EMTH403

Mathematical Foundation for Computer Science

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Lecture Outcomes

After this lecture, you will be able to

- understand what are path, cycles and connectivity in a graph.
- understand how to discard an isomorphism with the help of cycles.

Connectivity - Path

- Many problems can be modeled with paths formed by traveling along the edges of graphs.
- For instance, the problem of determining whether a message can be sent between two computers using intermediate links can be studied with a graph model.
- Problems of efficiently planning routes for mail delivery, garbage pickup, diagnostics in computer networks, and so on can be solved using models that involve paths in graphs.

Connectivity - Path

- Informally, a path is a sequence of edges that begins at a vertex of a graph and travels from vertex to vertex along edges of the graph.

- As the path travels along its edges, it visits the vertices along this path, that is, the endpoints of these edges.

Connectivity - Path

A formal definition of paths and related terminology is given below.

Connectivity - Path

Let n be a nonnegative integer and G an undirected graph.

A path of length n from u to v in G is a sequence of n edges $e_1, \ldots, e_n$ of G for which there exists a sequence $x_0 = u, x_1, \ldots, x_{n-1}, x_n = v$ of vertices such that e_i has, for $i = 1, \ldots, n$, the endpoints x_{i-1} and x_i .

Content Source: Discrete Mathematics and Its Applications. Seventh Edition. Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University. (and formerly AT&T Laboratories)

Connectivity - Path

- The path is a circuit if it begins and ends at the same vertex, that is, if u = v, and has length greater than zero.
- The path or circuit is said to pass through the vertices $x_1, x_2, \ldots, x_{n-1}$ or traverse the edges $e_1, e_2, \ldots, e_n$.
- A path or circuit is simple if it does not contain the same edge more than once.

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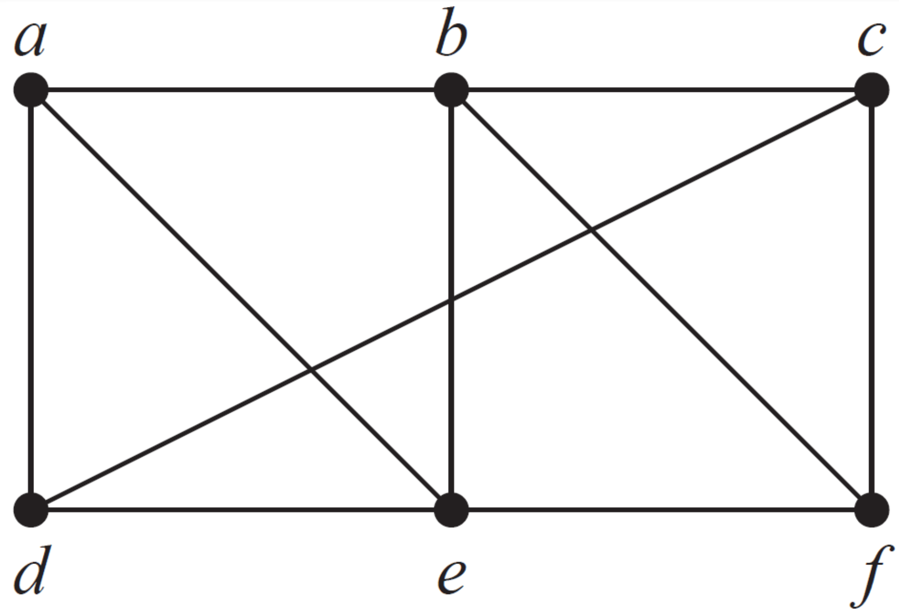
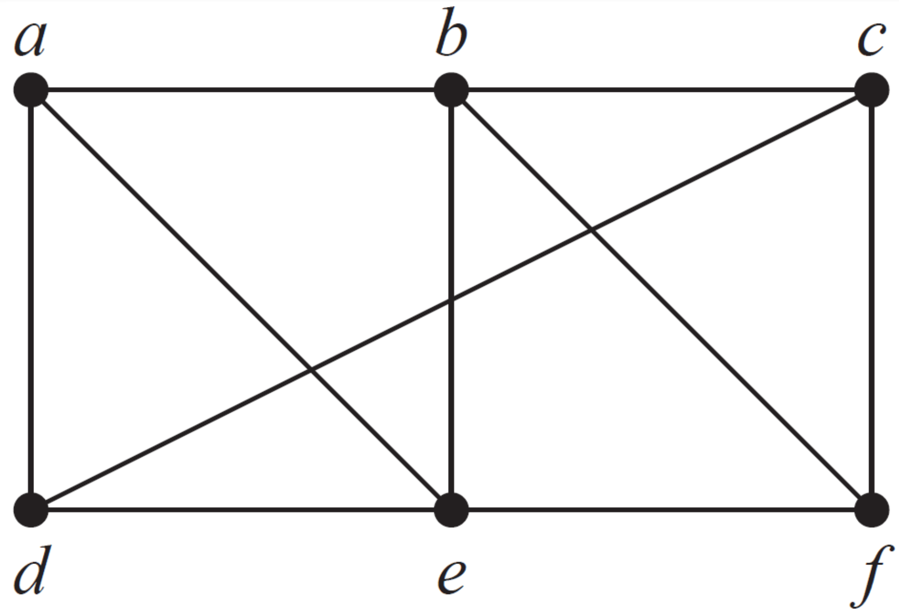
Connectivity - Path



