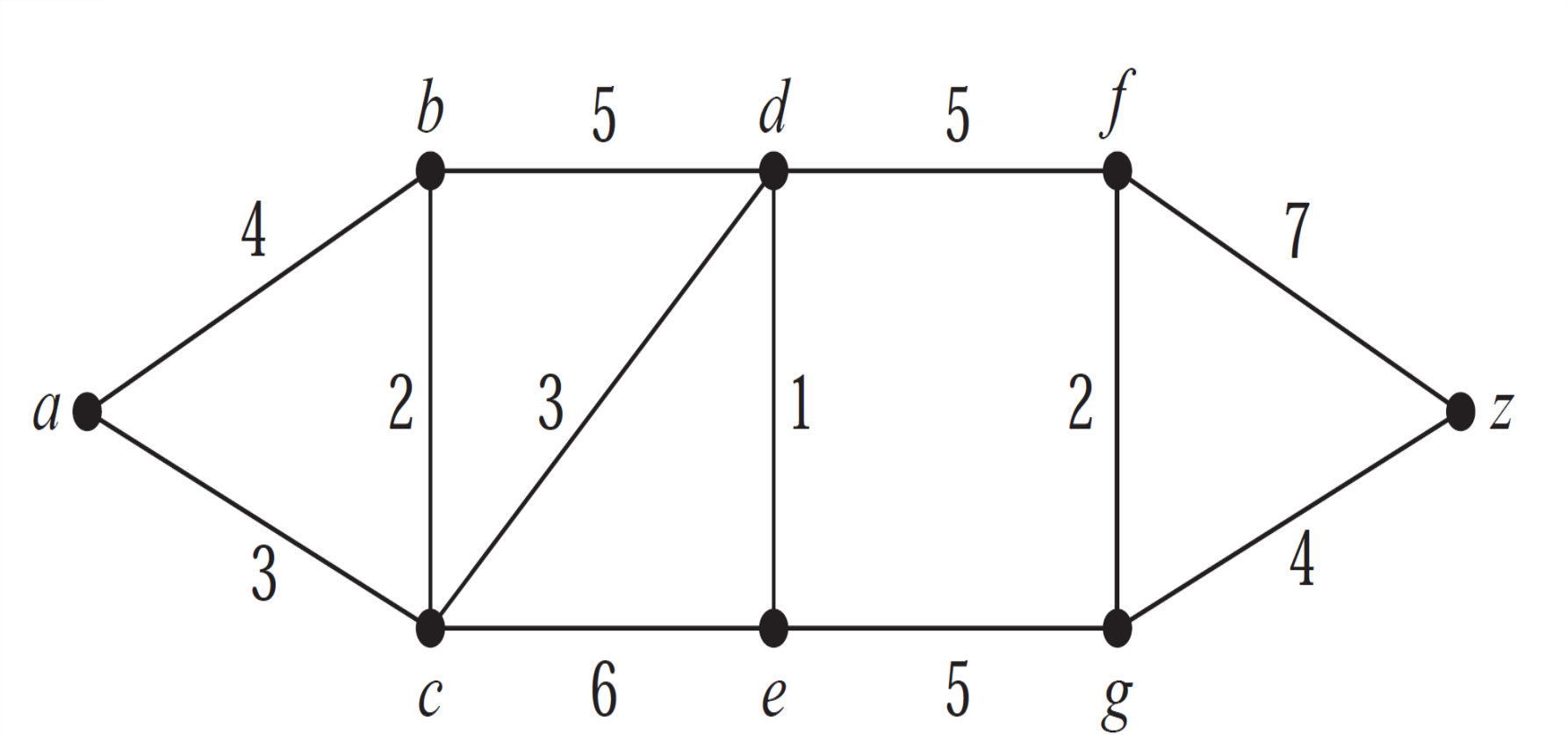
- We will present a greedy algorithm discovered by the Dutch mathematician Edsger Dijkstra in 1959.

Dijkstra's Algorithm

Find the length of a shortest path between a and z in the given weighted graph.



