

A hand is shown placing a blue L-shaped block on top of a colorful cube. The cube is constructed from various colored blocks (yellow, orange, pink, purple, blue, green, grey) and is sitting on a white wooden surface. Several other L-shaped blocks in different colors (green, blue, red, yellow) are scattered on the surface to the right. The background is a solid light blue.

# EMTH403

Mathematical Foundation  
for Computer Science

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# Lecture Outcomes



After this lecture, you will be able to

- understand the use of product rule in the basics of counting.
- understand the use of sum rule in the basics of counting.
- understand the use of subtraction rule in the basics of counting.

# Basics of Counting – Example 1

**Ques:-** How many different three-letter initials can people have?

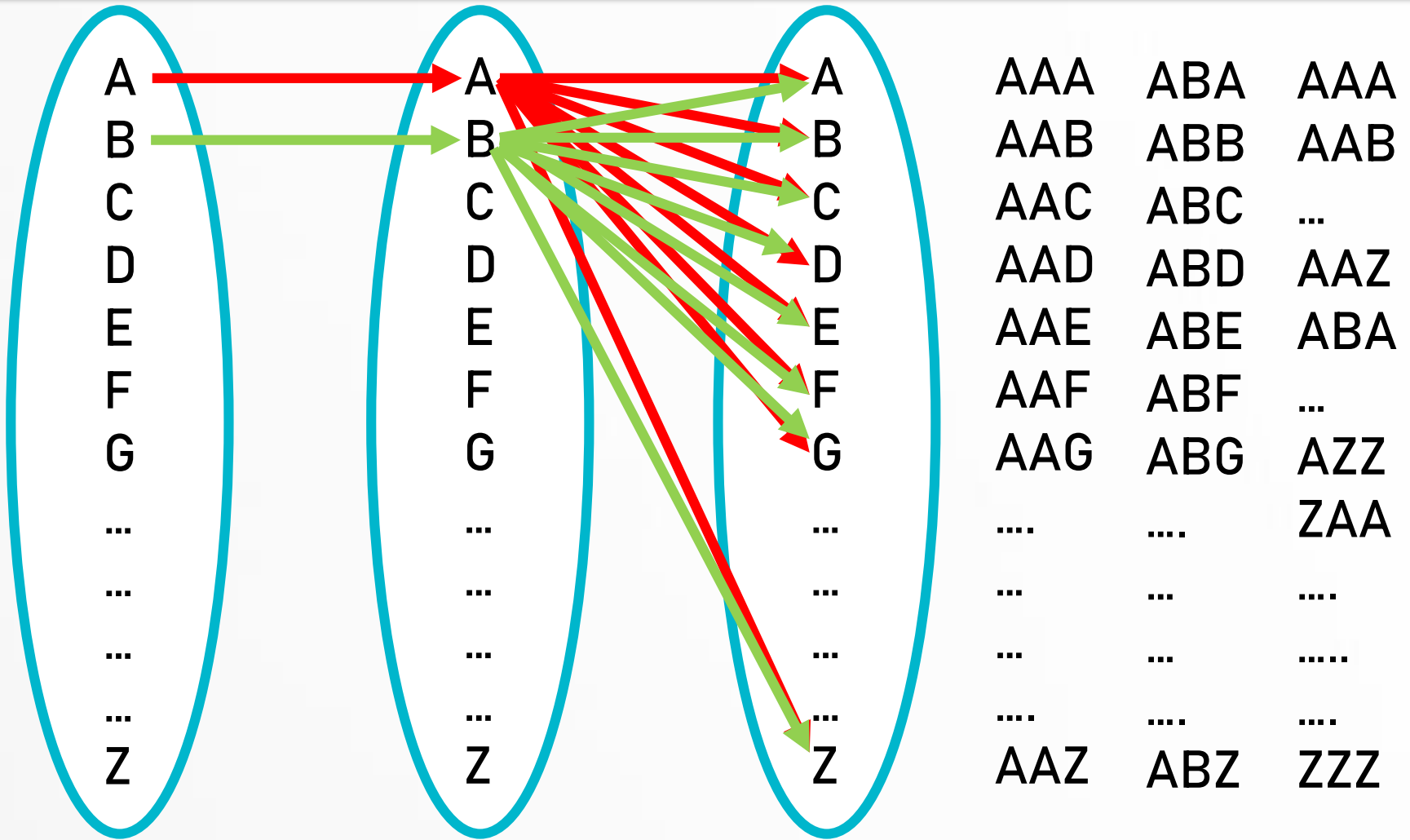
**Sol:-** first initial ( 26 ways),  
then the second initial ( 26 ways),  
and then the third initial ( 26 ways).

