Advanced Data Structures

ECAP770

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LOVELY PROFESSIONAL UNIVERSITY

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CONTENT

Unit 01: Introduction

CONTENTS

Objectives

After studying this unit, you will be able to:

- Describe basic concepts of data structure
- Learn Algorithm and its complexity
- Know Abstract data type
- Data structure types

Introduction

The static representation of a linear ordered list using an array wastes resources and, in some situations, causes overflows. We no longer want to pre-allocate memory to any linear list; instead, we want to allocate memory to elements as they are added to the list. This necessitates memory allocation that is dynamic.

Semantically data can exist in either of the two forms – atomic or structured. In most of the programming problems data to be read, processed and written are often related to each other. Data items are related in a variety of different ways. Whereas the basic data types such as integers, characters etc. can be directly created and manipulated in a programming language, the responsibility of creating the structured type data items remains with the programmers themselves. Accordingly, programming languages provide mechanism to create and manipulate structured data items.

A data structure is a type of storage that is used to organize and store data. It is a method of organizing data on a computer so that it may be easily accessible and modified.

It's critical to choose the correct data format for your project based on your requirements and project. If you wish to store data sequentially in memory, for example, you can use the Array data structure.

1.1 Data Structure

A data structure is a set of data values along with the relationship between the data values. Since, the operations that can be performed on the data values depend on what kind of relationships exists among them, we can specify the relationship amongst the data values by specifying the operations permitted on the data values. Therefore, we can say that a data structure is a set of values along with the set of operations permitted on them. It is also required to specify the semantics of the operations permitted on the data values, and this is done by using a set of axioms, which describes how these operations work, and therefore a data structure is made of:

1. A set of data values.

2. A set of functions specifying the operations permitted on the data values.

3. A set of axioms describing how these operations work.

Hence, we conclude that a data structure is a triple (D,F,A), where

1. D is a set of data values

2. F is a set of functions

3. A is a set of axioms

A triple (D, F, A) is referred to as an abstract data structure because it does not tell anything about its actual implementation. It does not tell anything about how these values will be physically represented in the computer memory and these functions will be actually implemented.

Therefore, every abstract data structure is required to be implemented, and the implementation of an abstract data structure requires mapping of the abstract data structure to be implemented into the data structure supported by the computer. For example, if the abstract data structure to be implemented is integer, then it can be implemented by mapping into bits which is a data structure supported by hardware. This requires that every integer data value is to be represented using suitable bit patterns and expressing the operations on integer data values in terms of operations for manipulating bits.

Data Structure mainly two types:

1. Linear type data structure

2. Non-linear type data structure

Linear data structure: A linear data structure traverses the data elements sequentially, in which onlyone data element can directly be reached. Ex: Arrays, Linked Lists

Arrays: An array is a collection of similar type of data items and each data item is called an element of the array.

The data type of the element may be any valid data type like char, int, float or double. – The individual elements of the array age are: – age[0], age[1], age[2], age[3], age[98], age[99].

Linked List: Linked list is a linear data structure which is used to maintain a list in the memory.

It can be seen as the collection of nodes stored at non-contiguous memory locations. Each node of the list contains a pointer to its adjacent node

Stack: Stack is a linear list in which insertion and deletions are allowed only at one end, called top.

A stack is an abstract data type, can be implemented in most of the programming languages. It is named as stack because it behaves like a real-world stack, for example: - piles of plates or deck of cards etc.

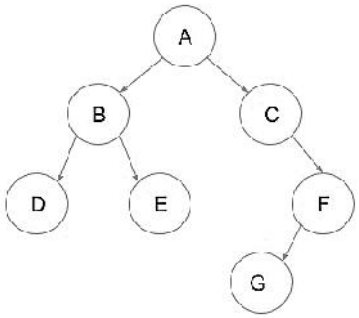
Queue is a linear list in which element can be inserted only at one end called rear and deleted only at other end called front.

It is abstract data structure, similar to stack. It is open at both end therefore if follows first-in-first-out (FIFO) technique for storing the data items.

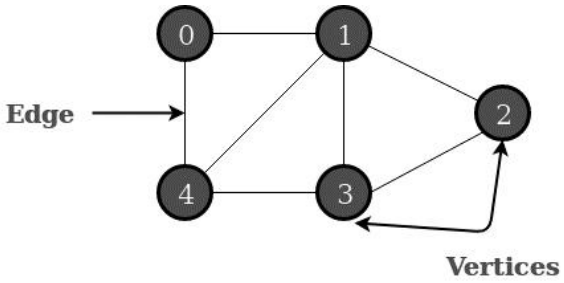
Non-linear data structure: Every data item is attached to several other data items in a way that is specific for reflecting relationships. The data items are not arranged in a sequential structure.

Ex: Trees, Graphs.

Trees: Trees are multilevel data structures with a hierarchical relationship among its elements known as nodes.



Graphs: Graphs can be defined as the pictorial representation of the set of elements (represented by vertices) connected by the links known as edges



Basic Terminology

Data: Data can be defined as an elementary value or the collection of values, for example, student's name and its id are the data about the student.

Group Items: Data items which have subordinate data items are called Group item, for example, name of a student can have first name and the last name.

Record: Record can be defined as the collection of various data items, for example, if we talk about the student entity, then its name, address, course and marks can be grouped together to form the record for the student.

Field: A File is a collection of various records of one type of entity, for example, if there are 60 students in the class, then there will be 20 records in the related file where each record contains the data about each student.

Need of Data Structures

As applications are getting complex and amount of data is increasing day by day, there may arises many problems:

Processor speed: To handle very large amount of data, high speed processing is required, but as the data is growing day by day to the billions of files per entity, processor may fail to deal with that much amount of data.

Data Search: Consider an inventory size of 100 items in a store, If our application needs to search for a particular item, it needs to traverse 100 items every time, results in slowing down the search process.

Multiple requests: If thousands of users are Searching the data simultaneously on a web server, then there are the chances that a very large server can be failed during that process To solve these problems data structures are used.

Basic Concept of Data

The memory (also called storage or core) of a computer is simply a group of bits (switches). At any instant of the computer's operation any particular bit in memory is either 0 or 1 (off or on).

The setting or state of a bit is called its value and that is the smallest unit of information. A set of bit values form data.

Some logical properties can be imposed on the data. According to the logical properties data can be segregated into different categories. Each category having unique set of logical properties is known as data type.

Data type are of two types:

1. Simple data type or elementary item like integer, character.

2. Composite data type or group item like array, structure, union.

Data structures are of two types:

1. *Primitive Data Structures*: Data can be structured at the most primitive level, wherethey are directly operated upon by machine-level instructions. At this level, data may becharacter or numeric, and numeric data may consist of integers or real numbers.

2. *Non-primitive Data Structures*: Non-primitive data structures can be classified as arrays,lists, and fi les.

An array is an ordered set which contains a fixed number of objects. No deletions or insertionsare performed on arrays i.e. the size of the array cannot be changed. At best, elements may bechanged.

A list, by contrast, is an ordered set consisting of a variable number of elements to whichinsertions and deletions can be made, and on which other operations can be performed. When alist displays the relationship of adjacency between elements, it is said to be linear; otherwise, it issaid to be non-linear.

A file is typically a large list that is stored in the external memory of a computer. Additionally, afile may be used as a repository for list items (records) that are accessed infrequently.

From a real world perspective, very often we have to deal with structured data items whichare related to each other. For instance, let us consider the address of an employee. We can takeaddress to be one variable of character type or structured into various fields, as shown below:

As shown above Address1 is unstructured address data. In this form you cannot access individual items from it. You can at best refer to the entire address at one time. While in the second from, i.e., Address2, you can access and manipulate individual fi elds of the address – House No., Street, PIN etc. Given hereunder are two instances of the address1 and address2 variables.

1.2 Data Structure Operations

The data appearing in our data structure is processed by means of certain operations. The particular data structure that one chooses for a given situation depends largely on the frequency with which specific operations are performed. The following four operations play a major role:

1. Traversing: Accessing each record exactly once so that certain items in the record may be

processed. (This accessing or processing is sometimes called 'visiting" the records.)

2. Searching: Finding the location of the record with a given key value, or finding the locations

of all records, which satisfy one or more conditions.

3. Inserting: Adding new records to the structure.

4. Deleting: Removing a record from the structure.

Sometimes two or more data structure of operations may be used in a given situation; e.g., we may want to delete the record with a given key, which may mean we first need to search for the location of the record.

1.3 Abstract Data Type

Before we move to abstract data type let us understand what data type is. Most of the languages support basic data types viz. integer, real, character etc. At machine level, the data is stored as strings containing 1's and 0's. Every data type interprets the string of bits in different ways and gives different results. In short, data type is a method of interpreting bit patterns.

Every data type has a fixed type and range of values it can operate on. For example, an integer variable can hold values between the min and max values allowed and carry out operations like addition, subtraction etc. For character data type, the valid values are defined in the character set and the operations performed are like comparison, conversion from one case to another etc.

There are fixed operations, which can be carried out on them. We can formally defi ne data types as a formal description of the set of values and operations that a variable of a given type may take. That was about the inbuilt data types. One can also create user defined data types, decide the range of values as well as operations to be performed on them. The first step towards creating a user defined data type or a data structure is to defi ne the logical properties. A tool to specify the logical properties of a data type is Abstract Data Type.

Data abstraction can be defined as separation of the logical properties of the organization of programs' data from its implementation. This means that it states what the data should be like.

It does not consider the implementation details. ADT is the logical picture of a data type; in addition, the specifications of the operations required to create and manipulate objects of this data type.

While defining an ADT, we are not concerned with time and space efficiency or any other implementation details of the data structure. ADT is just a useful guideline to use and implement the data type.

An ADT has two parts:

1. Value definition

2. Operation definition.

Value definition is again divided into two parts:

1. Definition clause

2. Condition clause

As the name suggests the definition clause states the contents of the data type and condition clause defines any condition that applies to the data type. Definition clause is mandatory while condition clause is optional.

In operation definition, there are three parts:

1. Function

2. Precondition

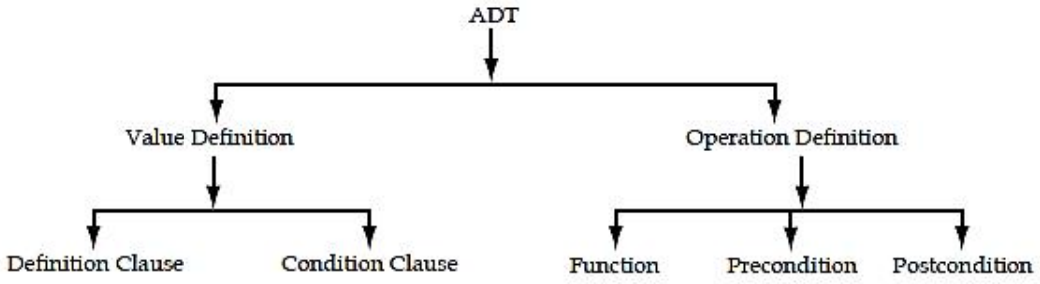
3. Postcondition

The *function* clause defines the role of the operation. If we consider the addition operation inintegers the function clause will state that two integers can be added using this function. Ingeneral, precondition specifies any restrictions that must be satisfied before the operation canbe applied.

This clause is optional. If we consider the division operation on integers then theprecondition will state that the divisor should not be zero. So any call for divide operation, whichdoes not satisfy this condition, will not give the desired output.

Precondition specifies any condition that may apply as a pre-requisite for the operation definition.There are certain operations that can be carried out if certain conditions are satisfied. For example,in case of division operation the divisor should never be equal to zero. Only if this condition issatisfied the division operation is carried out. Hence, this becomes a precondition. In that case &(ampersand) should be mentioned in the operation definition.

Postcondition specifies what the operation does. One can say that it specifies the state after theoperation is performed. In the addition operation, the post condition will give the addition of thetwo integers.



Component of ADT

As an example, let us consider the representation of integer data type as an ADT. We will consider only two operations addition and division.

Value Definition

1. Definition clause: The values must be in between the minimum and maximum values specified for the particular computer.

2. Condition clause: Values should not include decimal point.

Operations

1. add (a, b)

Function: add the two integers a and b.

Precondition: no precondition.

Postcondition: output = a + b

2. Div (a, b)

Function: Divide a by b.

Precondition: b != 0

Postcondition: output = a/b.

There are two ways of implementing a data structure viz. static and dynamic. In static implementation, the memory is allocated at the compile time. If there are more elements than the specified memory then the program crashes. In dynamic implementation, the memory is allocated as and when required during run time.

Any type of data structure will have certain basic operations to be performed on its data like insert, delete, modify, sort, search etc. depending on the requirement. These are the entities that decide the representation of data and distinguish data structures from each other.

Let us see why user defined data structures are essential. Consider a problem where we need to create a list of elements. Any new element added to the list must be added at the end of the list and whenever an element is retrieved, it should be the last element of the list. One can compare this to a pile of plates kept on a table. Whenever one needs a plate, the last one on the pile is taken and if a plate is to be added on the pile, it will be kept on the top. The description wants us to implement a stack. Let us try to solve this problem using arrays.

We will have to keep track of the index of the last element entered in the list. Initially, it will be set to –1. Whenever we insert an element into the list, we will increment the index and insert the value into the new index position. To remove an element, the value of current index will be the output

and the index will be decremented by one. In the above representation, we have satisfied the insertion and deletion conditions.

Using arrays we could handle our data properly, but arrays do allow access to other values in addition to the top most one. We can insert an element at the end of the list but there is no way to ensure that insertion will be done only at the end. This is because array as a data structure allows access to any of its values. At this point we can think of another representation, a list of elements where one can add at the end, remove from the end and elements other than the top one are not accessible. As already discussed, this data structure is called as STACK. The insertion operation is known as push and removal as pop. You can try to write an ADT for stacks.

Another situation where we would like to create a data structure is while working with complex numbers. The operations add, subtract division and multiplication will have to be created as per the rules of complex numbers. The ADT for complex numbers is given below. Only addition and multiplication operations are considered here, you can try to write the remaining operations.

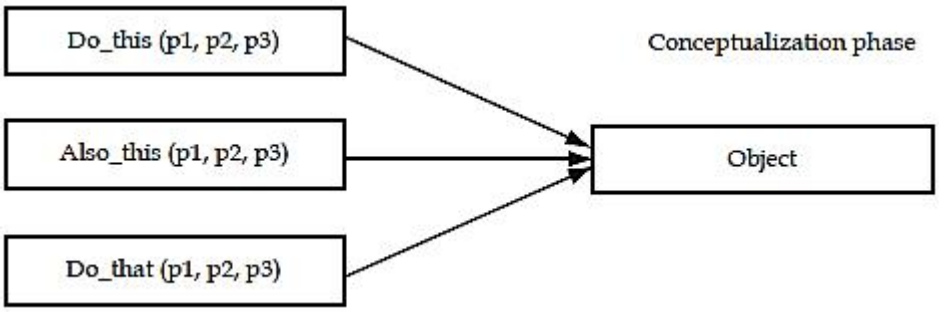
Abstract Data Type (ADT)

1. A framework for an object interface

2. What kind of stuff it'd be made of (no details)?

3. What kind of messages it would receive and kind of action it'll perform when properly

triggered?



From this we figure out

1. Object make-up (in terms of data)

2. Object interface (what sort of messages it would handle?)

3. How and when it should act when triggered from outside (public trigger) and by another

object friendly to it?

These concerns lead to an ADT – a definition for the object.

An Abstract Data Type (ADT) is a set of data items and the methods that work on them.

An implementation of an ADT is a translation into statements of a programming language, of the declaration that defines a variable to be of that ADT, plus a procedure in that language for each operation of the ADT. An implementation chooses a data structure to represent the ADT; each data structure is built up from the basic data types of the underlying programming language.

Thus, if we wish to change the implementation of an ADT, only the procedures implementing the operations would change. This change would not affect the users of the ADT.

Although the terms 'data type', 'data structure' and 'abstract data type' sound alike, they have different meanings. In a programming language, the data type of a variable is the set of values that the variable may assume. For example, a variable of type Boolean can assume either the value true or the value false, but no other value. An abstract data type is a mathematical model, together with various operations defined on the model. As we have indicated, we shall design algorithms in terms of ADTs, but to implement an algorithm in a given programming language.

we must find some way of representing the ADTs in terms of the data types and operators supported by the programming language itself. To represent the mathematical model underlying an ADT, we use data structures, which are a collection of variables, possibly of several data types, connected in various ways.

The cell is the basic building block of data structures. We can picture a cell as a box that is capable of holding a value drawn from some basic or composite data type. Data structures are created by giving names to aggregates of cells and (optionally) interpreting the values of some cells as representing relationships or connections (e.g., pointers) among cells.

1.4 Algorithm

Algorithm is set of rules/ instructions that step-by-step define how a work is to be executed upon in order to get the expected results.
systematic procedure that produces in a finite number of steps the answer to a question or the solution of a problem.
Computer algorithms work via input and output. They take the input and apply each step of the algorithm to that information to generate an output.
E.g. a search engine is an algorithm that takes a search query as an input and searches its database for items relevant to the words in the query. It then outputs the results.
Financial companies use algorithms in areas such as loan pricing, stock trading, asset-liability management, and many automated functions. For example, algorithmic trading, known as algo trading, is used for deciding the timing, pricing, and quantity of stock orders. Also referred to as automated trading or black-box trading, algo trading uses computer programs to buy or sell securities at a pace not possible for humans.
Computer algorithms make life easier by trimming the time it takes to manually do things. In the world of automation, algorithms allow workers to be more proficient and focused. Algorithms make slow processes more proficient. In many cases, especially in automation, algos can save companies money.

1.5 Characteristics of an Algorithm

- Well defined Input and output
- Clear and Unambiguous
- Finite-ness
- Feasible
- Language Independent

Input and output should be defined precisely.

Each step in the algorithm should be clear and unambiguous.

Algorithms should be most effective among many different ways to solve a problem.

An algorithm shouldn't include computer code. Instead, the algorithm should be written in such a way that it can be used in different programming languages.

The algorithm must be finite, i.e. it should not end up in an infinite loops or similar.

The algorithm must be simple, generic and practical, such that it can be executed upon will the available resources. It must not contain some future technology, or anything.

The Algorithm designed must be language-independent, i.e. it must be just plain instructions that can be implemented in any language, and yet the output will be same, as expected.

1.6 Types of Algorithms

Algorithms are categorized based on the concepts that they use to accomplish a task.

- Divide and conquer algorithms
- Brute force algorithms
- Greedy algorithms
- Backtracking algorithms
- Randomized algorithms

 Example:

Step 1: Start

Step 2: Declare variables num1, num2 and sum.

Step 3: Read values num1 and num2.

Step 4: Add num1 and num2 and assign the result to sum.

Sum = num1+num2

Step 5: Display sum

Step 6: Stop

1.7 Algorithm Complexity

Space Complexity

Time Complexity

Space Complexity: Space complexity of an algorithm refers to the amount of memory that this algorithm requires to execute and get the result. This can be for inputs, temporary operations, or outputs.

Fixed Part: This refers to the space that is definitely required by the algorithm. For example, input variables, output variables, program size, etc.

Variable Part: This refers to the space that can be different based on the implementation of the algorithm. For example, temporary variables, dynamic memory allocation, recursion stack space, etc.

Time Complexity: Time complexity of an algorithm refers to the amount of time that this algorithm requires to execute and get the result. This can be for normal operations, conditional if-else statements, loop statements, etc.

Constant time part: Any instruction that is executed just once comes in this part. For example, input, output, if-else, switch, etc.

Variable Time Part: Any instruction that is executed more than once, say n times, comes in this part. For example, loops, recursion, etc.

1.8 Asymptotic Notations

To measure the efficiency of an algorithm asymptotic analysis is used.

The efficiency of an algorithm depends on the amount of time, storage and other resources required to execute the algorithm.

Performance of algorithm is change with different type of inputs.

The study of change in performance of the algorithm with the change in the order of the input size is defined as asymptotic analysis.

Asymptotic notations are the mathematical notations used to describe the running time of an algorithm when the input tends towards a particular value or a limiting value.

Types of asymptotic notations

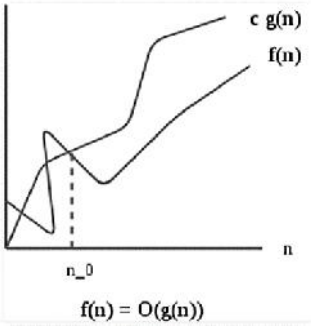
There are three major asymptotic notations

- Big-O notation
- Omega notation
- Theta notation

Big-O notation represents the upper bound of the running time of an algorithm. It gives the worst-case complexity of an algorithm.

O(n) is useful when we only have an upper bound on the time complexity of an algorithm.

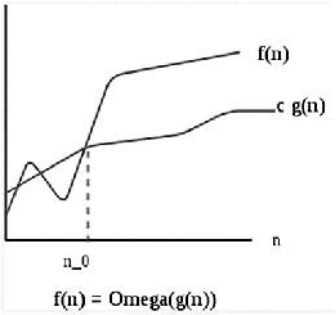
It is widely used to analyses an algorithm as we are always interested in the worst-case scenario.



O(g(n)) = { f(n): there exist positive constants c and n0 such that 0 ≤ f(n) ≤ cg(n) for all n ≥ n0 }

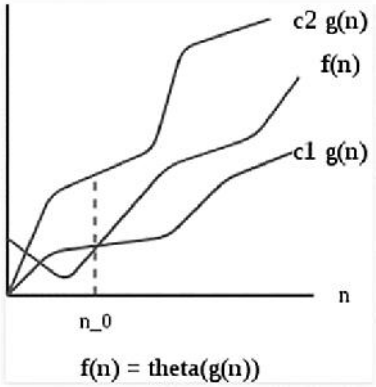
Omega notation represents the lower bound of the running time of an algorithm. It provides the best-case complexity of an algorithm.

Omega Notation can be useful when we have lower bound on time complexity of an algorithm.Omega notation is the least used notation among all three.



Ω (g(n)) = {f(n): there exist positive constants c and n0 such that 0 <= c*g(n) <= f(n) forall n >= n0}.

Theta notation encloses the function from above and below. It represents the upper and the lower bound of the running time of an algorithm, it is used for analysing the average-case complexity of an algorithm.



Θ(g(n)) = {f(n): there exist positive constants c1, c2 and n0 such that 0 <= c1*g(n) <= f(n) <= c2*g(n) for all n >= n0}

Properties of Asymptotic Notations

If f(n) is O(g(n)) then a*f(n) is also O(g(n)) ; where a is a constant.

General Properties

If f(n) is O(g(n)) then a*f(n) is also O(g(n)) ; where a is a constant.

Transitive Properties

If f(n) is O(g(n)) and g(n) is O(h(n)) then f(n) = O(h(n))

Reflexive Properties

If f(n) is given then f(n) is O(f(n))

Symmetric Properties

If f(n) is Θ(g(n)) then g(n) is Θ(f(n))

Transpose Symmetric Properties

If f(n) is O(g(n)) then g(n) is Ω (f(n))

Summary

- Data Structure is method or technique to data organization, management, and storageformat in the computer so we can perform operations on the stored data more efficiently.
- Data structure is a combination of one or more basic data types to form a single addressable data type.
- An algorithm is a finite set of instructions which, when followed, accomplishes a particular task, the termination of which is guaranteed under all cases, i.e. the termination is guaranteed for every input.
- The instructions must be unambiguous and the algorithm must produce the output within a finite number of executions of its instructions.
- Abstract data type (ADT) is a mathematical model with a collection of operations defined on that model. Although the terms 'data type', 'data structure' and 'abstract data type' sound alike, they have different meanings.

Keywords

- *Data*: Data can be defined as an elementary value or the collection of values, for example, student's name and its id are the data about the student.
- *Group Items*: Data items which have subordinate data items are called Group item, for example, name of a student can have first name and the last name.
- *Linear Data Structure*: A linear data structure traverses the data elements sequentially, in whichonly one data element can directly be reached.
- *Non-linear Data Structure*: Every data item is attached to several other data items in a way thatis specific for reflecting relationships. The data items are not arranged in a sequential structure.
- *Searching*: Finding the location of the record with a given key value, or finding the locations ofall records, which satisfy one or more conditions.
- *Traversing*: Accessing each record exactly once so that certain items in the record may beprocessed.

Self Assessment

1. Which is type of data structure.

A. Primitive
B. Non-primitive
C. Both primitive and non-primitive
D. None of above

2. Which of the following is linear data structure?

A. Trees
B. Arrays
C. Graphs
D. None of these

3. Which of the following is non-linear data structure?

A. Array
B. Linked lists
C. Stacks
D. None of these

4. User defined data type is also called?

A. Primitive
B. Identifier
C. Non-primitive
D. None of these

5. Stack is based on which principle

A. FIFO
B. LIFO
C. Push
D. None of the Above

6. What are the characteristics of an Algorithm.

A. Clear and Unambiguous
B. Finite-ness
C. Feasible
D. All of above

7. A procedure for solving a problem in terms of action and their order is called as

A. Program instruction
B. Algorithm
C. Process
D. Template

8. Algorithm can be represented as

A. Pseudocode
B. Flowchart
C. None of the above
D. Both Pseudocode and Flowchart

9. What are the different types of Algorithms?

A. Brute force algorithms
B. Greedy algorithms
C. Backtracking algorithms
D. All of these

10. Which is algorithm complexity.

A. Space Complexity
B. Time Complexity
C. Both space and time complexity
D. None of aboveone of these

11. Which one is asymptotic notations?

A. Big-O notation
B. Omega notation
C. Theta notation
D. All of above

12. Big-O Notation represents…

A. Space complexity
B. Upper bound of the running time of an algorithm
C. Lower bound of the running time of an algorithm
D. None of above

13. Omega Notation (Ω-notation) represents….

A. Upper bound of the running time of an algorithm
B. Space complexity
C. Lower bound of the running time of an algorithm
D. None of above

14. Which is property of Asymptotic Notations?

A. Reflexive
B. Symmetric
C. Transpose Symmetric
D. All of these

15. Abstract Data Type having.

A. Value definition
B. Operation definition
C. Both value and operation definition
D. None of above.

Answers for Self Assessment

1.	C	2.	B	3.	D	4.	C	5.	B
6.	D	7.	B	8.	D	9.	D	10.	C
11.	D	12.	B	13.	C	14.	D	15.	C

Review Question

1. Define data structure and its application.
2. What are the advantages of data structure?
3. Discuss abstract data type.
4. What is significance of space and time complexity in algorithm.
5. Explain different types of algorithms.
6. Discuss Asymptotic notations with example.
7. Define record and file.

Further Readings

- Data Structures and Algorithms; Shi-Kuo Chang; World Scientifi c.
- Data Structures and Efficient Algorithms, Burkhard Monien, Thomas Ottmann, Springer.
- Mark Allen Weles: Data Structure & Algorithm Analysis in C Second Adition. Addison-Wesley publishing
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Unit 02: Arrays vs Linked Lists

CONTENTS

Objectives

After studying this unit, you will be able to:

- Learn basic concepts of arrays
- Understand the basics of linked list
- Describe the types of array operations
- Discuss the operations of linked lists

Introduction

A data structure consists of a group of data elements bound by the same set of rules. The data elements also known as members are of different types and lengths. We can manipulate data stored in the memory with the help of data structures. The study of data structures involves examining the merging of simple structures to form composite structures and accessing definite components from composite structures. An array is an example of one such composite data structure that is derived from a primitive data structure.

An array is a set of similar data elements grouped together. Arrays can be one-dimensional or multidimensional. Arrays store the entries sequentially. Elements in an array are stored in continuous locations and are identified using the location of the first element of the array.

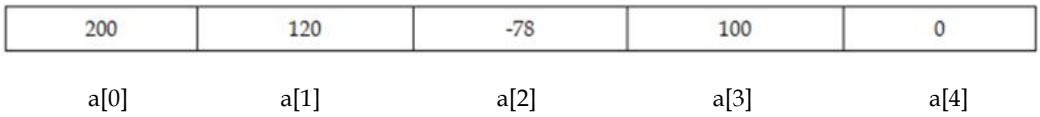
2.1 Arrays

An array is a data type, much like a variable as both array and variable hold information. However, unlike a variable, an array can hold several pieces of data called elements. Arrays can hold any type of data, which includes string, integers, Boolean, and so on. An array can also handle other variables as well as other arrays. An integer index identifies the individual elements of an array.

Arrays are allocated the memory in a strictly contiguous fashion. The simplest array is one-dimensional array which is a list of variables of same data type. An array of one-dimensional arrays

is called a two-dimensional array; array of two-dimensional arrays is three-dimensional array and so on.

The members of the array can be accessed using positive integer values (indicating their order in the array) called subscript or index.



The description of this array is listed below:

Name of the array : a

Data type of the array : integer

Number of elements : 5

Valid index values : 0, 1, 2, 3, 4

Value stored at the location a[0] : 200

Value stored at the location a[1] : 120

Value stored at the location a[2] : -78

Value stored at the location a[3] : 100

Value stored at the location a[4] : 0

Initializing an Array

We can initialize an array by assigning values to the elements during declaration. We can access the element by specifying its index. While initializing an array, the initial values are given sequentially separated by commas and enclosed in braces.

Example:

Consider the elements 10, 20, 30, and 40. The array can be represented as:

a[4]={10, 20, 30, 40}

The elements can be stored in an array as shown below:

a[0] = 10

a[1] = 20

a[2] = 30

a[3] = 40

The element 20 can be accessed by referencing a[1].

Now, consider n number of elements in an array. Hence, to access any element within the array, we use a[i], where i is the value between 0 to n-1.

The corresponding code used in C language to read n number of integers in an array is:

```
for(i= 0; i<n; i++)
{
scanf("%d",&a[i]);
}
```

Array Initialization in its Declaration

A variable is initialized in its declaration.

Example:

int value = 10;

Here, the value 10 is called an initializer.

Similar to a variable, we can initialize an array at the time of its declaration. The following example shows an array initialization.

Example:

int a[5] = {10, 11, 12, 13, 14};

In this declaration, a[0] is initialized to 10, a[1] is initialized to 11, and so on. There must be at least one initial value between braces. If the number of initialized array elements is lesser than the declared size, then the remaining array elements are assigned the value 0.

If we provide all the array elements during initialization, it is not necessary to specify the array size. Thecompiler automatically counts the number of elements and reserves the space in the memory for thearray.

Example:

int a[] = {10, 20, 30, 40};

Here the compiler reserves four spaces for array a.

2.2 Types of Arrays

The elements in an array are referred either by a single subscript or by two or more subscripts. Hence, the arrays are of two types namely, one-dimensional array and multidimensional array, based on the subscript referred. A two-dimensional array is also a type of multidimensional array. When the array is referred by a single subscript, then it is known as one-dimensional array or linear array. When the array is referred by two subscripts, it is known as a two-dimensional array. Some programming languages allow more than two or three subscripts and these arrays are known as multidimensional arrays.

According the number of subscripts required to access an array element, arrays can be of following types:

1. One-dimensional array
2. Multi-dimensional array

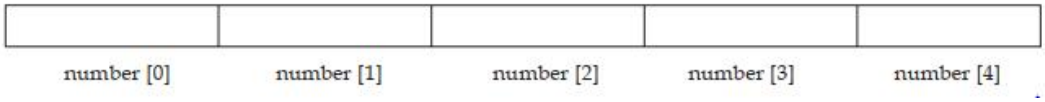
Linear Array

A linear or one-dimensional array is a structured collection of elements (often called array elements). It can be accessed individually by specifying the position of each element by an index value.

Example: If we want to store a set of five numbers by an array variable number. Then it will be accomplished in the following way:

int number [5];

This declaration will reserve five contiguous memory locations capable of storing an integer type value each, as shown below:



Now let us see how individual elements of linear array are accessed. The syntax for accessing an arraycomponent is:

ArrayName[IndexExpression]

The IndexExpression must be an integer value. The integer value can be of char, short int, long int, or

Boolean value because these are integral data types. The simplest form of index expression is a constant.

 Example:

If we consider an array number[25], then,

number[0] specifies the 1st component of the array

number[1] specifies the 2nd component of the array

number[2] specifies the 3rd component of the array

number[3] specifies the 4th component of the array

number[4] specifies the 5th component of the array

.

.

.

number[23] specifies the 2nd

To store and print values from the number array, we can perform the following:

```
for(int i=0; i< 25; i++)
{
number[i]=i; // Storing a number in each array element
printf("%d", number[i]); //Printing the value
}
```

Multidimensional Array

Multidimensional arrays are also known as "arrays of arrays." Programming languages often need to store and manipulate two or more dimensional data structures such as, matrices, tables, and so on. When programming languages use two subscripts they are known as two-dimensional arrays. One subscript denotes a row and the other denotes a column.

The declaration of two-dimension array is as follows:

data_typearray_name[row_size][column_size];

 Example:

int m[5][10]

Here, m is declared as a two dimensional array having 5 rows (numbered from 0 to 4) and 10 columns (numbered from 0 to 9). The first element of the array is m[0][0] and the last row last column is m[4][9]

Now let us discuss a three-dimensional array. A three-dimensional array is considered as an array of two-dimensional arrays.

 Example:

A three dimensional array is created as follows:

int bigArray[ ][ ][ ] = new int [10][10][4];

This will create an array named bigArray containing 400 integers. We can access any element of this array by using 3 indices.

 Example:

Suppose we want to assign a value 312 to the element at position 3 down, 7 across, and 2 in, then we write it as:

bigArray [2][6][1] = 312;

Initialization of Multidimensional Arrays

Like the one dimension arrays, two-dimensional arrays are also initialized by declaring a list of initial values enclosed in braces.

 Example:

int table[2][3]={0,0,0,1,1,1};

The table array initializes the elements of first row to 0 and the second row to

1. The initialization is done row by row. The above statement can be equivalently written as:

int table[2][3]={{0,0,0},{1,1,1}}

Three or four-dimensional arrays are more complicated. They can also be initialized by declaring a list of initial values enclosed in braces.

 Example:

int table[3][3][3]={1,2,3,4,5 6,7,8,…….27 };

This will create an array named table containing 27 integers. We can access any element of this array by using 3 indices.

The method to access table[1][1][1], is as shown below:

The values for array - table[3][3][3] are as follows:

{1, 2, 3}

{4, 5, 6}

{7, 8, 9}

{10, 11, 12}

{13, 14, 15}

{16, 17, 18}

{19, 20, 21}

{22, 23, 24}

{25, 26, 27}

The values in the array can be accessed using three for loops. The loop contains three variables i, j, and k respectively. This is as shown below:

```
for(i=0;i<3;i++)
{
for(j=0;j<3;j++)
for(k=0;k<3;k++)
{
printf("%d\t",table[i][j][k]);
```

```
}
printf("\n");
}
}
printf("%d", table[1][1][1]);
```

For every iteration of the i, j and k loops, the values printed are:

[0][0][0] = 1

[0][0][1] =2

[0][0][2] =3

[1][1][1] =14

2.3 Types of Array Operations

The operations performed on an array, are

1. Adding operation
2. Sorting operation
3. Searching operation
4. Traversing operation

Adding Operation

Adding elements into an array is known as insertion. The insertion of data elements is done at the end of an array. This is possible only if there is enough space in the array to add the additional elements. The elements can also be inserted in the middle of the array. Here, the average half of the array elements is moved to the next location to empty the block of memory, and to accommodate the new element.

Algorithm for Inserting an Element into an Array

Let a be an array of size N and I be the array index. Algorithm to insert an element in the MthPosition of the array a is as follows

1. Start

2. read a[N], I<-0
3. repeat for I=N to M (Decrement I by one)
4. a[I+1]<- a[I]
5. a[M]<-ELEMENT
6. M<-M+1
7. Stop

The below program illustrates the concept of inserting an element into a one-dimensional array.

Example:

```
#include<stdio.h>
#include<conio.h>
void main()
{
int n, i, data, po_indx, a[50]; //Variable declaration
```

```
clrscr();
printf("Enter number of elements in the array\n");
/*Get the number of elements to be added to the array from the user*/
scanf("%d", &n);
printf("\nEnter %d elements\n\n", n); //Print the number of elements
for(i=0;i<n;i++) //Iterations using for loop
scanf("%d",&a[i]); //Accepting the values in the array
printf("\nEnter a data to be inserted\n");
scanf("%d",&data); //Reads the data added by user
printf("\nEnter the position of the item \n");
scanf("%d",&po_indx); //Reads the position where the data is inserted
/* Checking if the position is greater than the size of the array*/
if(po_indx-1>n)
printf("\nposition not valid\n"); //If the condition is true this will be printed
else //If the condition is false the 'else' part will get executed
{
for(i=n;i>=po_indx;i--) //Iterations using for loop
a[i]=a[i-1]; //Value of a[i-1] is assigned to a[i]
/*Value of data will be assigned to [po_indx-1] position*/
a[po_indx-1]=data;
n=n+1; //Incrementing the value of n
printf("\nArray after insertion\n"); //Print the array list after insertion
for(i=0;i<n;i++) //Use for loop and
printf("%d\t",a[i]); //Print the final array after insertion
}
getch(); //Display characters on screen
}
```

Output:

```
Enter number of elements in the array
5
Enter 5 elements
15 20 32 45 62
Enter a data to be inserted
77
Enter the position of the item
2
Array after insertion
15 77 20 32 45 62
```

In this example:

1. First, the header files are included using #include directive.

2. Then, the index, array, and the variables are declared.

3. The program accepts the number of elements in the array.

4. Using a for loop, the values are accepted and stored in the array.

5. Then, the program accepts the data along with the position where it needs to be inserted.

6. If the position to be inserted is greater than the number of elements (po_indx-1>n) then the program displays "position is not valid". Otherwise, the program by means of a for loop, checks whether i>=po_indx is true and assigns the a[i-1] value to a[i].

7. Then data is assigned to a[po_indx-1].

8. Then, the program increments the number of elements and prints the array after insertion.

9. getch() prompts the user to press a key and the program terminates.

Sorting Operation

Sorting operation arranges the elements of a list in a certain order. Efficient sorting is important for optimizing the use of other algorithms that require sorted lists to work correctly.

Sorting an array efficiently is quite complicated. There are different sorting algorithms to perform the task of sorting, but here we will discuss only Bubble Sort.

Bubble Sort

Bubble sort is a simple sorting technique when compared to other sorting techniques. The bubble sort algorithm starts from the very first element of the data set. In order to sort elements in the ascending order, the algorithm compares the first two elements of the data set. If the first element is greater than the second, then the numbers are swapped.

This process is carried out for each pair of adjacent elements to the end of the data set until no swaps occur on the last pass. This algorithm's average and worst case performance is O (2n) as it is rarely used to sort large, unordered data sets.

Bubble sort can always be used to sort a small number of items where efficiency is not a high priority. Bubble sort may also be effectively used to sort a partially sorted list.

Algorithm for Sorting an Array

Let A be an array containing data with N elements. This algorithm sorts the elements in A as follows:

1. Start

2. Repeat Steps 3 and 4 for K= 1 to N-1

3. Set PTR :=1 [Initializes pass pointer PTR]

4. Repeat while PTR $\leq$ N – K: [Executes pass]

If A[PTR] > A[PTR+1], then:

Interchange A[PTR] and A[PTR +1]

[End of If structure]

Set PTR := PTR+1

[End of inner loop]

[End of Step 2 outer loop]

5. Exit

In the algorithm, there is an inner loop, which is controlled by the variable PTR, and an index K controls the outer loop. K is used as a counter and PTR is used as an index.

The below program illustrates the concept of sorting an array using bubble sort.

 Example:

```
#include <stdio.h>
#include <conio.h>
int A[8] = {55, 22, 2, 43, 12, 8, 32, 15}; //Declaring the array with 8 elements
int N = 8; //Size of the array
void BUBBLE (void); //BUBBLE Function declaration
void main()
{
int i; //Variable declaration
clrscr();
/*Printing the values in the array*/
printf("\n\nValues present in array A =");
for (i=0; i<8; i++) //Iterations using for loop
printf(" %d, ", A[i]); //Printing the array
BUBBLE(); //BUBBLE function is called
/*Printing the values from the array after sorting*/
printf("\n\nValues present in the array after sorting =");
for (i=0; i<8; i++) //Iterations
printf(" %d, ", A[i]); // Printing the array after sorting
getch(); // waits for a key to be pressed
}
void BUBBLE(void) //BUBBLE Function definition
{
int K, PTR, TEMP; //Declaration variables
for(K=0; K <= (N-2); K++) //Iterations
{
PTR = 0; //Assign 0 to variable PTR
while(PTR <= (N-K-1-1)) //Checking if PTR <= (N-K-1-1)
{
/* Checking if the element at A[PTR] is greater than A[PTR+1]*/
if(A[PTR] > A[PTR+1])
{
TEMP = A[PTR];
A[PTR] = A[PTR+1];
A[ PTR +1] = TEMP;
}
/*Increment the array index*/
PTR = PTR+1;
}
}
```

}

Output:

Values present in A[8] = 55, 22, 2, 43, 12, 8, 32, 15

Values present in A[8] after sorting = 2, 8, 12, 15, 22, 32, 43, 55

In this example:

1. First, the header files are included using #include directive.

2. Then, the array A is declared globally along with the array elements and the size.

3. Then, inside the main function the variable i is declared.

4. The values in the array are printed using a for loop.

5. Next, the Bubble function is called. The sorting operation is carried out and values present in the array are printed.

6. getch() prompts the user to press a key. Then the program terminates.

7. In The BUBBLE function the variables K, PTR and TEMP are declared as integers.

8. PTR is set to 0.

9. Within the while loop the adjacent array elements are compared. If the element at a lower position is greater than the element at the next position, both the elements are interchanged.

10. The array index is then incremented.

Searching Operation

Searching is an operation used for finding an item with specified properties among a collection of items.In a database, the items are stored individually as records, or as elements of a search space addressed by a mathematical formula or procedure. The mathematical formula or procedure may be the root of an equation containing integer variables.

Search operation is closely related to the concept of dictionaries. Dictionaries are a type of data structure that support operations such as, search, insert, and delete.

Computer systems are used to store large amounts of data. From these large amount of data, individual records are retrieved based on some search criterion. The efficient storage of data is an important issue to facilitate fast searching.

There are many different searching techniques or algorithms. The selection of algorithm depends on the way the information is organized in memory. Now, we will discuss linear searching technique.

Algorithm for Linear search

Let A be a linear array with N elements and ITEM be the given item of information. The search algorithm will find the location LOC of ITEM in A or sets LOC :=0 if the search fails. The algorithm is as follows:

1. Start

2. [Insert ITEM at the end of A.] Set A[N+1] :=ITEM

3. [Initialize counter.] Set LOC :=1

4. [Search for ITEM.]

(a) Repeat while A[LOC] ≠ ITEM:

(b) Set LOC := LOC + 1

[End of loop]

5. [Successful?] If LOC = N + 1, then: Set LOC := 0

6. Exit

Traversing Operation

Traversing an array refers to moving in inward and outward direction to access each element in an array. To traverse an array, one can use for loop. The array elements are accessed using an array index or a pointer of type similar to that of array elements. To access the elements using a pointer, the pointer must be initialized with the base address of the array. Traversing operation also involves printing the elements in an array.

Algorithm for Traversal Operation

Let X be an array of size N. You need to traverse through the array and perform the required operations on each element of the array. Let the required operation be OP. Here, i is the array index and the lower bound starts with 0. The algorithm for traversing a given array is as follows:

1. Start

2. read X[N], i=0

3. repeat for I = 0, 1, 2…..N

OP on X[i]

4. Stop

 Example:

```
#include<stdio.h>
#include<conio.h>
#define SIZE 20 //Define array size
void main()
{
float sum(float[], int); //Function declaration
float x[SIZE], Sum_total=0.0;
int i, n; //Variable declaration
clrscr();
printf("Enter the number of elements in array\n");
scanf(" %d", &n); //Reads the data added by user
printf("Enter %d elements:\n", n); //Printing the values in the array
for(i=0; i<n; i++) //Iterations using for loop
/* Input the elements of the array (Traverse operation)*/
scanf(" %f", &x[i]);
printf("The elements of array are:\n\n"); //Printing the elements of the array
for(i=0; i<n; i++) //Iterations using for loop
/*print the elements of array in floating point form(Traverse operation)*/
printf(" %.2f\t", x[i]);
/*Call the function sum and store the value returned in Sum_total*/
Sum_total = sum(x, n);
/*Printing the sum*/
printf("\n\nSum of the given array is: %.2f\n", Sum_total);
getch(); //wait until a key is pressed
}
```

```
float sum(float x[], int n) //Function declaration
{
int i; //Variable declaration
float total=0.0; //the variable total is set to 0.0
for(i=0; i<n; i++) //Iterations
total+=x[i]; //each element x[i] is added to the value of total
return(total); //Returning the total value
}
```

Output:

```
Enter the number of elements in array
5
Enter 5 elements
14 15 16 17 18
The elements of array are:
14.00 15.00 16.00 17.00 18.00
Sum of the given array is: 80.00
```

In this example:

1. First, the header files are included using #include directive.

2. Using the #define directive, the array size, SIZE, is set to 20.

3. In the main() function, the function sum and the variables are declared.

4. A for loop is used accept the elements of the array.

5. The next for loop prints the elements of the array.

6. The program calls the sum() function to add all the elements of the array. The value returned by the sum() function is stored in the variable Sum_total.

7. The program then prints the sum of the elements of the array.

8. getch() prompts the user to press a key to exit the program.

9. The function sum that accepts two arguments and returns a float value is defined. The function sum does the following steps:

(a) Initializes an integer variable i.

(b) Initializes a float variable total and assigns 0.0 to it.

(c) Adds the elements of the array using a for loop and the result is stored in total.

(d) Finally, it returns the value of total.

2.4 Linked list

Linked lists are the most common data structures. They are referred to as an array of connected objects where data is stored in the pointer fields. Linked lists are useful when the number of elements to be stored in a list is indefinite.

Concept of Linked Lists

An array is represented in memory using sequential mapping, which has the property that elements are fixed distance apart. But this has the following disadvantage. It makes insertion or

deletion at any arbitrary position in an array a costly operation, because this involves the movement of some of the existing elements.

When we want to represent several lists by using arrays of varying size, either we have to represent each list using a separate array of maximum size or we have to represent each of the lists using one single array. The first one will lead to wastage of storage, and the second will involve a lot of data movement.

So we have to use an alternative representation to overcome these disadvantages. One alternative is a linked representation. In a linked representation, it is not necessary that the elements be at a fixed distance apart. Instead, we can place elements anywhere in memory, but to make it a part of the same list, an element is required to be linked with a previous element of the list. This can be done by storing the address of the next element in the previous element itself. This requires that every element be capable of holding the data as well as the address of the next element. Thus every element must be a structure with a minimum of two fields, one for holding the data value, which we call a data field, and the other for holding the address of the next element, which we call link field.

Therefore, a linked list is a list of elements in which the elements of the list can be placed anywhere in memory, and these elements are linked with each other using an explicit link field, that is, by storing the address of the next element in the link field of the previous element.

This program uses a strategy of inserting a node in an existing list to get the list created. An insert function is used for this. The insert function takes a pointer to an existing list as the first parameter, and a data value with which the new node is to be created as a second parameter, creates a new node by using the data value, appends it to the end of the list, and returns a pointer to the first node of the list. Initially the list is empty, so the pointer to the starting node is NULL.

Therefore, when insert is called first time, the new node created by the insert becomes the start node. Subsequently, the insert traverses the list to get the pointer to the last node of the existing list, and puts the address of the newly created node in the link field of the last node, thereby appending the new node to the existing list. The main function reads the value of the number of nodes in the list. Calls iterate that many times by going in a while loop to create the links with the specified number of nodes.

2.5 Types of linked list

Single linked list

Double linked list

Circular linked list

A doubly-linked list is a linked data structure that consists of a set of sequentially linked records called nodes. Each node contains two fields, called links, that are references to the previous and to the next node in the sequence of nodes.

In the single linked list each node provides information about where the next node is in the list. It faces difficulty if we are pointing to a specific node, then we can move only in the direction of the links. It has no idea about where the previous node lies in memory. The only way to find the node which precedes that specific node is to start back at the beginning of the list. The same problem arises when one wishes to delete an arbitrary node from a single linked list. Since in order to easily delete an arbitrary node one must know the preceding node. This problem can be avoided by using Doubly Linked List, we can store in each node not only the address of next node but also the address of the previous node in the linked list. A node in Doubly Linked List has three fields

1. Data

2. Previous Link

3. Next Link

Implementation of Doubly Linked List

Structure of a node of Doubly Linked List can be defi ned as:

```
struct node
{
int data;
struct node *llink;
struct node *rlink;
}
```

Circular Linked List

Circular Linked List is another remedy for the drawbacks of the Single Linked List besides Doubly Linked List. A slight change to the structure of a linear list is made to convert it to circular linked list; link fi eld in the last node contains a pointer back to the fi rst node rather than a Null.

Representation of Linked List

Because each node of an element contains two parts, we have to represent each node through a structure.

While defi ning linked list we must have recursive defi nitions:

```
struct node
{
int data;
struct node * link;
}
```

Here, link is a pointer of struct node type i.e. it can hold the address of variable of struct node type. Pointers permit the referencing of structures in a uniform way, regardless of the organization of the structure being referenced. Pointers are capable of representing a much more complex relationship between elements of a structure than a linear order.

Initialization:

```
main()
{
struct node *p, *list, *temp;
list = p = temp = NULL;
.
.
.
}
```

 Example:

Program:

```
# include <stdio.h>
# include <stdlib.h>
struct node
{
int data;
struct node *link;
```

```
};
struct node *insert(struct node *p, int n)
{
struct node *temp;
/* if the existing list is empty then insert a new node as the
starting node */
if(p==NULL)
{
p=(struct node *)malloc(sizeof(struct node)); /* creates new
node data value passes
as parameter */
if(p==NULL)
{
printf("Error\n");
exit(0);
}
p-> data = n;
p-> link = p; /* makes the pointer pointing to itself because
it is a circular list*/
}
else
{
temp = p;
/* traverses the existing list to get the pointer to the last node
of it */
while (temp->link != p)
temp = temp->link;
temp-> link = (struct node *)malloc(sizeof(struct node)); /*
creates new node using
data value passes as
parameter and puts its
address in the link field
of last node of the
existing list*/
if(temp -> link == NULL)
{
printf("Error\n");
exit(0);
}
temp = temp->link;
```

```
temp-> data = n;
temp-> link = p;
}
return (p);
}
void printlist( struct node *p )
{
struct node *temp;
temp = p;
printf("The data values in the list are\n");
if(p!= NULL)
{
do
{
printf("%d\t",temp->data);
temp=temp->link;
} while (temp!= p);
}
else
printf("The list is empty\n");
}
void main()
{
int n;
int x;
struct node *start = NULL ;
printf("Enter the nodes to be created \n");
scanf("%d",&n);
while ( n -- > 0 )
{
printf( "Enter the data values to be placed in a node\n");
scanf("%d",&x);
start = insert ( start, x );
}
printf("The created list is\n");
printlist( start );
}
```

Deleting the Specified Node in Singly Linked List

To delete a node, first we determine the node number to be deleted (this is based on the assumption that the nodes of the list are numbered serially from 1 to n). The list is then traversed to get a pointer to the node whose number is given, as well as a pointer to a node that appears before the

node to be deleted. Then the link fi eld of the node that appears before the node to be deleted is made to point to the node that appears after the node to be deleted, and the node to be deleted is freed.

 Lab Exercise:

```
# include <stdio.h>
# include <stdlib.h>
struct node *delet( struct node *, int );
int length ( struct node * );
struct node
{
int data;
struct node *link;
};
struct node *insert(struct node *p, int n)
{
struct node *temp;
if(p==NULL)
{
p=(struct node *)malloc(sizeof(struct node));
if(p==NULL)
{
printf("Error\n");
exit(0);
}
p-> data = n;
p-> link = NULL;
}
else
{
temp = p;
while (temp->link != NULL)
temp = temp->link;
temp-> link = (struct node *)malloc(sizeof(struct node));
if(temp -> link == NULL)
{
printf("Error\n");
exit(0);
}
temp = temp->link;
temp-> data = n;
temp-> link = NULL;
}
return (p);
```

```
}
void printlist( struct node *p )
{
printf("The data values in the list are\n");
while (p!= NULL)
{
printf("%d\t",p-> data);
p = p->link;
}
}
void main()
{
int n;
int x;
struct node *start = NULL;
printf("Enter the nodes to be created \n");
scanf("%d",&n);
while ( n- > 0 )
{
printf( "Enter the data values to be placed in a node\n");
scanf("%d",&x);
start = insert ( start, x );
}
printf(" The list before deletion id\n");
printlist( start );
printf("% \n Enter the node no \n");
scanf( " %d",&n);
start = delet (start , n );
printf(" The list after deletion is\n");
printlist( start );
}
/* a function to delete the specified node*/
struct node *delet( struct node *p, int node_no )
{
struct node *prev, *curr ;
int i;
if (p == NULL )
{
printf("There is no node to be deleted \n");
}
```

```
else
{
if ( node_no> length (p))
{
printf("Error\n");
}
else
{
prev = NULL;
curr = p;
i = 1 ;
while ( i<node_no )
{
prev = curr;
curr = curr->link;
i = i+1;
}
if ( prev == NULL )
{
p = curr ->link;
free ( curr );
}
else
{
prev -> link = curr ->link ;
free ( curr );
}
}
}
return(p);
}
/* a function to compute the length of a linked list */
int length ( struct node *p )
{
int count = 0 ;
while ( p != NULL )
{
count++;
p = p->link;
}
```

return (count) ;

}

Inserting a Node after the Specified Node in a Singly Linked List

To insert a new node after the specified node, first we get the number of the node in an existinglist after which the new node is to be inserted. This is based on the assumption that the nodes ofthe list are numbered serially from 1 to n. The list is then traversed to get a pointer to the node,whose number is given. If this pointer is x, then the link field of the new node is made to point tothe node pointed to by x, and the link field of the node pointed to by x is made to point to the newnode. Figures 2.3 and 2.4 show the list before and after the insertion of the node, respectively.

Insertion in Linked List can happen at following places:

At the beginning of the linked list.

At the end of the linked list.

At a given position in the linked list.

Algorithm: Insertion at beginning

Step 1: IF PTR = NULL

Write OVERFLOW

Go to Step 7

[END OF IF]

Step 2: SET NEW_NODE = PTR

Step 3: SET PTR = PTR → NEXT

Step 4: SET NEW_NODE → DATA = VAL

Step 5: SET NEW_NODE → NEXT = HEAD

Step 6: SET HEAD = NEW_NODE

Step 7: EXIT

Deletion from a Linked List

Delete from beginning

Delete from end

Delete from middle/ given position

Find the previous node of the node to be deleted.

Change the next pointer of the previous node

Free the memory of the deleted node.

In case of first node deletion, we need to update the head of the linked list.

Algorithm: Deletion at beginning

Step 1: IF HEAD = NULL

Write UNDERFLOW

Go to Step 5

[END OF IF]

Step 2: SET PTR = HEAD

Step 3: SET HEAD = HEAD -> NEXT

Step 4: FREE PTR

Step 5: EXIT

Searching in linked list

Searching is performed to find the location of a particular element in the list. Traversing is performed in the list and make the comparison of every element of the list with the specified element. If the element is matched with any of the list element then the location of the element is returned from the function.

