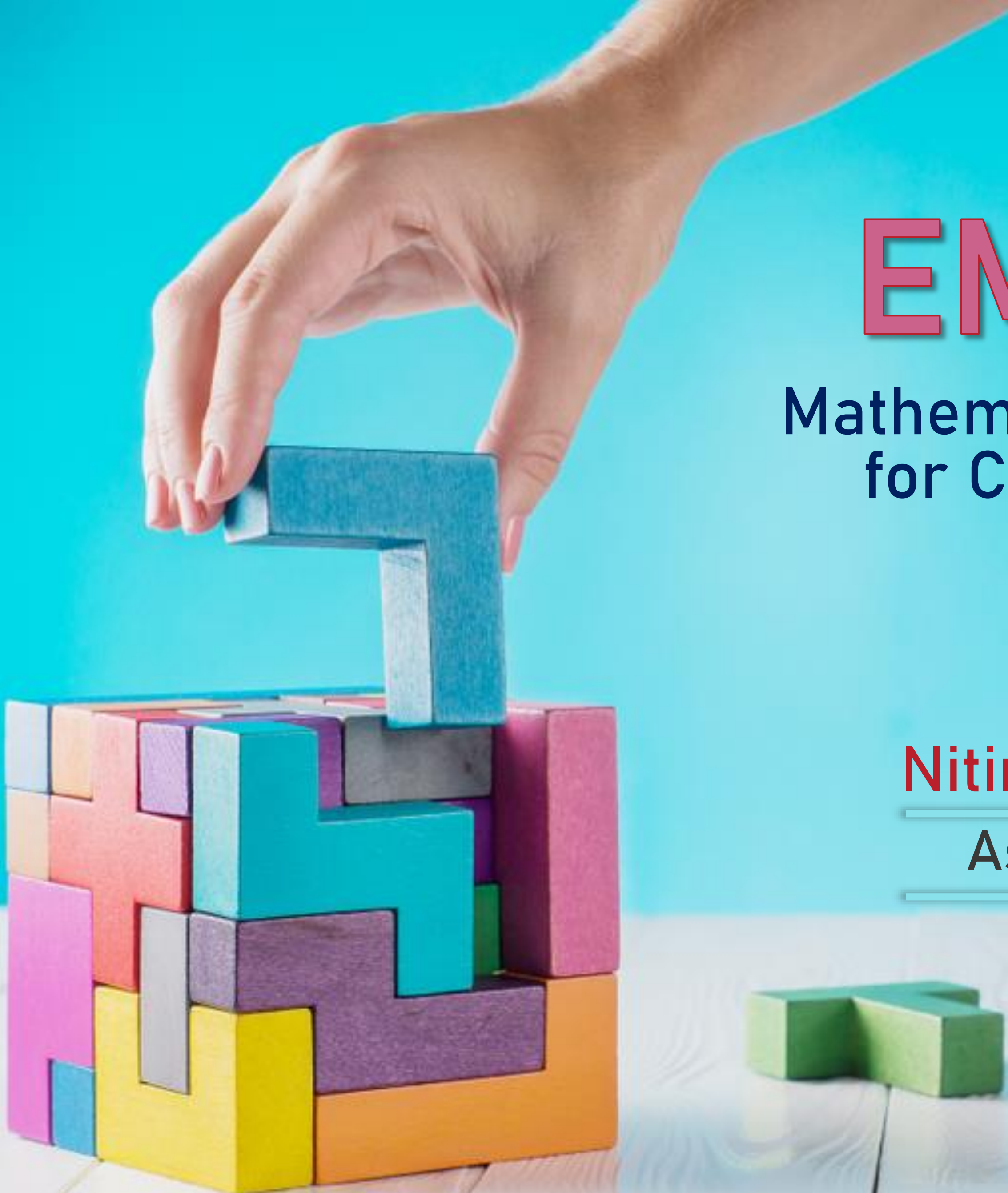


EMTH403

Mathematical Foundation for Computer Science

Nitin K. Mishra (Ph.D.)
Associate Professor

Lecture Outcomes

After this lecture, you will be able to

- understand different types of graphs.
- understand degree of a graph.

Applications of Graphs

Potentially anything (graphs can represent relations, relations can describe the extension of any predicate).

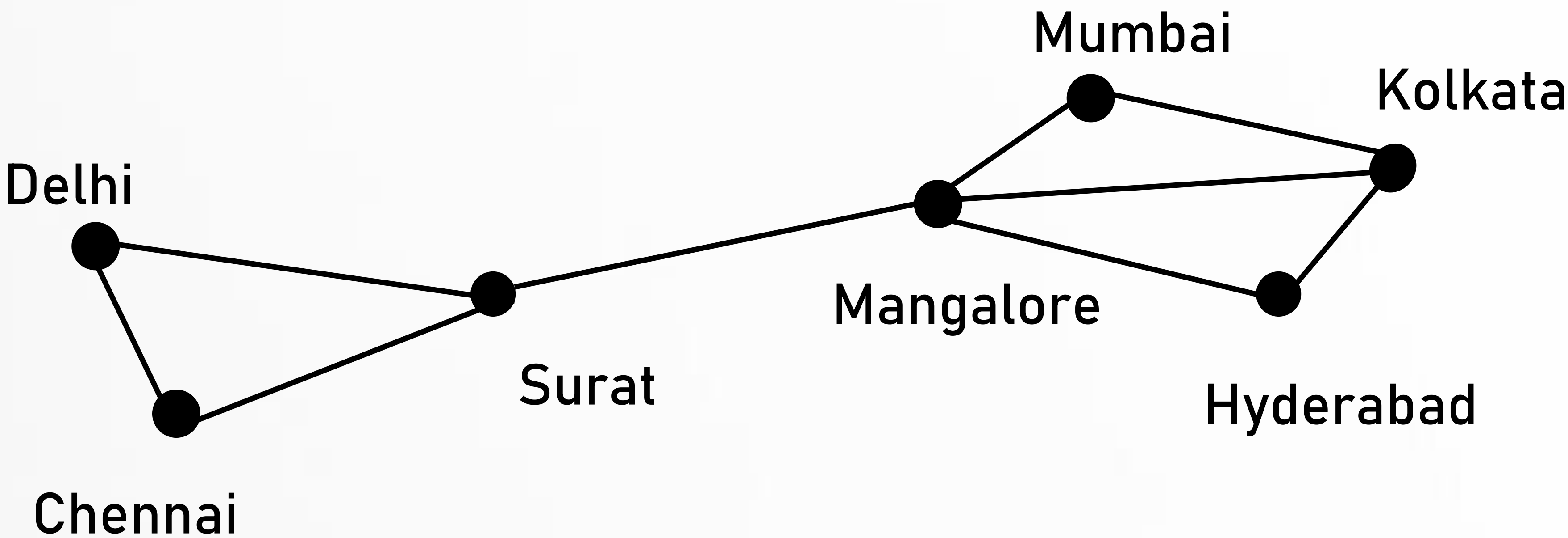
Apps in networking, scheduling, flow optimization, circuit design, path planning.

