



EMTH403

Mathematical Foundation for Computer Science

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Lecture Outcomes



After this lecture, you will be able to

- understand the use of product rule in the basics of counting.
- understand the use of sum rule in the basics of counting.
- understand the use of subtraction rule in the basics of counting.

Basics of Counting – Example 1

Ques:- How many different three-letter initials can people have?

Sol:- first initial (26 ways),

then the second initial (26 ways),

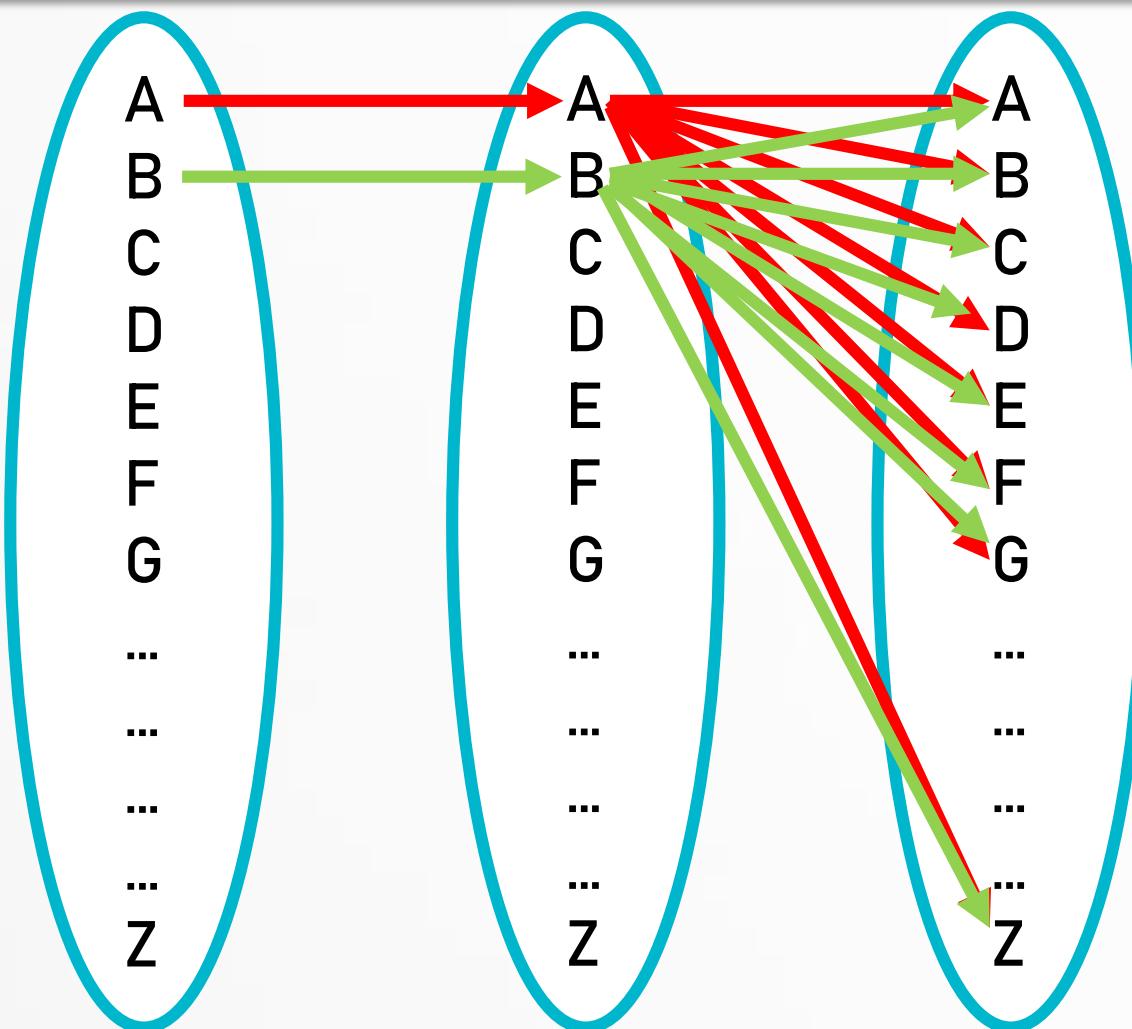
and then the third initial (26 ways).

Therefore, by the product rule there are

$$26 \cdot 26 \cdot 26 = 26^3 = 17,576$$

possible three-letter initials.

