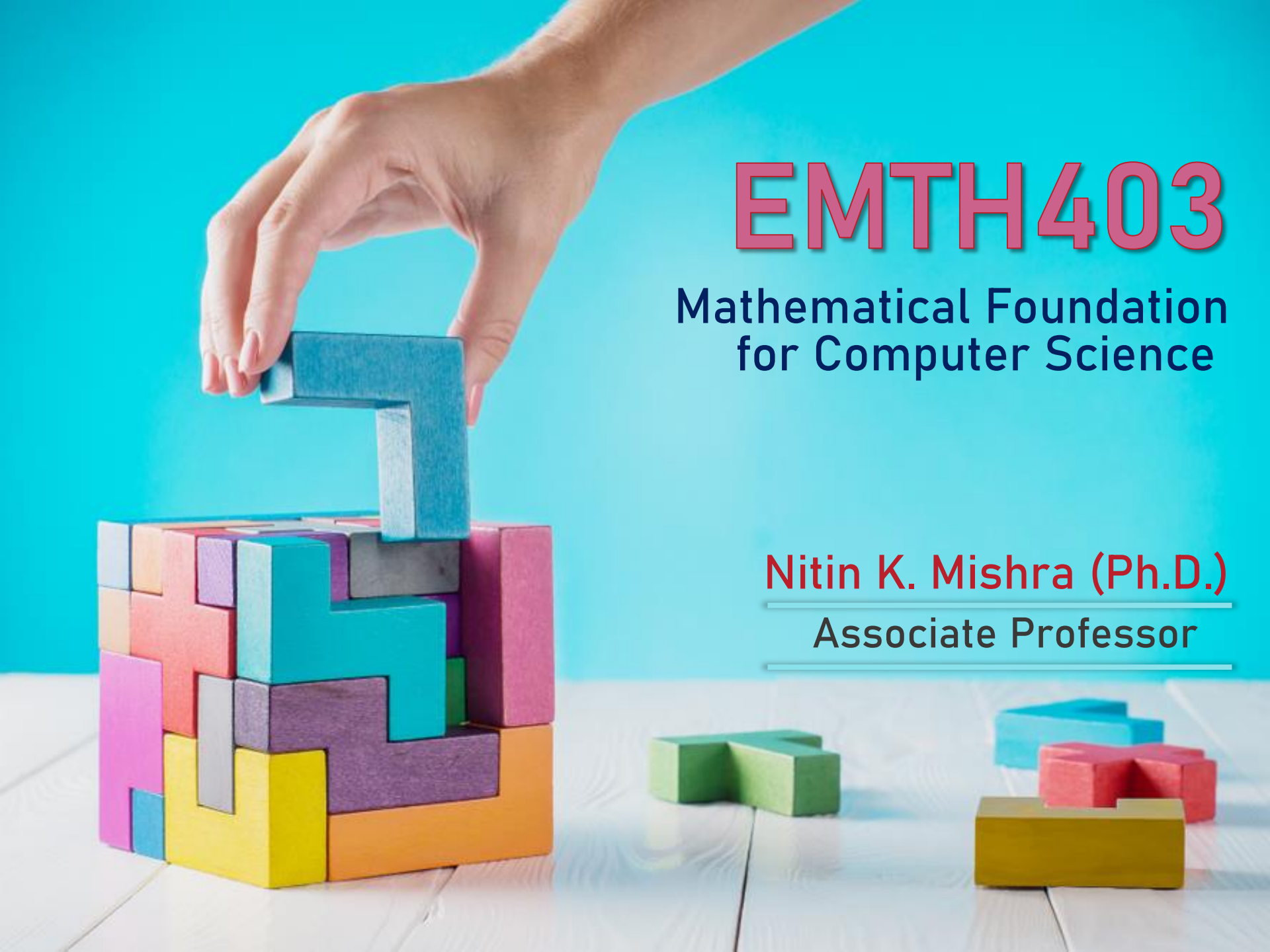

# EMTH403

## Mathematical Foundation for Computer Science

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# Lecture Outcomes

After this lecture, you will be able to

- understand what are Hamilton path and graphs in a directed and undirected graph.
- understand what are Hamilton circuit and graphs in a directed and undirected graph.

# Hamilton Paths and Circuits

A Hamilton path in a graph G is a path which visits every vertex in G exactly once.

A Hamilton circuit is a Hamilton path that returns to its start.

