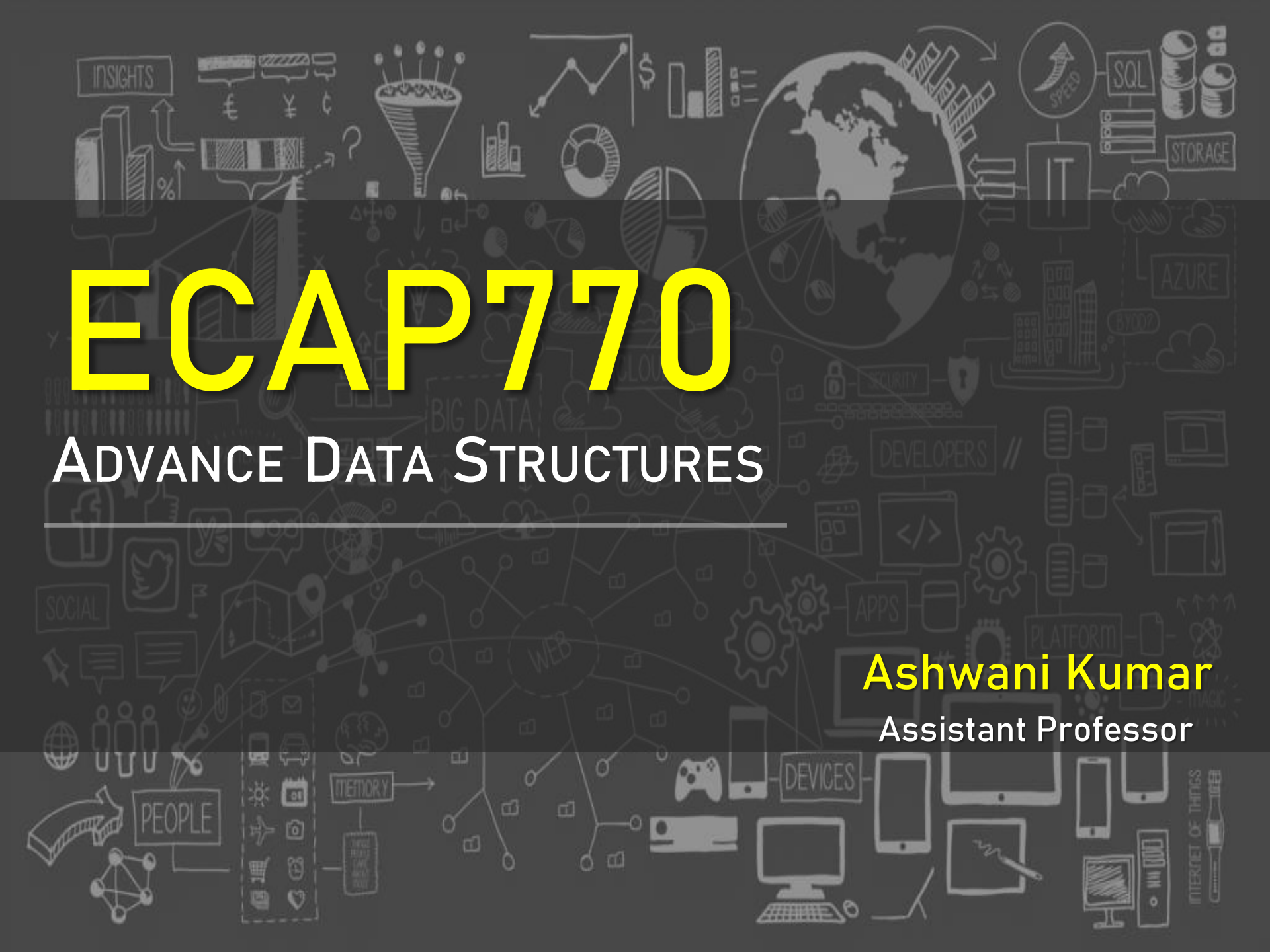

ECAP770

Advance Data Structures

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Learning Outcomes

After this lecture, you will be able to

- Understand spanning trees

Spanning Tree

- A spanning tree is a sub-graph of an undirected connected graph, which has all the vertices covered with minimum possible number of edges. If a vertex is missed, then it is not a spanning tree.
- A spanning tree does not have cycles and it cannot be disconnected.

Types of graphs

- Undirected graph: An undirected graph is a graph in which all the edges do not point to any particular direction.
- Connected graph: A connected graph is a graph in which a path always exists from a vertex to any other vertex.
- A graph is connected if we can reach any vertex from any other vertex by following edges in either direction.

Types of graphs

- Directed graph: A directed graph is defined as a graph in which set of V vertices and set of Edges, each connecting two different vertices, but it is not mandatory that node points in the opposite direction also.

Properties of Spanning Tree

- A undirected connected graph can have more than one spanning tree.
- All the possible spanning trees of a graph have the same number of edges and vertices.
- The spanning tree does not have any cycle / loops.
- Any connected and undirected graph will always have at least one spanning tree.

