EPEA516
Analytical Skills II

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Learning Outcomes

After this lecture, you will be able to

- define the concept of compound interest,
- explore the formulae of compound interest,
- differentiate between simple and compound interest.

Compound Interest (C.I.)

- Borrower & Lender
- Fix up - Unit of time
- Yearly/Half-yearly/Quarterly
- Amount (after first year/unit of time)

  - Principal (for the second year/unit of time)
- Compound Interest = Amount - Principal

Basic Formulae

- P or p = Principal
- R or r = Rate
- N or n = Number of years (T or t)
- C.I. = Compound interest
- A = Amount

Basic Formulae

- Compound Interest = Amount – Principals
- When interest is compounded annually
  - Amount = $P\left[1 + \frac{\text{Rate}}{100}\right]^{n}$
  - Compound Interest = $P\left[1 + \frac{R}{100}\right]^{n} - P$
  - Compound Interest = $P\left\{\left[1 + \frac{R}{100}\right]^{n} - 1\right\}$

Basic Formulae

- Rate of interest
  - $A = P\left[1 + \frac{R}{100}\right]^{n}$
  - $\frac{A}{P} = \left[1 + \frac{R}{100}\right]^{n}$
  - $\left[\frac{A}{P}\right]^{1/n} = \left[1 + \frac{R}{100}\right]$

Basic Formulae

- Rate of interest
  - $\left[\frac{A}{P}\right]^{1/n} - 1 = \frac{R}{100}$
  - $\left\{\left[\frac{A}{P}\right]^{1/n} - 1\right\} \times 100 = R$
  - Rate of interest (R) = $\left\{\left[\frac{A}{P}\right]^{1/n} - 1\right\} \times 100$

    $or \quad \left\{\left[\frac{A}{P}\right]^{1/n} - 1\right\}$% p.a.

Basic Formulae

- When interest is compounded Half-yearly

  - Amount = P $\left[1 + \frac{\frac{R}{2}}{100}\right]^{2n}$

  - Compound Interest = P $\left[1 + \frac{\frac{R}{2}}{100}\right]^{2n}$ – P

  - Compound Interest = P$\left\{\left[1 + \frac{\frac{R}{2}}{100}\right]^{2n} - 1\right\}$

  - Rate of interest (R) = 2 x 100 x$\left\{\left[\frac{A}{P}\right]^{1/2n} - 1\right\}$

Basic Formulae

- When interest is compounded Quarterly
  - Amount = $P\left[1+\frac{\frac{R}{4}}{100}\right]^{4n}$
  - Compound Interest = $P\left[1+\frac{\frac{R}{4}}{100}\right]^{4n} - P$
  - Compound Interest = $P\left\{\left[1+\frac{\frac{R}{4}}{100}\right]^{4n} - 1\right\}$
  - Rate of interest (R) = $4 \times 100 \times \left\{\left[\frac{A}{P}\right]^{1/4n} - 1\right\}$

Basic Formulae

- When interest is compounded annually but time is in fraction, $1\frac{1}{2}$ years.
  - Amount = $P\left[1+\frac{R}{100}\right]^{1} \times \left[1+\frac{\frac{1}{2}R}{100}\right]$
  - Compound Interest = $P\left[1+\frac{R}{100}\right]^{1} \times \left[1+\frac{\frac{1}{2}R}{100}\right] - P$
  - Compound Interest = $P\left\{\left[1+\frac{R}{100}\right]^{1} \times \left[1+\frac{\frac{1}{2}R}{100}\right] - 1\right\}$

Basic Formulae

- When rates are different for different years
  - R_1 %, R_2 %, R_3 %, R_4 %, …. for 1st, 2nd, 3rd, 4th year …. respectively.
  - Amount =

$$P \times \left[1 + \frac{R_1}{100}\right] \times \left[1 + \frac{R_2}{100}\right] \times \left[1 + \frac{R_3}{100}\right] \times \left[1 + \frac{R_4}{100}\right] \times \ldots\ldots$$

Basic Formulae

- Present worth of Rs. x due n years hence
  - Present Worth = $\dfrac{x}{\left[1+\dfrac{R}{100}\right]^{n}}$

