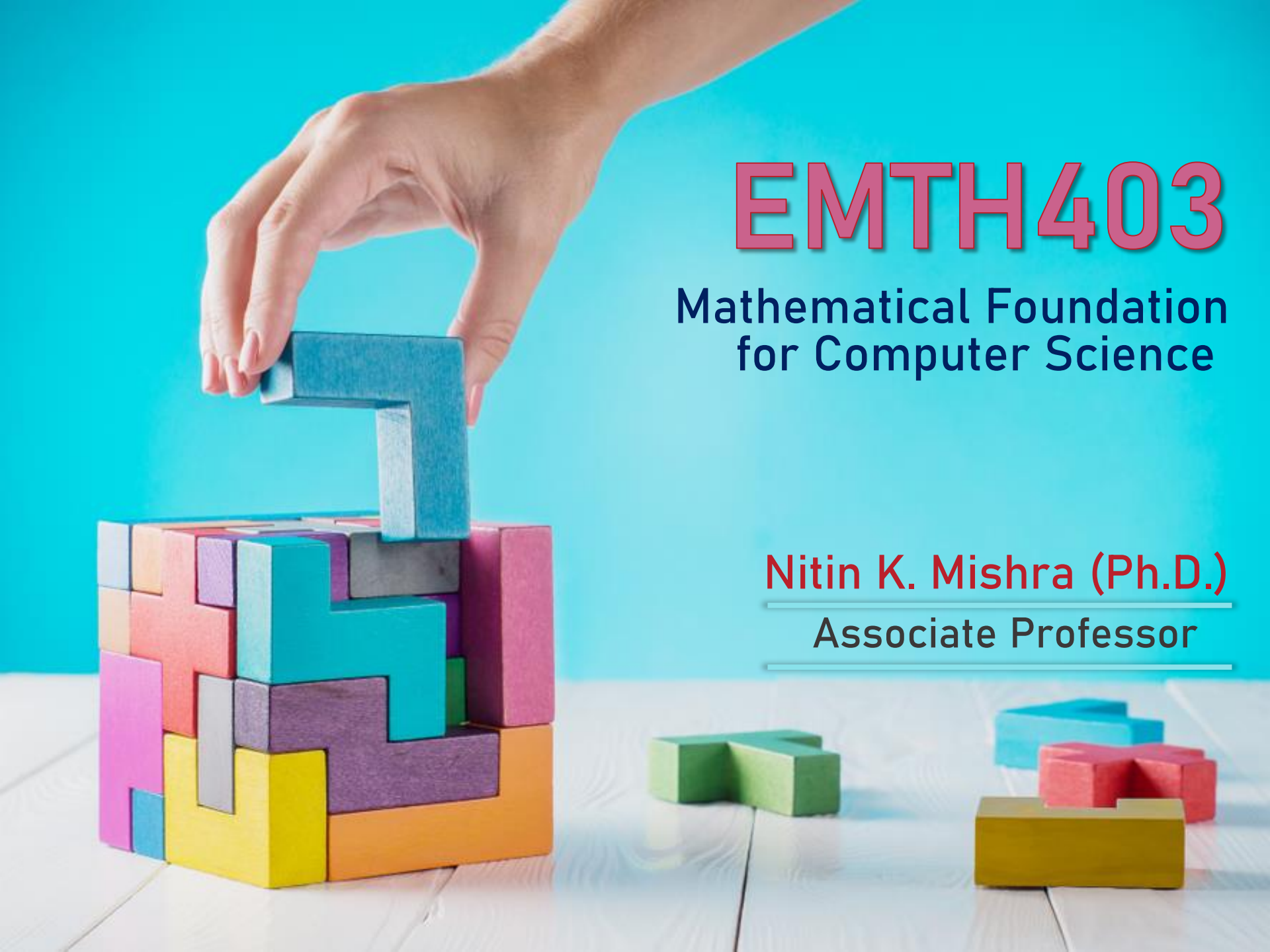


EMTH403

Mathematical Foundation for Computer Science

Nitin K. Mishra (Ph.D.)

