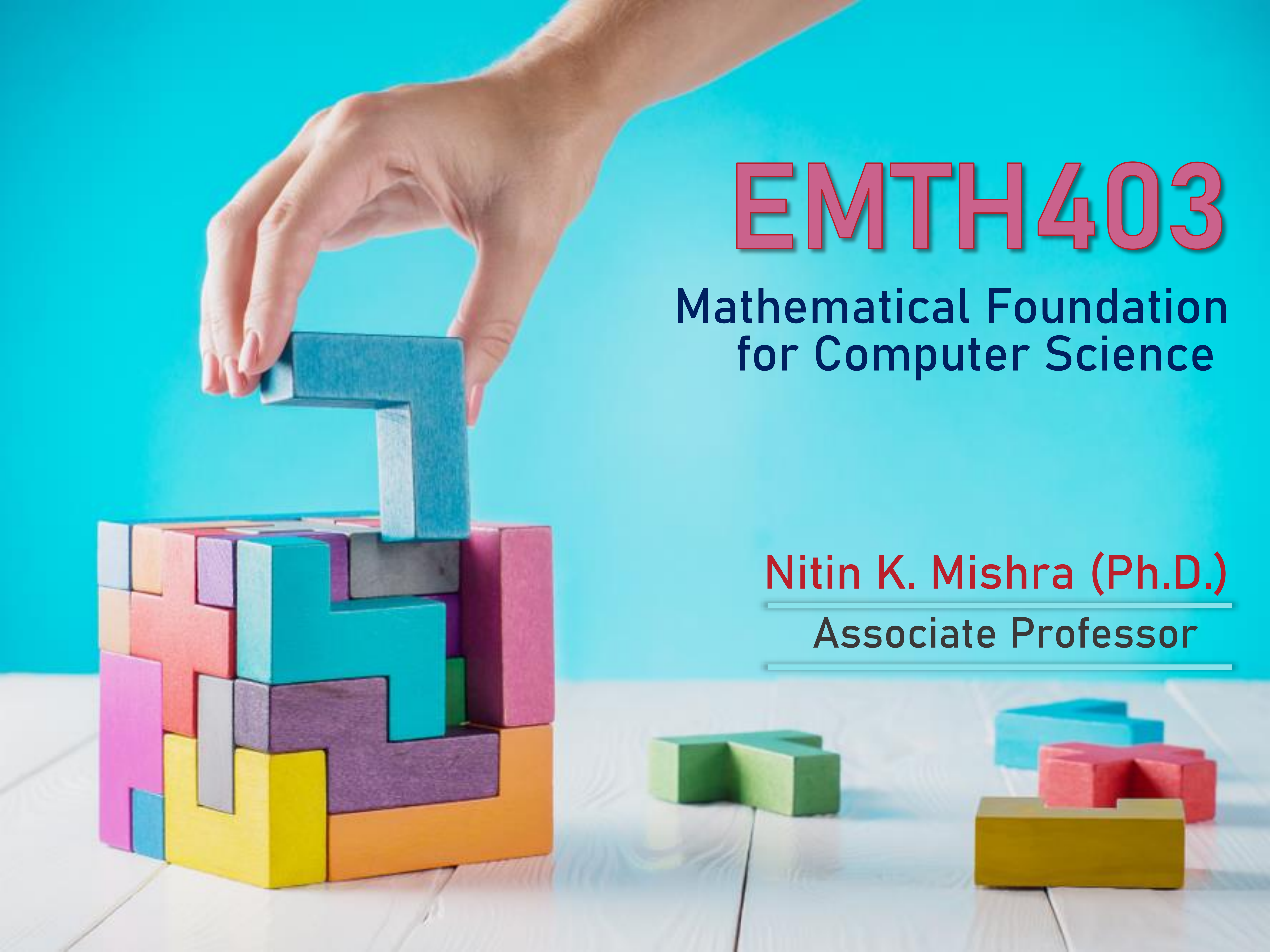


EMTH403

Mathematical Foundation for Computer Science

Nitin K. Mishra (Ph.D.)

Associate Professor

Lecture Outcomes

After this lecture, you will be able to

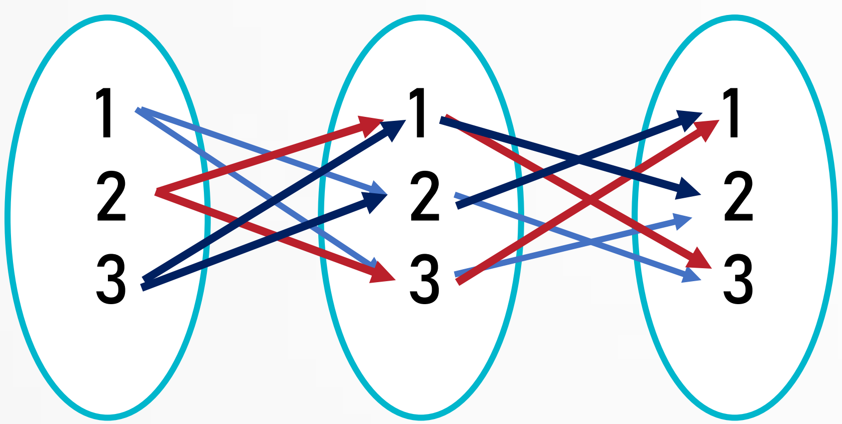
- understand the Division rule in the basics of counting.
- understand how to find total number of functions in the basics of counting.

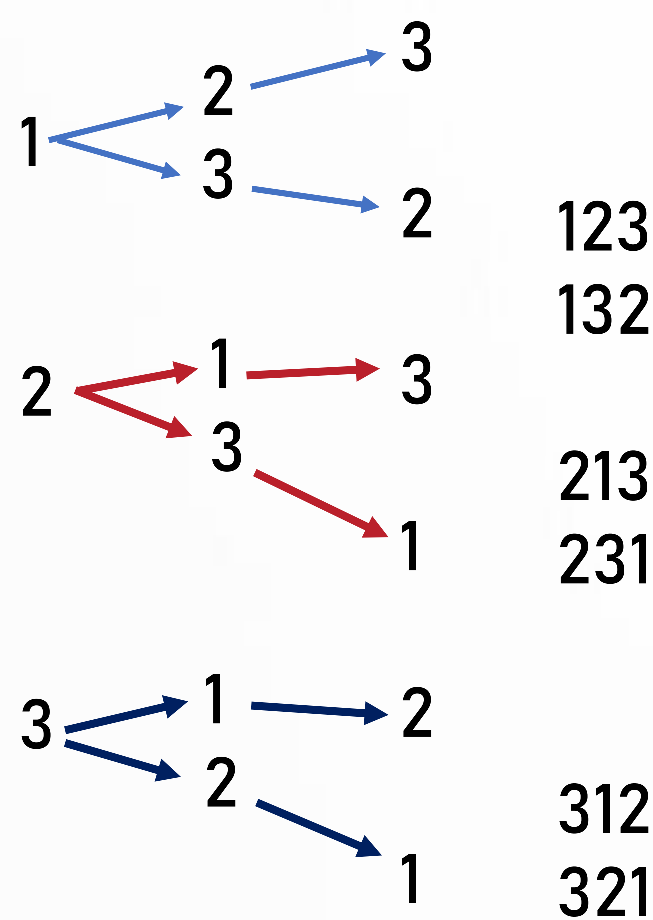
Basics of Counting – The Division Rule

There are n/d ways to do a task if it can be done using a procedure that can be carried out in n ways, and for every way w, exactly d of the n ways correspond to way w.

