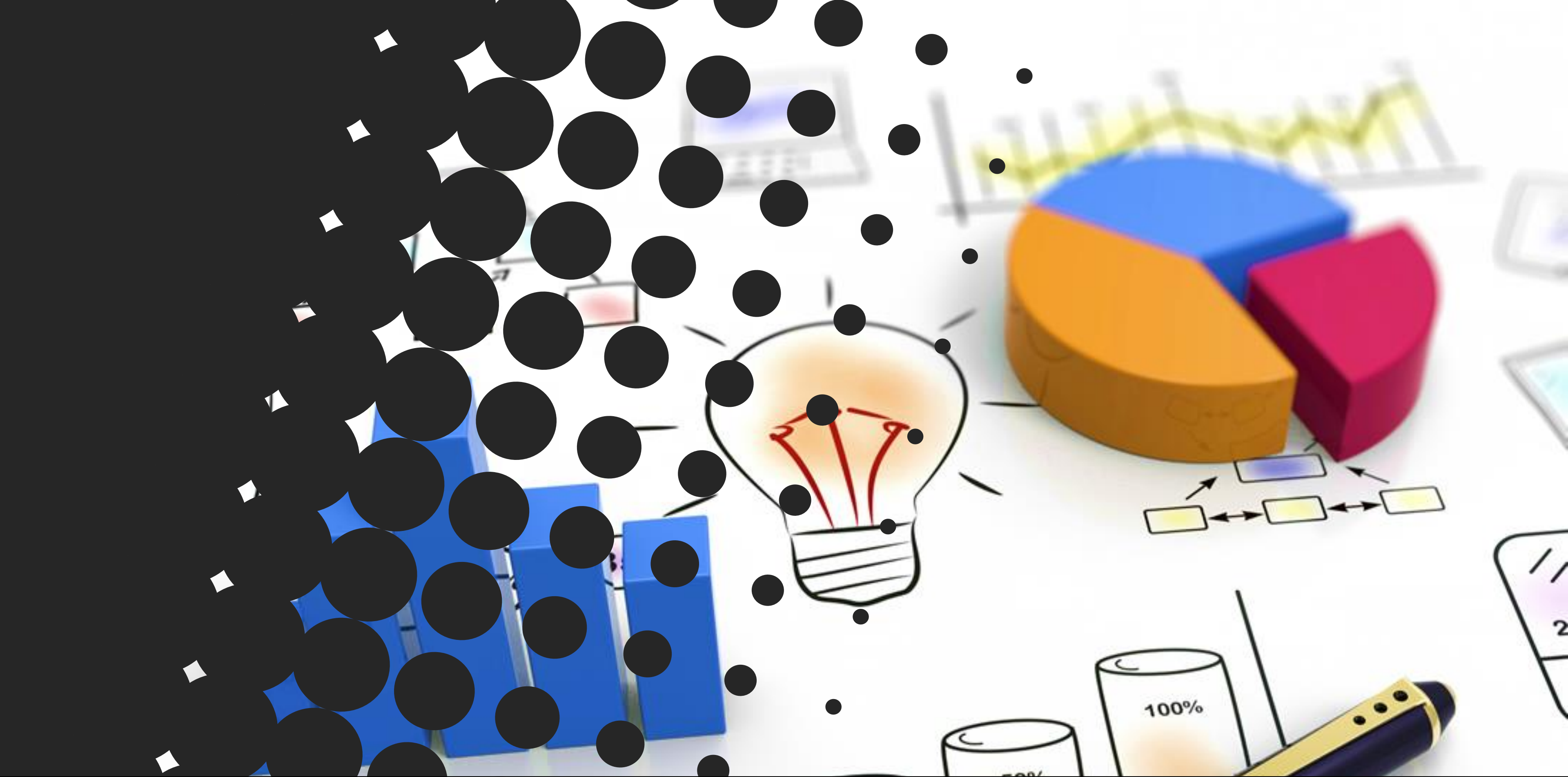

EPEA516
Analytical Skills II

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Learning Outcomes

After this lecture, you will be able to

- explore the formulae of time and work,
- solve problems based on time and work.