Associate Professor

Lecture Outcomes

After this lecture, you will be able to

- understand what are Euler path in a directed and undirected graph.
- understand what are Euler circuit in a directed and undirected graph.

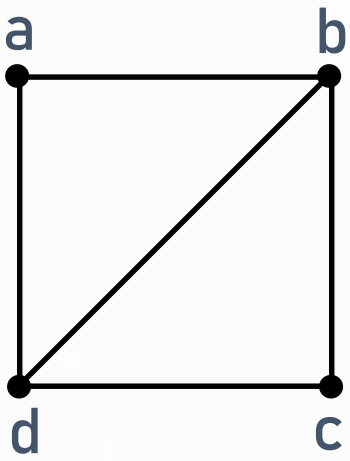
Euler Paths and Circuits

Necessary and Sufficient Conditions

A connected multigraph has a Euler circuit if and only if each of its vertices has an even degree.

A connected multigraph has a Euler path but not an Euler circuit if and only if it has exactly two vertices of odd degree.

Euler Paths and Circuits

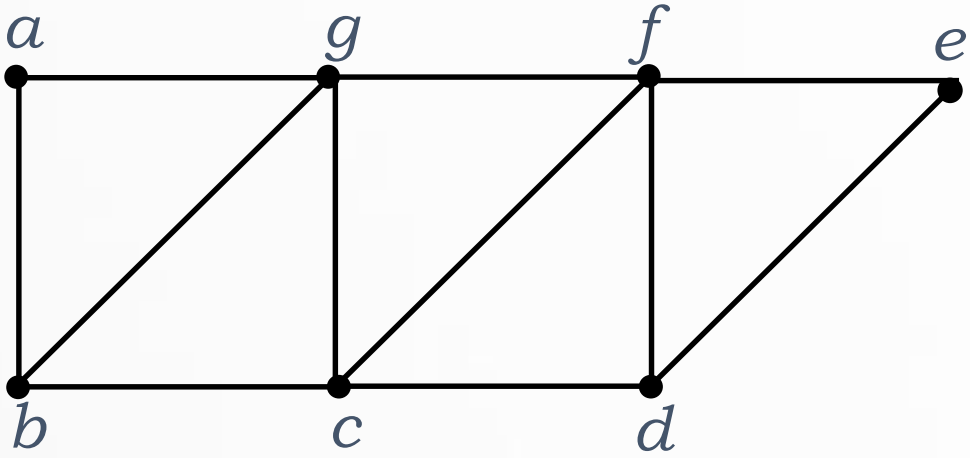


Ques:- Does the graph above has an Euler path?

Sol:- Yes

(d, a, b,c, d, b.)

Euler Paths and Circuits

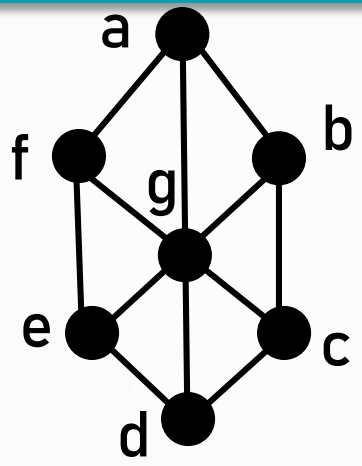


Ques:- Does the graph above has an Euler path?

Sol:- Yes

(b, a, g, f, e, d, c, g, b, c, f, d)

Euler Paths and Circuits

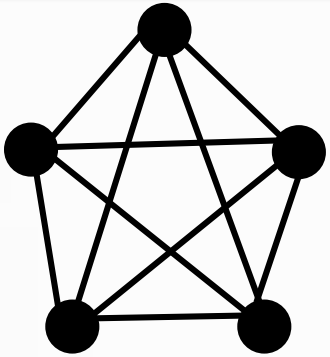


Ques:- Does the graph above has an Euler path?