More apps: Genealogy analysis, computer game-playing, program compilation, object-oriented design etc.

Simple Graph – Definition

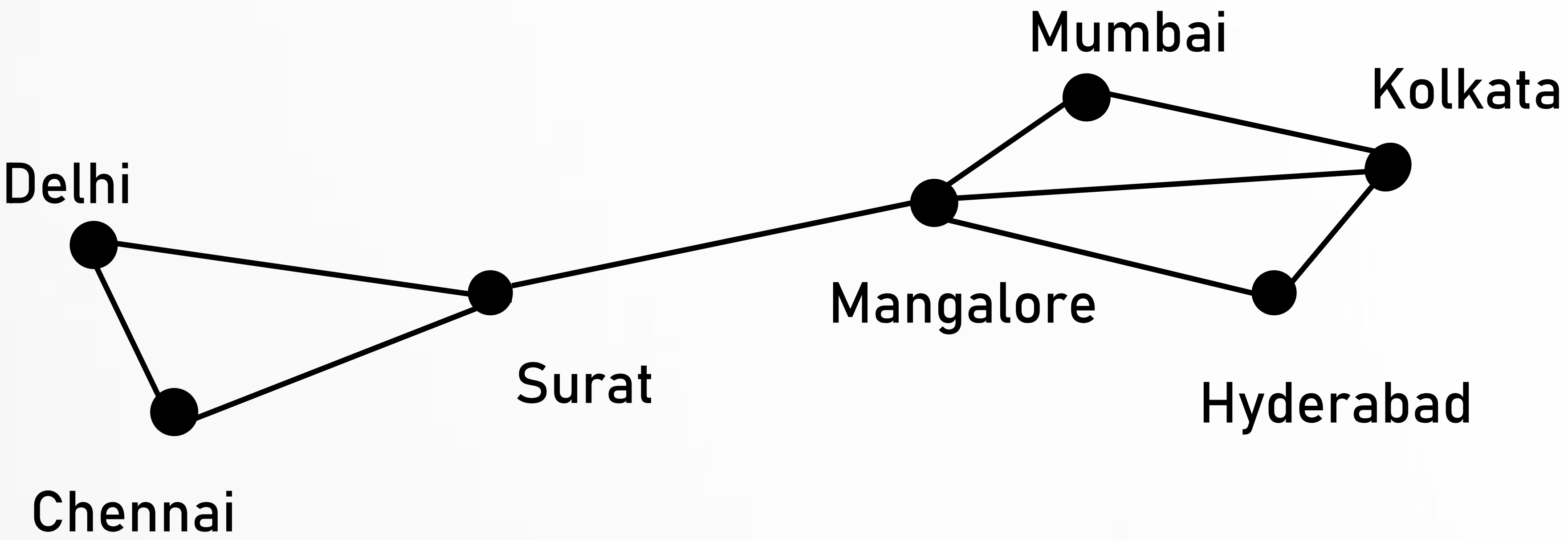
A simple graph G = (V, E) consists of V, a nonempty set of vertices, and E, a set of unordered pairs of distinct elements of V called edges.

Simple Graph – Example 1



How many vertices? How many edges?

