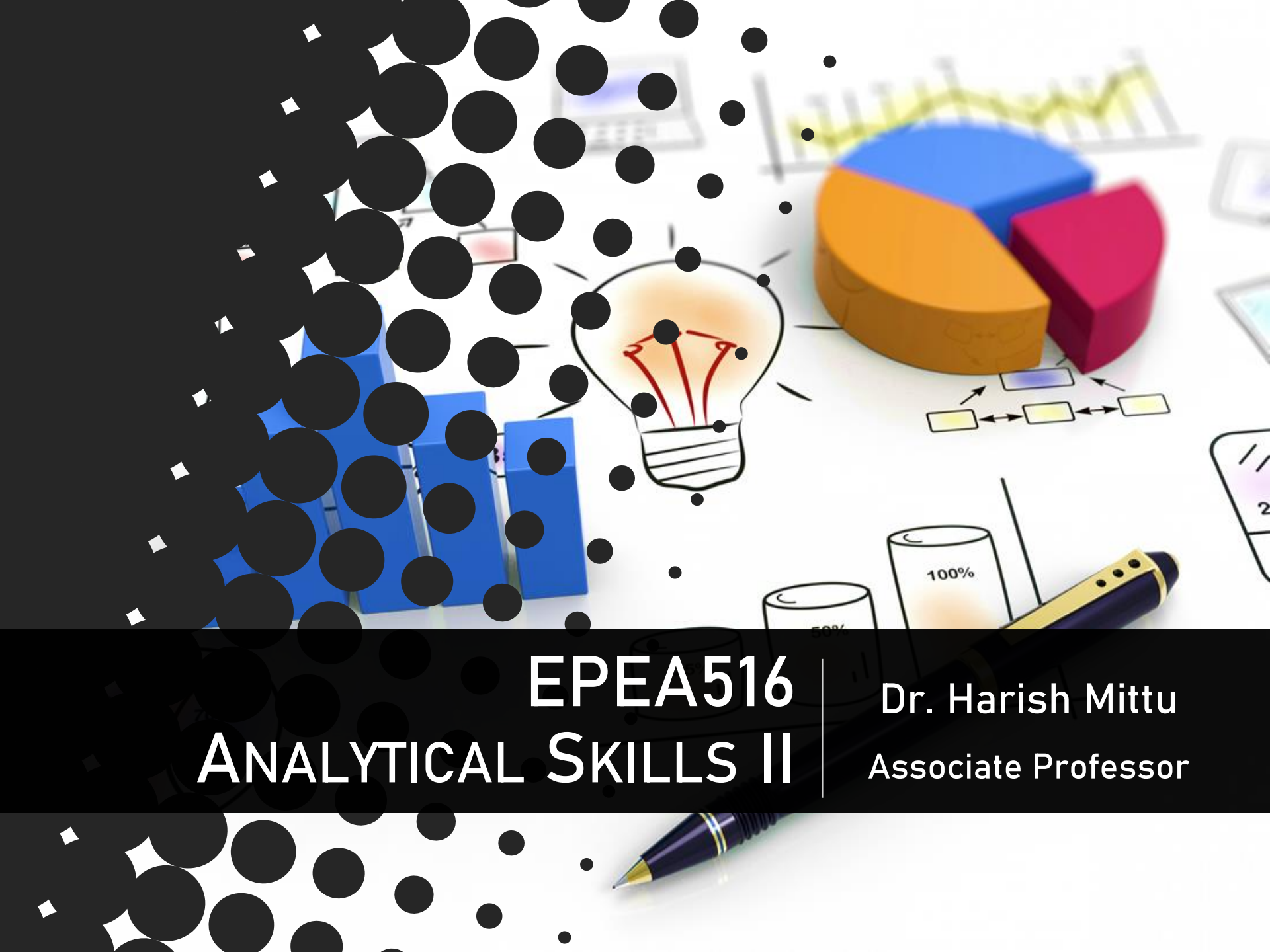

EPEA516
Analytical Skills II

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Learning Outcomes

After this lecture, you will be able to

- develop understanding about basics of relative speed, faster and slower trains,
- derive important facts and formulae relating to various problems on trains.

Basics and Important Facts

- If a train overtakes a pole/man/milestone, then the distance covered in overtaking is equal to the length of the train.
- Time taken by a train of length ‘a’ m to pass a pole/standing man/milestone/signal post is equal to the time taken by the train to cover ‘a’ m.

