



# EPEA516

## ANALYTICAL SKILLS II

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# Learning Outcomes



After this lecture, you will be able to

- solve problems based on time and amount computation.

# Problem 1

- A sum of money placed at C.I. doubles in 3 years. In how many years will it become four times?
- $n = 2$
- $t = 3$  years
- $m = 2$  (four times i.e.  $2 \times 2 = 2^2$  or  $n^m$ )
- $T = ?$

# Problem 1

- $n = 2$ ,  $t = 3$  years,  $m = 2$ , &  $T = ?$
- We know that if a certain sum becomes  $n$  times in  $t$  years at compound interest, then the same sum becomes  $n^m$  times in  $mt$  years.
- $T = m \times t$
- $T = 2 \times 3$
- $T = 6$  years

## Problem 2

- A sum of money doubles itself at C.I. in 15 years. In how many years will it become eight times?
- $n = 2$
- $t = 15$  years
- $m = 3$  ( Eight times i.e.  $2 \times 2 \times 2 = 2^3$  or  $n^m$  )
- $T = ?$



## Problem 2

- $n = 2$ ,  $t = 15$  years,  $m = 3$ , &  $T = ?$
- We know that if a certain sum becomes  $n$  times in  $t$  years at compound interest, then the same sum becomes  $n^m$  times in  $mt$  years.
- $T = m \times t$
- $T = 3 \times 15$
- $T = 45$  years

# Problem 3

- In how many years Rs.1,00,000 will become Rs.1,33,100 at compound interest rate of 10% p.a.
- $P = \text{Rs. } 1,00,000$
- $R = 10\% \text{ p.a.}$
- $A = \text{Rs. } 1,33,100$
- $n = ?$
- $A = P \left[ 1 + \frac{R}{100} \right]^n$

# Problem 3

- $P = \text{Rs. } 1,00,000$ ;  $R = 10\% \text{ p.a.}$ ;  $A = 1,33,100$ ; and  $n = ?$

- $A = P \left[ 1 + \frac{R}{100} \right]^n$

- $\frac{A}{P} = \left[ 1 + \frac{R}{100} \right]^n$

- $\frac{\overset{1331}{\cancel{1,33,100}}}{\cancel{1,00,000}} = \left[ 1 + \frac{\cancel{10}}{\cancel{100}} \right]^n$   
1000 10

- $\frac{11 \times 11 \times 11}{10 \times 10 \times 10} = \left[ 1 + \frac{1}{10} \right]^n$



# Problem 3

- $\frac{11 \times 11 \times 11}{10 \times 10 \times 10} = \left[1 + \frac{1}{10}\right]^n$

- $\left[\frac{11}{10}\right]^3 = \left[\frac{10 + 1}{10}\right]^n$

- $\left[\frac{11}{10}\right]^3 = \left[\frac{11}{10}\right]^n$

- $3 = n$

- $n = 3 \text{ years}$

## Problem 4

- Mehak invested an amount of Rs. 18,000 at compound interest rate 4% p.a. for a period of 3 years. What amount will she receive at the end of 3 years?
- $P = \text{Rs. } 18,000$
- $R = 4\% \text{ p.a.}$
- $n = 3 \text{ years}$
- $A = ?$
- $A = P \left[ 1 + \frac{R}{100} \right]^n$

## Problem 4

- $P = \text{Rs. } 18000$ ,  $R = 4\% \text{ p.a.}$ ,  $n = 3 \text{ years}$ , and  $A = ?$

- $A = P \left[ 1 + \frac{R}{100} \right]^n$

- $A = 18000 \times \left[ 1 + \frac{4}{100} \right]^3$

- $A = 18000 \times \left[ \frac{100 + 4}{100} \right]^3$

# Problem 4

- $A = 18000 \times \left[ \frac{100 + 4}{100} \right]^3$
- $A = 18000 \times \left[ \frac{104}{100} \right]^3$
- $A = \frac{18000 \cancel{\cancel{\cancel{0}}} \times 104 \times 104 \times 104}{100 \cancel{\cancel{0}} \times 100 \cancel{\cancel{0}} \times 100}$
- $A = \frac{18 \times 104 \times 104 \times 104}{1000}$
- $A = \frac{20247552}{1000}$
- $A = \text{Rs. } 20,247.552$

