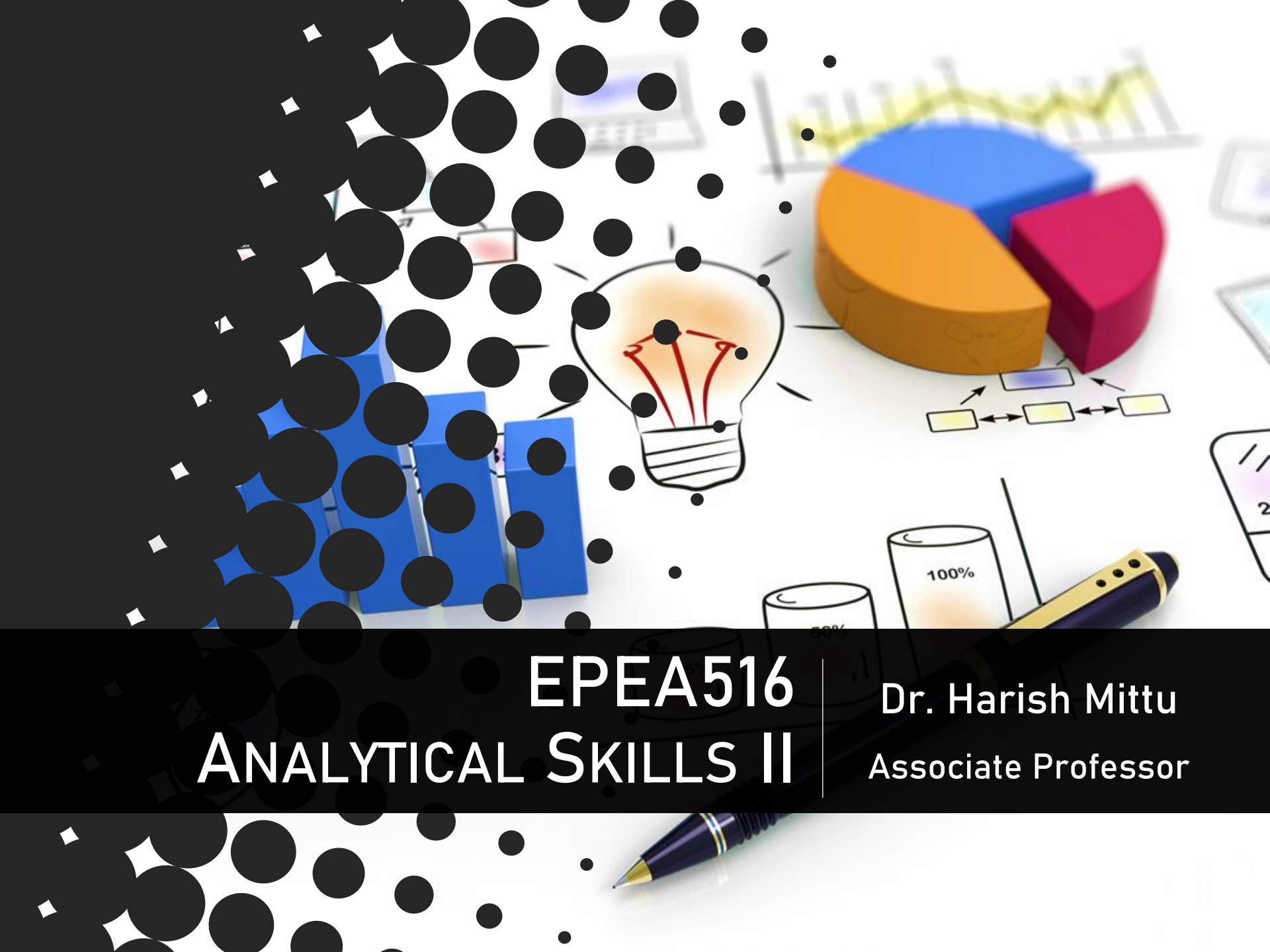

EPEA516
Analytical Skills II

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Learning Outcomes

After this lecture, you will be able to

- define the sequence and series,
- enlist different types of series,
- identify the pattern in the given series.

Sequence

- Set of Numbers
- Definite Order/Rule
- $a_1, a_2, a_3, a_4, \ldots\ldots, a_n$
- 1, 3, 5, 7, ……, 21 or 2, 4, 6, 8, 10, …..
- Finite/Infinite
- $\{a_n\}$

Series

- Sequence - Numbers
- Numbers – Terms
- Terms - Predefined Rule
- Careful Study
- Specific Pattern
- Next Term

Types of Series

- Number Series
- Special Series
- Alpha–Numeric Series

Types of Number Series

- Arithmetic Series
- Arithmetic Series – Second Order
- Arithmetic Series – Third Order
- Geometric Series
- Arithmetico-Geometric Series
- Geometrico-Arithmetic Series
- Series of Squares, Cubes, …..

