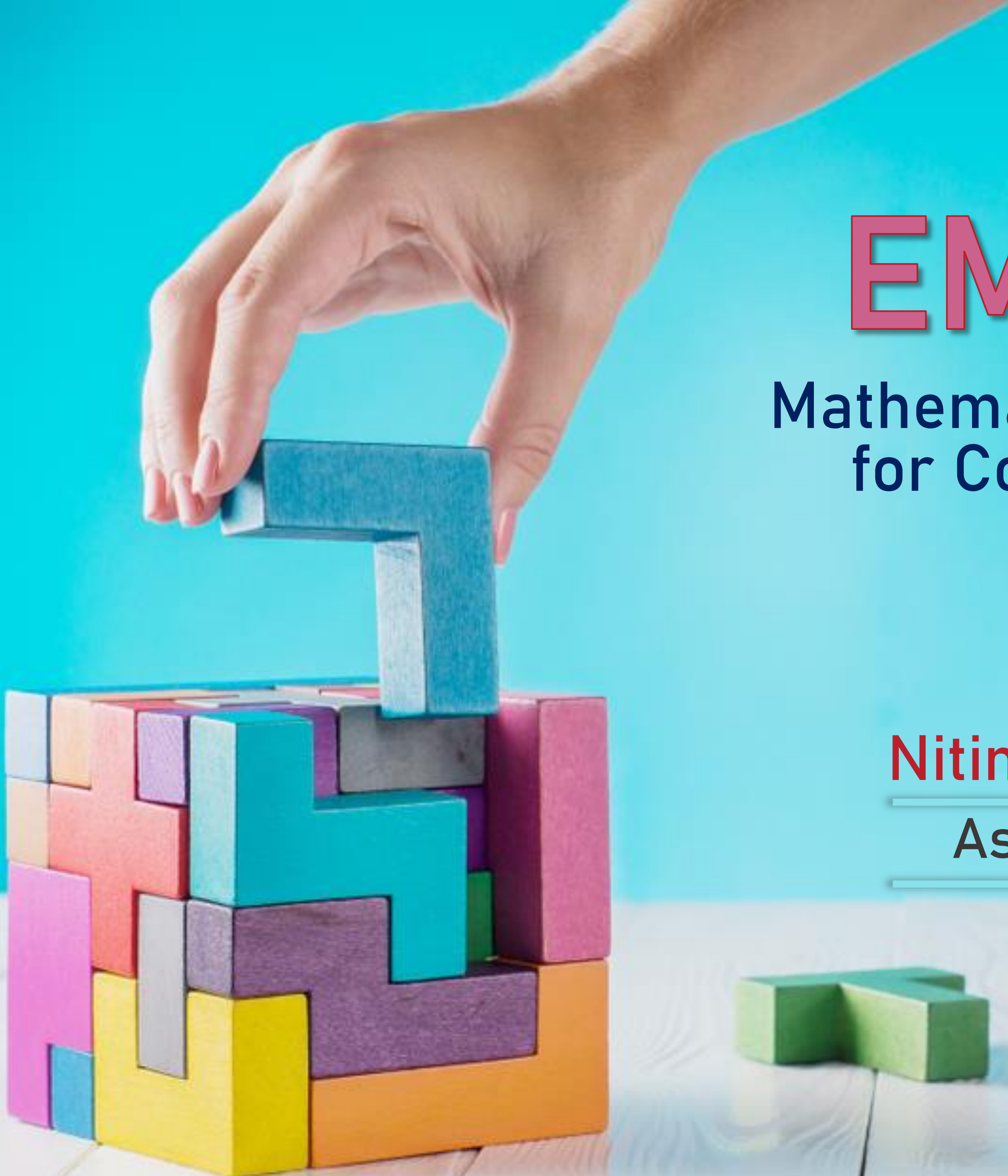

EMTH403

Mathematical Foundation for Computer Science

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Lecture Outcomes

After this lecture, you will be able to

- understand what are relations
- understand what are the different properties of relation
- understand what is reflexive, symmetric,

