

Gray, Thompson (1996)

Minimisation des révisions au filtre
symétrique sous contrainte

Contrainte

Filtre asymétrique
sans biais
 $\Rightarrow \mathbf{X}'_d \hat{\boldsymbol{\theta}} = \mathbf{e}_1$

Filtre asymétrique
de biais constant
 $\Rightarrow \mathbf{X}'_{d-1} \hat{\boldsymbol{\theta}} = \mathbf{e}_1$

$y_t = \sum_{j=0}^d \beta_j t^j + \xi_t + \varepsilon_t$, avec ε_t bruit
blanc $\mathbb{V}[\varepsilon_t] = \sigma^2$ et ξ_t non corrélé à ε_t ,
 $\boldsymbol{\Omega} = \mathbb{V}[(\xi_{t-h}, \dots, \xi_{t+h})]$

$$\min R(\boldsymbol{\theta}) = \mathbb{E}[(M_{\boldsymbol{\theta}^s} y_t - M_{\boldsymbol{\theta}} y_t)^2]$$

$$= I(\boldsymbol{\theta}, 0, y_t, M_{\boldsymbol{\theta}^s} y_t)$$

$$\text{s.c. } \mathbf{X}'_d \boldsymbol{\theta} = \mathbf{e}_1 \text{ ou } \mathbf{X}'_{d-1} \boldsymbol{\theta} = \mathbf{e}_1$$

Formule générale

$$\begin{cases} I(\boldsymbol{\theta}, q, y_t, u_t) = \mathbb{E}[(\Delta^q(M_{\boldsymbol{\theta}} y_t - u_t))^2] \\ J(\boldsymbol{\theta}, f, \omega_1, \omega_2) = \int_{\omega_1}^{\omega_2} f[\phi_{\boldsymbol{\theta}}(\omega), \varphi_{\boldsymbol{\theta}}(\omega)] d\omega \\ \hat{\boldsymbol{\theta}} \in \operatorname{argmin} \sum \alpha_i I(\boldsymbol{\theta}, q_i, y_t, u_t^{(i)}) + \beta_i J(\boldsymbol{\theta}, f_i, \omega_1^{(i)}, \omega_2^{(i)}) \\ \text{s.c. } \mathbf{C}\boldsymbol{\theta} = \mathbf{a} \end{cases}$$

$$\begin{aligned} F_g(\boldsymbol{\theta}) &= I(\boldsymbol{\theta}, 0, y_t, \mathbb{E}[L_{\boldsymbol{\theta}} y_t]) \\ S_g(\boldsymbol{\theta}) &= I(\boldsymbol{\theta}, q, y_t, \mathbb{E}[L_{\boldsymbol{\theta}} y_t]) \\ T_g(\boldsymbol{\theta}) &= J(\boldsymbol{\theta}, f: (\rho, \varphi) \mapsto \rho^2 \sin(\varphi)^2, 0, \omega_1) \\ y_t &= \sum_{j=0}^d \beta_j t^j + \varepsilon_t, \quad \varepsilon_t \stackrel{i.i.d.}{\sim} \mathcal{N}(0, \sigma^2) \end{aligned}$$

Guggemos et al (2018)

$$\begin{aligned} F_g(\boldsymbol{\theta}) &= \sum_j \theta_j^2 \\ S_g(\boldsymbol{\theta}) &= \sum_j (\Delta^q \theta_j)^2 \\ T_g(\boldsymbol{\theta}) &= \int_0^{\omega_1} \phi_{\boldsymbol{\theta}}^2(\omega) \sin^2(\varphi_a(\omega)) d\omega \\ \hat{\boldsymbol{\theta}} &\in \operatorname{argmin} \nu_1 F_g(\boldsymbol{\theta}) + \nu_2 S_g(\boldsymbol{\theta}) + (1 - \nu_1 - \nu_2) T_g(\boldsymbol{\theta}) \\ \text{s.c. } \mathbf{X}'_d \boldsymbol{\theta} &= \mathbf{e}_1 \end{aligned}$$

Wildi, McElroy (2019)

$$\begin{aligned} A_w(\boldsymbol{\theta}) &= 2 \int_0^{\omega_1} (\rho_s(\omega) - \rho_{\boldsymbol{\theta}}(\omega))^2 h(\omega) d\omega \\ T_w(\boldsymbol{\theta}) &= 8 \int_0^{\omega_1} \rho_s(\lambda) \rho_{\boldsymbol{\theta}}(\lambda) \sin^2\left(\frac{\varphi_{\boldsymbol{\theta}}(\omega)}{2}\right) h(\omega) d\omega \\ S_w(\boldsymbol{\theta}) &= 2 \int_{\omega_1}^{\pi} (\rho_s(\omega)^2 - \rho_{\boldsymbol{\theta}}(\omega))^2 h(\omega) d\omega \\ \min \nu_1 A_w + \nu_2 T_w + (1 - \nu_1 - \nu_2) S_w \end{aligned}$$

$$\sigma^2 = 0, \boldsymbol{\Omega} = \mathbf{K}^{-1}$$

$$\begin{cases} f_1(\rho, \varphi, \omega) = 2(\rho_s(\omega) - \rho)^2 h(\omega) \\ f_2(\rho, \varphi, \omega) = 8\rho_s(\omega) \rho \sin^2\left(\frac{\varphi}{2}\right) h(\omega) \\ A_w(\boldsymbol{\theta}) = J(\boldsymbol{\theta}, f_1, 0, \omega_1) \\ T_w(\boldsymbol{\theta}) = J(\boldsymbol{\theta}, f_2, 0, \omega_1) \\ S_w(\boldsymbol{\theta}) = J(\boldsymbol{\theta}, f_1, \omega_1, \pi) \\ R_w(\boldsymbol{\theta}) = J(\boldsymbol{\theta}, f_2, \omega_1, \pi). \end{cases}$$