Algorithm: Searching in linked list

Step 1: SET PTR = HEAD

Step 2: Set I = 0

STEP 3: IF PTR = NULL

WRITE "EMPTY LIST"

GOTO STEP 8

END OF IF

STEP 4: REPEAT STEP 5 TO 7 UNTIL PTR != NULL

STEP 5: if ptr → data = item

write i+1

End of IF

STEP 6: I = I + 1

STEP 7: PTR = PTR → NEXT

[END OF LOOP]

STEP 8: EXIT

Linked List Common Errors

Here is summary of common errors of linked lists. Read these carefully, and read them againwhen you have problem that you need to solve.

1. Allocating a new node to step through the linked list; only a pointer variable is needed.

2. Confusing the and the -> operators.

3. Not setting the pointer from the last node to 0 (null).

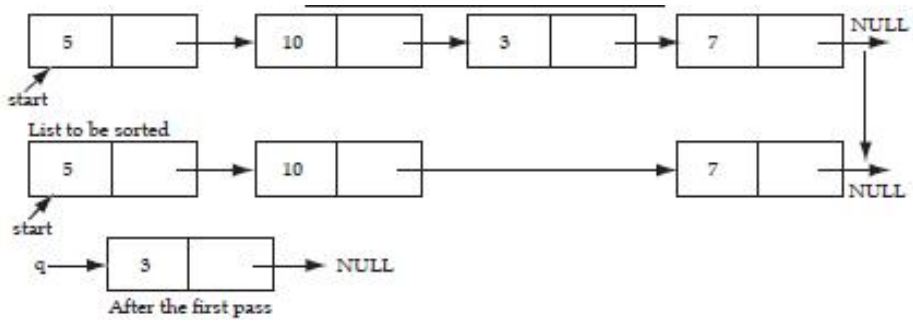
4. Not considering special cases of inserting/removing at the beginning or the end of thelinked list.

5. Applying the delete operator to a node (calling the operator on a pointer to the node)before it is removed. Delete should be done after all pointer manipulations are completed.

6. Pointer manipulations that are out of order. These can ruin the structure of the linked list.

Sorting and Reversing a Linked List

To sort a linked list, fi rst we traverse the list searching for the node with a minimum data value.Then we remove that node and append it to another list which is initially empty. We repeat thisprocess with the remaining list until the list becomes empty, and at the end, we return a pointerto the beginning of the list to which all the nodes are moved.



Sorting of linked list

To reverse a list, we maintain a pointer each to the previous and the next node, then we make thelink field of the current node point to the previous, make the previous equal to the current, and thecurrent equal to the next.

Arrays vs. Linked list

Array	Linked list
Data elements are stored in contiguous locations in memory.	New elements can be stored anywhere and a reference is created for the new element using pointers.
Insertion and Deletion operations are costlier since the memory locations are consecutive and fixed.	Insertion and Deletion operations are fast and easy in a linked list.
Memory is allocated during the **compile time** (*Static memory allocation*).	Memory is allocated during the **run-time** (*Dynamic memory allocation*).
Size of the array must be specified at the time of array declaration/initialization.	Size of a Linked list grows/shrinks as and when new elements are inserted/deleted.

Summary

- An array is a set of same data elements grouped together. Arrays can be one-dimensional ormultidimensional.
- A linear or one-dimensional array is a structured collection of elements (often called as array elements) that are accessed individually by specifying the position of each element with a single index value.
- Multidimensional arrays are nothing but "arrays of arrays". Two subscripts are used to refer to the elements.
- The operations that are performed on an array are adding, sorting, searching, and traversing.
- Traversing an array refers to moving in inward and outward direction to access each element in an array.
- Linked list is a technique of dynamically implementing a list using pointers. A linked list contains two fields namely, data field and link field.
- A singly-linked list consists of only one pointer to point to another node and the last node always points to NULL to indicate the end of the list.
- A doubly-linked list consists of two pointers, one to point to the next node and the other to point to the previous node.
- In a circular singly-linked list, the last node always points to the first node to indicate the circular nature of the list.
- A circular doubly-linked list consists of two pointers for forward and backward traversal and the last node points to the first node.

- Searching operation involves searching for a specific element in the list using an associated key.
- Insertion operation involves inserting a node at the beginning or end of a list.
- Deletion operation involves deleting a node at the beginning or following a given node or at the end of a list.

Keywords

Non-linear Data Structure: Every data item is attached to several other data items in a way thatis specific for reflecting relationships. The data items are not arranged in a sequential structure.

Searching: Finding the location of the record with a given key value, or fi nding the locations ofall records, which satisfy one or more conditions.

Traversing: Accessing each record exactly once so that certain items in the record may beprocessed.

Circular Linked List: A linear linked list in which the last element points to the fi rst element, thus,forming a circle.

Doubly Linked List: A linear linked list in which each element is connected to the two nearestelements through pointers.

Self-Assessment

1. Which one is correct statement?

A. Search in array is delete an element from array
B. Search in array is find an element from array
C. Search in array is insert an element in array
D. None of above

2. Which is not correct syntax?

A. abcname[ ];
B. abcname[10];
C. abcname[3] = {20,25,35};
D. abcname[5] = {15;20;28};

3. Searching is a process in which we find element in array

A. True
B. False

4. Operations can be performed on array

A. Sorting
B. Merging
C. Traversing
D. All of above

5. To merge two arrays, how many variables are required (minimum)?

A. 1
B. 2
C. 5
D. 3

6. Elements of arrays are stored in …….. memory locations

A. Random
B. Sequential
C. Both random and sequential
D. None of above

7. Which is correct statement?

A. Insertion at the given index of an array
B. Insertion after the given index of an array
C. Insertion before the given index of an array
D. All of above

8. Traversal is process of visit each element of an array

A. True
B. False

9. Which statement is incorrect?

1. int arr[MAX]= {10,12,15,20},i,val;
2. printf("the array element are \n′);
3. for(i=0;i<4;i++)
4. none of above

A. 1
B. 2
C. 3
D. 4

10. Array is___

A. A group of elements of same data type
B. Array elements are stored in memory in continuous or contiguous locations
C. An array contains more than one element
D. All of above

11. What are the advantages of arrays?

A. Objects of mixed data types can be stored
B. Easier to store elements of same data type
C. Elements in an array cannot be sorted

D. Index of first element of an array is 1

12. The index of the first element in an array is_____

A. 1
B. 2
C. -1
D. 0

13. Linked list consist of…

A. Data field
B. Link field
C. Both data and link field
D. None of above

14. What are the shortcomings of array?

A. Memory allocation
B. Memory efficiency
C. Execution time
D. All of above

15. What are the types of linked list?

A. Single
B. Double
C. Circular
D. All of above

16. Operations performed on Linked list are…

A. Insertion
B. Deletion
C. Search
D. All of above

17. Insertion in Linked List can happen at following places

A. At the beginning of the linked list.
B. At the end of the linked list.
C. At a given position in the linked list.
D. All of above

18. Linked list is considered as an example of _____ type of memory allocation

A. Static
B. Heap
C. Dynamic

D. Compile time

Answers for Self Assessment

1.	B	2.	D	3.	A	4.	D	5.	D
6.	B	7.	D	8.	A	9.	B	10.	D
11.	B	12.	D	13.	C	14.	D	15.	D
16.	D	17.	D	18.	C				

Review Questions

1. Define array and its types.
2. Give an example of multidimensional array.
3. Discuss any two types of array initialization methods with example.
4. Discuss different sorting methods.
5. Write a program to sort the elements of a linked list.
6. Differentiate between array and linked list with suitable example.
7. Discuss different operation performed with linked list.
8. Discuss advantages of linked list as compared to arrays.

Further Readings

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Unit 03: Stacks

CONTENTS

Objectives

After studying this unit, you will be able to:

- Learn fundamentals of stacks
- Explain the basic operations of stack
- Explain the implementation and applications of stacks

Introduction

Stacks are simple data structures and an important tool in programming language. Stacks are linear lists which have restrictions on the insertion and deletion operations. These are special cases of ordered list in which insertion and deletion is done only at the ends.

The basic operations performed on stack are push and pop. Stack implementation can be done in two ways - static implementation or dynamic implementation. Stack can be represented in the memory using a one-dimensional array or a singly linked list.

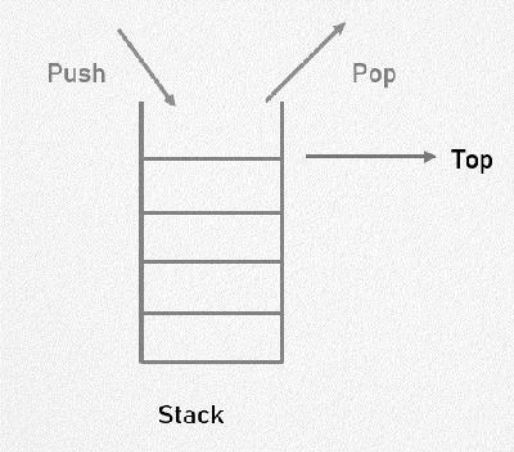
Stack is another linear data structure having a very interesting property. Unlike arrays and link lists, an element can be inserted and deleted not at any arbitrary position but only at one end. Thus, one end of a stack is sealed for insertion and deletion while the other end allows both the operations.

3.1 Stack Structure

The stack data structure is used to maintain records of a file in which the order among the records of file is not important. Figure 7.1 displays the structure of a stack where stack is like a hollow cylinder with a closed bottom end and an open top end. In the stack data structure, the records are added and deleted at the top end. Last-In-First-Out (LIFO) principle is followed to retrieve records from the stack. The records added last are accessed first.

A stack is a linear data structure in as much as its member elements are ordered as 1st, 2nd,…. and last. However, an element can be inserted in and deleted from only one end. The other end remains sealed. This open end to which elements can be inserted and deleted from is called stack top or top of the stack. Consequently, the elements are removed from a stack in the reverse order of insertion.

A stack is said to possess LIFO (Last In First Out) property. A data structure has LIFO property if the element that can be retrieved first is the one that was inserted last.



3.2 Basic Operations of Stack

The basic operations of stack are to:

1. Insert an element in the stack (Push operation)

2. Delete an element from the stack (Pop operation)

Push Operation

The procedure to insert a new element to the stack is called push operation. The push operation adds an element on the top of the stack. 'Top' refers to the element on the top of stack. Push makes the 'Top' point to the recently added element on the stack. After every push operation, the 'Top' is incremented by one. When the array is full, the status of stack is FULL and the condition is called stack overflow. No element can be inserted when the stack is full

Algorithm to Implement Push Operation on Stack

PUSH (STACK, n, top, item) /* n = size of stack*/

if (top = n) then STACK_FULL; /* checks for stack overflow */

else

{ top = top+1; /* increases the top by 1 */

STACK [top] = item ;} /* inserts item in the new top position */

end PUSH

Pop Operation

The procedure to delete an element from the top of the stack is called pop operation. After every pop operation, the 'Top' is decremented by one. When there is no element in the stack, the status of the stack is called empty stack or stack underflow. The pop operation cannot be performed when it is in stack underflow condition.

Algorithm to Implement Pop Operation in a Stack

POP (STACK, top, item)

if (top = 0) then STACK_EMPTY; /* check for stack underflow*/

else { item = STACK [top]; /* remove top element*/

top = top – 1; /* decrement stack top*/

}

end POP

3.3 Implementation of Stacks

There are two basic methods for the implementation of stacks – one where the memory is used statically and the other where the memory is used dynamically.

Array-based Implementation

A stack is a sequence of data elements. To implement a stack structure, an array can be used as it is a storage structure. Each element of the stack occupies one array element. Static implementation of stack can be achieved using arrays. The size of the array, once declared, cannot be changed during the program execution. Memory is allocated according to the array size. The memory requirement is determined before the compilation. The compiler provides the required memory. This is suitable when the exact number of elements is known. The static allocation is an inefficient memory allocation technique because if fewer elements are stored than declared, the memory is wasted and if more elements need to be stored than declared, the array cannot expand. In both the cases, there is inefficient use of memory.

The following pseudo-code shows the array-based implementation of a stack. In this, the elements of the stack are of type T.

```
struct stk
{ T array[max_size];
/* max_size is the maximum size */
int top = -1;
/* stack top initially given value -1 */
} stack;
void push(T e)
/*inserts an element e into the stack s*/
{
if (stack.top == max_size)
printf("Stack is full-insertion not possible");
else
{
stack.top = stack.top + 1;
stack.array[stack.top] = e;
}
}
T pop()
/*Returns the top element from the stack */
{
T x;
if(stack.top == -1)
printf("Stack is empty");
else
{
x = stack.array[stack.top];
stack.top = stack.top - l;
return(x);
}
```

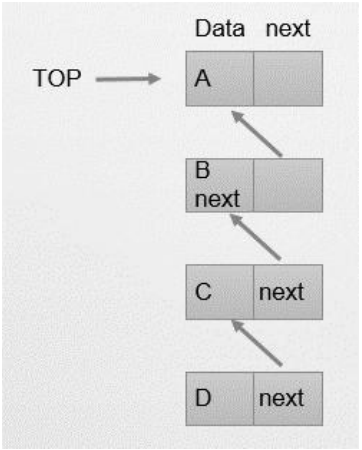
```
}
booleanempty()
/* checks if the stack is empty * /
{
boolean empty = false;
if(stack.top == -1)
empty = true else empty = false;
return(empty);
}
void initialise()
/* This procedure initializes the stack s * /
{
stack.top = -1;
}
```

The above implementation strategy is easy and fast since it does not have run-time overheads. At the same time it is not flexible since it cannot handle a situation when the number of elements exceeds max_size. Also, let us say, if max_size is derided statically to 100 and a stack actually has only 10 elements, then memory space for the rest of the 90 elements would be wasted.

Linked List Representation of Stacks

The array representation of stacks is easy and convenient. However, it allows the representation of only fixed sized stacks. The size of the stack varies during program application for different applications. Representing stack using linked list can solve this problem. A singly linked list can be used to represent any stack. In a singly linked list, the data field represents the ITEM and the LINK field points to the next item.

In linked list implementation of the stack, we need to create nodes and the nodes are maintained non-contiguously in the memory. Each node contains a pointer to its immediate successor node in the stack. Stack is said to be overflown if the space left in the memory heap is not enough to create a node.



Here the memory is used dynamically. For every push operation, the memory space for one element is allocated at run-time and the element is inserted into the stack. For every pop operation, the memory space for the deleted element is de-allocated and returned to the free space pool. Hence the shortcomings of the array-based implementation are overcome. But since, this allocates memory dynamically, the execution is slowed down.

The following pseudo-code is for the pointer-based implementation of a stack. Each element of the stack is of type T.

```
struct stk
{
T element;
struct stk *next;
};
struct stk *stack;
void push(struct stk *p, T e)
{
struct stk *x;
x = new(stk);
x.element = e;
x.next = NULL;
p = x;
}
```

Here the stack full condition is checked by the call to new which would give an error if no memory space could be allocated.

```
T pop(struct stk *p)
{
struct stk *x;
if (p == NULL)
printf("Stack is empty");
else
{ x = p;
x = x.next;
return(p.element);
}
booleanempty(sstructstk *p)
{
if (p == NULL)
return(true);
else
return(false);
}
void initialize(struct stk *p)
{
p = NULL;
}
```

3.4 Applications of Stacks

There are numerous applications of the stack data structure in computer algorithms. It is used to store return information in the case of function/procedure/subroutine calls. Hence, one would find a stack in architecture of any Central Processing Unit (CPU). In this section, we would just illustrate a few of them.

Expression Evaluation and Conversion

Parenthesis Checking

Backtracking

Function Call

String Reversal

Memory Management

Syntax Parsing

Parenthesis checker

Parenthesis checker is used for balanced Brackets in an expression. The balanced parenthesis means that when the opening parenthesis is equal to the closing parenthesis, then it is a balanced parenthesis.

(a+b*(c/d))

[10+20*(6+7)]

(x+y)/(c-d)

Balanced parenthesis

A = (50+25)

In the above expression there is one opening and one closing parenthesis means that both opening and closing brackets are equal; therefore, the above expression is a balanced parenthesis.

Unbalanced parenthesis

A= [(15+25)

The above expression has two opening brackets and one closing bracket, which means that both opening and closing brackets are not equal; therefore, the above expression is unbalanced.

Algorithm

- Initialize a character stack.
- Now traverse the expression string exp.
- If the current character is a starting bracket ('(' or '{' or '[') then push it to stack.
- If the current character is a closing bracket (')' or '}' or ']') then pop from stack and if the popped character is the matching starting bracket then balanced else brackets are not balanced.
- After complete traversal, if there is some starting bracket left in stack then not balanced

Expression conversion and evaluation

Arithmetic expressions can be represented in 3 forms:

Infix notation

Postfix notation (Reverse Polish Notation)

Prefix notation (Polish Notation)

Infix Notation

Infix Notation can be represented as:

operand1 operator operand1

Example: 15 + 26

a + b

Postfix Notation

Postfix Notation can be represented as

operand1 operand2 operator

Example: 15 29 +

a b +

Prefix notation

Prefix notation can be represented as

operator operand1 operand2

Example: + 10 20

+ a b

Infix notation is used most frequently in our day to day tasks. Machines find infix notations tougher to process than prefix/postfix notations. Hence, compilers convert infix notations to prefix/postfix before the expression is evaluated.

The precedence of operators needs to be taken care as per hierarchy

(^) > (*) > (/) > (+) > (-)

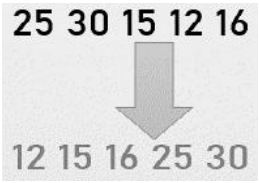
Brackets have the highest priority.

To evaluate an infix expression, We need to perform 2 steps:

- Convert infix to postfix
- Evaluate postfix

Sorting

A Sorting process is used to rearrange a given array or elements based upon selected algorithm/ sort function.



Quick Sort is used for sorting a list of data elements.Quicksort is a sorting algorithm based on the divide and conquer approach.An array is divided into subarrays by selecting a pivot element.During array dividing, the pivot element should be positioned in such a way that elements less than pivot are kept on the left side and elements greater than pivot are on the right side of the pivot.The left and right subarrays are also divided using the same approach. This process continues until each subarray contains a single element

There are many different versions of quick Sort that pick pivot in different ways.

Always pick first element as pivot.

Always pick last element as pivot

Pick a random element as pivot.

Pick median as pivot.

Algorithm

```
quickSort(arr, beg, end)
 if (beg < end)
pivotIndex = partition(arr,beg, end)
quickSort(arr, beg, pivotIndex)
quickSort(arr, pivotIndex + 1, end)
partition(arr, beg, end)
 set end as pivotIndex
pIndex = beg - 1
 for i = beg to end-1
 if arr[i] < pivot
   swap arr[i] and arr[pIndex]
pIndex++
 swap pivot and arr[pIndex+1]
return pIndex + 1
```

Tower of Hanoi

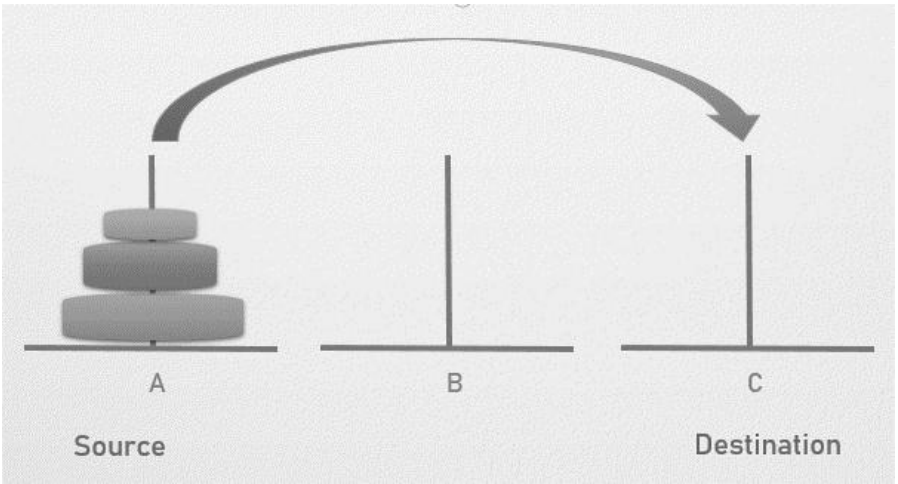
The Tower of Hanoi, is a mathematical problem which consists of three rods and multiple disks.Initially, all the disks are placed on one rod, one over the other in ascending order of size similar to a cone-shaped tower.

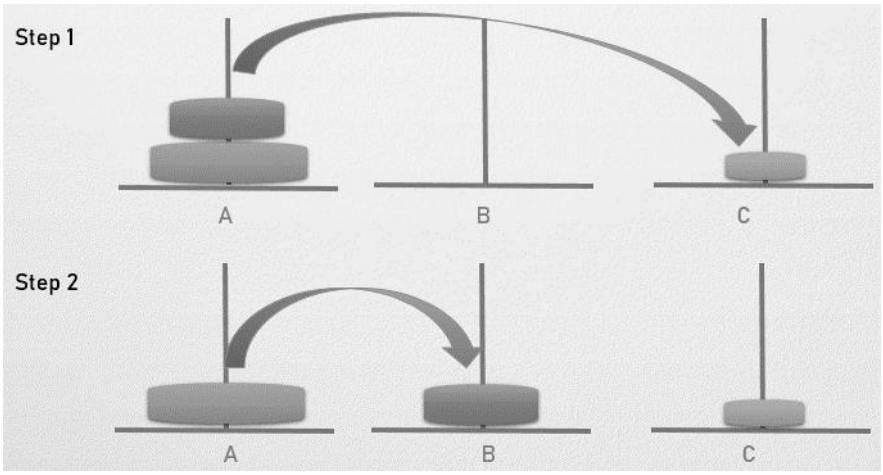
The objective of this problem is to move the stack of disks from the source to destination, following these rules:

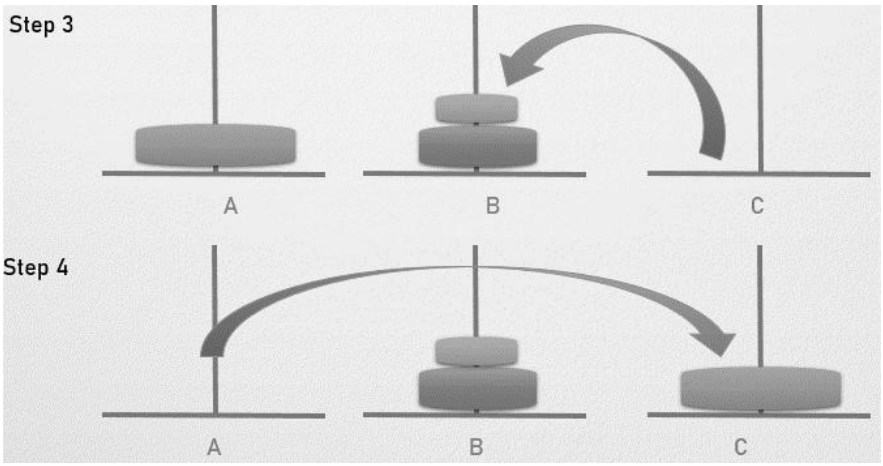
1. Only one disk can be moved at a time.

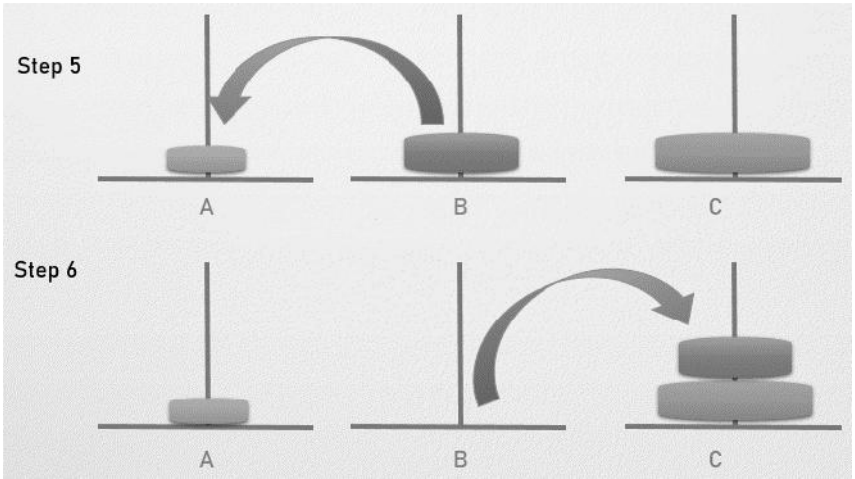
2. Only the top disk can be removed.

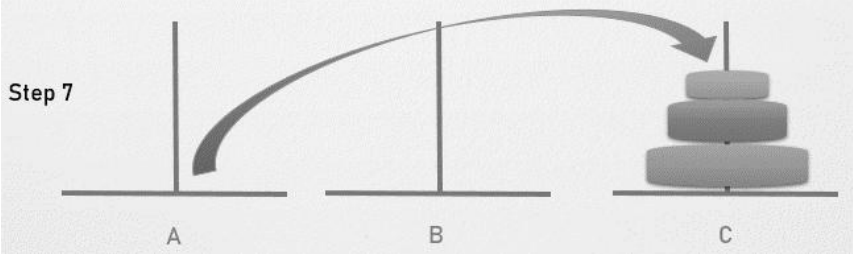
3. No large disk can sit over a small disk.




Step 1
A
B
C
Step 2
A
B
C


Step 3
A
B
C
Step 4
A
B
C


Step 5
A
B
C
Step 6
A
B
C



Iterative Algorithm

1. At First Calculate the number of moves required i.e. "pow(2,n) - 1" where "n" is number of discs.
2. If the number of discs i.e n is even then swap Destination Rod and Auxiliary Rod.
3. for i = 1 upto number of moves:

   Check if "i mod 3" == 1:

   Perform Movement of top disc between Source Rod and Destination Rod.

   Check if "i mod 3" == 2:

   Perform Movement of top disc between Source Rod and Auxiliary Rod.

   Check if "i mod 3" == 0:

   Perform Movement of top disc between Auxiliary Rod and Destination Rod.

Simulating Recursive Function using Stack

A recursive solution to a problem is often more expensive than a non-recursive solution, both in terms of time and space. Frequently, this expense is a small price to pay for the logical simplicity and self-documentation of the recursive solution. However, in a production program (such as a compiler, for example) that may be run thousands of times, the recurrent expense is a heavy burden on the system's limited resources.

Thus, a program may be designed to incorporate a recursive solution in order to reduce the expense of design and certifi cation, and then carefully converted to a non-recursive version to be put into actual day-to-day use. As we shall see, in performing such as conversion it is often possible to identify parts of the implementation of recursion that are superfluous in a particular application and thereby significantly reduce the amount of work that the program must perform.

Suppose that we have the statement

rout (x); where route is defi ned as a function by the header

rout(a); x is referred to as an argument (of the calling function), and a is referred to as a parameter(of the called function).

What happens when a function is called? The action of calling a function may be divided intothree parts:

1. Passing Arguments

2. Allocating and initializing local variables

3. Transferring control to the function.

1. ***Passing arguments***: For a parameter in C, a copy of the argument is made locally within the function, and any changes to the parameter are made to that local copy. The effect to this scheme is that the original input argument cannot be altered. In this method, storage for the argument is allocated within the data area of the function.

2. ***Allocating and initializing local variables***: After arguments have been passed, the local variables of the function are allocated. These local variables include all those declared directly in the function and any temporaries that must be created during the course of execution.

3. ***Transferring control to the function***: At this point control may still not be passed to the function because provision has not yet been made for saving the return address. If a function is

given control, it must eventually restore control to the calling routine by means of a branch. However, it cannot execute that branch unless it knows the location to which it must return. Since this location is within the calling routine and not within the function, the only way that the function can know this address is to have it passed as an argument. This is exactly what happens. Aside from the explicit arguments specified by the programmer, there is also a set of implicit arguments that contain information necessary for the function to execute and return correctly. Chief among these implicit arguments is the return address. The function stores this address within its own data area. When it is ready to return control to the calling program, the function retrieves the return address and branches to that location. Once the arguments and the return address have been passed, control may be transferred to the function, since everything required has been done to ensure that the function can operate on the appropriate data and then return to the calling routine safely.

Summary

- A stack is a linear data structure in which allocation and deallocation are made in a last-in-first-out (LIFO) method.
- The basic operations of stack are inserting an element on the stack (push operation) and deleting an element from the stack (pop operation).
- Stacks are represented in main memory by using one-dimensional array or a singly linked list.
- To implement a stack structure, an array can be used as its storage structure. Each element of the stack occupies one array element. Static implementation of stack can be achieved using arrays.
- Stack is used to store return information in the case of function/procedure/subroutine calls. Hence, one would fi nd a stack in architecture of any Central Processing Unit (CPU).
- In infix notation operators come in between the operands. An expression can be evaluated using stack data structure.

Keywords

LIFO: (Last In First Out) the property of a list such as stack in which the element which can be retrieved is the last element to enter it.

Pop: Stack operation retrieves a value form the stack.

Infix: Notation of an arithmetic expression in which operators come in between their operands.

Postfix: Notation of an arithmetic expression in which operators come after their operands.

Prefix: Notation of an arithmetic expression in which operators come before their operands.

Push: Stack operation which puts a value on the stack.

Stack: A linear data structure where insertion and deletion of elements can take place only at one end.

SelfAssessment

1. Which method is followed by Stack?

A. FILO
B. LIFO
C. Both FILO and LIFO
D. None of above

2. Which is not stack operation?

A. count()
B. peek()
C. getche()
D. display()

3. Which is not part of stack?

A. Overflow
B. Enqueue
C. Underflow
D. None of above

4. To returns the element at the given position which function is used?

A. isEmpty()
B. isFull()
C. peek()
D. display()

5. Stack implementation is done using……

A. Array
B. Linked list
C. Both using array and linked list
D. None of above

6. Parenthesis checker is used for

A. Balanced Brackets
B. Unbalanced Brackets
C. Both balanced and unbalanced
D. None of above

7. Which is incorrect statement?

A. (a+b*(c/d))
B. [10+20*(6+7)}]
C. (x+y)/(c-d)
D. None of above

8. Which is not Sorting type

A. Bubble
B. Merge
C. Insertion
D. Linear

9. Tower of Hanoi is associated with….

A. Queue
B. Stack
C. Array
D. None of above

10. Arithmetic expressions can be represented as

A. Infix notation
B. Postfix notation
C. Prefix notation
D. All of above

11. Prefix notation can be represented as

A. operator operand1 operand2
B. operand1 operand2 operator
C. operand1 operator operand1
D. None of above

12. Postfix notation can be represented as

A. operator operand1 operand2
B. operand1 operator operand1
C. operand1 operand2 operator
D. None of above

13. Stack implementation is done using_____

A. Statically
B. Dynamically
C. Both Statically and Dynamically
D. None of above

14. Which is not stack operation?

A. peek()
B. pop()
C. display()
D. printf()

15. Underflow condition occur when stack is_____

A. Full
B. Empty
C. Half
D. None of above

Answers for Self Assessment

1.	C	2.	C	3.	B	4.	C	5.	C
6.	C	7.	B	8.	D	9.	B	10.	D
11.	A	12.	C	13.	C	14.	D	15.	B

Review Questions

1 Explain how function calls may be implemented using stacks for return values.

2 What are the advantages of implementing a stack using dynamic memory allocation method?

3 Explain concept of tower of Hanoi.

4 what are the different methods for implementing stacks?

5 Give an example of push and pop operation using stack.

6 Write an algorithm to reverse an input string of characters using a stack.

Further Readings

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Web Links

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Unit 04: Queues

CONTENTS

Objectives

After studying this unit, you will be able to:

- Learn implementation of queues
- Explain priority queue
- Discuss applications of queues

Introduction

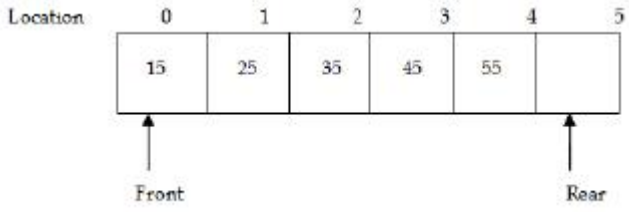
A queue is a linear list of elements that consists of two ends known as front and rear. We can delete elements from the front end and insert elements at the rear end of a queue. A queue in an application is used to maintain a list of items that are ordered not by their values but by their sequential value.

The queue abstract data type is also a widely used one with applications very common in real life. An example comes from the operating system software where the scheduler picks up the next process to be executed on the system from a queue data structure. In this unit, we would study the various properties of queues, their operations and implementation strategies.

4.1 Fundamentals of Queues

A queue is an ordered collection of items in which deletion takes place at one end, which is called the front of the queue, and insertion at the other end, which is called the rear of the queue. The queue is a 'First In First Out' system (FIFO). In a time-sharing system, there can be many tasks waiting in the queue, for access to disk storage or for using the CPU. The queues in a bank, or railway station counter are examples of queue. The first person in the queue is the first to be attended.

The two main operations in the queue are insertion and deletion of items. The queue has two pointers, the front pointer points to the first element of the queue and the rear pointer points to the last element of the queue.



4.2 Basic Operations of Queue

The basic operations of queue are insertion and deletion of items which are referred as enqueue and dequeue respectively. In enqueue operation, an item is added to the rear end of the queue. In dequeue operation, the item is deleted from the front end of the queue.

Insert at Rear End

To insert an item into the queue, first it should be verified whether the queue is full or not. If the queue is full, a new item cannot be inserted into the queue. The condition FRONT=NULL indicates that the queue is empty. If the queue is not full, items are inserted at the rear end. When an item is added to the queue, the value of rear is incremented by 1.

In queue we need to maintain two data pointers, front and rear. Operations on queue are comparatively difficult to implement than that of stacks

Step 1 − Check if the queue is full.

Step 2 − If the queue is full, produce overflow error and exit.

Step 3 − If the queue is not full, increment rear pointer to point the next empty space.

Step 4 − Add data element to the queue location, where the rear is pointing.

Step 5 − return success.

Algorithm: Enqueue operation

procedure enqueue(data)

  if queue is full

  return overflow

 endif

  rear ← rear + 1

 queue[rear] ← data

 return true

 end procedure

Delete from the Front End

To delete an item from the stack, first it should be verified that the queue is not empty. If the queue is not empty, the items are deleted at the front end of the queue. When an item is deleted from the queue, Dequeue operation include two tasks: access the data where front is pointing and remove the data after access.

Step 1 − Check if the queue is empty.

Step 2 − If the queue is empty, produce underflow error and exit.

Step 3 − If the queue is not empty, access the data where front is pointing.

Step 4 − Increment front pointer to point to the next available data element.

Step 5 − Return success. the value of the front is incremented by 1.

Algorithm: Dequeue operation

procedure dequeue

  if queue is empty

```
    return underflow
  end if
  data = queue[front]
  front ← front + 1
  return true
end procedure
```

 Example:

```
/*Program of queue using array*/
/*insertion and deletion in a queue*/
/*insertion and deletion in a queue*/
# include <stdio.h>
# define MAX 50
int queue_arr[MAX];
int rear = -1;
int front = -1;
void ins_delete();
void insert();
void display();
void main()
{
int choice;
while(1)
{
printf("1.Insert\n");
printf("2.Delete\n");
printf("3.Display\n");
printf("4.Quit\n");
printf("Enter your choice : \n");
scanf("%d",&choice);
switch(choice)
{
case 1 : insert();
break;
case 2 :ins_delete();
break;
case 3: ins_display();
break;
case 4: exit(1);
default:
```

```
printf("Wrong choice\n");
}/*End of switch*/
}/*End of while*/
}/*End of main()*/
void insert()
{
int added_item;
if (rear==MAX-1)
printf("Queue overflow\n");
else
{
if (front==-1) /*If queue is initially empty */
front=0;
printf("Enter an element to add in the queue : ");
scanf("%d", &added_item);
rear=rear+1;
queue_arr[rear] = added_item ;
}
} /*End of insert()*/
void ins_delete()
{
if (front == -1 || front > rear)
{
printf("Queue underflow\n");
return ;
}
else
{
printf("Element deleted from queue is : %d\n", queue_arr[front]);
front=front+1;
}
} /*End of delete() */
void display()
{
int i;
if (front == -1)
printf("Queue is empty\n");
else
{
printf("Elements in the queue:\n");
for(i=front;i<= rear;i++)
```

```
printf("%d ",queue_arr[i]);
printf("\n");
}
} /*End of display() */
```

Output:

1. Insert

2. Delete

3. Display

4. Quit

Enter your choice: 1

Enter an element to add in the queue: 25

Enter your choice: 1

Enter an element to add in the queue: 36

Enter your choice: 3

Elements in the queue: 25, 36

Enter your choice: 2

Element deleted from the queue is: 25

In this example:

1. The preprocessor directives #include are given. MAXSIZE is defined as 50 using the #define statement.

2. The queue is declared as an array using the declaration int queue_arr[MAX].

3. In the while loop, the different options are displayed on the screen and the value entered in the variable choice is accepted.

4. The switch case compares the value entered and calls the method corresponding to it. If the value entered is invalid, it displays the message "Wrong choice".

5. Insert method: The insert method inserts item in the queue. The if condition checks whether the queue is full or not. If the queue is full, the "Queue overflow" message is displayed. If the queue is not full, the item is inserted in the queue and the rear is incremented by 1.

6. Delete method: The delete method deletes item from the queue. The if condition checks whether the queue is empty or not. If the queue is empty, the "Queue underflow" message is displayed. If the queue is not empty, the item is deleted and front is incremented by 1.

7. Display method: The display method displays the contents of the queue. The if condition checks whether the queue is empty or not. If the queue is not empty, it displays all the items in the queue.

4.3 Types of Queue

Simple Queue

In a simple queue, insertion takes place at the rear and removal occurs at the front. It follows the FIFO (First in First out) rule.

Circular Queue

In a circular queue, the last element points to the first element making a circular link. In a circular queue, the rear end is connected to the front end forming a circular loop. An advantage of circular queue is that, the insertion and deletion operations are independent of one another. This prevents an interrupt handler from performing an insertion operation at the same time when the main function is performing a deletion operation.

Double ended queue

Double ended queue is also known as deque. It is a type of queue where the insertions and deletions happen at the front or the rear end of the queue. The various operations that can be performed on the double ended queue are:

1. Insert an element at the front end
2. Insert an element at the rear end
3. Delete an element at the front end
4. Delete an element at the rear end

 Example:

Program for Implementation of Circular Queue.

```
#include<stdio.h>
#include<conio.h>
#define SIZE 5
int Q_F(int COUNT)
{
return (COUNT==SIZE)? 1:0;
}
int Q_E(int COUNT)
{
return (COUNT==0)? 1:0;
}
void rear_insert(int item, int Q[], int *R, int *COUNT)
{
if(Q_F(*COUNT))
{
printf("Queue overflow");
return;
}
*R=(*R+1) % SIZE;
Q[*R]=num;
*COUNT+=1;
}
void front_delete(int Q[], int *F, int *COUNT)
{
if(Q_E(*COUNT))
{
printf("Queue underflow");
return;
}
printf("The deleted element is %d\n", Q[*F]);
*F=(*F+1) % SIZE;
```

```
*COUNT-=1;
}
void display(int Q[], int F, int COUNT)
{
int i,j;
if(Q_E(COUNT))
{
printf("Queue is empty\n");
return;
}
printf("The contents of the queue are:\n");
i=F;
for(j=1;j<=COUNT; j++)
{
printf("%d\n", Q[i]);
i=(i+1) % SIZE;
}
printf("\n");
}
void main()
{
int choice, num, COUNT, F, R, Q[20];
clrscr();
F=0;
R=-1;
COUNT=0;
for(;;)
{
printf("1. iInsert at front\n");
printf("2. Delete at rear end\n");
printf("3. Display\n");
printf("4. Exit\n");
scanf("%d", &choice);
switch(choice)
{
case 1: printf("Enter the number to be inserted\n");
scanf("%d", &num);
rear_insert(num, Q, &R, &COUNT);
break;
case 2: front_delete(Q, &F, &COUNT);
break;
```

```
case 3: display(Q, F, COUNT);
break;
default: exit(0);
}
}
}
```

Output:

```
1. Insert at rear end
2. Delete at front end
3. Display
4. Exit
1
Enter the number to be inserted
50
1. Insert at rear end
2. Delete at front end
3. Display
4. Exit
1
Enter the number to be inserted
60
1. Insert at rear end
2. Delete at front end
3. Display
4. Exit
3
The contents of the queue are 50 60
1. Insert at rear end
2. Delete at front end
3. Display
4. Exit
2
The element deleted is 50
```

 In this example:

1. The header files are included and a constant value 5 is defined for variable SIZE using #define statement. The SIZE defines the size of the queue.

2. A queue is created using an array named Q with an element capacity of 20. A variable named COUNT is declared to store the count of number elements present in the queue.

3. Four functions are created namely, Q_F(), Q_E(), rear_insert(), front_delete(),and display(). The user has to select an appropriate function to be performed.

4. The switch statement is used to call the rear_insert(), front_delete(), and display() functions.

5. When the user enters 1, rear_insert() function is called. In the rear_insert() function, the if loop checks if the count is full. If the condition is true, then the program prints a message "Queue is empty". Else, it checks for the value of R and assigns the element (num) entered by the user to R. Initially, when there are no elements in the queue, the value of R will be 0. After every insertion, the variable COUNT is incremented.

6. When the user enters 2, the front_delete() function is called. In this function, the if loop checks if the variable COUNT is empty. If the condition is true, then the program prints a message "Queue underflow". Else, the element in the 0th

7. When the user enters 3, the display() function is called. In this function, the if loop checks if the value of COUNT is 0. If the condition is true, the program prints a message "Queue is empty". Else, the value of F is assigned to the variable i. The for loop then displays the elements present in the queue. position will be deleted. The size of F is computed and the COUNT is set to 1.

8. When the user enters 4, the program terminates.

Priority Queue

In priority queue, the elements are inserted and deleted based on their priority. Each element is assigned a priority and the element with the highest priority is given importance and processed first. If all the elements present in the queue have the same priority, then the first element is given importance.

A priority queue is an abstract data type in which each element is associated with a priority value.Elements are served on the basis of their priority.An element with high priority is dequeued before an element with low priority.If two elements have the same priority, they are served according to their order in the queue.

The priority queue moves the highest priority elements at the beginning of the priority queue and the lowest priority elements at the back of the priority queue.It supports only those elements that are comparable. Priority queue in the data structure arranges the elements in either ascending or descending order.

Types of Priority Queue

Ascending Order Priority Queue

An ascending order priority queue gives the highest priority to the lower number in that queue

Example:

List: 5 6 20 22 10

Arrange these numbers in ascending order.

List 5 6 10 20 22

Descending Order Priority Queue

A descending order priority queue gives the highest priority to the highest number in that queue.

Example:

List: 5 6 35 22 10

Arrange these numbers

List: 35 22 10 6 5

Priority Queue Operations

Inserting an Element into the Priority Queue

Deleting an Element from the Priority Queue

Peeking from the Priority Queue (Find max/min)

Extract-Max/Min from the Priority Queue

Priority queue can be implemented using

Array

Linked list

Heap data structure

Binary search tree

Priority Queue Applications

Dijkstra's algorithm: To find shortest path in graph.

Prim's Algorithm: Prim's algorithm uses the priority queue to the values or keys of nodes and draws out the minimum of these values at every step.

Data Compression: Huffman codes use the priority queue to compress the data.

Operating Systems: load balancing and interrupt handling in an operating system

4.4 Implementation of Queues

There are two possible implementation strategies – one where the memory is used statically and the other where memory is used dynamically.

Queue can be implemented using:

Array

Linked List

Queue implementation Using Array

To represent a queue we require a one-dimensional array of some maximum size say n to hold the data items and two other variables front and rear to point to the beginning and the end of the queue.

Queue implemented using array stores only fixed number of data values. Two variables front andrear, that are implemented in queue. Front and rear variables point to the position from where insertions and deletions are performed in a queue.

Initially both front and rear are set to -1.For insert a new value into the queue, increment rear value by one and then insert at that position. For delete a value from the queue, then delete the element which is at front position and increment front value by one.

Enqueue operation

Enqueue() function is used to insert a new element into the queue. In a queue, the new element is always inserted at rear position. The enQueue() function takes one integer value as a parameter and inserts that value into the queue.

Algorithm: Enqueue operation

Step 1: IF REAR = MAX - 1

Write OVERFLOW

Go to step

[END OF IF]

Step 2: IF FRONT = -1 and REAR = -1

SET FRONT = REAR = 0

ELSE

SET REAR = REAR + 1

[END OF IF]

Step 3: Set QUEUE[REAR] = NUM

Step 4: EXIT

Dequeue operation

Dequeue() is a function used to delete an element from the queue. In a queue, the element is always deleted from front position. The Dequeue() function does not take any value as parameter.

Algorithm: Dequeue operation

Step 1: IF FRONT = -1 or FRONT > REAR

Write UNDERFLOW

ELSE

SET VAL = QUEUE[FRONT]

SET FRONT = FRONT + 1

[END OF IF]

Step 2: EXIT

Queue implementation Using Linked list

Due to the drawbacks of array. The array implementation cannot be used for the large scale applications where the queues are implemented.The alternative of array implementation is linked list implementation of queue. In a linked queue, each node of the queue consists of two parts i.e. data part and the link part. Each element of the queue points to its immediate next element in the memory.

In the linked queue, there are two pointers maintained in the memory i.e. front pointer and rear pointer. The front pointer contains the address of the starting element of the queue while the rear pointer contains the address of the last element of the queue.

Insert operation

There can be the two scenario of inserting this new node ptr into the linked queue.In the first scenario, we insert element into an empty queue. In this case, the condition front = NULL becomes true.In the second case, the queue contains more than one element. The condition front = NULL becomes false.

Algorithm

Step 1: Allocate the space for the new node PTR

Step 2: SET PTR -> DATA = VAL

Step 3: IF FRONT = NULL

SET FRONT = REAR = PTR

SET FRONT -> NEXT = REAR -> NEXT = NULL

ELSE

SET REAR -> NEXT = PTR

SET REAR = PTR

SET REAR -> NEXT = NULL

[END OF IF]

Step 4: END

Delete operation

Deletion operation removes the element that is first inserted among all the queue elements. The condition front == NULL becomes true if the list is empty. Otherwise, we will delete the element that is pointed by the pointer front.

Algorithm

Step 1: IF FRONT = NULL

Write " Underflow "

Go to Step 5

[END OF IF]

Step 2: SET PTR = FRONT

Step 3: SET FRONT = FRONT -> NEXT

Step 4: FREE PTR

Step 5: END

4.5 Applications of Queues

One major application of the queue data structure is in the computer simulation of a real-world situation. Queues are also used in many ways by the operating system, the program that schedules and allocates the resources of a computer system. One of these resources is the CPU (CentralProcessing Unit) itself. If you are working on a multi-user system and you tell the computerto run a particular program, the operating system adds your request to its "job queue". Whenyour request gets to the front of the queue, the program you requested is executed. Similarly, thevarious users for the system must share the I/O devices (printers, disks etc.). Each device has itsown queue of requests to print, read or write to these devices. The following subsection discussesone application of the queues – the priority queue. It is used in time-sharing multi-user systemswhere programs of high priority are processed first arid programs with the same priority forma standard queue.

In Operating systems:

a) Semaphores

b) FCFS (first come first serve) scheduling,

c) Spooling in printers

d) Buffer for devices like keyboard

In Networks:

a) Queues in routers/ switches

b) Mail Queues

Queues are used in operating systems for handling interrupts.

Queues are used as buffers in most of the applications like MP3 media player, CD player, etc

When a resource is shared among multiple consumers.

CPU scheduling,

Disk Scheduling.

Summary

- A queue is an ordered collection of items in which deletion takes place at the front and insertion at the rear of the queue.
- In a memory, a queue can be represented in two ways; by representing the way in which the elements are stored in the memory, and by naming the address to which the front and rear pointers point to.
- The different types of queues are double ended queue, circular queue, and priority queue.
- The basic operations performed on a queue include inserting an element at the rear end and deleting an element at the front end.
- A priority queue is a collection of elements such that each element has been assigned a priority. An element of higher priority is processed before any element of lower priority.
- Two elements with the same priority are processed according to the order in which they were inserted into the queue.

Keywords

FIFO: (First In First Out) The property of a linear data structure which ensures that the element retrieved from it is the first element that went into it.

Front: The end of a queue from where elements are retrieved.

Queue: A linear data structure in which the element is inserted at one end while retrieved from another end.

Rear: The end of a queue where new elements are inserted.

Dequeue: Process of deleting elements from the queue.

Enqueue: Process of inserting elements into queue.

SelfAssessment

1. Which technique is followed by queue?

A. FIFO
B. LIFO
C. Both LIFO and FIFO
D. None of above

2. Enqueue () operation is used to perform

A. Deletion
B. Insertion
C. Display
D. All of above

3. Which operation is part of queue

A. Peek
B. isFull
C. isEmpty
D. All of above

4. Queue implementation is done using

A. Array
B. Stack
C. Linked List
D. All of above

5. Which is not types of Queue

A. Circular
B. Simple
C. Complex
D. Priority

6. Queue is a_____ data structure.

A. Static
B. Linear
C. Dynamic
D. None of above

7. Front pointer and rear pointer are used in…

A. Implementation of Queue using Linked List
B. Implementation of Queue using array
C. Implementation of Queue using stack
D. All of above

8. Underflow condition represent.

A. It checks if the queue is full before enqueueing any element.
B. It checks if there exists any item before popping from the queue.
C. It checks whether all variables are declared
D. All of above

9. Overflow condition represent.

A. It checks if there exists any item before popping from the queue.
B. It checks whether all variables are initialized.
C. It checks if the queue is full before enqueueing any element.
D. All of above

10. Front pointer contains

A. Address of the starting element of the queue
B. Address of the last element of the queue.
C. Link for next element
D. All of above

11. Priority queue is a_____ data structure.

A. Static
B. Linear
C. Abstract
D. All of above

12. Priority queue types are

A. Ascending order
B. Descending order
C. Both ascending and descending order
D. None of above

13. Priority Queue Operations are

A. Deleting an Element from the Priority Queue
B. Peeking from the Priority Queue (Find max/min)
C. Extract-Max/Min from the Priority Queue
D. All of above

14. Priority Queue Implementation are performed using.

A. Linked list
B. Heap data structure
C. Binary search tree
D. All of above

15. Binary Heap can be divided into

A. Max heap
B. Min-heap.
C. Both max and min heap
D. None of above

Answers for Self Assessment

1.	A	2.	B	3.	D	4.	D	5.	C
6.	B	7.	A	8.	B	9.	C	10.	A
11.	B	12.	C	13.	D	14.	D	15.	C

Review Questions

1 "Using double ended queues is more advantageous than using circular queues. "Discuss

2 "Stacks are different from queues." Justify.

3 "Using priority queues is advantageous in job scheduling algorithms. "Analyze

4 Can a basic queue be implemented to function as a dynamic queue? Discuss

5 Describe the application of queue.

6 How will you insert and delete an element in queue?

7 Explain dynamic memory allocation advantages.

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Unit 05: Search Trees

CONTENTS

Objectives

After studying this unit, you will be able to:

- Discuss the basics of tree
- Learn concept of binary tree
- Know binary tree traversal
- Explain the representation of tree in memory

Introduction

We know that data structure is a set of data elements grouped together under one name. A data structure can be considered as a set of rules that hold the data together. Almost all computer programs use data structures. Data structures are an essential part of algorithms. We can use it to manage huge amount of data in large databases. Some modern programming languages emphasize more on data structures than algorithms.

There are many data structures that help us to manipulate the data stored in the memory, which we have discussed in the previous units. These include array, stack, queue, and linked-list.

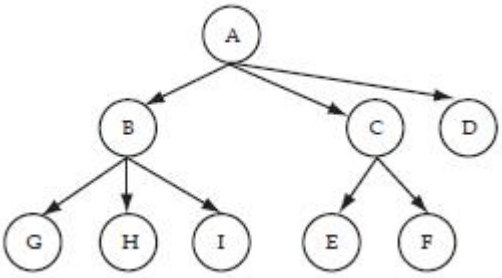
Choosing the best data structure for a program is a challenging task. Similar tasks may require different data structures. We derive new data structures for complex tasks using the already existing ones. We need to compare the characteristics of the data structures before choosing the right data structure. A tree is a hierarchical data structure suitable for representing hierarchical information. The tree data structure has the characteristics of quick search, quick inserts, and quick deletes.

5.1 Concept of Tree

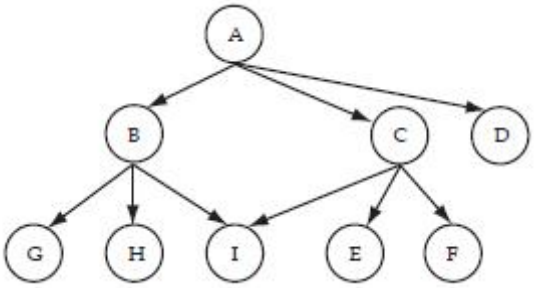
A tree is a set of one or more nodes T such that:

1. There is a specially designated node called root, and

2. Remaining nodes are partitioned into n >= 0 disjoint set of nodes T1, T2,…,Tn each of which is a tree.



This is a tree because it is a set of nodes {A, B, C, D, E, F, G, H, I}, with node A as a root node, and the remaining nodes are partitioned into three disjoint sets: {B, G, H, I}, {C, E, F} AND {D} respectively. Each of these sets is a tree individually because each of these sets satisfies the above properties.



This is not a tree because it is a set of nodes {A, B, C, D, E, F, G, H, I}, with node A as a root node, but the remaining nodes cannot be partitioned into disjoint sets, because the node I is shared.

Given below are some of the important definitions, which are used in connection with trees.

Degree of Node of a Tree: The degree of a node of a tree is the number of sub-trees havingthis node as a root, or it is a number of decedents of a node. If degree is zero then it is calledterminal node or leaf node of a tree.

Degree of a Tree: It is defined as the maximum of degree of the nodes of the tree, i.e. degreeof tree = max (degree (node i) for i = 1 to n).

Level of a Node: We defi ne the level of the node by taking the level of the root node to be 1,and incrementing it by 1 as we move from the root towards the sub-trees i.e. the level of allthe descendents of the root nodes will be 2. The level of their descendents will be 3 and soon. We then define depth of the tree to be the maximum value of level for node of a tree.

Root Node: The root of a tree is called a root node. A root node occurs only once in the whole tree.

Parent Node: The parent of a node is the immediate predecessor of that node**.**

Child Node: Child nodes are the immediate successors of a node.

Leaf Node: A node which does not have any child nodes is known as a leaf node.

Link: The pointer to a node in the tree is known as a link. A node can have more than one link.

Path: Every node in the tree is reachable from the root node through a unique sequence of links. This sequence of links is known as a path. The number of links in a path is considered to be the length of the path.

Levels: The level of a node in the tree is considered to be its hierarchical rank in the tree.

Height: The height of a non-empty tree is the maximum level of a node in the tree. The height of an empty tree (no node in a tree) is 0. The height of a tree containing a single node is 1. The longest path in the tree has to be considered to measure the height of the tree.

Height of a tree (h) = I_{max}+ 1, where I_{max} is the maximum level of a tree.

Siblings: The nodes which have the same parent node are known as siblings.

Graphs consist of a set of nodes and edges, just like trees. But for graphs, there are no rules for the connections between nodes. In graphs, there is no concept of a root node, nor a concept of parents and children. Rather, a graph is just a collection of interconnected nodes. All trees are graphs. A tree is a special case of a graph, in which the nodes are all reachable from some starting node.