In the simple graph shown in above, a, d, c, f , e is a simple path of length 4, because {a, d}, {d, c}, {c, f }, and {f, e} are all edges.

Connectivity - Path



However, d, e, c, a is not a path, because {e, c} is not an edge.

Note that b, c, f , e, b is a circuit of length 4 because {b, c}, {c, f }, {f, e}, and {e, b} are edges, and this path begins and ends at b.

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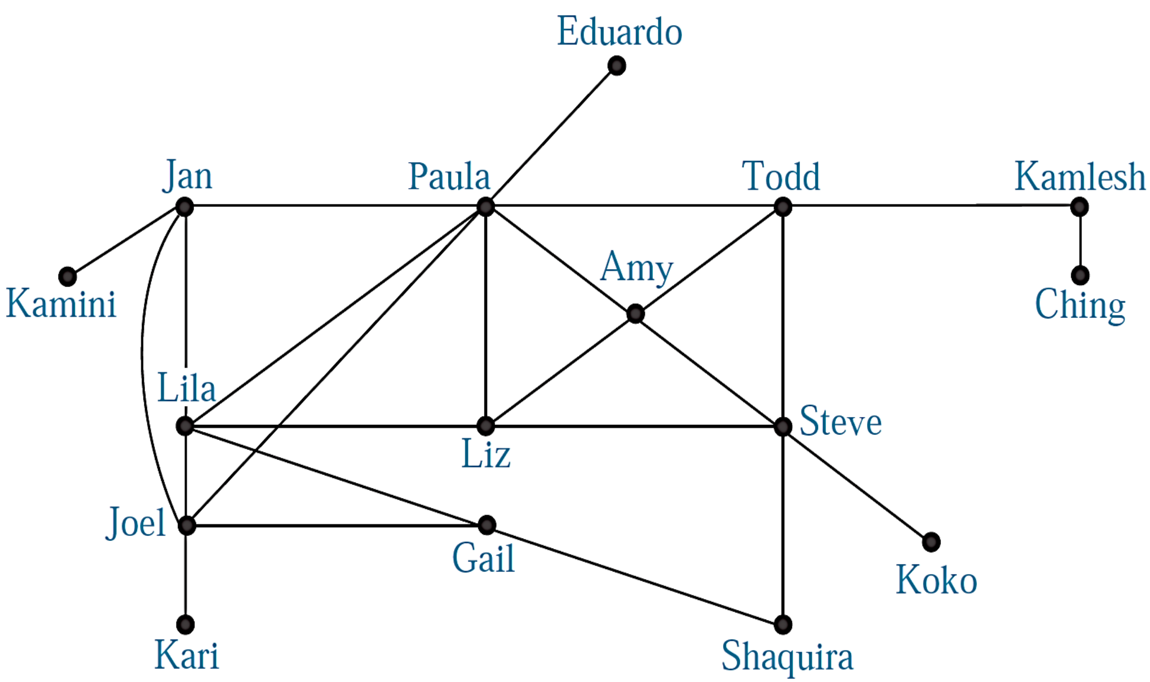
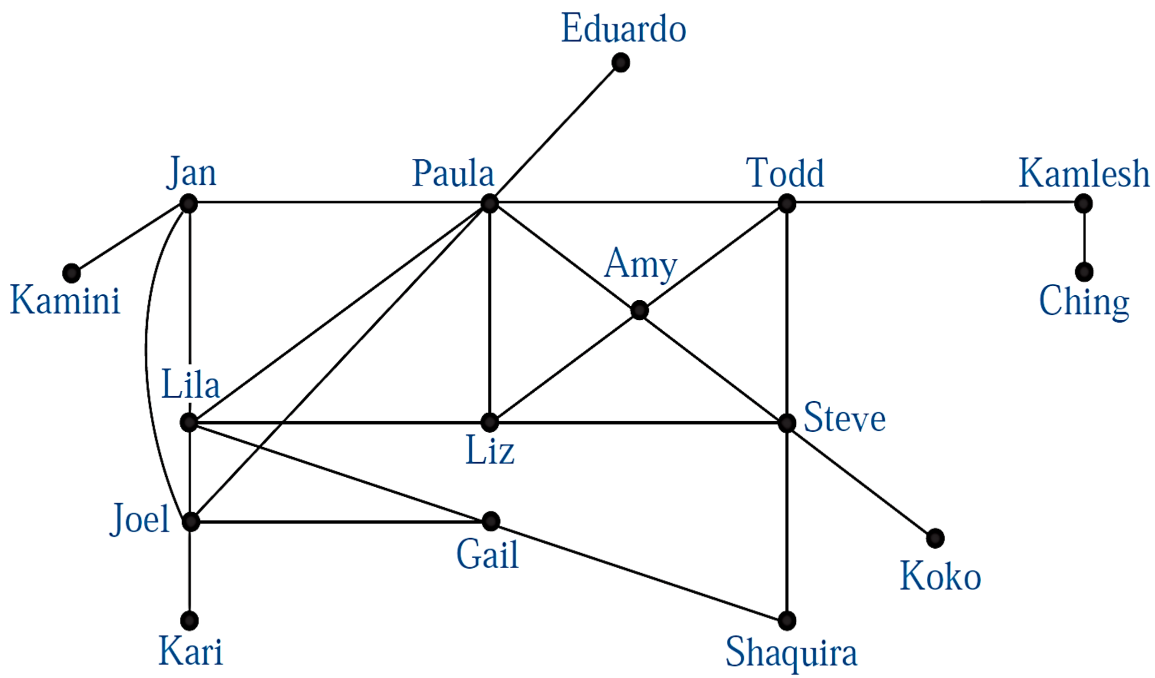
