

A hand is shown placing a blue L-shaped block onto a larger, colorful geometric structure composed of various other blocks. The structure is built on a light-colored wooden surface. In the background, there are more scattered blocks in green, blue, red, and yellow. The background is a solid light blue.

# EMTH403

Mathematical Foundation  
for Computer Science

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Associate Professor

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# Lecture Outcomes



After this lecture, you will be able to

- understand what are isomorphic graph
- understand what is an isomorphism.

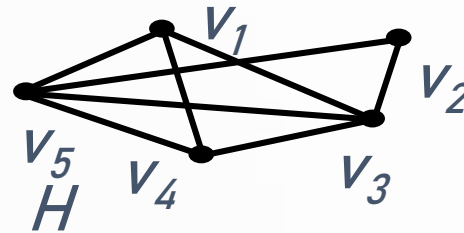
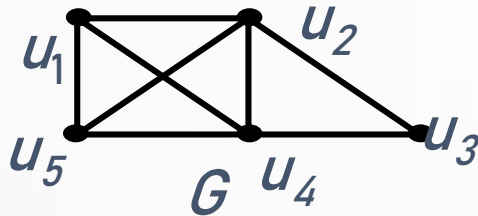
# Graph Isomorphism – Definition

The simple graphs  $G_1 = (V_1, E_1)$  and  $G_2 = (V_2, E_2)$  are isomorphic if there exists a one to-one and onto function  $f$  from  $V_1$  to  $V_2$  with the property that  $a$  and  $b$  are adjacent in  $G_1$  if and only if  $f(a)$  and  $f(b)$  are adjacent in  $G_2$ , for all  $a$  and  $b$  in  $V_1$ .

Such a function  $f$  is called an isomorphism.\* Two simple graphs that are not isomorphic are called non-isomorphic.

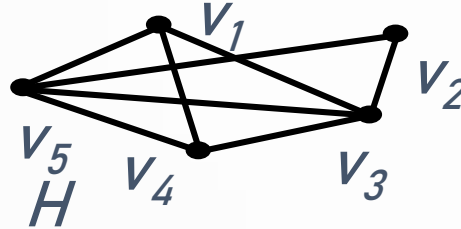
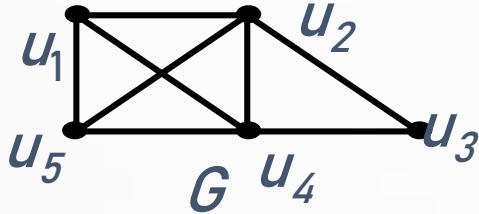
# Graph Isomorphism

- Are these two graphs isomorphic?



- They both have 5 vertices
- They both have 8 edges
- They have the same number of vertices with the same degrees: 2, 3, 3, 4, 4.

# Graph Isomorphism



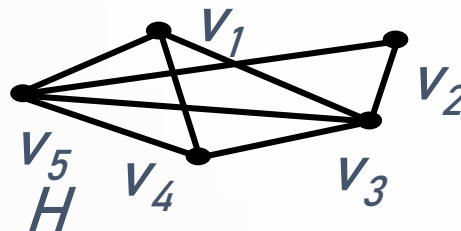
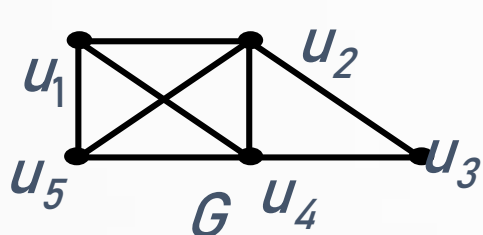
	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$
$u_1$	0	1	0	1	1
$u_2$	1	0	1	1	1
$u_3$	0	1	0	1	0
$u_4$	1	1	1	0	1
$u_5$	1	1	0	1	0

	$v_1$	$v_2$	$v_3$	$v_4$	$v_5$
$v_1$	0	0	1	1	1
$v_2$	0	0	1	0	1
$v_3$	1	1	0	1	1
$v_4$	1	0	1	0	1
$v_5$	1	1	1	1	0

	$v_1$	$v_3$	$v_2$	$v_5$	$v_4$
$v_1$					
$v_3$					
$v_2$					
$v_5$					
$v_4$					

- $G$  and  $H$  don't appear to be isomorphic.
- However, we haven't tried mapping vertices from  $G$  onto  $H$  yet.

