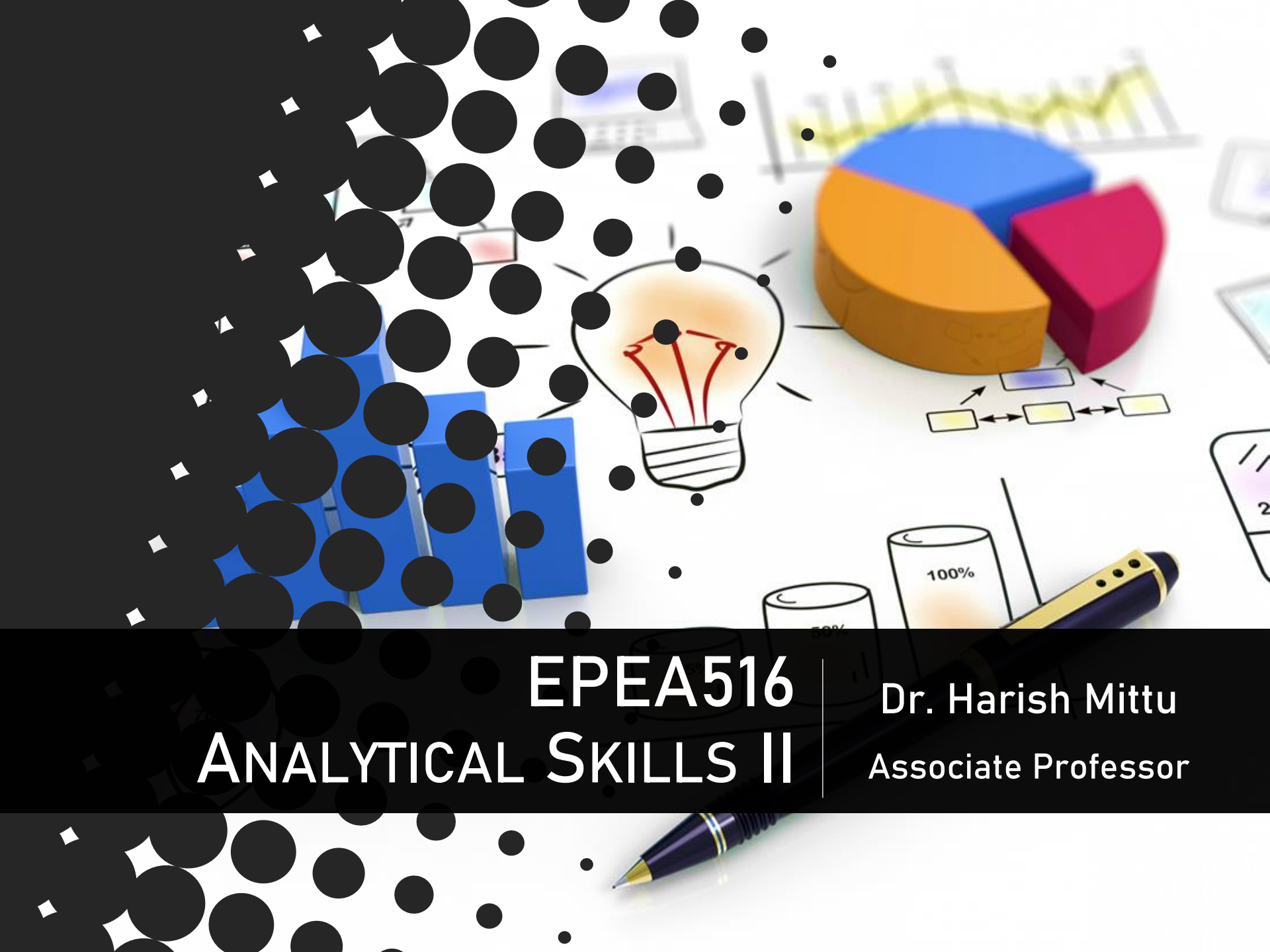

EPEA516
Analytical Skills II

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Learning Outcomes

After this lecture, you will be able to

- develop understanding about basics of pipes and cisterns,
- analyze various important facts and formulae relating to pipes and cisterns.