Connectivity – Path – Example



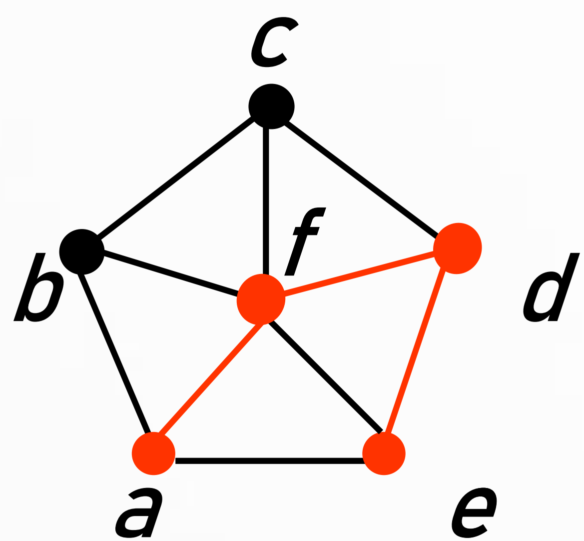
In an acquaintanceship graph there is a path between two people if there is a chain of people linking these people, where two people adjacent in the chain knows one another. For example, there is a chain of six people linking Kamini and Ching

Connectivity – Path – Example



Many social scientists have conjectured that almost every pair of people in the world are linked by a small chain of people, perhaps containing just five or fewer people.

