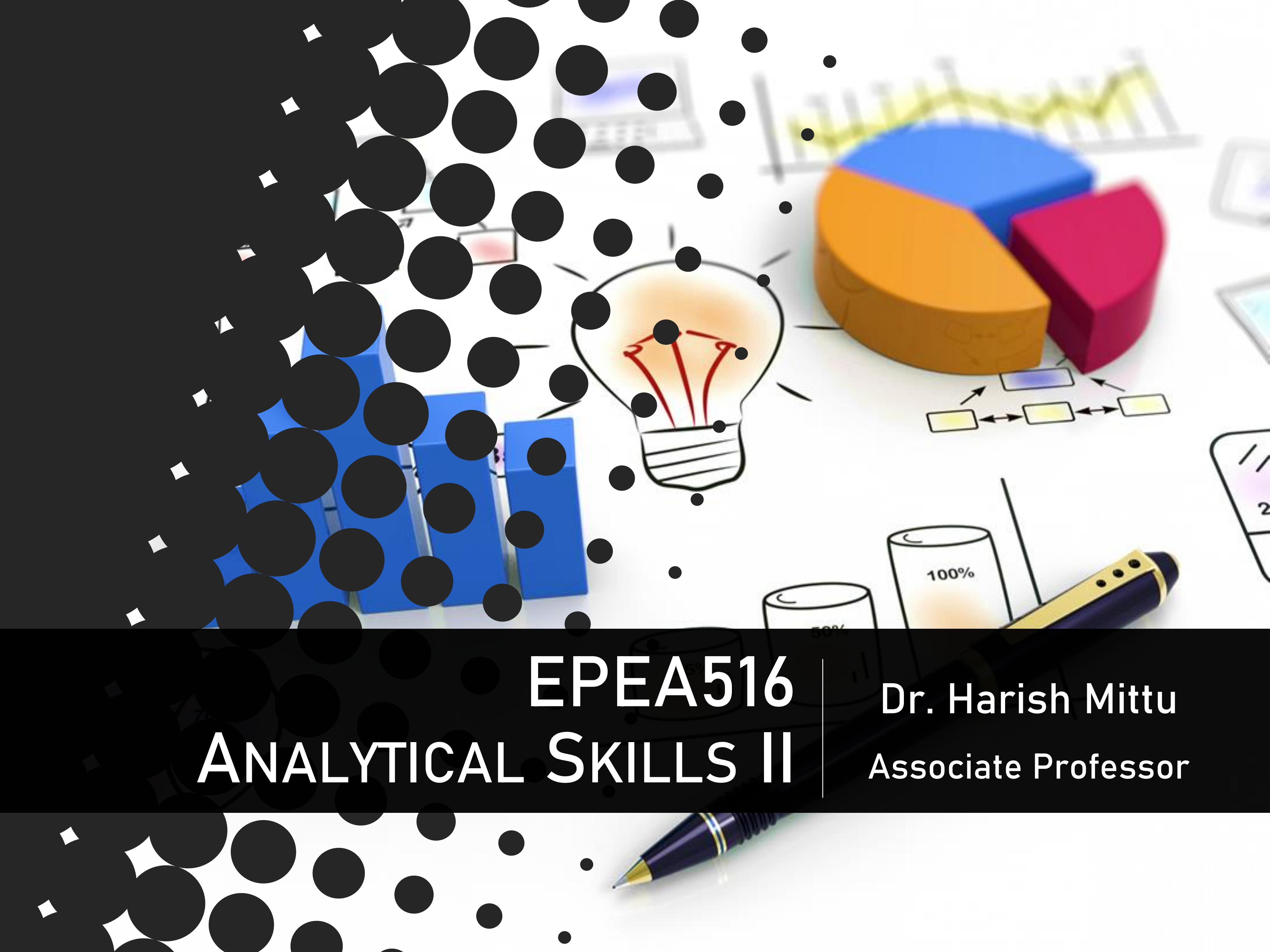

EPEA516
Analytical Skills II

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Learning Outcomes

After this lecture, you will be able to

- develop understanding about basics of speed, distance, time, average speed, and units of measurement,
- derive different units, important facts and formulae relating to speed, distance, time, and average speed.

