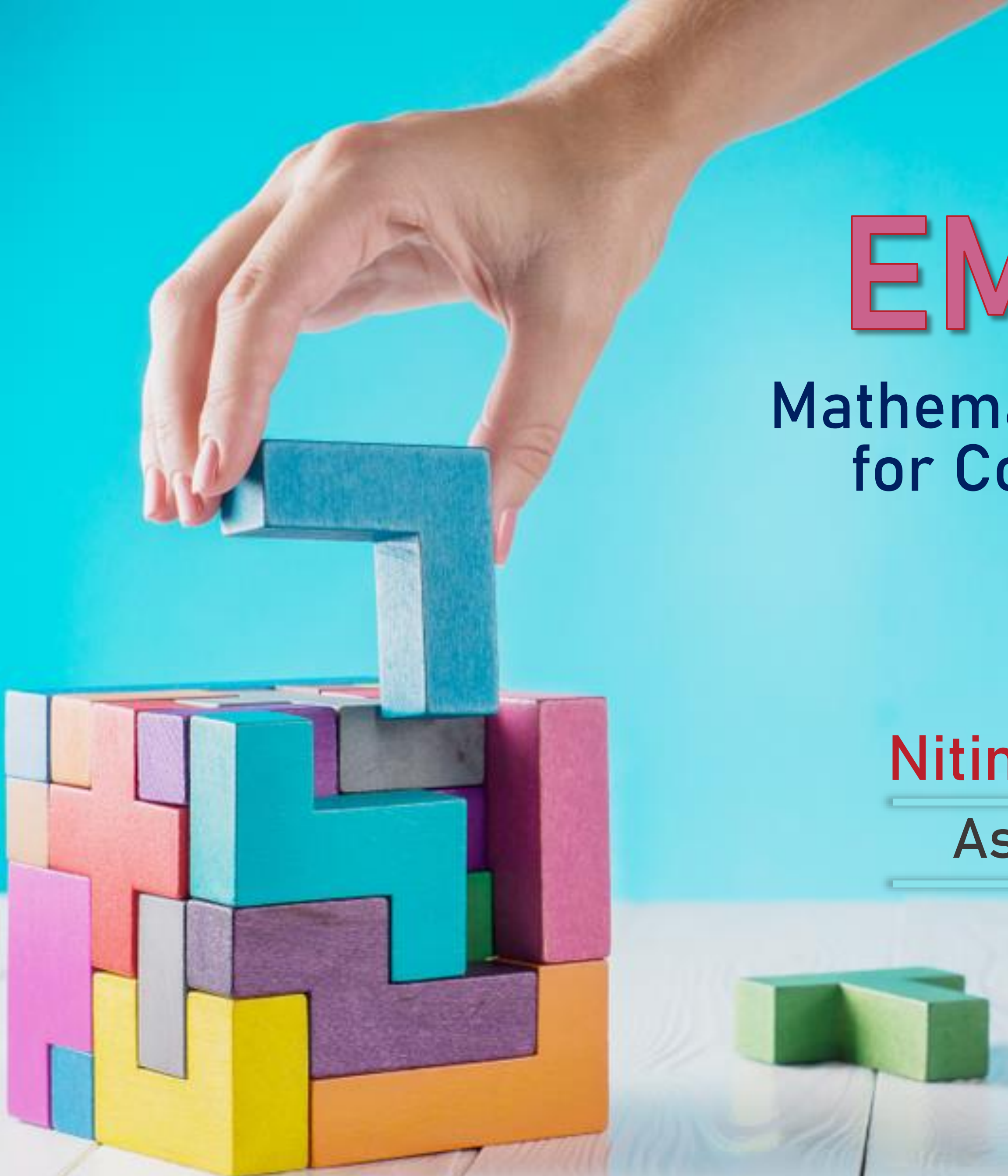


EMTH403

Mathematical Foundation for Computer Science

Nitin K. Mishra (Ph.D.)
Associate Professor

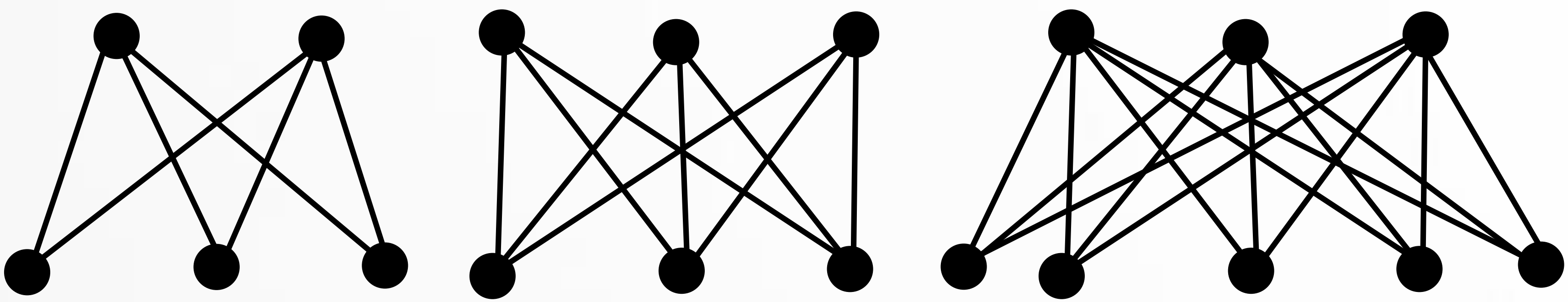
Lecture Outcomes

After this lecture, you will be able to

- understand what is complete graph and complete bipartite graph.
- understand what is a union and subgraphs.
- understand how to represent a graph in a matrix form.

Special Simple Graphs – Complete Graphs

The complete bipartite graphs $K_{2,3}$, $K_{3,3}$, and $K_{3,5}$ are displayed in Figure below.

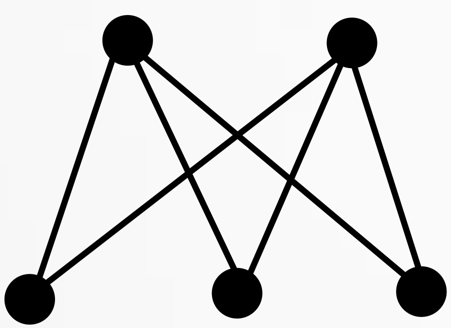