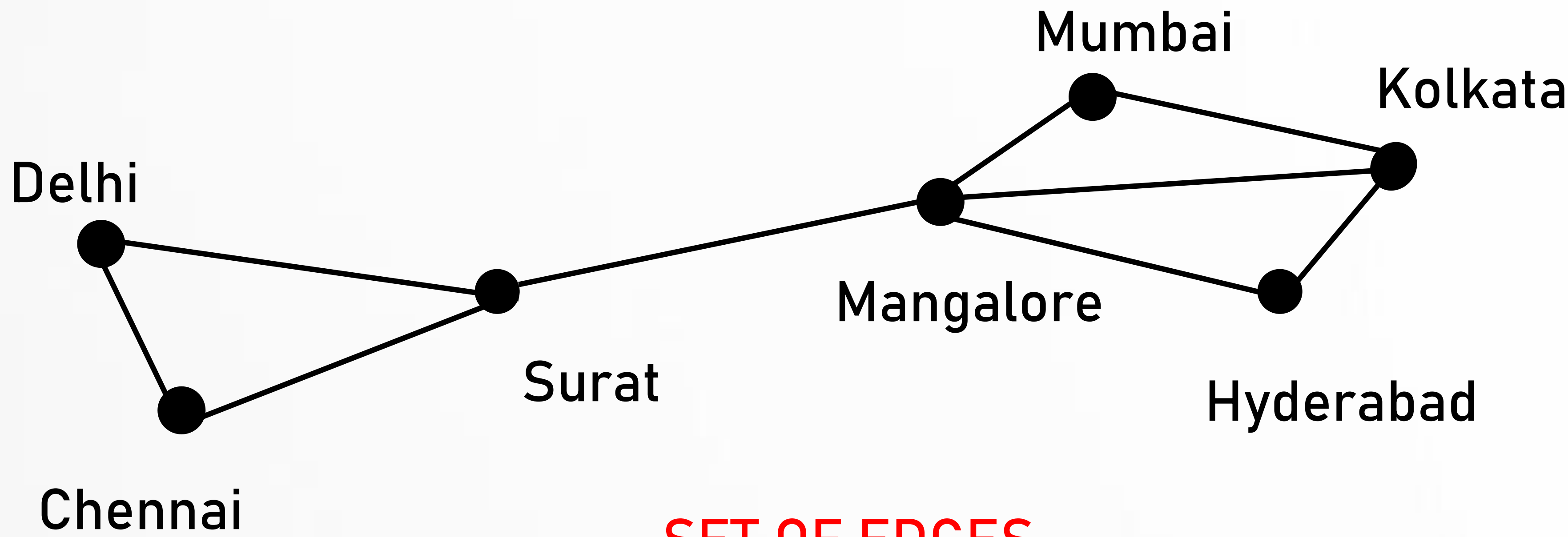
Simple Graph – Example 1



SET OF VERTICES

V = { Mangalore, Surat, Mumbai, Chennai, Kolkata, Delhi, Hyderabad }

Simple Graph – Example 1



SET OF EDGES

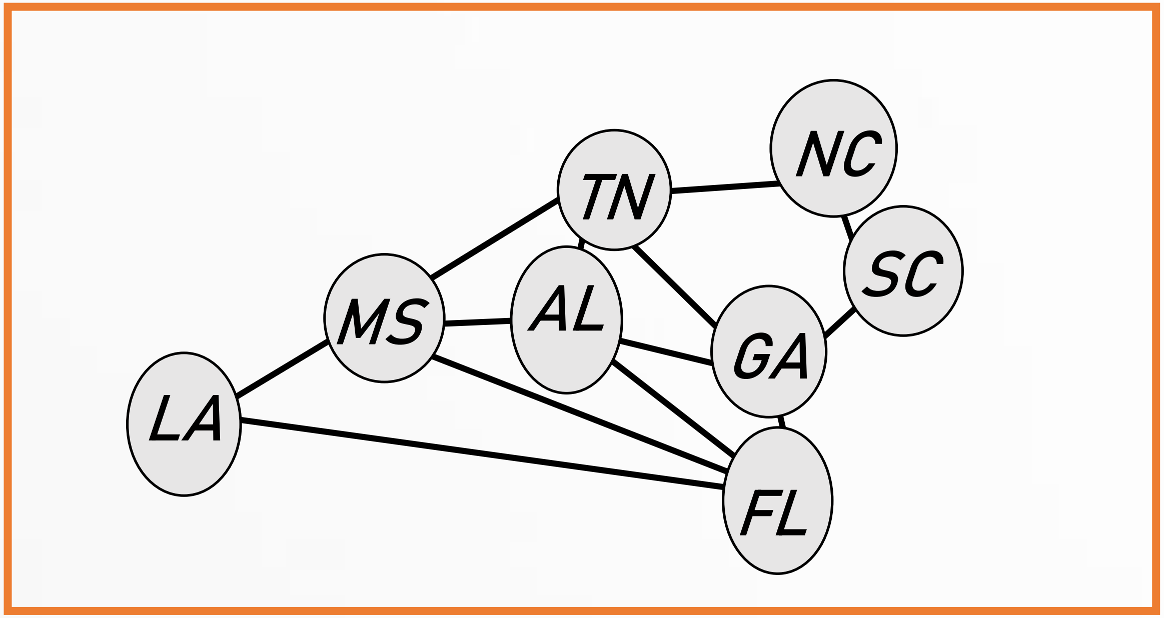
E = { {Delhi, Chennai}, {Delhi, Surat},
{Chennai, Surat}, {Surat, Mangalore},
{Mangalore, Mumbai}, {Mumbai, Kolkata},
{Kolkata, Hyderabad}, {Mangalore, Hyderabad},
{Mangalore, Kolkata} }

Simple Graph – Example 2

Let *V* be the set of states in the far

-southeastern U.S.:

I.e., *V* = {FL, GA, AL, MS, LA, SC, TN, NC}



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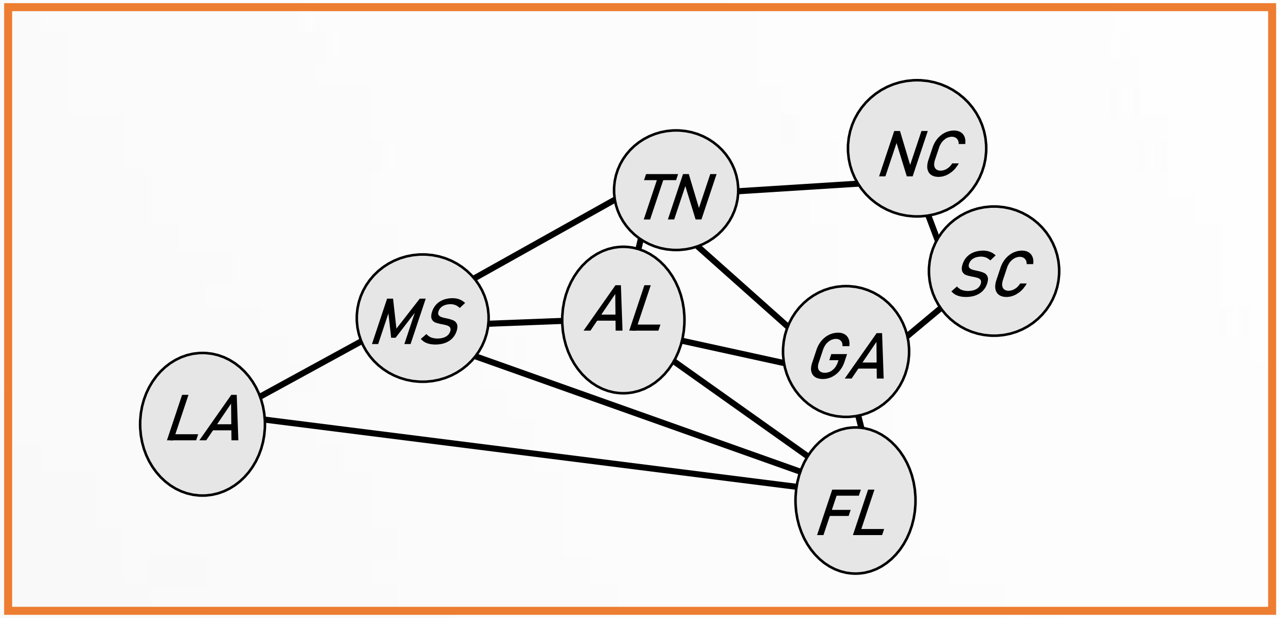
Simple Graph – Example 2

