EMTH403

Mathematical Foundation for Computer Science

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Associate Professor

Lecture Outcomes

After this lecture, you will be able to

- understand what are trees.
- understand what different parts of trees.

Definition:

- A tree is a connected undirected graph with no simple circuits.

- Recall: A circuit is a path of length >=1 that begins and ends a the same vertex.

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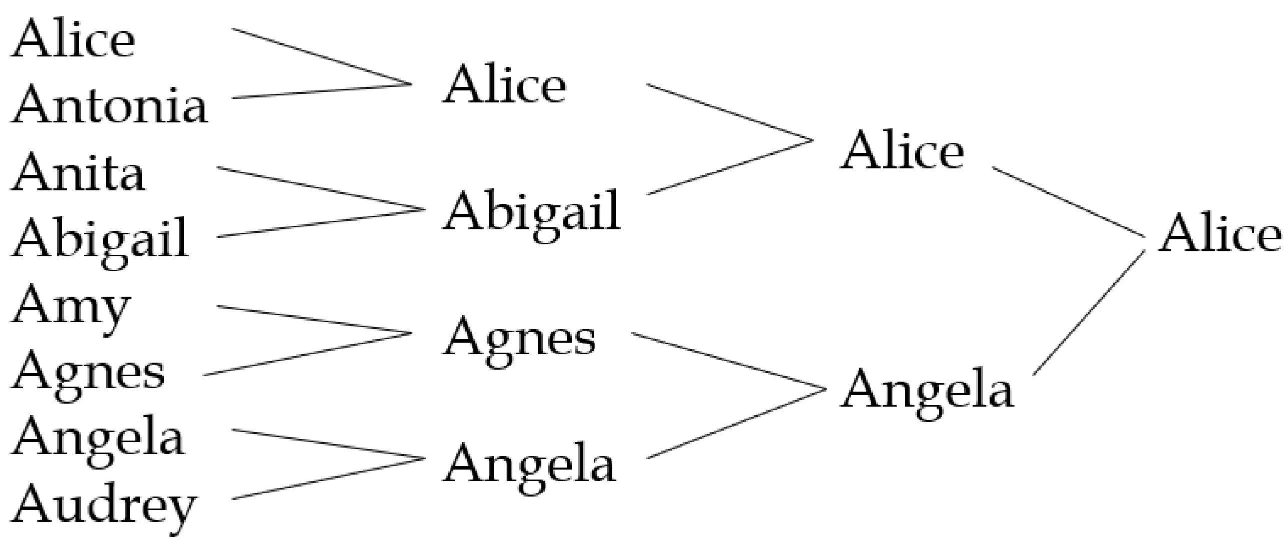
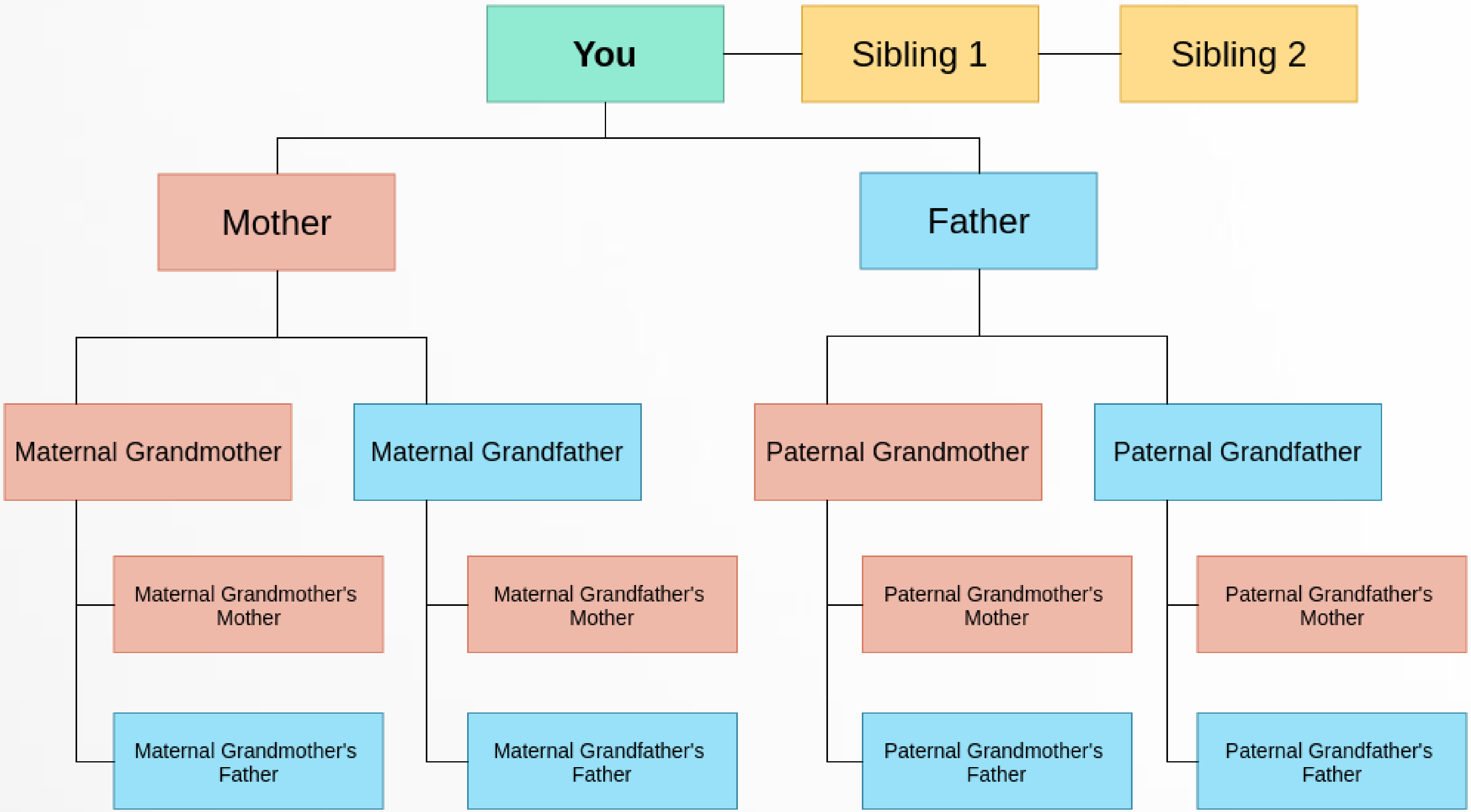
Tournament Trees

