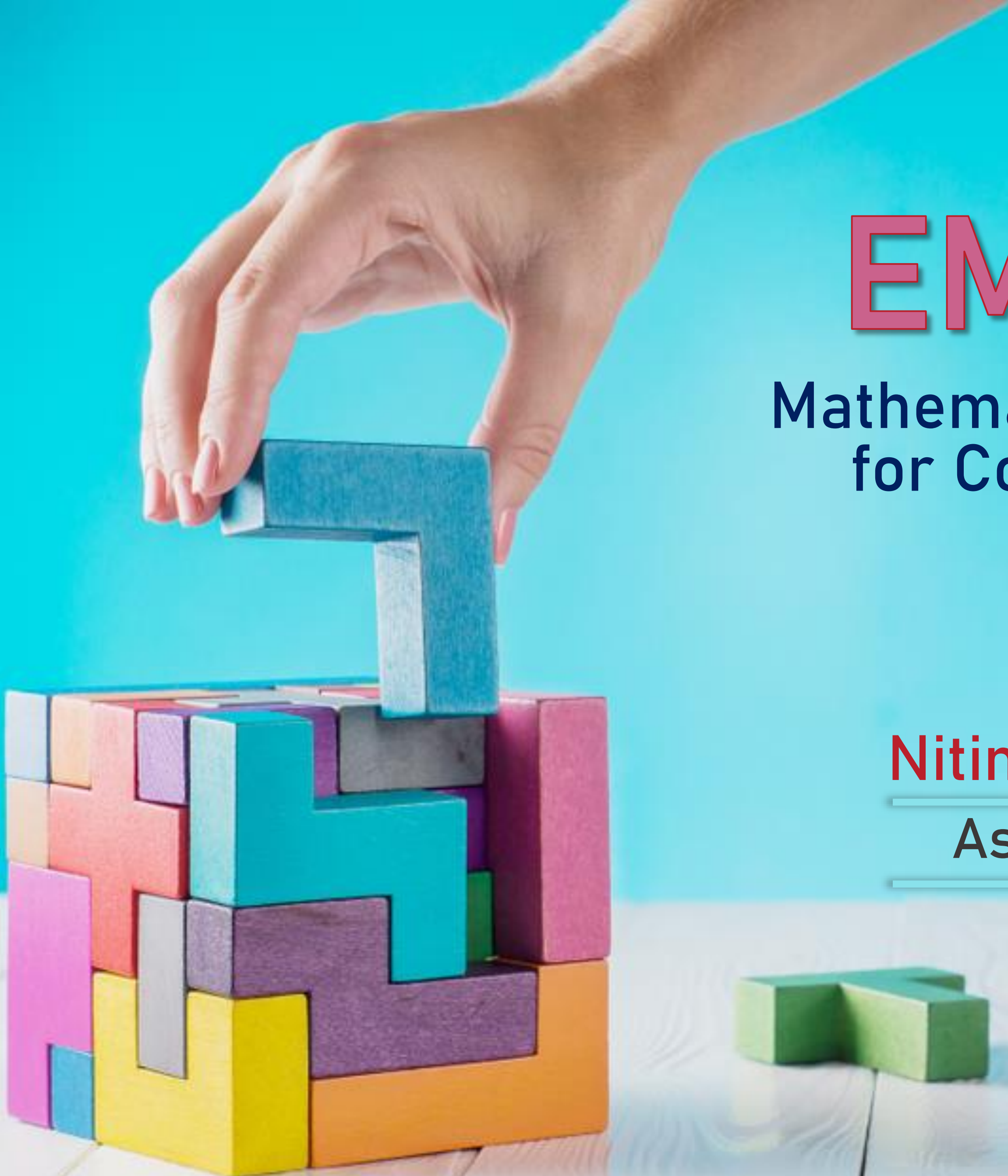


EMTH403

Mathematical Foundation for Computer Science

Nitin K. Mishra (Ph.D.)
Associate Professor

Lecture Outcomes

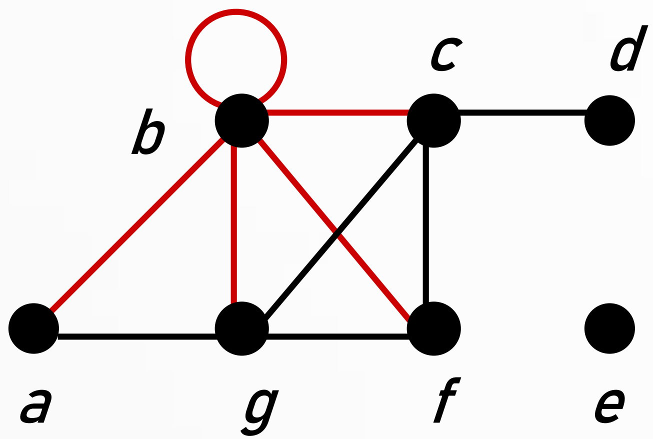
After this lecture, you will be able to

- understand degree of vertex in a graph.
- understand what is Handshaking theorem
- understand some special simple graphs.

Degree of a vertex – Definition

The degree of a vertex in an undirected graph is the number of edges incident with it, except that a loop at a vertex contributes twice to the degree of that vertex.

deg(b) = 6



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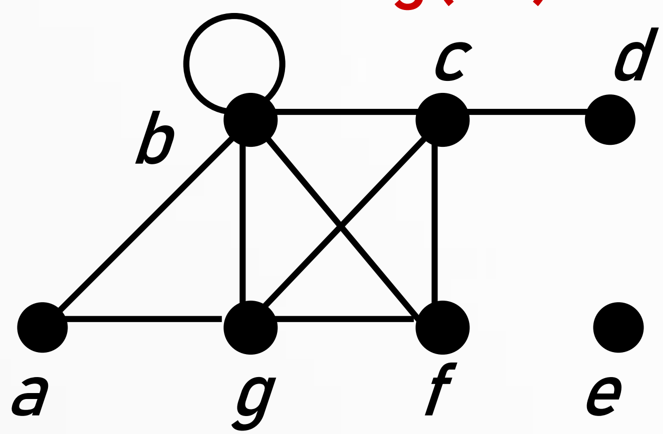
Degree of a vertex - Example