Basic Formulae

- The difference between the compound interest and the simple interest on a certain sum of money for 2 years at R% per annum
  - In terms of P and R
    - $\text{C.I.} - \text{S.I.} = P\left[\frac{R}{100}\right]^2$
  - In terms of S.I. and R
    - $\text{C.I.} - \text{S.I.} = \frac{R \times \text{S.I.}}{2 \times 100}$

Basic Formulae

Derivation-

- Let, given sum of money = Rs. P
- Simple interest on Rs. P for 2 years at R% per annum

$$= \frac{P \times R \times 2}{100}$$

- Compound interest on Rs. P for 2 years at R% per annum

$$= P\left\{\left[1 + \frac{R}{100}\right]^2 - 1\right\}$$

Basic Formulae

- $C.I. - S.I. = P\left\{\left[1 + \frac{R}{100}\right]^2 - 1\right\} - \frac{P \times R \times 2}{100}$

- $C.I. - S.I. = P\left\{\cancel{1} + \left[\frac{R}{100}\right]^2 + \cancel{\frac{2R}{100}} - \cancel{1} - \cancel{\frac{2R}{100}}\right\}$

- $C.I. - S.I. = P\left\{\left[\frac{R}{100}\right]^2\right\}$

Basic Formulae

- $\text{C.I.} - \text{S.I.} = P\left\{\left[\frac{R}{100}\right]^2\right\}$

- $\text{C.I.} - \text{S.I.} = P \times \frac{R}{100} \times \left\{\frac{R}{100}\right\}$

- $\text{C.I.} - \text{S.I.} = \frac{R}{100 \times 2} \times \left\{\frac{P \times R \times 2}{100}\right\}$

- $\text{C.I.} - \text{S.I.} = \frac{R \times \text{S.I.}}{100 \times 2}$ (Because $\text{S.I.} = \frac{P \times R \times 2}{100}$)

Basic Formulae

- The difference between the compound interest and the simple interest on a certain sum of money for 3 years at R% per annum
  - In terms of P and R
    - $$\text{C.I.} - \text{S.I.} = P\left\{\left[\frac{R}{100}\right]^3 + 3\left[\frac{R}{100}\right]^2\right\}$$
  - In terms of S.I. and R
    - $$\text{C.I.} - \text{S.I.} = \frac{\text{S.I.}}{3}\left\{\left[\frac{R}{100}\right]^2 + 3\left[\frac{R}{100}\right]\right\}$$

Basic Formulae

Derivation-

- Let, given sum of money = Rs. P
- Simple interest on Rs. P for 3 years at R% per annum

$$= \frac{P \times R \times 3}{100}$$

- Compound interest on Rs. P for 3 years at R% per annum

$$= P\left\{\left[1 + \frac{R}{100}\right]^3 - 1\right\}$$

Basic Formulae

- $C.I. - S.I. = P\left\{\left[1 + \frac{R}{100}\right]^3 - 1\right\} - \frac{P \times R \times 3}{100}$
- $C.I. - S.I. = P\left\{\cancel{1} + \left[\frac{R}{100}\right]^3 + 3\left[\frac{R}{100}\right]^2 + \cancel{\frac{3R}{100}} - \cancel{1} - \cancel{\frac{3R}{100}}\right\}$
- $C.I. - S.I. = P\left\{\left[\frac{R}{100}\right]^3 + 3\left[\frac{R}{100}\right]^2\right\}$

Basic Formulae

- $C.I. - S.I. = P\left\{\left[\frac{R}{100}\right]^3 + 3\left[\frac{R}{100}\right]^2\right\}$

- $C.I. - S.I. = P\left[\frac{R}{100}\right]\left\{\left[\frac{R}{100}\right]^2 + 3\left[\frac{R}{100}\right]\right\}$

- $C.I. - S.I. = \frac{P \times R \times 3}{100} \times \frac{1}{3} \times \left\{\left[\frac{R}{100}\right]^2 + 3\left[\frac{R}{100}\right]\right\}$

Basic Formulae

- $C.I. - S.I. = \frac{P \times R \times 3}{100} \times \frac{1}{3} \times \left\{ \left[\frac{R}{100}\right]^2 + 3\left[\frac{R}{100}\right] \right\}$

- $C.I. - S.I. = S.I. \times \frac{1}{3} \times \left\{ \left[\frac{R}{100}\right]^2 + 3\left[\frac{R}{100}\right] \right\}$

(Because $S.I. = \frac{P \times R \times 2}{100}$)

- $C.I. - S.I. = \frac{S.I.}{3} \times \left\{ \left[\frac{R}{100}\right]^2 + 3\left[\frac{R}{100}\right] \right\}$

Basic Formulae

- If a certain sum becomes n times in t years at compound interest, then the same sum becomes n^m times in mt years.