$K_{2,3}$ $K_{3,3}$ $K_{3,5}$

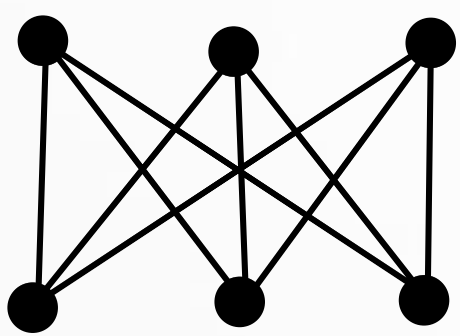
Content Source: Discrete Mathematics and Its Applications. Seventh Edition. Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University. (and formerly AT&T Laboratories)

Special Simple Graphs – Complete Bipartite Graphs

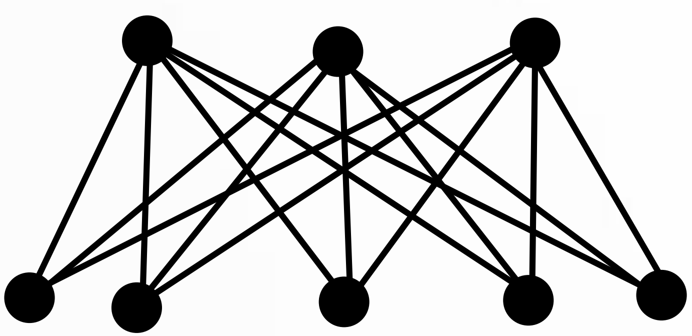
A complete bipartite graph $K_{m,n}$ is a graph that has its vertex set partitioned into two subsets of m and n vertices, respectively with an edge between two vertices if and only if one vertex is in the first subset and the other vertex is in the second subset.