Arithmetic Series

- Difference b/w Any Two Consecutive Terms - Same
- Common Difference
- Successive Number

  = Previous Number (+) or (-) Fixed Number

- Example - 1, 4, 7, 10, 13, ...
- $T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = \ldots = 3$ (Common Difference)
- Example - 13, 10, 7, 4, 1, ...
- $T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = \ldots = -3$ (Common Difference)

Arithmetic Series – Second Order

- Difference b/w Successive Terms – Arithmetic Series
- Example – 1, 3, 7, 13, ...
- $T_2 - T_1 = 3 - 1 = 2$
- $T_3 - T_2 = 7 - 3 = 4$
- $T_4 - T_3 = 13 - 7 = 6$
- 2, 4, 6, ...
- $a_2 - a_1 = a_3 - a_2 = \ldots = 2$ (Common Difference)
- $T_5 = (13 + 8) = 21$

Arithmetic Series – Third Order

- Difference b/w Successive Terms – Arithmetic Series of Second Order
- Example – 2, 9, 17, 28, 44, ……
- $T_2 - T_1$ = 9–2=7; $T_3 - T_2$= 17–9=8; $T_4 - T_3$ = 28–17=11; $T_5 - T_4$ = 44–28=16
- 7, 8, 11, 16, …
- $a_2 - a_1$ = 7 – 8 = 1; $a_3 - a_2$ = 11 – 8 = 3; $a_4 - a_3$ = 16 – 11 = 5
- 1, 3, 5, ……
- $t_2 - t_1 = t_3 - t_2$ = … = 2 (Common Difference)
- T_6 = (44 + 23) = 67

Geometric Series

- Ratio of Any Two Consecutive Terms - Same
- Common Ratio
- Successive Number

  = Previous Number (x) or (÷) Fixed Number

- Example - 4, 8, 16, 32, ...
- $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = 2$ (Common Ratio)
- Example - 64, 32, 16, 8, ...
- $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = \frac{1}{2}$ (Common Ratio)

Arithmetico-Geometric Series

- Successive Term

  = (Previous term + Fixed Number) x (Another Fixed Number)

- Example - 1, 9, 33, 105, ...
- T_2 - T_1 = 9 – 1 = 8; T_3 - T_2 = 33 – 9 = 24; T_4 - T_3 = 105 – 33 = 72
- 8, 24, 72, ... (Geometric Series)
- T_2 = 9 = (1 + 2) x 3; T_3 = 33 = (9 + 2) x 3; T_4 = 105 = (33 + 2) x 3
- Arithmetico-Geometric Series
- T_5 = (105 + 2) × 3 = 107 x 3 = 321

Geometrico-Arithmetic Series

- Successive Term

= (Previous term (x)/(÷) Fixed Number) (+)/(-) (Another Fixed Number)

- Example - 2, 5, 17, 65, …
- T_2 - T_1 = 5 – 2 = 3; T_3 - T_2 = 17 – 5 = 12; T_4 - T_3 = 65 – 17 = 48
- 3, 12, 48, … (Geometric Series, Common Ratio = 4)
- T_2 = 5 = (2 x 4) - 3; T_3 = 17 = (5 x 4) - 3; T_4 = 65 = (17 x 4) - 3
- Geometrico-Arithmetic Series
- T_5 = (65 x 4) - 3 = 260 – 3 = 257

Series of Squares, Cubes ……

- Simple powers of natural numbers i.e. squares, cubes, …. or their combinations.
- Example – 4, 9, 16, 25, 36, 49, 64, ……..
- $2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, \ldots\ldots\ldots$
- 2, 3, 4, 5, 6, 7, 8, ……..
- Common Difference = 1
- $T_8 = (8 + 1)^2 = (9)^2 = 81$

Series of Squares, Cubes ……

- Example - $\frac{1}{8}, \frac{4}{27}, \frac{9}{64}, \frac{16}{125}, \frac{25}{216}, \ldots\ldots\ldots$
- $\frac{1^2}{2^3}, \frac{2^2}{3^3}, \frac{3^2}{4^3}, \frac{4^2}{5^3}, \frac{5^2}{6^3}, \ldots\ldots\ldots$
- $\frac{n^2}{(n+1)^3}$
- $T_6 = \frac{6^2}{7^3} = \frac{216}{343}$

