



EMTH403

Mathematical Foundation for Computer Science

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Lecture Outcomes



After this lecture, you will be able to

- understand what are isomorphic graph
- understand what is an isomorphism.

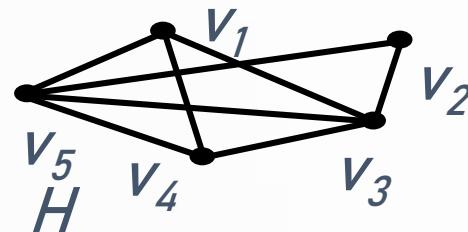
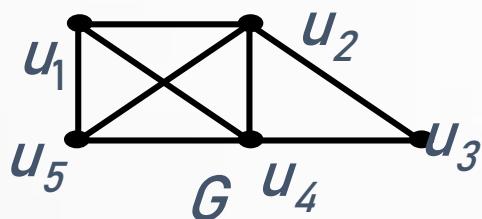
Graph Isomorphism - Definition

The simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic if there exists a one to-one and onto function f from V_1 to V_2 with the property that a and b are adjacent in G_1 if and only if $f(a)$ and $f(b)$ are adjacent in G_2 , for all a and b in V_1 .

Such a function f is called an isomorphism.* Two simple graphs that are not isomorphic are called non-isomorphic.

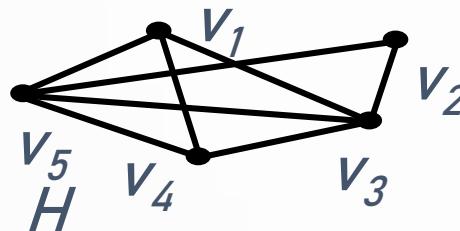
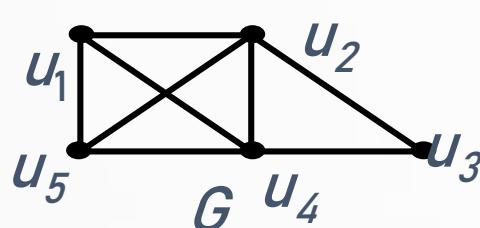
Graph Isomorphism

- Are these two graphs isomorphic?



- They both have 5 vertices
- They both have 8 edges
- They have the same number of vertices with the same degrees: 2, 3, 3, 4, 4.

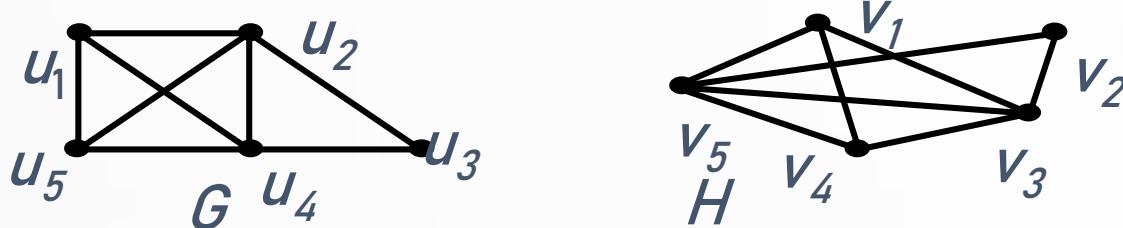
Graph Isomorphism



	u_1	u_2	u_3	u_4	u_5		v_1	v_2	v_3	v_4	v_5		
u_1	0	1	0	1	1		v_1	0	0	1	1	1	
u_2	1	0	1	1	1		v_2	0	0	1	0	1	
u_3	0	1	0	1	0		v_3	1	1	0	1	1	
u_4	1	1	1	0	1		v_4	1	0	1	0	1	
u_5	1	1	0	1	0		v_5	1	1	1	1	0	

- G and H don't appear to be isomorphic.
- However, we haven't tried mapping vertices from G onto H yet.