$K_{2,3}$



$K_{3,3}$



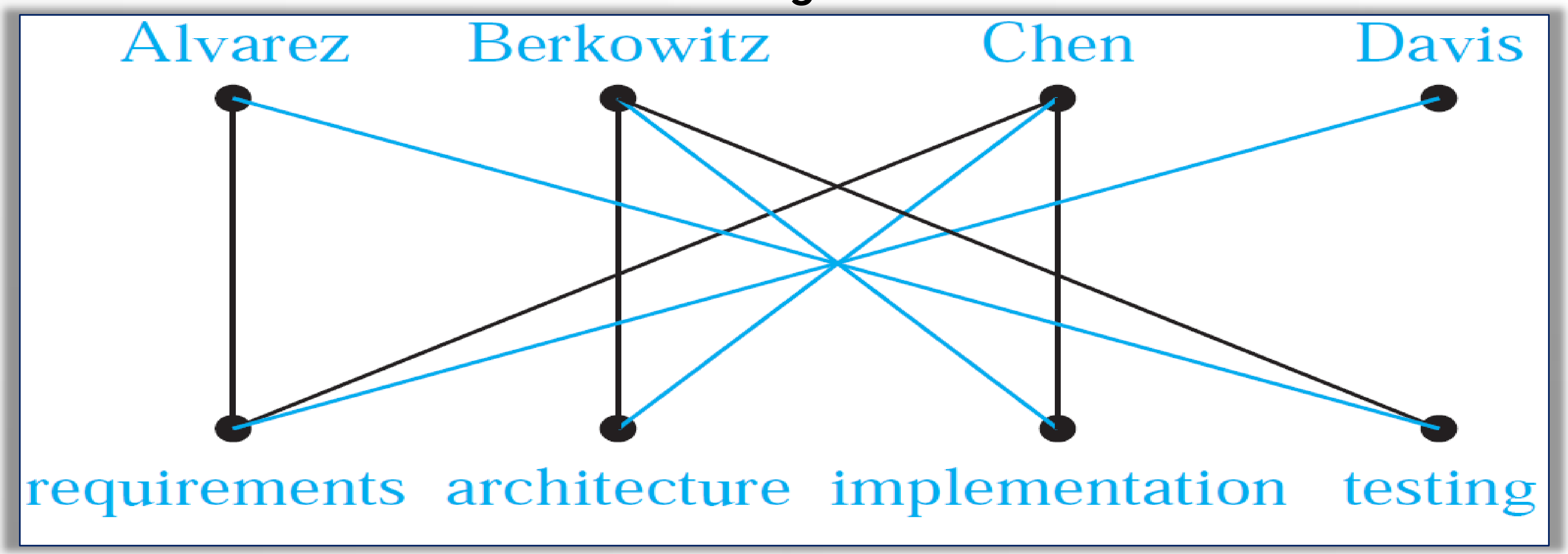
$K_{3,5}$

Content Source: Discrete Mathematics and Its Applications. Seventh Edition. Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University. (and formerly AT&T Laboratories)

Special Simple Graphs – Complete Bipartite Graphs

Complete Bipartite Graphs can be used to model many types of applications that involve matching the elements of one set to elements of another

Job Assignments



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Special Simple Graphs – Complete Bipartite Graphs

Complete Bipartite Graphs can be used to model many types of applications that involve matching the elements of one set to elements of another

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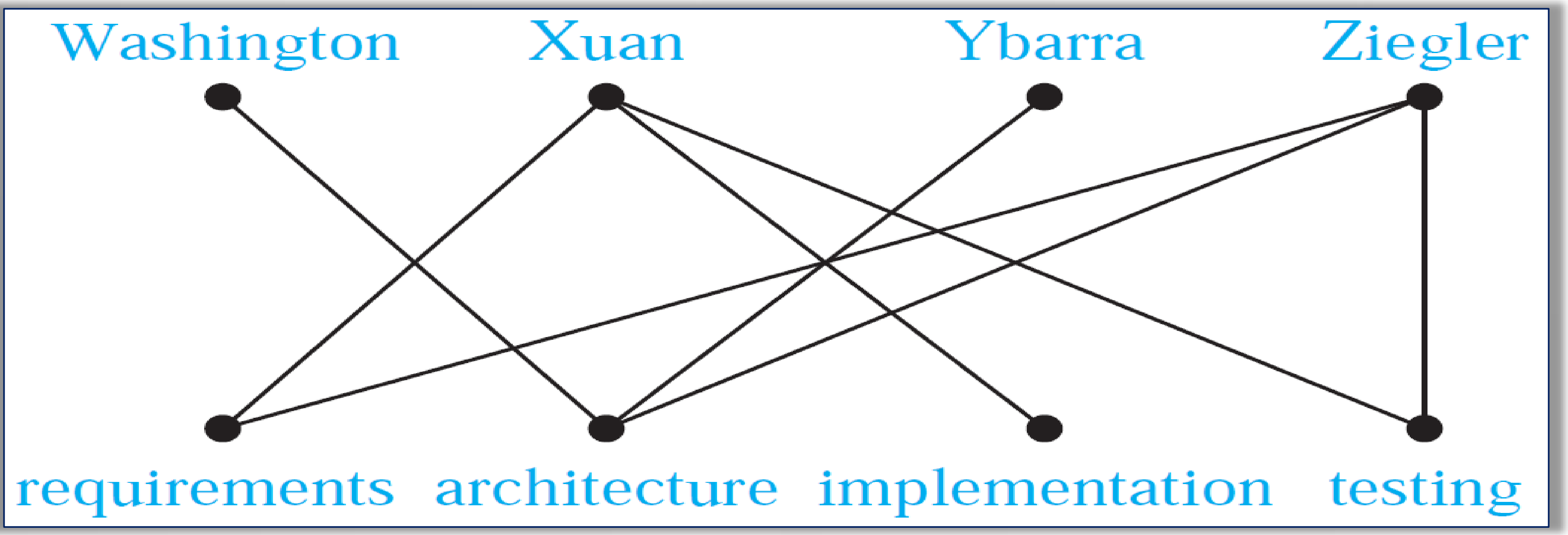
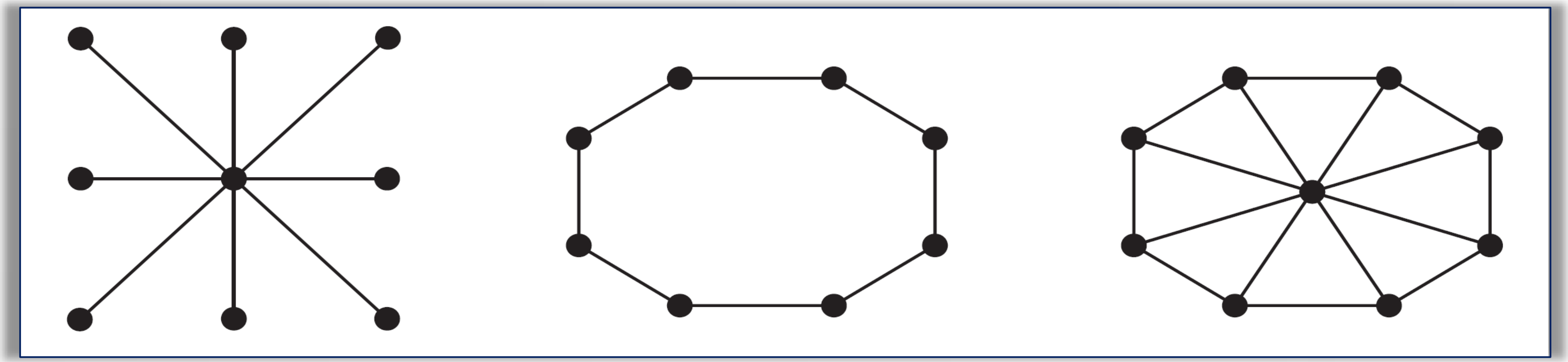