Representation of Tree in Graphs

A graph G consists of a set of objects V = {v1, v2, v3 …} called vertices (points or nodes) and a set of objects E = {e1, e2, e3 ….} called edges (lines or arcs).

The set V (G) is called the vertex set of G and E (G) is the edge set.

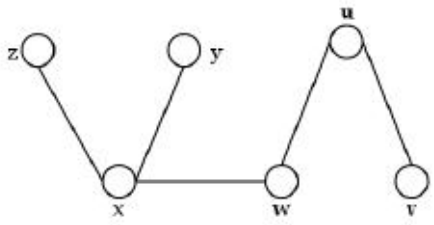
The graph is denoted as G = (V, E)

Let G be a graph and {u, v} an edge of G. Since {u, v} is 2-element set, we write {v, u} instead of {u, v}.

This edge can be represented as uv or vu.

If e = uv is an edge of a graph G, then u and v are adjacent in G and e joins u and v.

Consider given graph



This graph G is defined by the sets:

V (G) = {u, v, w, x, y, z} and E(G) = { uv, uw, wx, xy, xz}

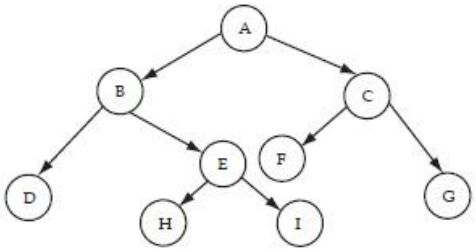
Every graph has a diagram associated with it. The vertex u and an edge e are incident with each other as are v and e. If two distinct edges e and f are incident with a common vertex, then they are adjacent edges.

5.2 Binary Tree

A binary tree is a special tree where each non-leaf node can have atmost two child nodes. Most important types of trees which are used to model yes/no, on/off, higher/lower, i.e., binary decisions are binary trees.

A tree data structure in which every node has a maximum of two child nodes is known as a binary tree. It is the most commonly used non-linear data structure. A binary tree could either have only a root node or two disjoint binary trees called the left sub-tree or the right sub-tree. An empty tree could also be a binary tree.

Recursive Definition: "A binary tree is either empty or a node that has left and right sub-trees that are binary trees. Empty trees are represented as boxes (but we will almost always omit the boxes)".



Binary Tree Structure

In a formal way, we can define a binary tree as a finite set of nodes which is either empty or partitioned in to sets of T0, Tl, Tr, where T0 is the root and Tl and Tr are left and right binary trees, respectively.

So, for a binary tree we find that:

1. The maximum number of nodes at level i will be 2i−1

2. If k is the depth of the tree then the maximum number of nodes that the tree can have is

$2^k - 1 = 2^{k-1} + 2^{k-2} + \ldots + 2^0$

Types of Binary Tree

There are two main binary tree and these are:

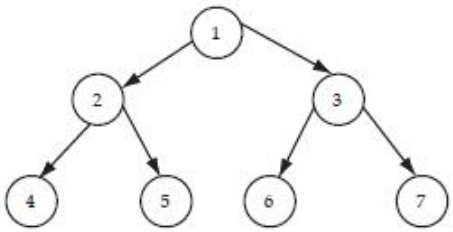
1. Full binary tree

2. Complete binary tree

Full Binary Tree

A full binary tree is a binary of depth k having 2k − 1 nodes. If it has < 2k − 1, it is not a full binary tree.

For k = 3, the number of nodes = 2k − 1 = 23 − 1 = 8 − 1 = 7. A full binary tree with depth k = 3 is shown in figure.



A Full Binary Tree

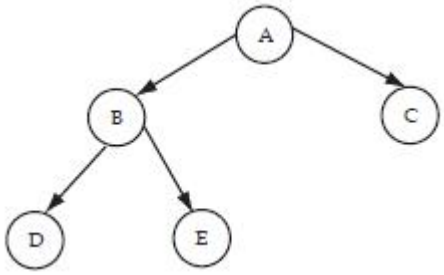
We use numbers from 1 to 2k − 1 as labels of the nodes of the tree.

If a binary tree is full, then we can number its nodes sequentially fom 1 to 2k−1, starting from the root node, and at every level numbering the nodes from left to right.

Complete Binary Tree

A complete binary tree of depth k is a tree with n nodes in which these n nodes can be numbered sequentially from 1 to n, as if it would have been the fi rst n nodes in a full binary tree of depth k.

A complete binary tree with depth k = 3 is shown in Figure



A Complete Binary Tree

Properties of a Binary Tree

Main properties of binary tree are:

1. If a binary tree contains n nodes, then it contains exactly n – 1 edges;

2. A Binary tree of height h has 2h – 1 nodes or less.

3. If we have a binary tree containing n nodes, then the height of the tree is at most n and at least ceiling log2 (n + 1).

4. If a binary tree has n nodes at a level l then, it has at most 2n nodes at a level l + 1

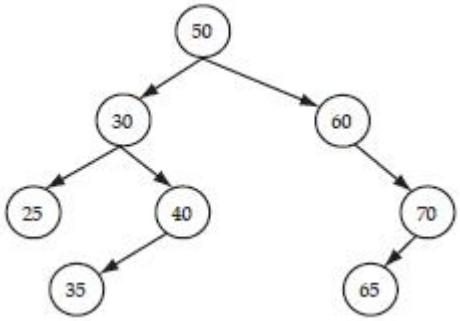
5. The total number of nodes in a binary tree with depth k (root has depth zero) is N = 20 + 21 + 22 + …….+ 2k = 2k+1 – 1.

5.3 Binary Search Tree

A binary search tree is a binary tree which may be empty, and every node contains an identifier and

1. Identifier of any node in the left sub-tree is less than the identifier of the root

2. Identifier of any node in the right sub-tree is greater than the identifier of the root and the left sub-tree as well as right sub-tree both are binary search trees.



Binary Search Tree

A binary search tree is basically a binary tree, and therefore it can be traversed is inorder, preorder, and postorder. If we traverse a binary search tree in inorder and print the identifiers contained in the nodes of the tree, we get a sorted list of identifiers in the ascending order.

A binary search tree is an important search structure. For example, consider the problem of searching a list. If a list is an ordered then searching becomes faster, if we use a contiguous list and binary search, but if we need to make changes in the list like inserting new entries and deleting old entries. Then it is much slower to use a contiguous list because insertion and deletion in a contiguous list requires moving many of the entries every time. So we may think of using a linked list because it permits insertions and deletions to be carried out by adjusting only few pointers, but in a linked list there is no way to move through the list other than one node at a time hence permitting only sequential access. Binary trees provide an excellent solution to this problem. By making the entries of an ordered list into the nodes of a binary search tree, we find that we can search for a key in O(n log n) steps.

Creating a Binary Search Tree

We assume that every node a binary search tree is capable of holding an integer data item and the links which can be made pointing to the root of the left and the right sub-tree respectively.

Therefore the structure of the node can be defi ned using the following declaration:

```
struct tnode
{
int data;
tnode *lchid;
tnode *rchild;
}
```

To create a binary search tree we use a procedure named insert which creates a new node with the data value supplied as a parameter to it, and inserts into an already existing tree whose root

pointer is also passed as a parameter. The procedure accomplishes this by checking whether the tree whose root pointer is passed as a parameter is empty. If it is empty then the newly created node is inserted as a root node. If it is not empty then it copies the root pointer into a variable temp1, it then stores value of temp1 in another variable temp2, compares the data value of the node pointed to by temp1 with the data value supplied as a parameter, if the data value supplied as a parameter is smaller than the data value of the node pointed to by temp1 then it copies the left link of the node pointed by temp1 into temp1 (goes to the left), otherwise it copies the right link of the node pointed by temp1 into temp1(goes to the right). It repeats this process till temp1 becomes nil.

When temp1 becomes nil, the new node is inserted as a left child of the node pointed to by temp2 if data value of the node pointed to by temp2 is greater than data value supplied as parameter. Otherwise the new node is inserted as a right child of node pointed to by temp2. Therefore the insert procedure is

```
void insert(tnode *p, int val)
{
tnode *temp1, *temp2;
if (p == NULL)
{
p = new(tnode);
p->data = val;
p->1child = NULL;
p->rchild = NULL;
}
else
{
temp1 = p;
while(temp1 != NULL)
{
temp2 = temp1;
if(temp1->data > val)
temp1 = temp1->1eft;
else
temp1 = temp1->right;
}
if(temp2->data > val)
{
temp2->left = new(tnode);
temp2 = temp2->left;
temp2->data = val;
temp2->left = NULL;
temp2->right= NULL;
}
else
{
temp2->right = new(tnode);
```

```
temp2 = temp2->right;
temp2->data = val;
temp2->left = NULL;
temp2->right = NULL;
}
}
}
```

5.4 Binary Search Tree Operations

The four main operations that we perform on binary trees are:

1. Searching

2. Insertion

3. Deletion

4. Traversal

Searching in Binary Search Trees

In searching, the node being searched is called as key node. We first match the key node with the root node. If the value of the key node is greater than the current node, then we search for it in the right subtree, else we search in the left sub-tree. We continue this process until we find the node or until no nodes are left. The pseudo code for searching a binary search tree is as follows:

Pseudocode for a Binary Search Tree

```
find(X, node){
if(node = NULL)
return NULL
if(X = node:data)
return node
else if(X<node:data)
return find(Y,node:leftChild)
else if(X>node:data)
return find(X,node:rightChild)
}
```

Inserting in Binary Search Trees

To insert a new element in an existing binary search tree, first we compare the value of the new nodewith the current node value. If the value of the new node is lesser than the current node value, we insertit as a left sub-node. If the value of the new node is greater than the current node value, then we insert itas a right sub-node. If the root node of the tree does not have any value, we can insert the new node asthe root node.

Algorithm for Inserting a Value in a Binary Search Tree

1. Read the value for the node that needs to be created and store it in a node called NEW.

2. At first, if (root! =NULL) then root = NEW.

3. If (NEW->value < root->value) then attach NEW node as a left child node of root, else attach NEWnode as a right child node of root.

4. Repeat steps 3 and 4 for creating the desired binary search tree completely.

When inserting any node in a binary search tree, it is necessary to look for its proper position in thebinary search tree. The new node is compared with every node of the tree. If the value of the nodewhich is to be inserted is more than the value of the current node, then the right sub-tree is considered,else the left sub-tree is considered. Once the proper position is identified, the new node is attached asthe left or right child node. Let us now discuss the pseudo code for inserting a new element in a binarysearch tree.

Pseudocode for Inserting a Value in a Binary Search Tree

```
//Purpose: Insert data object X into the Tree
//Inputs: Data object X (to be inserted), binary-search-tree node
//Effect: Do nothing if tree already contains X;
// otherwise, update binary search tree by adding a new node containing data object X
insert(X, node){
if(node = NULL){
node = new binaryNode(X,NULL,NULL)
return
}
if(X = node:data)
return
else if(X<node:data)
insert(X, node:leftChild)
else // X>node:data
insert(X, node:rightChild)
}
```

Deleting in Binary Search Trees

If the node to be deleted has no children, we can just delete it. If the node to be deleted has one child,

then the node is deleted and the child is connected directly to the parent node.

There are mainly three cases possible for deletion of any node from a binary search tree. They are:

1. Deletion of the leaf node
2. Deletion of a node that has one child
3. Deletion of a node that has two children

We can delete an existing element from a binary search tree using the following pseudocode:

Pseudocode for Deleting a Value from a Binary Search Tree

```
//Purpose: Delete data object X from the Tree
//Inputs: Data object X (to be deleted), binary-search-tree node
//Effect: Do nothing if tree does not contain X;
// else, update binary search tree by deleting the node containing data object X
delete(X, node){
if(node = NULL) //nothing to do
return
if(X<node:data)
delete(X, node:leftChild)
```

```
else if(X>node:data)
delete(X, node:rightChild)
else { // found the node to be deleted. Take action based on number of node children
if(node:leftChild = NULL and node:rightChild = NULL){
delete node
node = NULL
return
}
else if(node:leftChild = NULL){
tempNode = node
node = node:rightChild
delete tempNode}
else if(node:rightChild = NULL){
tempNode = node
node = node:leftChild
delete tempNode
}
else { //replace node:data with minimum data from right sub-tree
tempNode = findMin(node.rightChild)
node:data = tempNode:data
delete(node:data,node:rightChild)
}
}
```

Pseudocode for Finding Minimum Value from a Binary Search Tree

```
//Purpose: return least data object X in the Tree
//Inputs: binary-search-tree node node
// Output: bst-node n containing least data object X, if it exists; NULL otherwise
findMin(node)
{
if(node = NULL) //empty tree
return NULL
if(node:leftChild = NULL)
return node
return findMin(node:leftChild)
}
```

Deleting a node with one child

Step 1 - Find the node to be deleted using search operation

Step 2 - If it has only one child then create a link between its parent node and child node.

Step 3 - Delete the node using free function and terminate the function.

Deleting a node with two children

Step 1 - Find the node to be deleted using search operation

Step 2 - If it has two children, then find the largest node in its left subtree (OR) the smallest node in its right subtree.

Step 3 - Swap both deleting node and node which is found in the above step.

Step 4 - Then check whether deleting node came to case 1 or case 2 or else goto step 2

Step 5 - If it comes to case 1, then delete using case 1 logic.

Step 6- If it comes to case 2, then delete using case 2 logic.

Step 7 - Repeat the same process until the node is deleted from the tree.

Binary Search Tree time complexities

Search Operation - O(n)

Insertion Operation - O(1)

Deletion Operation - O(n)

Application of a Binary Search Tree

1. A prominent data structure used in many systems programming applications for representing and managing dynamic sets.

2. Average case complexity of Search, Insert, and Delete Operations is O(log n), where n is the number of nodes in the tree.

One of the applications of a binary search tree is the implementation of a dynamic dictionary.

A dictionary is an ordered list which is required to be searched frequently, and is also required to be updated (insertions and deletions) frequently. Hence can be very well implemented using a binary search tree, by making the entries of dictionary into the nodes of binary search tree. A more effi cient implementation of a dynamic dictionary involves considering a key to be a sequence of characters, and instead of searching by comparison of entire keys, we use these characters to determine a multi-way branch at each step, this will allow us to make a 26-way branching according the fi rst letter, followed by another branch according to the second letter and so on.

Summary

- Search trees are data structures that support many dynamic-set operations such as searching, finding the minimum or maximum value, inserting, or deleting a value.
- In a binary search tree, for a given node n, each node to the left has a value lesser than n and each node to the right has a value greater than n.
- The time taken to perform operations on a binary search tree is directly proportional to the height of the tree.
- Binary trees provide an excellent solution to this problem. By making the entries of an ordered list into the nodes of a binary tree, we shall fi nd that we can search for a target key in O(log n) steps, just as with binary search, and we shall obtain algorithms for inserting and deleting entries also in time O(log n).

Keywords

- ***Binary Search Tree***: A binary search tree is a binary tree which may be empty, and every node contains an identifier.
- ***Searching***: Searching for the key in the given binary search tree, start with the root node and compare the key with the data value of the root node.
- ***Degree of a tree***: The highest degree of a node appearing in the tree.

- ***Inorder***: A tree traversing method in which the tree is traversed in the order of left-tree, node andthen right-tree.
- ***Level of a node***: The number of nodes that must be traversed to reach the node from the root.
- ***Root node***: The node in a tree which does not have a parent node.
- ***Tree***: A two-dimensional data structure comprising of nodes where one node is the root and rest of the nodes form two disjoint sets each of which is a tree.

Self Assessment

1. Tree is a _____ hierarchical data structure.

A. Linear
B. Nonlinear
C. Abstract
D. All of above

2. Data access is quick and easier in

A. Nonlinear data structure
B. Linear data structure
C. Both Linear and Nonlinear
D. None of above

3. Types of trees are

A. Binary Tree
B. Binary Search Tree
C. AVL Tree
D. All of above

4. Binary search tree is also called.

A. Ordered
B. Sorted
C. Both ordered and sorted binary tree
D. None of above

5. value of the nodes in the left sub-tree is less than the value of the root is

A. General Tree
B. Binary Tree
C. Binary Search Tree
D. None of above

6. Types of Binary Trees are

A. Full binary tree
B. Complete binary tree

C. Perfect binary tree
D. All of above

7. Which is not Binary Tree operation?

A. Search
B. Peek
C. Insertion
D. Deletion

8. Binary Search Tree applications are

A. In multilevel indexing in the database.
B. For dynamic sorting.
C. It is used to implement various searching algorithms.
D. All of above

9. Binary Search Tree time complexity for search operation is

A. O(n)
B. O(1)
C. O(2)
D. None of above

10. Binary Search Tree time complexity for insert operation is

A. O(0)
B. O(1)
C. O(2)
D. None of above

11. What is the worst case time complexity for Delete operation?

A. O(0)
B. O(1)
C. O(n)
D. None of above

12. Which is incorrect statement about Binary search tree.

A. The left and right sub-trees should also be binary search trees
B. In order sequence gives decreasing order of elements
C. The left child is always lesser than its parent
D. The right child is always greater than its parent

13. To arrange binary tree in ascending order…

A. Pre order traversal only
B. Post order traversal only

C. In order traversal only

D. None of above

14. Which of the following is not component of binary tree

A. Data element

B. Pointer to right subtree

C. Super tree

D. Pointer to left subtree

15. Which of the following is not a type of tree data structure

A. General Tree

B. Primary Tree

C. Binary Tree

D. Binary Search Tree

E.

Answers for self Assessment

1.	B	2.	A	3.	D	4.	C	5.	C
6.	D	7.	B	8.	D	9.	A	10.	B
11.	C	12.	B	13.	C	14.	C	15.	B

Review Questions

1. define tree with suitable example.

2. Draw a binary tree with six child node.

4. Discuss degree of tree.

5. Explain representation of tree.

6. what are the application of tree.

7. Discuss time complexity of Binary search tree.

Further Readings

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Unit 06: Tree Data Structure 1

CONTENTS

Objectives

After studying this unit, you will be able to:

- State about AVL tree
- Describe balancing operations
- Discuss B-tree
- Learn B-tree properties and operations

Introduction

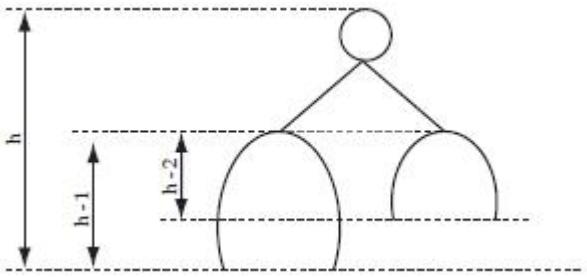
One of the most essential data structures is the tree, which is used to conduct operations like insertion, deletion, and searching of items efficiently. Construction of a well-balanced tree for sorting all data is not practicable when working with a huge number of data, though. As a result, only valuable data is saved as a tree, and the actual volume of data used changes over time as new data is inserted and old data is deleted. It is possible to conduct traversals, insertions, and deletions without utilizing either stack or recursion in some circumstances where the NULL link to a binary tree to special links is referred to as threads.

6.1 AVL Tree

An AVL tree is another balanced binary search tree. AVL Tree is invented in 1962. It takes its name from the initials of its inventors – Adelson, Velskii and Landis. An AVL tree has the following properties:

1. The sub-trees of every node differ in height by at most one level.

2. Every sub-tree is an AVL tree.

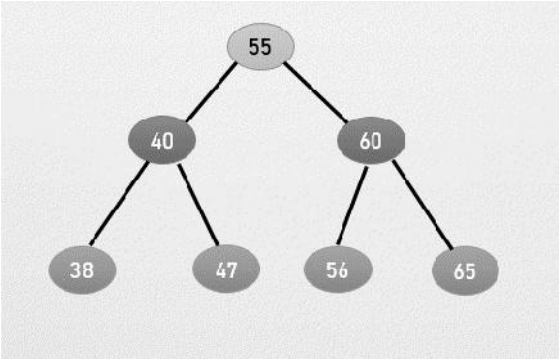


Here, the height of the tree is h. Height of one subtree is h–1 while that of another subtree of the same node is h–2, differing from each other by just 1. Therefore, it is an AVL tree.

AVL Tree is defined as height balanced binary search tree. In AVL tree each node is associated with a balance factor which is calculated by subtracting the height of its right sub-tree from that of its left sub-tree.

Why AVL Tree

AVL tree controls the height of the binary search tree. The time taken for all operations in a binary search tree of height h is O(h). For skewed BST it can be extended to O(n) (worst case).By limiting this height to log n, AVL tree imposes an upper bound on each operation to be O(log n) where n is the number of nodes.



Balance Factor

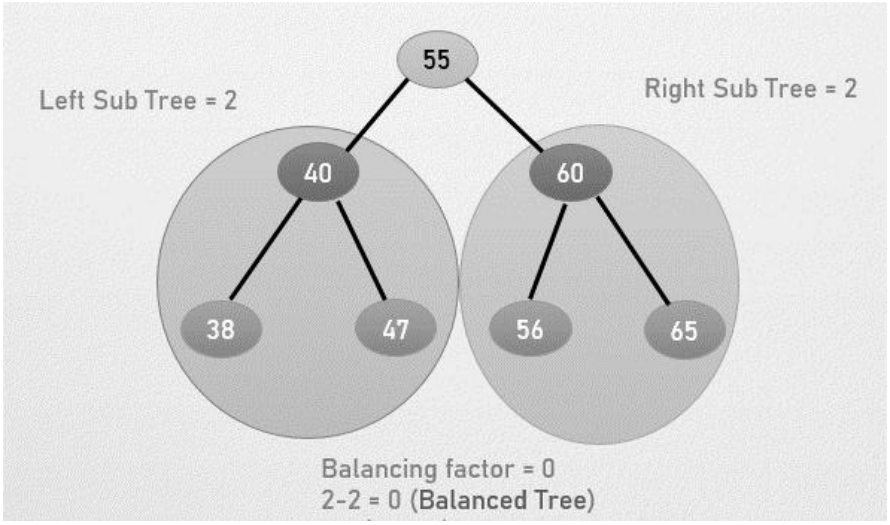
Balance factor of a node in an AVL tree is the difference between the height of the left sub tree and that of the right sub tree of that node.

Balance Factor = (Height of Left Sub tree - Height of Right Sub tree) or (Height of Right Sub tree - Height of Left Sub tree). Balance factor value are: -1, 0 or 1.

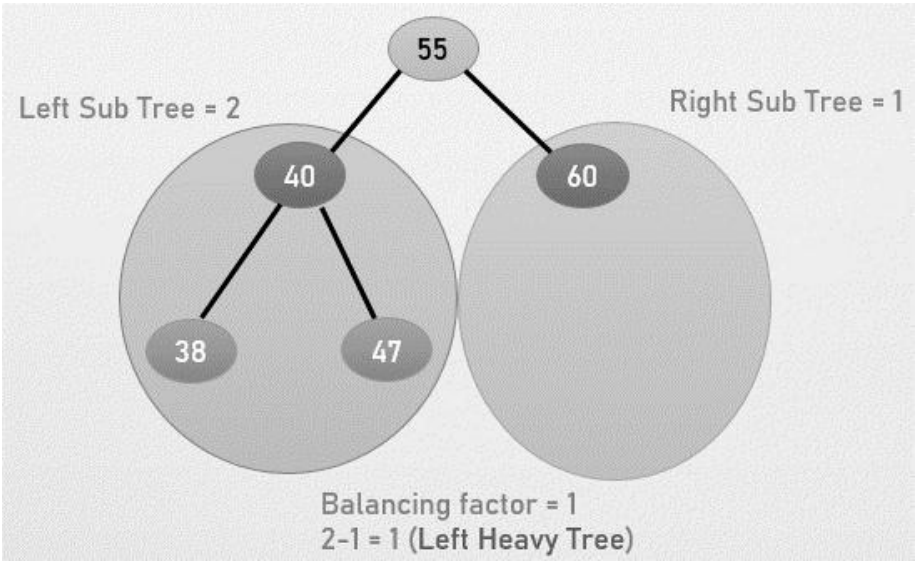
If balance factor of any node is 1, it means that the left sub-tree is one level higher than the right sub-tree.

If balance factor of any node is 0, it means that the left sub-tree and right sub-tree contain equal height.

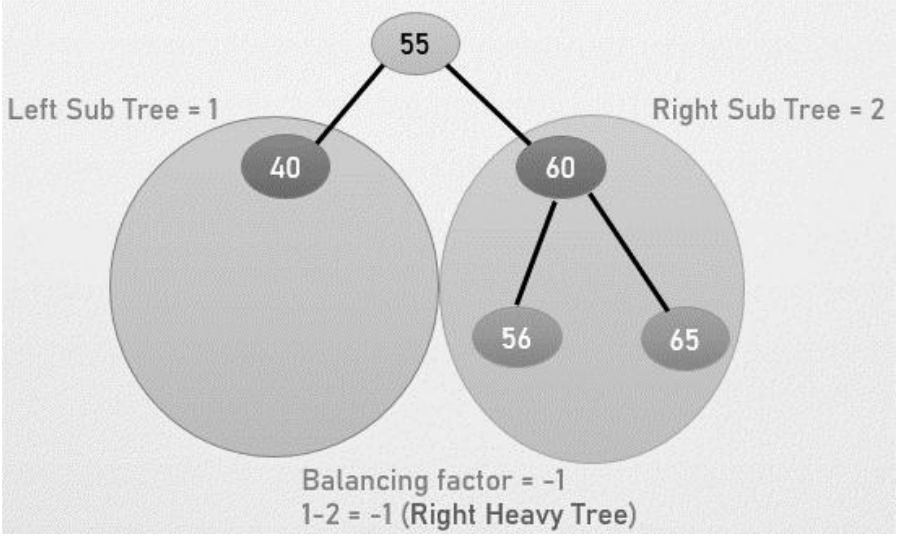
If balance factor of any node is -1, it means that the left sub-tree is one level lower than the right sub-tree.



Balanced Tree



AVL tree (Left Heavy Tree)



AVL Tree (Right Heavy Tree)

AVL Tree Rotations for balancing

Rotations are performed in AVL tree only in case if Balance Factor is other than -1, 0, and 1.

Left rotation

Right rotation

Left-Right rotation

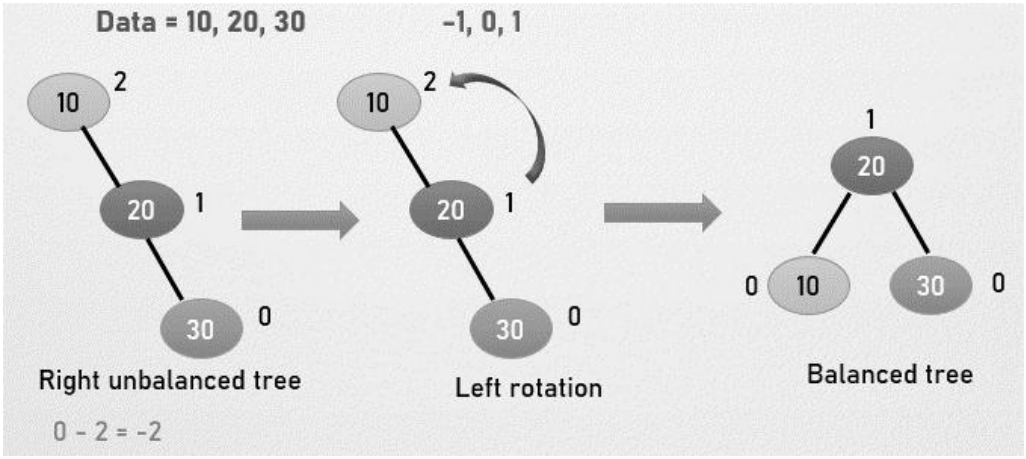
Right-Left rotation

The Left rotation and Right rotation are **single rotations**.

Left-Right rotation and Right-Left rotation are **double rotations**.

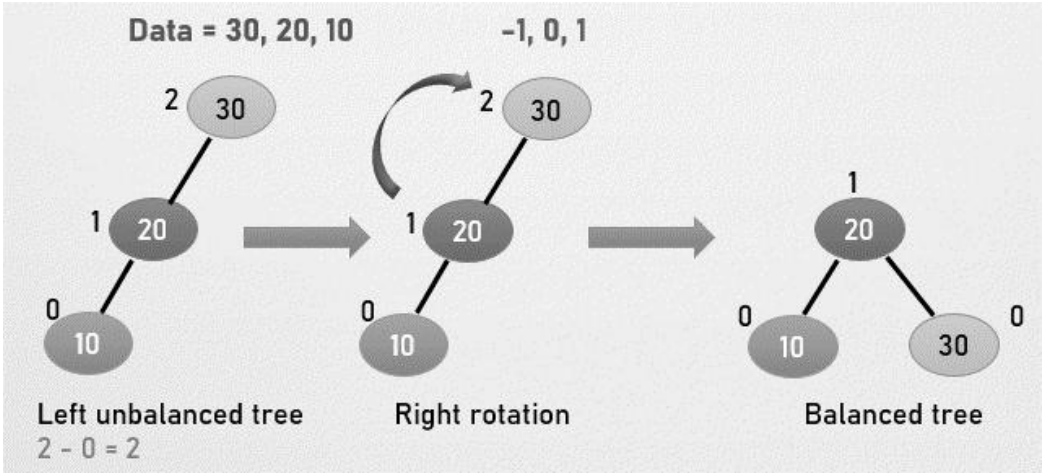
Left rotation

If a tree becomes unbalanced, when a node is inserted into the right subtree of the right subtree, then we perform a single left rotation.



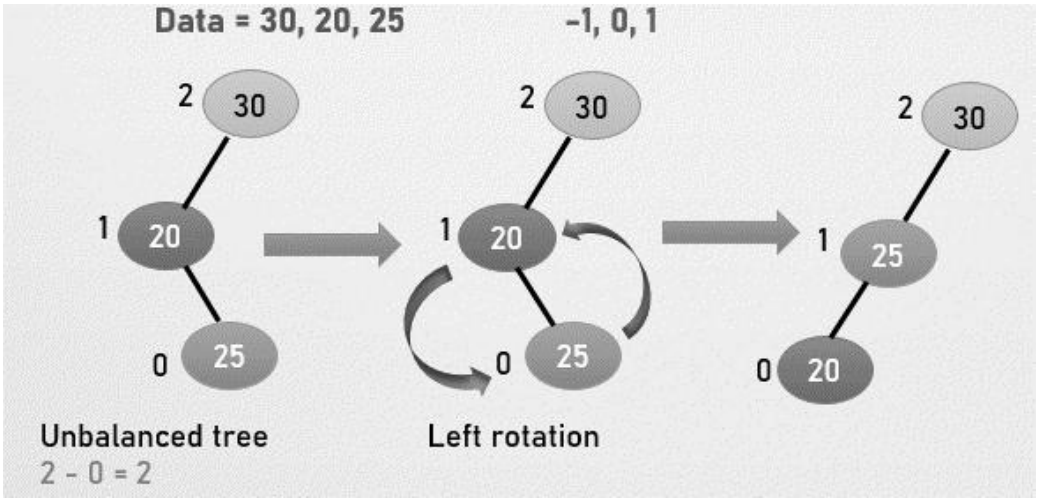
Right rotation

AVL tree may become unbalanced, if a node is inserted in the left subtree of the left subtree. The tree then needs a right rotation.

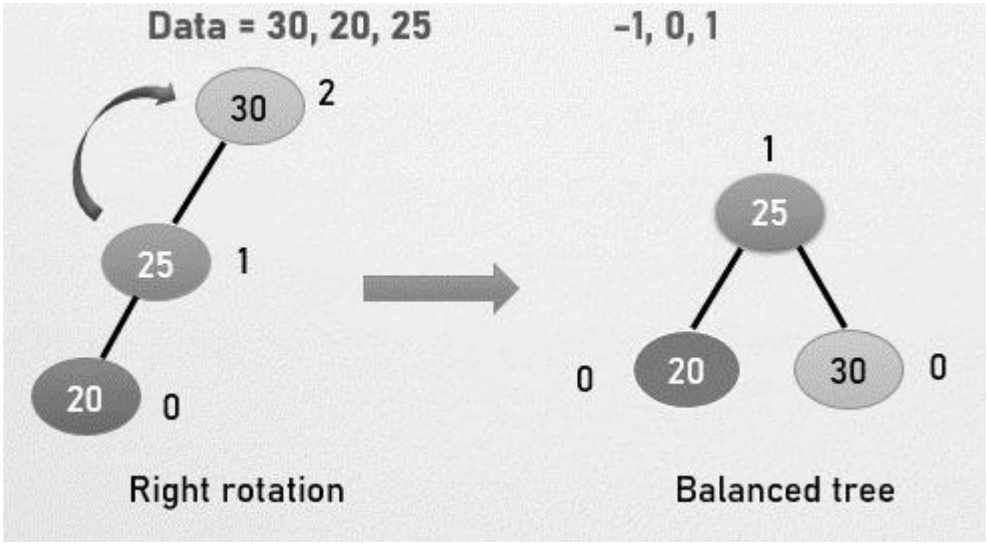


Left-Right rotation

Double rotations are slightly complex rotations. To understand them better, we should take note of each action performed while rotation. A left-right rotation is a combination of left rotation followed by right rotation.



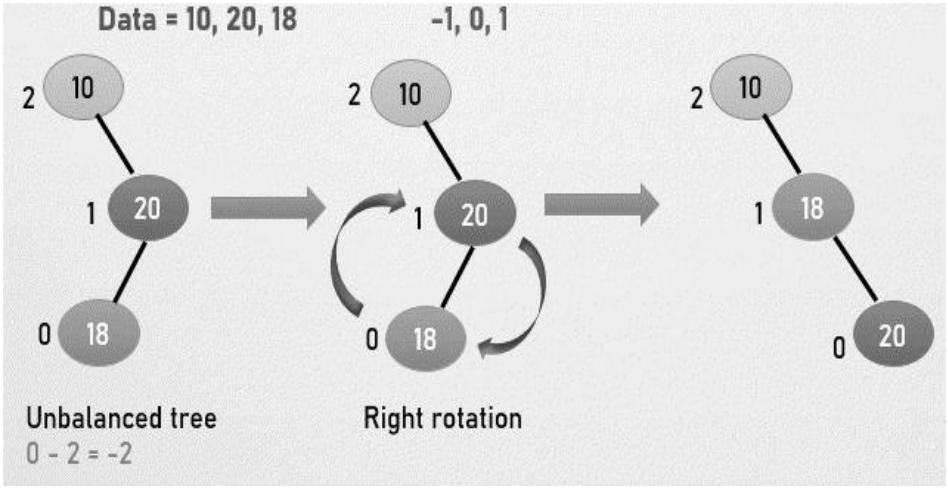
Step - 1



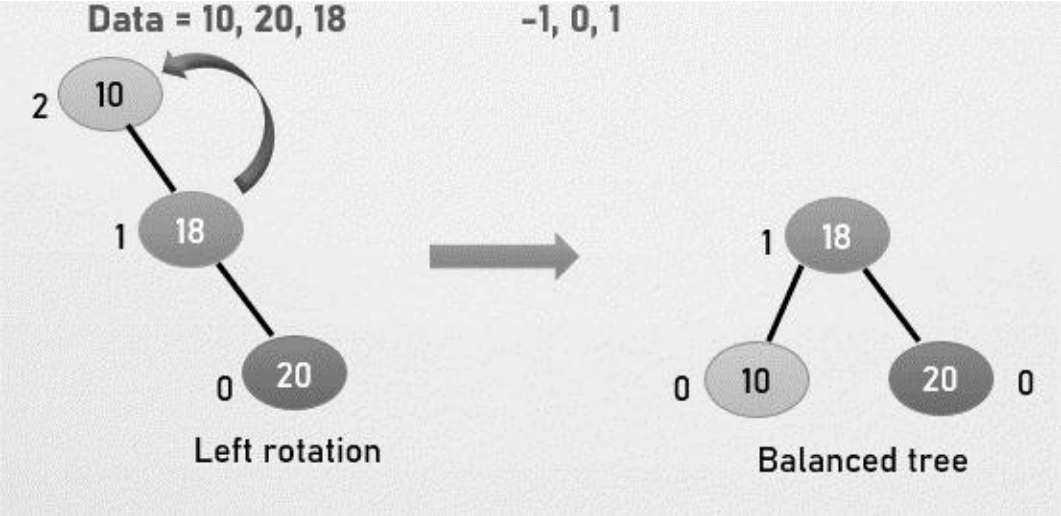
Step – 2

Right-Left rotation

It is a combination of right rotation followed by left rotation.



Step – 1



Step – 2

6.2 AVL Operation

Insertion into AVL Trees

To implement AVL trees, you need to maintain the height of each node. You insert into an AVL tree by performing a standard binary tree insertion. When you're done, you check each node on the path from the new node to the root. If that node's height hasn't changed because of the insertion, then you are done. If the node's height has changed, but it does not violate the balance property, then you continue checking the next node in the path. If the node's height has changed and it now violates the balance property, then you need to perform one or two rotations to fix the problem, and then you are done.

Insertion in AVL is same as binary search tree. Insertion may lead to violation in the tree property and therefore the tree may need balancing. The tree can be balanced by applying rotations.

Deletion

When you delete a node, there are three things that can happen to the parent:

1. Its height is decremented by one.

2. Its height doesn't change and it stays balanced.

3. Its height doesn't change, but it becomes imbalanced.

You handle these three cases in different ways:

1. The parent's height is decremented by one. When this happens, you check the parent's parent: you keep doing this until you return or you reach the root of the tree.

2. The parent's height doesn't change and it stays balanced. When this happens you may return – deletion is over.

3. The parent's height doesn't change, but it becomes imbalanced. When this happens, you have to rebalance the subtree rooted at the parent. After rebalancing, the subtree's height may be one smaller than it was originally. If so, you must continue checking the parent's parent.

To rebalance, you need to identify whether you are in a zig-zig situation or a zig-zag situation and rebalance accordingly.

Complexities of Different Operations on an AVL Tree

The rotation operations (left and right rotate) take constant time as only a few pointers are being changed there. Updating the height and getting the balance factor also takes constant time.

Insertion	Deletion	Search
O(log n)	O(log n)	O(log n)

6.3 Applications of AVL Trees

AVL trees are applied in the following situations:

1. There are few insertion and deletion operations

2. Short search time is needed

3. Input data is sorted or nearly sorted

AVL tree structures can be used in situations which require fast searching. But, the large cost of rebalancing may limit the usefulness.

Consider the following:

1. A classic problem in computer science is how to store information dynamically so as to allow for quick look up. This searching problem arises often in dictionaries, telephone directory, symbol tables for compilers and while storing business records etc. The records are stored in a balanced binary tree, based on the keys (alphabetical or numerical) order.

The balanced nature of the tree limits its height to O (log n), where n is the number of inserted records.

2. AVL trees are very fast on searches and replacements. But, have a moderately high cost for addition and deletion. If application does a lot more searches and replacements than it does addition and deletions, the balanced (AVL) binary tree is a good choice for a data structure.

3. AVL tree also has applications in fi le systems.

Advantages of AVL tree

The height of the AVL tree is always balanced. The height never grows beyond log N, where N is the total number of nodes in the tree.

Search time complexity is better as compared to Binary Search trees.

AVL trees have self-balancing capabilities.

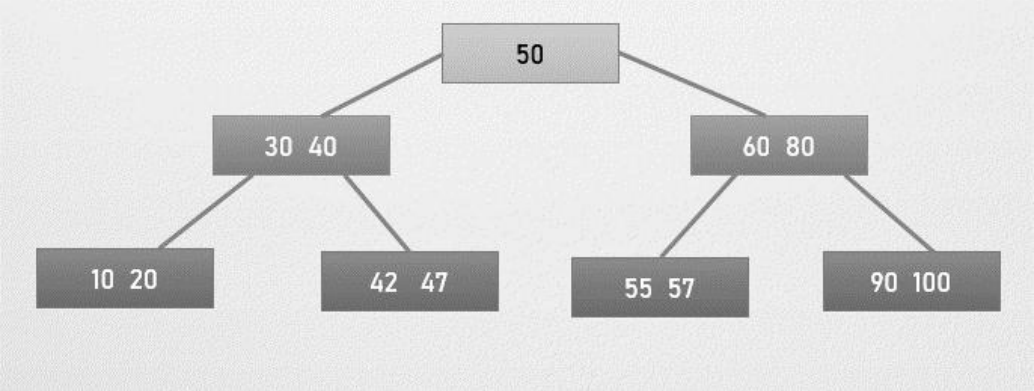
6.4 B-tree

A B-tree is a tree data structure that keeps data sorted and allows insertions and deletions that is logarithmically proportional to fi le size. It is commonly used in databases and fi le systems.

In B-trees, internal nodes can have a variable number of child nodes within some pre-defined range. When data is inserted or removed from a node, its number of child nodes changes. In order to maintain the pre-defi ned range, internal nodes may be joined or split. Because a range of child nodes is permitted, B-trees do not need re-balancing as frequently as other self balancing search trees, but may waste some space, since nodes are not entirely full. The lower and upper bounds on the number of child nodes are typically fixed for a particular implementation.

A B-tree is kept balanced by requiring that all leaf nodes are at the same depth. This depth will increase slowly as elements are added to the tree, but an increase in the overall depth is infrequent, and results in all leaf nodes being one more hop further removed from the root.

B-trees are balanced trees that are optimized for situations when part or the entire tree must be maintained in secondary storage such as a magnetic disk. Since disk accesses are expensive (time consuming) operations, a b-tree tries to minimize the number of disk accesses.



Structure of B-trees

Unlike a binary-tree, each node of a b-tree may have a variable number of keys and children. The keys are stored in non-decreasing order. Each key has an associated child that is the root of a subtree containing all nodes with keys less than or equal to the key but greater than the preceding key. A node also has an additional rightmost child that is the root for a subtree containing all keys greater than any keys in the node.

A b-tree has a minimum number of allowable children for each node known as the minimization factor. If t is this minimization factor, every node must have at least t – 1 keys. Under certain circumstances, the root node is allowed to violate this property by having fewer than t – 1 keys.

Every node may have at most 2t – 1 keys or, equivalently, 2t children. Since each node tends to have a large branching factor (a large number of children), it is typically necessary to traverse relatively few nodes before locating the desired key. If access to each node requires a disk access, then a b-tree will minimize the number of disk accesses required.

The minimization factor is usually chosen so that the total size of each node corresponds to a multiple of the block size of the underlying storage device. This choice simplifies and optimizes disk access. Consequently, a b-tree is an ideal data structure for situations where all data cannot reside in primary storage and accesses to secondary storage are comparatively expensive (or time consuming).

Why B Tree

B-Trees is used to reduce the number of disk accesses.Most of the tree operations (search, insert, delete, max, min) require O(h) disk accesses where h is the height of the tree. B-tree is a fat tree. The height of B-Trees is kept low by putting maximum possible keys in a B-Tree node.

Data structures like binary search tree, avl tree, red-black tree, etc. can store only one key in one node. If you have to store a large number of keys, then the height of such trees becomes very large and the access time increases.

The height of the B-tree is low so total disk accesses for most of the operations are reduced significantly compared to balanced Binary Search Trees like AVL Tree, Red-Black Tree,etc.

B Tree properties

B-Tree of Order m has the following properties:

1 - All leaf nodes must be at same level.

2 - All nodes except root must have at least [m/2]-1 keys and maximum of m-1 keys.

3 - All non leaf nodes except root (i.e. all internal nodes) must have at least m/2 children.

4 - If the root node is a non leaf node, then it must have at least 2 children.

5 - A non leaf node with n-1 keys must have n number of children.

6 - All the key values in a node must be in Ascending Order.

6.5 Operations on B-trees

The algorithms for the search, create, and insert operations are shown below. Note that these algorithms are single pass; in other words, they do not traverse back up the tree. Since b-trees strive to minimize disk accesses and the nodes are usually stored on disk, this single-pass approach will reduce the number of node visits and thus the number of disk accesses. Simpler double-pass approaches that move back up the tree to fi x violations are possible.

Since all nodes are assumed to be stored in secondary storage (disk) rather than primary storage (memory), all references to a given node be bepreceeded by a read operation denoted by Disk-Read. Similarly, once a node is modifi ed and it is no longer needed, it must be written out to secondary storage with a write operation denoted by Disk-Write. The algorithms below assume that all nodes referenced in parameters have already had a corresponding Disk-Read operation. New nodes are created and assigned storage with the Allocate-Node call. The implementation details of the Disk-Read, Disk-Write, and Allocate-Node functions are operating system and implementation dependent.

Search Operation

The search operation on a b-tree is analogous to a search on a binary tree. Instead of choosing between a left and a right child as in a binary tree, a b-tree search must make an n-way choice. The correct child is chosen by performing a linear search of the values in the node. After finding the value greater than or equal to the desired value, the child pointer to the immediate left of that value is followed. If all values are less than the desired value, the rightmost child pointer is followed. Of course, the search can be terminated as soon as the desired node is found. Since the running time of the search operation depends upon the height of the tree, B-Tree-Search is O(logt n).

B-Tree-Search(x, k)

i<- 1

while i<= n[x] and k >keyi[x]

do i<- i + 1

if i<= n[x] and k = keyi[x]

then return (x, i)

if leaf[x]

then return NIL

else Disk-Read(ci[x])

return B-Tree-Search(ci[x], k)

Search algorithm

Let the key (the value) to be searched by "X".

Start searching from the root and recursively traverse down.

If X is lesser than the root value, search left sub tree, if X is greater than the root value, search the right sub tree.

If the node has the found X, simply return the node.

If the X is not found in the node, traverse down to the child with a greater Key.

If X is not found in the tree, we return NULL.

Insertion Operation

Insertions in B-Tree performed only at the leaf node level.Inserting operation performed with two steps: searching the appropriate node to insert the element and splitting the node if required.

Insertion algorithm

Check whether tree is Empty.

If tree is Empty, then create a new node with new key value and insert it into the tree as a root node

If tree is not empty, then, find the appropriate leaf node at which the node can be inserted.

If the leaf node contain less than m-1 keys then insert the element in the increasing order.

Else, if the leaf node contains m-1 keys, then follow the following steps.

-Insert the new element in the increasing order of elements.

-Split the node into the two nodes at the median.

-Push the median element upto its parent node.

-If the parent node also contain m-1 number of keys, then split it too by following thesame steps.

Deletion operation

In case of deletion from B-Tree user need to follow more rule as compared to search and insertion.

Three case for deletion from B-Tree

If the key is in the leaf node

If the key is in an internal node

If the key is in a root node

Key is in the leaf node, case-1

Target is in the leaf node, more than min keys.

Deleting this will not violate the property of B Tree

Target is in leaf node, it has min key nodes

Deleting this will violate the property of B Tree

Target node can borrow key from immediate left node, orimmediate right node (sibling)

The sibling will say yes if it has more than minimum number of keys

The key will be borrowed from the parent node, the max value will betransferred to a parent, the max value of the parent node will betransferred to the target node, and remove the target value

Target is in the leaf node, but no siblings have more than min number of keys Search for key

Merge with siblings and the minimum of parent nodes

Total keys will be now more than min

The target key will be replaced with the minimum of a parent node

Key is in an internal node, case-2

Either choose, in- order predecessor or in-order successor

In case the of in-order predecessor, the maximum key from its left sub tree will be selected

In case of in-order successor, the minimum key from its right sub tree will be selected

If the target key's in-order predecessor has more than the min keys, only then it can replace the target key with the max of the in-order predecessor

If the target key's in-order predecessor does not have more than min keys, look for in-order successor's minimum key.

If the target key's in-order predecessor and successor both have less than min keys, then merge the predecessor and successor.

Key is in a root node, Case- 3

Replace with the maximum element of the in-order predecessor sub tree

If, after deletion, the target has less than min keys, then the target node will borrow max value from its sibling via sibling's parent.

The max value of the parent will be taken by a target, but with the nodes of the max value of the sibling.

Summary

- AVL tree controls the height of the binary search tree. The time taken for all operations in a binary search tree of height h is O (h).
- In AVL tree each node is associated with a balance factor which is calculated by subtracting the height of its right sub-tree from that of its left sub-tree.
- B-trees are balanced trees that are optimized for situations when part or the entire tree must be maintained in secondary storage such as a magnetic disk.
- A B-tree is a specialized multiway tree designed especially for use on disk. In a B-tree each node may contain a large number of keys. The number of subtrees of each node, then, may also be large.
- A B-tree is designed to branch out in this large number of directions and to contain a lot of keys in each node so that the height of the tree is relatively small.
- This means that only a small number of nodes must be read from disk to retrieve an item.
- The goal is to get fast access to the data, and with disk drives this means reading a very small number of records. Note that a large node size (with lots of keys in the node) also fits with the fact that with a disk drive one can usually read a fair amount of data at once.

Keywords

B-Tree Algorithms: A B-tree is a data structure that maintains an ordered set of data and allows efficient operations to fi nd, delete, insert, and browse the data.

B-trees: B-trees are balanced trees that are optimized for situations when part or the entire tree must be maintained in secondary storage such as a magnetic disk.

SelfAssessment

1. AVL Tree is invented in

A. 1955
B. 1966
C. 1962
D. None of above

2. Which statement is true about AVL tree?

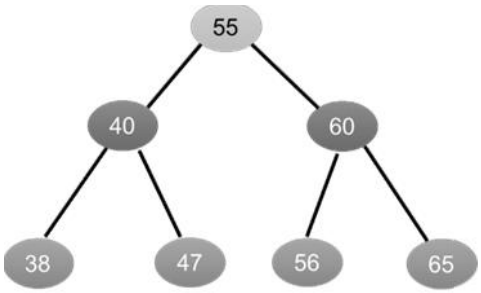
A. AVL tree controls the height of the binary search tree.
B. The time taken for all operations in a binary search tree of height h is O(h).
C. For skewed BST it can be extended to O(n) (worst case).
D. All of above

3. Balance Factor values in AVL tree is

A. -1
B. 0

C. 1
D. All of above

4. The balance factor in diagram is



A. 1
B. 0
C. 2
D. None of above

5. AVL Tree Rotations are

A. Right rotation
B. Left-Right rotation
C. Right-Left rotation
D. All of above

6. Left rotation and Right rotation are

A. Double rotations
B. Single rotation
C. Triple rotation
D. None of above

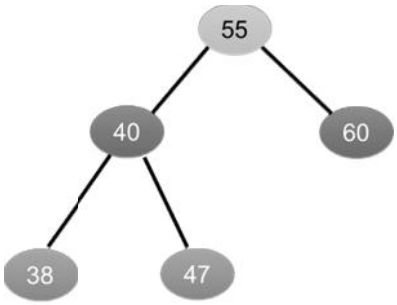
7. Which statement is true about AVL tree?

A. AVL tree is a self-balancing Binary Search Tree.
B. AVL Tree is defined as height balanced binary search tree.
C. In AVL tree each node is associated with a balance factor.
D. All of above

8. Left-Right rotation and Right-Left rotation are

A. Single rotation
B. Triple rotation
C. Double rotations
D. None of above

9. The balance factor in diagram is



A. 1
B. 0
C. 2
D. None of above

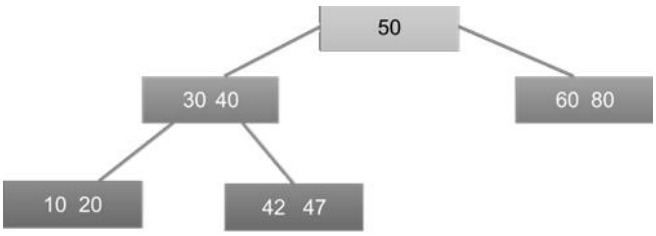
10. B Tree properties are

A. All leaf nodes must be at same level
B. All non-leaf nodes except root
C. A non-leaf node with n-1 keys must have n number of children
D. All of above

11. Which statement is true about B-tree?

A. Each node can contain more than one key
B. Each node can have more than two children.
C. A B Tree of order m can have at most m-1 keys and m children.
D. All of above

12. Diagram represents correct B-tree



A. True
B. False

13. Which is not B-tree operation

A. Search
B. Insert
C. Data manipulation
D. Delete

14. Insertion Operation can performed in B-tree at

A. Root node
B. Leaf node
C. Both root and leaf node
D. None of above

15. What are the different cases for deletion from B-Tree?

A. If the key is in the leaf node
B. If the key is in an internal node
C. If the key is in a root node
D. All of above

Answers for Self Assessment

1.	C	2.	D	3.	D	4.	B	5.	D
6.	B	7.	D	8.	C	9.	A	10.	B
11.	D	12.	B	13.	C	14.	B	15.	D

Review Questions

1. define AVL tree and its advantages.
2. How AVL tree is different from B-tree.
3. Describe the deletion of an item from b-trees.
4. Describe of structure of B-tree. Also explain the operation of B-tree.
5. Explain insertion of an item in b-trees.
6. Differentiate between Left Heavy Tree and right Heavy Tree with example.
7. Discuss different AVL tree rotations with suitable diagram.

Further Readings

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Unit 07: Tree Data Structure 2

CONTENTS

Objectives

After studying this unit, you will be able to:

- State about red-black trees
- Learn splay tree and its operations
- Discuss 2-3 trees and its properties
- Learn operationson 2-3 trees

Introduction

Recall that, for binary search trees, although the average-case times for the lookup, insert, and delete methods are all O(log N), where N is the number of nodes in the tree, the worst-case time is O(N). We can guarantee O(log N) time for all three methods by using a balanced tree -- a tree that always has height O(log N)-- instead of a binary search tree.

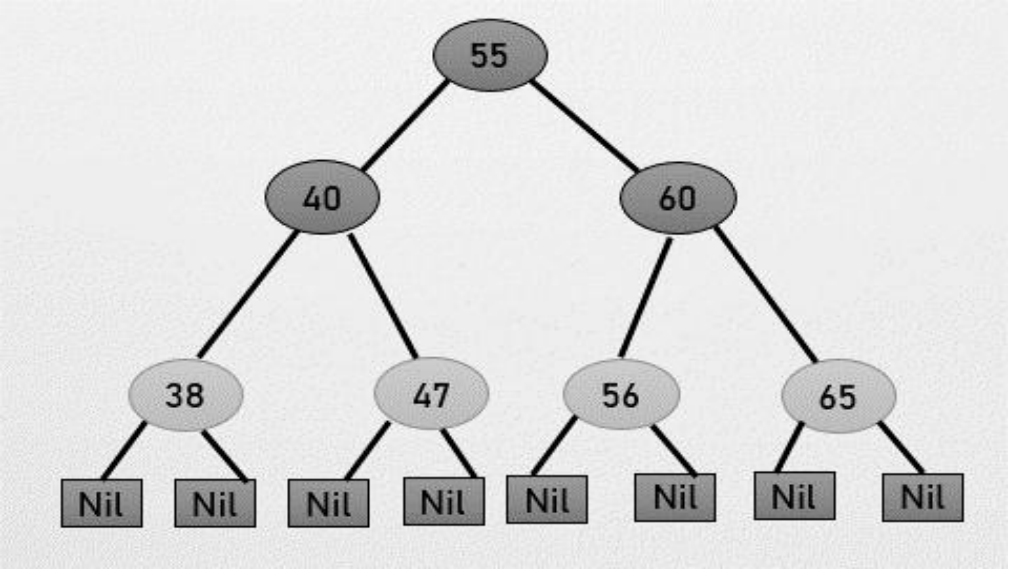
A number of different balanced trees have been defined, including AVL trees, 2-4 trees, and B trees. Here we will look at yet another kind of balanced tree called a red-black tree. The important idea behind all of these trees is that the insert and delete operations may restructure the tree to keep it balanced. So lookup, insert, and delete will always be logarithmic in the number of nodes but insert and delete may be more complicated than for binary search trees.