Connectivity – Path



W_5

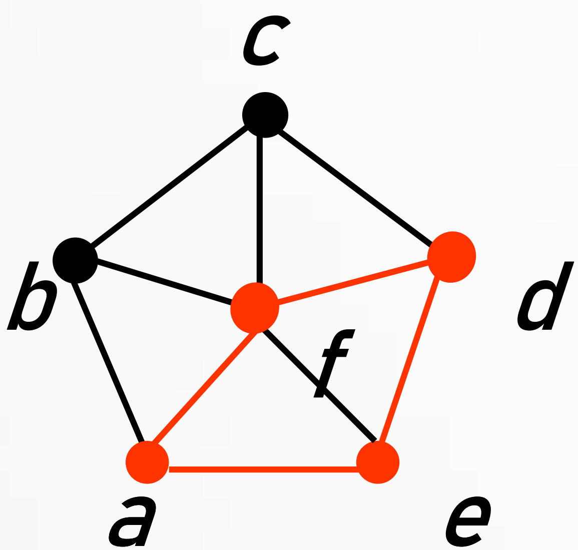
This path passes through vertices f and d in that order.

Connectivity – Path

A path of length n from u to v in an undirected graph is a sequence of edges $e_1, e_2, \ldots, e_n$ of the graph such that edge e_1 has endpoints x_o and x_1, edge e_2 has endpoints x_1 and x_2, . . . and edge e_n has endpoints x_{n-1} and x_n,

where $x_0 = u$ and $x_n = v$.

Connectivity – Path



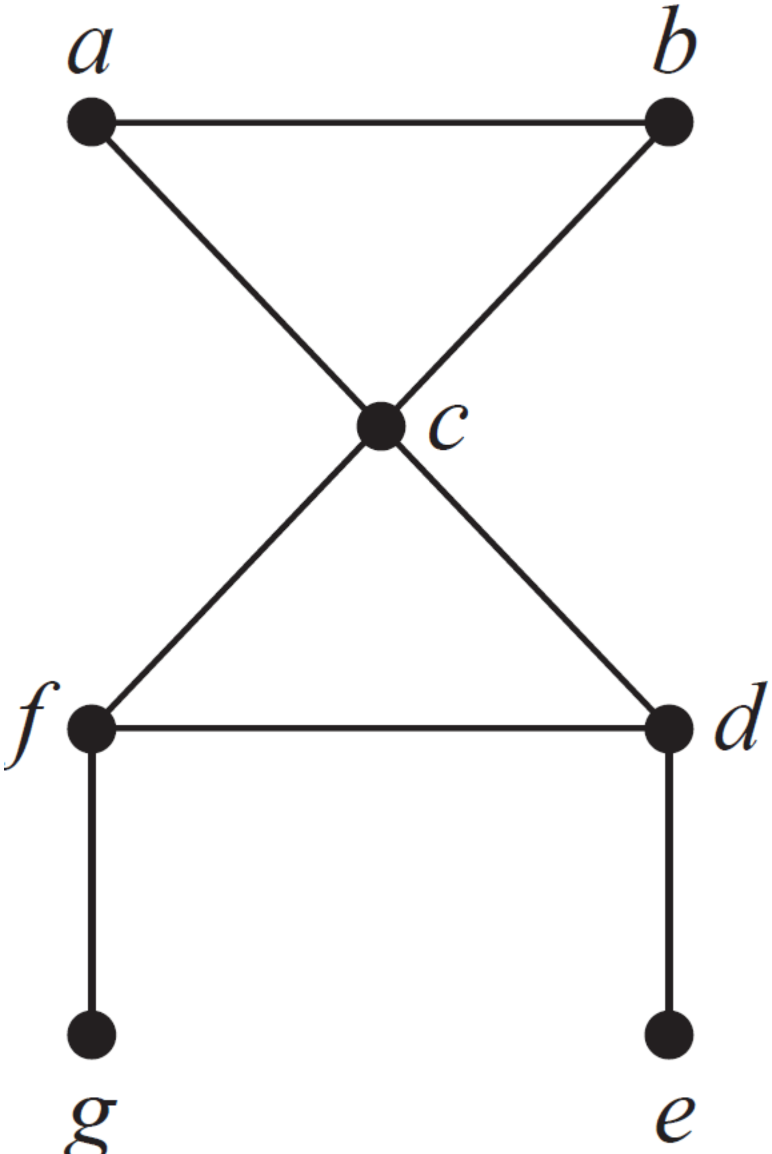
W_5

This path passes through vertices f, d, e, in that order. It has length 4.