Let E={{ u, v}|u adjoins v}

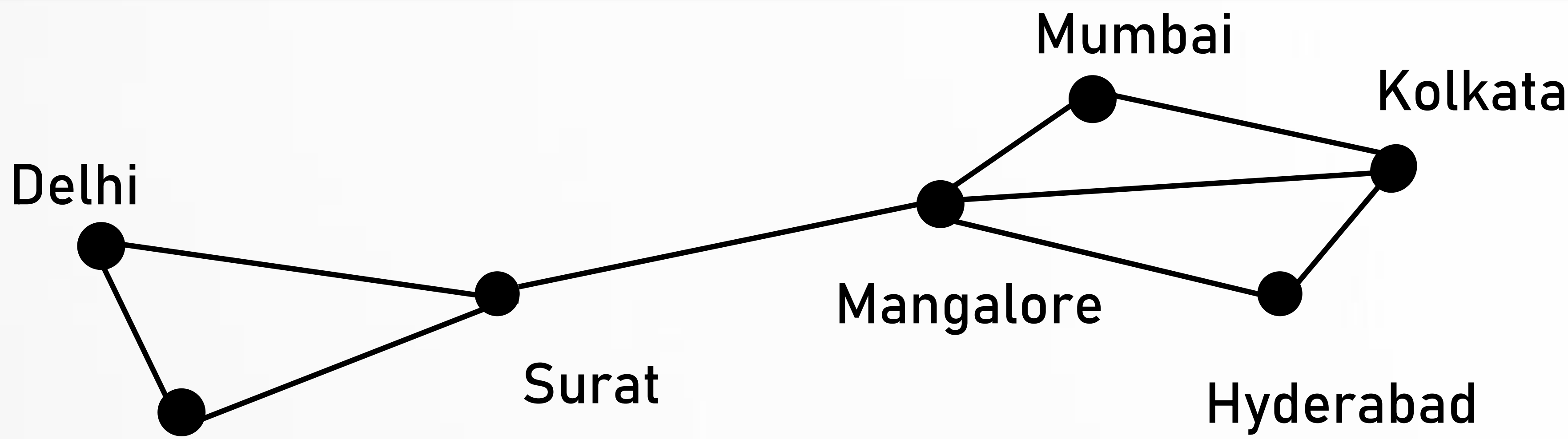
={{FL,GA},{FL,AL},{FL,MS}, {FL,LA},{GA,AL},{AL,MS},

{MS,LA},{GA,SC},{GA,TN}, {SC,NC},{NC,TN},{MS,TN},

{MS,AL}}



Simple Graph – Example 1



The network is made up of computers and telephone lines between computers.

There is at most 1 telephone line between 2 computers in the network.

Each line operates in both directions. No computer has a telephone line to itself.

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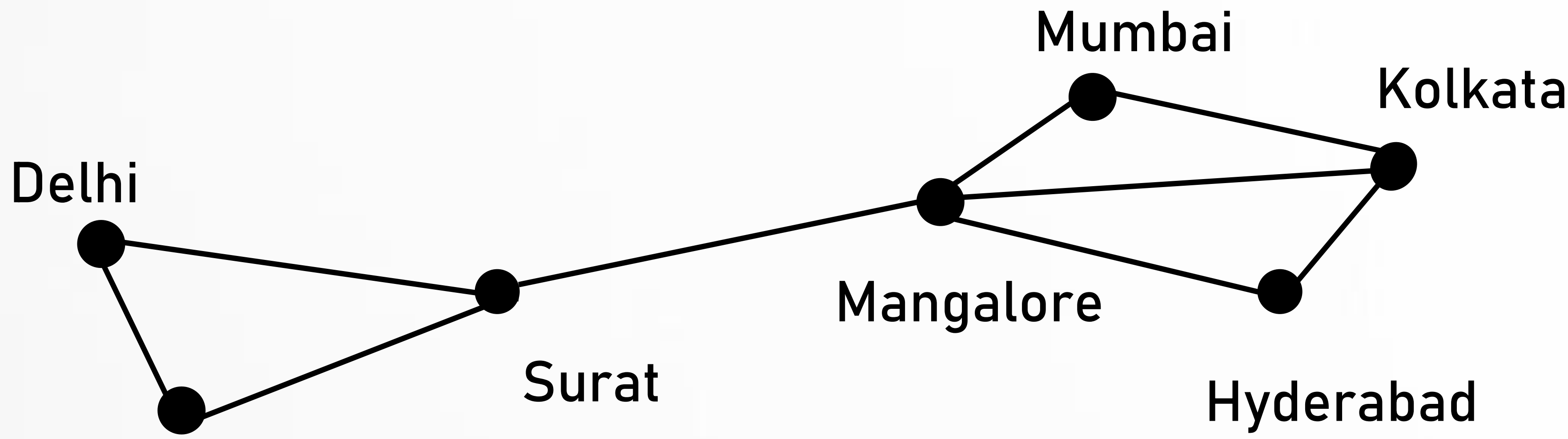
Simple Graph – Example 1