Introduction

- Timely Completion – Task/Work
- Complete Task – Earlier/Later
- Manpower – Increased/Decreased
- Time & Manpower– Inversely Proportional
- Time & Work/Men & Work - Directly Proportional

Formulae

- If ‘A’ can do a piece of work in ‘n’ days, then at a uniform rate of working ‘A’ will finish $\frac{1}{n}$ th work in one day.
- For Example
- If Amit can do a piece of work in 18 days then at a uniform rate of working Amit will finish $\frac{1}{18}$ th work in one day.

Formulae

- If $\frac{1}{n}$ th of a work is done by 'A' in one day, then 'A' will take 'n' days to complete the full work.
- For Example
- If $\frac{1}{10}$ th of a work is done by Rohit in one day, then Rohit will take 10 days to complete the full work.

Formulae

- If 'A' does $\frac{1}{n}$ th of a work in one hour then to complete the full work, 'A' will take n hrs.
- For Example
- If Varun does $\frac{1}{10}$ th of a work in one hour then to complete the full work, Varun will take 10 hrs.

Formulae

- If ‘A’ does three times faster work than ‘B’, then ratio of work done by A and B is 3:1 and ratio of time taken by A and B is 1:3.
- For Example
- If Karan does five times faster work than Vishal, then ratio of work done by Karan and Vishal is 5:1 and ratio of time taken by Karan and Vishal is 1:5.

Formulae

- A, B and C can do a piece of work in T_1, T_2 and T_3 days, respectively. If they have worked for D_1, D_2 and D_3 days, respectively, then
- Amount of work done by A in 1 day = $\frac{1}{T_1}$
- Amount of work done by B in 1 day = $\frac{1}{T_2}$
- Amount of work done by C in 1 day = $\frac{1}{T_3}$

Formulae

- Amount of work done by A in D_1 days = $\frac{D_1}{T_1}$
- Amount of work done by B in D_2 days = $\frac{D_2}{T_2}$
- Amount of work done by C in D_3 days = $\frac{D_3}{T_3}$
- Amount of work done by A, B and C together

$$= \frac{D_1}{T_1} + \frac{D_2}{T_2} + \frac{D_3}{T_3}$$

=1 (If the work is complete.)

Example 1

- A, B and C can do a piece of work in 6, 8, and 10 days, respectively. If they have worked for 3, 4, and 5 days, respectively, then calculate the amount of work done by A, B and C together.

- Amount of work done by A in 3 days = $\frac{\cancel{3}}{\cancel{6}} = \frac{1}{2}$

- Amount of work done by B in 4 days = $\frac{\cancel{4}}{\cancel{8}} = \frac{1}{2}$

- Amount of work done by C in 5 days = $\frac{\cancel{5}}{\cancel{10}} = \frac{1}{2}$

- Amount of work done by A, B and C together = $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{\cancel{3}}{\cancel{2}}$ 1.5

  = 1.5

Formulae

- If ‘A’ can do a piece of work in ‘X’ days and ‘B’ can do the same work in Y days, then both of them working together will do the same work in $\frac{XY}{X+Y}$ days.
- Derivation-
- A can do a piece of work in X days and B can do the same work in Y days.
- A’s 1 day’s work = $\frac{1}{X}$ and B’s 1 day’s work = $\frac{1}{Y}$

Formulae

- A's 1 day's work = $\frac{1}{X}$ and B's 1 day's work = $\frac{1}{Y}$

- Then, (A + B)'s 1 day's work $= \frac{1}{X} + \frac{1}{Y}$

$$= \frac{Y + X}{XY}$$

$$= \frac{X + Y}{XY}$$

- A and B together can complete the work in $\frac{XY}{X + Y}$ days

Example 2

- A can finish a piece of work by working alone in 6 days; B, while working alone, can finish the same work in 12 days. If both of them work together, then in how many days, the work will be completed?
- Given, X = 6 and Y = 12
- A and B will complete the work (by working together) in

$$\frac{XY}{X+Y} \text{ days}$$

Example 2

- A and B will complete the work (by working together) in

$$= \frac{XY}{X+Y} \text{ days}$$

$$= \frac{6 \times 12}{6 + 12} \text{ days}$$

$$= \frac{\cancel{72}}{\cancel{18}} \text{ days}$$

$$= 4 \text{ days}$$

Formulae

- If A, B and C, while working alone, can complete a work in X, Y and Z days, then they will together complete the work in $\frac{XYZ}{XY+YZ+ZX}$ days.
- Derivation –
- A, B and C, while working alone, can complete a work in X, Y and Z days.
- A's 1 day's work = $\frac{1}{X}$; B's 1 day's work = $\frac{1}{Y}$; C's 1 day's work= $\frac{1}{Z}$