# Problem 5

- Rohit invested an amount of Rs. 1,000 at 4% p.a. compounded half-yearly for a period of 2 years. What amount will he receive at the end of 2 years?
- $P = \text{Rs. } 1,000$
- $R = 4\% \text{ p.a. } (\frac{4}{2} \% = 2 \% \text{ half-year})$
- $n = 2 \text{ years (4 half-years)}$
- $A = ?$
- $A = P \left[ 1 + \frac{\frac{R}{2}}{100} \right]^{2n}$



# Problem 5

- $P = \text{Rs. } 1,000; \frac{R}{2} = 2\%; 2n = 2 \times 2 = 4; \text{ and } A = ?$

- $A = P \left[ 1 + \frac{\frac{R}{2}}{100} \right]^{2n}$

- $A = 1000 \times \left[ 1 + \frac{2}{100} \right]^4$

- $A = 1000 \times \left[ \frac{100 + 2}{100} \right]^4$

- $A = 1000 \times \left[ \frac{102}{100} \right]^4$

# Problem 5

- $A = 1000 \times \left[ \frac{102}{100} \right]^4$
- $A = \cancel{1000}^{\cancel{100}} \times \frac{102}{\cancel{100}} \times \frac{102}{\cancel{100}} \times \frac{102}{100} \times \frac{102}{100}$
- $A = \frac{108243216}{100000}$
- $A = \text{Rs. } 1082.43 \text{ (approx.)}$

# Problem 6

- Neha invested an amount of Rs. 24,000 at 12% p.a. compounded quarterly for a period of 1 year. What amount will she receive at the end of 1 year?
- $P = \text{Rs. } 24,000$
- $R = 12\% \text{ p.a. } (\frac{12}{4} \% = 3 \% \text{ per quarter})$
- $n = 1 \text{ year (4 quarters)}$
- $A = ?$
- $A = P \left[ 1 + \frac{\frac{R}{4}}{100} \right]^{4n}$

# Problem 6

- $P = \text{Rs. } 24,000; \frac{R}{4} = 3\%; 4n = 4 \times 1 = 4; \text{ and } A = ?$

- $A = P \left[ 1 + \frac{\frac{R}{4}}{100} \right]^{4n}$

- $A = 24000 \times \left[ 1 + \frac{3}{100} \right]^4$

- $A = 24000 \times \left[ \frac{100 + 3}{100} \right]^4$

- $A = 24000 \times \left[ \frac{103}{100} \right]^4$

# Problem 6

- $A = 24000 \times \left[ \frac{103}{100} \right]^4$

- $A = 24\cancel{000} \times \frac{103}{\cancel{100}} \times \frac{103}{\cancel{100}} \times \frac{103}{100} \times \frac{103}{100}$

- $A = \frac{2701221144}{100000}$

- $A = \text{Rs. } 27012.21 \text{ (approx.)}$



# Problem 7

- Find the amount on Rs. 4,000 at 10% p.a. for 2 years 6 months, compounded annually.