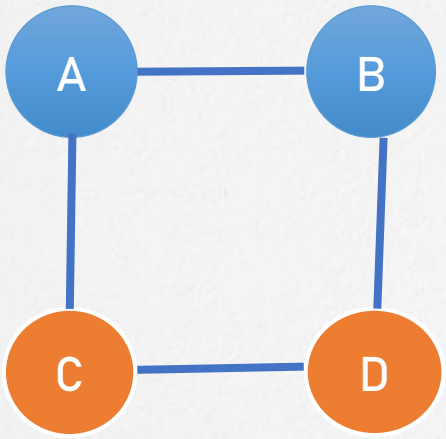
Properties of Spanning Tree

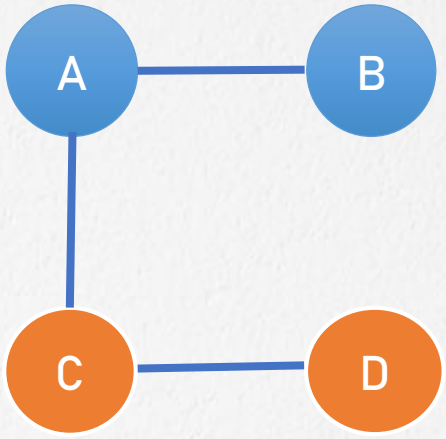
- Spanning tree is always minimally connected. Removing one edge from the spanning tree will make the graph disconnected
- A spanning tree is maximally acyclic. Adding one edge to the spanning tree will create a cycle or loop.

Mathematical Properties of Spanning Tree

- Spanning tree has n-1 edges, where n is the number of nodes (vertices).
- A complete graph can have maximum n^{n-2} number of spanning trees.
- From a complete graph, by removing maximum e - n + 1 edges, we can construct a spanning tree.

Example: Spanning Tree

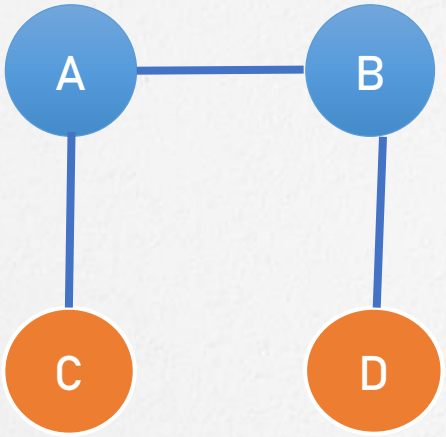


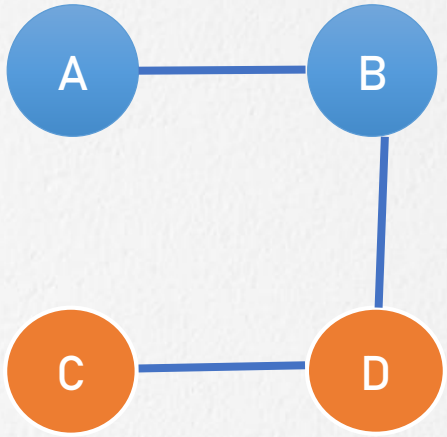


Undirected connected graph

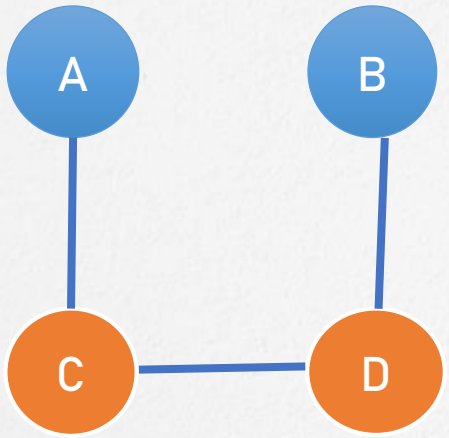
If we have n = 4, the maximum number of possible spanning trees is equal to $4^{4-2} = 16$. Thus, 16 spanning trees can be formed from a complete graph with 4 vertices.