It is a circuit because it begins and ends at the same vertex.

It is called simple because it does not contain the same edge more than once.

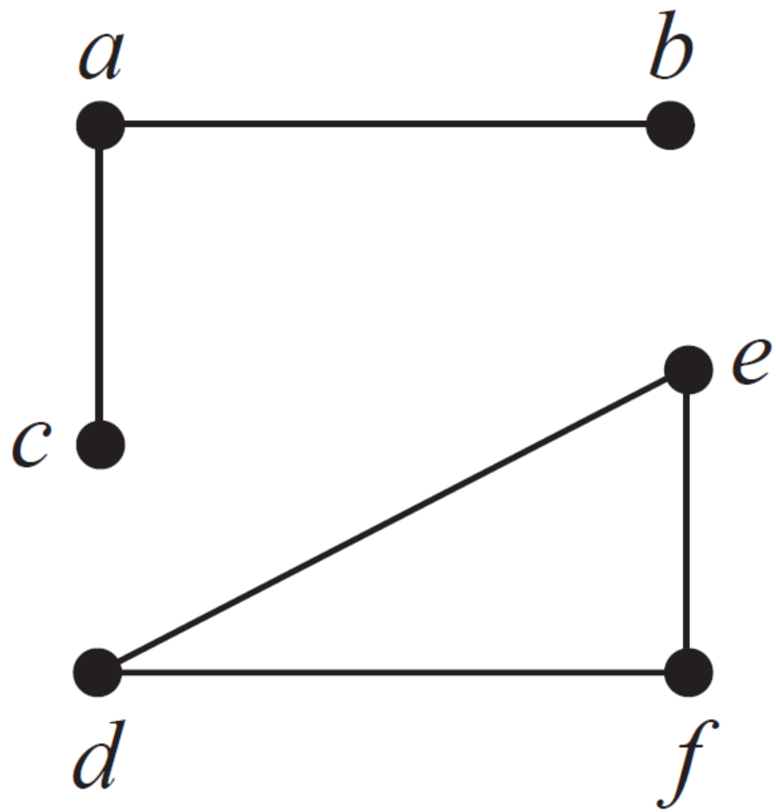
Connectivity – Path



- An undirected graph is called connected if there is a path between every pair of distinct vertices of the graph.

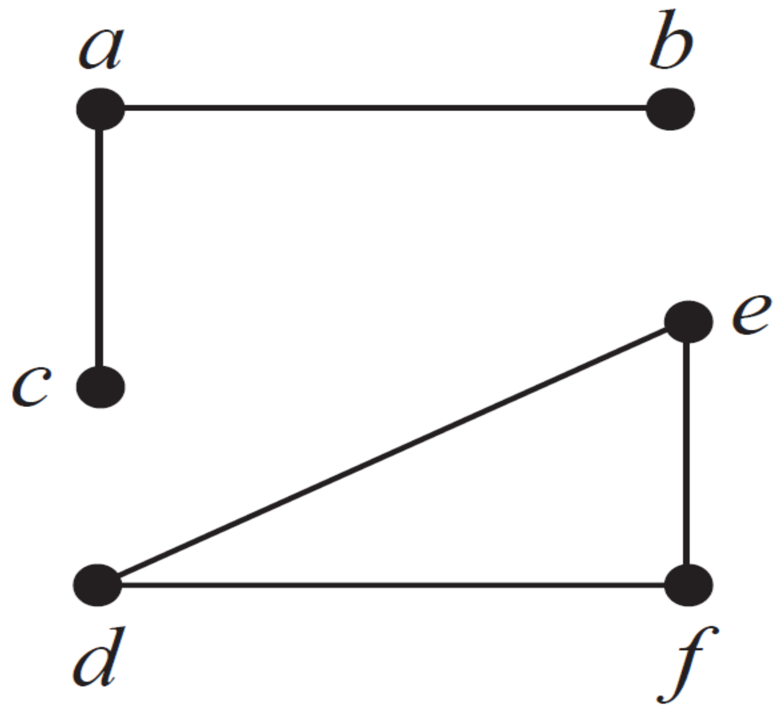
Connectivity – Path

- An undirected graph that is not connected is called disconnected.