Find the degree of all the other vertices.

deg(a)   deg(c)   deg(f)   deg(g)

deg(*c*) = 4

deg(*b*) = 6



deg(*d*) = 1

deg(*a*) = 2

deg(*e*) = 0

deg(*g*) = 4

deg(*f*) = 3

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Degree of a vertex - Example

Find the degree of all the other vertices.

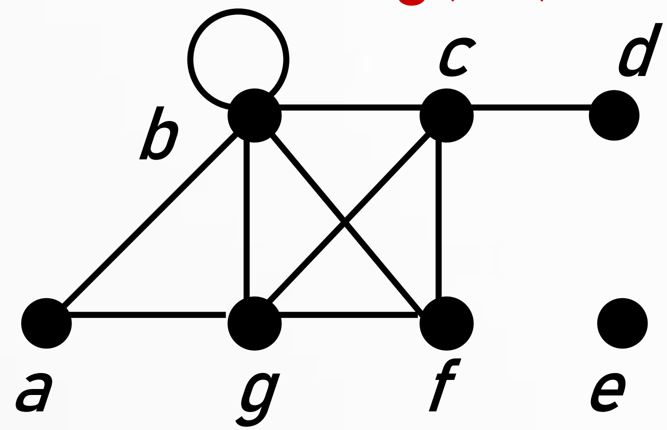
deg(a) = 2,deg(c) = 4,deg(f) = 3, deg(g) = 4

TOTAL of degrees = 2 + 4 + 3 + 4 + 6 + 1 + 0 = 20

deg(*c*) = 4

deg(*b*) = 6

deg(*d*) = 1



deg(*a*) = 2

deg(*e*) = 0

deg(*g*) = 4 deg(*f*) = 3

Handshaking Theorem

Let G = (V, E) be an undirected graph G with e edges. Then

$$\sum_{v \in V} \deg(v) = 2e$$

"The sum of the degrees over all the vertices equals twice the number of edges."

