

A hand is shown placing a blue L-shaped block on top of a colorful cube. The cube is constructed from various colored blocks (yellow, orange, pink, purple, blue, green, grey) and is sitting on a white wooden surface. The background is a solid light blue.

EMTH403

Mathematical Foundation
for Computer Science

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Lecture Outcomes

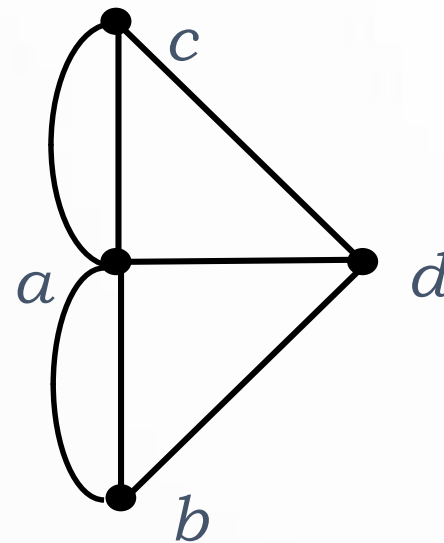
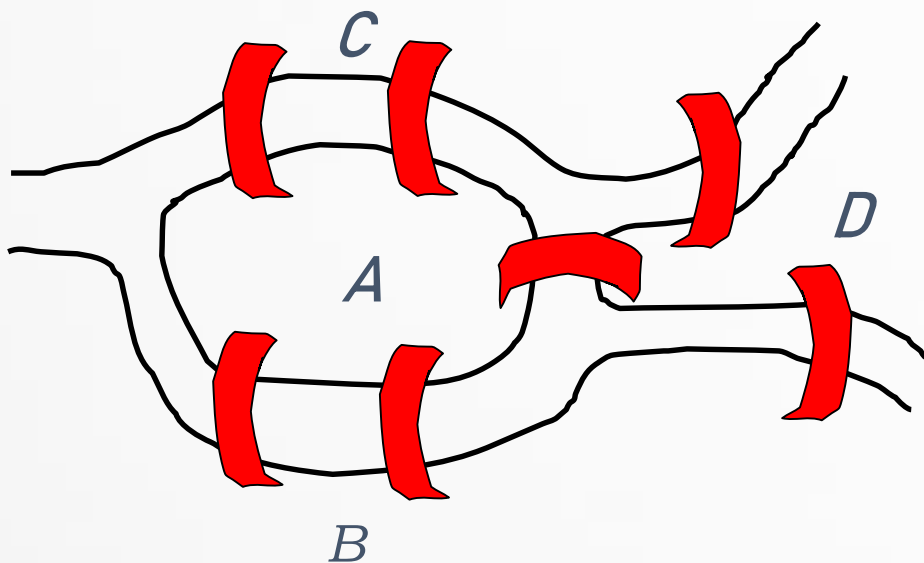


After this lecture, you will be able to

- understand what are Euler path in an undirected graph
- understand what are Euler circuit in an undirected graph

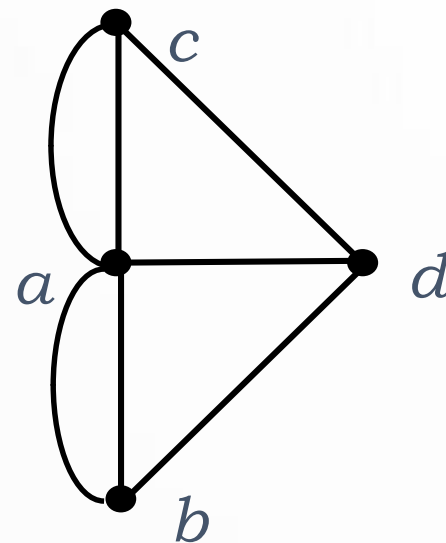
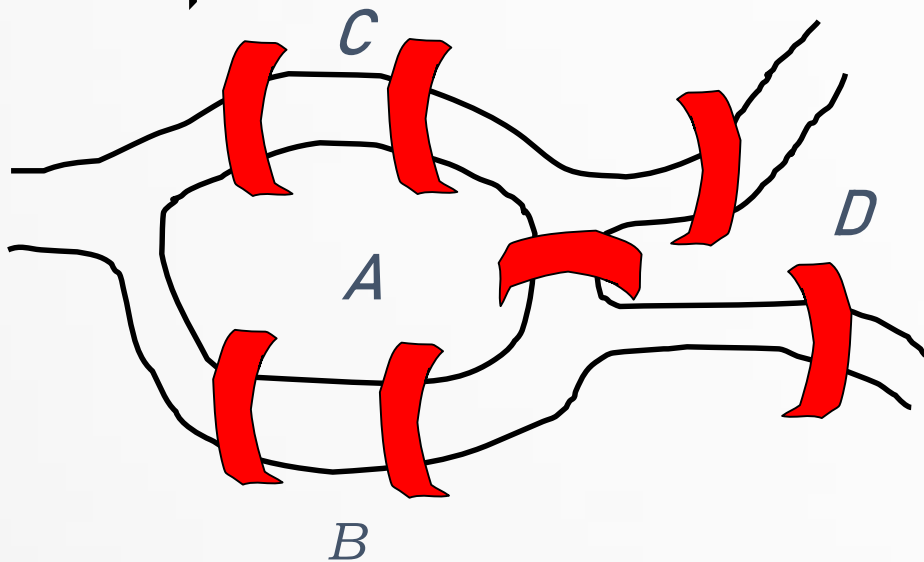
Euler Paths and Circuits