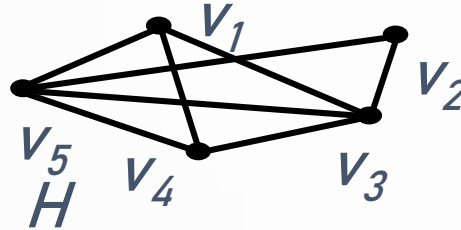
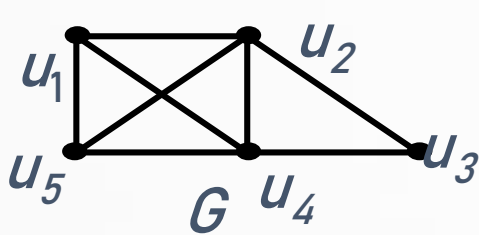
# Graph Isomorphism



Start with the vertices of degree 2 since each graph only has one:

$$\deg(u_3) = \deg(v_2) = 2 \quad \text{therefore} \quad f(u_3) = v_2$$

# Graph Isomorphism



Now consider vertices of degree 3

$$\deg(u_1) = \deg(u_5) = \deg(v_1) = \deg(v_4) = 3$$

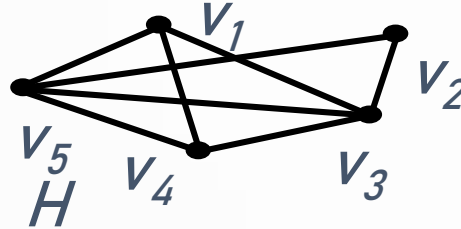
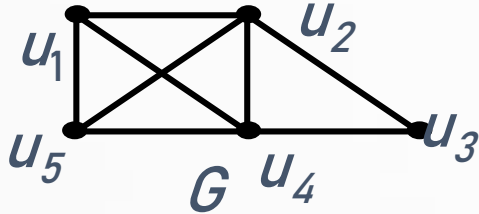
therefore we must have either one of

$$f(u_1) = v_1 \text{ and } f(u_5) = v_4$$

$$f(u_1) = v_4 \text{ and } f(u_5) = v_1$$

Along with  $f(u_3) = v_2$

# Graph Isomorphism



Now try vertices of degree 4:

$$\deg(u_2) = \deg(u_4) = \deg(v_3) = \deg(v_5) = 4$$

therefore we must have one of:

$$f(u_2) = v_3 \text{ and } f(u_4) = v_5 \quad \text{or}$$

$$f(u_2) = v_5 \text{ and } f(u_4) = v_3$$

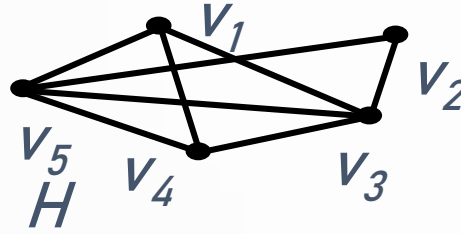
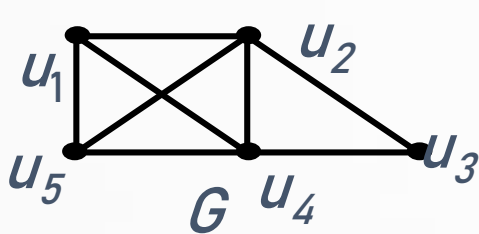
Along with  $f(u_3) = v_2$