7.1 Red-Black Tree

A Red Black Tree is a self-balancing binary search tree with one extra attribute for each node: the colour, which is either red or black.It was invented in 1972 by Rudolf Bayer.Colours in tree are used to ensure that the tree remains balanced during insertion and deletion.

A Red Black Tree is a self-balancing binary search tree in which each node has a red or black colour. The red black tree satisfies all of the features of the binary search tree, but it also has several additional properties. A Red-Black tree's height is O(Logn), where (n is the number of nodes in the tree).Red-black trees are one of many search-tree schemes that are "balance" in order to guarantee that basic dynamic-set operations take O(lg n) time in the worst case.

BST operations take O(h) time where h is the height of the BST.Cost of these operations may become O(n) for a skewed Binary tree. If we make sure that the height of the tree remains O(log n) after every insertion and deletion, then an upper bound of O(log n) for all these operations. The height of a Red-Black tree is always O(log n) where n is the number of nodes in the tree.



Red-black tree

7.2 Red-Black Tree Properties

- Red - Black Tree must be a Binary Search Tree.
- The root of the tree is always black.
- The children of Red colored node must be colored BLACK. (There should not be two consecutive RED nodes)
- Every new node must be inserted with RED color.
- Every leaf (e.i. NULL node) must be colored BLACK.
- In all the paths of the tree, there should be same number of BLACK colored nodes.

Each node has the following attributes:

Color

Key

Left Child

Right Child

Parent (except root node)

Three Invariants

A red/black tree is a binary search tree in which each node is colored either red or black. At the interface, we maintain three invariants:

Ordering Invariant This is the same as for binary search trees: all the keys to left of a node are smaller, and all the keys to the right of a node are larger than the key at the node itself.

Height Invariant The number of black nodes on every path from the root to each leaf is the same. We call this the black height of the tree.

Color Invariant No two consecutive nodes are red. The balance and color invariants together imply that the longest path from the root to a leaf is at most twice as long as the shortest path. Since insert and search in a binary search tree have time proportional to the length of the path from the root to the leaf, this guarantees O (log(n)) times for these operations, even if the tree is not perfectly balanced. We therefore refer to the height and color invariants collectively as the balance invariant.

7.3 Red-Black Tree Operations

Insertion

Deletion

Search

Recolor and Rotation performed on insertion and deletion operation as per Red-Black Tree properties.

Insertion operation

The insertion operation in Red Black Tree is similar to the Binary Search Tree.Every new node must be inserted with the color RED.

After every insertion operation, we need to check all the properties of Red-Black Tree. If all the properties are satisfied then we go to next operation otherwise we perform the following operation to make it Red Black Tree.

1. Recolor

2. Rotation

3. Rotation followed by Recolor

Steps for Red-black tree insertion operations

Check whether tree is Empty.

If tree is Empty then insert the newNode as Root node with color Black and exit from the operation.

If tree is not empty then insert the newNode as leaf node with color Red.

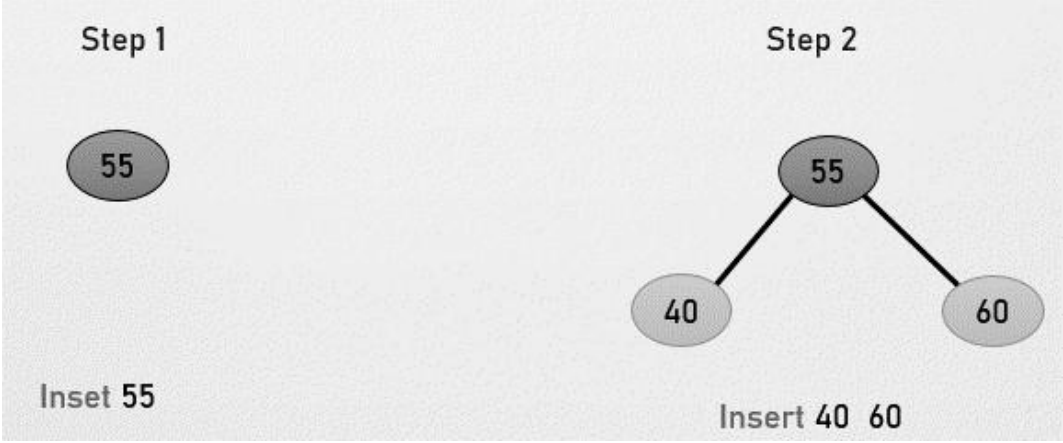
If the parent of newNode is Black then exit from the operation.

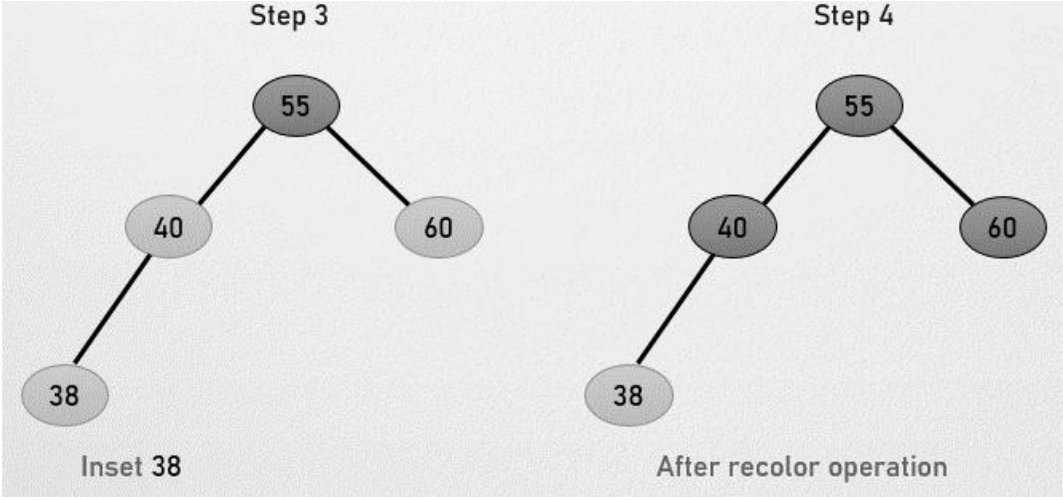
If the parent of newNode is Red then check the color of parentnode's sibling of newNode.

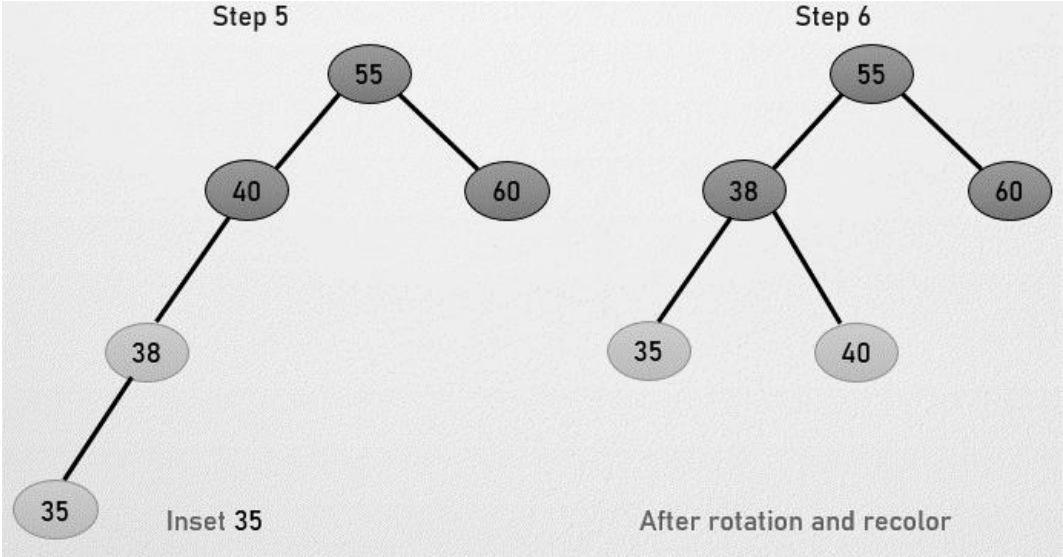
If it is colored Black or NULL then make suitable Rotation and Recolor it.

If it is colored Red then perform Recolor. Repeat the same until tree becomes Red Black Tree.

Red-black tree insertion operation







Algorithm to insert a node

Following steps are followed for inserting a new element into a red-black tree:

Let y be the leaf (ie. NIL) and x be the root of the tree.

Check if the tree is empty (ie. whether x is NIL). If yes, insert newNode as a root node and color it black.

Else, repeat steps following steps until leaf (NIL) is reached.

Compare newKey with rootKey.

If newKey is greater than rootKey, traverse through the right subtree.

Else traverse through the left subtree.

Assign the parent of the leaf as a parent of newNode.

If leafKey is greater than newKey, make newNode as rightChild.

Else, make newNode as leftChild.

Assign NULL to the left and rightChild of newNode.

Assign RED color to newNode.

Deletion operation

The deletion operation in Red-Black Tree is similar to the BST. In deletion operation, we need to check with the Red-Black Tree properties. If any of the properties are violated then make suitable operations like Recolor, Rotation and Rotation followed by Recolor to make it Red-Black Tree.

Algorithm to delete a node

Save the color of nodeToBeDeleted in origrinalColor.

If the left child of nodeToBeDeleted is NULL

Assign the right child of nodeToBeDeleted to x.

Transplant nodeToBeDeleted with x.

Else if the right child of nodeToBeDeleted is NULL

Assign the left child of nodeToBeDeleted into x.

Transplant nodeToBeDeleted with x.

Else

Assign the minimum of right subtree of noteToBeDeleted into y.

Save the color of y in originalColor.

Assign the rightChild of y into x.

If y is a child of nodeToBeDeleted, then set the parent of x as y.

Else, transplant y with rightChild of y.

Transplant nodeToBeDeleted with y.

Set the color of y with originalColor.

If the originalColor is BLACK, call DeleteFix(x).

Red-black tree applications

- To implement Java packages.
- To implement Standard Template Libraries (STL) in C++.
- It is used in K-mean clustering algorithm for reducing time complexity.
- To implement finite maps
- It is used to implement CPU Scheduling in Linux.
- In MySQL Red-Black tree is used for indexes on tables.

Time complexity in big O notation

Sr. No.	Algorithm	Time Complexity
1.	Search	O(log n)
2.	Insert	O(log n)
3.	Delete	O(log n)

 **Lab Exercise**:

Implementation of red-black tree

```
#include <iostream>
using namespace std;
struct Node {
 int data;
 Node *parent;
 Node *left;
 Node *right;
 int color;
};
typedef Node *NodePtr;
class RedBlackTree {
  private:
 NodePtr root;
 NodePtr TNULL;
 void initializeNULLNode(NodePtr node, NodePtr parent) {
  node->data = 0;
  node->parent = parent;
  node->left = nullptr;
  node->right = nullptr;
  node->color = 0;
 }
 // Preorder
 void preOrderHelper(NodePtr node) {
  if (node != TNULL) {
   cout << node->data << " ";
   preOrderHelper(node->left);
   preOrderHelper(node->right);
  }
 }
 // Inorder
 void inOrderHelper(NodePtr node) {
  if (node != TNULL) {
   inOrderHelper(node->left);
```

```
    cout << node->data << " ";
    inOrderHelper(node->right);
  }
}
// Post order
void postOrderHelper(NodePtr node) {
  if (node != TNULL) {
    postOrderHelper(node->left);
    postOrderHelper(node->right);
    cout << node->data << " ";
  }
}
NodePtr searchTreeHelper(NodePtr node, int key) {
  if (node == TNULL || key == node->data) {
    return node;
  }
  if (key < node->data) {
    return searchTreeHelper(node->left, key);
  }
  return searchTreeHelper(node->right, key);
}
// For balancing the tree after deletion
void deleteFix(NodePtr x) {
  NodePtr s;
  while (x != root && x->color == 0) {
    if (x == x->parent->left) {
      s = x->parent->right;
      if (s->color == 1) {
        s->color = 0;
        x->parent->color = 1;
        leftRotate(x->parent);
        s = x->parent->right;
      }
      if (s->left->color == 0 && s->right->color == 0) {
        s->color = 1;
        x = x->parent;
      } else {
        if (s->right->color == 0) {
          s->left->color = 0;
          s->color = 1;
```

```
        rightRotate(s);
        s = x->parent->right;
       }
       s->color = x->parent->color;
       x->parent->color = 0;
       s->right->color = 0;
       leftRotate(x->parent);
       x = root;
      }
     } else {
      s = x->parent->left;
      if (s->color == 1) {
       s->color = 0;
       x->parent->color = 1;
       rightRotate(x->parent);
       s = x->parent->left;
      }
      if (s->right->color == 0 && s->right->color == 0) {
       s->color = 1;
       x = x->parent;
      } else {
       if (s->left->color == 0) {
        s->right->color = 0;
        s->color = 1;
        leftRotate(s);
        s = x->parent->left;
       }
       s->color = x->parent->color;
       x->parent->color = 0;
       s->left->color = 0;
       rightRotate(x->parent);
       x = root;
      }
     }
    }
    x->color = 0;
}
void rbTransplant(NodePtr u, NodePtr v) {
    if (u->parent == nullptr) {
     root = v;
```

```
  } else if (u == u->parent->left) {
    u->parent->left = v;
  } else {
    u->parent->right = v;
  }
  v->parent = u->parent;
}
void deleteNodeHelper(NodePtr node, int key) {
  NodePtr z = TNULL;
  NodePtr x, y;
  while (node != TNULL) {
    if (node->data == key) {
      z = node;
    }
    if (node->data <= key) {
      node = node->right;
    } else {
      node = node->left;
    }
  }
  if (z == TNULL) {
    cout << "Key not found in the tree" << endl;
    return;
  }
  y = z;
  int y_original_color = y->color;
  if (z->left == TNULL) {
    x = z->right;
    rbTransplant(z, z->right);
  } else if (z->right == TNULL) {
    x = z->left;
    rbTransplant(z, z->left);
  } else {
    y = minimum(z->right);
    y_original_color = y->color;
    x = y->right;
    if (y->parent == z) {
      x->parent = y;
    } else {
      rbTransplant(y, y->right);
```

```
      y->right = z->right;
      y->right->parent = y;
    }
    rbTransplant(z, y);
    y->left = z->left;
    y->left->parent = y;
    y->color = z->color;
  }
  delete z;
  if (y_original_color == 0) {
    deleteFix(x);
  }
}
// For balancing the tree after insertion
void insertFix(NodePtr k) {
  NodePtr u;
  while (k->parent->color == 1) {
    if (k->parent == k->parent->parent->right) {
      u = k->parent->parent->left;
      if (u->color == 1) {
        u->color = 0;
        k->parent->color = 0;
        k->parent->parent->color = 1;
        k = k->parent->parent;
      } else {
        if (k == k->parent->left) {
          k = k->parent;
          rightRotate(k);
        }
        k->parent->color = 0;
        k->parent->parent->color = 1;
        leftRotate(k->parent->parent);
      }
    } else {
      u = k->parent->parent->right;

      if (u->color == 1) {
        u->color = 0;
        k->parent->color = 0;
        k->parent->parent->color = 1;
```

```
      k = k->parent->parent;
     } else {
      if (k == k->parent->right) {
       k = k->parent;
       leftRotate(k);
      }
      k->parent->color = 0;
      k->parent->parent->color = 1;
      rightRotate(k->parent->parent);
     }
    }
    if (k == root) {
     break;
    }
   }
   root->color = 0;
}
void printHelper(NodePtr root, string indent, bool last) {
   if (root != TNULL) {
    cout << indent;
    if (last) {
     cout << "R----";
     indent += "   ";
    } else {
     cout << "L----";
     indent += "|  ";
    }
    string sColor = root->color ? "RED" : "BLACK";
    cout << root->data << "(" << sColor << ")" << endl;
    printHelper(root->left, indent, false);
    printHelper(root->right, indent, true);
   }
}
 public:
RedBlackTree() {
   TNULL = new Node;
   TNULL->color = 0;
   TNULL->left = nullptr;
   TNULL->right = nullptr;
   root = TNULL;
```

```
}
void preorder() {
  preOrderHelper(this->root);
}
void inorder() {
  inOrderHelper(this->root);
}
void postorder() {
  postOrderHelper(this->root);
}
NodePtr searchTree(int k) {
  return searchTreeHelper(this->root, k);
}
NodePtr minimum(NodePtr node) {
  while (node->left != TNULL) {
    node = node->left;
  }
  return node;
}
NodePtr maximum(NodePtr node) {
  while (node->right != TNULL) {
    node = node->right;
  }
  return node;
}
NodePtr successor(NodePtr x) {
  if (x->right != TNULL) {
    return minimum(x->right);
  }
  NodePtr y = x->parent;
  while (y != TNULL && x == y->right) {
    x = y;
    y = y->parent;
  }
  return y;
}
NodePtr predecessor(NodePtr x) {
  if (x->left != TNULL) {
    return maximum(x->left);
  }
```

```
  NodePtr y = x->parent;
  while (y != TNULL && x == y->left) {
    x = y;
    y = y->parent;
  }
  return y;
}
void leftRotate(NodePtr x) {
  NodePtr y = x->right;
  x->right = y->left;
  if (y->left != TNULL) {
    y->left->parent = x;
  }
  y->parent = x->parent;
  if (x->parent == nullptr) {
    this->root = y;
  } else if (x == x->parent->left) {
    x->parent->left = y;
  } else {
    x->parent->right = y;
  }
  y->left = x;
  x->parent = y;
}
void rightRotate(NodePtr x) {
  NodePtr y = x->left;
  x->left = y->right;
  if (y->right != TNULL) {
    y->right->parent = x;
  }
  y->parent = x->parent;
  if (x->parent == nullptr) {
    this->root = y;
  } else if (x == x->parent->right) {
    x->parent->right = y;
  } else {
    x->parent->left = y;
  }
  y->right = x;
  x->parent = y;
```

```
}
// Inserting a node
void insert(int key) {
  NodePtr node = new Node;
  node->parent = nullptr;
  node->data = key;
  node->left = TNULL;
  node->right = TNULL;
  node->color = 1;
  NodePtr y = nullptr;
  NodePtr x = this->root;
  while (x != TNULL) {
    y = x;
    if (node->data < x->data) {
      x = x->left;
    } else {
      x = x->right;
    }
  }
  node->parent = y;
  if (y == nullptr) {
    root = node;
  } else if (node->data < y->data) {
    y->left = node;
  } else {
    y->right = node;
  }
  if (node->parent == nullptr) {
    node->color = 0;
    return;
  }
  if (node->parent->parent == nullptr) {
    return;
  }
  insertFix(node);
}

NodePtr getRoot() {
  return this->root;
}
```

```
  void deleteNode(int data) {
    deleteNodeHelper(this->root, data);
  }
  void printTree() {
    if (root) {
      printHelper(this->root, "", true);
    }
  }
};
int main() {
  RedBlackTree bst;
  bst.insert(55);
  bst.insert(40);
  bst.insert(65);
  bst.insert(60);
  bst.insert(75);
  bst.insert(57);
  bst.printTree();
  cout << endl
<< "After deleting" << endl;
  bst.deleteNode(40);
  bst.printTree();
}
```

7.4 Splay Trees

Splay trees are binary search trees that achieve our goals by being self-adjusting in a quite remarkable way: Every time we access a node of the tree, whether for insertion or retrieval, we perform radical surgery on the tree, lifting the newly accessed node all the way up, so that it becomes the root of the modified tree. Other nodes are pushed out of the way as necessary to make room for this new root. Nodes that are frequently accessed will frequently be lifted up to become the root, and they will never drift too far from the top position. Inactive nodes, on the other hand, will slowly be pushed farther and farther from the root.

It is possible that splay trees can become highly unbalanced, so that a single access to a node of the tree can be quite expensive. Later in this section, however, we shall prove that, over a long sequence of accesses, splay trees are not at all expensive and are guaranteed to require not many more operations even than AVL trees. The analytical tool used is called amortized algorithm analysis, since, like insurance calculations, the few expensive cases are averaged in with many less expensive cases to obtain excellent performance over a long sequence of operations.

Splay Trees were invented by Sleator and Tarjan. This data structure is essentially a binary tree with special update and access rules. It has the property to adapt optimally to a sequence of tree operations. More precisely, a sequence of m operations on a tree with initially n nodes takes time O (n ln (n) + m ln (n)).

Splaying

Splaying is a process in which a node is transferred to the root by performing suitable rotations. In a splay tree, whenever we access any node (searching, inserting or deleting a node), it is splayed to the root.

7.5 Operations

Splay trees support the following operations. We write S for sets, x for elements and k for key values.

splay(S, k) returns an access to an element x with key k in the set S. In case no such element exists, we return an access to the next smaller or larger element.

split(S, k) returns (S_1,S_2), where for each x in S_1 holds: key[x] <= k , and for each y in S_2 holds: k < key[y].

join(S_1,S_2) returns the union S = S_1 + S_2. Condition: for each x in S_1 and each y in S_2: x

<= y.

insert(S,x) augments S by x.

delete(S,x) removes x from S.

Each split, join, delete and insert operation can be reduced to splay operations and modifications of the tree at the root which take only constant time. Thus, the run time for each operation is essentially the same as for a splay operation.

The most important tree operation is splay(x), which moves an element x to the root of the tree. In case x is not present in the tree, the last element on the search path for x is moved instead. The run time for a splay(x) operation is proportional to the length of the search path for x. While searching for x we traverse the search path top-down. Let y be the last node on that path. In a second step, we move y along that path by applying rotations as described later.

The time complexity of maintaining a splay tree is analyzed using an Amortized Analysis. Consider a sequence of operations op_1, op_2, ..., op_m. Assume that our data structure has a potential. One can think of the potential as a bank account. Each tree operation op_i has actual costs proportional to its running time. We're paying for the costs c_i of op_i with its amortized costs a_i. The difference between concrete and amortized costs is charged against the potential of the data structure. This means that we're investing in the potential if the amortized costs are higher that the actual costs, otherwise we're decreasing the potential.

Thus, we're paying for the sequence op_1, op_2, ..., op_m no more than the initial potential plus the sum of the amortized costs a_1 + a_2 + ... + a_m.

The trick of the analysis is to defi ne a potential function and to show that each splay operation has amortized costs O (ln (n)). It follows that the sequence has costs O (m ln (n) + n ln (n))

7.6 Rotations in Splay Tree

Zig rotation / Right rotation

Zag rotation / Left rotation

Zig zag / Zig followed by zag

Zag zig / Zag followed by zig

Zig zig / two right rotations

Zag zag / two left rotations

Factors for selecting a type of rotation

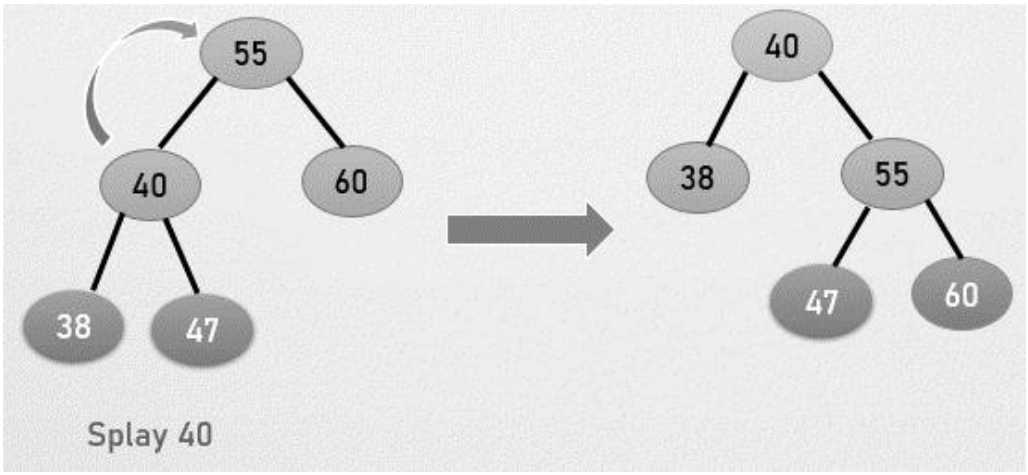
Does the node which we are trying to rotate have a grandparent?

Is the node left or right child of the parent?

Is the node left or right child of the grandparent?

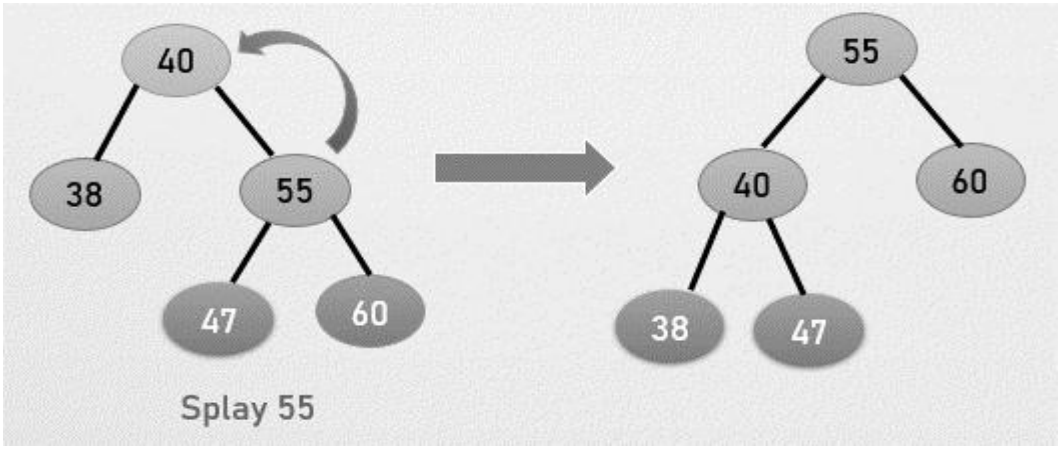
Zig Rotation

The Zig Rotation in splay tree is like single right rotation in AVL Tree rotations.In zig rotation, every node moves one position to the right from its current position.



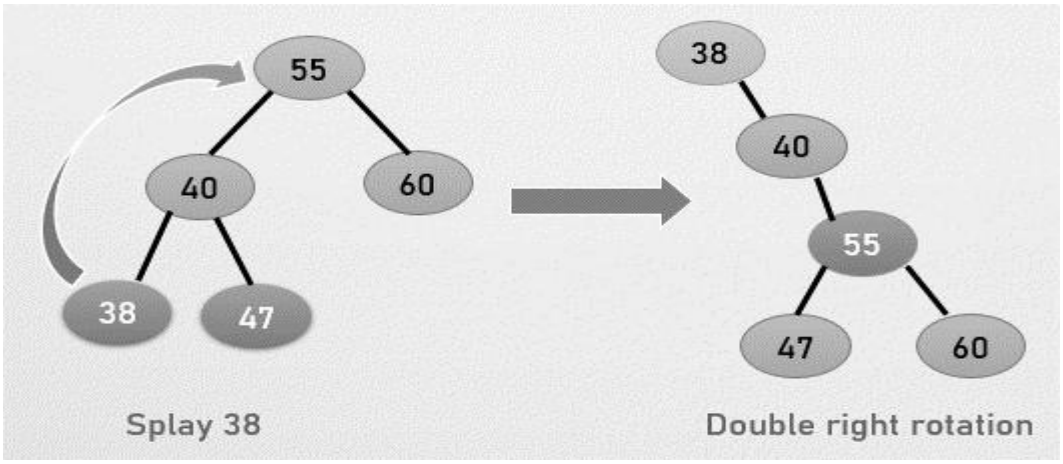
Zag Rotation

In zag rotation, every node moves one position to the left from its current position.

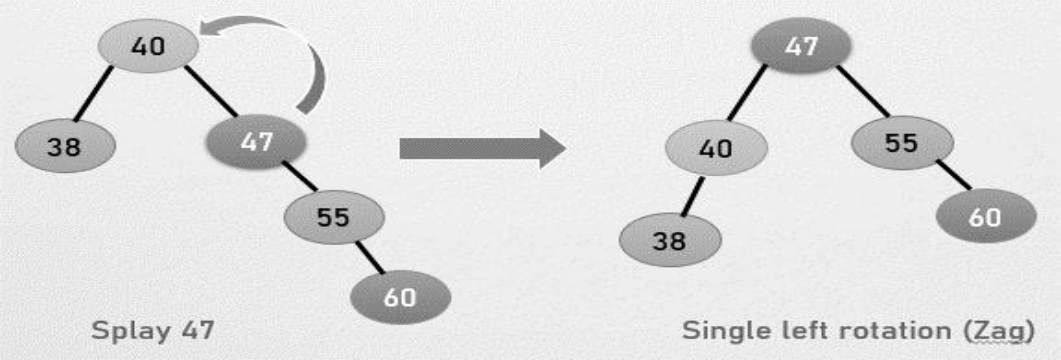


Zig-Zig Rotation

The Zig-Zig Rotation in splay tree is a double zig rotation. In zig-zig rotation, every node moves two positions to the right from its current position.



Zig-Zag Rotation



Advantages of Splay Trees

Splaying ensures that frequently accessed elements stay near the root of the tree so that they are easily accessible.

The average case performance of splay trees is comparable to other fully-balanced trees: O(log n).Splay trees do not need bookkeeping data; therefore, they have a small memory footprint.

Disadvantages

A splay tree can arrange itself linearly. Therefore, the worst-case performance of a splay tree is O(n).

Multithreaded operations can be complicated since, even in a read-only configuration, splay trees can reorganize themselves.

7.7 2-3 Trees

A 2-3 tree is another type of tree which has 2 types of nodes, 2-node and 3-node.A 2-3 tree data structure is a specific form of a B tree where every node with children has either two children and one data element or three children and two data elements.It is a self-balancing tree. It is balanced with every leaf node at equal distance from the root node.

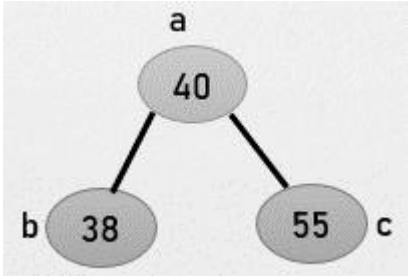
2-Node

2-Node: A node with a single data element that has two child nodes.

1. Every value appearing in the child (b) 38 must be ≤ (a) 40.

2. Every value appearing in the child (c) 55 must be ≥ (a) 40.

3. The length of the path from the root of a 2-node to every leaf in its child must be the same.



3-Node

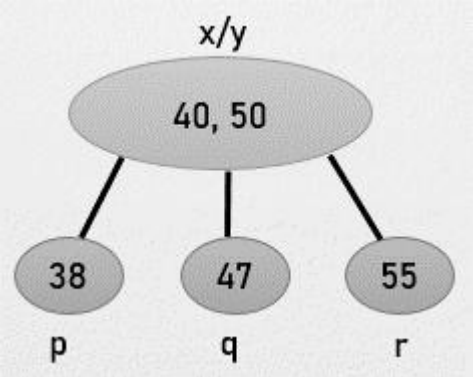
3-Node: A node with two data elements that has three child nodes.

1. Every value appearing in child P must be ≤ X.

2. Every value appearing in child Q must be in between X and Y.

3. Every value appearing in child R must be ≥ Y.

4. The length of the path from the root of a 3-node to every leaf in its child must be the same.



7.8 Properties of 2-3 Trees

- Every internal node in the tree is a 2-node or a 3-node i.e it has either one value or two values.
- A node with one value is either a leaf node or has exactly two children. Values in left sub tree < value in node < values in right sub tree.
- Data stored in sorted manner.
- Insertion operation performed in leaf node.
- A node with two values is either a leaf node or has exactly 3 children. It cannot have 2 children.
- Values in left sub tree < first value in node < values in middle sub tree < second value in node < value in right sub tree.
- All leaf nodes are at the same level.

7.9 2-3 Tree Operations

Insertion

Deletion

Search

Insertion operation

If the tree is empty, create a node and put value into the node

Otherwise find the leaf node where the value belongs.

If the leaf node has only one value, put the new value into the node

If the leaf node has more than two values, split the node and promote the median of the three values to parent.

If the parent then has three values, continue to split and promote, forming a new root node if necessary

Search operation

If Tree is empty, return False (data item cannot be found in the tree).

If current node contains data value which is equal to data, return True.

If we reach the leaf-node and it doesn't contain the required key value, return False.

Recursive Calls

If data < currentNode.leftVal, we explore the left sub tree of the current node.

Else if currentNode.leftVal < data < currentNode.rightVal, we explore the middle sub tree of the current node.

Else if data data > currentNode.rightVal, we explore the right sub tree of the current node.

Deletion process

There are three cases in deletion process

1. When the record is to be removed from a leaf node containing two records.

In this case, the record is simply removed, and no other nodes are affected.

2. When the only record in a leaf node is to be removed.

3. When a record is to be removed from an internal node.

In both the second and the third cases, the deleted record is replaced with another that can take its place while maintaining the correct order of 2-3 tree.

Summary

- A Red Black Tree is a self-balancing binary search tree in which each node has a red or black colour.
- The red black tree satisfies all of the features of the binary search tree, but it also has several additional properties.
- Splay trees are self-adjusting binary search trees in which every access for insertion or retrieval of a node, lifts that node all the way up to become the root, pushing the other nodes out of the way to make room for this new root of the modified tree. Hence, the frequently accessed nodes will frequently be lifted up and remain around the root position; while the most infrequently accessed nodes would move farther and farther away from the root.
- A 2-3 tree data structure is a specific form of a B tree where every node with children has either two children and one data element or three children and two data elements.

Keywords

AVL tree	2-3 tree
Zag rotation / Left rotation	Zig zag / Zig followed by zag
Zag zig / Zag followed by zig	Zig zig / two right rotations
Zag zag / two left rotations	2-Node, 3-Node

Self Assessment

1. Red Black Tree invented in

A. 1960

B. 1972

C. 1976

D. None of above

2. The height of a Red-Black tree is

A. O(1)

B. O(log n)

C. O(0)

D. None of above

3. The color of the root in red black tree is

A. Red
B. Black
C. Green
D. All of above

4. Red-Black Tree must be

A. AVL tree
B. BST
C. Binary tree
D. All of above

5. What is color of newly inserted node in red-black tree

A. Red
B. Black
C. Blue
D. None of above

6. Recolor and Rotation performed in

A. Insertion
B. Deletion
C. Both insertion and deletion
D. Search

7. 90-10 rule is part of

A. BST
B. Binary tree
C. Splay tree
D. All of above

8. Left rotation is also called

A. Zig rotation
B. Zag rotation
C. Zag-zag rotation
D. None of above

9. Two right rotations is equal to

A. Zag zag
B. Zag zig
C. Zig zig
D. All of above

10. What are factors for selecting a type of rotation

A. Does the node which we are trying to rotate have a grandparent?
B. Is the node left or right child of the parent?
C. Is the node left or right child of the grandparent?
D. All of above

11. What are the operations performed on Splay Tree

A. Insertion
B. Deletion
C. Search
D. All of above

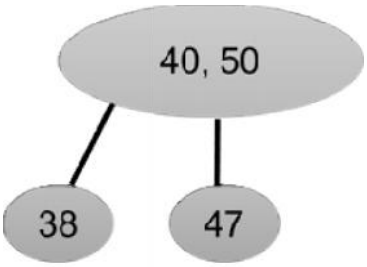
12. 2-node is

A. A node with a double data element that has two child nodes.
B. A node with a single data element that has two child nodes.
C. A node with a single data element that has one child nodes.
D. All of above

13. 3-node is

A. A node with two data elements that has three child nodes
B. A node with three data elements that has three child nodes
C. A node with two data elements that has two child nodes
D. None of above

14. Is diagram represent correct 3-node tree?



A. True
B. False

15. Properties of 2-3 Trees are

A. Data stored in sorted manner
B. Every internal node in the tree is a 2-node or a 3-node
C. Insertion operation performed in leaf node
D. All of above

Answers for Self Assessment

1. B 2. B 3. B 4. C 5. A

6.	C	7.	C	8.	B	9.	C	10.	D
11.	D	12.	B	13.	A	14.	B	15.	D

Review Questions

1. Discuss red black tree properties.
2. Define recolor and rotation process.
3. Differentiate between zig-zag rotation.
4. Discuss concept of 2-node and 3-node with suitable diagram.
5. Explain splay operation in splay trees.
6. "The time complexity of maintaining a splay tree is analyzed using an Amortized Analysis."Explain
7. "A splay tree does not keep track of heights and does not use any balance factors like anAVL tree". Explain

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Unit 08: Heaps

CONTENTS

Objectives

After studying this unit, you will be able to:

- Understand basics of heap
- Learn max and min heap
- Discuss operations on heap
- Learn priority queue

Introduction

The heap data structure is a complete binary tree where each node of the tree has an orderly relationship with its successors. Binary search trees are totally ordered, but the heap data structure is only partially ordered. It is suitable for inserting and deleting minimum value operations.

Heap is an array object that is considered as a complete binary tree. Each node of the tree corresponds to an element of the array that stores the value in the node. The tree is completely filled at all levels except possibly the lowest, which is filled from the left upwards to a point. Heap data structures are suitable for implementing priority queues. The heap serves as a foundation of a theoretically important sorting algorithm called heap sort, which we will discuss after defining the heap.

8.1 Heap

A heap is a specialized tree-based data structure that satisfies the heap property: if B is a child node of A, then key (A) ≥ key(B). This implies that an element with the greatest key is always in the root node, and so such a heap is sometimes called a max-heap. (Alternatively, if the comparison is reversed, the smallest element is always in the root node, which results in a min-heap.) The heap is one maximally-efficient implementation of an abstract data type called a priority queue. Heaps are crucial in several efficient graph algorithms.

A heap is a storage pool in which regions of memory are dynamically allocated. For example, in C++ the space for a variable is allocated essentially in one of three possible places: Global variables are allocated in the space of initialized static variables; the local variables of a procedure are allocated in the procedure's activation record, which is typically found in the processor stack; and dynamically allocated variables are allocated in the heap. In this unit, the term heap is taken to mean the storage pool for dynamically allocated variables.

I consider heaps and heap-ordered trees in the context of priority queue implementations. While it may be possible to use a heap to manage a dynamic storage pool, typical implementations do not. In this context, the technical meaning of the term heap is closer to its dictionary defi nition–"a pile of many things."

A binary tree has the heap property *iff*

1. It is empty or

2. The key in the root is larger than that in either child and both subtrees have the heap property.

A heap can be used as a priority queue: the highest priority item is at the root and is trivially extracted. But if the root is deleted, you are left with two sub-trees and you must efficiently re-create a single tree with the heap property.

The value of the heap structure is that you can both extract the highest priority item and insert a new one in O(logn) time.

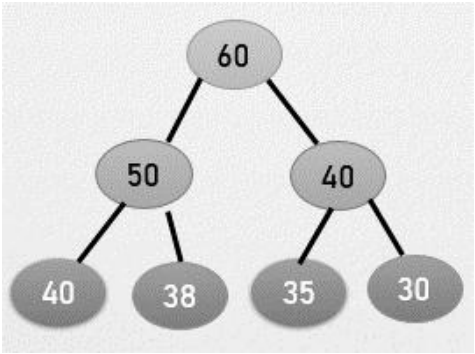
A heap can be defined as binary trees with keys assigned to its nodes (one key per node). The two types of heaps are:

1. Max heaps

2. Min heaps

Max heaps

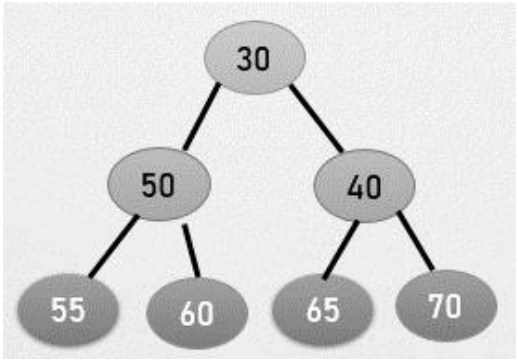
The key present at the root node must be greatest or equal to the keys present at all of its children.The same property must be true for all sub-trees in that Binary Tree.



Max heap

Min heaps

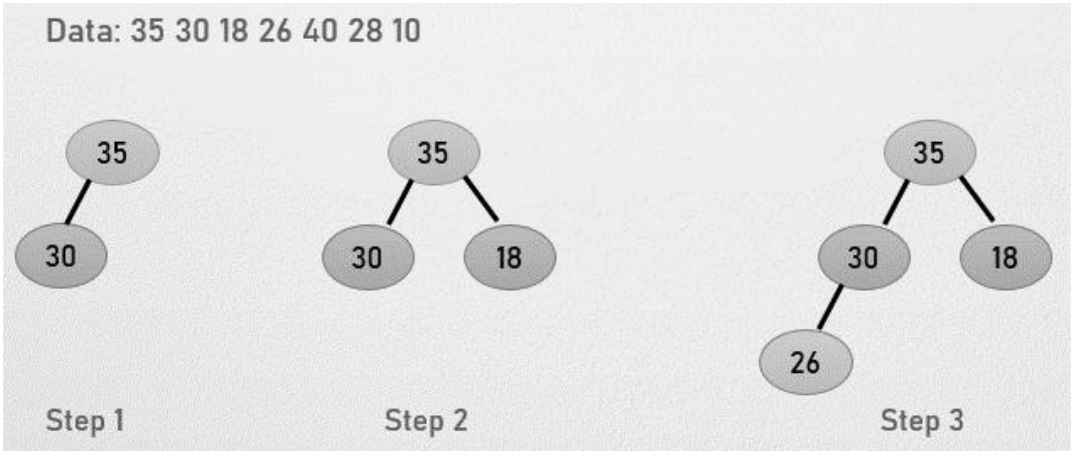
The key present at the root node must be minimum or equal to the keys present at all of its children. The same property must be true for all sub-trees in that Binary Tree.

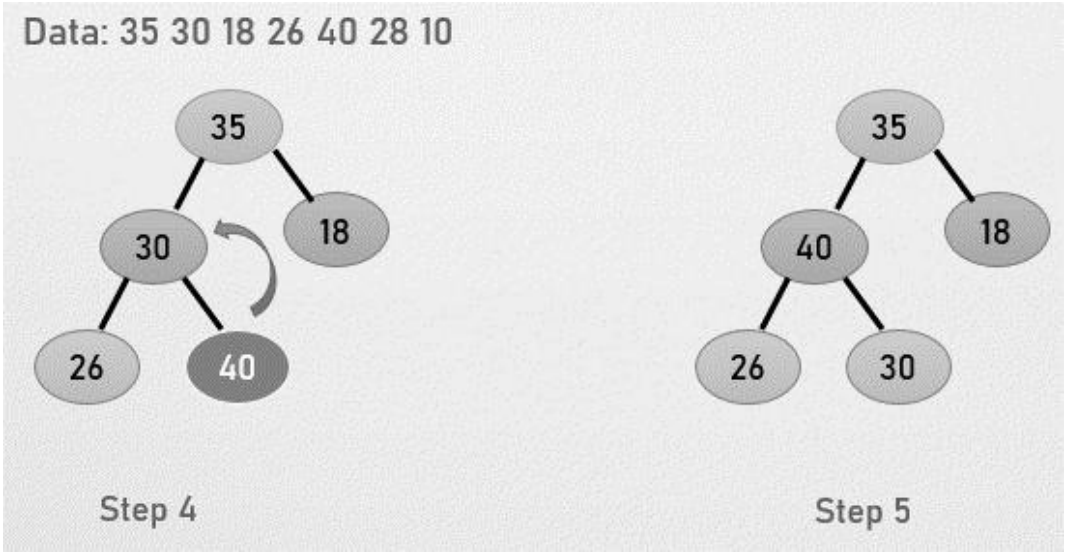


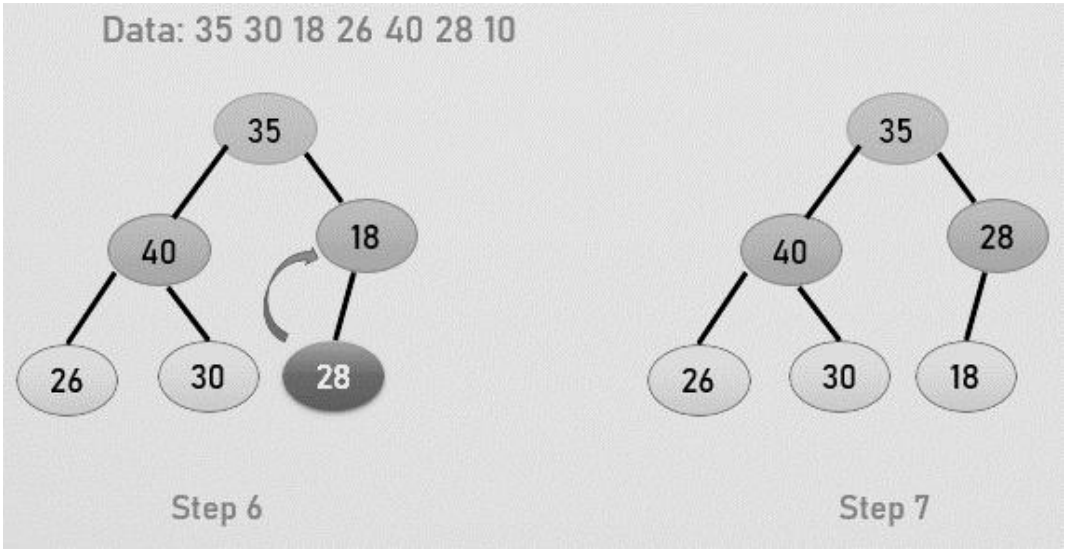
Heap Tree construction

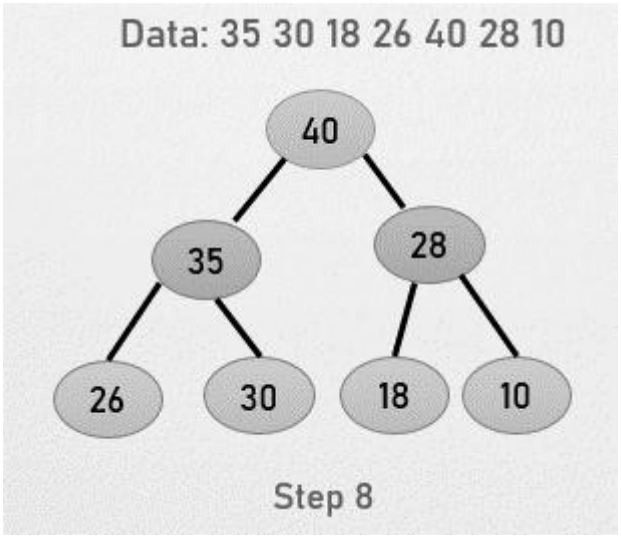
1. Create a new node at the end of heap.
2. Assign new value to the node.
3. Compare the value of this child node with its parent.
4. If value of parent is less than child, then swap them.
5. Repeat step 3 & 4 until Heap property holds.

Example:**Max heap**



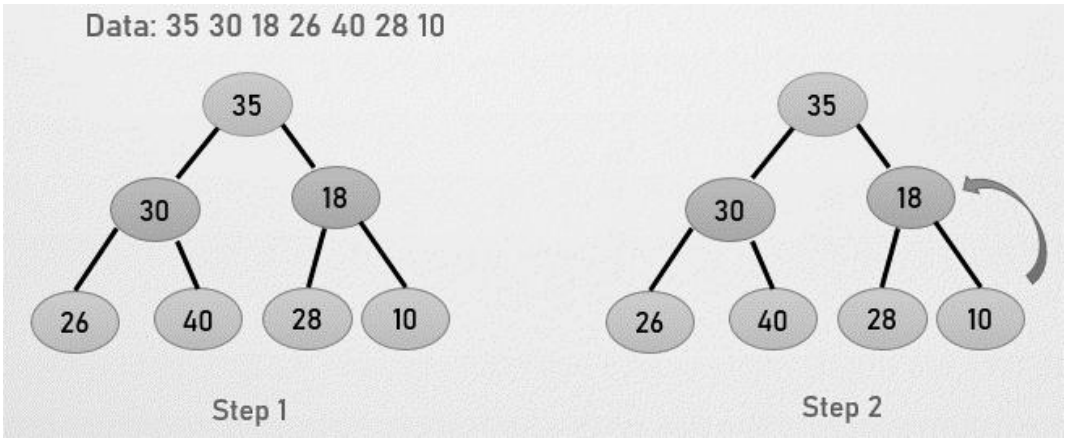


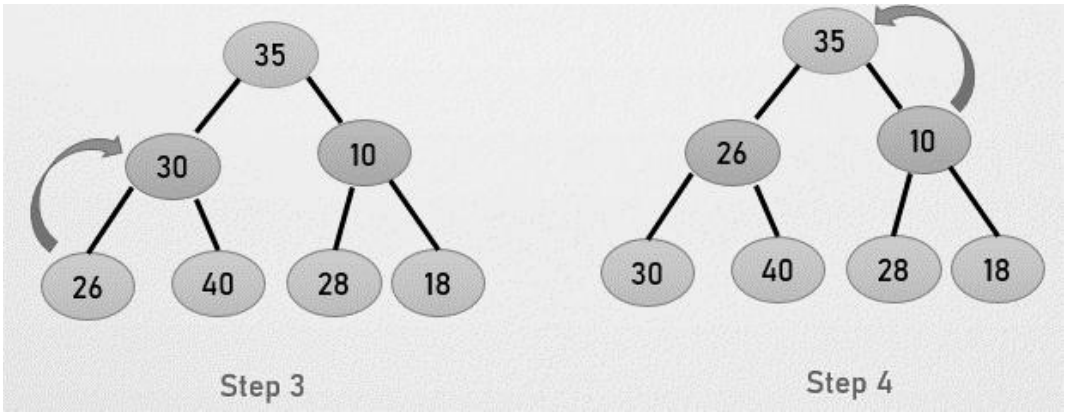


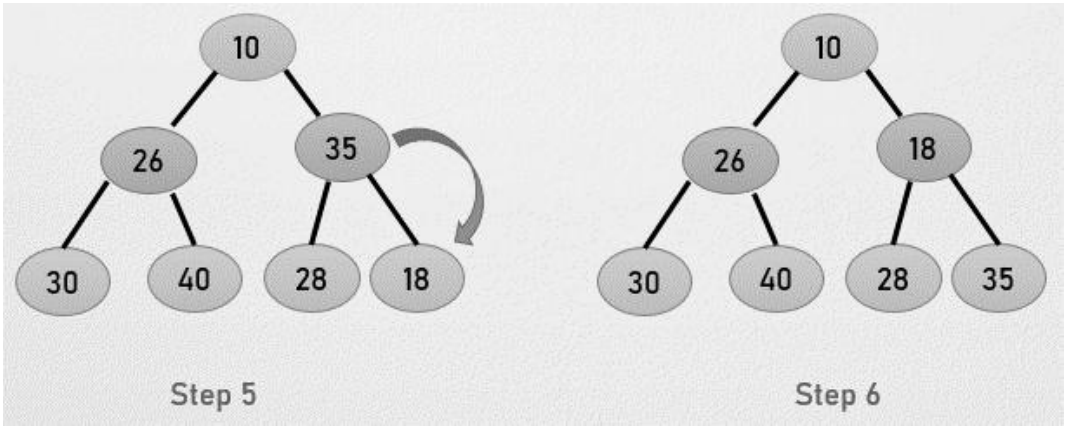


8.2 Heapify Method (Min Heap)

Complexity: O(n)







8.3 Deletion Operation on Heap Tree

There are two methods for deletion in heap tree.

Method 1

1. Remove root node.
2. Move the last element of last level to root.
3. Compare the value of this child node with its parent.

4. If value of parent is less than child, then swap them.

5. Repeat step 3 & 4 until Heap property holds.

Method 2

1. Select the element to be deleted.

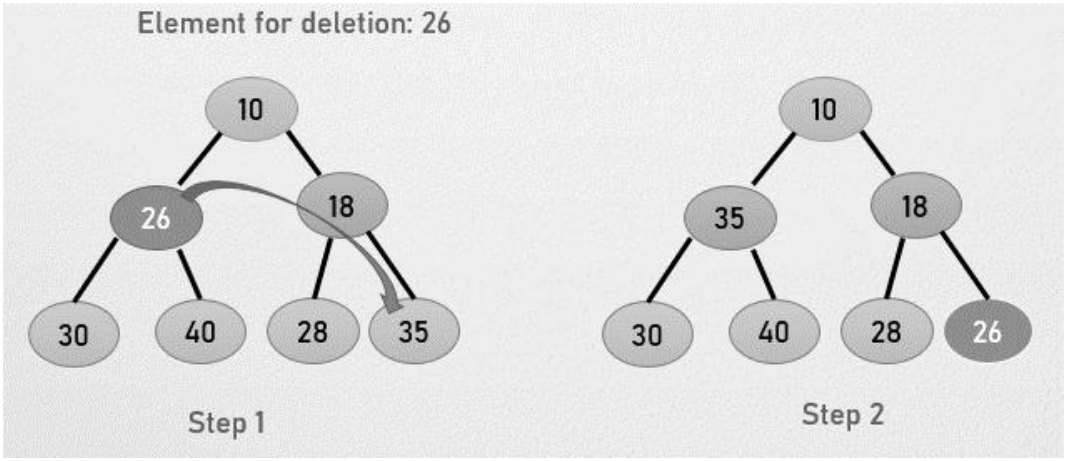
2. Swap it with the last element.

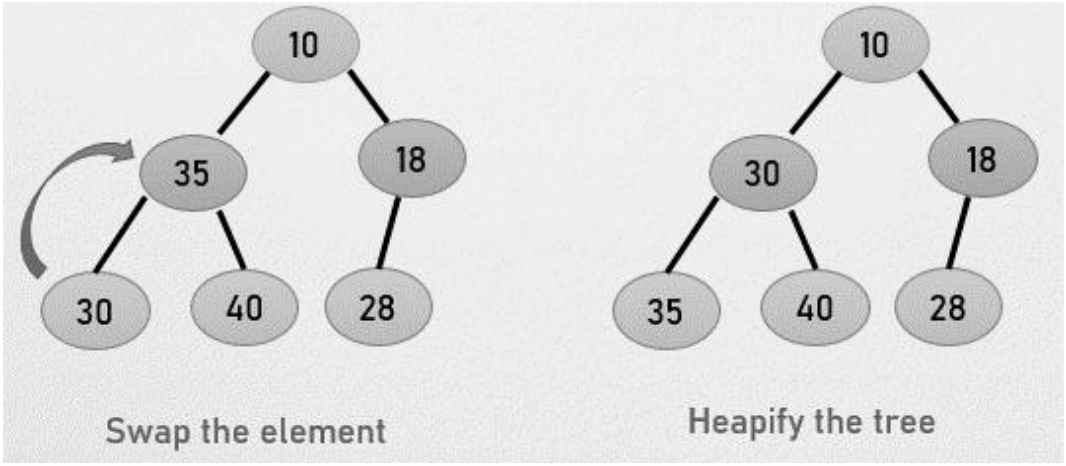
3. Remove the last element.

