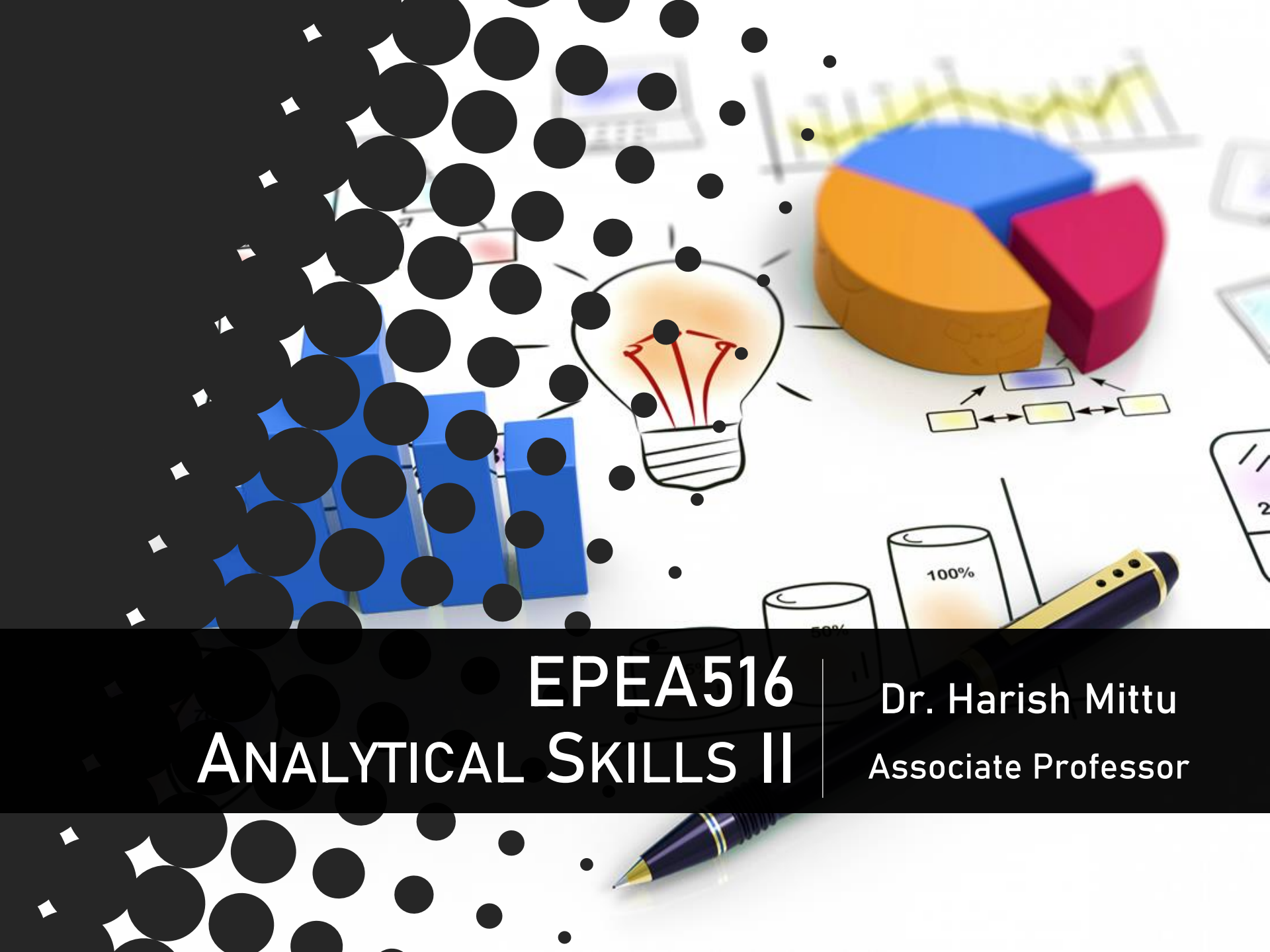

EPEA516
Analytical Skills II

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Learning Outcomes

After this lecture, you will be able to

- understand chain rule,
- enlist types of chain rule,
- apply chain rule for solving practical problems.