Therefore, by the product rule there are

$$26 \cdot 26 \cdot 26 = 26^3 = 17,576$$

possible three-letter initials.

# Basics of Counting – Example 1



$$26 \times 26 \times 26 = 17,576$$

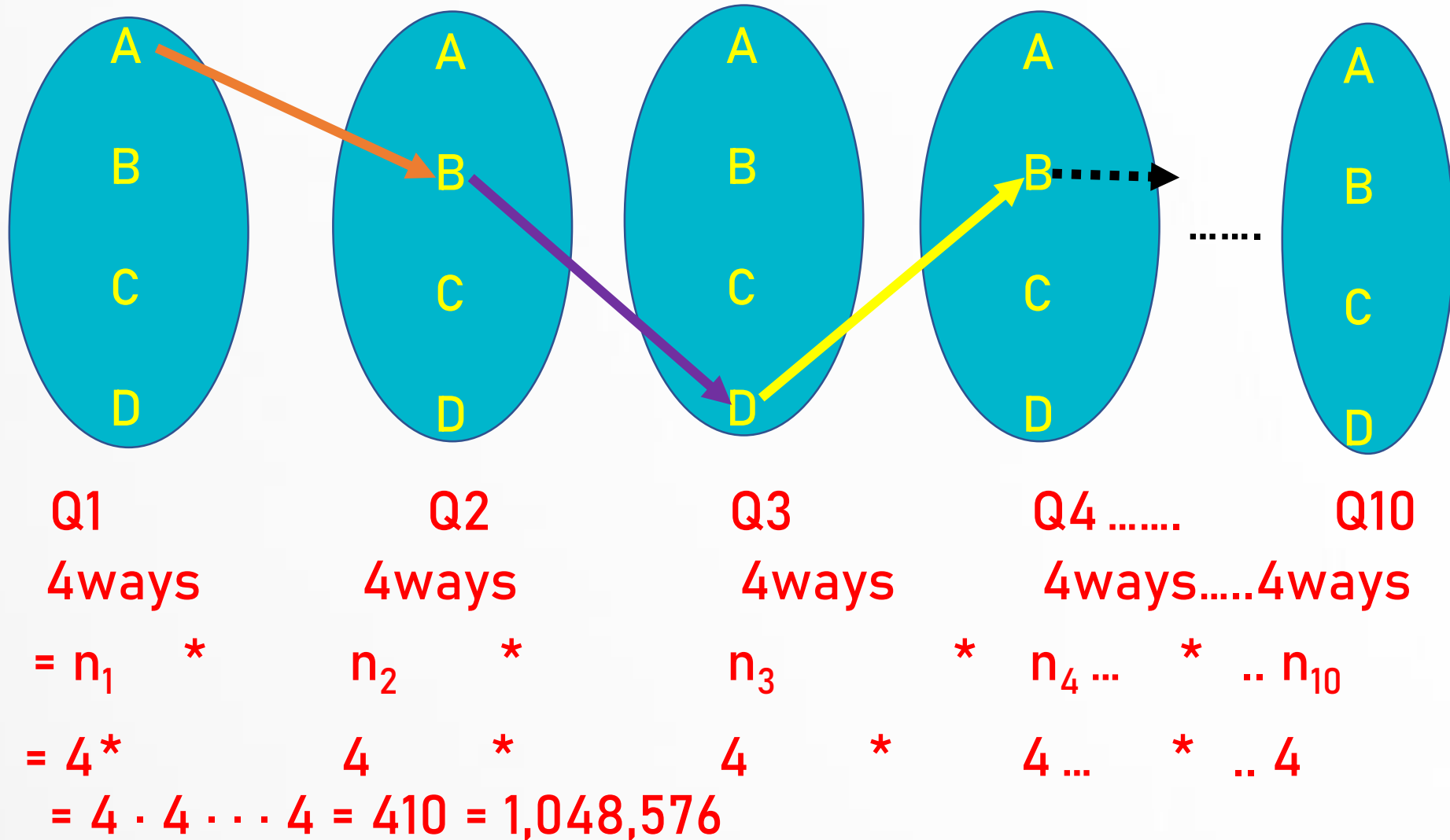
# Basics of Counting – Example 2

**Ques:-** A multiple-choice test contains 10 