Speed, Distance, & Time

- Length of Path
- Distance Covered
- Unit Time Interval
- $\dfrac{\text{Distance covered by the object}}{\text{Time the object has taken to cover that distance}}$
- Speed = $\dfrac{\text{Distance Travelled}}{\text{Time Taken}}$

Speed, Distance, & Time

- D = Distance
- T = Time
- S = Speed

$$\text{Speed (S)} = \frac{\text{Distance (D)}}{\text{Time (T)}}$$

$$\text{Speed} \times \text{Time} = \text{Distance}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Speed, Distance, & Time

- Speed = $\frac{\text{Distance}}{\text{Time}}$
- If Distance = Constant then $\boxed{\text{Speed } \alpha \ \frac{1}{\text{Time}}}$
- If Speed = Constant then $\boxed{\text{Distance } \alpha \text{ Time}}$
- If Time = Constant then $\boxed{\text{Speed } \alpha \text{ Distance}}$

Units of Measurement

- Distance – Kilometre (Km)
- Time - Hours (h)
- Speed - Kilometre per hour (Km/h)

- Distance – Metre (m)
- Time - Seconds (s)
- Speed - Metre per seconds (m/s)

Conversion of Units

- $1 \text{ Km/h} = \dfrac{1 \text{ Kilometre}}{1 \text{ hour}}$

$$= \frac{1000 \text{ metre}}{60 \text{ minutes}} \quad [1\text{km} = 1000 \text{ m} \ \& \ 1\text{h} = 60 \text{ min.}]$$

$$= \frac{1000 \text{ metre}}{60 \times 60 \text{ seconds}} \quad [1\text{min.} = 60 \text{ seconds}]$$

$$= \frac{{}^{5}\cancel{1000} \text{ metre}}{\underset{18}{\cancel{3600}} \text{ seconds}}$$

- $\boxed{1 \text{ Km/h} = \frac{5}{18} \text{ m/s}}$

- $\boxed{1 \text{ m/s} = \frac{18}{5} \text{ Km/h}}$

Conversion of Units

- $1\ Km/h = \frac{5}{18}\ m/s$ and $1\ m/s = \frac{18}{5}\ Km/h$

- 36 Km/h $= \overset{2}{\cancel{36}} \times \frac{5}{\cancel{18}}\ m/s$

  $= 10\ m/s$

- 20 m/s $= \overset{4}{\cancel{20}} \times \frac{18}{\cancel{5}}\ Km/h$

  $= 72\ Km/h$

Conversion of Units

- 1 mile = 1.6093 km

  = 1.6093 x 1000 m [1km = 1000 m]

  = 1609.30 m

- 1 km $= \frac{1}{1.6093}$ mile = 0.621 mile
- 1 yard = 0.9144 m
- 1 m $= \frac{1}{0.9144}$ yards = 1.0936 yards
- 1 m = 39.4 inches

Average Speed

- Total Distance Covered
- Total Time Taken
- $$\boxed{\text{Average Speed} = \frac{\text{Total Distance Covered}}{\text{Total Time Taken}}}$$

Average Speed

- If ‘A’ covers a distance ‘x’ Km at ‘a’ Km/h and, then ‘y’ Km at ‘b’ Km/h, then calculate the average speed during the whole journey.

- Time taken to travel ‘x’ Km at ‘a’ Km/h, $T_1 = \frac{x}{a}$

- Time taken to travel ‘y’ Km at ‘b’ Km/h, $T_2 = \frac{y}{b}$

- Total Time = $T_1 + T_2$

$$= \frac{x}{a} + \frac{y}{b}$$

Average Speed

- Total Time $= T_1 + T_2 = \dfrac{x}{a} + \dfrac{y}{b}$

$$= \frac{(bx + ay)}{ab} \text{ or } \frac{(ay + bx)}{ab}$$

- Total Distance $= (x + y)$ Km
- Average Speed $= \dfrac{\text{Total Distance Covered}}{\text{Total Time Taken}}$
- Average Speed $= \dfrac{(x + y)}{\dfrac{(ay + bx)}{ab}}$
- Average Speed $= \boxed{\dfrac{ab(x + y)}{(ay + bx)} \text{ Km/h}}$