# Travelling Salesman Problem

A Hamilton circuit or path may be used to solve practical problems that require visiting “vertices”, such as:

road intersections

pipeline crossings

communication network nodes

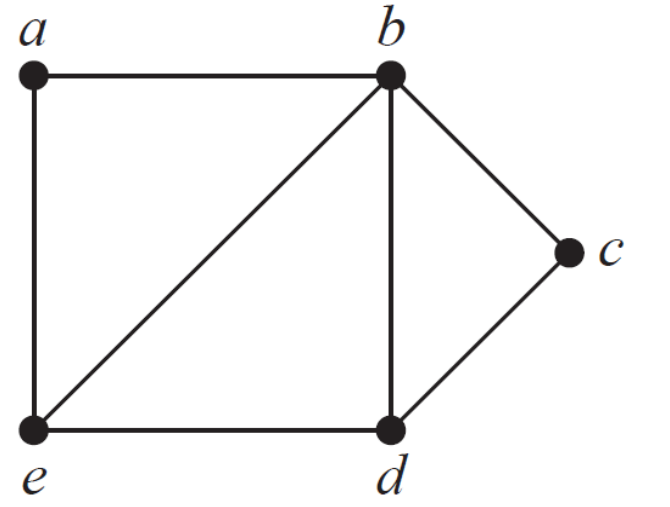
A classic example is the Travelling Salesman Problem – finding a Hamilton circuit.

# Finding Hamilton Circuits

Unlike the Euler circuit problem, finding Hamilton circuits is hard.

There is no simple set of necessary and sufficient conditions, and no simple algorithm.

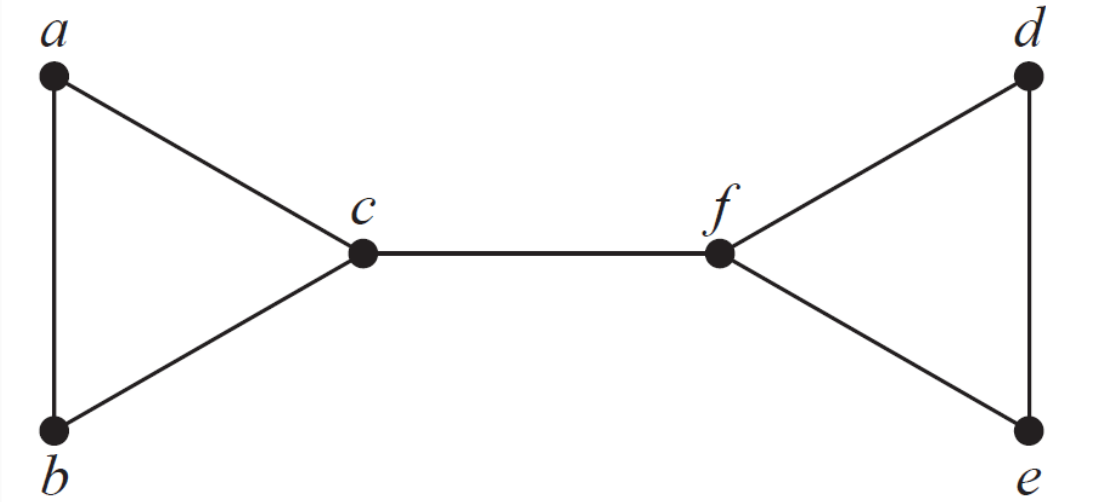
# Hamilton circuit?




Ques:- Does the graph above has an Hamilton Circuit?

Sol:- a, b, c, d, e, a is a Hamilton circuit.

