

Gray, Thompson (1996)

$$y_t = \underbrace{\sum_{j=0}^d \beta_j t^j}_{=\mu_t} + \xi_t + \varepsilon_t, \quad \begin{cases} \varepsilon_t & \text{bruit blanc, } \mathbb{V}[\varepsilon_t] = \sigma^2 \\ \xi_t & \text{non corrél      } \varepsilon_t \end{cases}$$

Crit  res :  $\begin{cases} F_{GT}(\boldsymbol{\theta}) = \mathbb{E}[(\hat{\mu}_t - \mu_t)^2] \\ S_{GT}(\boldsymbol{\theta}) = \mathbb{E}[(\Delta^{d+1} \hat{\mu}_t)^2] \end{cases}, \quad \hat{\mu}_t = M_{\boldsymbol{\theta}} y_t$

$$\hat{\boldsymbol{\theta}} \in \operatorname{argmin} \alpha F_{GT}(\boldsymbol{\theta}) + (1 - \alpha) S_{GT}(\boldsymbol{\theta})$$

s.c.  $\mathbf{X}'_d \boldsymbol{\theta} = \mathbf{e}_1$

$$\alpha = 1; \mathbb{V}[(\xi_{t-h}, \dots, \xi_{t+h})] = \mathbf{K}^{-1}; \sigma^2 = 0$$

$$F_{GT}(\boldsymbol{\theta}) = I(\boldsymbol{\theta}, 0, y_t, \mu_t)$$

$$S_{GT}(\boldsymbol{\theta}) = I(\boldsymbol{\theta}, d+1, y_t, 0)$$

Formule g  n  rale

$$\begin{cases} I(\boldsymbol{\theta}, q, y_t, u_t) = \mathbb{E}[(\Delta^q (M_{\boldsymbol{\theta}} y_t - u_t))^2] \\ J(\boldsymbol{\theta}, f, \omega_1, \omega_2) = \int_{\omega_1}^{\omega_2} f[\phi_{\boldsymbol{\theta}}(\omega), \varphi_{\boldsymbol{\theta}}(\omega)] d\omega \end{cases}$$

$$\hat{\boldsymbol{\theta}} \in \operatorname{argmin} \sum \alpha_i I(\boldsymbol{\theta}, q_i, y_t, u_t^{(i)}) + \beta_i J(\boldsymbol{\theta}, f_i, \omega_1^{(i)}, \omega_2^{(i)})$$

s.c.  $\mathbf{C} \boldsymbol{\theta} = \mathbf{a}$

$$F_g(\boldsymbol{\theta}) = I(\boldsymbol{\theta}, 0, y_t, \mathbb{E}[M_{\boldsymbol{\theta}} y_t])$$

$$S_g(\boldsymbol{\theta}) = I(\boldsymbol{\theta}, q, y_t, \mathbb{E}[M_{\boldsymbol{\theta}} y_t])$$

$$y_t = \sum_{j=0}^p \beta_j t^j + \varepsilon_t, \quad \varepsilon_t \stackrel{i.i.d}{\sim} \mathcal{N}(0, \sigma^2)$$

$$\xi_t = 0$$

$$q = d+1$$

Guggemos et al (2018)

Crit  res :  $\begin{cases} F_g(\boldsymbol{\theta}) = \sum_{j=-h}^h \theta_j^2 \\ S_g(\boldsymbol{\theta}) = \sum_{j=-h}^h (\Delta^q \theta_j)^2 \end{cases}$

$$\hat{\boldsymbol{\theta}} \in \operatorname{argmin} \alpha F_g(\boldsymbol{\theta}) + (1 - \alpha) S_g(\boldsymbol{\theta})$$

s.c.  $\mathbf{X}'_d \boldsymbol{\theta} = \mathbf{e}_1$

$$q = 2, \alpha = 0$$

$$\alpha = 1$$

$$q = 3, \alpha = 0$$

LOESS

Nadaraya-Watson  
ou estimateur par  
noyaux

Filtre Epanechnikoff

Filtre Macaulay

Henderson

$$\kappa_j^{TC} = \left(1 - \left|\frac{j}{h+1}\right|^3\right)^3$$

$$d = 0$$

$$\kappa_j^E = 1 - \left|\frac{j}{h+1}\right|^2$$

$$\kappa_j^U = 1$$

$$\kappa_j^H = \left[1 - \frac{j^2}{(h+1)^2}\right] \left[1 - \frac{j^2}{(h+2)^2}\right] \times \left[1 - \frac{j^2}{(h+3)^2}\right]$$

$$y_t = \sum_{j=0}^d \beta_j t^j + \varepsilon_t,$$

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_{t-h} \\ \vdots \\ \varepsilon_{t+h} \end{pmatrix} \sim \mathcal{N}(0, \mathbf{K})$$

Crit  re minimisation :  
 $I(\boldsymbol{\theta}, 0, y_t, g_t)$

Proietti et Luati (2008)

$$y_t = \sum_{j=0}^d \beta_j t^j + \varepsilon_t, \quad \varepsilon_t \text{ bruit blanc}$$

$$\boldsymbol{\kappa} = (\kappa_{-h}, \dots, \kappa_h) \text{ noyaux, } \mathbf{K} = \operatorname{diag}(\kappa_{-h}, \dots, \kappa_h)$$

$$(\hat{\beta}_0, \dots, \hat{\beta}_d) \in \operatorname{argmin} (\mathbf{y} - \mathbf{X}_p \hat{\boldsymbol{\beta}})' \mathbf{K} (\mathbf{y} - \mathbf{X}_d \hat{\boldsymbol{\beta}})$$

$$\Rightarrow \hat{\boldsymbol{\theta}} \mathbf{y} = \hat{\beta}_0 = \mathbf{K} \mathbf{X}_d (\mathbf{X}'_d \mathbf{K} \mathbf{X}_d)^{-1} \mathbf{e}_1 \mathbf{y}$$

Dagum et Bianconcini (2008) — RKHS

$$f_0(t) \text{ noyau continu, } P_i \text{ polyn  mes orthonormaux de } \mathbb{L}^2(f_0) \text{ et } K_p(t) = \sum_{i=0}^{d-1} P_i(t) P_i(0) f_0(t).$$

$$\hat{\theta}_i = \frac{K_d(i/b)}{\sum_{j=-h}^h K_d(j/b)}$$

$f_0$  = version con-  
tinue de  $\kappa_i$   
 $b = h+1$

Th  or  me d'Henderson (1916)