Basics of Counting – Example 1



AAA	ABA	AAA
AAB	ABB	AAB
AAC	ABC	...
AAD	ABD	AAZ
AAE	ABE	ABA
AAF	ABF	...
AAG	ABG	AZZ
...	...	ZAA
...
...
AAZ	ABZ	ZZZ

$$26 \times 26 \times 26 = 17,576$$

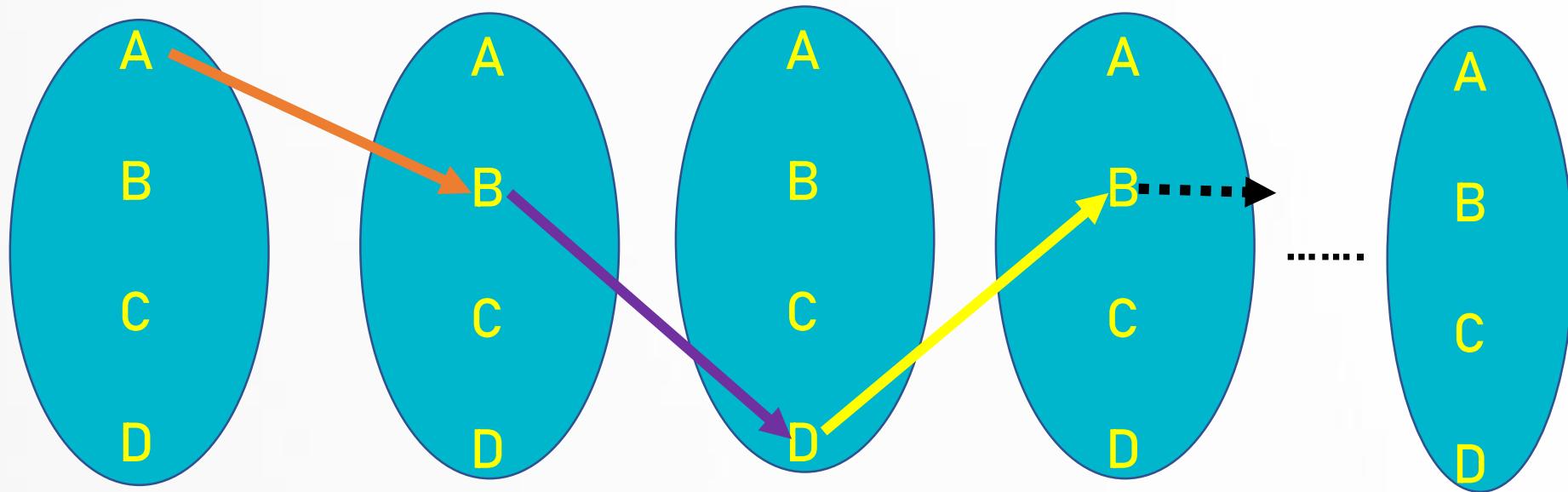
Basics of Counting – Example 2

Ques:- A multiple-choice test contains 10 questions. There are four possible answers for each question. In how many ways can a student answer the questions on the test if the student answers every question?

$$\text{Sol:- } 4 \cdot 4 \cdots 4 = 4^{10} = 1,048,576$$

Possible ways are a, b, c, b, b, d, a, b, d, b etc.