Derivation-

- Let the sum of money = Rs. P

- $nP = P\left[1 + \frac{R}{100}\right]^t$

- $n = \left[1 + \frac{R}{100}\right]^t$ ………… (1)

Basic Formulae

- Let the sum become n^m times in T years.

- $n^m = \left[1 + \frac{R}{100}\right]^T$

- $n = \left[1 + \frac{R}{100}\right]^{T/m}$ ………… (2)

- From (1) and (2)

- $\left[1 + \frac{R}{100}\right]^t = \left[1 + \frac{R}{100}\right]^{T/m}$

Basic Formulae

- $\left[1 + \frac{R}{100}\right]^{t} = \left[1 + \frac{R}{100}\right]^{T/m}$
- $t = \frac{T}{m}$
- T = mt years
- Sum becomes n^{m} times in mt years.

Basic Formulae

- If a certain sum becomes n times in t years, then the rate of compound interest is

$$R = 100[(n)^{1/t} - 1]$$

Basic Formulae

- If a certain sum of money at compound interest amounts to Rs. x in A years and to Rs. y in B years, then the rate of interest per annum is R = $\left\{\left[\frac{y}{x}\right]^{1/(B-A)} - 1\right\}$ x 100 %

Derivation –

- Let Principal = Rs. P
- Rate of interest = R% p.a.
- A. T. Q., x = P $\left[1 + \frac{R}{100}\right]^{A}$ and y = P $\left[1 + \frac{R}{100}\right]^{B}$

Basic Formulae

- $y = P\left[1 + \frac{R}{100}\right]^{B}$
- $x = P\left[1 + \frac{R}{100}\right]^{A}$
- $\frac{y}{x} = \frac{P\left[1 + \frac{R}{100}\right]^{B}}{P\left[1 + \frac{R}{100}\right]^{A}}$
- $\frac{y}{x} = \left[1 + \frac{R}{100}\right]^{B-A}$
- $1 + \frac{R}{100} = \left[\frac{y}{x}\right]^{1/(B-A)}$

Basic Formulae

- $1 + \frac{R}{100} = \left[\frac{y}{x}\right]^{1/(B-A)}$
- $\frac{R}{100} = \left[\frac{y}{x}\right]^{1/(B-A)} - 1$
- $R = \left\{\left[\frac{y}{x}\right]^{1/(B-A)} - 1\right\} \times 100$

Basic Formulae

- If a loan of Rs. P at R% compound interest per annum is to be repaid in n equal yearly instalments, then the value of each instalment is given by per annum is

$$\frac{P}{\left[\frac{100}{100+R}\right]^{1}+\left[\frac{100}{100+R}\right]^{2}+\left[\frac{100}{100+R}\right]^{3}+\ldots+\left[\frac{100}{100+R}\right]^{n}}$$

Basic Formulae

Derivation-

- Principal for Rs. X due at end of first year at R% = $\frac{100X}{100 + R}$

- Principal for Rs. X due at end of second year at R% = $\left[\frac{100}{100 + R}\right]^2 X$

....

- Principal for Rs. X due at end of nth year at R% = $\left[\frac{100}{100 + R}\right]^n X$

Basic Formulae

- $\left[\frac{100}{100+R}\right]^1 X + \left[\frac{100}{100+R}\right]^2 X + \left[\frac{100}{100+R}\right]^3 X + \cdots + \left[\frac{100}{100+R}\right]^n X = P$
- $\left\{\left[\frac{100}{100+R}\right]^1 + \left[\frac{100}{100+R}\right]^2 + \left[\frac{100}{100+R}\right]^3 + \cdots + \left[\frac{100}{100+R}\right]^n\right\} X = P$
- $X = \frac{P}{\left[\frac{100}{100+R}\right]^1 + \left[\frac{100}{100+R}\right]^2 + \left[\frac{100}{100+R}\right]^3 + \ldots + \left[\frac{100}{100+R}\right]^n}$

