# EMTH403

## Mathematical Foundation for Computer Science

Nitin K. Mishra (Ph.D.)

Associate Professor

# Lecture Outcomes

After this lecture, you will be able to

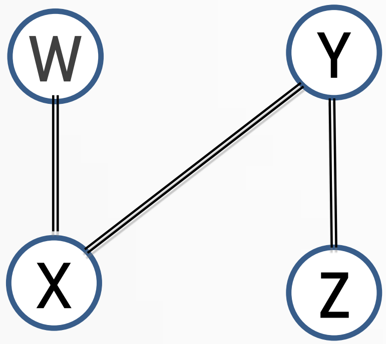
- understand what are isomorphic graph
- understand what is an isomorphism.

# Graph Isomorphism - Definition

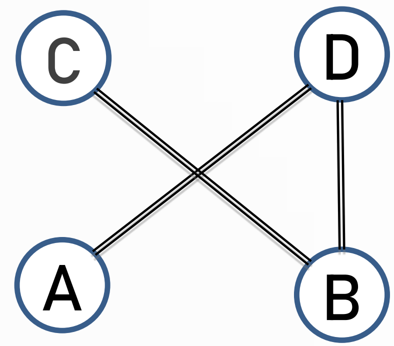
The simple graphs $G_1 = (V_1,E_1)$ and $G_2 = (V_2,E_2)$ are *isomorphic* if there exists a one to-one and onto function $f$ from $V_1$ to $V_2$ with the property that $a$ and $b$ are adjacent in $G1$ if and only if $f(a)$ and $f(b)$ are adjacent in $G_2$, for all $a$ and $b$ in $V_1$.

Such a function $f$ is called an *isomorphism*.∗ Two simple graphs that are not isomorphic are called *non-isomorphic*.

# Graph Isomorphism – 1st Mapping




Graph G




Graph G'

Degree 1

| | |
|---|---|
| W | C |
| Z | A |

Degree 2

| | |
|---|---|
| X | B |
| Y | D |

There exist a 1-1 and onto (bijective) function $f_1$: G $\rightarrow$ G' such that

$f_1$ (W) = C

$f_1$ (Y) = D

$f_1$ (Z) = A

$f_1$ (X) = B

$\text{Deg}(W) \neq \text{Deg}(B)$

$1 \neq 2$

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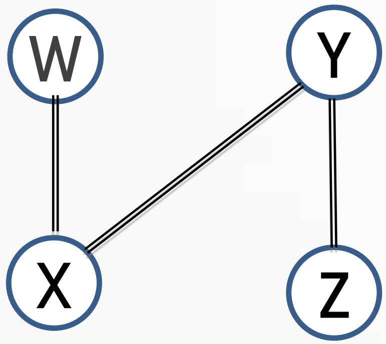
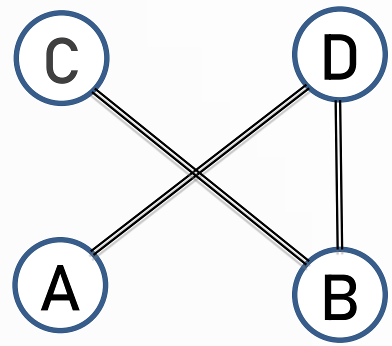
# Graph Isomorphism – 1st Mapping




Graph G




Graph G'

Degree 1

W C

Z A

Degree 2

X B

Y D

$Deg(X) \neq Deg(C)$

$2 \neq 1$

There exist a 1-1 and onto (bijective) function $f_1: G \rightarrow G'$ such that

$f_1(W) = C$

$f_1(Y) = D$

$f_1(Z) = A$

$f_1(X) = B$

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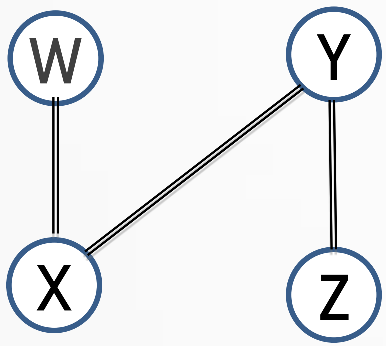
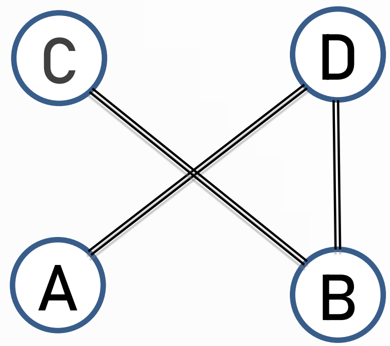
# Graph Isomorphism – 1st Mapping




Graph G




Graph G'

Degree 1

W C

Z A

Degree 2

X B

Y D

$f_1(W) = C$

$f_1(Y) = D$

$f_1(Z) = A$

$f_1(X) = B$

There exist a 1-1 and onto (bijective) function $f_1$: $G \rightarrow G'$ such that