Basics of Counting – Example 2



Q1

4ways

$$= n_1 * \dots$$

$$= 4 *$$

$$= 4 \cdot 4 \cdot \dots \cdot 4 = 4^{10} = 1,048,576$$

Q2

4ways

$$= n_2 * \dots$$

$$= 4 *$$

Q3

4ways

$$= n_3 * \dots$$

$$= 4 *$$

Q4

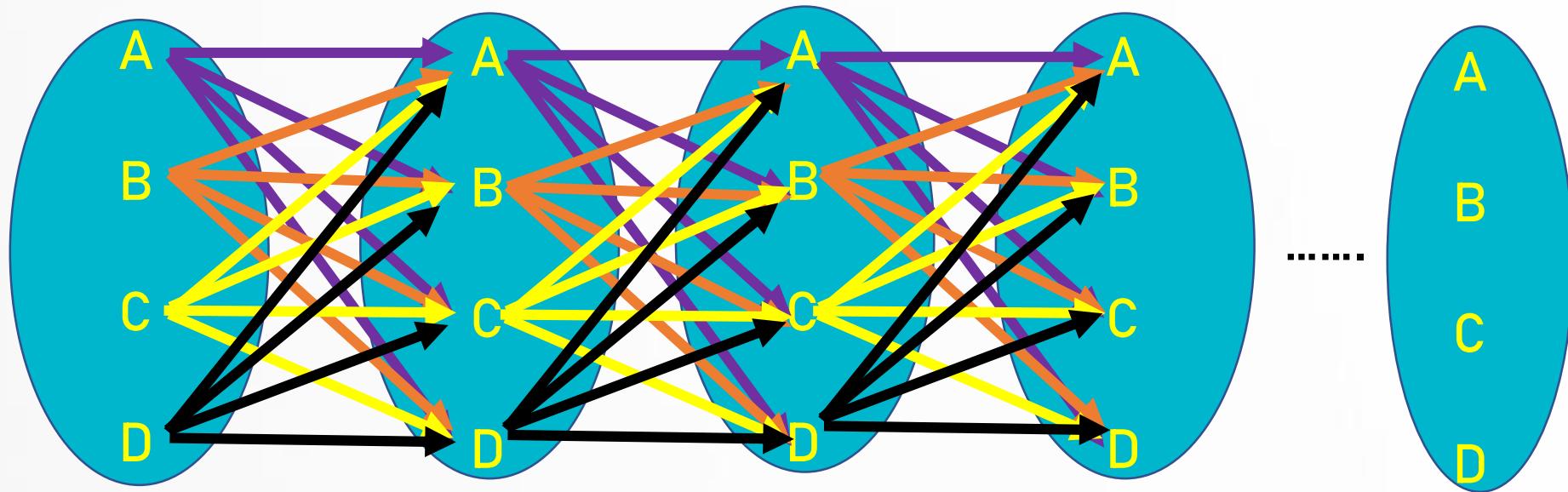
4ways.....4ways

$$= n_4 \dots * \dots n_{10}$$

$$= 4 \dots * \dots 4$$

Q10

Basics of Counting – Example 2



Q1

4ways

$$= n_1 * \dots$$

$$= 4 *$$

$$= 4 \cdot 4 \cdot \dots \cdot 4 = 4^{10} = 1,048,576$$

Q2

4ways

$$= n_2 * \dots$$

$$= 4 *$$