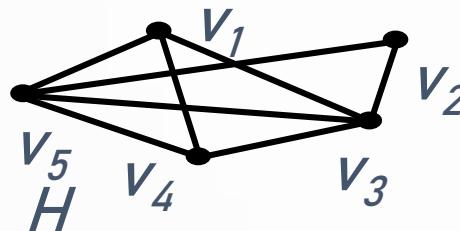
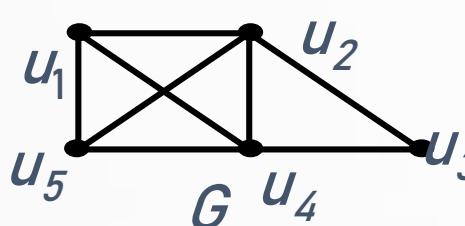
Graph Isomorphism



Start with the vertices of degree 2 since each graph only has one:

$$\deg(u_3) = \deg(v_2) = 2 \text{ therefore } f(u_3) = v_2$$

Graph Isomorphism



Now consider vertices of degree 3

$$\deg(u_1) = \deg(u_5) = \deg(v_1) = \deg(v_4) = 3$$

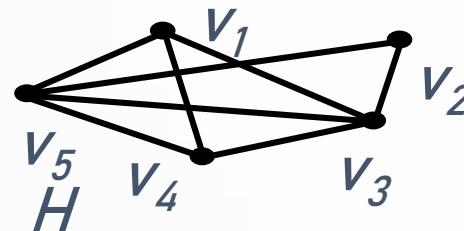
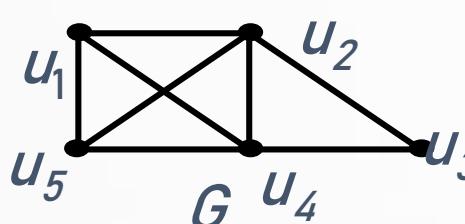
therefore we must have either one of

$$f(u_1) = v_1 \text{ and } f(u_5) = v_4$$

$$f(u_1) = v_4 \text{ and } f(u_5) = v_1$$

Along with $f(u_3) = v_2$

Graph Isomorphism



Now try vertices of degree 4:

$$\deg(u_2) = \deg(u_4) = \deg(v_3) = \deg(v_5) = 4$$

therefore we must have one of:

$$f(u_2) = v_3 \text{ and } f(u_4) = v_5 \quad \text{or}$$

$$f(u_2) = v_5 \text{ and } f(u_4) = v_3$$

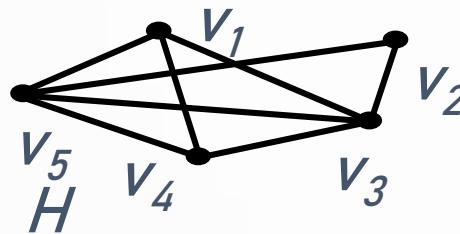
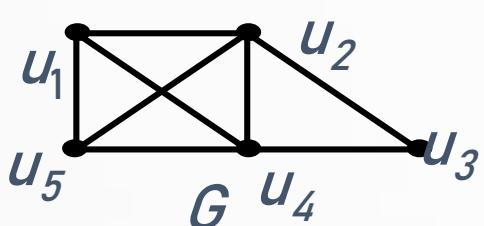
Along with $f(u_3) = v_2$

And along with $f(u_1) = v_1$ and $f(u_5) = v_4$

OR

$f(u_1) = v_4$ and $f(u_5) = v_1$

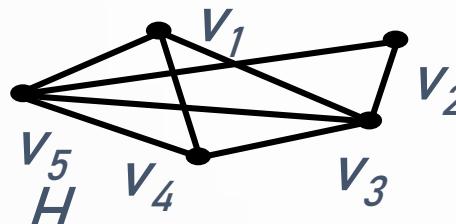
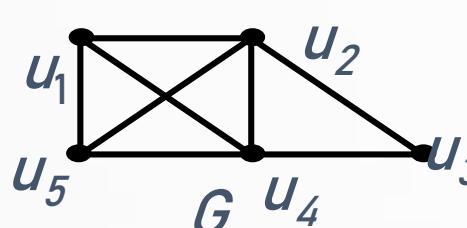
Graph Isomorphism



