EMTH403

Mathematical Foundation for Computer Science

Nitin K. Mishra (Ph.D.)

Associate Professor

Lecture Outcomes

After this lecture, you will be able to

- understand the Division rule in the basics of counting.
- understand how to find total number of functions in the basics of counting.

Permutation – Example 1

Ques:- In how many ways can we select three students from a group of five students to stand in line for a picture?

Sol:- By the product rule, there are

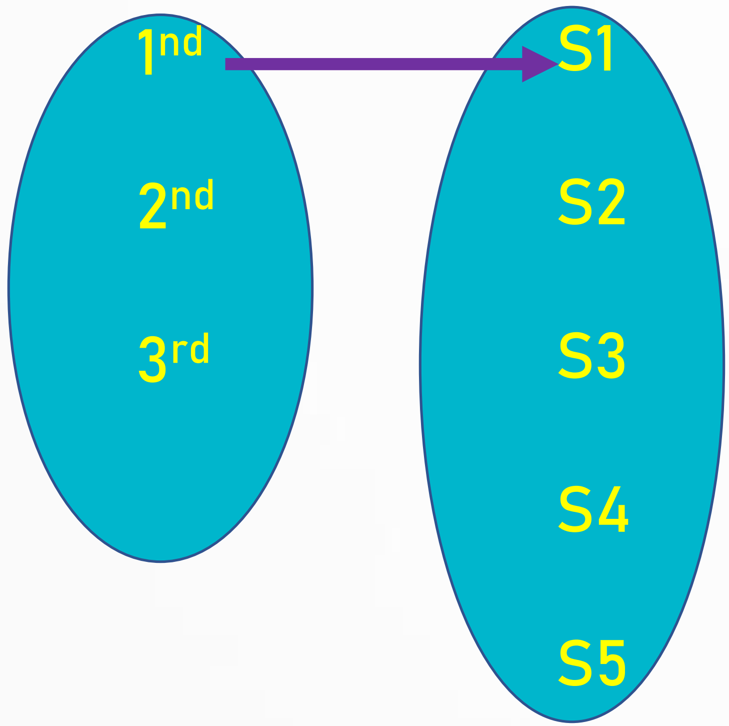
$5 \cdot 4 \cdot 3 = 60$ ways

Permutation – Example 1

5 Choices

4 Choices

3 Choices



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Permutation – Example 1

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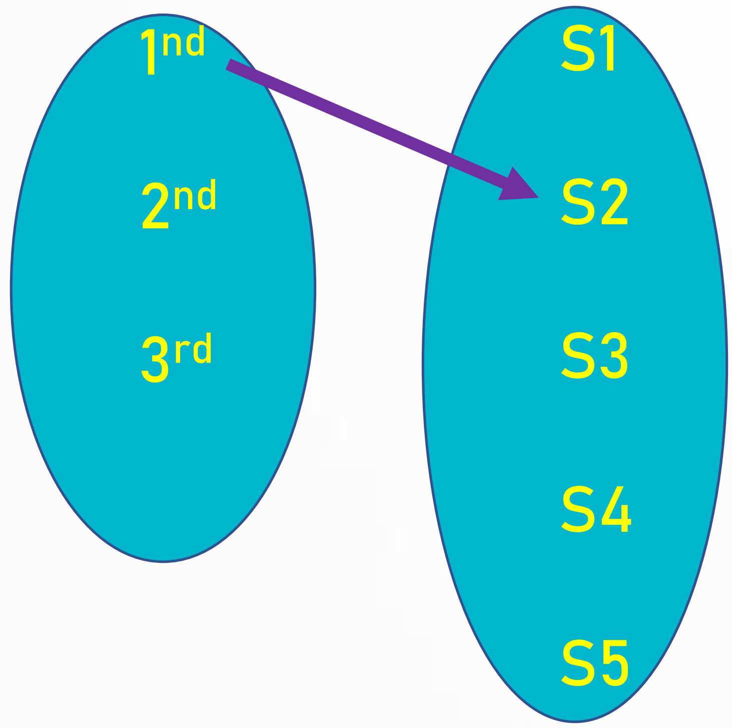