Q3

4ways

$$= n_3 * \dots$$

$$= 4 *$$

Q4

4ways.....4ways

$$= n_4 \dots * \dots n_{10}$$

$$= 4 \dots * \dots 4$$

Q10

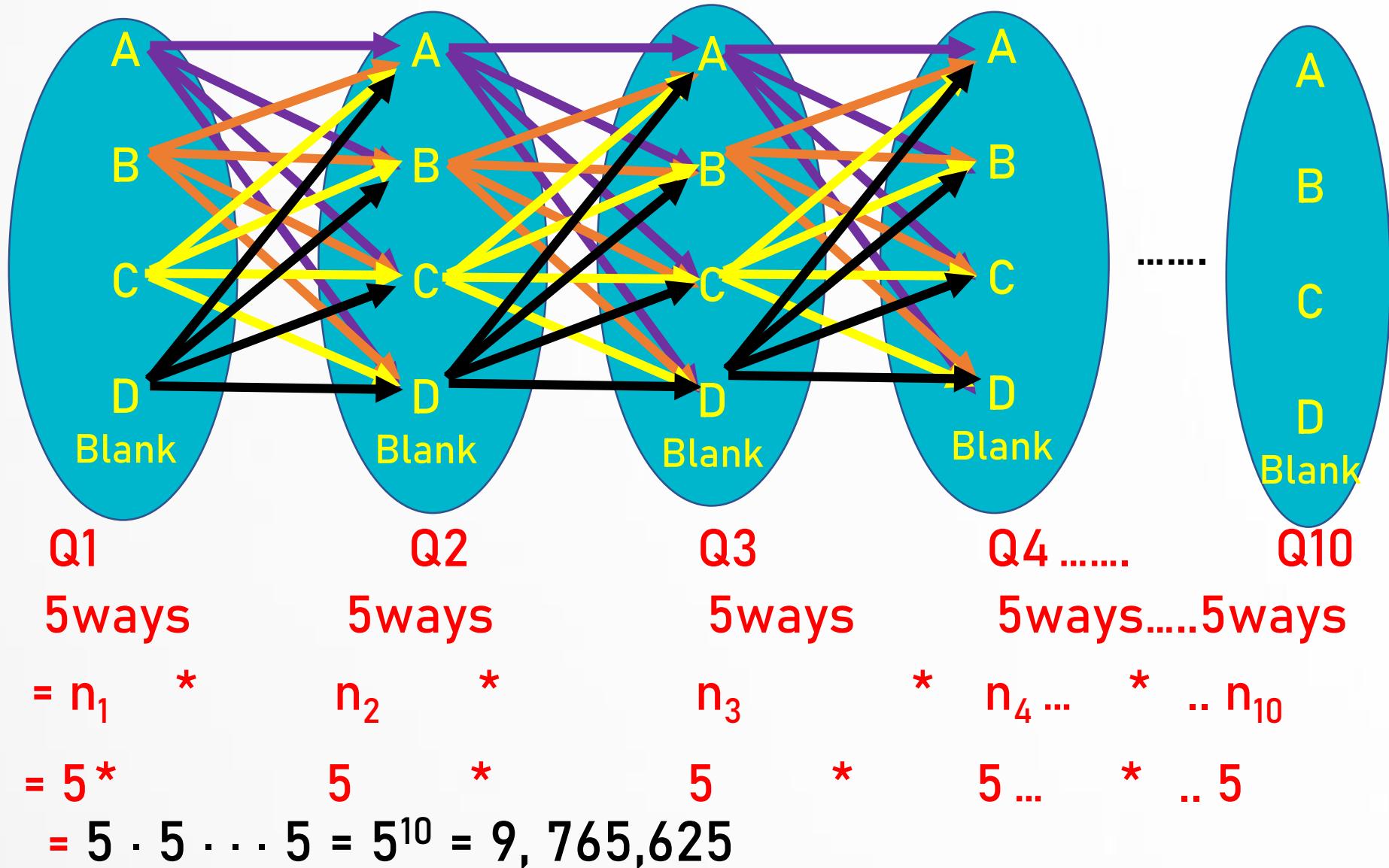
Basics of Counting – Example 3

Ques:- A multiple-choice test contains 10 questions. There are four possible answers for each question. In how many ways can a student answer the questions on the test if the student can leave answers blank?

Sol:- $5 \cdot 5 \cdots 5 = 5^{10} = 9,765,625$

Possible ways are a, b, c, b, blank, d, a, b, d, b etc.

