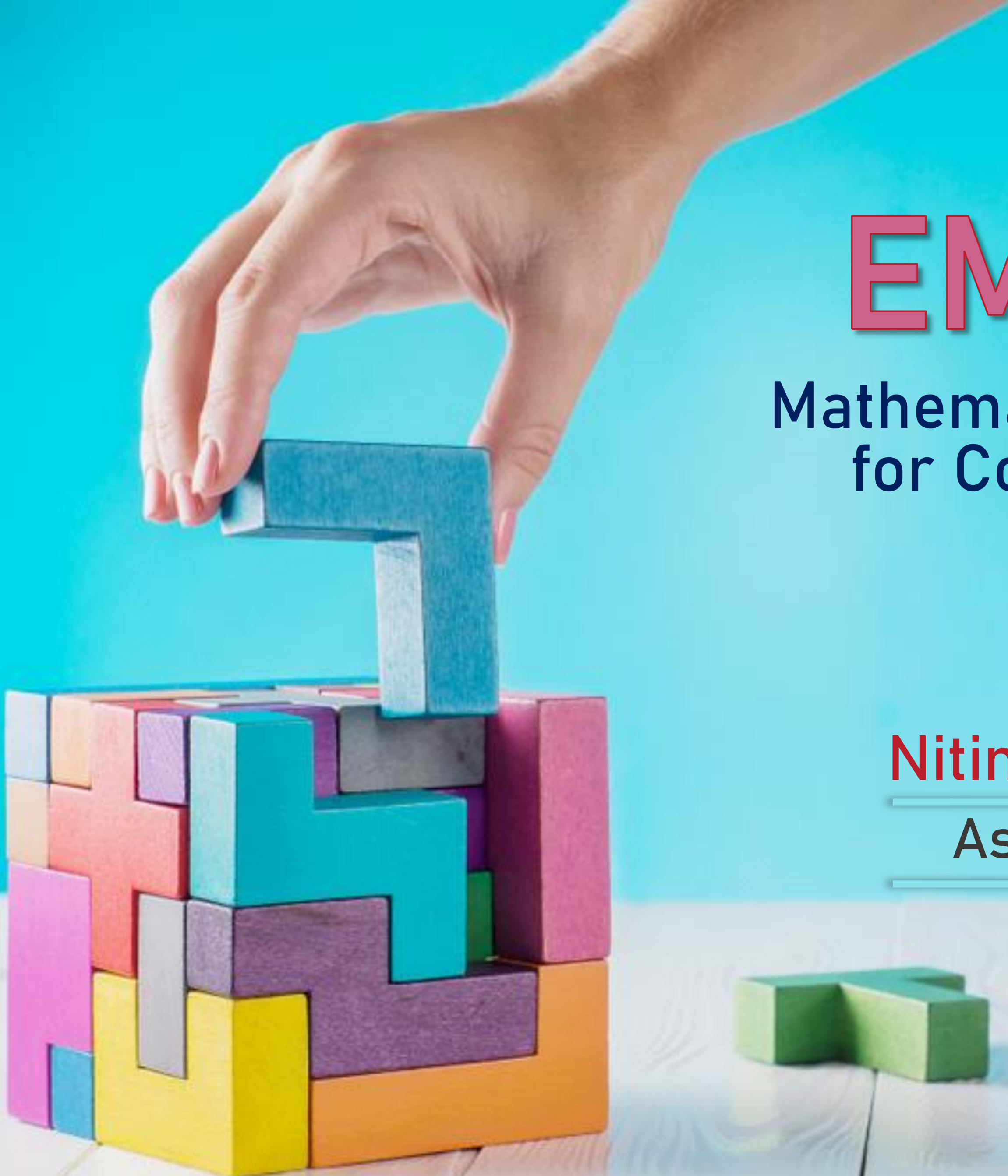

EMTH403

Mathematical Foundation for Computer Science

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Lecture Outcomes

After this lecture, you will be able to

- understand what are duals of Boolean expression.
- understand what is a Complemented Lattice.
- understand what is Boolean Algebra.

Duality

To explain the relationship between the two identities in each pair we use the concept of a dual.

The dual of a Boolean expression is obtained by interchanging Boolean sums and Boolean products and interchanging 0s and 1s.

Content Source: Discrete Mathematics and Its Applications. Seventh Edition. Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University. (and formerly AT&T Laboratories)

Duality

Ques:- Find the duals of $x(y + 0)$ and $\bar{x} \cdot 1 + (\bar{y} + z)$.

Sol:- Interchanging $\cdot$ signs and $+$ signs and interchanging 0s and 1s in these expressions produces their duals.