Basics of Counting – The Division Rule

$$\lfloor 3 = 3 * 2 * 1 = 6$$
$$\lfloor 4 = 4 * 3 * 2 * 1 = 24$$
$$\lfloor 5 = 5 * 4 * 3 * 2 * 1 = 120$$





3 choices, 2 Choices, 1 Choice

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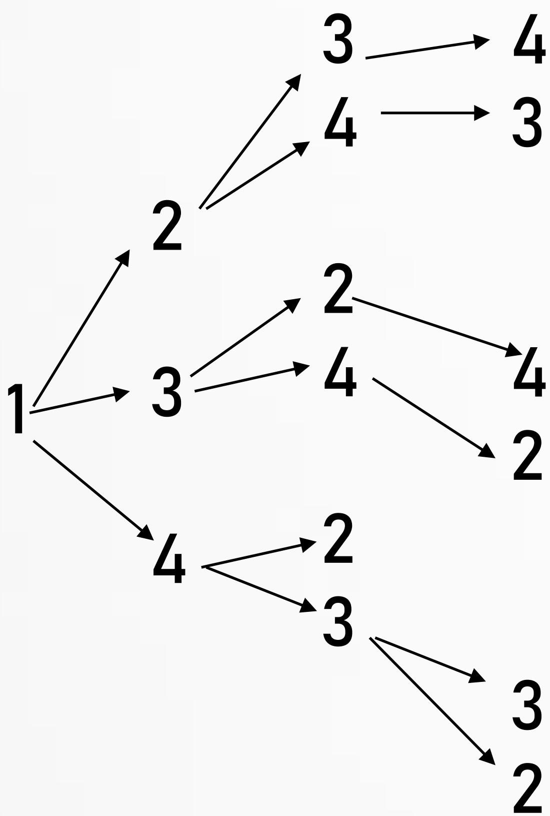
Basics of Counting – The Division Rule



1234	Similarly	Similarly	Similarly
1243	2134	3124	4123
1324	2143	3142	4132
1342	2314	3214	4213
1423	2341	3241	4231
1432	2431	3412	4321
	2413	3421	4312

4choices,3 choices, 2 Choices, 1 Choice

$$4 * 3 * 2 * 1 = 24 = \lfloor 4$$

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Basics of Counting – The Division Rule

$$\frac{\lfloor 4}{4} = \frac{4*3*2*1}{4} = 3*2*1 = 6$$

$$\frac{\lfloor 5}{5} = \frac{5*4*3*2*1}{5} = 4*3*2*1 = 24$$

$$\frac{\lfloor 7}{7} = \frac{7*6*5*4*3*2*1}{7} = 720$$

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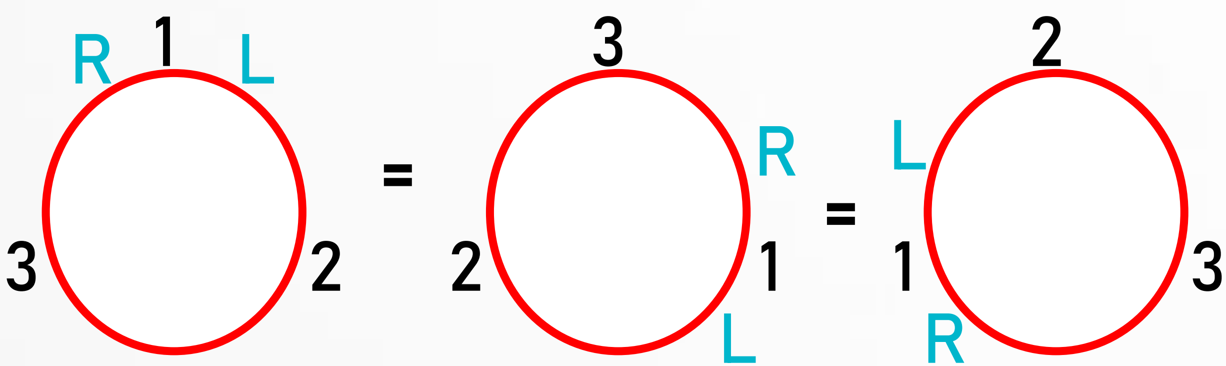
Basics of Counting – The Division Rule

Example: Of the 3!= 6 permutations of three objects, the (3-1)!= 2 distinct circular permutations are {1,2,3} and {1,3,2}.

123
132



213
231

312
321

$$\frac{\lfloor 3}{3} = \frac{3 * 2 * 1}{3} = 2 * 1 = 2$$

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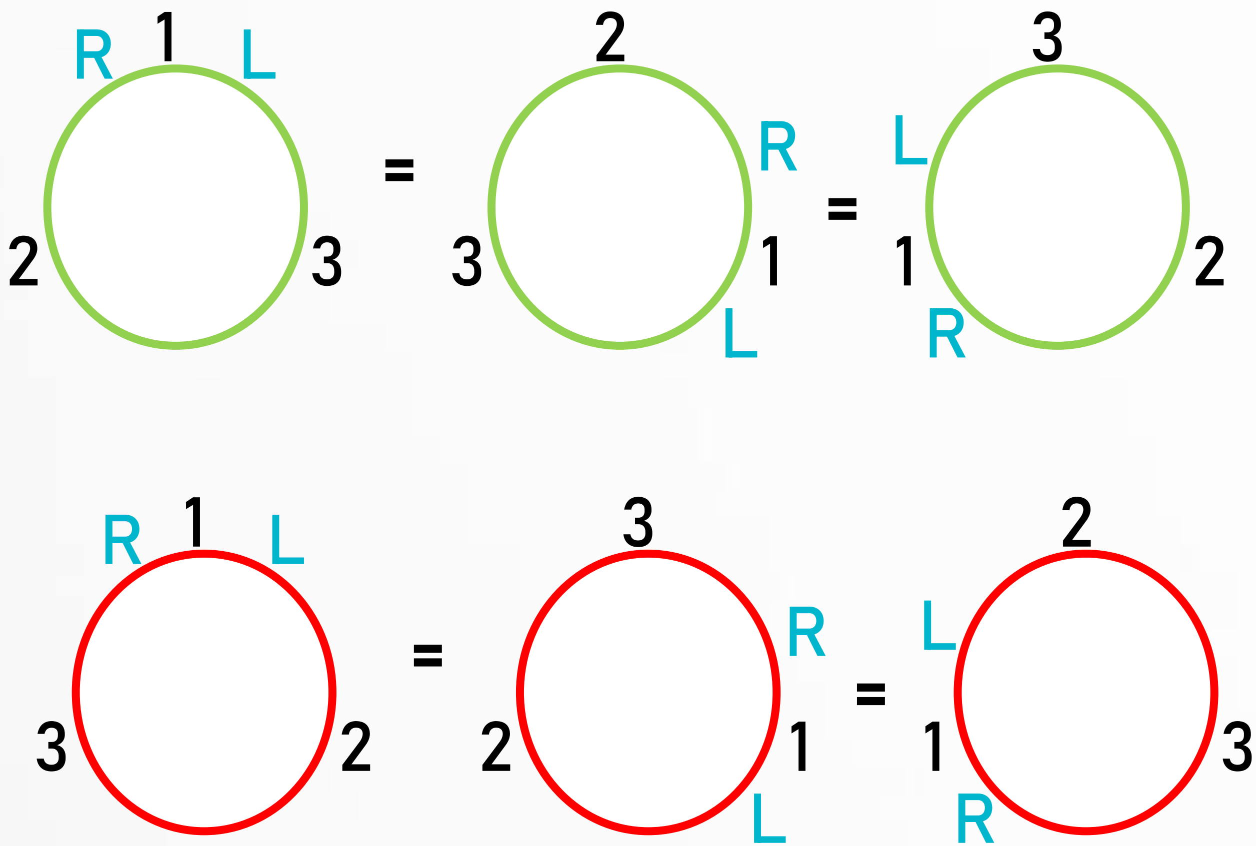
Basics of Counting – The Division Rule