Conclusion

- Compound Interest = Amount – Principal
- When interest is compounded annually
  - Compound Interest = $P\left\{\left[1 + \frac{\text{Rate}}{100}\right]^{n} - 1\right\}$
  - Rate of interest (R) = $\left\{\left[\frac{A}{P}\right]^{1/n} - 1\right\}$ % p.a.
- When interest is compounded Half-yearly
  - Compound Interest = $P\left\{\left[1 + \frac{\frac{R}{2}}{100}\right]^{2n} - 1\right\}$

Conclusion

- When interest is compounded Quarterly
  - Compound Interest = $P\left\{\left[1+\frac{\frac{R}{4}}{100}\right]^{4n}-1\right\}$
- When interest is compounded annually but time is in fraction, $1\frac{1}{2}$ years.
  - Compound Interest = $P\left\{\left[1+\frac{R}{100}\right]^{1} \times \left[1+\frac{\frac{1}{2}R}{100}\right]-1\right\}$

Conclusion

- When rates are different for different years
  - Compound Interest =

$$P\left\{\left[1+\frac{R_1}{100}\right] x \left[1+\frac{R_2}{100}\right] x \left[1+\frac{R_3}{100}\right] x \left[1+\frac{R_4}{100}\right] x \;\ldots\ldots\; - 1\right\}$$

- Present worth of Rs. x due n years hence
  - Present Worth = $\dfrac{x}{\left[1+\frac{R}{100}\right]^{n}}$

Conclusion

- For 2 years at R% per annum
  - In terms of P and R
    - $\text{C.I.} - \text{S.I.} = P\left[\frac{R}{100}\right]^2$
  - In terms of S.I. and R
    - $\text{C.I.} - \text{S.I.} = \frac{R \times \text{S.I.}}{2 \times 100}$

Conclusion

- For 3 years at R% per annum
  - In terms of P and R
    - $\text{C.I.} - \text{S.I.} = P\left\{\left[\frac{R}{100}\right]^3 + 3\left[\frac{R}{100}\right]^2\right\}$
  - In terms of S.I. and R
    - $\text{C.I.} - \text{S.I.} = \frac{\text{S.I.}}{3}\left\{\left[\frac{R}{100}\right]^2 + 3\left[\frac{R}{100}\right]\right\}$

Conclusion

- If a certain sum becomes n times in t years at compound interest, then the same sum becomes n^m times in mt years.

- If a certain sum becomes n times in t years, then the rate of compound interest is

$$R = 100[(n)^{1/t} - 1]$$

Conclusion

- If a certain sum of money at compound interest amounts to Rs. x in A years and to Rs. y in B years, then the rate of interest per annum is

$$R = \left\{ \left[\frac{y}{x}\right]^{1/(B-A)} - 1 \right\} \times 100\ \%$$

Conclusion

- If a loan of Rs. P at R% compound interest per annum is to be repaid in n equal yearly instalments, then the value of each instalment is

$$\frac{P}{\left[\frac{100}{100 + R}\right]^{1} + \left[\frac{100}{100 + R}\right]^{2} + \left[\frac{100}{100 + R}\right]^{3} + \cdots + \left[\frac{100}{100 + R}\right]^{n}}$$

Summary

- Concept and Formulae
  - Compound Interest
  - Yearly
  - Half-yearly
  - Quarterly
  - Compounded annually but time is in fractions
  - Different rate of interest for different years
  - Present worth of Rs. x due n years hence

Summary

- C.I. – S.I. on a certain sum of money
  - for 2 years at R% per annum
  - for 3 years at R% per annum
- The sum becomes n^m times in mt years if it becomes n times in t years at compound interest, then Rate of compound interest if a certain sum becomes n times in t years.

Summary

- Rate of interest per annum if a certain sum of money at compound interest amounts to Rs. x in A years and to Rs. y in B years.
- Value of each instalment if a loan of Rs. P at $R\%$ compound interest per annum is to be repaid in n equal yearly instalments.

That's all for now…