# Hamilton Path?

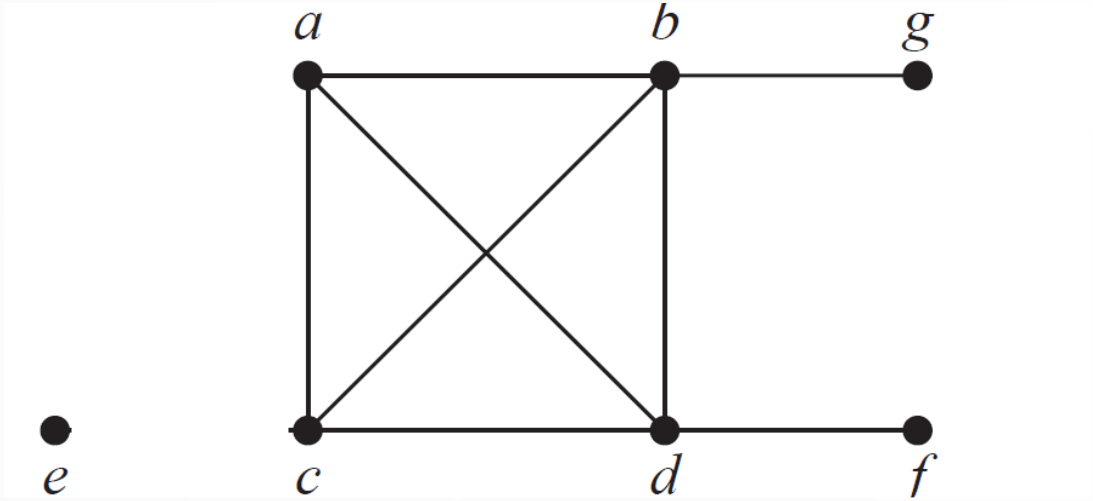



Ques:- Does the graph above has an Hamilton Path?

Sol:- This graph has the Hamilton path a, b, c, f, d, e.

This simple path hits each vertex once.

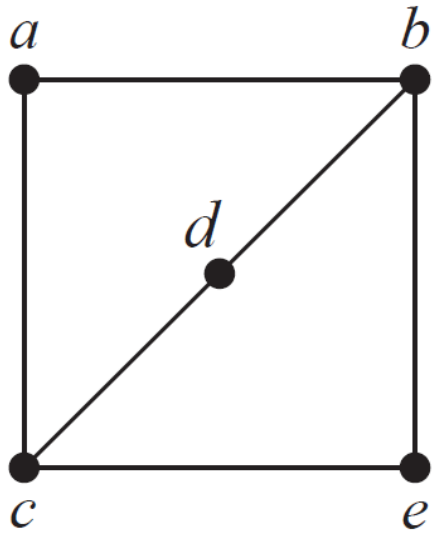
# Hamilton circuit?




Ques:- Does the graph above has an Hamilton Circuit?

Sol:- There is no Hamilton circuit because of the cut edges ({ c, e}, for instance). Once a purported circuit had reached vertex e, there would be nowhere for it to go.

# Hamilton Circuit

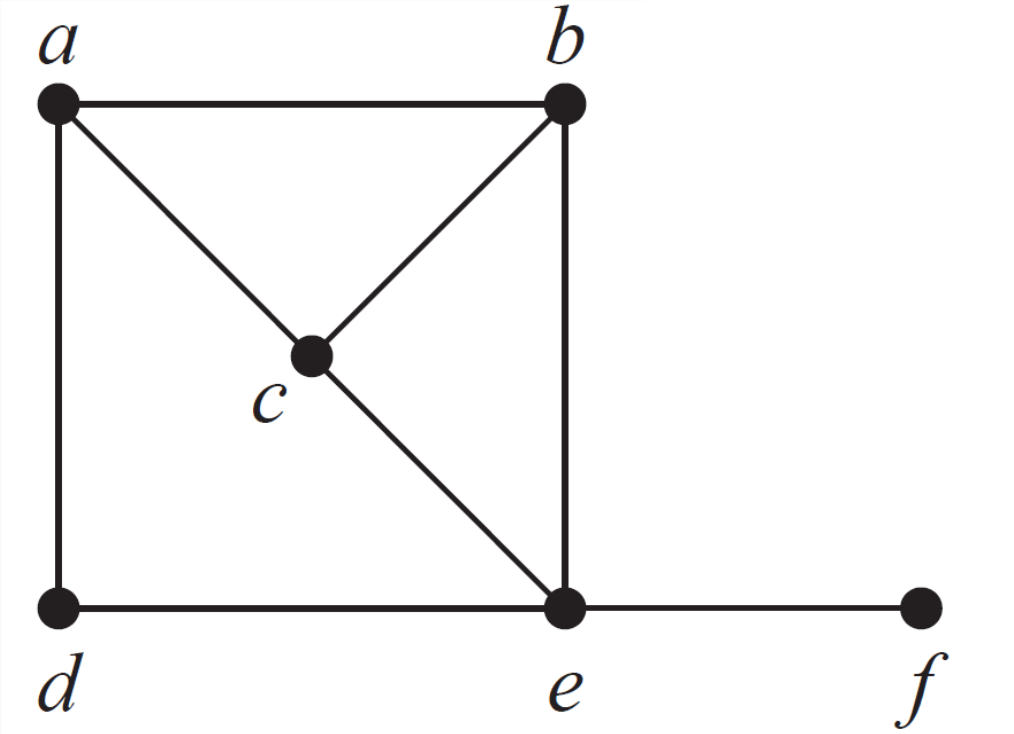



Ques:- Does the graph above has an Hamilton Circuit?