4. Heapify the tree.

Example:

Deletion operation on heap





8.4 Applications of Heaps

Priority queue implementation

Heap sort

Order statistics

Priority queue

In priority queue key is associated with every element.The element with highest priority will be moved to the front of the queue and one with lowest priority will move to the back of the queue.Queue returns the element according to priority.However, if elements with the same priority occur, they are served according to their order in the queue.

One of the most important applications of priority queues is in discrete event simulation. Simulation is a tool which is used to study the behavior of complex systems. The fi rst step in simulation is modeling. You construct a mathematical model of the system I wish to study. Then you write a computer program to evaluate the model.

The systems studied using discrete event simulation have the following characteristics: The system has a state which evolves or changes with time. Changes in state occur at distinct points in simulation time. A state change moves the system from one state to another instantaneously. State changes are called events.

A priority queue is a queue with items having an orderable characteristic called priority. The objects having the highest priority are always removed first from the priority queues. A priority queue can be obtained by creating a heap. First call a function that creates an ascending heap.

After creating the heap, delete the root node and call a function to recreate the heap for the remaining elements. This method helps in implementing an ascending priority queue. In the same way, we can implement a descending priority queue.

A ***max-priority*** queue returns the element with maximum key first. A max-heap is used for a max-priority queue.

A ***min-priority*** queue returns the element with the smallest key first. A min-heap is used for a min-priority queue.

Ascending order priority queue: In ascending order priority queue, a lower priority number is given as a higher priority in a priority.

Descending order priority queue: In descending order priority queue, a higher priority number is given as a higher priority in a priority.

8.5 Priority Queue Operations

Insert

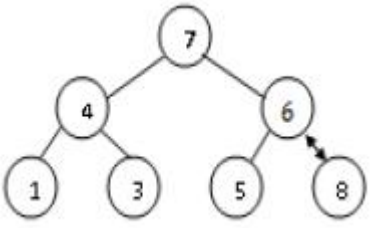
Delete

Peeking from the Priority Queue (Find max/min)

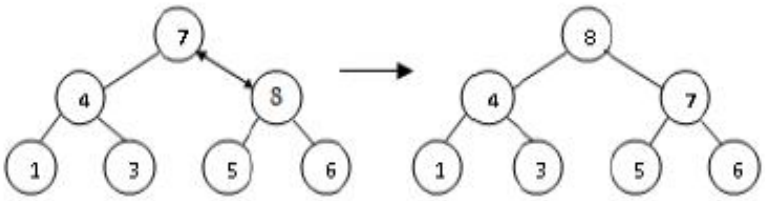
Extract-Max/Min from the Priority Queue

Insertion operation

To insert a new key into a heap, add a new node with key K after the last leaf of the existing heap. Then, shift K up to its suitable place in the new heap. Consider inserting value 8 into the heap shown in the figure



Compare 8 with its parent key. Stop if the parent key is greater than or equal to 8. Else, swap these two keys and compare 8 with its new parent (Refer to figure 14.8). This swapping continues until 8 is not greater than its last parent or it reaches the root. In this algorithm too, we can shift up an empty node until it reaches its proper position, where it acquires the value 8.



This insertion operation does not require more key comparisons than the heap's height. Since the height of a heap with n nodes is about log2n, the time efficiency of insertion is in O(log n).

Insertion operation: algorithm

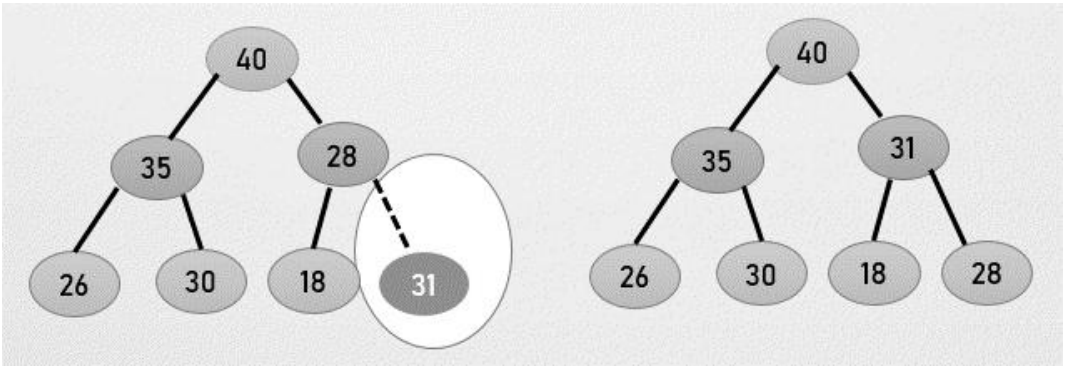
If there is no node,

create a newNode.

else (a node is already present)

insert the newNode at the end (last node from left to right.)

Heapify the array



Delete operation

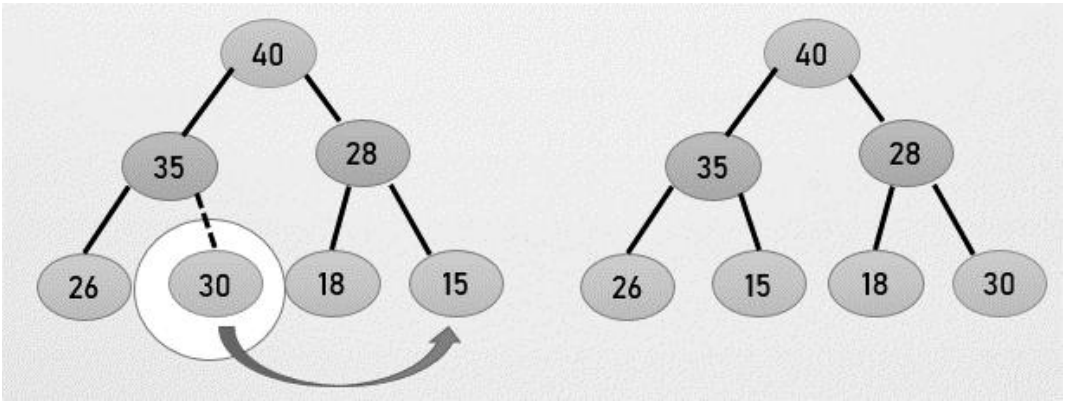
If nodeToBeDeleted is the leafNode

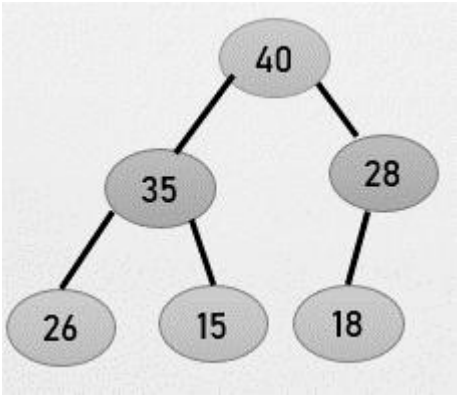
remove the node

Else swap nodeToBeDeleted with the lastLeafNode

remove noteToBeDeleted

heapify the array



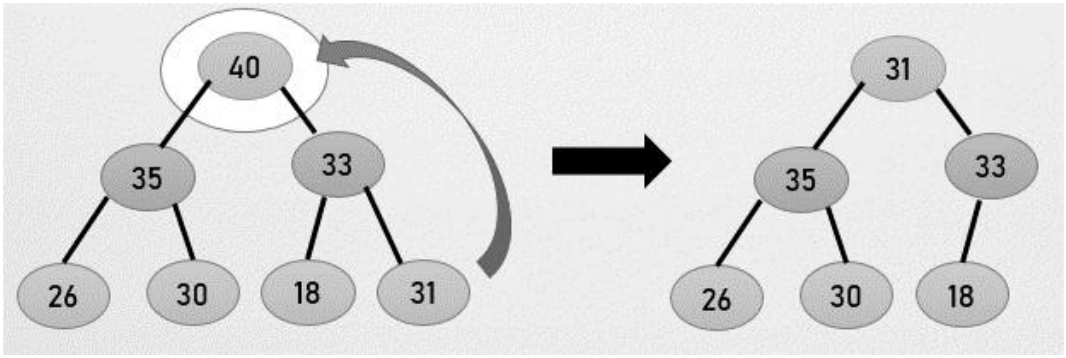


Delete operation

Deleting the Root of a Heap/ Extract Maximum

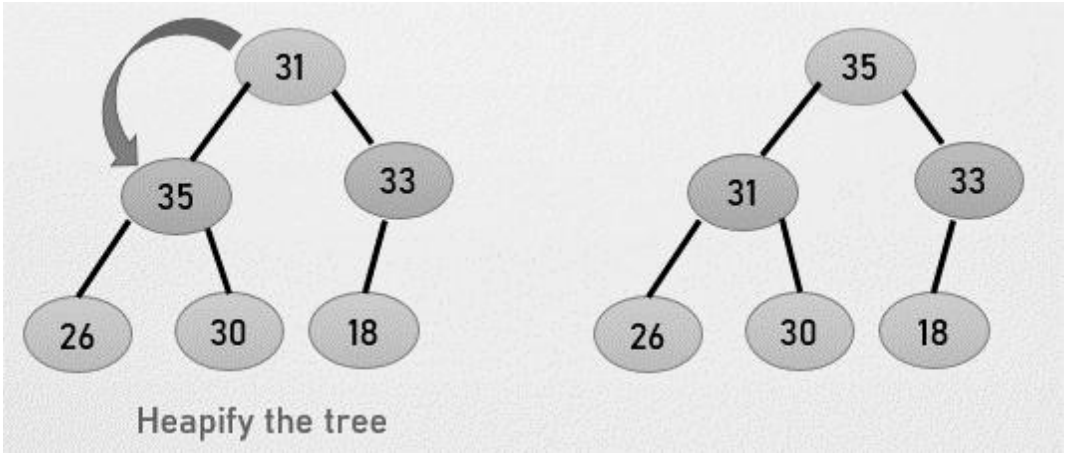
The following steps show the method to delete the root key from a heap in the figure

Step 1: Exchange the root's key with the last key K of the heaps as shown in the figure



Step 2: Decrease the heap's size by 1

Step 3: "Heapify" the smaller tree by shifting K down the tree as we did in the bottom-up heap construction algorithm. That is, verify the parental dominance for K and if it holds, we complete the process. If not, swap K with the largest of its children and repeat this operation until the parental dominance condition holds for K in its new position.



We can determine the efficiency of deletion by the number of key comparisons required to "heapify" the tree after the swap is done, and the size of the tree is decreased by 1. Since it does not need more key comparisons than twice the heap's height, the time efficiency of deletion is in O(log n).

Applications of Priority Queue

Dijkstra's algorithm for shortest path.

Load balancing and interrupt handling in an operating system.

Data compression in Huffman code.

Summary

- A heap is a partially sorted binary tree. Although a heap is not completely in order, it conforms to a sorting principle: every node has a value less (for the sake of simplicity, I will assume that all orderings are from least to greatest) than either of its children.
- The heap data structure is a complete binary tree where each node of the tree relates to an element of the array that stores the value in the node.
- The two principal ways to construct a heap are by using the bottom-up heap construction algorithm and the top-down heap construction algorithm
- A heap is used to implement heapsort. Heapsort is a comparison-based sorting algorithm which has a worst-case of O (n log n) runtime.
- A priority queue is a queue with items having an orderable characteristic called priority. The objects having the highest priority are always removed first from the priority queues.
- Priority queue can be attained by creating a heap.

Keywords

- ***Ascending Heap***: It is a complete binary tree in which the value of each node is greater than or equal to the value of its parent.
- ***Heapify***: Heapify is a procedure for manipulating heap data structures.
- ***N-ary Tree***: An n-ary tree is either an empty tree, or a non-empty set of nodes which consists of a root and exactly N sub-trees. The degree of each node of an N-ary tree is either zero or N.
- ***Heap***: A heap is a specialized tree-based data structure that satisfi es the heap property: if B is a child node of A, then key(A) ≥ key(B).
- ***Binary Heap***: A binary heap is a heap-ordered binary tree which has a very special shape called a complete tree.
- ***Discrete Event Simulation***: One of the most important applications of priority queues is in discrete event simulation.

Self Assessment

1. Heap satisfy following properties

A. Structural property
B. Ordering property
C. Both Structural and Ordering property
D. None of above

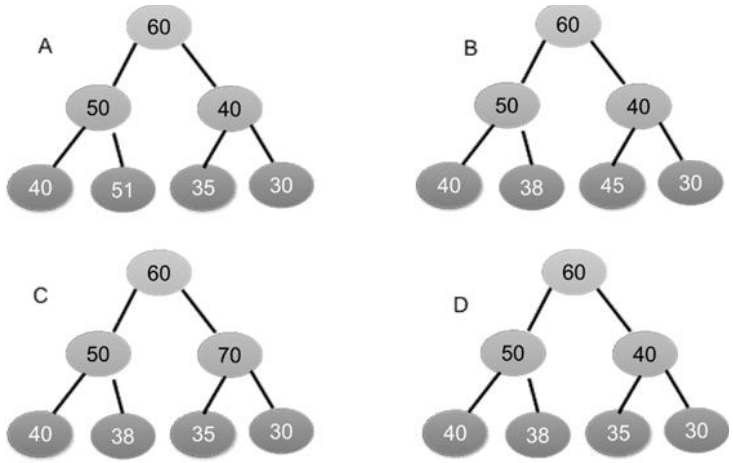
2. What are the types of Heap

A. Max-Heap
B. Min-Heap

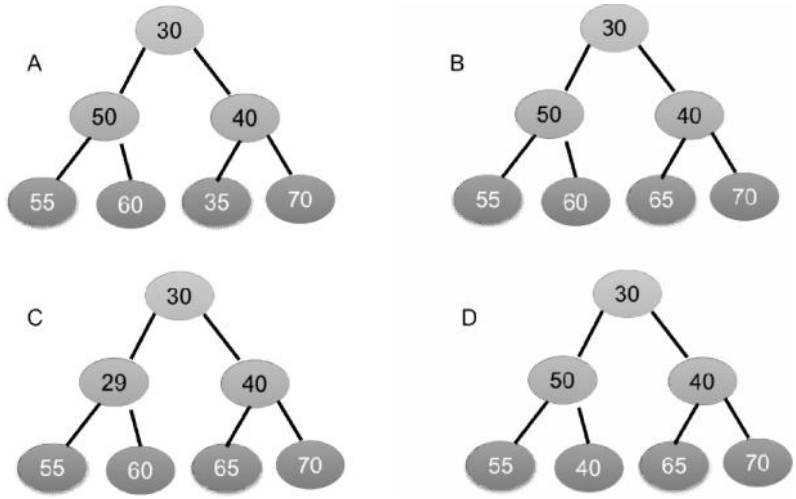
C. Both Max-Heap and Min-Heap

D. None of above

3. Which is correct option for Max-Heap?



4. Which is correct option for Min-Heap?



5. In which heap the root node must be greatest among the keys present at all of its children?

A. Max-heap

B. Min-heap

C. Both A and B

D. None of the above

6. Heap can be used to perform___

A. A decreasing order array

B. Normal Array

C. Priority queue

D. Stack

7. What is the complexity of adding an element to the heap?

A. O(log n)
B. O(log h)
C. Both O(log n) and O(log h)
D. None of above

8. In the worst case, the time complexity of inserting a node in a heap would be

A. O(logN)
B. O(1)
C. O(H)
D. None of above

9. Applications of heap are___

A. Priority queue implementation
B. Heap sort
C. Order statistics
D. All of above

10. Priority queue types are__

A. Max
B. Min
C. Descending order
D. All of above

11. Which is not Priority queue operation?

A. Delete
B. Peeking from the Priority Queue
C. Isfull
D. Extract-Max/Min from the Priority Queue

12. What are the methods used to implement Priority Queue?

A. Arrays,
B. Linked list,
C. Heap data structure and binary search tree
D. All of above

13. Which is most efficient way to implementing the priority queue?

A. Arrays,
B. Linked list,
C. Heap data structure
D. Binary search tree

14. What are the applications of Priority Queue

A. Dijkstra's algorithm for shortest path
B. It is used in prim's algorithm
C. It is used in heap sort
D. All of above

15. What is the time complexity to insert a node based on key in a priority queue?

A. O(nlogn)
B. O(n)
C. O(1)
D. O(n2)

Answers for Self Assessment

1.	C	2.	C	3.	D	4.	B	5.	A
6.	C	7.	C	8.	A	9.	D	10.	D
11.	C	12.	D	13.	D	14.	D	15.	B

Review Questions

1. Discuss heap properties.
2. "A heap can be implemented as an array by recording its elements in top-down left-to-right manner". Describe in detail.
3. "Binary search property is different from heap property". Justify.
4. Describe priority heap with example.
5. What are the applications of Priority Queue?
6. Represent the max heap and min heap for the data 3, 8, 20, 28, 42, 54.
7. Differentiate between max heap and min heap with example.

Further Readings

- Burkhard Monien, Data Structures and Effi cient Algorithms, Thomas Ottmann,
  Springer.
- Kruse, Data Structure & Program Design, Prentice Hall of India, New Delhi.
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- RG Dromey, How to Solve it by Computer, Cambridge University Press.
- Lipschutz. S. (2011). Data Structures with C. Delhi: Tata McGraw hill
- Reddy. P. (1999). Data Structures Using C. Bangalore: Sri Nandi Publications
- Samantha. D (2009). Classic Data Structures. New Delhi: PHI Learning Private Limited

Web Links

www.en.wikipedia.org

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www.webopedia.com

https://www.programiz.com/dsa/heap-data-structure

https://www.javatpoint.com/heap-data-structure

https://www.tutorialspoint.com/data_structures_algorithms/heap_data_structure.htm

Unit 09: More on Heaps

CONTENTS

Objectives

After studying this unit, you will be able to:

- Understand basics of heap sort
- Learn binomial heaps
- Discuss fibonacci heaps

Introduction

A heap is a complete binary tree, and a binary tree is one in which each node can have no more than two children. A complete binary tree is one in which all levels except the last, i.e., the leaf node, are completely filled and all nodes are justified to the left.

The elements of a heap sort are processed by generating a min-heap or max-heap with the items of the provided array. The ordering of an array in which the root element reflects the array's minimal or maximum element is known as min-heap or max-heap.Heapsort is a well-liked and efficient sorting method. The idea behind heap sort is to remove elements from the heap part of the list one by one and then insert them into the sorted part.

9.1 Heap Sort

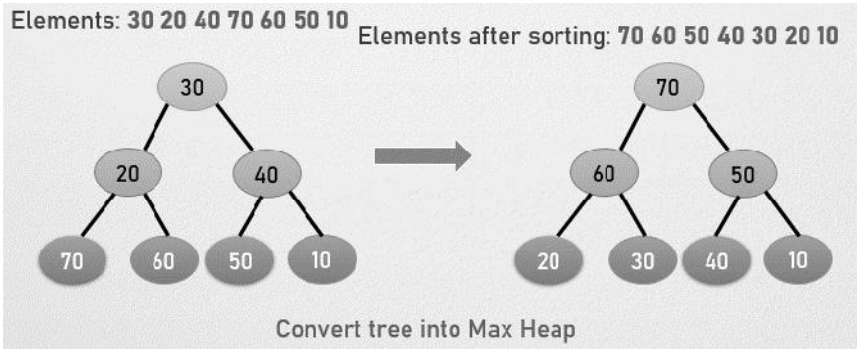
Heap sort is sorting technique based upon Binary Heap data structure.It is comparison-based sorting technique.It processes the elements by creating the min heap or max heap using the elements of the array. Heap sort basically recursively performs two main operations -

Build a heap H, using the elements of array.

Repeatedly delete the root element of the heap formed in 1st phase.

Steps for Heap Sort

- Construct a Binary Tree from the list of Elements.
- Transform the Binary Tree into Max Heap / Min Heap.
- Delete the root element from Max Heap / Min Heap
- Reducing the size of heap by 1
- Heapify the root of the tree.
- Put the deleted element into the Sorted list.
- Repeat the same until Min Heap becomes empty.



Algorithm

HeapSort(arr)

BuildMaxHeap(arr)

for i = length(arr) to 2

swap arr[1] with arr[i]

heap_size[arr] = heap_size[arr] ? 1

MaxHeapify(arr,1)

End

BuildMaxHeap(arr)

BuildMaxHeap(arr)

heap_size(arr) = length(arr)

for i = length(arr)/2 to 1

MaxHeapify(arr,i)

End

MaxHeapify(arr,i)

MaxHeapify(arr,i)

L = left(i)

R = right(i)

if L ? heap_size[arr] and arr[L] > arr[i]

largest = L

else

largest = i

if R ? heap_size[arr] and arr[R] > arr[largest]

largest = R

if largest != i

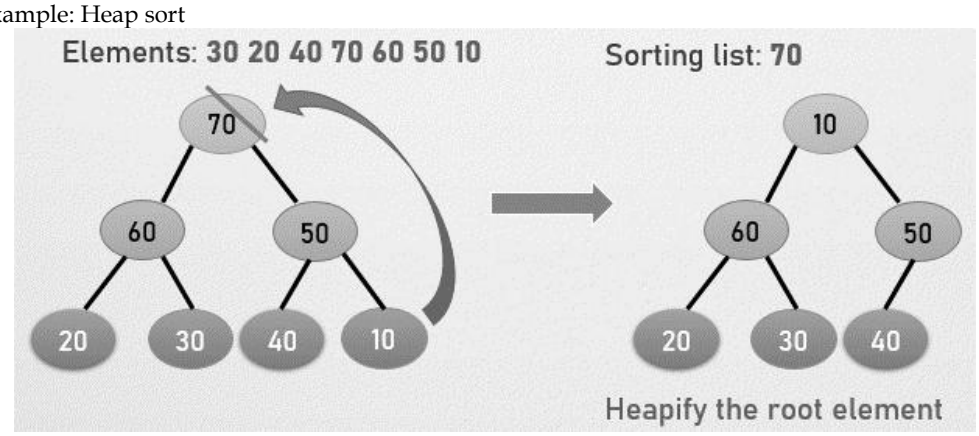
swap arr[i] with arr[largest]

MaxHeapify(arr,largest)

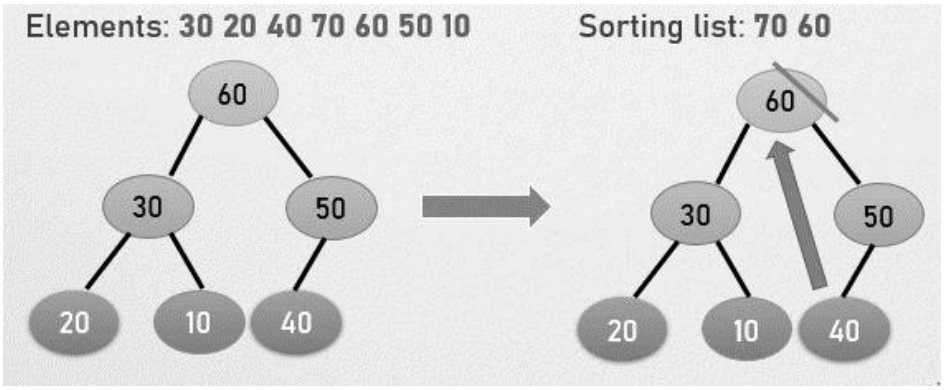
End

Heap sort first converts the initial array into a heap. The heapsort algorithm uses 'heapify' method to complete the task. The heapify algorithm, as given in the above code, receives a binary tree as input and converts it to a heap. Then, the root is compared with its two children, and the larger child is swapped with it. This may result in one of the left or right sub-trees losing the heap property. As a result, the heapify algorithm is recursively applied to the suitable sub-tree rooted at the node whose value was swapped with the root. This process continues until a leaf node is reached, or until the heap property is satisfied in the sub-tree.

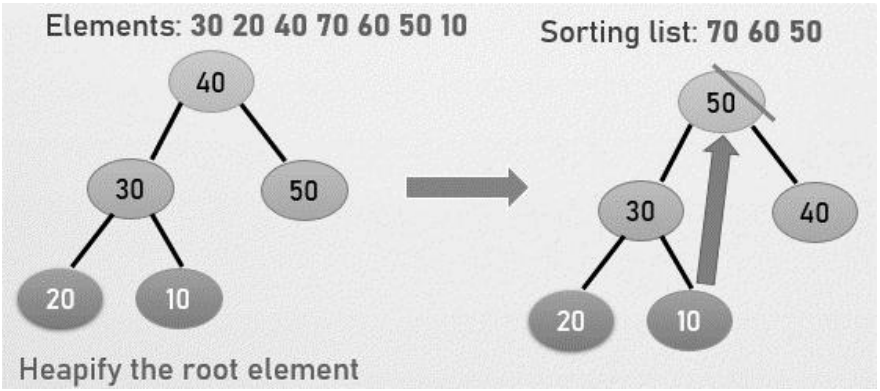
Example: Heap sort



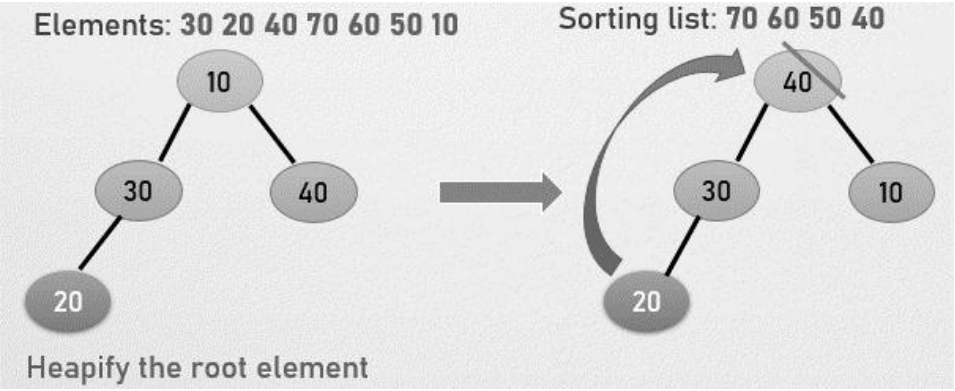
Step 1



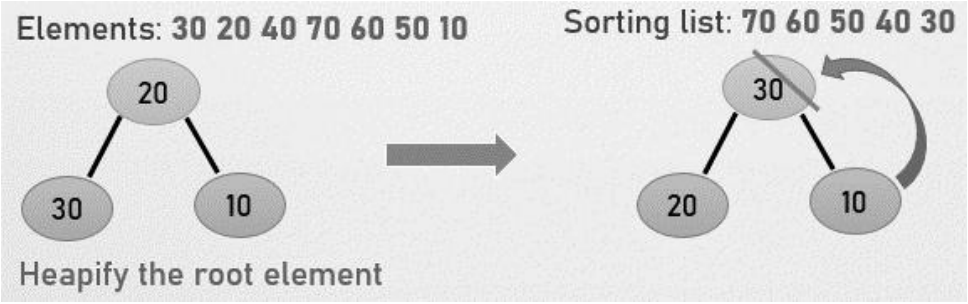
Step 2



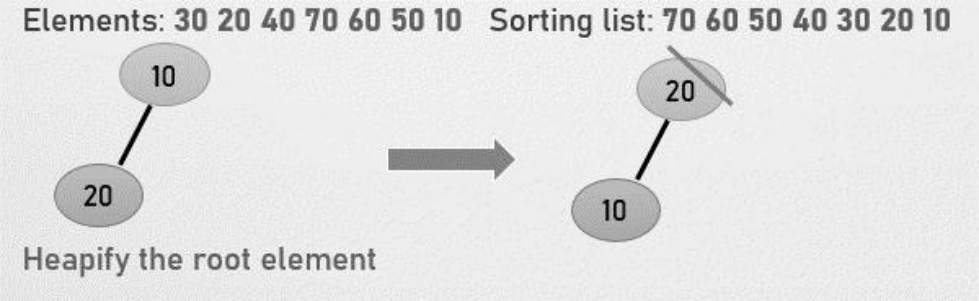
Step 3



Step 4



Step 5



Step 6

Lab Exercise: Heap sort implementation

```
#include <iostream>
using namespace std;
void heapify(int a[], int n, int i)
{
   int largest = i; // Initialize largest as root
   int left = 2 * i + 1; // left child
   int right = 2 * i + 2; // right child
   // If left child is larger than root
   if (left < n && a[left] > a[largest])
      largest = left;
```

```
   // If right child is larger than root
   if (right < n && a[right] > a[largest])
      largest = right;
   // If root is not largest
   if (largest != i) {
      // swap a[i] with a[largest]
      int temp = a[i];
      a[i] = a[largest];
      a[largest] = temp;

      heapify(a, n, largest);
   }
}
/*Function to implement the heap sort*/
void heapSort(int a[], int n)
{

   for (int i = n / 2 - 1; i >= 0; i--)
      heapify(a, n, i);
   // One by one extract an element from heap
   for (int i = n - 1; i >= 0; i--) {
      /* Move current root element to end*/
      // swap a[0] with a[i]
      int temp = a[0];
      a[0] = a[i];
      a[i] = temp;

      heapify(a, i, 0);
   }
}
/* function to print the array elements */
void printArr(int a[], int n)
{
   for (int i = 0; i < n; ++i)
   {
      cout<<a[i]<<" ";
   }

}
int main()
{
```

```
    int a[] = {47, 9, 22, 42, 27, 25, 0};
    int n = sizeof(a) / sizeof(a[0]);
    cout<<"Before sorting array elements are - \n";
    printArr(a, n);
    heapSort(a, n);
    cout<<"\nAfter sorting array elements are - \n";
    printArr(a, n);
    return 0;
}
```

9.2 Complexity of the Heap Sort

Worst Case: O(n log n)

Best Case: O(n log n)

Average Case: O(n log n)

9.3 Heap Sort Applications

Systems concerned with security and embedded systems such as Linux Kernel use Heap Sort because of the O(n log n) upper bound on Heapsort's running time and constant O(1) upper bound on its auxiliary storage.

Although Heap Sort has O(n log n) time complexity even for the worst case, it doesn't have more applications (compared to other sorting algorithms like Quick Sort, Merge Sort). However, its underlying data structure, heap, can be efficiently used if we want to extract the smallest (or largest) from the list of items without the overhead of keeping the remaining items in the sorted order. For e.g Priority Queues.

K sorted array

K largest or smallest elements in an array

9.4 Advantages of HeapSort

Efficiency: As the number of objects to sort grows, the time required to conduct Heap sort grows logarithmically, whereas alternative methods may grow exponentially slower. This is a very efficient sorting algorithm.

Memory usage: Memory usage is modest because it requires no additional memory space to work other than what is required to keep the initial list of objects to be sorted.

Simplicity: Because it does not involve difficult computer science concepts like recursion, it is easier to understand than other equally efficient sorting algorithms.

9.5 Binomial Heap

A binomial Heap is a collection of Binomial Trees that satisfies the heap properties, i.e., min heap. It supports quicker merging of two heaps in O(log n).A min heap is a heap in which each node has a value lesser than the value of its child nodes.

A Binomial tree is a tree in which B_k is an ordered tree defined recursively, where k is defined as the order of the binomial tree.The binomial tree B0 consists of a single node. The binomial tree B_k consists of two binomial trees Bk-1 that are linked together, the root of one is the leftmost child of the root of the other.

If the binomial tree is represented as B0 then the tree consists of a single node.In general terms, B_k consists of two binomial trees, i.e., Bk-1 and Bk-1 are linked together in which one tree becomes the left sub tree of another binomial tree.

Binomial Tree B0

If B0, k= 0, there would be only one node in the tree

B0

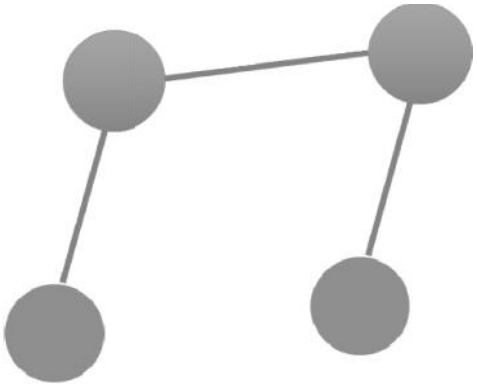
Binomial Tree B1

If B1, k= 1, means k-1 equal to 0. Therefore, there would be two binomial trees of B0 in which one B0 becomes the left sub tree of another B0.

B1

Binomial Tree B2

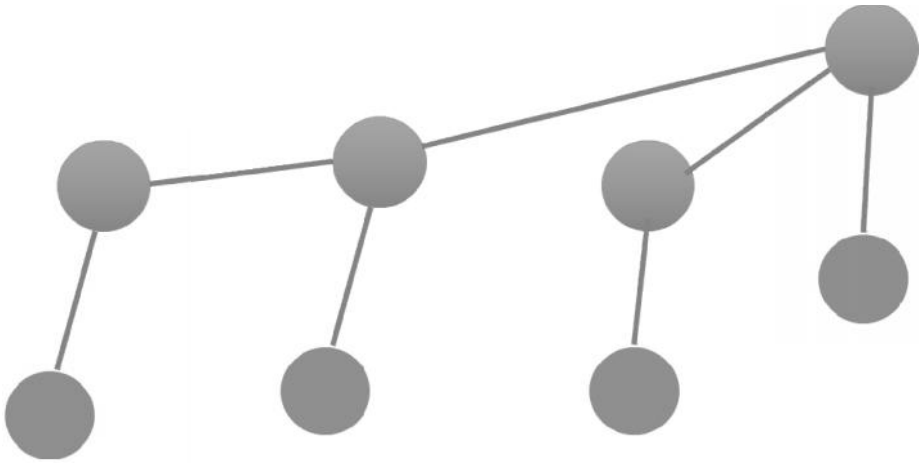
If B2, k= 2, means k-1 equal to 1. Therefore, there would be two binomial trees of B1 in which one B1 becomes the left sub tree of another B1.



B2

Binomial Tree B3

If B3 , k= 3, means k-1 equal to 2. Therefore, there would be two binomial trees of B2 in which one B2 becomes the left sub tree of another B2.



9.6 Operations of Binomial Heap

- Union of two binomial heap
- Finding the minimum key
- Creating a new binomial heap
- Inserting a node
- Extracting minimum key
- Decreasing a key
- Deleting a node

Union of two binomial heap

Merging in a heap can be done by comparing the keys at the roots of two trees, and the root node with the larger key will become the child of the root with a smaller key than the other. The time complexity for finding a union is O (logn).

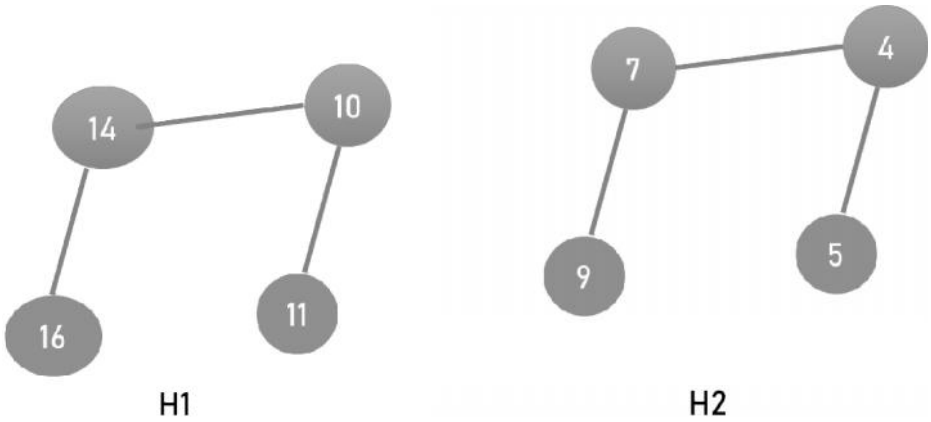
Case 1: If degree[x] is not equal to degree[next x] then move pointer ahead.

Case 2: if degree[x] = degree[next x] = degree[sibling(next x)] then

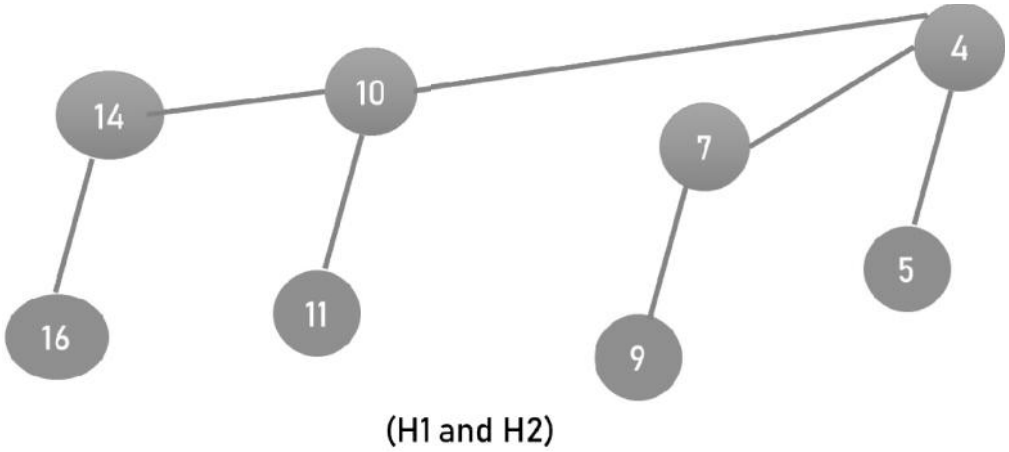
Move pointer ahead.

Case 3: If degree[x] = degree[next x] but not equal to degree[sibling[next x]] and key[x] < key[next x] then remove [next x] from root and attached to x.

Case 4: If degree[x] = degree[next x] but not equal to degree[sibling[next x]] and key[x] > key[next x] then remove x from root and attached to [next x].



Comparison of root keys of H1 and H2



Find minimum

To find the minimum element of the heap, find the minimum among the roots of the binomial trees.It requires O(Logn) time. It can be optimized to O(1) by maintaining a pointer to minimum key root.

Decrease Key

We compare the decreases key with it parent and if parent's key is more, we swap keys and recur for the parent. Swap process stop when we either reach a node whose parent has a smaller key or we hit the root node. Time complexity of decrease Key() is O(Logn).

Extract minimum key

First find this element, remove it from its binomial tree, and obtain a list of its sub trees. Transform this list of sub trees into a separate binomial heap by reordering them from smallest to largest order. Then merge this heap with the original heap.This operation requires O(Logn) time.

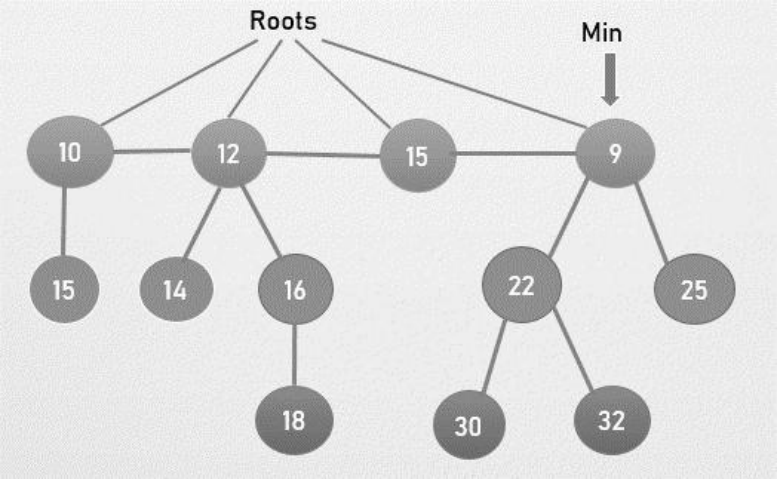
9.7 Fibonacci Heap

A Fibonacci heap is a circular doubly linked list, with a pointer to the minimum key, but the elements of the list are not single keys. Instead, we collect keys together into structures called binomial heaps. Binomial heaps are trees that satisfy the heap property — every node has a smaller key than its children.

Fibonacci heap data structure is collection of trees which follow min heap or max heap property. In a Fibonacci heap, a node can have more than two children or no children at all.

Properties of a Fibonacci Heap

- A pointer is maintained at the minimum element node.
- The trees within a Fibonacci heap are unordered but rooted.
- It is a set of min heap-ordered trees.
- It consists of a set of marked nodes.



The child nodes of a parent node are connected to each other through a circular doubly linked list.Deleting a node from the tree takes O(1) time.The concatenation of two such lists takes O(1) time.

Fibonacci heaps have a faster amortized running time than other heap types. Fibonacci heaps have a less rigid structure as compared to binomial heaps.Fibonacci heaps are used to implement the priority queue element in Dijkstra's algorithm.The reduced time complexity of Decrease-Key has importance in Dijkstra and Prim algorithms. With Binary Heap, time complexity of these algorithms is O(VLogV + ELogV). If Fibonacci Heap is used, then time complexity is improved to O(VLogV + E).

9.8 Operations on a Fibonacci Heap

- Insertion
- Find Min
- Union
- Extract Min
- Decrease a key
- Delete a node

Insertion

- Create a new node for the element.
- Check if the heap is empty.
- If the heap is empty, set the new node as a root node and mark it min.
- Else, insert the node into the root list and update min.

Algorithm: Insertion

```
insert(H, x)
  degree[x] = 0
```

p[x] = NIL

child[x] = NIL

left[x] = x

right[x] = x

mark[x] = FALSE

concatenate the root list containing x with root list H

if min[H] == NIL or key[x] < key[min[H]]

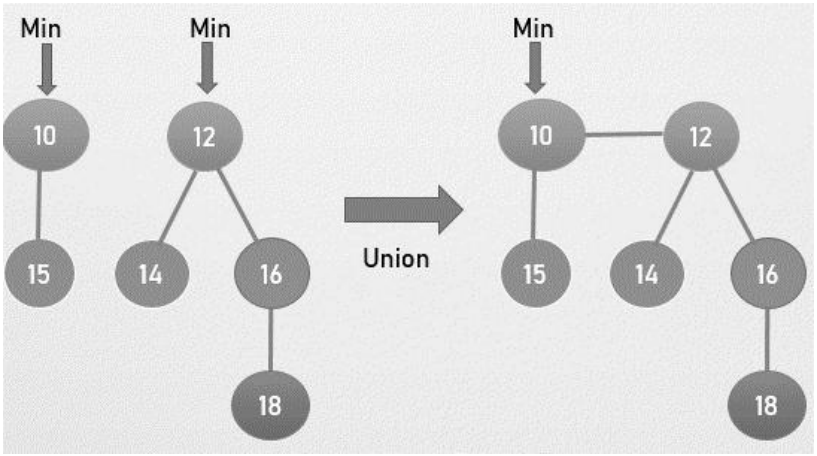
then min[H] = x

n[H] = n[H] + 1

Union

Steps for Union of two Fibonacci heaps.

- Concatenate the roots of both the heaps.
- Update min by selecting a minimum key from the new root lists.



Extract Min

In extract min minimum value is removed from the heap and the tree is re-adjusted.

Steps for Extract Min

- Delete the min node.
- Set the min-pointer to the next root in the root list.
- Create an array of size equal to the maximum degree of the trees in the heap before deletion.
- Do the following (steps 5-7) until there are no multiple roots with the same degree.
- Map the degree of current root (min-pointer) to the degree in the array.
- Map the degree of next root to the degree in array.
- If there are more than two mappings for the same degree, then apply union operation to those roots such that the min-heap property is maintained (i.e. the minimum is at the root).

Decrease Key

In decreasing a key operation, the value of a key is decreased to a lower value.

Decrease the value of the node 'x' to the new chosen value.

CASE 1 - If min heap order not violated,

Update min pointer if necessary.

CASE 2 - If min heap order violated and parent of 'x' is unmarked,

Cut off the link between 'x' and its parent.

Mark the parent of 'x'.

Add tree rooted at 'x' to the root list and update min pointer if necessary.

CASE 3 - If min heap order is violated and parent of 'x' is marked,

Cut off the link between 'x' and its parent p[x].

Add 'x' to the root list, updating min pointer if necessary.

Cut off link between p[x] and p[p[x]].

Add p[x] to the root list, updating min pointer if necessary.

If p[p[x]] is unmarked, mark it.

Else, cut off p[p[x]] and repeat steps 4.2 to 4.5, taking p[p[x]] as 'x'.

Deleting a Node

This process makes use of decrease-key and extract-min operations. The following steps are followed for deleting a node.

Let k be the node to be deleted.

Apply decrease-key operation to decrease the value of k to the lowest possible value (i.e. -∞).

Apply extract-min operation to remove this node.

 **Lab Exercise**: Operations on a Fibonacci

```
#include <cmath>
#include <cstdlib>
#include <iostream>
using namespace std;
struct node {
 int n;
 int degree;
 node *parent;
 node *child;
 node *left;
 node *right;
 char mark;
 char C;
};
class FibonacciHeap {
  private:
 int nH;

 node *H;
  public:
 node *InitializeHeap();
 int Fibonnaci_link(node *, node *, node *);
```

```
  node *Create_node(int);
  node *Insert(node *, node *);
  node *Union(node *, node *);
  node *Extract_Min(node *);
  int Consolidate(node *);
  int Display(node *);
  node *Find(node *, int);
  int Decrease_key(node *, int, int);
  int Delete_key(node *, int);
  int Cut(node *, node *, node *);
  int Cascase_cut(node *, node *);
  FibonacciHeap() { H = InitializeHeap(); }
};
// Initialize heap
node *FibonacciHeap::InitializeHeap() {
  node *np;
  np = NULL;
  return np;
}
// Create node
node *FibonacciHeap::Create_node(int value) {
  node *x = new node;
  x->n = value;
  return x;
}
// Insert node
node *FibonacciHeap::Insert(node *H, node *x) {
  x->degree = 0;
  x->parent = NULL;
  x->child = NULL;
  x->left = x;
  x->right = x;
  x->mark = 'F';
  x->C = 'N';
  if (H != NULL) {
    (H->left)->right = x;
    x->right = H;
    x->left = H->left;
    H->left = x;
    if (x->n < H->n)
      H = x;
```

```
  } else {
    H = x;
  }
  nH = nH + 1;
  return H;
}
// Create linking
int FibonacciHeap::Fibonnaci_link(node *H1, node *y, node *z) {
  (y->left)->right = y->right;
  (y->right)->left = y->left;
  if (z->right == z)
    H1 = z;
  y->left = y;
  y->right = y;
  y->parent = z;
  if (z->child == NULL)
    z->child = y;
  y->right = z->child;
  y->left = (z->child)->left;
  ((z->child)->left)->right = y;
  (z->child)->left = y;
  if (y->n < (z->child)->n)
    z->child = y;
  z->degree++;
}
// Union Operation
node *FibonacciHeap::Union(node *H1, node *H2) {
  node *np;
  node *H = InitializeHeap();
  H = H1;
  (H->left)->right = H2;
  (H2->left)->right = H;
  np = H->left;
  H->left = H2->left;
  H2->left = np;
  return H;
}
// Display the heap
int FibonacciHeap::Display(node *H) {
  node *p = H;
  if (p == NULL) {
```

```
    cout << "Empty Heap" << endl;
    return 0;
  }
  cout << "Root Nodes: " << endl;
  do {
    cout << p->n;
    p = p->right;
    if (p != H) {
      cout << "-->";
    }
  } while (p != H && p->right != NULL);
  cout << endl;
}
// Extract min
node *FibonacciHeap::Extract_Min(node *H1) {
  node *p;
  node *ptr;
  node *z = H1;
  p = z;
  ptr = z;
  if (z == NULL)
    return z;
  node *x;
  node *np;
  x = NULL;
  if (z->child != NULL)
    x = z->child;
  if (x != NULL) {
    ptr = x;
    do {
      np = x->right;
      (H1->left)->right = x;
      x->right = H1;
      x->left = H1->left;
      H1->left = x;
      if (x->n < H1->n)
        H1 = x;
      x->parent = NULL;
      x = np;
    } while (np != ptr);
  }
```

```
  (z->left)->right = z->right;
  (z->right)->left = z->left;
  H1 = z->right;

  if (z == z->right && z->child == NULL)
    H = NULL;
  else {
    H1 = z->right;
    Consolidate(H1);
  }
  nH = nH - 1;
  return p;
}
// Consolidation Function
int FibonacciHeap::Consolidate(node *H1) {
  int d, i;
  float f = (log(nH)) / (log(2));
  int D = f;
  node *A[D];
  for (i = 0; i <= D; i++)
    A[i] = NULL;
  node *x = H1;
  node *y;
  node *np;
  node *pt = x;
  do {
    pt = pt->right;
    d = x->degree;
    while (A[d] != NULL)
    {
      y = A[d];

      if (x->n > y->n)
      {
        np = x;
        x = y;
        y = np;
      }
      if (y == H1)
        H1 = x;
      Fibonnaci_link(H1, y, x);
```

```
    if (x->right == x)
      H1 = x;
    A[d] = NULL;
    d = d + 1;
   }
   A[d] = x;
   x = x->right;
  }
  while (x != H1);
  H = NULL;
  for (int j = 0; j <= D; j++) {
   if (A[j] != NULL) {
    A[j]->left = A[j];
    A[j]->right = A[j];
    if (H != NULL) {
     (H->left)->right = A[j];
     A[j]->right = H;
     A[j]->left = H->left;
     H->left = A[j];
     if (A[j]->n < H->n)
       H = A[j];
    } else {
     H = A[j];
    }
    if (H == NULL)
     H = A[j];
    else if (A[j]->n < H->n)
     H = A[j];
   }
  }
}

// Decrease Key Operation
int FibonacciHeap::Decrease_key(node *H1, int x, int k) {
  node *y;
  if (H1 == NULL) {
   cout << "The Heap is Empty" << endl;
   return 0;
  }
  node *ptr = Find(H1, x);
  if (ptr == NULL) {
```

```
    cout << "Node not found in the Heap" << endl;
    return 1;
  }
  if (ptr->n < k) {
    cout << "Entered key greater than current key" << endl;
    return 0;
  }
  ptr->n = k;
  y = ptr->parent;
  if (y != NULL && ptr->n < y->n) {
    Cut(H1, ptr, y);
    Cascase_cut(H1, y);
  }
  if (ptr->n < H->n)
    H = ptr;
  return 0;
}
// Cutting Function
int FibonacciHeap::Cut(node *H1, node *x, node *y)
{
  if (x == x->right)
    y->child = NULL;
  (x->left)->right = x->right;
  (x->right)->left = x->left;
  if (x == y->child)
    y->child = x->right;
  y->degree = y->degree - 1;
  x->right = x;
  x->left = x;
  (H1->left)->right = x;
  x->right = H1;
  x->left = H1->left;
  H1->left = x;
  x->parent = NULL;
  x->mark = 'F';
}
// Cascade cut
int FibonacciHeap::Cascase_cut(node *H1, node *y) {
  node *z = y->parent;
  if (z != NULL) {
    if (y->mark == 'F') {
```

```
    y->mark = 'T';
   } else
   {
    Cut(H1, y, z);
    Cascase_cut(H1, z);
   }
  }
}
// Search function
node *FibonacciHeap::Find(node *H, int k) {
  node *x = H;
  x->C = 'Y';
  node *p = NULL;
  if (x->n == k) {
   p = x;
   x->C = 'N';
   return p;
  }
  if (p == NULL) {
   if (x->child != NULL)
    p = Find(x->child, k);
   if ((x->right)->C != 'Y')
    p = Find(x->right, k);
  }
  x->C = 'N';
  return p;
}

// Deleting key
int FibonacciHeap::Delete_key(node *H1, int k) {
  node *np = NULL;
  int t;
  t = Decrease_key(H1, k, -5000);
  if (!t)
   np = Extract_Min(H);
  if (np != NULL)
   cout << "Key Deleted" << endl;
  else
   cout << "Key not Deleted" << endl;
  return 0;
}
```

```
int main() {
 int n, m, l;
 FibonacciHeap fh;
 node *p;
 node *H;
 H = fh.InitializeHeap();
 p = fh.Create_node(7);
 H = fh.Insert(H, p);
 p = fh.Create_node(3);
 H = fh.Insert(H, p);
 p = fh.Create_node(17);
 H = fh.Insert(H, p);
 p = fh.Create_node(24);
 H = fh.Insert(H, p);
 fh.Display(H);
 p = fh.Extract_Min(H);
 if (p != NULL)
  cout << "The node with minimum key: " << p->n << endl;
 else
  cout << "Heap is empty" << endl;
 m = 26;
 l = 16;
 fh.Decrease_key(H, m, l);
 m = 16;
 fh.Delete_key(H, m);
}
```

Complexities

Insertion	O(1)
Find Min	O(1)
Union	O(1)
Extract Min	O(log n)
Decrease Key	O(1)
Delete Node	O(log n)

Summary

- Heap sort is sorting technique based upon Binary Heap data structure. It is comparison-based sorting technique.
- The elements of a heap sort are processed by generating a min-heap or max-heap with the items of the provided array.

- Advantages of heapsortare Efficiency, Memory usage and Simplicity
- A binomial Heap is a collection of Binomial Trees that satisfies the heap properties, i.e., min heap.
- A Binomial tree is a tree in which Bk is an ordered tree defined recursively, where k is defined as the order of the binomial tree.
- Fibonacci heap data structure is collection of trees which follow min heap or max heap property. In a Fibonacci heap, a node can have more than two children or no children at all.

Keywords

Max heap	Min heap
Binomial Heap	Heap sort
Extract Min	Decrease Key

Self Assessment

1. Heap sort is___

A. It is based upon Binary Heap data structure.
B. It is comparison-based sorting technique.
C. It processes the elements by creating the min heap or max heap using the elements of the array.
D. All of above

2. Elements arranged in descending order __

A. Max heap
B. Min heap
C. Both Max and Min heap
D. None of above

3. Heapify is part of ___

A. Max heap
B. Min heap
C. Both Max and Min heap
D. None of above

4. Elements arranged in ascending order_

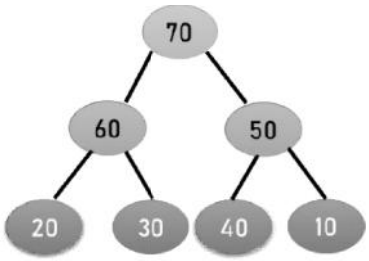
A. Max heap
B. Min heap
C. Both Max and Min heap
D. None of above

5. Complexity of the Heap Sort in worst case is___

A. (log 1)
B. (log n)

C. (n log n)
D. None of above

6. Given graph is example of ____



A. Min heap
B. Max heap
C. Both max and min heap
D. None of above

7. Binomial Heap is a collection of ___

A. Binary trees.
B. AVL trees.
C. Binomial trees.
D. None of above

8. In binomial tree B1 what are the numbers of nodes.

A. 0
B. 1
C. 2
D. 3

9. Operations of Binomial Heap __

A. Finding the minimum key
B. Creating a new binomial heap
C. Inserting a node
D. All of above

10. What is value of K in binomial tree B3?

A. 1
B. 2
C. 3
D. None of above

11. Properties of a Fibonacci Heap are___

A. It is a set of min heap-ordered trees.
B. It consists of a set of marked nodes.

C. The trees within a Fibonacci heap are unordered but rooted.
D. All of above

12. Which is not Fibonacci Heap operation.

A. Union
B. Extract Min
C. Peek
D. Decrease a key

13. A pointer is maintained in Fibonacci Heap at the _____ element node

A. Maximum
B. Minimum
C. Both minimum and maximum
D. None of above

14. The child nodes of a parent node are connected to each other through_____

A. Doubly linked list
B. Singly linked list
C. Circular doubly linked list
D. None of above

15. Deleting a node from the tree in Fibonacci Heap takes_____time.

A. (1)
B. (0)
C. (log n)
D. None of above

Answers for Self Assessment

1.	D	2.	A	3.	C	4.	B	5.	C
6.	B	7.	C	8.	C	9.	D	10.	C
11.	D	12.	D	13.	B	14.	C	15.	C

Review Questions

1. What are the steps for heap sort operation?
2. Write algorithm for heap sort.
3. Explain complexity of heap sort.
4. Define binomial Heap with suitable example.
5. Discuss different operations of binomial heap
6. Describe insert and union operations in Fibonacci Heap.
7. Explain different cases of Decrease Key.