Pipes & Cisterns

- To Fill/Emptying –Tank/Cistern
- Inlet
- Pipe – To Fill Tank/Cistern
- Outlet
- Pipe – Emptying Tank/Cistern

Pipes & Cisterns

- Time taken to fill a cistern - Positive
- Time taken to empty a cistern – Negative
- Amount of Work Done (Filling/Emptying a Cistern)

– Unity

Inlet – Part of Tank Filled

- Time taken by an Inlet to fill the empty tank = x hours
- Part of the tank filled by an Inlet in 1 hour = $\frac{1}{x}$
- Example
  - If an inlet takes 3 hours to fill the tank completely then the part of tank filled in one hour is $\frac{1}{3}$.

Inlet – Part of Tank Filled

- Example
- Time taken by an inlet to fill the tank = 15 minutes
- $\frac{1}{15}$th part of the tank is filled in = 1 minute
- 1 part of the tank is filled in $= \frac{1}{\frac{1}{15}}$ = 15 minutes
- $\frac{1}{3}$rd part of the tank is filled in $= \frac{1}{3} \times \cancel{15}^{5}$

  = 5 minutes

Outlet – Part of Tank Emptied

- Time taken by an outlet to empty the full tank = y hours
- Part of the tank emptied by an outlet in 1 hour = $\frac{1}{y}$
- Example
  - If a pipe can empty the full tank in 5 hours then the part of tank emptied in one hour is $\frac{1}{5}$.

Outlet – Part of Tank Emptied

- Example
- Time taken by an outlet to empty the tank = 35 minutes
- $\frac{1}{35}$th part of the tank is emptied in = 1 minute
- 1 part of the tank is emptied in $= \frac{1}{\frac{1}{35}}$ = 35 minutes
- $\frac{7}{5}$th part of the tank is emptied in $= \frac{7}{5} \times \cancel{35}^{7}$

  = 49 minutes

Inlet & Outlet – Net Part of Tank Filled

- Both inlet and outlet are open.
- Time taken by an Inlet to fill empty tank = x hours
- Part of the tank filled by an Inlet in 1 hour = $\frac{1}{x}$
- Time taken by an outlet to empty the full tank = y hours
- Part of the tank emptied by an outlet in 1 hour = $\frac{1}{y}$
- If y > x then net part of the tank filled in 1 hour = $\frac{1}{x} - \frac{1}{y}$

Inlet & Outlet – Net Part of Tank Filled

- An inlet can fill a cistern in 4 hours and an outlet can empty it in 8 hours. If both the inlet and outlet are opened simultaneously, find the time (in hours) to fill the cistern.
- x = 4 hours and y = 8 hours

$$8 \text{ hours} > 4 \text{ hours}$$

- If y > x then net part of the cistern filled in 1 hour = $\frac{1}{x} - \frac{1}{y}$
- Part of the cistern filled in 1 hour = $\frac{1}{4} - \frac{1}{8} = \frac{2-1}{8} = \frac{1}{8}$
- Time taken to fill the cistern = 8 hours

Inlet & Outlet – Net Part of Tank Emptied

- Both inlet and outlet are open.
- Time taken by an Inlet to fill empty tank = x hours
- Part of the tank filled by an Inlet in 1 hour = $\frac{1}{x}$
- Time taken by an outlet to empty the full tank = y hours
- Part of the tank emptied by an outlet in 1 hour = $\frac{1}{y}$
- If x > y then net part of the tank emptied in 1 hour = $\frac{1}{y} - \frac{1}{x}$