Sol:- There is no Hamiltonian circuit in this graph.

If there were one, then it would have to include all the edges of the graph, because it would have to enter and exit vertex a, enter and exit vertex d, and enter and exit vertex e. But then vertex c would have been visited more than once, a contradiction.

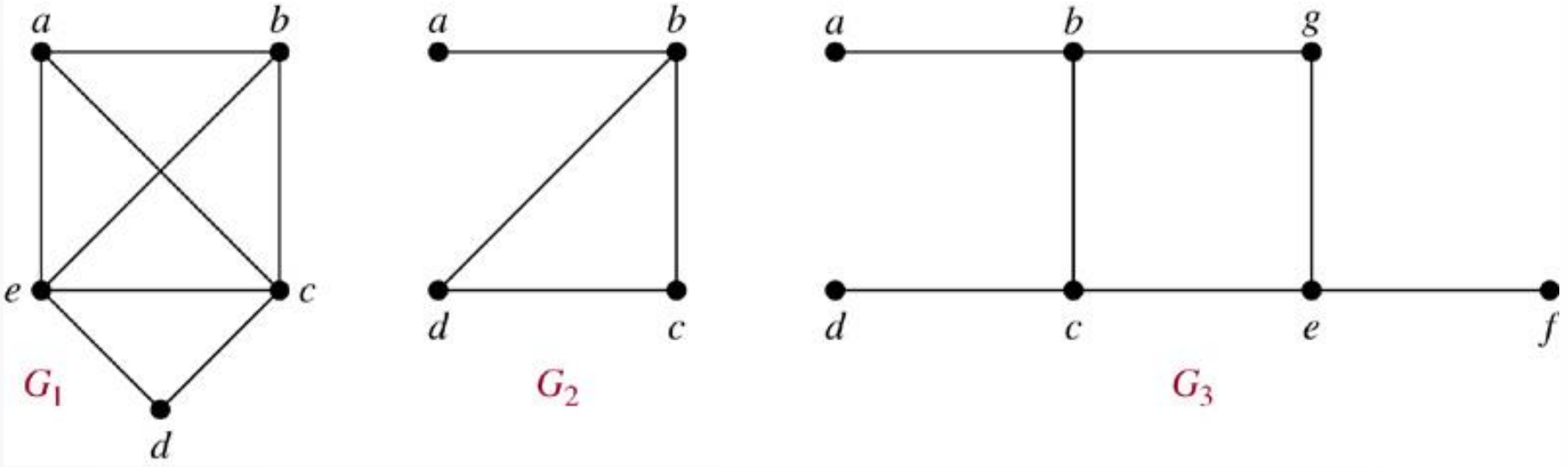
# Hamilton Path




Ques:- Does the graph above has an Hamilton Path?

Sol:- This graph has the Hamilton path f, e, d, a, b, c.

