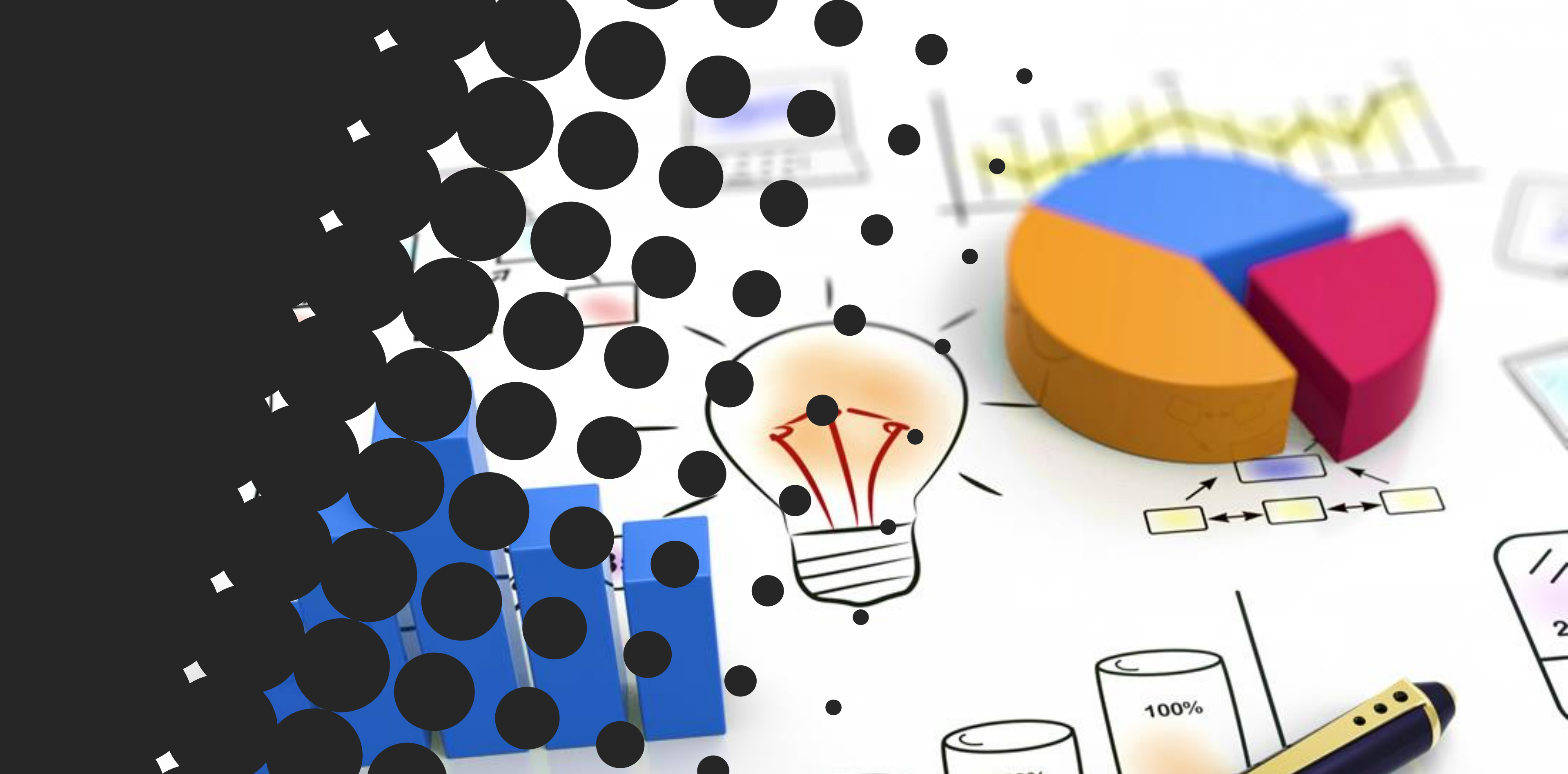

EPEA516
ANALYTICAL SKILLS II

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Learning Outcomes

After this lecture, you will be able to

- solve problems based on arithmetic and geometric progression.