The duals are $x + (y \cdot 1)$ and $(x + 0)(yz)$, respectively.

Boolean algebra

A **Boolean algebra** is a set B with two binary operations $\vee$ and $\wedge$, elements 0 and 1, and a unary operation $\bar{}$ such that **these properties hold** for all x, y, and z in B:

$$\left.\begin{aligned} x \vee 0 &= x \\ x \wedge 1 &= x \end{aligned}\right\}$$ Identity laws

$$\left.\begin{aligned} x \vee \bar{x} &= 1 \\ x \wedge \bar{x} &= 0 \end{aligned}\right\}$$ Complement laws

$$\left.\begin{aligned} (x \vee y) \vee z &= x \vee (y \vee z) \\ (x \wedge y) \wedge z &= x \wedge (y \wedge z) \end{aligned}\right\}$$ Associative laws

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Boolean algebra

A Boolean algebra is a set B with two binary operations $\vee$ and $\wedge$, elements 0 and 1, and a unary operation $\bar{\ }$ such that these properties hold for all x, y, and z in B:

$$\left.\begin{aligned} x \vee y &= y \vee x \\ x \wedge y &= y \wedge x \end{aligned}\right\}$$ Commutative laws

$$\left.\begin{aligned} x \vee (y \wedge z) &= (x \vee y) \wedge (x \vee z) \\ x \wedge (y \vee z) &= (x \wedge y) \vee (x \wedge z) \end{aligned}\right\}$$ Distributive laws

Boolean algebra

Boolean algebras may also be defined using the notion of a lattice, discussed earlier.

Lattice L is a partially ordered set in which every pair of elements x, y has a least upper bound, denoted by lub(x, y) and a greatest lower bound denoted by glb(x, y).

Boolean algebra

Given two elements x and y of L, we can define two operations $\vee$ and $\wedge$ on pairs of elements of L by

$x \vee y = \text{lub}(x, y)$ and $x \wedge y = \text{glb}(x, y)$.

Boolean algebra

For a lattice L to be a Boolean algebra, it must have two properties.

First, it must be complemented.

For a lattice to be complemented it must have a least element 0 and a greatest element 1

and for every element x of the lattice there must exist an element x such that $x \vee x = 1$ and $x \wedge x = 0$.

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Boolean algebra

For a lattice L to be a Boolean algebra, it must have two properties.

First, it must be complemented.

Second, it must be distributive.

This means that for every x, y, and z in L, $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$ and $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$.

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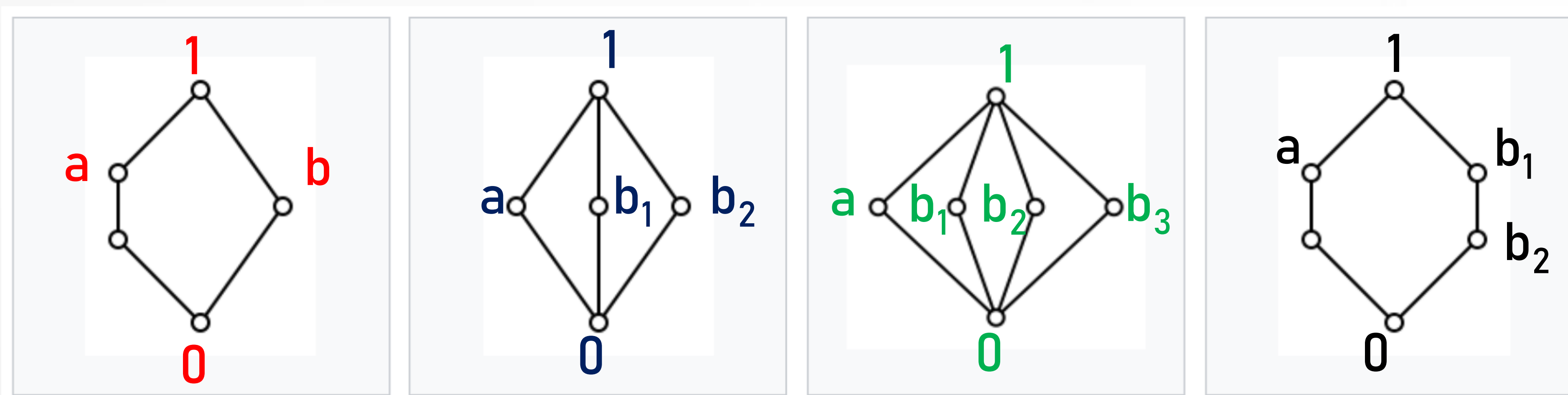
Complemented Lattice

A complemented lattice is a bounded lattice (with least element 0 and greatest element 1), in which every element a has a complement, i.e. an element b satisfying a ∨ b = 1 and a ∧ b = 0. Complements need not be unique.