U_3	V_2	U_3	V_2	U_3	V_2	U_3	V_2
U_1	V_1	U_1	V_1	U_1	V_1	U_1	V_1
U_5	V_4	U_5	V_4	U_5	V_4	U_5	V_4
U_2	V_3	U_2	V_3	U_2	V_3	U_2	V_3
U_4	V_5	U_4	V_5	U_4	V_5	U_4	V_5

- There are four possible bijections which are as follows:
 - $f(u_1) = v_1, f(u_2) = v_3, f(u_3) = v_2, f(u_4) = v_5, f(u_5) = v_4$
 - $f(u_1) = v_4, f(u_2) = v_3, f(u_3) = v_2, f(u_4) = v_5, f(u_5) = v_1$
 - $f(u_1) = v_1, f(u_2) = v_5, f(u_3) = v_2, f(u_4) = v_3, f(u_5) = v_4$
 - $f(u_1) = v_4, f(u_2) = v_5, f(u_3) = v_2, f(u_4) = v_3, f(u_5) = v_1$

Graph Isomorphism

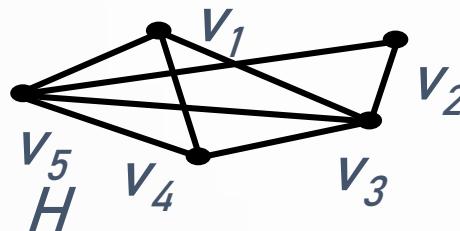
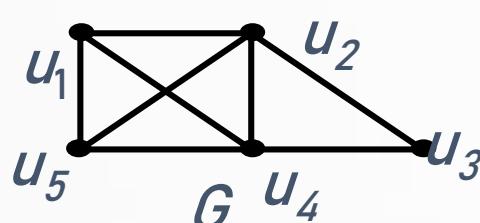


	u_1	u_2	u_3	u_4	u_5		v_1	v_2	v_3	v_4	v_5	
u_1	0	1	0	1	1		v_1	0	0	1	1	1
u_2	1	0	1	1	1		v_2	0	0	1	0	1
u_3	0	1	0	1	0		v_3	1	1	0	1	1
u_4	1	1	1	0	1		v_4	1	0	1	0	1
u_5	1	1	0	1	0		v_5	1	1	1	1	0

$$f(u_1) = v_1, f(u_2) = v_3, f(u_3) = v_2, f(u_4) = v_5, f(u_5) = v_4$$

Therefore the two graphs are isomorphic as their adjacency matrices are equal for the above given bijection.