Special Simple Graphs – Complete Bipartite Graphs

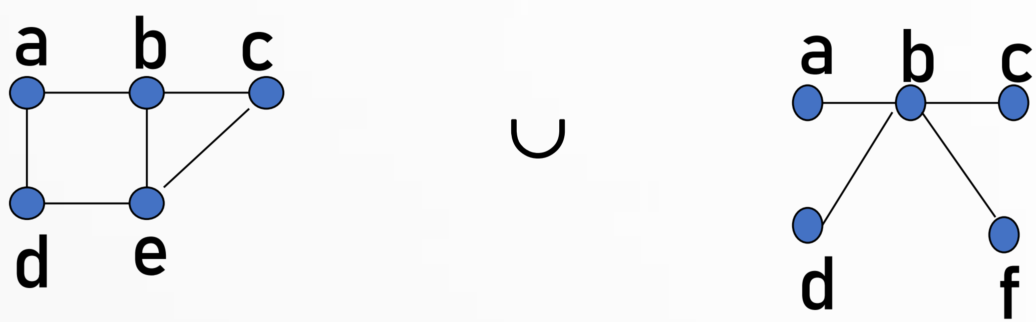
Local Area Networks

The various computers in a building, such as minicomputers and personal computers, as well as peripheral devices such as printers and plotters, can be connected using a local area network.



Union of 2 Simple graphs – Definition

The **union** of 2 simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the simple graph with vertex set $V = V_1 \cup V_2$ and edge set $E = E_1 \cup E_2$. The union is denoted by $G_1 \cup G_2$.



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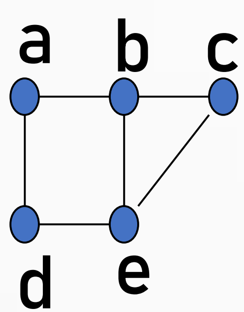
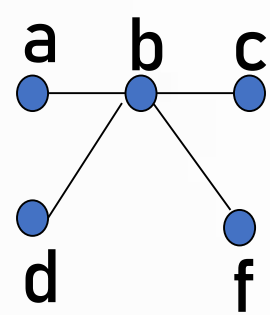
Union of 2 Simple graphs – Definition

The *union* $G_1 \cup G_2$ of two simple graphs $G_1=(V_1, E_1)$ and $G_2=(V_2,E_2)$ is the simple graph $(V_1 \cup V_2, E_1 \cup E_2)$.