123
231
312

132
213
321

$$\frac{\lfloor 3}{3} = \frac{3*2*1}{3} = 2*1=2$$

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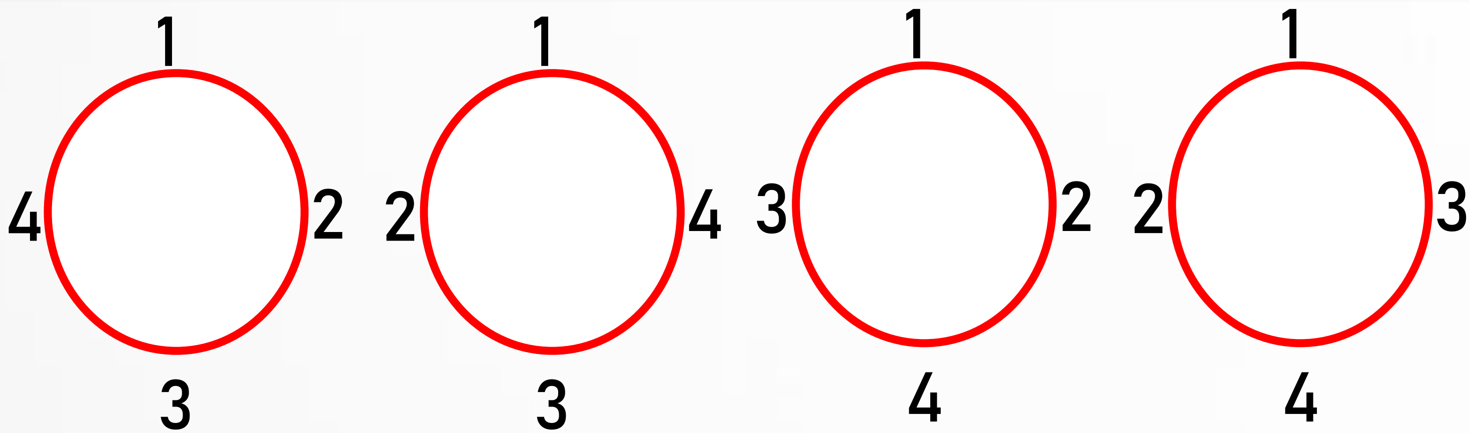
Basics of Counting – The Division Rule

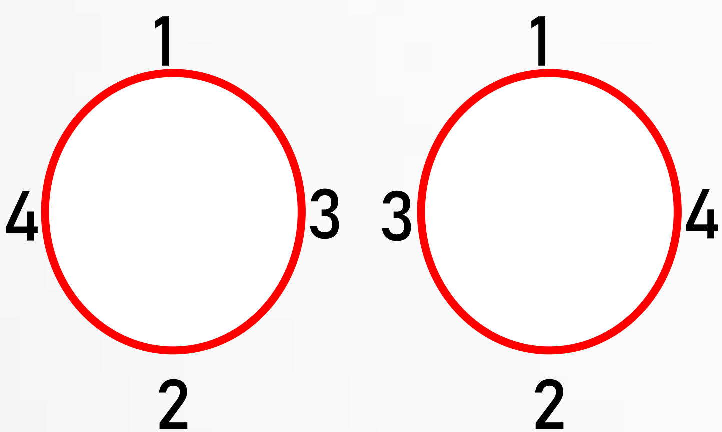
Ques:- How many different ways are there to seat four people around a circular table, where two seating's are considered the same when each person has the same left neighbor and the same right neighbor?

Ans:- There are 4! = 24 ways to order the given four people for these seats.

Because there are four ways to choose the person for seat 1, by the division rule there are "24" /4 = 6 different seating arrangements of four people around the circular table.

Basics of Counting – The Division Rule





	Repeated	Repeated	Repeated
1234	2134	3124	4123
1243	2143	3142	4132
1324	2314	3214	4213
1342	2341	3241	4231
1423	2431	3412	4321
1432	2413	3421	4312

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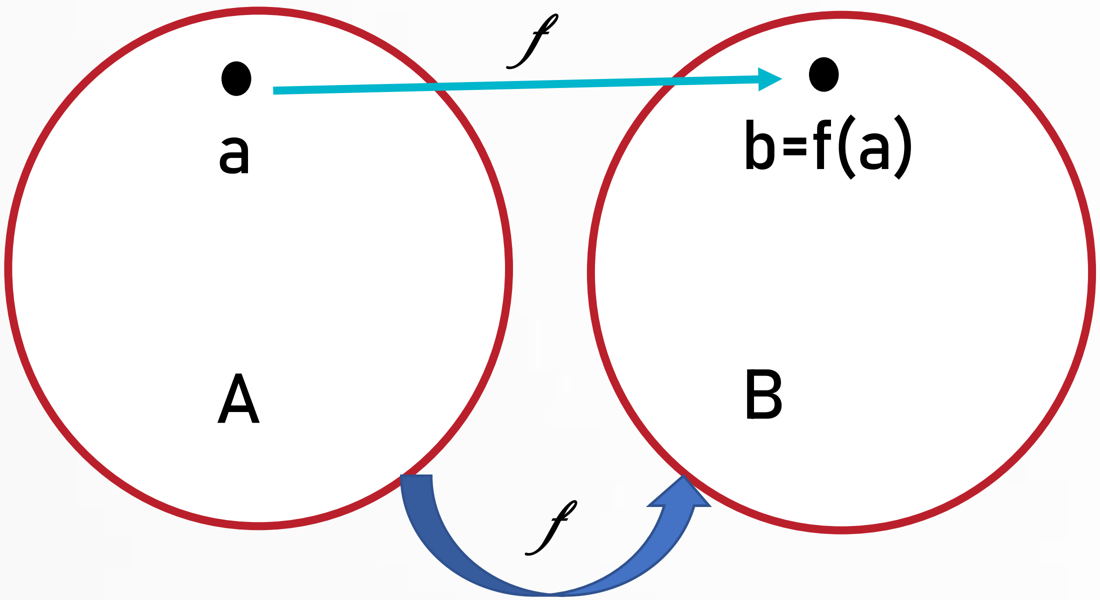
Basics of Counting – The Division Rule



The function f Maps A to B

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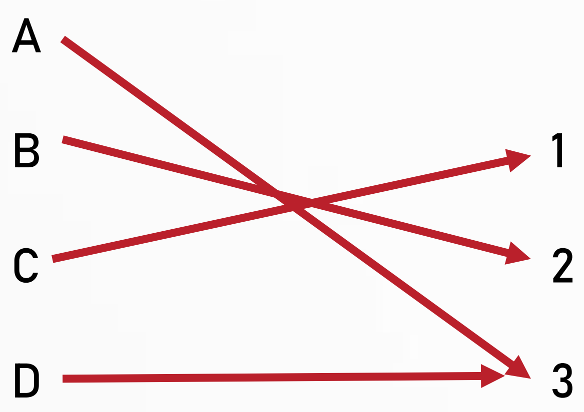
Different Types of Correspondences