Connectivity – Path

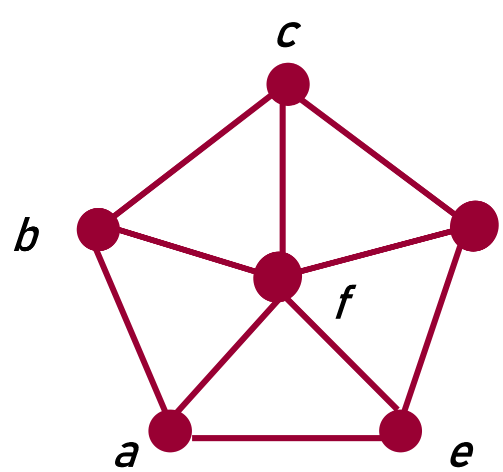
- We say that we disconnect a graph when we remove vertices or edges, or both, to produce a disconnected subgraph.



Connectedness in Undirected Graphs

Definition:- An undirected graph is called connected if there is a path between every pair of distinct vertices of the graph.

IS THIS GRAPH CONNECTED?



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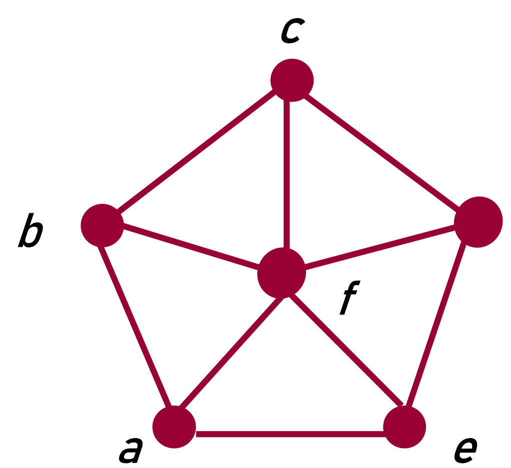
Connectedness in Undirected Graphs

Theorem:- There is a simple path between every pair of distinct vertices of a connected undirected graph.



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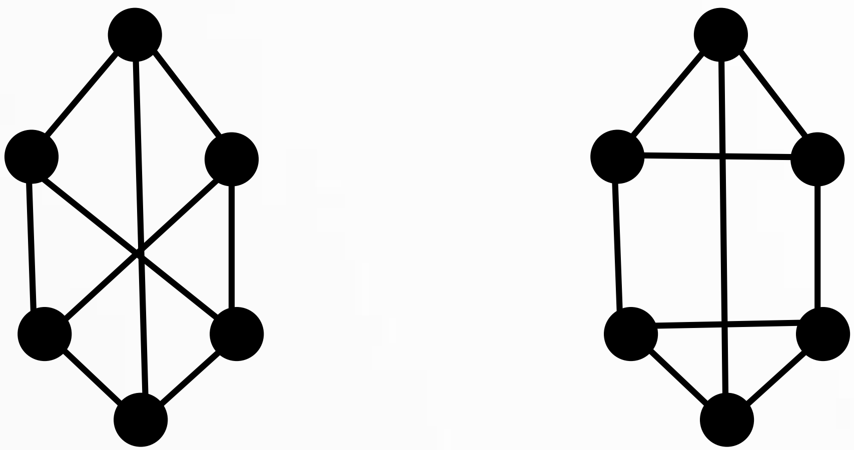
Graph Isomorphism - Definition

The simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are *isomorphic* if there exists a one to-one and onto function f from V_1 to V_2 with the property that a and b are adjacent in $G1$ if and only if $f(a)$ and $f(b)$ are adjacent in G_2, for all a and b in V_1.

Such a function f is called an *isomorphism*.∗ Two simple graphs that are not isomorphic are called *non-isomorphic*.

Graph Isomorphism

Ques:- Are the pairs of graphs are isomorphic?



Ans:- Hint: Check Cycles

Not isomorphic, the right-hand side graph contains a triangle, but the left-hand side graph does not.

