EMTH403

Mathematical Foundation for Computer Science

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Lecture Outcomes

After this lecture, you will be able to

- understand what is a pigeonhole principle.

- understand how to find total number of functions in the basics of counting.

The Pigeonhole Principle – Example 1

Ques:- Suppose that a flock of 13 pigeons flies into a set of 12 pigeonholes to roost.

Ans:- Because there are 13 pigeons but only 12 pigeonholes, a least one of these 12 pigeonholes must have at least two pigeons in it.

The Pigeonhole Principle

The Pigeonhole Principle – Theorem 1

If k is a positive integer and k + 1 or more objects

are placed into k boxes, then there is at least one

box containing two or more of the objects.

The Pigeonhole Principle – Example 2

Among any group of 367 people, there must be at least two with the same birthday, because there are only 366 possible birthdays.

The Pigeonhole Principle – Example 3

In any group of 27 English words, there must be at least two that begin with the same letter, because there are 26 letters in the English alphabet.

The Pigeonhole Principle – Example 4

Ques:- How many students must be in a class to guarantee that at least two students receive the same score on the final exam, if the exam is graded on a scale from 0 to 100 points?

Sol:- There are 101 possible scores on the final. The pigeonhole principle shows that among any 102 students there must be at least 2 students with the same score.

Combination – Example 1

A combination is a selection of objects in which order is not important. For instance, in a drawing for three identical prizes you would use combinations because the order of the winners would not matter.

If the prizes were different, then you would use permutations, because the order would matter. Count the possible combinations of 2 letters chosen from the list A, B, C, D.

Combination – Example 1

Sol:– List all of the permutations of two letters from the list A, B, C, D. Because order is not important in a combination, cross out any duplicate pairs.

AB   AC   AD   BA   BC   BD

CA   CB   CD   DA   DB   DC

There are 6 possible combinations of 2 letters from the list A, B, C, D.

Combination – Example 2

Count the possible combinations of three letters chosen from the list A, B, C, D, E.

Total possible combinations = 10

ABC   ABD   ABE   ACD   ACE   ADE   BCD   BCE   BDE
        CDE

ACB   ADB
BAC   BDA
BCA   BAD
CBA   DAB
CAB   DBA

The Pigeonhole Principle

The number of r-combinations of a set with n elements, where n is a nonnegative integer and r is an integer with 0 ≤ r ≤ n, equals

$$C(n, r) = \frac{n!}{r! \, (n - r)!}.$$

Combination – Example 3

Example: Use the formula to calculate the binomial coefficients

$_{10}C_5$, $_{10}C_0$ $0! = 1$

$_{10}C_5$

$$= \frac{10!}{(10-5)! \cdot 5!} = \frac{10!}{5! \cdot 5!} = \frac{(10 \cdot 9 \cdot 8 \cdot 7 \cdot 6) \cdot \cancel{5!}}{\cancel{5!} \cdot 5!}$$

$$= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$= 252$$

$_{10}C_0$

$$= \frac{10!}{(10-0)! \cdot 0!} = \frac{\cancel{10!}}{\cancel{10!} \cdot 0!} = \frac{1!}{0!} = \frac{1}{1}$$