An Onto function

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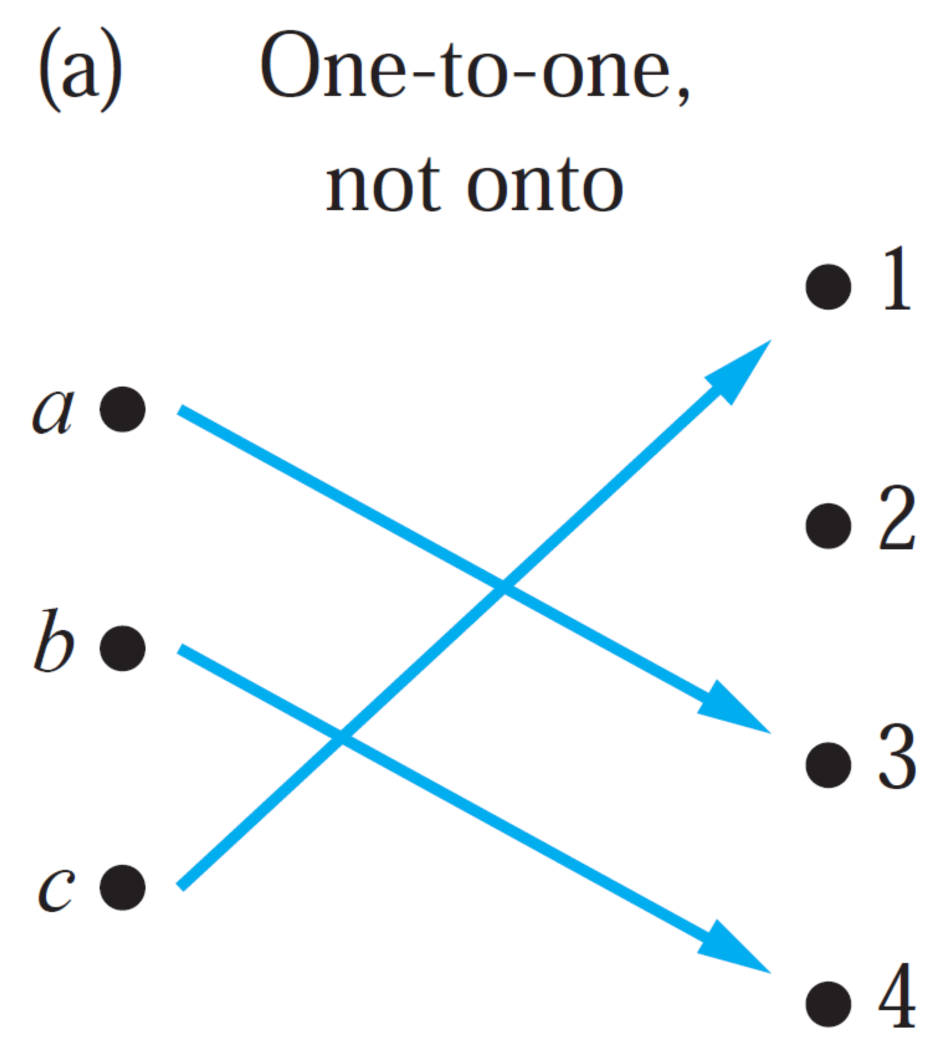
Different Types of Correspondences



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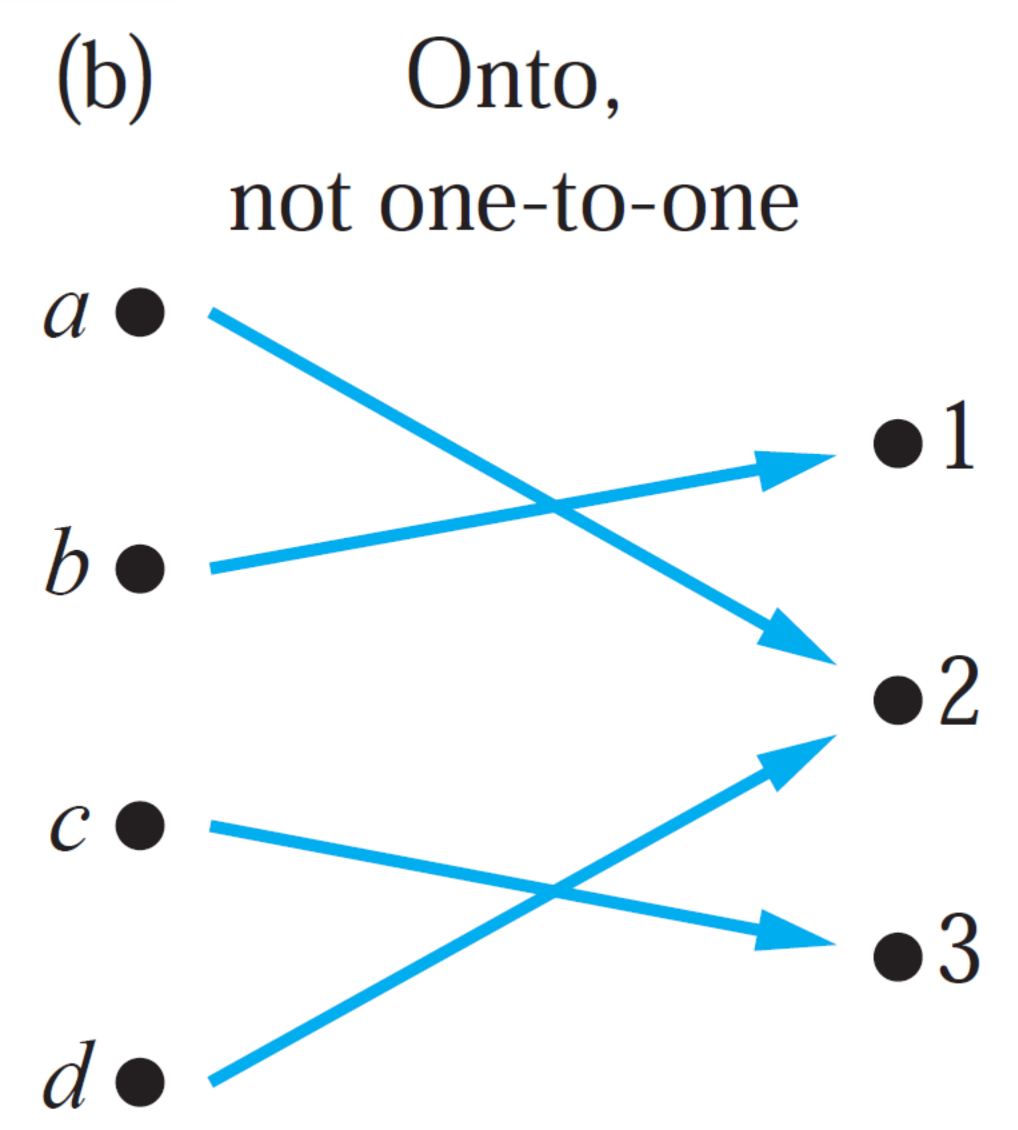
Different Types of Correspondences



