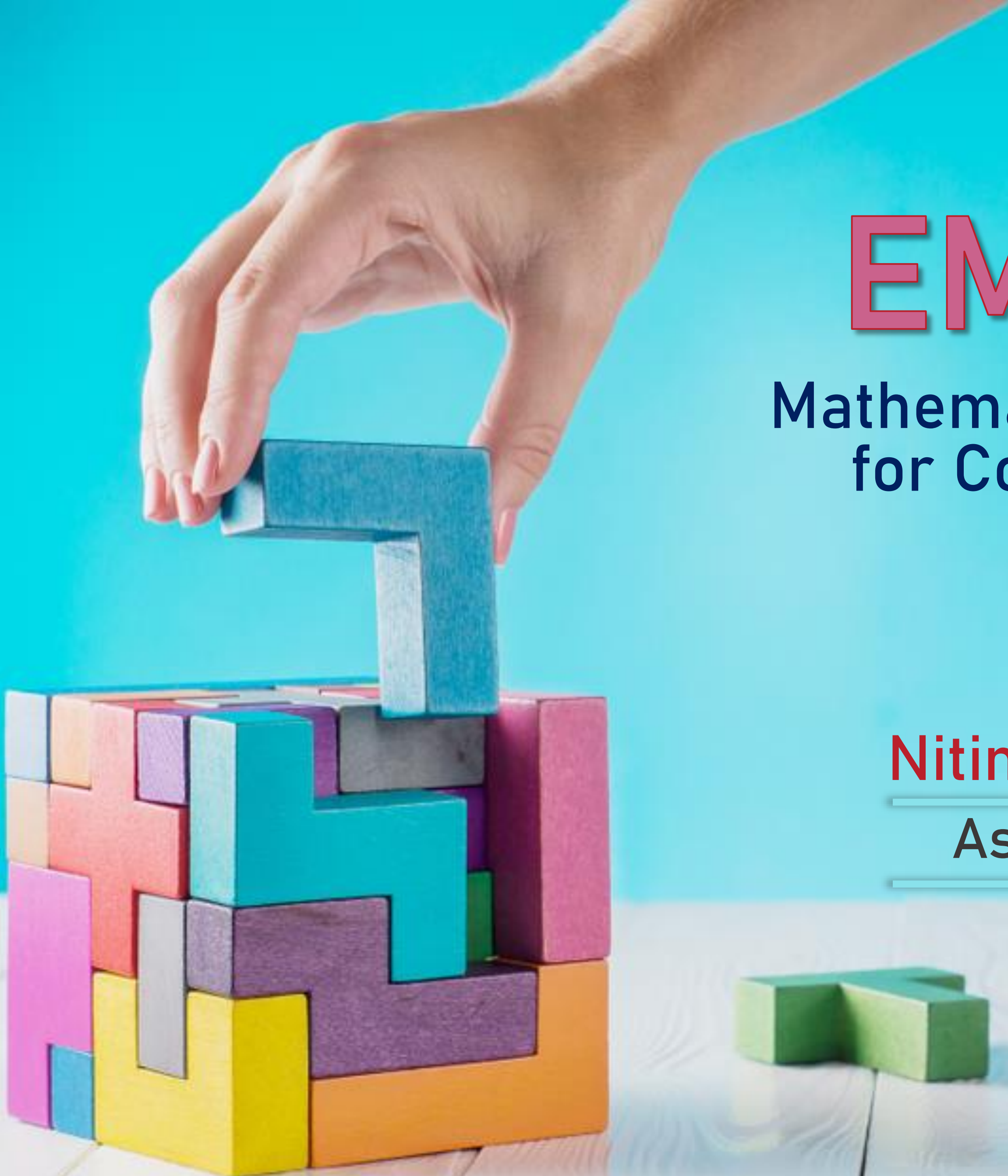

EMTH403

Mathematical Foundation for Computer Science

Nitin K. Mishra (Ph.D.)

Associate Professor

Lecture Outcomes

After this lecture, you will be able to

- understand what truth value of the existential quantification.
- express the compound proposition with existential quantification in English.
- understand how to write compound proposition with existential quantification using disjunctions, conjunctions, and negations

Quantifiers

Many mathematical statements assert that there is an element with a certain property.

Such statements are expressed using existential quantification.

With existential quantification, we form a proposition that is true if and only if P(x) is true for **at least one value** of x in the domain.

Existential Quantifier

The existential quantification of $P(x)$ is the proposition. "There exists an element x in the domain such that $P(x)$."

We use the notation $\exists x P(x)$ for the existential quantification of $P(x)$. Here $\exists$ is called the existential quantifier.