Basics and Important Facts

- If a train overtakes a bridge/tunnel/platform/another train, then the distance covered is equal to the sum of the two lengths i.e., sum of the length of the train and length of bridge/tunnel/platform/another train.
- Time taken by a train of length ‘a’ m to pass a stationary object/bridge/tunnel/platform/another train of length ‘b’ m is time taken by train to cover (a + b) m.

Basics, Important Facts and Formulae

- When two trains are moving in opposite direction
  - Relative Speed is equal to the sum of the speed of both trains.
  - Speed of faster train + Speed of Slower train
  - Suppose two trains or two bodies are moving in opposite directions at a m/s and b m/s.
  - Relative Speed = (a + b) m/s

Basics, Important Facts and Formulae

- When two trains are moving in same direction
  - Relative Speed is equal to the difference of the speed of both trains.
  - Speed of faster train – Speed of Slower train
  - Suppose two trains are moving in the same direction at a m/s and b m/s (where a > b).
  - Relative Speed = (a – b) m/s

Basics, Important Facts and Formulae

- Relative Speed
  - If two trains of lengths ‘x’ Km and ‘y’ Km are travelling in the same direction at ‘a’ Km/h and ‘b’ Km/h (a > b) respectively, then
  - Relative Speed = (a – b) Km/h
  - Time taken by Faster Train to cross Slower Train

$$= \left\{ \frac{x + y}{a - b} \right\} \text{ hours}$$

Basics, Important Facts and Formulae

- Relative Speed
  - If two trains of length ‘x’ Km and ‘y’ Km are travelling in the opposite directions at ‘a’ Km/h and ‘b’ Km/h, then
  - Relative Speed = (a + b) Km/h
  - Time taken by Trains to cross each other

$$= \left\{ \frac{x + y}{a + b} \right\} \text{ hours}$$

Basics, Important Facts and Formulae

- Speed – Faster & Slower Train
  - Two trains of lengths ‘x’ m and ‘y’ m run on parallel tracks. When running in the same direction, the faster train passes the slower one in t_1 seconds, but when they are running in opposite directions with the same speeds as earlier, they pass each other in t_2 seconds.
  - Speed of Faster Train = $\left\{\frac{x + y}{2}\right\} . \left\{\frac{t_1 + t_2}{t_1 t_2}\right\}$ m/s
  - Speed of Slower Train = $\left\{\frac{x + y}{2}\right\} . \left\{\frac{t_1 - t_2}{t_1 t_2}\right\}$ m/s

Important Facts and Formulae

- A train starts from a place at ‘a’ Km/h and another fast train starts from the same place after ‘T’ hours at ‘b’ Km/h in the same direction.
- The distance from the starting place at which both the trains will meet

$$= \left\{ \frac{abT}{b - a} \right\} \text{Km}$$

- Time after which the two trains will meet

$$= \left\{ \frac{aT}{b - a} \right\} \text{hours}$$

Important Facts and Formulae

- The distance between two stations ‘A’ and ‘B’ = ‘x’ Km
- A train starts from ‘A’ to ‘B’ at ‘a’ Km/h.
- T hours later another train starts from ‘B’ to ‘A’ at ‘b’ Km/h.
- Distance from A, at which both the trains will meet

$$= a\left\{\frac{x + bT}{a + b}\right\} \text{ Km}$$

- Time after which the two trains will meet

$$= \left\{\frac{x + bT}{a + b}\right\} \text{ hours}$$

Important Facts and Formulae

- If the speeds of two trains are in the ratio a : b and they are moving in opposite directions on the parallel track. If the first train crosses a pole in t_1 seconds and second train in $t_2$seconds, then the time taken by two trains to cross each other completely

$$\frac{t_1 a + t_2 b}{(a + b)} \text{ seconds}$$

Important Facts and Formulae

- If two trains, A (moving with speed 'a') and B (moving with speed 'b'), start at the same time from two points P and Q towards each other, and after crossing they take t_1 and t_2 seconds in reaching Q and P respectively, then

$$\frac{\text{a (i.e., Speed of A)}}{\text{b(i.e., Speed of B)}} = \frac{\sqrt{t_2}}{\sqrt{t_1}}$$