Dagum et Bianconcini (2008) — RKHS

$f_0(t)$ noyau continu, P_i polynômes orthonormaux de
 $\mathbb{L}^2(f_0)$ et $K_p(t) = \sum_{i=0}^{p-1} P_i(t) P_i(0) f_0(t)$.

$$\hat{\theta}_i = \frac{K_p(i/b)}{\sum_{j=-h}^q K_p(j/b)}$$

b choisi optimalement pour minimiser :

- l'erreur quadratique moyenne de révision ($b_{q,\gamma}$)
- l'accuracy A_w ($b_{q,G}$)
- la smoothness S_w ($b_{q,s}$)
- la timeliness T_w ($b_{q,\varphi}$)

$$\begin{aligned} \min R(\boldsymbol{\theta}), A_w(\boldsymbol{\theta}), S_w(\boldsymbol{\theta}), \\ \text{ou } T_w(\boldsymbol{\theta}) \\ \text{s.c. } \theta_i = K_p(i/b) \text{ et } \\ \sum \theta_i = 1 \end{aligned}$$

$$\begin{aligned} \min R(\boldsymbol{\theta}) + \alpha_T T_g(\boldsymbol{\theta}) \\ \text{s.c. linéaires} \end{aligned}$$

Extension en intégrant la
timeliness

Minimisation sous contrainte des
révisions + $\alpha_T T_g(\boldsymbol{\theta})$

$$\alpha_T = 0$$

$$d = 1$$

Proietti et Luati (2008)

LC/Musgrave

$y_t = \gamma_0 + \delta t + \varepsilon_t$,
 ε_t bruit blanc et $\boldsymbol{\theta}$ préserve con-
stantes et dépend de $|\delta/\sigma|$

$$\begin{cases} \mathbf{U} = \mathbf{X}_0 \\ \mathbf{Z} = \mathbf{x}_1 \\ \mathbf{D} = \sigma^2 \mathbf{I} \end{cases}$$

Modèle général

$\mathbf{y} = \mathbf{U}\boldsymbol{\gamma} + \mathbf{Z}\boldsymbol{\delta} + \boldsymbol{\varepsilon}$, $\boldsymbol{\varepsilon} \sim \mathcal{N}(0, \mathbf{D})$ et $[\mathbf{U} \ \mathbf{Z}] \subset \mathbf{X}$
Minimisation des révisions à $\boldsymbol{\theta}^s$ sous contrainte :
 $\mathbf{U}'_p \boldsymbol{\theta} = \mathbf{U}'_p \boldsymbol{\theta}^s$, $\mathbf{U} = \begin{bmatrix} \mathbf{U}_p \\ \mathbf{U}_f \end{bmatrix}$ avec \mathbf{U}_p de $h+1+q$ lignes

$$d = 2$$

QL

$y_t = \gamma_0 + \gamma_1 t + \delta t^2 + \varepsilon_t$,
 ε_t bruit blanc et $\boldsymbol{\theta}$ préserve ten-
dances linéaires et dépend de
 $|\delta/\sigma|$

$$\begin{cases} \mathbf{U} = \mathbf{X}_1 \\ \mathbf{Z} = \mathbf{x}_2 \\ \mathbf{D} = \sigma^2 \mathbf{I} \end{cases}$$

$$d = 3$$

CQ

$y_t = \gamma_0 + \gamma_1 t + \gamma_2 t^2 + \delta t^3 + \varepsilon_t$
 ε_t bruit blanc et $\boldsymbol{\theta}$ préserve ten-
dances quadratiques et dépend
de $|\delta/\sigma|$

$$\begin{cases} \mathbf{U} = \mathbf{X}_2 \\ \mathbf{Z} = \mathbf{x}_3 \\ \mathbf{D} = \sigma^2 \mathbf{I} \end{cases}$$

DAF

$y_t = \gamma_0 + \gamma_1 t + \gamma_2 t^2 + \gamma_3 t^3 + \varepsilon_t$
 ε_t bruit blanc et $\boldsymbol{\theta}$ préserve ten-
dances cubiques

$$\begin{cases} \mathbf{D} = \mathbf{K}^{-1} \\ \mathbf{U} = \mathbf{X}_3 \\ \mathbf{Z} = \mathbf{0} \end{cases}$$

$$\sigma^2 = 0, \boldsymbol{\Omega} = \mathbf{K}^{-1}, \\ d = 3$$