NOTE: This applies even if multiple edges and loops are present.

Handshaking Theorem

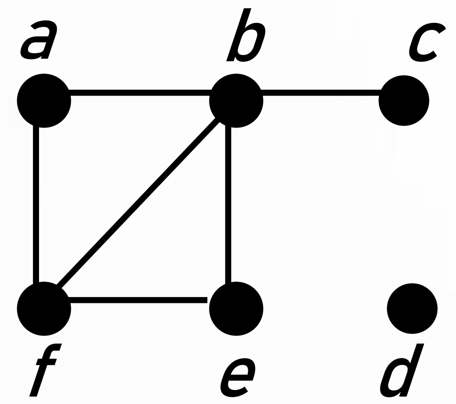
Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated and pendant vertices.

Vertices = 6

Edges = 6

The degree of each vertex is the number of edges incident to it.

deg(a) = 2



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Handshaking Theorem

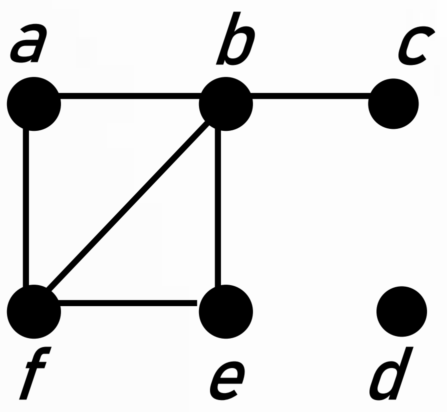
Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated and pendant vertices.

deg(b) = 4,

deg(c) = 1 (and hence c is pendant),

deg(d) = 0 (and hence d is isolated),

deg(e) = 2,



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Handshaking Theorem

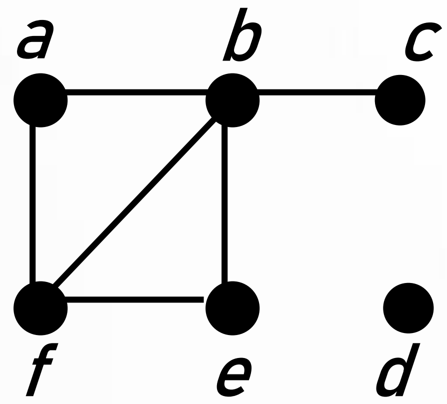
Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated and pendant vertices.

deg(f)= 3.

Note that the sum of the degrees is

2 + 4 + 1+0 + 2 + 3 = 12,

which is twice the number of edges.



Handshaking Theorem

Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated and pendant vertices.

