EMTH403

Mathematical Foundation for Computer Science

Nitin K. Mishra (Ph.D.)

Associate Professor

Lecture Outcomes

After this lecture, you will be able to

- understand what is a biconditional Statements
- learn what is equivalence of formulas
- understand what is duality law.

BICONDITIONALS

Let p and q be propositions. The biconditional statement $p \Leftrightarrow q$ is the proposition

"p if and only if q."

The biconditional statement $p \Leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise.

TABLE 6 **The Truth Table for the Biconditional $p \leftrightarrow q$.**

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

BICONDITIONALS

Biconditional statements are also called **bi-implications.**

TABLE 6 **The Truth Table for the Biconditional $p \leftrightarrow q$.**

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

BICONDITIONALS – Example 1

Let p be the statement "You can take the flight," and let q be the statement "You buy a ticket."

Then p ⇔ q is the statements "You can take the flight **if and only if** you buy a ticket."

TABLE 6 The Truth Table for the Biconditional $p \leftrightarrow q$.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

BICONDITIONALS – Example 1

This statement is true if p and q are either **both true or both false**, that is, if you buy a ticket and can take the flight or if you do not buy a ticket and you cannot take the flight.

TABLE 6 The Truth Table for the Biconditional $p \leftrightarrow q$.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Content Source: Discrete Mathematics and Its Applications. Seventh Edition. Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University. (and formerly AT&T Laboratories)

BICONDITIONALS – Example 1

It is false when p and q have opposite truth values, that is, when you do not buy a ticket, but you can take the flight (such as when you get a free trip) and when you buy a ticket but you cannot take the light (such as when the airline bumps you).

TABLE 6 The Truth Table for the Biconditional $p \leftrightarrow q$.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

BICONDITIONALS – Example 2

Ques:- Let p and q be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore," respectively. Express $p \Leftrightarrow \neg q$ compound proposition as an English sentence.

Ans:- Swimming at the New Jersey shore is allowed **if and only if** sharks have **not** been spotted near the shore.

Truth Tables of Compound Propositions

Construct the truth table of the compound proposition

$(p \vee \neg q) \rightarrow (p \wedge q)$.

TABLE 1 The Truth Table for the Negation of a Proposition.

p	$\neg p$
T	F
F	T

The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$ is

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Content Source: Discrete Mathematics and Its Applications. Seventh Edition. Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University. (and formerly AT&T Laboratories)

Truth Tables of Compound Propositions

Construct the truth table of the compound proposition

$(p \vee \neg q) \rightarrow (p \wedge q)$.

The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$ is

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

TABLE 3 The Truth Table for the Disjunction of Two Propositions.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Content Source: Discrete Mathematics and Its Applications. Seventh Edition. Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University. (and formerly AT&T Laboratories)

Truth Tables of Compound Propositions

Construct the truth table of the compound proposition

$(p \vee \neg q) \rightarrow (p \wedge q)$.

The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$ is

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

TABLE 2 The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth Tables of Compound Propositions

Construct the truth table of the compound proposition

$(p \lor \neg q) \rightarrow (p \land q)$.

The Truth Table of $(p \lor \neg q) \rightarrow (p \land q)$ is

p	q	$\neg q$	$p \lor \neg q$	$p \land q$	$(p \lor \neg q) \rightarrow (p \land q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

TABLE 5 The Truth Table for the Conditional Statement $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Content Source: Discrete Mathematics and Its Applications. Seventh Edition. Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University. (and formerly AT&T Laboratories)

Tautology

DEFINITION:- A compound proposition that is **always true**, no matter what the truth values of the propositional variables that occur in it, is called a tautology.

Example of a Tautology

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

Contradiction

DEFINITION:- A compound proposition that is **always false** is called a contradiction.

Example of a Contradiction.

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

Contingency

DEFINITION:- A compound proposition that is **neither a tautology nor a contradiction** is called a contingency. As we can see in the truth table for conditional statement $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Content Source: **Discrete Mathematics** and **Its Applications**. Seventh Edition. **Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University.** (and formerly AT&T Laboratories)

Equivalence of formulas

The compound propositions p and q are called **logically equivalent** if $p \Leftrightarrow q$ is a tautology.

TABLE 6 The Truth Table for the Biconditional $p \leftrightarrow q$.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

The notation $p \equiv q$ denotes that p and q are logically equivalent.

Equivalence of formulas

Remark 1: The symbol ≡ is not a logical connective, and p ≡ q is not a compound proposition but rather is the statement that p ⇔ q is a tautology.

TABLE 6 The Truth Table for the Biconditional $p \leftrightarrow q$.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Content Source: Discrete Mathematics and Its Applications. Seventh Edition. Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University. (and formerly AT&T Laboratories)

Equivalence of formulas

Remark 2: The symbol $\Leftrightarrow$ is sometimes used instead of $\equiv$ to denote logical equivalence.

One way to determine whether two compound propositions are equivalent is to use a truth table.

In particular, the compound propositions p and q are equivalent if and only if the columns giving their truth values agree.

Equivalence of formulas

Following Example illustrates this method to establish an extremely important and useful logical equivalence, namely, that of $\neg(p \vee q)$ with $\neg p \wedge \neg q$.

This logical equivalence is one of the two De Morgan laws, shown in the following Table, named after the English mathematician Augustus De Morgan, of the mid-nineteenth century.

Equivalence of formulas – Example 1

De Morgan's Laws.

1. $\neg(p \land q) \equiv \neg p \lor \neg q$
2. $\neg(p \lor q) \equiv \neg p \land \neg q$

Ques:- Show that $\neg(p \lor q)$ and $\neg p \land \neg q$ are logically equivalent.

The truth table for $\neg(p \lor q)$ and $\neg p \land \neg q$ is:-

p	q	$p \lor q$	$\neg(p \lor q)$	$\neg p$	$\neg q$	$\neg p \land \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Content Source: Discrete Mathematics and Its Applications. Seventh Edition. Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University. (and formerly AT&T Laboratories)

Equivalence of formulas – Example 2

Show that $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent.

Truth Tables for $\neg p \vee q$ and $p \rightarrow q$ is

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Content Source: Discrete Mathematics and Its Applications. Seventh Edition. Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University. (and formerly AT&T Laboratories)

Duality law

The dual of a compound proposition that contains only the logical operators ∨, ∧, and ¬ is the compound proposition obtained by replacing each ∨ by ∧, each ∧ by ∨, each T by F, and each F by T.

The dual of s is denoted by s^*.

Duality law – Example 1

Ques:- Find the dual of the compound proposition

$p \wedge \neg q \wedge \neg r$.

Ans:- $p \vee \neg q \vee \neg r$ (replacing each $\vee$ by $\wedge$, each $\wedge$ by $\vee$)

Duality law – Example 2

Ques:- Find the dual of the compound proposition $(p \land q \land r) \lor s$.

Ans:- $(p \lor q \lor r) \land s$ (replacing each $\lor$ by $\land$, each $\land$ by $\lor$)

Content Source: **Discrete Mathematics** and **Its Applications**. Seventh Edition. **Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University.** (and formerly AT&T Laboratories)

Duality law – Example 3

Ques:- Find the dual of the compound proposition $(p \vee F) \wedge (q \vee T)$.

Ans:- $(p \wedge T) \vee (q \wedge F)$ (replacing each $\vee$ by $\wedge$, each $\wedge$ by $\vee$, each T by F, and each F by T.)

That's all for now...