Graph Isomorphism

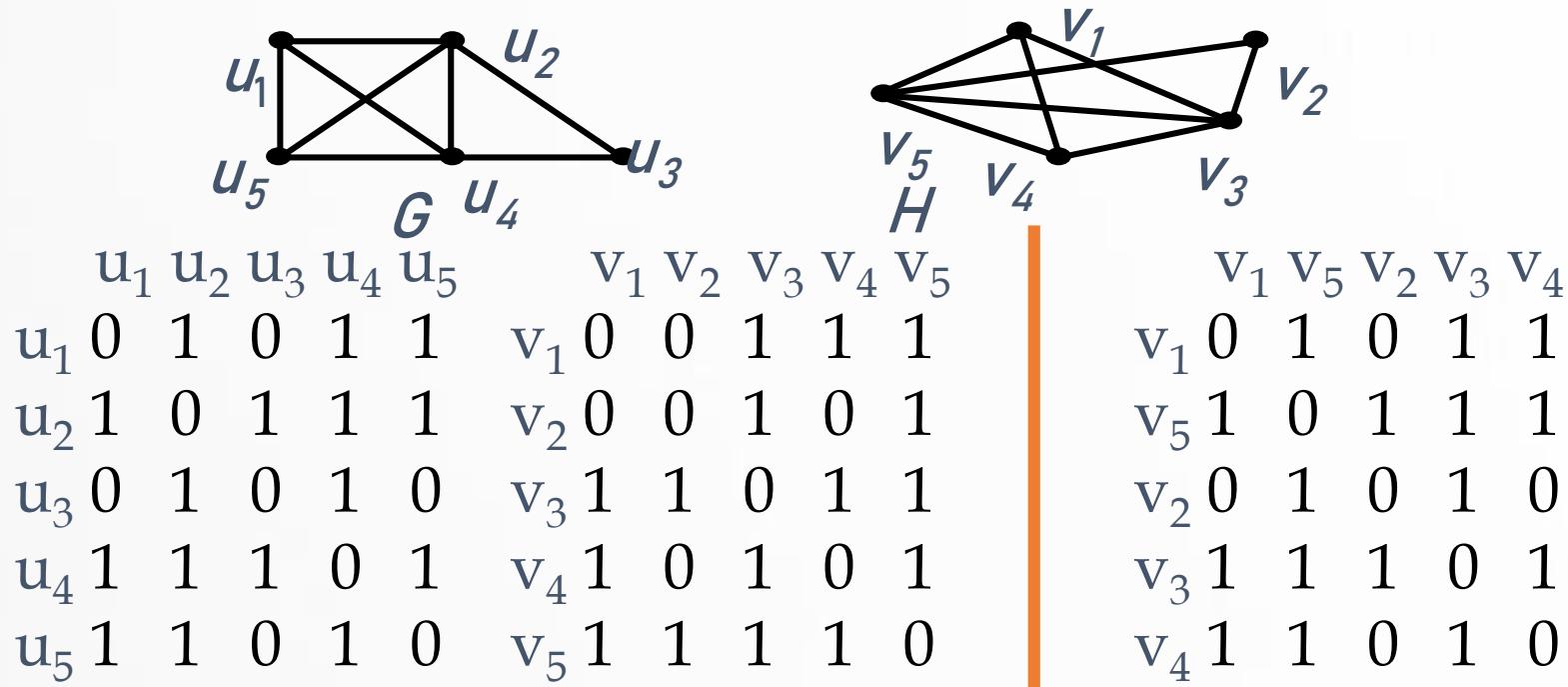


	u_1	u_2	u_3	u_4	u_5		v_1	v_2	v_3	v_4	v_5		v_4	v_3	v_2	v_5	v_1		
u_1	0	1	0	1	1		v_1	0	0	1	1	1		v_4	0	1	0	1	1
u_2	1	0	1	1	1		v_2	0	0	1	0	1		v_3	1	0	1	1	1
u_3	0	1	0	1	0		v_3	1	1	0	1	1		v_2	0	1	0	1	0
u_4	1	1	1	0	1		v_4	1	0	1	0	1		v_5	1	1	1	0	1
u_5	1	1	0	1	0		v_5	1	1	1	1	0		v_1	1	1	0	1	0

$$f(u_1) = v_4, f(u_2) = v_3, f(u_3) = v_2, f(u_4) = v_5, f(u_5) = v_1$$

Therefore the two graphs are isomorphic as their adjacency matrix are equal for the given bijection.

Graph Isomorphism

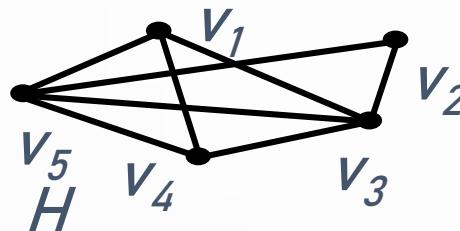
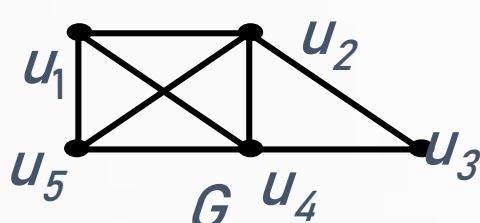


It turns out that

$$f(u_1) = v_1, f(u_2) = v_5, f(u_3) = v_2, f(u_4) = v_3, f(u_5) = v_4$$

Therefore the two graphs are isomorphic as their adjacency matrix are equal for the given bijection.

