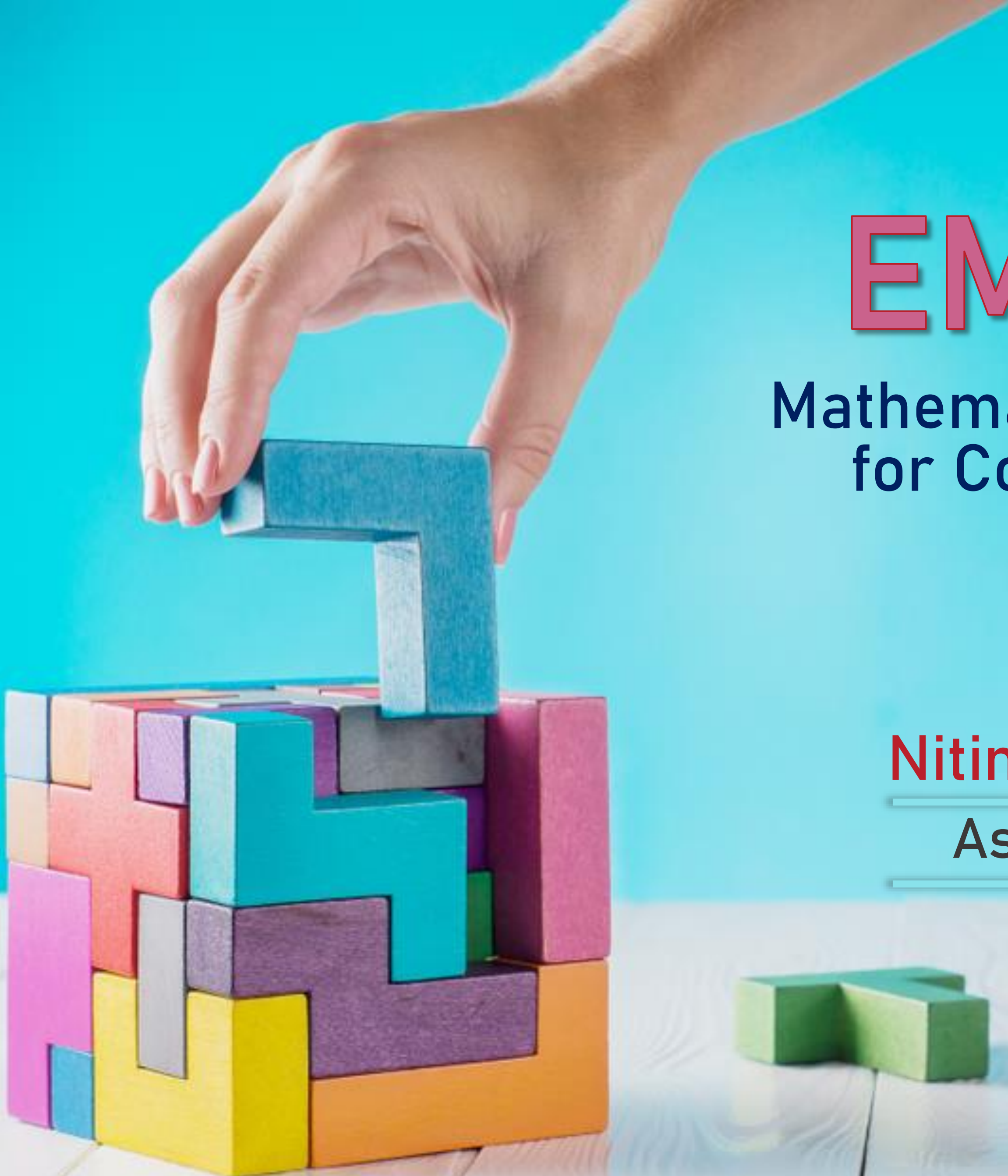


EMTH403

Mathematical Foundation for Computer Science

Nitin K. Mishra (Ph.D.)

Associate Professor

Lecture Outcomes

After this lecture, you will be able to

- understand some more applications of Eulers theorem for planar graphs.
- understand 3 corollary for planar graphs.

Euler's Formula (Cont.)