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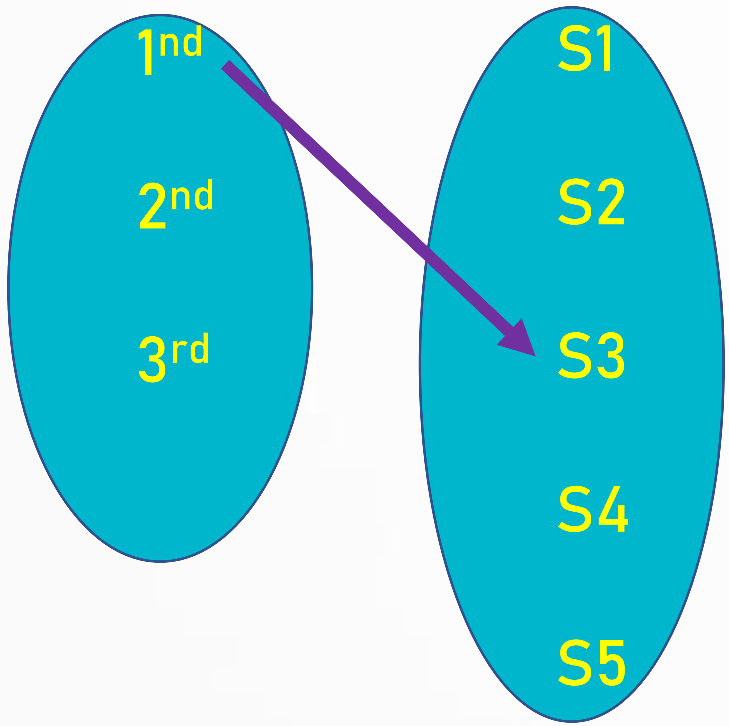

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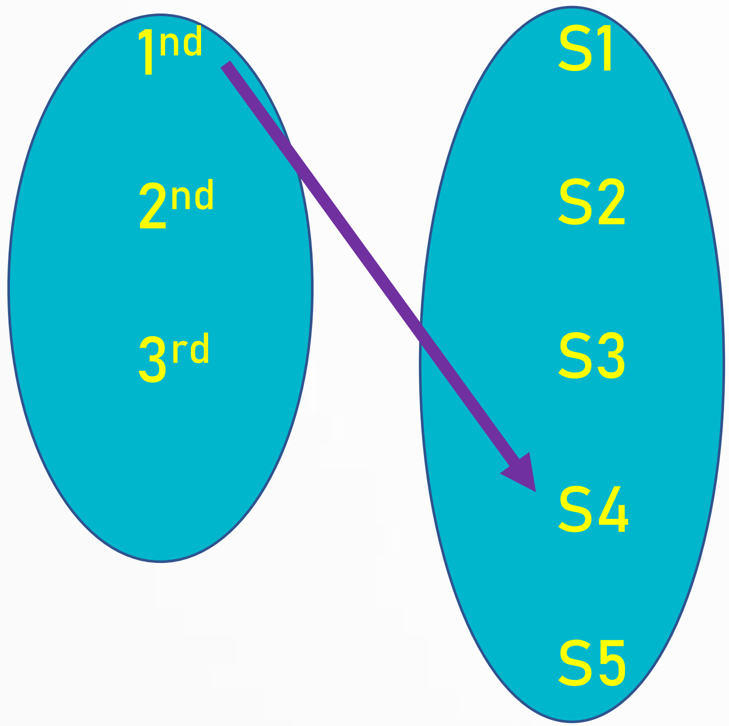

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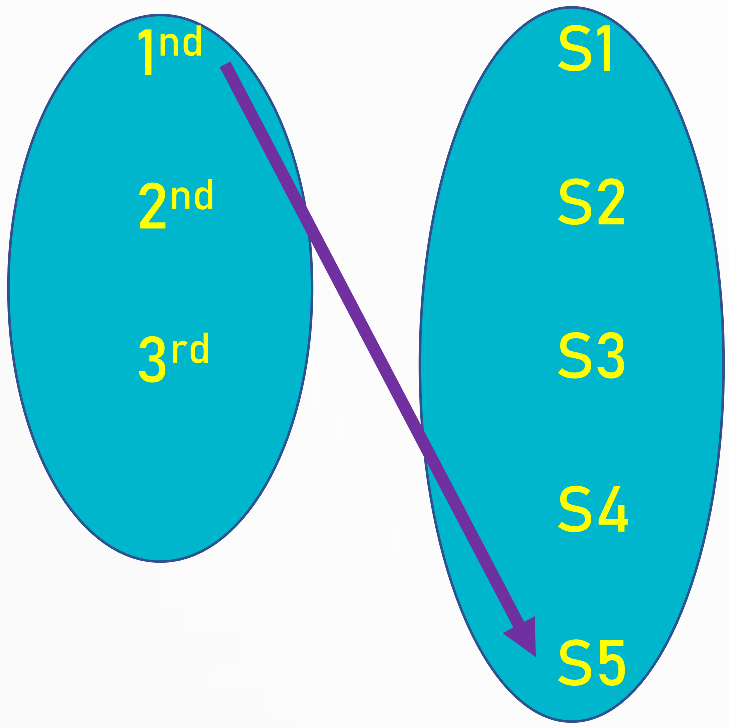