Inlet & Outlet – Net Part of Tank Emptied

- An inlet can fill a cistern in 20 hours and an outlet can empty it in 10 hours. If both the inlet and outlet are opened simultaneously, find the time (in hours) to emptied the cistern.
- x = 20 hours and y = 10 hours

  20 hours > 10 hours
- If x > y then net part of the cistern emptied in 1 hour = $\frac{1}{y} - \frac{1}{x}$
- Part of the cistern emptied in 1 hour = $\frac{1}{10} - \frac{1}{20} = \frac{2-1}{20} = \frac{1}{20}$
- Time taken to emptied the cistern = 20 hours

Inlet & Outlet – Time Taken to Fill/Empty Cistern

- Two pipes A, and B can fill/empty a cistern in x and y hours, while working alone. Calculate the time taken to fill/empty the cistern if both the pipes are opened together.
- Part of the cistern filled/emptied by pipe A alone in 1 hour

$$= \frac{1}{x}$$

- Part of the cistern filled/emptied by pipe B alone in 1 hour

$$= \frac{1}{y}$$

Inlet & Outlet – Time Taken to Fill/Empty Cistern

- Part of the cistern filled/emptied by both the pipes A & B in 1 hour

$$= \frac{1}{x} + \frac{1}{y}$$

$$= \frac{x + y}{xy}$$

- Both the pipes A & B together will fill/empty the cistern in

$$= \frac{xy}{x + y} \text{ hours}$$

Inlet & Outlet – Time Taken to Fill Cistern

- Three pipes A, B, and C can fill a cistern in x, y and z hours respectively, while working alone. Calculate the time taken to fill the cistern if all the three pipes are opened together.
- Part of the cistern filled by pipe A alone in 1 hour = $\frac{1}{x}$
- Part of the cistern filled by pipe B alone in 1 hour = $\frac{1}{y}$
- Part of the cistern filled by pipe C alone in 1 hour = $\frac{1}{z}$

Inlet & Outlet – Time Taken to Fill Cistern

- Part of the cistern filled by all the three pipes i.e., A, B & C in 1 hour

$$= \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

$$= \frac{yz + xz + xy}{xyz}$$

- Three pipes together will fill the cistern

$$= \frac{xyz}{yz + xz + xy} = \frac{xyz}{xy + yz + zx} \text{ hours}$$

Inlet & Outlet – Time Taken to Fill Cistern

- Two pipes, A and B, can separately fill a cistern in x and y hours respectively, while a third pipe C can empty it in z hours. Three pipes together will fill the cistern in

$$= \frac{xy\,(-z)}{xy + y(-z) + (-z)x} \text{ hours}$$

$$= \frac{-xyz}{xy - yz - zx} \text{ hours}$$

Inlet & Outlet – Time Taken to Fill Cistern

- Two pipes, A and C, can separately fill a cistern in x and z hours respectively, while a third pipe B can empty it in y hours. Three pipes together will fill the cistern

$$= \frac{x\,(-y)\,z}{x(-y) + (-y)z + zx} \text{ hours}$$

$$= \frac{-xyz}{-xy - yz + zx} \text{ hours}$$

Inlet & Outlet – Time Taken to Fill Cistern

- Two pipes, B and C, can separately fill a cistern in y and z hours respectively, while a third pipe A can empty it in x hours. Three pipes together will fill the cistern

$$= \frac{(-x)yz}{(-x)y + yz + z(-x)} \text{ hours}$$

$$= \frac{-xyz}{-xy + yz - zx} \text{ hours}$$

Inlet & Outlet – Time Taken to Empty Cistern

- Two pipes, A and B, can fill a cistern in x hours and y hours, respectively. There is also an outlet C. If all the three pipes are opened together, the tank is full in z hours. Calculate the time taken by an outlet C to empty the full cistern.
- Part of the cistern filled by pipe A alone in 1 hour = $\frac{1}{x}$
- Part of the cistern filled by pipe B alone in 1 hour = $\frac{1}{y}$
- Part of cistern filled by all the three pipes in 1 hour = $\frac{1}{z}$

