EMTH403

Mathematical Foundation for Computer Science

Nitin K. Mishra (Ph.D.)
Associate Professor

Lecture Outcomes

After this lecture, you will be able to

- understand what is antisymmetric relation.
- understand what is transitive relation.

Properties of Relations - Antisymmetric

A relation R on a set A such that for all a, b $\in$ A, if (a, b) $\in$ R and (b, a) $\in$ R, then a = b is called antisymmetric.

Similarly, the relation R on the set A is antisymmetric if

$\forall a \forall b(((a, b) \in R \wedge (b, a) \in R) \rightarrow (a = b))$.

Properties of Relations – Antisymmetric Example 1

Ques:- Consider the following relations on {1, 2, 3, 4}:

Is it antisymmetric($\forall a \forall b(((a, b) \in R \wedge (b, a) \in R) \rightarrow (a = b))$)?

R4 = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)}

Ans:- R4 is antisymmetric.

Content Source: Discrete Mathematics and Its Applications. Seventh Edition. Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University. (and formerly AT&T Laboratories)

Properties of Relations – Antisymmetric Example 2

Ques:- Consider the following relations on {1, 2, 3, 4}:

Is it antisymmetric ($\forall a \forall b(((a, b) \in R \land (b, a) \in R) \rightarrow (a = b)).$) ?

R5 = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)}

Ans:- R5 is antisymmetric.

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Properties of Relations – Antisymmetric Example 3

Ques:- Consider the following relations on {1, 2, 3, 4}:

Is it antisymmetric($\forall a \forall b(((a, b) \in R \land (b, a) \in R) \rightarrow (a = b))$.) ?

R6 = {(3, 4)}

Ans:- R6 is antisymmetric.

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Properties of Relations – Antisymmetric Example 4

Ques:- Consider the following relations on {1, 2, 3, 4}:

Is it antisymmetric ($\forall a \forall b(((a, b) \in R \wedge (b, a) \in R) \rightarrow (a = b)).$) ?

R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)}.

Ans:- One should verify that the relation is not antisymmetric.

This is done by finding a pair (a, b) with $a \neq b$ such that (a, b) and (b, a) are both in the relation.

Properties of Relations – Antisymmetric Example 5

Ques:- Consider the following relations on {1, 2, 3, 4}:

Is it antisymmetric($\forall a \forall b(((a, b) \in R \wedge (b, a) \in R) \rightarrow (a = b))$.) ?

R2 = {(1, 1), (1, 2), (2, 1)}.

Ans:- One should verify that the relation is not antisymmetric. This is done by finding a pair (a, b) with $a \neq b$ such that (a, b) and (b, a) are both in the relation.

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Properties of Relations – Antisymmetric Example 6

Ques:- Consider the following relations on {1, 2, 3, 4}:

Is it antisymmetric($\forall a \forall b(((a, b) \in R \land (b, a) \in R) \rightarrow (a = b)).$) ?

R3 = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)}

Ans:- One should verify that the relation is not antisymmetric. This is done by finding a pair (a, b) with $a \neq b$ such that (a, b) and (b, a) are both in the relation.

Properties of Relations – Antisymmetric Example 7

Ques:- Is the "divides" relation on the set of positive integers antisymmetric?

Ans:- It is antisymmetric, for if a and b are positive integers with $a \mid b$ and $b \mid a$, then $a = b$.

Properties of Relations - Transitive

A relation R on a set A is called transitive if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for all $a, b, c \in A$.

Remark: Using quantifiers we see that the relation R on a set A is transitive if we have

$\forall a \forall b \forall c(((a, b) \in R \land (b, c) \in R) \rightarrow (a, c) \in R).$

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Properties of Relations – Transitive Example 1

Ques:- Is the following relation transitive?

R4 = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)}.

Ans:- R4 is transitive, because (3, 2) and (2, 1), (4, 2) and (2, 1), (4, 3) and (3, 1), and (4, 3) and (3, 2) are the only such sets of pairs, and (3, 1), (4, 1), and (4, 2) belong to R4.

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Properties of Relations – Transitive Example 2

Ques:- Is the following relation transitive?

R5 = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)}.

Ans:- R5 is transitive

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Properties of Relations – Transitive Example 3

Ques:- Is the following relation transitive?

R6 = {(3, 4)}.

Ans:- R6 is transitive

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Properties of Relations – Transitive Example 4

Ques:- Is the following relation transitive?

R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)}.

Ans:- R1 is not transitive because (3, 4) and (4, 1) belong to R1, but (3, 1) does not

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Properties of Relations – Transitive Example 5

Ques:- Is the following relation transitive?

R2 = {(1, 1), (1, 2), (2, 1)}.

Ans:- R2 is not transitive because (2, 1) and (1, 2) belong to R2, but (2, 2) does not.

Properties of Relations – Transitive Example 6

Ques:- Is the following relation transitive?

R3 = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)}

Ans:- R3 is not transitive because (4, 1) and (1, 2) belong to R3, but (4, 2) does not.

Properties of Relations – Transitive Example 7

Ques:- Is the "divides" relation on the set of positive integers transitive?

Ans:- Suppose that a divides b and b divides c. Then there are positive integers k and l such that b = ak and c = bl. Hence, c = a(kl), so a divides c. It follows that this relation is transitive.

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That's all for now…