- A common form of tree used in everyday life is the tournament tree, used to describe the outcome of a series of games, such as a tennis tournament.
- A1



Application



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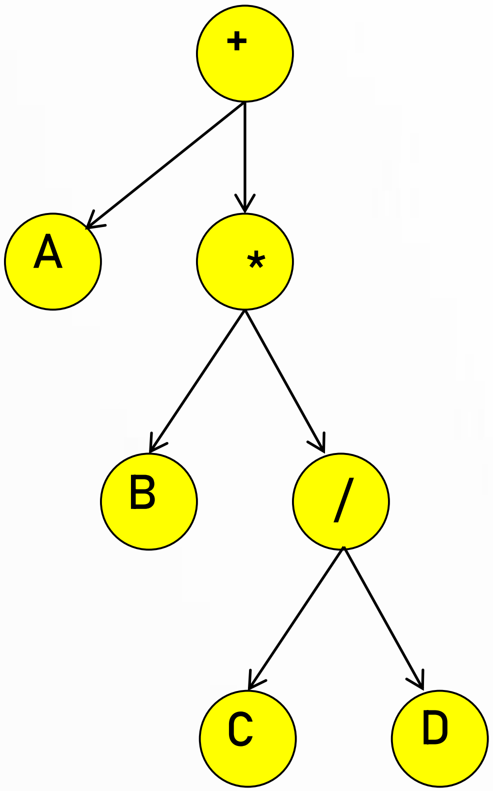
Application: Arithmetic Expression Trees

Example Arithmetic Expression:

A + (B * (C / D))

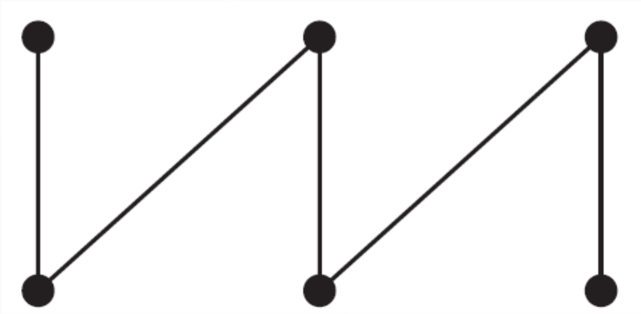


Theorem

- An undirected graph is a tree if and only if there is a unique simple path between any two of its vertices.

Trees

Which of these graphs are trees?



This graph is connected and has no simple circuits, so it is a tree.

Trees

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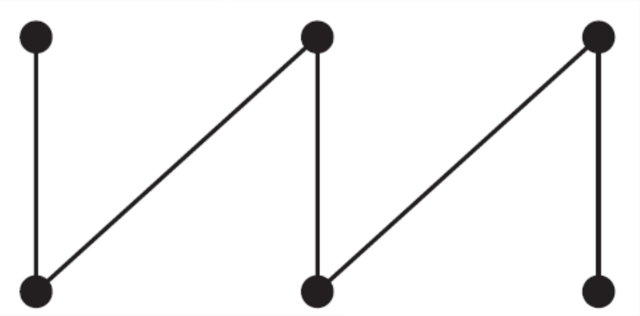

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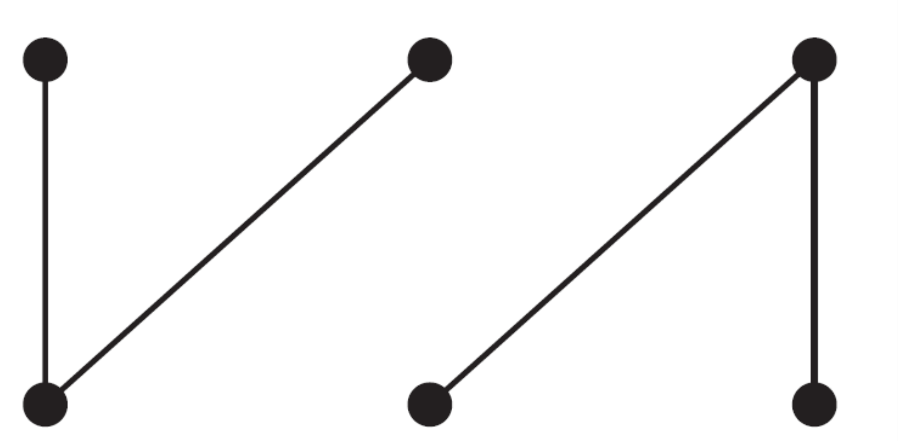

This graph is not connected, so it is not a tree.