Inlet & Outlet – Time Taken to Empty Cistern

- Part of the cistern emptied by pipe C in 1 hour

$$= \frac{1}{x} + \frac{1}{y} - \frac{1}{z}$$

$$= \frac{yz + xz - xy}{xyz}$$

- An outlet C can empty the full cistern

$$= \frac{xyz}{yz + xz - xy} \text{ hours}$$

Inlet & Outlet – Time Taken to Empty Cistern

- A cistern takes x hours to be filled by a pipe. But, due to a leak, it is filled in y hours. Calculate the amount of time in which the leak can empty the full tank.
- Part of the cistern filled by pipe in 1 hour = $\frac{1}{x}$
- Part of the cistern emptied by pipe in 1 hour = $\frac{1}{y}$
- Net part of the cistern emptied in 1 hour $= \frac{1}{x} - \frac{1}{y}$

$$= \frac{y - x}{xy}$$

Inlet & Outlet – Time Taken to Empty Cistern

- Net part of the cistern emptied in 1 hour $= \frac{1}{x} - \frac{1}{y}$

$$= \frac{y - x}{xy}$$

- Amount of time in which the leak can empty the full tank

$$= \frac{xy}{y - x}$$

Inlet & Outlet – Capacity of Cistern

- A cistern has a leak which can empty it in x hours.
- A pipe which admits y litres of water per hour into the cistern is turned on and now the cistern is emptied in z hours.
- Capacity of cistern $= \frac{xyz}{z - x}$ litres

Inlet – Time Taken to Fill Cistern

- Let an inlet A is ‘m’ times faster than the other inlet B.
  - Let inlet B can fill a cistern in x hours.
  - If both the inlets A & B are opened together then the time taken to fill the cistern = $\frac{x}{m+1}$ hours
  - Let inlet A can fill a cistern in y hours.
  - If both the inlets A & B are opened together then the time takento fill the cistern = $\{\frac{m}{m+1}\}y$ hours

Inlet – Time Taken to Fill Cistern

- Let an inlet A is ‘m’ times faster & takes ‘x’ minutes less than the other inlet B.
  - If both the inlets A & B are opened together then the

    time taken to fill the cistern $= \frac{mx}{(m-1)^2}$ hours
  - Inlet A will fill the cistern $= \frac{x}{m-1}$ hours
  - Inlet B will fill the cistern $= \frac{mx}{m-1}$ hours

Conclusion

- If y > x then net part of the tank filled in 1 hour $= \frac{1}{x} - \frac{1}{y}$
- If x > y then net part of the tank filled in 1 hour $= \frac{1}{y} - \frac{1}{x}$
- Both the pipes A & B together will fill/empty the cistern in

$$= \frac{xy}{x + y} \text{ hours}$$

- Three pipes together will fill the cistern

$$= \frac{xyz}{xy + yz + zx} \text{ hours}$$

Conclusion

- Three pipes together will fill the cistern in
  - $\frac{-xyz}{xy - yz - zx}$ hours
  - $\frac{-xyz}{-xy - yz + zx}$ hours
  - $\frac{-xyz}{-xy + yz - zx}$ hours

Conclusion

- Two pipes, A and B, can fill a cistern in x hours and y hours, respectively. There is also an outlet C. An outlet C can empty the full cistern = $\frac{xyz}{yz + xz - xy}$ hours
- A cistern takes x hours to be filled by a pipe. But, due to a leak, it is filled in Y hours. Amount of time in which the leak can empty the full tank = $\frac{xy}{y - x}$

Conclusion

- A cistern has a leak which can empty it in x hours. A pipe which admits y litres of water per hour into the cistern is turned on and now the cistern is emptied in z hours. Capacity of cistern $= \frac{xyz}{z - x}$ litres

Summary

- Pipes & Cisterns
  - Basics
  - Facts
  - Formulae

That’s all for now…