Chain Rule

- Compare Every Item
- Term - Found Out

Chain Rule

- Types
  - Direct Proportion
  - Indirect Proportion

Chain Rule

- Direct Proportion
  - Two Quantities - Directly Proportional
  - Increase/Decrease - One
  - Increases/Decreases - Other
  - Same Extent

Direct Proportion

- Two Quantities
- Directly Proportional
- Amount of Work Done α Number of Men Working
- Less Men → Less Work
- More Men → More Work

Chain Rule

- Indirect Proportion
  - Two Quantities - Indirectly Proportional
  - Increase/Decrease - One
  - Decreases/Increases- Other
  - Same Extent

Indirect Proportion

- Two Quantities
- Inversely Proportional
- Number of Persons α $\frac{1}{\text{Time taken to finish a work}}$
- Less Men  More Time
- More Men  Less Time

Fourth Proportion

- Fourth proportion to A, B, and C
- A : B :: C : X
- X – Fourth Proportion
- Compare Every Item - Term (Found Out)

Example 1

- If 30 women can do a certain piece of work in 21 days, in how many days will 15 women do it?
- Solution
- Women = 30  Days = 21
- Women = 15
- Number of Persons α $\frac{1}{\text{Days taken to finish a work}}$
- Less Women (30 to 15)  More Days/Time

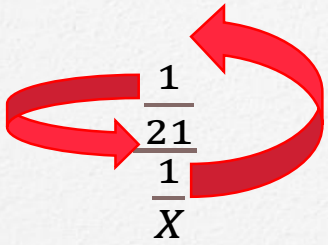
Example 1 - Indirect Proportion

Number of days = X (Say)

$$30 \propto \frac{1}{21} \text{ and } 15 \propto \frac{1}{X}$$

$$\frac{1}{30} : \frac{1}{15} :: 21 : X$$

$$\frac{30}{15} = \frac{\frac{1}{21}}{\frac{1}{X}}$$



$$\frac{30}{15} = \frac{X}{21}$$

Example 1 - Indirect Proportion

$$\frac{X}{21} = \frac{30}{15}$$

$$X = \frac{\overset{2}{\cancel{30}} \times 21}{\cancel{15}_{1}}$$

$$X = 42$$

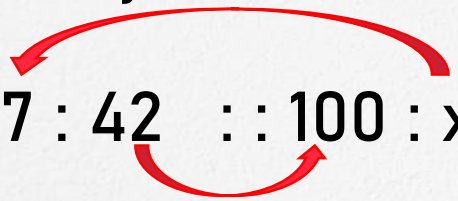
Example 2

- A kitchen requires 100 kgs of wheat for a week. How many kgs of wheat will it require for 42 days?
- Let the required quantity be ‘x’ kg.
- A Week = 7 days
- More Days – More Quantity (Direct Proportion)

7 : 42 : : 100 : x

Example 2

- Let the required quantity be ‘x’ kg.
- A Week = 7 days
- More Days – More Quantity (Direct Proportion)



$$7 : 42 :: 100 : x$$

$$\frac{1}{\cancel{7}}\, x = \frac{\cancel{42}^{6}}{} \times 100$$

$$x = 600 \text{ Kg}$$

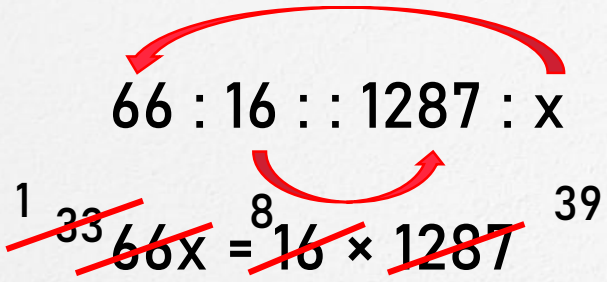
Example 3

- The price of 5.5 dozen pens is Rs. 1287. What is the price of 16 such pens?
- Let the cost of 16 pens be Rs. x.
- 5.5 dozen pens = (5.5 × 12) pens = 66 pens
- Less Pens - Less Cost (Direct Proportion)