Dijkstra's Algorithm



Dijkstra's Algorithm

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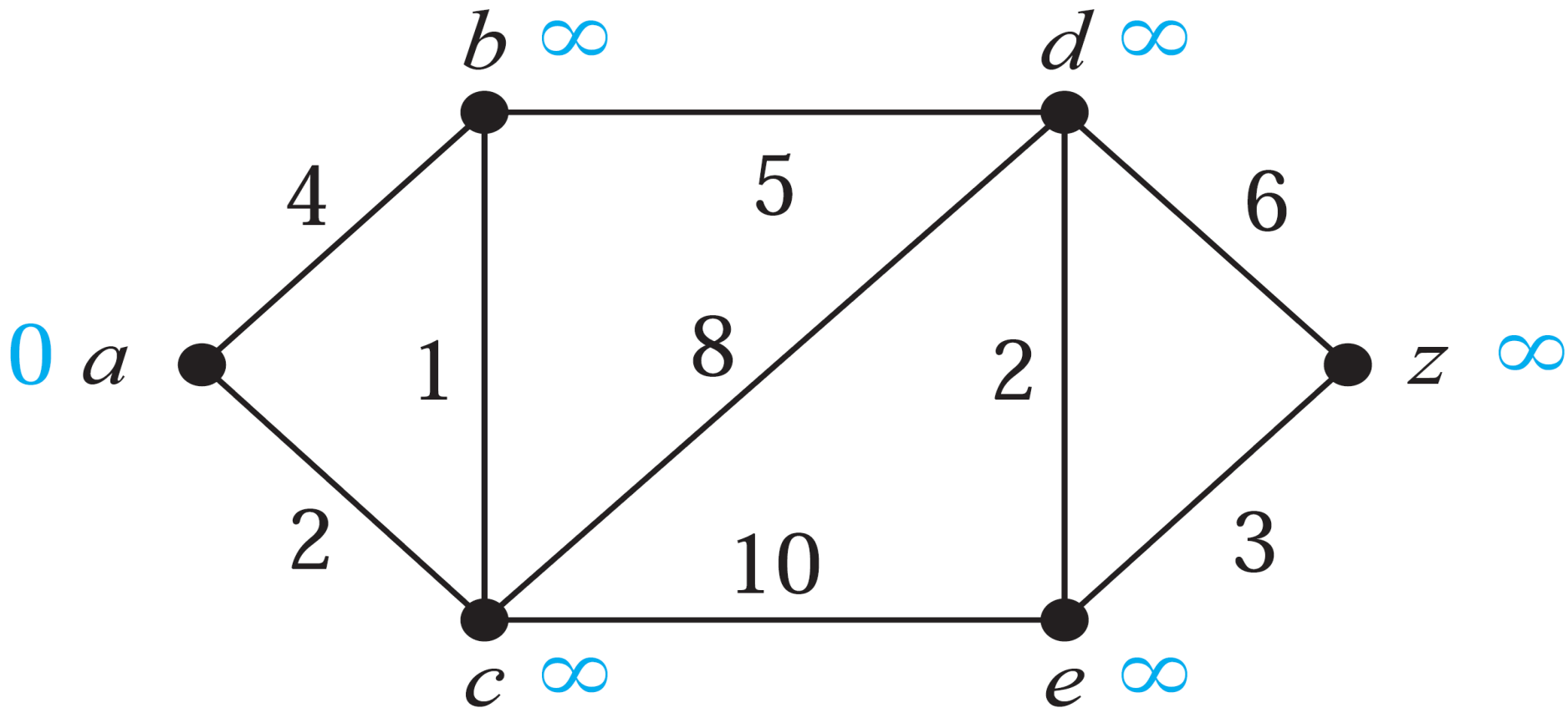