questions. There are four possible answers for each question. In how many ways can a student answer the questions on the test if the student answers every question?

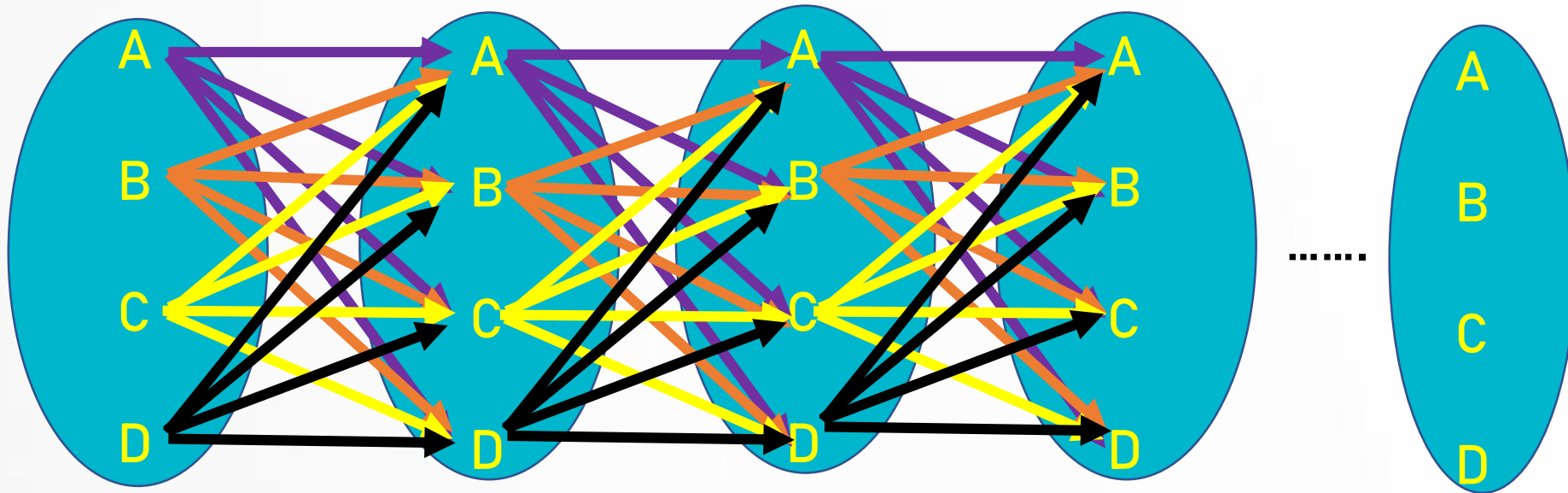
**Sol:-**  $4 \cdot 4 \cdot \dots \cdot 4 = 4^{10} = 1,048,576$

Possible ways are a, b, c, b, b, d, a, b, d, b etc.