Sol:- No

six vertices of odd degree

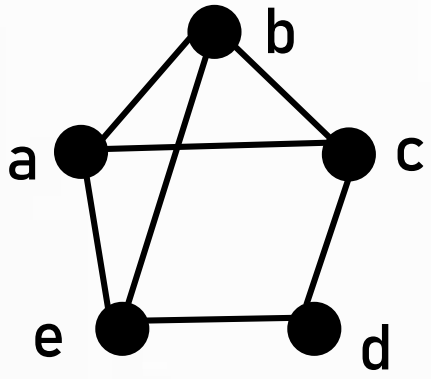
Euler Paths and Circuits



Ques:- Does the graph above has an Euler path?

Sol:- No

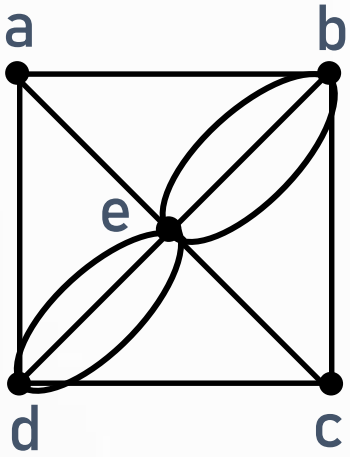
Euler Paths and Circuits



Ques:- Does the graph above has an Euler circuit or an Euler path?

Sol:- Since there are four vertices of odd degree (a, b, c, and e) and 4 > 2, this graph has neither an Euler circuit nor an Euler path.

