ECAP770

Advance Data Structures

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Learning Outcomes

After this lecture, you will be able to

- Understand warshall's algorithm

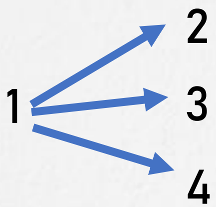
Floyd-Warshall Algorithm

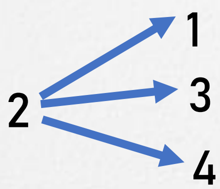
- The Floyd Warshall Algorithm is used for solving the All Pairs Shortest Path problem.
- The problem is to finding the shortest path between all the pairs of vertices in a weighted directed Graph.

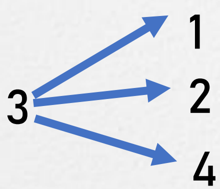
Floyd-Warshall Algorithm

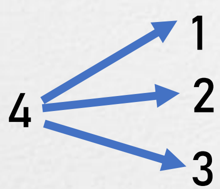
- This algorithm works for both the directed and undirected weighted graphs.
- Floyd-Warhshall algorithm follows the dynamic programming approach to find the shortest paths.
- Floyd-Warhshall algorithm is also called as Floyd's algorithm, Roy-Floyd algorithm, Roy-Warshall algorithm, or WFI algorithm.

Floyd-Warshall Algorithm

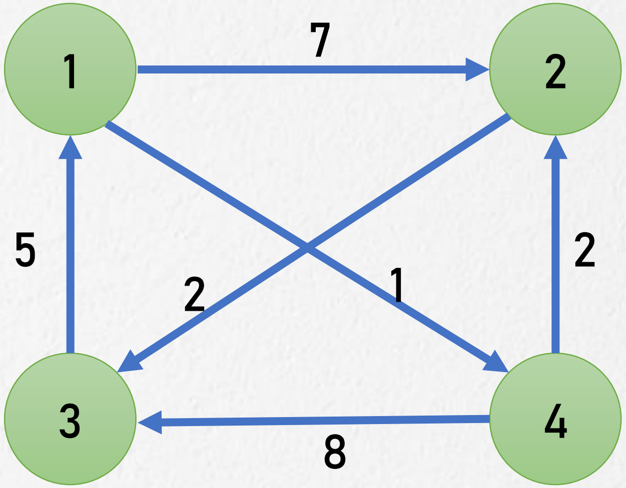




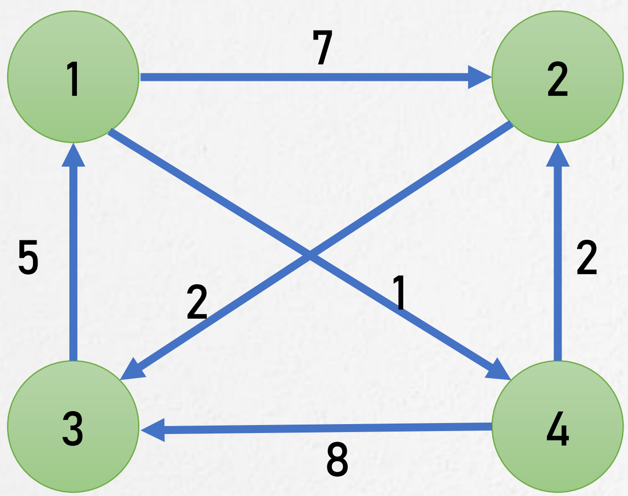




All pair shortest path

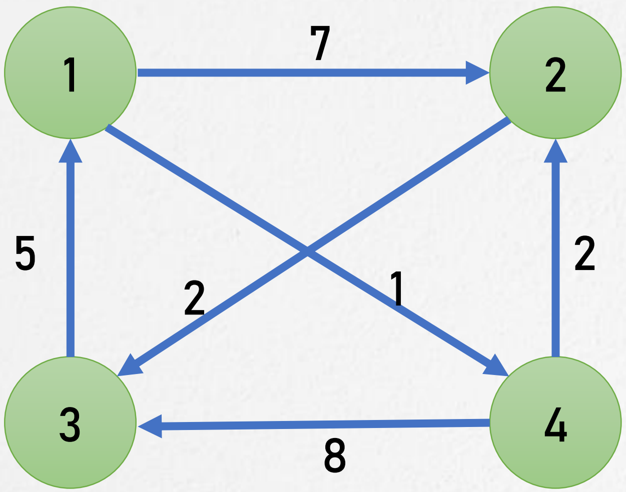


Floyd-Warshall Algorithm



$$D^0 = \begin{array}{c} \\ 12 \\ 3 \\ 4 \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 7 & \infty & 1 \\ \infty & 0 & 2 & \infty \\ 5 & \infty & 0 & \infty \\ \infty & 2 & 8 & 0 \end{bmatrix} \end{array}$$