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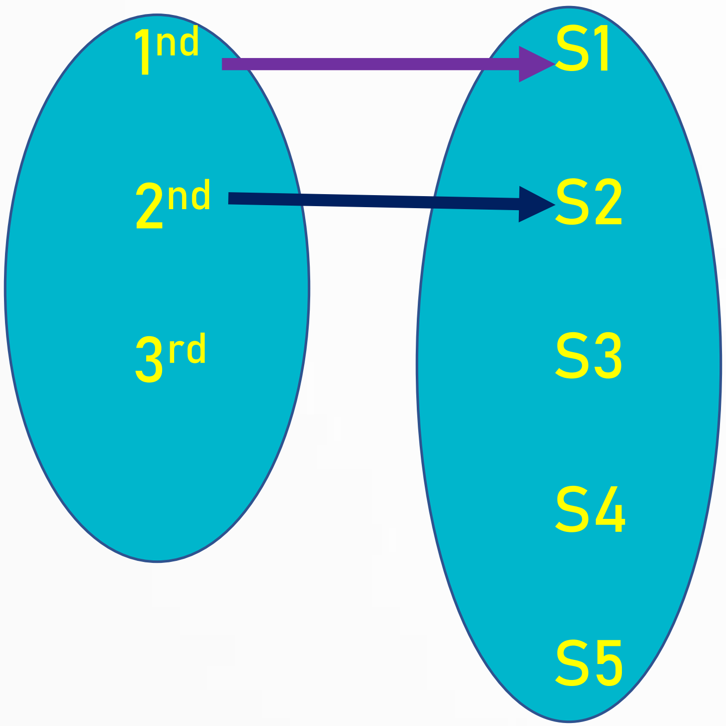

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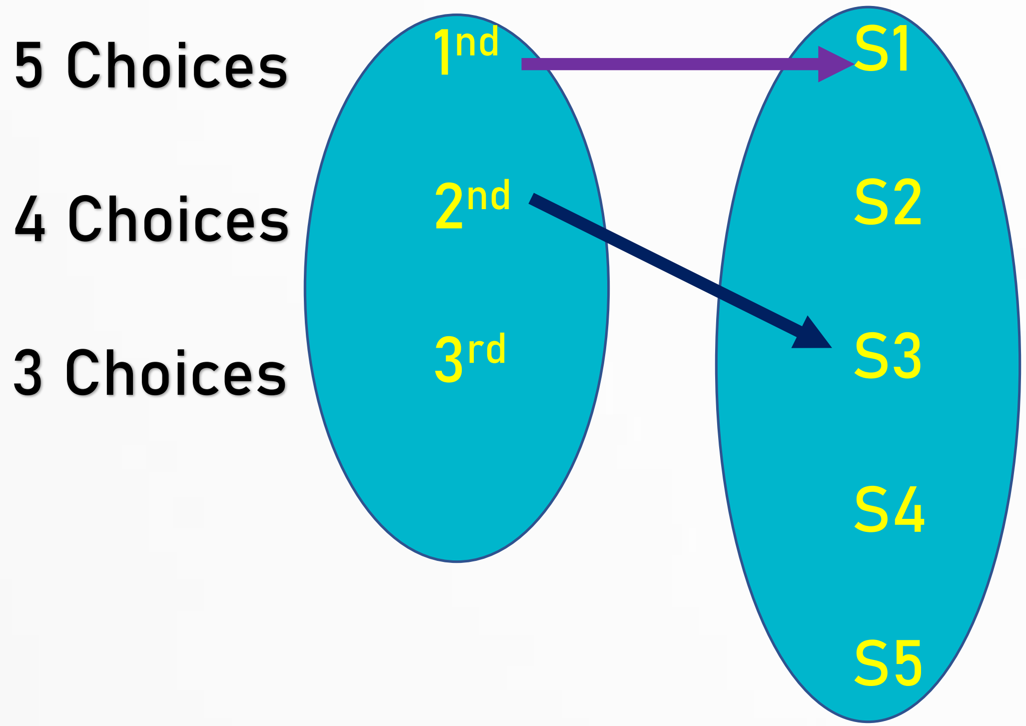