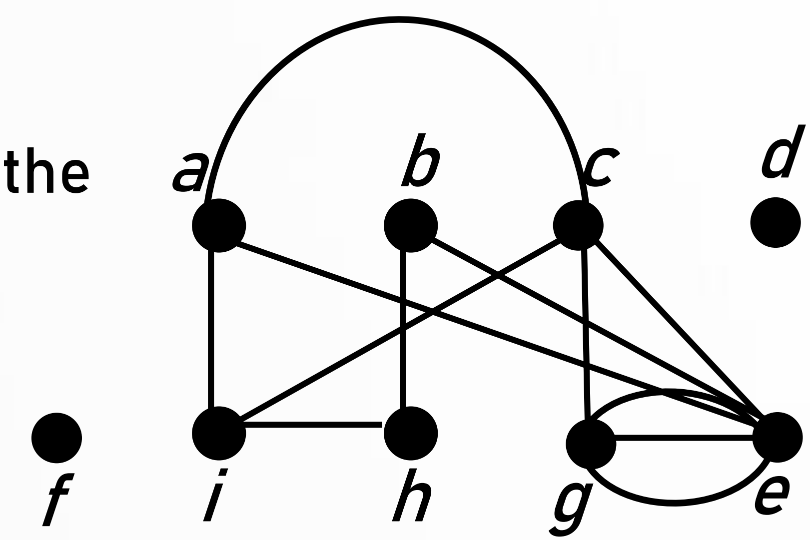
Vertices = 9

Edges = 12

The degree of each vertex is the number of edges incident to it.

deg(a) = 3,

deg(b) = 2,



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Handshaking Theorem

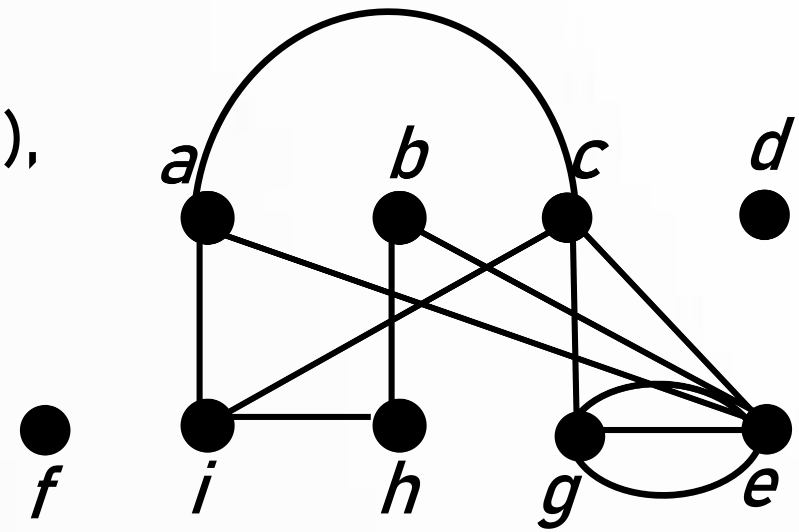
Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated and pendant vertices.

deg(c) = 4,

deg(d) = 0 (and hence d is isolated),

deg(e) = 6,

deg(f) = 0(and hence f is isolated),



Handshaking Theorem

Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated and pendant vertices.

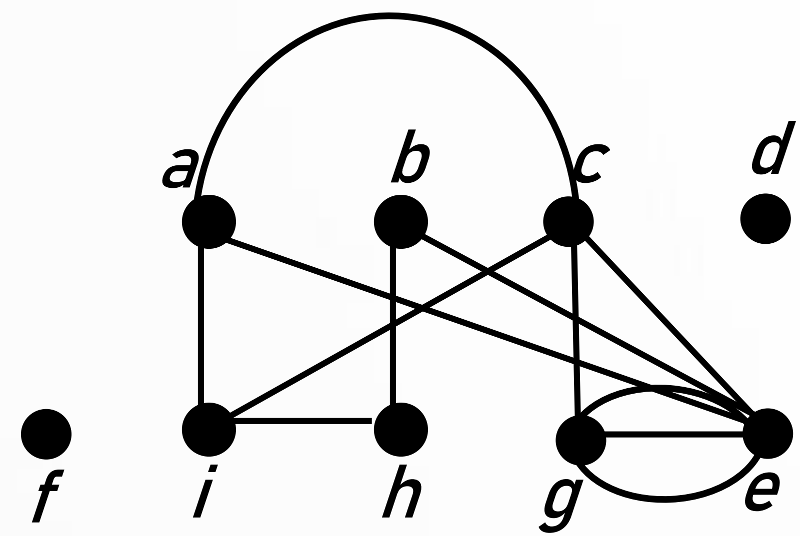
deg(g) = 4,

deg(h) = 2, and

deg(i) = 3.