Average Speed

- If a person goes from point ‘A’ to point ‘B’ at ‘a’ Km/h and comes back from point ‘B’ to point ‘A’ at ‘b’ Km/h, then calculate the average speed during the whole journey.
- Let, Distance = x Km
- Time taken to travel ‘x’ Km at ‘a’ Km/h, $T_1 = \frac{x}{a}$
- Time taken to travel ‘x’ Km at ‘b’ Km/h, $T_2 = \frac{x}{b}$
- Total Time = $T_1 + T_2$

$$= \frac{x}{a} + \frac{x}{b}$$

Average Speed

- Total Time $= T_1 + T_2$ $= \frac{x}{a} + \frac{x}{b}$

  $= \frac{x(b + a)}{(ab)}$ or $\frac{x(a + b)}{(ab)}$

- Total Distance $= (x + x)$ Km $= (2x)$ Km
- Average Speed $= \frac{\text{Total Distance Covered}}{\text{Total Time Taken}}$
- Average Speed $= \dfrac{(2\cancel{x})}{\dfrac{\cancel{x}(a + b)}{(ab)}}$
- $\boxed{\text{Average Speed} = \frac{2ab}{(a + b)} \text{ Km/h}}$

Total Distance

- If the a person covers a distance in 'T' hours, the first half at 'a' Km/h and second half at 'b' Km/h, then the total distance covered by the person is
- Average Speed $= \frac{2ab}{(a + b)}$ Km/h
- Total Distance = Average Speed x Time
- Total Distance $= \frac{2ab}{a + b} \times T$
- Total Distance $= \frac{2abT}{a + b}$ Km

Distance

- A person goes certain distance point 'A' to point 'B' at a speed of 'a' Km/h and returns back point 'B' to point 'A' at a speed of 'b' Km/h. If s/he takes 'T' hours in all, then calculate the distance between points 'A' and 'B'
- Let, the distance between points 'A' and 'B' = x Km.
- Time taken during onward journey , $T_1 = \frac{x}{a}$
- Time taken during return journey, $T_2 = \frac{x}{b}$
- Total Time = $T_1 + T_2$

Distance

- Total Time, $T = T_1 + T_2 = \frac{x}{a} + \frac{x}{b}$

$$T = \frac{x(b + a)}{(ab)} \text{ or } \frac{x(a + b)}{(ab)}$$

$$T = \frac{x(a + b)}{(ab)}$$

$$x = \frac{T(ab)}{(a + b)}$$

- Distance between A and B i.e., $x = \frac{T(ab)}{(a + b)}$ hours

$$\boxed{x = \text{Total Time Taken} \times \frac{\text{Product of Two Speeds}}{\text{Sum of Two Speeds}}}$$

Ratio of Speeds of Two Persons

- If two persons, A and B, start at the same time from two points P and Q towards each other, and after crossing they take T_1 and T_2 hours in reaching Q and P respectively, then calculate the ratio of their speeds.
- Let, Total distance between P and Q = d Km

  Speed of person A = a Km/h

  Speed of person B = b Km/h
- Relative speed of persons A and B = (a + b) Km/h

(Since persons A and B are moving in opposite directions)

Ratio of Speeds of Two Persons

- Persons, A and B, will meet after $\frac{d}{a + b}$ hours
- Distance travelled by A in $\frac{d}{a + b}$ hours = PO = $\frac{ad}{a + b}$ Km [D = S x T]
- Distance travelled by B in $\frac{d}{a + b}$ hours = QO = $\frac{bd}{a + b}$ Km
- Time taken by A to travel QO, $T_1 = \frac{\frac{bd}{a + b}}{a}$ [$T = \frac{D}{S}$]
- Time taken by B to travel PO, $T_2 = \frac{\frac{ad}{a + b}}{b}$

Ratio of Speeds of Two Persons

$$\frac{\frac{ad}{a+b}}{b} = T_2 \quad \text{and} \quad \frac{\frac{bd}{a+b}}{a} = T_1$$

$$\frac{\dfrac{\frac{a\cancel{d}}{\cancel{a+b}}}{b}}{\dfrac{\frac{b\cancel{d}}{\cancel{a+b}}}{a}} = \frac{T_2}{T_1}$$

$$\left(\frac{a}{b}\right)^2 = \frac{T_2}{T_1}$$

$$\frac{a}{b} = \frac{\sqrt{T_2}}{\sqrt{T_1}}$$

$$\boxed{\frac{a \text{ (i.e., Speed of A)}}{b \text{(i.e., Speed of B)}} = \frac{\sqrt{T_2}}{\sqrt{T_1}}}$$

Average Speed - Journey

- If a body travels x_1, x_2, x_3, … , x_n metres with different speeds a_1, a_2, a_3, … , a_n m/s in time t_1, t_2, t_3, … , t_n seconds respectively, then the average speed of the body throughout the journey is given by

- Average Speed = $\dfrac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$

- $\boxed{\text{Average Speed} = \dfrac{x_1 + x_2 + x_3 + \ldots + x_n}{t_1 + t_2 + t_3 + \ldots + t_n}}$