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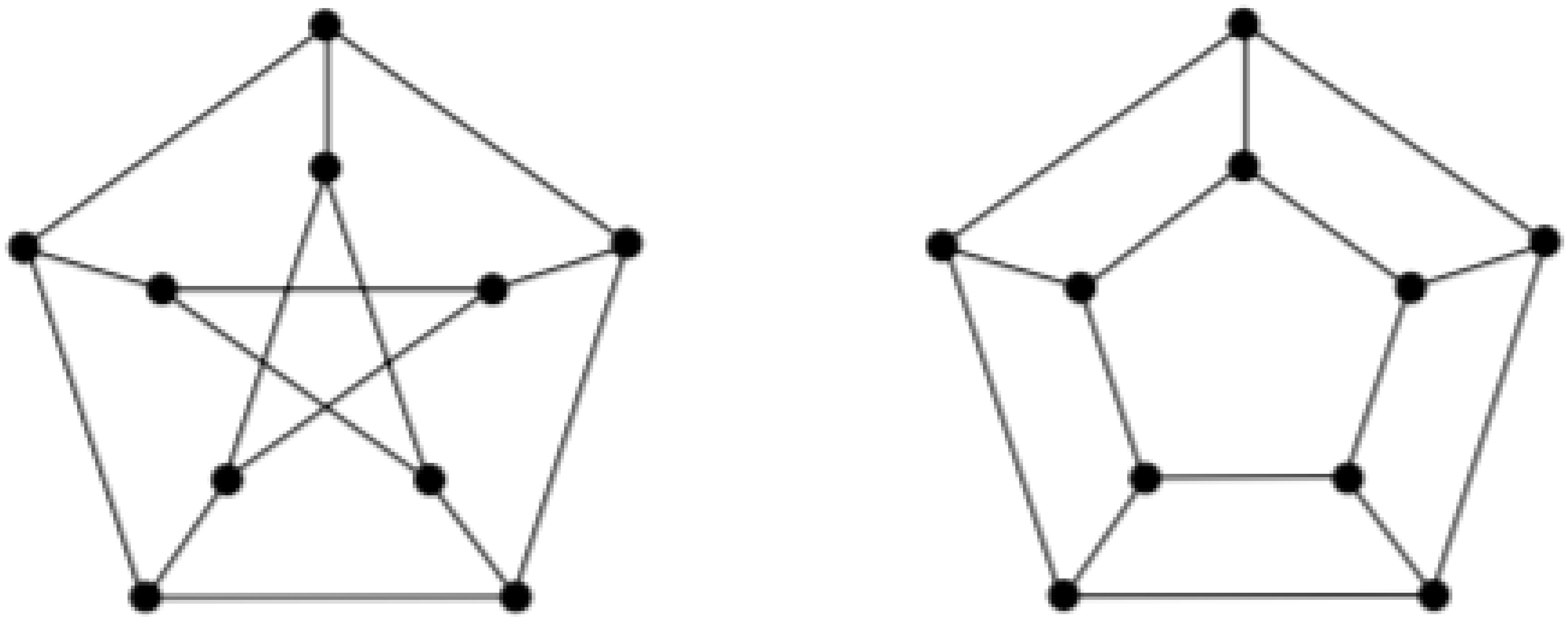
Graph Isomorphism

Ques:- Are the pairs of graphs are isomorphic?