  66 : 16 : : 1287 : x

Example 3

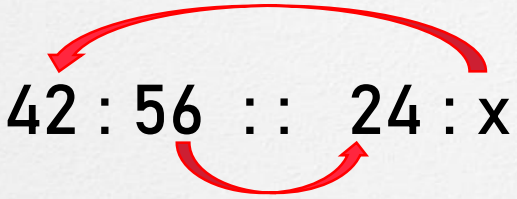
- Let the cost of 16 pens be Rs. x.
- 5.5 dozen pens = (5.5 × 12) pens = 66 pens
- Less Pens - Less Cost (Direct Proportion)



x = Rs. 312

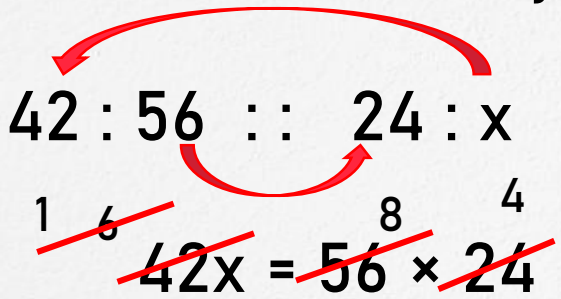
Example 4

- 56 men can complete a piece of work in 24 days. In how many days can 42 men complete the same piece of work?
- Let the required number of days = x
- Less Men - More Days (Indirect Proportion)



Example 4

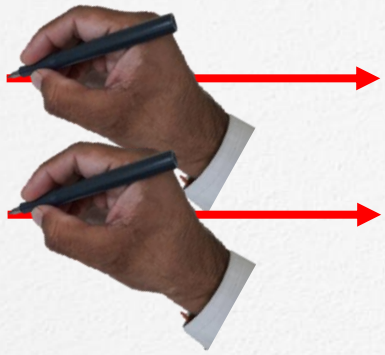
- Let the required number of days = x
- Less Men – More Days (Indirect Proportion)



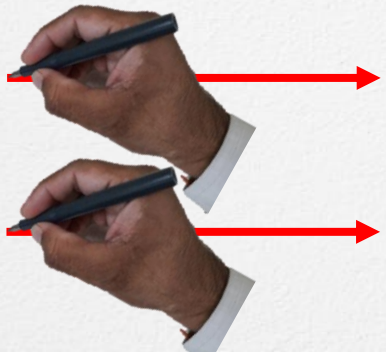
x = 32 Days

Example 5

- If 10 men can build a wall 35 meters long in 7 days , what length of a similar wall can be built by 18 men in 4 days?
- Solution
- Men = 10 → Length = 35 m
- Men = 18 → More Length



- Days = 7 → Length = 35 m
- Days = 4 → Less Length



Example 5 - Direct Proportion

Number of Persons (More) α Length of Wall (More)

(Direct Proportion)

10 : 18 : : 35 : X (Assumed length of Wall)

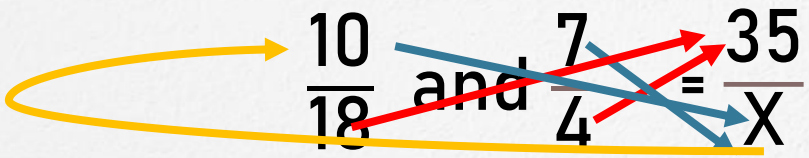
Number of Days (Less) α Length of Wall (Less)

(Direct Proportion)

7 : 4 : : 35 : X

Example 5 - Direct Proportion

$$10 : 18 \\ 7 : 4 \Big\} :: 35 : X$$



$$X = \frac{\cancel{35}^{1} \cdot \cancel{18}^{\cancel{5}} \cdot \cancel{4}^{2}}{\cancel{10}^{\cancel{5}} \cdot \cancel{7}^{1}} = 18 \cdot 2$$

$$X = 36$$

Example 6

- If 6 men working 4 hours a day can reap a field in 10 days, in how many days will 8 men reap the field, working 2 hours a day?
- Solution
- Men = 6 → Days = 10
- Men = 8 → Less days