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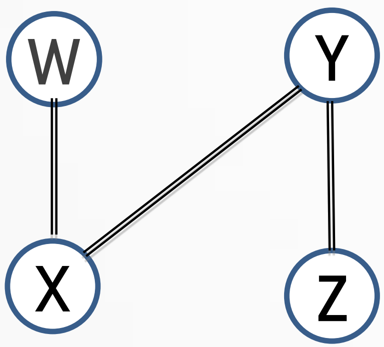
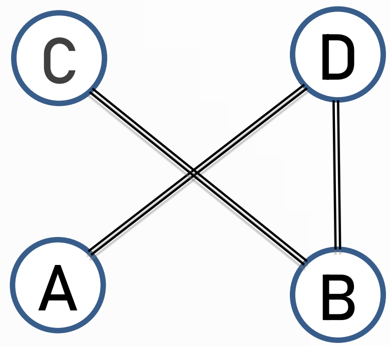
# Graph Isomorphism – 2nd Mapping




Graph G




Graph G'

Degree 1

W C

Z A

Degree 2

X B

Y D

$f_2$ (W) = A

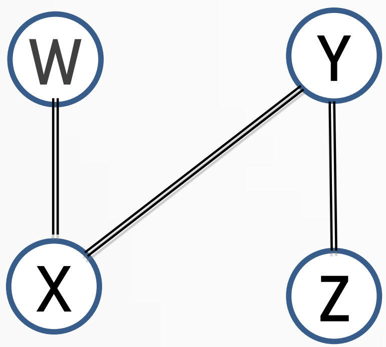
$f_2$ (Y) = D

$f_2$ (Z) = C

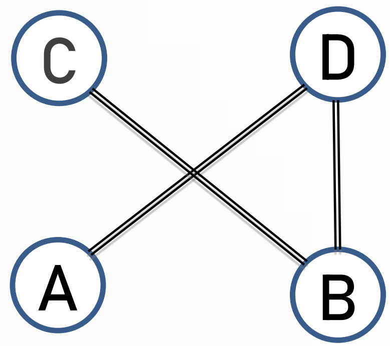
$f_2$ (X) = B

There exist a 1-1 and onto (bijective)
function $f_2$: G $\rightarrow$ G' such that

# Graph Isomorphism – $3^{rd}$ Mapping




Graph G




Graph G'

Degree 1

| | |
|---|---|
| W | C |
| Z | A |

Degree 2

| | |
|---|---|
| X | B |
| Y | D |

There exist a 1-1 and onto (bijective)
function $f_3$: G $\rightarrow$ G' such that

$f_3$ (W) = C

$f_3$ (Y) = B

$f_3$ (Z) = A

$f_3$ (X) = D

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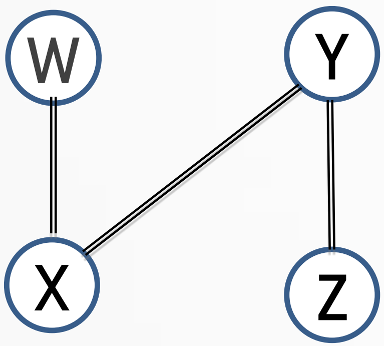
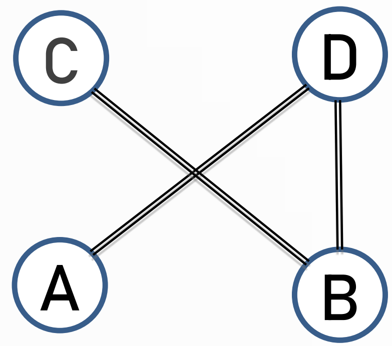
# Graph Isomorphism – $4^{th}$ Mapping




Graph G




Graph G'

Degree 1

W C

Z A

Degree 2

X B

Y D

There exist a 1-1 and onto (bijective) function $f_4: G \rightarrow G'$ such that

$f_4(W) = A$

$f_4(Y) = B$

$f_4(Z) = C$

$f_4(X) = D$

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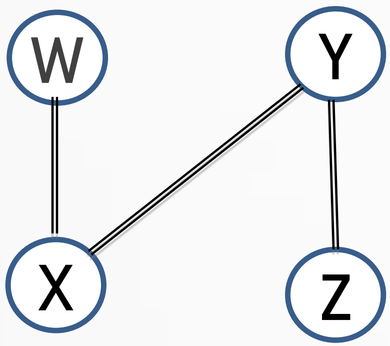
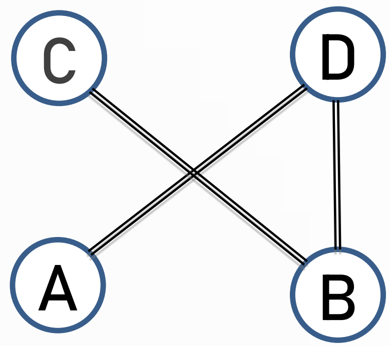
# Is a bijection $f_1$ an isomorphism also?