# Hamilton Circuit

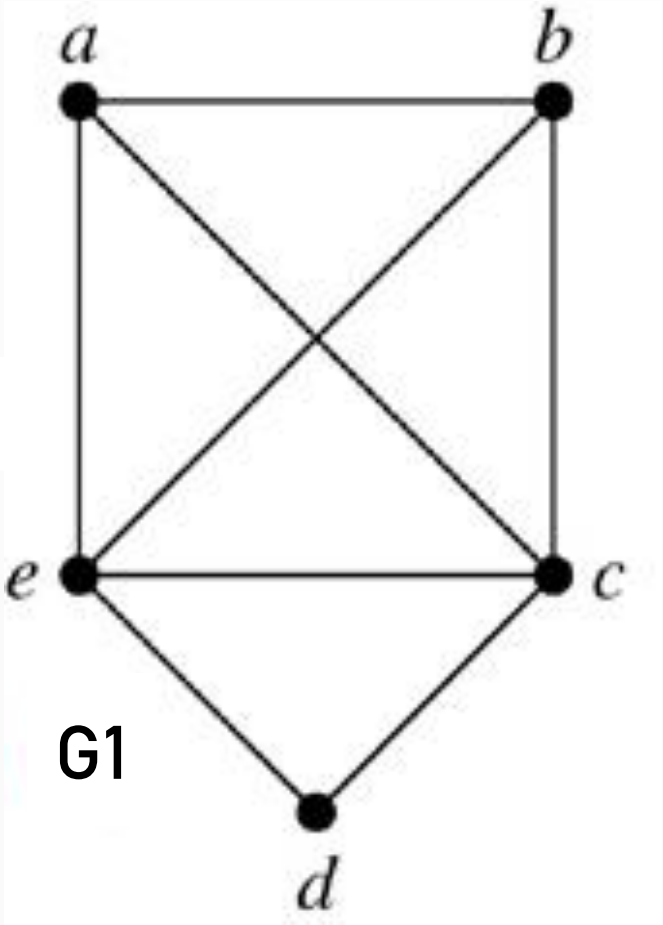



Ques:- Which of these three figures has a Hamilton circuit?

Or, if no Hamilton circuit, a Hamilton path?

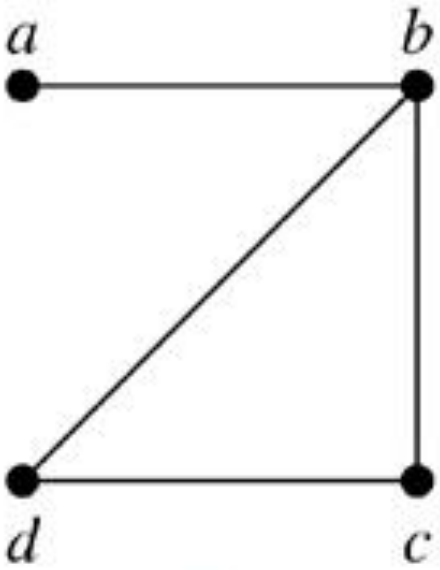
Sol:- Lets do it one by one.

# Hamilton Circuit




Sol:- $G_1$ has a Hamilton circuit: a, b, c, d, e, a

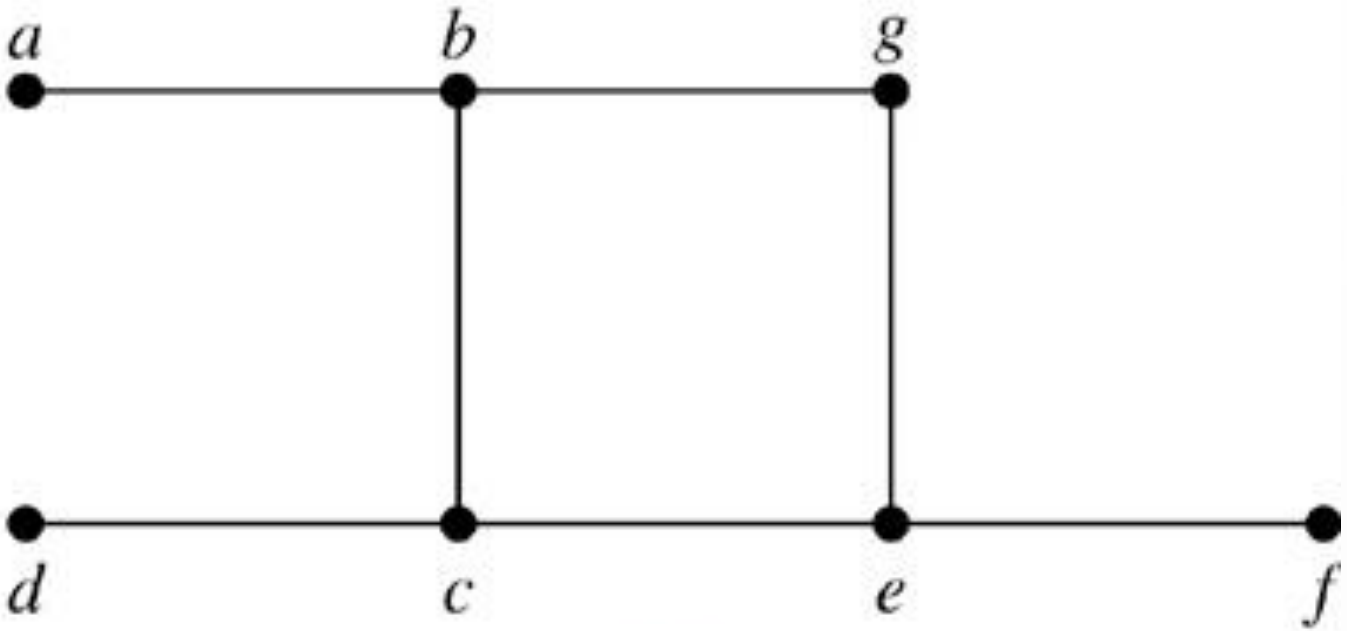
# Hamilton Circuit




G2

Sol:- $G_2$ does not have a Hamilton circuit, but does have a Hamilton path: a, b, c, d