Problem 1

- Is the sequence given by T_{n} = 12n - 9 form an A. P.?
- T_{n} = 12n - 9
- $T_{\mathrm{n}-1}$ = 12(n-1) - 9
- $T_{\mathrm{n}-1}$ = 12n – 12 - 9
- $T_{\mathrm{n}-1}$ = 12n - 21
- T_{n} - $T_{\mathrm{n}-1}$ = (12n - 9) – (12n – 21)

  = $\cancel{12n}$ - 9 – $\cancel{12n}$ + 21

  = 12

Problem 2

- Find the 15^{th} term of the sequence 12, 7, 2, - 3, - 8, ...
- First term = a = 12
- Common Difference = d = - 5
- n = 15

$$T_n = a + (n - 1)\, d$$

$$T_{15} = 12 + (15 - 1)\,(-5)$$

$$T_{15} = 12 - 70$$

$$T_{15} = -\,58$$

Problem 3

- Find the sum of the series 5 + 9 + 13 + … to 50 terms.
- a = 5, d = 9 – 5 = 4, and n = 50

$$S_n = \frac{n}{2}\{2a + (n - 1)\, d\}$$

$$S_{50} = \frac{50}{2}\{2 \times 5 + (50 - 1) \times 4\}$$

$$S_{50} = 25\{10 + (49) \times 4\}$$

$$S_{50} = 25\{10 + 196\}$$

$$S_{50} = 25\{206\}$$

$$S_{50} = 5150$$

Problem 4

- Find the sum of 30 terms of an A.P., whose first and last terms are 25 and 216 respectively.
- a = 25, l = 216, and n = 30

$$S_n = \frac{n}{2}\{a + l\}$$

$$S_{30} = \frac{30}{2}\{25 + 216\}$$

$$S_{30} = 15\{241\}$$

$$S_{30} = 3615$$

Problem 5

- Find the sum of 20 terms of an A.P., whose common difference and last term are 7 and 93 respectively.

- d = 7, l = 93, and n = 20

$$S_n = \frac{n}{2}[2l - (n - 1) d]$$

$$S_{20} = \frac{20}{2}[2 \times 93 - (20 - 1) \times 7]$$

$$S_{20} = 10 [186 - 133]$$

$$S_{20} = 10 [53]$$

$$S_{20} = 530$$

Problem 6

- If sum of n terms of an A. P. is given by S_n = 100 n^2 + 25, then find its nth term.
- $S_n = 100\ n^2 + 25$
- $S_{n-1} = 100\ (n-1)^2 + 25$

$$\boldsymbol{T_n = S_n - S_{n-1}}$$

$$T_n = 100\ n^2 + 25 - [100\ (n-1)^2 + 25]$$

$$T_n = 100\ n^2 + 25 - [100(n^2 - 2n + 1) + 25]$$

$$T_n = 100\ n^2 + 25 - [100n^2 - 200n + 100 + 25]$$

$$T_n = \cancel{100\ n^2} + \cancel{25} - \cancel{100n^2} + 200n - 100 - \cancel{25}$$

$$T_n = 200n - 100$$

Problem 7

- Determine x so that x + 2, 4x - 6 and 3x - 2 are the three consecutive terms of an A.P.
- Three numbers a, b, c are in A.P. iff b - a = c - b iff a + c = 2b
- x + 2, 4x – 6, and 3x – 2

a b c

$$x + \cancel{2} + 3x - \cancel{2} = 2(4x - 6)$$

$$4x = 8x - 12$$

$$12 = 8x - 4x$$

Problem 7

$$12 = 8x - 4x$$

$$\cancel{4}x = \cancel{12}\ 3$$

$$x = 3$$

Three consecutive terms are $x + 2$, $4x - 6$ and $3x - 2$

i.e., $3 + 2$, $4(3) - 6$ and $3(3) - 2$

i.e., 5, 6 and 7

Problem 8

- Find the 7th term from the end of the A. P. of 50 terms whose first term and common difference are 5 and 8 respectively.