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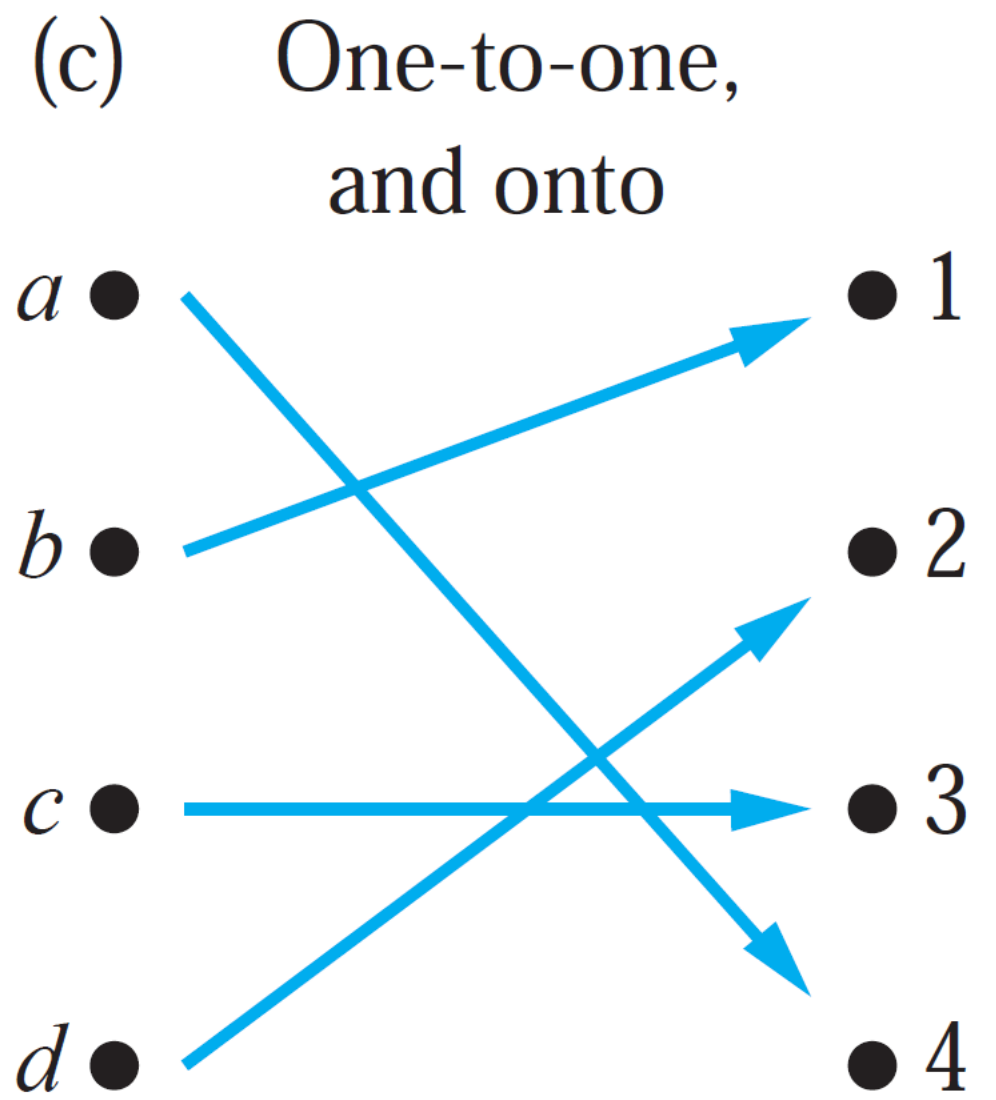
Different Types of Correspondences



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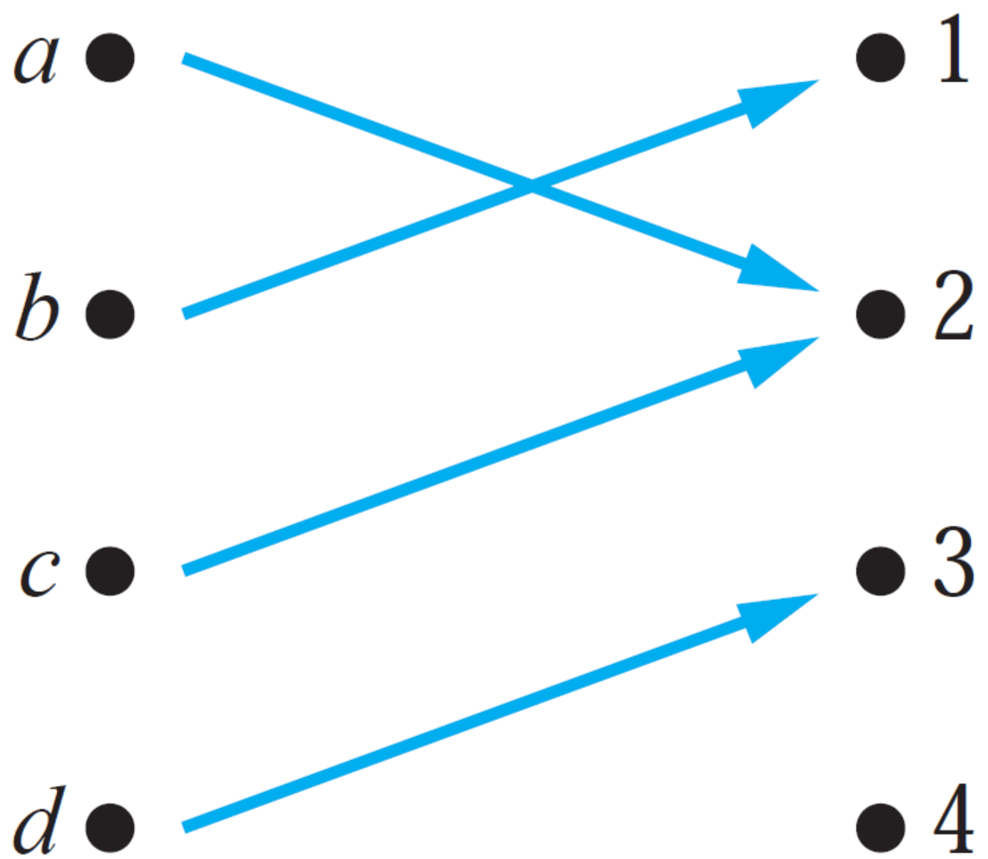
Different Types of Correspondences