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Trees

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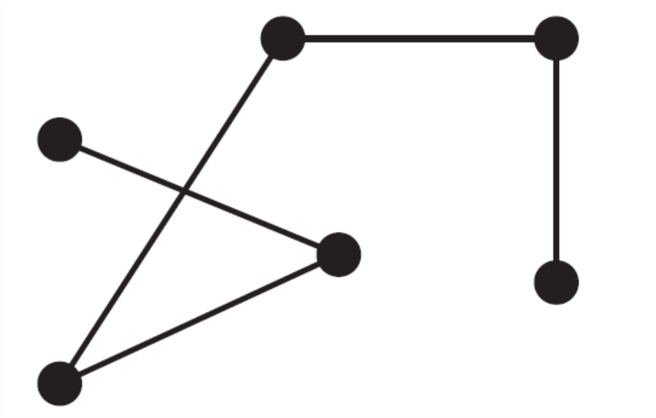

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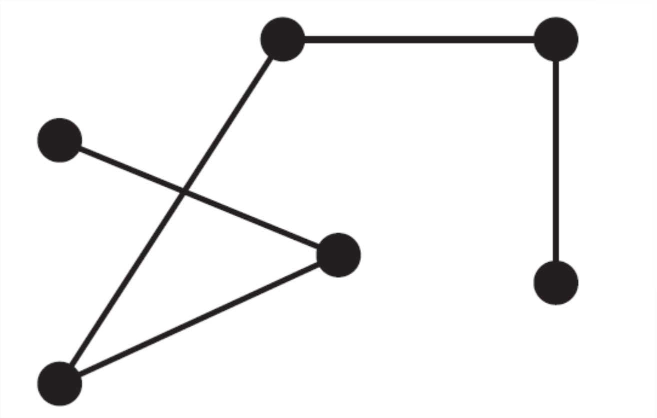

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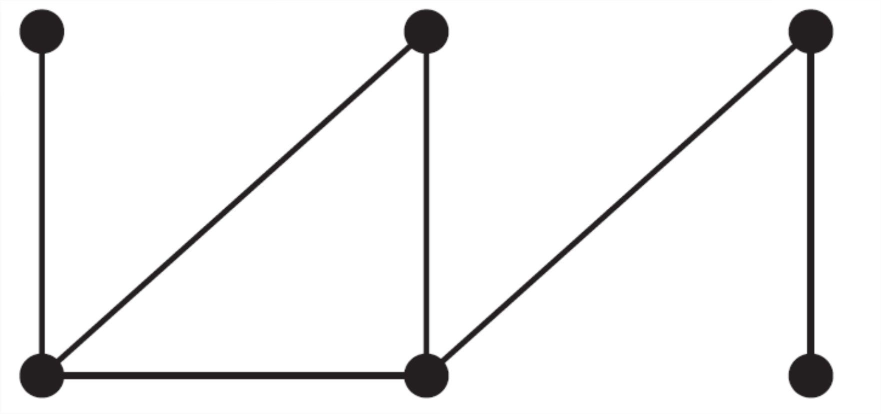

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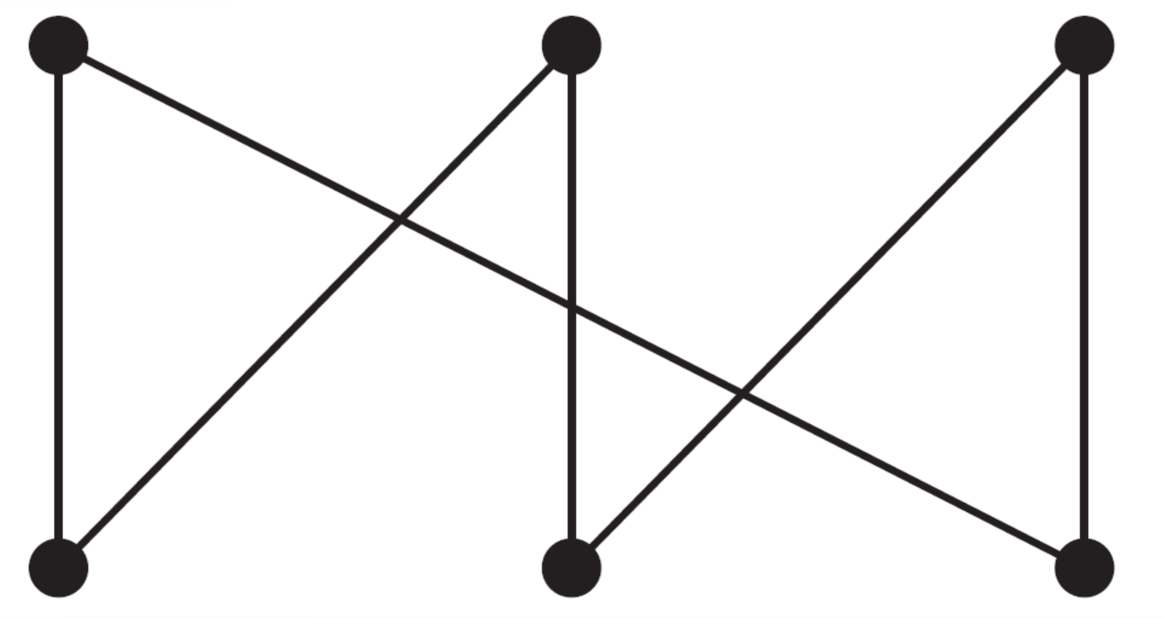

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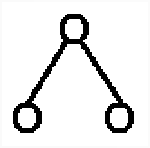
Trees

Definition in terms of Number of Edges and Nodes:

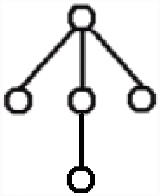
A tree is a finite connected graph such that:

no. of Nodes = no. of Edges + 1

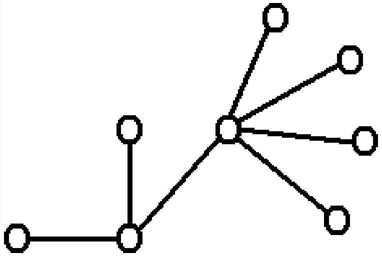
For Example:



3 nodes
2 edges



5 nodes
4 edges



8 nodes
7 edges

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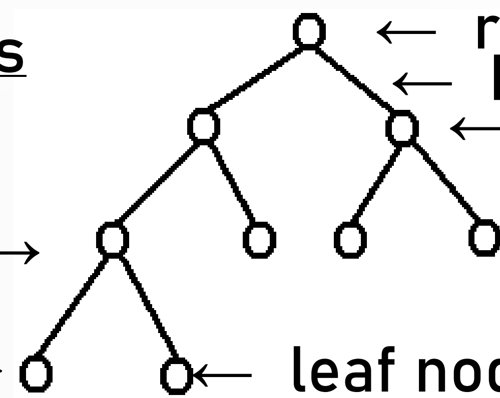
Trees