The Seven bridges of Königsberg, Prussia (now called Kaliningrad and part of the Russian republic)



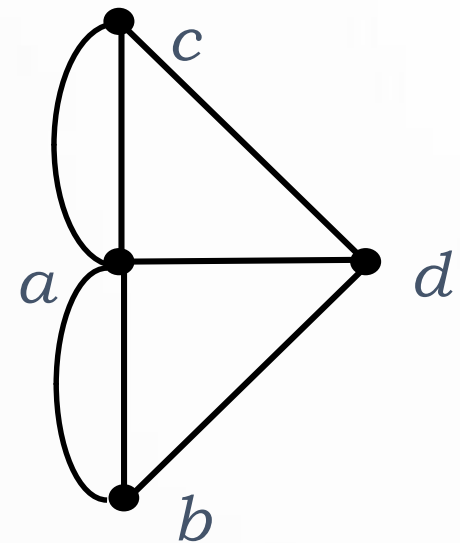
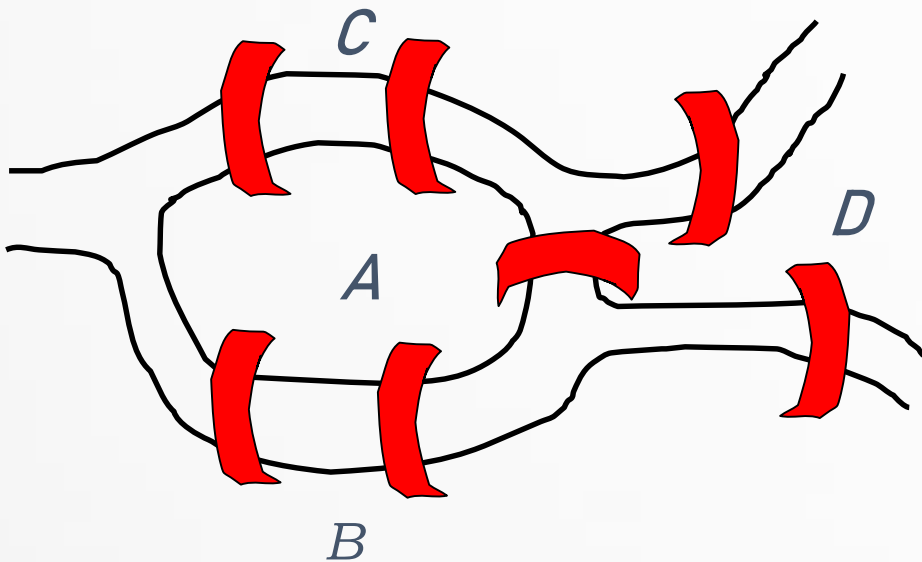
Euler Paths and Circuits

The townspeople wondered whether it was possible to start at some location in the town, travel across all the bridges once without crossing any bridge twice, and return to the starting point .



Euler Paths and Circuits

The Swiss mathematician Leonhard Euler solved this problem. His solution, published in 1736, may be the first use of graph theory.



Applications

Euler paths and circuits can be used to solve many practical problems.

For example, many applications ask for a path or circuit that traverses each street in a neighborhood, each road in a transportation network, each connection in a utility grid, or each link in a communications network exactly once.

Finding an Euler path or circuit in the appropriate graph model can solve such problems.

Applications

For example, if a postman can find an Euler path in the graph that represents the streets the postman needs to cover, this path produces a route that traverses each street of the route exactly once.