Binary Relation

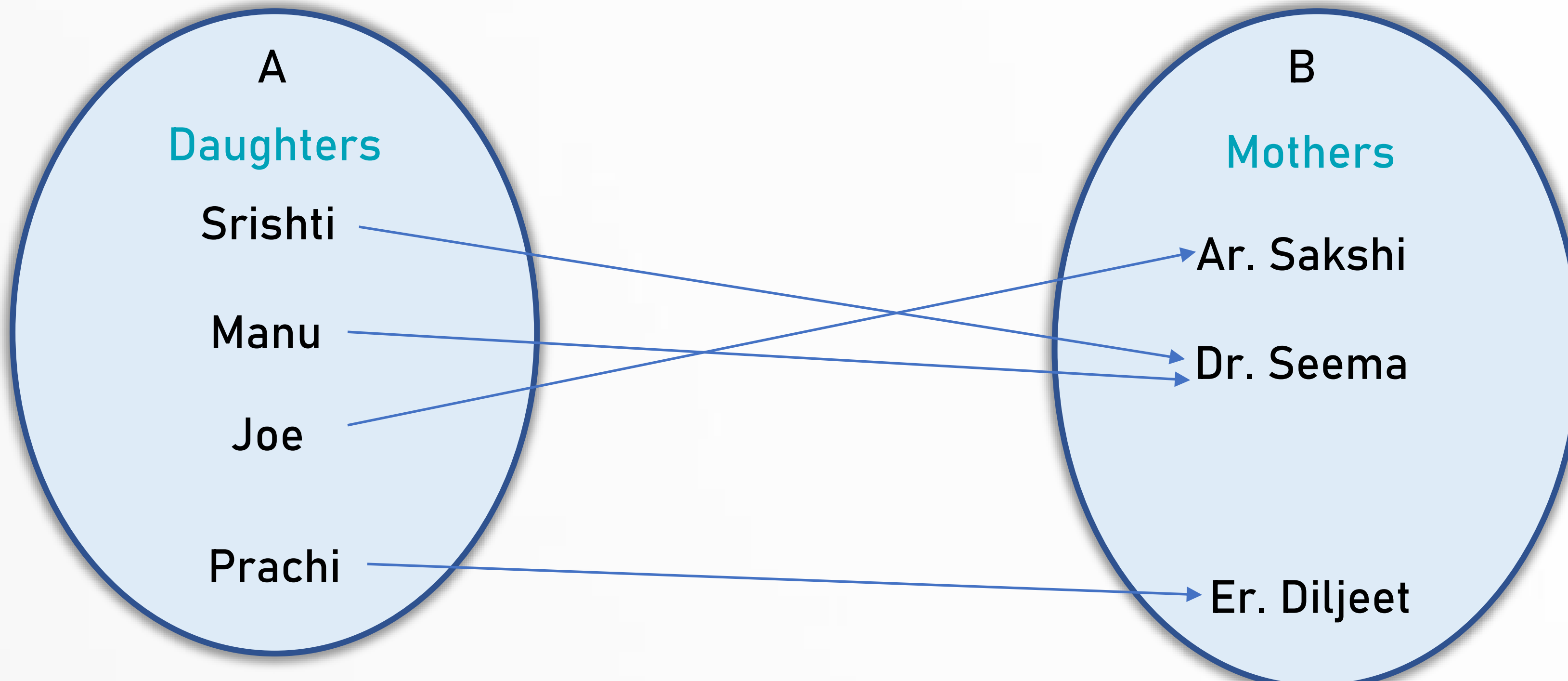
Prerequisite:- Conditional Statement

Let A and B be sets. A binary relation from A to B is a subset of A × B.

TABLE 5 The Truth Table for the Conditional Statement $p \to q$.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Relation – Example



Relation={(Srishti, Dr. Seema), (Manu, Dr. Seema), (Joe, Ar. Sakshi), (Prachi, Er. Diljeet)}

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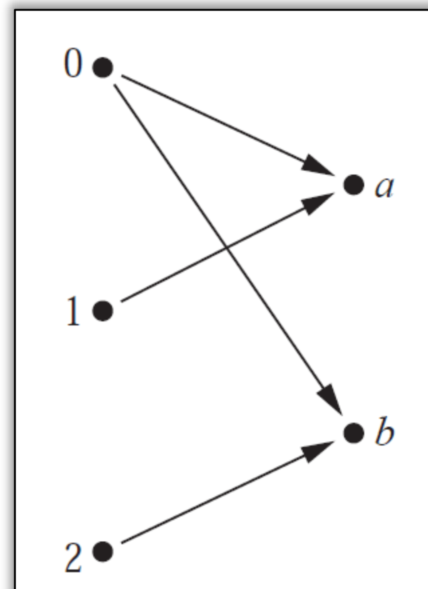
Binary Relation

Let A = {0, 1, 2} and B = {a, b}. Then {(0, a), (0, b), (1, a), (2, b)} is a relation from A to B.

This means, for instance, that 0R a, but $\not{R}$ that 1 b.

Relations can be represented graphically, as shown in Figure 1, using arrows to represent ordered pairs.



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Binary Relation

Another way to represent this relation is to use a table, given below.

R	a	b
0	×	×
1	×	
2		×

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Relation

A relation on a set A is a relation from A to A.

In other words, a relation on a set A is a subset of $A \times A$.

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Relation

Ques:- Let A be the set {1, 2, 3, 4}. Which ordered pairs are in the relation R = {(a, b) | a divides b}?

Ans:- Because (a, b) is in R if and only if a and b are positive integers not exceeding 4 such that a divides b, we see that

R = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (4, 4)}.