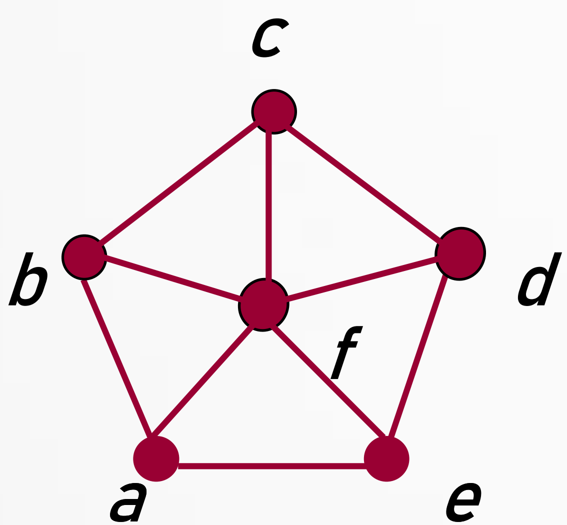
$\cup$



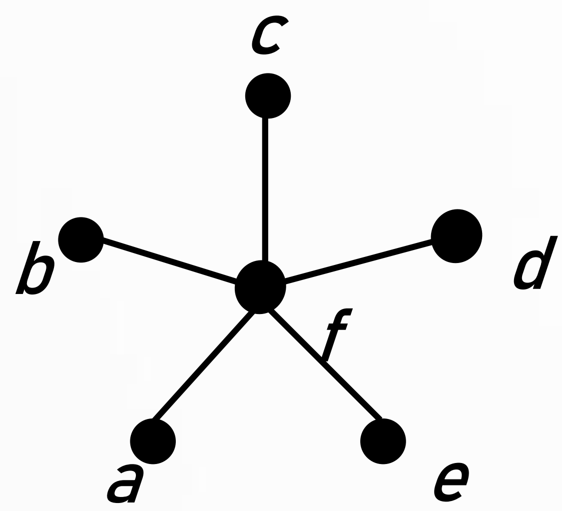
Content Source: **Discrete Mathematics** and **Its Applications**. Seventh Edition. **Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University.** (and formerly AT&T Laboratories)