- Hours = 4 → Days = 10
- Hours = 2 → More days

Example 6 - Indirect Proportion

$$\text{Number of Men (More)} \propto \frac{1}{\text{Number of days (Less)}}$$
(Indirect Proportion)

8 : 6 : : 10 : X (Assumed days)

$$\text{Number of Hours (Less)} \propto \frac{1}{\text{Number of days (More)}}$$
(Indirect Proportion)

2 : 4 : : 10 : X

Example 6 - Indirect Proportion

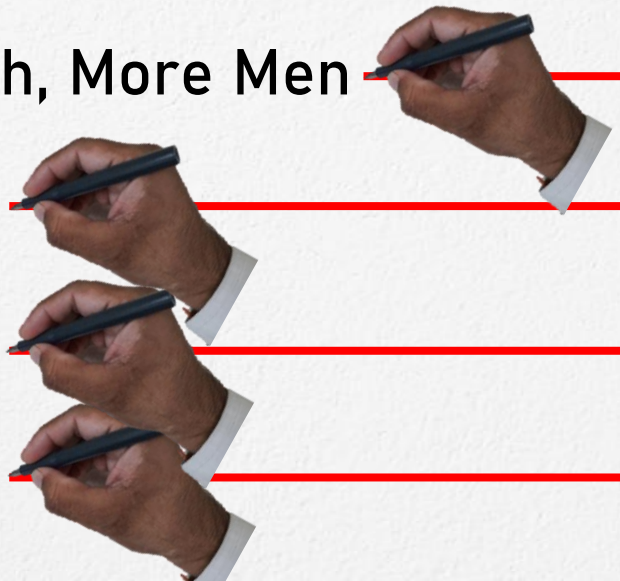
$$\left.\begin{matrix} 8 : 6 \\ 2 : 4 \end{matrix}\right\} :: 10 : X$$

$$X = \frac{\overset{5}{\cancel{10}} \cdot \overset{3}{\cancel{6}} \cdot \overset{1}{\cancel{4}}}{\underset{1}{\cancel{8}} \cdot \underset{1}{\cancel{2}}} = 5 \cdot 3$$

$$X = 15$$

Example 7

- If 8 men working 9 hours a day can build a wall 18m long, 2m wide and 12m high in 10 days, how many men will be required to build a wall 32m long, 5m wide and 9m high, working 6 hours a day in 8 days?
- Solution
- More length and breadth, More Men → Direct
- Less height, Less Men → Direct
- Less days, More Men → Indirect
- Less hour, More Men → Indirect



Example 7

Length of Wall(More) α Men(More)

(Direct Proportion)

18 : 32 : : 8 : X (Assumed days)

Breadth of Wall(More) α Men(More)

(Direct Proportion)

2 : 5 : : 8 : X

Height of Wall(Less) α Men(Less)

(Direct Proportion)

12 : 9 : : 8 : X

Example 7

$$\text{Number of Days (Less)} \ \alpha \ \frac{1}{\text{Number of Men (More)}}$$
(Indirect Proportion)

8 : 10 : : 8 : X (Assumed days)

$$\text{Number of Hours (More)} \ \alpha \ \frac{1}{\text{Number of Men (Less)}}$$
(Indirect Proportion)

6 : 9 : : 8 : X

Example 7

$18 : 32$

$2 : 5$

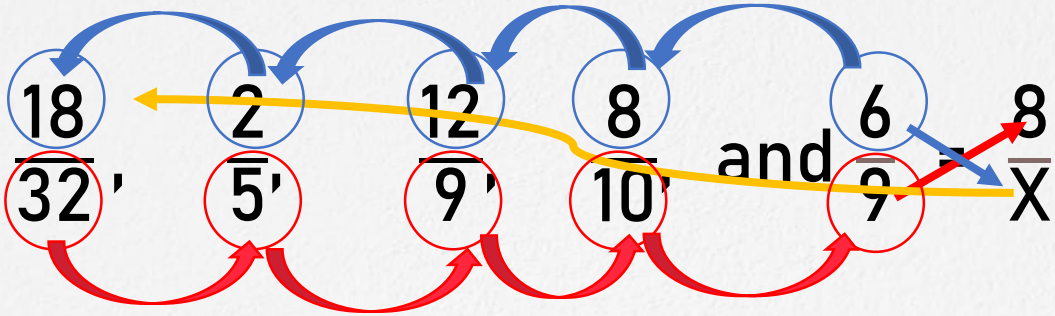
$12 : 9$ $:: 8 : X$

$8 : 10$

$6 : 9$

$\frac{18}{32}, \frac{2}{5}, \frac{12}{9}, \frac{8}{10},$ and $\frac{6}{9} = \frac{8}{X}$