And along with  $f(u_1) = v_1$  and  $f(u_5) = v_4$

OR

$$f(u_1) = v_4 \text{ and } f(u_5) = v_1$$

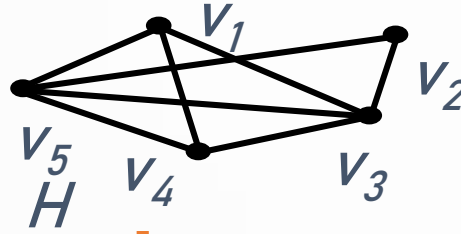
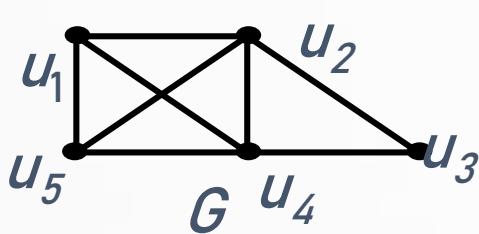
# Graph Isomorphism



$U_3$	$V_2$	$U_3$	$V_2$	$U_3$	$V_2$	$U_3$	$V_2$
$U_1$	$V_1$	$U_1$	$V_1$	$U_1$	$V_1$	$U_1$	$V_1$
$U_5$	$V_4$	$U_5$	$V_4$	$U_5$	$V_4$	$U_5$	$V_4$
$U_2$	$V_3$	$U_2$	$V_3$	$U_2$	$V_3$	$U_2$	$V_3$
$U_4$	$V_5$	$U_4$	$V_5$	$U_4$	$V_5$	$U_4$	$V_5$

- There are four possible bijections which are as follows:
  - $f(u_1) = v_1, f(u_2) = v_3, f(u_3) = v_2, f(u_4) = v_5, f(u_5) = v_4$
  - $f(u_1) = v_4, f(u_2) = v_3, f(u_3) = v_2, f(u_4) = v_5, f(u_5) = v_1$
  - $f(u_1) = v_1, f(u_2) = v_5, f(u_3) = v_2, f(u_4) = v_3, f(u_5) = v_4$
  - $f(u_1) = v_4, f(u_2) = v_5, f(u_3) = v_2, f(u_4) = v_3, f(u_5) = v_1$

# Graph Isomorphism



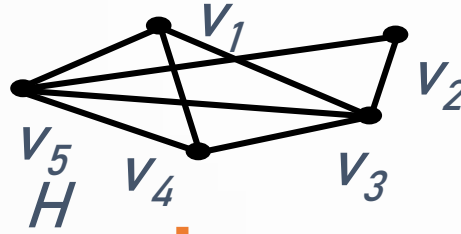
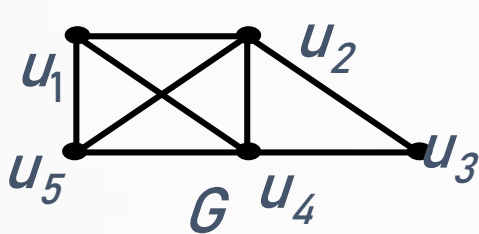
	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$		$v_1$	$v_2$	$v_3$	$v_4$	$v_5$
$u_1$	0	1	0	1	1	$v_1$	0	0	1	1	1
$u_2$	1	0	1	1	1	$v_2$	0	0	1	0	1
$u_3$	0	1	0	1	0	$v_3$	1	1	0	1	1
$u_4$	1	1	1	0	1	$v_4$	1	0	1	0	1
$u_5$	1	1	0	1	0	$v_5$	1	1	1	1	0

	$v_1$	$v_3$	$v_2$	$v_5$	$v_4$
$v_1$	0	1	0	1	1
$v_3$	1	0	1	1	1
$v_2$	0	1	0	1	0
$v_5$	1	1	1	0	1
$v_4$	1	1	0	1	0

$$f(u_1) = v_1, f(u_2) = v_3, f(u_3) = v_2, f(u_4) = v_5, f(u_5) = v_4$$

Therefore the two graphs are isomorphic as their adjacency matrices are equal for the above given bijection.