Defining Parts of a Tree:

Botanical Generic

Generic Familial Relations



← root -------tart node

← branch--edge or arc

← branch node –

-- internal node

predecessor

ancestor – parent – father →

successor

descendent – child – son →

← leaf node ---- external node

brothers or siblings

vertex

is another name for node

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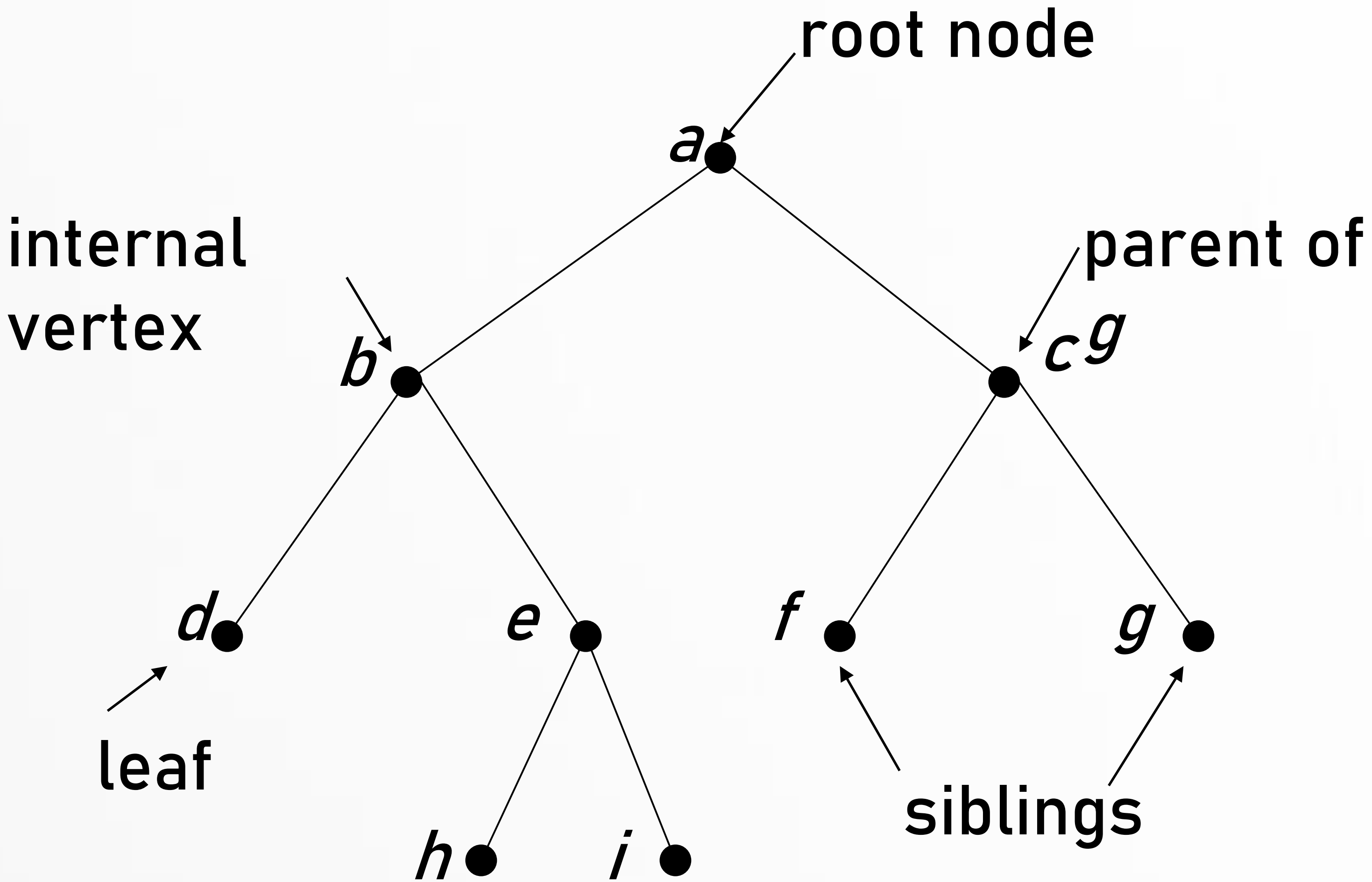
Trees



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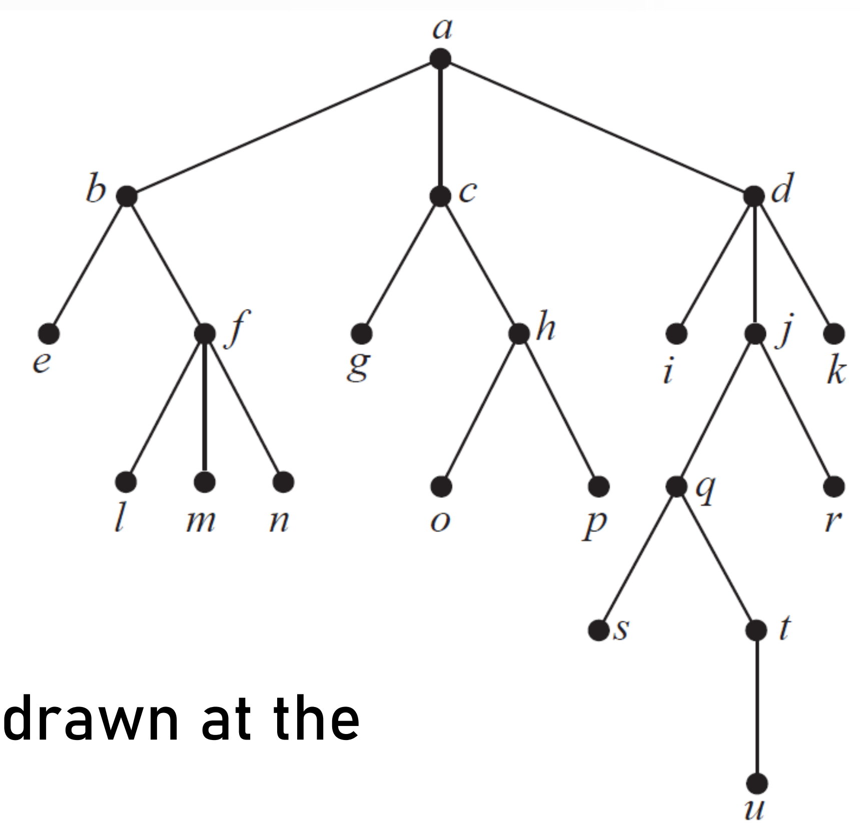
Trees