Formulae

- A's 1 day's work = $\frac{1}{X}$; B's 1 day's work = $\frac{1}{Y}$; C's 1 day's work= $\frac{1}{Z}$
- (A + B + C)'s 1 day's work = $\frac{1}{X} + \frac{1}{Y} + \frac{1}{Z}$

$$= \frac{YZ+ZX+XY}{XYZ} \text{ or } \frac{XY+YZ+ZX}{XYZ}$$

- A, B and C together can complete the work in

$$\frac{XYZ}{XY+YZ+ZX} \text{ days}$$

Example 3

- A, B and C can complete a piece of work in 10, 15 and 18 days. In how many days, would all of them complete the same work working together?

- Here, X = 10, Y = 15 and Z = 18

- Therefore, the work will be completed in $= \dfrac{XYZ}{XY + YZ + ZX}$

$$= \frac{10 \,.\, 15 \,.\, 18}{10 \,.\, 15 + 15 \,.\, 18 + 18 \,.\, 10} \text{ days}$$

$$= \frac{2700}{150 + 270 + 180} \text{ days}$$

Example 3

$$= \frac{2700}{150 + 270 + 180} \text{ days}$$

$$= \frac{2700}{600}$$

$$= 4\frac{1}{2} \text{ days}$$

Formulae

- Two persons, A and B, working together, can complete a piece of work in X days. If A, working alone, can complete the work in Y days, then B, working alone, will complete the work in $\frac{XY}{Y-X}$ days.
- Derivation -
- A and B together can complete the work in X days.
- (A + B)'s 1 day's work = $\frac{1}{X}$
- Similarly, A's 1 day's work = $\frac{1}{Y}$

Formulae

- Therefore, B's 1 day's work = $\frac{1}{X} - \frac{1}{Y} = \frac{Y - X}{XY}$
- B alone can complete the work in $\frac{XY}{Y - X}$ days.

Example 4

- A and B, working together, take 15 days to complete a piece of work. If A alone can do this work in 20 days, then how long would B take to complete the same work?
- Here, X = 15 and Y = 20
- B alone will complete the work in = $\frac{XY}{Y - X}$ days

$$= \frac{15 \cdot 20}{20 - 15}$$

$$= \frac{\cancel{300}}{\cancel{5}} = 60 \text{ days}$$

Formulae

- If A and B, working together, can finish a piece of work in X days, B and C in Y days, C and A in Z days, then
- (a) A, B and C working together, will complete the job in

$$\frac{2XYZ}{XY + YZ + ZX} \text{ days.}$$

- (b) A alone will complete the job in $\frac{2XYZ}{XY + YZ - ZX}$ days.
- (c) B alone will complete the job in $\frac{2XYZ}{-XY + YZ + ZX}$ days.
- (d) C alone will complete the job in $\frac{2XYZ}{XY - YZ + ZX}$ days.

Formulae

- Derivation-
- (A + B)'s 1 day's work = $\frac{1}{X}$
- (B + C)'s 1 day's work = $\frac{1}{Y}$
- (C + A)'s 1 day's work = $\frac{1}{Z}$
- [(A + B) + (B + C) + (C + A)]'s 1 day's work = $\frac{1}{X} + \frac{1}{Y} + \frac{1}{Z}$
- 2 (A + B + C)'s 1 day's work = $\frac{1}{X} + \frac{1}{Y} + \frac{1}{Z}$

Formulae

- 2 (A + B + C)'s 1 day's work $= \frac{1}{X} + \frac{1}{Y} + \frac{1}{Z}$

- (A + B + C)'s 1 day's work $= \frac{1}{2}\left(\frac{1}{X} + \frac{1}{Y} + \frac{1}{Z}\right)$

$$= \frac{XY + YZ + ZX}{2XYZ}$$

- A, B and C, working together, will complete the work in

$$\frac{2XYZ}{XY + YZ + ZX} \text{ days}$$

Formulae

- Also, A's 1 day's work = (A + B + C)'s 1 day's work

  – (B + C)'s 1 day's work

$$= \frac{XY + YZ + ZX}{2XYZ} - \frac{1}{Y}$$

$$= \frac{XY + YZ + ZX - 2ZX}{2XYZ}$$

$$= \frac{XY + YZ - ZX}{2XYZ}$$

- So, A alone can do the work in $\frac{2XYZ}{XY + YZ - ZX}$ days.
- Similarly, B alone can do the work in $\frac{2XYZ}{-XY + YZ + ZX}$ days &

  C alone can do the work in $\frac{2XYZ}{XY - YZ + ZX}$ days.