- $\boxed{\text{Average Speed} = \dfrac{a_1 t_1 + a_2 t_2 + a_3 t_3 + \ldots + a_n t_n}{t_1 + t_2 + t_3 + \ldots + t_n}}$

Change in Time – Cover Same Distance

- If the new speed is $\frac{m}{n}$ of the original speed, then calculate the change in time taken to cover the same distance.
- Let Distance = d, Original Time = t, and Speed = a
- $a = \frac{d}{t}$ or $t = \frac{d}{a}$ [S = D/T or T = D/S]
- New Speed = $\frac{m}{n}$ x Original Speed = $\frac{m}{n} \times a$
- Let New Time = T $= \dfrac{d}{\frac{m}{n} \times a}$ [T = D/S]

$$T = \frac{nd}{m \times a}$$

Change in Time – Cover Same Distance

- $t = \dfrac{d}{a}$ and $T = \dfrac{nd}{m \times a}$

- $T - t = \dfrac{nd}{m \times a} - \dfrac{d}{a}$

- $T - t = \left(\dfrac{n}{m} - 1\right) \dfrac{d}{a}$

- Change in Time = $\left(\dfrac{n}{m} - 1\right) \times t$ $\left[t = \dfrac{d}{a} \right]$

- $\boxed{\text{Change in Time} = \left(\dfrac{n}{m} - 1\right) \times \text{Original Time}}$

Usual/Original Time

- If a person changes his/her speed to $\frac{m}{n}$ of its usual/original speed & late by T min., then usual time taken by him/her is

$$\boxed{\text{Usual Time} = \frac{mT}{n - m} \text{ if } \frac{m}{n} < 1}$$

$$\boxed{\text{Usual Time} = \frac{mT}{m - n} \text{ if } \frac{m}{n} > 1}$$

Relation – Speed, Time and Distance

- A body covers a distance 'x' in time 't_1' with speed 'a', but when it travels with speed 'b' covers the same distance in time 't_2'.
- $a = \frac{x}{t_1}$ and $b = \frac{x}{t_2}$
- $a\ t_1 = x$ and $b\ t_2 = x$
- $a\ t_1 = b\ t_2$

$$\boxed{\frac{a}{t_2} = \frac{b}{t_1}}$$

Relation – Speed, Time and Distance

- $a = \frac{x}{t_1}$ and $b = \frac{x}{t_2}$
- $a - b = \frac{x}{t_1} - \frac{x}{t_2} = x\left(\frac{1}{t_1} - \frac{1}{t_2}\right)$
- $a - b = x\left(\frac{t_2 - t_1}{t_1 t_2}\right)$
- $\frac{a - b}{t_2 - t_1} = \frac{x}{t_1 t_2}$
- $\frac{a - b}{t_2 - t_1} = \left(\frac{x}{t_1}\right).\left(\frac{x}{t_2}\right).\left(\frac{1}{x}\right)$
- $\frac{a - b}{t_2 - t_1} = \frac{a.b}{x}$ or $\frac{a.b}{x} = \frac{a - b}{t_2 - t_1}$

$$\boxed{\frac{\text{Product of Speed}}{x} = \frac{\text{Difference of Speed}}{\text{Difference of Time}}}$$

Relation – Speed, Time and Distance

- $a = \frac{x}{t_1}$ and $b = \frac{x}{t_2}$
- $\frac{\text{Product of speed}}{x} = \frac{ab}{x}$
- $\frac{\text{Product of speed}}{x} = \frac{a}{\frac{x}{b}}$
- $\boxed{\frac{\text{Product of speed}}{x} = \frac{a}{t_2}}$ $\left[b = \frac{x}{t_2} \text{ or } \frac{x}{b} = t_2\right]$
- $\frac{a}{t_2} = \frac{b}{t_1}$ and $\frac{\text{Product of Speed}}{x} = \frac{\text{Difference of Speed}}{\text{Difference of Time}}$
- $\boxed{\frac{\text{Product of speed}}{d} = \frac{a}{t_2} = \frac{b}{t_1} = \frac{\text{Difference of speed}}{\text{Difference of Time}}}$

Ratio of Speed & Time

- If the ratio of the speeds of A and B is a : b, then the ratio of the times taken by them to cover the same distance.