Note,

that the sum of the degrees is



3 + 2 + 4 + 0 + 6 + 0 + 4 + 2 + 3 = 24,

which is twice the number of edges.

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Handshaking Theorem

Ques:- How many edges are there in a graph with 10 vertices each of degree six?

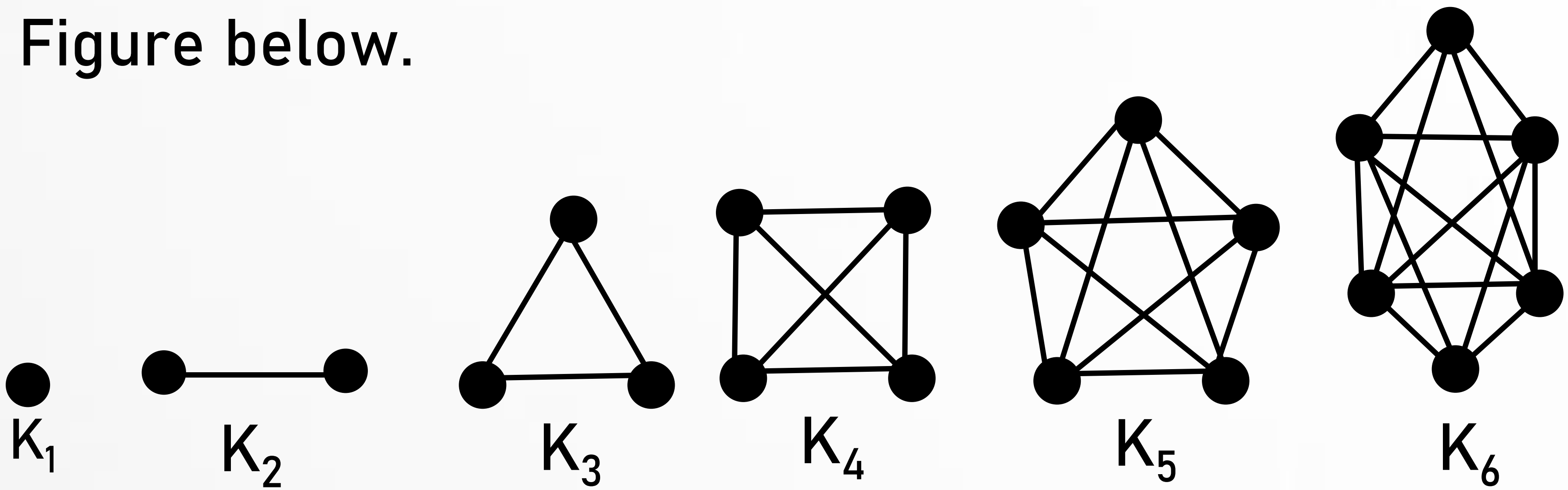
Sol:- Because the sum of the degrees of the vertices is $6 \cdot 10 = 60$, it follows that $2m = 60$ where m is the number of edges. Therefore, $m = 30$.

Hint:- $\sum_{v \in V} \deg(v) = 2e$

Special Simple Graphs – Complete Graphs

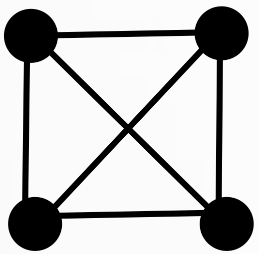
A complete graph on n vertices, denoted by K_n, is a simple graph that contains exactly one edge between each pair of distinct vertices.

The graphs K_n, for n = 1, 2, 3, 4, 5, 6, are displayed in Figure below.



Special Simple Graphs – Complete Graphs

A simple graph for which there is at least one pair of distinct vertex not connected by an edge is called noncomplete.