Graph Isomorphism



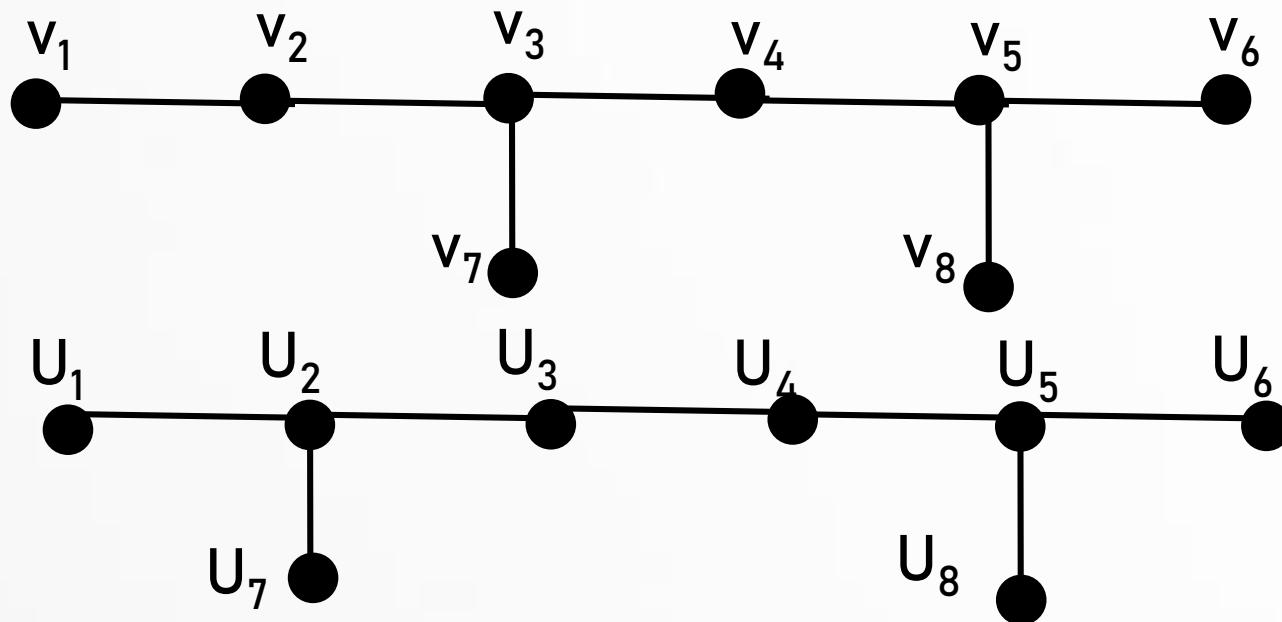
	u_1	u_2	u_3	u_4	u_5		v_1	v_2	v_3	v_4	v_5		v_4	v_5	v_2	v_3	v_1		
u_1	0	1	0	1	1		v_1	0	0	1	1	1		v_4	0	1	0	1	1
u_2	1	0	1	1	1		v_2	0	0	1	0	1		v_5	1	0	1	1	1
u_3	0	1	0	1	0		v_3	1	1	0	1	1		v_2	0	1	0	1	0
u_4	1	1	1	0	1		v_4	1	0	1	0	1		v_3	1	1	1	0	1
u_5	1	1	0	1	0		v_5	1	1	1	1	0		v_1	1	1	0	1	0