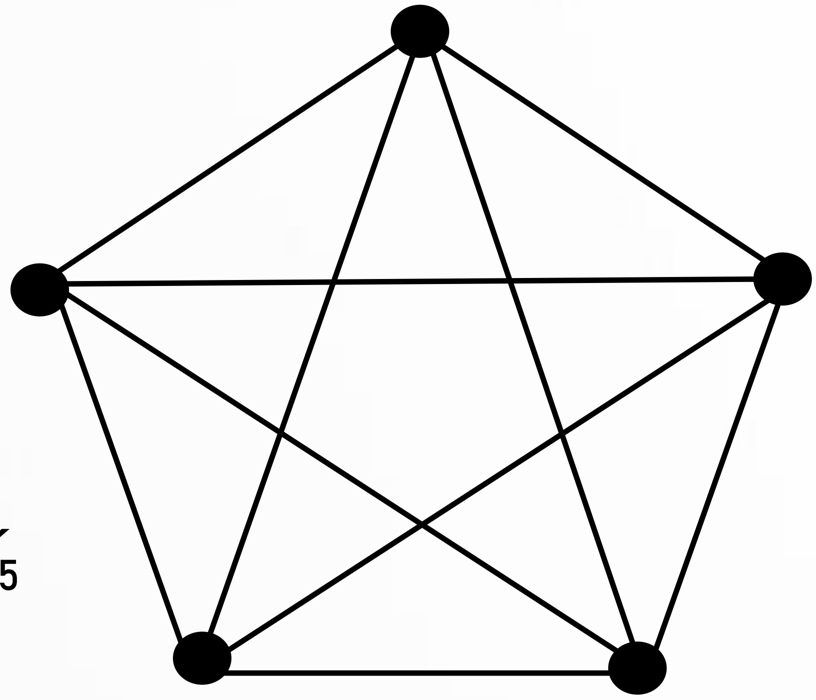
Corollary 1: If G is a connected planar simple graph with e edges and v vertices where $v \geq 3$, then $e \leq 3v - 6$.

Is K_5 planar?

Yes No

Answer: No

K_5



Euler's Formula (Cont.)

K_5 has 5 vertices and 10 edges.

We see that $v \geq 3$.

So, if K_5 is planar, it must be true that $e \leq 3v - 6$.

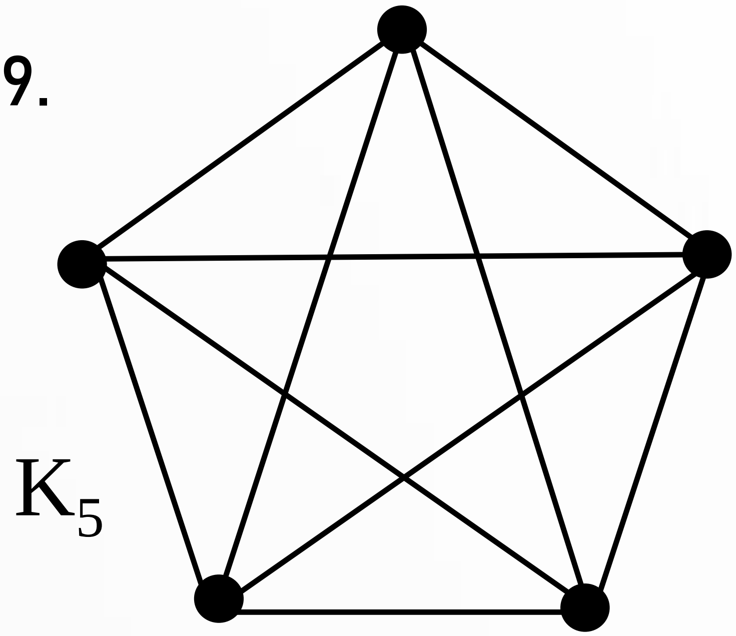
$3v - 6 = 3*5 - 6 = 15 - 6 = 9$.

So e must be ≤ 9.

But $e = 10$.

10 is not ≤ 9.

So, K_5 is nonplanar.



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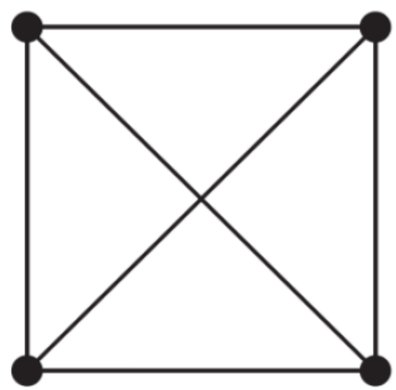
Euler's Formula (Cont.)

Corollary 1: If G is a connected planar simple graph with e edges and v vertices where $v \geq 3$, then $e \leq 3v - 6$.

Is K_4 planar?