Dijkstra's Algorithm

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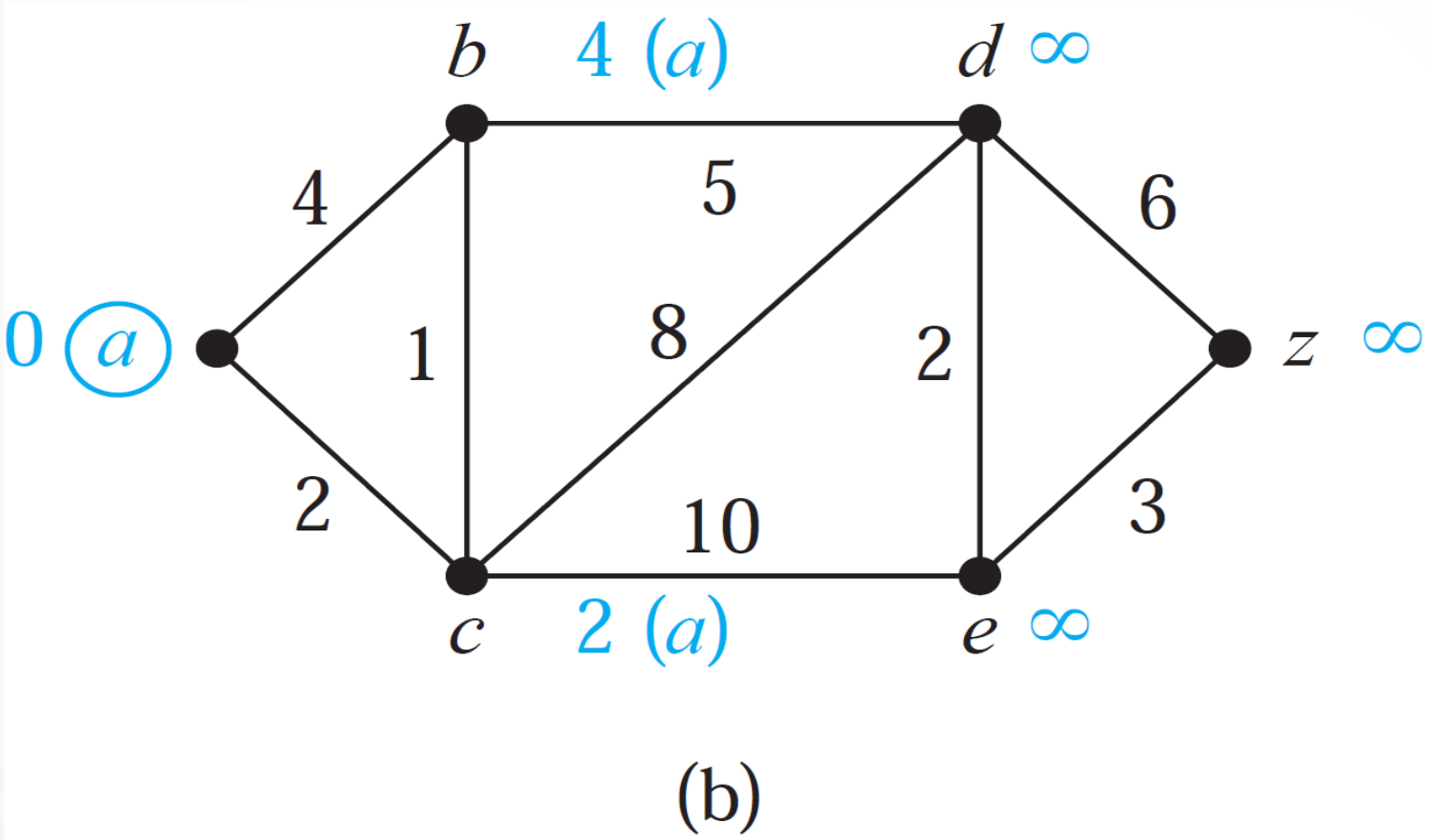
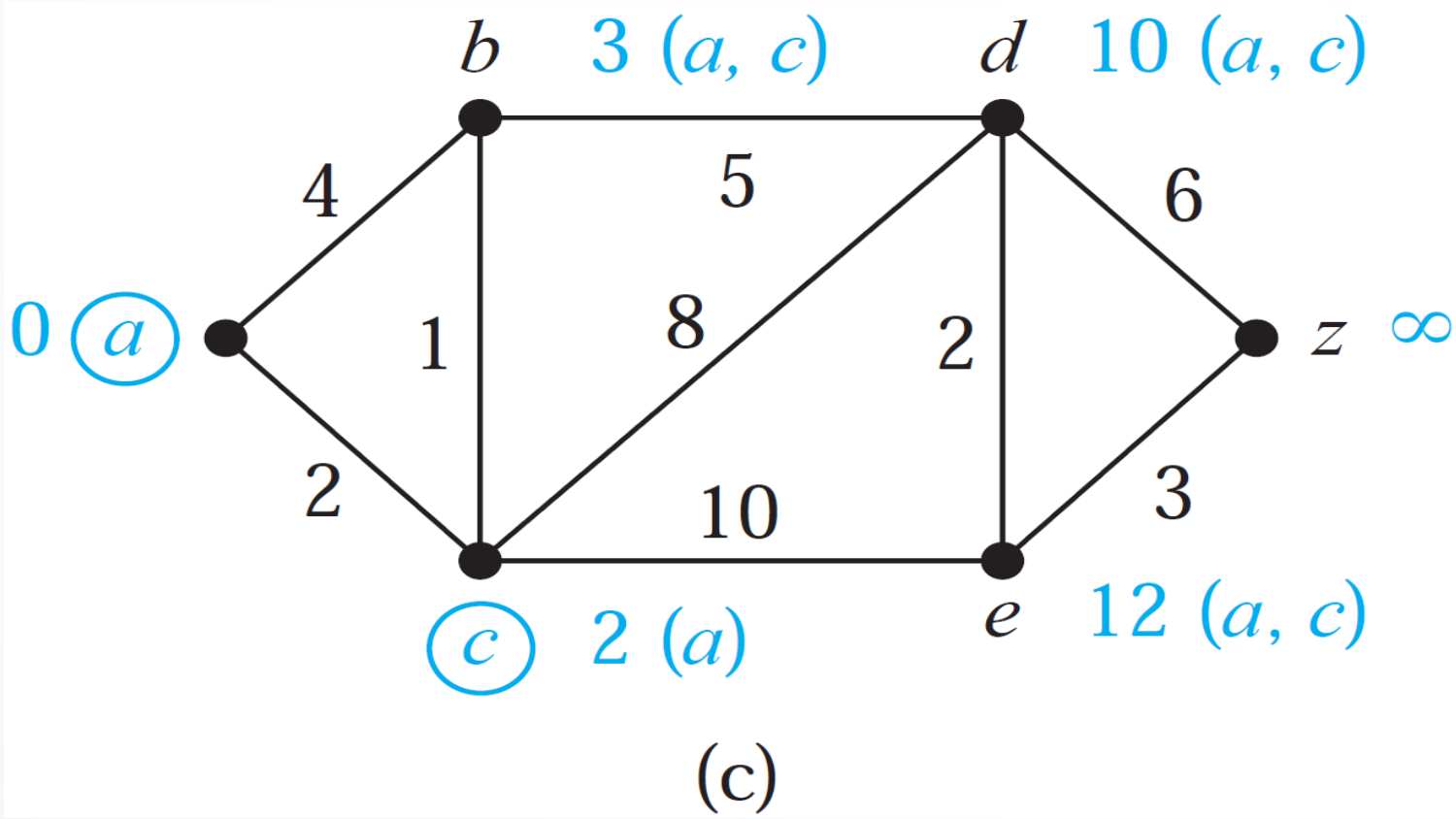