$$f(u_1) = v_4, f(u_2) = v_5, f(u_3) = v_2, f(u_4) = v_3, f(u_5) = v_1$$

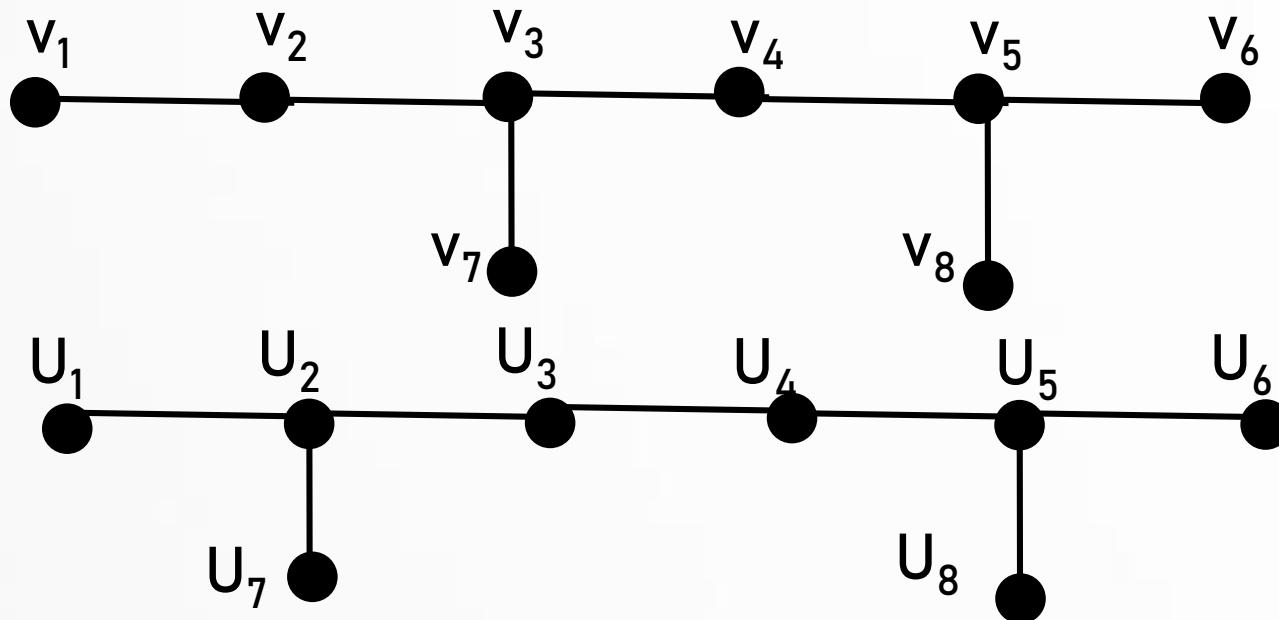
Therefore the two graphs are isomorphic as their adjacency matrix are equal for the given bijection.

Graph Isomorphism

Ques:- Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.?



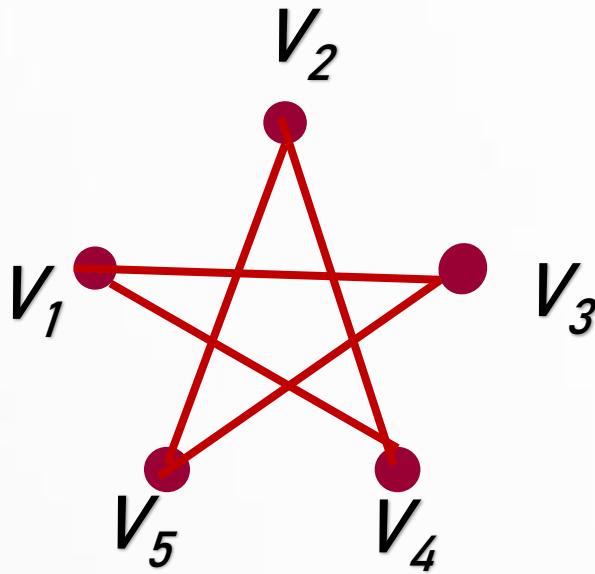
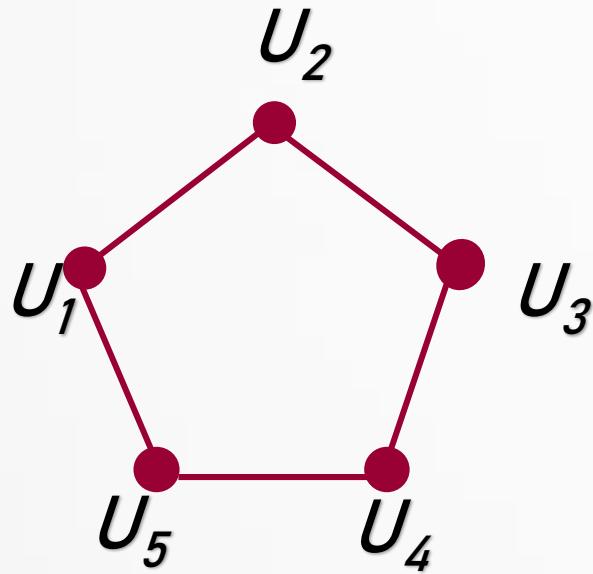
Graph Isomorphism