$$S_1 = \frac{d}{t_1} \quad \text{and} \quad S_2 = \frac{d}{t_2}$$

$$\frac{S_1}{S_2} = \frac{\frac{\cancel{d}}{t_1}}{\frac{\cancel{d}}{t_2}}$$

$$\frac{a}{b} = \frac{t_2}{t_1}$$

$$\frac{a}{b} = \frac{t_2}{t_1}$$

$$\boxed{\frac{t_1}{t_2} = \frac{b}{a} \quad \text{or } t_1 : t_2 = b : a \text{ or } t_1 : t_2 = \frac{1}{a} : \frac{1}{b}}$$

Conclusion

- $\boxed{\text{Speed} = \dfrac{\text{Distance Travelled}}{\text{Time Taken}}}$
- $\boxed{\text{Speed x Time} = \text{Distance}}$
- $\boxed{\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}}$
- $\boxed{1\ \text{Km/h} = \dfrac{5}{18}\ \text{m/s}\ \&\ 1\ \text{m/s} = \dfrac{18}{5}\ \text{Km/h}}$
- $\boxed{\text{Average Speed} = \dfrac{\text{Total Distance Covered}}{\text{Total Time Taken}}}$

Conclusion

- If A covers a distance x Km at a Km/h and, then y Km at b Km/h, then the average speed during the whole journey is $\boxed{\frac{ab(x + y)}{(ay + bx)} \text{ Km/h}}$
- If a person goes from point 'A' to point 'B' at 'a' Km/h and comes back from point 'B' to point 'A' at 'b' Km/h, then the average speed during the whole journey is given by $\boxed{\frac{2ab}{(a + b)} \text{ Km/h}}$

Conclusion

- A person goes certain distance point ‘A’ to point ‘B’ at a speed of x Km/h and returns back point ‘B’ to point ‘A’ at a speed of y Km/h. If s/he takes T hours in all, then the distance between points ‘A’ and ‘B’ is

$$\boxed{T\left(\frac{xy}{x+y}\right) \text{ hours}}$$

or

$$\boxed{\text{Total Time Taken} \times \frac{\text{Product of Two Speeds}}{\text{Sum of Two Speeds}}}$$

Conclusion

- If two persons, A and B, start at the same time from two points P and Q towards each other, and after crossing they take T_1 and T_2 hours in reading Q and P respectively, then

$$\boxed{\frac{\text{a (i.e., Speed of A)}}{\text{b(i.e., Speed of B)}} = \frac{\sqrt{T_2}}{\sqrt{T_1}}}$$

- If the new speed is $\frac{m}{n}$ of the original speed, then the change in time taken to cover the same distance is given by

$$\boxed{\left(\frac{n}{m} - 1\right) \times \text{Original Time}}$$

Conclusion

- If the a person changes his/her speed to $\frac{m}{n}$ of its usual/original speed & late by T min., then usual time taken by him/her is

$$\boxed{\text{Usual Time} = \frac{mT}{n-m} \text{ if } \frac{m}{n} < 1} \ \& \ \boxed{\text{Usual Time} = \frac{mT}{m-n} \text{ if } \frac{m}{n} > 1}$$

- If the a person covers a distance in T hours, the first half at a km/h and second half at b, then the total distance covered by the person is $\boxed{\frac{2abT}{a+b}}$

Conclusion

- If a body travels $x_1, x_2, x_3, \ldots, x_n$ metres with different speeds $a_1, a_2, a_3, \ldots, a_n$ m/s in time $t_1, t_2, t_3, \ldots, t_n$ seconds respectively, then the average speed of the body throughout the journey is given by

$$\boxed{\frac{x_1 + x_2 + x_3 + \ldots + x_n}{t_1 + t_2 + t_3 + \ldots + t_n} \text{ or } \frac{a_1t_1 + a_2t_2 + a_3t_3 + \ldots + a_nt_n}{t_1 + t_2 + t_3 + \ldots + t_n}}$$

- A body covers a distance 'x' in time 't_1' with speed 'a', but when it travels with speed 'b' covers the same distance in time 't_2'.

$$\boxed{\frac{\text{Product of speed}}{d} = \frac{a}{t_2} = \frac{b}{t_1} = \frac{\text{Difference of Speed}}{\text{Difference of Time}}}$$

Conclusion

- If the ratio of the speeds of two persons X and Y is a : b, then the ratio of the times taken by them to cover the same distance is

$$\boxed{\frac{1}{a} : \frac{1}{b} \text{ or } b : a}$$

Summary

- Basic Concepts
  - Speed, Distance, Time, & Average Speed
  - Units of Measurement
- Conversion of Units
- Speed, Distance, Time, & Average Speed
  - Important Facts
  - Formulae

That’s all for now…