# Graph Isomorphism



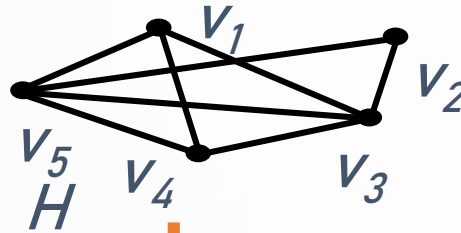
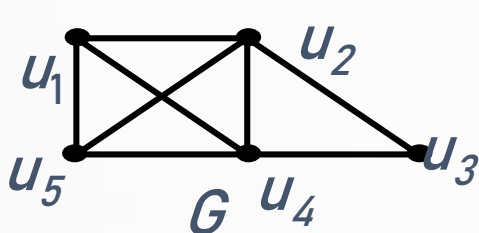
	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$		$v_1$	$v_2$	$v_3$	$v_4$	$v_5$
$u_1$	0	1	0	1	1	$v_1$	0	0	1	1	1
$u_2$	1	0	1	1	1	$v_2$	0	0	1	0	1
$u_3$	0	1	0	1	0	$v_3$	1	1	0	1	1
$u_4$	1	1	1	0	1	$v_4$	1	0	1	0	1
$u_5$	1	1	0	1	0	$v_5$	1	1	1	1	0

	$v_4$	$v_3$	$v_2$	$v_5$	$v_1$
$v_4$	0	1	0	1	1
$v_3$	1	0	1	1	1
$v_2$	0	1	0	1	0
$v_5$	1	1	1	0	1
$v_1$	1	1	0	1	0

$$f(u_1) = v_4, f(u_2) = v_3, f(u_3) = v_2, f(u_4) = v_5, f(u_5) = v_1$$

Therefore the two graphs are isomorphic as their adjacency matrix are equal for the given bijection.

# Graph Isomorphism



	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$		$v_1$	$v_2$	$v_3$	$v_4$	$v_5$
$u_1$	0	1	0	1	1	$v_1$	0	0	1	1	1
$u_2$	1	0	1	1	1	$v_2$	0	0	1	0	1
$u_3$	0	1	0	1	0	$v_3$	1	1	0	1	1
$u_4$	1	1	1	0	1	$v_4$	1	0	1	0	1
$u_5$	1	1	0	1	0	$v_5$	1	1	1	1	0

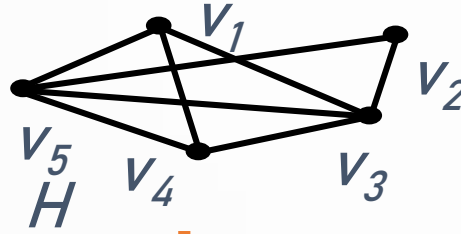
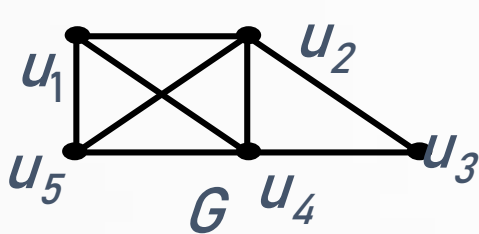
	$v_1$	$v_5$	$v_2$	$v_3$	$v_4$
$v_1$	0	1	0	1	1
$v_5$	1	0	1	1	1
$v_2$	0	1	0	1	0
$v_3$	1	1	1	0	1
$v_4$	1	1	0	1	0

It turns out that

$$f(u_1) = v_1, f(u_2) = v_5, f(u_3) = v_2, f(u_4) = v_3, f(u_5) = v_4$$

Therefore the two graphs are isomorphic as their adjacency matrix are equal for the given bijection.