$$= 1$$

Combination – Example 4

Example: Use the formula to calculate the binomial coefficients
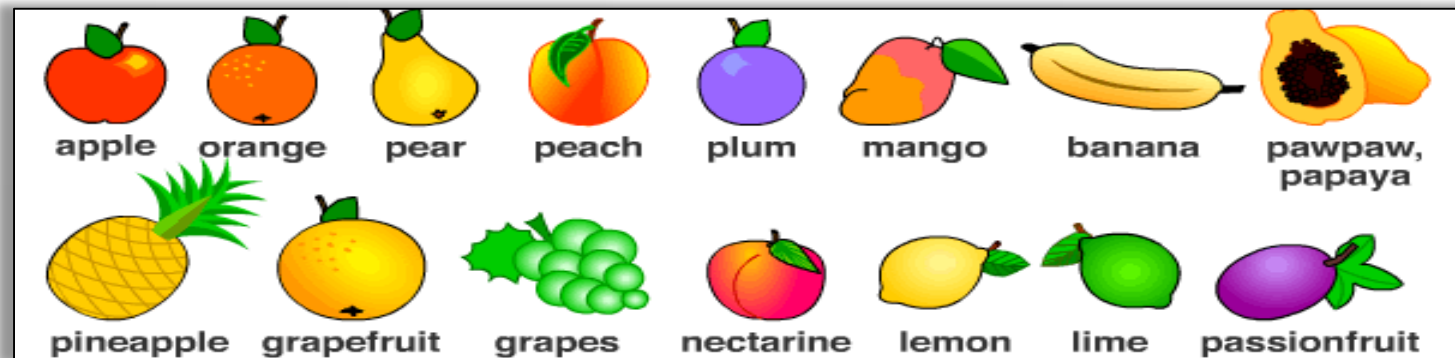$\binom{50}{48}$ and $\binom{12}{1}$.   0!=1

$$\binom{50}{48}$$

$$= \frac{50!}{(50-48)! \; \bullet \; 48!} \quad = \frac{50!}{2! \; \bullet \; 48!} = \frac{(50 \; \bullet \; 49) \; \bullet \; 48!}{2! \; \bullet \; 48!}$$

$$= \frac{50 \; \bullet \; 49}{2 \; \bullet \; 1} = 1225$$

Example: Use the formula to calculate the binomial coefficients $\binom{50}{48}$ and $\binom{12}{1}$.   0!=1

$$\binom{12}{1} = \frac{12!}{(12-1)! \cdot 1!} \quad = \frac{12!}{1! \cdot 1!} = \frac{12 \cdot 11!}{11! \cdot 1!}$$

$$= \frac{12}{1} = 12$$

Ques:- How many ways can a combination of 4 different fruit can be chosen from a set of 15?

Combination – Example 7

Ques:- How many ways can a combination of 4 different fruit can be chosen from a set of 15?

Sol:- $C(n, r) = \dfrac{n!}{r! \, (n-r)!}$

$= \dfrac{15!}{4! \; 11!} = \dfrac{1307674368000}{958003200} = 1365$

There are 1365 different ways 4 fruits can be chosen.

Combination – Example 8

Ques:- Count the possible combinations of 2 letters chosen from the list A, B, C, D.

Sol:- AB   AC   AD   BA   BC   BD

        CA   CB   CD   DA   DB   DC

{AB}, {AC}, {AD}, {BC}, {BD}, and {CD}.

There are 6 possible combinations of 2 letters from the list A, B, C,

D. $C(n, r) = \dfrac{n!}{r! \, (n - r)!} = \dfrac{4!}{2! \, 2!} = \dfrac{24}{2.2} = 6$

Combination – Example 9

Ques:- How many different committees of three students can be formed from a group of four students?