Dijkstra's Algorithm

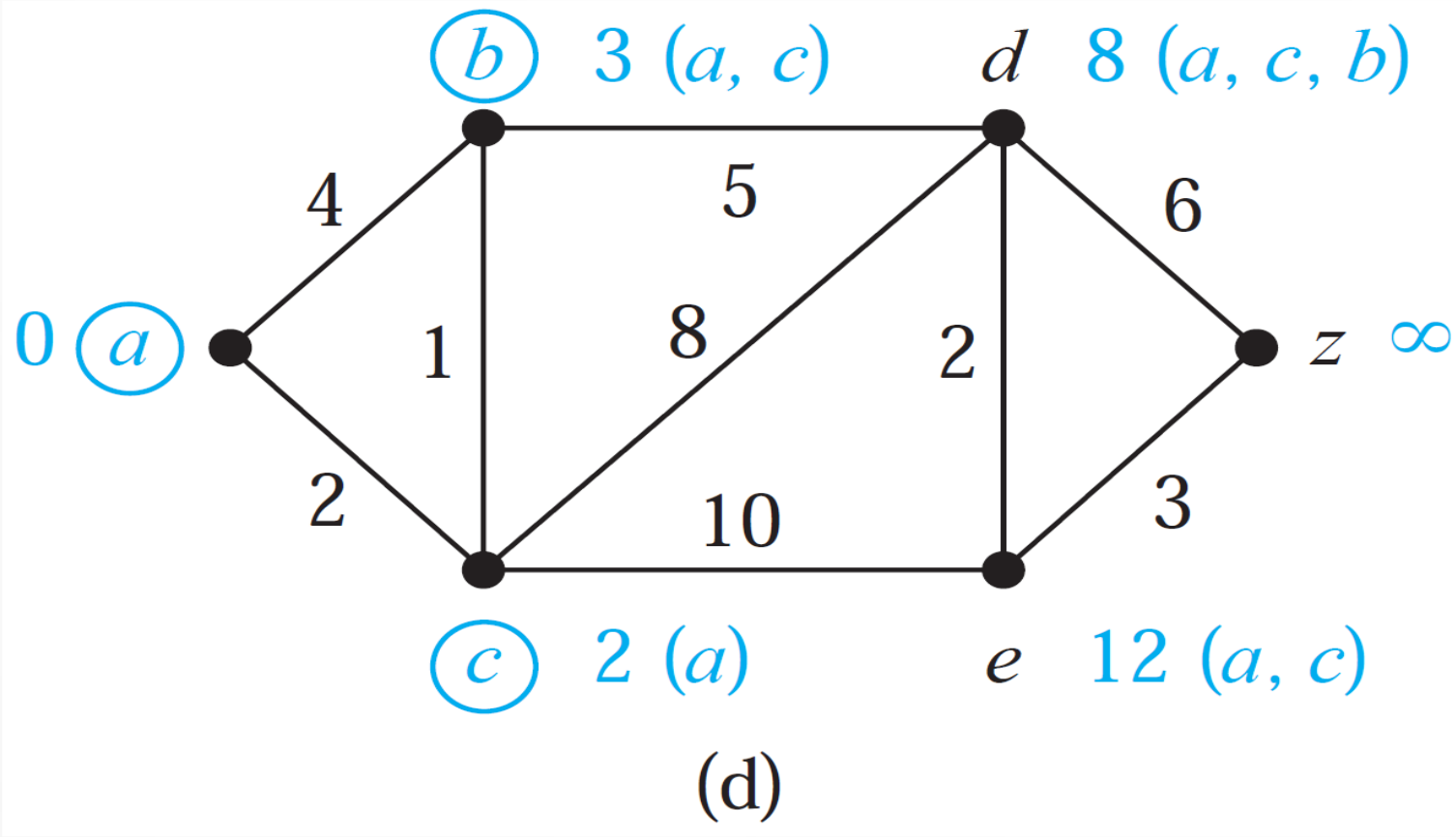
Find the length of a shortest path between a and z in the given weighted graph.



(c)

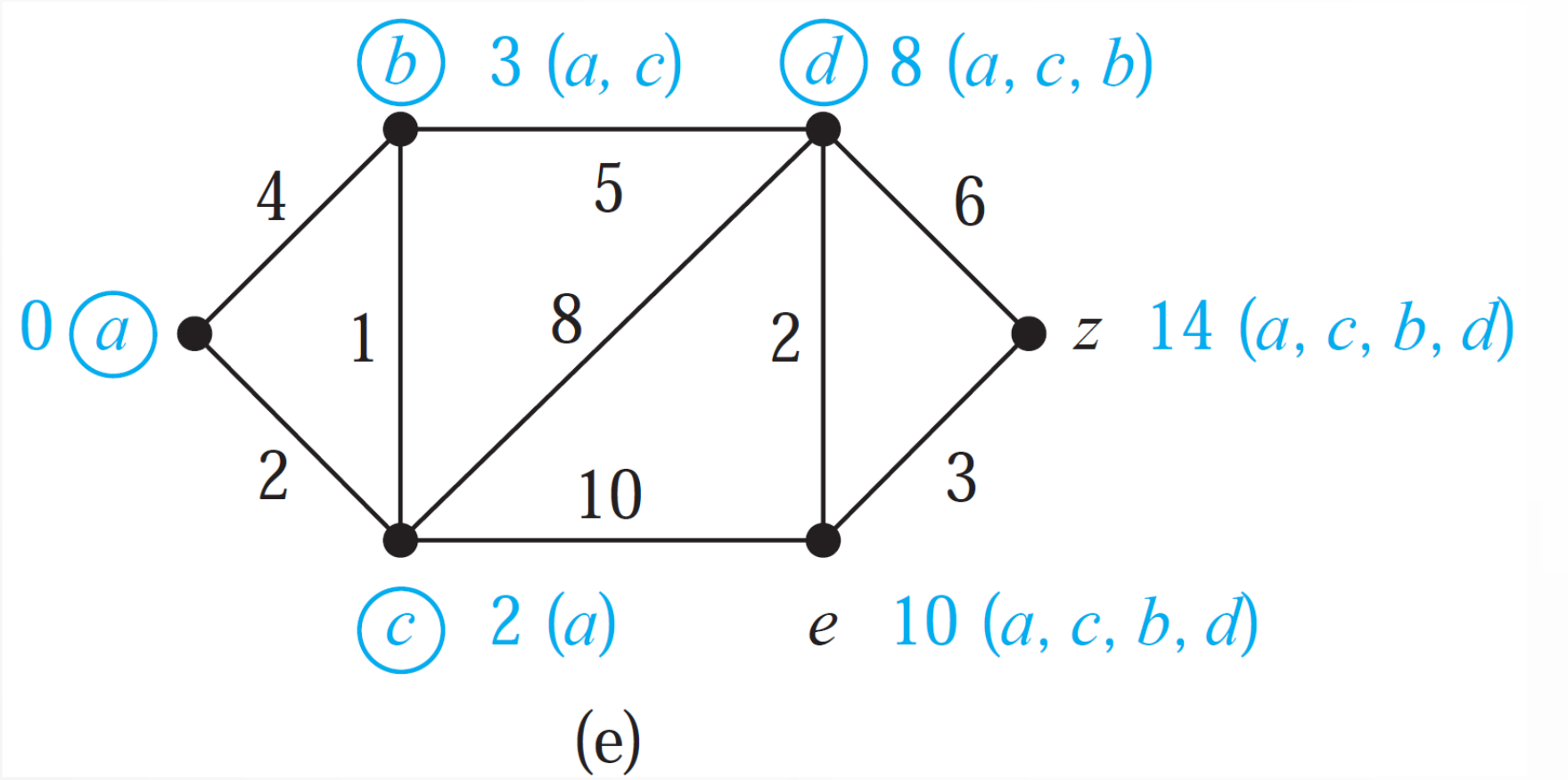
Dijkstra's Algorithm

Find the length of a shortest path between a and z in the given weighted graph.