Further Readings

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http://staff.ustc.edu.cn/~csli/graduate/algorithms/book6/chap20.htm

http://www.cs.toronto.edu/~anikolov/CSC265F18/binomial-heaps.pdf

Unit 10: Graphs

CONTENTS

Objectives

After studying this unit, you will be able to:

- Understand basics of graphs
- Learn basic graph terminology
- Discuss adjacency matrix and linked adjacency chains
- learnspanning trees

Introduction

In this unit, we introduce you to an important mathematical structure called Graph. Graphs have found applications in subjects as diverse as Sociology, Chemistry, Geography and Engineering Sciences. They are also widely used in solving games and puzzles. In computer science, graphs are used in many areas one of which is computer design. In day-to-day applications, graphs find their importance as representations of many kinds of physical structure.

We use graphs as models of practical situations involving routes: the vertices represent the cities and edges represent the roads or some other links, specially in transportation management, Assignment problems and many more optimization problems. Electric circuits are another obvious example where interconnections between objects play a central role. Circuit's elements like transistors, resistors, and capacitors are intricately wired together. Such circuits can be represented and processed within a computer in order to answer simple questions like "Is everything connected together?" as well as complicated questions like "If this circuit is built, will it work?"

10.1 Graphs

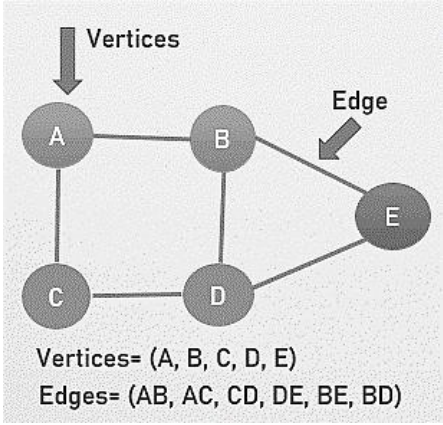
A Graph G consists of a set V of vertice (nodes) and a set E of edges (arcs). We write G=(V,E). V

is a fi nite and non empty set of vertices. E is a set of pairs of vertices; these pairs are called edges.

Therefore

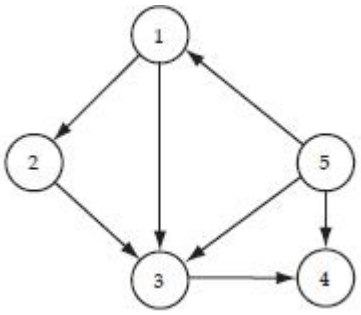
V(G), read as V of G, is set of vertices,and E(G), read as E of G, is set of edges.

An edge e = (v,w), is a pair of vertices v and w, and is said to be incident with v and w.It is a pictorial representation of a set of objects where objects are connected by links.A graph may be pictorically represented as given in Figure



In an undirected graph, pair of vertices representing any edge is unordered. Thus (v,w) and (w,v) represent the same edge. In a directed graph each edge is an ordered pair of vertices, i.e. each edge is represented by a directed pair. If e = (v,w), then v is tail or initial vertex and w is head or fi nal vertex. Subsequently (v,w) and (w,v) represent two different edges.

A directed graph may be pictorically represented as given in Figure



Directed graph

The direction is indicated by an arrow. The set of vertices for this graph remains the same as that

of the graph in the earlier example, i.e.

V(G) = {1,2,3,4,5}

However the set of edges would be

E(G) = {(1,2), (2,3), (3,4), (5,4), (5,1), (1,3), (5,3)}

10.2 Graph Terminology

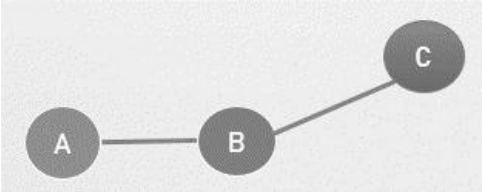
A good deal of nomenclature is associated with graphs. Most of the terms have straight forward definitions, and it is convenient to put them in one place even though we would not be using some of them until later.

- Vertices
- Edges
- Path
- Closed path
- Degree of the Node
- Adjacent Nodes/ Adjacency

Vertices: Each node of the graph is represented as a vertex.

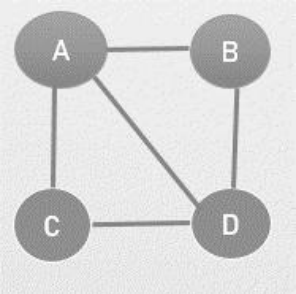
Edge: it is used to represent the relationships between various nodes in a graph. An edge between two nodes expresses a one-way or two-way relationship between the nodes.

Path: Path represents a sequence of edges between the two vertices. E.g. ABC



Closed Path: A path will be called as closed path if the initial node is same as terminal node.

Degree of the Node: A degree of a node is the number of edges that are connected with that node. Degree of A=3.



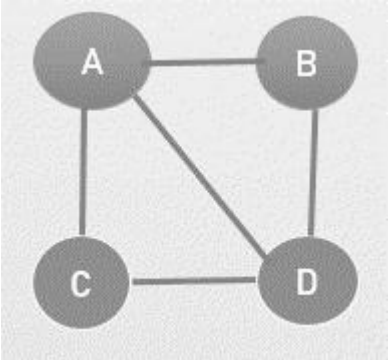
Adjacent Nodes/ Adjacency: if two nodes are connected to each other through an edge are called as neighbors or adjacent nodes.

10.3 Types of Graphs

- Undirected Graph
- Directed Graph
- Weighted Graph
- Un-weighted Graph
- Complete Graph
- Finite Graph
- Trivial Graph
- Multi Graph
- Pseudo Graph
- Connected Graph
- Labeled Graphs
- Disconnected Graph

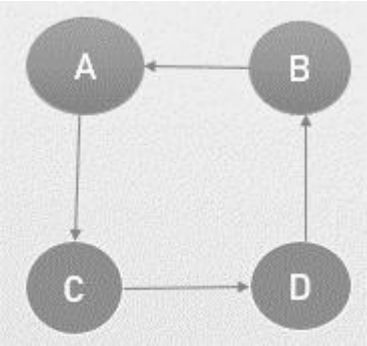
Undirected graph

An undirected graph nodes are connected and all the edges are bi-directional i.e. the edges do not point in any specific direction.



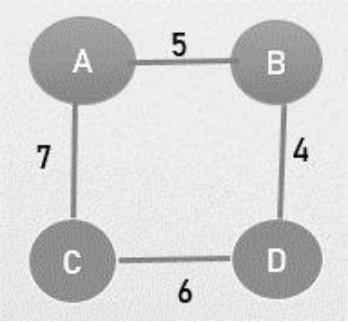
Directed graph

A directed graph is a graph in which all the edges are uni-directional i.e. the edges point in a single direction. It is also called a digraph.



Weightedgraph

In weighted graph edges or path have values or cost. All the values seen associated with the edges are called weights.

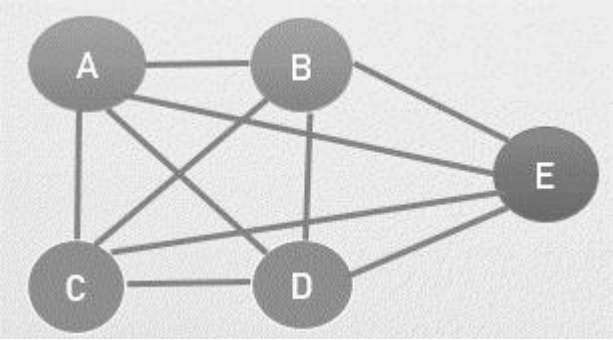


Un-weighted graph

In un-weighted graph there is no value or weight associated with the edge. By default, all the graphs are un-weighted.

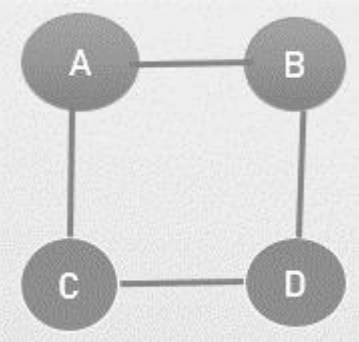
Complete graph

A complete graph is the one in which every node is connected with all other nodes. A complete graph contain n(n-1)/2 edges where n is the number of nodes in the graph.



Finite graph

The graph G=(V, E) is called a finite graph if the number of vertices and edges in the graph is limited in number

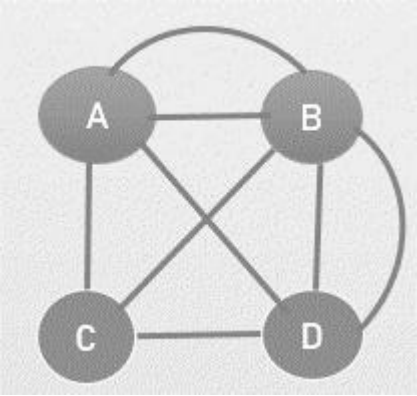


Trivial Graph

A graph G= (V, E) is trivial if it contains only a single vertex and no edges.

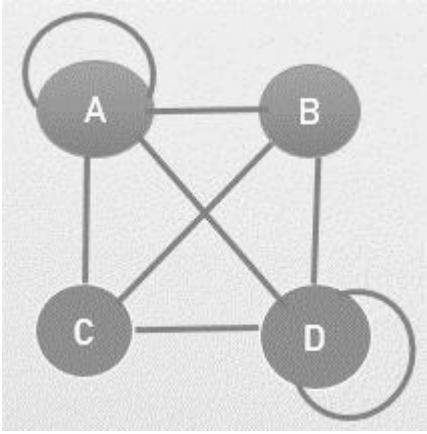
Multi Graph

If there are numerous edges between a pair of vertices in a graph G= (V, E), the graph is referred to as a multi graph. There are no self-loops in a Multi graph.



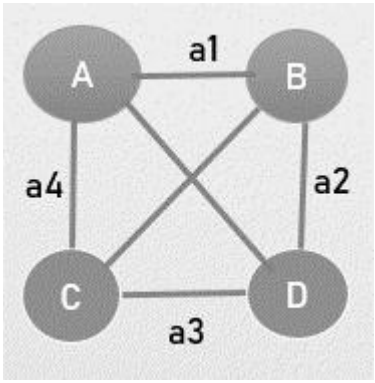
Pseudo graph

If a graph G= (V, E) contains a self-loop besides other edges, it is a pseudo graph.



Labeled graph

A graph G=(V, E) is called a labeled graph if its edges are labeled with some name or data.



10.4 Representations of Graphs

Graph is a mathematical structure and fi nds its application in many areas of interest in which problems need to be solved using computers. Thus, this mathematical structure must be represented as some kind of data structures. Two such representations are, commonly used.

There are various ways to represent a graph. A simple representation is given by an adjacency list, which specifies all vertices adjacent to each vertex of the graph. This list can be implemented as a table, in which case it is called a star representation, which can be forward or reverse.

Another representation is a matrix, which comes in two forms: an adjacency matrix and an incidence matrix. An adjacency matrix of graph G = (V,E) is a binary $|V| \times |V|$ matrix such that each entry of this matrix.

These are:

1. Adjacent Matrix

2. Adjacency List representation.

The choice of representation depends on the application and function to be performed on the graph.

Adjacent Matrix

Two vertices is called adjacent or neighbor if it support at least one common edge.A finite graph can be represented in the form of a square matrix.Boolean value (0,1) of the matrix indicates if there is a direct path between two vertices.

It is also called 2D matrix that is used to map the association between the graph nodes.If a graph has n number of vertices, then the adjacency matrix of that graph is n x n, and each entry of the matrix represents the number of edges from one vertex to another.

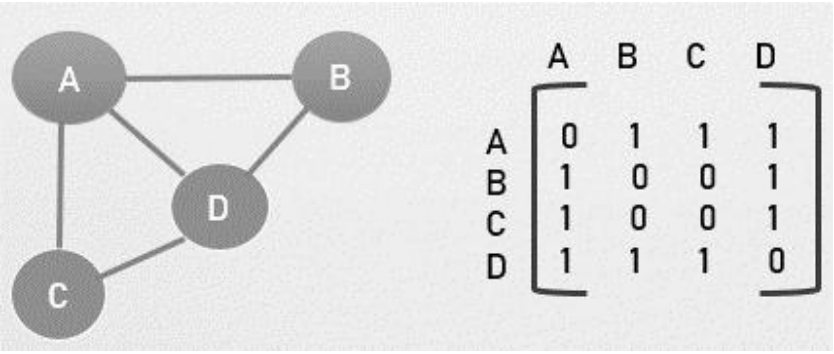
Adjacency Matrix Representation

The adjacency matrix A for a graph G = (V,E) with n vertices, is an n × n matrix of bits, such that A

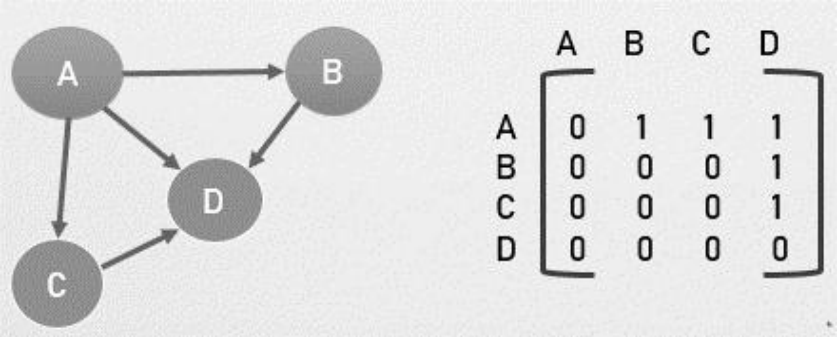
A_ij = 1, iff there is an edge from vi to vj and

A_{ij} = 0, if there is no such edge.

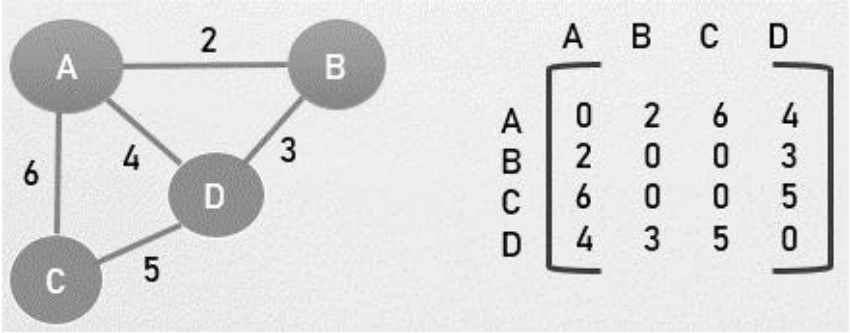
Undirected Graph Representation



Directed Graph Representation



Undirected Weighted Graph

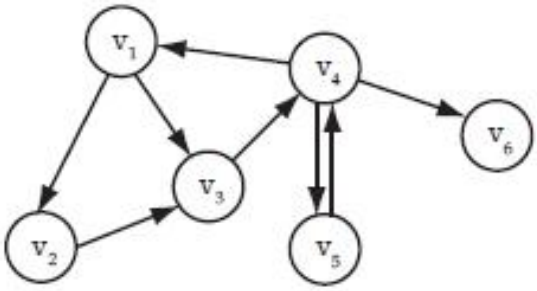


Applications: Adjacency Matrix

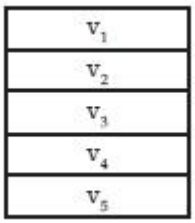
- Navigation tasks
- It is used to represent finite graphs
- Creating routing table in networks

Adjacency List Representation

In this representation, we store a graph as a linked structure. We store all the vertices in a list and then for each vertex, we have a linked list of its adjacent vertices.

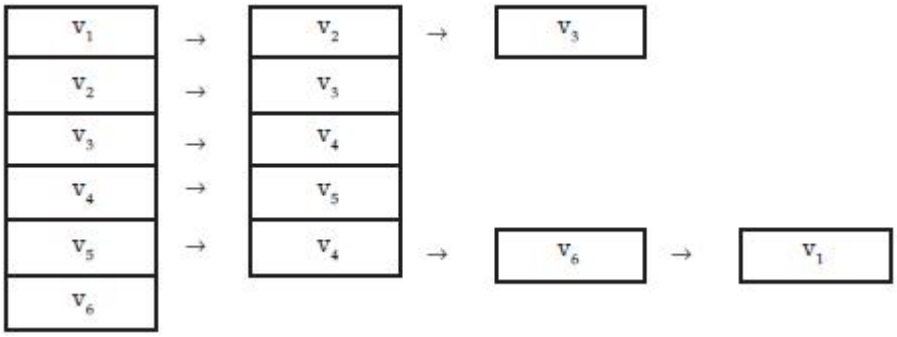


The adjacency list representation needs a list of all of its nodes, i.e.



And for each node a linked list of its adjacent nodes.

Therefore we shall have



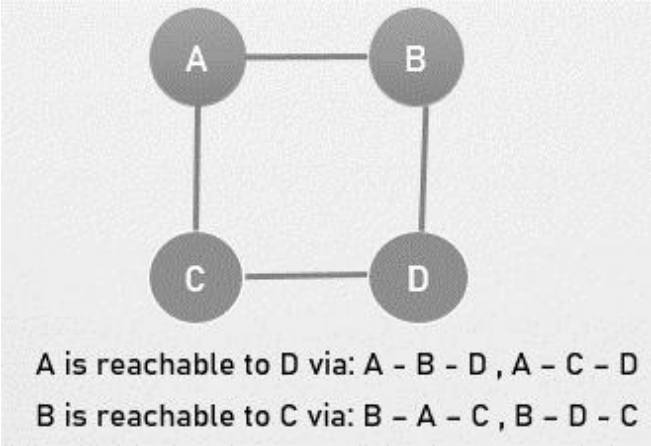
Adjacency List Structure for Graph

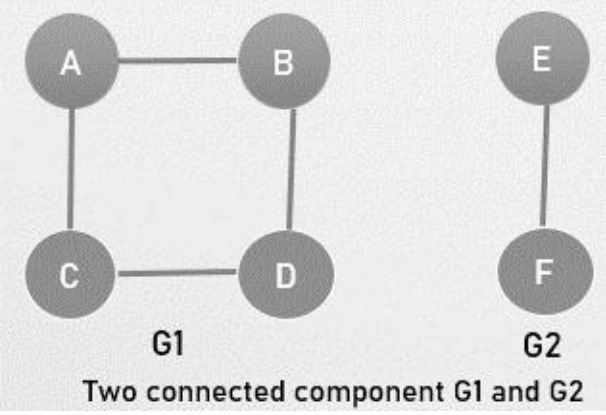
The adjacency list representation is better for sparse graphs because the space required is O(V + E), as contrasted with the O(V2) required by the adjacency matrix representation.

10.5 Connected Components

Component of an undirected graph is a sub graph in which each pair of nodes is connected with each other via a path.Every vertex of the graph lies in a connected component that consists of all the vertices that can be reached from that vertex, together with all the edges that join those vertices.

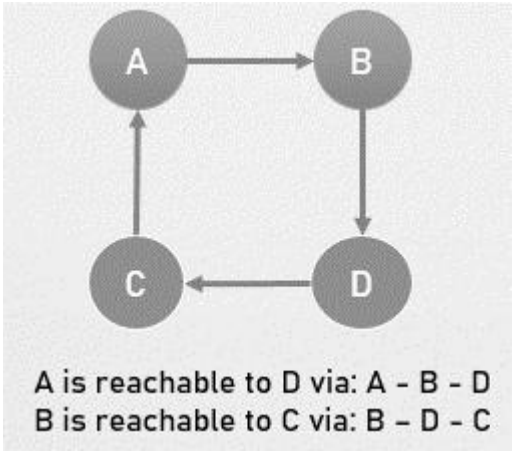
We can use traversal algorithm, depth-first or breadth-first, to find the connected components of an undirected graph.

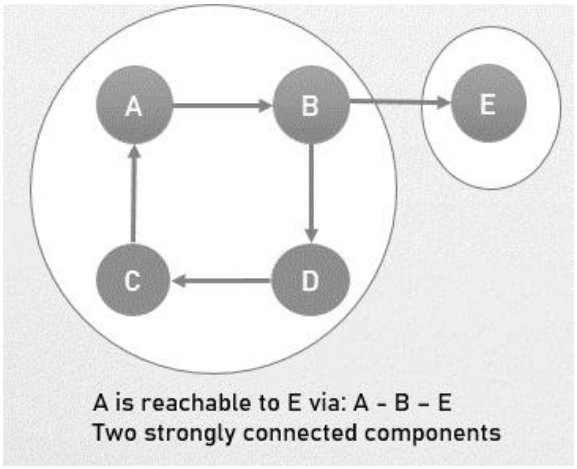




Strongly Connected Component

For a directed graph, a strongly connected component has a directed path between any two nodes.





10.6 Spanning Tree

A spanning tree is a tree that connects all the vertices of a graph with the minimum possible number of edges. Thus, a spanning tree is always connected. Also, a spanning tree never contains a cycle. A spanning tree is always defined for a graph and it is always a subset of that graph. Thus, a disconnected graph can never have a spanning tree.

A spanning tree is a sub-graph of an undirected connected graph, which has all the vertices covered with minimum possible number of edges. If a vertex is missed, then it is not a spanning tree.A spanning tree does not have cycles and it cannot be disconnected.

Every undirected and connected graph has a minimum of one spanning tree. Consider a graph having V vertices and E number of edges. Then, we will represent the graph as G(V, E). Its spanning tree will be represented as G′(V, E′) where E′ ⊆ E and the number of vertices remain the same. So, a spanning tree G′ is a subgraph of G whose vertex set is the same but edges may be different.

Spanning Trees Terminologies

Undirected graph: An undirected graph is a graph in which all the edges do not point to any particular direction.

Connected graph: A connected graph is a graph in which a path always exists from a vertex to any other vertex. A graph is connected if we can reach any vertex from any other vertex by following edges in either direction.

Directed graph: A directed graph is defined as a graph in which set of V vertices and set of Edges, each connecting two different vertices, but it is not mandatory that node points in the opposite direction also.

Properties of Spanning Tree

- An undirected connected graph can have more than one spanning tree.
- All the possible spanning trees of a graph have the same number of edges and vertices.
- The spanning tree does not have any cycle / loops.
- Any connected and undirected graph will always have at least one spanning tree.
- Spanning tree is always minimally connected. Removing one edge from the spanning tree will make the graph disconnected
- A spanning tree is maximally acyclic. Adding one edge to the spanning tree will create a cycle or loop.

Mathematical Properties of Spanning Tree

- Spanning tree has n-1 edges, where n is the number of nodes (vertices).
- A complete graph can have maximum nn-2 number of spanning trees.
- From a complete graph, by removing maximum e - n + 1 edges, we can construct a spanning tree.

Example:If we have n = 4, the maximum number of possible spanning trees is equal to 4^{4-2} = 16. Thus, 16 spanning trees can be formed from a complete graph with 4 vertices.

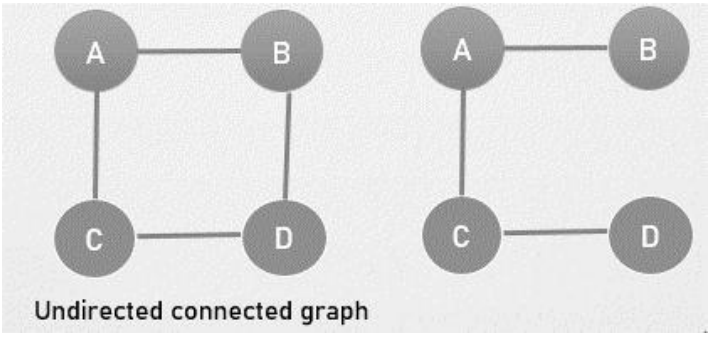


Fig. a

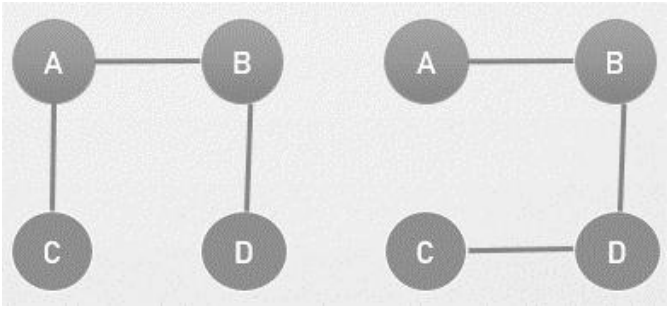


Fig. b

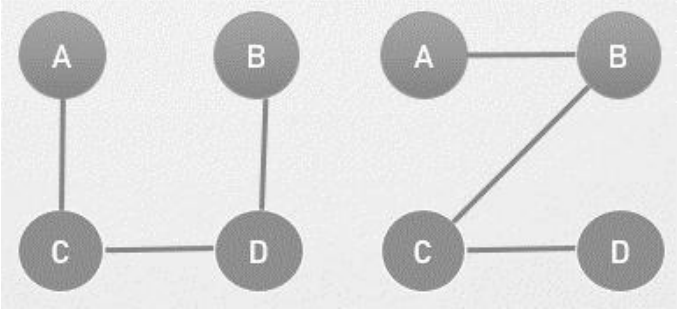


Fig. c

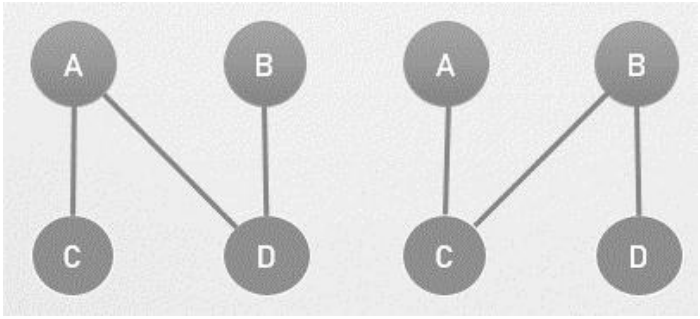


Fig. d

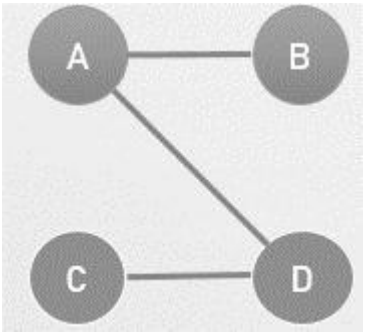


Fig. e

Application of Spanning Tree

It is used to find a minimum path to connect all nodes in a graph

Computer Network Routing Protocol

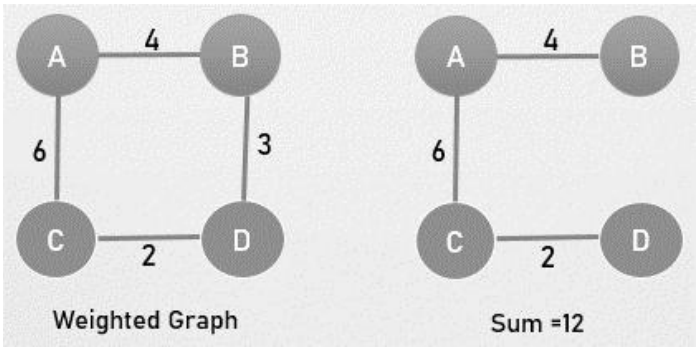
Cluster Analysis

Minimum Spanning Tree

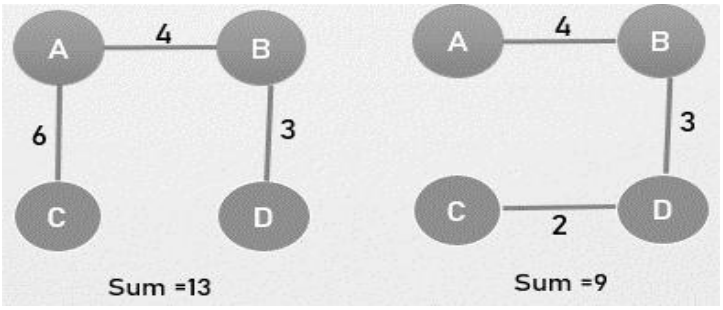
A minimum spanning tree is defined for a weighted graph. A spanning tree having minimum weight is defined as a minimum spanning tree. This weight depends on the weight of the edges. In real-world applications, the weight could be the distance between two points, cost associated with the edges or simply an arbitrary value associated with the edges.

A minimum spanning tree is a spanning tree in which the sum of the weight of the edges is as minimum as possible.

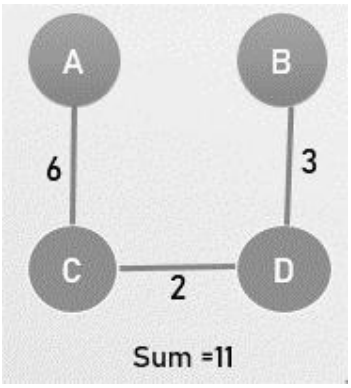
Example: Minimum Spanning Tree



A



B C



D

Minimum spanning tree = C, sum =9.

Summary

- A Graph G consists of a set V of vertice (nodes) and a set E of edges (arcs)
- An undirected graph nodes are connected and all the edges are bi-directional i.e. the edges do not point in any specific direction.
- If there are numerous edges between a pair of vertices in a graph G= (V, E), the graph is referred to as a multi graph. There are no self-loops in a Multi graph.
- Two vertices is called adjacent or neighbor if it support at least one common edge. A finite graph can be represented in the form of a square matrix.
- Every undirected and connected graph has a minimum of one spanning tree.

Keywords

Vertices	Edges
Path	Closed path
Degree of the Node	Spanning tree

Self Assessment

1. Graph is collection of ___

A. Vertices
B. Edges
C. Both vertices and edges

D. None of above

2. Which is not part of graph.

A. Path
B. Extract Min
C. Edge
D. Closed path

3. Types of Graph are____

A. Pseudo
B. Trivial
C. Disconnected
D. All of above

4. A complete graph is _____

A. Connected with all other nodes
B. Connected with bidirectional nodes
C. Connected with directional nodes
D. None of above

5. Graphs are commonly represent using ___

A. Adjacency Matrix
B. Adjacency List
C. Both Adjacency Matrix and Adjacency List
D. None of above

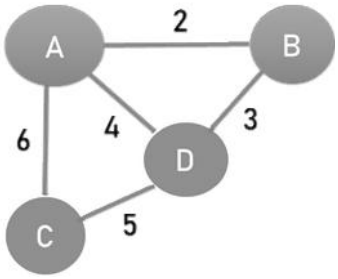
6. Two vertices is called adjacent.

A. If it there is no common edge
B. If it support at least two common edge
C. If it support at least one common edge
D. None of above

7. Applications of adjacency matrix are____

A. Navigation tasks
B. It is used to represent finite graphs
C. Creating routing table in networks
D. All of above

8. Image represent a _____



A. Undirected Weighted Graph
B. Directed Graph Representation
C. Undirected Graph Representation
D. None of above

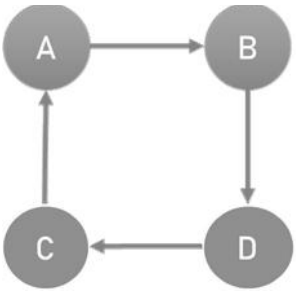
9. To find the connected components of an undirected graph___

A. Depth-first search algorithm
B. Dijkstra's Algorithm
C. Centrality Algorithms
D. None of above

10. Strongly connected components are____

A. Undirected path between any two nodes
B. Directed path between any two nodes
C. It support at least two common edge
D. None of above

11. Graph represents ____



A. Undirected Connected Component
B. Bidirectional Connected Component
C. Strongly Connected Component
D. All of above

12. Which is not type of graph ___

A. Connected
B. Directed
C. Centrality
D. Bidirectional

13. Spanning tree is ____

A. Have more than one spanning tree
B. Have the same number of edges and vertices
C. Does not have any cycle / loops
D. All of above

14. Mathematical properties of spanning tree____

A. Has n-1 edges, n = number of nodes
B. Complete graph can have maximum nn-2 number of spanning trees
C. All of above

15. Minimum Spanning Tree is _____

A. The sum of the weight of the edges is as minimum as possible
B. The sum of the weight of the edges is as maximum as possible
C. The sum of the weight of the edges is as average as possible
D. None of above

Answers for Self Assessment

1.	C	2.	B	3.	D	4.	A	5.	C
6.	C	7.	D	8.	A	9.	A	10.	B
11.	C	12.	C	13.	D	14.	D	15.	A

Review Questions

1. Define graph and its different types.
2. Discuss edge and vertices with example.
3. How to find degree of node?
4. Differentiate between directed and weighted graph with example.
5. Give an example of Adjacency List representation.
6. How spanning tree is different from minimum spanning tree?
7. What are the applications of spanning tree?

Further Readings

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Unit 11: More on Graphs

CONTENTS

Objectives

After studying this unit, you will be able to:

- Understand breadth first search
- Learn depth first search
- Discuss network flow problems andwarshall's algorithm
- learntopological sort

Introduction

Graph traversal entails visiting each vertex and edge in a predetermined order. You must verify that each vertex of the graph is visited exactly once when utilizing certain graph algorithms. The sequence in which the vertices are visited is crucial, and it may be determined by the algorithm or question you're working on.It's critical to keep track of which vertices have been visited throughout a traversal. Marking vertices is the most popular method of tracking them.

In Graph traversal visiting every vertex and edge exactly once in a well-defined order.In graph algorithms, you must ensure that each vertex of the graph is visited exactly once. The order in which the vertices are visited may depend upon the algorithm or type of problem going to solve.

Two common elementary algorithms for tree-searching are

– Breadth-first search (BFS)

– Depth-first search (DFS).

Both of these algorithms work on directed or undirected graphs. Many advanced graph algorithms are based on the ideas of BFS or DFS. Each of these algorithms traverses edges in the graph, discovering new vertices as it proceeds. The difference is in the order in which each algorithm discovers the edges.

11.1 Breadth First Search (BFS)

Breadth first search is a graph traversal algorithm that starts traversing the graph from root node and explores all the neighbouring nodes. Then, it selects the nearest node and explore all the unexplored nodes. The algorithm follows the same process for each of the nearest node until it finds the goal.

For using BFS algorithm user should know about data structure queue and its relevant operations like en-queue and de-queue.

Algorithm: Breadth First Search

Step 1: SET STATUS = 1 (ready state)

for each node in G

Step 2: Enqueue the starting node A

and set its STATUS = 2

(waiting state)

Step 3: Repeat Steps 4 and 5 until

QUEUE is empty

Step 4: Dequeue a node N. Process it

and set its STATUS = 3

(processed state).

Step 5: Enqueue all the neighbours of

N that are in the ready state

(whose STATUS = 1) and set

their STATUS = 2

(waiting state)

[END OF LOOP]

Step 6: EXIT

Example: Breadth First Search

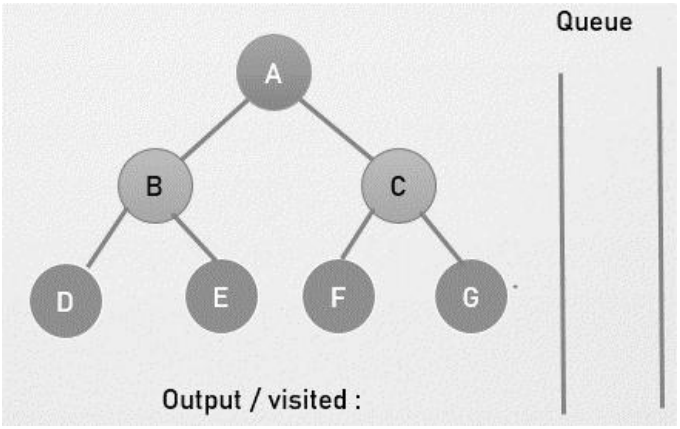
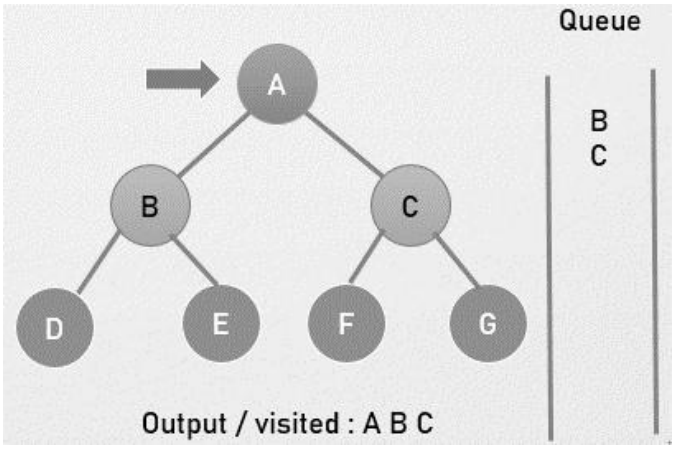


Fig (a)



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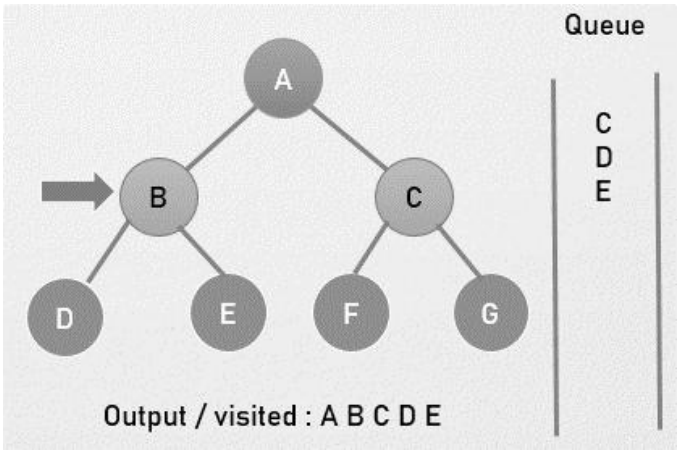


Fig (c)

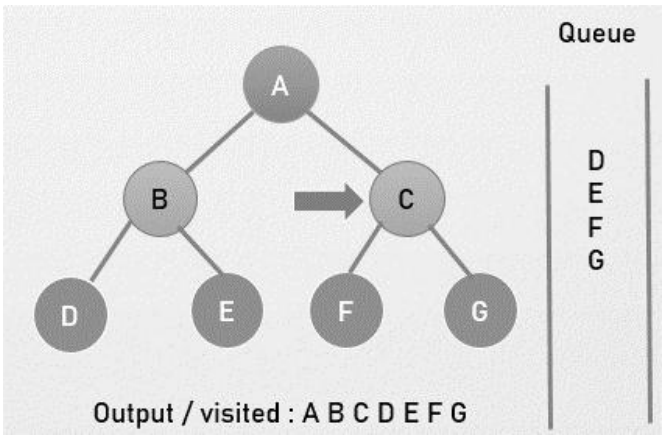


Fig (d)

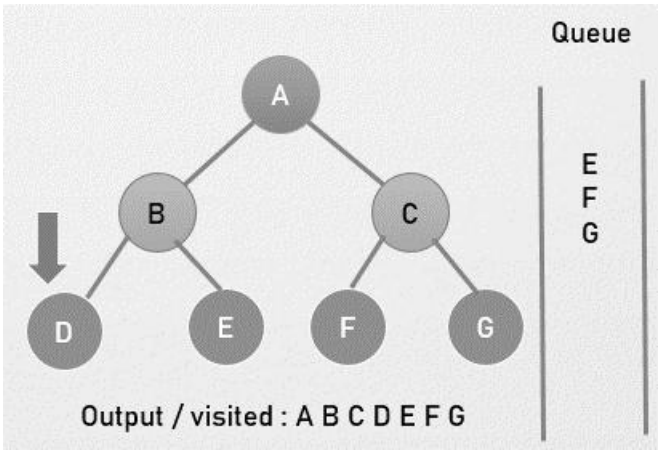


Fig (e)

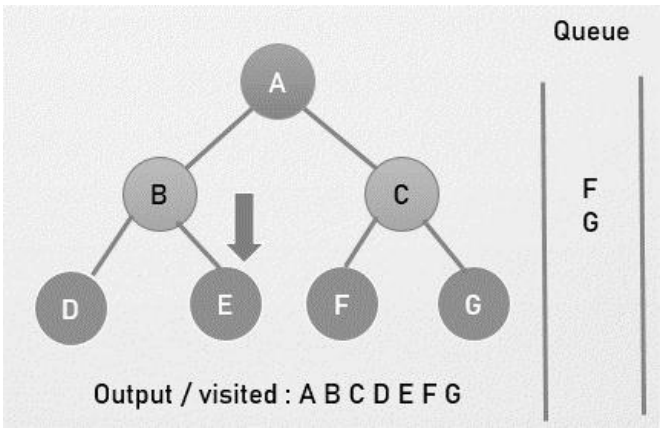


Fig (f)

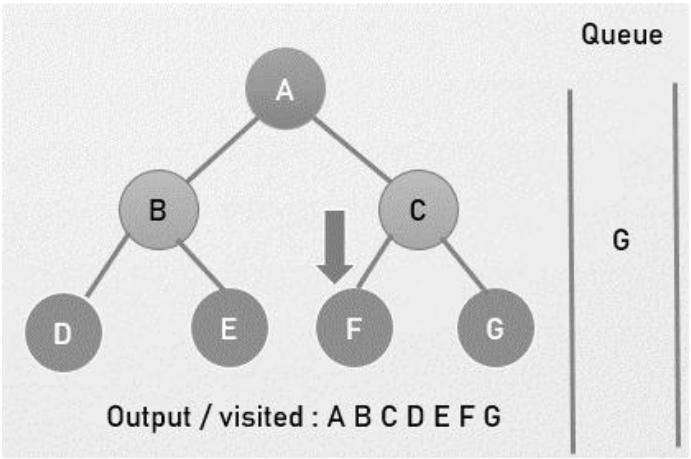


Fig (g)

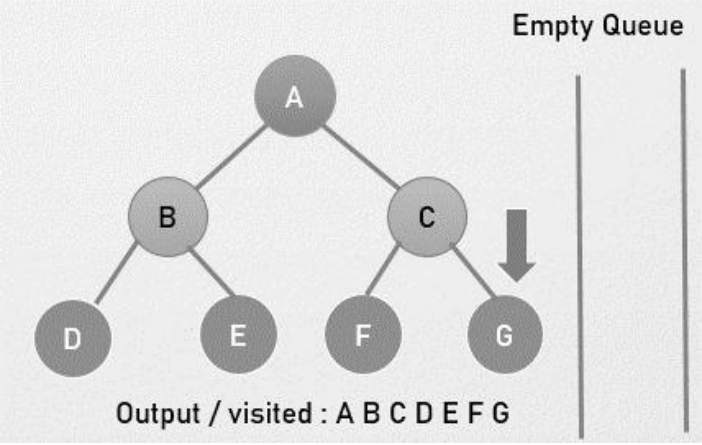


Fig (h)

Algorithm Complexity

The time complexity of the BFS algorithm is represented in the form of O(V + E), where V is the number of nodes and E is the number of edges.

The space complexity of the algorithm is O(V).

BFS Applications

- Path finding algorithms
- To build index by search index
- Cycle detection in an undirected graph
- For GPS navigation
- In minimum spanning tree
- Social networking websites

11.2 Depth First Search

Depth first search is another way of traversing graphs, which is closely related to preorder traversal of a tree. Recall that preorder traversal simply visits each node before its children. It is most easy to program as a recursive routine.

DFS traversal is a recursive algorithm for searching all the vertices/ nodes of a graph or tree using stack data structure.In Depth First Search (DFS) algorithm traverses a graph in a depth ward motion.The DFS algorithm use the concept of backtracking.Depth-first search (DFS): Finds a path between two vertices by exploring each possible path as far as possible before backtracking.Often implemented recursively. Many graph algorithms involve visiting or marking vertices.

Steps for DFS

Step 1 − Visit the adjacent unvisited vertex. Mark it as visited. Display it. Push it in a stack.

Step 2 − If no adjacent vertex is found, pop up a vertex from the stack. (It will pop up all the vertices from the stack, which do not have adjacent vertices.)

Step 3 − Repeat Rule 1 and Rule 2 until the stack is empty.

For using DFS algorithm user should know about data structure Stack (Last In First Out) and its relevant operations like Push and Pop.

Algorithm: Depth First Search

Step 1: SET STATUS = 1 (ready state) for each node in G

Step 2: Push the starting node A on the stack and set its STATUS = 2 (waiting state)

Step 3: Repeat Steps 4 and 5 until STACK is empty

Step 4: Pop the top node N. Process it and set its STATUS = 3 (processed state)

Step 5: Push on the stack all the neighbours of N that are in the ready state (whose STATUS = 1) and set their

STATUS = 2 (waiting state)

[END OF LOOP]

Step 6: EXIT

Example: Depth First Search

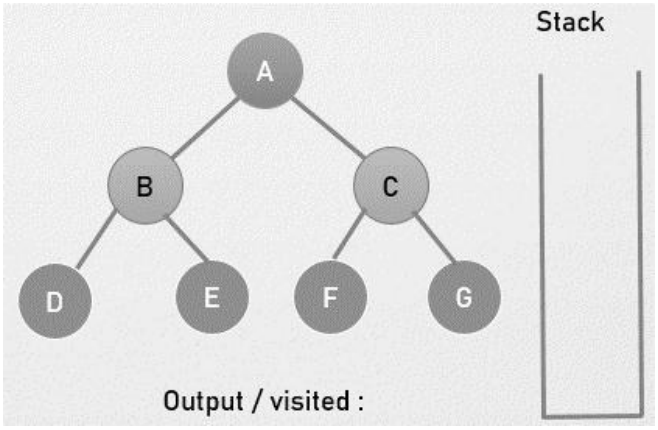


Fig (a)

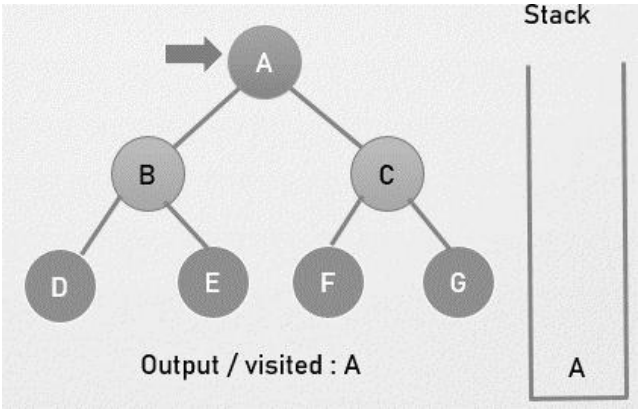


Fig (b)

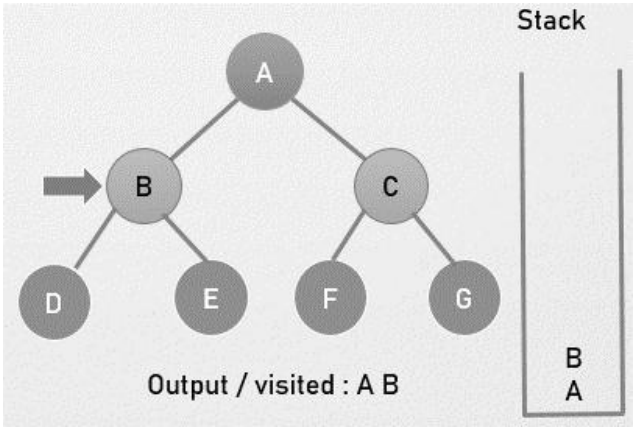


Fig (c)

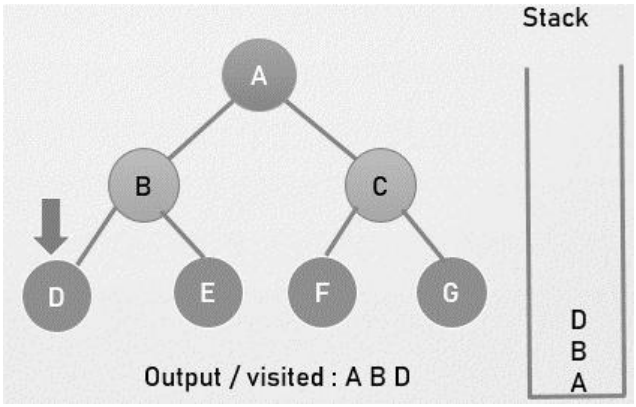


Fig (d)

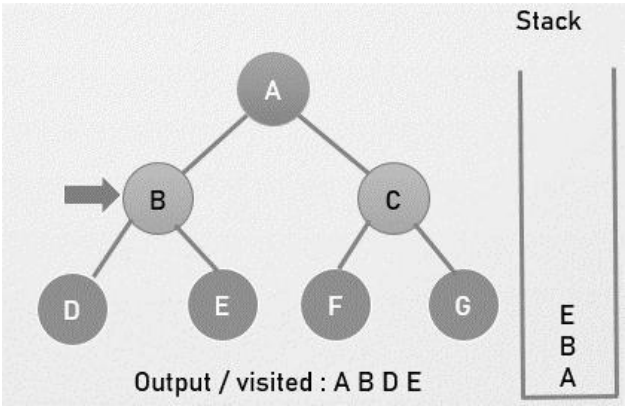


Fig (e)

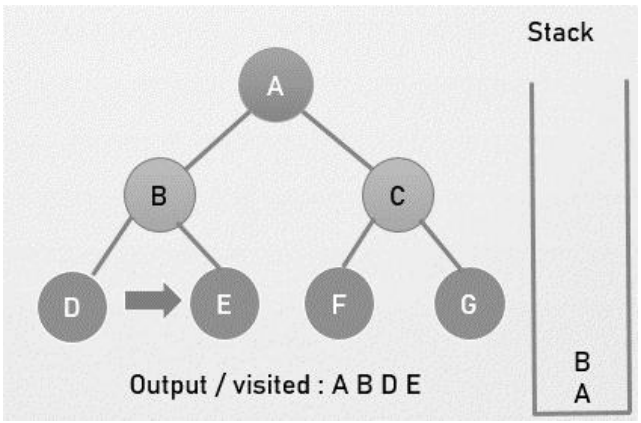


Fig (f)

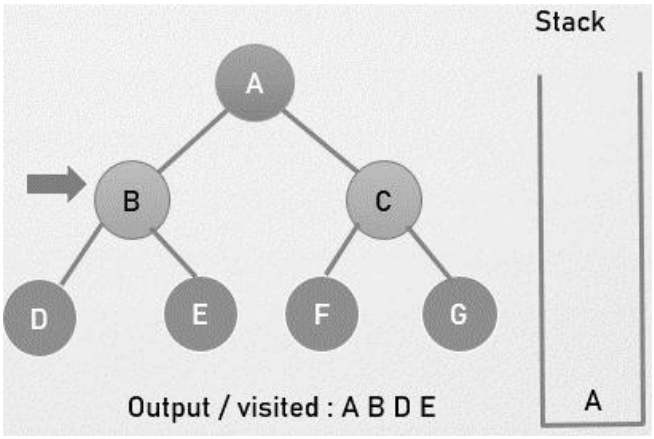


Fig (g)

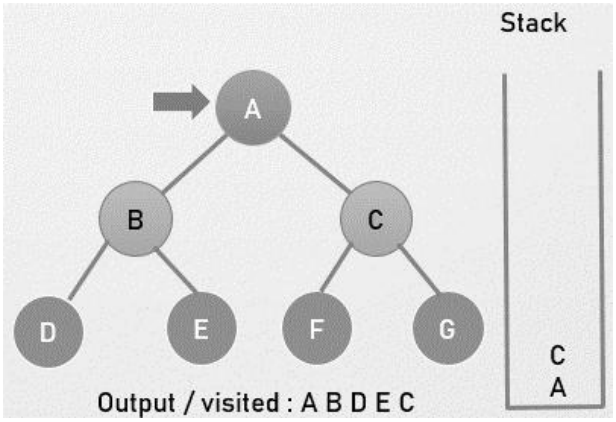


Fig (h)

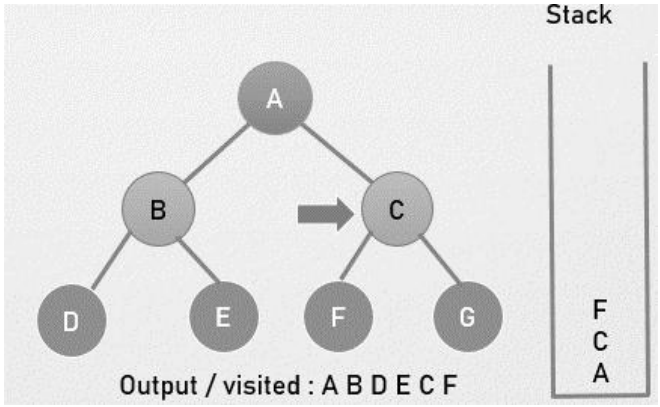


Fig (i)

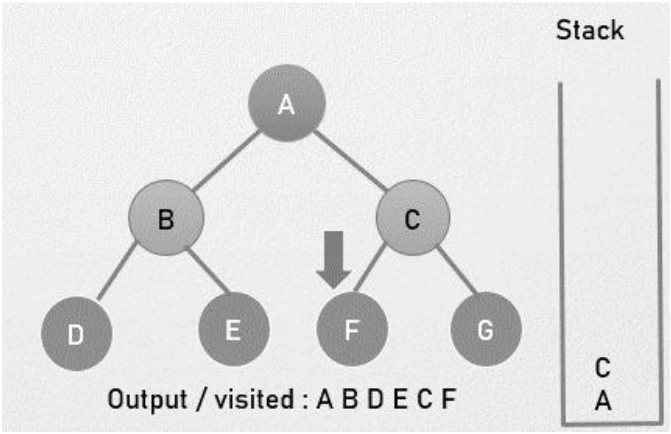


Fig (j)

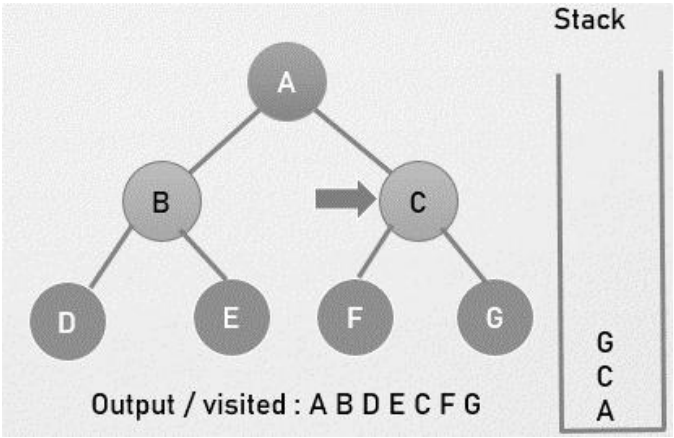


Fig (k)

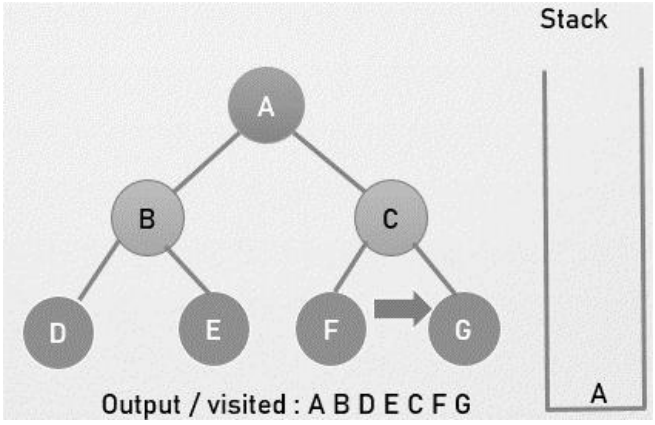


Fig (l)

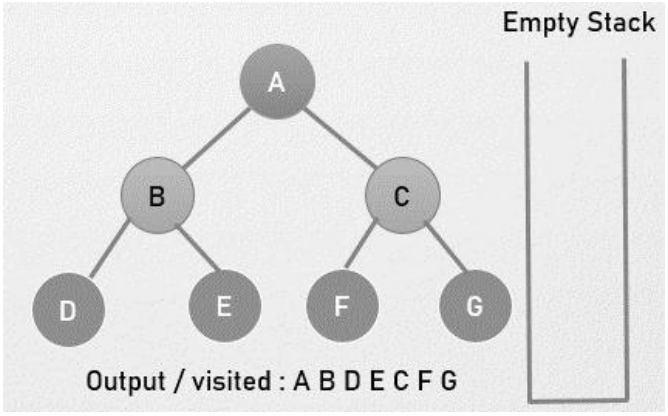


Fig (m)

Algorithm Complexity

Time complexity: O(V + E), where V is the number of vertices and E is the number of edges in the graph.