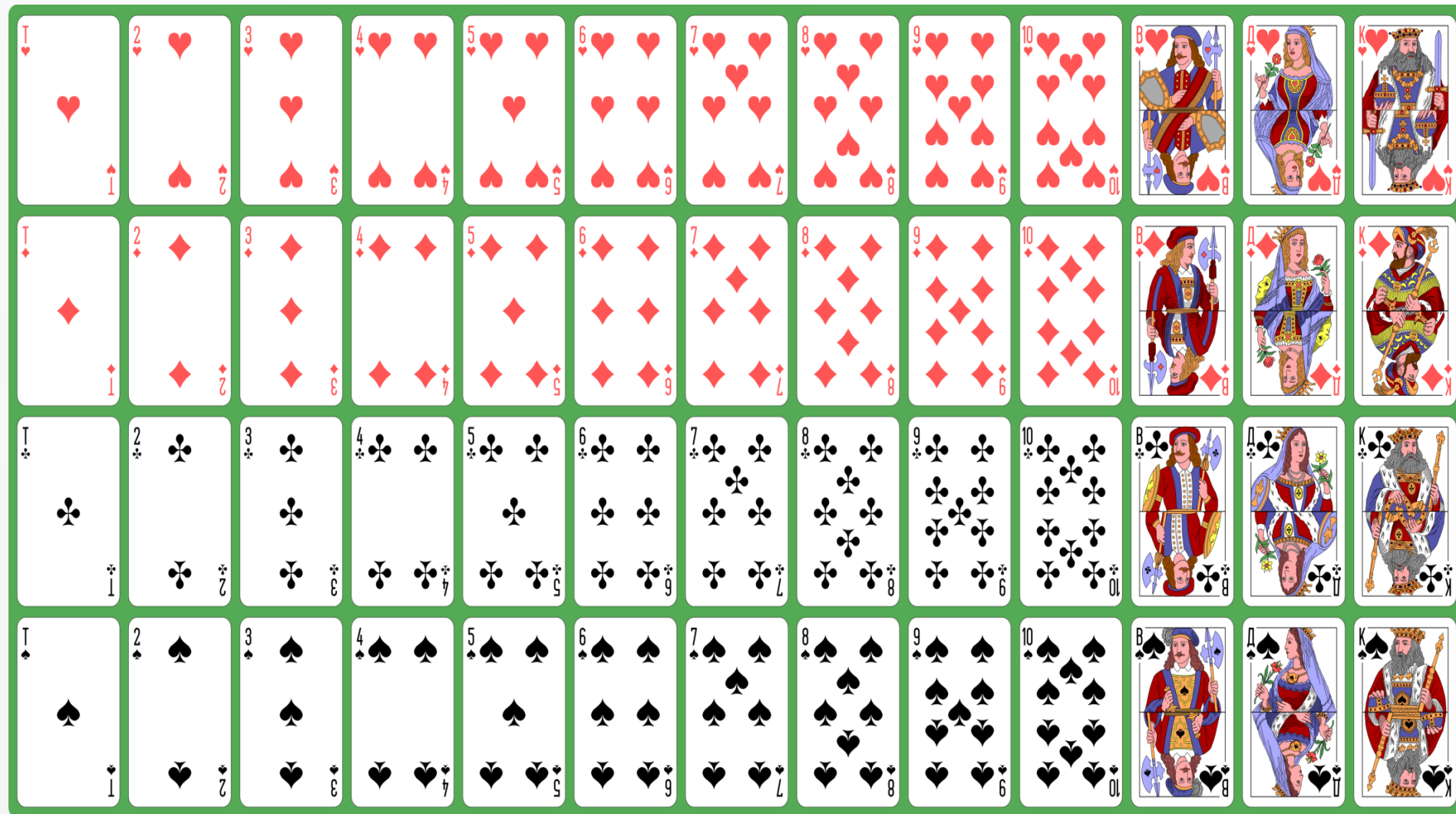Sol:- $C(n, r) = \dfrac{n!}{r! \, (n - r)!} = \dfrac{4!}{1! \; 3!} = \dfrac{24}{1.6} = 6.$
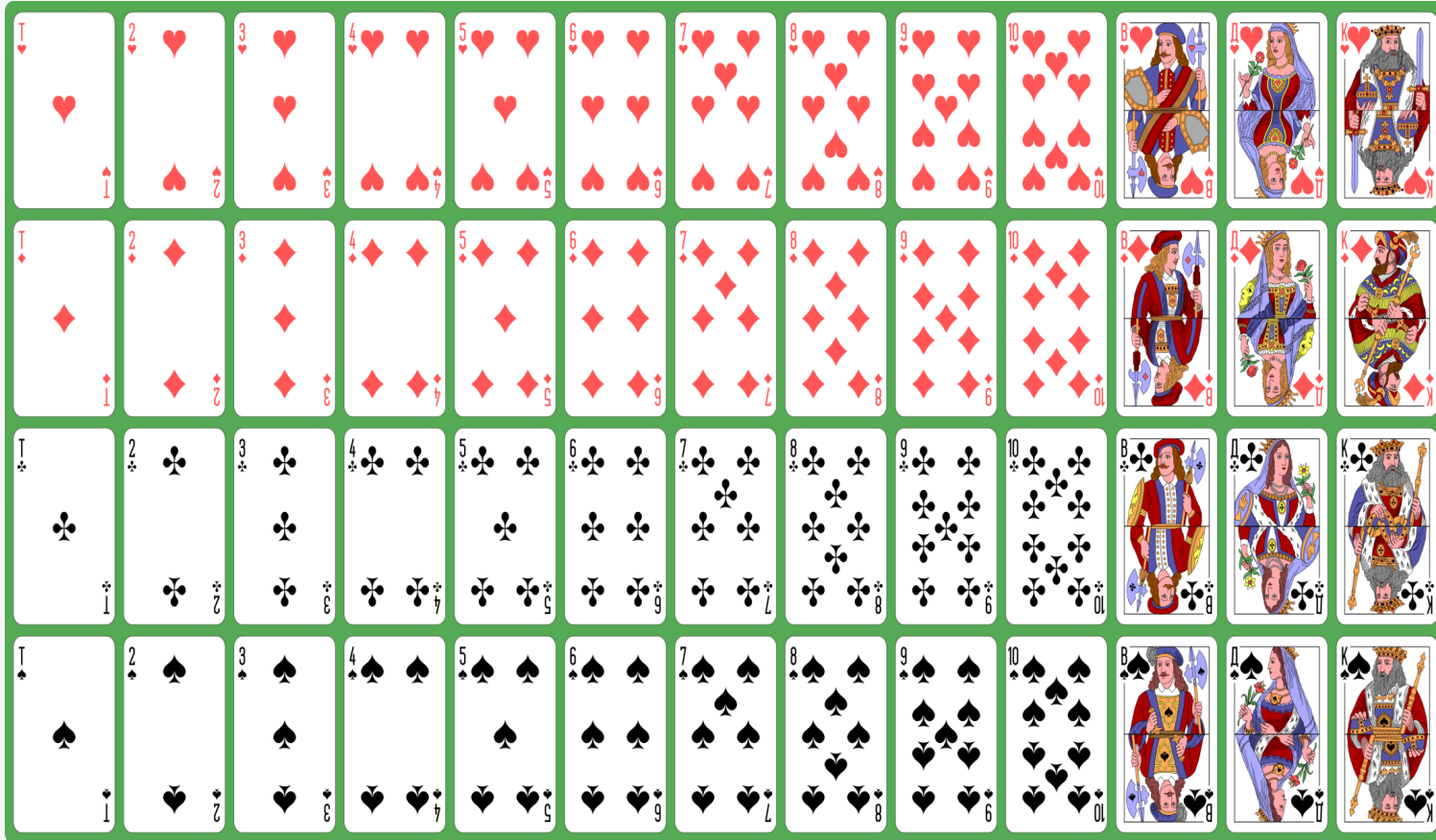
Committee 1 = {S1, S2, S3}

Committee 2 = {S1, S2, S4}

Committee 3 = {S1, S3, S4}

Committee 4 = {S2, S3, S4}

Combination – Example 10

Total = 52
Black = 26
Red = 26

Heart = 13, Diamond = 13, Club = 13, Spade = 13

King = 4, Queen =4, Jack = 4

1 to 10 4 each.

Combination – Example 10

Ques:- How many hands of five cards can be dealt from a standard deck of 52 cards?

Sol:- Because the order in which the five cards are dealt from a deck of 52 cards does not matter, there

are   C(52, 5) = = $\dfrac{52!}{5!\ 47!}$ = $\dfrac{52.51.50.49.48.47!}{5.4.3.2.1.\ 47!}$ =

6 2,598,960.

That's all for now...