Graph G




Graph G'

Degree 1

W C

Z A

Degree 2

X B

Y D

There exist a 1-1 and onto (bijective) function $f_1$: G $\rightarrow$ G' such that

$f_1$ (W) = C
$f_1$ (Y) = D
$f_1$ (Z) = A
$f_1$ (X) = B

Answer: Yes the bijection $f_1$: G $\rightarrow$ G' is an isomorphism from G to G'.

Therefore the graph G is isomorphic to G' and there exist an isomorphism $f_1$: G $\rightarrow$ G'.

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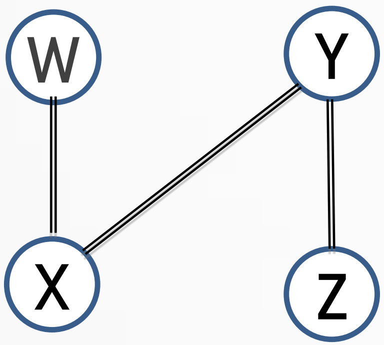
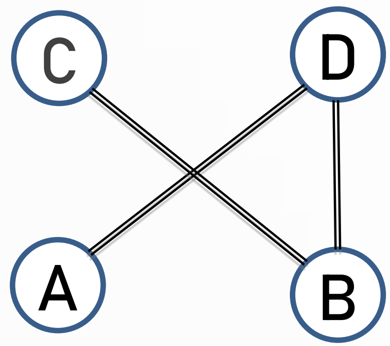
# How many Isomorphism are possible from isomorphic graph G to G”?




Graph G




Graph G’

Degree 1

W C

Z A

Degree 2

X B

Y D

There exist a 1-1 and onto (bijective)
function $f_2$: G $\rightarrow$ G’ such that

$f_2$ (W) = A
$f_2$ (Y) = D
$f_2$ (Z) = C
$f_2$ (X) = B

The graph G is isomorphic to G’ as in previous slide but $f_2$: G $\rightarrow$ G’ is not isomorphism .

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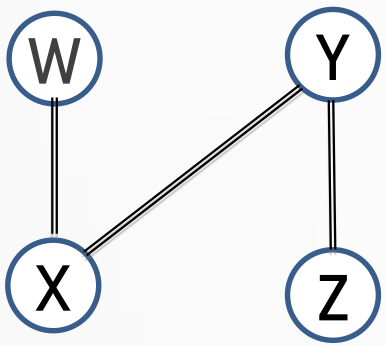
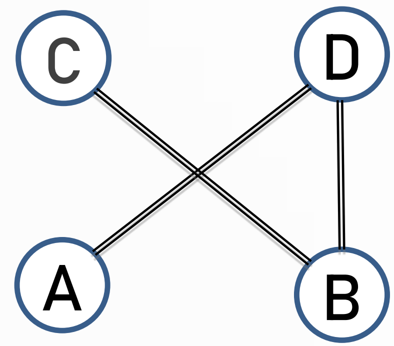
# How many Isomorphism are possible from isomorphic graph G to G”?




Graph G




Graph G’

There exist a 1-1 and onto (bijective) function $f_3$: $G \rightarrow G'$ such that

$f_3$ (W) = C
$f_3$ (Y) = B
$f_3$ (Z) = A
$f_3$ (X) = D

Degree 1

W C

Z A

Degree 2

X B

Y D

The graph G is isomorphic to G’ as in previous slide but $f_3$: $G \rightarrow G'$ is not isomorphism .

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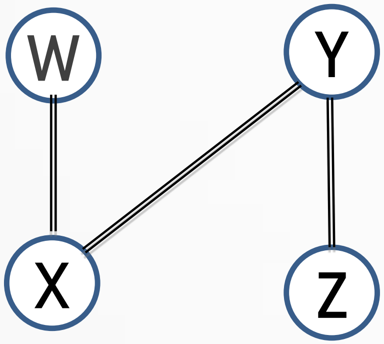
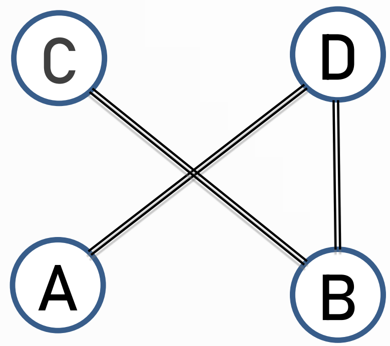
# How many Isomorphisms are possible from isomorphic graph G to G"?




