EMTH403

Mathematical Foundation for Computer Science

Nitin K. Mishra (Ph.D.)

Associate Professor

Lecture Outcomes

After this lecture, you will be able to

- understand what are the basics of counting.
- understand how to use product rule.

Basics of Counting

We must count objects to solve many different types of problems.

For instance, counting is used to determine the complexity of algorithms.

to determine whether there are enough telephone numbers or Internet protocol addresses to meet demand.

Basics of Counting

In mathematical biology, especially in sequencing DNA.

Counting techniques are used extensively when probabilities of events are computed.

Basics of Counting

Suppose that a password on a computer system consists of six, seven, or eight characters.

Each of these characters must be a digit or a letter of the alphabet.

Content Source: **Discrete Mathematics** and **Its Applications**. Seventh Edition. **Kenneth H. Rosen. Monmouth University. Prepared by Jerrold W. Grossman, Oakland University.** (and formerly AT&T Laboratories)

Basics of Counting

Each password must contain at least one digit.

How many such passwords are there?

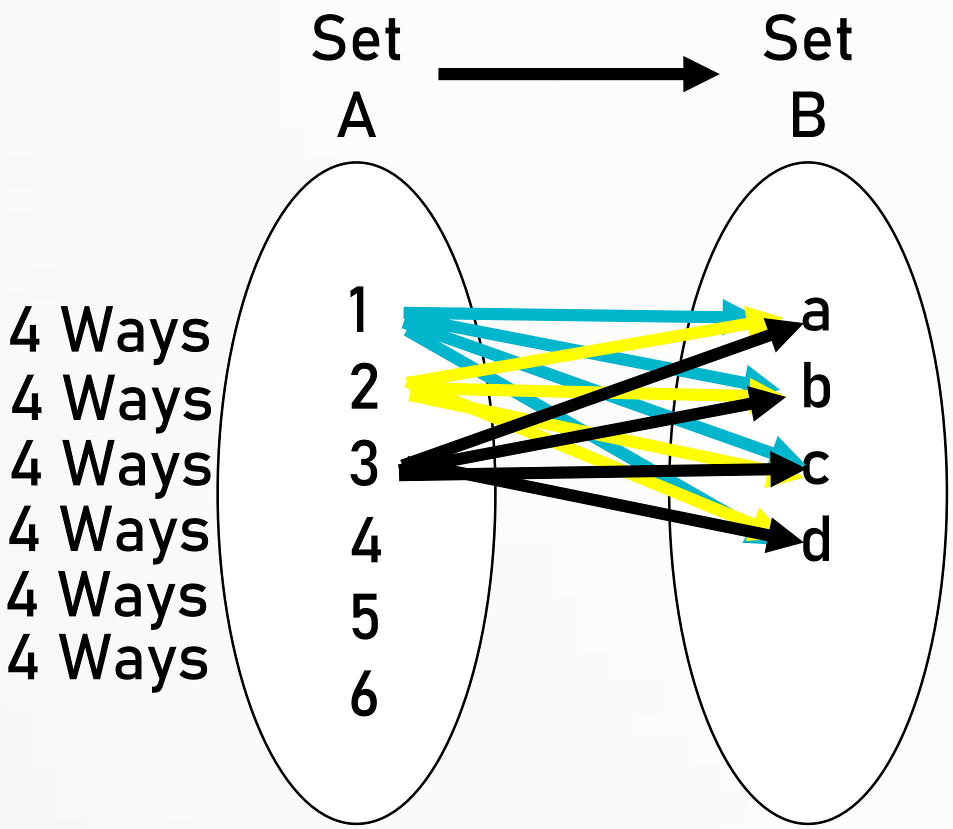
Counting problems arise throughout mathematics and computer science.

The Product Rule

Suppose that a procedure can be broken down into a sequence of two tasks.

If there are n_1 ways to do the first task and for each of these ways of doing the first task, there are n_2 ways to do the second task, then there are $n_1 * n_2$ ways to do the procedure.

The Product Rule



{1a, 1b, 1c, 1d,
2a, 2b, 2c, 2d,
3a, 3b, 3c, 3d,
4a, 4b, 4c, 4d,
5a, 5b, 5c, 5d,
6a, 6b, 6c, 6d}=

4+4+4+4+4+4=

4*6=24

n(A)*n(B)=24

The Product Rule – Example 1

Ques:- There are 32 microcomputers in a computer center. Each microcomputer has 24 ports. How many different ports to a microcomputer in the center are there?

Sol:- $32 \cdot 24 = 768$ ports.