Quantifiers

- A domain must always be specified when a statement $\exists x P(x)$ is used.
- Furthermore, the meaning of $\exists x P(x)$ changes when the domain changes.
- Without specifying the domain, the statement $\exists x P(x)$ has no meaning.

Content Source: Discrete Mathematics and Its Applications. Seventh Edition. Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University. (and formerly AT&T Laboratories)

Existential Quantifier

- Besides the phrase "there exists," we can also express existential quantification in many other ways, such as by using the words "for some," "for at least one," or "there is."

- The existential quantification $\exists x P(x)$ is read as

  "There is an x such that P(x),"

  "There is at least one x such that P(x),"

  or

  "For some xP(x)."

Existential Quantifier

The meaning of the existential quantifier is summarized in the second row of Table. We illustrate the use of the existential quantifier in following examples.

Statement	*When True?*	*When False?*
$\forall x P(x)$	$P(x)$ is true for every x.	There is an x for which $P(x)$ is false.
$\exists x P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every x.

Content Source: **Discrete Mathematics** and **Its Applications**. Seventh Edition. **Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University.** (and formerly AT&T Laboratories)

Existential Quantifier – Example - 1

Ques:- Let $P(x)$ denote the statement "$x > 3$." What is the truth value of the quantification $\exists x P(x)$, where the domain consists of all real numbers?

Ans:- Because "$x > 3$" is sometimes true—for instance, when $x = 4$—the existential quantification of $P(x)$, which is $\exists x P(x)$, is true.

Existential Quantifier

- Observe that the statement $\exists x P(x)$ is false if and only if there is no element x in the domain for which $P(x)$ is true.
- That is, $\exists x P(x)$ is false if and only if $P(x)$ is false for every element of the domain.

Existential Quantifier – Example - 2

Ques:- Let $Q(x)$ denote the statement "$x = x + 1$." What is the truth value of the quantification $\exists x Q(x)$, where the domain consists of all real numbers?

Ans:- Because $Q(x)$ is false for every real number x, the existential quantification of $Q(x)$, which is $\exists x Q(x)$, is false.

Existential Quantifier

Remark:

Generally, an implicit assumption is made that all domains of discourse for quantifiers are non-empty.

Existential Quantifier

If the domain is empty, then $\exists x Q(x)$ is false whenever $Q(x)$ is a propositional function because when the domain is empty, there can be no element x in the domain for which $Q(x)$ is true.

Existential Quantifier

Remark: When all elements in the domain can be listed—say, $x_1, x_2, \ldots, x_n$—the existential quantification $\exists x P(x)$ is the same as the disjunction

$P(x_1) \vee P(x_2) \vee \cdots \vee P(x_n)$,

because this disjunction is true if and only if at least one of $P(x_1), P(x_2), \ldots, P(x_n)$ is true

Existential Quantifier – Example - 3

Ques:- What is the truth value of $\exists x P(x)$, where $P(x)$ is the statement "$x^2 > 10$" and the universe of discourse consists of the positive integers not exceeding 4?

Ans:- Because the domain is {1, 2, 3, 4}, the proposition $\exists x P(x)$ is the same as the disjunction $P(1) \lor P(2) \lor P(3) \lor P(4)$.

Because $P(4)$, which is the statement "$4^2 > 10$," is true, it follows that $\exists x P(x)$ is true.

Existential Quantifier – Example - 4

Ques:- Determine the truth value of each of the statement $\exists n(n = -n)$ if the domain consists of all integers.

Ans:- This statement is true, since $0 = -0$.

Content Source: **Discrete Mathematics** and **Its Applications**. Seventh Edition. **Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University.** (and formerly AT&T Laboratories)

Existential Quantifier – Example - 5

Ques:- Determine the truth value of the statement $\exists n(2n = 3n)$ if the domain consists of all integers.

Ans:- Since $2 \cdot 0 = 3 \cdot 0$, this is true.

Existential Quantifier – Example - 6

Ques:- Determine the truth value of statement $\exists n(n^2 = 2)$ if the domain for all variables consists of all integers.

Ans:- There are two real numbers that satisfy $n^2 = 2$, namely $\pm\sqrt{2}$, but there do not exist any integers with this property, so the statement is false.

Existential Quantifier – Example - 7

Ques:- Let P(x) be the statement "$x = x^2$." If the domain consists of the integers, what is the truth value of $\exists x P(x)$?

Ans:- T (let $x = 1$)

Existential Quantifier – Example - 8

Ques:- Determine the truth value of the statement $\exists n(n^2 < 0)$ if the domain for all variables consists of all integers.