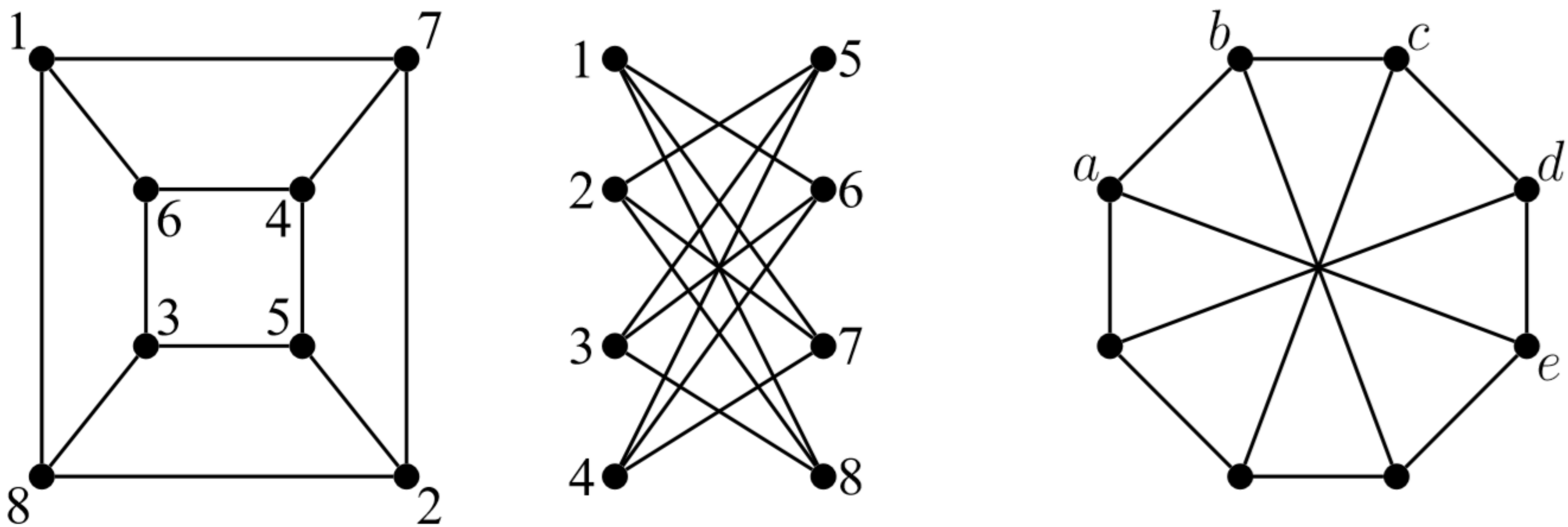
Ans:- Hint: Check Cycles

Not isomorphic.

First does not contain a cycle of length 4, while the right graph contains a cycle of length 4.

Graph Isomorphism

Ques:- Determine which pairs of graphs below are isomorphic. Justify your answer!



Ans:- The first two graphs are isomorphic.

On the other hand, the third graph contains an odd cycle on 5 vertices a,b,c,d,e, thus, this graph is not isomorphic to the first two.

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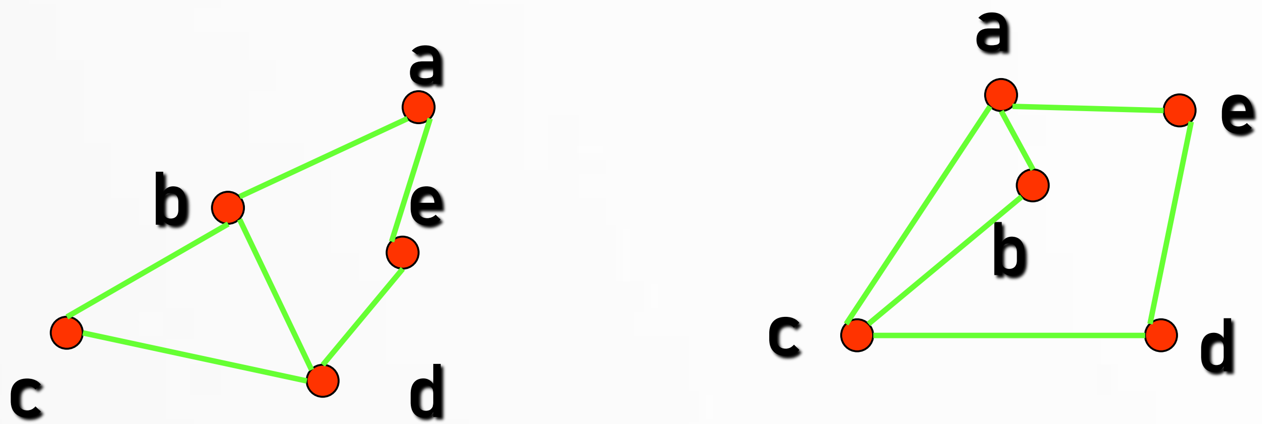
Graph Isomorphism

Ques:- Are the following two graphs isomorphic?



Sol:- Yes, they are isomorphic, because they can be arranged to look identical. You can see this if in the right graph you move vertex b to the left of the edge {a, c}. Then the isomorphism f from the left to the right graph is: f(a) = e, f(b) = a, f(c) = b, f(d) = c, f(e) = d.

That's all for now...