Example 7 - Direct Proportion



$$X = \frac{8 \cdot 32 \cdot 5 \cdot 9 \cdot 10 \cdot 9}{18 \cdot 2 \cdot 12 \cdot 8 \cdot 6}$$

$$X = 50$$

Example 8

- The cost of 16 packets of salt, each weighing 900 grams is Rs. 28. What will be the cost of 27 packets, if each packet weighs 1 kg?
- Let the required cost = Rs. x.
- More Packets - More Cost (Direct Proportion)
- More Weight - More Cost (Direct Proportion)
- Packets - 16 : 27
- Weight - 900 : 1000

Example 8

Packets 16 : 27

Weights 900 : 1000

} :: 28 : X

$$\frac{16}{27} \text{ and } \frac{900}{1000} = \frac{28}{X}$$

$$X = \frac{\cancel{28}^{7} \cdot \cancel{27}^{3} \cdot \cancel{1000}^{\cancel{10}^{5}}}{\cancel{16}^{2} \cdot \cancel{900}^{\cancel{9}^{\cancel{8}}}} = \frac{105}{2}$$

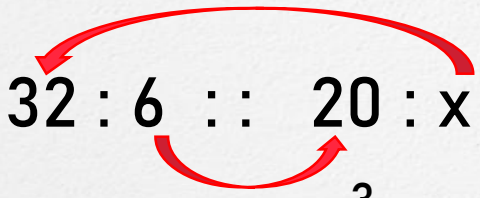
$$X = 52.50$$

Example 9

- 3 men or 6 women can do a piece of work in 20 days. In how many days will 12 men and 8 women do the same work?
- 3 men or 6 women – 20 Days
- 12 men and 8 women – ? Days
- More Women - Less Days (Indirect Proportion)
- 3 men or 6 women – 20 Days
- 3 x 4 men or 6 x 4 women Or 12 men or 24 women

Example 9

- 12 men or 24 women
- 12 men and 8 women or (24 + 8) women = 32 women
- Let the required number of days = x
- 6 women – 20 Days
- More Women - Less Days (Indirect Proportion)



$32 : 6 :: 20 : x$

$4 \quad 8 \quad 3 \quad 5$

$32x = 6 \times 20$ or $x = \frac{15}{4}$ or $x = 3\frac{3}{4}$ days

Example 10

- 64 persons can dig a trench 50 m long, 2 m wide and 2 m deep in 5 days, working 12 hours daily. In how many days, working 8 hours daily, will 80 persons dig another trench 75 m long, 4 m wide and 3 m deep?
- More Persons - Less Days (Indirect Proportion)
- More Length - More Days (Direct Proportion)
- More Width - More Days (Direct Proportion)
- More Depth - More Days (Direct Proportion)
- Less Working Hours - More Days (Indirect Proportion)

Example 10

Let the required number of days be x

Persons	80 : 64	: : 5 : X
Length	50 : 75	
Width	2 : 4	
Depth	2 : 3	
Working Hours	8 : 12	

$$\frac{80}{64}, \frac{50}{75}, \frac{2}{4}, \frac{2}{3}, \text{ and } \frac{8}{12} = \frac{5}{X}$$