Dijkstra's algorithm

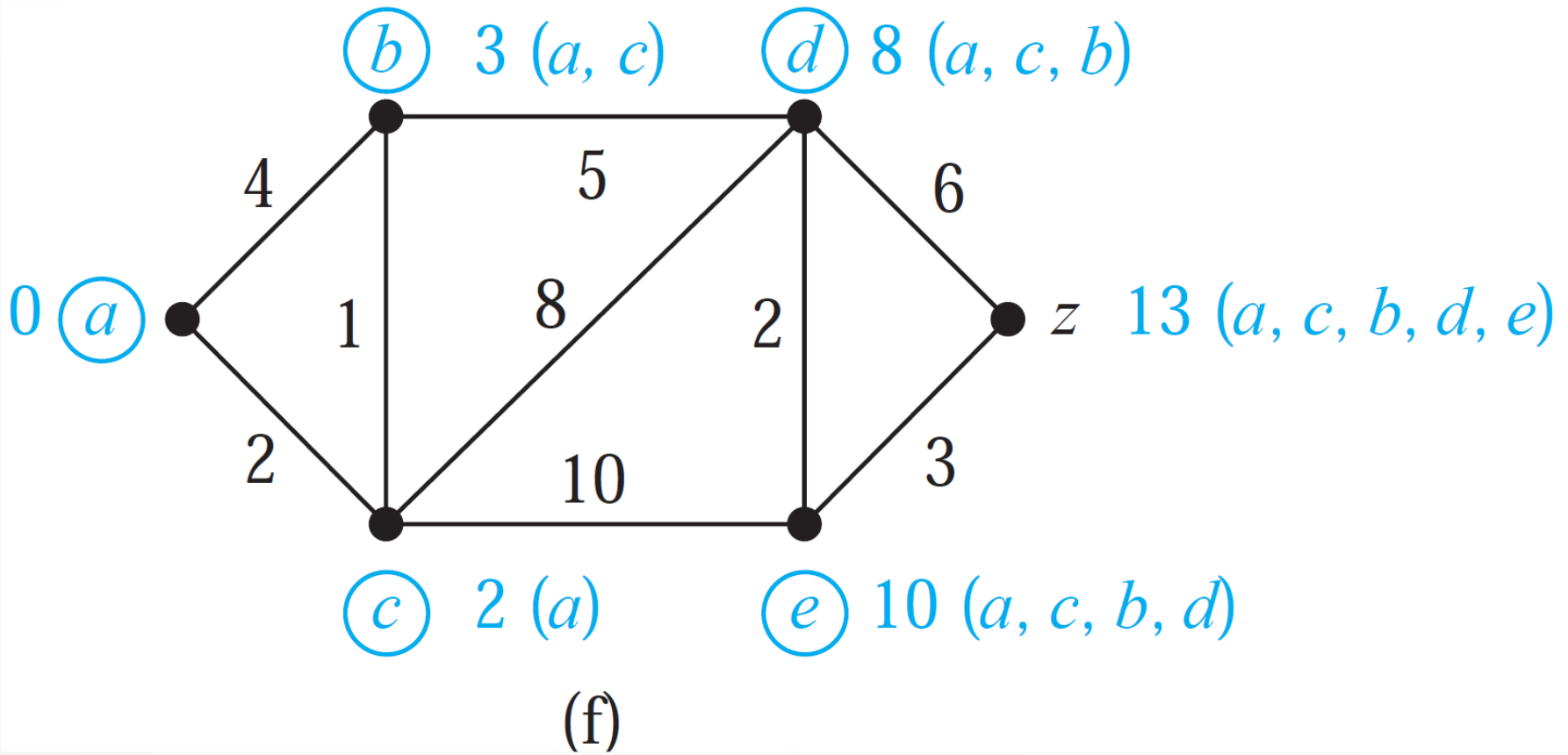
Find the length of a shortest path between a and z in the given weighted graph.



(e)