K_4

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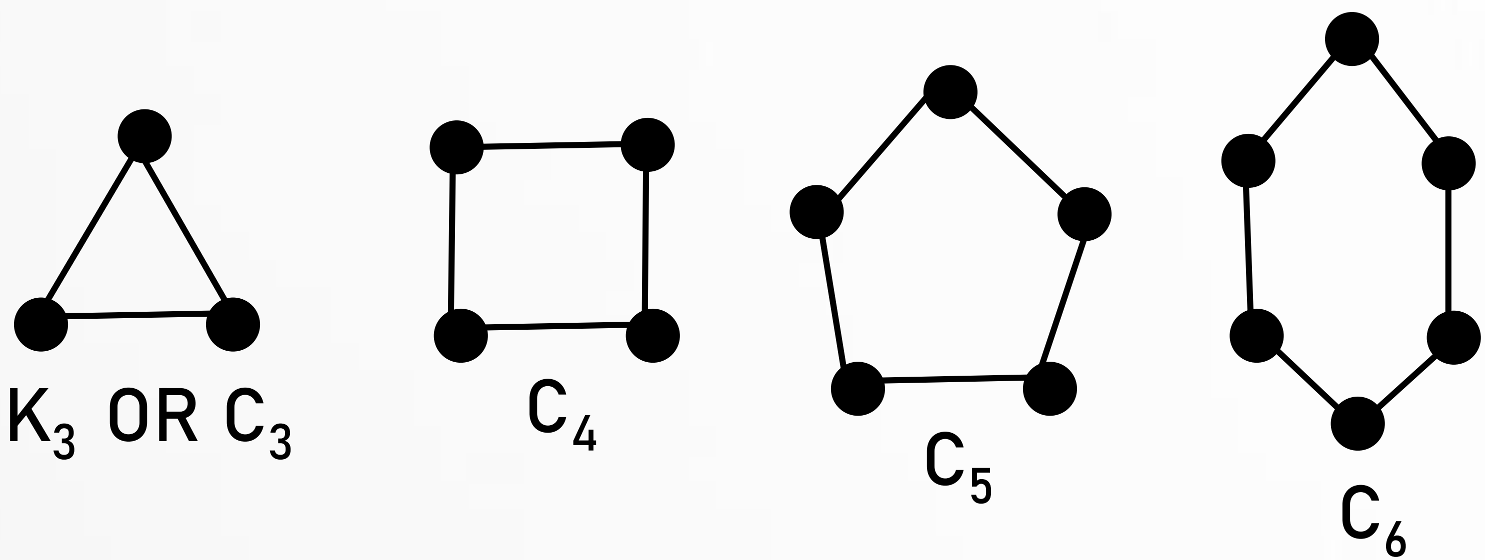
Special Simple Graphs – Cycles

A cycle C_n, $n \geq 3$, consists of n vertices $v_1, v_2, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots, \{v_{n-1}, v_n\}$, and $\{v_n, v_1\}$. The cycles C_3, C_4, C_5, and C_6 are displayed in Figure below.



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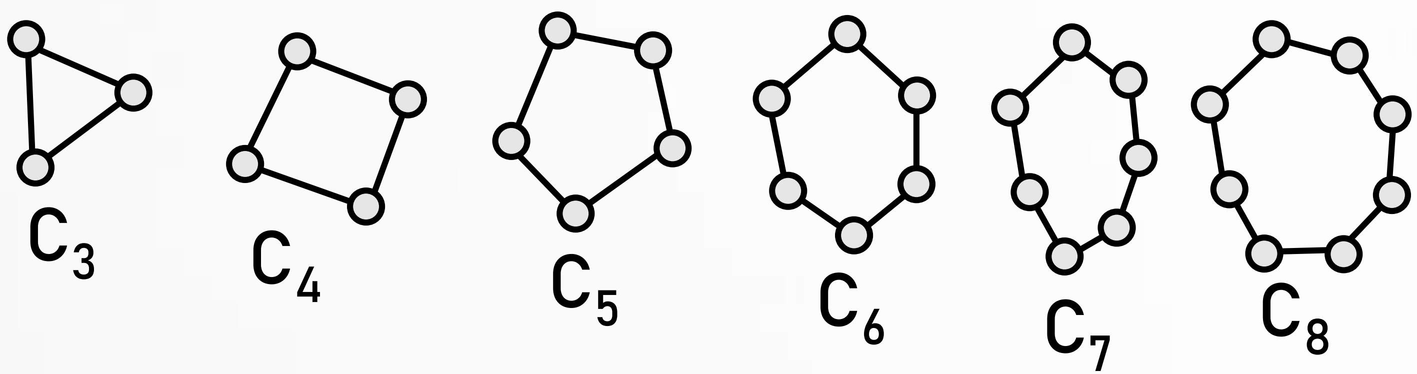
Special Simple Graphs - Cycles