Example 10

$\frac{80}{64}, \frac{50}{75}, \frac{2}{4}, \frac{2}{3}$, and $\frac{8}{12} = \frac{5}{X}$

$$X = \frac{5 \cdot \overset{8}{\cancel{64}} \cdot 75 \cdot \cancel{4} \cdot 3 \cdot 12}{\underset{10}{\cancel{80}} \cdot \underset{10}{\cancel{50}} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{8}}$$

$$X = \frac{\overset{27}{\cancel{2700}}}{\cancel{100}}$$

$X = 27$

Example 11

- The normal dosage of a particular medicine is ‘t’ tablets per day for each patient. A hospital’s current supply of these tablets will last ‘p’ patients for ‘d’ days. If the recommended dosage increases by 20% and the number of patients decreases by one-third, then for how many days will the hospital’s supply last?
- More Dosage – Less Days (Indirect Proportion)
- Less Patients – More Days (Indirect Proportion)

Example 11

- The normal dosage of a particular medicine is ‘t’ tablets per day for each patient. A hospital’s current supply of these tablets will last ‘p’ patients for ‘d’ days. If the recommended dosage increases by 20% and the number of patients decreases by one-third, then for how many days will the hospital’s supply last?
- New dosage = 120% of t $= \frac{\cancel{120}^{6}}{\cancel{100}_{5}} \times t = \frac{6}{5} t$
- Number of patients $= 1 - \frac{1}{3} p = \frac{3-1}{3} p = \frac{2}{3} p$
- Let the required number of days = X

Example 11

Dosage $\frac{6}{5}t : t$

Patients $\frac{2}{3}p : p$

$:: d : X$

$$\frac{\frac{6}{5}t}{t} \text{ and } \frac{\frac{2}{3}p}{p} = \frac{d}{X}$$

$$X = \frac{d \cdot t \cdot p}{\frac{6}{5}t \cdot \frac{2}{3}p} \quad \text{or } X = \frac{5d}{4}$$

Example 12

- A garrison had provisions for a certain number of days. After 10 days, $\frac{1}{5}$ of the men desert and it is found that the provisions will now last just as long as before. How long was that?
- Initially, let ‘x’ men having food for ‘y’ days.
- After 10 days, x men had food for (y – 10) days.
- $(x - \frac{1}{5}x) = \frac{4}{5}$ x men had food for ‘y’ days.

Example 12

- Initially, let ‘x’ men having food for ‘y’ days.
- After 10 days, x men had food for (y – 10) days.
- $(x - \frac{1}{5}x) = \frac{4}{5}$ x men had food for ‘y’ days.
- $x\ (y - 10) = (\frac{4}{5}\ x).\ (y)$ or $5xy - 50x = 4xy$
- $5xy - 4xy = 50x$ or $xy - 50x = 0$
- $x\ (y - 50) = 0$
- $x = 0$ or $y - 50 = 0$ or $y = 50$

Example 13

- A team of workers was employed by a contractor who undertook to finish 360 pieces of an article in a certain number of days. Making four more pieces per day than was planned, they could complete the job a day ahead of schedule. How many days did they take to complete the job?
- Let ‘x’ days will be taken by team to finish 360 pieces.
- Number of pieces made by team in each day = $\frac{360}{x}$

Example 13

- Let ‘x’ days will be taken by team to finish 360 pieces.
- Number of pieces made by team in each day = $\frac{360}{x}$
- More Number of Pieces/Day - Less days

(Indirect Proportion)

- $(\frac{360}{x} + 4) : \frac{360}{x} : : x : (x - 1)$

Example 13

- $\left(\frac{360}{x} + 4\right) : \frac{360}{x} :: x : (x - 1)$
- $\left(\frac{360}{x} + 4\right).(x - 1) = \frac{360}{x}.x$
- $360 - \frac{360}{x} + 4x - 4 = 360$ Or $-\frac{360}{x} + 4x = 4$ or $4\left(-\frac{90}{x} + x\right) = 4$
- $-\frac{90}{x} + x = 1$ or $-90 + x^2 = x$ or $x^2 - x - 90 = 0$ or $(x - 10).(x + 9) = 0$
- $x = 10$ or -9
- $x = 10$ (Because $x = -9$ not possible)