- $P = \text{Rs. } 4,000$

- $R = 10 \% \text{ p.a.}$

- $n \frac{1}{m} = 2 \text{ years } 6 \text{ months} = 2 \frac{\cancel{6}}{\cancel{12}_2} \text{ years} = 2 \frac{1}{2} \text{ years}$

- $A = ?$

- $$A = P \left\{ \left[ 1 + \frac{R}{100} \right]^n \times \left[ 1 + \frac{1}{m} \frac{R}{100} \right] \right\}$$

# Problem 7

- $P = \text{Rs. } 4,000$ ;  $R = 10 \%$ ;  $n = 2$ ;  $\frac{1}{m} = \frac{1}{2}$ ; and  $A = ?$

- $A = P \left\{ \left[ 1 + \frac{R}{100} \right]^n \times \left[ 1 + \frac{\frac{1}{m} R}{100} \right] \right\}$

- $A = 4000 \times \left\{ \left[ 1 + \frac{10}{100} \right]^2 \times \left[ 1 + \frac{\frac{1}{2} \times 10}{100} \right] \right\}$

- $A = 4000 \times \left\{ \left[ 1 + \frac{1}{10} \right]^2 \times \left[ 1 + \frac{1}{20} \right] \right\}$

# Problem 7

- $A = 4000 \times \left\{ \left[ 1 + \frac{1}{10} \right]^2 \times \left[ 1 + \frac{1}{20} \right] \right\}$

- $A = 4000 \times \left\{ \left[ \frac{10 + 1}{10} \right]^2 \times \left[ \frac{20 + 1}{20} \right] \right\}$

- $A = 4000 \times \left\{ \left[ \frac{11}{10} \right]^2 \times \left[ \frac{21}{20} \right] \right\}$

- $A = 4000 \times \left\{ \frac{121}{100} \times \frac{21}{20} \right\}$

# Problem 7

- $A = 4000 \times \left\{ \frac{121}{100} \times \frac{21}{20} \right\}$

- $A = 4000 \times \overset{1.2705}{\left\{ \frac{\cancel{2541}}{\cancel{2000}} \right\}}$

- $A = 4000 \times \{1.2705\}$

- $A = \text{Rs. } 5,082$

# Problem 8

- Amit invests Rs. 5,000 in a bond which gives interest at 2%p.a. during the first year, 5% p.a. during the second year and 10% p.a. during the third year. How much does he get at the end of the third year.
- $P = \text{Rs. } 5,000$
- $R_1 = 2\% \text{ p.a.}$
- $R_2 = 5\% \text{ p.a.}$
- $R_3 = 10\% \text{ p.a.}$
- $A = ?$



# Problem 8

- $P = \text{Rs. } 5,000$ ;  $R_1 = 2\% \text{ p.a.}$ ;  $R_2 = 5\% \text{ p.a.}$ ;  $R_3 = 10\% \text{ p.a.}$  &

$A = ?$

- $A = P \left\{ \left[ 1 + \frac{R_1}{100} \right] \times \left[ 1 + \frac{R_2}{100} \right] \times \left[ 1 + \frac{R_3}{100} \right] \right\}$

- $A = 5000 \times \left\{ \left[ 1 + \frac{\cancel{2}}{\cancel{100}} \right] \times \left[ 1 + \frac{\cancel{5}}{\cancel{100}} \right] \times \left[ 1 + \frac{\cancel{10}}{\cancel{100}} \right] \right\}$   
50                      20                      10

- $A = 5000 \times \left\{ \left[ 1 + \frac{1}{50} \right] \times \left[ 1 + \frac{1}{20} \right] \times \left[ 1 + \frac{1}{10} \right] \right\}$

# Problem 8

- $A = 5000 \times \left\{ \left[ 1 + \frac{1}{50} \right] \times \left[ 1 + \frac{1}{20} \right] \times \left[ 1 + \frac{1}{10} \right] \right\}$

- $A = 5000 \times \left\{ \left[ \frac{50 + 1}{50} \right] \times \left[ \frac{20 + 1}{20} \right] \times \left[ \frac{10 + 1}{10} \right] \right\}$

- $A = 5000 \times \left\{ \left[ \frac{51}{50} \right] \times \left[ \frac{21}{20} \right] \times \left[ \frac{11}{10} \right] \right\}$

- $A = 5000 \times \left\{ \left[ \frac{11781}{10000} \right] \right\}$

- $A = 5000 \times \{1.1781\}$

# Problem 8

- $A = 5000 \times \{1.1781\}$

- $A = \text{Rs. } 5890.50$

# Problem 9

- If a sum of Rs. 13040 is to be paid back in two equal annual instalments at  $3\frac{3}{4}\%$  p.a., what is the amount of each instalment?
- $P = \text{Rs. } 13,040$
- $R = 3\frac{3}{4}\% \text{ p.a.} = \frac{15}{4}\% \text{ p.a.}$
- No. of instalments = 2
- Amount of each instalment = ?