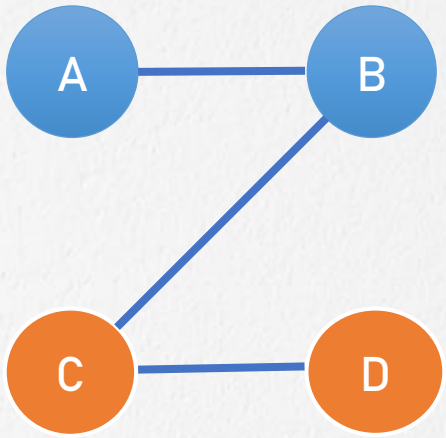
Example: Spanning Tree



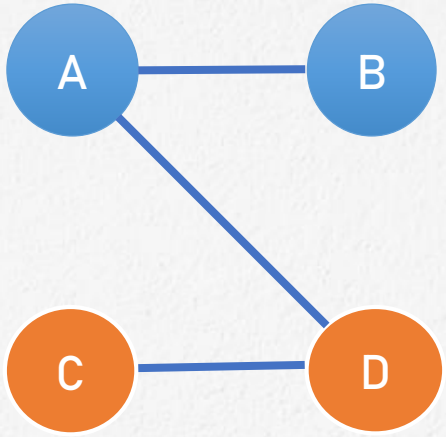


Example: Spanning Tree





Example: Spanning Tree



Application of Spanning Tree

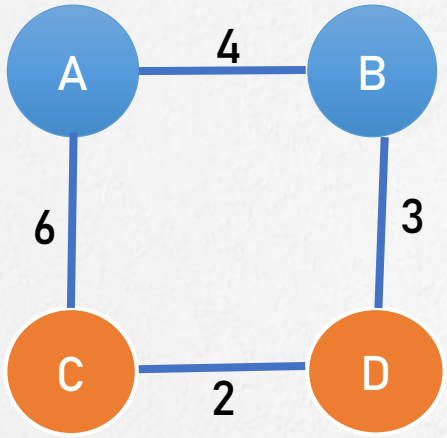
It is used to find a minimum path to connect all nodes in a graph

- Computer Network Routing Protocol
- Cluster Analysis

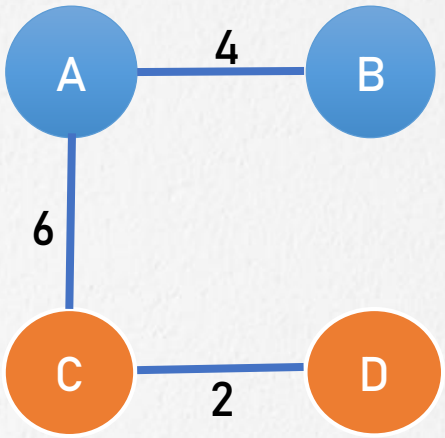
Minimum Spanning Tree

- A minimum spanning tree is a spanning tree in which the sum of the weight of the edges is as minimum as possible.

Minimum Spanning Tree



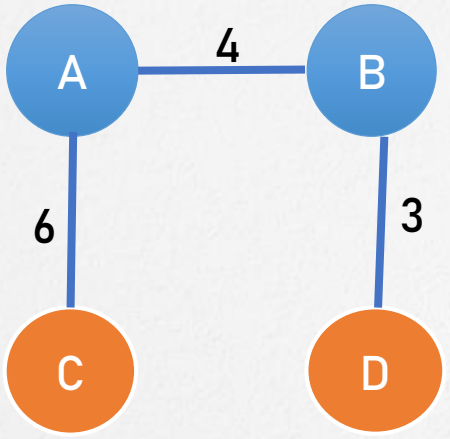
Weighted Graph



Sum =12

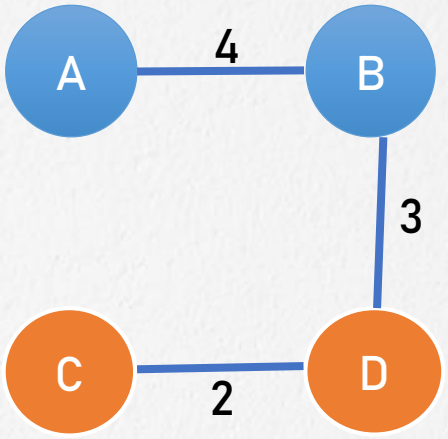
(A)

Minimum Spanning Tree



Sum =13

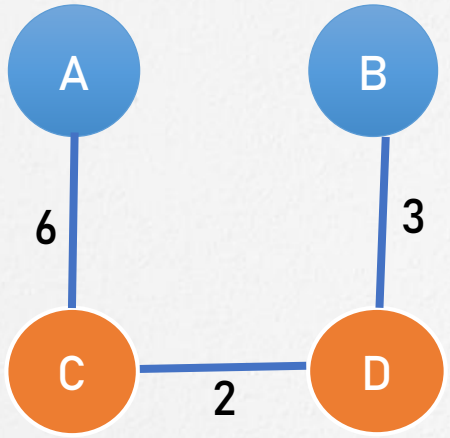
(B)



Sum =9

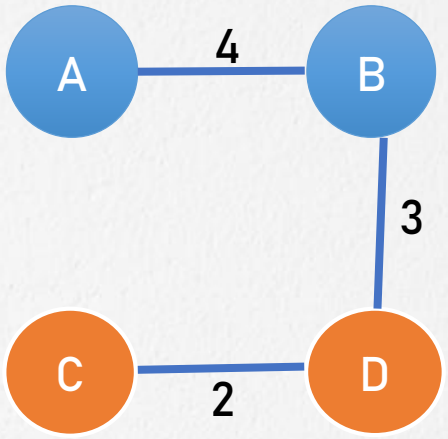
(C)

Minimum Spanning Tree



Sum =11

(D)



Sum =9

(C)

Minimum spanning tree

That's all for now...