# Basics of Counting – Example 2



# Basics of Counting – Example 2



Q1

4ways

$$= n_1 *$$

$$= 4 *$$

Q2

4ways

$$n_2 *$$

$$4 *$$

Q3

4ways

$$n_3 *$$

$$4 *$$

Q4 .....

4ways....4ways

$$n_4 ... * .. n_{10}$$

$$4 ... * .. 4$$

Q10

$$= 4 \cdot 4 \cdot \dots \cdot 4 = 4^{10} = 1,048,576$$

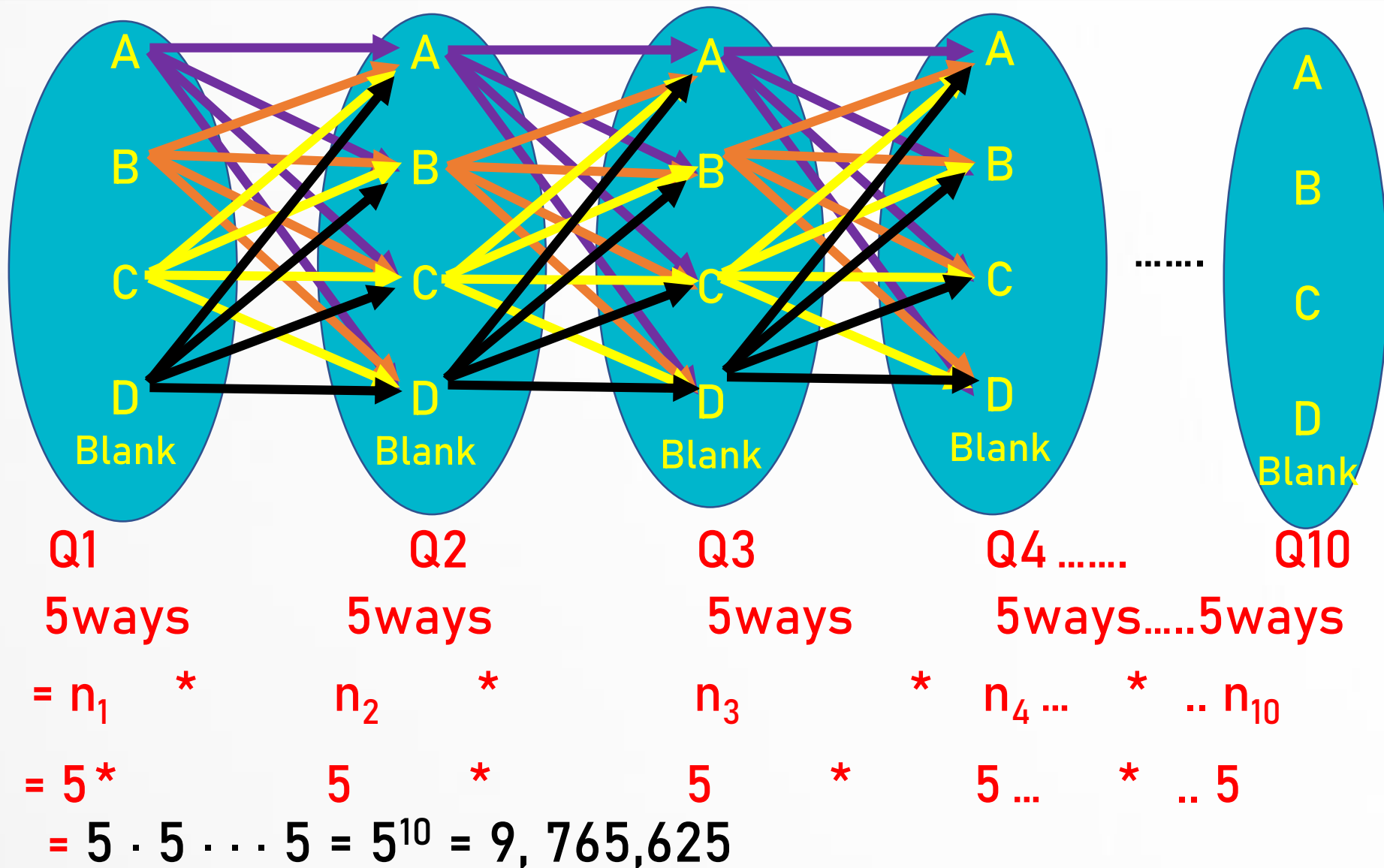
# Basics of Counting – Example 3

**Ques:-** A multiple-choice test contains 10 questions. There are four possible answers for each question. In how many ways can a student answer the questions on the test if the student can leave answers blank?