These graphs are not isomorphic. In the first graph the vertices of degree 3 are adjacent to a common vertex. This is not true of the second graph.

Graph Isomorphism

Ques:- Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.



Graph Isomorphism

Ans:- These graphs are isomorphic, since each is the 5-cycle.

One isomorphism is

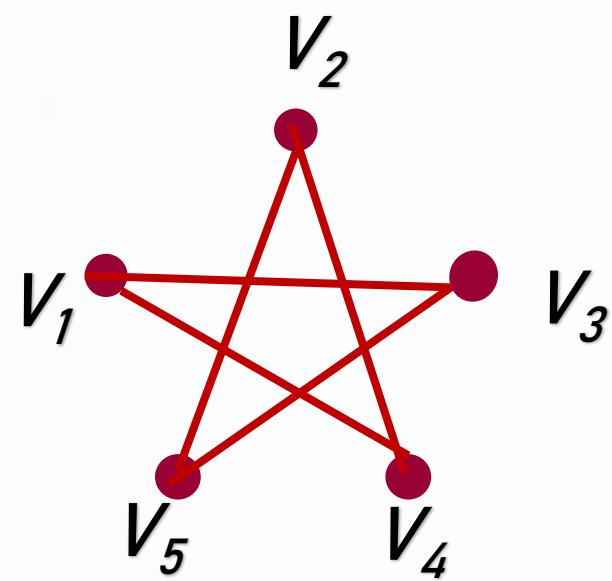
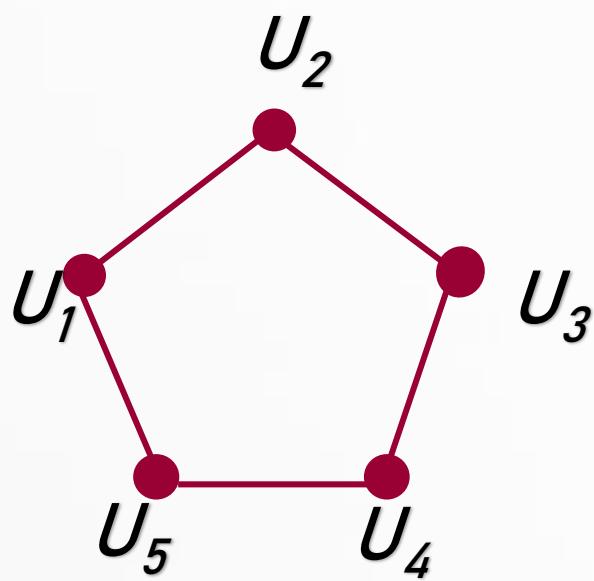
$$f(u_1) = v_1,$$

$$f(u_2) = v_3,$$

$$f(u_3) = v_5,$$

$$f(u_4) = v_2,$$

$$\text{and } f(u_5) = v_4.$$



That's all for now...