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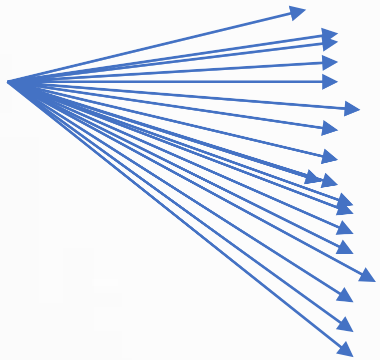
The Product Rule – Example 1

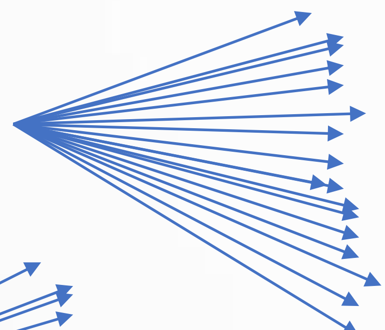
32 micro computers, 24 ports

First micro computer



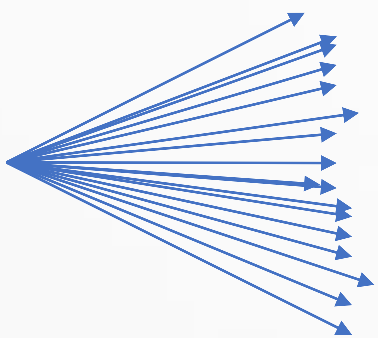
24 ports

Second micro computer



24 ports

Similarly 32nd micro computer



24 ports

Therefore

total ports = 24+24+24+24+…….+32 times

=24*32 = 768 Ports

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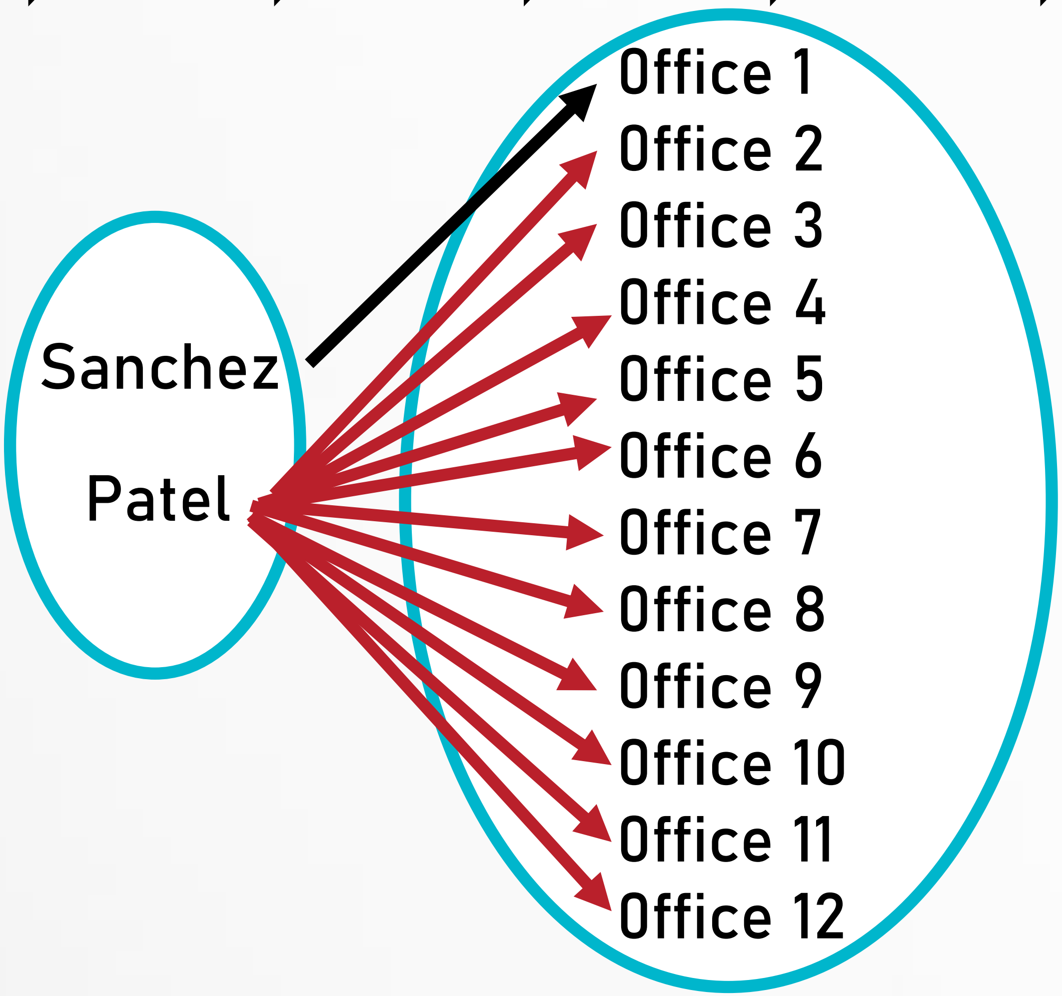
The Product Rule – Example 2

Ques:- A new company with just two employees, Sanchez and Patel, rents a floor of a building with 12 offices. How many ways are there to assign different offices to these two employees?