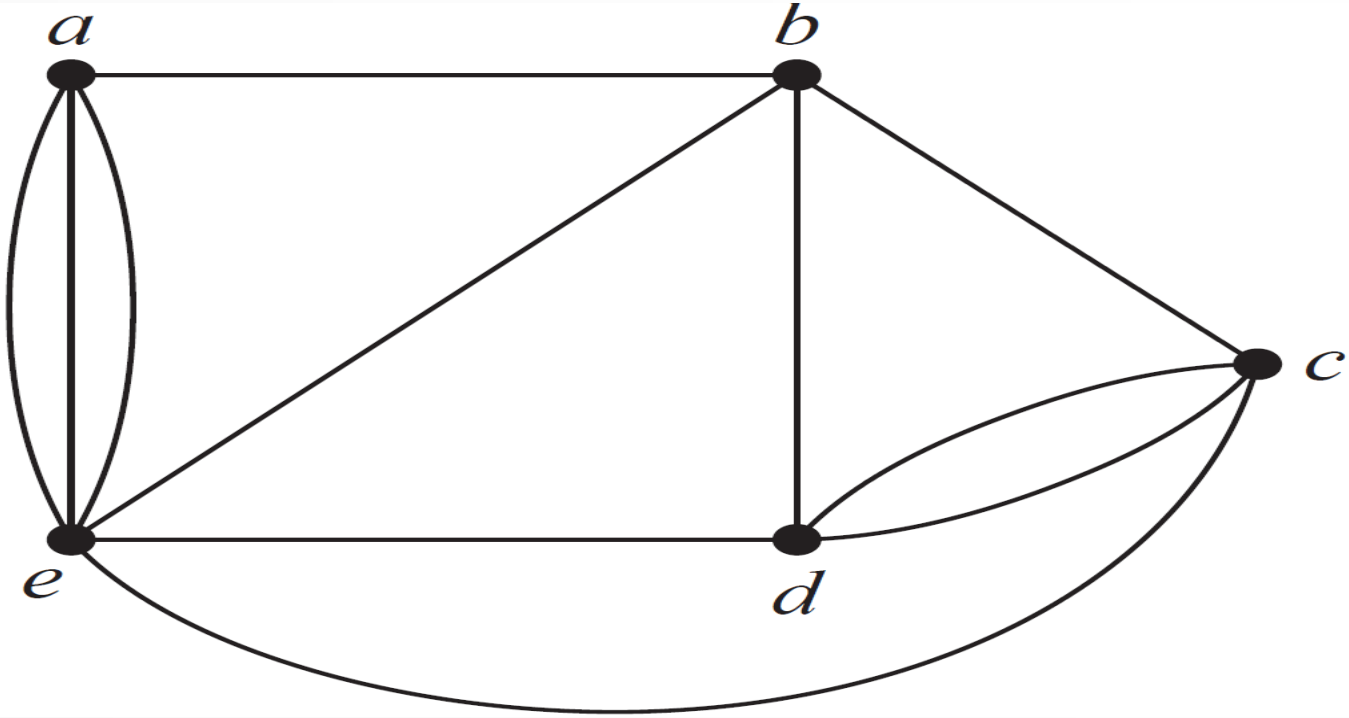
Euler Paths and Circuits



Ques:- Does the graph above has an Euler circuit or an Euler path?

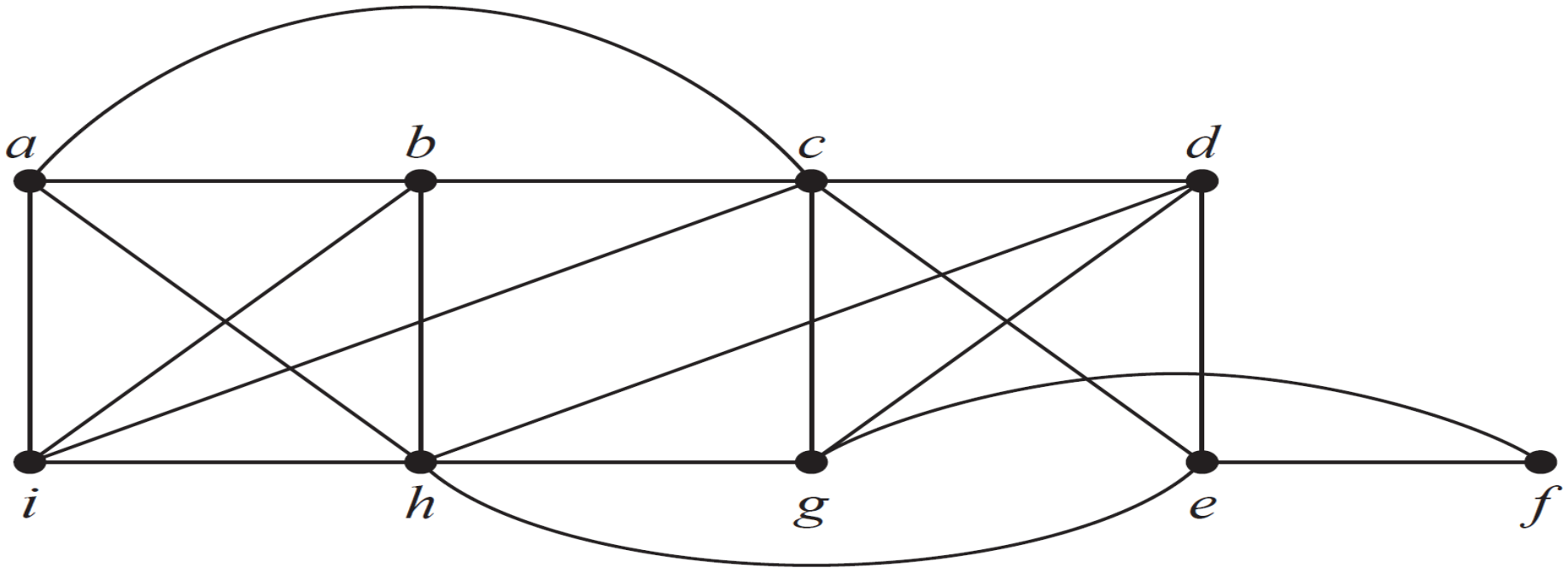
Sol:- Since there are two vertices of odd degree (a and d), this graph has no Euler circuit, but it does have an Euler path starting at a and ending at d. We can find such a path by inspection, One such path is a, e, c, e, b, e, d, b, a, c, d.