Yes No



K_4

Answer: Yes

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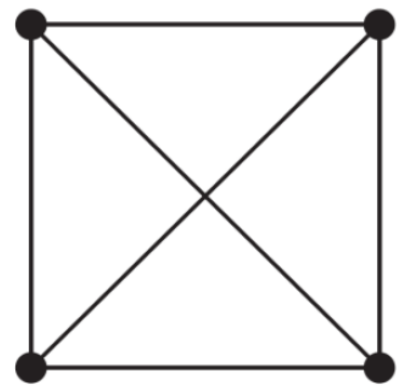
Euler's Formula (Cont.)

K_4 has 4 vertices and 6 edges.

We see that $v \geq 3$.

So, if K_4 is planar, it must be true that $e \leq 3v - 6$.

$3v - 6 = 3*4 - 6 = 12 - 6 = 6$.



K_4

So e must be ≤ 6.

But $e = 6$.

6 is ≤ 6.

So, K_4 is planar.

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Euler's Formula (Cont.)

Corollary 2: If G is a connected planar simple graph, then G must have a vertex of degree not exceeding 5.

If G has one or two vertices, it is true;

thus, we assume that G has at least three vertices.

If the degree of each vertex were at least 6, then by Handshaking Theorem, $2e \geq 6v$, i.e., $e \geq 3v$,

but this contradicts the inequality from Corollary 1: 1 $e \leq 3v - 6$.

$$2e = \sum_{v \in V} \deg(v)$$

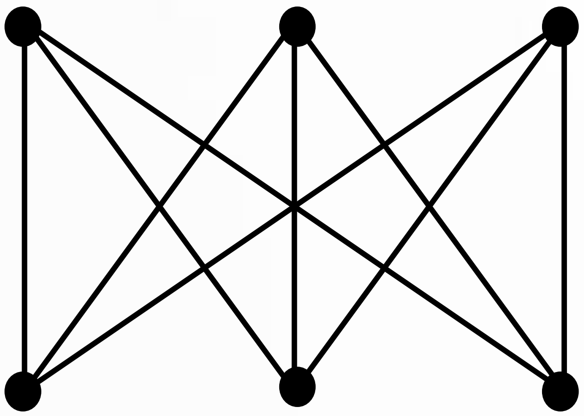
Euler's Formula (Cont.)

Corollary 3: If a connected planar simple graph has e edges and v vertices with $v \geq 3$ and no circuits of length 3, then $e \leq 2v - 4$.

Is $K_{3,3}$ planar?

Yes No

Answer: No



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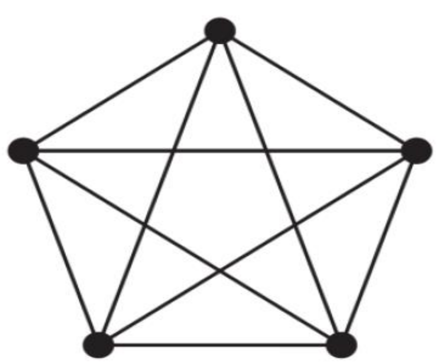
Theorem 1 (Kuratowski, 1930)

A graph is planar if and only if it does not contain a subgraph that is a K_5 or $K_{3,3}$ configuration.

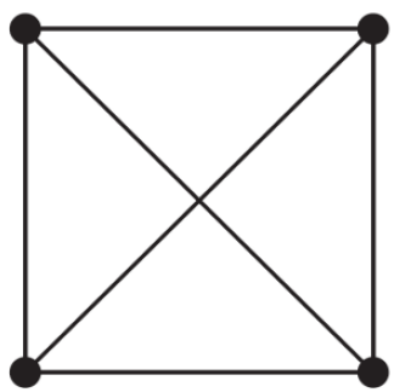
Euler's Formula (Cont.)

Which of these nonplanar graphs have the property that the removal of any vertex and all edges incident with that vertex produces a planar graph? a) K_5

If we remove a vertex from K_5 , then we get K_4 , which is clearly planar.