D^1

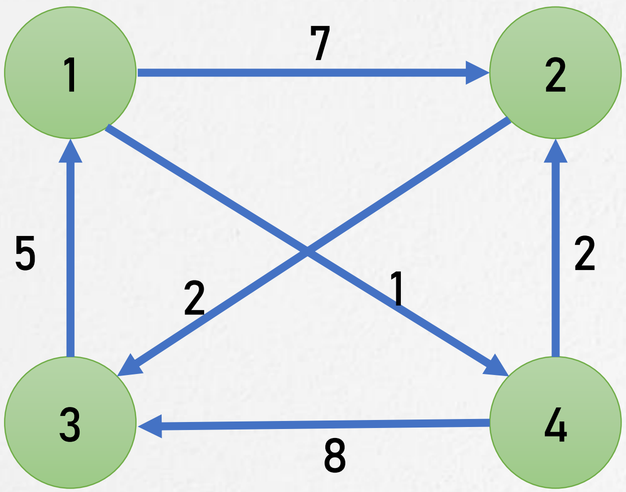


$$D^0 = \begin{array}{c} \\ 12 \\ 3 \\ 4 \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 7 & \infty & 1 \\ \infty & 0 & 2 & \infty \\ 5 & \infty & 0 & \infty \\ \infty & 2 & 8 & 0 \end{bmatrix} \end{array}$$

2 to 3= (2-1),(1-3) == 2
2 to 4= (2-1),(1-4) == ∞
3 to 2= (3-1),(1-2) == 5+7=>12
3 to 4= (3-1),(1-4) == 5+1=> 6
4 to 2= (4-1),(1-4) == 2
4 to 3= (4-1),(1-3) == 8

$$D^1 = \begin{array}{c} \\ 12 \\ 3 \\ 4 \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 7 & \infty & 1 \\ \infty & 0 & 2 & \infty \\ 5 & 12 & 0 & 6 \\ \infty & 2 & 8 & 0 \end{bmatrix} \end{array}$$

D^2

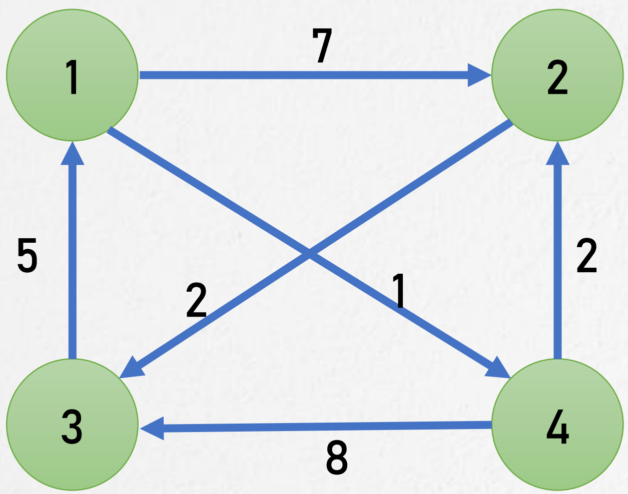


$$D^1 = \begin{array}{c} \\ 12 \\ 3 \\ 4 \\ \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 0 & 7 & \infty & 1 \\ \infty & 0 & 2 & \infty \\ 5 & 12 & 0 & 6 \\ \infty & 2 & 8 & 0 \end{array} \right] \end{array}$$

1 to 3= (1-2),(2-3) == 7+2=>9
1 to 4= (1-2),(2-4) == 1
3 to 1= (3-2),(2-1) == 5
3 to 4= (3-2),(2-4) == 6
4 to 1= (4-2),(2-4) == ∞
4 to 3= (4-2),(2-3) == 2+2=>4

$$D^2 = \begin{array}{c} \\ 12 \\ 3 \\ 4 \\ \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 0 & 7 & 9 & 1 \\ \infty & 0 & 2 & \infty \\ 5 & 12 & 0 & 6 \\ \infty & 2 & 4 & 0 \end{array} \right] \end{array}$$