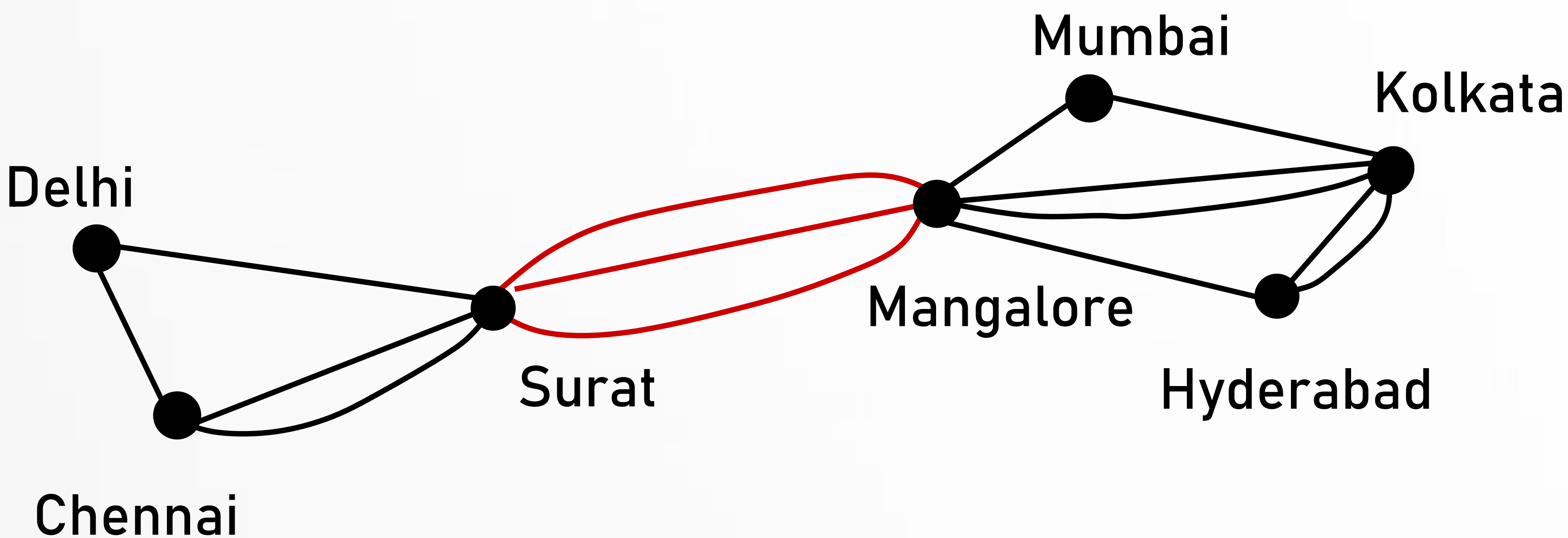
These are undirected edges, each of which connects two distinct vertices, and no two edges connect the same pair of vertices.

A Non-Simple Graph - Definition

In a multigraph $G = (V, E)$ two or more edges may connect the same pair of vertices.

A Multigraph

There can be multiple telephone lines between two computers in the network.



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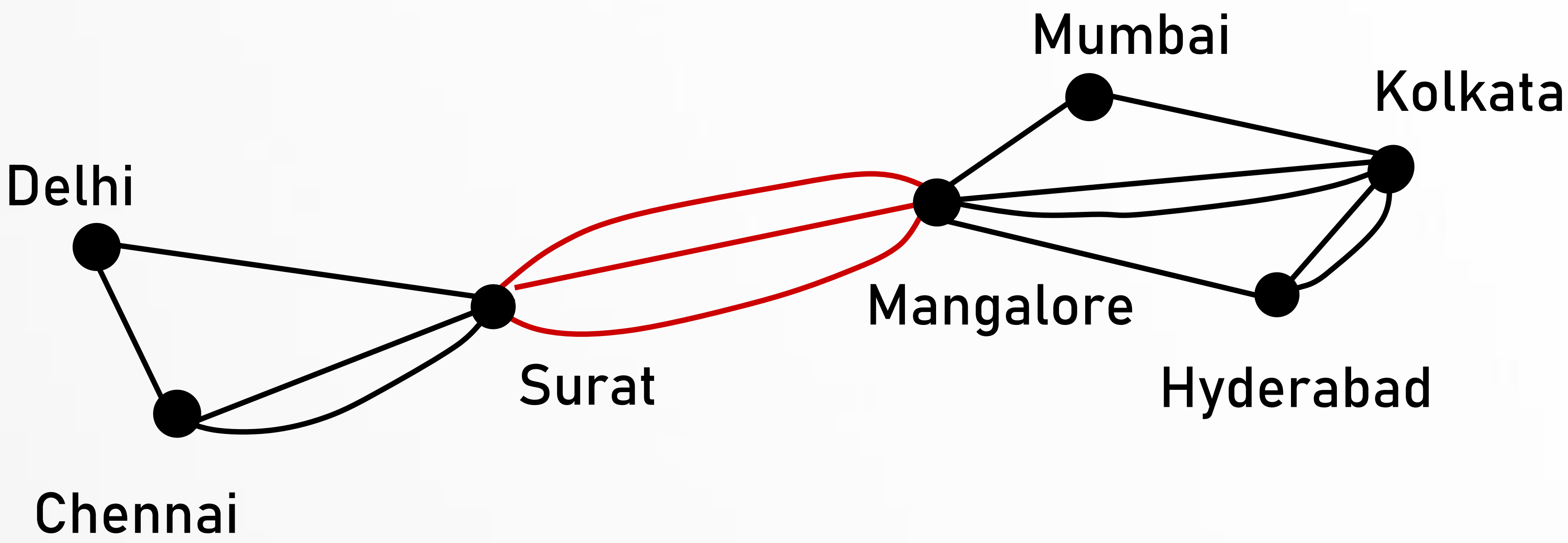
Multiple Edges

Two edges are called multiple or parallel edges if they connect the same two distinct vertices.