Space Complexity: O(V).

DFS Applications

- Mapping Routes and Network Analysis.
- Path Finding.
- Cycle detection in graphs.
- Topological Sorting.
- Solving puzzle.

11.3 Network Flow Problem

Network flow is an advanced branch of graph theory. The problem resolves around a special type of weighted directed graph with two special vertices: the source vertex, which has no incoming edge, and the sink vertex, which has no outgoing edge. By convention, the source vertex is usually labelled s and the sink vertex labelled t.

In graph theory, a flow network is a directed graph involving a source(s) and a sink(t) and several other nodes connected with edges. Each edge has an individual capacity which is the maximum limit of flow that edge could allow.

Network Flow Problems

Problem1: Given a flow network G = (V, E), the maximum flow problem is to find a flow with maximum value.

Problem 2: The multiple source and sink maximum flow problem is similar to the maximum flow problem, except there is a set {s1,s2,s3.....sn} of sources and a set {t1,t2,t3.....tn} of sinks.

For any non-source and non-sink node, the input flow is equal to output flow.

For any edge(Ei) in the network, 0 ≤ flow (Ei) ≤capacity(Ei).

Total flow out of the source node is equal total to flow in to the sink node.

Net flow in the edges follows skew symmetry

Problem: Maximize the total amount of flow from s to t. subject to two constraints

– Flow on edge e doesn't exceed c(e)

– For every node v ≠ s, t, incoming flow is equal to outgoing flow

Types of network flow problems

Minimum-cost flow problem: in which the edges have costs as well as capacities and the goal is to achieve a given amount of flow (or a maximum flow) that has the minimum possible cost.

Multi-commodity flow problem: in which one must construct multiple flows for different commodities whose total flow amounts together respect the capacities.

Nowhere-zero flow: a type of flow studied in combinatorics in which the flow amounts are restricted to a finite set of nonzero values.

Maximum flow problem.

Algorithms for network flow problem

The ***Ford–Fulkerson algorithm***, a greedy algorithm for maximum flow that is not in general strongly polynomial

Dinic's algorithm, a strongly polynomial algorithm for maximum flow

The ***Edmonds–Karp algorithm***, a faster strongly polynomial algorithm for maximum flow

The ***network simplex algorithm***, a method based on linear programming but specialized for network flow

The ***out-of-kilter algorithm*** for minimum-cost flow

The ***push–relabel maximum flow algorithm***, one of the most efficient known techniques for maximum flow

11.4 Ford-Fulkerson Algorithm

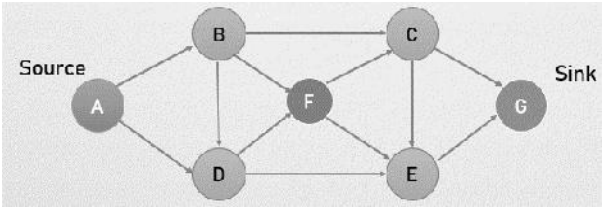
It was developed by L. R. Ford, Jr. and D. R. Fulkerson in 1956. A simple and practical max-flow algorithm. Objective: find valid flow paths until there is none left, and add them up.

Terminology: Ford-Fulkerson Algorithm

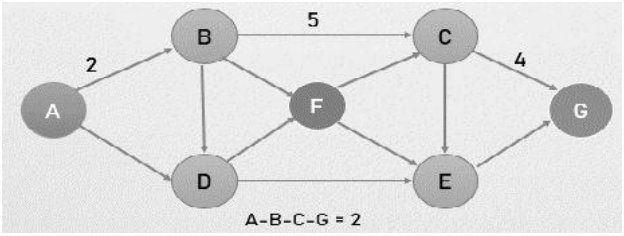
Source	Sink
Bottleneck capacity	Flow
Augmenting path	Residual capacity

The ***source*** vertex has all outward edges, no inward edges.

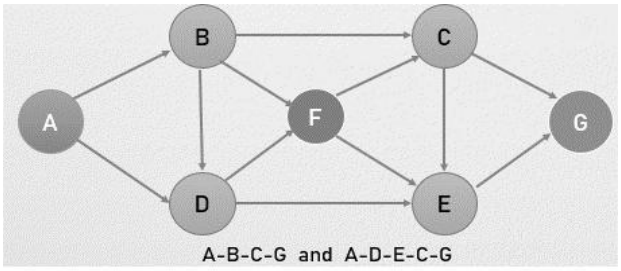
Sink have all inward edges, no outward edges.



Bottleneck capacity of a path is the minimum capacity of any edge on the path.

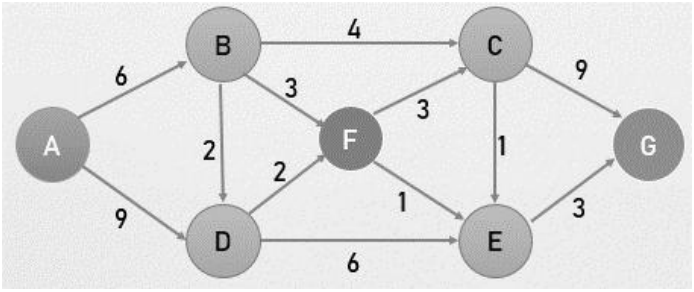


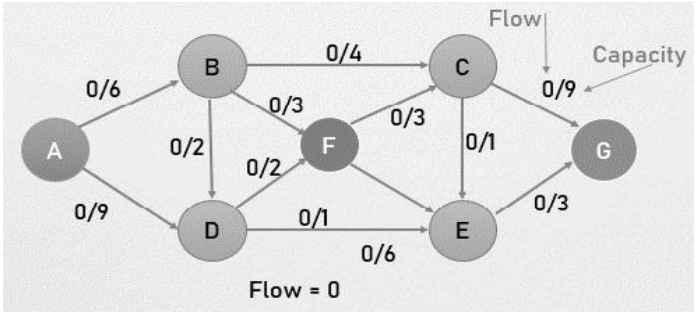
An ***augmenting path*** is a simple path from source to sink which do not include any cycles and that pass only through positive weighted edges.



Residual capacity: which is equal to original capacity of the edge minus current flow. Residual capacity is basically the current capacity of the edge.

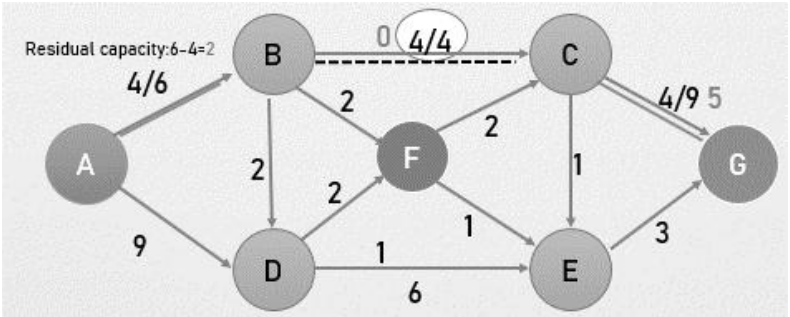
Example: Ford-Fulkerson Algorithm





Path: A-B-C-G

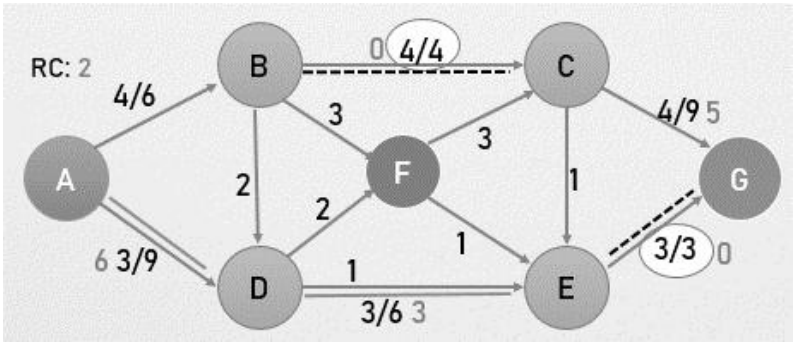
Flow = 4



Augmenting path	Bottleneck capacity
A-B-C-G	4

Path: A-D-E-G

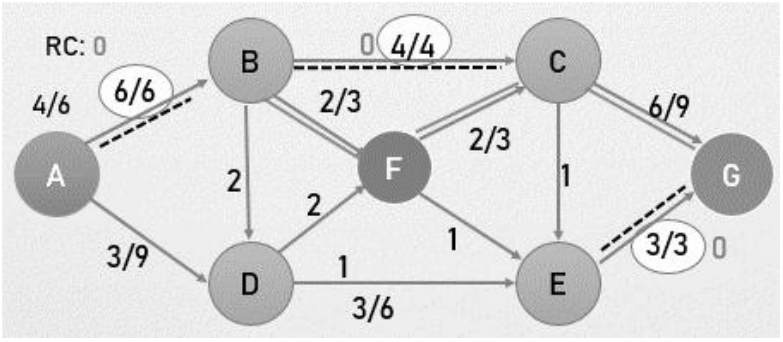
Flow= 4+3



Augmenting path	Bottleneck capacity
A-B-C-G	4
A-D-E-G	3

Path: A-B-F-C-G

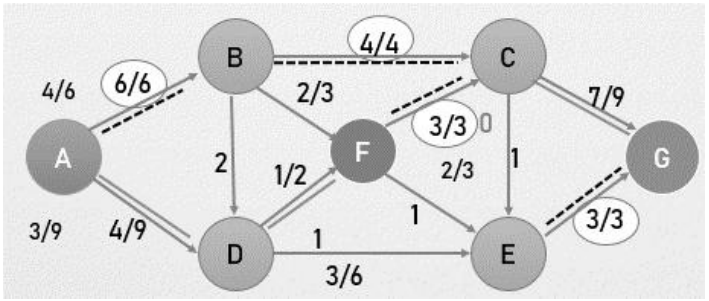
Flow = 4+3+2



Augmenting path	Bottleneck capacity
A-B-C-G	4
A-D-E-G	3
A-B-F-C-G	2

Path: A-D-F-C-G

Flow = 4+3+2+1 = 10



Augmenting path	Bottleneck capacity
A-B-C-G	4
A-D-E-G	3
A-B-F-C-G	2
A-D-F-C-G	1

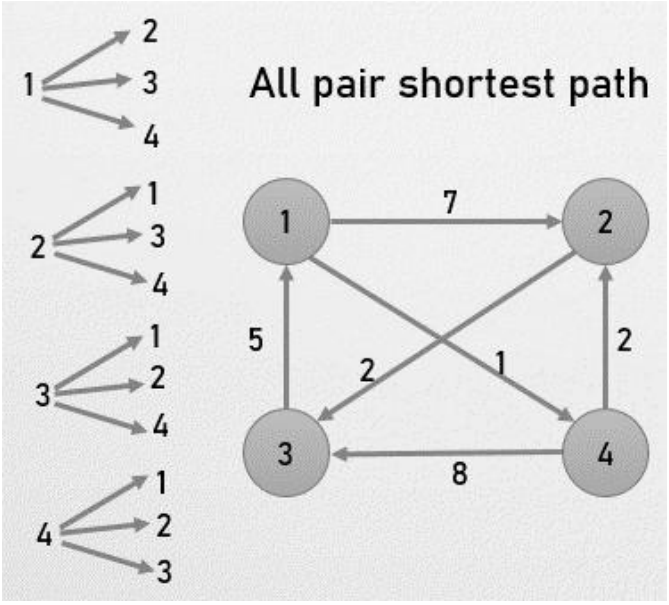
Ford-Fulkerson Applications

- Circulation with demands
- Water distribution pipeline
- Bipartite matching problem

11.5 Floyd-Warshall Algorithm

The Floyd Warshall Algorithm is used for solving the All Pairs Shortest Path problem. The problem is to finding the shortest path between all the pairs of vertices in a weighted directed Graph.

This algorithm works for both the directed and undirected weighted graphs.Floyd-Warhshall algorithm follows the dynamic programming approach to find the shortest paths.Floyd-Warhshall algorithm is also called as Floyd's algorithm, Roy-Floyd algorithm, Roy-Warshall algorithm, or WFI algorithm.



Floyd-Warshall Algorithm

n = no of vertices

A = matrix of dimension n*n

for k = 1 to n

  for i = 1 to n

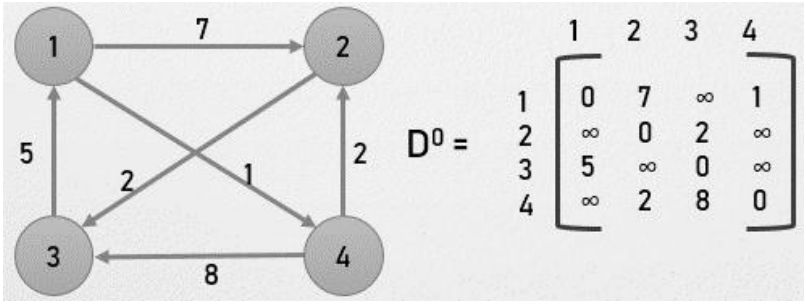
    for j = 1 to n

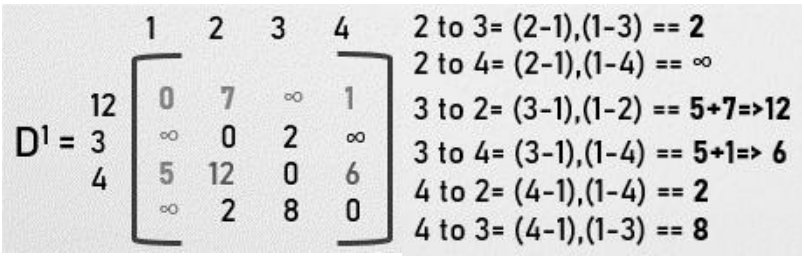
Ak[i, j] = min (Ak-1[i, j], Ak-1[i, k] + Ak-1[k, j])

return A

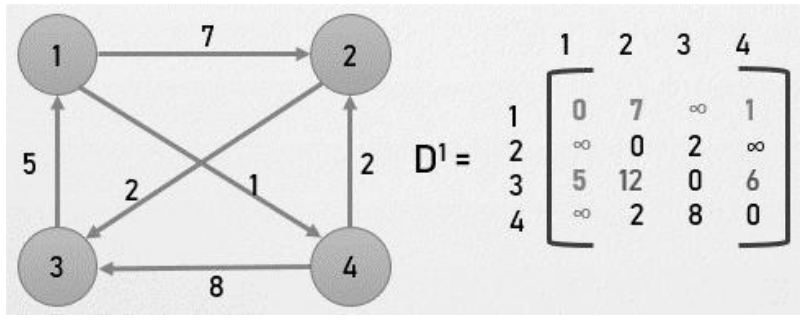
Example: Floyd Warshall Algorithm

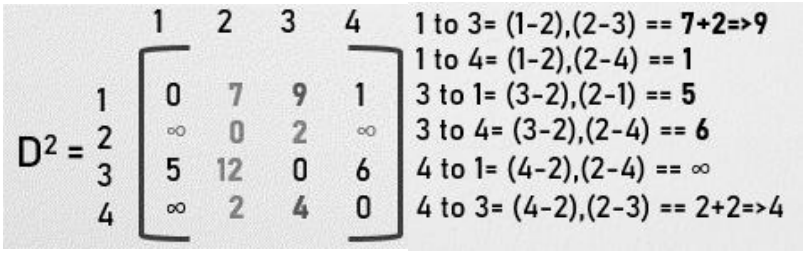
D1


1
2
3
4
7
5
2
1
2
8
D0 =
1 2 3 4
1 0 7 ∞ 1
2 ∞ 0 2 ∞
3 5 ∞ 0 ∞
4 ∞ 2 8 0

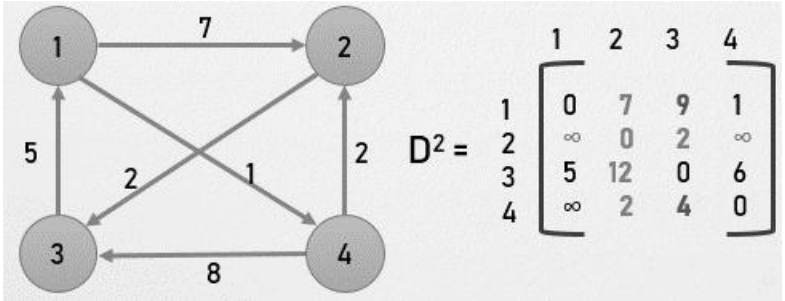

D1 =
1 2 3 4
12 0 7 ∞ 1
3 ∞ 0 2 ∞
4 5 12 0 6
∞ 2 8 0
2 to 3= (2-1),(1-3) == 2
2 to 4= (2-1),(1-4) == ∞
3 to 2= (3-1),(1-2) == 5+7=>12
3 to 4= (3-1),(1-4) == 5+1=> 6
4 to 2= (4-1),(1-4) == 2
4 to 3= (4-1),(1-3) == 8

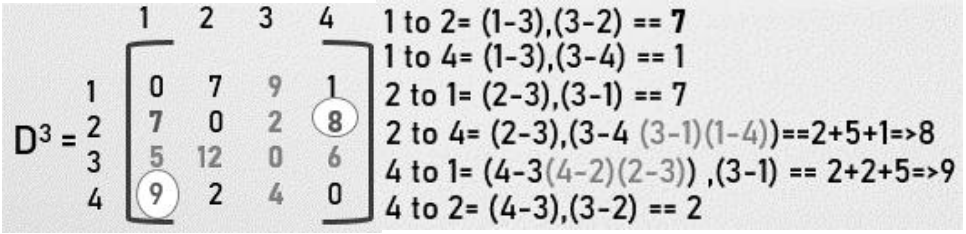
D2


1
2
3
4
7
5
2
1
2
8
D1 =
1 2 3 4
1 0 7 ∞ 1
2 ∞ 0 2 ∞
3 5 12 0 6
4 ∞ 2 8 0

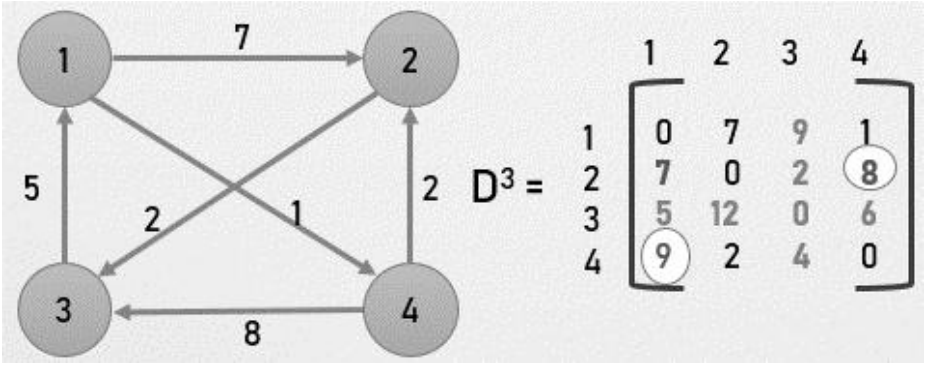

D2 =
1 2 3 4
1 0 7 9 1
2 ∞ 0 2 ∞
3 5 12 0 6
4 ∞ 2 4 0
1 to 3= (1-2),(2-3) == 7+2=>9
1 to 4= (1-2),(2-4) == 1
3 to 1= (3-2),(2-1) == 5
3 to 4= (3-2),(2-4) == 6
4 to 1= (4-2),(2-4) == ∞
4 to 3= (4-2),(2-3) == 2+2=>4

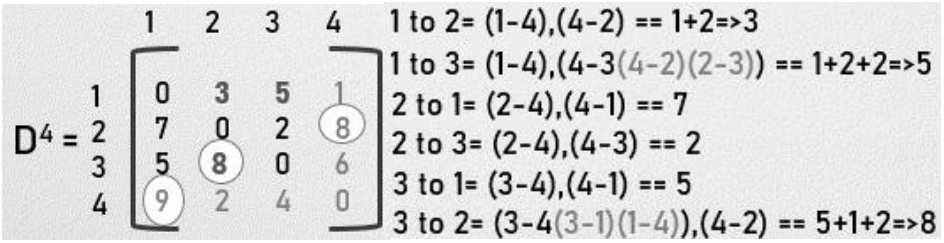
D3

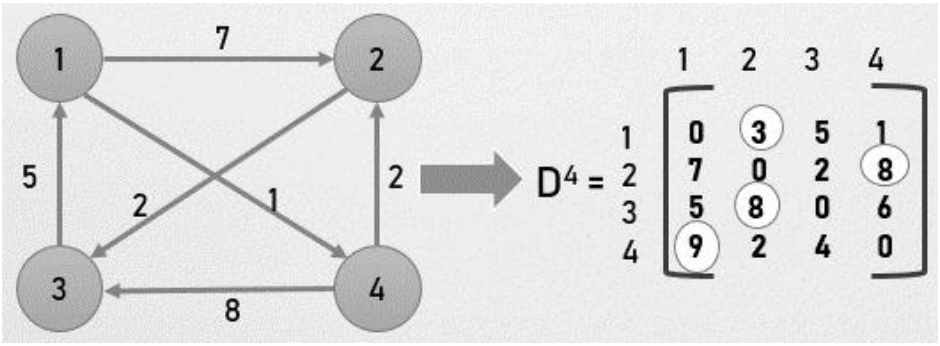

1
2
3
4
7
5
2
1
2
8
D2 =
1 2 3 4
1 0 7 9 1
2 ∞ 0 2 ∞
3 5 12 0 6
4 ∞ 2 4 0


D3 =
1 2 3 4
1 0 7 9 1
2 7 0 2 8
3 5 12 0 6
4 9 2 4 0
1 to 2= (1-3),(3-2) == 7
1 to 4= (1-3),(3-4) == 1
2 to 1= (2-3),(3-1) == 7
2 to 4= (2-3),(3-4 (3-1)(1-4))==2+5+1=>8
4 to 1= (4-3(4-2)(2-3)) ,(3-1) == 2+2+5=>9
4 to 2= (4-3),(3-2) == 2

D4







All pair shortest path

Complexity

Time complexity = O(|V|3)

Space complexity = O(|V|2)

Applications: Floyd Warshall Algorithm

To find the transitive closure of directed graphs

To find the Inversion of real matrices

To find the shortest path is a directed graph

For testing whether an undirected graph is bipartite

11.6 Topological Sort

Topological Sort is a linear ordering of the vertices in such a way that if there is an edge in the DAG (directed acyclic graph) going from vertex 'u' to vertex 'v', then 'u' comes before 'v' in the ordering.

Topological Sorting is possible if and only if the graph is a Directed Acyclic Graph.There may exist multiple different topological orderings for a given directed acyclic graph.

Steps for topological sort

Step 1: Find the indegree for every vertex.

Step 2: Place te vertices whose indegree is 0 on the empty queue.

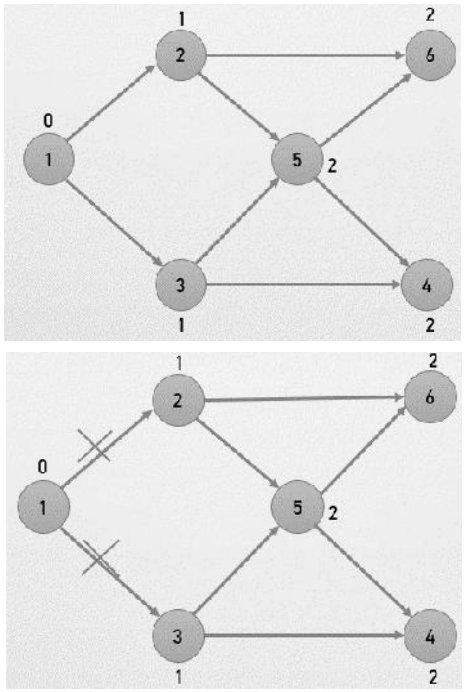
Step 3: Dequeue the vertex V and decrement the indegree"s of all its adjacent vertices.

Step 4: Enqueue the vertex on the queue, if its degree falls to zero.

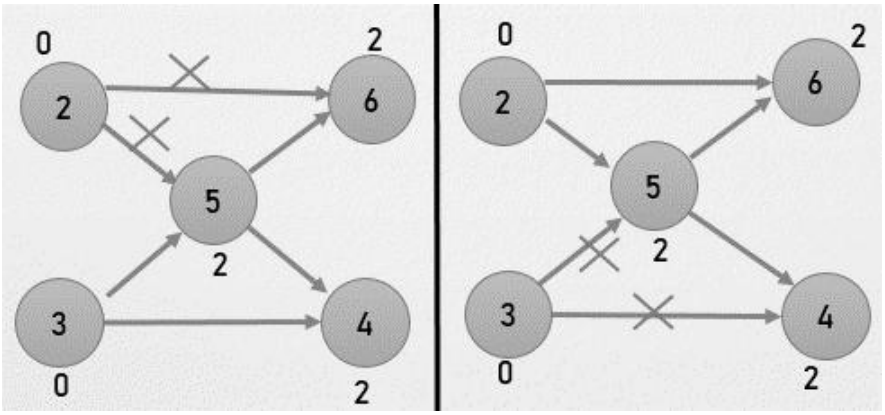
Step 5: Repeat from step 3 until the queue becomes empty.

Step 6: The topological ordering is the order in which the vertices dequeued.

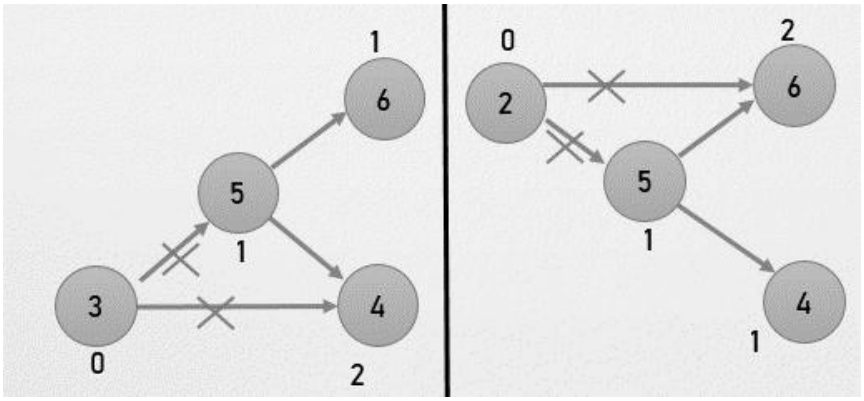
Example: Topological sort



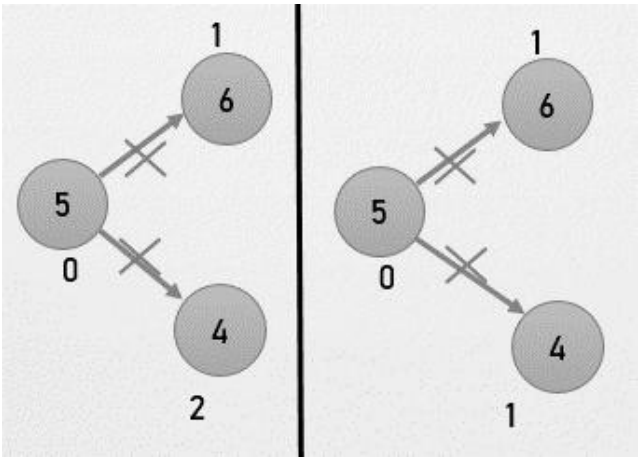
In-degree of vertex Visited vertex: 1



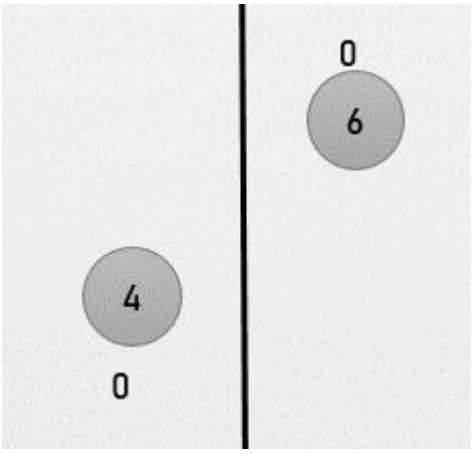
Visited vertex: 1 2 Visited vertex: 1 3



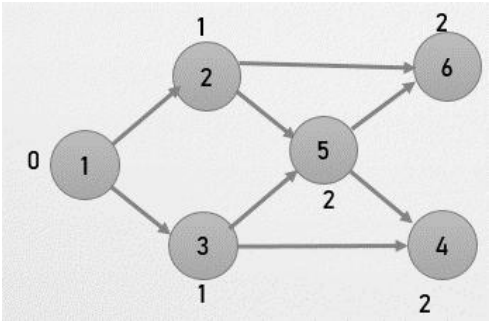
Visited vertex: 1 2 3 Visited vertex: 1 3 2



Visited vertex: 1 2 3 5Visited vertex: 1 3 2 5



Visited vertex: 1 2 3 5 6 4Visited vertex: 1 3 2 5 4 6



Visited vertex: 1 2 3 5 6 4

Visited vertex: 1 3 2 5 4 6

Visited vertex: 1 2 3 5 4 6

Visited vertex: 1 3 2 5 6 4

Applications of Topological Sort

Instruction Scheduling

Determining the order of compilation tasks to perform in make files

Scheduling jobs from the given dependencies among jobs

Data Serialization

Summary

- Graphs provide in excellent way to describe the essential features of many applications.
- Graphs are mathematical structures and are found to be useful in problem solving. They may be implemented in many ways by the use of different kinds of data structures.
- Graph traversals, Depth first as well as Breadth First, are also required in many applications.
- Breadth first search is a graph traversal algorithm that starts traversing the graph from root node and explores all the neighboring nodes.
- DFS traversal is a recursive algorithm for searching all the vertices/ nodes of a graph or tree using stack data structure.
- The Floyd Warshall Algorithm is used for solving the All Pairs Shortest Path problem

Keywords

Network Flow	BFS
DFS	Floyd Warshall Algorithm
Topological sort	network simplex algorithm

Self Assessment

1. Which is not Graph traversal algorithm___

A. Breadth First Search
B. Depth First Search
C. Euclidean algorithm
D. Dijkstra's Algorithm

2. Time complexity of the BFS algorithm is ____

A. Log n
B. (V + E)
C. (log 1)
D. log n

3. Breadth First Search algorithm use ____ data structure.

A. Stack
B. Queue
C. Linked list
D. All of above

4. Depth First Search applications are___

A. Path Finding.
B. Cycle detection in graphs.
C. Topological Sorting.
D. All of above

5. Space complexity of the BFS algorithm is ____

A. Log n
B. (V + E)
C. (V)
D. log n

6. Breadth First Search algorithm use ____ data structure.

A. Queue
B. Linked list
C. Stack
D. All of above

7. Network flow Problems defines___

A. To find a flow with minimum value.
B. To find a flow with maximum value.
C. To find a flow with average value.
D. All of above

8. Which is not network flow problem____

A. Multi-commodity
B. Minimum-cost
C. Travelling salesman problem
D. Nowhere-zero

9. Algorithm used for network flow problem are ____

A. Dinic's algorithm
B. Edmonds–Karp algorithm
C. Out-of-kilter
D. All of above

10. Floyd Warshall Algorithm is used for ___

A. To find a flow with average value.
B. Travelling salesman problem
C. All Pairs Shortest Path problem.
D. All of above

11. Time complexity of Floyd-Warshall Algorithm ____

A. log n
B. O(|V|3)
C. O(|V|2)
D. All of above

12. Space complexity of Floyd-Warshall Algorithm ____

A. (log 2)
B. log 1
C. O(|V|3)
D. O(|V|2)

13. Topological Sorting is possible if ___

A. Graph is a Directed cyclic Graph
B. Graph is a Directed Acyclic Graph
C. Graph is an undirected Acyclic Graph.
D. All of above

14. Applications of topological sort are ____

A. Data Serialization
B. Instruction Scheduling
C. Scheduling jobs from the given dependencies among jobs
D. All of above

15. The In-degree of starting vertex in topological sort graph is ____

A. 1
B. 0
C. 2
D. 4

Answers for Self Assessment

1.	C	2.	B	3.	B	4.	D	5.	C
6.	C	7.	B	8.	C	9.	D	10.	C
11.	B	12.	D	13.	B	14.	D	15.	B

Review Questions

1. Discuss graph traversal with example.
2. Why queue and stack data structure is used with BFS and DFS?
3. Give an example of BFS with example.
4. Describe different types of network flow problem.
5. What are the applications of topological sort?
6. Discuss all pair shortest path problem.
7. DefineBottleneck capacity and Augmenting path.

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Unit 12: Hashing Techniques

CONTENTS

Objectives

After studying this unit, you will be able to:

- Learn basics of hashing
- Understand linear list representation
- Learn hash table representation
- Discuss hash functions and its methods

Introduction

The search time of each algorithm depend on the number n of elements of the collection S of the data. A searching technique called Hashing or Hash addressing which is essentially independent of the number n.

Hashing is the transformation of a string of characters into a usually shorter fixed-length value or key that represents the original string. Hashing is used to index and retrieve items in a database because it is faster to find the item using the shorter hashed key than to find it using the original value. It is also used in many encryption algorithms.

A Hash Function is a Unary Function that is used by Hashed Associative Containers: it maps its argument to a result of type sized. A Hash Function must be deterministic and stateless. That is, the return value must depend only on the argument, and equal arguments must yield equal results.

12.1 Hashing

In many applications we require to use a data object called symbol table. A symbol table is nothing but a set of pairs (name, value). Where value represents collection of attributes associated with

thename, and this collection of attributes depends upon the program element identified by the name.For example, if a name x is used to identify an array in a program, then the attributes associatedwith x are the number of dimensions, lower bound and upper bound of each dimension, and theelement type. Therefore a symbol table can be thought of as a linear list of pairs (name, value),and hence you can use a list of data object for realizing a symbol table. A symbol table is referredto or accessed frequently either for adding the name, or for storing the attributes of the name, orfor retrieving the attributes of the name. Therefore, accessing efficiency is a prime concern whiledesigning a symbol table. Hence the most common way of getting a symbol table implementedis to use a hash table. Hashing is a method of directly computing the index of the table by usingsome suitable mathematical function called hash function. The hash function operates on thename to be stored in the symbol table, or whose attributes are to be retrieved from the symboltable. If h is a hash function and x is a name, then h(x) gives the index of the table where x alongwith its attributes can be stored. If x is already stored in the table, then h(x) gives the index of thetable where it is stored to retrieve the attributes of x from the table. There are various methods ofdefining a hash function like a division method. In this method, you take the sum of the values ofthe characters, divide it by the size of the table, and take the remainder. This gives us an integervalue lying in the range of 0 to (n – 1) if the size of the table is n. The other method is a mid-square method. In this method, the identifi er is first squared and then the appropriate numberof bits from the middle of square is used as the hash value. Since the middle bits of the squareusually depend on all the characters in the identifi er, it is expected that different identifiers willresult into different values. The number of middle bits that you select depends on the table size.Therefore if r is the number of middle bits that you use to form hash value, then the table sizewill be 2r. Hence when you use this method the table size is required to be power of 2. Anothermethod is folding in which the identifi er is partitioned into several parts, all but the last partbeing of the same length. These parts are then added together to obtain the hash value.

To store the name or to add attributes of the name, you compute hash value of the name, andplace the name or attributes as the case may be, at that place in the table whose index is the hashvalue of the name. For retrieving the attribute values of the name kept in the symbol table, I applythe hash function to the name to obtain index of the table where you get the attributes of thename. Hence you find that no comparisons are required to be done. Hence the time required forthe retrieval is independent of the table size. Therefore, retrieval is possible in a constant amountof time, which will be the time taken for computing the hash function. Therefore, hash table seemsto be the best for realization, of the symbol table, but there is one problem associated with thehashing, and it is of collisions. Hash collision occurs when the two identifiers are mapped intothe same hash value. This happens because a hash function defines a mapping from a set of valididentifiers to the set of those integers, which are used as indices of the table. Therefore, you seethat the domain of the mapping defined by the hash function is much larger than the range of themapping, and hence the mapping is of many to one nature. Therefore, when I implement a hashtable a suitable collision handling mechanism is to be provided which will be activated whenthere is a collision.

Collision handling involve finding out an alternative location for one of the two collidingsymbols. For example, if x and y are the different identifiers and if h(x) = h(y), x and y are thecolliding symbols. If x is encountered before y, then the ith entry of the table will be used foraccommodating symbol x, but later on when y comes there is a hash collision, and thereforeyou have to fi nd out an alternative location either for x or y. This means you find out a suitablealternative location and either accommodate y in that location, or you can move x to that location. and place y in the ith location of the table. There are various methods available to obtain an alternative location to handle the collision. They differ from each other in the way search is made for an alternative location. The following are the commonly used collision handling techniques.

Terminology: Hash table

Data bucket – Data buckets are memory locations where the records are stored. It is also known as unit of storage.

Hash index – It is an address of the data block. A hash function could be a simple mathematical function to even a complex mathematical function.

Linear Probing – Linear probing is a fixed interval between probes. In this method, the next available data block is used to enter the new record, instead of overwriting on the older record.

Double Hashing –Double hashing is a computer programming method used in hash tables to resolve the issues of has a collision.

Quadratic probing– It helps you to determine the new bucket address. It helps you to add Interval between probes by adding the consecutive output of quadratic polynomial to starting value given by the original computation.

Time complexity

Time complexity in linear search is O(n)

Time complexity in binary search is O(log n)

Time complexity in hashing is O(1)

12.2 Steps to Implement Hashing

An element is converted into an integer by using a hash function. This element can be used as an index to store the original element, which falls into the hash table.

The element is stored in the hash table where it can be quickly retrieved using hashed key.

hash = hash func(key)

index = hash % array size

The hash is independent of the array size and it is then reduced to an index (a number between 0 and array size − 1) by using the modulo operator (%).

Operations of a hash table

Search − Searches an element in a hash table.

Insert − inserts an element in a hash table.

Delete − Deletes an element from a hash table.

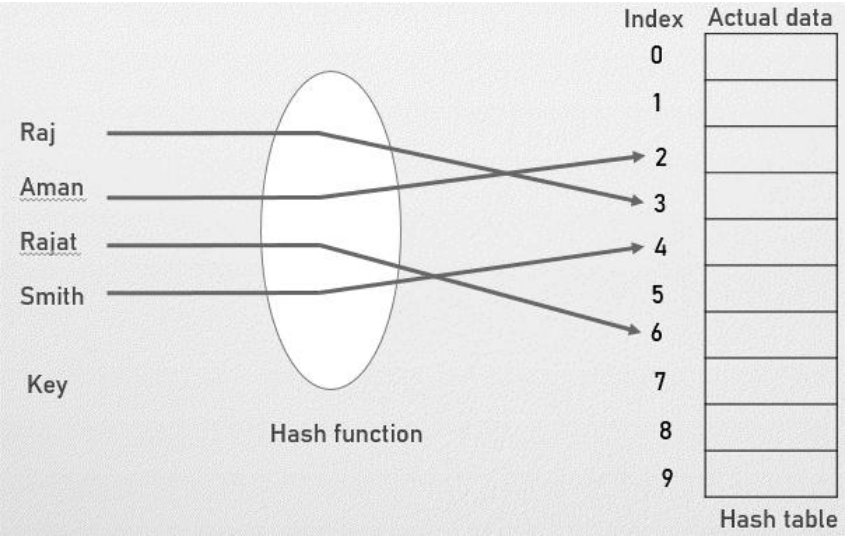
12.3 Hash Table

A Hash table is a data structure that stores information, and the information has basically two components.

- key and value.

The hash table can be implemented with the help of an associative array

It uses a hash function to compute an index into an array of buckets or slots from which the desired value can be found.It is an array of list where each list is known as bucket.It contains value based on the key.Hash table is synchronized and contains only unique elements.



12.4 Hash Function

A hash function is any function that can be used to map a data set of an arbitrary size to a data set of a fixed size, which falls into the hash table. The values returned by a hash function are called hash values, hash codes, hash sums, or hashes. An efficient hash function should be built such that the index value of the added item is distributed equally across the table.

An effective collision resolution technique should be created to generate an alternate index for a key whose hash index corresponds to a previously inserted position in a hash table.

Characteristics of hash function

Uniform Distribution: For distribution throughout the constructed table.

Fast: The generation of hash should be very fast, and should not produce any considerable overhead.

Less collisions: Collisions occur when pairs of elements are mapped to the same hash value. These should be avoided.

Some of the methods of defining hash function are discussed below:

1. ***Modular arithmetic***: In this method, first the key is converted to integer, then it is divided by the size of index range, and the remainder is taken to be the hash value. The spread achieved depends very much on the modulus. If modulus is power of small integers like 2 or 10, then many keys tend to map into the same index, while other indices remain unused. The best choice for modulus is often but not always is a prime number, which usually has the effect of spreading the keys quite uniformly.

2. ***Truncation***: This method ignores part of key, and use the remainder part directly as hash value. (considering non-numeric fi elds as their numerical code) If the keys for example are eight digit numbers and the hash table has 1000 entries, then the first, second, and fifth digit from right might make hash value. So 62538194 maps to 394. It is a fast method, but often fails to distribute keys evenly.

3. ***Folding***: In this method, the identifi er is partitioned into several parts all but the last part being of the same length. These parts are then added together to obtain the hash value. For example an eight digit integer can be divided into groups of three, three, and two digits. The groups are the added together, and truncated if necessary to be in the proper range of indices. Hence 62538149 maps to, 625 + 381 + 94 = 1100, truncated to 100. Since all information in the key can affect the value of the function, folding often achieves a better spread of indices than truncation.

4. ***Mid square method***: In this method, the identifi er is squared (considering non-numeric fi elds as their numerical code), and then the appropriate number of bits from the middle depend on all the characters in the identifi er, it is expected that different identifiers will result in different values. The number of middle bits that we select depends on table size. Therefore if r is the number of middle bits used to form hash value, then the table size will be 2r, hence when you use mid square method the ta Division methodble size should be a power of 2.

12.5 Division Method

The hash function can be described asthe hash function divides the value k by M and then uses the remainder obtained.The division method involves mapping a key k into one of m slots by taking the remainder of k divided by m as expressed in the hash function.

h(K) = k mod M

Here,

k is the key value, and

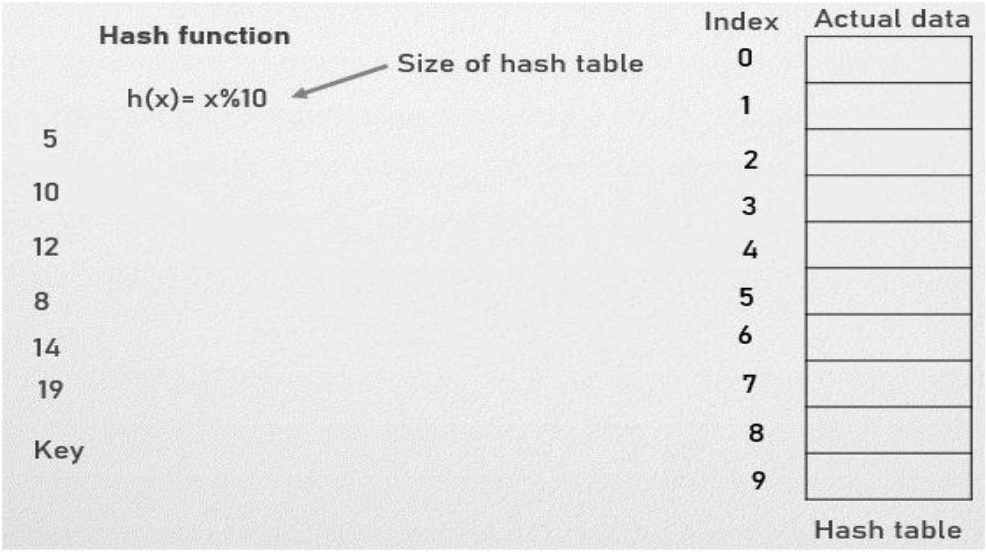
M is the size of the hash table.

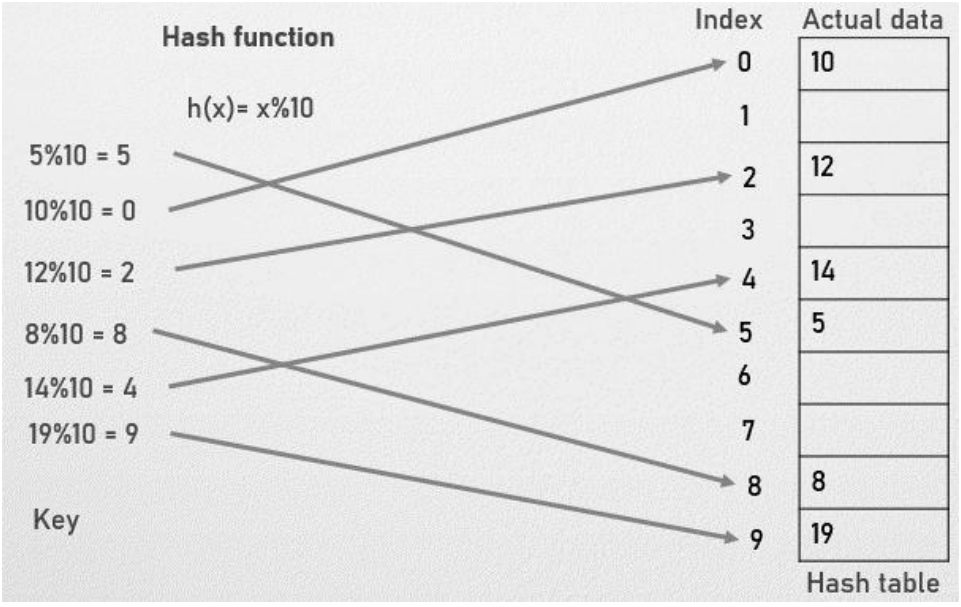
Example:

m=30; k=80

h(k) = k mod m = 20

Example:





Division method: pros

Any key will indeed map to one of m slots (as we want from a hash function, mapping a key to one of m slots)

Fast and simple

Division method: cons

Need to avoid certain values of m to avoid bias (as in the even number example)

12.6 Mid Square Method

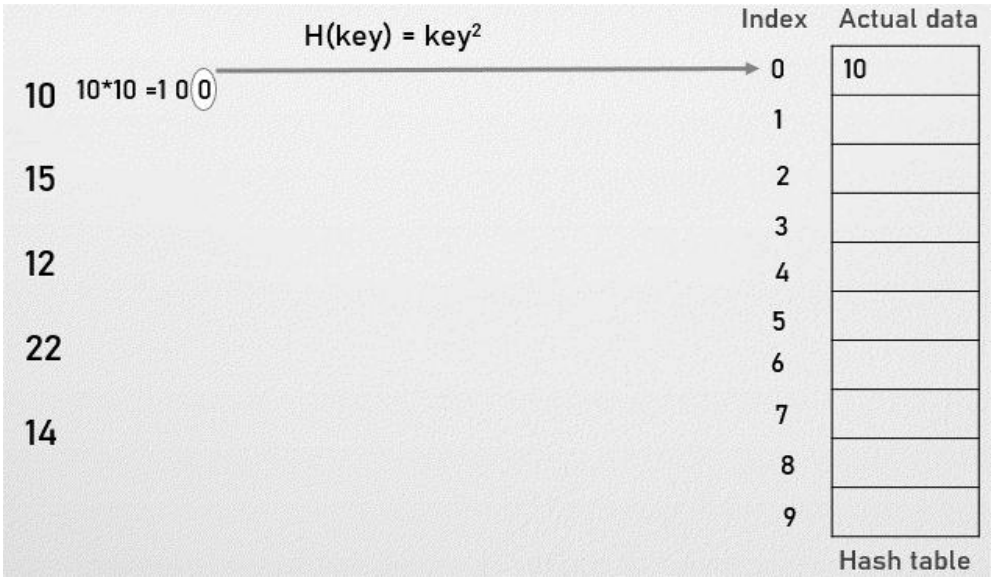
In this technique, squares the key value. Then, some digits from the middle are extracted. These extracted digits form a number which is taken as the new number for address.

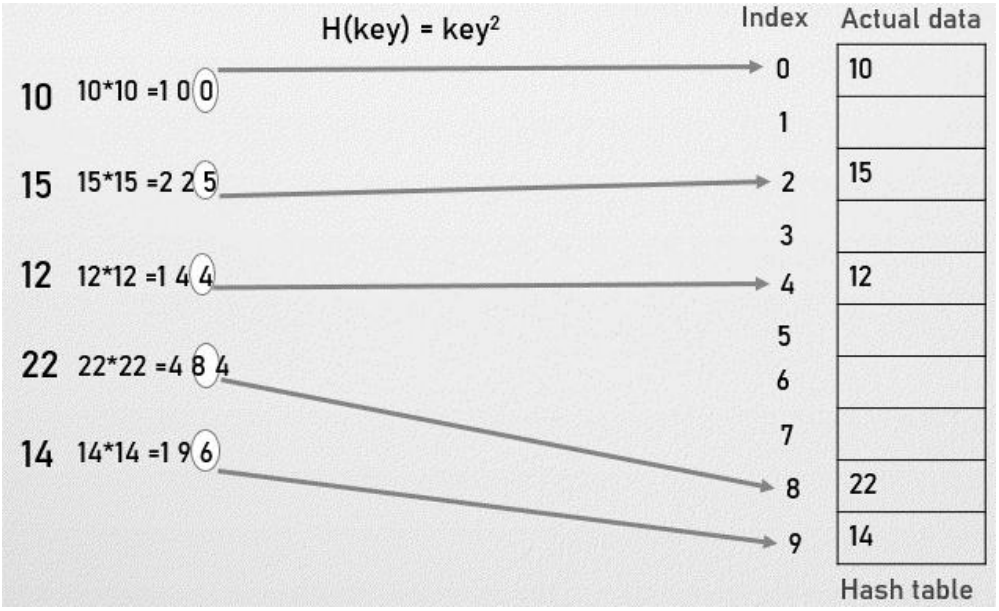
Limitation:

The size of key2 is too large.

E.g. 2025*2025= 4100625

Example:





12.7 Digit Folding Method

The folding method for constructing hash functions begins by dividing the item into equal-size pieces (the last piece may not be of equal size). These pieces are then added together to give the resulting hash value.

Fold shift

Fold boundary

-The left and right numbers are folded on fixed boundary between them and the centre.

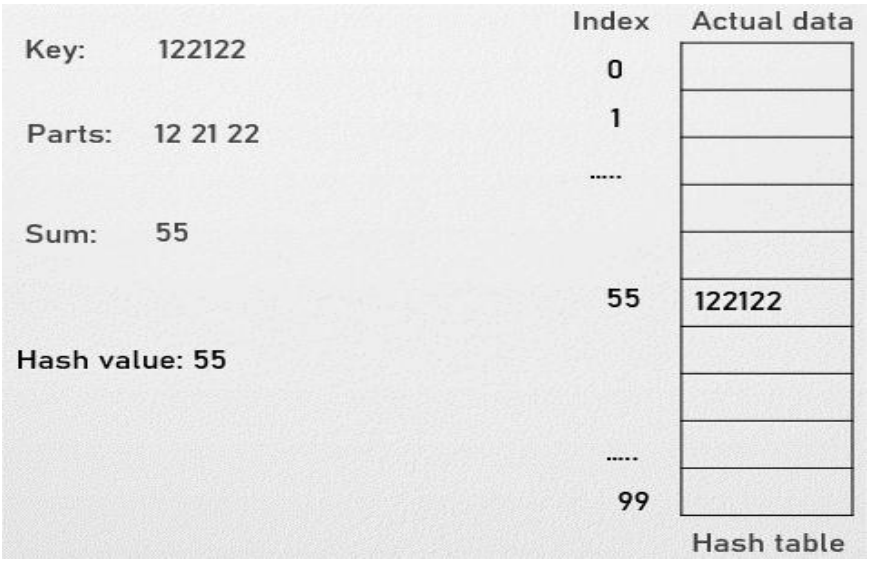
-The two outside values are then reversed.

Case 1: if hash table size is 100 (0-99) and sum in 3 digits, then we will ignore last carry, if any.

Or

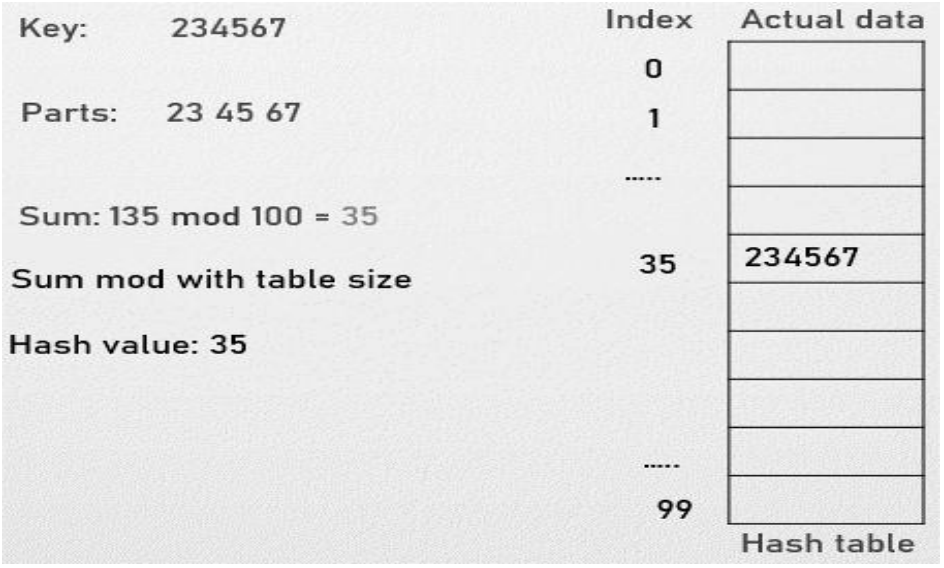
Case 2: if hash table size is 100 (0-99) and sum in 3 digits, then we need to perform the extra step of dividing by 100(size of table) and keeping the remainder.