# Problem 9

- $P = \text{Rs. } 13,040$ ;  $R = \frac{15}{4} \% \text{ p.a.}$ ; No. of instalments = 2; and  
Amount of each instalment = ?

- Amount of each instalment  $= \frac{P}{\left[ \frac{100}{100 + R} \right]^1 + \left[ \frac{100}{100 + R} \right]^2}$

$$= \frac{13040}{\left[ \frac{100}{100 + \frac{15}{4}} \right]^1 + \left[ \frac{100}{100 + \frac{15}{4}} \right]^2}$$



# Problem 9

- Amount of each instalment

$$\begin{aligned} &= \frac{13040}{\left[ \frac{100}{100 + \frac{15}{4}} \right]^1 + \left[ \frac{100}{100 + \frac{15}{4}} \right]^2} \\ &= \frac{13040}{\left[ \frac{100}{\frac{400 + 15}{4}} \right]^1 + \left[ \frac{100}{\frac{400 + 15}{4}} \right]^2} \\ &= \frac{13040}{\left[ \frac{100 \times 4}{415} \right]^1 + \left[ \frac{100 \times 4}{415} \right]^2} \\ &= \frac{13040}{\left[ \frac{400}{415} \right]^1 + \left[ \frac{400}{415} \right]^2} \end{aligned}$$

# Problem 9

- Amount of each instalment

$$= \frac{13040}{\left[\frac{400}{415}\right]^1 + \left[\frac{400}{415}\right]^2}$$

$$= \frac{13040}{\left[\frac{400}{415}\right] \times \left[1 + \frac{400}{415}\right]}$$

$$= \frac{13040}{\left[\frac{400}{415}\right] \times \left[\frac{815}{415}\right]}$$

$$= \frac{13040 \times 415 \times 415}{400 \times 815}$$

$$= \text{Rs. } 6,889$$

# Conclusion

- If a certain sum becomes  $n$  times in  $t$  years at compound interest, then the same sum becomes  $n^m$  times in  $mt$  years.

$$T = m \times t$$

- If  $A$ ,  $P$ , and  $R$  is given.

$$\frac{A}{P} = \left[ 1 + \frac{R}{100} \right]^n$$

# Conclusion

- When interest is compounded annually

- $A = P \left[ 1 + \frac{R}{100} \right]^n$

- When interest is compounded half-yearly

- $A = P \left[ 1 + \frac{\frac{R}{2}}{100} \right]^{2n}$

- When interest is compounded quarterly

- $A = P \left[ 1 + \frac{\frac{R}{4}}{100} \right]^{4n}$

# Conclusion

- When interest is compounded annually but time is in fraction,  $n\frac{1}{m}$  years.

- $$A = P \left\{ \left[ 1 + \frac{R}{100} \right]^n \times \left[ 1 + \frac{\frac{1}{m}R}{100} \right] \right\}$$

- When rates are different for different years

- $$A = P \left\{ \left[ 1 + \frac{R_1}{100} \right] \times \left[ 1 + \frac{R_2}{100} \right] \times \left[ 1 + \frac{R_3}{100} \right] \right\}$$

- Amount of each instalment =

$$\frac{P}{\left[ \frac{100}{100 + R} \right]^1 + \left[ \frac{100}{100 + R} \right]^2}$$



# Summary

- Computation of
  - Time
  - Amount

**That's all for now...**