Dijkstra's Algorithm

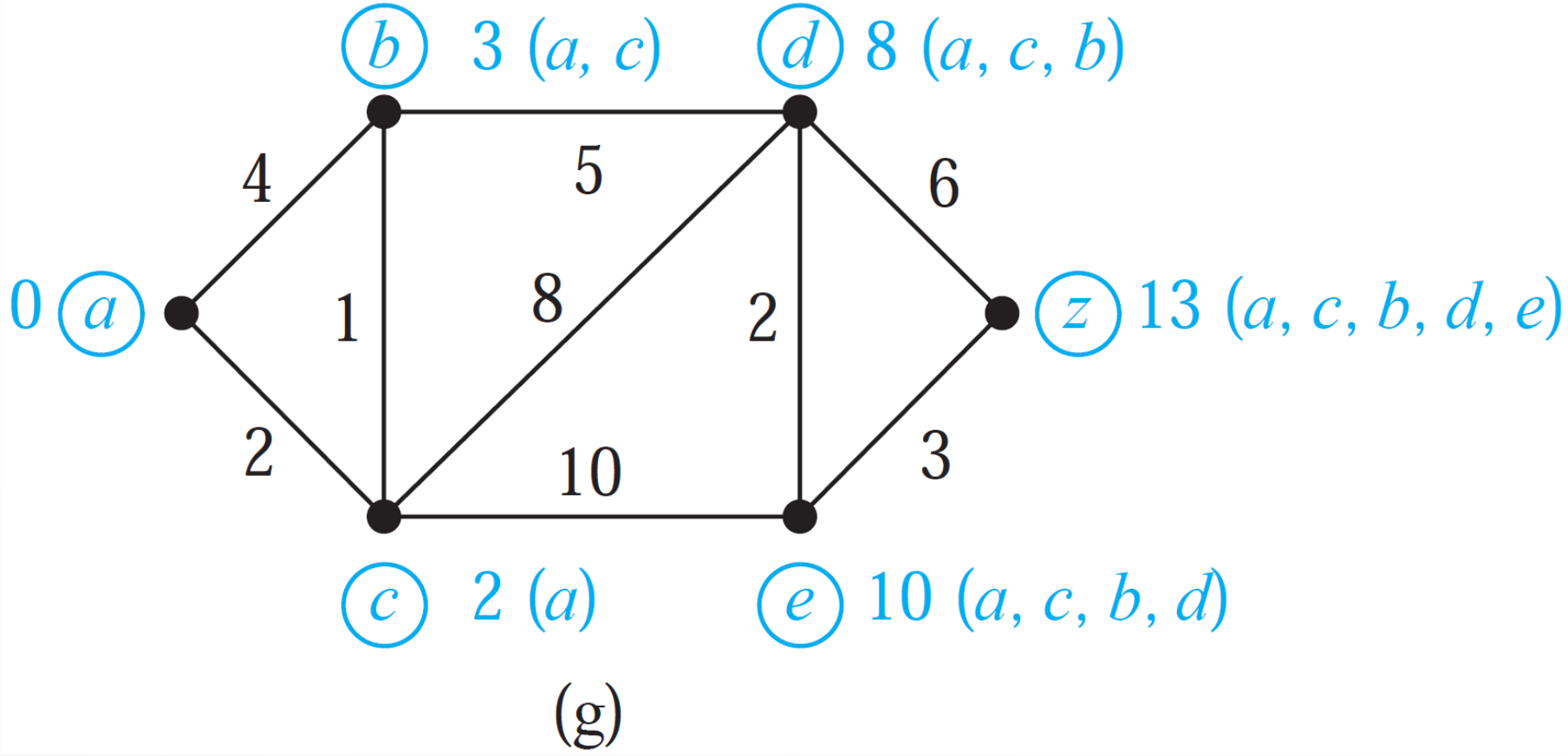
Find the length of a shortest path between a and z in the given weighted graph



(f)

Dijkstra's Algorithm

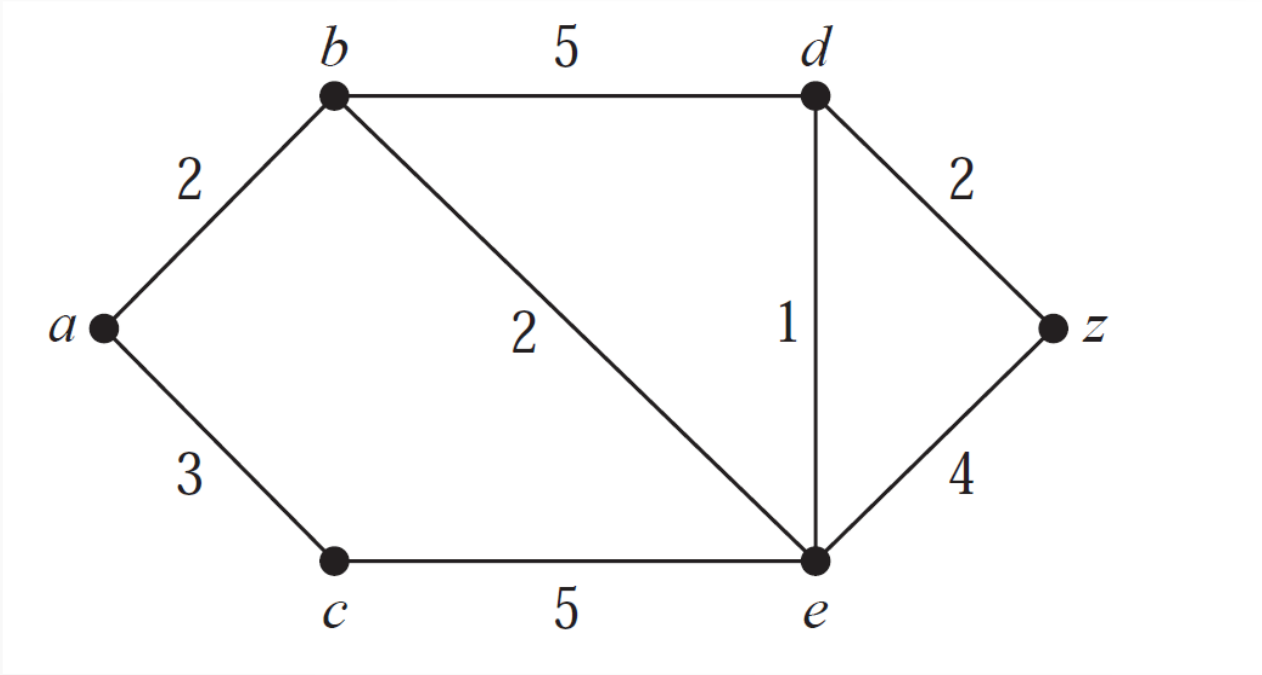
Find the length of a shortest path between a and z in the given weighted graph.