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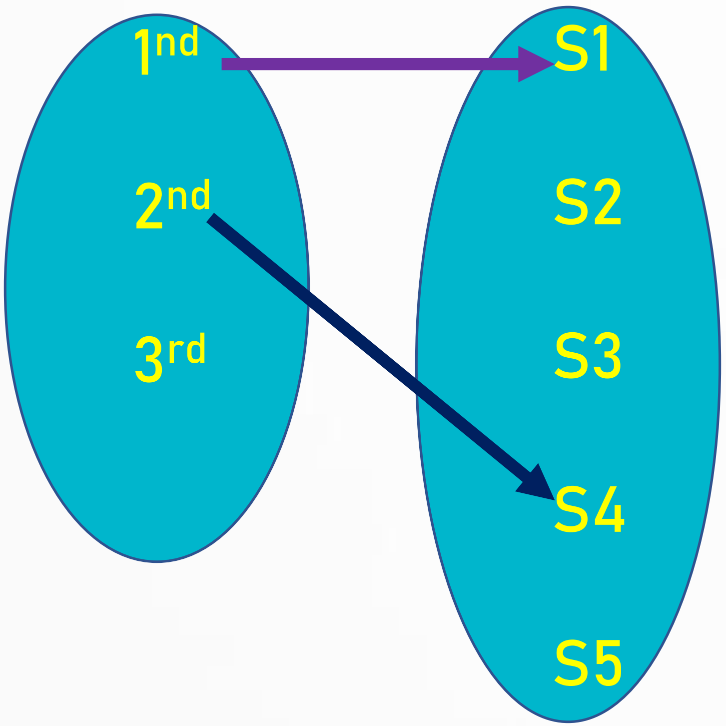

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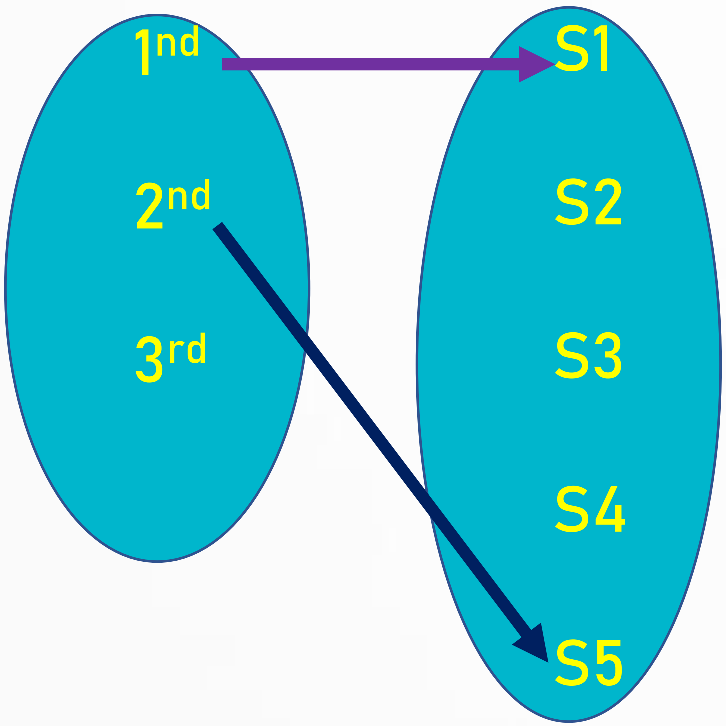

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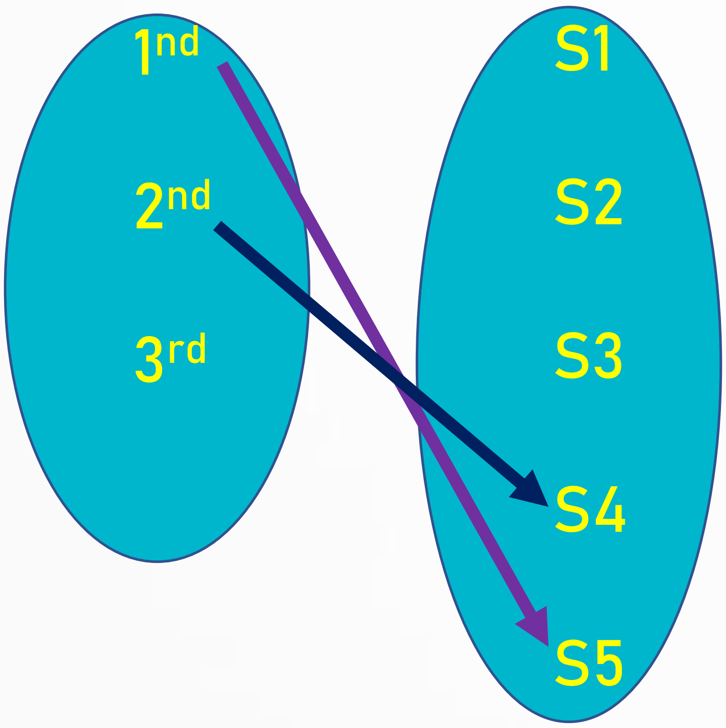