If no Euler path exists, some streets will have to be traversed more than once.

Applications

The problem of finding a circuit in a graph with the fewest edges that traverses every edge at least once is known as the Chinese postman problem in honor of Guan Meigu, who posed it in 1962.

Euler Theorem

A connected multigraph with at least two vertices has an Euler circuit if and only if each of its vertices has even degree.

Euler Paths and Circuits

An Euler path is a path using every edge of the graph G exactly once.

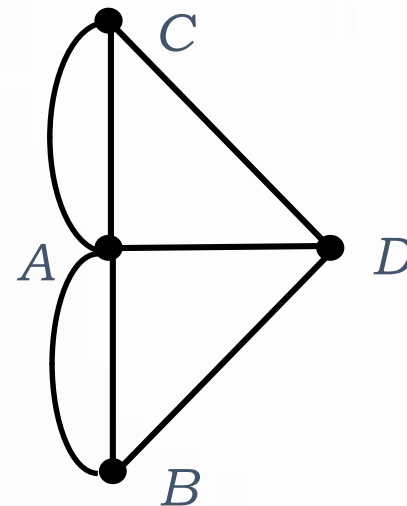
An Euler circuit is an Euler path that returns to its start.

Euler Paths and Circuits

An Euler path is a path using every edge of the graph G exactly once.

Ques:- Does this graph have an Euler circuit?

Ans:- No.



Euler Paths and Circuits

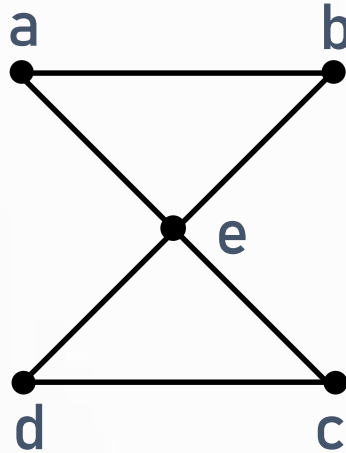
Necessary and Sufficient Conditions

A connected multigraph has a Euler circuit if and only if each of its vertices has an even degree.

A connected multigraph has a Euler path but not an Euler circuit if and only if it has exactly two vertices of odd degree.

Euler Paths and Circuits - Table

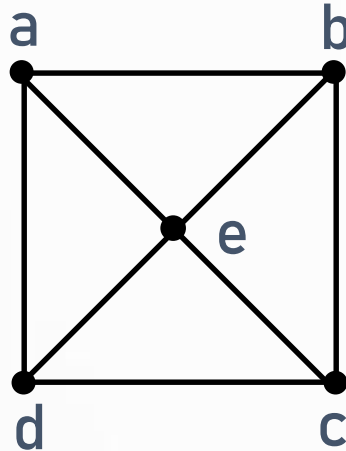
Euler Paths and Circuits



Ques:- Does the graph above has an Euler circuit?

Sol:- No

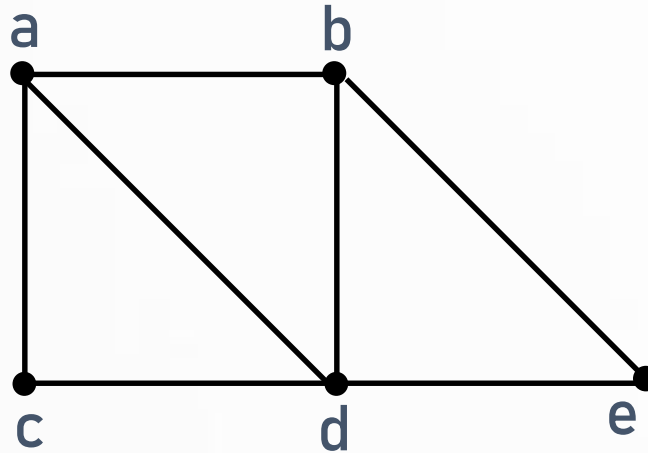
Euler Paths and Circuits



Ques:- Does the graph above has an Euler circuit?

Sol:- No

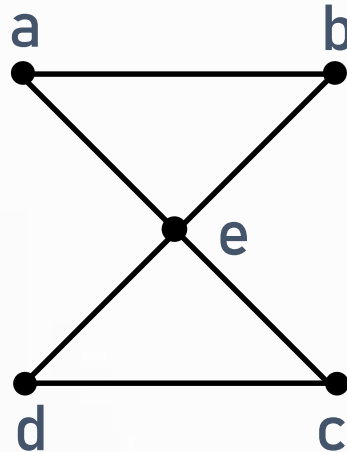
Euler Paths and Circuits



Ques:- Does the graph above has an Euler circuit?