Euler Paths and Circuits



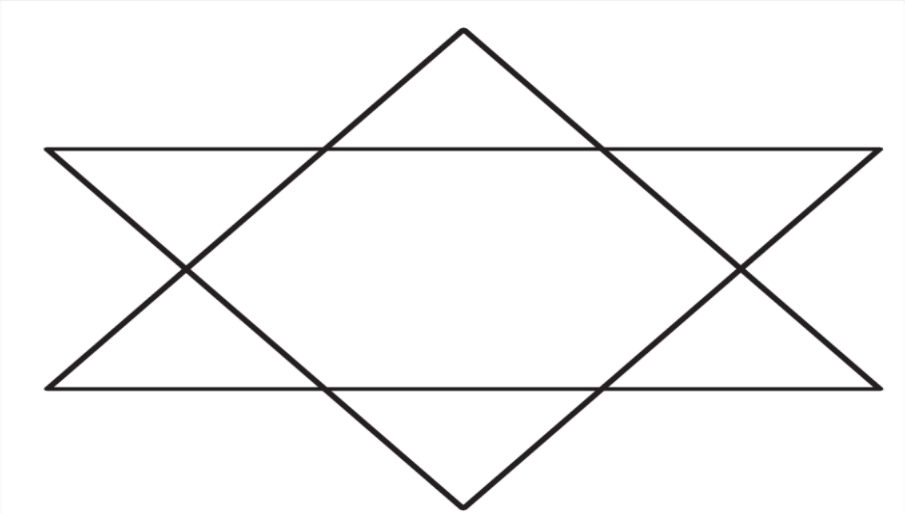
Sol:- All the vertex degrees are even, so there is an Euler circuit. We can find such a circuit by inspection, One such circuit is a, b, c, d, c, e, d, b, e, a, e, a.

Euler Paths and Circuits



Sol:- All the vertex degrees are even, so there is an Euler circuit. We can find such a circuit by inspection, One such circuit is a, b, c, d, e, f, g, h, i, a, h, b, i, c, e, h, d, g, c, a.

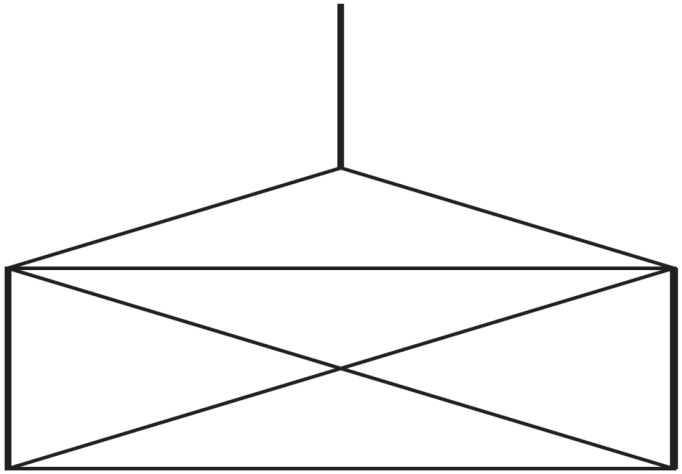
Euler Paths and Circuits



Ques:- Determine whether the picture shown can be drawn with a pencil in a continuous motion without lifting the pencil or retracing part of the picture.

Sol:- The graph in the current exercise has all vertices of even degree; therefore it has an Euler circuit and can be so traced.