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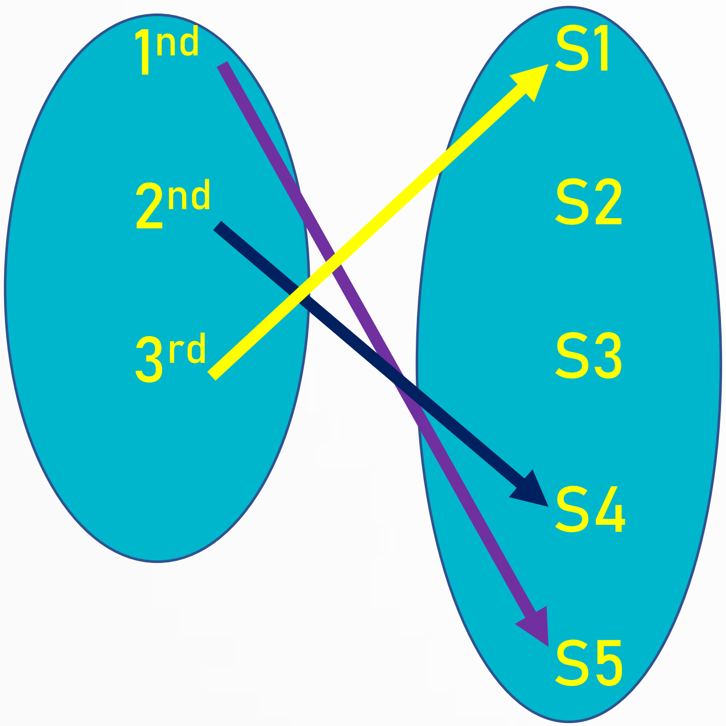

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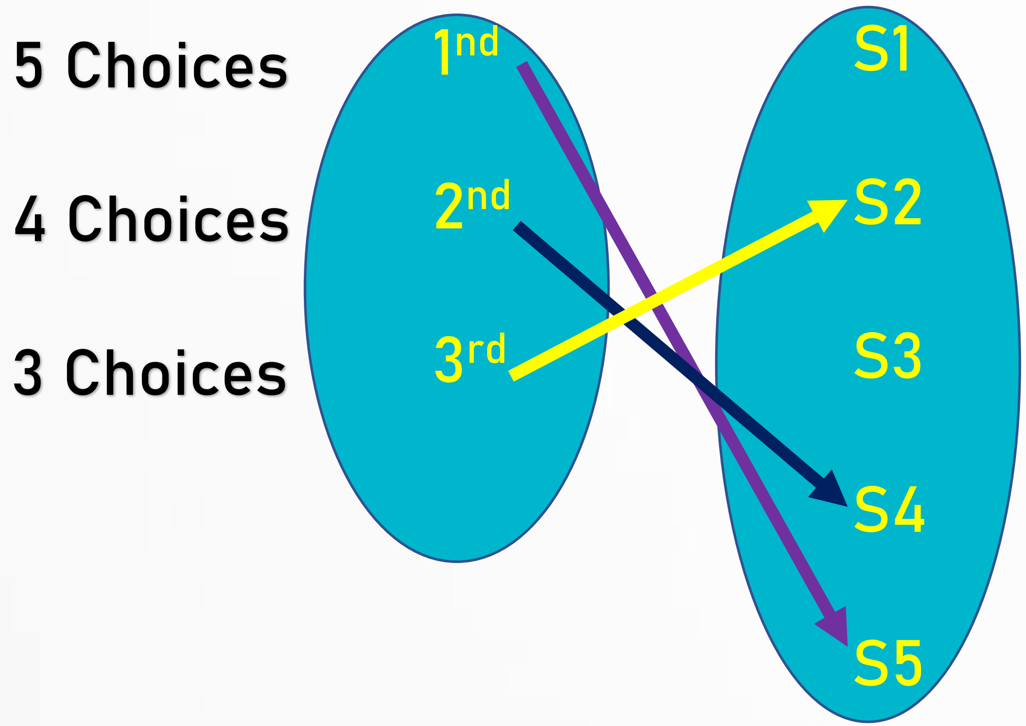