Union of 2 Simple graphs – Definition

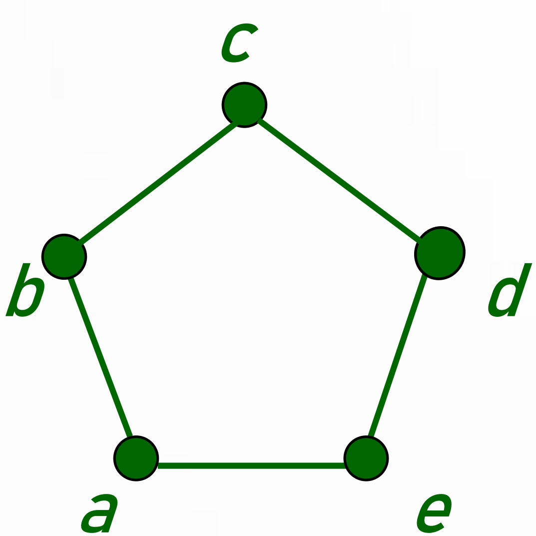
W_5 is the union of S_5 and C_5



W_5

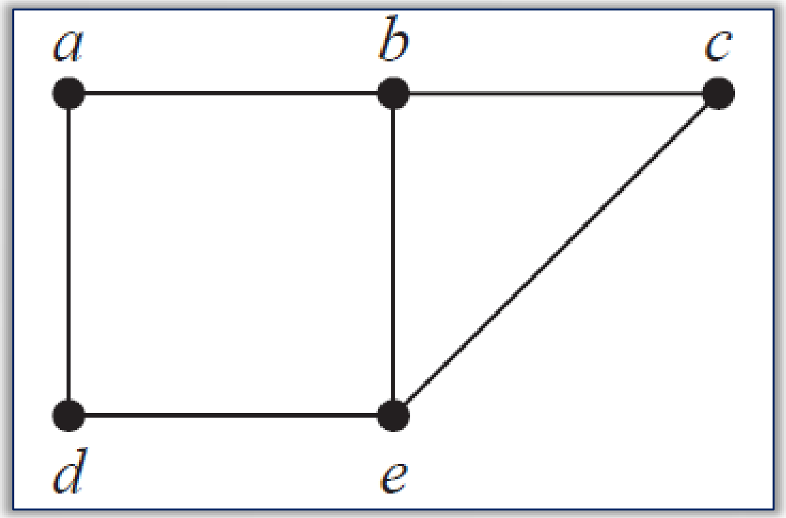


S_5

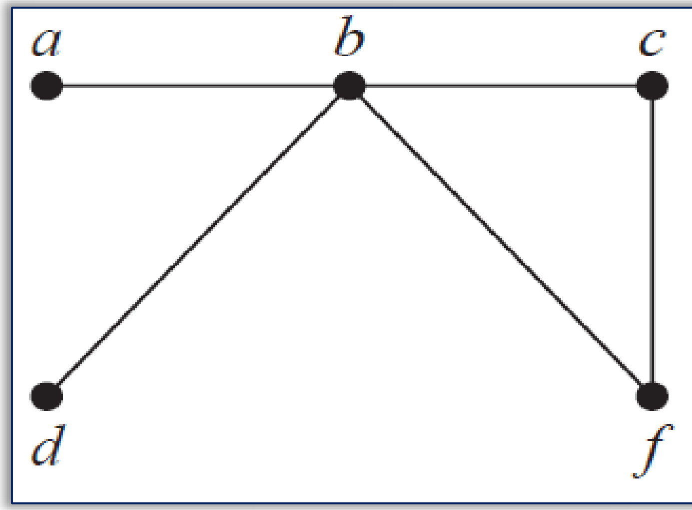


C_5

Union of 2 Simple graphs – Definition

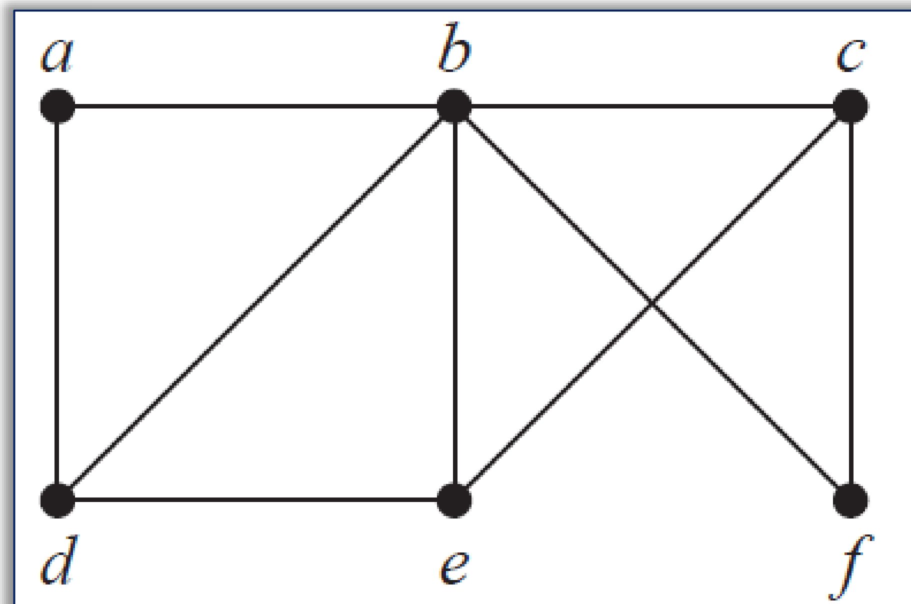


G_1



G_2

G_2 U G_2



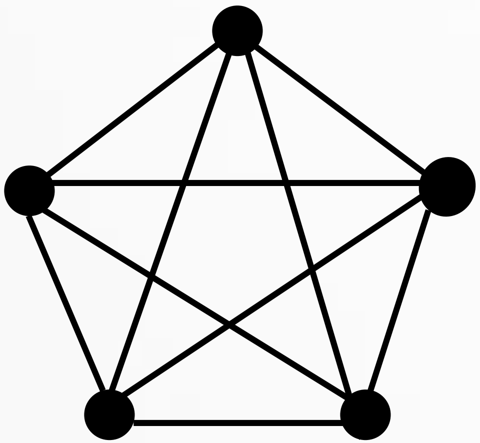
Subgraph - Definition

A **subgraph** of a graph $G = (V, E)$ is a graph

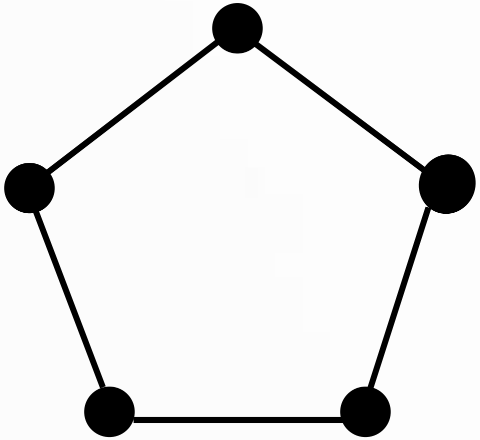
$H = (W, F)$ where $W \subseteq V$ and $F \subseteq E$.