Example 5

- A and B can do a piece of work in 12 days, B and C in 15 days, C and A in 20 days. How long would each of them would take separately to complete the same work?
- Here, X = 12, Y = 15 and Z = 20
- A alone can do the work in $\frac{2XYZ}{XY + YZ - ZX}$

$$= \frac{2 \,.\, 12 \,.\, 15 \,.\, 20}{12 \,.\, 15 + 15 \,.\, 20 - 20 \,.\, 12} \text{ days}$$

$$= \frac{\cancel{7200}}{\cancel{240}} = 30 \text{ days}$$

Example 5

- B alone can do the work in $\dfrac{2XYZ}{-XY + YZ + ZX}$ days

$$= \frac{2 \cdot 12 \cdot 15 \cdot 20}{-12 \cdot 15 + 15 \cdot 20 + 20 \cdot 12} \text{ days}$$

$$= \frac{\cancel{7200}}{\cancel{360}}$$

$$= 20 \text{ days}$$

- C alone can do the work in $\dfrac{2XYZ}{XY - YZ + ZX}$ days

$$= \frac{2 \cdot 12 \cdot 15 \cdot 20}{12 \cdot 15 - 15 \cdot 20 + 20 \cdot 12} \text{ days}$$

$$= \frac{\cancel{7200}}{\cancel{120}}$$

$$= 60 \text{ days}$$

Formulae

- (a) If A can complete a work in X days and B is k times efficient than A, then the time taken by both A and B, working together, to complete the work is $\frac{X}{k + 1}$
- (b) If A and B, working together, can complete a work in X days and B is k times efficient than A, then the time taken by
- (i) A, working alone, to complete the work is $(k + 1)\ X$
- (ii) B, working alone, to complete the work is $(\frac{k + 1}{k})X$

Example 6

- A can do a piece of work in 24 days. If B works twice as fast as A, then how long would they take to complete the work working together?
- Here, X = 24 & k = 2
- Time taken by A and B, working together, to complete the work = $\frac{X}{k + 1}$ days $= \frac{24}{2 + 1}$ days

$$= \frac{\cancel{24}}{\cancel{3}} \text{ days}$$

$$= 8 \text{ days}$$

Formulae

- If A working alone takes 'a' days more than A and B; B working alone takes 'b' days more than A and B together, then the number of days taken by A and B, working together, to finish a job is given by $\sqrt{ab}$.

Example 7

- A alone would take 8 hours more to complete the job than if both A and B worked together. If B worked alone, he took $4\frac{1}{2}$ hours more to complete the job than A and B worked together. What time would they take if both A and B worked together?
- Here, $a = 8$

$$b = 4\frac{1}{2}$$

$$= \frac{9}{2}$$

Example 7

- Here, a = 8 and b = $4\frac{1}{2} = \frac{9}{2}$
- Time taken by A and B, working together, to complete the job

$$= \sqrt{ab} \text{ days}$$

$$= \sqrt{\cancel{8}^{\,4} \times \frac{9}{\cancel{2}}}$$

$$= \sqrt{4 \times 9}$$

$$= \sqrt{36}$$

$$= 6 \text{ days}$$

Formulae

- If A is 'k' times more efficient than B and is, therefore, able to complete a work in 'l' days less than B, then
- (a) A and B, working together, can finish the work in $\frac{kl}{k^2 - 1}$ days.
- (b) A, working alone, can finish the work in $\frac{l}{k - 1}$ days.
- (c) B, working alone, can finish the work in $\frac{kl}{k - 1}$ days.