Another Non-Simple Graph

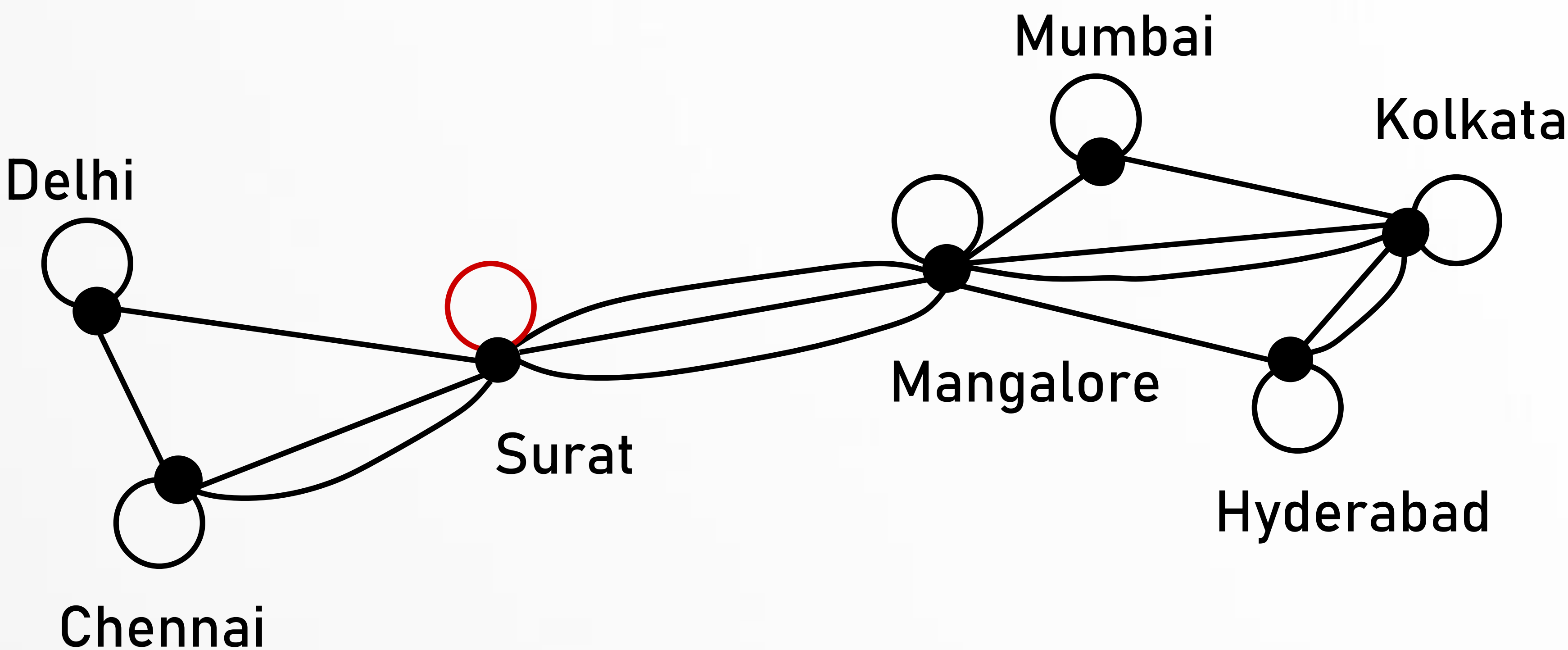
In a multigraph G = (V, E) two or more edges may connect the same

Definition:- In a pseudograph G = (V, E) two or more edges may connect the same pair of vertices, and in addition, an edge may connect a vertex to itself.

Pair of vertices.

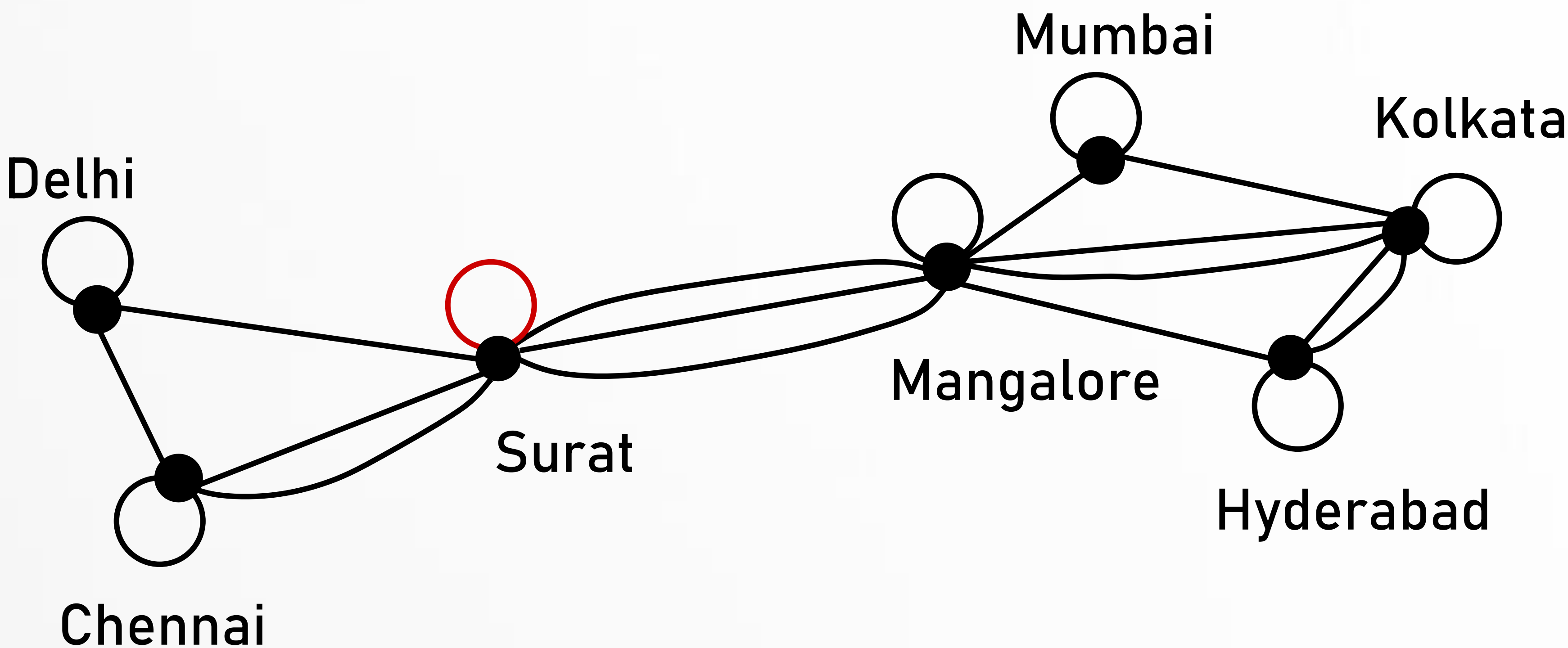
A Pseudograph

There can be telephone lines in the network from a computer to itself (for diagnostic use).