Important Facts and Formulae

- A train travels a certain distance at a speed of ‘a’ Km/h without stoppages and with stoppages it covers the same distance at a speed of ‘b’ Km/h. Calculate the stoppage time per hour.
- Let, the distance travelled = x Km.
- Time taken by the train without stopping = $\frac{x}{a}$ hour
- Time taken by the train with stoppage = $\frac{x}{b}$ hour
- Total Stoppage Time = $\frac{x}{b} - \frac{x}{a}$

Important Facts and Formulae

- Total Stoppage Time $= \frac{x}{b} - \frac{x}{a}$

$$= \frac{ax - bx}{ab}$$

$$= \frac{(a - b)x}{ab} \text{hour}$$

- Stoppage Time/Hour $= \frac{\frac{(a - b)x}{ab}}{\frac{x}{b}}$

$$= \frac{a - b}{a} \text{ hour} = \frac{\text{Difference of Speed}}{\text{Speed without Stoppages}}$$

Important Facts and Formulae

- Two trains of lengths ‘x’ m and ‘y’ m run on parallel tracks. When running in the same direction, the faster train passes the slower one in t_1 seconds, but when they are running in opposite directions with the same speeds as earlier, they pass each other in t_2 seconds. Calculate speed of faster and slower train.

Important Facts and Formulae

- Length of Faster Train = ‘x’ m
- Length of Slower Train = ‘y’ m
- When running in the same direction
- Faster train passes the slower one = t_1 seconds
- When running in opposite directions
- Two trains pass each other = t_2 seconds
- Let, speed of faster train = a m/s
- Speed of slower train = b m/s

Important Facts and Formulae

- Total distance covered when two trains cross each other

$$= x + y$$

- When the two trains are running is the same direction.
- Relative Speed = (a – b) m/s & Relative Speed = $\frac{x + y}{t_1}$ m/s

$$(a - b) = \frac{x + y}{t_1} \qquad \ldots(1)$$

- When two trains are running in the opposite directions.
- Relative Speed = (a – b) m/s & Relative Speed = $\frac{x + y}{t_2}$ m/s

$$(a - b) = \frac{x + y}{t_2} \qquad \ldots(2)$$

Important Facts and Formulae

$$(a - b) = \frac{x + y}{t_1} \quad \ldots(1) \qquad \& \qquad (a + b) = \frac{x + y}{t_2} \quad \ldots(2)$$

(1) + (2)

$$2a = (x + y)\left\{\frac{1}{t_1} + \frac{1}{t_2}\right\}$$

$$a = \left(\frac{x + y}{2}\right)\left\{\frac{t_2 + t_1}{t_1 t_2}\right\}$$

Speed of Faster Train = a

$$= \left(\frac{x + y}{2}\right)\left\{\frac{t_1 + t_2}{t_1 t_2}\right\}$$

(2) – (1)

$$2b = (x + y)\left\{\frac{1}{t_2} - \frac{1}{t_1}\right\}$$

$$b = \left(\frac{x + y}{2}\right)\left\{\frac{t_1 - t_2}{t_1 t_2}\right\}$$

Speed of Slower Train = b

$$= \left(\frac{x + y}{2}\right)\left\{\frac{t_1 - t_2}{t_1 t_2}\right\}$$

Important Facts and Formulae

- If the two trains are of equal length.
- Let x = y = l
- Speed of Faster Train = a = $\left(\frac{x + x}{2}\right)\left\{\frac{t_1 + t_2}{t_1 t_2}\right\}$

$= \left(\frac{2x}{2}\right)\left\{\frac{t_1 + t_2}{t_1 t_2}\right\}$ or $\boxed{(x)\left\{\frac{t_1 + t_2}{t_1 t_2}\right\}}$

- Speed of Slower Train = b = $\left(\frac{x + x}{2}\right)\left\{\frac{t_1 - t_2}{t_1 t_2}\right\}$

$= \left(\frac{2x}{2}\right)\left\{\frac{t_1 - t_2}{t_1 t_2}\right\}$ or $\boxed{(x)\left\{\frac{t_1 - t_2}{t_1 t_2}\right\}}$

Important Facts and Formulae

- Two trains start simultaneously from stations ‘A’ and ‘B’ towards each other with speeds a Km/h and b Km/h, respectively. When they meet it is found that the second train had travelled ‘x’ Km more than the first. Calculate distance between two stations.
- Speed of first train = a Km/h & second train = b Km/h
- Let, Distance between the two stations = ‘d’ Km
- Let, Distance travelled by first train = ‘y’ Km
- Distance travelled by second train = (y + x) Km