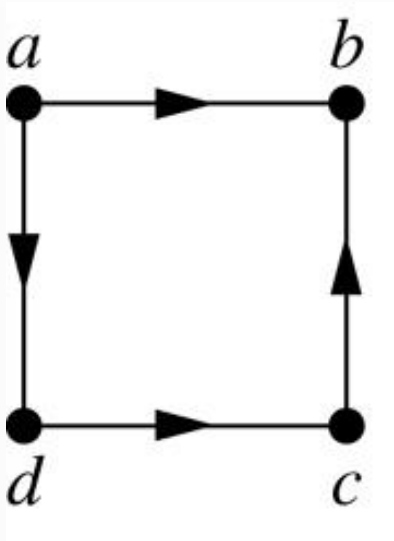
Euler Paths and Circuits



Ques:- Determine whether the picture shown can be drawn with a pencil in a continuous motion without lifting the pencil or retracing part of the picture.

Sol:- This graph has 4 vertices of odd degree; therefore it has no Euler path or circuit and cannot be so traced.

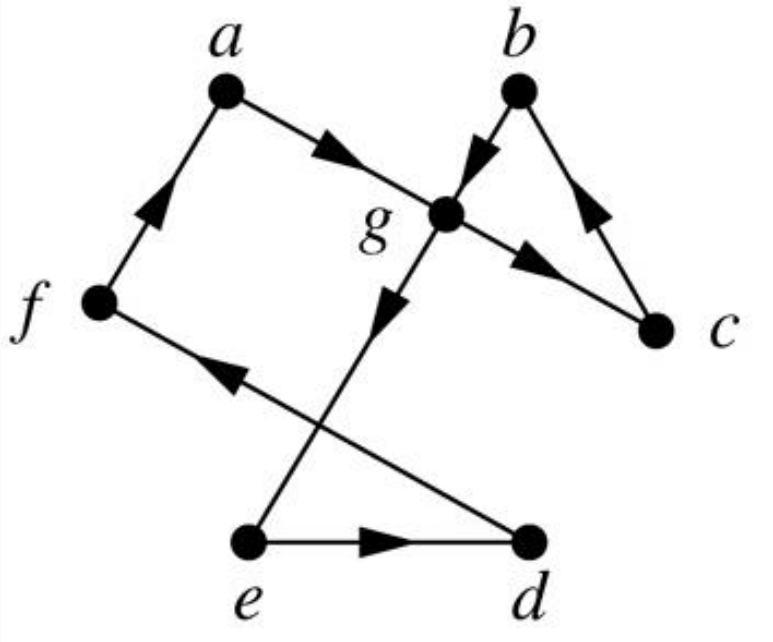
Euler Circuit in Directed Graphs



Ques:- Does the graph above has an Euler circuit?

Sol:- No

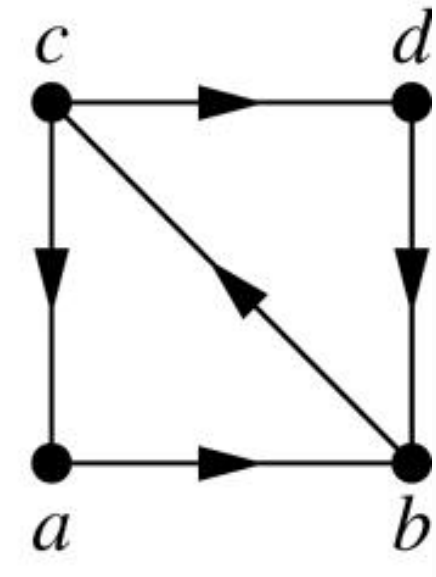
Euler Circuit in Directed Graphs



Ques:- Does the graph above has an Euler circuit?