- n^{th} term from the end = $(m - n + 1)^{th}$ term from beginning

$$T_{m-n+1} = a + (m - n)\, d$$

- a = 5, d = 8, m = 50, and n = 7

$$T_{50-7+1} = 5 + (50 - 7) \times 8$$

$$T_{44} = 5 + (43) \times 8$$

$$T_{44} = 5 + 344$$

$$T_{44} = 349$$

Problem 9

- Is the sequence 3, -6, 12, -24, ……. form a G. P.?
- T_1 = 3, T_2 = -6, T_3 = 12, T_4 = -24, T_5 = 48, ….
- $\frac{T_2}{T_1} = \frac{-6}{3} = -2$
- $\frac{T_3}{T_2} = \frac{12}{-6} = -2$
- $\frac{T_4}{T_3} = \frac{-24}{12} = -2$
- $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = -2$ (G.P.)

Problem 10

- Find the $\boxed{9^{th}}$ term of the sequence $\boxed{2,}$ 4, 8, 16, 32 ....
- First term = a = 2
- Common ratio = r = $\frac{4}{2}$ = 2
- n = 9

$$T_n = ar^{n-1}$$

$$T_9 = (2)\ 2^{9-1}$$

$$T_9 = (2)\ 2^{8} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$T_9 = 512$$

Problem 11

- Find the sum of $\boxed{7}$ terms of the series $\boxed{\frac{1}{4}} + \frac{1}{2} + 1 \ldots\ldots\ldots$

- $a = \frac{1}{4}$, $r = \frac{\frac{1}{2}}{\frac{1}{4}}$ or $\frac{1}{\frac{1}{2}}$ = 2 (> 1), and n = 7

$$S_n = \frac{a(r^n - 1)}{(r - 1)} = \frac{\frac{1}{4}(2^7 - 1)}{(2 - 1)}$$

$$S_n = \frac{\frac{1}{4}(128 - 1)}{1}$$

$$S_n = \frac{127}{4}$$

Problem 12

- Find the sum of $\boxed{12}$ terms of a G.P., whose first term, common ratio, and last terms are $\boxed{3,}$ $\boxed{(-2)}$ and $\boxed{48}$ respectively.
- a = 3, r = -2, l = 48, and n = 12

$$S_n = \frac{lr - a}{r - a}$$

$$S_n = \frac{48 \text{ x } (-2) - 3}{-2 - 3}$$

$$S_n = \frac{(-96) - 3}{-5} = \frac{\cancel{-}99}{\cancel{-}5}$$

$$S_n = \frac{99}{5}$$

Problem 13

- Find the sum of $\boxed{5}$ terms of a G.P., whose first term, and common ratio are $\boxed{3}$ and $\boxed{(-2)}$ respectively.

- a = 3, r = -2 (< 1), and n = 5

$$S_n = \frac{a(1 - r^n)}{(1 - r)}$$

$$S_5 = \frac{3(1 - (-2)^5)}{(1 - (-2))}$$

$$S_5 = \frac{3(1 - (-32))}{(1 - (-2))} = \frac{3(1 + 32)}{(1 + 2)} = \frac{\cancel{3}(33)}{(\cancel{3})}$$

$$S_5 = 33$$

Problem 14

- Find the sum of the infinite sequence $\boxed{9,}$ – 3, 1, $\frac{-1}{3}$, ...
- a = 9, r = $\frac{\cancel{-3}}{{}_{3}\cancel{9}}$ = $\frac{-1}{3}$ where $\left|\frac{-1}{3}\right| < 1$ i.e., $\frac{-1}{3} < r < \frac{1}{3}$

$$S_\infty = \frac{a}{(1 - r)}$$

$$S_\infty = \frac{9}{(1 -(\frac{-1}{3}))}$$

$$S_\infty = \frac{9}{(1 + \frac{1}{3})} = \frac{9}{(\frac{3 + 1}{3})} = \frac{9 \text{ x } 3}{4}$$

$$S_\infty = \frac{27}{4}$$

Problem 15

- Determine x so that x, x + 2, and x + 6 are the three consecutive terms of a G.P.