**Sol:-**  $5 \cdot 5 \cdot \dots \cdot 5 = 5^{10} = 9,765,625$

Possible ways are a, b, c, b, blank, d, a, b, d, b etc.

# Basics of Counting – Example 2



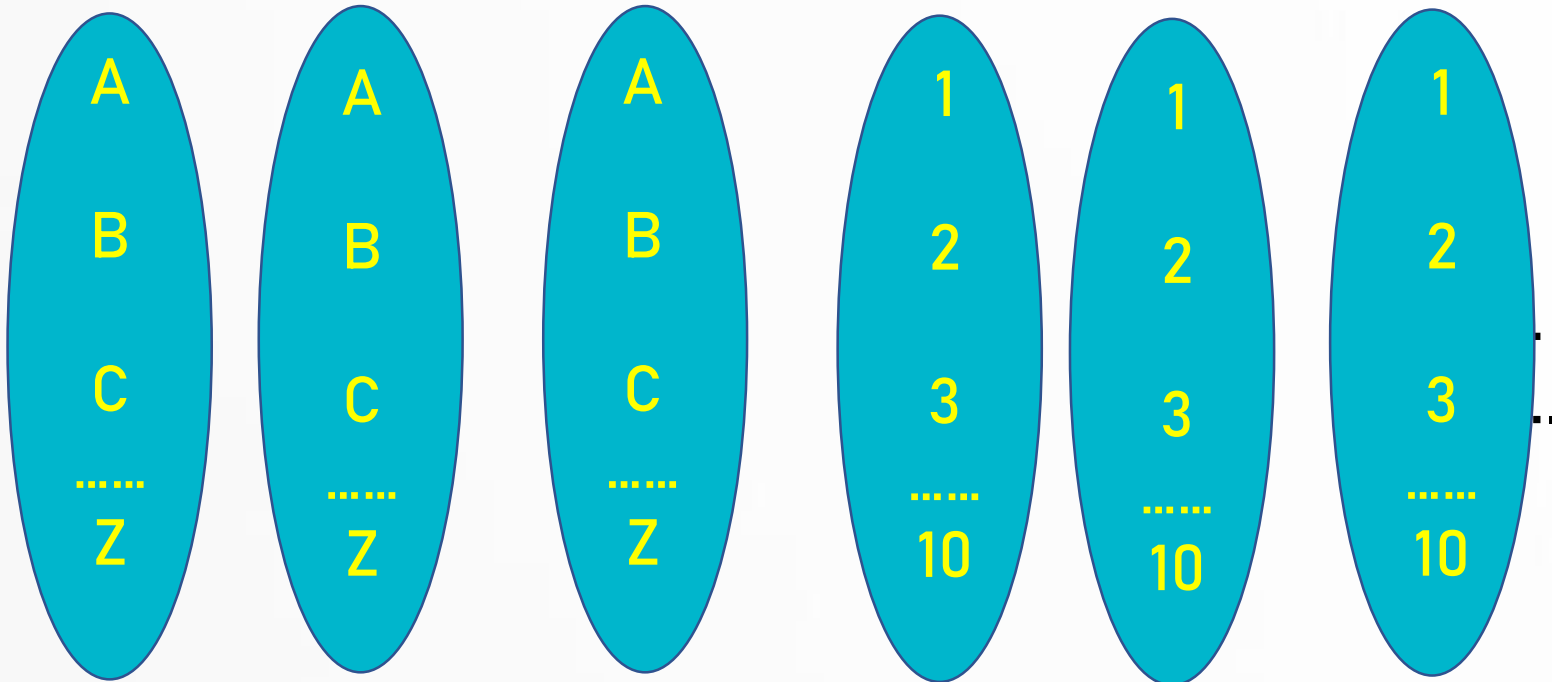
# Basics of Counting – Example 2

**Ques:-** How many different license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits ?

**Sol:-** There are 26 choices for each of the three uppercase English letters and ten choices for each of the three digits. Hence, by the product rule there are a total of