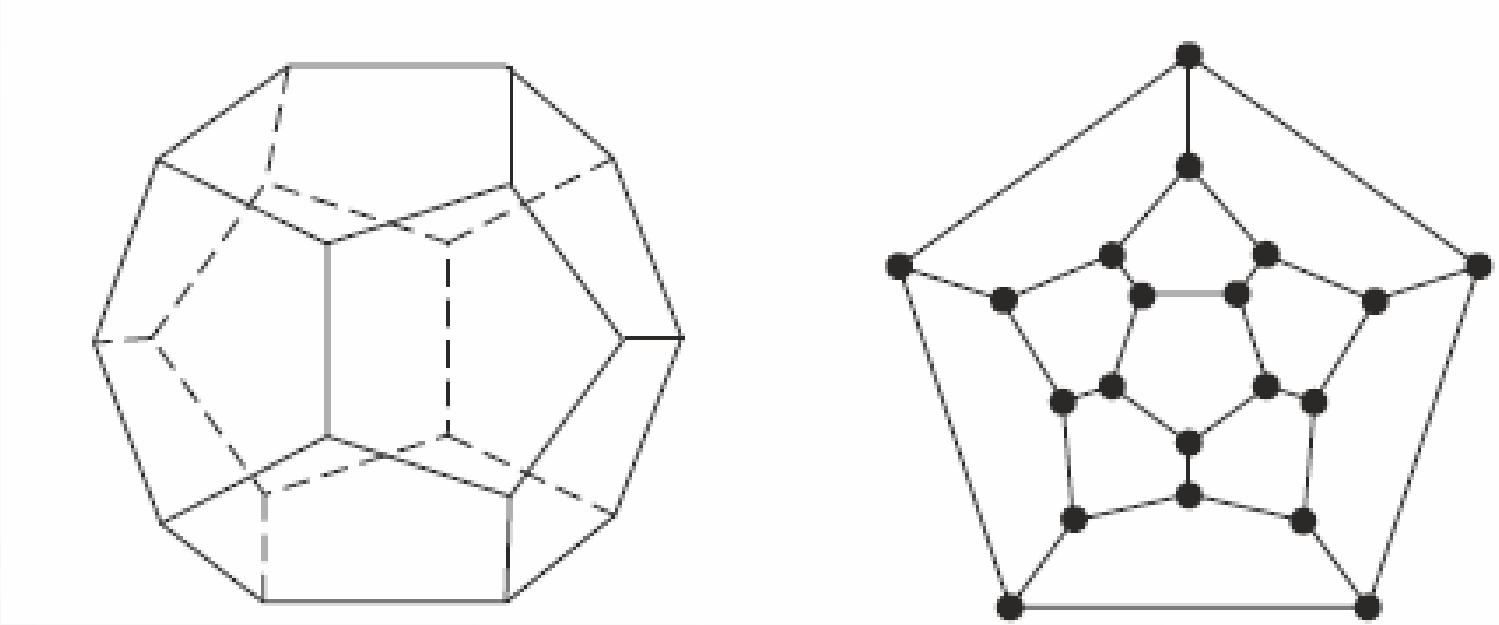
# Hamilton Circuit




G3

Sol:- $G_3$ has neither.