# Graph Isomorphism



	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$		$v_1$	$v_2$	$v_3$	$v_4$	$v_5$
$u_1$	0	1	0	1	1	$v_1$	0	0	1	1	1
$u_2$	1	0	1	1	1	$v_2$	0	0	1	0	1
$u_3$	0	1	0	1	0	$v_3$	1	1	0	1	1
$u_4$	1	1	1	0	1	$v_4$	1	0	1	0	1
$u_5$	1	1	0	1	0	$v_5$	1	1	1	1	0

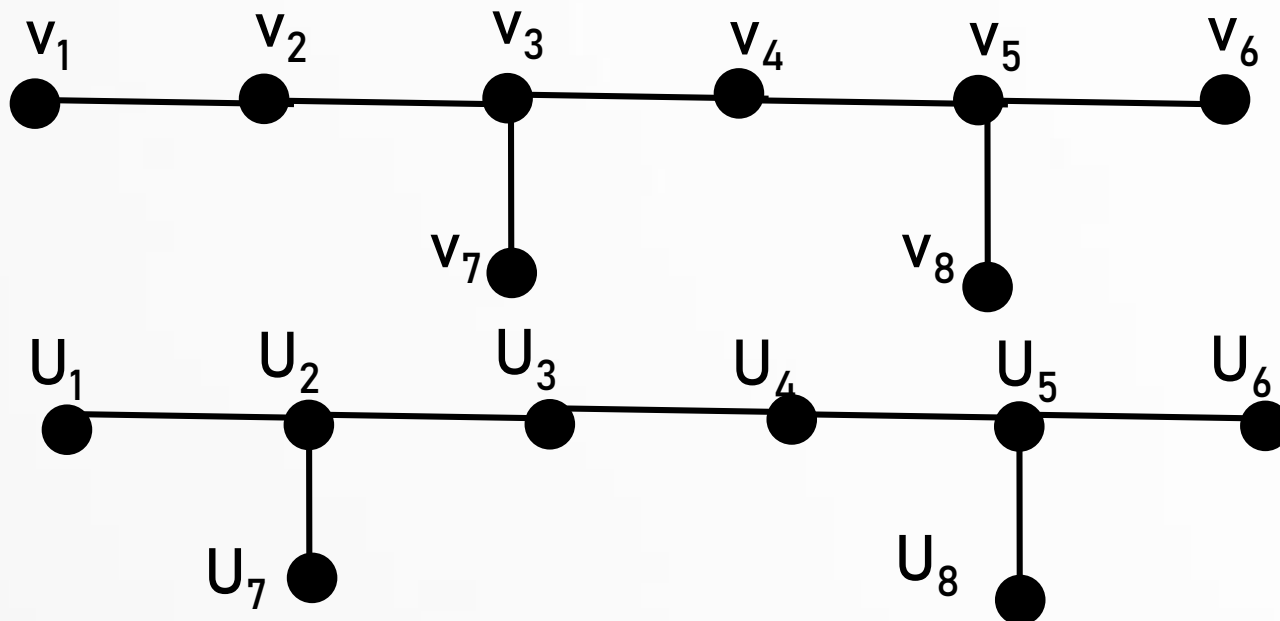
	$v_4$	$v_5$	$v_2$	$v_3$	$v_1$
$v_4$	0	1	0	1	1
$v_5$	1	0	1	1	1
$v_2$	0	1	0	1	0
$v_3$	1	1	1	0	1
$v_1$	1	1	0	1	0