$$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 17,576,000 \text{ possible license plates.}$$

# Basics of Counting – Example 2



26ways 26ways 26ways 10way 10ways 10ways

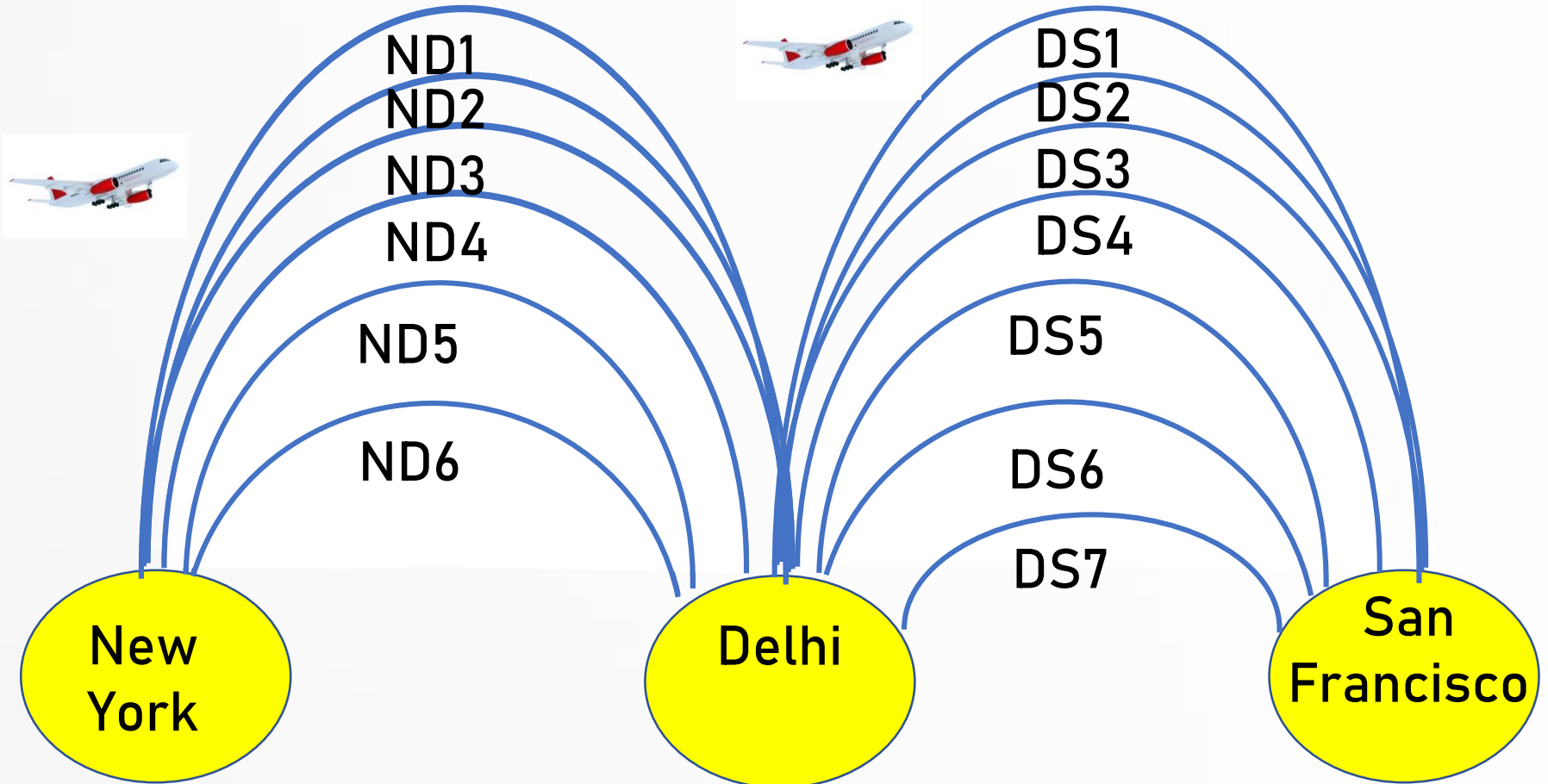
$$\begin{aligned} &= n_1 * n_2 * n_3 * n_4 * n_5 * n_6 \\ &= 26 * 26 * 26 * 10 * 10 * 10 \\ &= 17,576,000 \end{aligned}$$