Subgraph - Example

C_5 is a subgraph of K_5

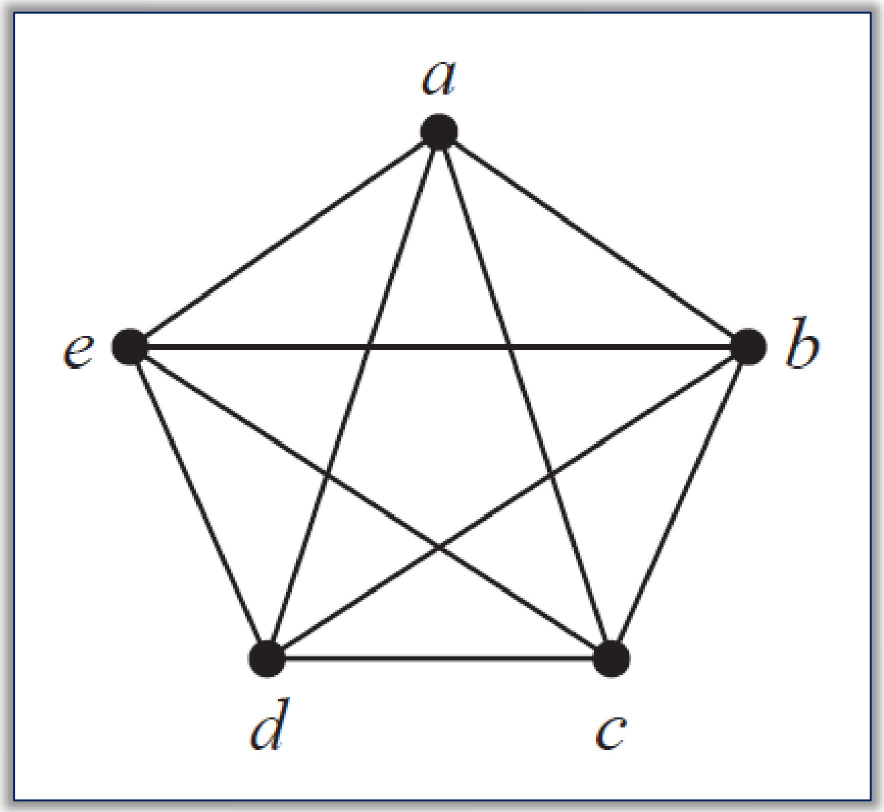


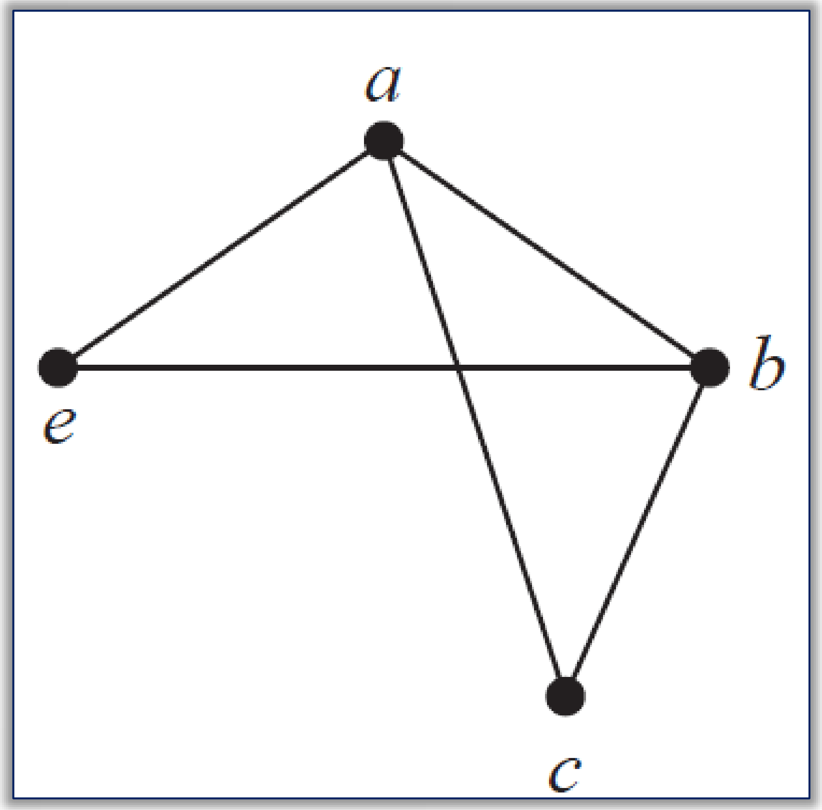
K_5



C_5

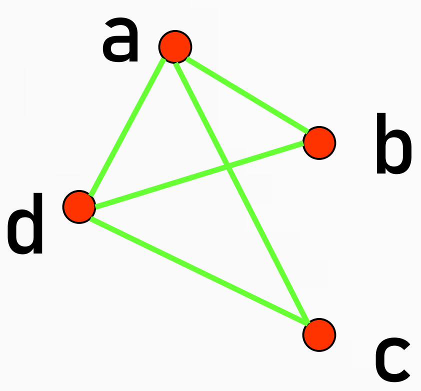
Subgraph - Example

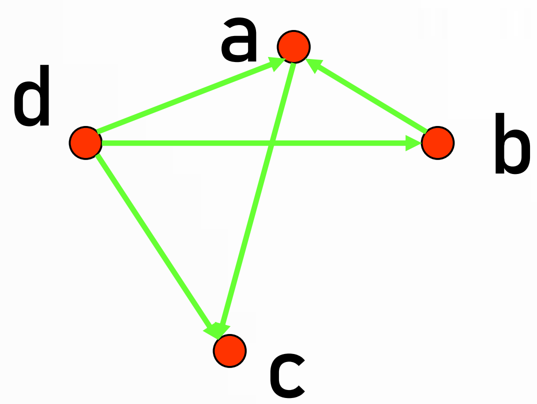




A subgraph of K_5

Representing Graphs





Vertex	Adjacent Vertices
a	b, c, d
b	a, d
c	a, d
d	a, b, c

Initial Vertex	Terminal Vertices
a	c
b	a
c	
d	a, b, c

Representing Graphs- Definition

Let $G = (V, E)$ be a simple graph with $|V| = n$. Suppose that the vertices of G are listed in arbitrary order as $v_1, v_2, \ldots, v_n$.

The adjacency matrix A (or A_G) of G, with respect to this listing of the vertices, is the $n \times n$ zero-one matrix with 1 as its (i, j) entry when v_i and v_j are adjacent, and 0 otherwise.

In other words, for an adjacency matrix $A = [a_{ij}]$,

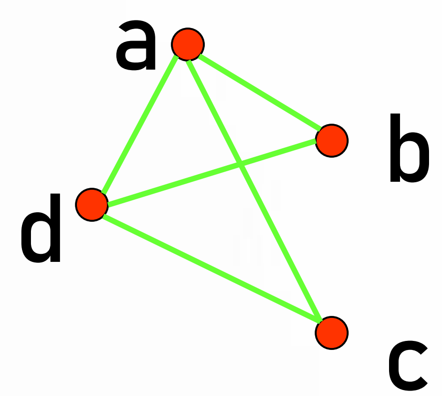
$a_{ij} = 1$ if $\{v_i, v_j\}$ is an edge of G,

$a_{ij} = 0$ otherwise.

Content Source: Discrete Mathematics and Its Applications. Seventh Edition. Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University. (and formerly AT&T Laboratories)

Representing Graphs - Example

What is the adjacency matrix A_G for the following graph G based on the order of vertices a, b, c, d ?