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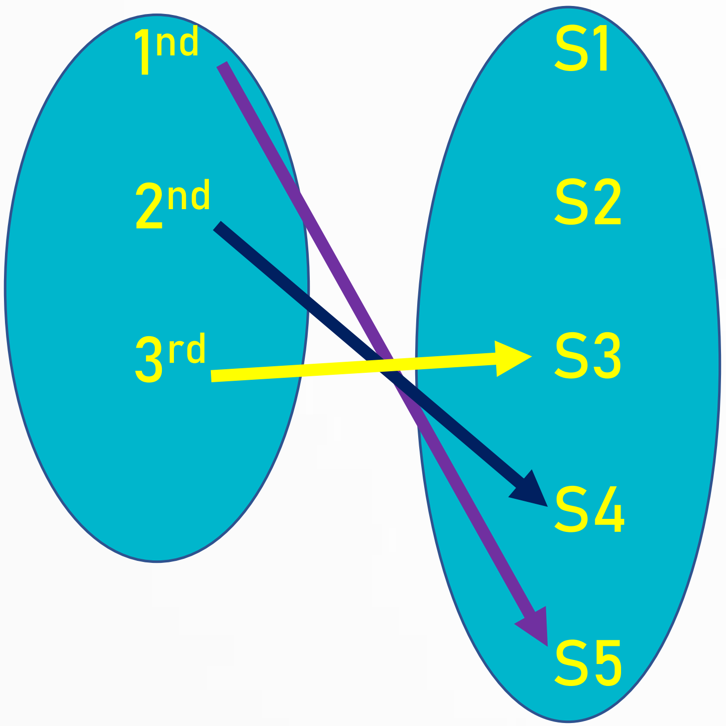

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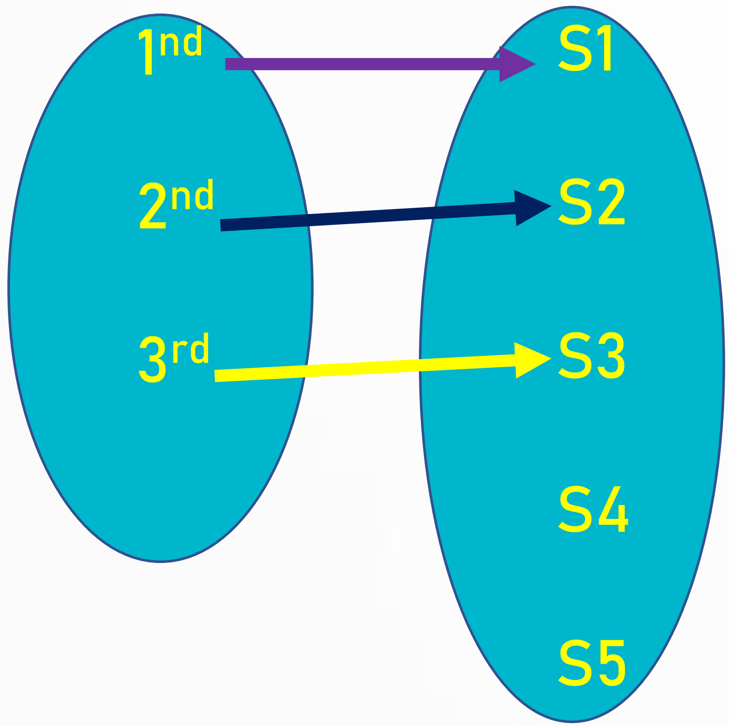

Other Cases

1^{ST}	2^{ND}	3^{RD}
S1	S2	S3
S1	S2	S4
S1	S2	S5
S1	S3	S2
S1	S3	S4
S1	S3	S5
S1	S4	S1
S1	S4	S2
S1	S4	S3
......		
S5	S4	S3

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Permutation – Example 1

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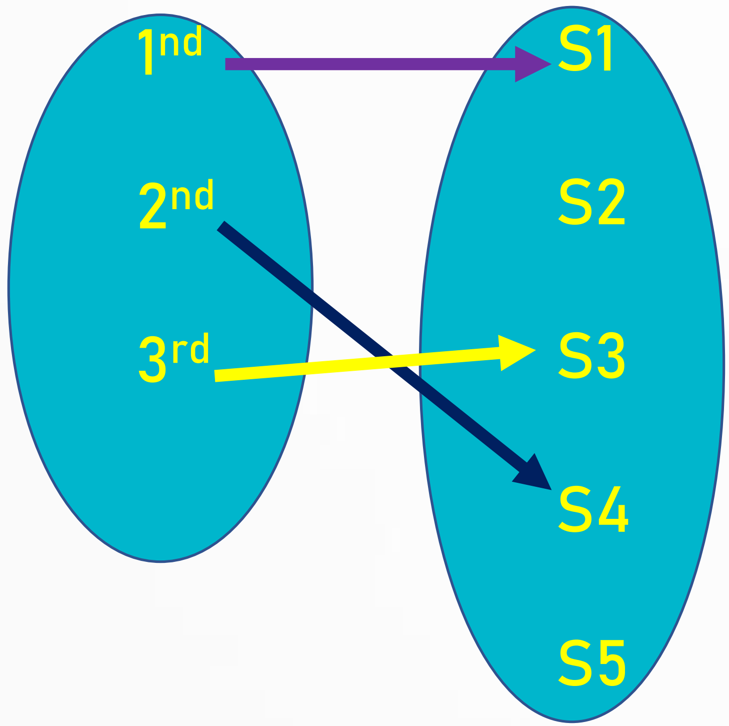