Ques:- How many edges are there in C_n?

Ans:- n



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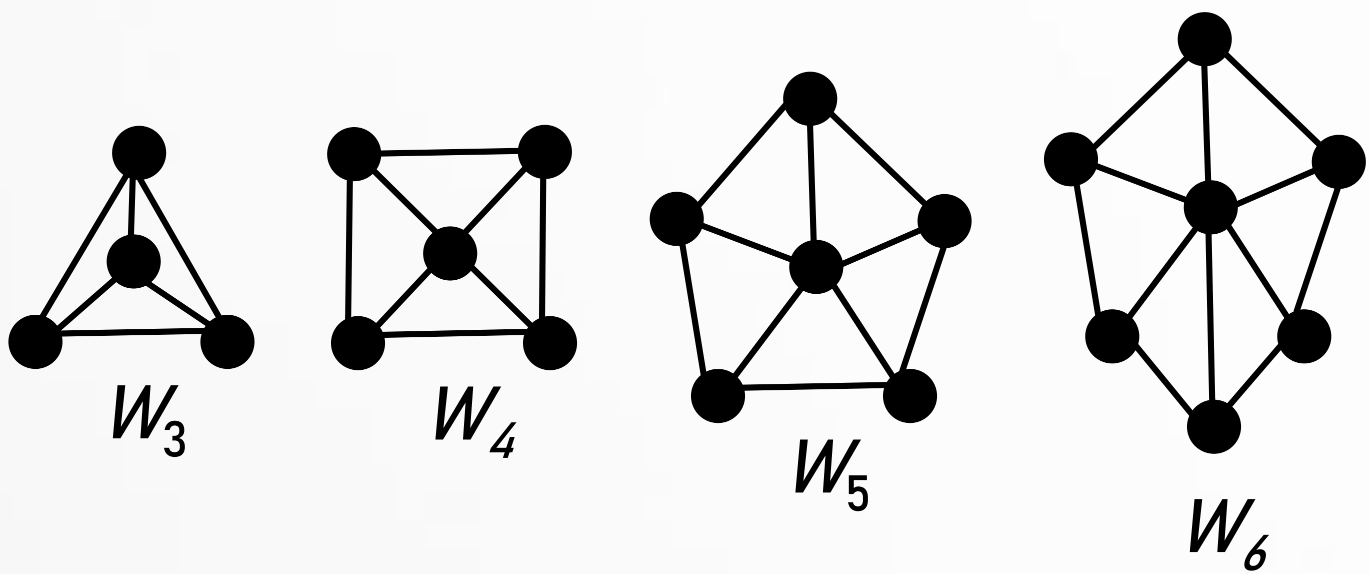
Special Simple Graphs – Wheels