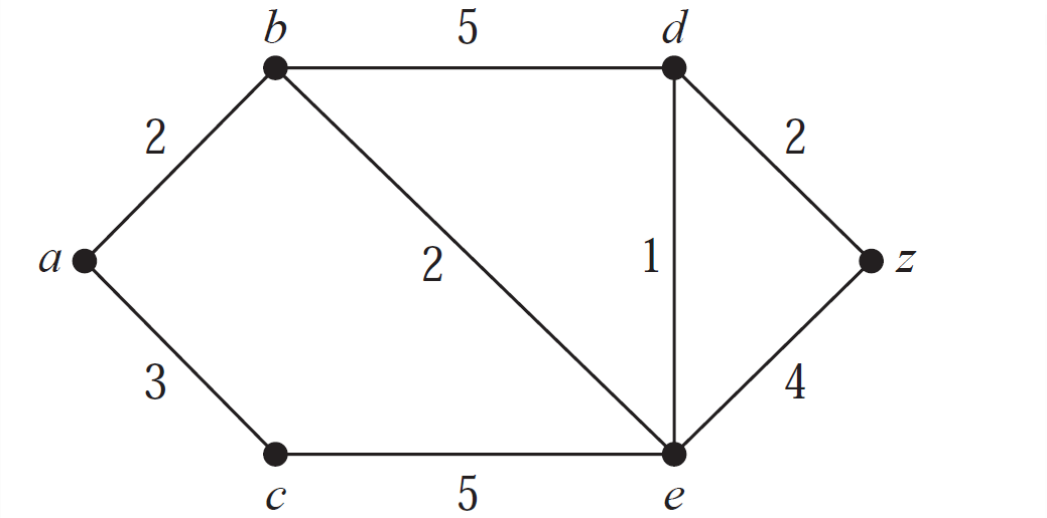
(g)

Dijkstra's Algorithm

Find the length of a shortest path between a and z in the given weighted graph.



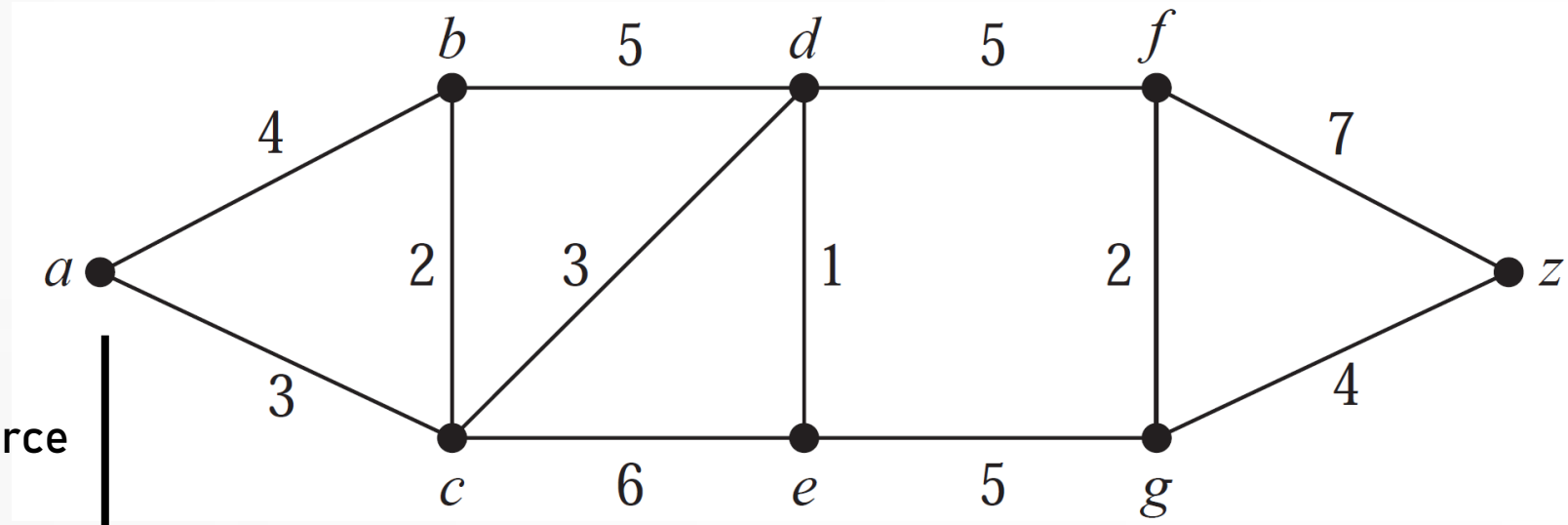
Dijkstra's Algorithm



Source	Path					
	a	b	c	d	e	z
a	---	2(a)	3(a)	∝	∝	∝
a,b	---	---	3(a)	7(a, b)	4(a, b)	∝
a,b,c	---	---	----	7(a,b)	4(a, b)	∝
a,b,c,e	---	---	----	5(a,b,e)	----	8(a,b,e)
a,b,c,e,d	---	---	----	-------	----	7(a,b,e,d)