Folding method: case 1

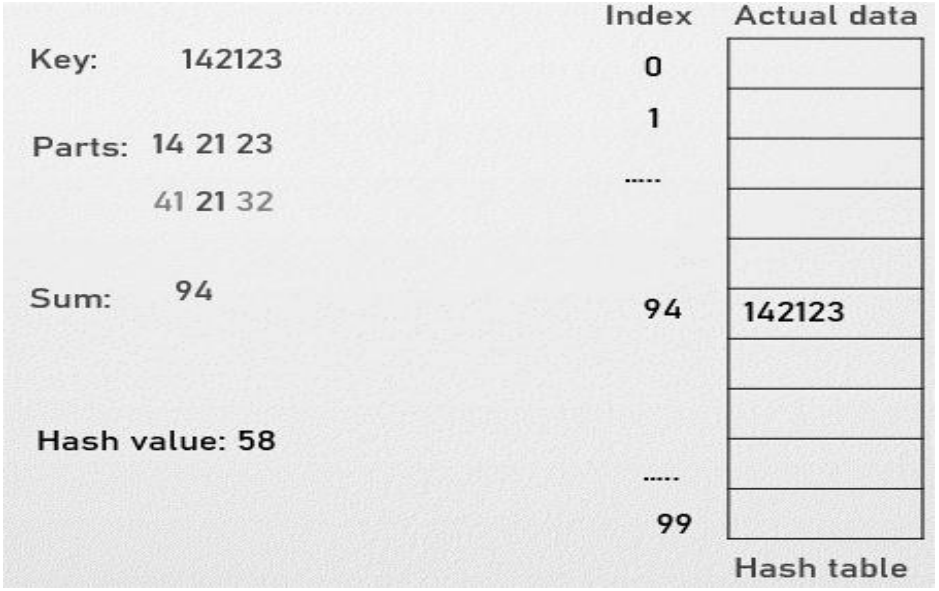


Folding method: case 2

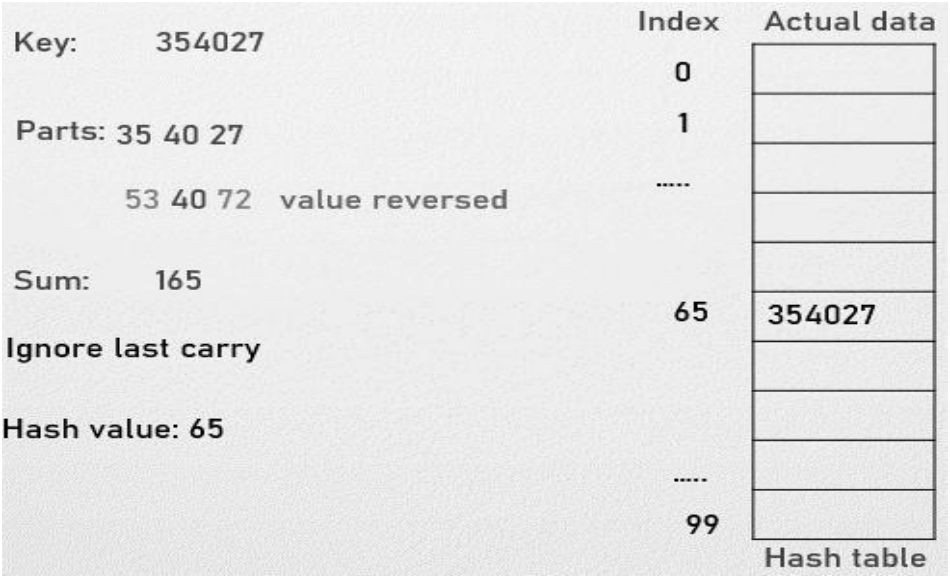
Folding method: case 3



Fold boundary: case 1



Fold boundary: case 2



12.8 Multiplication Method

h(key) = floor(table size *(key *A)) % size

Or

h(k) = floor(n(kA mod 1))

K= key

A is constant value between 0 and 1

Knuth recommends A = 0.6180339887... (Golden Ratio)

1) Choose constant

2) Multiply key k by A

3) Extract fractional part of k A (this gives us a number between 0 and 1)

4) Multiply fractional part by m and take floor of the multiplication (this transforms a number between 0 and 1, to a discrete number between 0 and m-1 that we can map to slot in the hash table)

Example: Multiplication Method

k=34

Size of table = 100

A=0.618033

h(34) = floor (100(34 * 0.618033)) % 100

=floor (3461.80) %100

=3461 %100

=61

Pros:Indeed maps any key into 1 of m slots, as we expect from a hash function

Choice of m not as critical as in division method

There is also a bit implementation (not discussed)

Cons:

Slower than division method

Summary

- Hash functions are mostly used in hash tables, to quickly locate a data record (for example, a dictionary defi nition) given its search key (the headword).
- Specifically, the hash function is used to map the search key to the index of a slot in the table where the corresponding record is supposedly stored.
- A hash function is any function that can be used to map a data set of an arbitrary size to a data set of a fixed size, which falls into the hash table.
- The folding method for constructing hash functions begins by dividing the item into equal-size pieces (the last piece may not be of equal size).

Keywords

Hash function	Division method
Mid square method	Fold boundary
Hash table	Hashes
Modular arithmetic	

Self Assessment

1. Time complexity in hashing is ___

A. (log o)
B. (log n)
C. (1)
D. None of above

2. The values returned by a hash function are called ____

A. Hash values
B. Hash codes
C. Hash sums
D. All of above

3. Methods for calculating the hash function are____

a) Division method
b) Folding method
c) Mid square method
d) All of above

4. Hashing is process of__

A. Encrypt data
B. Converting an input of any length into a fixed size string
C. Calculate mean of number
D. None of above

5. Hash table components are_

A. Key
B. Value
C. Both key and value
D. None of above

6. Which operator is used in hashing ___

A. +
B. *
C. %
D. /

7. Which is not characteristics of hash function ____

A. Less collisions
B. Uniform Distribution
C. Static
D. All of above

8. Which is not a methods for calculating the hash function ____

A. Folding
B. Mid square
C. Square
D. Division

9. (h(x)= x%10), 10 represent in statement __

A. Size of hash function
B. Size of hash key
C. Size of hash table
D. None of above

10. (h(x)= x%10), x represent in statement _

A. Key value
B. Hash table size
C. Hash function
D. None of above

11. Fold shift and Fold boundary methods used in ___

A. Division method
B. Mid square method
C. Multiplication Method
D. None of above

12. h(k) = floor(n(kA mod 1)), statement represent which method ____

A. Division method
B. Multiplication Method
C. Mid square method
D. Pairing method

13. Knuth recommends value for A is ____

A. 0.61803
B. 0.62347
C. 0.71803
D. None of above

14. Golden ratio is recommended by __

A. Smith K
B. Knuth
C. George S
D. None of above

15. In multiplication method the value of A is between _

A. 2 and 4
B. 2 and 3
C. 0 and 1
D. None of above

Answers for Self Assessment

l.	C	2.	D	3.	D	4.	B	5.	C
6.	C	7.	C	8.	C	9.	C	10.	A
11.	D	12.	B	13.	A	14.	B	15.	C

Review Question

1. Discuss hashing.
2. What is the significance of hashing in data structure?
3. Define hash function with suitable example.
4. Give an example mid square method.
5. Differentiate between division method and multiplication method.
6. Define Linear Probing and data bucket.

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Unit 13: Collision Resolution

CONTENTS

Objectives

After studying this unit, you will be able to:

- Discus separate chaining
- Understand open addressing-linear probing
- Learn quadratic probing

Introduction

The implementation of hash tables is frequently called hashing. Hashing is a technique used for performing insertions, deletions, and finds in constant average time. Tree operations that require any ordering information among the elements are not supported efficiently. Thus, operations such as findMin, findMax, and the printing of the entire table in sorted order in linear time are not supported.

Note that straightforward hashing is not without its problems, because for almost all hash functions, more than one key can be assigned to the same position. For example, if the hash function h1 applied to names returns the ASCII value of the first letter of each name (i.e., h1(name) = name[0]), then all names starting with the same letter are hashed to the same position. This problem can be solved by finding a function that distributes names more uniformly in the table. For example, the function h_2 could add the first two letters (i.e., h_2(name) = name[0] + name[1]), which is better than h1. But even if all the letters are considered (i.e., h_3(name) = name[0] + · · · + name[strlen(name) – 1]), the possibility of hashing different names to the same location still exists. The function h3 is the best of the three because it distributes the names most uniformly for the three defined functions, but it also tacitly assumes that the size of the table has been increased. If the table has only 26 positions, which is the number of different values returned by h1, there is no improvement using h3 instead of h1. Therefore, one more factor can contribute to avoiding conflicts between hashed keys, namely, the size of the table. Increasing this size may lead to better hashing, but not always! These two factors—hash function and table size—may minimize the number of collisions, but they cannot completely eliminate them. The problem of collision has to be dealt with in a way that always guarantees a solution.

13.1 Collision Resolution

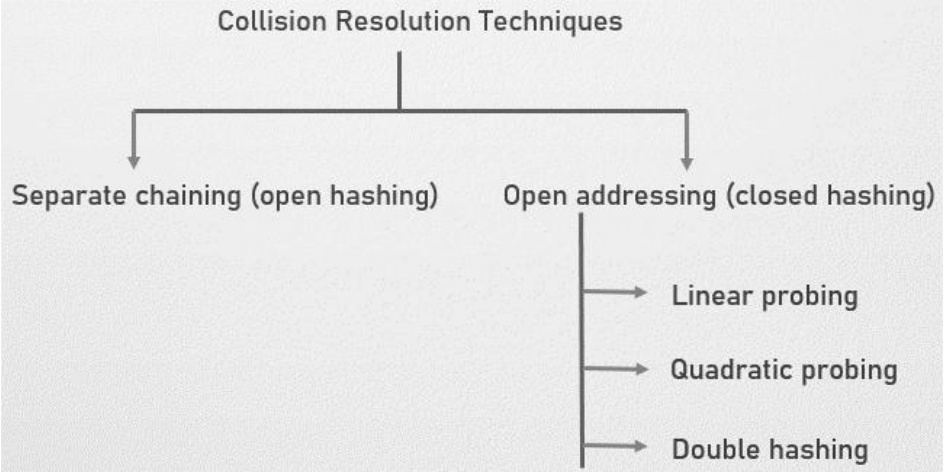
When two keys or hash values compete with a single hash table slot, then Collision occur.To resolve collision we use collision resolution techniques.Collisions can be reduced with a selection of a good hash function.

Collision Resolution Techniques

There are two types of collision resolution techniques.

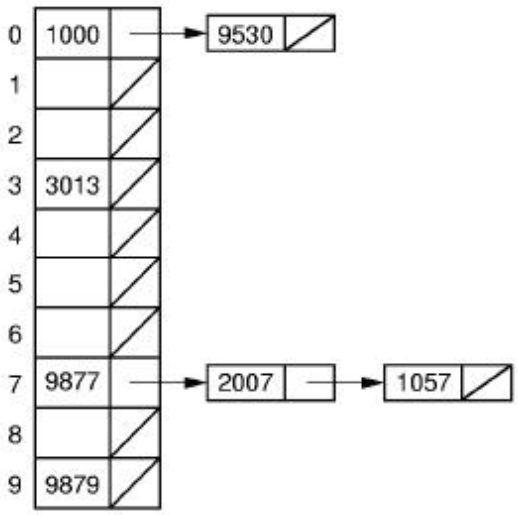
Separate chaining (open hashing)

Open addressing (closed hashing)



Open Hashing

The simplest form of open hashing defines each slot in the hash table to be the head of a linked list. All records that hash to a particular slot are placed on that slot's linked list. The figure below illustrates a hash table where each slot stores one record and a link pointer to the rest of the list.



Records within a slot's list can be ordered in several ways: by insertion order, by key value order, or by frequency-of-access order. Ordering the list by key value provides an advantage in the case of an unsuccessful search, because I know to stop searching the list once you encounter a key that is greater than the one being searched for. If records on the list are unordered or ordered by frequency, then an unsuccessful search will need to visit every record on the list.

Given a table of size M storing N records, the hash function will (ideally) spread the records evenly among the M positions in the table, yielding on average N/M records for each list. Assuming that the table has more slots than there are records to be stored, you can hope that few slots will contain

more than one record. In the case where a list is empty or has only one record, a search requires only one access to the list. Thus, the average cost for hashing should be Θ(1). However, if clustering causes many records to hash to only a few of the slots, then the cost to access a record will be much higher because many elements on the linked list must be searched.

Open hashing is most appropriate when the hash table is kept in main memory, with the lists implemented by a standard in-memory linked list. Storing an open hash table on disk in an efficient way is difficult, because members of a given linked list might be stored on different disk blocks. This would result in multiple disk accesses when searching for a particular key value, which defeats the purpose of using hashing.

Let:

1. U be the universe of keys:

(a) Integers

(b) Character strings

(c) Complex bit patterns

2. B the set of hash values (also called the buckets or bins). Let B = {0, 1,..., m - 1}where m > 0 is a positive integer.

A hash function h: U → B associates buckets (hash values) to keys.

Two main issues:

Collisions

If x1 and x2 are two different keys, it is possible that h(x1) = h(x2). This is called a collision. Collision resolution is the most important issue in hash table implementations.

Hash Functions

Choosing a hash function that minimizes the number of collisions and also hashes uniformly is another critical issue.

Closed Hashing

1. All elements are stored in the hash table itself

2. Avoids pointers; only computes the sequence of slots to be examined.

3. Collisions are handled by generating a sequence of rehash values.

h: U × U → {0, 1, 2,..., m - 1}

universe of primary keys probe number

4. Given a key x, it has a hash value h(x,0) and a set of rehash values

h(x, 1), h(x,2), . . . , h(x, m-1)

5. I require that for every key x, the probe sequence

< h(x,0), h(x, 1), h(x,2), . . . , h(x, m-1)>

be a permutation of <0, 1, ..., m-1>.

This ensures that every hash table position is eventually considered as a slot for storing a record with a key value x.

Search (x, T)

Search will continue until you fi nd the element x (successful search) or an empty slot (unsuccessful search).

Delete (x, T)

1. No delete if the search is unsuccessful.

2. If the search is successful, then put the label DELETED (different from an empty slot).

Insert (x, T)

1. No need to insert if the search is successful.

2. If the search is unsuccessful, insert at the fi rst position with a DELETED tag.

13.2 Separate Chaining

In this technique, a linked list is created from the slot in which collision has occurred, after which the new key is inserted into the linked list. This linked list of slots looks like a chain, so it is called separate chaining.

Separate chaining, is to keep a list of all elements that hash to the same value. We can use the Standard Library list implementation. If space is tight, it might be preferable to avoid their use (since these lists are doubly linked and waste space).

Performance of Chaining

Load factor α = n/m

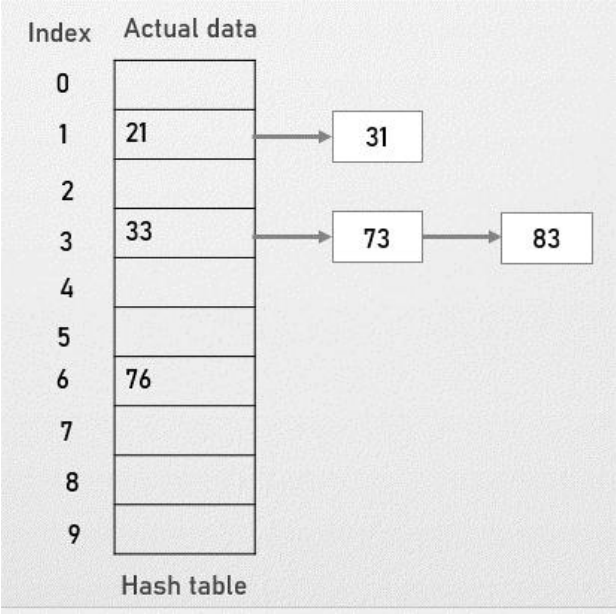
m = Number of slots in hash table

n = Number of keys to be inserted in hash table

Expected time to search = O(1 + α)

Expected time to delete = O(1 + α)

Time to insert = O(1)



Time complexity

For Searching

In worst case, all the keys might map to the same bucket of the hash table.

In such a case, all the keys will be present in a single linked list.

Sequential search will have to be performed on the linked list to perform the search.

Worst case complexity for searching is O(n).

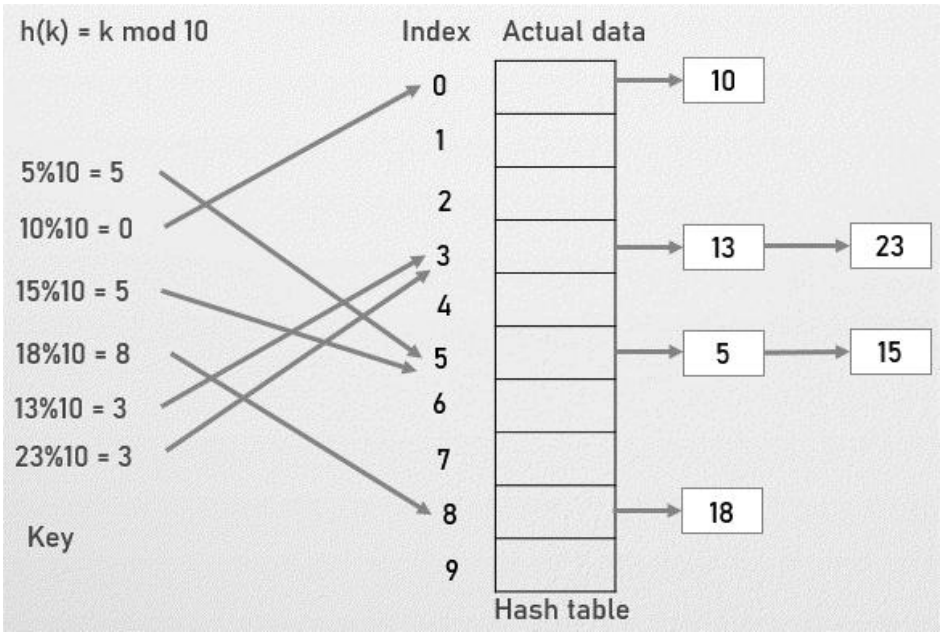
For Deletion

In worst case, the key might have to be searched first and then deleted.

In worst case, time taken for searching is O(n).

Worst case complexity for deletion is O(n).

Example: Separate chaining



Advantages of separate chaining

It is easy to implement.

The hash table never fills full, so we can add more elements to the chain.

It is mostly used when it is unknown how many and how frequently keys may be inserted or deleted.

It is less sensitive to the function of the hashing.

Disadvantages of separate chaining

The cache performance of chaining is not good.

The memory wastage is too much in this method.

It requires more space for element links.

If the chain becomes long, then search time can become O(n) in the worst case.

13.3 Open Addressing

The open addressing technique requires a hash table with fixed and known size. All elements are stored in the hash table itself. The size of the table must be greater than or equal to the total number of keys.During insertion, if a collision is encountered, alternative cells are tried until an empty bucket is found.

In case of collision:

Probing is performed until an empty bucket is found.

Once an empty bucket is found, the key is inserted.

Probing is performed in accordance with the technique used for open addressing.

Operations in Open Addressing

Insert (k): Keep probing until an empty slot is found. Once an empty slot is found, insert k.

Search (k): Keep probing until slot's key doesn't become equal to k or an empty slot is reached.

Delete: The key is first searched and then deleted. After deleting the key, that particular bucket is marked as "deleted".

Closed Hashing methods:

- Linear Probing
- Quadratic probing
- Double hashing

13.4 Linear Probing

In linear probing fixed sized hash table is used and when hash collision situation occur then, we linearly traverse the table in a cyclic manner to find the next empty slot.In this approach searches are performed sequentially so it's known as linear probing.

In this, when the collision occurs, we perform a linear probe for the next slot, and this probing is performed until an empty slot is found. In linear probing, the worst time to search for an element is O(table size). The linear probing gives the best performance of the cache but its problem is clustering. The main advantage of this technique is that it can be easily calculated.

Let hash(x) be the slot index computed using a hash function and n be the table size

If slot hash(x) % n is full, then we try (hash(x) + 1) % n

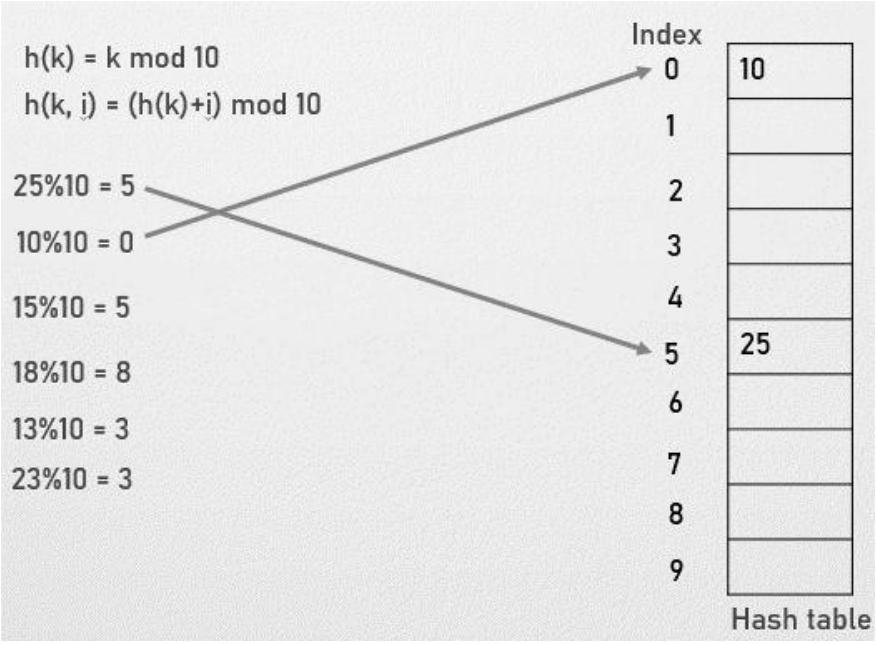
If (hash(x) + 1) % n is also full, then we try

(hash(x) + 2) % n

If (hash(x) + 2) % n is also full, then we try

(hash(x) + 3) % n so on…

Example: Linear Probing



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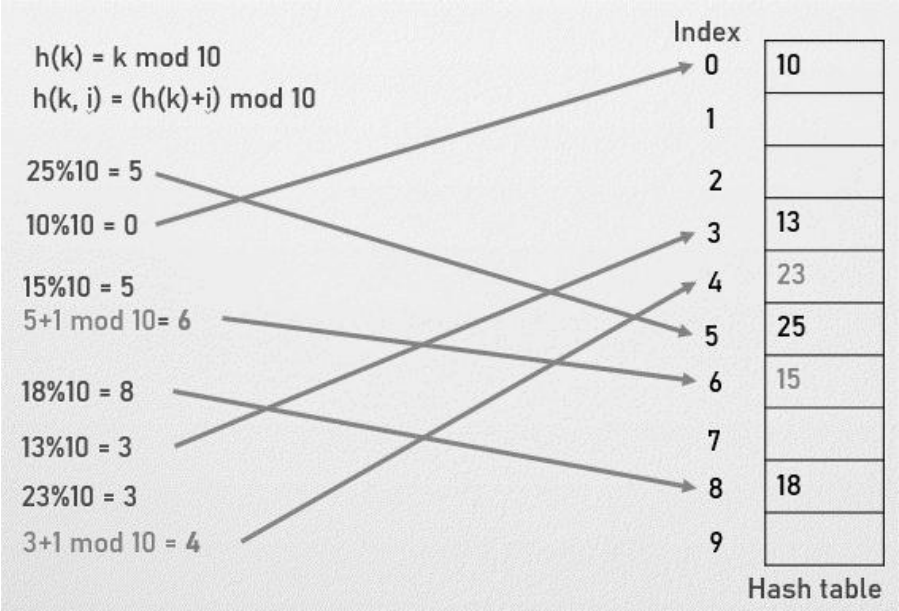


b

Advantage

It is easy to compute.

Disadvantage

The clustering is major problem with linear probing

Many consecutive elements form groups.

It takes too much time to find an empty slot.

Time complexity

Worst time to search for an element is O(table size).

13.5 Quadratic Probing

Quadratic probing is a collision resolution method that eliminates the primary clustering problem of linear probing. Quadratic probing is what you would expect—the collision function is quadratic.

It is an open-addressing scheme.Here we look for i^2 slot in ith iteration if the given hash value x collides in the hash table.It is used to eliminate the primary clustering problem of linear probing.

In quadratic probing the sequence is that $H+1^2$, $H+2^2$, $H+3^2$,....$H+K^2$

The hash function for quadratic probing is

hi(X) = (Hash(X) + $F(i)^2$) % Table Size for i = 0, 1, 2, 3,...etc.

If the slot hash(x) % S is full, then we try

(hash(x) + 1*1) % S.

If (hash(x) + 1*1) % S is also full, then we try (hash(x) + 2*2) % S.

If (hash(x) + 2*2) % S is also full, then we try (hash(x) + 3*3) % S.

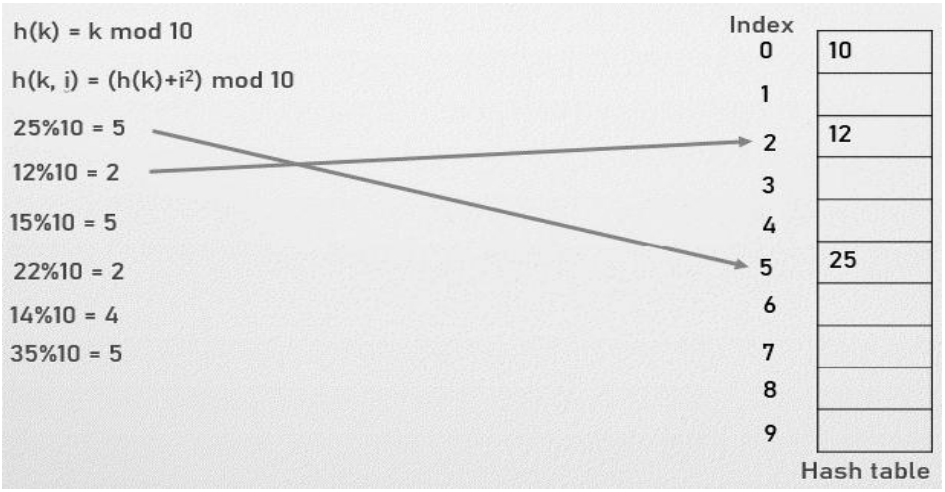
This process continue until an empty slot is found.

In Quadratic Probing to get slot your table size must meet these requirements:

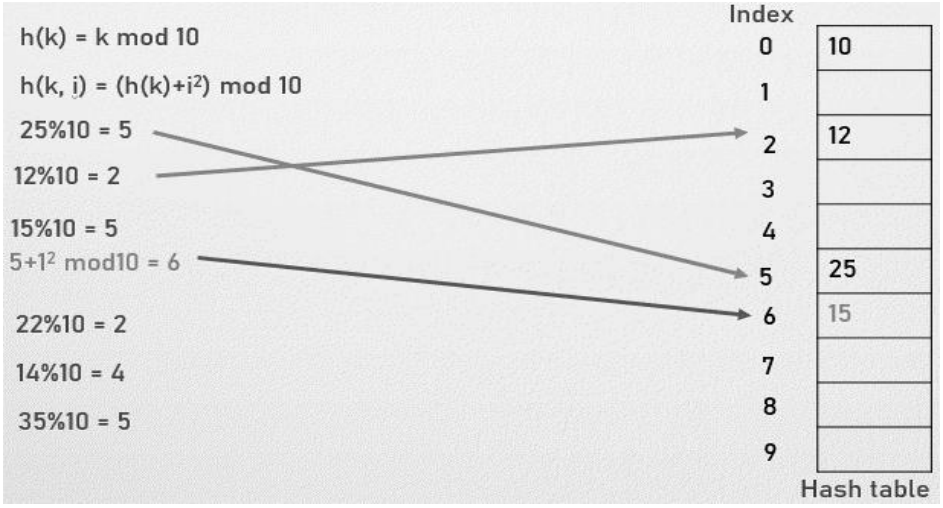
-Be a prime number

-never be more than half full (even by one element)

Example : Quadratic probing



a



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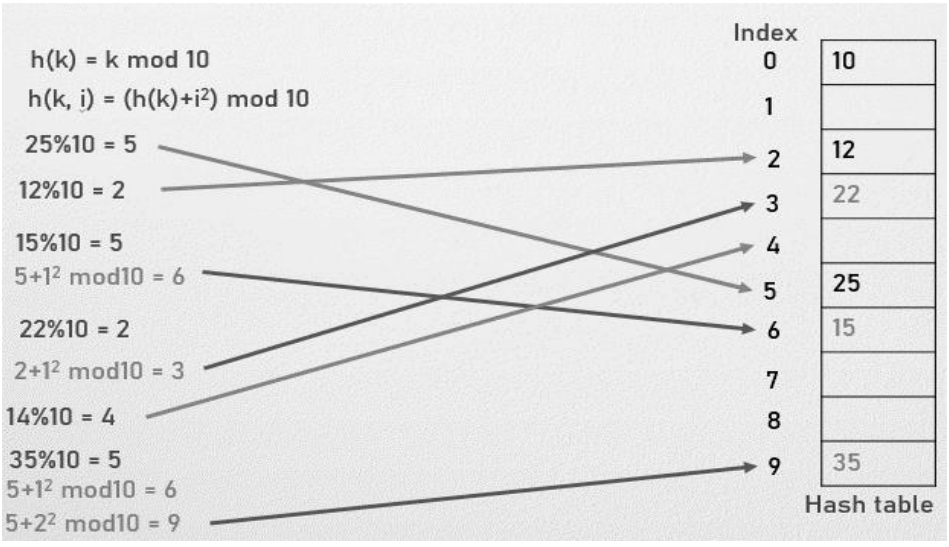

c

Advantage:

Primary clustering problem resolved

Disadvantage:

Secondary clustering

No guarantee for finding slots

Separate Chaining Vs. Open Addressing

Separate Chaining	Open Addressing
Keys are stored inside the hash table as well as outside the hash table.	All the keys are stored only inside the hash table. No key is present outside the hash table.
The number of keys to be stored in the hash table can even exceed the size of the hash table.	The number of keys to be stored in the hash table can never exceed the size of the hash table.
Deletion is easier.	Deletion is difficult.
Extra space is required for the pointers to store the keys outside the hash table.	No extra space is required.
Cache performance is poor. This is because of linked lists which store the keys outside the hash table.	Cache performance is better. This is because here no linked lists are used.
Some buckets of the hash table are never used which leads to wastage of space.	Buckets may be used even if no key maps to those particular buckets.

Summary

- When two keys or hash values compete with a single hash table slot, then Collision occur.
- To resolve collision we use collision resolution techniques. Collisions can be reduced with a selection of a good hash function.
- Hash functions are mostly used in hash tables, to quickly locate a data record (for example, a dictionary definition) given its search key (the headword).
- Specifically, the hash function is used to map the search key to the index of a slot in the table where the corresponding record is supposedly stored.
- In linear probing fixed sized hash table is used and when hash collision situation occur then, we linearly traverse the table in a cyclic manner to find the next empty slot.
- Quadratic probing is a collision resolution method that eliminates the primary clustering problem of linear probing.

Keywords

Separate chaining	Quadratic probing
Linear Probing	Open Addressing
Hash function	Load factor

Self Assessment

1. Which is part of collision resolution technique ___

A. Separate chaining
B. Open addressing
C. Double hashing
D. All of above

2. Open addressing includes ____

A. Linear probing
B. Quadratic probing
C. Double hashing
D. All of above

3. In Load factor $\alpha = n/m$, m represent ____

A. Number of keys
B. Number of slots in hash table
C. Hash function
D. None of above

4. In Load factor $\alpha = n/m$, n represent __

A. Number of slots in hash table
B. Hash function
C. Number of keys to be inserted in hash table
D. None of above

5. Worst-case complexity for searching in Separate chaining is _

A. Log (1)
B. (n)
C. log (0)
D. None of above

6. Worst-case complexity for deletion in Separate chaining is _

A. (n)
B. log (1)
C. log (2)
D. None of above

7. Advantages of separate chaining.

A. It is less sensitive to the function of the hashing
B. The hash table never fills full, so we can add more elements to the chain
C. It is easy to implement
D. All of above

8. Quadratic probing is part of __

A. Open hashing
B. Closed hashing
C. Linear probing
D. None of above

9. Double hashing is part of ___

A. Open hashing
B. Closed hashing
C. Linear probing
D. All of above

10. Which is not part of open addressing.

A. Linear probing
B. Quadratic probing
C. Open hashing
D. All of above

11. Operations in Open Addressing are ___

A. Insert
B. Delete
C. Search
D. All of above

12. In open addressing __

A. All elements are stored in the hash table itself
B. The size of the table must be greater than or equal to the total number of keys.
C. During insertion, if a collision is encountered, alternative cells are tried until an empty bucket is found.
D. All of above

13. Clustering is major problem in _

A. Linear probing
B. Quadratic probing
C. Double hashing
D. None of above

14. Which one is most relevant to Quadratic Probing?

A. h(k) = k mod 10
B. h(k, i) = (h(k)+i2) mod 10
C. h(k, i) = (h(k)+i) mod 10
D. None of above

15. Which one is not relevant to Linear Probing?

A. h(k, i) = (h(k)+i) mod 10
B. h(k) = k mod 10
C. h(k, i) = (h(k)+i2) mod 10
D. None of above

Answers for Self Assessment

l.	D	2.	D	3.	B	4.	C	5.	B
6.	A	7.	D	8.	B	9.	B	10.	C
11.	D	12.	D	13.	A	14.	B	15.	C

Review Questions

1. Discuss significance of collision resolution.
2. Differentiate between open hashing and closed hashing.
3. Explain cluster problem and its solution in hashing.
4. What are the advantages of quadratic probing?
5. Give an example of linear probing.
6. Discuss load factor in hashing.

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Unit 14: More on Hashing

CONTENTS

Objectives

After studying this unit, you will be able to:

- Discus Double hashing techniques
- Understand load factor
- Learn rehashing

Introduction

When two keys or hash values compete with a single hash table slot, then Collision occur.To resolve collision we use collision resolution techniques. Collisions can be reduced with a selection of a good hash function.

The open addressing technique requires a hash table with fixed and known size. All elements are stored in the hash table itself. The size of the table must be greater than or equal to the total number of keys.During insertion, if a collision is encountered, alternative cells are tried until an empty bucket is found.

In case of collision:Probing is performed until an empty bucket is found.Once an empty bucket is found, the key is inserted.Probing is performed in accordance with the technique used for open addressing.

14.1 Double Hashing

Double Hashing is a hashing collision resolution technique in open addressed Hash tables.In double hashing, there are two hash functions.The second hash function is used to provide an offset value in case the first function causes a collision.Second hash function used to remove the collision when you encountered the collision.

Double hashing uses two hash functions, one to find the initial location to place the key and a second to determine the size of the jumps in the probe sequence. The ith probe is

$$h(k, i) = (h1(k) + i * h2(k)) \bmod m.$$

Keys that hash to the same location, are likely to hash to a different jump size, and so will have different probe sequences. Thus, double hashing avoids secondary clustering by providing as many as m2 probe sequences. How do we ensure every location is checked? Since each successive probe is offset by h2(k), every cell is probed if h2(k) is relatively prime to m. Two possible ways to ensure

h2(k) is relatively prime to m are, either make m = 2k and design h2(k) so it is always odd, or make m prime and ensure h2(k) < m. Of course, h2(k) cannot equal zero.

Double hashing can be performed using:

(hash1(key) + i * hash2(key)) mod TABLE_SIZE

Here hash1() and hash2() are hash functions

First hash function:

hash1(key) = key mod TABLE_SIZE

Second hash function is :

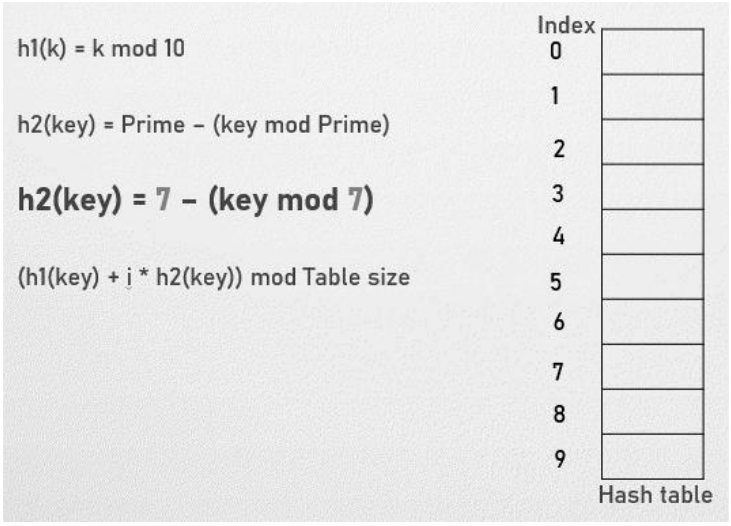
hash2(key) = PRIME – (key mod PRIME)

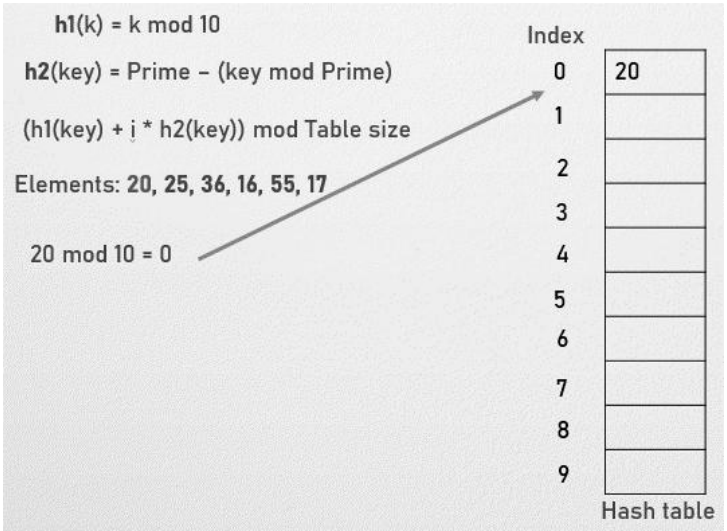
Where PRIME is a prime smaller than the TABLE_SIZE

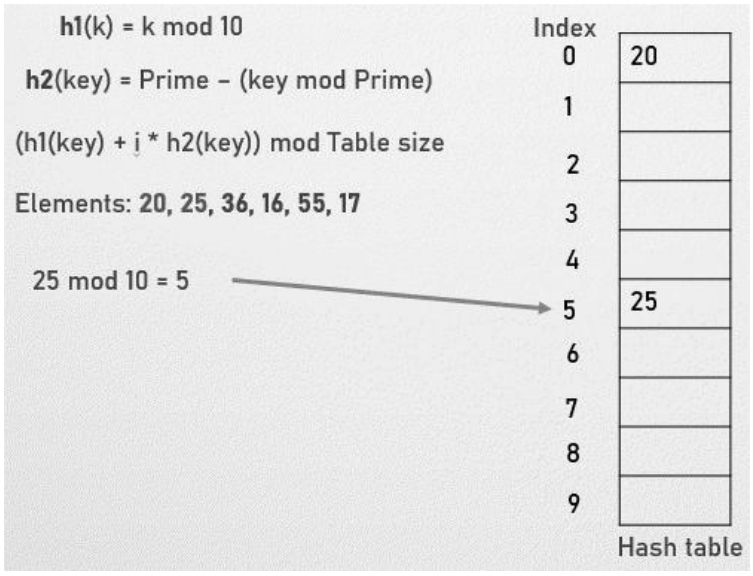
A good second Hash function is:It must never evaluate to zero. Must make sure that all cells can be probed.

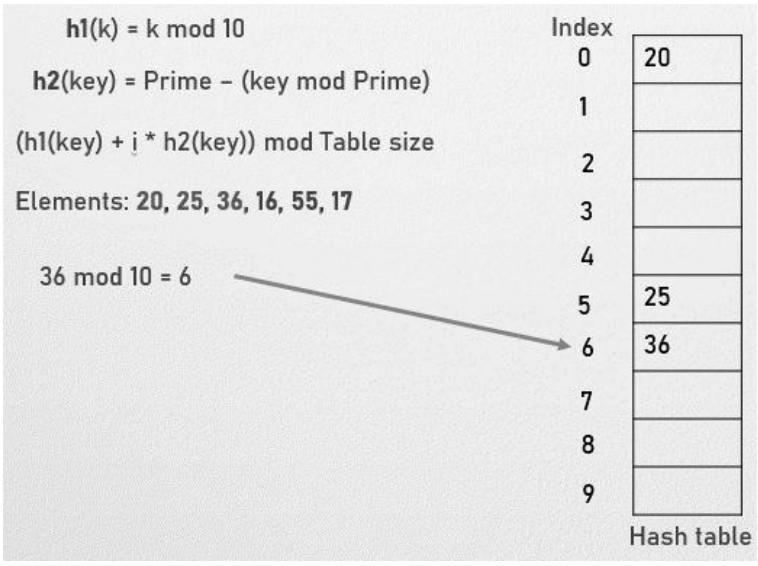
Example: Double hashing

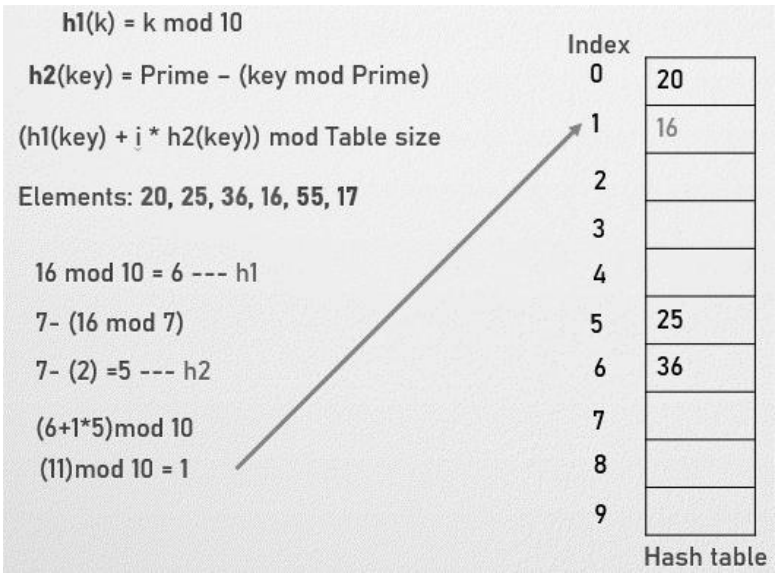
Elements: 20, 25, 36, 16, 55, 17 table size= 10 prime number = 7

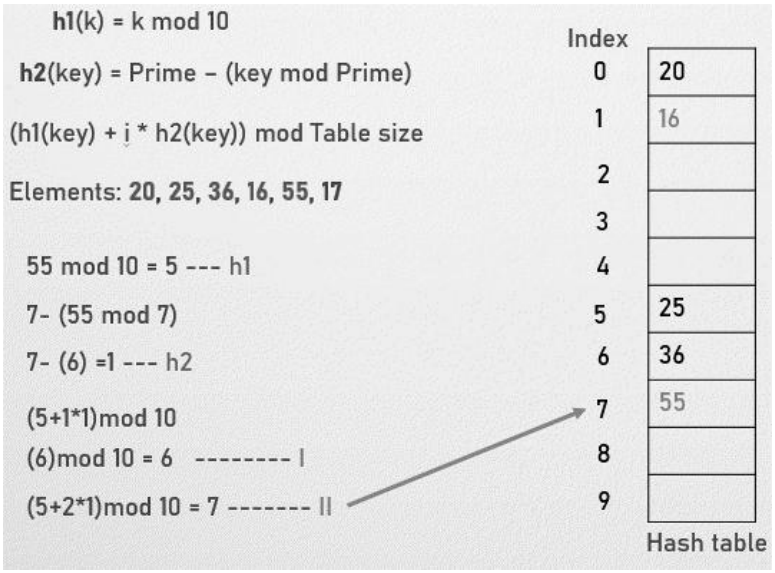


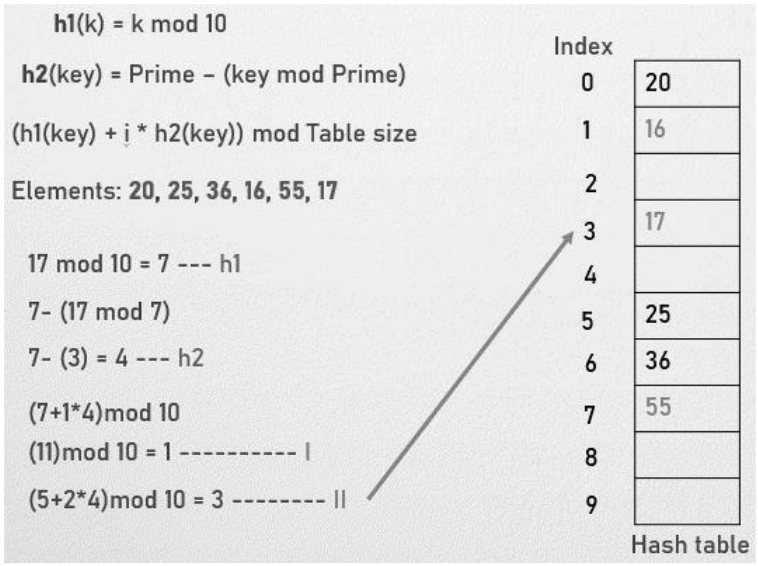



h1(k) = k mod 10
h2(key) = Prime – (key mod Prime)
(h1(key) + i * h2(key)) mod Table size
Elements: 20, 25, 36, 16, 55, 17
25 mod 10 = 5
Index
0
1
2
3
4
5
6
7
8
9
20
25
Hash table


h1(k) = k mod 10
h2(key) = Prime – (key mod Prime)
(h1(key) + i * h2(key)) mod Table size
Elements: 20, 25, 36, 16, 55, 17
36 mod 10 = 6
Index
0
1
2
3
4
5
6
7
8
9
20
25
36
Hash table


h1(k) = k mod 10
h2(key) = Prime – (key mod Prime)
(h1(key) + i * h2(key)) mod Table size
Elements: 20, 25, 36, 16, 55, 17
16 mod 10 = 6 --- h1
7- (16 mod 7)
7- (2) =5 --- h2
(6+1*5)mod 10
(11)mod 10 = 1
Index
0
1
2
3
4
5
6
7
8
9
20
16
25
36
Hash table





Double hashing highlights

- Computational cost is higher.
- No primary clustering.
- No secondary clustering.
- Double hashing can find the next free slot faster than the linear probing approach.
- Double hashing is used for uniform distribution of records throughout a hash table.
- Double hashing is useful if an application requires a smaller hash table.

14.2 Rehashing

This is another method of collision handling. In this method you fi nd an alternative empty location by modifying the hash function, and applying the modifi ed hash function to the colliding symbol. For example, if x is symbol and h(x) = i, and if the ith location is already occupied, then I modify the hash function h to h1, and fi nd out h1(x), if h1(x) =j, and jth location is empty, then I accommodate x in the jth location. Otherwise you once again modify h1 to some h2 and repeat the process till the collision gets handled. Once the collision gets handled we revert back to the original hash function before considering the next symbol.

It is process of re-calculating the hash code of already stored entries.The Hash table provides Constant time complexity of insertion and searching, provided the hash function is able to distribute the input load evenly. In case of Collision, the time complexity can go up to O(N) in the worst case. Rehashing of a hash map is done when the number of elements in the map reaches the maximum threshold value.

When load factor increases to more then its predefined value, complexity increases.To overcome this problem, size of array is increased and all the values are hashed again and stored in new double size array to maintain a low load factor and complexity.

Load factor

Load factor is number of element (n) divide by number of bucket (m).

Load factor (λ) =n/m

- λ < 1 i.e. m>n

if λ < 1 then no need to apply rehashing

if λ > 1 then we need to increase number of buckets

Increase in bucket size is known as rehashing.The Load Factor decides "when to increase the size of the hash Table."

Rehashing steps

-Increase number of buckets.

-Modify hash function

Hash function before rehashing : x mod m

after rehashing x mod m′

-apply changed hash function to existing elements.

m′ calculation

m′ = closet prime number of 2m

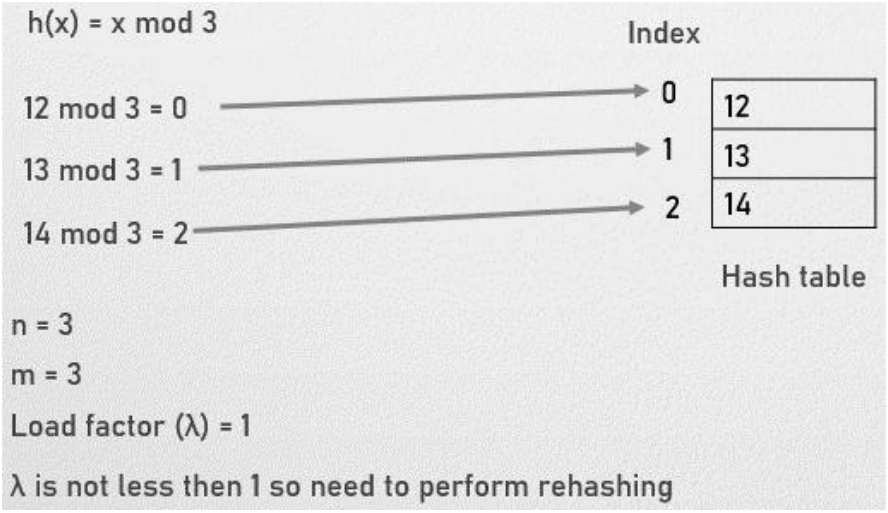
Example:

m=3 m′= 2(3) = 6

Closet prime number = 5 or 7.

Example: Rehashing

Elements: 12, 13, 14 table size = 3



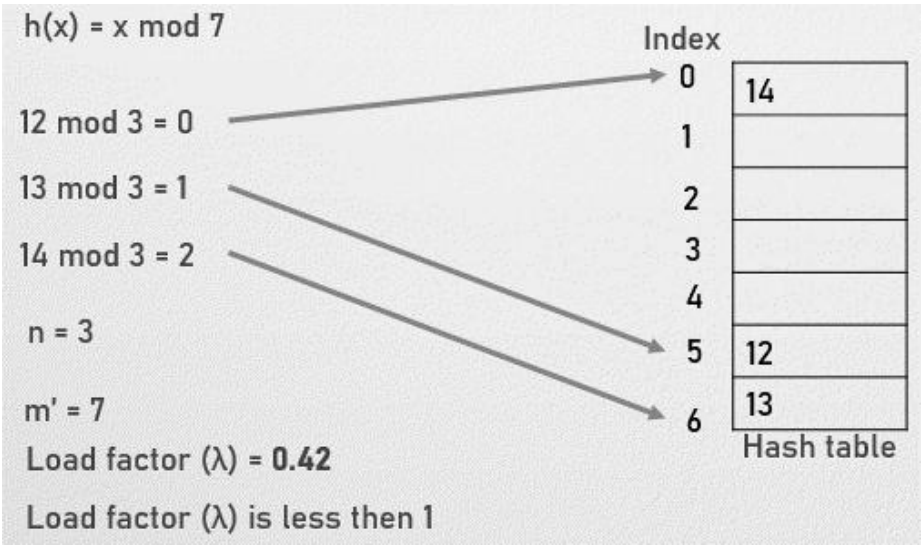
m′ = 2n

m′ = 2x3 => 6

Nearest prime numbers are 5 and 7

m′ = 7

x mod m′ => x mod 7



A Comparison of Rehashing Methods

Linear Probing	m distinct probe sequences	Primary clustering
Quadratic Probing	m distinct probe sequences	No primary clustering; but secondary clustering
Double Probing	m^2 distinct probe sequences	No primary clustering No secondary clustering

Complexity of Rehashing

Time complexity – O(n)

Space complexity – O(n)

Summary

- Rehashing schemes use a second hashing operation when there is a collision.
- The open addressing technique requires a hash table with fixed and known size.
- Double Hashing is a hashing collision resolution technique in open addressed Hash tables.
- Rehashing is process of re-calculating the hash code of already stored entries.
- The load factor in HashMap is basically a measure that decides when exactly to increase the size of the HashMap to maintain the same time complexity of O(1).
- The Load Factor decides "when to increase the size of the hash Table."

Keywords

Rehashing

Hash map

Double hashing

Clustering

Load factor

Open addressing

Prime number

Self Assessment

1. Which statement is correct about open addressing?

A. It requires a hash table with fixed and known size.
B. All elements are stored in the hash table itself
C. The size of the table must be greater than or equal to the total number of keys.
D. All of above

2. In double hashing prime value is __

A. Less than table size
B. Greater than table size
C. Equal to table size
D. None of above

3. In double hashing how many hash functions used.

A. 4
B. 2
C. 1
D. 3

4. Which statement is correct about double hashing?

A. The second hash function is used to provide an offset value in case the first function causes a collision.
B. In double hashing, there are two hash functions.
C. Second hash function used to remove the collision when you encountered the collision.
D. All of above

5. Which hash function used in double hashing?

A. (h1(key) + h2(key)) mod Table size
B. (h1(key) + i * (key)) mod Table size
C. (h1(key) + i * h2(key)) mod Table size
D. None of above

6. In which situation second hash function is used in double hashing?

A. In case table size is smaller
B. In case the first function causes a collision
C. In case first hash function provide zero value
D. None of above

7. In statement- h2(key) = 7 – (key mod 7), 7 represent _

A. Table size
B. Key value
C. Prime number

D. None of above

8. Which statement is correct for double hashing technique?

A. No primary clustering
B. No secondary clustering
C. Double hashing can find the next free slot faster than the linear probing approach
D. All of above

9. Which statement is correct about rehashing?

A. In which the table is resized
B. It is process of re-calculating the hash code of already stored entries
C. It is a collision resolution technique
D. All of above

10. In Load factor =n/m, m represents _

A. Number of element
B. Number of key values
C. Number of bucket
D. None of above

11. In which situations rehashing is required.

A. When table is completely full.
B. With quadratic probing when the table is filled half.
C. When insertions fail due to overflow.
D. All of above

12. The value of Load factor in rehashing should be__

a) Less than 1
b) Greater then 1
c) Equal to 0
d) All of above

13. In Load factor =n/m, n represents _

A. Number of bucket
B. Number of element
C. Number of key values
D. None of above

14. Table size is 3 and elements are 12, 13, and 14. Load factor is_

A. 3
B. 2
C. 1

D. 0

15. What is notation for load factor?

A. λ
B. ∞
C. μ
D. Ω

Answers for Assessment

1.	D	2.	A	3.	B	4.	D	5.	C
6.	B	7.	C	8.	D	9.	D	10.	C
11.	D	12.	A	13.	B	14.	C	15.	A

Review Questions

1. What are the conditions for double hashing?
2. Discuss double hashing technique with example.
3. What are the two hash functions used in double hashing?
4. What is significance of load factor in hashing?
5. What are the advantages of double hashing?
6. Discuss steps for rehashing.
7. What is time and space complexity of rehashing?

Further Readings

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