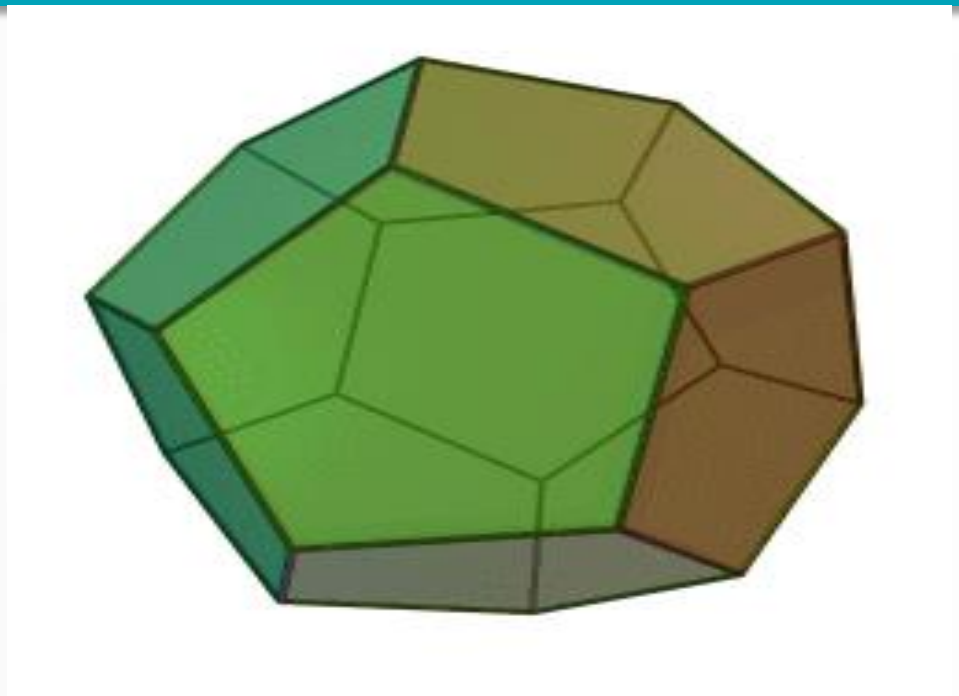
# Hamilton Circuit



Dodecahedron puzzle and it equivalent graph

**Ques:-** Is there a circuit in this graph that passes through each vertex exactly once?

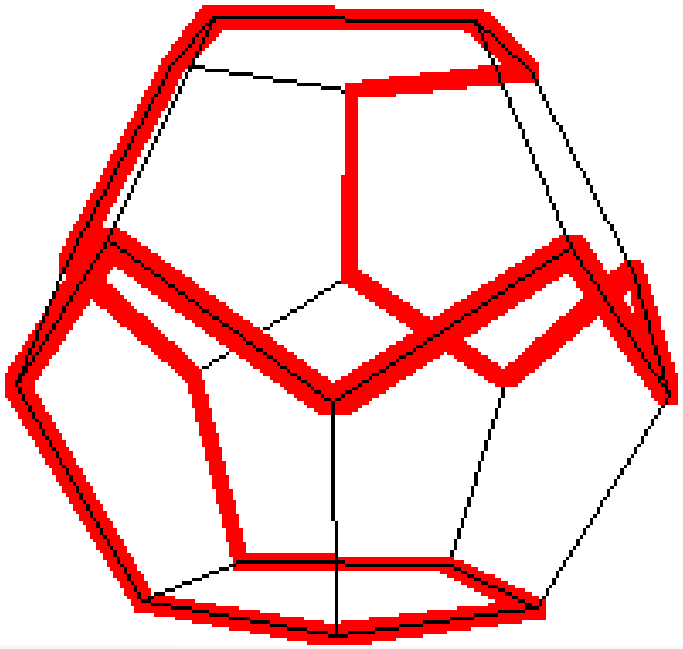
# Hamilton Circuit

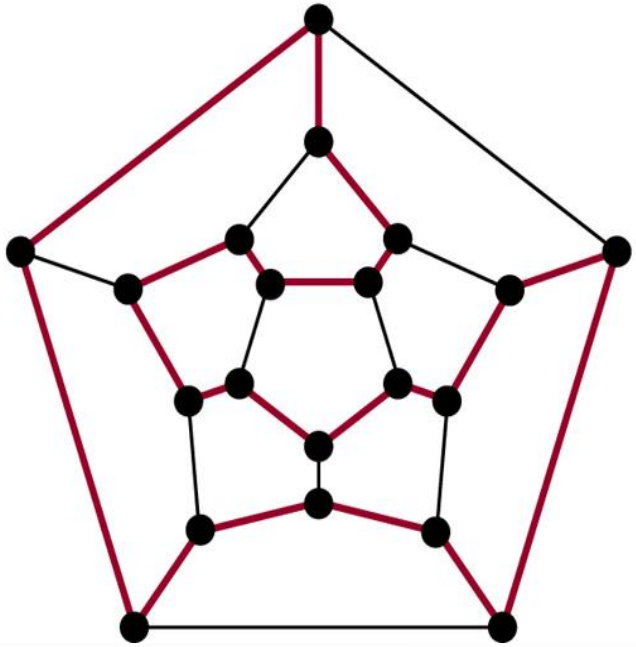


Dodecahedron is a polyhedron with twelve flat faces

**Ques:-** Is there a circuit in this graph that passes through each vertex exactly once?

# Hamilton Paths and Circuits





Yes; this is a circuit that passes through each vertex exactly once.

