EMTH403

Mathematical Foundation for Computer Science

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Lecture Outcomes

After this lecture, you will be able to

- understand what are Predicates.
- understand what are the statement function, variables
- understand binary, ternary and n – ary predicate .

Predicates

- In this section we will introduce a more powerful type of logic called **predicate logic.**
- We will see how predicate logic can be used to express the meaning of a wide range of statements in **mathematics and computer science** in ways that permit us to **reason and explore relationships between objects.**

Predicates

- To understand predicate logic, we first need to introduce the concept of a predicate.
- **Afterward, we will introduce the notion of quantifiers**, which enable us to reason with statements that assert that a certain property holds for all objects of a certain type and with statements that assert the existence of an object with a particular property.

Predicates

The statement “x is greater than 3” has two parts. The **first part**, the variable x, is the subject of the statement.

The **second part**—the predicate, “is greater than 3”—refers to a property that the subject of the statement can have.

Predicates

Statements involving variables, such as

"$x > 3$," "$x = y + 3$," "$x + y = z$,"

and

"computer x is under attack by an intruder,"

and

"computer x is functioning properly,"

Predicates

We can denote the statement “x is greater than 3” by P(x), where P denotes the predicate “is greater than 3” and x is the variable.

The statement P(x) is also said to be the value of the propositional function P at x.

Predicates

Once a value has been assigned to the variable x, the statement P(x) becomes a proposition and has a truth value.

Propositional function – 1 Variable Example – 1

Ques:- Let P(x) denote the statement "x > 3." What is the truth values of P(4)?

Ans:- We obtain the statement P(4) by setting x = 4 in the statement "x > 3."

Hence, P(4), which is the statement "4 > 3," is true.

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Propositional function – 1 Variable Example - 2

Ques:- Let P(x) denote the statement “x > 3.” What is the truth values of P(2)?

Ans:- P(2), which is the statement “2 > 3,” is false.

Propositional function – 1 Variable Example - 3

Ques:- Let A(x) denote the statement "Computer x is under attack by an intruder." Suppose that of the computers on campus, only CS2 and MATH1 are currently under attack by intruders. What are truth values of A(CS1)?

Ans:- We obtain the statement A(CS1) by setting x = CS1 in the statement "Computer x is under attack by an intruder." Because CS1 is not on the list of computers currently under attack, we conclude that A(CS1) is **false**.

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Propositional function – 1 Variable Example - 4

Ques:- Let A(x) denote the statement “Computer x is under attack by an intruder.” Suppose that of the computers on campus, only CS2 and MATH1 are currently under attack by intruders. What are truth values of A(CS2), and A(MATH1)?

Ans:- Because CS2 and MATH1 are on the list of computers under attack, we know that A(CS2) and A(MATH1) are **true**.

Propositional function – 1 Variable Example - 5

Ques:-Let P(x) denote the statement "$x \leq 4$." What is the truth value of P(0) ?

Ans:-T, since $0 \leq 4$

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Propositional function – 1 Variable Example - 6

Ques:- Let P(x) denote the statement "$x \leq 4$." What is the truth value of P(4)?

Ans:- T, since $4 \leq 4$.

Propositional function – 1 Variable Example - 7

Ques:- Let $P(x)$ denote the statement "$x \leq 4$."

What is the truth value of $P(6)$?

Ans:- F, since $6 \nleq 4$

Propositional function – 1 Variable Example - 8

Ques:- Let P(x) be the statement "$x = x^2$." If the domain consists of the integers, What is the truth value of P(0)?

Ans:- T, since $0 = 0^2$

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Propositional function – 1 Variable Example - 9

Ques:- Let P(x) be the statement "$x = x^2$." If the domain consists of the integers, What is the truth value of P(1)?

Ans:- T, since $1 = 1^2$

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Propositional function – 1 Variable Example - 10

Ques:- Let P(x) be the statement "$x = x^2$." If the domain consists of the integers, What is the truth value of P(2)?

Ans:- F, since $2 \neq 2^2$

Propositional function – 1 Variable Example - 11

Ques:- Let P(x) be the statement "$x = x^2$." If the domain consists of the integers, What is the truth value of P(−1)?

Ans:- F, since $-1 \neq (-1)^2$

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Propositional function – 2 Variables Example - 1

Ques:- Let Q(x, y) denote the statement “x = y + 3.” What are the truth values of the propositions Q(1, 2)?

Ans:- To obtain Q(1, 2), set x = 1 and y = 2 in the statement Q(x, y). Hence, Q(1, 2) is the statement “1 = 2 + 3,” which is **false**.

Propositional function – 2 Variables
Example - 2

Ques:- Let $Q(x, y)$ denote the statement “$x = y + 3$.” What are the truth values of the propositions $Q(3, 0)$?

Ans:- The statement $Q(3, 0)$ is the proposition “$3 = 0 + 3$,” which is **true.**

Propositional function – 2 Variables Example - 3

Ques:- Let A(c, n) denote the statement “Computer c is connected to network n,” where c is a variable representing a computer and n is a variable representing a network. Suppose that the computer MATH1 is connected to network CAMPUS2, but not to network CAMPUS1. What is the values of A(MATH1, CAMPUS1)?

Ans:- Because MATH1 is not connected to the CAMPUS1 network, we see that A(MATH1, CAMPUS1) is **false.**

Propositional function – 2 Variables Example - 4

Ques:- Let A(c, n) denote the statement "Computer c is connected to network n," where c is a variable representing a computer and n is a variable representing a network. Suppose that the computer MATH1 is connected to network CAMPUS2, but not to network CAMPUS1. What is the values of A(MATH1, CAMPUS2)?

Ans:- However, because MATH1 is connected to the CAMPUS2 network, we see that A(MATH1, CAMPUS2) is **true**.

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Propositional function – 2 Variables Example - 5

Ques:- Let Q(x, y) denote the statement “x is the capital of y.” What is the truth value of Q(Delhi, India)?

Ans:- This is true

Propositional function – 3 Variables Example - 1

Ques:- What is the truth values of the propositions R(1, 2, 3) where R(x, y, z) denote the statement “x + y = z.”?

Ans:- The proposition R(1, 2, 3) is obtained by setting x = 1, y = 2, and z = 3 in the statement R(x, y, z). We see that R(1, 2, 3) is the statement “1 + 2 = 3,” which is **true**.

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Propositional function – 3 Variables Example - 2

Ques:- What is the truth values of the propositions R(0, 0, 1) where R(x, y, z) denote the statement “x + y = z.”?

Ans:- That R(0, 0, 1), is the statement “0 + 0 = 1,” is false.

Propositional function – n Variables Example

A statement of the form $P(x_1, x_2, \ldots, x_n)$ is the value of the **propositional function** P at the n-tuple $(x_1, x_2, \ldots, x_n)$, and P is also called an n-place predicate or a **n- ary predicate.**

That's all for now...