(d) Neither one-to-one nor onto



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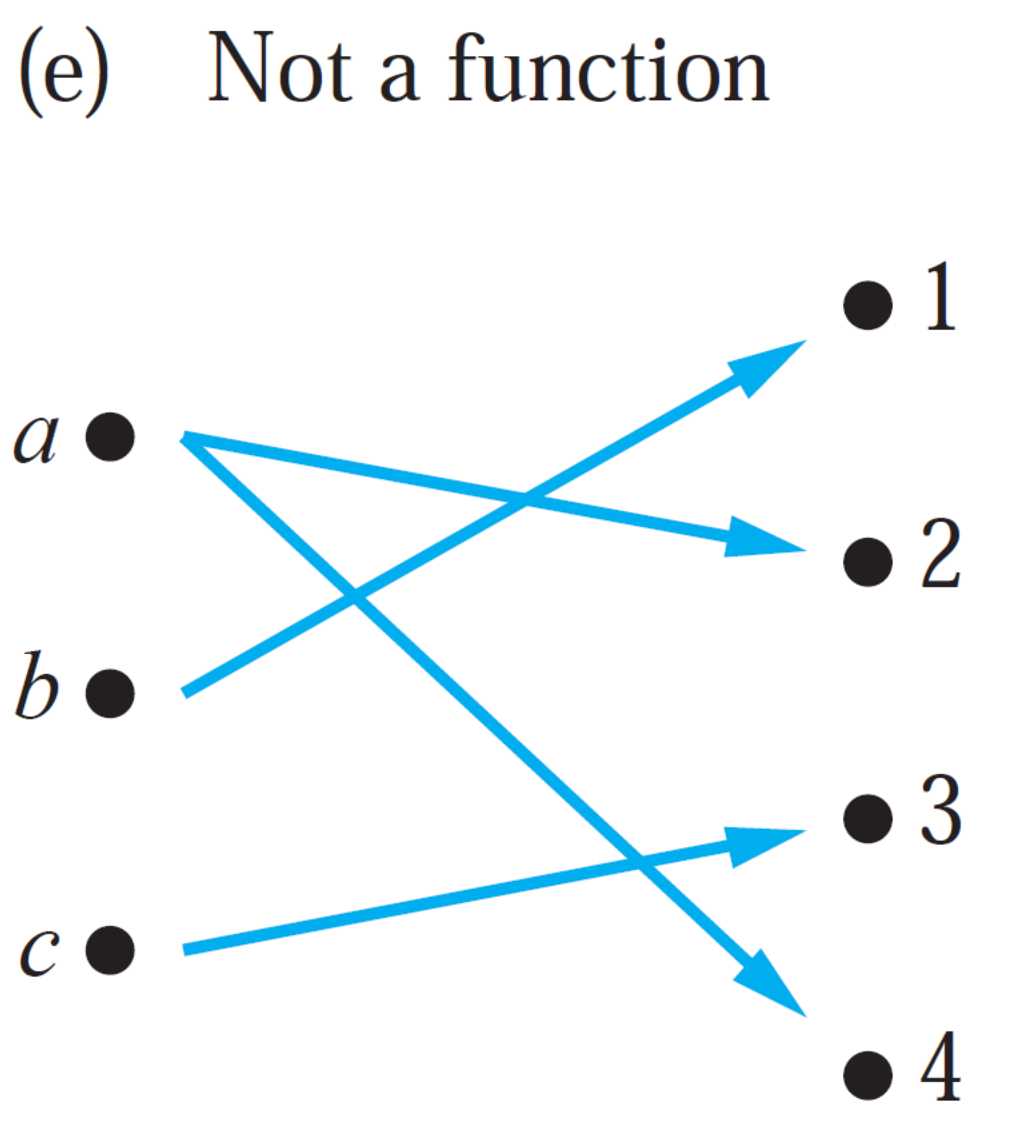
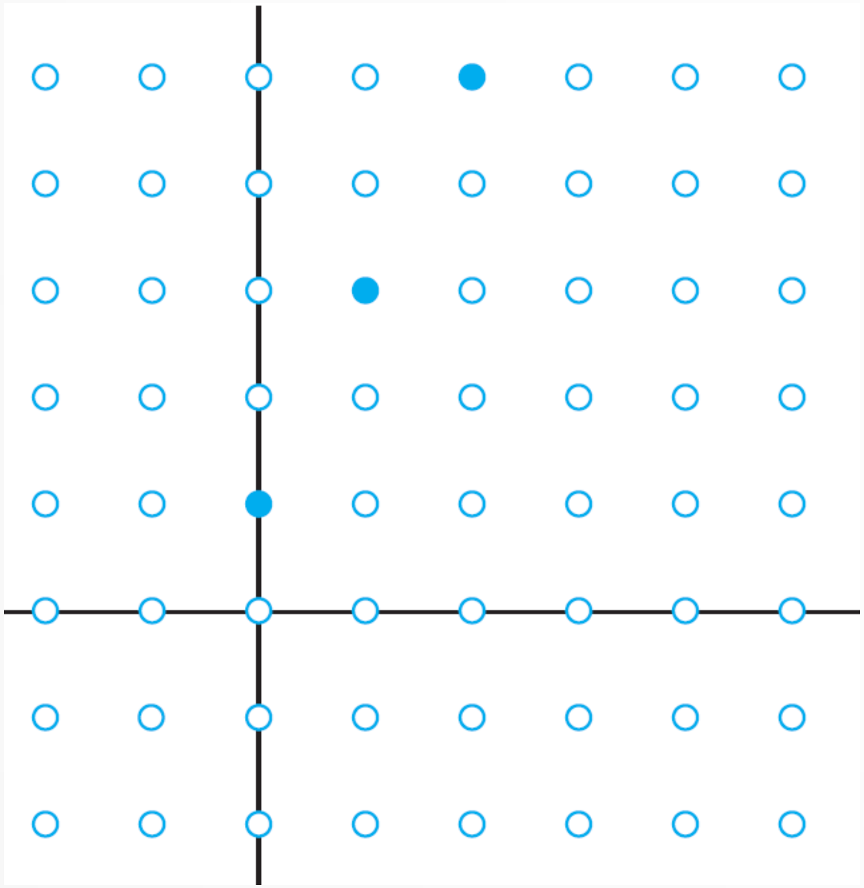
Different Types of Correspondences