Example 8

- A is thrice as good a workman as B and takes 10 days less to complete a piece of work than B takes. Find out time in which B alone can complete the work.
- Here, k = 3 and l = 10
- Time taken by B, working alone, to complete the work

$$= \frac{kl}{k - 1} \text{ days}$$

$$= \frac{3 \times 10}{3 - 1} \text{ days}$$

Example 8

- Here, k = 3 and l = 10.
- Time taken by B, working alone, to complete the work

$$= \frac{3 \times 10}{3 - 1} \text{ days}$$

$$= \cancel{\frac{30}{2}} \text{ days}$$

$$= 15 \text{ days}$$

Formulae

- If A can complete $\frac{a}{b}$ part of work in X days, then $\frac{c}{d}$ part of the work will be done in $\frac{b}{a} . \frac{c}{d} . X$ days

  or

  $$\frac{b . c . X}{a . d} \text{ days}$$

Example 9

- A completes $\frac{3}{4}$th of a work in 12 days. In how many days he would complete $\frac{1}{8}$th of the work?
- Here, a = 3, b = 4, X = 12, c = 1 and d = 8
- Number of days required to complete $\frac{1}{8}$ of the work

$$= \frac{b\,.\,c\,.\,X}{a\,.\,d} \text{ days} \qquad = \frac{\cancel{4} \times 1 \times \cancel{12}^{\cancel{4}\ 2}}{\cancel{3} \times \cancel{8}_{\cancel{2}}}$$

$$= 2 \text{ days}$$

Formulae

- There are two groups of people with same level of efficiency. In one group, N_1 persons can do W_1 works in T_1 time and in the other group, N_2 persons can do W_2 works in T_2 time. The relationship between the two groups is given by $N_1W_2T_1 = N_2W_1T_2$

Example 10

- If 10 persons can complete $\frac{2}{5}$ of a work in 8 days, then find out the number of persons required to complete the remaining work in 12 days.

- Given, $N_1 = 10, W_1 = \frac{2}{5}, T_1 = 8$

$$N_2 = ?, W_2 = \frac{3}{5}, T_2 = 12$$

$$N_1 W_2 T_1 = N_2 W_1 T_2$$

Example 10

- $N_1 = 10, W_1 = \frac{2}{5}, T_1 = 8$ and $N_2 = ?, W_2 = \frac{3}{5}, T_2 = 12$

$$N_1 W_2 T_1 = N_2 W_1 T_2$$

$$10 \times \frac{3}{5} \times 8 = N_2 \times \frac{2}{5} \times 12$$

$$\frac{10 \times \cancel{3} \times \cancel{8}}{\cancel{5}} = \frac{N_2 \times \cancel{2} \times \cancel{12}}{\cancel{5}}$$

$$N_2 = 10$$

Formulae

- There are two groups of people with same efficiency. In group, N_1 persons can do W_1 works in T_1 time working H_1 hours a day and N_2 persons can do W_2 works in T_2 time working H_2 hours a day. The relationship between the two groups is given by

$$N_1 W_2 T_1 H_1 = N_2 W_1 T_2 H_2$$

Example 11

- If 10 persons can cut 20 trees in 3 days by working 12 hours a day. Then, in how many days can 24 persons cut 32 trees by working 4 hours a day?

- Given, $N_1 = 10, W_1 = 20, T_1 = 3, H_1 = 12$

$N_2 = 24, \ W_2 = 32, \ T_2 = ?, H_2 = 4$

$N_1 W_2 T_1 H_1 = N_2 W_1 T_2 H_2$

$$\cancel{10} \times \cancel{32} \times \cancel{3} \times \overset{6}{\cancel{12}} = \overset{\cancel{8}}{\cancel{24}} \times \overset{\cancel{2}}{\cancel{20}} \times T_2 \times \cancel{4}$$

$T_2 = 6$ days

Formulae

- If ‘a’ men and ‘b’ women can do a piece of work in ‘n’ days, then ‘c’ men and ‘d’ women can do the work in

$$\frac{nab}{bc + ad} \text{ days}$$

Example 12

- 12 men and 15 women can do a work in 14 days. In how many days, 7 men and 5 women would complete the work?
- Given, a = 12, b = 15, n = 14, c = 7 and d = 5
- Required number of days $= \dfrac{nab}{bc + ad}$ days

$$= \frac{14 \times 12 \times 15}{15 \times 7 + 12 \times 5} \text{ days}$$

$$= \frac{2520}{105 + 60} \text{ days}$$

Example 12

- Required number of days $= \dfrac{2520}{105 + 60}$ days

$$= \frac{2520}{165} \text{ days}$$

$$= 15\frac{3}{11} \text{ days}$$

or

$$= 15.27 \text{ (approx.)}$$

Conclusion

- Formulae, Shortcuts, & Examples
  - Computation of Time
  - Computation of Work

Summary

- Time & Work
    - Computation of Work Done

That's all for now…