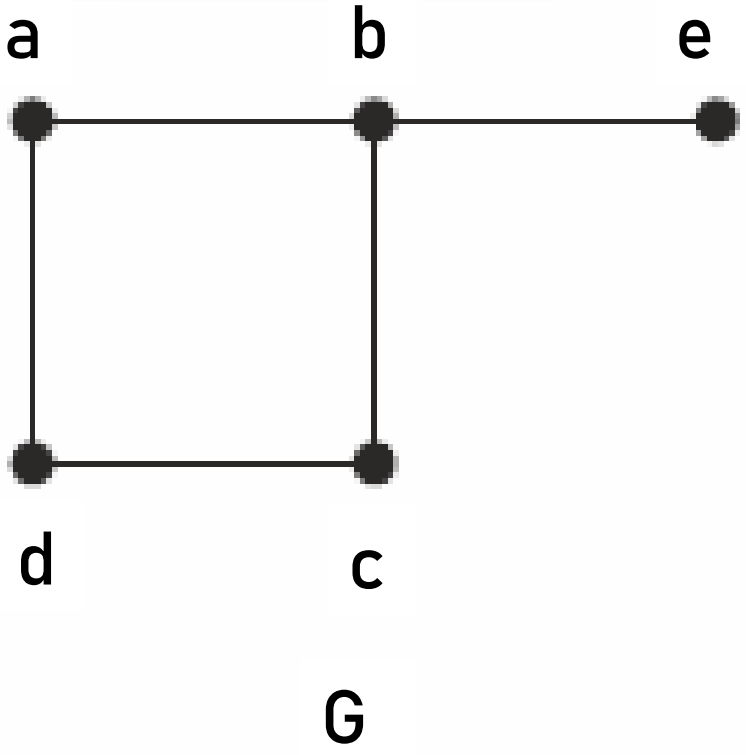
# Properties to look for …

No vertex of degree 1

If a node has degree 2, then both edges incident to it must be in any Hamilton circuit.

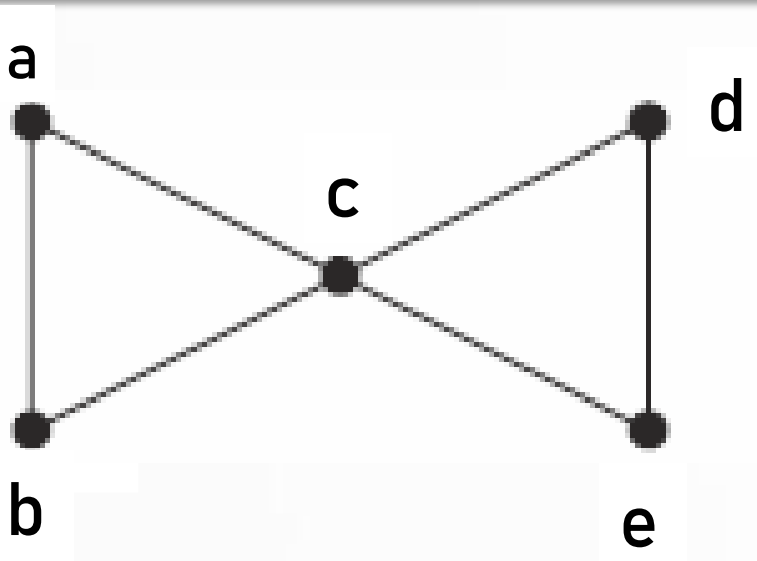
No smaller circuits contained in any Hamilton circuit (the start/endpoint of any smaller circuit would have to be visited twice).

# Hamilton Circuit




Sol:- There is no Hamilton circuit in G because G has a vertex of degree one: e.

# Hamilton circuit?




Sol:- Because the degrees of the vertices a, b, d, and e are all two, every edge incident with these vertices must be part of any Hamilton circuit. No Hamilton circuit can exist in H, for any Hamilton circuit would have to contain four edges incident with c, which is impossible.

# Summary

| Property | Euler | Hamilton |
|---|---|---|
| Repeated visits to a given node allowed? | Yes | No |
| Repeated traversals of a given edge allowed? | No | No |

# That's all for now…