D^3

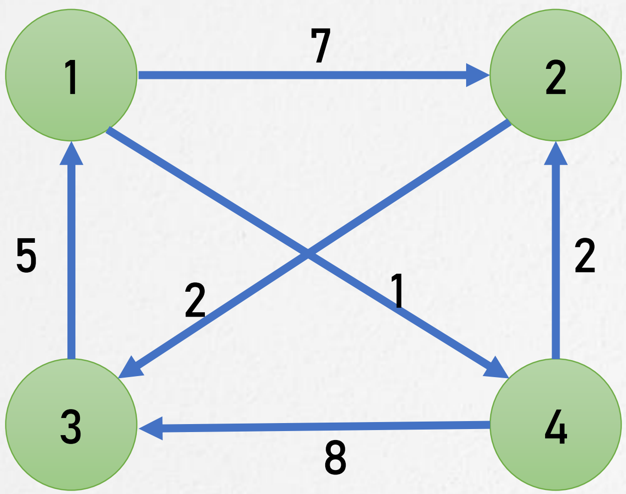


$$D^2 = \begin{array}{c} \\ 12 \\ 3 \\ 4 \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 0 & 7 & 9 & 1 \\ \infty & 0 & 2 & \infty \\ 5 & 12 & 0 & 6 \\ \infty & 2 & 4 & 0 \end{array}\right] \end{array}$$

1 to 2= (1-3),(3-2) == 7
1 to 4= (1-3),(3-4) == 1
2 to 1= (2-3),(3-1) == 7
2 to 4= (2-3),(3-4 (3-1)(1-4))==2+5+1=>8
4 to 1= (4-3(4-2)(2-3)) ,(3-1) == 2+2+5=>9
4 to 2= (4-3),(3-2) == 2

$$D^3 = \begin{array}{c} \\ 12 \\ 3 \\ 4 \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 0 & 7 & 9 & 1 \\ 7 & 0 & 2 & 8 \\ 5 & 12 & 0 & 6 \\ 9 & 2 & 4 & 0 \end{array}\right] \end{array}$$

D^4



$$D^3 = \begin{array}{c} \\ 12 \\ 3 \\ 4 \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 7 & 9 & 1 \\ 7 & 0 & 2 & 8 \\ 5 & 12 & 0 & 6 \\ 9 & 2 & 4 & 0 \end{bmatrix} \end{array}$$

1 to 2= (1-4),(4-2) == 1+2=>3

1 to 3= (1-4),(4-3(4-2)(2-3)) == 1+2+2=>5

2 to 1= (2-4),(4-1) == 7

2 to 3= (2-4),(4-3) == 2

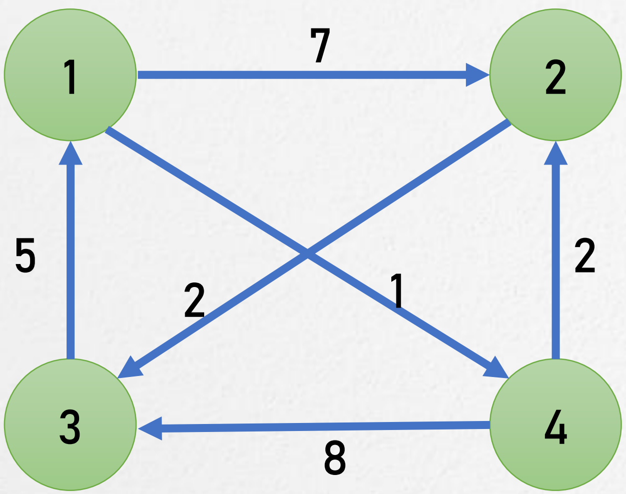
3 to 1= (3-4),(4-1) == 5

3 to 2= (3-4(3-1)(1-4)),(4-2) == 5+1+2=>8

$$D^4 = \begin{array}{c} \\ 12 \\ 3 \\ 4 \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & 5 & 1 \\ 7 & 0 & 2 & 8 \\ 5 & 8 & 0 & 6 \\ 9 & 2 & 4 & 0 \end{bmatrix} \end{array}$$

Floyd-Warshall Algorithm

All pair shortest path



$$D^4 = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 12 \\ 3 \\ 4 \\ \end{array} & \begin{bmatrix} 0 & 3 & 5 & 1 \\ 7 & 0 & 2 & 8 \\ 5 & 8 & 0 & 6 \\ 9 & 2 & 4 & 0 \end{bmatrix} \end{array}$$

Complexity

- Time complexity = O(|V|3)
- Space complexity = O(|V|2)

That’s all for now…