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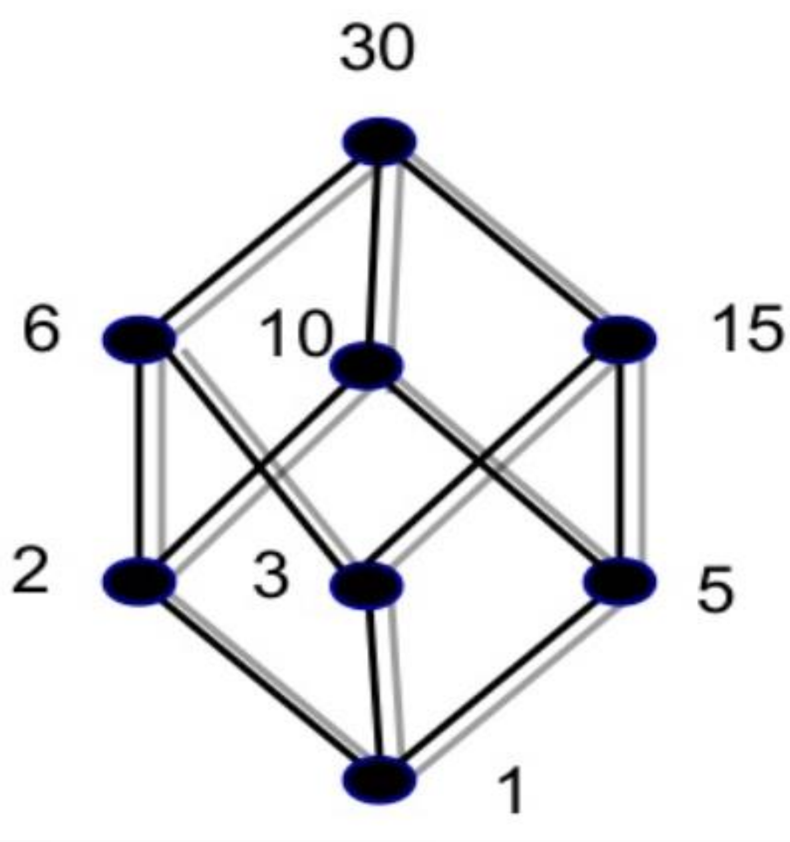
Complemented Lattice

D_{30} is a complemented lattice.

Element	Its Complement
1	30
2	15
3	10
5	6
6	5
10	3
15	2
30	1



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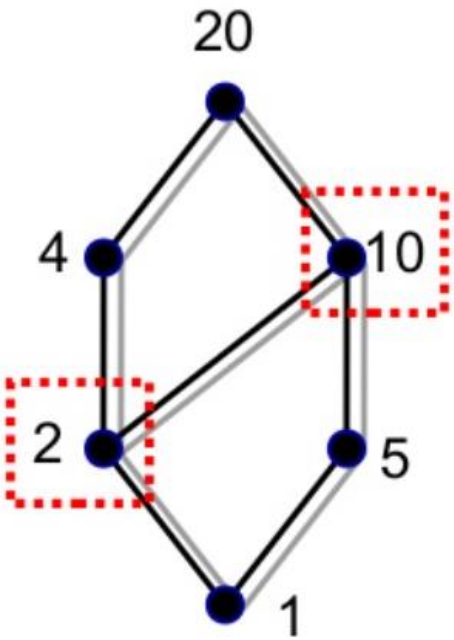
Complemented Lattice

D_{20} is not complemented lattice.



D_{20}

Element	Its Complement
1	20
2	10
4	5
5	4
10	2
20	1

$$2 \wedge 10 \neq 0 \quad (2 \wedge 10 = 2)$$

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Boolean algebra

For a lattice L to be a Boolean algebra, it must have two properties.

First, it must be complemented.