Basics of Counting – Example 2



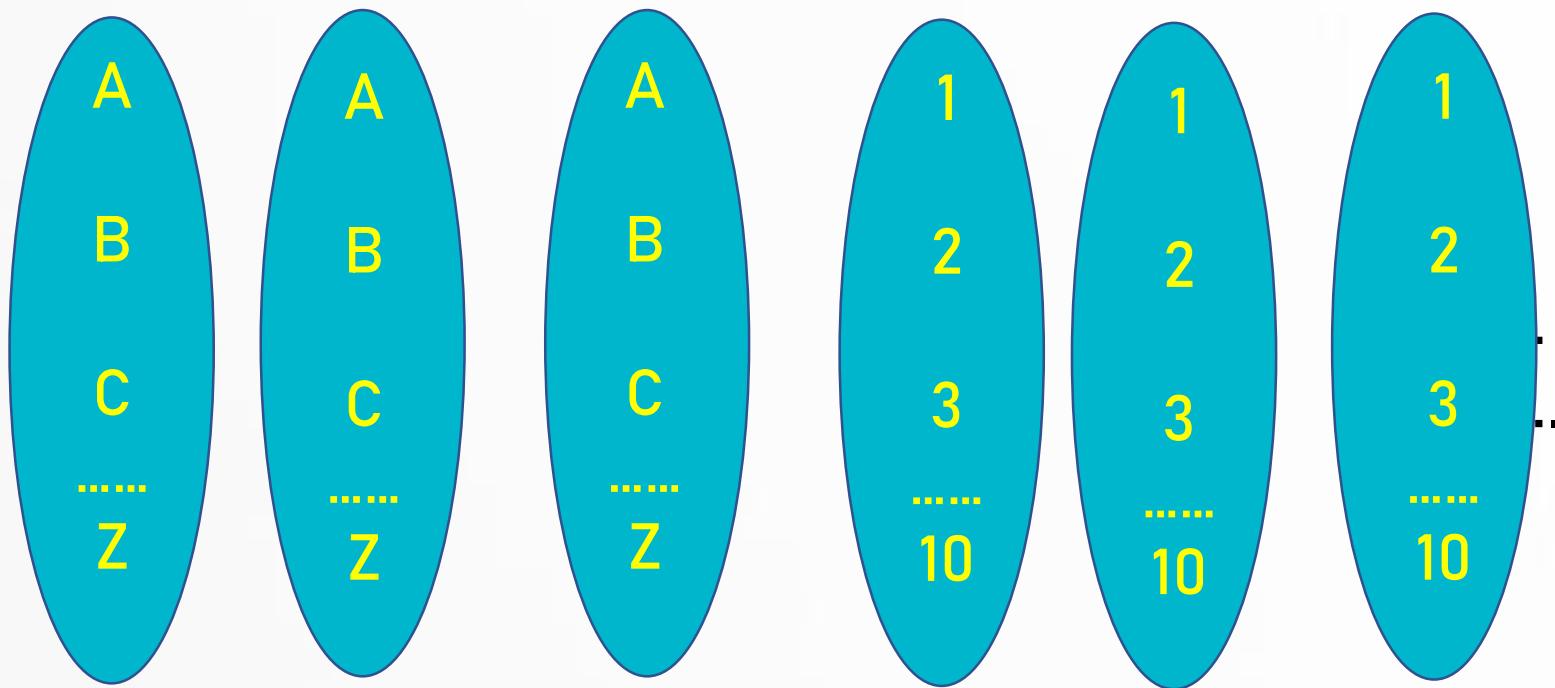
Basics of Counting – Example 2

Ques:- How many different license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits ?

Sol:- There are 26 choices for each of the three uppercase English letters and ten choices for each of the three digits. Hence, by the product rule there are a total of

$$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 17,576,000 \text{ possible license plates.}$$