K_5



K_4

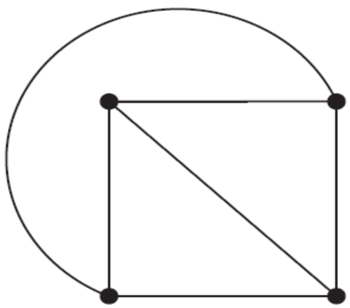


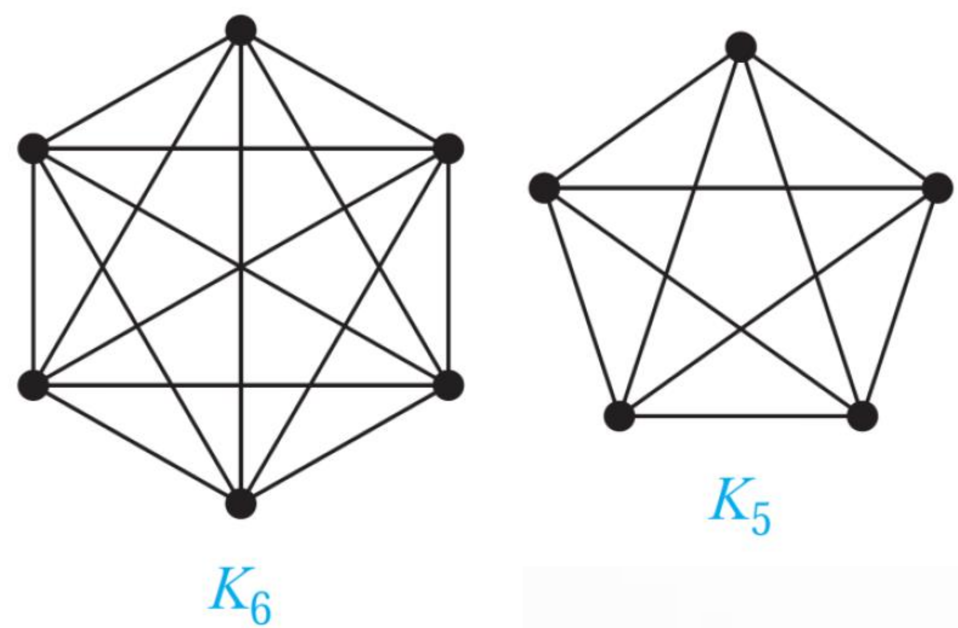
FIGURE 3 K_4 **Drawn with No Crossings.**

Euler's Formula (Cont.)

Which of these nonplanar graphs have the property that the removal of any vertex and all edges incident with that vertex produces a planar graph?

b) K_6

If we remove a vertex from K_6 , then we get K_5 , which is not planar.(As shown in previous slide no. 49)

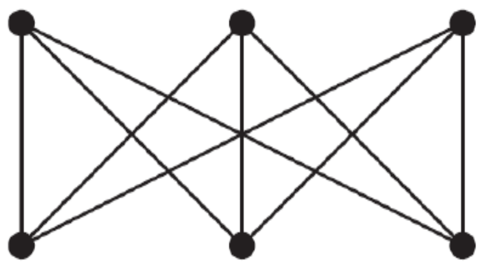


Euler's Formula (Cont.)

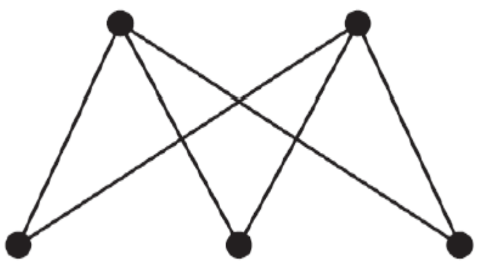
Which of these nonplanar graphs have the property that the removal of any vertex and all edges incident with that vertex produces a planar graph?

c) $K_{3,3}$

If we remove a vertex from $K_{3,3}$, then we get $K_{3,2}$, which is clearly planar.



$K_{3,3}$



$K_{2,3}$

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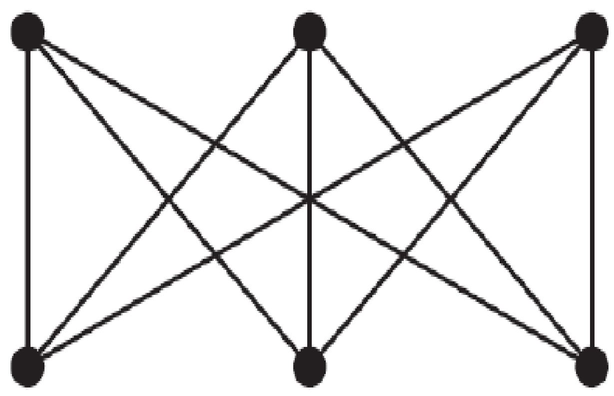
Euler's Formula (Cont.)

Which of these nonplanar graphs have the property that the removal of any vertex and all edges incident with that vertex produces a planar graph?

d) $K_{3,4}$

The answer is no, since we can remove a vertex in the part of size 4 to leave $K_{3,3}$, which is not planar. (As discussed earlier in slide 30)



That's all for now...