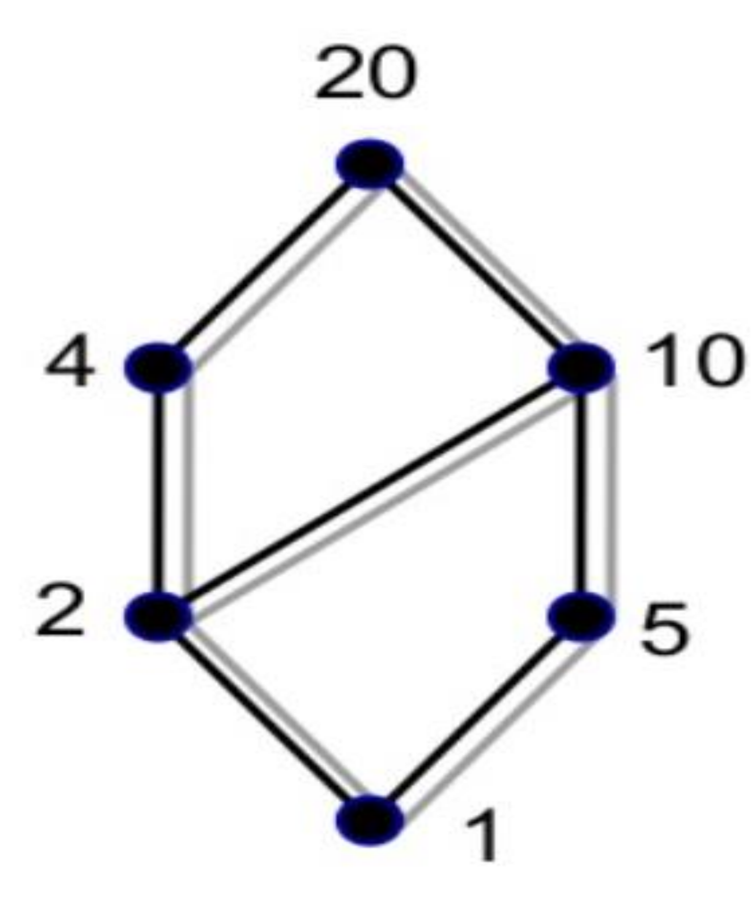
Second, it must be distributive.

This means that for every x, y, and z in L, $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$ and $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$.

Boolean algebra

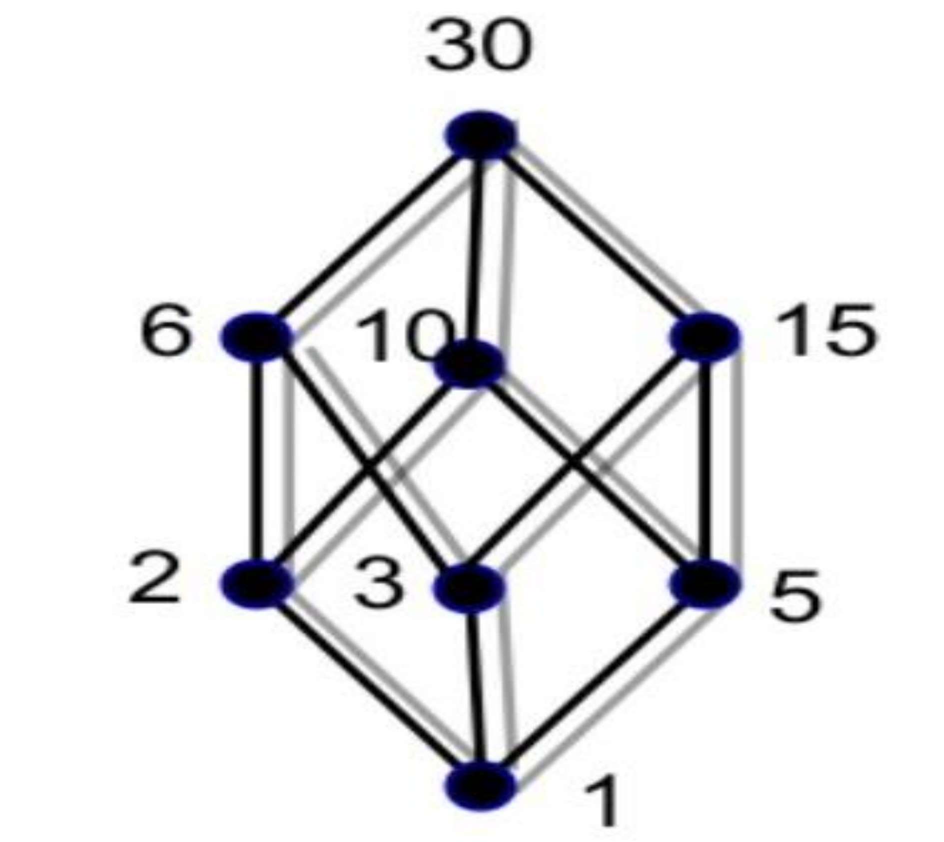
Consider the lattices D_{20} of all positive integer divisors of 20.