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S1	S3	S5
S1	S4	S1
S1	S4	S2
S1	S4	S3
……		
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Permutation – Example 1

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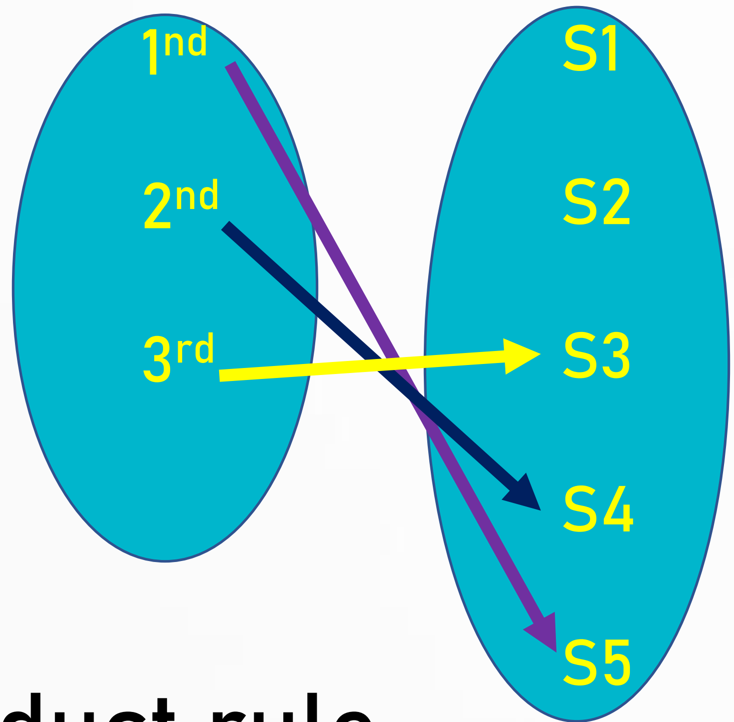

By the product rule, there are

$5 \cdot 4 \cdot 3 = 60$ ways

1ST	2ND	3RD
S1	S2	S3
S1	S2	S4
S1	S2	S5
S1	S3	S2
S1	S3	S4
S1	S3	S5
S1	S4	S1
S1	S4	S2
S1	S4	S3
……		
S5	S4	S3

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Permutation – Example 2

Ques:- In how many ways can we arrange all five of these students in a line for a picture?

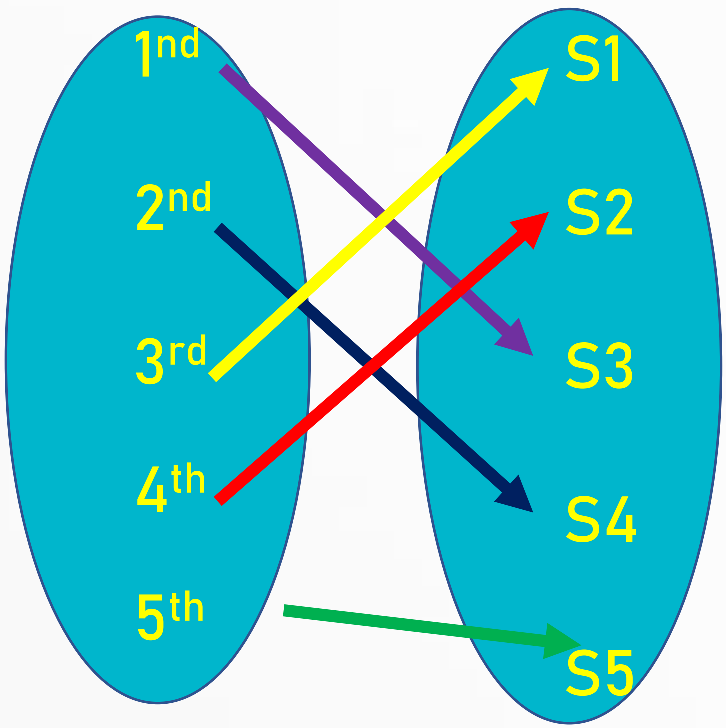
5 Choices

4 Choices

3 Choices

2 Choices

1 Choices



1ST	2ND	3RD	4th	5th
S1	S2	S3	S4	S5
S1	S2	S3	S5	S4
……			….	
S3	S4	S1	S2	S5
……			……	
S5	S4	S3	S5	S4

Sol:- By the product rule, there are
$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ ways

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Permutation – Example 2

Ques:- In how many ways can we arrange all five of these students in a line for a picture?