Sol:- No

Euler Paths and Circuits

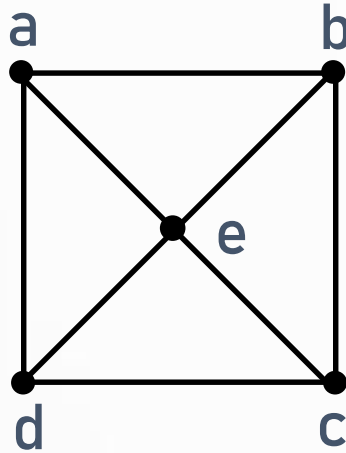


Ques:- Does the graph above has an Euler path?

Sol:- Yes

(a, e, c, d, e, b, a)

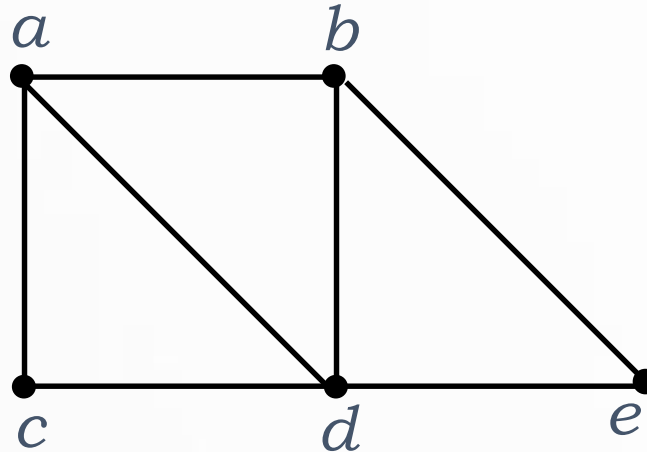
Euler Paths and Circuits



Ques:- Does the graph above has an Euler path?

Sol:- No

Euler Paths and Circuits

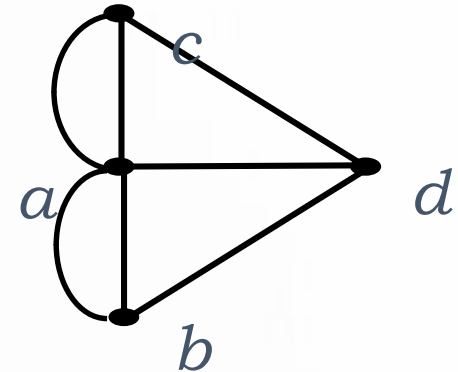
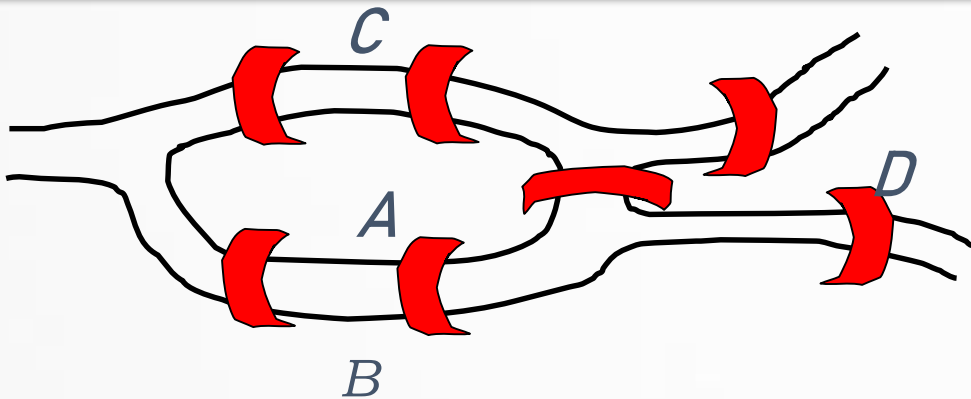


Ques:- Does the graph above has an Euler path?

Sol:- Yes

(a, c, d, e, b, d, a, b)

Euler Paths and Circuits



Ques:- Suppose that in addition to the seven bridges of Königsberg there were two additional bridges, connecting regions B and C and regions B and D, respectively. Could someone cross all nine of these bridges exactly once and return to the starting point?

Sol:- No, an Euler circuit does not exist in the graph modeling this hypothetical city either. Vertices A and B have odd degree.

That's all for now...