

# Introduction to AI with Prolog

## Recursion

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## Goals of the lesson

- ▶ Introduce recursive definitions in Prolog
- ▶ Discuss the mismatches between the declarative meaning of a Prolog program, and its procedural meaning

Predicates can be defined recursively. It means that one or more rules in predicate's definition refers to itself.

The most common example is a description of family relationships.

```
ancestor(Anc,Dec) :- child(Dec,Anc).  
ancestor(Anc,Dec) :- child(Dec,Par), ancestor(Anc,Par).  
child(jane, peter).  
child(mary, kate).  
child(sam, mary).  
child(jason, jane).  
child(ann, jason).
```

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## Basis

```
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ancestor(Anc,Dec) :- child(Dec,Par), ancestor(Anc,Par).  
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## Step

```
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ancestor(Anc,Dec) :- child(Dec,Par), ancestor(Anc,Par).  
child(jane, peter).  
child(mary, kate).  
child(sam, mary).  
child(jason, jane).  
child(ann, jason).
```

Next, let us define natural numbers, having a definition of zero and a rule of succession. This representation is variously called successor arithmetic, successor notation and also **Peano arithmetic**.

$$\text{num}(0) .$$
$$\text{num}(s(N)) :- \text{num}(N) .$$

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```
num(0).
```

```
num(s(N)) :- num(N).
```

```
sum(0, N, N).
```

```
sum(s(M), N, s(S)) :- sum(M, N, S).
```

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$$\text{num}(0).$$
$$\text{num}(s(N)) \text{ :- num}(N).$$
$$\text{sum}(0, N, N).$$
$$\text{sum}(s(M), N, s(S)) \text{ :- sum}(M, N, S).$$

Our numbers will be something like:  $0, s(0), s(s(0)), s(s(s(0))), \dots$ . Not very pleasant if you want to compute something, but sometimes used in automated provers.



Suppose we want to compute factorial (in normal numbers this time). We already know about recursion, and after a while will end up with a program similar to this one:

$f(0, 1).$

$f(N, F) :- N > 0, M \text{ is } N - 1, f(M, Acc), F \text{ is } Acc * N.$

Binary predicate  $f/2$  takes two arguments: natural number  $N$  and factorial of  $N$ . The table below shows possible queries.

$f(4, F).$	$F = 24.$
$f(5, 120).$	$\text{true}.$
$f(X, 120).$	$\text{ERROR}$

A recursive function is **tail recursive** when recursive call is the last thing executed by the function. It works in Prolog just well. Let us rewrite our factorial function, and make it tail recursive.

```
f(N,N,F,F) :- !.  
f(N,M,Acc,F) :- M1 is M + 1, Acc1 is Acc * M1, f(N,M1,Acc1,F).  
factorial(N,F) :- f(N,1,1,F).
```

As we can see predicate `f` takes 4 arguments now: natural number `N`, computation step `M`, result computed on step `M`, and total result `F`. Below are some examples.

<code>factorial(4, F).</code>	<code>F = 24.</code>
<code>factorial(5, 120).</code>	<code>true.</code>
<code>factorial(X, 120).</code>	<code>X = 5.</code>

Recall our family program which as we know works just fine:

```
ancestor(Anc,Dec) :- child(Dec,Anc).  
ancestor(Anc,Dec) :- child(Dec,Par), ancestor(Anc,Par).  
child(jane, peter).  
child(mary, kate).  
child(sam, mary).  
child(jason, jane).  
child(ann, jason).
```

But what happens if we swap goals in the second rule?

```
ancestor(Anc,Dec) :- child(Dec,Anc).  
ancestor(Anc,Dec) :- ancestor(Anc,Par), child(Dec,Par).  
child(jane, peter).  
child(mary, kate).  
child(sam, mary).  
child(jason, jane).  
child(ann, jason).
```

If we pose query `ancestor(jane, ann)` we will get a correct answer (`true`), and then an error message which means that Prolog is looping.