Answer these questions about the rooted tree illustrated.

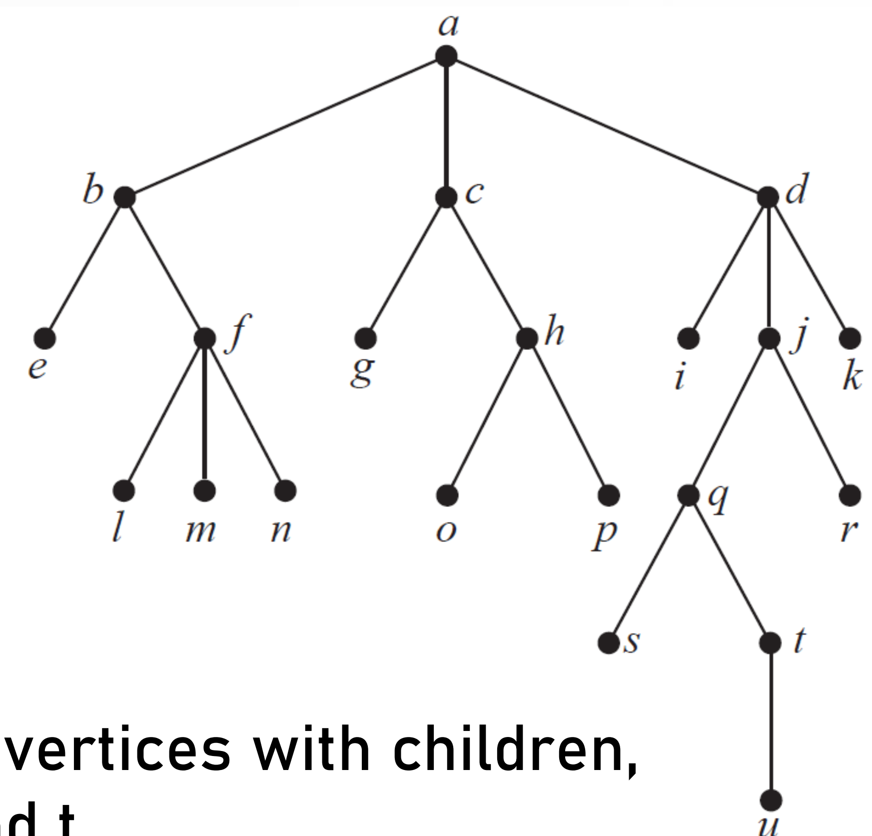
Which vertex is the root?



Vertex *a* is the root, since it is drawn at the top.

Trees

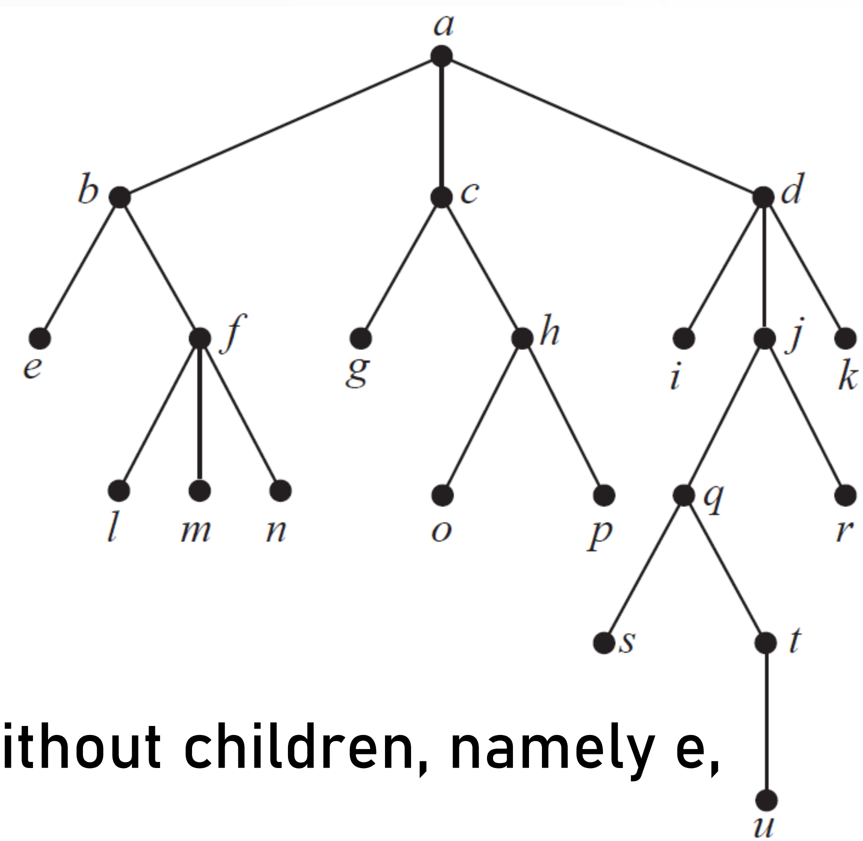
Which vertices are internal?