Ans:- By the product rule, there are $12 \cdot 11 = 132$ ways to assign offices to these two employees.

The Product Rule – Example 2

12 Offices = {Office 1, Office 2, Office 3, Office 4, Office 5, Office 6, Office 7, Office 8, Office 9, Office 10, Office 11, Office 12}



S1P1 NOT
S1P2 or
S1P3 or
S1P4 or
S1P5 or
S1P6 or
S1P7 or
S1P8 or
S1P9 or
S1P10 or
S1P11 or
S1P12.

S2P1 or
S2P2 NOT
S2P3 or
S2P4 or
S2P5 or
S2P6 or
S2P7 or
S2P8 or
S2P9 or
S2P10 or
S2P11 or
S2P12.

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The Product Rule – Example 2

S1P1 NOT
S1P2 or
S1P3 or
S1P4 or
S1P5 or
S1P6 or
S1P7 or
S1P8 or
S1P9 or
S1P10 or
S1P11 or
S1P12.

S2P1 or
S2P2 NOT
S2P3 or
S2P4 or
S2P5 or
S2P6 or
S2P7 or
S2P8 or
S2P9 or
S2P10 or
S2P11 or
S2P12.

With S3 11 ways

With S4 11 ways

Total= 11+

11+11+11+11+11+11+……

……….12 times

Total= 11*12=132 ways

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The Product Rule – Example 3

Ques:- There are 18 mathematics majors and 325 computer science majors at a college. In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?

Ans:- 5850

The Product Rule – Example 3

Ques:- There are 18 mathematics majors and 325 computer science majors at a college.

Ans:- 5850

The Product Rule – Example 4

Ques:- The chairs of an auditorium are to be labeled with an uppercase English letter followed by a positive integer not exceeding 100. What is the largest number of chairs that can be labeled differently?

Sol:- 26 uppercase English letters, and then assigning to it one of the 100 possible integers.

The product rule shows that there are $26 \cdot 100 = 2600$ different ways that a chair can be labeled

The Product Rule – Example 4

Ques:- The chairs of an auditorium are to be labeled with an uppercase English letter followed by a positive integer not exceeding 100. What is the largest number of chairs that can be labeled differently?

The Product Rule – Example 5

Ques:- How many different bit strings of length seven are there?

$$\underline{2}*\underline{2}*\underline{2}*\underline{2}*\underline{2}*\underline{2}*\underline{2}= 2^7$$

$$=n_1*n_2*n_3*n_4*n_5*n_6*n_7$$

Sol:- Each of the seven bits can be chosen in two ways, because each bit is either 0 or 1.

Therefore, the product rule shows there are a total of 27 = 128 different bit strings of length seven.

The Product Rule – For 2 and 3 strings

Total possible bit string of length 2 = 2^2 = 4
=Like 00, 01, 10, 11.

Total possible bit string of length 3 = 2^3 = 8.
=Like 000, 100,
001, 101
010, 110
011, 111.

The Product Rule

Total possible bit string of length 4 = 2^4 = 16
=Like 0000. 0001, 0010, 0100, 1000 etc.

Total possible bit string of length 7 = 2^7 = 128.
=Like 0000001, 0000010, 0000100, 0001000, 0010000 etc.

The Product Rule – Example 6

Ques:- How many bit strings are there of length eight?

A) 2^7

B) 2^8

C) 2^9

D) 2^{10}

ANS:-

2^8

The Product Rule – Example 7

Ques:- How many bit strings of length ten both begin and end with a 1?

Sol:- A bit string is determined by choosing the bits in the string, one after another, so the product rule applies.

We want to count the number of bit strings of length 10, except that we are not free to choose either the first bit or the last bit (they are mandated to be 1 's

The Product Rule – Example 7

Ques:- How many bit strings of length ten both begin and end with a 1?

Sol:- Therefore there are 8 choices to make, and each choice can be made in 2 ways (the bit can be either a 1 or a 0). Thus the product rule tells us that there are $2^8 = 256$ such strings.

The Product Rule – Example 7

Ques:- How many bit strings of length ten both begin and end with a 1?

Sol:-.

That's all for now…