$$f(u_1) = v_4, f(u_2) = v_5, f(u_3) = v_2, f(u_4) = v_3, f(u_5) = v_1$$

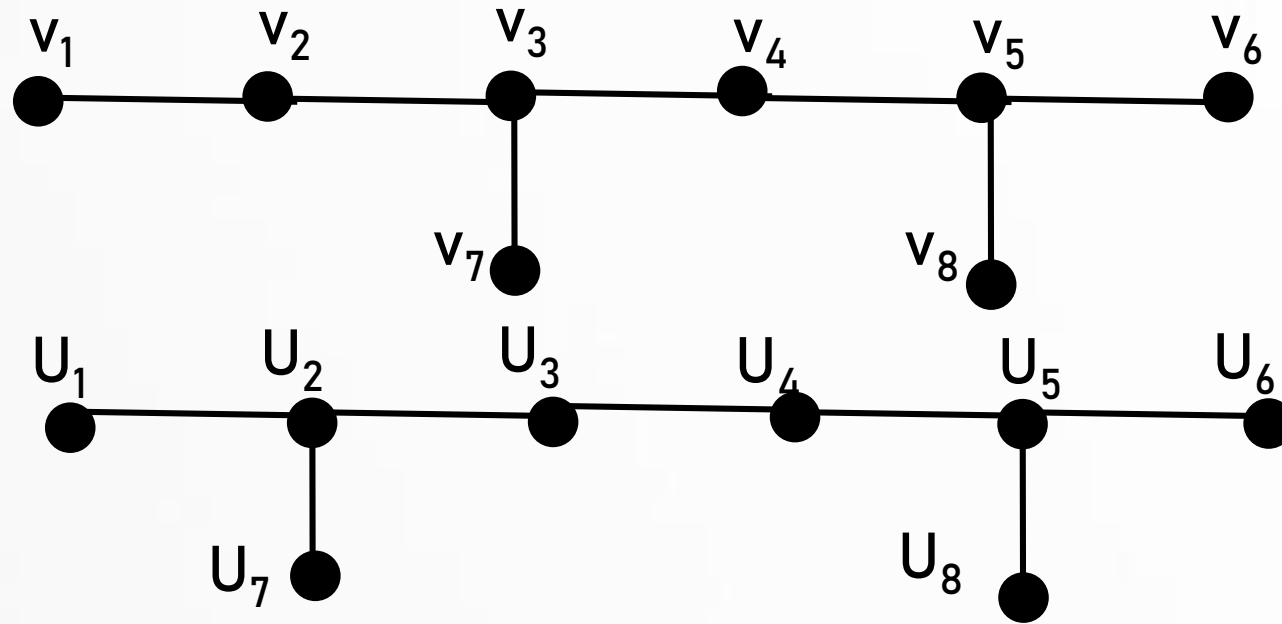
Therefore the two graphs are isomorphic as their adjacency matrix are equal for the given bijection.

# Graph Isomorphism

Ques:- Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.?



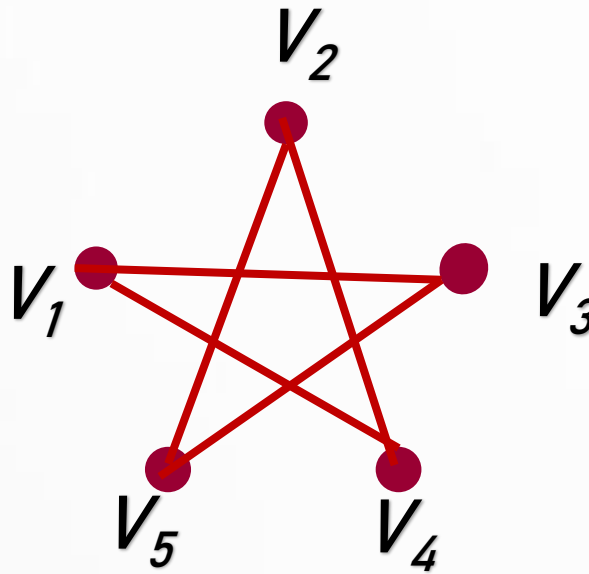
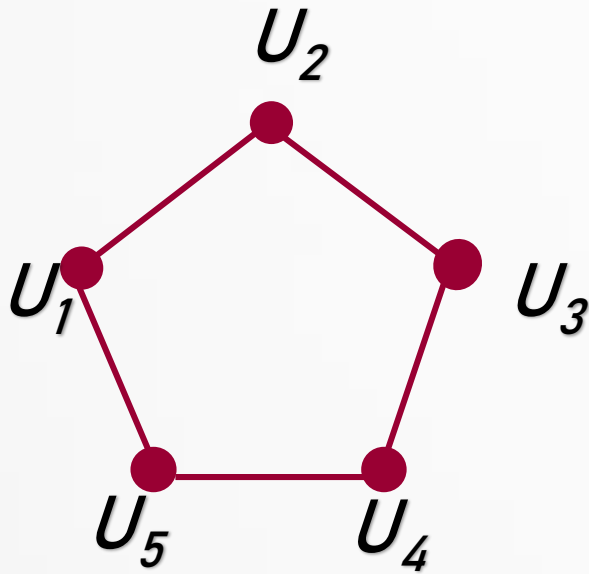
# Graph Isomorphism