- Three numbers a, b, c are in G.P. iff $\frac{b}{a} = \frac{c}{b}$ iff $b^2 = ac$

x, x + 2, and x + 6

a b c

$$(x + 2)^2 = x\,(x + 6)$$

$$\cancel{x^2} + 4x + 4 = \cancel{x^2} + 6x$$

Problem 15

$$\cancel{x^2} + 4x + 4 = \cancel{x^2} + 6x$$

$$4 = 6x - 4x$$

$$\cancel{2}x = \cancel{4}\ 2$$

$$x = 2$$

Three consecutive terms are x, x + 2, and x + 6

i.e., 2, 2 + 2, and 2 + 6

i.e., 2, 4, and 8

Problem 16

- Find the 5^{th} term from the end of the G. P. of 15 terms whose first term and common ratio are 9 and 3 respectively.
- n^{th} term from the end = $(m - n + 1)^{th}$ term from beginning
- nth term from the end = T_{m-n+1} = ar^{m-n}

{Because $T_n = ar^{n-1}$}

$$T_{m-n+1} = ar^{m-n}$$

- a = 9, r = 3, m = 15, and n = 5

Problem 16

$$T_{m-n+1} = ar^{m-n}$$

- a = 9, r = 3, m = 15, and n = 5

$$T_{15-5+1} = 9 \times (3)^{15-5}$$

$$T_{11} = 9 \times (3)^{10}$$

$$T_{11} = 9 \times 59049 \text{ or } (3)^2 \times (3)^{10}$$

$$T_{11} = 531441 \text{ or } (3)^{12}$$

Problem 17

- Find the 10^{th} term from the end of the G. P. with last term $\frac{512}{729}$ and common ratio $\frac{2}{3}$.
- n^{th} term from the end in terms of last term 'l' & common ratio 'r' is $T_n = \frac{l}{r^{n-1}}$

$$l = \frac{512}{729},\ r = \frac{2}{3},\ \text{and } n = 10$$

$$T_{10} = \frac{\frac{512}{729}}{\left(\frac{2}{3}\right)^{10-1}}$$

Problem 17

- l = $\frac{512}{729}$, r = $\frac{2}{3}$, and n = 10

$$T_{10} = \frac{\frac{512}{729}}{\left(\frac{2}{3}\right)^{10-1}} = \frac{\frac{(2)^9}{(3)^6}}{\frac{(2)^9}{(3)^9}}$$

$$T_{10} = \frac{(2)^{9-9}}{(3)^{6-9}} = \frac{(2)^0}{(3)^{-3}}$$

$$T_{10} = 1 \text{ x } (3)^3$$

$$T_{10} = 27$$

Conclusion

- Verification of Sequence
  - A.P. or G.P.
- Compute n term of Progression
  - In A.P.
    - $\boxed{T_n = a + (n - 1)\, d}$
    - $\boxed{T_n = S_n - S_{n-1}}$
  - In G.P.
    - $\boxed{T_n = ar^{n-1}}$
- Three consecutive terms of an A.P. and G.P.

Conclusion

- n^{th} term from the end of a G.P. having m terms is $(m - n + 1)^{th}$ term from beginning i.e.,

$$\boxed{T_{m-n+1} = ar^{m-n}}$$

- n^{th} term from the end of an A.P. having m terms is $(m - n + 1)^{th}$ term from beginning i.e.,

$$\boxed{T_{m-n+1} = a + (m - n)\, d}$$

- n^{th} term from the end of a G.P. in terms of last term 'l' & common ratio 'r' is $\boxed{T_n = \dfrac{l}{r^{n-1}}}$

Conclusion

- Sum of n terms of A.P.
  - $\boxed{S_n = \frac{n}{2} \{2a + (n - 1) d\}}$
  - $\boxed{S_n = \frac{n}{2} \{a + l\}}$
  - $\boxed{S_n = \frac{n}{2} [2l - (n - 1) d]}$

Conclusion

- Sum of n terms of G.P.
  - $\boxed{S_n = \dfrac{lr - a}{r - a}}$
  - $\boxed{S_n = \dfrac{a(r^n - 1)}{(r - 1)} \text{ when } r > 1}$
  - $\boxed{S_n = \dfrac{a(1 - r^n)}{(1 - r)} \text{ when } r < 1}$
- Sum of infinite terms of G.P.
  - $\boxed{S_\infty = \dfrac{a}{(1 - r)} \text{ when } |r| < 1 \text{ i.e., } -1 < r < 1}$

Summary

- Problems
  - Arithmetic Progression
  - Geometric Progression

That's all for now…