Sol:- Yes (a, g, c, b, g, e, d, f, a)

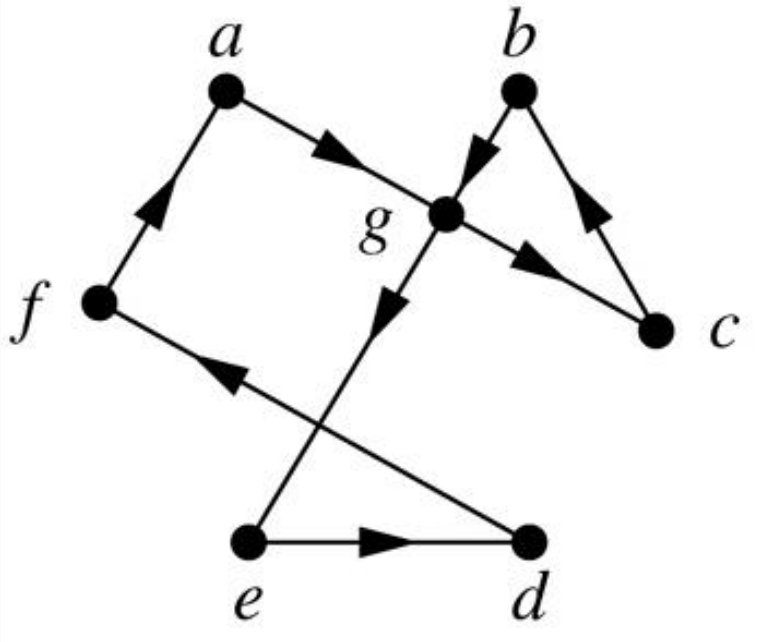
Euler Circuit in Directed Graphs



Ques:- Does the graph above has an Euler circuit?

Sol:- No

Euler Path in Directed Graphs

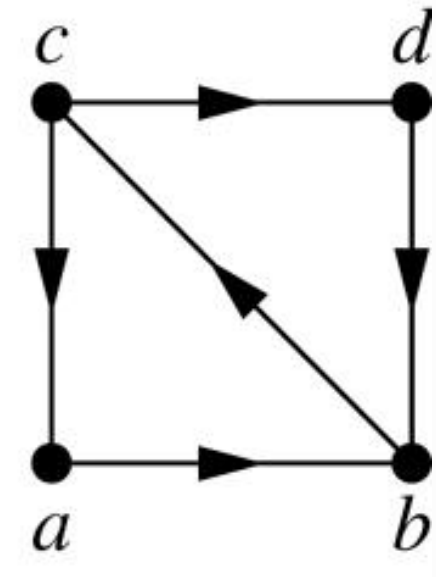


Ques:- Does the graph above has an Euler path?

Sol:- Yes

(a, g, c, b, g, e, d, f, a)

Euler Path in Directed Graphs



Ques:- Does the graph above has an Euler path?

Sol:- Yes

(c, a, b, c, d, b)

Euler Paths and Circuits

Ques:- For which values of n do the graphs K_n have an Euler path but no Euler circuit?

Sol:- Clearly K_2 has an Euler path but no Euler circuit.

Euler Paths and Circuits

Ques:- For which values of n do the graphs K_n have an Euler path but no Euler circuit?

Sol:- For odd $n > 2$ there is an Euler circuit (since the degrees of all the vertices are $n - 1$, which is even), whereas for even $n > 2$ there are at least 4 vertices of odd degree and hence no Euler path.

Euler Paths and Circuits

Ques:- For which values of n do the graphs K_n have an Euler path but no Euler circuit?

Sol:- Thus for no n other than 2 is there an Euler path but not an Euler circuit.

Euler Paths and Circuits

Ques:- For which values of n do the graph C_n have an Euler path but no Euler circuit?

Sol:- Since C_n has an Euler circuit for all n, there are no values of n meeting these conditions.

Euler Paths and Circuits

Ques:- For which values of n do the graph W_n have an Euler path but no Euler circuit?

Sol:- A wheel has at least 3 vertices of degree 3 (around the rim), so there can be no Euler path.

Euler Paths and Circuits

Ques:- For which values of n do the graph Q_n have an Euler path but no Euler circuit?

Sol:- The same argument applies here as applied in part (a). In more detail, Q_1 (which is the same as K_2) is the only cube with an Euler path but no Euler circuit, since for odd $n > 1$ there are too many vertices of odd degree, and for even $n > 1$ there is an Euler circuit.

That's all for now…