Basics of Counting – Example 2



26ways 26ways 26ways 10way 10ways 10ways

$$= n_1 * n_2 * n_3 * n_4 * n_5 * n_6$$

$$= 26 * 26 * 26 * 10 * 10 * 10$$

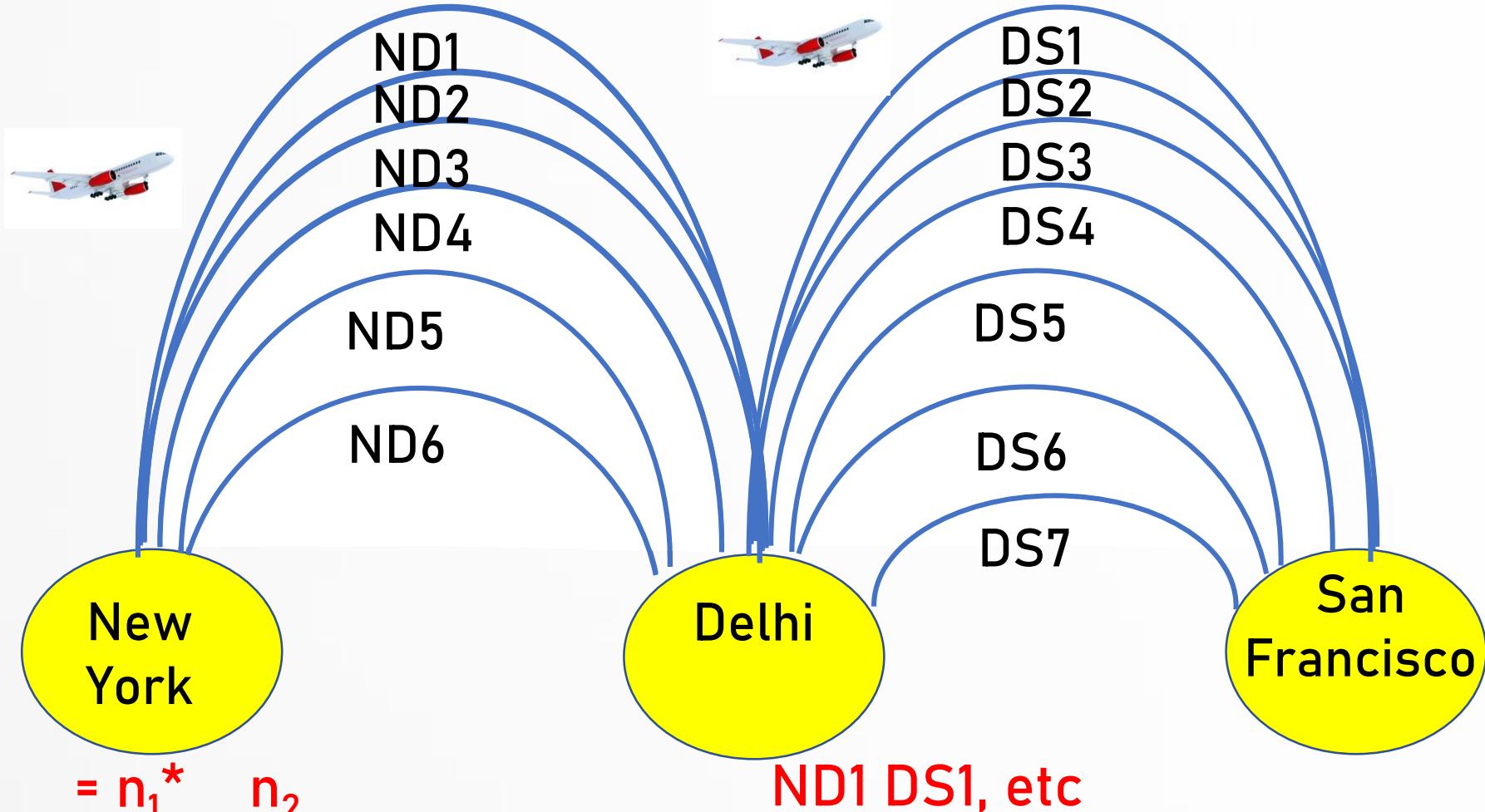
$$= 17,576,000$$

Basics of Counting - Example

Ques:- Six different airlines fly from New York to Delhi and seven fly from Delhi to San Francisco.

How many different pairs of airlines can you choose on which to book a trip from New York to San Francisco via Delhi, when you pick an airline for the flight to Delhi and an airline for the continuation flight to San Francisco?

Basics of Counting – Example



$$= n_1 * n_2$$

$$= 6 * 7$$

$$= 42$$

ND1 DS1, etc

The Sum Rule

If a task can be done either in one of n_1 ways or in one of n_2 ways, Where none of the set of n_1 ways is the same as any of the set of n_2 ways, then there are $n_1 + n_2$ ways to do the task.

The Sum Rule

Ques:- There are 18 mathematics majors and 325 computer science majors at a college. In how many ways can one representative be picked who is either a mathematics major or a computer science major?

Sol:- 343

The Sum Rule

Ques:- A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 19 possible projects, respectively. No project is on more than one list. How many possible projects are there to choose from?

Sol:- The student can choose a project by selecting a project from the first list, the second list, or the third list.

Because no project is on more than one list, by the sum rule there are $23 + 15 + 19 = 57$ ways to choose a project.

The Sum Rule

List 1

Project 1
Project 2
Project 3
Project 4
Project 5
Project 6
Project 7
Project 8
Project 9
Project 10

Project 11
Project 12
Project 13
Project 14
Project 15
Project 16
Project 17
Project 18
Project 19
Project 20
Project 21
Project 22
Project 23

List 2

Project 1
Project 2
Project 3
Project 4
Project 5
Project 6
Project 7
Project 8
Project 9
Project 10

List 3

Project 11
Project 12
Project 13
Project 14
Project 15
Project 16
Project 17
Project 18
Project 19
Project 10

$$= n_1 + n_2 + n_3$$

$$= 23 + 15 + 19 = 57 \text{ ways}$$

The Subtraction Rule

If a task can be done in either n_1 ways or n_2 ways, then the number of ways to do the task is $n_1 + n_2$ minus the number of ways to do the task that are common to the two different ways

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|.$$

The Subtraction Rule

Ques:- How many bit strings of length eight either start with a 1 bit or end with the two bits 00?

Sol:- bit string of length eight that begins with a 1+ a bit string of length eight ending with the two bits 00 - bit string of length eight starting with a 1 and that ends with the two bits 00

$$= 2^7 + 2^6 - 2^5 = 128 - 64 - 32 \text{ ways.}$$

= the number of ways to construct a bit string of length eight that begins with a 1 or that ends with 00, equals $128 + 64 - 32 = 160$.

That's all for now...