The internal vertices are the vertices with children, namely a, b, c, d, f, h , j , q, and t.

Trees

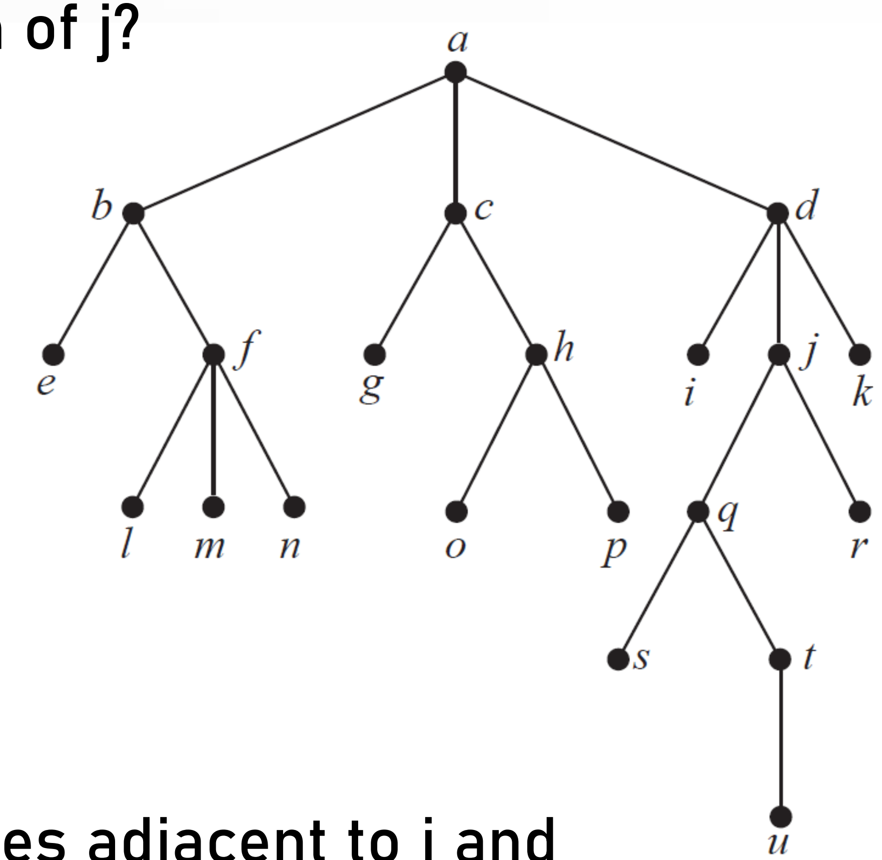
Which vertices are leaves?



The leaves are the vertices without children, namely e, g, i, k, l, m, n, o, p, r, s, and u.

Trees

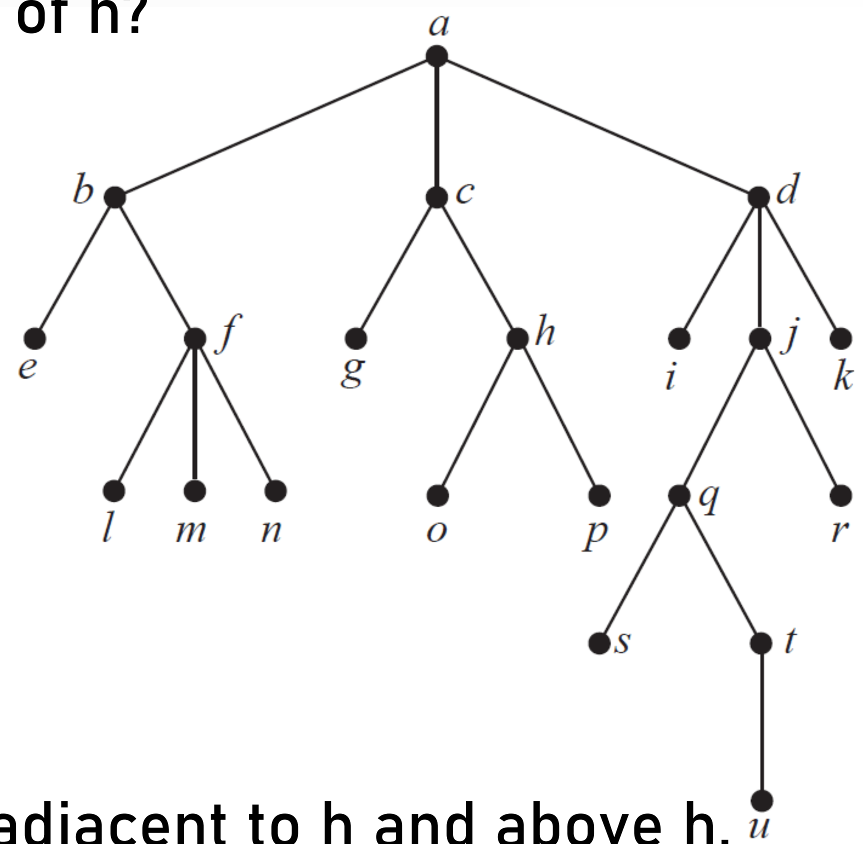
Which vertices are children of j?



The children of j are the vertices adjacent to j and below j, namely q and r.