Graph G




Graph G'

Degree 1

W C

Z A

Degree 2

X B

Y D

There exist a 1-1 and onto (bijective) function $f_4$: G $\rightarrow$ G' such that

$f_4$(W) = A
$f_4$(Y) = B
$f_4$(Z) = C
$f_4$(X) = D

The graph G is isomorphic to G' and $f_4$: G $\rightarrow$ G' is an isomorphism .

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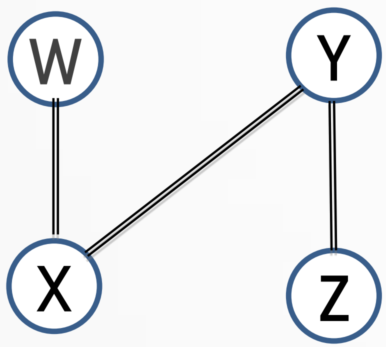
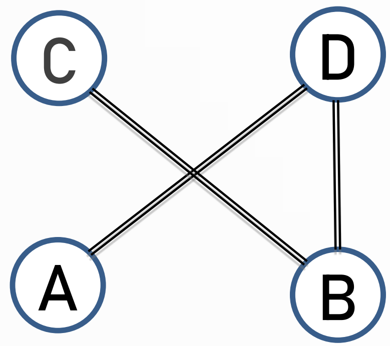
# How many Isomorphisms are possible from isomorphic graph G to G''?




Graph G




Graph G'

Degree 1

W C

Z A

Degree 2

X B

Y D

$f_4(W) = A$
$f_4(Y) = B$
$f_4(Z) = C$
$f_4(X) = D$

$f_1(W) = C$
$f_1(Y) = D$
$f_1(Z) = A$
$f_1(X) = B$

The graph G is isomorphic to G'. $f_1$: G $\rightarrow$ G' and $f_4$: G $\rightarrow$ G' are the only two isomorphism .

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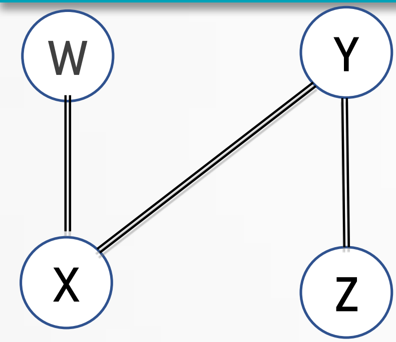
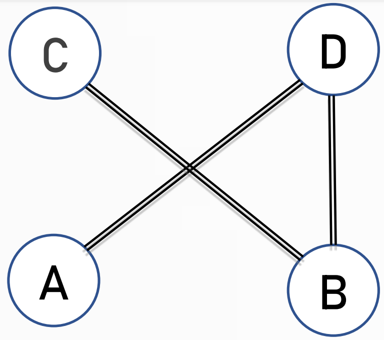
# Graph Isomorphism




Graph G




Graph G'

Adjacency matrix of graph G'

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

$f_1(W) = C$
$f_1(Y) = D$
$f_1(Z) = A$
$f_1(X) = B$

Adjacency matrix of graph G

$$\begin{array}{c} \\ Z \\ X \\ W \\ Y \end{array} \begin{array}{c} \begin{array}{cccc} Z & X & W & Y \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

The resultant matrix corresponds to matrix of graph G' and hence G and G' are isomorphic to each other.

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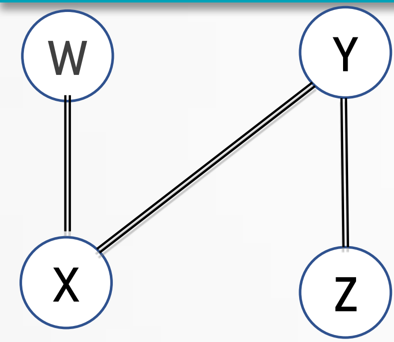
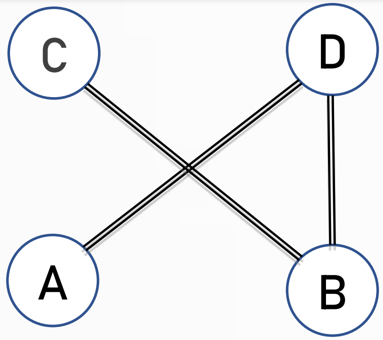
# Graph Isomorphism




Graph G




Graph G'

Adjacency matrix of graph G'

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Adjacency matrix of graph G

$$\begin{array}{c} \\ Z \\ X \\ W \\ Y \end{array} \begin{array}{c} \begin{array}{cccc} Z & X & W & Y \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

$f_4(W) = A$
$f_4(Y) = B$
$f_4(Z) = C$
$f_4(X) = D$

The resultant matrix corresponds to matrix of graph G' and hence G and G' are isomorphic to each other.

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That's all for now…