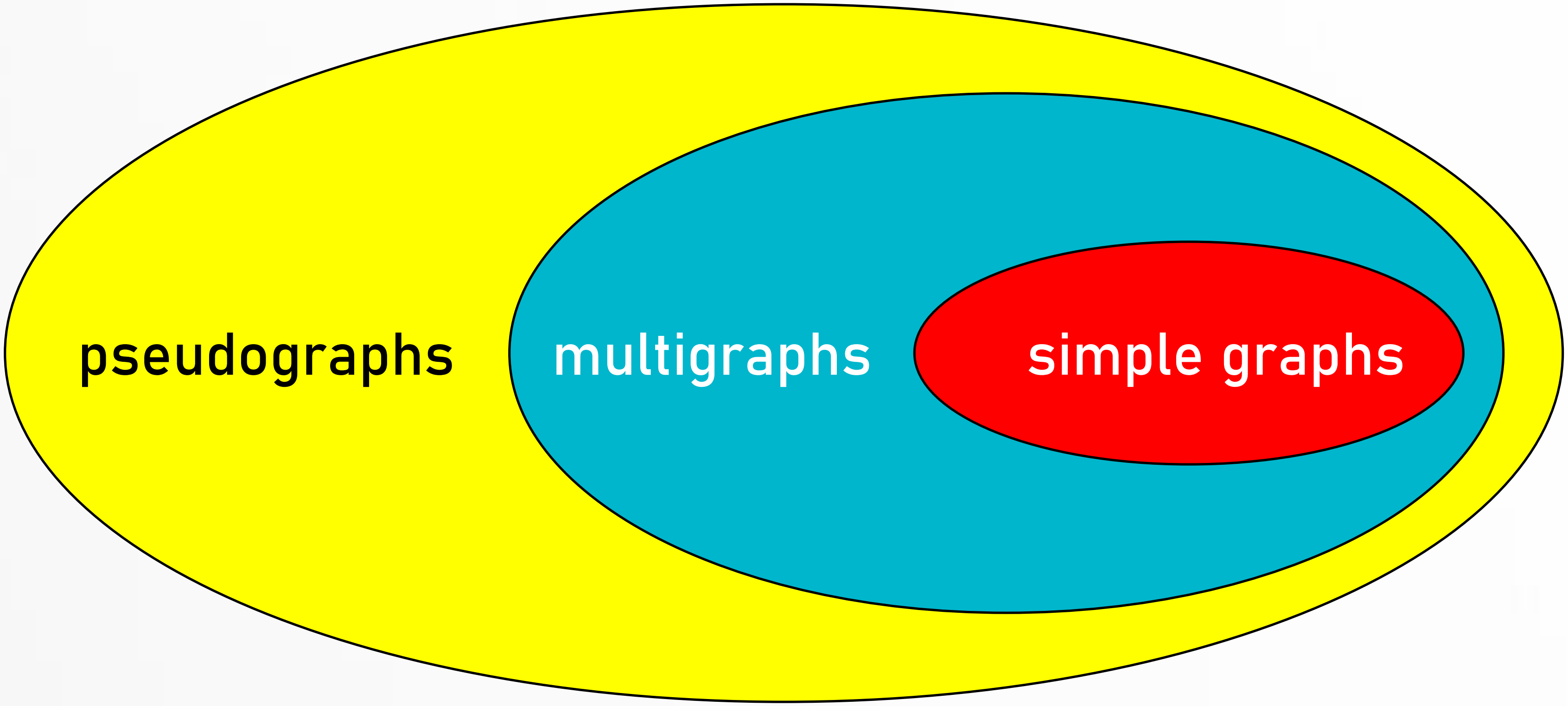


Loops

An edge is called a *loop* if it connects a vertex to itself.



Undirected Graphs



A Directed Graph – Definition

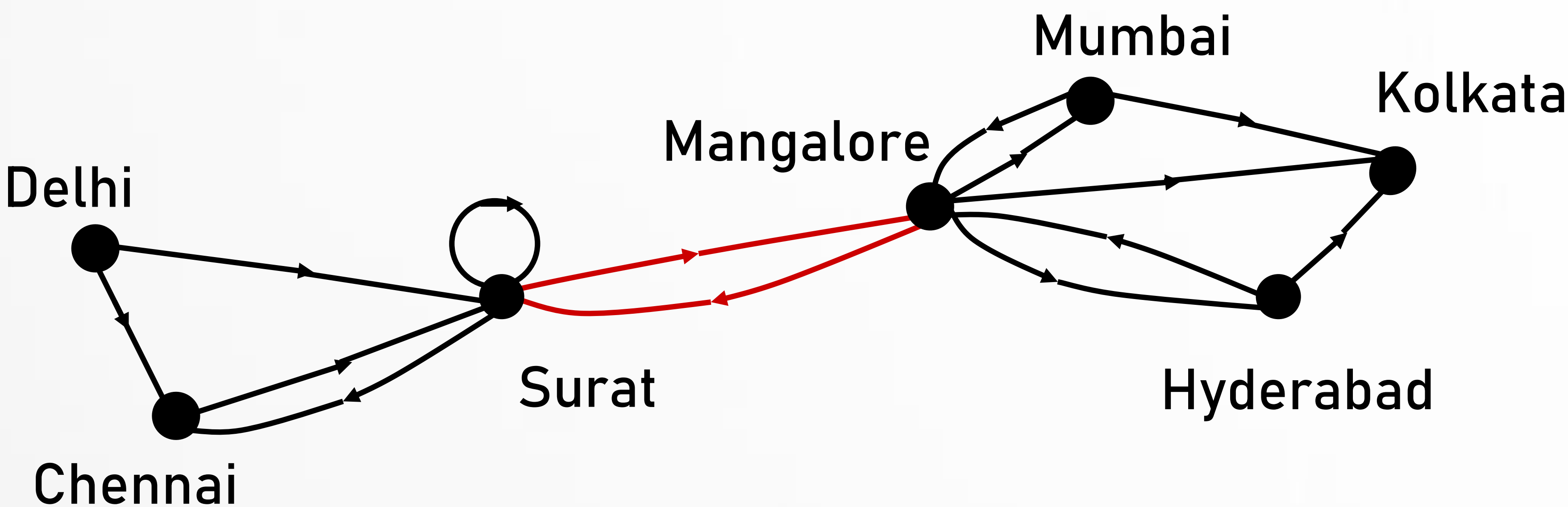
In a directed graph $G = (V, E)$ the edges are ordered pairs of (not necessarily distinct) vertices.