Shortest Path from a to z = abedz, Length=7

Dijkstra's Algorithm

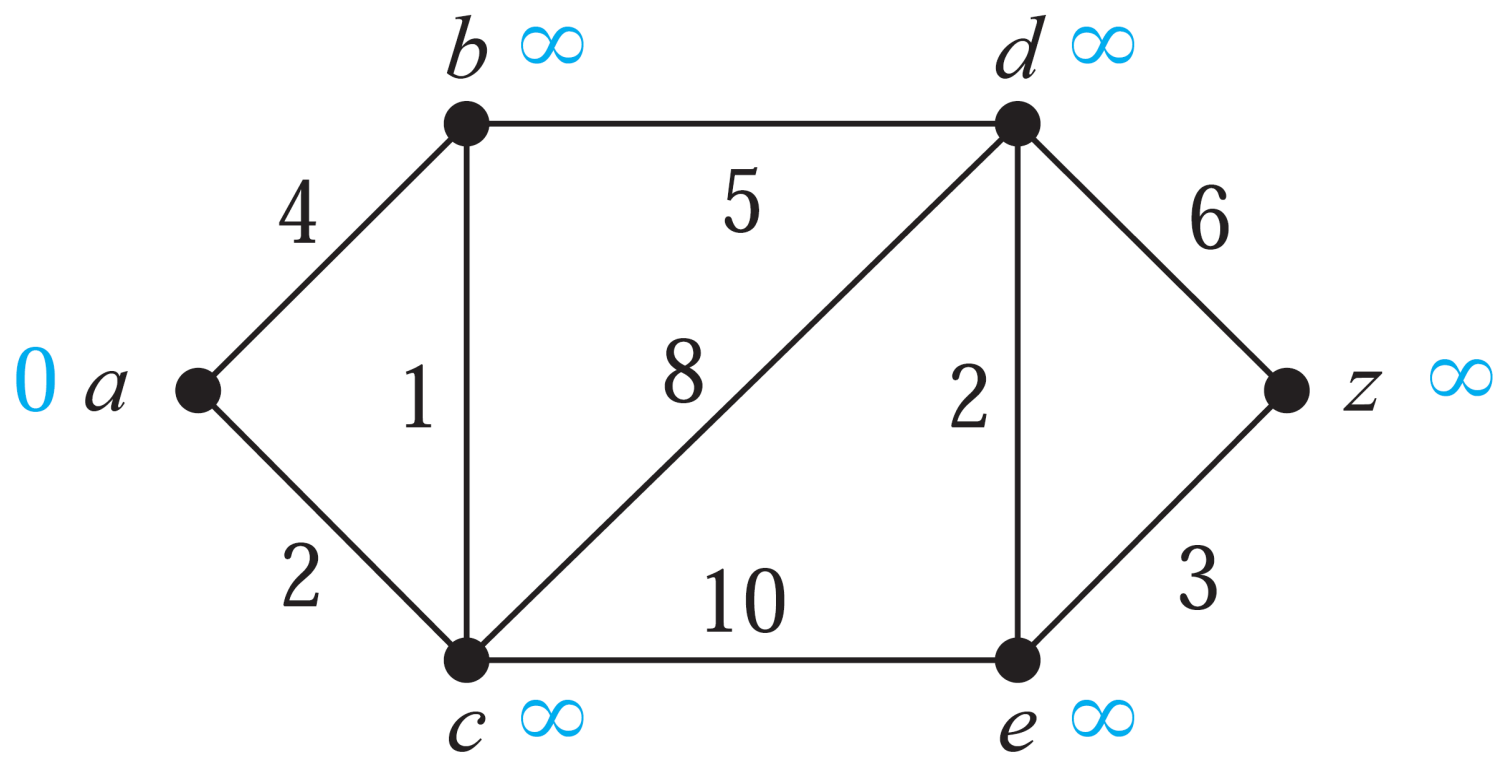


Source	a	b	c	d	e	f	g	z
a	0	4(a)	3(a)	∝	∝	∝	∝	∝
a,c	--	4(a)	---	6(a,c)	9(a,c)	∝	∝	∝
a,c,b	--	---	---	6(a,c)	9(a,c)	∝	∝	∝
a,c,b,d	--	---	---	---	7(a,c,d)	11(a,c,d)	∝	∝
a,c,b,d,e	--	---	---	---	---	11(a,c,d)	12(a,c,d,e)	∝
a,c,b,d,e,f	--	---	---	---	---	---	12(a,c,d,e)	18(acdf)
a,c,b,d,e,f,g	--	---	---	---	---	---	---	16(acdeg)

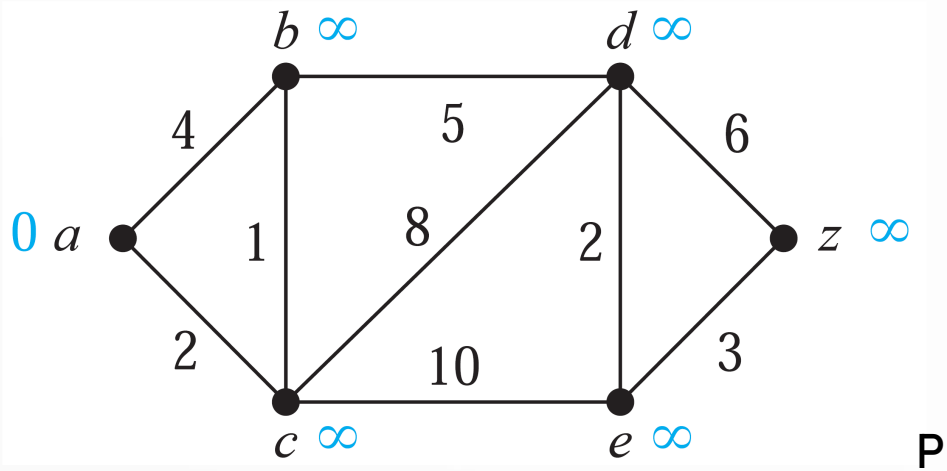
Shortest Path from a to z = acdegz, Length=16

Dijkstra's Algorithm

Find the length of a shortest path between a and z in the given weighted graph.



Dijkstra's Algorithm



Source	a	b	c	d	e	z
						Path
a	---	4(a)	2(a)	∝	∝	∝
a,c	---	3(a,c)	----	10(a, c)	12(a,c)	∝
a,c,b	---	----	----	8(a,c,b)	12(a,c)	∝
a,c,b,d	---	----	----	------	10(a,c,b,d)	14(a,c,b,d)
a,c,b,d,e	---	----	----	------	--------	13(a,c,b,d,e)

Shortest Path from a to z = a,c,b,d,e, Length=13

That's all for now…