D_{20} is not a boolean algebra.



Boolean algebra

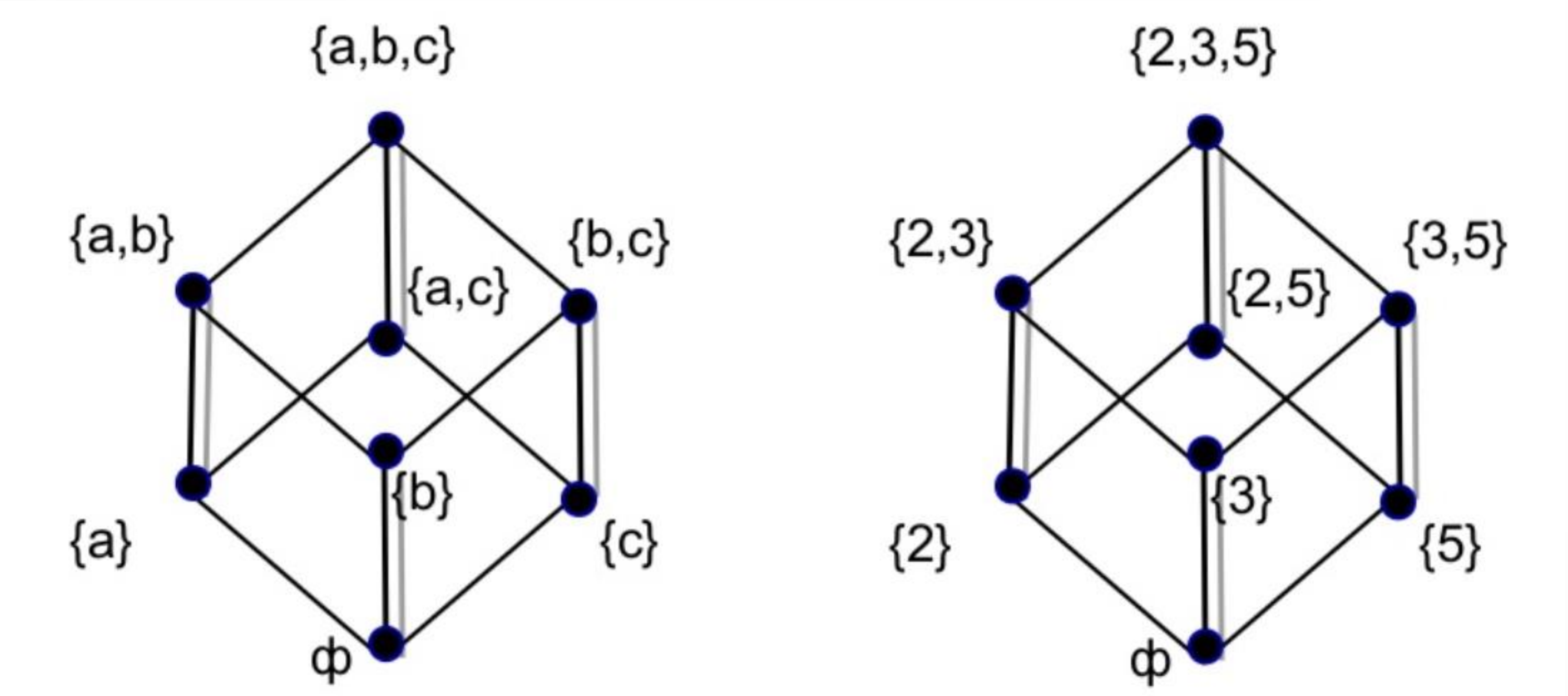
Consider the lattices D_{30} of all positive integer divisors of 30.



D_{30} is a Boolean algebra.

Boolean algebra

S = {a, b, c} and T = {2, 3, 5} consider the Hasse diagram of the two lattices (P(S), ⊆) and (P(T), ⊆).



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Boolean algebra

Ques:- Show that in a Boolean algebra, the idempotent laws $x \vee x = x$ and $x \wedge x = x$ hold for every element x.

Sol:- By the domination, distributive, and identity laws,

$x \vee x = (x \vee x) \wedge 1 = (x \vee x) \wedge (x \vee \bar{x}) = x \vee (x \wedge \bar{x}) = x \vee 0 = x.$

Similarly, $x \wedge x = (x \wedge x) \vee 0 = (x \wedge x) \vee (x \wedge \bar{x}) = x \wedge (x \vee \bar{x}) = x \wedge 1 = x.$

That's all for now...