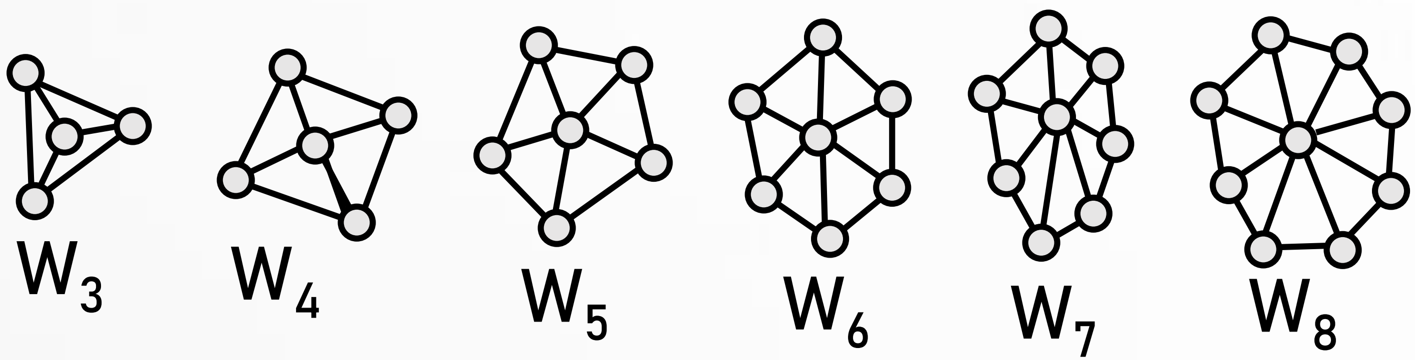
We obtain a wheel W_n when we add an additional vertex to a cycle C_n, for $n \geq 3$, and connect this new vertex to each of the n vertices in C_n, by new edges. The wheels W_3, W_4, W_5, and W_6 are displayed in Figure below.



Special Simple Graphs - Wheels

Ques:- How many edges are there in W_n?

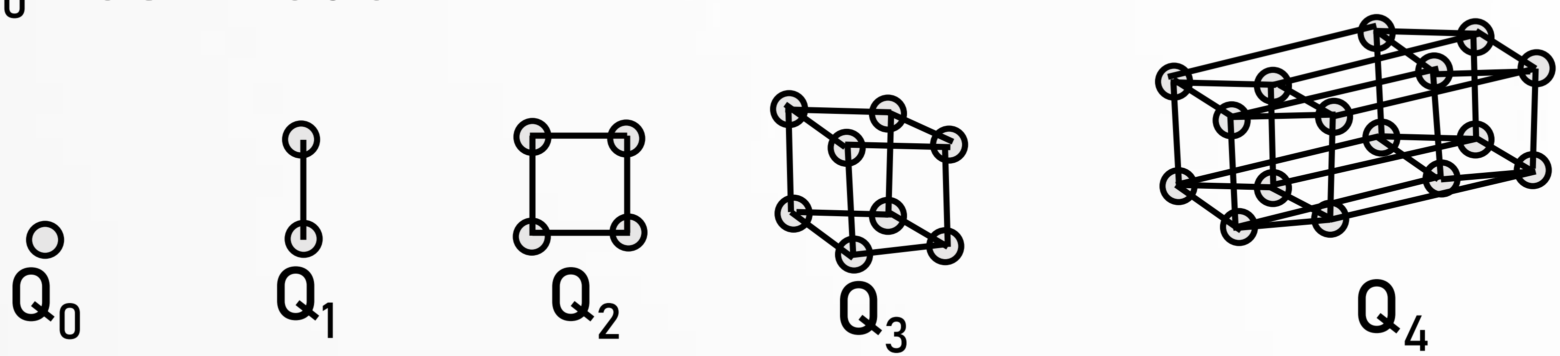
Ans:- 2n



Special Simple Graphs – *n*-cubes (hypercubes)

For any $n \in \mathbf{N}$, the hypercube Q_n is a simple graph consisting of two copies of Q_{n-1} connected together at corresponding nodes.

Q_0 has 1 node.



Number of vertices: 2^n.
Number of edges: 2n

That's all for now...