Function - Example



$n \longrightarrow 2n+1$

$0 \longrightarrow 1$

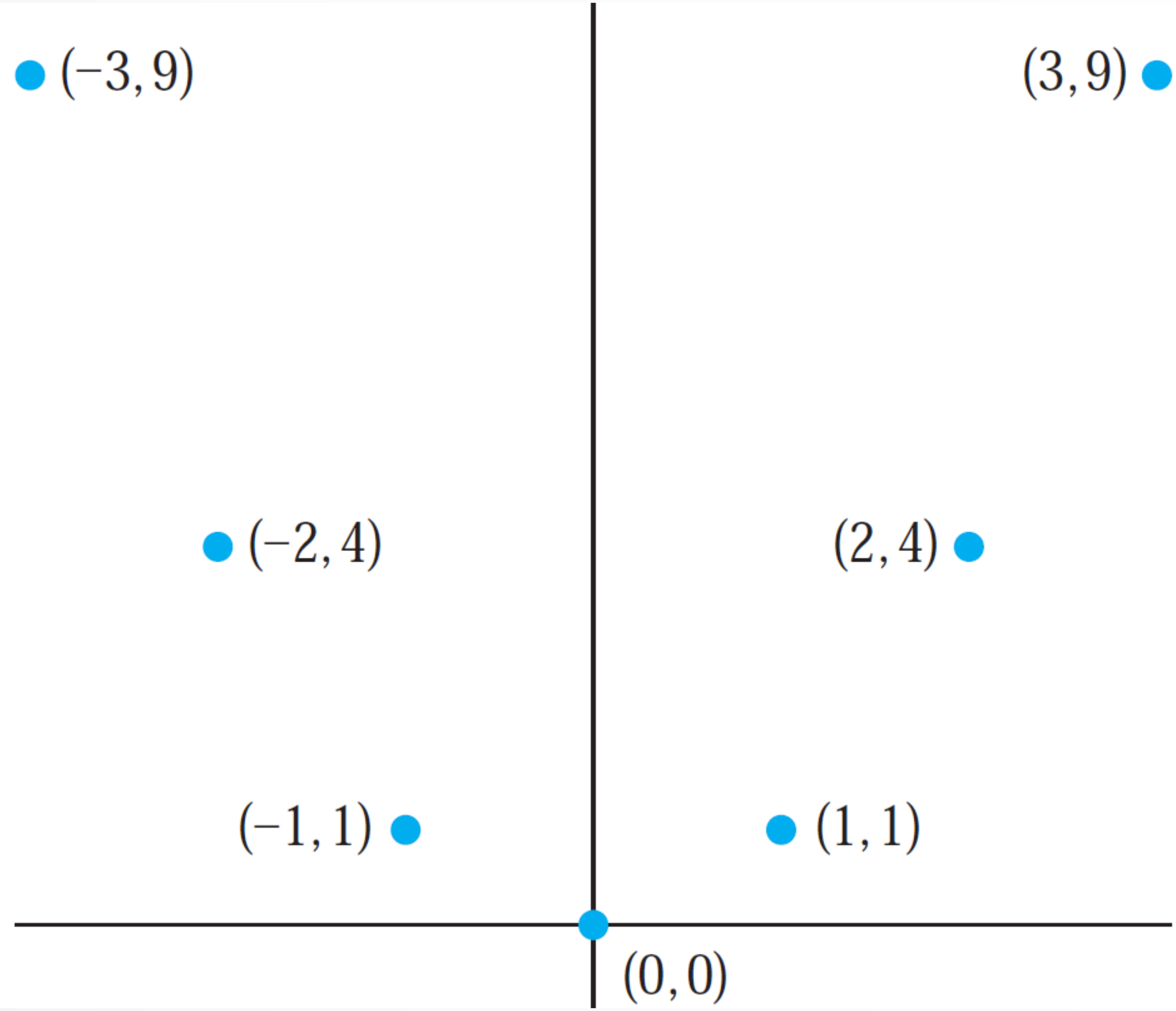
$1 \longrightarrow 3$

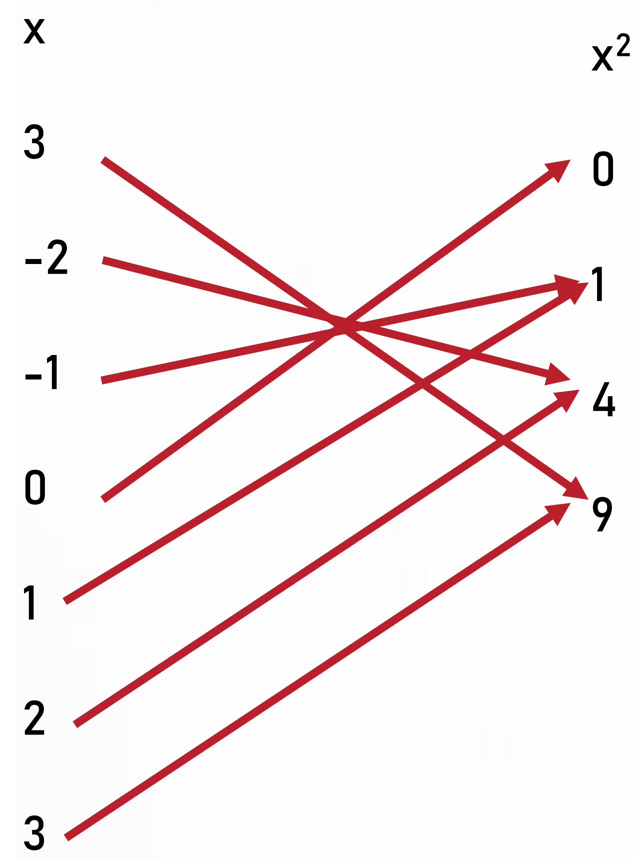
$2 \longrightarrow 5$

$3 \longrightarrow 7$

The Graph of f (n) = 2n + 1 from Z to Z.

Function - Example





The Graph of f (x) = x^2 from Z to Z.

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Counting Functions

Ques:- How many functions are there from a set with m elements to a set with n elements?

Ans:- By the product rule there are $n \cdot n \cdot \cdot \cdot \cdot \cdot n = n^m$ functions from a set with m elements to one with n elements.

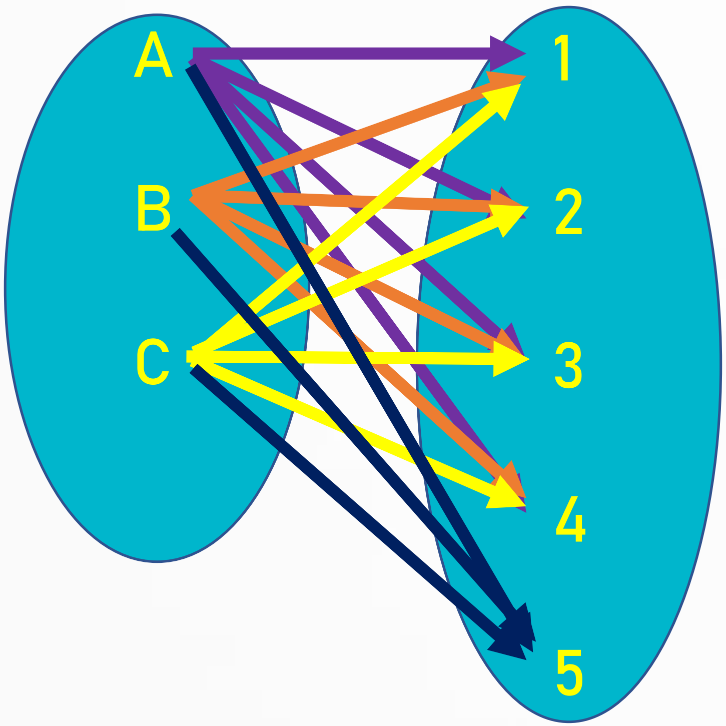
For example, there are $5^3 = 125$ different functions from a set with three elements to a set with five elements.

Counting Functions

5 Choices

5 Choices

5 Choices



There are $5^3 = 125$ different functions from a set with three elements to a set with five elements.

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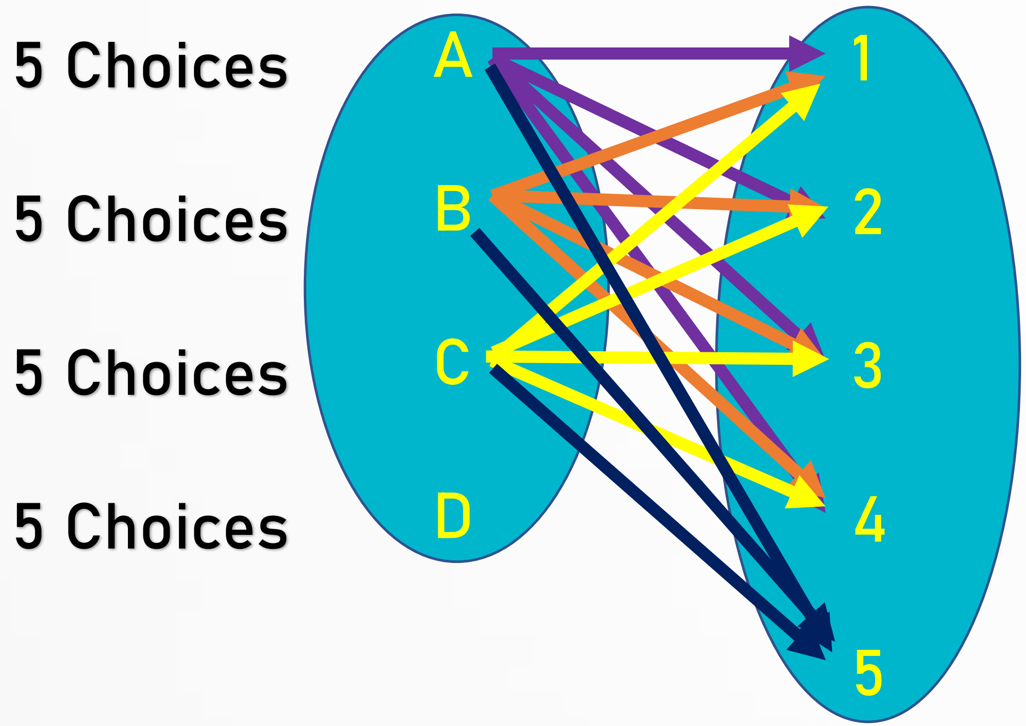
Counting Functions

Ques:- How many functions are there from a set with 4 elements to a set with 5 elements?

Ans:- There are $5^4 = 625$ different functions from a set with four elements to a set with five elements.

Counting Functions



There are $5^4 = 125$ different functions from a set with four elements to a set with five elements.

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Counting Functions

N Choices

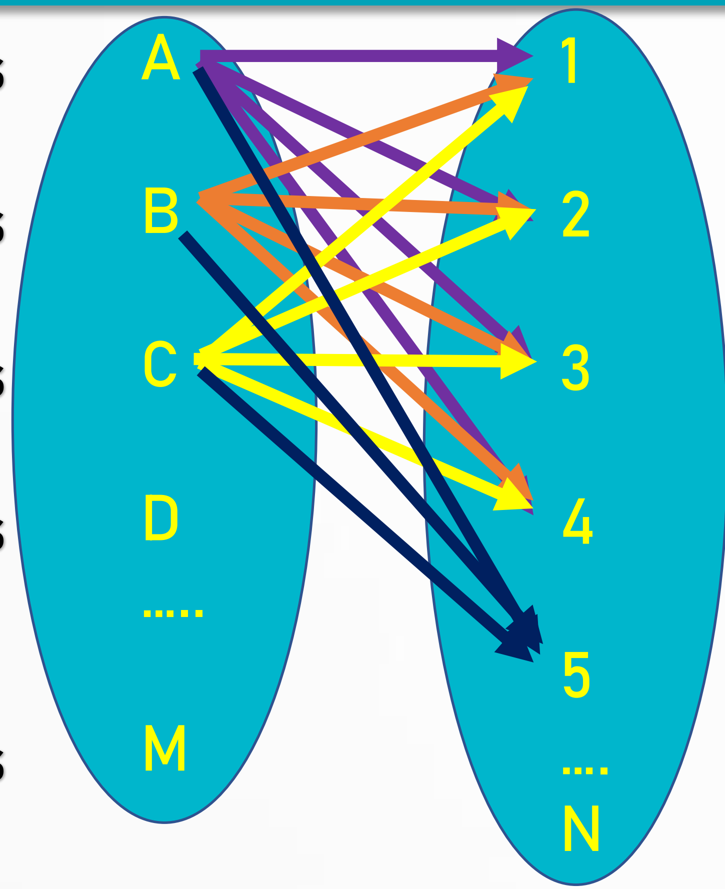
N Choices

N Choices

N Choices

....