Trees

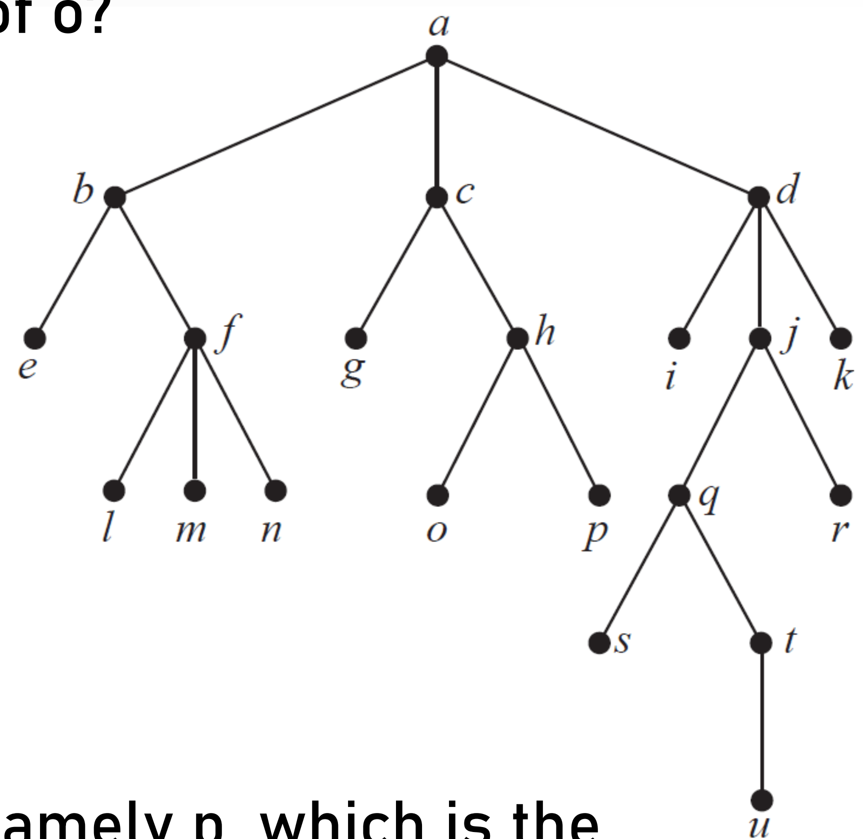
Which vertex is the parent of h?



The parent of h is the vertex adjacent to h and above h, namely c.

Trees

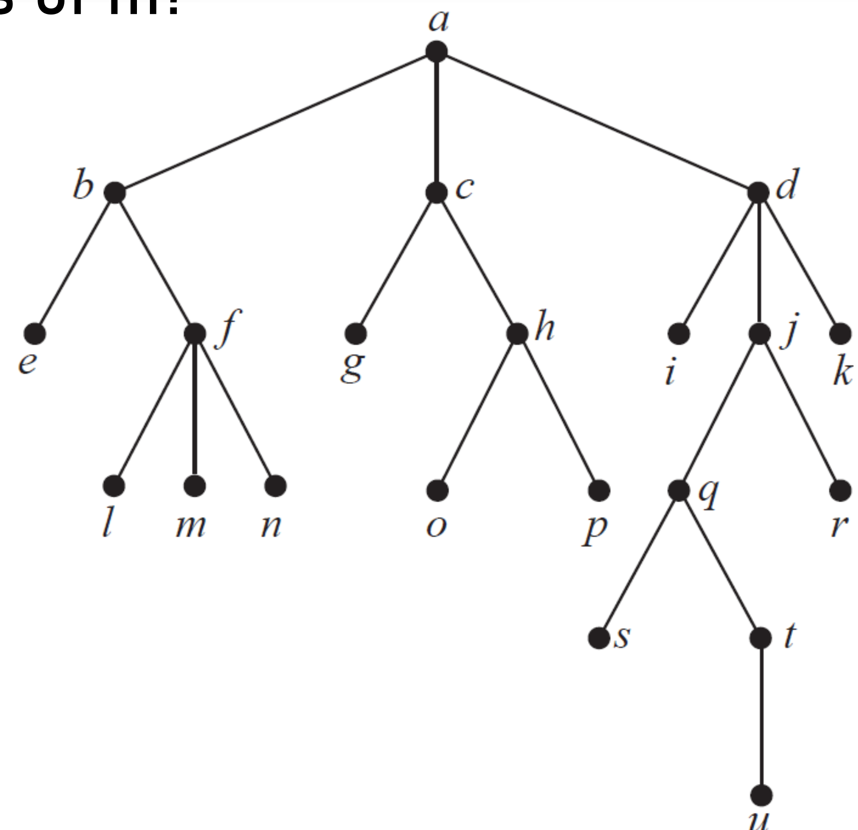
Which vertices are siblings of o?



Vertex o has only one sibling, namely p, which is the other child of o's parent, h.

Trees

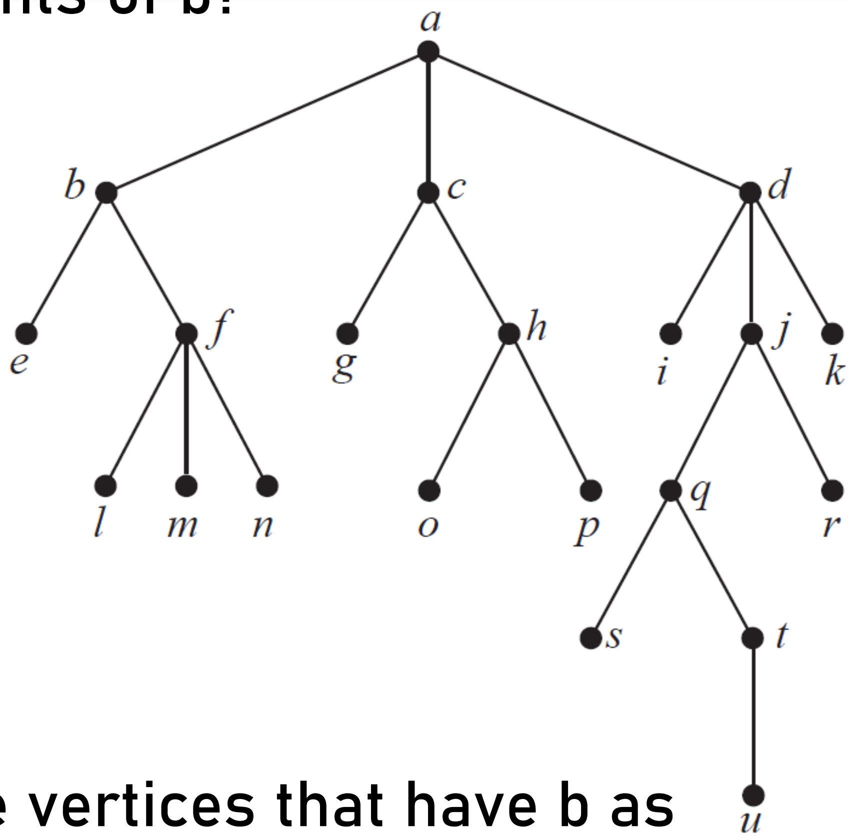
Which vertices are ancestors of m?