Solution:

$$A_G = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Note: Adjacency matrices of undirected graphs are always symmetric.

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Representing Graphs – Definition

Let G = (V, E) be an undirected graph with |V| = n. Suppose that the vertices and edges of G are listed in arbitrary order as $v_1, v_2, \ldots, v_n$ and $e_1, e_2, \ldots, e_m$, respectively.

The incidence matrix of G with respect to this listing of the vertices and edges is the $n \times m$ zero-one matrix with 1 as its (i, j) entry when edge e_j is incident with v_i, and 0 otherwise.

In other words, for an incidence matrix $M = [m_{ij}]$,

$m_{ij} = 1$ if edge e_j is incident with v_i

$m_{ij} = 0$ otherwise.

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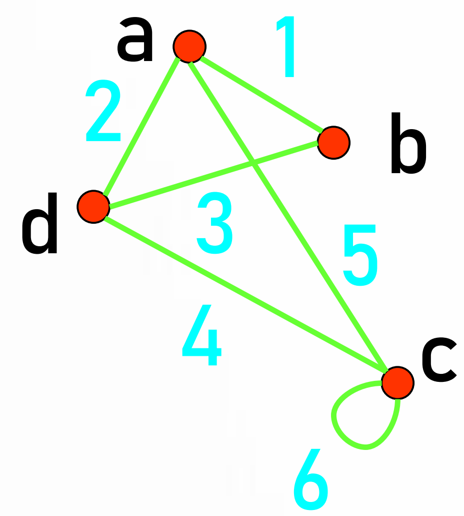
Representing Graphs - Example

What is the incidence matrix M for the following graph G based on the order of vertices a, b, c, d and edges 1, 2, 3, 4, 5, 6?



Solution:

$$M = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Note: Incidence matrices of directed graphs contain two 1s per column for edges connecting two vertices and one 1 per column for loops.

That's all for now…