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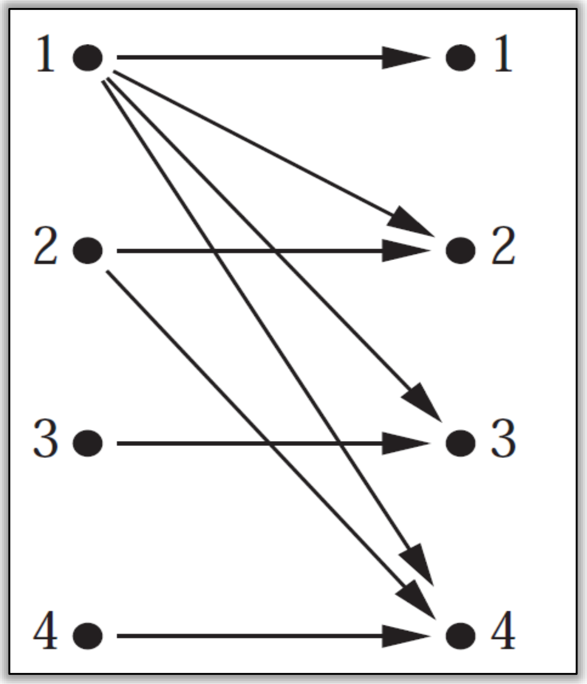
Relation

The pairs in this relation are displayed in graphically form. R = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (4, 4)}.



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Relation

The pairs in this relation are displayed both in tabular form. R = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (4, 4)}.

R	1	2	3	4
1	×	×	×	×
2		×		×
3			×	
4				×

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Properties of Relations

1. Reflexive
2. Symmetric
3. Antisymmetric
4. Transitive

Properties of Relations – Reflexive

A relation R on a set A is called reflexive if $(a, a) \in R$ for every element $a \in A$.

Remark: Using quantifiers we see that the relation R on the set A is reflexive if $\forall a((a, a) \in R)$, where the universe of discourse is the set of all elements in A.

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Properties of Relations – Reflexive

We see that a relation on A is reflexive if every element of A is related to itself.

Properties of Relations – Reflexive Example

Ques:- Consider the following relations on {1, 2, 3, 4}. Which of these relations are reflexive?

R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)}.

R2 = {(1, 1), (1, 2), (2, 1)}.

R3 = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)}.

R4 = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)}.

R5 = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)}.

R6 = {(3, 4)}.

Properties of Relations – Reflexive Example

R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)}.

R2 = {(1, 1), (1, 2), (2, 1)}.

R3 = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)}.

R4 = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)}.

R5 = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)}.

R6 = {(3, 4)}.

The relations R3 and R5 are reflexive because they both contain all pairs of the form (a, a), namely, (1, 1), (2, 2), (3, 3), and (4, 4). The other relations are not reflexive.

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Properties of Relations – Reflexive Example 2

Ques:- Is the “divides” relation on the set of positive integers reflexive?

Ans:- Because $a \mid a$ whenever a is a positive integer, the “divides” relation is reflexive.

Properties of Relations - Symmetric

A relation R on a set A is called symmetric if $(b, a) \in R$ whenever $(a, b) \in R$, for all $a, b \in A$.

Remark: Using quantifiers, we see that the relation R on the set A is symmetric if $\forall a \forall b((a, b) \in R \rightarrow (b, a) \in R)$.

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Properties of Relations – Symmetric Example 1

Ques:- Consider the following relations on {1, 2, 3, 4}:

R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)}. Is it symmetric?

Ans:- R1 is not symmetric, because a pair (a, b) belongs to R1 but (b, a) is not.

Properties of Relations – Symmetric Example 2

Ques:- Consider the following relations on {1, 2, 3, 4}:

R2 = {(1, 1), (1, 2), (2, 1)}. Is it symmetric?

Ans:- R2 is symmetric

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Properties of Relations – Symmetric Example 3

Ques:- Consider the following relations on {1, 2, 3, 4}:

R3 = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)}. Is it symmetric?

Ans:- For R3, it is necessary to check that both (1, 2) and (2, 1) belong to the relation, and (1, 4) and (4, 1) belong to the relation.

Properties of Relations – Symmetric Example 4

Ques:- Consider the following relations on {1, 2, 3, 4}:

R4 = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)}. Is it symmetric?

Ans:- One should verify that the relation is not symmetric. This is done by finding a pair (a, b) such that it is in the relation but (b, a) is not

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Properties of Relations – Symmetric Example 5

Ques:- Consider the following relations on {1, 2, 3, 4}:

R5 = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)}. Is it symmetric?

Ans:- One should verify that the relation is not symmetric. This is done by finding a pair (a, b) such that it is in the relation but (b, a) is not.

Properties of Relations – Symmetric Example 6

Ques:- Consider the following relations on {1, 2, 3, 4}:

R6 = {(3, 4)}. Is it symmetric?

Ans:- One should verify that the relation is not symmetric. This is done by finding a pair (a, b) such that it is in the relation but (b, a) is not.

Properties of Relations – Symmetric Example 7

Ques:- Is the "divides" relation on the set of positive integers symmetric?

Ans:-

This relation is not symmetric because $1 \mid 2$, but $2 \nmid 1$.

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That's all for now...