N Choices



There are M^N different functions from a set with N elements to a set with M elements.

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Counting One-to-One Functions

Ques:- How many one-to-one functions are there from a set with m elements to one with n elements?

Ans:- By the product rule, there are $n(n - 1)(n - 2) \cdots (n - m + 1)$ one-to-one functions from a set with m elements to one with n elements.

For example, there are $5 \cdot 4 \cdot 3 = 60$ one-to-one functions from a set with three elements to a set with five elements.

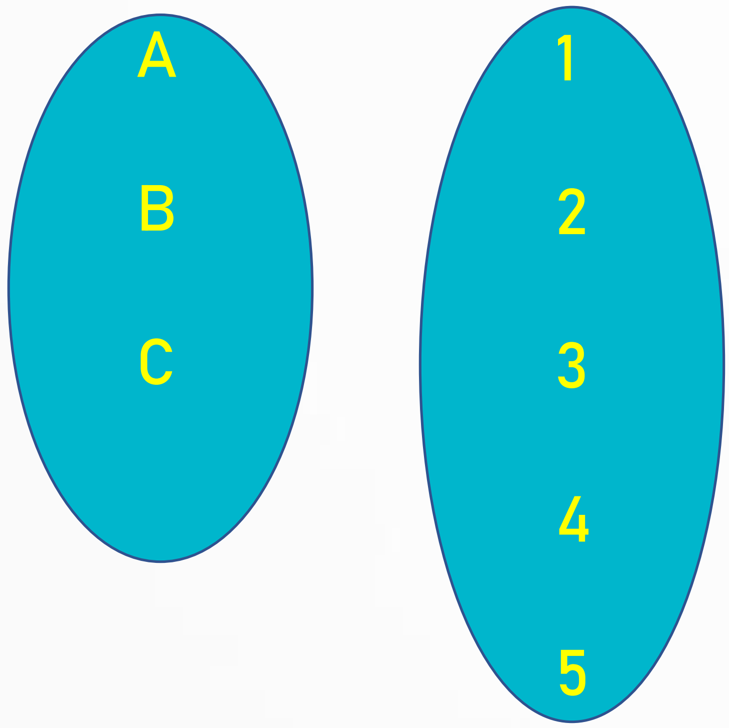
Counting One-to-One Functions

Cannot Repeat in 1-1 Mapping/Function

5 Choices

4 Choices

3 Choices



There are $5 \cdot 4 \cdot 3 = 60$ different functions from a set with three elements to a set with five elements.

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Counting One-to-One Functions

N Choices

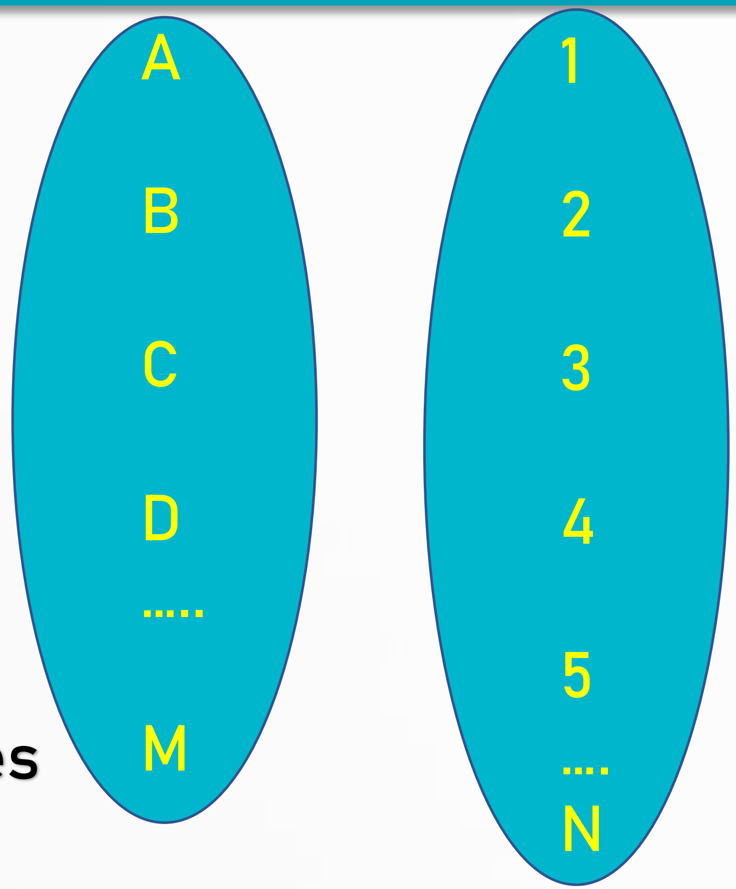
N-1 Choices

N-2 Choices

N-3 Choices

....

N-(M-1) Choices



There are $N(N - 1)(N - 2) \cdots (N - M + 1)$ different functions from a set with N elements to a set with M elements.

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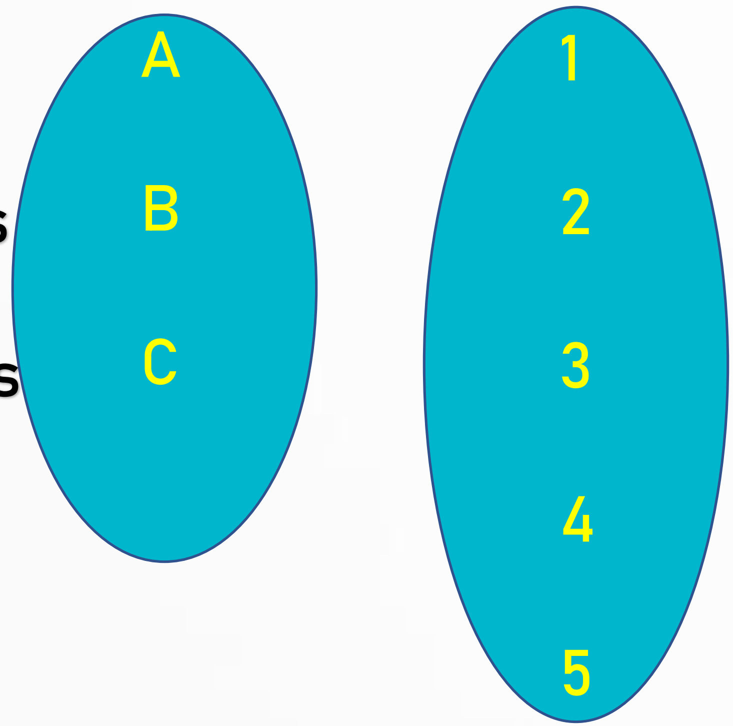
Counting One-to-One Functions

Cannot Repeat in 1-1 Mapping/Function

5 Choices

5-1 Choices

5-2 Choices



There are 5 · (5-1) · (5-2) = 5 · 4 · 3 = 60 different functions from a set with three elements to a set with five elements.

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That’s all for now…