# Basics of Counting – Example

**Ques:-** Six different airlines fly from New York to Delhi and seven fly from Delhi to San Francisco.

How many different pairs of airlines can you choose on which to book a trip from New York to San Francisco via Delhi, when you pick an airline for the flight to Delhi and an airline for the continuation flight to San Francisco?

# Basics of Counting – Example



$$= n_1 * n_2$$

$$= 6 * 7$$

$$= 42$$

# The Sum Rule

If a task can be done either in one of  $n_1$  ways or in one of  $n_2$  ways, Where none of the set of  $n_1$  ways is the same as any of the set of  $n_2$  ways, then there are  $n_1 + n_2$  ways to do the task.

# The Sum Rule

**Ques:-** There are 18 mathematics majors and 325 computer science majors at a college. In how many ways can one representative be picked who is either a mathematics major or a computer science major?

**Sol:-** 343

# The Sum Rule

**Ques:-** A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 19 possible projects, respectively. No project is on more than one list. How many possible projects are there to choose from?

**Sol:-** The student can choose a project by selecting a project from the first list, the second list, or the third list.

Because no project is on more than one list, by the sum rule there are  $23 + 15 + 19 = 57$  ways to choose a project.

# The Sum Rule

## List 1

Project 1  
Project 2  
Project 3  
Project 4  
Project 5  
Project 6  
Project 7  
Project 8  
Project 9  
Project 10  
Project 11  
Project 12  
Project 13  
Project 14  
Project 15  
Project 16  
Project 17  
Project 18  
Project 19  
Project 20  
Project 21  
Project 22  
Project 23

## List 2

Project 1  
Project 2  
Project 3  
Project 4  
Project 5  
Project 6  
Project 7  
Project 8  
Project 9  
Project 10  
Project 11  
Project 12  
Project 13  
Project 14  
Project 15

## List 3

Project 1  
Project 2  
Project 3  
Project 4  
Project 5  
Project 6  
Project 7  
Project 8  
Project 9  
Project 10  
Project 11  
Project 12  
Project 13  
Project 14  
Project 15  
Project 16  
Project 17  
Project 18  
Project 19

$$= n_1 + n_2 + n_3$$

$$= 23 + 15 + 19 = 57 \text{ ways}$$

# The Subtraction Rule

If a task can be done in either  $n_1$  ways or  $n_2$  ways, then the number of ways to do the task is  $n_1 + n_2$  minus the number of ways to do the task that are common to the two different ways

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|.$$

# The Subtraction Rule

**Ques:-** How many bit strings of length eight either start with a 1 bit or end with the two bits 00?

**Sol:-** bit string of length eight that begins with a 1+ a bit string of length eight ending with the two bits 00 - bit string of length eight starting with a 1 and that ends with the two bits 00

$$= 2^7 + 2^6 - 2^5 = 128 - 64 = 64 \text{ ways.}$$

= the number of ways to construct a bit string of length eight that begins with a 1 or that ends with 00, equals  $128 + 64 - 64 = 128$ .

That's all for now...