The ancestors of m are all the vertices on the unique simple path from m back to the root, namely f, b and a.

Trees

Which vertices are descendants of b?

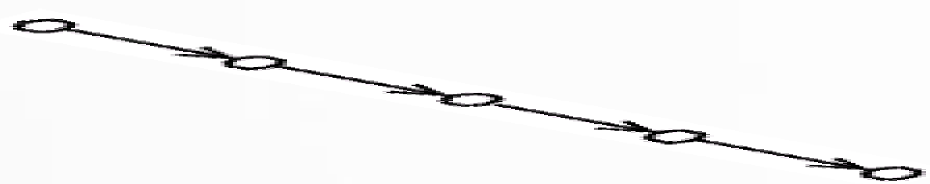


The descendants of b are all the vertices that have b as an ancestor, namely e, f, l, m, and n.

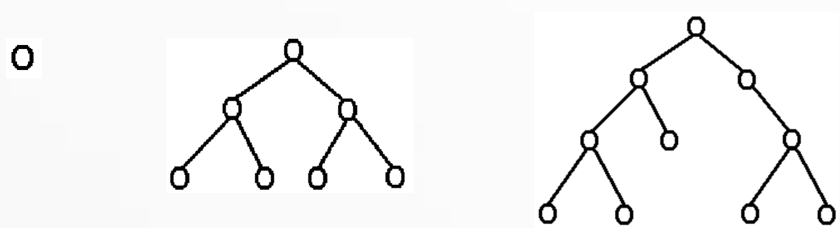
Trees

Kinds of Trees – in terms of *branching factor* (outdegree).

Unary Trees – outdegree for every node ≤ 1. (a singly linked list)



Binary Trees – outdegree for every node ≤ 2.



Ternary Trees – outdegree for every node ≤ 3.

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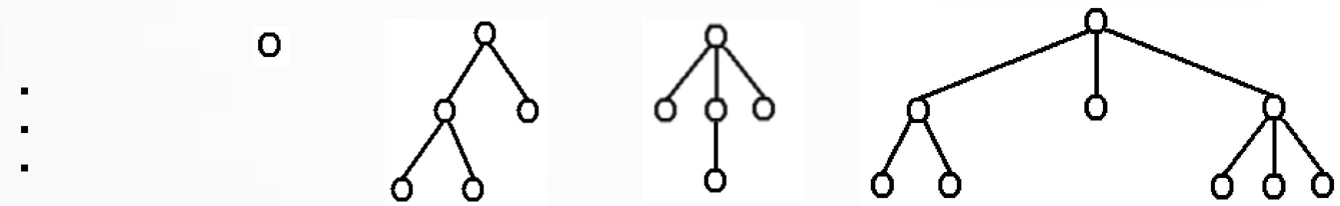
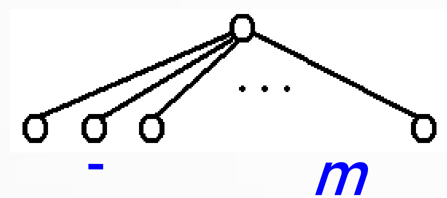
Trees

Ternary Trees – outdegree for every node ≤ 3.



m - ary Trees – outdegree for every node ≤ m.



Note: The indegree for every node in a directed tree, except the root, is 1.

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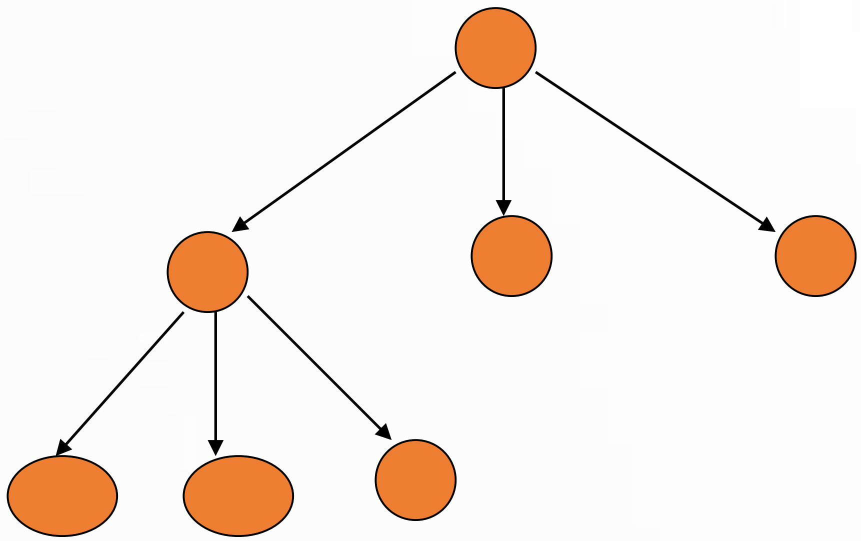
Trees - Example: 3-ary tree



Theorem: A full m - ary tree with i internal vertices contains n = mi +1 vertices

Proof: i internal vertices have m children. Therefore, we have mi vertices. Since the root is not a child we have n = mi + 1 vertices.

That's all for now...