A Directed Graph - Definition

Some telephone lines in the network may operate in only one direction.

Those that operate in two directions are represented.

By pairs of edges in opposite directions.

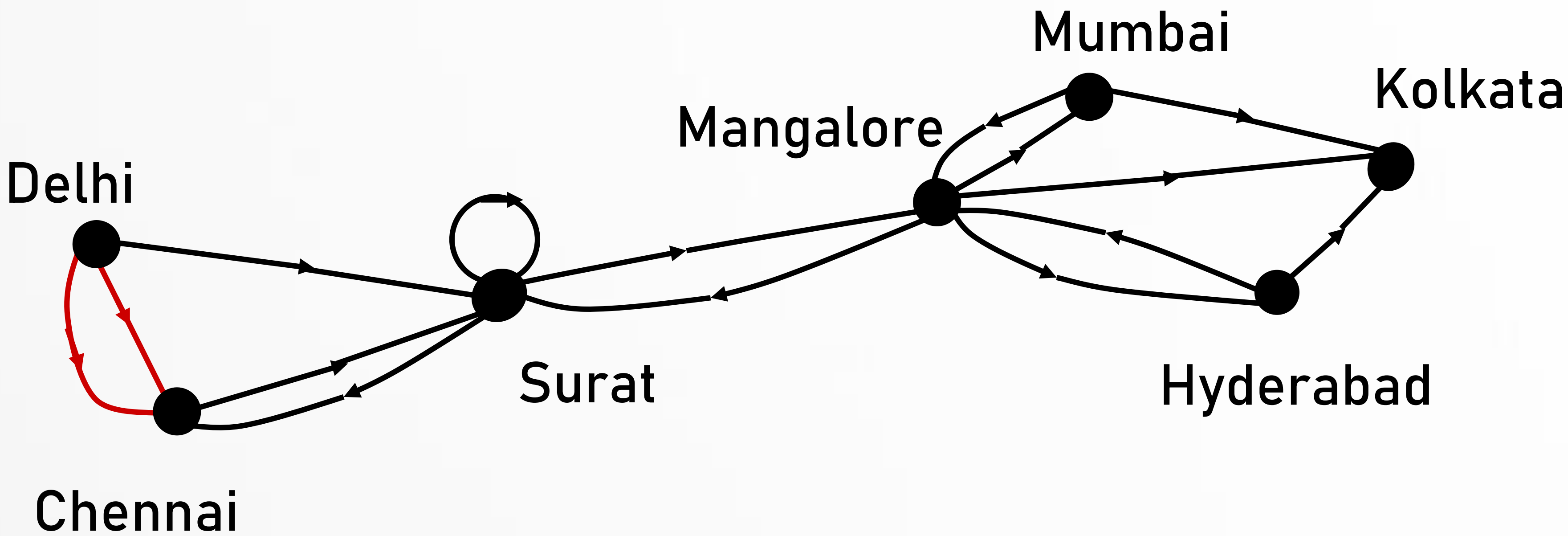


A Directed Multigraph - Definition

In a directed multigraph $G = (V, E)$ the edges are ordered pairs of (not necessarily distinct) vertices, and in addition there may be multiple edges.

A Directed Multigraph

There may be several one-way lines in the same direction from one computer to another in the network.



Types of Graphs

Type	Edges	Multiple Edges Allowed?	Loops Allowed?
Simple graph	Undirected	NO	NO
Multigraph	Undirected	YES	NO
Pseudograph	Undirected	YES	YES
Directed graph	Directed	NO	YES
Directed multigraph	Directed	YES	YES

Adjacent Vertices (Neighbours) – Definition

Two vertices, u and v in an undirected graph G are called adjacent (or neighbors) in G, if $\{u, v\}$ is an edge of G.

An edge e connecting u and v is called incident with vertices u and v, or is said to connect u and v. The vertices u and v are called endpoints of edge $\{u, v\}$.

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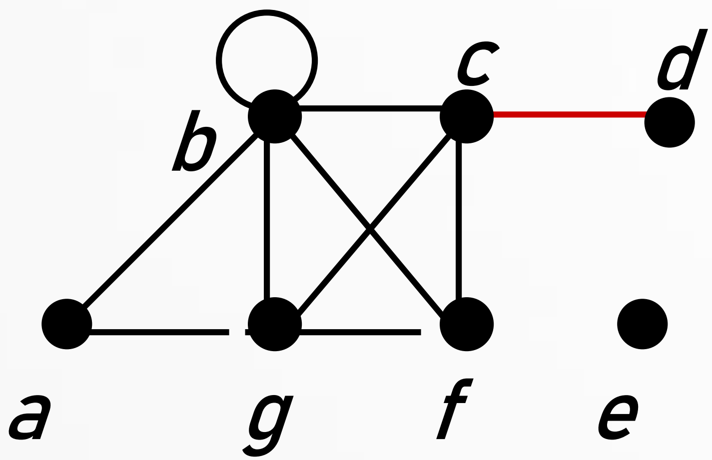
Degree of a vertex - Definition

The degree of a vertex in an undirected graph is the number of edges incident with it, except that a loop at a vertex contributes twice to the degree of that vertex.



deg(d) = 1

That's all for now...