These graphs are not isomorphic. In the first graph, the vertices of degree 3 are adjacent to a common vertex. This is not true of the second graph.

# Graph Isomorphism

Ques:- Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.



# Graph Isomorphism

Ans:- These graphs are isomorphic, since each is the 5-cycle.

One isomorphism is

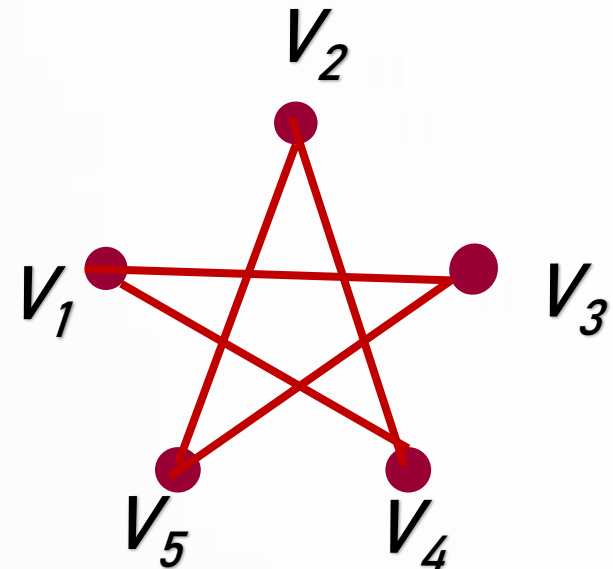
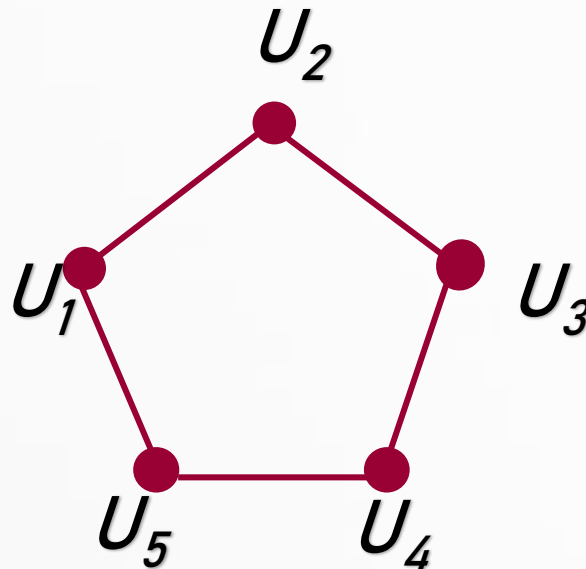
$$f(u_1) = v_1,$$

$$f(u_2) = v_3,$$

$$f(u_3) = v_5,$$

$$f(u_4) = v_2,$$

$$\text{and } f(u_5) = v_4.$$



That's all for now...