Double Series

- Two series - single series
- Alternating terms - independent series
- Example - 1, 2, 4, 6, 7, 18, 10, 54, ...
- 1, 4, 7, 10, ... - Arithmetic Series
- Common Difference = 4 – 1 = 7 – 4 = 10 – 7 = 3
- 2, 6, 18, 54, ... - Geometric Series
- Common Ratio = $\frac{6}{2} = \frac{18}{6} = \frac{54}{18} = 3$
- Next Term = 13 (i.e. 10 + 3)

Finding Wrong Term - Series

- All terms of a given number series are similar except one.
- All terms follow the same pattern except one.
- Wrong Term
- Example - 5, 10, 17, 24, 37, 50, 65.
- $5 = 2^2 + 1$; $10 = 3^2 + 1$; $17 = 4^2 + 1$; $26 = 5^2 + 1$;

  $37 = 6^2 + 1$; $50 = 7^2 + 1$; $65 = 8^2 + 1$

Finding Missing Term - Series

- Number Series
- A Blank Space/Question Mark
- Find Missing Term
- Example - 49, 56, 64, 72, ?, 90, 100
- $49 = 7^2$; $56 = 7^2 + 7$; $64 = 8^2$; $72 = 8^2 + 8$; ? ;

  $90 = 9^2 + 9$; $10 = 10^2$
- $T_5 = 81 = 9^2$

Types of Special Series

- Series of Date/Time
- Numbers followed by their L.C.M. or H.C.F
- Numbers Followed by their Product
- By Use of Digit Sum

Series of Date/Time

- Example-
  - 3 - 9 - 2022, 13 - 9 - 2022, 23 - 9 - 2022, & 2 - 10 – 2022.
  - Differs by 10 days.
  - September, 2022 - 30 days
  - 2 - 10 - 2004 replaced by 3 - 10 – 2022
- Example –
  - 1:00, 2:25, 3:40, 5:15, 6:40
  - Difference - 1 hour 25 min.
  - 3:40 should be replaced by 3:50.

Numbers Followed by Their L.C.M. or H.C.F

- Example – 1, 2, 3, 6, 4, 5, 6, 60, 5, 6, 7, ?
  - L.C.M. of 1, 2 & 3 = 6
  - L.C.M. of 4, 5 & 6 = 60
  - L.C.M. of 5, 6 & 7 = 210
- Example – 8, 4, 4, 7, 8, 1, 3, 9, 3, 2, 1, ?
  - H.C.F. of 8 and 4 = 4
  - H.C.F. of 7 and 8 = 1
  - H.C.F. of 3 and 9 = 3
  - H.C.F. of 2 and 1 = 1

Numbers Followed by Their Product

- Example – 1, 3, 3, 9, 27, 243, ?
- 1 = 1
- 1 × 3 = 3
- 3 × 3 = 9
- 3 × 9 = 27
- 9 × 27 = 243
- Next Number = Product of Previous Two Numbers
- Next Number = 27 × 243 = 6561

Use of Digit Sum

- Example – 11, 13, 17, 25, 32, ?
- 13 = 11 + (1 + 1)
- 17 = 13 + (1 + 3)
- 25 = 17 + (1 + 7)
- 32 = 25 + (2 + 5)
- Next Number =

  Previous Number + Sum of the digits of pervious number
- Next Number = 32 + (3 + 2) = 37

Alpha–Numeric Series

- Letters & Numbers
- Two-line Series - Number Series & Alphabet Series
- Same Pattern/Rule
- Example - 2, 7, 17, 37, 77,

  3, a, b, c, d,
- 7 = 2 x 2 + 3; 17 = 7 x 2 + 3; 37 = 17 x 2 + 3; 77 = 37 x 2 + 3
- a = 3 x 2 + 3 = 9; b = 9 x 2 + 3 = 21; c = 21 x 2 + 3 = 45;

  d = 45 x 2 + 3 = 93

Conclusion

- Sequence
  - Set of Numbers
  - Definite Order/Rule
- Series
  - Sequence of Numbers
  - Predefined Rule

Conclusion

- Number Series
  - Arithmetic Series
  - Arithmetic Series – Second Order
  - Arithmetic Series – Third Order
  - Geometric Series
  - Arithmetico-Geometric Series
  - Geometrico-Arithmetic Series
  - Series of Squares, Cubes, ...

Conclusion

- Some Special Series
  - Series of Date/Time
  - Numbers followed by their L.C.M. or H.C.F
  - Numbers Followed by their Product
  - By Use of Digit Sum
- Alpha–Numeric Series

Summary

- Sequence
- Series
- Type of Series

That’s all for now…