Sol:- By the product rule, there are

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ ways

OR

$5 \cdot (5-1) \cdot (5-2) \cdot (5-3) \cdot (5-4) = 120$ ways

OR

$n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot (n-4) = 120$ ways

Where $n=5$

OR

$n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot (n-(r-1)) = 120$ ways

where $n=5$, where $r=5$.

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Permutation – Example 2

Ques:- In how many ways can we arrange all five of these students in a line for a picture?

Sol:- N students and r positions

$$\text{Total ways} = n(n-1)(n-2)\cdots(n-(r-1))$$

$$\text{Total ways} = n(n-1)(n-2)\cdots(n-r+1)$$

Permutation - Theorem

If n is a positive integer and r is an integer with $1 \leq r \leq n$, then there are

$$P(n, r) = n(n - 1)(n - 2) \cdot\cdot\cdot (n - r + 1)$$

r-permutations of a set with n distinct elements.

Cor.:- If n and r are integers with $0 \leq r \leq n$, then

$$P(n, r) = \frac{n!}{(n - r)!}.$$

Permutation – Example 3

Ques:- How many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 100 different people who have entered a contest?

$$\textit{Total ways} = n(n-1)(n-2)\cdots(n-(r-1))$$

where n=100, r=3.

$$\textit{Total ways} = n(n-1)(n-2)\cdots(n-r+1)$$

Permutation – Example 3

Ques:- How many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 100 different people who have entered a contest?

Sol:- It is, the number of 3-permutations of a set of 100 elements.

Consequently, the answer is

$$P(100, 3) = \frac{n!}{(n-r)!} = \frac{100!}{(100-3)!} = 100 \cdot 99 \cdot 98 = 970{,}200.$$

Permutation – Example 4

Ques:- Suppose that there are eight horses in a race. The winner receives a gold medal, the second place finisher receives a silver medal, and the third-place finisher receives a bronze medal. How many different ways are there to award these medals, if all possible outcomes of the race can occur and there are no ties?

Permutation – Example 4

Sol:- The number of different ways to award the medals is the number of 3-permutations of a set with eight elements.

Hence, there are $P(8, 3) = 8 \cdot 7 \cdot 6 = 336$ possible ways to award the medals.

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Permutation – Example 5

Ques:- How many permutations of the letters ABCDEFGH contain the string ABC ?

Sol:- the letters ABC must occur as a block, we can find the answer by finding the number of permutations of six objects, namely, the block ABC and the individual letters D, E, F, G, and H.

Permutation – Example 5

ABCDEFGH

OR

ABCDEFHG

OR

DABCEFHG

OR

DEABCFHG

OR

DEFHGABC etc.

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Permutation – Example 5

Ques:- How many permutations of the letters ABCDEFGH contain the string ABC ?

Sol:- Because these six objects can occur in any order, there are $P(6, 6) = \frac{n!}{(n-r)!} = \frac{6!}{(6-6)!} = 6! = 720$ permutations of the letters ABCDEFGH in which ABC occurs as a block.

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Permutation – Example 6

Ques:- How many permutations of {a, b, c, d, e, f, g} end with a?

Sol:- If we want the permutation to end with a, then we may as well forget about the a, and just count the number of permutations of { b, c, d, e, f, g}.

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Permutation – Example 6

b c d e f g a

OR

b c d e g f a

OR

b c d f e g a

OR

c b d e f g a etc.

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Permutation – Example 6

Ques:- How many permutations of {a, b, c, d, e, f, g} end with a?

Sol:- Each permutation of these 6 letters, followed by a, will be a permutation of the desired type, and conversely. Therefore the answer is P(6, 6) = $\frac{n!}{(n-r)!}$ = $\frac{6!}{(6-6)!}$ = 6! = 720.

Permutation – Example 7

Ques:- How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other?

Sol:- First position the men and then consider possible positions for the women.

Permutation – Example 7

W1, W2, W3, W4, W5

_ M1_M2_M3_M4_M5_M6_M7_M8_

_ 1_ 2_ 3_ 4_ 5_ 6_ 7_ 8_

Positions= 9

Women= 5

$$P(9, 5) = \frac{n!}{(n-r)!} = \frac{9!}{(9-5)!} = 98765$$

$$P(8, 8) = \frac{n!}{(n-r)!} = \frac{8!}{(8-8)!} = 8!$$

$$P(8, 8) \cdot P(9, 5) = 8! \cdot \frac{9!}{4!} = 609{,}638{,}400$$

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That's all for now…