Ans:- Squares can never be negative; therefore this statement is false.

Existential Quantifier – Example - 1

Ques:- Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express the quantification $\exists x P(x)$ in English.

Ans:- There is a student who spends more than five hours every weekday in class.

Content Source: Discrete Mathematics and Its Applications. Seventh Edition. Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University. (and formerly AT&T Laboratories)

Existential Quantifier – Example - 2

Ques:- Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express the quantifications $\exists x \neg P(x)$ in English.

Ans:- There is a student who does not spend more than five hours every weekday in class.

Content Source: **Discrete Mathematics** and **Its Applications**. Seventh Edition. **Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University.** (and formerly AT&T Laboratories)

Existential Quantifier – Example - 3

Ques:- Translate the statement $\exists x(C(x) \rightarrow F(x))$ into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.

Ans:- This statement is that there exists an x in the domain such that if x is a comedian then x is funny. In English, this might be rendered, "There exists a person such that ifs/he is a comedian, then s/he is funny."

Content Source: **Discrete Mathematics** and **Its Applications**. Seventh Edition. **Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University.** (and formerly AT&T Laboratories)

Existential Quantifier – Example - 4

Ques:- Translate the statement $\exists x(C(x) \land F(x))$ into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.

Ans:- This statement is that there exists an x in the domain such that x is a comedian and x is funny. In English, this might be rendered, "There exists a funny comedian" or "Some comedians are funny" or "Some funny people are comedians."

Content Source: Discrete Mathematics and Its Applications. Seventh Edition. Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University. (and formerly AT&T Laboratories)

Existential Quantifier – Example - 1

Ques:- Let $P(x)$ be the statement "x can speak Russian" and let $Q(x)$ be the statement "x knows the computer language C++." Express the sentence "There is a student at your school who can speak Russian and who knows C++." in terms of $P(x)$, $Q(x)$, quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

Ans:- We assume that this sentence is asserting that the same person has both talents. Therefore, we can write $\exists x(P(x) \land Q(x))$.

Content Source: Discrete Mathematics and Its Applications. Seventh Edition. Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University. (and formerly AT&T Laboratories)

Existential Quantifier – Example - 2

Ques:- Let $P(x)$ be the statement "x can speak Russian" and let $Q(x)$ be the statement "x knows the computer language C++." "There is a student at your school who can speak Russian but who doesn't know C++." in terms of $P(x)$, $Q(x)$, quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

Ans:- Since "but" really means the same thing as "and" logically, this is $\exists x(P(x) \land \neg Q(x))$

Existential Quantifier – Example - 3

Ques:- Suppose that the domain of the propositional function P(x) consists of the integers 0, 1, 2, 3, and 4. Write out each of the proposition $\exists x P(x)$ using disjunctions, conjunctions, and negations.

Ans:- We want to assert that P(x) is true for some x in the universe, so either P(0) is true or P(1) is true or P(2) is true or P(3) is true or P(4) is true. Thus the answer is $P(0) \lor P(1) \lor P(2) \lor P(3) \lor P(4)$.

Existential Quantifier – Example - 4

Ques:- Suppose that the domain of the propositional function $P(x)$ consists of the integers 0, 1, 2, 3, and 4. Write out the proposition $\exists x \neg P(x)$ using disjunctions, conjunctions, and negations.

Ans:- $\neg P(0) \vee \neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4)$

Existential Quantifier – Example - 5

Ques:- Suppose that the domain of the propositional function P(x) consists of the integers 1, 2, 3, 4, and 5. Express the statement **∃xP(x)** without using quantifiers, instead using only negations, disjunctions, and conjunctions.

Ans:- We want to assert that P(x) is true for some x in the universe, so either P(1) is true, or P(2) is true, or P(3) is true, or P(4) is true, or P(5) is true. Thus, the answer is

P(1) V P(2) V P(3) V P(4) V P(5).

Existential Quantifier – Example - 6

Ques:- Suppose that the domain of the propositional function $P(x)$ consists of the integers 0, 1, 2, 3, and 4. Write out the proposition $\neg\forall x P(x)$ using disjunctions, conjunctions, and negations.

Ans:- $\neg(P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge P(4))$

Existential Quantifier – Example - 7

Ques:- Suppose that the domain of the propositional function P(x) consists of the integers 1, 2, 3, 4, and 5. Express the statement $\neg\exists x P(x)$ without using quantifiers, instead using only negations, disjunctions, and conjunctions.

Ans:- This is just the negation of part

$\neg(P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5))$

That's all for now…