Important Facts and Formulae

- $d = y + y + x = 2y + x$
- $t_1 = \frac{y}{a}$ and $t_2 = \frac{y + x}{b}$
- $t_1 = t_2$ (Since the time taken by both the trains is same)
- $\frac{y}{a} = \frac{y + x}{b}$
- $by = ay + ax$
- $by - ay = ax$ or $(b - a)y = ax$
- $y = \frac{ax}{(b - a)}$

Important Facts and Formulae

- $y = \frac{ax}{(b - a)}$
- $d = 2\frac{ax}{(b - a)} + x$
- $d = \frac{2ax + x(b - a)}{(b - a)}$
- $d = \frac{2ax + bx - ax}{(b - a)} = \frac{ax + bx}{(b - a)}$
- $d = \frac{(a + b)x}{(b - a)}$ Km

Conclusion

- Distance covered by a train to overtake a pole/man/milestone = Length of Train
- Distance covered by a train to overtake a bridge/tunnel/platform/another train = Length of the train + Length of bridge/tunnel/platform/another train

Conclusion

- Time taken by a train of length ‘a’ m to pass a pole/standing man/milestone/signal post = Time taken by the train to cover ‘a’ m
- Time taken by a train of length ‘a’ m to pass a stationary object/bridge/tunnel/platform/another train of length ‘b’ m = Time taken by train to cover (a + b) m

Conclusion

- When two trains are moving in opposite direction
  - Relative Speed = Sum of Speed of both Trains
  - Time taken by Trains to cross each other

$$= \left\{\frac{x + y}{a + b}\right\} \text{ hours}$$

- When two trains are moving in same direction
  - Relative Speed = Difference of Speed of both Trains
  - Time taken by Faster Train to cross Slower Train

$$= \left\{\frac{x + y}{a - b}\right\} \text{ hours}$$

Conclusion

- Two trains of lengths ‘x’ m and ‘y’ m run on parallel tracks. When running in same direction, faster train passes slower one in t_1 seconds, but when they are running in opposite directions with same speeds as earlier, they pass each other in t_2 seconds.
  - Speed of Faster Train = $\{\frac{x + y}{2}\} . \{\frac{t_1 + t_2}{t_1 t_2}\}$ m/s
  - Speed of Slower Train = $\{\frac{x + y}{2}\} . \{\frac{t_1 - t_2}{t_1 t_2}\}$ m/s
- If the two trains are of equal length.
  - Speed of Faster Train = (x) $\{\frac{t_1 + t_2}{t_1 t_2}\}$
  - Speed of Slower Train = (x) $\{\frac{t_1 - t_2}{t_1 t_2}\}$

Conclusion

- If distance between two stations ‘A’ and ‘B’ is ‘x’ Km, A train starts from ‘A’ to ‘B’ at ‘a’ Km/h. ‘T’ hours later another train starts from ‘B’ to ‘A’ at ‘b’ Km/h, then distance from A, at which both the trains will meet

$$= a \left\{ \frac{x + bT}{a + b} \right\} \text{Km}$$

- Time after which the two trains will meet

$$= \left\{ \frac{x + bT}{a + b} \right\} \text{hours}$$

Conclusion

- If the speeds of two trains are in the ratio a : b and they are moving in opposite directions on the parallel track. If the first train crosses a pole in t_1 seconds and second train in t_2 seconds, then the time taken by two trains to cross each other completely is $\frac{t_1 a + t_2 b}{(a + b)}$ seconds.
- If two trains, A (moving with speed 'a') and B (moving with speed 'b'), start at the same time from two points P and Q towards each other, and after crossing they take t_1 and t_2 seconds in reaching Q and P respectively, then $\frac{a}{b} = \frac{\sqrt{t_2}}{\sqrt{t_1}}$

Conclusion

- A train travels a certain distance at a speed of ‘a’ Km/h without stoppages and with stoppages it covers the same distance at a speed of ‘b’ Km/h, then the stoppage time per hour is $\frac{\text{Difference of Speed}}{\text{Speed without Stoppages}}$
- Two trains start simultaneously from stations ‘A’ and ‘B’ towards each other with speeds a Km/h and b Km/h, respectively. When they meet, it is found that the second train had travelled ‘x’ Km more than the first. Distance between the two stations is $\frac{(a + b)x}{(b - a)}$ Km

Summary

- Basics
  - Relative Speed
  - Faster & Slower Trains
- Problems on Trains
  - Important Facts
  - Formulae

That’s all for now…