Example 14

- The cost of 5 kgs of apples is Rs. 450. The cost of 12 dozen mangoes is Rs. 4,320 and the cost of 4 kgs of oranges is Rs. 240. What is the total cost of 8 kg of apples, 8 dozens of mangoes and 8 kg of oranges?
- Cost of 5 kg apples = Rs. 450
- Cost of 1 kg apples = Rs. $\frac{450}{5}$
- Cost of 8 kg apples = Rs. $\frac{\overset{90}{\cancel{450}}}{\cancel{5}}$. 8 = Rs. 720

Example 14

- Cost of 12 dozen mangoes = Rs. 4320
- Cost of 1 dozen mangoes = Rs. $\frac{4320}{12}$
- Cost of 8 dozen mangoes = Rs. $\frac{\cancel{4320}^{360}}{\cancel{12}} \times 8$ = Rs. 2880
- Cost of 4 kg oranges = Rs. 240
- Cost of 1 kg orange = Rs. $\frac{240}{4}$
- Cost of 8 kg orange = Rs. $\frac{\cancel{240}^{60}}{\cancel{4}} \times 8$ = Rs. 480
- Total cost = Rs. (720 + 2880 + 480) = Rs. 4080

Example 15

- 2 men and 7 boys can do a piece of work in 14 days; 3 men and 8 boys can do the same in 11 days. In how many days 8 men and 6 boys can do the three times the amount of this work ?
- More Boys - Less Days (Indirect Proportion)
- More Work - More Days (Direct Proportion)
- (2 × 14) men + (7 × 14) boys
- or (3 × 11) men + (8 × 11) boys

Example 15

- (2 × 14) men + (7 × 14) boys
- (3 × 11) men + (8 × 11) boys

33 men + 88 boys
-(28 men + 98 boys)

= 5 men + 10 boys

- 5 men ≡ 10 boys or 1 man ≡ 2 boys
- (2 men + 7 boys) ≡ (2 × 2 + 7) boys = 11 boys
- (8 men + 6 boys) ≡ (8 × 2 + 6) boys = 22 boys
- Let the required number of days be x.
- Boys - 22 : 11
- Work - 1 : 3

} : : 14 : X

Example 15

Boys 22 : 11

Work 1 : 3

} : : 14 : X

$$\frac{22}{11} \text{ and } \frac{1}{3} = \frac{14}{X}$$

$$X = \frac{\cancel{11} \cdot 3 \cdot \overset{7}{\cancel{14}}}{\underset{2}{\cancel{22}} \cdot 1}$$

$$X = 21$$

Example 16

- 12 men and 18 boys, working $7\frac{1}{2}$ hours a day, can do a piece of work in 60 days. If a man works equal to 2 boys, then how many boys will be required to help 21 men to do twice the work in 50 days, working 9 hours a day?
- Less Days - More Boys (Indirect Proportion)
- More Hrs/Day - Less Boys (Indirect Proportion)
- More Work - More Boys (Direct Proportion)

Example 15

- 1 man ≡ 2 boys
- (12 men + 18 boys) ≡ (12 × 2 + 18) boys = 42 boys
- Let the required number of boys = x
- 21 men + x boys ≡ (21 × 2 + x) boys = (42 + x) boys
- Days - 50 : 60
- Hours/Day - 9 : $7\frac{1}{2} = \frac{15}{2}$
- Work - 1 : 2

} : : 42 : 42 + x

Example 16

- Days - 50 : 60
- Hours/Day - 9 : $7\frac{1}{2} = \frac{15}{2}$
- Work - 1 : 2

$$: : 42 : 42 + x$$

$$\frac{50}{60}, \frac{9}{\frac{15}{2}} \text{ and } \frac{1}{2} = \frac{42}{42 + x}$$

$$(42 + x) = \frac{42 \,.\, 60. \,15. \,2}{50. \,9. \,1. \,2}$$

$$42 + x = 84$$

$$x = 42$$

Conclusion

- Chain Rule
  - Compare Every Item
  - Term - Found Out
- Direct Proportion
  - Two Quantities - Directly Proportional
  - Increase/Decrease (One)- Increase/Decrease (Other)
- Indirect Proportion
  - Two Quantities - Indirectly Proportional
  - Increase/Decrease (One) - Decreases/Increases (Other)

Summary

- Time & Work
    - Chain Rule

That’s all for now…